THE USE OF NUMERICAL OPTMISATION TECHNIQUES IN COMPUTATIONAL FIRE ENGINEERING MODELS: A STUDY THROUGH EVACUATION MODELLING ANALYSES

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A thesis submitted in partial fulfilment of the requirements of the University of Greenwich for the Degree of Doctor of Philosophy

May 2011

DEDICATIONS

This thesis is dedicated, first of all, to the Universe's Creator. It is also dedicated to all the good spirits, in the material and spiritual worlds on Earth, which through their efforts; try to make this planet a better place for everyone to live. And this thesis is kindly and humanly dedicated to the following people:

My beloved wife, “*minha bela princesa*”, Sara Lynne Machado-Jones, who has been showing me, with her good sense of humour and natural happiness, that material life is beautiful and it is still worth to put into practice what we preach.

My two blessed and lovely sons: Philip Richard Machado-Jones and Samuel George Machado-Jones, which have been giving me inspiration to finish this thesis. They became my strongest reason in this existence to be a better person.

My father in the material world of this existence, João Marcelo Lopes Tavares, who taught me in this life that mathematics can be enjoyable and loved; and be as beautiful as the human beings with the chromosome XX .

My mother in the material world of this existence, Maria Inês Feijó Machado Tavares, who have always taught and still has been teaching me, by her behaviour and words, that love (i.e., light) is the only absolute variable in the Cosmos, as the magnificent Albert Einstein defined in his Theory of Relativity.

My brothers in the material world of this existence: “V. Ex.a.” (*Vossa Excelência*, *Juiz de Direito*) Gustavo Machado Tavares and Dr. Leonardo Machado Tavares, M.D. for their true friendship.

And finally, to all my friends from this existence and previous ones as well, in the spiritual and material worlds, which, surely, have been always helping me to achieve my sublime objectives in both aspects, moral and intellectual, in order to evolve myself as a whole in this immortal and long journey called life.

DECLARATION

"I certify that this work has not been accepted in substance for any degree, and is not concurrently being submitted for any degree other than that of (name of research degree) being studied at the University of Greenwich. I also declare that this work is the result of my own investigations except where otherwise identified by references and that I have not plagiarised the work of others".

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ABSTRACT

THE USE OF NUMERICAL OPTMISATION TECHNIQUES IN COMPUTATIONAL FIRE ENGINEERING MODELS: A STUDY THROUGH EVACUATION MODELLING ANALYSES

Evacuation models have been playing an important function in the transition process from prescriptive fire safety codes to performance-based ones over the last three decades. In fact, such models became also useful tools in different tasks within fire safety engineering field, such as fire risks assessment and fire investigation. However, there are some difficulties in this process when using these models. For instance, during the evacuation modeling analysis, a common problem faced by fire safety engineers concerns the number of simulations which needs to be performed. In other terms, which fire designs (i.e., scenarios) should be investigated using the evacuation models? This type of question becomes more complex when specific issues such as the optimal positioning of exits within an arbitrarily structure needs to be addressed.

In the other hand, numerical optimisation techniques have been applied to a range of different fields such as structural analysis. These techniques have shown to be a powerful tool for designers, saving their time and consequently reducing costs during the process.

For this reason, the emphasis, throughout this study, is to develop a methodology that enables the optimisation of fire safety analysis of structural designs. In other words, the current research was primarily intended to demonstrate and develop this combination of fire engineering tools and techniques such as the Design of Experiments (DoE) and numerical optimisation techniques. For this purpose, a Computational Fire Engineering (CFE) tool combined with Numerical Optimisation Techniques and associated statistical methods (i.e., Design of Experiments (DoE) and Response Surface Models (RSM)) are used. The study is focused on evacuation modelling; nevertheless the methodology proposed here could equally be applied to CFD-based fire simulation tools. While the approach that has been developed is intended to be generally applicable, the techniques have been explored and

demonstrated using the buildingEXODUS computational package. This fire engineering simulation tool is used worldwide, to improve the fire safety in building designs.

This study therefore intended, besides to develop a numerical methodology to allow the efficient optimisation of fire safety aspects of structural designs, to understand how the core variables impact the evacuation efficiency.

For instance, a common problem faced by fire safety engineers, in the field of evacuation analysis, is the optimal positioning of exits within an arbitrarily complex structure. This problem is usually addressed through time consuming and expensive trial and error. While a solution is usually found, to this problem, it is seldom the optimal solution, resulting in a compromise in building performance and safety.

The methodology explored in this thesis, as applied to CFE, was initially based around a relatively small set of physical variables. This approach evolved and was subsequently expanded to include more complex behavioural, procedural and environmental parameters. The methodology has also been further developed and applied to evacuation simulation.

This integrated approach is intended to help fire safety engineers and designers to develop optimal designs (i.e., safe designs) in an optimised manner. In reality, this was the motivation of this study: to introduce numerical optimisation techniques and associated concepts, well known within the operational research field, as an approach for a more efficient and systematic procedure when developing and/or improving fire safety designs.

Post comparisons between the outputs obtained, using these different DoE techniques, have been also performed in order to analyze which technique is most suitable for the optimisation of structural designs.

This thesis describes a number of analyses (of a variety of structural designs) that have been used to evaluate the application of optimisation techniques into the CFE context. This included the use of the buildingEXODUS simulation tool, as mentioned previously, followed by the application of a variety of optimisation techniques (both gradient and non-gradient based numerical optimisation techniques) as well as different types of DoE (such as Latin Hypercube, Central Composite Design (CCD) and also a Random approach) in order to improve the designs according to a number of different variables. These variables have initially included physical modifications to the geometry.

The proposed methodological approach developed in this thesis is demonstrated on a variety of practical problems. These problems are represented by 4 case studies which vary from complexity to the nature of the variables. These case studies involved both types of problems, namely: unconstrained and constrained.

The results obtained have shown to be satisfactory, i.e., global minima and local minima closest to the global minima region were found. For all the cases, a gradient-based algorithm (i.e., the Fletcher-Reeves numerical optimisation technique) and non gradient-based algorithm (i.e., the Particle Swarm Optimisation numerical optimisation technique) were used to find the optimal solution. And as mentioned before, different DoE techniques were also applied.

Important issues within building fire safety design were found and discussed in this thesis. For example, it was shown that the positioning of the exits can have a stronger impact on the design evacuation time rather than the exits' widths for some scenarios. Furthermore, the level of life safety for buildings should also consider the exits' locations within enclosures.

The analysis revealed that this methodology seems to be a very powerful tool for evacuation modelling analysis.

This systematic methodology to efficiently optimise evacuation safety aspects of structural designs should be also extended to more complex designs, such as larger enclosures and open spaces.

This methodology is also intended to be applied to problems found in the field of fire simulation, such as: the sizing and positioning of smoke extraction vents and the modelling of cable fires.

ACKNOWLEDGEMENTS

At first place, I would like to thank God, or like “some scientists” say '*Mother Nature'*, for my life, my health in the material and spiritual aspects. Therefore, I would like to thank The Creator, as the great astrophysicist Hugh Ross defines God, for all the good things which I have been blessed with, including this opportunity of doing my Ph.D.

I would like to thank my first supervisor, Professor Edwin Richard Galea, for have accepted me as his Ph.D. student at the University of Greenwich and for all his academic support during this research. What I have learnt within the Fire Safety Engineering Group (FSEG) during these years is priceless. I am indebted grateful for his wisdom expressed in his valuable comments and suggestions about this research. I cannot express my gratitude into words.

I would like also to thank my second supervisor, Dr. John Ewer, for his useful help when preparing this thesis. I am very grateful for his comments and remarks about the chapters of this thesis and also for his patience in the early stages of my Ph.D. studies when I was learning fire modelling.

I would like to thank the Alβan Programme for the financial support for my Ph.D. studies conceded in October of 2003 until August of 2006. And also to the University of Greenwich for my Ph.D. bursary conceded in September of 2006.

I would like to thank my colleagues from the FSEG for their company and some of my technician colleagues from the Computing and Mathematical Sciences (CMS) department for our nice football matches in Greenwich Park. I have great memories from those days, despite some bizarre and comic moments from our unskilled talents as footballers. I would like to thank all my ex-flatmates here in London, which thought me directly and indirectly that sharing a house/flat is an “interesting” experiment in terms of human behaviour. I would like to thank all the good people from the St. Pius Church for their beautiful charity work and also to the *Spiritist*/Christian Spiritualist movement in the UK for receiving me so kindly every Sunday and every time that I needed.

There are so many people that I would like to thank and that I should thank; however I would not be able to enumerate all of them here on these paragraphs and to express all my gratitude to all of them neither. Nevertheless, I would mention few names which represent all of the others: Professor Dr. Cristo Rei; Pai Francisco;

Jutahy; Urubatan; Mauricio Alves da Silva; Homero Rodrigues Machado; Raimundo Tavares; Maria Cristina Duarte Tavares; Dr. Steve Gwynne; Dr. Stoyan Stoyanov; Professor Vitally Strusevich; Dr. Dayse Duarte; Professor Silvio Romero; Bruno Williams; Mrs. Cynthia Parry-Jones and Mr. Richard J. Parry-Jones; Annelie; Karine Delaveau; Professor Allan Kardec; Chico Xavier; Emmanuel; Dr. Bezerra de Menezes; Professor Dr. Divaldo Pereira Franco; Professor Dr. Raul Teixeira Mrs. Françoise Barkshire, Mrs. Lauren Quinton and everyone from the CMS and many others...and I must not forget: Beethoven and the Beatles for their great music which helped me to relax when writing up this thesis.

I would never forget to thank my parents for their love and their moral and financial supports; to my brothers for their love and friendship and to my wife for her love and good positive energy.

And finally to Jesus; his love gave me strength to live abroad away from my family and friends for doing my Ph.D.; and gave me peace and serenity to be able to accept changes in my actual existence. But as the thoughtful and inspired Sir George Harrison, MBE said once, “the whole life is a change: from the morning to the evening; from spring to winter everything is a change from birth to death; life is change”. Therefore, here I am and I expect that this research might bring some new perspectives, “change”, on the way Computational Fire Engineering (CFE) modelling analyses, in special evacuation modelling analyses, are conducted.

CONTENTS

1. Introduction

1.1 Motivation and Relevance of the Research

A common problem faced by fire safety engineers in the field of evacuation analysis is the optimal positioning of exits within an arbitrarily complex structure in order to minimise evacuation times. To a certain extent, fire safety building codes provide guidelines for the positioning of exits however; these constraints are more concerned with ensuring a certain (unquantified) level of life safety rather than minimising evacuation times to provide a maximum level of safety. For an arbitrarily complex room, ignoring constraints imposed by regulations such as minimising travel distances and avoiding dead-end corridors, to know where to locate the exits is not an easy task. In reality, this problem is usually addressed through time consuming and expensive trial and error experimental trials/modelling.

While a solution is usually found to this problem, it is seldom the optimal solution, resulting in a compromise in building performance and safety. For instance, there is no clear guidance regarding where to place an exit in order to produce minimum evacuation times. Is it better to have two exits of X m width or one exit of 2X m width? If two exits are required, what is the optimal relative positioning of these exits? The analysis required to address these questions grows in difficulty as the available options - and hence complexity - of the scenario increases. This issue gets even more unfeasible for assessing more complex situations involving many more exits, exits of varying size and complex shaped compartments.

For instance, attempting to address this issue while dealing with only two exits may be manageable, but what if there were 10 exits, each of varying dimensions? How would the engineer know he had found the optimal or near optimal solution?

Understanding the effect of these building design variables on the objective of minimising the time it takes for an evacuation is very important for the engineer.

A possible answer to this problem may be found by using numerical optimisation techniques.

In reality, numerical optimisation techniques have been applied in a range of different fields (such as structural analysis, aerospace design, product reliability and maintainability, information and communications technologies, medical studies and many others) and have shown to be a powerful tool for designers, saving their time and consequently reducing costs during the process.

Therefore, this is the main objective of this study: to investigate, develop and recommend a systematic methodology to efficiently optimise evacuation safety aspects of structural designs. Such an approach will be of particular interest to practical fire engineers as it allows the fire engineer rapidly and efficiently optimise their design.

Furthermore, this study explored the concept of combining numerical optimisation techniques and associated concepts, such as DoE (Design of Experiments) techniques and RSM (Response Surface Modelling) with evacuation simulation provided by the buildingEXODUS simulation tool. The buildingEXODUS tool was chosen because it is a well known and widely used evacuation model system that is directly associated with the assessment of safety within buildings. Through this research, as mentioned previously, a systematic methodology has been developed to allow the efficient optimisation of the evacuation safety aspects of a range of structural designs. This approach was pioneer and is intended to be generally applicable.

The current study focused on evacuation modelling; nevertheless the methodology proposed in this Thesis, could equally be applied to CFD based fire simulation tools.

In the next section, the objectives of this study are discussed.

1.2 Objectives

1.2.1 General Objectives

According to what was mentioned previously, the main objective of this study is to develop a new methodology to help to ensure fire safety in enclosed environments. This methodology is based on applying numerical optimization techniques to computational fire engineering (CFE) modelling. In summary, this study intended to present how the concepts inherent in the classical optimization theory field, such as Design of Experiments (DoE) techniques and Response Surface Models (RSM), for instance, can be applied to the CFE context; and then generating a combined analysis between the used numerical optimization techniques and the used of CFE models.

For this purpose, a number of wide ranging case studies were created in order to demonstrate and validate this approach.

The results obtained from this combined analysis provided a new perspective on how to use CFE models in order to develop a safe design in terms of fire safety; and hence establishing a link between the operational research field and the CFE field.

Post comparisons between the outputs obtained using these different DoE techniques have also been performed in order to analyse which is the most suitable for this problem.

The methodology explored in this thesis evolved during the research. It was initially based around a relatively small set of physical variables and was subsequently expanded to include more complex behavioural, procedural and environmental parameters, as the technique has been developed and applied to evacuation simulation.

This thesis involved the analyses of a variety of structural designs in order to calibrate the optimisation technique. This included the use of the buildingEXODUS simulation tool, as mentioned before, and then the application of a variety of DoE techniques (i.e., LatinHypercube, Central Composite Design etc.) and optimisation techniques (both gradient and non-gradient based numerical optimisation techniques) in order to improve the designs according to a number of different variables. These variables initially included physical modifications to the geometry.

In summary, the general objective of this study was:

- To develop an analytical methodology which would provide an optimised analysis of designs in terms of fire safety of the occupants.

In the next section, the specific objectives are presented.

1.2.2 Specific Objectives

The specific objectives reached in this study are described below:

- To identify and understand how the important variables impact the evacuation process behaviour (i.e., to investigate the manner in which the core variables interact to control evacuation efficiency);
- To suggest a method which can help modellers (in the DoE research area) to define how many and which design points should be picked up for unconstrained and constrained problems.

In the next section, the outline of this thesis is presented.

1.3 Thesis Overview

This thesis is divided into 12 chapters (including this one).

Chapter 2 is an overview of evacuation modelling. It discusses the importance of evacuation modelling within the Fire Safety Engineering context. The chapter does also discuss the way evacuation models are being understood by the Fire Safety Engineering community and proposes a new categorization for classifying these models. The chapter is finalized with a review of the main evacuation models, their features and capabilities.

Chapter 3 is a review of Design of Experiments, Response Surface Models and Numerical Optimisation Techniques in general. In this chapter, the classical concepts found in the optimisation theory are presented and discussed in more detail.

Chapter 4 presents the problem analysed in this study. It presents the whole concept and the methodology developed and applied in this study. This chapter also introduces some of the basic concepts of optimisation theory and briefly discusses the nature of design variables.

Chapter 5 presents a problem involving a square room with one and two exits and a rectangular room with two exits, in which the best location for the exits is found to minimise the evacuation times. In other terms, the optimal location for positioning the exits is determined by finding the minimum evacuation time. The optimal solution for all these cases is found by data analysis. This data analysis consists in simply running evacuation simulations using several different scenarios for finding the scenario which gives the minimum evacuation time. This method has been called in this thesis as "brute force method".

Chapter 6 presents the same case studies presented in Chapter 5, but solved through the use of numerical optimisation techniques. The case studies are solved as being unconstrained and constrained problems.

Chapter 7 presents a case study in which composes the set of the additional more complex scenarios of this thesis. The case study presented in this chapter is a constrained problem, in where the best location for two exits is found for a "L-shaped" room. This case is similar to the square room with two exits, however, the geometry analysed is more complex given its shape which enables more exit locations combinations. Besides that, based on the increase of exit locations combinations, more small evacuation times are found. In optimisation terms, more local minima regions are found and with that, the response surface is more challenging for the optimisation methodology. This problem is solved through brute force method and through numerical optimisation analysis.

Chapter 8 presents the case study. This case study is based on a “compartmented room”; i.e., multiple connected compartments. These multiple rooms have the same overall square floor plan as presented previously in chapters 5 and 6, with the same area and the same number of people. However, in this case, there is a smaller room that occupies one corner of the large square area. For this case study, two scenarios were analysed: scenario (1) has one exit from the smaller room and one exit from the bigger room; and scenario (2) has one exit from the smaller room and two exits from the bigger room. Therefore, this problem is solved as an unconstrained problem for the scenario 1) and as a constrained problem for the scenario 2). The results are obtained through brute force method and through the use of numerical optimisation techniques.

Chapter 9 presents the final case study. This case study is based also on a more complex geometry where the nature of the problem has also increased in complexity. This case study is based on a rectangular shaped room which attempts to represent an aircraft geometry. For this reason, a continuum space is taken into consideration and also multiple exits with varying sizes. The results are obtained through brute force method and also through the use of numerical optimisation techniques.

Chapter 10 presents and discusses a proposal for a generic method for picking up design points for unconstrained and constrained problems. This method is denominated here as the “modified CCD” and is validated based on its successful use in the case studies presented in the previous chapters, namely, 6,7,8 and 9. This method is a hybrid DoE method which can be applied for both unconstrained and constrained problems. It was developed through data analyses based on the previous studies, which included evacuation simulations of a variety of different types of scenario. The modified CCD method has shown itself to be more efficient when compared to the random DoE technique.

Chapter 11 reports some guidelines and recommendations on the use of numerical optimisation techniques for evacuation analysis.

This thesis concludes with a discussion of the contributions to knowledge and the achievements of this research, in Chapter 12. The limitations of this study as well as some suggestions for future work are also covered in this discussion.

2. Overview of Evacuation Modelling

"Traditional prescriptive design guides and regulations are increasingly challenged by complex demands. Designers and regulators are turning to performance-based analysis and regulations, which have been facilitated by the new generation of people movement models"
Galea, E.R. 2003

2.1 Evacuation Modelling – the state of the art

"Over many years, the world has experienced fires in enclosed environments. These fires have caused both direct and indirect losses" **[1]**. These events are more critical in terms of human fatalities. Table 2.1 presents data regarding the number of fatalities associated to fires in buildings **[1]**.

Therefore, fire safety can be measured in terms of fatalities. For this reason, in order to avoid and/or reduce human fatalities, the Fire Safety Engineering (FSE) community has been making efforts to assure the safety of occupants. This task is not easy, especially due to the growing complexity of the architectural designs, which can introduce more fire risks **[2]**. This is a possible reason why, especially during these last three decades, the fire safety codes are being changed from a prescriptive approach to a performance-based one; because the prescriptive codes are not able to address all of the fire safety issues for complex geometries.

Table 2.1: Number of fatalities from fires in buildings around the world

Event	Number of Fatalities	Number of Injuries
Grand Hotel in Las Vegas, the USA (1980)	84	679
Gothenburg in Sweden (1998)	63	180
Rhode Island in the USA (2003)	Over 100	35
Kumbakonam in India (2004)	90 (children)	100
Asunción in Paraguay (2004)	Over 300	100
Andraus building in São Paulo, Brazil (1974)	16	320
Joelma building in São Paulo, Brazil (1972)	189	320

Based on this perspective, some countries like some North-European countries (Sweden and the UK), New Zealand, Australia, Japan, Canada and the USA are in advanced stage of development and implementation of the performance-based codes **[2]**. In fact, the evacuation and fire models have been playing an important function in this process; given that they help to assure that the solutions proposed by performance-based codes are feasible and are able to address fire safety issues properly. This worldwide movement toward performance-based codes has created a demand for computer evacuation models that will provide an estimate of the evacuation time for a building" **[3]**. Fire engineers often use evacuation models to assess buildings and their ability to provide sufficient time for the occupants to evacuate safely in the event of a fire or other emergency **[4]**. This viewpoint is also re-enforced by other authors: "the complex demands on design spaces challenge traditional prescriptive design guides and regulations. Designers and regulators are consequently turning to performance-based analysis and regulations facilitated by the new generation of people movement models" **[5]** (i.e., evacuation models). In fact, the development of such models became an important field of research and work within the FSE industry. This field is commonly denominated by such specialists as Computational Fire Engineering (CFE), see Figure 2.1.

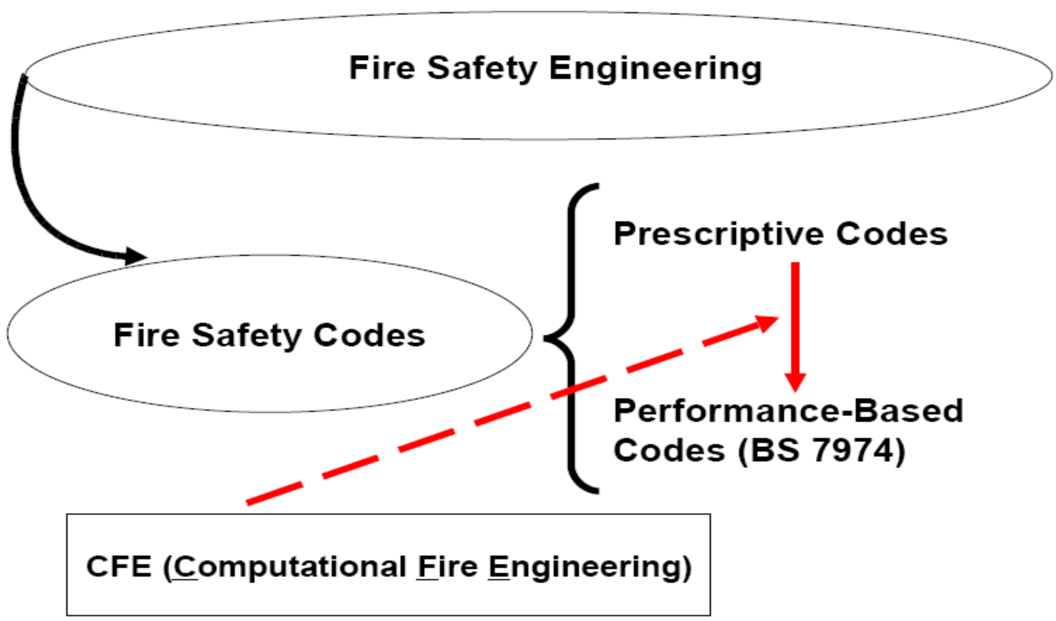


Figure 2.1: The use of CFE models to address fire safety issues [6]

In Figure 2.1, the BS 7974 (British Standard), part 6, is shown, once this fire safety code follows a performance-based approach and the use of evacuation models has been enabling its validation.

Within this context, particularly, evacuation modelling became a popular area of research within the FSE community. There are several regular conferences around the world dedicated to this field. On these conferences, different topics within the evacuation processes' context are studied and discussed. New evacuation models are developed and presented and the improvements on the existent evacuation models are presented as well. Scientific journals are also another source of presenting these achievements on evacuation modelling.

As previously mentioned, for the last few decades, the reality is that evacuation models have been used to address fire safety issues within complex structures, where the prescriptive codes, generally, do not provide clear guidance. For this reason, these models have been largely applied for estimating the RSET (Required Safe Egress Time), instead of the use of hand calculations approach. In fact, evacuation models are important tools for the evaluation of engineered designs, because such evaluations require the estimate of safe evacuation time for the occupants **[7]**. (The concept of evacuation time and other core time-lines are discussed in more detail in chapter 3 of this thesis).

In other terms, it could be said that there are essentially two methods available for calculating evacuation times; the more traditional, hand calculation approach or with the use of evacuation models. The estimation of the evacuation times using the hand calculation approach often follows the equations provided the Society of Fire Protection Engineers (SFPE) Handbook (section 3: hazard calculations, chapter 13, movement of people: the evacuation timing and chapter 14: emergency movement) **[8]**. Although it is possible to get a good indication of the total evacuation times in relatively low populated enclosed environments, using the hand calculation approach, the introduction of significant areas of congestion in highly populated buildings and structures, means that a more appropriate method of calculation is to use one of the many evacuation models available. Therefore, evacuation models have became useful tools within the FSE community.

Furthermore, evacuation models have been developed largely over the last few decades. They are being used in a wide field of applications, such as: crowd dynamics in open spaces, pedestrian movement in assemblies, human behaviour in evacuation process (i.e., commonly called also as egress process) during emergency situations in enclosed environments and etc. (And beyond the FSE community, evacuation models have been the object of study in many other fields of knowledge such as Risks Assessment/Safety Sciences, Crowd Management, Operation Research, Artificial Intelligence/Computer Modelling, and many others). Therefore, evacuation models have become important information sources and analysis tools for understanding of the evacuation processes in general. At the time of writing this Thesis, there are over 40 different evacuation models. These models can be used for different types of enclosed environments, such as: buildings, aircraft, ships and trains. Many existing evacuation models are either computationally inefficient, or are missing some crucial human behaviours in crowds **[9]**. In published literature, there are only a few reviews of various evacuation models **[10,11]**. Their work is internationally well-known and readily available to the fire safety industry. This body of work has probably influenced many others to carry on with the reviewing of evacuation models.

Some other reviews provide a set of information with specific details about evacuation models in general **[12-15]**.

An accurate review of 22 evacuation models is made by Gwynne, S. et al **[14]**. In this review, they established that the evacuation models fall into two categories: those which only consider human movement and those which attempt to link movement with behaviour.

The first category of evacuation models only attempt to model the interaction between the building occupants and the structure, referred to as "occupants-structure" models; and for this reason are commonly called as "ball-bearing" models. These models, as the name suggest, do not consider the interaction between occupants, referred to as "occupant-occupant", for instance. Thus, consequently, they do not take into account important aspects associated with the psychological attributes of the occupants. And for this reason, some other authors also referred to this approach of evacuation modelling as environmental determinism **[16]**. Aspects such as pre-movement times are not considered. There are only a few evacuation models which can be classified as "ball-bearing" models, such as the EVACNET **[17]** and the MAGNETMODEL **[18]**.

Nevertheless, these types of evacuation models do have some advantages:

- quick and easy to set-up and run the simulations;
- validated and proven to work for certain cases, where the scenarios do not have complex issues to be addressed.

And the main disadvantages are as follows:

- for complex geometries, they might underestimate the evacuation time;
- do not provide realistic representation of the interactions: “occupants-occupants” and “occupants-fire”;
- the models are essentially deterministic and do not take sufficient account of uncertainties caused by several factors. (these uncertainties should be quantified in terms of probabilities based on the probability distribution of movement of occupants and movement of smoke etc.).

The second category of evacuation models takes into account not only the physical characteristics of the enclosure but treats the individual as an active agent taking into consideration his/her response to stimuli such as the various hazards which can cause the individual to tire; and individual behaviours, such as personal reaction times, exit preference etc. **[14]**. This means that these types of evacuation models do consider all the possible and important interactions during evacuation processes: “occupants-structure”; “occupants-occupants” and “occupants-fire” (in case of an occurrence of fire). An example of this type of evacuation model is the EXODUS computational simulation package **[19]**, which is the evacuation model that was selected and used in this research and is described in some detail in the next section of this chapter.

Furthermore, these evacuation models take into account physical and psychological attributes of the occupants for the evacuation time calculations, and are therefore more realistic. As a consequence of the increasingly complicated human behaviour included in the models, simulation times can be significantly increased over simple models, especially in complicated geometries with large populations. The models rely heavily on the modeller having a good understand of human behaviour of the population that is being simulated, and as such, these evacuation models should be continually updated and improved in order to take into account new collected data from recent researches in the field. Depending on the actual model used, it is usually possible to take into consideration the impact of the fire hazards (in case of occurrence of a fire event) on the occupant's movement and decision-making. The SIMULEX evacuation model **[20]** like EXODUS is also capable to incorporating these aspects into the model. Another evacuation model which enables this kind of interaction is the EGRESS model **[21]**.

These models, when used properly, can produce realistic and accurate results. For this, the evacuation modelling process should be planed and conducted carefully as shown in Figure 2.2.

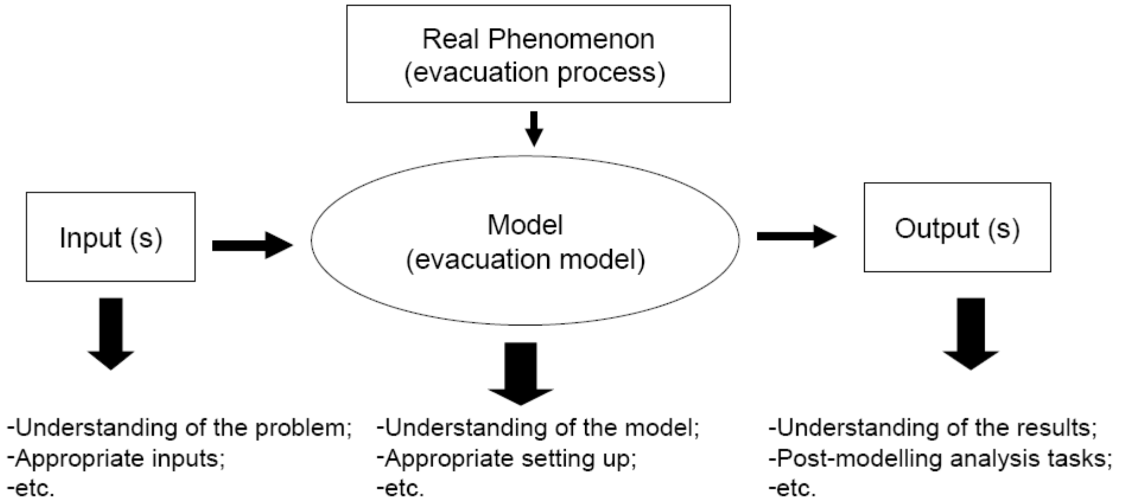


Figure 2.2: The evacuation modelling process [6]

From Figure 2.2, it should be noted that, before starting the evacuation modelling process, it is necessary to have a good understanding of the phenomenon to be modelled, i.e., the evacuation process. Once this is understood, it is also important (for a particular problem, i.e., the scenario which is going to be analysed) to always use appropriate data. It is also crucial to know how to set up the scenario correctly (i.e. accurately representing the scenario to be modelled) in the evacuation model. And finally, it is important to know how to interpret the results. Therefore, a good understanding of all these steps, that are inherent in the whole evacuation modelling process, is required for obtaining accurate results.

This is relevant to mention, because even for a robust and accurate evacuation model, given that it is only a model, it will never represent 100% the reality (in this case, the evacuation process), see Figure 2.3. This is also true for fire models; and most models in general.

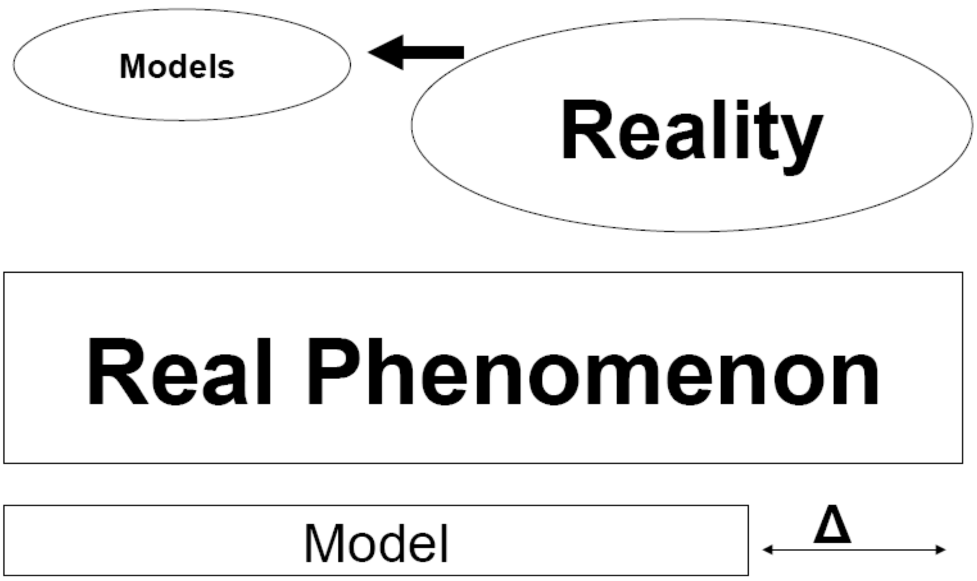


Figure 2.3: Model "versus" Reality [6]

Figure 2.3, indicates that it is possible to observe the difference between the real phenomenon (from "reality") and the model, as the error (Δ); once a model is an attempt to represent the "real world" (or reality). The error is due to deficiencies in the completeness of the model and unknowns in the specification of the scenario. Both these factors will generally be present in all but the simplest of models.

Following this sequence, a logical diagram is presented in Figure 2.4 which shows the sources of error during a calculation of the evacuation time through evacuation models.

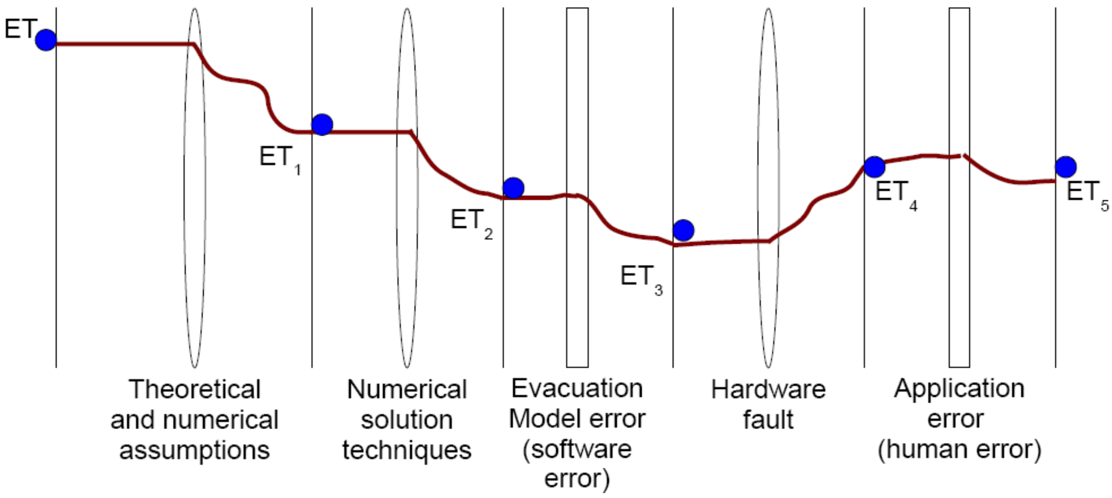


Figure 2.4: Symbolic representation of sources of error during a calculation (ET = real evacuation time; ET$_5$ = evacuation time calculated; where ET - ET$_5$ = Δ: "the error")

Despite these sources of error, the overall error can be reduced as long as the specialist's judgement can be improved. In other words, according to what was mentioned previously, these evacuation models – when properly used – can produce realistic and accurate results, with only minor error, hence the model can satisfactorily represent the real phenomenon in question: i.e. the evacuation process. And this is the state of the art regarding evacuation modelling.

In summary, it can be said that the second type of evacuation models (i.e., those which represent not only the occupant's movement, but also their behaviour) should be always used for complex evacuation analysis, where the main interactions, mentioned before, must be considered, see Figure 2.5.

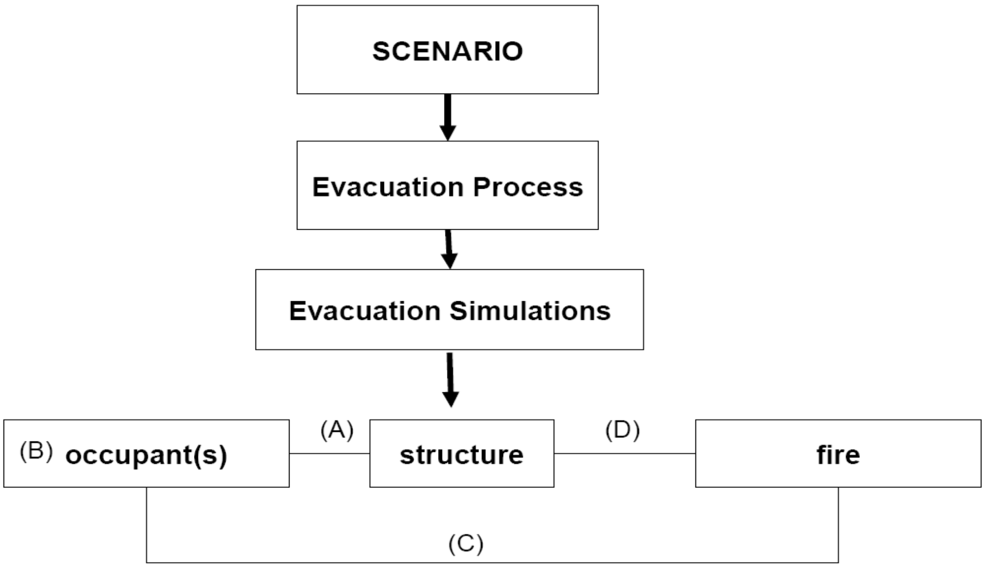


Figure 2.5: The main interactions during an evacuation process [6]

In Figure 2.5, (A) represents the interaction occupants-structure; (B), the interaction occupants-occupants; (C), the interaction occupants-fire (in case of fire events) and (D), the interaction fire-structure (for this a fire model would be used).

Each of the many evacuation models have their particular advantages and disadvantages, as mentioned previously. But regarding to the second type, in general terms, what makes them different from each other is the way that they represent the geometry of the structure, the occupant's characteristics and other factors. In other words, how they represent the main interactions, will then define the basic features of each evacuation model.

Also, the manner that their algorithms work, will determine how accurate the evacuation model is. For example, even for those evacuation models in which the pre-movement times are considered, some of them assume that the whole population will have the same behaviour, which will undermine the effects due to individual behaviour.

It is also extremely important that research data should be conducted in order to insert new data into the evacuation models and consequently updating and improving them **[22]**. An example of such types of research data is found in the study that conducted by Tavares, R.M. et al **[23]**. This issue is crucial for validation of the evacuation models.

The next section of this chapter gives a more detailed discussion about some of the main evacuation models.

2.2 Review of some Evacuation Models

2.2.1 General Comments on Evacuation Models

As mentioned in the first section of this chapter, there are only a few evacuation models' reviews available in published literature. Depending on the author, the criteria used to classify these evacuation models can vary widely. Some authors made generic and qualitative analyses of these models **[24],** where their analyses were more focused on the psychological and sociological aspects.

For instance, evacuation models were classified into two different types: "ball-bearing" models (which constitute the physical science approaching for evacuation modelling) and psychological models (which constitute the social science approaching for evacuation modelling) **[24]**. Regarding the first approach of evacuation models, there is an interesting book which explores with further details **[25]**. In this book, the author believes that physics finds its place in a science of society and makes some parallels with different mathematical, physical and even sociological theories. His work might bring some light to this kind of evacuation modelling approaching. Also regarding to this issue, a valuable contribution was made in a Ph.D. thesis **[26]**. In this research work, some issues on human and social behaviours for evacuation modelling analysis are well addressed through structured mind maps and logical diagrams that are rich in details which cover important topics necessary for a better understanding of these issues.

It might be also important to mention that some researchers have been researching evacuation modelling under the occupants' behaviour's perspective **[27]**; and for this reason, their work is, generally, restricted to the psychological and sociological aspects.

Therefore, this study will not follow the approach of describing the evacuation models. And besides, as it will be better explained in the next chapter of this thesis, this study is not focused on the human behaviour's aspect of evacuation processes; and on interactions with active fire protection measures such as fire detectors, sprinklers, communication systems and ventilation systems which can provide extra-time for evacuation and increase life safety.

In the other hand, more critical and specific evacuation models' reviews in terms of the occupant's motion and its relation with the occupant's decision-making within the structure geometry's boundaries (i.e., the environment context) have been done by other researchers. In these reviews, some of the evacuation models' algorithms as well as some of the relevant equations are explained **[28,29]**.

And as mentioned in the previous section of this chapter, given that there are over 40 evacuation models at the time of writing this Thesis; this large number of evacuation models means that detailed and comparative analysis of all of the available models is far beyond the scope of this thesis. Furthermore, for logical reasons, some of the main existent evacuation models are discussed here, namely: EVACNET; MAGNETMODEL; EGRESS; SIMULEX etc. In the section 2.3 of this chapter, the EXODUS evacuation model, which is the model used in this study, is also presented with more details.

Despite of the qualitative characteristic of some evacuation models' reviews, some of them present useful information. For example, a helpful list of features and capabilities which the evacuation models might have is given in some reviews **[13]**. In this thesis, for academic reasons, only the relevant features and capabilities from this list are used for a later comparison between the mentioned evacuation models. These features and capabilities are as follow:

- “modelling method”;
- occupant behaviour;
- occupant movement;
- use of fire data;
- use of CAD drawings;
- validation studies;
- limitations.

It is important to observe here that what has been established as being modelling methods **[13]** is different from what this study established. (And besides that, the “modelling method” should not be seen as a feature and neither as a capability, once the “modelling method” is responsible to define the features and capabilities of any evacuation model, see Figure 2.6. In reality, it seems that some other evacuation models' reviews make this same confusion. Probably the reason for that is because many of these reviews are mainly focused on human behaviour's issues, instead of analysing the whole context). Here instead of following this concept of modelling method, this study took as reference the categorization made by other authors **[12]**. In reality, they made a useful and logical categorization of the evacuation models. They classified them into 4 main categories according to the modelling method, namely: flow-based; cellular automata; agent-based; activity-based models and those models that incorporate social scientific processes (like the EXODUS model, for instance). Another useful document was published in 2007 **[30]** which contains a valuable and well detailed guideline on evacuation models, which also describes similarly the evacuation models as the two mentioned studies previously. Therefore, for practical purposes, this thesis will follow a similar approach.

Figure 2.6 should make this idea clearer.

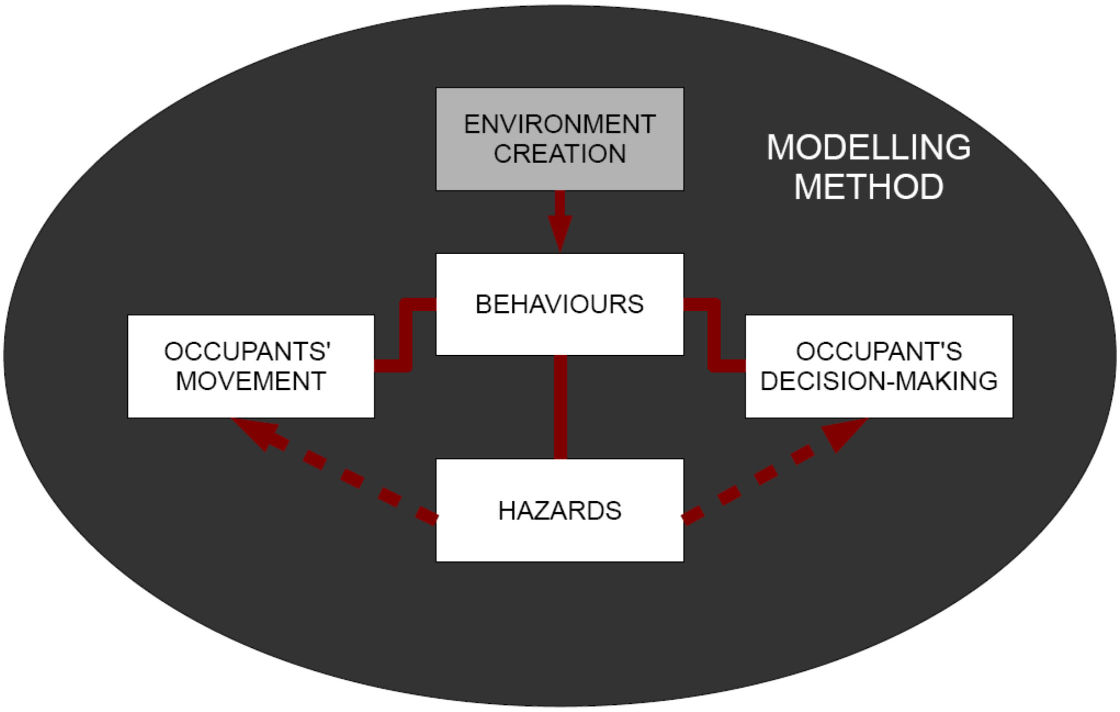


Figure 2.6: The generic context of evacuation models [6]

As Figure 2.6 shows, the modelling method commands how the evacuation models work. In reality, all the other components of an evacuation model rely on the modelling method. For this reason, the tasks during the modelling process, which will represent the real "behaviours" of a typical evacuation process are driven by the modelling method. This is why the modelling method is extremely important within this context; and therefore, the evacuation models should be classified according to their modelling method. The features and capabilities of the evacuation models should not be used as the criteria to classify them, once these features and capabilities are results of how the evacuation models work; consequently, are results of the modelling method **[6]**. Also according, based on that, probably, the evacuation models should be classified in the same way that fire models are classified: according to the modelling method. In the fire modelling field, the models are classified into two types: zone models and field models (i.e., CFD – Computational Fluids Dynamics – based modes) **[6]**. The literature in this field is rich in examples of this logical classification **[31-34]**.

It is also possible to see from Figure 2.6 that the main interactions during an evacuation process (see Figure 2.5) are defined here as well.

For a more specific and systematic understanding, the modelling method can be divided as follow:

- Macroscopic Approach;
- Microscopic Approach;
- Effect-based simulation Approach.

In the macroscopic approach, the occupants are modelled as part of the field. In this approach, the flow-based models are included.

In the microscopic approach, the occupants can be modelled in three different ways: occupants modelled separately; occupants modelled as particle-based (where the occupants are passive-particles subject to forces; i.e., cellular-automata) and occupants modelled as agent-based (where the occupants have objectives to reach and independent decision-makings and actions; this can also include forces). As mentioned, in this approach, the cellular automata and the agent-based models are included.

In the effect-based simulation approach, the activity-based models and those models that incorporate social scientific processes are included. The evacuation models which follow this modelling method tend to be more realistic. This is based on the fact that these models integrate all the main interactions (i.e., occupants-occupants; occupants-structure etc.), giving the flexibility for the modelled occupants to change their decision-makings and actions according to the environment's conditions (i.e., hazards etc.) while the evacuation modelling is progressing.

Regarding environment creation (i.e., the structure creation), usually the evacuation models are capable of building the floor plan in one of two ways: (1) interactive entry of the structure geometry's boundaries and (2) importing a CAD

(Computational Aid Design) file directly. The first option is time consuming for all but the simplest of geometries, but, for some scenarios, it might provide a more accurate definition. In the second option, the process is faster, however the accuracy of the design relies on the quality of the CAD file (usually a 2D floor plan .DXF file).

When considering behaviours, there are three components: occupant's movement; occupant's decision-making and hazards.

As the name suggests, these behaviours will determine how the occupants' behaviour, during the evacuation process, takes into account the main interactions mentioned before. It is clear to perceive that, for a robust evacuation model, all of these behaviours are inter-connected amongst themselves; and consequently they will impact on each other's performance. In this way, a good representation of real evacuation processes can be achieved since, in reality, this is the way how evacuation processes actually happen.

In relation to an occupant's movement, it can be said that this will depend on how the space is defined by the evacuation model (i.e., discrete or continuous spaces). If it is a discrete space, the occupants "jump" from cell to cell (i.e., commonly called nodes). If it is a continuous space, the occupants can be placed anywhere along the space. In the occupant's movement behaviour, attributes such as: position, speed, direction, acceleration and etc. are defined. In addition to these, all the individual physical characteristics which influence the motion (i.e., weight, age, mobility etc.) are also defined.

The occupant's decision-making is probably the most difficult behaviour to be addressed and this is the key-aspect of a good evacuation model. The occupant's decision-making influences the occupant's movement, in the same way that it is influenced by the occupant's movement as well as the hazards. In the occupant's decision-making behaviour, the psychological attributes such as: familiarity, response times, etc. are defined. As an attempt to address the complex issue of human behaviour, a good evacuation model must be capable of providing the flexibility to change this behaviour during the evacuation modelling process. For instance, depending on the changes of the external stimuli during the evacuation process, the occupants might need to change their actions (e.g. if there is jamming in a specific corridor, then the occupants might need to choose an alternative root, etc.).

Hazards influence both the occupant's movement and the occupant's decision-making behaviours through the definition of the physiological indexes, such as narcotic gases, temperature, toxic gases, etc. These indexes have a strong impact on the occupants' mobility; on the individual behaviour and on collective behaviour during real evacuation process. For this reason, a good evacuation model is capable of allowing the user to insert these indexes manually and/or through importing data files generated from fire models.

These and more specific aspects are discussed in the next sections, where some evacuation models are discussed.

2.2.2 The Evacuation Models

In the next paragraphs of this section, some of the existent evacuation models are presented and discussed in terms of their features and capabilities. It is relevant to mention that this review is mainly based on the available published material on the subject and also, that references written in other languages, other than English, were not consulted.

2.2.2.1 EVACNET

The EVACNET evacuation model **[17]** is used for buildings and its structure is based on a coarse network. The EVACET is designed to determine the optimal building evacuation plan. In other terms, the aim of this model is to minimize the time to evacuate the building. For this reason, this model is also referred as an “optimisation model”.

Therefore, in order to optimise the evacuation paths, this model considers the occupants as a “big mass of people”, treating them homogeneously. It does not treat each occupant as an individual; furthermore the individual behaviours are not taken into account. For this reason, the EVANET evacuation model is considered as a flow-based model, since it does not attempt to model the occupants' behaviour and its relation with the motion; but only the occupant's movement. Therefore, once this evacuation model is a flow-based model, it is based on the macroscopic approach, as defined before; thus for this reason it is also called a ‘macroscopic model’

Briefly, the EVACNET system represents each space within the enclosure (i.e., rooms, staircases, hall etc.) as nodes. Based on this, the node type is defined according to what type of space the node is representing, see Table 2.2.

Table 2.2: Node Type for the EVACNET evacuation model

Node Type	Node Type Description
WP	Work Place
HA	Hall
SW	Stairwell
LA	Landing
LO	Lobby
ES	Escalator

Each node has a maximum node capacity (i.e., the maximum number of people the node can hold) and a initial node occupancy (i,.e., number of people at the node).

The links between two nodes are represented by the “edges” and the exits are represented by the “destination nodes”.

Each edge has a maximum edge capacity (i.e., the maximum number of people can travel through this edge simultaneously) and an edge travel time (i.e., how long it takes to travel through this edge).

The Figure 2.7 shows an example of how the EVACNET represents a particular built environment.

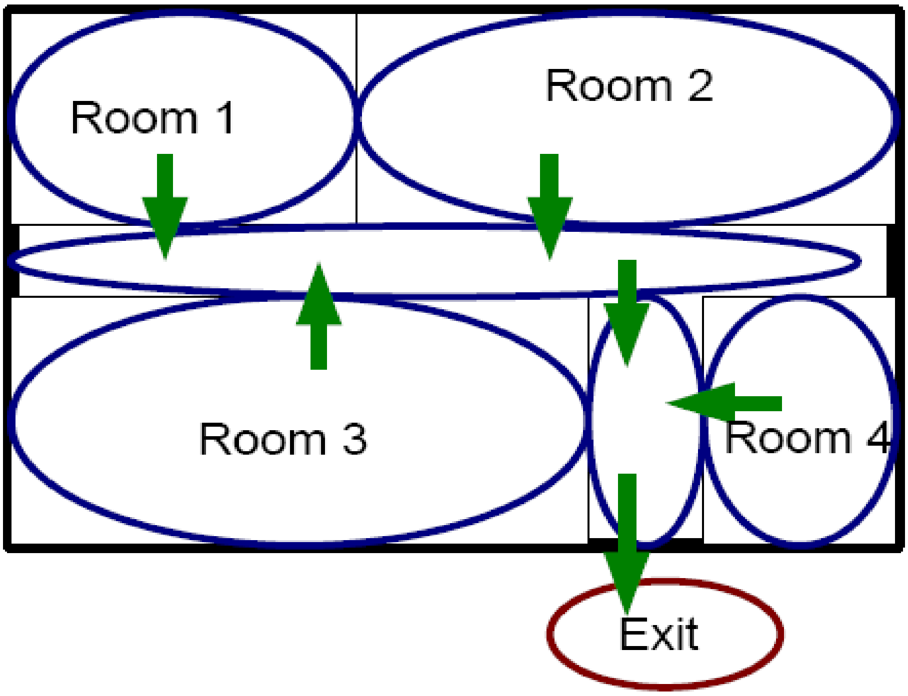


Figure 2.7: The generic context of evacuation models

From Figure 2.7, the ellipses (room and corridor) represent the nodes for each space. The arrows represent the edges which connect each space. And the ellipse outside the enclosure represents the destination node (i.e., the exit).

The EVACNET software does not use CAD drawings and cannot use fire data.

The outputs obtained from evacuation simulations, performed using this evacuation model, are:

- destination allocation;
- arc movement summaries;
- floor clearing times;
- evacuation profiles and snap shots;
- bottleneck identification.

The Figure 2.8 shows the principal menu of the EVACNET evacuation model.

```
                     EVACNET +
         BUILDING EVACUATION ANALYSIS PROGRAM
                 MASTER OPTION LIST

CODE   REQUESTED ACTION
----   ---------------------
EN   - ENTER NODE DEFINITIONS
EA   - ENTER ARC DEFINITIONS
LN   - LIST NODES
LA   - LIST ARCS
DN   - DELETE NODES
DA   - DELETE ARCS
SYS  - DEFINE OR REDEFINE SYSTEM ATTRIBUTES
SAVE - SAVE CURRENT MODEL
RM   - RETRIEVE DEFINED MODEL
RUN  - RUN MODEL
EXAM - EXAMINE RESULTS
QUIT - TERMINATE EXECUTION OF EVACNET
HELP - WHENEVER YOU HAVE QUESTIONS

ENTER CODE OF REQUESTED ACTION
```

Figure 2.8: Principal menu of the EVACNET evacuation model

There are a set of validation references for this evacuation model, as listed in the references further in this chapter. The software is also freely available for the public. Despite these validation studies and its free availability, based on its limitations (e.g., it is a ball-bearing model, it is a coarse network model, etc.), this evacuation model should not be recommended for complex evacuation modelling analysis, in which detailed information (i.e., information needed to set-up and run the model) is required.

Table 2.3 presents a summary of the EVACNET.

Table 2.3: EVACNET evacuation model's summary

FEATURES	EVACNET
Modelling method	Macroscopic (flow-based model)
Computer language	FORTRAN
Type of Node	Coarse
Use of CAD drawings	NO
Inclusion of Fire data	NO
Human Behaviour	NO

2.2.2.2 MAGNETMODEL

The MAGNETMODEL evacuation model **[35-38]** is used for buildings and its structure is based on fine network. The MAGNETMODEL is a particle-based model and uses the cellular automata approach as its modelling method. This evacuation model simulates the occupant's movement as a magnetized object in a magnetic field. Each occupant has a "positive pole" as well as the obstacles inside of the enclosures (i.e., walls, columns, furniture etc.); while the occupant's targets during the evacuation (i.e., the exits; specific areas, like corridors, floors etc.) have "negative poles". With this physical principle, the occupants will move their target and during their movements, they will avoid the obstacles. In summary, the functions which drive the Magnetic model were taken directly from physics.

This evacuation model does consider the occupant as an individual; nevertheless, only in terms of the physical aspects. The psychological aspects, for instance, are not included in their behaviours. Despite this not being a flow-based model, this evacuation model can be considered as a ball-bearing model, since the occupants' decision-making behaviours (in terms of cognitive aspects) are not addressed in a realistic manner.

Therefore, in order to optimise the evacuation paths, this model considers the occupants as a "big mass of people", treating them homogeneously. It does not treat each occupant as an individual; furthermore the individual behaviours are not taken into account.

The MAGNETMODEL is considered a Cellular Automata (CA) model. For this reason, it is based on a microscopic approach, as defined previously in this chapter. Therefore, based on that, it is also called as a 'microscopic model'.

There are several published materials about cellular automata in different fields of the knowledge. Therefore, it is not intended here in this thesis to discuss about CA in specific details.

But in simple terms, it can be said that the main difference between evacuation models based on CA and those which are based on other modelling method is the 'discretization' of the space. Therefore, the MAGNETMODEL simulates the occupants as being entities (i.e., automata) in cells. The occupant is represented as a circle placed in a cell, see Figure 2.9.

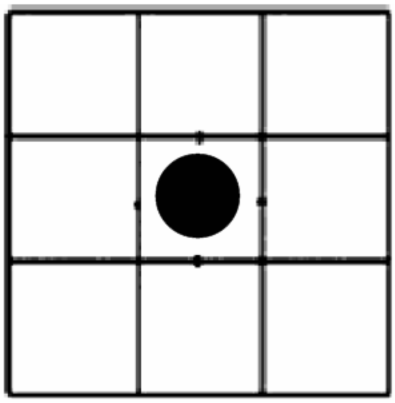


Figure 2.9: Representation of an occupant within a CA-based model, like the MAGNET model

The space is modelled as ‘grid cell’ and the occupants’ movement is based on the transition from each cell to another cell until reaching the target. Each cell can be occupied by only one occupant. The velocity of each occupant during the movement will depend, for instance, on the cell availability.

The application of magnetic models and equations of motion in the magnetic field cause the occupant’s movement. For instance, if a force (i.e., magnetic force) from another pole influences the occupant, this occupant will then move faster. The occupant’s movement speed is directly proportional to the force. In other words, this speed increases as the force continues to influence on the occupant until the occupant reaches an upper limit of speed. And as mentioned previously, an occupant and another occupant repulse each other. This is also applied to an occupant in relation to an obstacle.

This relation is based on the Coulomb's law (see equation 2.1). This is why the MAGNETMODEL is commonly called as Magnetic Force Model.

$$F = \frac{K_c Q_1 Q_2}{r^2} . \hat{v} \qquad \text{(eq. 2.1)}$$

Where:

F – electrostatic force vector;

K_c – Coulomb's constant;

Q_1 – charge on which the force acts;

Q_2 – acting charge;

r^2 – distance vector between the two charges;

vˆ - unit vector pointing in the direction of r

This is the force responsible for the occupants' motion. There is also another force which is responsible for avoiding collisions between occupants and with obstacles, see Figure 2.10.

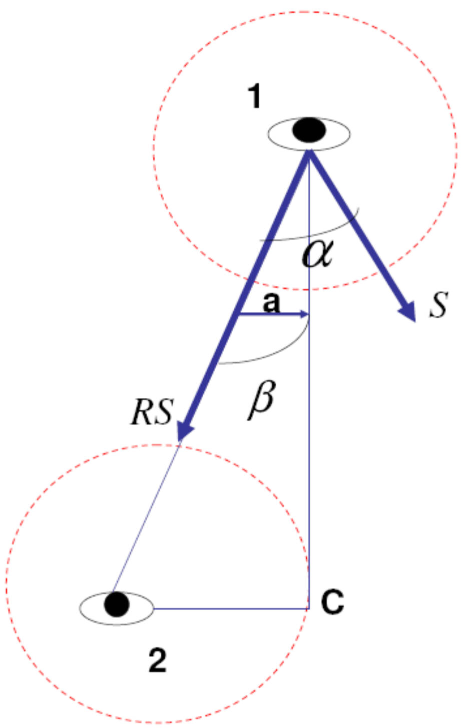


Figure 2.10: Representation of an additional force taken into account in the MAGNETMODEL to avoid collision between the occupants

Based on Figure 2.10, the acceleration “**a”** is calculated as follows:

$$\mathbf{a} = \mathbf{S}.\cos(\alpha)\mathbf{.}\tan(\beta) \qquad \text{(eq. 2.2)}$$

Where:

a - acceleration acts on occupant 1 to change the direction of **RS** to the direction of line 1C;

S - speed of occupant 1;

α (alpha) - angle between **RS** and **S**;

β (beta) - angle between **RS** and the line 1C;

RS - relative speed of occupant 1 to occupant 2.

In theory, the MAGNETMODEL is an interesting and logical model, however when analysed deeply, it becomes easy to identify its weakness for simulating real evacuation processes. The reason for the weakness is based on the arbitrary setting of the magnetic load. For instance, the occupants’ movement cannot be based purely on a magnetic force, once an occupant’s speed would increase towards an exit without limit according to the Coulomb’s Law and this is completely unrealistic.

Therefore, the validation of the MAGNETMODEL can just be done by visual inspection. And for this reason, there are no real practical phenomena which can be validated using this evacuation model.

Table 2.4 presents a summary of the MAGNETMODEL.

Table 2.4: MAGNETMODEL evacuation model's summary

FEATURES	MAGNETMODEL
Modelling method	Microscopic (particle-based/agent-based model using CA)
Computer language	unknown
Type of Node	Coarse
Use of CAD drawings	NO
Inclusion of Fire data	NO
Human Behaviour	YES (but in a global perspective)

2.2.2.3 Helbing's Social Force Model

This evacuation model is commonly called as Social Force Model; but given that it was designed mainly by Helbing, D., it is also common to be called as "Helbing's Social Force Model" **[39-42]**.

Similarly to the MAGNETMODEL, this evacuation model is also a 'force type model'; since it assumes that forces will cause the occupants' movement. Therefore, for the same reasons explained in the MAGNETMODEL, the Helbing's Social Force Model is a microscopic model. This model is considered to be the best among all microscopic models that has been developed. In the next paragraphs, a general description of this evacuation model is made.

The Helbing's Social Force Model assumes that the occupants are subjected to 'social forces' which cause their movements. The sum of these forces influences the occupants and with that, acceleration (i.e., $d\mathbf{v} / dt$) is developed, as equation 2.3 explains:

$$m \frac{d\mathbf{v}_i}{dt} = m \frac{v_0 \mathbf{e}_i - \mathbf{v}_i}{\tau} + \sum_{j \neq i} \mathbf{f}_{ij} + \mathbf{f}_b + \xi_i(t) \qquad \text{(eq. 2.3)}$$

Where:

x_i (t) – position of occupant i at time t;

v_i (t) – velocity of occupant i at time t = $d\mathbf{x}_i$ (t)/ dt;

m – mass of occupant;

m/τ – friction coefficient;

v_o - intended velocity with which it tend to move in the absence of interaction;

e_i - direction into which pedestrian i is driven $\in\{(1,0),(0,1)\}$;

$\xi_i(t)$ - the fluctuation of individual velocities;

f_{ij} - the repulsive interaction between pedestrian i and j;

f_b - the interaction with the boundaries.

The principle of this evacuation model, as mentioned before, is similar to the MAGENTMODEL. The occupants have targets to reach (i.e., exits; specific areas within the enclosure etc.) and the motivation to reach these targets causes the intended velocity of motion. Based on that, each occupant's movement is then directed to a specific destination. The direction is a vector from a specific position to the target point. Besides this index, there is also the ideal speed within this model which is calculated.

Therefore, the intended velocity is assumed to be the result of the product between the ideal speed and the vector of direction.

Within the Helbing's Social Force Model is also possible to insert the speed limitation (i.e., lower and upper values) in order to make the speed more realistic. With this, the problem found in the MAGNETMODEL is avoided, since the speed will not then increase continuously towards the target.

In this evacuation model, the two types of interaction are also considered in the calculations, namely: the interaction between the occupant(s) and the interaction between the occupant(s) and the obstacle(s).

In fact, the Helbing's Social Force Model successfully describes interesting emerging phenomena; however it is too simple to give a good insight into the behaviour of a particular occupant.

Table 2.5 presents a summary of the Helbing's Social Force Model.

Table 2.5: Helbing's Social Force Model's summary

FEATURES	Helbing's Social Force Model
Modelling method	Microscopic (particle-based/agent-based model using CA)
Computer language	unknown
Type of Node	Coarse
Use of CAD drawings	NO
Inclusion of Fire data	NO
Human Behaviour	YES (it does include individual behaviour, but accurately)

2.2.2.4 EGRESS

The EGRESS evacuation model **[43-48]** is a CA-based model and for this reason is a microscopic model. EGRESS use Artificial Intelligence (AI) to model the occupants' behaviour in terms of decision-making; and it is also capable of including hazard conditions to influence the decision-making process.

The EGRESS evacuation model represents the enclosure's space using a hexagonal grid, see Figure 2.11. This model allows simulating evacuation processes from a plot plan of any desired structure with metric dimensions of up to several square kilometres. Furthermore, it has been used for many different cases, such as offshore oil and gas installations, ships, railway stations, chemical plant, aircraft, trains etc.

Figure 2.11: Representation of an enclosure space using the EGRESS evacuation model

The mean speed of travel (in a given direction) is obtained from the probabilities of moving in certain directions towards the goal **[29]**. The probabilities consist of: a) the probability of moving one cell closer to the target; b) the probability of moving one cell further away from the target; c) the probability of moving to a cell that is the same distance away from the target; d) the probability of staying in the same location.

The EGRESS evacuation model has been validated based on evacuation trials and there are several publications about it as indicated in the references of this chapter.

EGRESS is an evacuation model which is capable of simulating the impact of hazard conditions on the occupants' movement and decision-making. It can also identify and simulate the effects of bottlenecks and congestion on the occupants' movement and decision-making.

Nevertheless, despite this, like the evacuation models discussed previously, it does not address the cognitive issues (i.e., the occupants' behaviour in terms of psychological aspects).

Table 2.6 presents a summary of the EGRESS evacuation model.

Table 2.6: EGRESS evacuation model's summary

FEATURES	EGRESS
Modelling method	Microscopic (agent-based model using CA)
Computer language	C++
Type of Node	coarse
Use of CAD drawings	NO
Inclusion of Fire data	YES
Human Behaviour	YES (but not accurately)

2.2.2.5 SIMULEX

The SIMULEX evacuation model **[49-53]** is well-known in the FSE community. This evacuation model is an agent-based mode and, for the same reason explained previously in this chapter, is considered as a microscopic model.

SIMULEX is capable to simulate a large number of occupants in complex geometries. It determines the flow rate based on the occupants' body size (of which four options are possible, see table 2.7) and population density. The occupants' bodies are represented by an elliptical body size shape defined by one main circle and two smaller circles bounding each shoulder, see Figure 2.12.

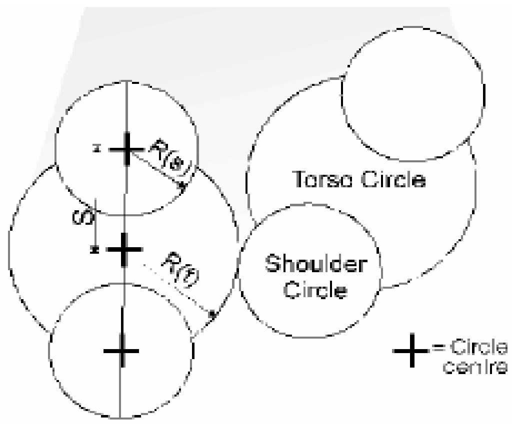


Figure 2.12: Representation of body types according to the SIMULEX evacuation model

Table 2.7: Type dimensions for the SIMULEX evacuation model

Body Type	Average (m)	Male (m)	Female (m)	Child (m)
R(t)	0.25	0.27	0.24	0.21
R(s)	0.15	0.17	0.14	0.12
S	0.10	0.11	0.09	0.07

This evacuation model allows users to create a plan layout that includes multiple floor plans connected by stairs in .DXF format directly from commercially available CAD programs as an input for the building. Occupants can be added either one by one or as groups at any location on the 2D floor plans, see Figure 2.13. SIMULEX also allows the creation of a 3D model of a building. The space is used as continuous for the occupants' movement, however it is 'discretized' for calculating and storing a distance map, see Figure 2.14. The distance map is used to direct occupants to the target (i.e., the closest exit and/or a specific area within the enclosure). The walk speed of each occupant depends on the distance to the occupant ahead.

SIMULEX uses the fine network approach (it generates automatically a 0.2mX0.2m spatial mesh, overlaid onto the .DXF floor plans) which provides a good visual display of the evacuation simulation. As a result of that, it can require a large amount of time to compute depending on the scenario.

In terms of occupants' movement, SIMULEX can represent some important occupants-occupants and occupants-structure interactions, such as: overtaking, sideways stepping, body rotation and small degrees of back stepping.

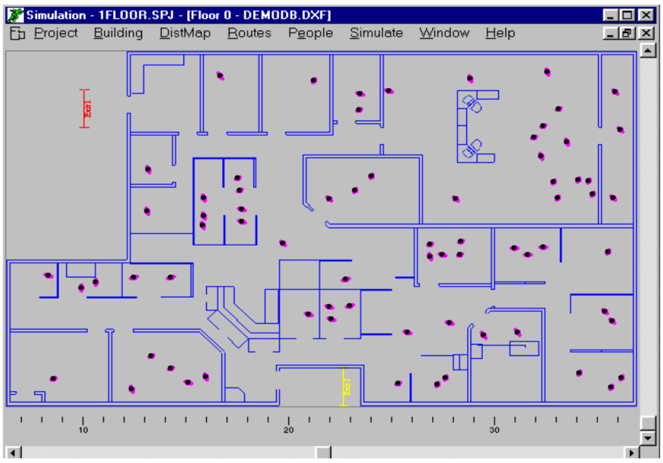


Figure 2.13: Representation of an enclosure space in 2D using the SIMULEX evacuation model

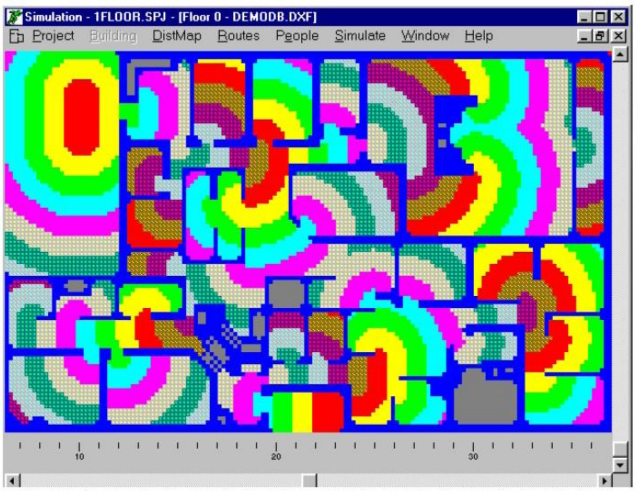


Figure 2.14: Distance map calculated by the SIMULEX evacuation model

In general terms, SIMULEX is a robust model. It enables the user to address a set of important issues in order to simulate in a realistic way evacuation processes. For instance, in SIMULEX, it is possible to insert the response times for each occupant; it also allows the occupants to make their own decision based on the occupants-occupants and also on the occupants-structure interactions. The occupants can decide their own walk speeds and these walk speeds are reduced as the occupants get closer together. SIMULEX takes into consideration factors such as gender, age etc.

Therefore, this evacuation model has been constantly updated through research. For example, the algorithms for the movement of occupants are based on real-life data, collected by using computer-based techniques for the analysis of human movement, observed in real life footage.

Nevertheless, SIMULEX does have some limitations. For instance, way finding and environmental conditions are not considered. And also, even in the latest versions of SIMULEX, where relevant information and constraints in terms of social behaviour are provided, the social interaction and emergent group response are not addressed yet in the model.

Table 2.8 presents a summary of the SIMULEX evacuation model.

Table 2.8: SIMULEX evacuation model's summary

FEATURES	SIMULEX
Modelling method	Microscopic (agent-based model)
Computer language	C/C++
Type of Node	fine
Use of CAD drawings	YES
Inclusion of Fire data	YES
Human Behaviour	YES

2.2.2.6 EXIT89

THE EXIT89 evacuation model **[54-58]** was developed by the support given by the NFPA (National Fire Protection Association) in the USA. Like SIMULEX, the EXIT89 evacuation model is also an agent-based model.

This evacuation model is very popular, since it can also model the impact of fire products (i.e., smoke for instance) on the occupants during the evacuation process. The fire data can be defined directly in the model or from an output file generated from a zone fire model: CFAST (which is very popular because it is available for free from NIST – the National Institute of Standards and Technology, USA) **[59]**.

The EXIT89 defines the space by rectangles which are connected among themselves by exits and other elements of connection (i.e., corridors etc.), see Figure 2.15.

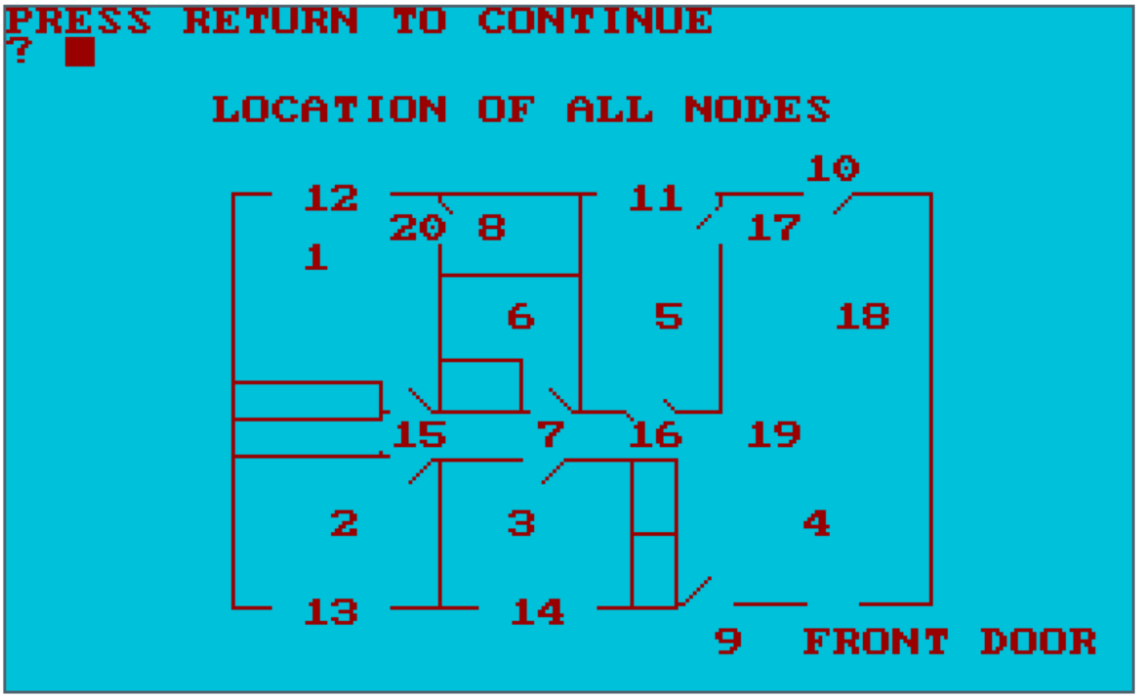


Figure 2.15: Representation of an enclosure space using the EXIT89 evacuation model

In each rectangle, a specific number of occupants is then defined. The occupants have their own walk speed, which varies according to the level of congestion and also the influence of the fire products – in those cases where they are considered.

EXIT89 enables the evacuation simulation to start instantaneously, but the simulation can also be started after a certain time (i.e., the fire products inputs can be used to determine the time of activation of fire detection systems, like fire alarms).

In fact, EXIT89 is an evacuation model that allows the user to address a set of important issues, such as:

- accounting for occupants with a range of mobilities, including disabled occupants and young children;
- delay times;
- choice of routing options;
- choice of walking speeds;
- contra flows;
- travel both up and down stairwells;
- etc.

The EXIT89 evacuation model can predict good results for complex geometries with a large population. It has been used to simulate evacuation processes in high-rise buildings.

Despite its' features, the EXIT89 model has the same sort of shortcomings as SIMULEX, in terms of social interaction and emergent group response **[12]**.

Another characteristic of this model, which can be seen as a "limitation" is the fact that the output file generated by the EXIT89 only prints out a summary showing floor clearing times, stairway clearing times and last time each exit was used and how many people used each exit.

Table 2.9 presents a summary of the EXIT89 evacuation model.

Table 2.9: EXIT89 evacuation model's summary

FEATURES	EXIT89
Modelling method	Microscopic (agent-based model)
Computer language	FORTRAN
Type of Node	coarse
Use of CAD drawings	NO
Inclusion of Fire data	YES
Human Behaviour	YES

2.2.2.7 STEPS

The STEPS (Simulation of Transient Evacuation and Pedestrian) evacuation software was developed by Mott MacDonald Group **[60]**. The STEPS model is an agent-based model.

STEPS is used to model an occupants' movement under both normal and emergency situations (i.e., evacuation process). Therefore, given its worldwide application in crowd management and pedestrian behaviour, STEPS is quite often denominated as a 'pedestrian model' rather than evacuation model. Therefore, in general terms, STEPS can be defined as a people movement simulation program that also models evacuation.

This 'evacuation model' uses coarse nodes to represent the space. Despite this, STEPS produces real-time 3D simulations which make it easy to interpret the results by both non-specialists and experts in the FSE field. In reality, this is one of the best features of STEPS, due to the nature of the graphical display, see Figure 2.16.

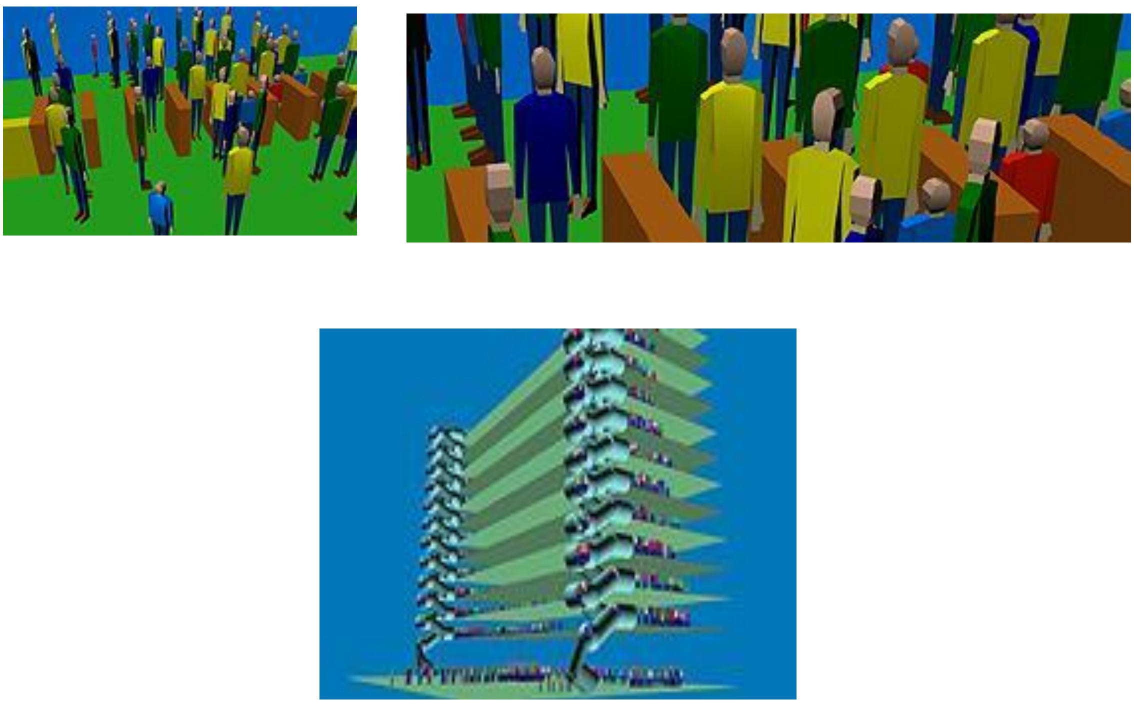


Figure 2.16: 3D representation using STEPS evacuation model

With this feature, important issues found during evacuation processes can be identified, such as: bottlenecks; congestions; preferred exits; queuing etc.

In STEPS, each occupant uses one cell at any given time and moves in the desired direction if the next cell is empty. Each occupant has its own characteristics, such as patience factor and familiarity behaviour.

The mechanism in which STEPS simulates the occupants' movement within enclosures is very similar to the manner how SIMULEX and EXIT89 model movement.

According to Mott MacDonald Group, STEPS main capabilities can be summarized as follows:

- efficient handling of large and complex models;
- direct import of 2D and 3D CAD models;
- 3D interactive (virtual reality) graphical user interface.

Another useful feature of STEPS is that it can be used in conjunction, in real-time 3D simulation, with fire modelling outputs obtained from CFD (Computational Fluid Dynamics) fire models.

STEPS has been widely used for large-scale and complex scenarios, such as metro stations, airports, shopping malls etc.

Despite its application in practical and real situations, there are very few available publications about this model.

The table 2.10 presents a summary of the STEPS evacuation model.

Table 2.10: STEPS evacuation model's summary

FEATURES	STEPS
Modelling method	Microscopic (agent-based model)
Computer language	unknown
Type of Node	coarse
Use of CAD drawings	YES
Inclusion of Fire data	YES
Human Behaviour	YES

2.3 Description of the EXODUS model

The basis of the model has frequently been described in other publications **[61-65]**; and so will only be briefly described here.

The EXODUS evacuation model was designed to simulate circulation and evacuation of large numbers of people from complex enclosures.

The software takes into consideration occupants-occupants, occupants-structure and occupants-fire interactions. The model tracks the trajectory of each individual as they move around the geometry. In evacuation applications involving fire, the model can also predict when occupants will be affected by fire hazards such as heat, smoke and toxic gases. The software has been written in C++ using Object Orientated techniques utilising rule base technology to control the simulation. Thus, the behaviour and movement of each individual is determined by a set of heuristics or rules. For additional flexibility these rules have been categorised into five interacting sub-models, the **OCCUPANT, MOVEMENT, BEHAVIOUR, TOXICITY** and **HAZARD** sub-models. These sub-models operate on a region of space defined by the **GEOMETRY** of the enclosure, see Figure 2.17. The version of the software used to simulate evacuation from the built environment is known as buildingEXODUS. (This is the version that has been used for this study).

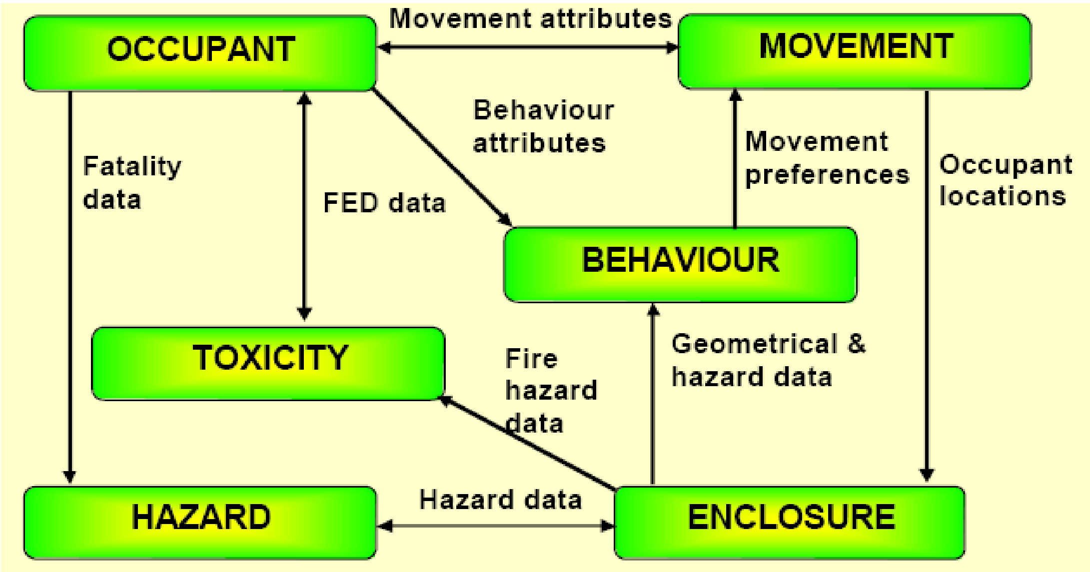


Figure 2.17: Core EXODUS evacuation model

The Occupant sub-model allows the nature of the occupant population to be specified. The population can consist of a range of people with different movement abilities, reflecting age, gender and physical disabilities as well as different levels of knowledge of the enclosure's layout, response times etc. On the basis of an individual's personal attributes, the Behaviour Sub-model determines the occupant's response to the current situation, and passes its decision on to the Movement Sub-model. The behaviour model considers such behaviours as; determining the occupants' initial response, conflict resolution, overtaking, etc. In addition a number of localised decision-making processes are available to each individual according to the conditions in which they find themselves and the information available to them. This includes the ability to customise their travel path according to the levels of congestion around them, the environmental conditions, the social relationships within the population and interaction with signage.

Behaviour exhibited by people during egress and evacuation situations can be quite complex, even in relatively simple situations involving simple structures. For example, room occupants may; move in groups and at the speed of the slowest member of the group, attempt to re-unite separated groups prior to egress, select an exit for which they are most familiar, follow the movement of other unrelated room occupants, recommit to different exits during the egress and so on. (In this study, as it will be better explained in the chapters where the study cases are presented, in order to simplify the analysis and isolate issues associated with the enclosure configuration and exit location these complex behaviours are greatly simplified. The behavioural response imposed on the population is such that occupants will elect to move towards their nearest exit and furthermore, that the occupants know the location of their nearest exit. While this behaviour may be considered simple it is nevertheless reasonable for our purpose. Indeed, this type of assumption is not very dissimilar to the type of assumptions implicit in most building regulations and used in many performance-based evacuation analyses).

In terms of operation, The EXODUS evacuation model can be summarized in four modes of operation, which are: a) geometry mode; b) population mode; c) scenario mode and d) simulation mode.

a) Geometry mode - allows constructing the enclosure to be analysed, where several methods are available:

- construct using interactive tools provided within the model;
- construct using a .DXF file from a CAD package;
- import library case.

b) Population mode - is used to generate the occupant(s), where a range of interactive tools is provided (i.e., definition of psychological and physical attributes for each occupant and/or groups of occupants) and also a library of populations is available.

c) Scenario mode - controls the scenario specification (e.g., exit capabilities, fire hazards etc.).

d) Simulation mode - allows the evacuation simulations to be performed, where the output data can be specified.

The EXODUS evacuation model defines the space using fine nodes. The geometry when defined consists of a 2D grid of nodes, see Figure 2.18. These nodes are linked by arcs. The occupants move from one node to another along these arcs. These nodes are 0.5mX0.5m and each occupant is located in a node at time, see Figure 2.19. In summary, it uses the same principle used in SIMULEX, for instance.

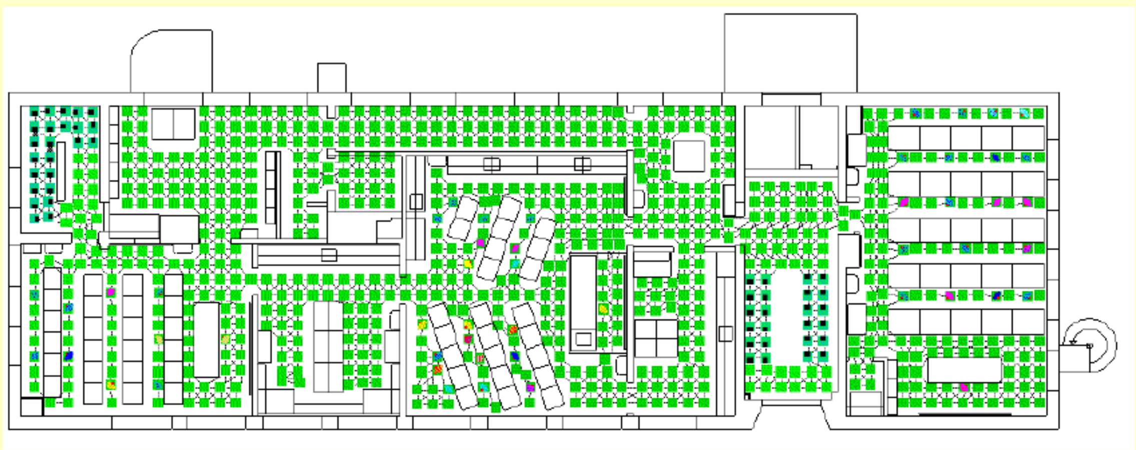


Figure 2.18: Representation of an enclosure using the EXODUS evacuation model

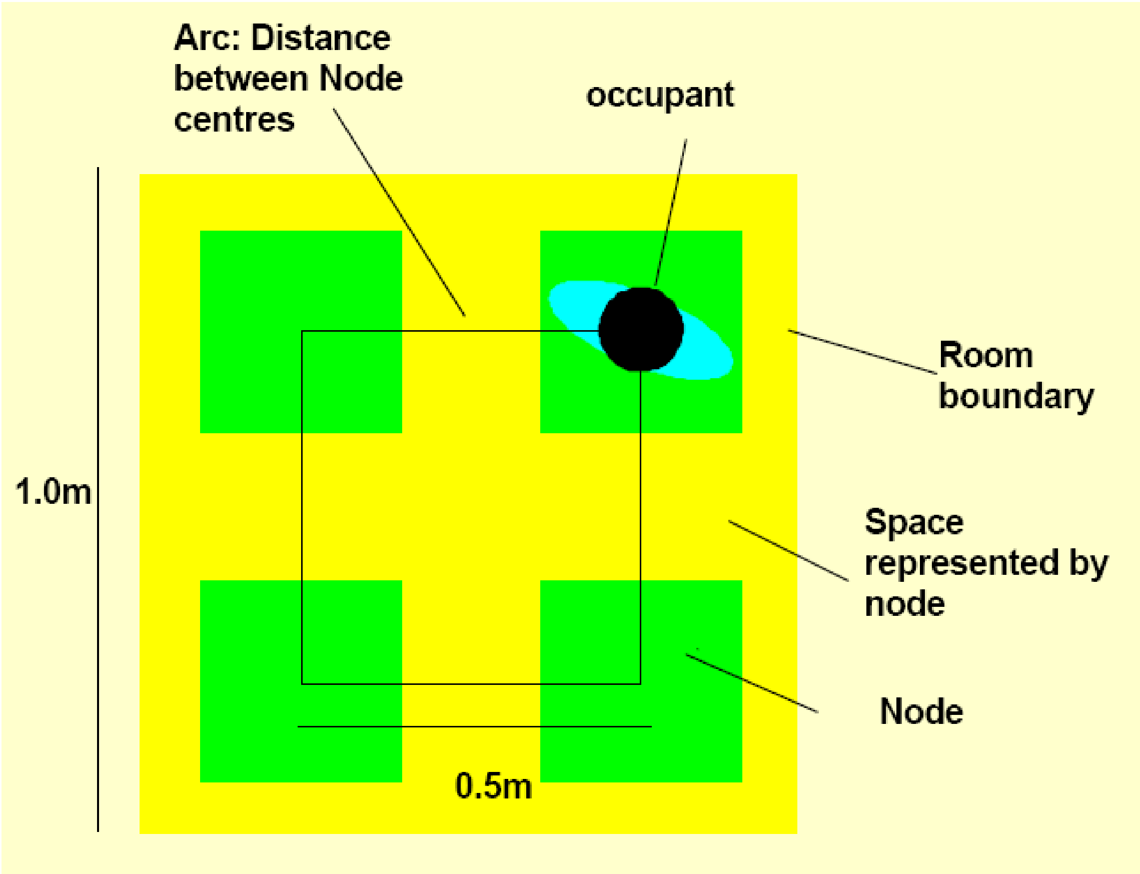


Figure 2.19: Core EXODUS evacuation model

There are in total nine different types of nodes which are defined by different colours according to their specific function (i.e., attractor, seat, free space etc.).

Once the evacuation simulations are finished, it is possible to replay the simulations and also, the EXODUS evacuation model allows the construction of virtual reality files based on the scenarios simulated, see Figure 2.20 This is a powerful tool, similar to the STEPS model, because it enables the designer to identify areas within the geometry that might have congestions, bottlenecks and etc., see Figure 2.21.

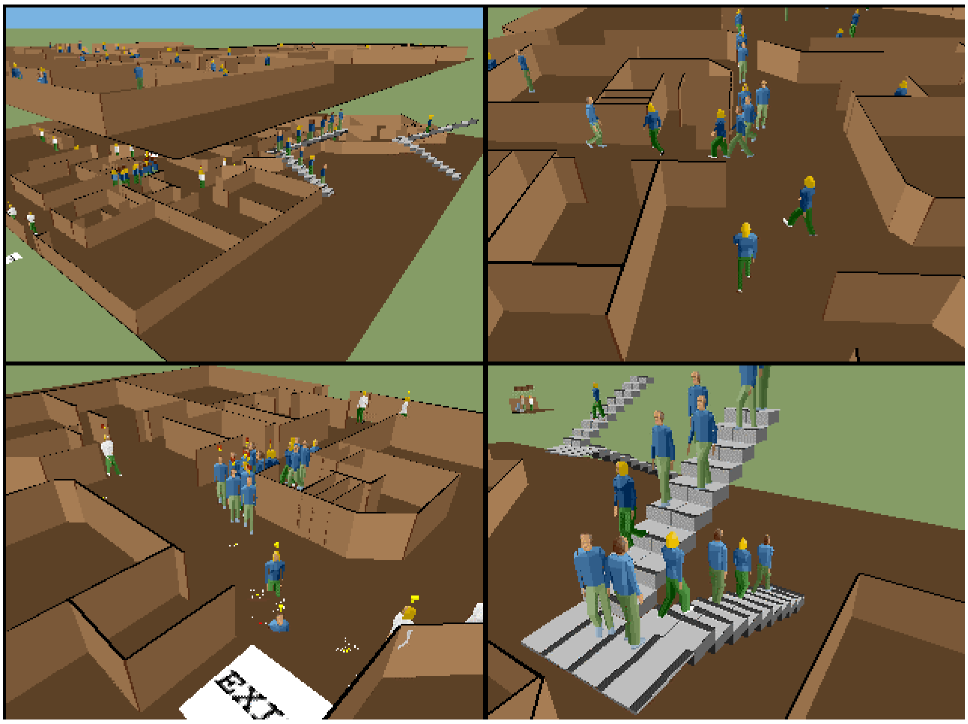


Figure 2.20: 3D representation of an evacuation simulation using the EXODUS evacuation model

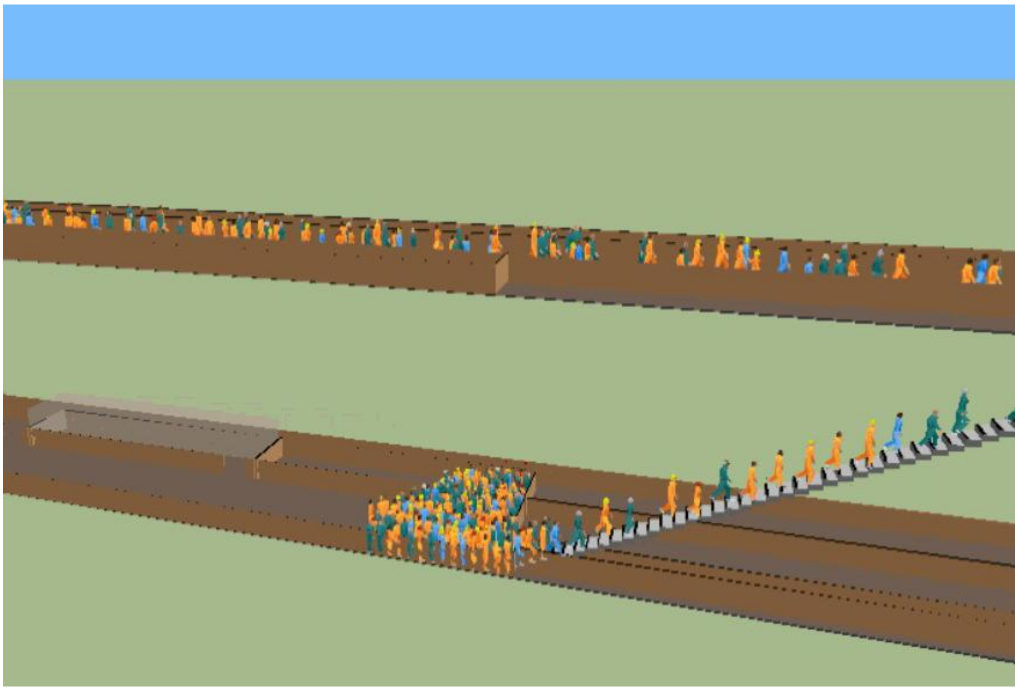


Figure 2.21: 3D representation of an evacuation simulation in a hypothetic underground station using the EXODUS evacuation model

Figure 2.21 shows a 3D graphic display of an evacuation simulation performed in a hypothetical underground station, where fire products were also taken into account. It is possible to see congestion formed towards the staircases.

In terms, of occupants' attributes, EXODUS takes into consideration, aspects such as: physical (i.e., age, gender, agility etc.); psychological (such as patience, drive, response time etc.) etc.

The EXODUS evacuation model is one the models that provides tools to represent the occupants' decision-making process in the initial stages of a fire emergency situation. For example, within the model, it is possible to define psychological attributes such as patience, familiarity, response time and others, all of which might influence the reaction and subsequent behaviour of the population. Therefore, EXODUS includes the representation of the time people take to become aware of the fire, the time taken to decide to start the evacuation movement and the time taken to choose an egress path.

Furthermore, this evacuation model is considered as an effect-based simulation model, as defined previously in this chapter.

This evacuation model has been constantly updated through academic research, where, for instance, data collected from real scale experiments has been inserted into the model.

The EXODUS evacuation model, like SIMULEX and STEPS, is used worldwide, is well known and is commonly used for diverse real scenarios.

There are several publications about this model.

EXODUS can, when well used, produce accurate results; however a single run of EXODUS does not provide sufficient grounds to base firm conclusions concerning evacuation efficiency. For this reason, it is recommended that the user should perform parametric studies which might involve repeated runs of the model varying key-model parameters and then identifying trends from the results.

The table 2.11 presents a summary of all the evacuation models described in this chapter.

Table 2.11: EXODUS evacuation models' summary

FEATURES	EXODUS
Modelling method	Effect-based simulation model
Computer language	C++
Type of Node	fine
Use of CAD drawings	YES
Inclusion of Fire data	YES
Human Behaviour	YES

And in the next chapter, the problem proposed and studied in this thesis is presented and discussed.

3. Mathematical and Statistical Review

"Mathematics is the queen of the sciences"
Carl Friedrich Gauss
1777-1855

"Pure mathematics is, in its way, the poetry of logical ideas"
Albert Einstein
1879 -1955

"If people do not believe that mathematics is simple,
it is only because they do not realize
how complicated life is"
John Louis von Neumann
1903- 1957

This chapter consists of a review of Design of Experiments, Response Surface Models and Numerical Optimisation Techniques.

As mentioned previously, the problems investigated in this thesis had the same key question: for an arbitrarily complex structure, containing an arbitrarily large population; is there an optimal location for the exits that will minimise egress times?

In order to solve these problems, this thesis has presented and discussed a systematic methodology which combines the use of numerical optimisation techniques and evacuation simulation analysis. This methodology has been found to provide the best solution in terms of giving an optimal or near optimal solution (i.e., determination of the best exit position to minimise the egress time). Such an approach is of particular interest to practical fire engineers as it allows the fire engineer to rapidly and efficiently optimise their design. The summary of this methodological approach is described in the next paragraphs.

The methodology has largely been discussed with further details in chapter 3 and 4 and also throughout this thesis; therefore, it is presented here in summary form. The methodology uses four main steps, namely:

STEP 1: Identify the Objective Function (OF)

The OF is the function that is to be optimised, in these problems the OF is the Total Evacuation Time (ET). The main objective is to minimise the ET.

STEP 2: Define Design Variables (DV) and constraints

For an OF there may be many DV (DV1, DV2, DV3 …) e.g. the number of exits, exit location, exit width, shape of the enclosure, number of occupants, etc. Thus the OF may be multi-dimensional.

STEP 3: Construct Response Surface (RS) describing the OF

The RS is a multi-dimensional surface in DV space. Each point on the RS is determined by running the evacuation software for each unique set of DVs. This requires many hundreds of evacuation scenarios to be simulated ensuring that the entire design space is covered. As this would be impractical, an approximation to the RS is developed by running only a handful of scenarios for selected KEY values of the DV. The KEY values for the DV required to produce a "reasonable" approximation to the RS are determined using DoE techniques. These techniques identify strategic combinations of DV which provide a good coverage of the design space and which hopefully will produce a reasonable approximation to the RS. Examples of DoE techniques are the Latin Hypercube and Central Composite Design (CCD). DoE techniques can only be used in unconstrained problems (i.e., differently from constrained problems, in unconstrained problems, there are no restrictions imposed by conditions expressed by inequalities and/or equations. These restrictions, or conditions, represent constraints to the way the problem will be solved; for this reason the objective function becomes dependent to these constraints. As a result of that, the design space becomes modified, since the design variable values become restricted to certain values within the design variables' domains. This is better explained in the previous chapter of this thesis; especially in chapter 3.). In constrained problems, random selection procedures must be used.

Each strategic combination of DVs identified by the DoE technique defines a scenario, which is then run using the evacuation model. The RSM is determined by fitting a mathematical function to the surface of points (ET, DV1, DV2, ….) in the design space. Examples of RSM which were used in this study are the full-quadratic and high order polynomial approximations.

- **STEP 4: Determine the minimum of the RSM**

Numerical optimisation techniques are used to find the minimum of the multi-dimensional RSM. There are many optimisation algorithms which can broadly be categorised into two families, the classical gradient-based methods and the stochastic-based methods. Examples of optimisation techniques include: Fletcher-Reeves – a gradient method - and Particle Swarm Optimisation (PSO) – a stochastic method.

Once the minimum of the RSM is found it represents the set of DV required to produce the OPTIMAL solution.

3.1 DoE – Design of Experiments Techniques

"Experiments are performed by people in nearly all walks of life. The basic reason for running most experiments is to find out something that is not known"
Virgil L. Anderson, Robert A. McLean

"The only source of knowledge is experience"
Albert Einstein
1879-1955

The DoE techniques compose one of the steps within the optimisation strategy used in this study. These steps are better discussed in the next Chapter.

As an introduction, before discussing DoE techniques, it is helpful to go back to the concepts of design variables and design space.

Following this sequence, it is already known that for this study, the system to be analysed is an evacuation process. (It is important to observe that it is common in the optimisation field to use the terms "system" and "process" interchangeably). Therefore, given that an evacuation process is a system, there are factors which influence the behaviour of this system (i.e., system dynamics). In other words, these factors determine the performance of this system and consequently the result(s) generated from it (i.e., the output(s)). This is why the result(s) of the system can explain how significant a specific factor and/or the interaction amongst factors is for the system dynamics.

Furthermore, the output of the system can measure this impact; i.e., the influence of these factors and/or their interaction on the system. Hence, the evacuation time can be used to measure the response of the evacuation process to any factors which might influence the evacuation.

In the optimisation field, such influences, instead of being called "factors" (which is commonly a statistical terminology), are called "design variables" (which is an optimisation terminology). The design variables taken into consideration for this study are: exit location; exit width and relative distance between exits. (It should be noted that these design variables will not be discussed further in the Chapter and the interested reader is referred to the previous Chapter).

Another important point which should be remembered is that the design variables are completely independent from each other, since their values have no impact on the value of any other design variables. The response (i.e., the evacuation time) is not a design variable, because it is a dependent variable, since its value will depend on the values and interaction between all of the other design variables.

This is similar to the mathematical principle of defining a function. "A function is an association between two or more variables, in which, for every value of each of the independent variables (or arguments), there is exactly one value of the dependent variable in a specified set called the domain of the function" **[66, 67]**.

It is a fundamental requirement to know the relationship between the different design variables in order to define their relation to the system.

Another basic, but also important issue is to define the nature of the design variables; i.e. to determine if they are discrete or continuous. It is relevant to mention here that, for a specific system, all the design variables should have the same nature.

Furthermore, once this understanding of the design variable is complete, they should then be defined.

Given a response θ which is a function of two design variables: α and β. These design variables will have their own lower (for instance, 0 for both design variables) and upper limits (for instance, 4 for both design variables) in which their values will vary. Consequently, the design space is then defined; where the combination of the values of both design variables will correspond to a unique value for the response, see Figure 3.1.

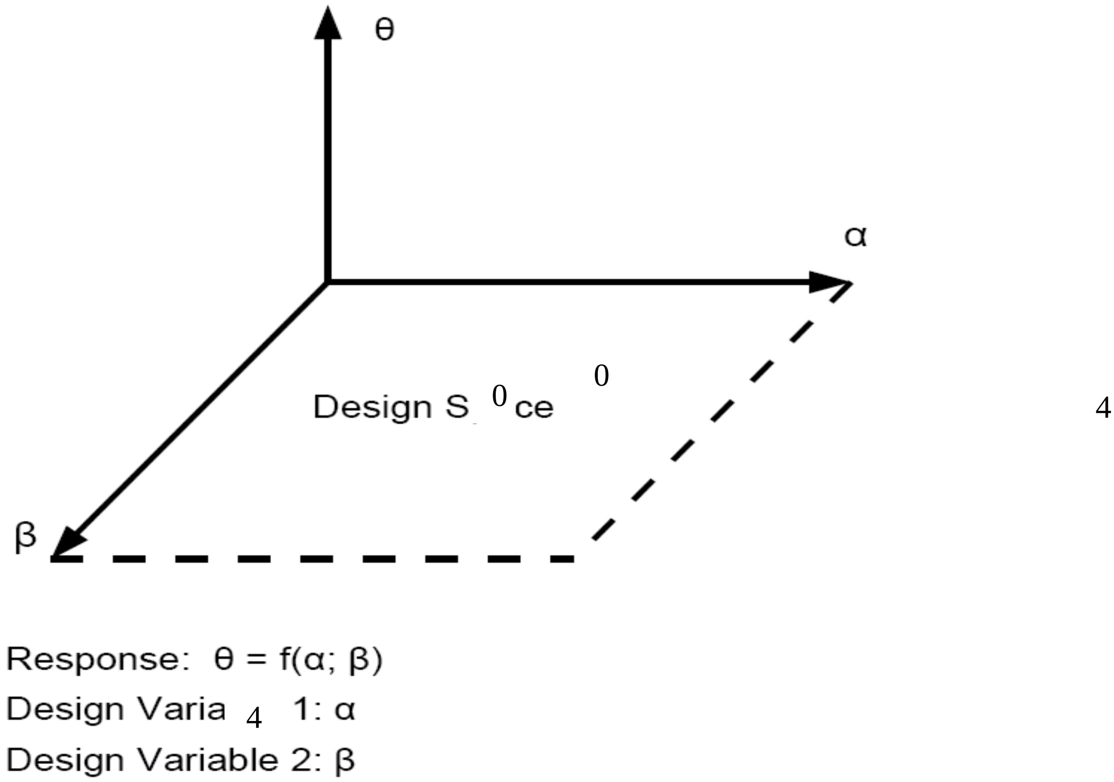


Figure 3.1: Design Space

The coordinates will then compose the response surface, as Figure 3.2 presents.

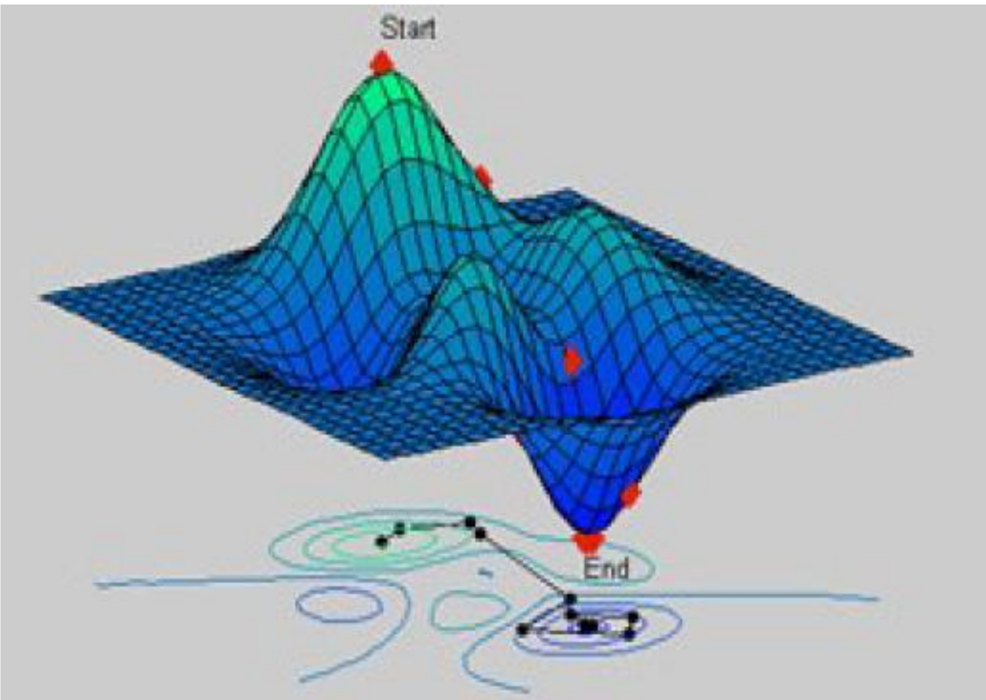


Figure 3.2: Hypothetical Response Surface (using MATLAB)

Once the concepts of design variables and design space are defined, the concepts behind the DoE techniques can be discussed. First of all, it is important to define what a design is. Design can be simply defined as “the art or process of deciding how something will look, work etc.” (definition taken from Oxford Advanced Learner's Dictionary, 2000). Therefore, it can be concluded that design of experiments is the process of deciding how experiments will be defined.

At this point, it is important to understand the definition of experiments. An experiment can be stated simply as a test. In practical life, designers (or investigators) perform experiments in order to discover something (i.e., the behaviour; the dynamics) about a particular process (i.e., system). Therefore, an experiment can be defined as a test or series of tests in which purposeful changes are made to the input variables of a process or system so that it is possible to observe and identify the reasons for changes that may be observed in the output response **[68]**.

Therefore, when an experiment, or a set of experiments, is defined (i.e., designed) a methodological process is then established. This "process" is named Design of Experiments, commonly known as DoE. An overview of the use of DoE techniques is provided in Figure 3.3.

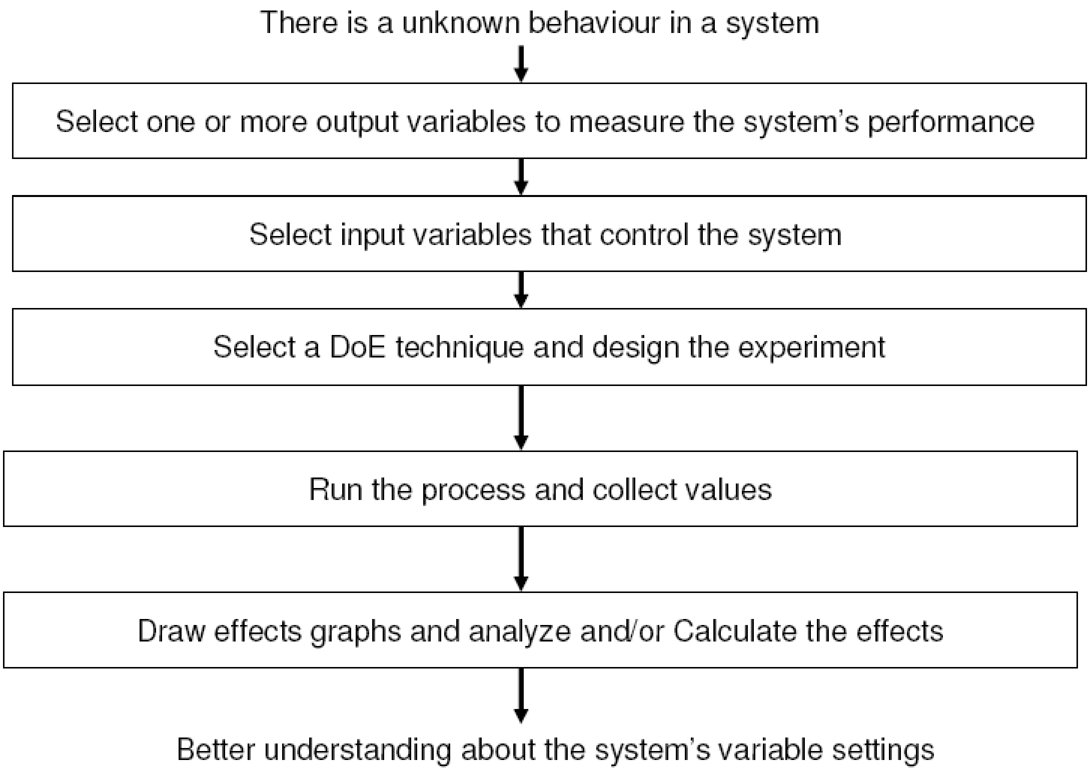


Figure 3.3: Overview of the use of DoE techniques

The common situation is when the designer does not know the exact underlying relationship between responses and design variables (of a specific system), but wants to know how the responses are influenced by these design variables **[69]**. (This Thesis is concerned with investigating how certain design variables impact the evacuation process). The problem of design of experiments is to establish what pattern of design points will best reveal aspects of the situation of interest (i.e., the investigated system)

[70]. DoE techniques are numerical methods used to obtain a set of design points that are spread throughout the current design space **[71]**. These techniques are responsible to plan experiments for collecting appropriate data which may be analysed by statistical methods resulting in valid and objective conclusions **[72]**. In other words, DoE techniques refer to the process of planning, designing and analysing the experiment so that valid and objective conclusions can be drawn effectively and efficiently **[73]**.

In this case, it is often helpful to approximate the underlying relationship with an empirical model, which is called a response surface model (RSM) or curve fit, **[74]**, (which is discussed in more detail in the next section of this chapter). For creating the RSM, it is needed to know the value of the responses for some combinations of design variables. Each combination of design variables could be viewed as a point in the *n*-dimensional *design space*, where n is the total number of design variables **[69]**. The particular arrangement of points in the design space is known as an experimental design or design of experiments **[75]**.

In reality, the problem of experimental design or design of experiments (DoE) is encountered in many fields, particularly in the development and optimization of manufacturing processes. There are many examples where DoE is encountered, such as: the manufacturing of engines in the car industry; the production of wafers in the electronics industry; the synthesis of compounds in the pharmaceutical industry; and many others. Another field in which the DoE techniques are applied is the optimization of analytical instruments. For instance, there are many DoE applications found in the scientific literature describing the optimization of spectrophotometers and chromatographic equipment.

Furthermore, there are sets of examples in which the DoE techniques are applied to biomedical experiments in the literature **[76]**. There are also examples in the literature of efforts of applying the DoE techniques to psychology and related behavioural studies; explaining technical terms encountered within the DoE techniques under the cognitive perspective **[77]**. The DoE techniques were also used for evaluating the production methods for milk **[78]**.

The DoE techniques were originally developed in the field of 'mathematical statistics'; when experiments with real systems, for instance, have been frequently subjected to the design and analysis techniques. Most developments of the DoE techniques have been in biological disciplines, in particular in agriculture, and also in medicine and psychology **[79]**. The main relevant principles of DoE techniques were established in the 1920s and 1930s **[79]**. In the 1930s, the DoE techniques were focussed on agricultural experiments **[80]**. Since the 1950s, DoE techniques were concentrated on chemical experimentation **[70]**. And since the 1990s, Taguchi's designs became very popular in industrial quality control. (Taguchi's DoE technique is discussed further in this chapter).

It is important to observe that DoE techniques do not tell which design variables should be studied; neither do DoE techniques tell which range of input values should be considered. In summary, the DoE techniques do tell which combinations of values of the design variables should be used for estimating the main effects of the variables and interactions between variables.

With this in mind, the key question that the designer might need to answer is: does a specific design variable or a set of design variables influence the response, and if so, how? Furthermore, the main purpose of DoE techniques is to help the designer to create an experimental design, analyse the characteristics of this design, create the response surface model for this design, and later, to analyse the characteristics of the response surface model.

Besides, it is important to determine accurate settings of design variables (what he also names "controllable variables") in order to reduce the effects of what he names "uncontrollable variables" (i.e., unknown factors) **[81]**. These variables "are difficult to control during an experiment and they are responsible for variability in product performance or product performance inconsistency" **[81]**. The Figure 3.4 shows the illustration for this explanation.

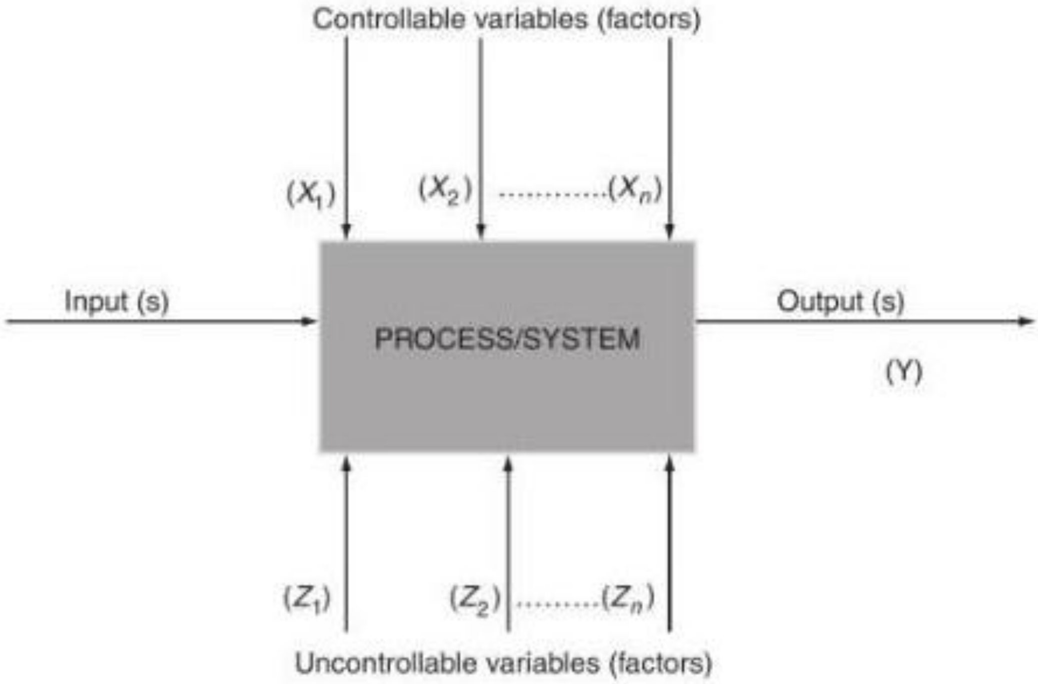


Figure 3.4: General model of a process/system [81]

At the time of writing this Thesis, there are a large (and increasing) number of DoE techniques. In fact, these techniques are based on logical algorithms which define the nature of the combinations between the design variables (i.e., these combinations are the *design points*). In other words, the number of design points needed and the location of these design points in the design space will be defined by the DoE techniques. And, in simple terms, what differentiates the various DoE techniques is how their algorithms work. In the next paragraphs, some DoE techniques are described.

In summary, DoE techniques help to define the place within the design space, where the design variable(s) should be.

"*Blessed are the 'placemakers': for they shall be called the helpers of the God of Maths*"

Adaptation from the Bible; Matt. 5:9

In the next paragraphs, some DoE techniques are presented and discussed, including those used in this study.

3.1.1 DoE – Review of some DoE techniques

"The best time to design an experiment is when it is finished; the worst time is at the beginning."
BOX et al (1978)

Here, a simple description of some DoE techniques is presented. The DoE techniques described here are: Factorial Designs; Central Composite Design; Latin Hypercube Design; Placket-Burman Design; Taguchi Design and Random Design.

Before starting to describe these DoE techniques, it is important to introduce the concepts of "design matrix" and "model matrix". An useful description of these two concepts is available in the literature **[82]**; therefore, this thesis follows this approach.

The design matrix characterizes the experimental design (i.e., the design of experiment). In this matrix, there will be as many columns as there are design variables; and also, there will be as many rows as there are points (i.e., design points) in the experimental design. Therefore, each row of the matrix represents a particular combination of the design variables for the corresponding point in design space (i.e., the design point). Each column of the design matrix represents the values of the particular design variable at all the points of the experimental design. For instance, considering the following problem: 2 design variables X and Y. For both design variables, the lower limit is -1 and the upper limit is 1. Considering the full factorial design in two variables, the design matrix is represented in Table 3.1. And the geometric illustration for this case is shown in Figure 3.5.

Table 3.1: Design matrix for a full factorial design in two variables

Design Point	**X**	**Y**
Design point 1	1	1
Design point 2	1	-1
Design point 3	-1	1
Design point 4	-1	-1

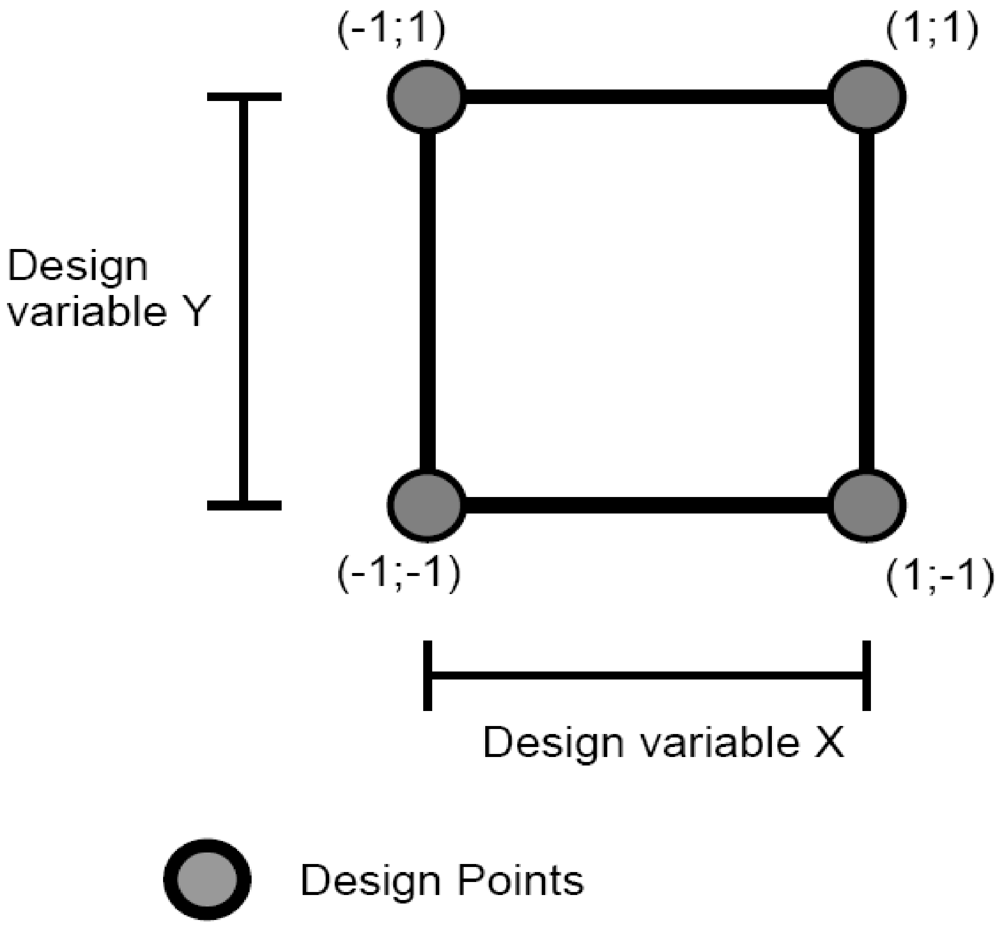


Figure 3.5: A Full factorial design in two variables (2^2 design)

Taking the same example and using now a $2^{(3-1)}$ fractional factorial design in three design variables, where the third design variable, Z, has the same upper and lower limits. The design matrix is shown in Table 3.2 and the geometric representation is shown in Figure 3.6.

Table 3.2: Design matrix for a fractional factorial design in three variables

Design Point	**X**	**Y**	**Z**
Design point 1	1	1	-1
Design point 2	1	-1	1
Design point 3	-1	1	1
Design point 4	-1	-1	-1

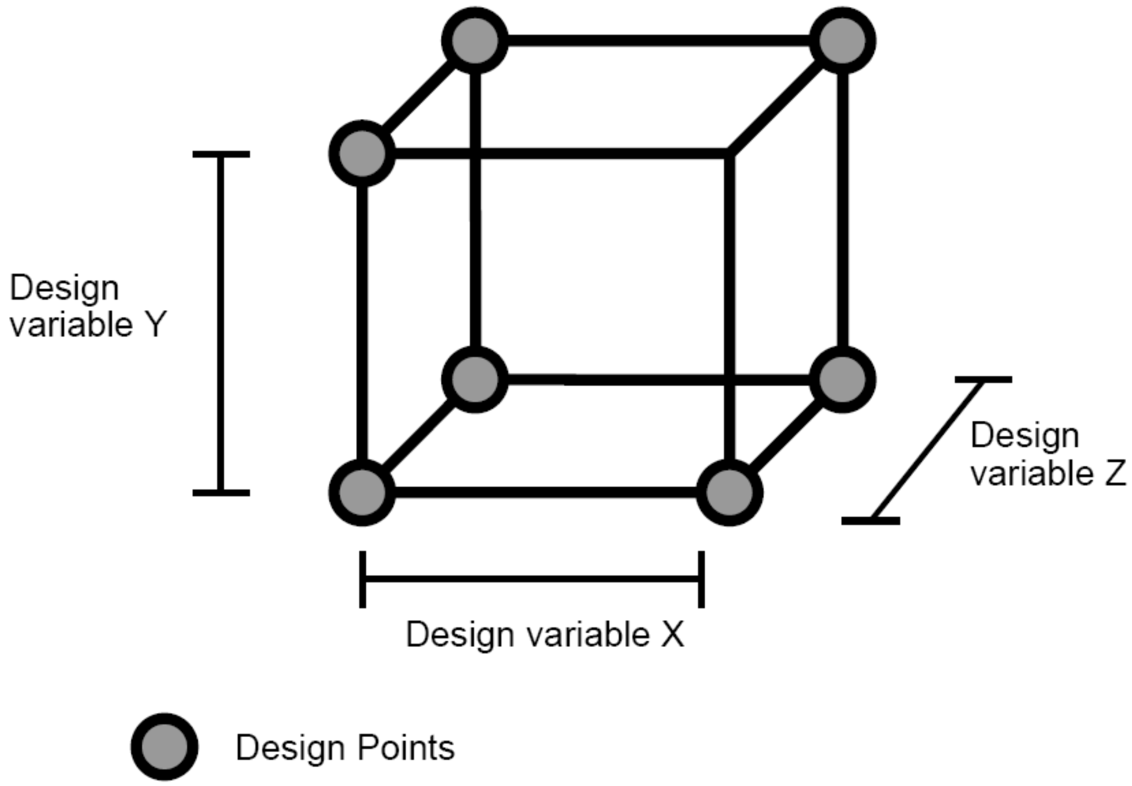


Figure 3.6: A Fractional factorial design in three variables ($2^{(3-1)}$ design)

The other relevant concept which must be understood is the model matrix. The model matrix helps to better understand the various efficiency measures of an experimental design. The model matrix is similar to the design matrix. As a consequence, there will be as many rows in the model matrix as there are points in the experimental design (consequently, the same number of rows as in the design matrix). Also, there will be as many columns in the model matrix as there are terms in the Response Surface Model (RSM). It should be noted that the RSM will be explained in more detail, later in this chapter. Each row of the model matrix represents the particular combination of the terms in the RSM (not the design variables as in the design matrix) for the corresponding point in the design space. Each column of the model matrix represents the values of the particular term in the RSM at all the corresponding points of the experimental design. The concept of model matrix will be explained further when the concepts of RSM are discussed.

In the following sections, some DoE techniques are presented and discussed.

3.1.1.2 Factorial Designs

Many experiments involve the analysis of the impact of two or more design variables. Factorial designs are the most efficient DoE technique for this type of experiment **[83]**. (This name factorial designs or factorial experiments was first coined by Fisher, R.A. **[84,85]**.

In practical terms, factorial design means that in each complete trial or replication of the experiment, all possible combinations within the limits (lower and upper) of the design variables are investigated. These values, defined by all the combinations (within the limits), are commonly called “levels”.

For instance, if there are **'*c*'** levels for a design variable **C** and **'*d*'** levels for a design variable D, so it follows that each replicate contains all **'*cd*'** treatment combinations. When the design variables are arranged in a factorial design, they are often said to be “crossed”. In fact, this is one of the main advantages of factorial designs because, instead of arranging the design variables at a time (i.e. this methodology is commonly called a “one-factor-at-a-time” experiment and is also known as “pseudo scientific”), it arranges the design variables in a manner which reveals the interactions between them. It is important to know these interactions, since they might have influence on the response.

Factorial designs can be classified into three main designs types according to the levels taken into account, namely: Full Factorial Design; Three-Level Factorial Design and Fractional Factorial Design. These types of design are described in the following sections.

Full Factorial Designs

Considering the multidimensional box in the design space shown in Figure 3.8, in which the designer is interested in estimating the relationship between a response and the design variables within this box. This box is defined by the upper and lower limits on the design variables. Therefore, for this case, it is useful to estimate the responses at the vertices of this multidimensional box. This kind of arrangement of points in the design space is denominated as a “full factorial experimental design” or simply “full factorial design”.

In summary, the design points defined using the full factorial design are the vertices of a “hyper cube”, which can be in any *n*-dimensional design space, defined by the minimum (i.e., lower) and the maximum (i.e., upper) values (i.e., limits) of each design variable. These design points are also called factorial points.

The name factorial is used because the design variables involved in the RSM are often referred to as factors in statistics, as mentioned previously in this chapter.

A very important case for factorial design is the one described in Figure 3.7, where each of the design variables has only two possible values (i.e., two levels). In this case, the designs are called 2^n factorial designs (where n is the total number of design variables). In order terms, for these cases, the design is called full factorial design.

In summary, the full factorial design is the DoE technique in which the method to create the design points is based on this simple formula 2^n. And all the design points include all possible combinations of the minimum and maximum values of the design variables.

Three-Level Factorial Designs

When each design variable is evaluated at three levels, the DoE technique is then called a 3^n factorial design. In other words, all possible combinations of three discrete values of the design variable are used. Therefore, the three-level factorial design is the DoE technique in which the method to create the design points is based on the simple formula 3^n.

Fractional Factorial Designs

For cases in which the number of design points is high, factorial designs grow rapidly (because their formulas obey an exponential order), which would tend to make them impractical to use. For instance, a case with 10 design variables using a two-level full factorial design will consist of 2^{10} = 1,024 design points. The same case, instead using a three-level factorial design, would consist of 3^{10} = 59,049 design points.

Based on these problems, in order to make the factorial designs more practical and feasible to use, the fractional factorial design was proposed. The fractional factorial design establishes that it is possible to estimate the response not at all the vertices of the multidimensional box, but only at a subset of the vertices. The formula used for this experimental design is $2^{(n-k)}$, where n is the total number of design variables and k is an integer number smaller than n.

In general terms, one of the ways for constructing such design is: firstly, the design matrix for the full factorial design in (n-k) design variables is built. Secondly, the k columns are added to this design matrix.

3.1.1.3 Central Composite Design - CCD

This DoE technique is commonly used to fit the produced response data to a non-linear RSM, such as a second order polynomial **[86]**. For this reason, in order to define the coefficients in a polynomial with quadratic terms, for instance, the DoE technique must provide at least three levels for each design variable. Therefore, in the multidimensional box defined by the design variables limits, the CCD consists of three different types of design points, namely: factorial points; central point and axial points.

The factorial points are the same points described in the factorial designs; either from a full factorial design or from a fractional factorial design.

The central point, as the name suggests, is a single point at the centre of the design space (i.e., the centre of the multidimensional box).

The axial points, as the name suggests, are the points located on the axes of the coordinate system and located symmetrically with respect to the central point.

Based on this arrangement, the formula used to define the design points for the CCD is $1+2n+2^n$. Figures 3.7 and 3.8 illustrate the design points defined using the CCD DoE technique for hypothetical cases.

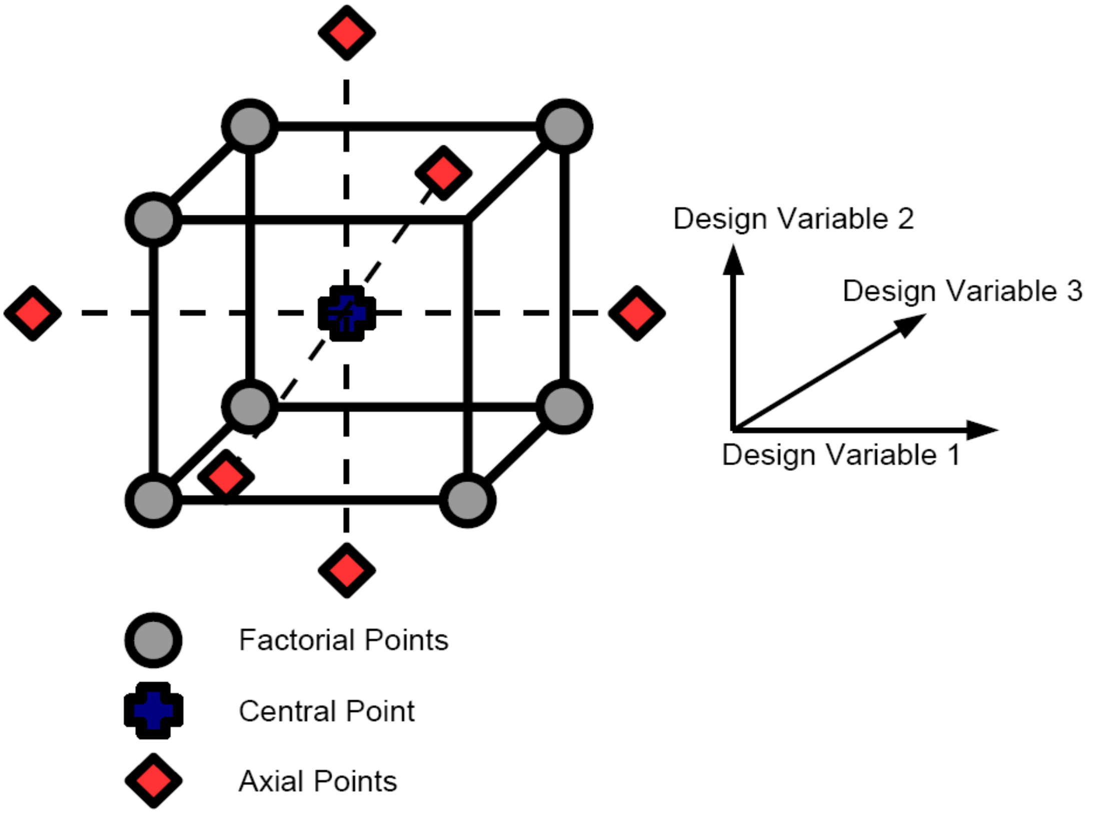


Figure 3.7: CCD with three design variables

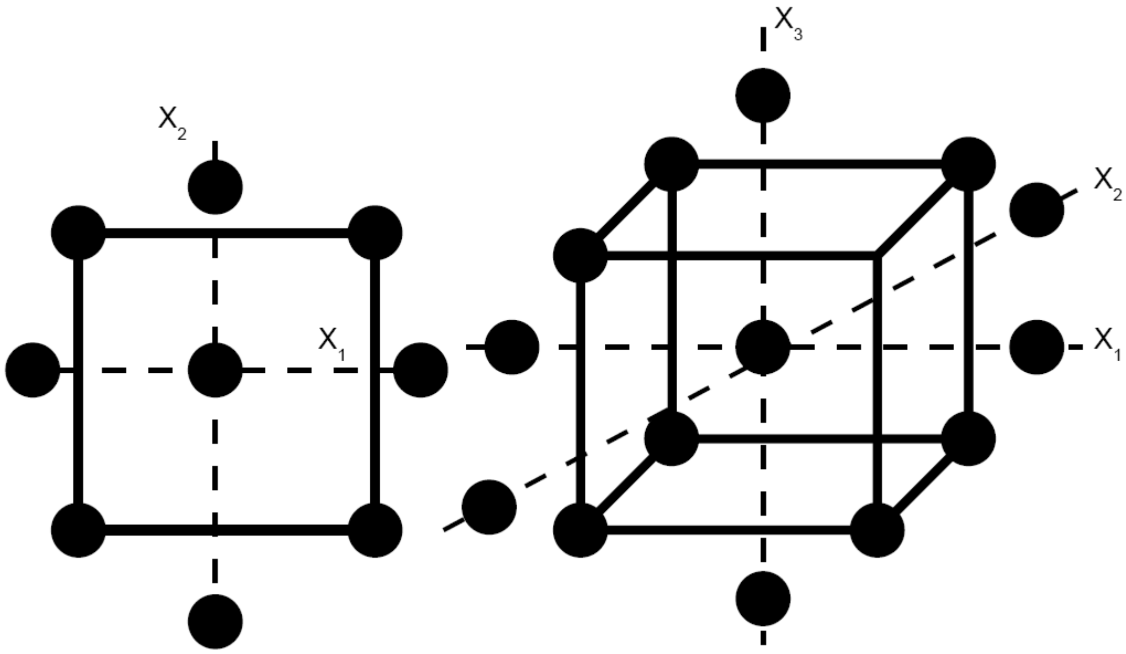


Figure 3.8: CCD for two and three design variables

One of the advantages of using this DoE technique is that it only requires a small number of design points. The other advantage is the fact that this technique selects the points which are located in the centre of the edges of the design space; which allows it to cover the important regions of the design space.

The disadvantage of the CCD (similar to the disadvantage of the full factorial design) is that this DoE technique is only efficient for a rather small number of design variables **[82]**. For problems with 2, 3 or 4 design variables, the CCD can predict good design points (i.e., well located in the design space).

The table 3.3 presents a comparison between the factorial designs and the CCD in terms of design points produced using them.

Table 3.3: Design matrix for a fractional factorial design in three variables

Number of Design Variables	Full Factorial Design	Three-Level Factorial Design	Central Composite Design
n	**2^n**	**3^n**	**$1+2n+2^n$**
2	4	9	9
3	8	27	15
4	16	81	25
5	32	243	43
6	64	729	77
7	128	2187	143
8	256	6561	273

3.1.1.4 Latin Hypercube Design - LHD

This DoE technique was introduced in 1979 for numerically evaluating a multiple integral **[87]**. Latin Hypercube Designs are considered by many specialists in this field, as the most popular class of DoE technique. There are several publications about this experimental plan and some of them are listed in the references of this chapter.

By definition, the LHD specify the sample points so that as much of the design space is sampled as uniformly possible, with the minimum number of response evaluations. This DoE technique places the input levels for each design variable on a uniform grid. Then it places the levels across the design variables by randomly permuting the column for each design variable.

An ideal DoE technique should use only a few important design variables. Furthermore, when it plans the design of these design variables, replication is not required. This can be achieved by using the LHD **[88]**.

In reality, the LHD has the property that by projecting an *n*-point design on to any design variable, it will be possible to get *n different* levels for that design variable. This feature makes LHD highly suited for computer experimentation and this why it is very popular with computational experiments.

The LHD uses a common rule a uniform probability distribution to define its special design (i.e., to select the design points). For this reason, this DoE technique is based on statistical criterion, which might provide some consistence for the generated results. The algorithm in this case, for instance, works like this: the set of values obtained for the first design variable are combined randomly (but into equally likely combinations) with the set of values of the second design variable. These pairs are combined again with the set of values of the third design variable and so on; until it is possible to construct a matrix.

In summary, the advantage of using the LHD is that it selects *n* equally spaced values for each design variable and it randomly scrambles the order of the values for each design variable. For instance, considering a case with two design variables A and B. For each design variable, lower and higher values are taken into consideration. For design variable A, A_H and A_L are these values; for design variable B, B_H and B_L are these values. These design variables are sampled with a resolution of 5; which means that 5 values of each design variable are sampled. It is important to notice that the same value of any design variable is never tested twice and the cells within the Hypercube are themselves chosen randomly, see Figure 3.9.

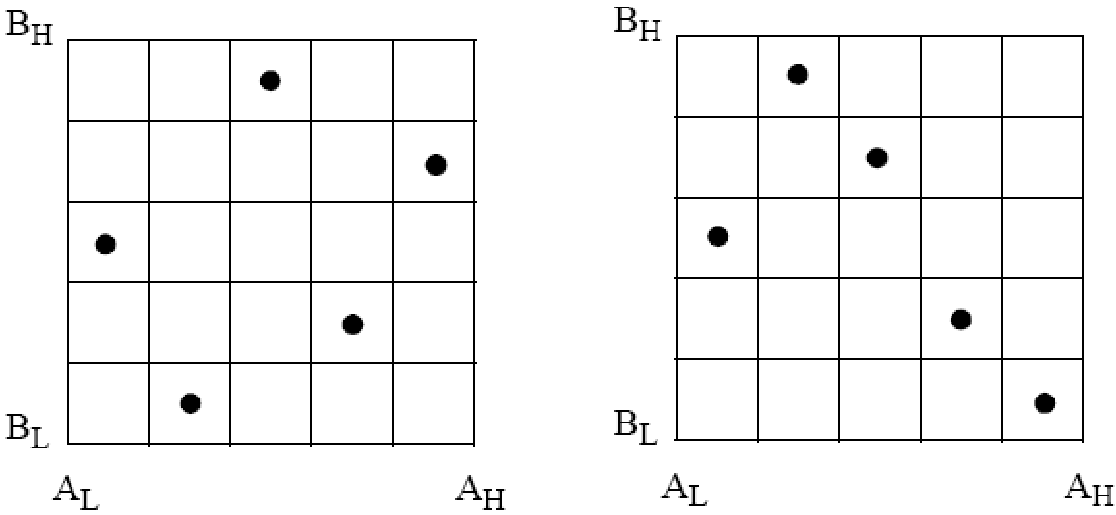


Figure 3.9: Hypothetical case for two possible samplings using the LHD of a two dimensional design space with a "discretization" level of five

From figure 3.11, it is possible to see that the design points that were selected are well spread through the design space and also, once the experiment is repeated using this DoE technique, different design points are selected.

3.1.1.5 Placket-Burman Design

This DoE technique has been largely used in bio-pharmaceutical and biotechnological studies, **[89, 90].** The Placket-Burman DoE technique is a two-level design and it plans the design based on the formula: $K = n + 1$. This DoE technique has a limitation since it can be only used when K is a multiple of 4.

This DoE technique defines signs for the values of the design variables. The “-” sign corresponds to the minimum value and the “+” sign corresponds to the maximum value, as specified by the range of the design variable. All the other rows of the design matrix are built by shifting the previous one.

The Figure 3.10 presents an example using the Plackett-Burman design in 12 runs for up to 11 design variables.

	Pattern	*X*1	*X*2	*X*3	*X*4	*X*5	*X*6	*X*7	*X*8	*X*9	*X*10	*X*11
1	+++++++++++	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1
2	-+-+++---+-	-1	+1	-1	+1	+1	+1	-1	-1	-1	+1	-1
3	--+-+++---+	-1	-1	+1	-1	+1	+1	+1	-1	-1	-1	+1
4	+--+-+++---	+1	-1	-1	+1	-1	+1	+1	+1	-1	-1	-1
5	-+--+-+++--	-1	+1	-1	-1	+1	-1	+1	+1	+1	-1	-1
6	--+--+-+++-	-1	-1	+1	-1	-1	+1	-1	+1	+1	+1	-1
7	---+--+-+++	-1	-1	-1	+1	-1	-1	+1	-1	+1	+1	+1
8	+---+--+-++	+1	-1	-1	-1	+1	-1	-1	+1	-1	+1	+1
9	++---+--+-+	+1	+1	-1	-1	-1	+1	-1	-1	+1	-1	+1
10	+++---+--+-	+1	+1	+1	-1	-1	-1	+1	-1	-1	+1	-1
11	-+++---+--+	-1	+1	+1	+1	-1	-1	-1	+1	-1	-1	+1
12	+-+++---+--	+1	-1	+1	+1	+1	-1	-1	-1	+1	-1	-1

Figure 3.10: An example using the Plackett-Burman design in 12 runs for up to 11 design variables.

3.1.1.6 Taguchi Design

As previously mentioned, in this chapter, this DoE technique became very popular in industrial quality control since the 1990's, especially in the USA **[91]**. Nevertheless, this technique has actually been used since early 1960's, in Japan, by companies such as Sony and Toyota, in their TQM (i.e., Total Quality Management) approach.

This DoE technique uses orthogonal arrays for searching the design space. Orthogonal arrays are special matrices used as design matrices in fractional factorial designs. These arrays allow the effects of several design variables to be determined efficiently.

Hence, for each design point in the design space (called an "inner array" by Taguchi), the design metric (called the "S/N ratio" by Taguchi) as a function of the experimental arrangement of the *DP*s xj (i.e., Taguchi calls this the design parameter, which is the design variable, whose value is chosen during the design process) is:

$$S/N(\vec{x}) \equiv -10\log\left[\int_{Pr(\vec{p}|\vec{x})}(PP(\vec{x},\vec{p})-\tau)^2 dPr(\vec{p}|\vec{x})\right]$$

$$S/N(\vec{x}_j) \equiv -10\log\left[\sum_{i=1}^{m}(PP(\vec{x}_j,\vec{p}_i)-\tau)^2\times 1/m\right]$$

Where *PP* is the one performance parameter being considered, *m* is the number of noise parameter arrangements *pi*, and τ is the desired target value. The points in the noise space *pi* (called the "outer array" by Taguchi) are chosen using a factorial method (called "orthogonal arrays" by Taguchi). Given that the noise consists entirely of probabilistic uncertainty, Taguchi's method is an experimental approximation to an exact expression of:

This DoE technique is well structured in mathematical principles as shown in the previous paragraphs. In general, for the Taguchi Orthogonal Arrays, the maximum number of variables to be used with corresponding design is determined by the relationship: n = (N -1)/2, where n is the maximum number of design variables and N is the number of points in the design. One disadvantage of this technique is the fact that it only works well for a small number of design variables. For more than 4 design variables this technique is not recommended.

3.1.1.7 Random Design

Standard response surface (this is discussed in more detail in the next section of this chapter) designs, such as those defined by the CCD or factorial designs, for instance, are widely used because they are quite general and flexible designs **[92].** Nevertheless, there are situations where these standard response surface designs would not be applicable. For these kinds of situation, the complexity involved requires alternative “DoE techniques”, such as “computer-generated designs” – which is a new terminology that is starting to be used.

A typical example of a situation in which a computer-generated design may be appropriate, is when the experimental region is irregular. In other words, when the design space has been modified. This is very common when the problem is constrained (this will be better explained in this chapter, when numerical optimisation techniques are discussed).

The literature presents a simple example which can illustrate this kind of situation **[92]**. The example is as follows: A designer is investigating the properties of a particular adhesive. The adhesive is applied to two parts and is then cured at an elevated temperature. Over the ranges of these two design variables, taken as -1 to +1 on the usual coded variable scale, the designer knows that if too little adhesive is applied and/or the cure temperature is too low, the parts will not bond satisfactorily. In terms of the coded variables, this leads to a constraint on the design variables:

$$-1.5 < x_1 + x_2 \qquad \text{(eq.3.3)}$$

Where x_1 is the application amount of the adhesive and x_2 is the temperature. Therefore, if the temperature is too high and too much adhesive is applied, the parts will either be damaged by heat stress or an inadequate bond will result. This leads to another constraint on the design variable levels:

$$x_1 + x_2 < 1 \qquad \text{(eq.3.4)}$$

The Figure 3.11 shows the design region (once it is a 2D problem); i.e., experimental region that results from applying these constraints.

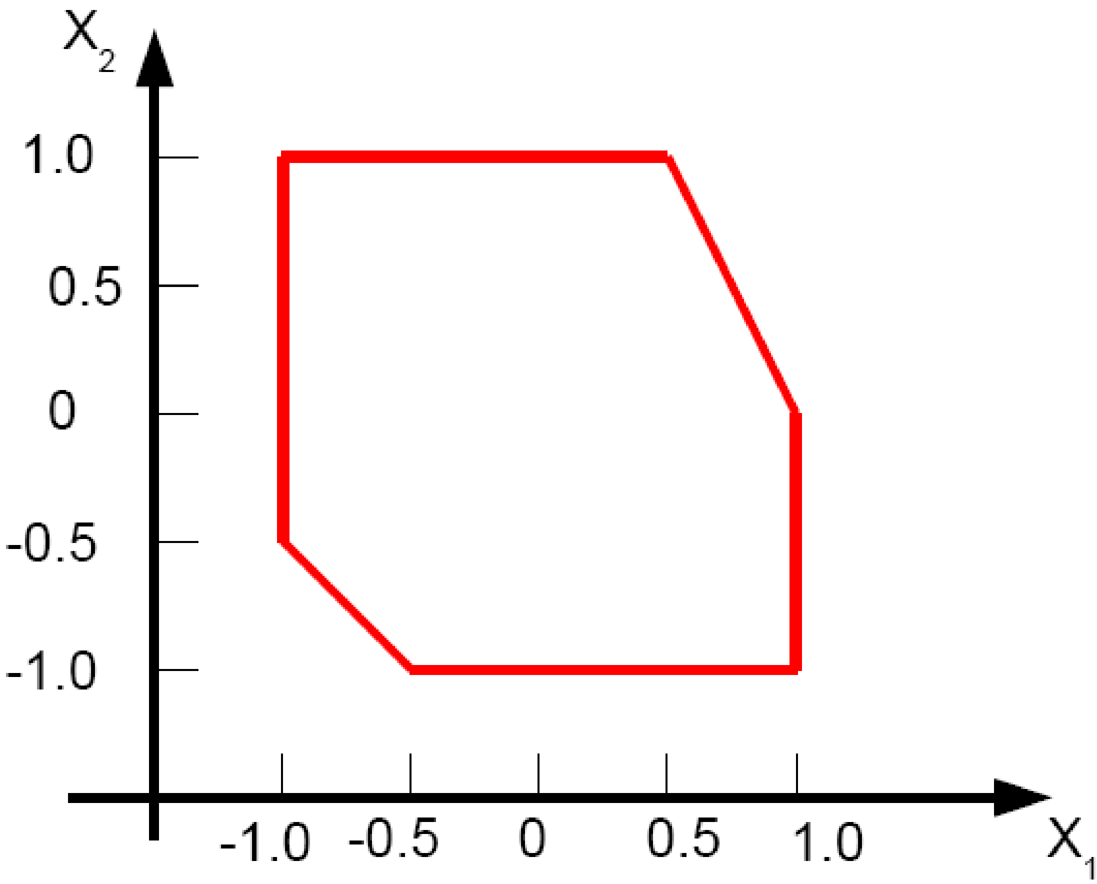


Figure 3.11: Example of a constrained design region in two design variables

Therefore, for this type of situation, where the design space or design region is modified (once the levels freedom is restricted), alternative techniques must be applied.

One of the alternative DoE techniques, which is one type of computer-generated design, is the Random Design. As the name suggests, this DoE technique picks the design points randomly throughout the design space. In this technique, no statistical criterion is used for selection of design points. This technique is also suitable when the number of design variables is high.

The values of the design variables are calculated by mapping the result of a random number generator in the range defined by the minimum and maximum values of the design variables. The random number generator used to construct the random design in VisualDOC computational package (which was the one used here in this thesis for the constrained problems) makes use of a uniform statistical distribution to create new random points. This means that every design point in the design space has an equal probability of being chosen as the next random point.

3.1.1.10 Final comments on DoE Techniques

In general terms, the DoE techniques have different features and the best choice is strongly dependent on the characteristics of the performed experiment either real or computational. Besides, the selection of the DoE technique will impact the way in which the response values of the design points are fitted by the RSM. Therefore, the DoE techniques' selection is strongly associated with the selection of the RSM. This relation will be better analysed in the next section of this chapter, where the Response Surface Models are presented and discussed.

Stoyanov, S. [**93]** made an accurate and objective comparison of the DoE techniques in which he also provided a useful table showing some of the computational DoE packages.

3.2 Response Surface Models (RSM)

The use of RSM has become a popular tool for multi-disciplinary optimisation, **[94, 95]**. The response surface models methodology was proposed as a statistical tool for finding the key-conditions of a chemical process, at which some response was optimized **[96]**.

The RSM use mathematical models (i.e., response surfaces) to approximate the objective functions of a system in the design space **[97]**. Subsequently, the optimum search is performed on the response surfaces **[97]**. There have been many different RSM proposed, including polynomials, *adaptive splines*, radial basis functions etc. **[98]**. In fact, the use of the RSM constitutes a very important task for the whole optimisation problem. The reason for that is because depending on how the curve fit is defined, the objective function is going to be represented by this fitting. Therefore, even if the DoE technique and the numerical optimisation techniques were selected accurately, if the RSM is not well defined, then the final optimum solution might not represent the real best solution.

In other words, if the RSM chosen to fit the data does not cover the design space of the problem, then when the numerical optimisation technique is applied, the result found from its use may not be realistic. For example: if considering a minimization problem, for instance, the search algorithm from the numerical optimisation technique would find the local minima region instead of the global minima; and in some cases, it might even find a region that is far from the optimum solution (see Figure 3.12). In summary, the RSM is responsible for "arranging" all the design points previously generated and this is arrangement is crucial for the solving of the problem.

The development of Response Surface Models, RSM, called as "approximation functions" **[99]**, has become a separate problem within the classical optimisation field. Since this is a large topic in its own right, only the basics concepts will be discussed here.

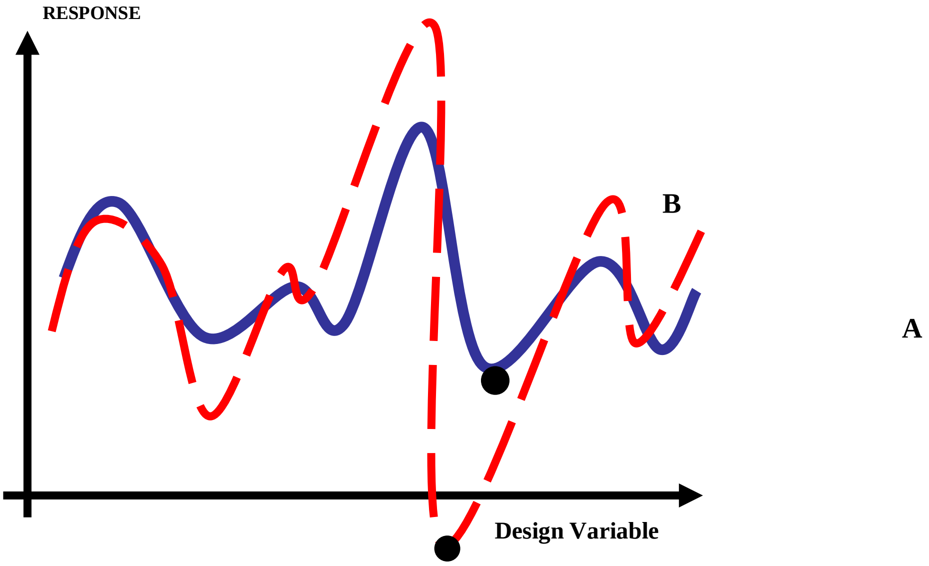


Figure 3.12: Response Curve for a hypothetical problem in which the objective function depends on 1 design variable

From Figure 3.12, the curve A represents the real response curve and the curve B represents the curve fitting proposed by the RSM (i.e., the virtual curve). It is clearly possible to see that, for this hypothetical example, the chosen RSM is not appropriate. The dark dots, which represent the global minima regions for both curves, are far away from each other. If the numerical optimisation techniques were applied for this case, a negative solution would be found; and this does not correspond to the real solution, which is positive.

Another important issue relating to the use of RSM, is the prior step needed to select the DoE technique. According to the previous discussion, the selection of the DoE technique is fundamental, because it will define the design points which will represent the design space. If this representation is not accurate (i.e. if the design points are not well located within the design space), as consequence, even if the RSM was selected properly, it will not represent the real design space and subsequently, it will not give the real solution, see Figure 3.13.

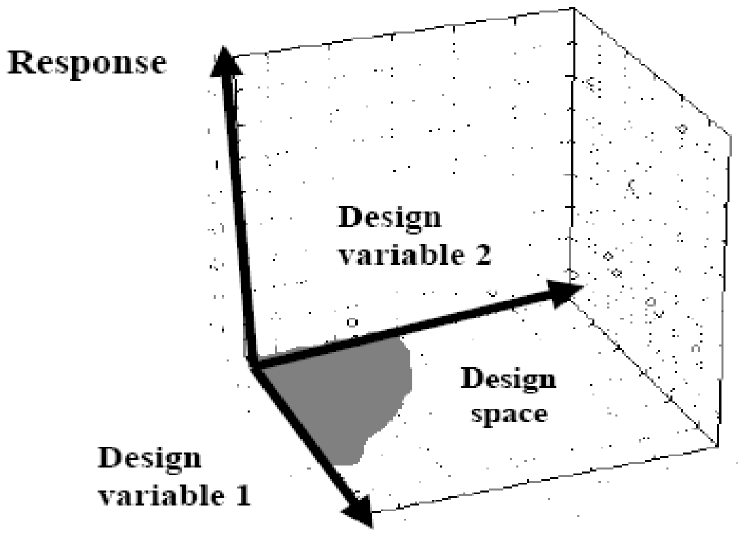


Figure 3.13: Hypothetical case in which the design points were selected by an inappropriate DoE technique

The grey region represents the region in which the design points were selected using a DoE technique. It shows clearly that this selection was inappropriate and will not represent the full design space realistically. The RSM will be based on this region and consequently it will not represent the real response surface. Finally the numerical optimisation techniques, when applied, will not find the real “optimal solution”.

Furthermore, as mentioned in the previous section of this chapter, this is why the selection of the DoE technique will impact the way in which the response values of the design points are fitted by the RSM. Therefore, the DoE techniques' selection is strongly associated with the selection of the RSM.

In addition, once the objective function is built, based on the RSM, and the curve fitting is accurate (i.e., the R^2 value is sufficiently close to 1; this will be explained in detail in this section), then it is possible to use the numerical optimisation techniques to solve the problem. Otherwise, it will be necessary to change the DoE technique in order to rebuild the design space and/or to use another RSM.

There are many definitions for RSM, for instance: “RSM is a collection of statistical and mathematical methods that are useful for modelling and analysing engineering problems. In this technique, the main objective is to optimize the response surface that is influenced by various process parameters. Response surface methodology also quantifies the relationship between the controllable input parameters and the obtained response surfaces" **[100]**.

The development of RSM follows a methodological set of procedures **[101]**. These procedures are adopted (and adapted) here in this thesis and summarized as follows:

- (i) The design of a series of experiments for the adequate and reliable measurement of the response of interest (i.e., as defined by the DoE techniques);
- (ii) The development of a mathematical model for the response surface with the best fit;
- (iii) Representing the direct and interactive effects of the process parameters through two and three dimensional plots, when possible.

The first procedure (i) was explained in the previous section of this chapter, when the DoE techniques were presented and discussed. Here in this section, the other two procedures, (ii) and (iii), are briefly presented and discussed.

The development of a mathematical model for the response surface (when there are more than one design variable considered), see Figure 3.14, or for a response region (when there is only one design variable considered), see Figure 3.15, is essentially a regression analysis task.

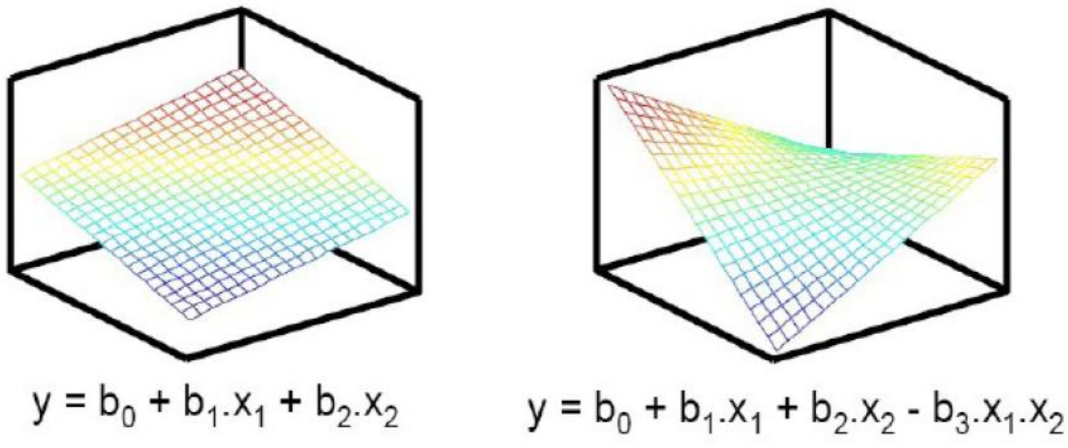


Figure 3.14: Two hypothetical response surfaces

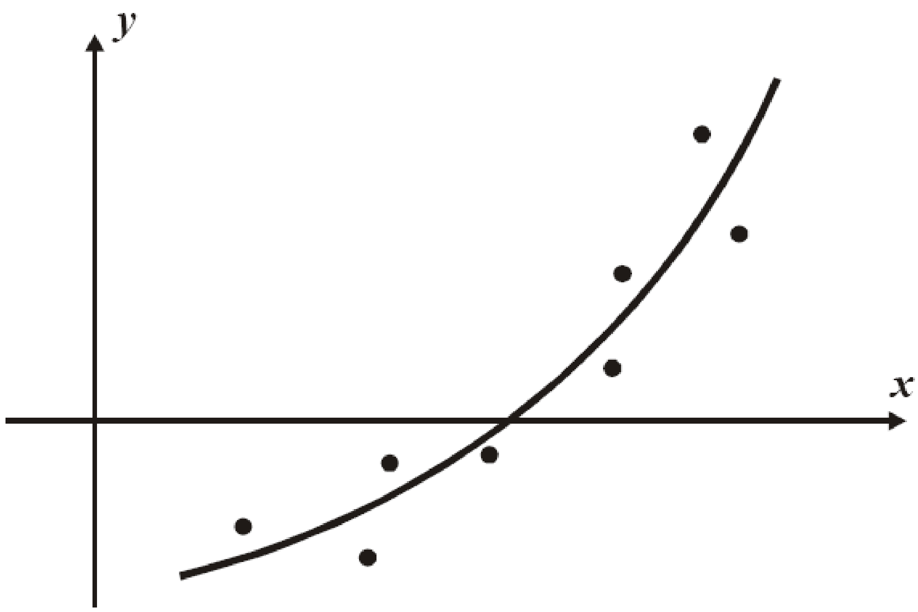


Figure 3.15: Hypothetical response region

In general terms, there are a wide variety of model types for performing a regression analysis. Before starting to describe some of these “regression models”, it is relevant to consider the concept of model. According to what was mentioned in chapter 2 of this thesis, it can be said that a model is a representation of some phenomenon. Following this logic, a mathematical model is a mathematical expression of some phenomenon. Therefore, a model commonly describes relationships between variables (the independent variables – design variables and the dependent variable – the response). In simple terms, these models can be classified into two types: deterministic models and probabilistic models.

Deterministic models hypothesize exact relationships and are suitable when the estimation (i.e., prediction) error is acceptable. For instance, in physics, many models are based on this approach to describe real phenomena. The second law of Newton is a good example of it; when he said that force is “exactly” mass times acceleration (i.e., $F = m \cdot a$).

Probabilistic models, on the other hand, hypothesize two components, namely: a deterministic component in nature and a random error. In these models, the “unknown factor” is taken into consideration (i.e., random error). For example, for a specific human being from planet Earth, happiness in the material world (Y) is 20 times the socio-economic status (X) plus random error (ε):

$$Y = 20X + \varepsilon \qquad \text{(eq.3.5)}$$

The random error may be due to other factors other than the socio-economic status and the error term could have a bigger effect on the response than the design variable X.

In summary, there three types of probabilistic models: regression models; correlation models and “other models”.

The regression models are used to answer the main question: what is the relationship between the variables? Furthermore, an equation is used to answer this question. This equation is composed by the dependent variable (i.e., the response) and the independent variables (i.e., the design variables). Therefore, regression models are used mainly used for estimation and prediction of the dependent variable.

When specifying the model, the variables are defined (i.e., which are already defined when using the DoE techniques, once the data generated from these techniques will be used in the regression model) and the nature of the relationship is also defined (i.e., the function form: linear or non-linear).

The Figure 3.16 summarizes the types of regression models.

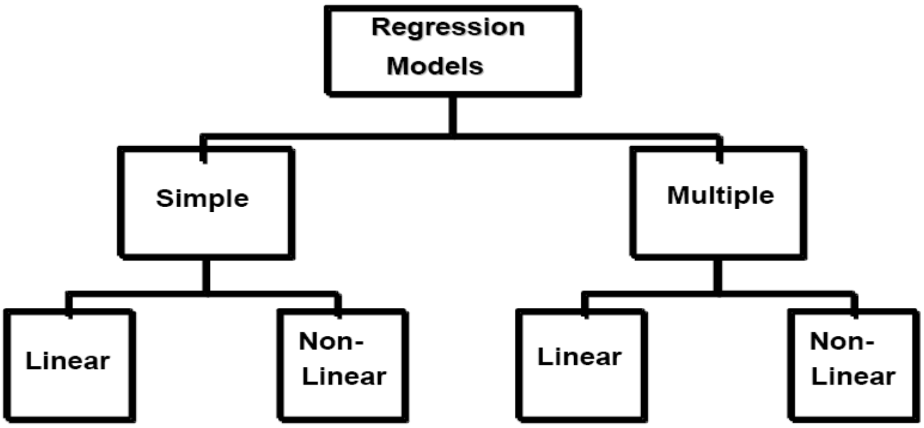


Figure 3.16: Types of regression models

The simple regression models are used when the effect of a single design variable is taken into consideration for the response. And the multiple regression models (the term multiple regression was first used by Pearson in 1908, **[102]**) are used when the effect of more than one design variable is taken into consideration for the response. In this study, as it will be seen later, most of the problems used the multiple regression model, since more than one design variable was considered; apart from the first study case in chapter 5, where the simple regression model was used, given that just one design variable was considered.

The linear regression model is very popular for both simple and multiple regression models. As the name suggests, the relationship between the variables is assumed to be linear. The general equation for the linear regression model is:

$$Y = \beta_0 + \beta_1 X_i + \varepsilon_i \qquad \text{(eq.3.6)}$$

Where:

- Y is the dependent variable (response);
- β_0 is the coefficient (Y – intercept);

- β_1 is the coefficient (slope);
- X is the independent variable (design variable);
- ε is the random error.

These coefficients (or parameters) represent the difference between the estimated curve and the actual curve defined by the data. In other words, this difference is the random error. The smaller this error, the better and more accurate is the curve fitting, see Figure 3.17. Hence, the best fitting will mean that the difference between actual values and predicted values are a minimum. This accuracy is assured by the regression model.

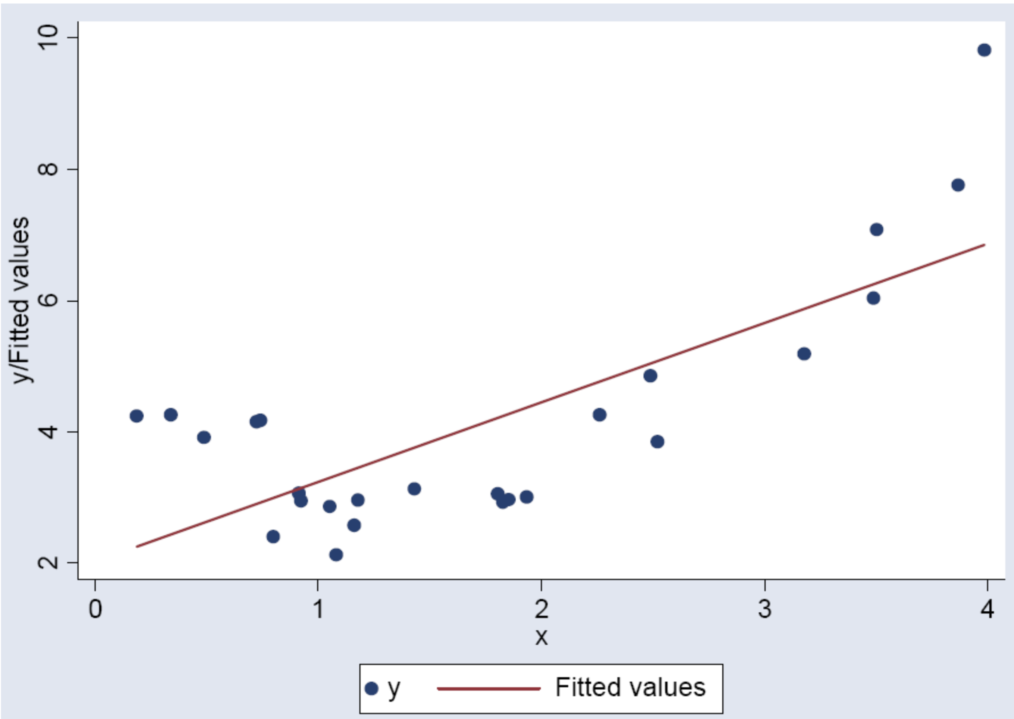


Figure 3.17: Example of a curve fitting

There are many different methods for estimating these coefficients (i.e., regression coefficients). This process of estimating these coefficients, and consequently "finding" a better curve fitting, is also commonly called the "building model process". The Least Square Method is a classical method for estimating such coefficients. Another method which has become popular is the General Stepwise Regression Method, which has been used for almost all the cases analysed in this study.

Before starting to describe these two regression methods, it is important to explain how the function which will define the fitting of these points is developed. The most widely used response surface approximation model functions are the first and the second-order polynomials:

$$Y = \beta_0 + \sum_{i=1,i<j}^{k-1} \sum_{j=2}^{k} \beta_{ij} x_i x_j + \sum_{i=1}^{k} \beta_i x_i^2 + \varepsilon$$

(eq.3.7)

As mentioned before, the β terms are coefficients, which are defined in the model function, and are obtained using regression methods (which are explained in detail later in this chapter).

Generally, eq. 3.7 can be rewritten in matrix form:

$$Y = bX + E \qquad \text{(eq. 3.8)}$$

Where:

- Y is the matrix of the measured values (the response);
- X is the matrix of the independent variables (i.e., design variables);
- b is the matrix of the coefficients;
- E is the matrix of the errors.

Therefore, the solution of eq. 3.8 can be obtained by the matrix approach:

$$b = (X^T X)^{-1} X^T Y \qquad \text{(eq. 3.9)}$$

Where:

- X^T is the transpose of the matrix X
- $(X^T X)^{-1}$ is the inverse of the matrix $X^T X$

For instance, if *m* different design points (defined by the DoE techniques) are considered in order to obtain the response values Y; as a consequence, the design variable values for all design points will be used to build the *m x n* design matrix D Є R^{mxn} (where *n* is the number of design variables) and the *m*-dimensional vector of the response data (obtained through experiments; real laboratory or computational simulations), $Y \in R^m$. It is also important to mention that, in order to build a robust RSM, the number of design points must be at least equal to (or, ideally, greater than) the dimension of the RSM terms (i.e., the number of coefficients β from the model function).

As mentioned previously, the first and second-order polynomials are largely used as model functions for response surface approximations. Nevertheless, for some cases, they do not give satisfactory results for fitting the curve. As an alternative, the higher order polynomials can also provide a good RSM approximation and improve the fitting accuracy. For this reason, the higher-order polynomials require more design points. The multiple regression function is also another alternative which is similar to the higher-order polynomials. Most of the study cases investigated in this thesis used both of these model functions for response surface approximations.

General Stepwise Regression Method

This regression method is more recent and has been referenced by many authors **[103]** and it was used in this study. The STATISTICA computational package provides a simple summary of the steps for developing a model function through the general stepwise regression method:

- identifying an initial model (i.e., the model function to be used);
- iteratively "stepping" : repeatedly altering the model at the previous step by adding or removing a predictor variable in accordance with the "stepping criteria" (usually the step criteria adopted is the coefficient of determination, the R-square index: R^2)which indicates how accurate the regression is. This coefficient varies from 0 to 1 (i.e., $0 < R^2 < 1$). Furthermore, for an accurate regression, the value of R^2 should be close to 1;
- terminating the search when stepping is no longer necessary given the stepping criteria, or when a specified maximum number of steps has been reached.

This method involves several complex formulations, since it is based on hybrid algorithms, which are based on a number of different methods, including the Least Squared Method. For this reason, the use of a powerful computational package might be required, since in some cases it is impractical to compute the indexes' calculations by hand as can be done using the Least Squared Method.

The General Stepwise Regression Method can provide accurate regressions. This is the method that was in the study cases, discussed in this thesis. The computational packages used for it were MATLAB and the STATISTICA.

In summary, regarding the Response Surface Models it can be said: they are "subsequent generalizations led to these methods being used to develop approximating functions that surrogate for long running computer codes" **[104]**. In fact, there are some publications which provide modern perspectives about RSM in light of emerging computational power **[105-107]**. As it could be seen along this section, there are several ways to develop a specific RSM, from different model functions to different regression methods to define the regression coefficients which are going to be inserted into these functions.

"There's more ways to the woods than one"
John Barth

3.3 Numerical Optimisation Techniques

The Numerical Optimisation Techniques are used when the function is built, using a specific RSM, based on the design points suggested by a DoE technique.

Therefore, according to what was explained in the previous chapter, in general terms, the optimisation problem involves an objective function (i.e., the dependent variable or response) which is dependent on the independent variables (i.e., the design variables); and this function needs to be minimized or maximized. The standard mathematical statement for a classical optimisation problem is as follows:

Maximize or minimize: $\boxed{OBJ(X, Y, Z \ldots n)}$

In case of constraints, subjected to:

$$\boxed{\begin{array}{l} g_j(X) = 0 \\ h_k(X) \le 0 \\ X^L \le X \le X^U \end{array}} \qquad \boxed{\begin{array}{l} (j = 1, m) \\ (k = 1, L) \end{array}}$$

Where:

The objective function is: $\boxed{OBJ(X, Y, Z \ldots n)}$

The equalities constraints are:

$$\boxed{g_j(X) = 0}$$

The inequalities constraints are:

$$\boxed{h_k(X) \le 0}$$

The number of equalities constraints: $\boxed{m}$

The number of inequalities constraints: $\boxed{L}$

The Lower and Upper bounds of the design variables are:

$$\boxed{X^L} \qquad \boxed{X^U}$$

In summary, the numerical optimisation techniques are used for finding the "optimum design". In other words, these techniques aim to determine the best feasible combination of the design variables which will be represented by the design points within the design space or design region. In mathematical terms, the optimal solution is represented by the global (i.e., absolute) minimum value or global (i.e., absolute) maximum value. It is important to mention that, given that finding the exact minimum or maximum values is a challenging task, in some cases, the global minima and maxima regions are also considered good results, see Figure 3.18

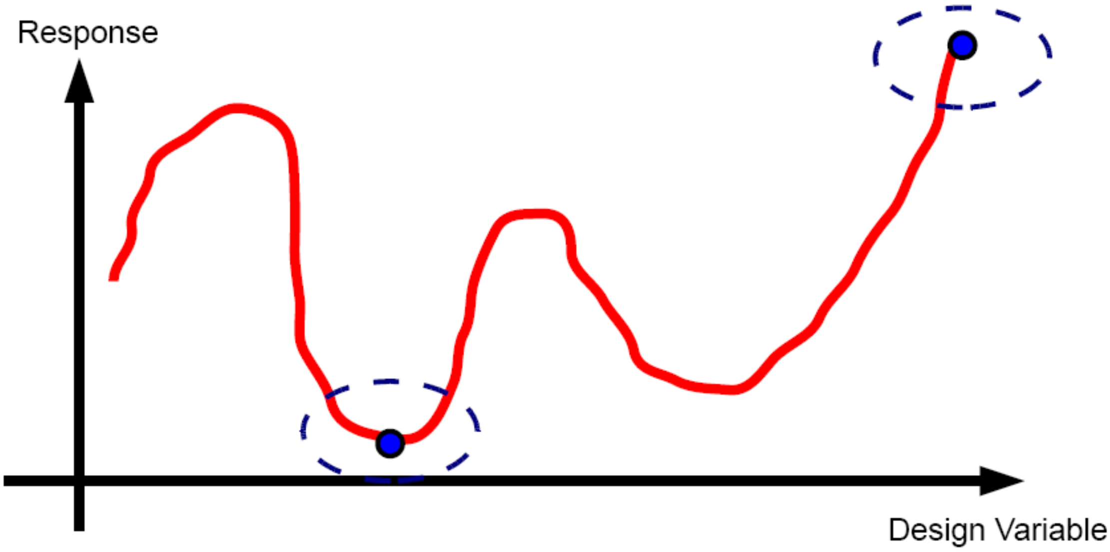


Figure 3.18: Global minima and maxima regions within a design region

The Figure 3.18 shows the global minimum and maximum values in the curve, represented by the dots; and also the global minima and maxima regions in the curve, represented by the ellipses.

In cases with two dimensions, the objective function can be thought as a surface in two dimensions (this is approach is often used in level curves in the study of topography), see Figure 3.19. Figure 3.20 presents a computational representation of such graphs.

For cases with higher dimensions, this argument can be extended in an analogous way.

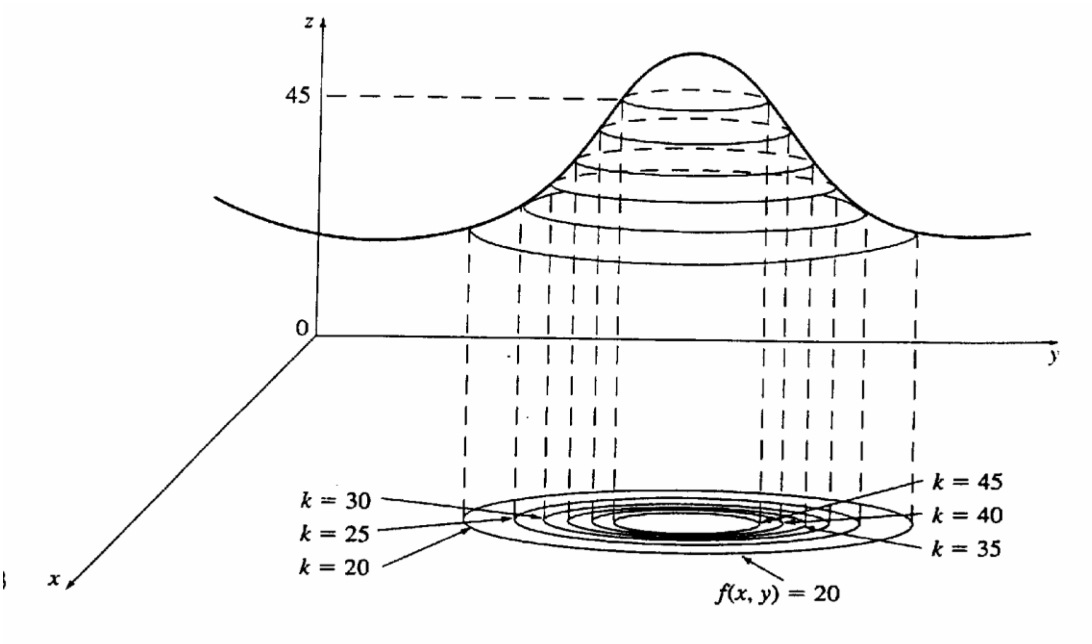


Figure 3.19: Example of a surface

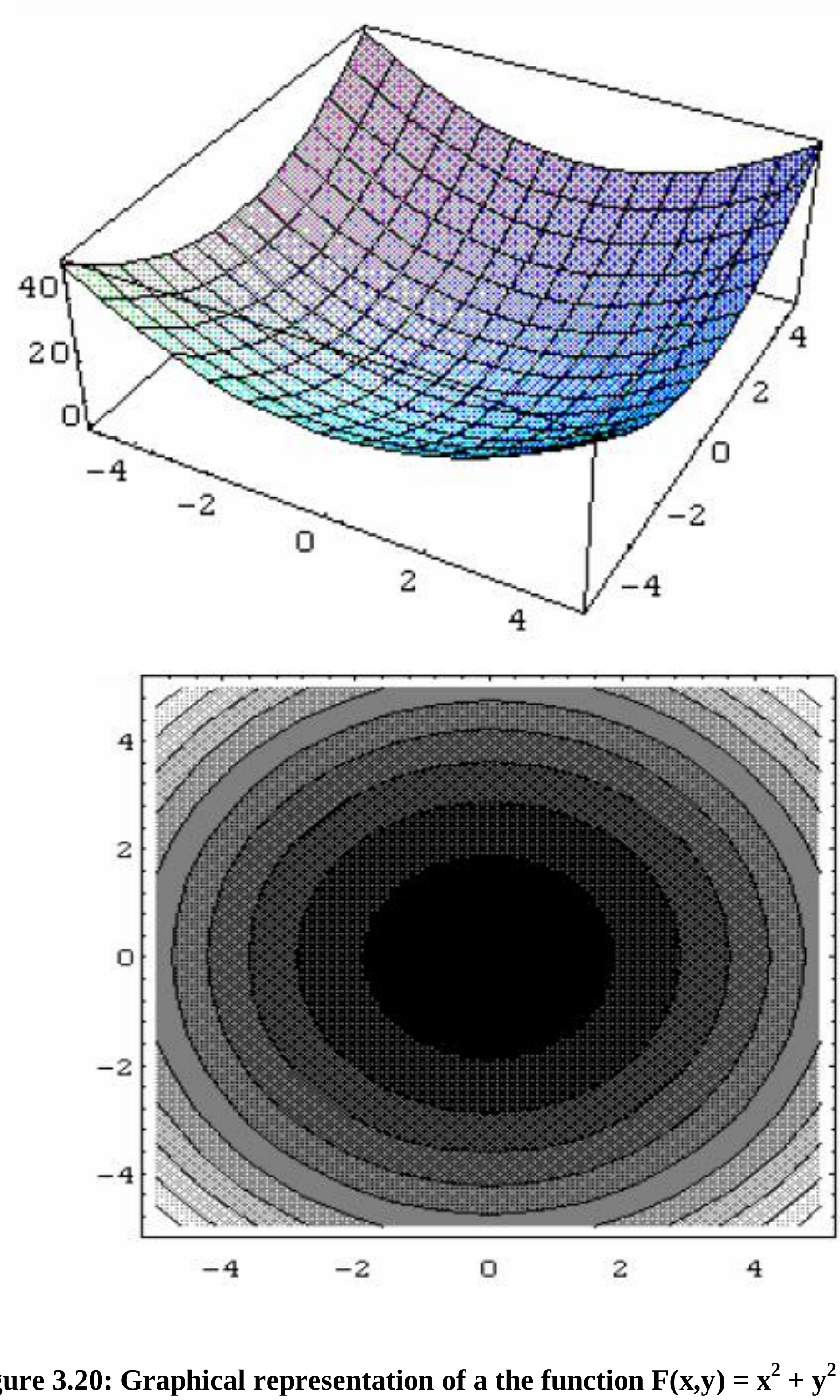


Figure 3.20: Graphical representation of a the function $F(x,y) = x^2 + y^2$ using the computational package MATHEMATICA

In figure 3.20, it is possible to see that the global minima region is located in the middle of the surface. This becomes clear when looking at the top view of the graph where it is possible to see the black circumferential region in the middle of it; representing the global minima region.

In general, the behaviour of the objective function is unknown "*a priori*". Several local minima (or maxima) in the objective function may exist over a specific design space.

Mathematically speaking, the minimum or maximum values of a function are the 'critical points'. These techniques tend to be complex and require considerable numerical effort **[108]**. In reality, these techniques (also commonly referred to as methods) are based on algorithms which search the minimum or maximum values over the design space. These algorithms are commonly called 'search algorithms' and are different according to the numerical optimisation technique used. Since the study cases analysed in this thesis involve minimization problems.

Before starting to explain the numerical optimisation techniques it is important to note how the optimisation problems are classified and also to be aware of the different ways for solving optimisation problems.

It was stated before, that the optimisation problems, based on the general form, are classified according to the existence or not of constraints. Nevertheless, in a more detailed analysis within the operational research field, the optimisation problems are classified into main four types which are special forms of the general problem, namely: unconstrained problems; linear programming (LP) problems; quadratic programming (QP) problems and nonlinear programming (NLP) problems. Therefore, the numerical optimisation techniques are developed and used based on this classification. The brief definitions of each type of problem are described in the following paragraphs.

Unconstrained problems – this type of optimisation problem, as the name suggest, has an objective function with no constraints. The objective function must be nonlinear, since the minimum of an objective function for an unconstrained problem is $-\infty$.

Linear Programming (LP) problems – this type of optimisation problem has an objective function with constraints and all of these constraints are linear functions.

Quadratic Programming (QP) problems – as the name suggests, this type of optimisation problem has an objective function which is a quadratic function and the constraints, similarly to the LP problems, are linear functions.

Nonlinear Programming (NLP) problems – this type of optimisation problems has an objective function with constraints in which one or more are nonlinear functions.

The Figure 3.21 summarizes what was described in the previous paragraphs regarding the types of optimisation problems.

In this thesis, since the study cases were based on problems with nonlinear functions, both with- and without- constraints; it can be said that the optimisation problems, analysed here, are: unconstrained problems and nonlinear programming (NLP) problems.

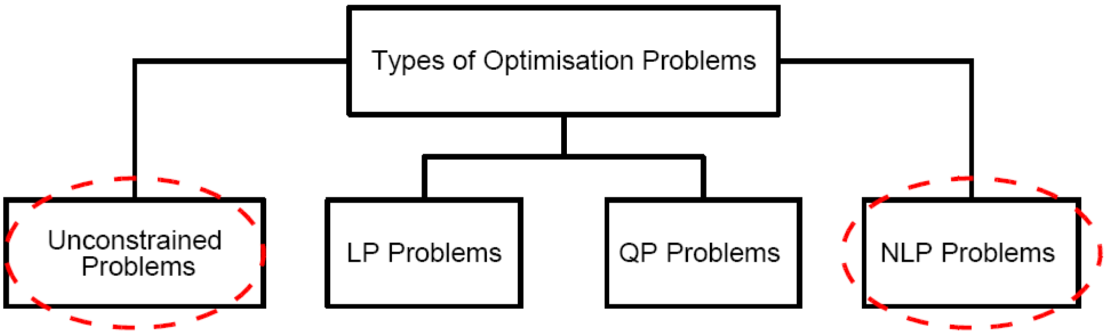


Figure 3.21: Types of optimisation problems

Regarding the ways of finding the solution for optimisation problems, based on more specific operational research studies, they are generally classified into six main types, namely: graphical optimisation; optimality criteria methods; numerical methods for unconstrained methods; interior point methods for LP and QP problems; the simplex method and numerical methods for general nonlinear programming (NLP) problems. The brief definitions of each classification type, are described in the following paragraphs.

Graphical optimisation (see Figure 3.22) – this 'method' can be used for reasonably simple problems in which no more than two design variables are involved. It can be helpful since it allows one to see the solution graphically over the design space. However, given that the results are obtained through graphs, this method is not able to provide accurate answers.

Optimality criteria methods – these 'methods' are based on fundamental calculus concepts for objective functions with one design variable. Optimality criteria are the conditions that any function must satisfy at its minimum point. The optimality criteria methods are also called "indirect methods". These methods are based on elementary calculus concepts which involve the calculation of first and second derivatives and also matrices' operations (such as, Hessian matrix; Karush-Kuhn-Tucker conditions; Taylor series; Lagrange multipliers etc.). These concepts can be extended to more complex functions, which enable the development of feasible conditions for elementary constrained and unconstrained optimisation problems. Nevertheless, these methods are only recommended for functions with few design variables and constraints (in those cases that have constraints). The reason for this is based on the fact that the numerical operations for solving the problem involve systems of equations and matrices; and this condition might become impractical in problems where the objective function has a large number of design variables and/or constraints.

Numerical methods for unconstrained problem – there are several numerical methods for solving unconstrained problems and all of them have their strengths and weaknesses **[108]**. Therefore, essentially, the difference between them is based on how their algorithm works (i.e., how the searching of the local and/or global maxima and/or minima is going to proceed along the response surface). These methods will be further explained, later in this chapter.

Interior point methods for LP and QP problems – these methods are considered to be recent in origin. They are becoming useful for solving general nonlinear programming (NLP) problems which generate QP problems as intermediate problems.

The simplex method – this method is well known, once its algorithm solves linear programming (LP) problems efficiently. This method is also considered very robust, since it can solve problems with several design variables and constraints.

Numerical methods for general nonlinear programming (NLP) problems – there are large numbers of this type of method in the literature. Each method has its own advantages and disadvantages. Also, since it is frequent used for nonlinear problems, there is no particular method which is clearly better than others.

The Figure 3.23 summarizes what was described in the previous paragraphs regarding the methods for solving optimisation problems.

In this thesis, since the study cases were based on problems with nonlinear functions with no constraints and with constraints; the methods used were numerical methods for unconstrained problem and numerical methods for general nonlinear programming (NLP) problems; they are marked with the red circles in Figure 3.23.

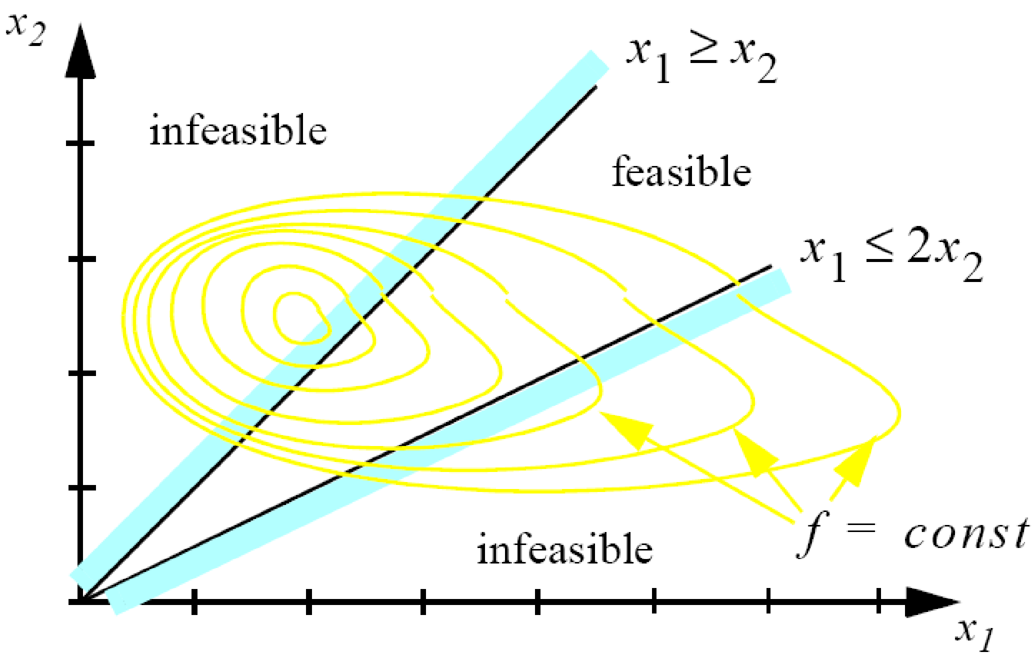


Figure 3.22: Example of a graphical optimisation for finding the optimal solution in a simple design space

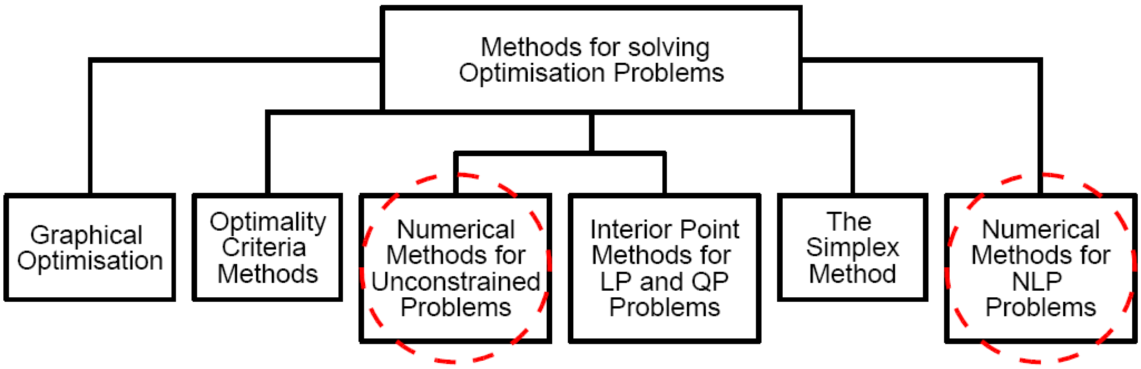


Figure 3.23: Methods for solving optimisation problems

There have been many optimisation techniques proposed in the literature and as previously mentioned, all of them have their advantages and disadvantages. Nevertheless, there is no criterion to define which numerical optimisation technique is clearly better than the others.

In the next paragraphs, some numerical optimisation techniques are briefly presented and discussed. The numerical optimisation techniques used in this study are explained in more detail.

These techniques, which are going to be presented are also called as “direct search methods”, since they do not use matrix operations or other more complicated calculus operation. This is different from the indirect search methods (i.e., the optimality criteria methods). Instead, they use robust algorithms which search directly the optimal solution along the design space.

There are generally two categories of algorithms applied in the numerical optimisation techniques. One category is the classical gradient-based methods (i.e. the algorithm is based on the gradients of the objective function and the constraints), such as the Fletcher-Reeves method and the Sequential Quadratic Programming. Another category is the stochastic-based methods (i.e., non gradient-based methods), such as Genetic Algorithm (GA) and Particle Swarm Optimisation (PSO). Essentially, the difference between the methods is firstly determined by having an algorithm that uses the gradient of the objective function. The second difference comes from how their algorithm actually works. In other words, the categorisation is determined by how the starting point for the searching is executed, see figure 3.24.

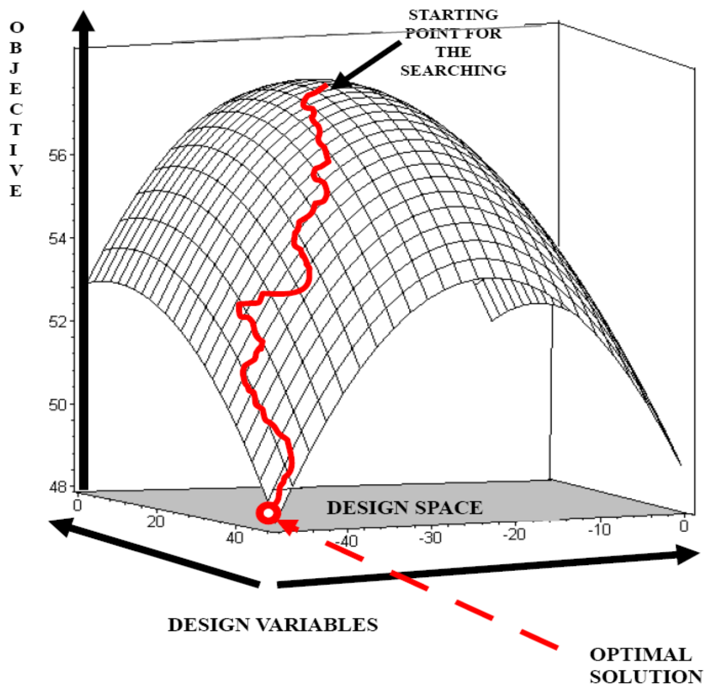


Figure 3.24: General optimal solution searching along a response surface

A well known gradient-based numerical optimisation technique is the Fletcher-Reeves technique. This method can be used for both constrained and unconstrained problems and its algorithm is based on information from the first derivatives of the objective function. It is also classified as a conjugate gradient method, because the search for the optimum region follows a “line direction”, which is obtained by selecting the successive direction vectors as a conjugate version of the successive gradients obtained as the method progresses **[109]**.

In summary, for gradient-based methods, we have the main algorithm:

$$\boxed{x^{k+1} = x^k + \alpha^k \nabla F^k}$$

Where:

$\boxed{x^k, x^{k+1}}$ - values of the design variables in the k and k+1 interaction

$\boxed{F(x)}$ - objective function to be minimized (or maximized)

$\boxed{\nabla F}$ - gradients of the objective function, constituting the direction of the search

$\boxed{\alpha}$ - the size of the step in the direction of the searching

The gradient direction is defined by the following equation:

$$d_k = S^{(k)} = -\nabla F(\underline{\mathbf{x}}^{(k)})$$

So, the new conjugate direction (**d**k+1) is calculated according to:

$$\mathbf{d}k+1 = \mathbf{-\ g}k+1 + \beta k.\mathbf{d}k$$

and βk, based on the Fletcher-Reeves method, is given by:

$$\boxed{\beta_k = \frac{(g_{k+1})^T (g_{k+1})}{(g_k)^T (g_k)}}$$

Furthermore, this algorithm defined by the Fletcher-Reeves method, works through 3 main steps:

- 1) Starting at any $\mathbf{x}_0$ define $\mathbf{d}_0 = -\mathbf{g}_0$, where **g** is the column vector of gradients of the objective function at point f(**x**) ;

- 2) Using **dk,** find the new point $\mathbf{x}_{k+1} = \mathbf{x}_k + \alpha_k \mathbf{d}_k$ **,** where ak is found using a line search that minimizes $f(\mathbf{x}_k + \alpha_k \mathbf{d}_k)$;
- 3) Calculate the new conjugate gradient direction $\mathbf{d}_{k+1}$, according to:

$\mathbf{d}_{k+1} = - \mathbf{g}_{k+1} + \beta_k \mathbf{d}_k$

One of the advantages of this method is that it converges faster than many of the other techniques and can be used for large nonlinear optimisation problems. Therefore, another main advantage of this method is that the search process makes good progress because it is based on gradients. The formula to determine the new direction along the search is simple.

The Sequential Quadratic Programming is also a numerically efficient method, but it is limited to finding the local minimum or maximum.

Conversely, non gradient-based numerical optimisation techniques are based on stochastic algorithms.

The Genetic Algorithm (GA) is also a widely used and well known technique. The GA is capable of finding the global minimum (or maximum) when local minima (or maxima) exist. It is only suitable for problems in which the design variables are non continuous in nature; which can be a disadvantage for some cases. The Particle Swarm Optimization (PSO), which is a widely used non-gradient-based method, is based on a simplified social model that is closely tied to swarming theory. It was developed by Dr. Eberhart and Dr. Kennedy in 1995 and it was inspired by the social behaviour of bird flocking or fish schooling. The algorithm of the PSO technique is very robust. The principle is that the design variables are understood as particles with associated velocities. The method is somewhat analogous to that of Fletcher-Reeves, however instead of using the gradients of the objective function to control the search algorithm, the vectors are represented by uniform random numbers between 0 and 1.

Basically, one fundamental criterion – which is taken into account for selecting which technique should be used – is the cost of performing the technique. For instance, usually, the computational costs for the stochastic-based methods are expensive **[110]**.

The numerical optimisation techniques used for the case studies, which were investigated in this thesis, were the Fletcher-Reeves and the PSO techniques, since it was determined that the results (using these two numerical optimisation techniques) were consistent and satisfactory. The computational package used for these calculations was VisualDOC.

3.4 Concluding Comments

This chapter presented a brief review of the techniques and methods that are used for design optimisation, namely: Design of Experiments (DoE) Techniques, Response Surface Models (i.e., RSM) and numerical optimisation techniques.

In conclusion, it can be said that the selection of an appropriate DoE technique is fundamental in order to properly represent the design space.

In addition, the selection of an appropriate RSM is needed, in order to correctly and realistically represent the real response surface (or response region).

Finally, the selection of a robust numerical optimisation technique is also extremely important for obtaining high quality and representative results.

4. The Problem: The Evacuation Process as a Multi-variable Optimisation Problem

4.1 Introduction

This chapter begins with simple definitions of the evacuation process and safe design. It then offers a brief discussion about the relationship between the two concepts.

Once these definitions have been presented, this chapter proceeds to describe the basic concepts inherent in classical optimisation theory. In this section, concepts such as: design variables, constraints, and the objective function, are discussed.

Finally, the general statement of the problem that has been analysed in this study is presented as well as the methodology that has been adopted in this study.

Therefore, in summary, this chapter explains how the evacuation time, which is the metric of how efficient the evacuation process is defined as a multi-variable function.

4.1.1. Evacuation Process and Safe Design

First of all, it is important to define the relation between evacuation processes and safe designs (specifically, in terms of fire safety). These two concepts are fundamentally linked to one other.

The Evacuation Process can be simply stated as being the escape movement that the occupant(s) of an enclosure make under emergency situations, such as fires, earthquakes, flooding, explosions, terrorist attacks and so on.

Safe Design, given a particular enclosure's environment configuration, is the design which could provide a successful evacuation process (i.e., allows complete evacuation with no injuries and no deaths) of its occupants in case of an emergency situation, as defined previously. In terms of fire safety, the emergency situation is the fire. As mentioned in section 2.1 of Chapter 2, the effectiveness of the fire safety strategy can be measured in terms of number of fatalities; therefore in a safe design, the probability of a successful evacuation process (i.e., no fatalities) is very high.

In this specific case where the emergency situation is a fire, the safe design is commonly established numerically using the following condition:

$$RSET < ASET \qquad \text{(eq. 4.1)}$$

Where:

RSET means the Required Safe Egress Time;

ASET means the Available Safe Egress Time.

The Figure 4.1 presents a set of timeliness which helps the reader to understand this inequality.

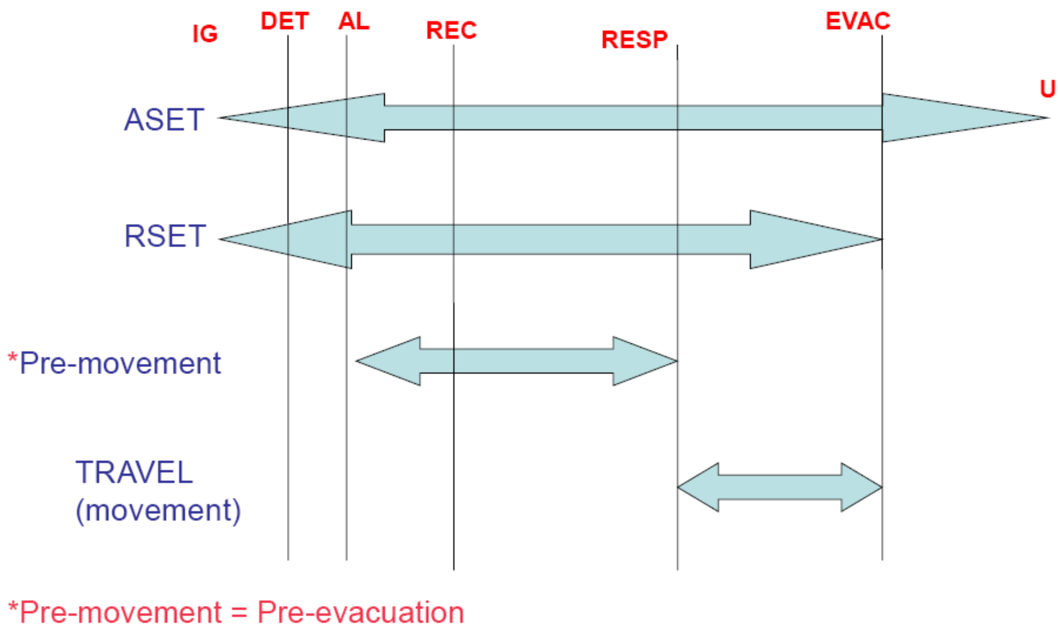


Figure 4.1: The important timeliness during an evacuation process

From Figure 4.1, these are the meanings:

IG – Ignition (the point in time at which the fire starts);

DET – Detection (the time when the detection systems are activated; i.e., sprinklers and etc.) ;

AL – Alarm (the time when the alarm is sounded);

REC – Recognition (the time when the occupants recognize that an emergency situation is taking place);

RESP – Response (the time when the occupants respond to the situation for starting the escape movement);

EVAC – Evacuation (the time when the occupants start to evacuate);

UC – Untenable Conditions (the time after which the fire products; i.e., smoke, heat, toxic gases, narcotic gases, irritant gases etc. would kill or incapacitate any remaining occupants).

The pre-movement time is also known as pre-evacuation time.

The difference between the ASET and the RSET is what the FSE community calls it the "safety margin".

Therefore, Figure 4.1 shows clearly that to have a successful evacuation process, RSET should not exceed ASET.

In practical terms, the RSET should be within the *pre-flashover* period. The Figure 4.2 illustrates this.

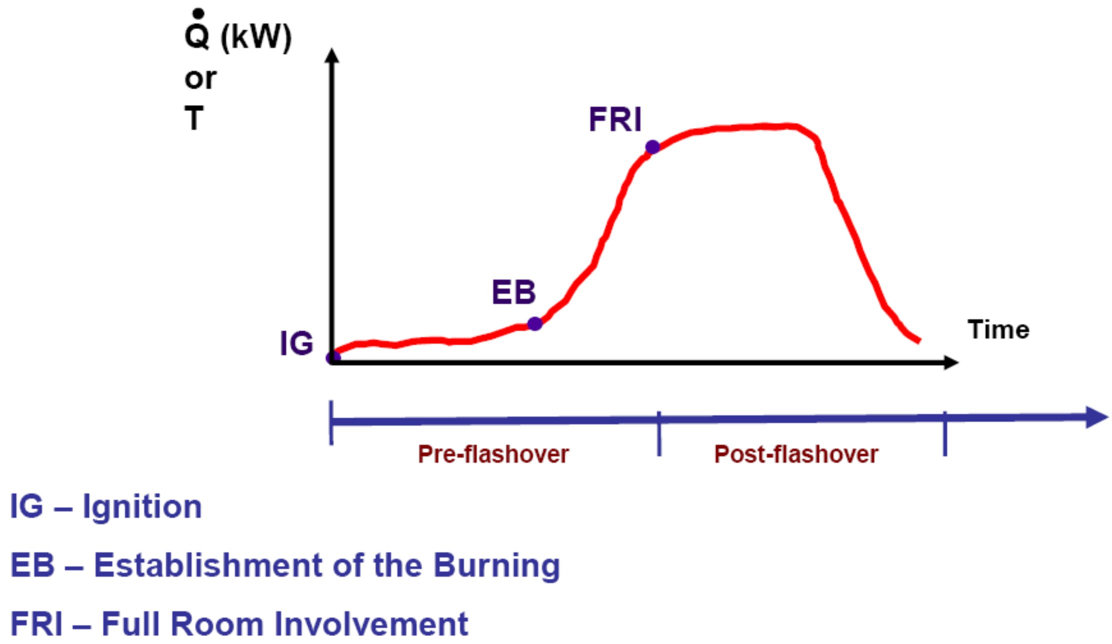
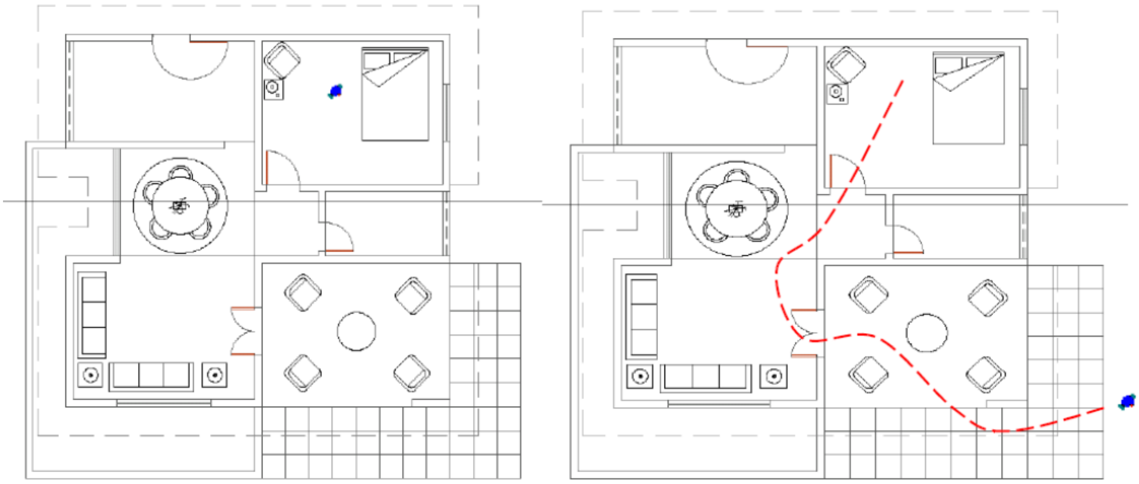


Figure 4.2: Typical curve for a fire in an enclosure

Therefore, the occupants should be able to escape before the FRI point is reached, when the flashover occurs and the untenable conditions are already reached. It is important to mention here that this reasoning only applies to occupants within a compartment and in other parts of a building; the flashover in one compartment could still leave time for people to escape to safe places.

The Figure 4.3 shows a simple evacuation process within an enclosed environment, i.e., a small bungalow. The evacuation process is then finished when the occupant of the bungalow is completely outside of it safely (alive and with no injuries). The dashed red line represents the occupant's path during the evacuation process. Obviously, in this simple hypothetical case, the safety margin (i.e., ASET – RSET) is assumed to be large enough for successful escape and consequently the RSET had to be significantly shorter than the ASET.



Occupant's position before the evacuation process

Occupant's position after the evacuation process

Figure 4.3: Evacuation Process in a hypothetical case

This form of consideration can easily be extended to other types of emergency situations, as has been mentioned previously. The ASET would be applied to the *timeline* associated with the corresponding emergency situation and this would subsequently be compared with the RSET. For instance, if instead of a fire, an earthquake takes place near to the analysed enclosure, then the time that the enclosure's structure would take until collapses would be the critical factor needed to determine the ASET.

In summary, consider the following outcome statement linking the Safe Design and the Evacuation Process:

Safe design = Successful evacuation process = No fatalities

4.2 The Defining Safe Design Process - DSDP

As discussed in the previous chapter, the CFE (Computational Fire Engineering) models are used to estimate these two main timelines. Often this will involve the use of evacuation models to estimate the RSET and using fire models to estimate the ASET. (In more complex analyses, a combination of these two models should be used, since the fire products will affect the occupants' movement and their decision-making behaviours and consequently, will affect the RSET). It also is relevant to mention here that there is a further level of complexity whereby the occupants' movement can affect the fire modelling (e.g. by opening doors in the fire simulation). This level of analysis is beyond the scope of this thesis and it is extremely costly in terms of computation.

Thus, to assure that a given design could provide this condition between these two timelines, the designers must attempt to consider a set of aspects, called "criteria", including, for instance, the safety of the occupants. In reality, the classical design concept usually attempts to satisfy some basic criteria, such as: comfort, functionality, maintenance, cost/benefits and aesthetics. However, when defining the safe design, another criterion should be considered as well, namely: the safety of the occupants.

In practice, these criteria can be conflicting and the development of a safe design is not an easy task for designers. Based on this, the challenge is to combine these criteria satisfactorily in a way that the safe design can be achieved. For this task, the first set of main questions is: how can a safe design be developed whilst satisfying these criteria, given their often conflicting nature? And, how can the real constraints be managed and incorporated into the design? The development of a safe design can be understood as a "*multicriteria* decision-making problem", since more than one criterion must be taken into account as the figure 4.4 shows.

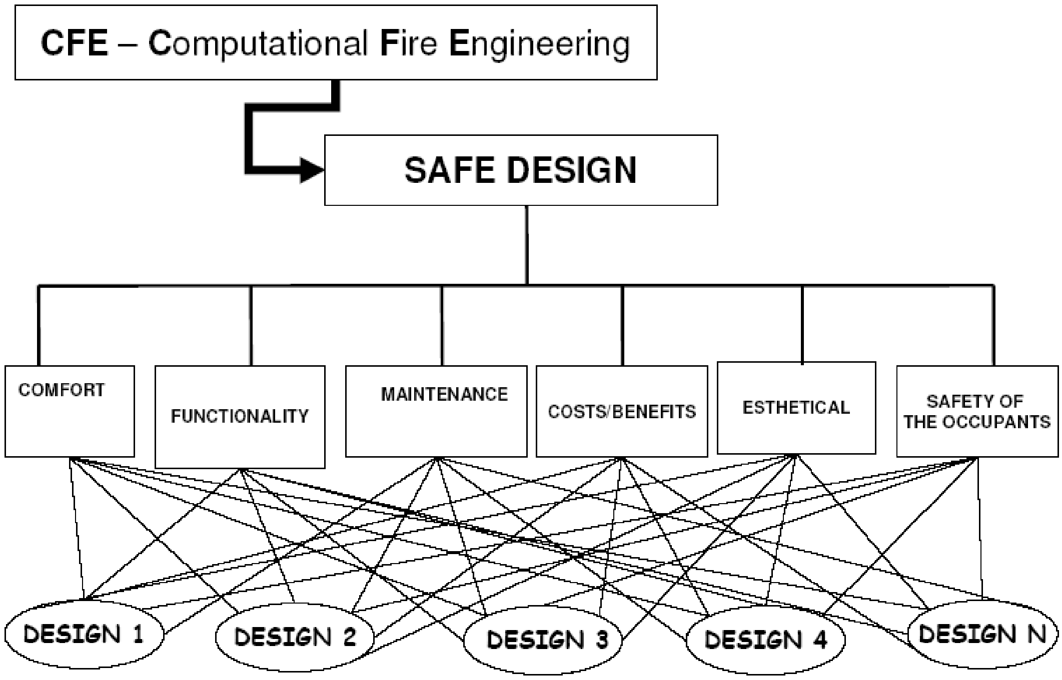


Figure 4.4: Defining a safe design using the CFE models

From figure 4.4, it is possible to observe that, during the development of the safe design, several "designs" (i.e., scenarios) are analysed, considering the criteria, mentioned previously.

This process can take a considerable amount of time, depending on the nature and specifics of the case. And, as mentioned before, this is also why the CFE models, particularly, the evacuation models have been used to help the designers to accurately and efficiently develop a safe design.

Nevertheless, even with the use of CFE models, this design process can still be time consuming. Depending on the complexity of the design, this process could take too long and could, consequently cost a substantial amount of money, which might not have been budgeted-for or available in the total development cost. Figures 4.5 and 4.6 show what is described as the Defining Safe Design Process (DSDP).

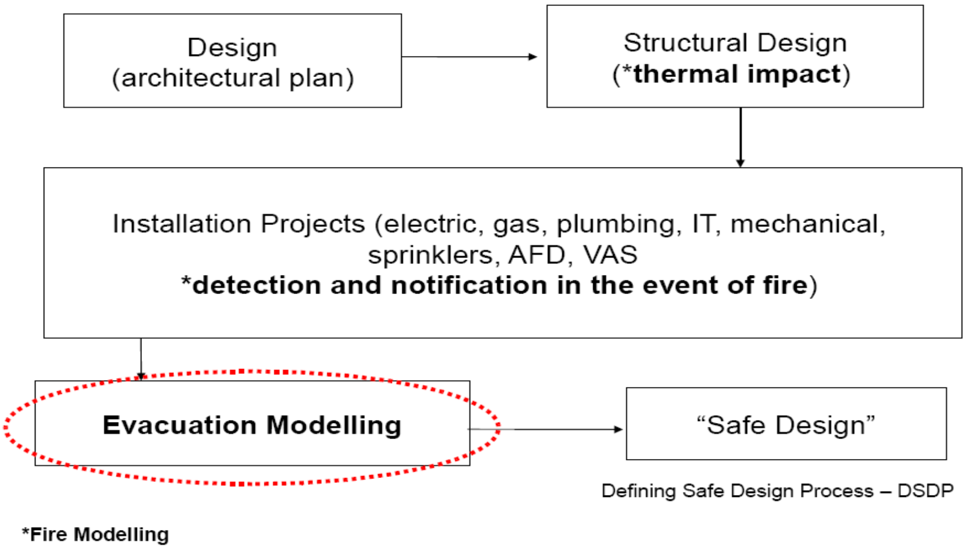


Figure 4.5: Defining Safe Design Process – DSDP

From Figure 4.5, it is possible to observe that the evacuation modelling process, depending on the case, can take too long in order to define the safe design.

The Figure 4.6 presents this in a more detailed structure.

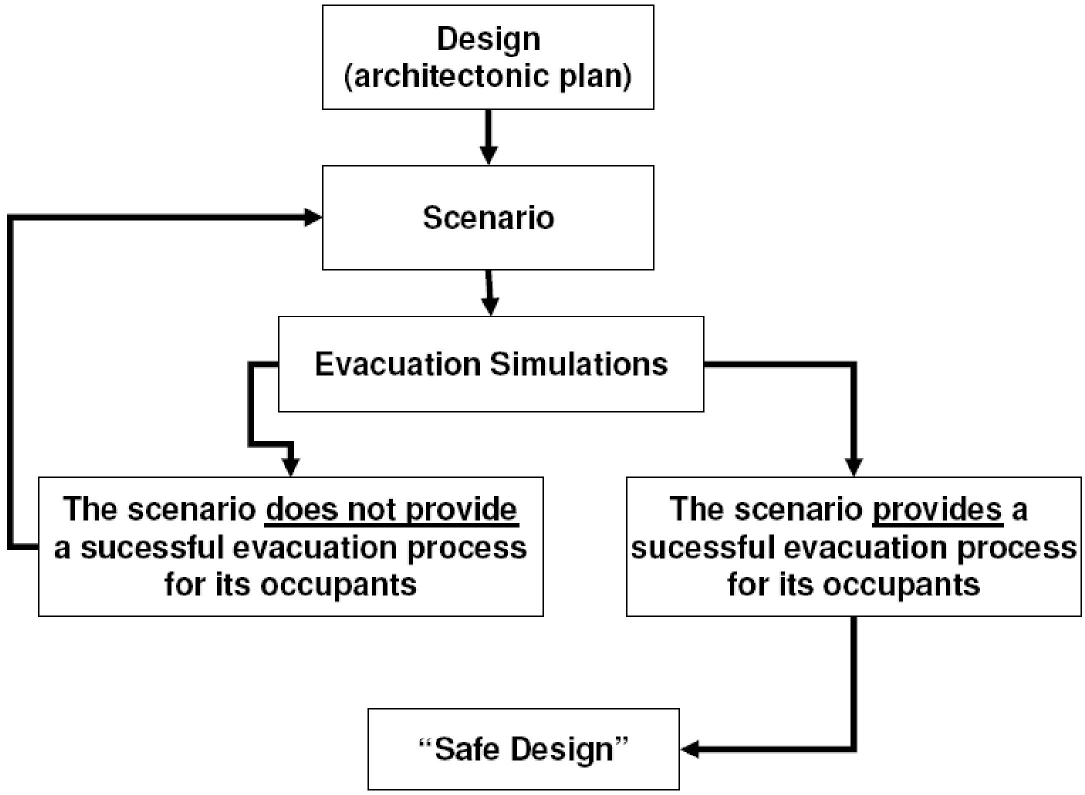


Figure 4.6: Defining Safe Design Process – DSDP

Putting this design process into practice, the first block event, represents the design defined by the architectural plan/design. At this stage, it is unclear if the design would or would not provide a successful evacuation process for its occupants in case of emergency situations. (This process can be also applied to existing designs). The scenario(s) is (are) then defined in order to check and/or define the safe design. At this stage, evacuation modelling is used to test each scenario. The number of scenarios which will need to be analysed and simulated will vary from case to case.

From this perspective, it is possible to understand that in some cases, this process will be straight forward; however in other cases, this might not be possible. Indeed, there are some additional questions which are implicit and might be done (these questions are inserted into the second main set of requirements): is it possible to reduce the number of simulations? what can be done to reduce the number of simulations, without compromising the safety? for how long should the simulations be run to assure this?

These kinds of questions are likely to occur during the DSDP and the designers will have to deal with them. Based on that, this study presents a new approach which aims to help to answer the questions that were stated before:

The first set of questions:

- How can a safe design be developed, that satisfies these criteria, given their potentially conflicting nature?
- How can real constraints be handled?

And the second set of questions:

- Is it possible to reduce the numbers of simulations?
- What can be done in order to reduce the number of simulations, without compromising the safety?
- Until when should the simulations be run to assure this?

Furthermore, in reality, the question which could cover these two sets of questions is: “how can the DSDP be optimised”?

First of all, a good understanding of the manner in which the design variables interact to control the evacuation efficiency is an important issue to be addressed.

In reality, given that time is the basic measure of the evacuation process (Engineering Guide to Human Behaviour in Fire, SFPE, 2002) **[111]**, the evacuation time is taken then as an index of how successful the evacuation process would be in an enclosure. Following on from this, it is assumed that lowering the evacuation time will make the design safer. In the next section of this chapter, this issue is discussed in some detail.

4.3 The Evacuation Time

The evacuation time can be seen as an index of the efficiency of the evacuation process. In other words, the evacuation time is the variable used to measure the evacuation process' performance. In fact, when analysed more thoroughly, the evacuation time also brings in key-concepts (to the evacuation process' context) such as congestion (C) and flow rate (FR).

In a safe design, when an evacuation process takes place, the ideal situation would be that there is little congestion (and preferably no congestion) which would enable good movement of the occupants (i.e., the flow rate is high). And as consequence of this, the evacuation time would be reduced. Therefore, the evacuation time is an important index of how efficient the evacuation process is within a specific design; in other words, it can indicate how safe the design is. The ideal condition is:

$$C \downarrow \ FR \uparrow \ ET \downarrow$$

(RSET < ASET)

Furthermore, this condition, in technical terms, should be the main objective of fire safety engineers when performing the evacuation analysis: to reduce the congested areas that will, in turn, improve the flow rate and consequently allow the occupants to evacuate faster and more safely.

Based on this consideration, this study has taken the evacuation time as the main metric that is used to analyse the evacuation process.

In the literature, many authors have suggested formulas for the calculation of the evacuation time. For instance, the following equation (which is similar to the Figure 4.1) **[111]**:

$$t_{ev} = t_d + t_a + t_o + t_i + t_t + t_q \qquad \text{(eq. 4.2)}$$

Where:

- t_{ev} is the time to evacuate (i.e., evacuation time);
- t_d is the time from the ignition point until detection of the fire;
- t_a is the time from detection of the fire until the alarm is sounded;
- t_o is the time from alarm until the time occupants make a decision to respond;
- t_i is the time for the occupants to investigate the fire, collect belongings, fight the fire and etc.;
- t_t is the travel time or the movement time, (which is the actual time required to follow an escape route until reaching a safe place, like an assembly point, including way-finding);

– t_q is the queuing time at corridors, exits and/or other places/obstacles in the enclosure.

The evacuation time is summarized as being a simple equation as follows **[112]**:

$$ET = t_1 + t_2 \quad \text{(eq. 4.3)}$$

Where:

– ET is the evacuation time;
– t_1 is the time to start the movement;
– t_2 is the time (for everyone) to move and pass through the exits.

The time to start the movement is also commonly called the pre-evacuation time (i.e., pre-movement time) **[113]**. In fact, this value is a variable which depends on the psychological attributes of the occupants. These attributes have been researched by many specialists, in the FSE community, dedicated to address human behaviour in fire situations. This issue is important and one of the complexities of the evacuation process. Nevertheless, this study is not dedicated to cover the time to start the movement, as mentioned in the previous chapter. Instead, the focus of this study is the time for the occupants to move to, and pass through the exits. Therefore, this study intended to investigate some of the important factors which influence this time. (This is explained more thoroughly later in this chapter). Given this, the equation 4.3 can be re-written to give another equation:

$$ET = t_M + t_E \quad \text{(eq. 4.4)}$$

Where:

– ET is the evacuation time;
– t_M is the time spent during the movement;
– t_E is the time spent towards the exits.

Clearly, in this equation, the interaction occupants-structure is the main consideration. The interaction occupants-occupants is also being considered, but only in terms of the physical aspects (i.e., in terms of how the occupants interact with each other during their movements).

The time spent during the movement is influenced mainly by: the lay-out of the enclosure, the travel distance, the number of occupants and the features of the enclosure's geometry.

The time spent moving towards the exits is influenced mainly by: the exit location, exit width and the number of occupants.

In reality, one of the issues which make the DSDP so complex is the determination of the optimal positioning of exits around the perimeter of the design geometry. This solution is typically found through trial and error exploration of all the possible significant exit locations.

Similarly to the existing problem for Evacuation Modelling, Fire Modelling needs to determine optimal sizing and locating vents extractors, this is also a common problem faced by fire safety engineers in evacuation analysis: the optimal positioning of exits within an arbitrarily complex structure in order to minimise evacuation times. To a certain extent, building codes provide many guidelines for the positioning of exits; however the guidelines do not give a clear indication of how to minimize the evacuation time. For example, given an arbitrarily complex room shape and ignoring constraints imposed by regulations (such as minimising travel distances and avoiding dead-end corridors), where should the exits be placed in order to minimise the evacuation time? Indeed, for an arbitrarily shaped room with a given number of exits, does the distribution of exits around the perimeter have an impact on the evacuation time?

This problem becomes more difficult as the number of available options increases and hence the complexity of the evacuation scenario increases. It can reasonably be expected that, for a given population size, the solution of the problem will be dependent on the shape and size of the compartment and the number and relative size of the available exits. For a specified problem, the engineer could examine several possible exit location options and select the configuration which produces the smallest evacuation time, but this would not necessarily produce the optimal configuration or the global minimum evacuation time. Using this approach, the engineer would have to examine every significant combination of exit location to be sure that the global minimum had been found. For an arbitrarily complex shaped room with a large number of exits of varying size, the number of possible permutations of exit size and location would measure in the hundreds if not thousands.

Therefore, the question now is: how would the engineer find the best solution and how would the engineer know that an optimal or near optimal solution had been found? A possible answer to this problem may be found using optimisation theory.

Numerical optimisation techniques have been applied in a range of different fields such as structural analysis and have been shown to be powerful tools for designers, saving time and reducing costs. The use of classical optimisation theory concept and its associated fields (such as Design of Experiments, DoE, and Response Surface Models, RSM) are explored here and used in the context of evacuation simulation analysis.

Therefore, this is one of the main objectives of this study, to present and discuss a systematic methodology that will allow the efficient optimisation of the evacuation safety aspects of structural designs. Such an approach will be of particular interest to practicing fire safety engineers as it allows the fire engineer rapidly and efficiently optimise their design.

In the next section, the optimisation theory is discussed briefly.

4.4 Optimisation Theory

There is a vast field of activities/tasks in the everyday world which can usually be described as systems; from actual physical systems such as chemical power plants to theoretical entities, like economic models. In general terms, systems are a collection of objects connected through any form of interaction or interdependence **[114]**. The efficiency of these "systems" often requires an attempt at the optimisation of a set of indices which measure the performance (i.e., the response behaviour) of the system. These indices, when quantifiable, are represented by algebraic variables. Then, values for these variables must be found which maximize the gain or profit and minimize the waste or loss of the system. Other authors say that this process of maximization and/or minimization of the system is known as optimisation **[115]**. In other terms, optimisation is the process by which the optimal, or optimum, solution to a problem is produced. (The word optimum has come from the Latin word "*optimus*", which means best). Finding the optimal solution of a certain problem, based on the analysed system features, generically follows a methodology, which involves several tasks **[74]**, see figure 4.7.

In general terms, the optimisation problem involves an objective function (i.e., the dependent variable; response; output; merit function) which needs to be minimized or maximized. The general formulation of a classical optimisation problem is as follows:

Maximize or minimize: *OBJ (X, Y, Z ... n)*

Subjected to:

$$g_j(X) = 0 \qquad (j = 1, m)$$
$$h_k(X) \leq 0 \qquad (k = 1, L)$$
$$X^L \leq X \leq X^U$$

In this study, evacuation processes are analysed using this perspective. The objective function is the evacuation time and the problem is to minimize this function.

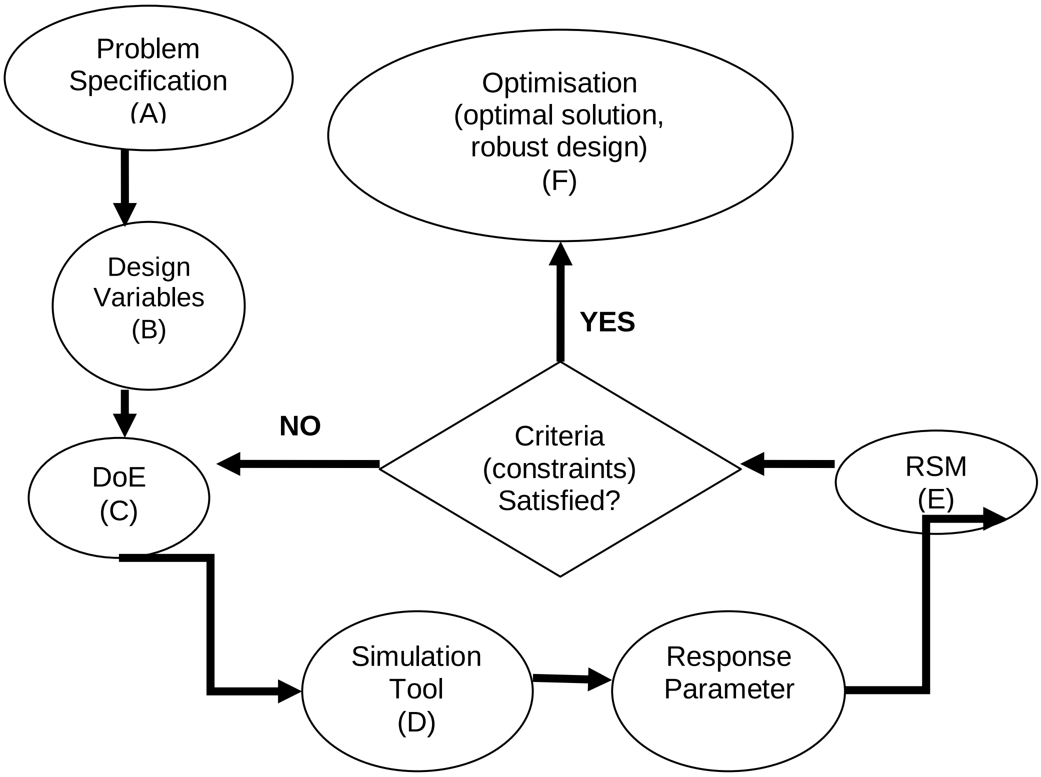


Figure 4.7: Optimisation Strategy [69]

Figure 4.7 presents the methodology adopted in this study. In this particular study, this methodology follows a set of steps which involve, amongst other computational packages, the use of computer simulation software to simulate the evacuation process.

Some of these steps, such as, the problem specification (step A) and the design variables (step B) are described in the following paragraphs of this chapter. The details of the simulation tool (step D) were discussed in section 2.3 of chapter 2, where the EXODUS evacuation model was described. It should be noted that the Design of Experiments Techniques (i.e., DoE techniques, step C); Response Surface Models (i.e., RSM, step E) and optimisation (step F), were discussed in the previous chapter.

In optimization problems, the first step is to define the design variables which impact the performance of the system. Once these have been defined, data must be produced in order to analyse the relationships between them and the consequent influence that these relations have on the system. This data is produced from laboratory experiments and/or computational simulation packages. In this study, the data was produced from a computational simulation package as described previously. For this, the obtained data, which represents numerical values for the design variables, are organized into a *design space* through a response surface. Therefore, a response surface model is developed with the purpose of describing this data. Thus, finally, given that the *response surface* is described by a function (which composes the objective function), numerical optimisation techniques are then applied to optimize this function (i.e., maximize and/or minimize).

In summary, the optimisation strategy follows this basic sequence: i) use of DoE techniques for generating data; ii) use of RSM for generating the response surfaces, enabling later data analysis; iii) use of numerical optimisation techniques for solving the problem. The figure 4.8 presents this methodology in a systematic approach.

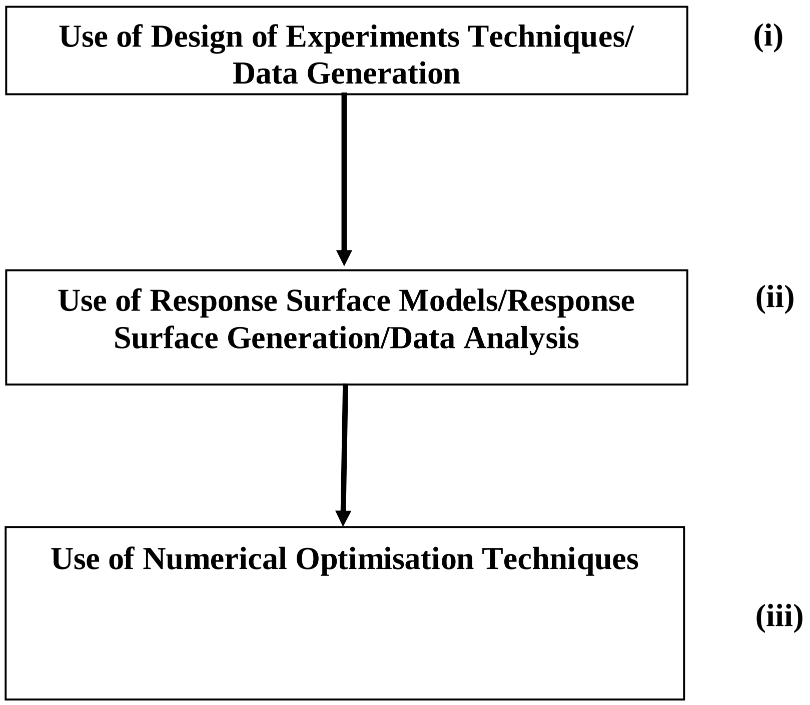


Figure 4.8: Summary of the Optimisation Strategy used in this study

In this study, evacuation processes, are analysed through the use of evacuation models.

In the next section, this is discussed further.

4.5 The problem specification: Classical Optimisation Problem "versus" Evacuation Processes

In this section, the problem specification (step A from Figure 4.7) is defined. Therefore, following questions should be addressed, namely:

- What is the system?
- Which variable will be taken as the objective function (i.e., to represent quantitatively the system)?
- Which variable(s) will be taken as the design variable(s) of the function?
- Is the problem a minimization problem or a maximization problem?
- Is the problem constrained or unconstrained?

According to what was mentioned in the previous section, optimisation problems involve an objective function which needs to be minimized or maximized. In this study, the objective function is the evacuation time and the requirement is to minimize it for a safe design. In this study, the design variables (i.e., the independent variables; factors; inputs), are the exit locations (or exit positions).

Based on these considerations, the questions are then answered:

For this study, the system is the evacuation process.

The design variable taken into account for this study was the exit(s) location, as mentioned previously in this chapter.

The problem here is a minimization problem, since it is assumed that as the evacuation time becomes lower, the design becomes correspondingly safer.

Regarding the nature of the problem (i.e., if the problem has constraints or not), in this study, both problems were considered: with constraints and with no constraints. These problems will be presented and discussed with further details, later in this Thesis.

The Table 4.1 summarizes this information.

Table 4.1: Summary of the problem

Optimisation Theory Concepts	Fire Safety Engineering Concepts
System	Evacuation Process
Objective Function	Evacuation Time
Design Variable	Exit(s) Location

In Table 4.1, the concepts found in the optimisation theory were associated with the concepts used in the FSE field.

Therefore, given their nature, evacuation processes are complex systems in nature. A complex system is any system which has more than one variable influencing its behaviour over time **[116]**. In fact, when the variables (which might influence, and those which definitely influence the evacuation process' performance along a period of time) are observed more carefully, this concept can be applied to evacuation analysis.

For instance, there are several variables which do have direct and/or indirect impact on the evacuation process performance, such as: exit(s) width; exit(s) locations; number of exits; relative distance between two exits; number of occupants; response times; shape of the room; type of fuel package within the enclosure; location of the fuel package; lay-out of the enclosure, etc.

In reality, it is correct to say that the evacuation time can be seen as a multi-variable function, since it describes the behaviour of a complex system, namely: the evacuation process. Based on that, mathematically speaking, the problem analysed here is described as follows:

Evacuation Time (ET) = objective function and can be written:

$$ET = f\,(x_1, x_2, x_3 \ldots x_n),$$

Where $x_1, x_2, x_3 \ldots x_n$ represent the design variables.

Following this thinking, these variables are correlated and influence the evacuation process; and this influence is measured by the evacuation time. An important aspect of this study is gaining an understanding in these variables and their interactions as well as the consequent impact of them and their relations on the evacuation time. The Figure 4.9 illustrates this.

As mentioned in chapter 1 of this thesis, this study intended to develop an analytical methodology which can provide an optimised analysis of building/structure designs in terms of fire safety of the occupants; combining the use of concepts of optimisation with the use of evacuation modelling analysis. At the same time, it has also investigated the manner in which the core variables interact to control evacuation efficiency.

For example, the relation between ET (i.e., evacuation time) and EL (i.e., exit(s) location) is investigated in this study.

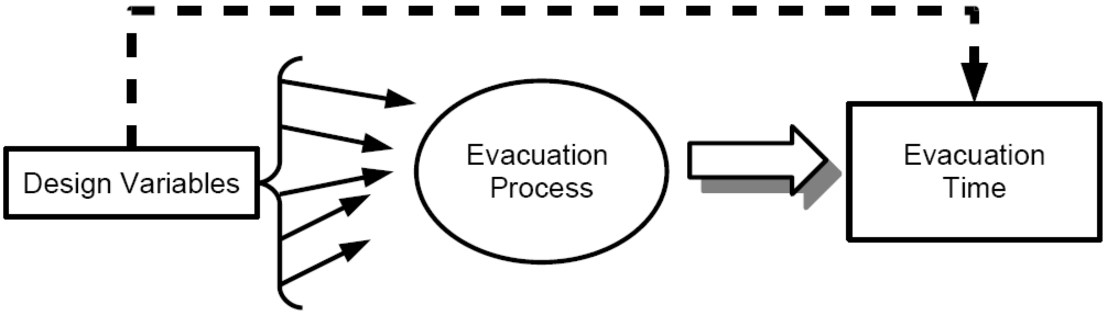


Figure 4.9: The design variables influence on the evacuation process

In fact, the Figure 4.9 can be visualized differently using a simple block diagram in which the steps shown in Figure 4.7 can also be associated, as Figure 4.10 presents.

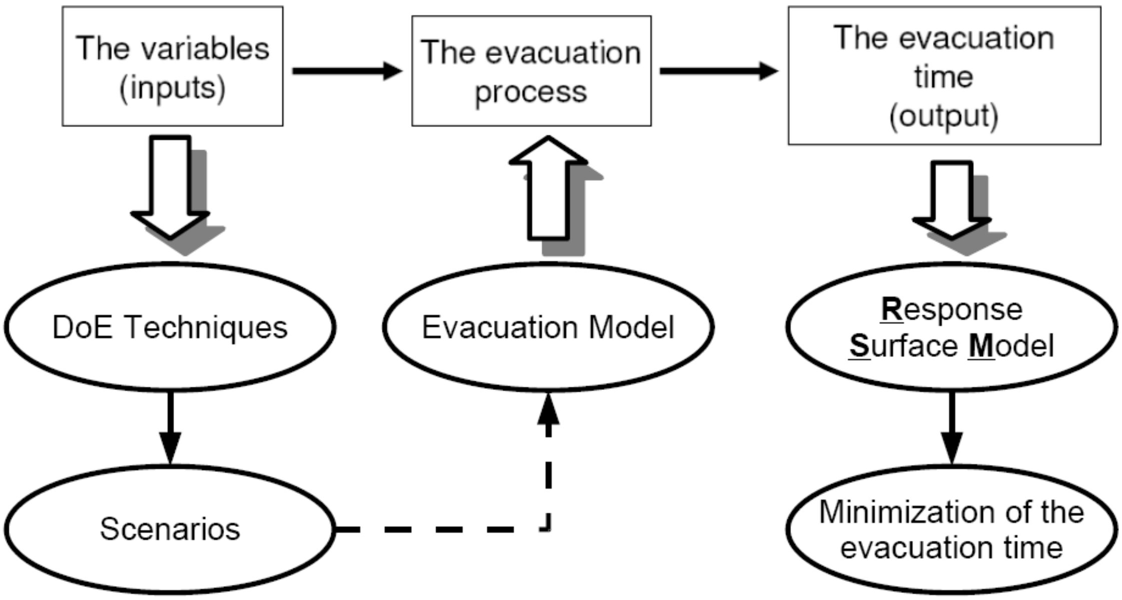


Figure 4.10: Block diagram associated with specific tasks from the optimisation process

Therefore, the problem to be investigated in this study can be stated very simply as follows, for an enclosure of given size and shape, containing an arbitrarily large population, is there an optimal location for the exit(s) that will minimise the evacuation times? If so, where is (are) the optimal location(s)?

This is the question which this study is attempting to answer through the use of the concepts within the optimisation theory combined with the use of evacuation modelling analysis.

For this purpose, hypothetical study cases have been built in order to investigate and help to answer this question.

The optimal solution is understood to be the global minima region of the objective function (i.e., the evacuation time) in the response surface. However, given that finding the global minima region precisely is not an easy task, if a local minima region that is near to the global minima region is found, then it is assumed that this is also a good result. These and more specific issues concerning to the optimisation theory and the techniques within this field are discussed in more details in the next chapter as well as the following chapters, where the study cases are analysed.

In the next section, the design variables are briefly described.

4.6 The Design Variables

In this section, the design variables (step B from Figure 4.7) are defined. As mentioned before, in addition to investigating the use of optimisation theory for evacuation applications, this study is also interested in investigating the fundamental relationship between design variables and evacuation, i.e., amongst the design variables themselves and between them and the evacuation process. This is also a relevant aspect of this study.

According to what was explained previously, the exit(s) location is the design variable considered in this study. (The relative distance, for cases where multiple exits were placed on the perimeter of the enclosure, was also considered). In reality, the exit width was also investigated, however, once the relation between evacuation time and exit width is known; this issue was considered for validating the methodology (i.e., the use of numerical optimisation techniques with evacuation modelling analysis) than to investigate the relationship itself.

It might be relevant to observe that these design variables come from the interaction occupants-structure. And on the other hand, that the design variables were found to be independent of one another; i.e., their values do not impact each others values. For instance, if the width of the exit is increased, it will not affect the exit location and vice-versa. This is a fundamental concept of a design variable. Therefore, it is very important to have a good understanding of the nature and the relationship between the design variables in order to construct a robust model of the problem.

Otherwise, if this understanding is not clear, there is the risk of variables which are not design variables be mistakenly included as design variables and this would have a critical impact on the whole optimisation process. (These concepts are also discussed in the next chapter).

The three design variables (i.e., exit width, exit location and relative distance between exits) are aspects of the design which directly and/or indirectly influence the evacuation process. This can be observed through the evacuation time values.

The relation between evacuation time and exit width is already well known. As the exit becomes wider, so the evacuation time is reduced and life safety is increased; but only until further widening will have no further impact on the evacuation time because the flow rate through the exit is already at a maximum. The reason for that is based on the fact that when the exit becomes wider; the flux of people passing through the exit (i.e., the flow rate) becomes higher and consequently evacuation process can be completed in shorter times. This relation is logical and already well known; so, for this reason, it will not be discussed in further details in this study.

However, the relations between evacuation time and exit location and also between the relative distance between exits are not fully understood. For this reason, this study gave some consideration to these relationships. For instance, there is no clear guidance regarding:

- Where to place an exit in order to produce minimum evacuation times?
- Is it better to have two exits of X m width or one exit of 2X m width?
- If there are two exits, what is the optimal relative positioning of these exits?

The investigation of these fundamental questions is not a simple task. Nevertheless, defining the optimal positioning of exits within an arbitrarily complex structure is one of the key-issues within evacuation modelling analysis.

In fact, the exit location constitutes an important aspect to look at in terms of evacuation efficiency, because it substantially impacts the evacuation time. For instance, in aircraft, it was found that, depending on where the exits are located, the evacuation time might increase or decrease substantially **[117]**.

People have got the natural tendency to go to the closest exit (i.e., nearest exits, based on the travel distance measures) which sometimes does not mean that this will produce the optimum evacuation times (i.e., the lowest values of evacuation times). The optimum evacuation times might be obtained not based on the travel distances, but based on other aspects, such as exit location.

Thus, in conclusion, the exit location influences the evacuation time values. For this reason, the exit location can be understood as being a design variable in relation to the evacuation time; and the evacuation time can be understood as being the dependent variable, since its values depend on the exit location's values and not the other around.

Based on this, it is possible to conclude that the exit location plays a relevant role for the evacuation processes in general. Despite this, there are few studies on this relationship between exit location and evacuation time. Therefore, this study was also intended to bring greater understanding to this issue. The key-question now is: how and why does the exit location impact the evacuation process? In order to explain this relationship, it is proposed, in this study, to analyse these and other issues using the perspective of optimisation theory.

In the next paragraphs, the nature of these design variables is described in generic terms. Further details about the design variables are given in the next chapter as well as in subsequent chapters where the study cases are presented and discussed.

Exit Width (EW)

The exit width is a continuous variable that can vary from 1.0 to 2.5 and these values are measured in meters (m).

$$1.0 \le EW \le 2.5$$

Exit Location (EL)

The exit location is a continuous variable. The exit location can move from the corner of the room to the middle of the wall. The EL values can vary from 0 (i.e., the lower value) to some maximum value (i.e., the upper value) which represents the other extreme of the perimeter of the enclosure’s geometry. Therefore, EL can go from 0 to the middle of the wall minus the width of the door. The Figure 4.11 illustrates this.

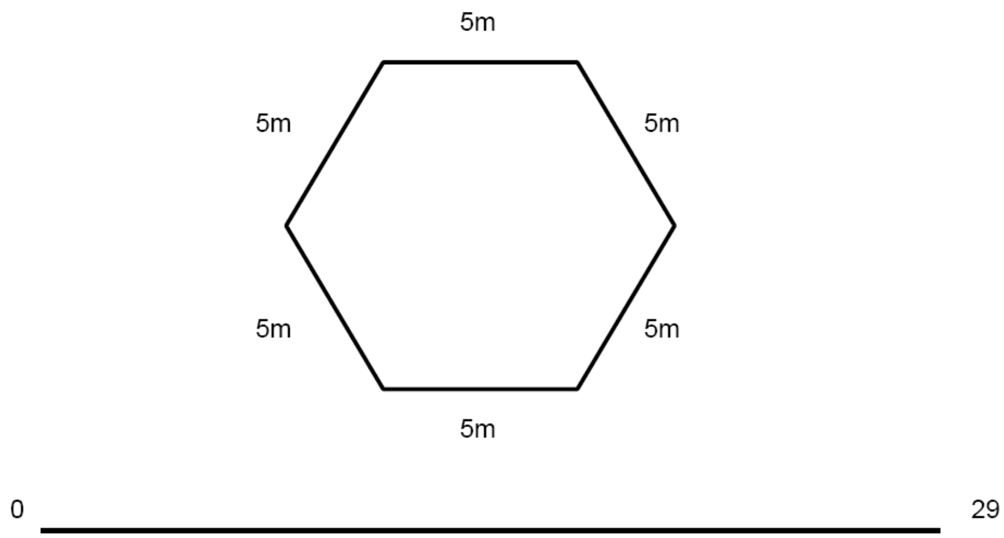


Figure 4.11: Representation of the domain for the exit location in a hypothetical enclosure

In Figure 4.11, it is possible to see that the perimeter of the enclosure is 30m. But assuming that the exit width is 1m; then the upper bound is assumed to be 29 rather than 30. Thus, the exit location can be placed anywhere along this domain (between 0 and 29). The number 0 (which represents the lower bound) is representing the starting point (i.e., any corner along the wall). The domain is graphically represented by the line. In this study, the orientation is anti-clockwise. For instance: orientation usually implies how it is directed/pointing, whereas anti-clockwise is how the edges are traversed.

0 – This means that the exit is located in the corner (the starting point);

1 – This means that the exit, from its left edge, is located 1m away from the corner;

1.5 – This means that the exit, from its left edge, is located 1.5m away from the corner;

3 – This means that the exit, from its left edge, is located 3m away from the corner.

For each study case, the domain is presented and discussed; but in general terms, this is how this study is conducted in terms of the exit location and similarly for the cases, where there are more than one exits.

Relative distance between exits

The relative distance between exits is also a continuous variable. As explained in the previous paragraphs, the same methodology used for one exit case is also used when there is more than one exit.

5 Identifying Optimal Solutions to Unconstrained and Constrained Problems through "Brute Force Method"

5.1 Introduction

This chapter presents a problem involving a square room with one and two exits and a rectangular room with two exits, in which the best location for the exits is found to minimise the evacuation times. In other terms, the optimal location for positioning the exits is determined by finding the minimum evacuation time.

The optimal solution for all these cases is found by data analysis. This data analysis consists in simply running evacuation simulations using several different scenarios for finding the scenario which gives the minimum evacuation time. This method has been called in this thesis as "brute force method".

5.2 The square room with 1 exit

5.2.1 The problem

The problem to be investigated can be stated very simply as follows: for a room of given size, containing an arbitrarily large population, is there an optimal location for the exit that will minimise the evacuation times?

For simplicity, this first hypothetical case is limited to a simple geometry, namely: a square room containing one exit. The square room used in the analysis has an area of 100 m^2 and the exit width can vary between 1.0m to 2.5m. The population of the room consists of 200 people producing a population density of 2 people/m^2. This means that 50% of the area is populated, which gives 50% of freedom for the occupants' movement during the evacuation process. This density is realistic to assembly enclosures, such as a cinema screen or a meeting hall. In fact, this is the type of enclosure analyzed in this thesis: assemblies.

In order to answer the question, the evacuation process is analysed through "brute force method" (i.e., data analysis). The exit locations, for the case of 1 single exit, are measured in terms of the distance from the left edge of the exit to the corner of the wall. The distance to the door is taken along the perimeter. It is also important to mention here that this simple symmetrical case only needs one edge to be analysed (i.e., for the one exit case) since the other three edges should give identical evacuation behaviours.

The details on how the problem was defined and the nature of the design variables, for instance, are further explained in the following sections of this chapter.

5.2.2 The scenarios

5.2.2a The simulations

The approach adopted involves the use of computer simulation software to simulate the evacuation process for each relevant exit configuration.

To determine the evacuation times for the various configurations, the buildingEXODUS evacuation software was used. The basis of the model has been described in other publications as mentioned in section 2.3 of chapter 2.

The model parameters used for the simulations performed for this study case are described in the following paragraphs.

As mentioned previously in this thesis, for the simulations, the influence of response time was removed by assuming that the entire population reacts instantly. This means that all the simulated occupants react immediately at the start of the simulation. The population was randomly generated with a maximum travel speed varying between 1.2 m/s and 1.5 m/s. The combination of instant response times, travel speeds and relatively short travel distances combine to produce large areas of congestion around the exits almost immediately. The unit flow rate on the exits was capped to a maximum value of 1.33 occupants/m/sec.

In order to simplify the analysis and isolate issues associated with room configuration and exit location these complex behaviours are greatly simplified. The behavioural response imposed on the population is such that occupants will elect to move towards their nearest exit and furthermore, that the occupants know the location of their nearest exit. While this behaviour may be considered simple it is nevertheless reasonable for the purpose of this study. Indeed, this type of assumption is not very dissimilar to the type of assumptions implicit in most building regulations and used in many performance-based evacuation analyses. The evacuation simulations were conducted assuming ideal conditions of zero response times and population behaviour such that occupants would move to their nearest exits. Both assumptions are made to simplify the analysis and to isolate issues associated with exit location.

The simulations were repeated a total of 600 times for each scenario. The reason for that is because it was found that from 600 times onwards, the values for the evacuation times start to become constant. Less than 600 times, the results have shown to be inconsistent. With this measure, it was attempted to avoid biased data. Thus all the results presented here represent an average over 600 simulations. At the start of each simulation, the starting location of the population was also randomised. This ensured that the population was distributed throughout the confines of the geometry.

This helps to remove any bias from the overall results, caused by the population starting positions.

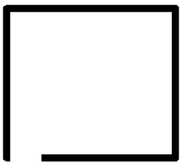
Several software specific parameters, the Cumulative Wait Time (CWT), Personal Evacuation Time (PET), Distance travelled (Dist) and Total Evacuation Time (TET) will often be referred to in this analysis and so each of these parameters will now be briefly defined. The CWT, PET and Dist are informative personal parameters that are determined, by the software, for each individual within a simulation. The CWT is a measure of the total amount of time that a person loses to congestion. For this application, this includes the time queuing to pass through the exit and the amount of time lost due to being obstructed by slower moving occupants. The PET is a measure of the time each individual requires to evacuate. This is measured from the start of the simulation to the point at which the individual exits the compartment. The Dist is a measure of the distance travelled by the occupant during the evacuation. Each of these parameters can be averaged over a simulation to produce average values for the simulation population. As mentioned before, given that each particular scenario is repeated 600 times, an average of the averages can also be determined. These are represented as AV CWT, AV PET and AV Dist.

Unlike the CWT, PET and Dist, the TET is not a personal parameter but represents the total evacuation time for a particular simulation. As each particular scenario is repeated 600 times, the average TET for the scenario can be determined and is represented by AV TET.

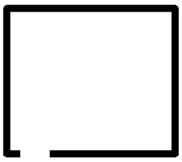
For the single exit scenario, it is possible to identify at most six unique exit locations representing the nearest corner of the door being located (i) 0 m, (ii) 1.0 m, (iii) 2.0 m, (iv) 3.0 m, (v) 4.0 m from the front left room corner and (vi) centrally located (see Figure 5.1). Note that not all of these exit locations will produce unique sub-cases for all the exit widths for example, for the 2.0 m case, sub-cases (iv) and (vi) are identical. Any other exit location is considered a non-significant variation of these six cases.

Given the simplicity of the geometry (i.e., a regular and symmetrical shape – square room), the domain of the design variable is:

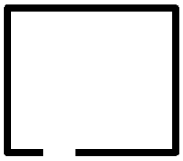
$$0 \le EP \le \text{ the middle of the wall}$$



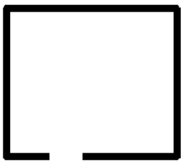
(i) Position 1: Exit located in the corner of the South Wall



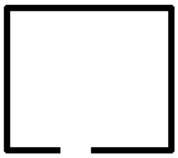
(ii) Position 2: Exit edge located 1.0m from the corner



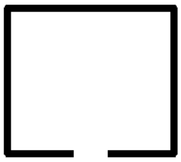
(iii) Position 3: Exit edge located 2.0m from the corner



(iv) Position 4: Exit edge located 3.0m from the corner



(v) Position 5: Exit edge located 4.0m from the corner



(vi) Position 6: Exit located in the middle of the South Wall

Figure 5.1: Exit locations for a square room with a single exit case

5.2.3 Results obtained through data analysis

A total of 16,800 simulations were performed (i.e., 600 per scenario, *n* different exit positions and *m* different door widths, giving *m* x *n* x 600 = 16,800). The results are discussed in the following sections.

5.2.3a The 1.0m wide exit cases

For this case, a total of 6,600 simulations were performed. The investigated scenarios were:

- the exit is in the corner;
- the exit is located 1.0m from the corner;
- the exit is located 2.0m from the corner;
- the exit is located 3.0m from the corner;
- the exit is located 4.0m from the corner;
- the exit is located 5.0m from the corner;
- the exit is located in the middle of the wall.

It was observed that, with a 1.0m exit, the compartment empties in approximately 167 seconds and while with a 2.0m exit, for instance (as shown in this chapter), the compartment empties in 87 seconds, approximately half the time of the narrower exit. Also as expected, the AV CWT for these cases with the 1.0m exit are considerably larger then those for the wider exits, indicating that the occupants of the 1.0m wide door cases experienced considerably more congestion than those in the wider door cases. The Table 5.1 presents the summary of this data.

Table 5.1 Evacuation data for room with 200 occupants and a single 1.0 m exit

Exit Location	AV TET (sec)	AV CWT (sec)	AV Dist (m)	AV PET (sec)
Position 1 (corner of wall)	165.4	75.0	8.9	83.1
Position 2	167.4	76.6	8.3	84.3
Position 3	167.1	76.6	8.1	84.1
Position 4	167.2	76.7	8.0	84.2
Position 5	167.2	76.8	8.0	84.2
Position 6 (middle of wall)	167.2	76.7	8.1	84.2

When the evacuation time is considered for the different cases (i.e., different exit location = exit positions), it is possible to note that with the exception of Position 1, all the cases produce virtually identical egress statistics and that Position 1 produces slightly shorter evacuation times.

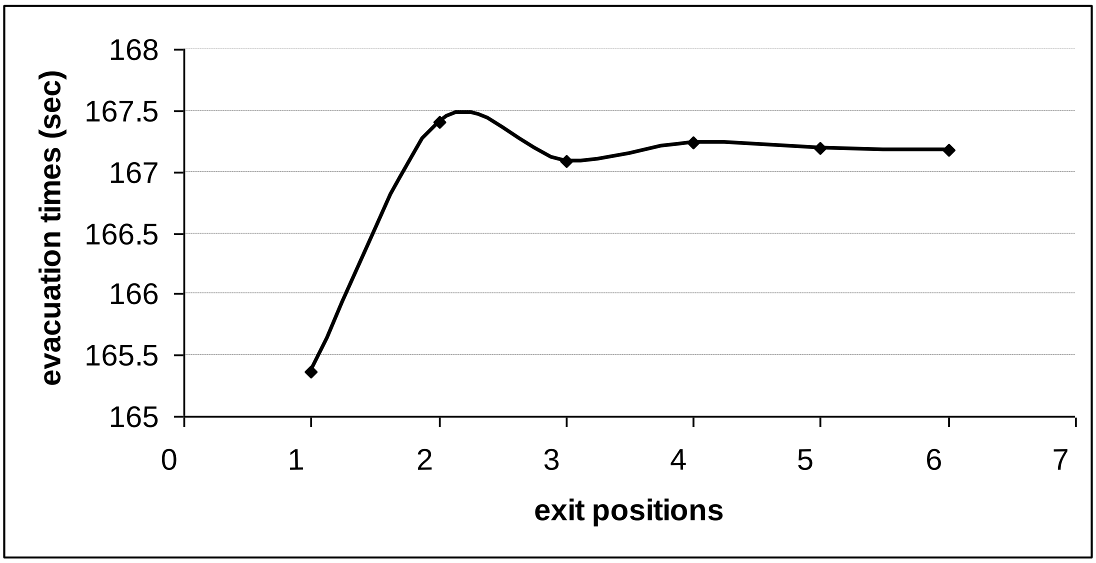


Figure 5.2: Average TET for the 1.0 m exit cases

The AV TET for the exit located in Position 1 is some 1.1% smaller than the average AV TET for the other exit locations. This is clearly shown in Figure 5.2. The shorter evacuation times are produced despite the fact that, on average, the occupants travelled further in the Position 1 scenario than in the other cases (see AV Dist in Table

5.1). This can be explained by the fact that having slightly further to travel to the exit would help with reducing the congestion since some of the occupants are still travelling – and hence not adding to the congestion.

However, the advantage offered by the corner location is half that for the smaller 1.0 m exit. Furthermore, if the population of the room is decreased to 100, it was found that the 1.0 m exit located in the corner provides only a 0.8% advantage over exits located away from the corner (see Table 5.2). Both these observations support the explanation suggested above.

Table 5.2: Evacuation data for room with 100 occupants and a single 1.0 m exit

Exit Location	AV TET (sec)	AV CWT (sec)	AV Dist (m)	AV PET (sec)
Position 1 (corner of wall)	83.3	34.5	8.6	42.4
Position 6 (middle of wall)	84.0	35.5	7.7	42.8

The above analysis suggests that for a crowd of given size, an exit placed in the corner of a square room will produce slightly better egress efficiencies than the same sized exit placed away from the corner. In these examples a small compartment was used which effectively minimised the influence of travel distance on evacuation efficiency. The relatively small room implies relatively short travel distances and hence travel times and this combined with the instant response times generates large areas of congestion around the exits very early in the evacuation. In the case of the corner located exit, this maximises the influence of the corner wall on evacuation efficiency. Thus for small compartments, the time involved in travelling to the exit (or the exit queue) does not exert a significant influence on the overall evacuation times or evacuation process' efficiencies.

As the size of the compartment increases, while the population size remains constant, the time required to reach the exit increases in significance while the advantage offered by the corner located exit decreases. This is due to the decrease in effective crowd size formed at the exit at any one time resulting from the greater staggered arrival times. As the compartment size continues to increase, eventually the staggered arrival times resulting from the increased travel time (i.e. distance) will dominant the evacuation process. While no attempt was made to determine the critical compartment size for a population of 200 people, a 30m X 30m compartment was found to provide an advantage for the corner located exit while a 60m X 60m compartment provided a slight advantage for the exit located in the centre of the wall.

5.2.3b The 1.5m wide exit cases

For this case, a total of 4,200 simulations were performed. The investigated scenarios were:

- the exit is in the corner;
- the exit is located 1.0m from the corner;
- the exit is located 2.0m from the corner;
- the exit is located 3.0m from the corner;
- the exit is located 4.0m from the corner;
- the exit is located 5.0m from the corner;
- the exit is located in the middle of the wall.

Similarly to the cases with 1.0m wide exit, it was observed the same trend behaviour. The Table 5.3 and Figure 5.3 present this data.

Table 5.3 Evacuation data for room with 200 occupants and a single 1.5 m exit

Exit Location	AV TET (sec)	AV CWT (sec)	AV Dist (m)	AV PET (sec)
Position 1 (corner of wall)	113.4	49.06	8.71	57.02
Position 2	114.0	49.74	8.23	57.34
Position 3	114.0	49.9	8.02	57.35
Position 4	114.0	49.95	8.04	57.41
Position 5	113.9	49.89	8.09	57.4
Position 6	113.8	49.67	8.32	57.35
Position 7 (middle of wall)	113.9	49.79	8.19	57.37

Figure 5.3: Average TET for the 1.5 m exit cases

For this case, the average evacuation time for the exits which are not in the corner is 113.9sec and the average evacuation time in the corner is 113.4 sec. The difference between the evacuation time for the corner exit and the average evacuation time for the others exits which are not in the corner is now 0.4%.

5.2.3c The 2.0m wide exit cases

For this case, a total of 4,200 simulations were performed. The investigated scenarios were:

- the exit is in the corner;
- the exit is located 1.0m from the corner;
- the exit is located 2.0m from the corner;
- the exit is located 3.0m from the corner;
- the exit is located 4.0m from the corner;
- the exit is located 5.0m from the corner;
- the exit is located in the middle of the wall.

Also similarly to the cases with 1.0m wide exit, it was observed the same trend behaviour. The Table 5.4 and Figure 5.4 present this data.

Table 5.4 Evacuation data for room with 200 occupants and a single 2.0 m exit

Exit Location	AV TET (sec)	AV CWT (sec)	AV Dist (m)	AV PET (sec)
Position 1 (corner of wall)	86.7	35.8	8.6	43.7
Position 2	87.2	36.3	8.2	43.8
Position 3	87.2	36.4	8.0	43.9
Position 4	87.2	36.5	8.0	43.9
Position 5	87.2	36.4	8.1	43.9
Position 6 (middle of wall)	87.2	36.4	8.1	43.9

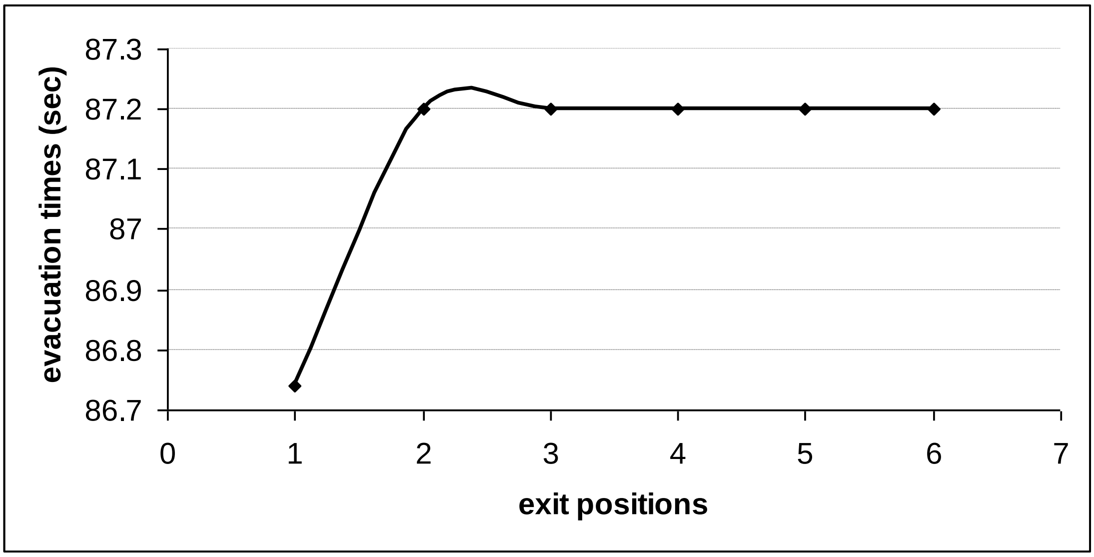


Figure 5.4: Average TET for the 2.0 m exit cases

When the exit width is increased to 2.0 m (see Table 5.4 and Figure 5.4), it is possible to note that the corner exit still provides an advantage over the other exit locations, producing the minimum evacuation time. The AV TET for the exit located in Position 1 is some 0.6% smaller than the average AV TET for the other exit locations. This is clearly shown in Figure 5.4.

5.2.3d The 2.5m wide exit cases

For this case, also a total of 4,200 simulations were performed. The investigated scenarios were:

- the exit is in the corner;
- the exit is located 1.0m from the corner;
- the exit is located 2.0m from the corner;
- the exit is located 3.0m from the corner;
- the exit is located 4.0m from the corner;
- the exit is located 5.0m from the corner;
- the exit is located in the middle of the wall.

It was observed that these cases exhibited similar trends in their behaviour, as were found in the previous cases. The Table 5.5 and Figure 5.5 present this data.

Table 5.5 Evacuation data for room with 200 occupants and a single 2.5 m exit

Exit Location	AV TET (sec)	AV CWT (sec)	AV Dist (m)	AV PET (sec)
Position 1 (corner of wall)	70.4	27.53	8.57	35.39
Position 2	70.7	27.96	8.15	35.51
Position 3	70.7	28.12	7.98	35.54
Position 4	70.8	28.12	8.02	35.58
Position 5	70.8	28.02	8.17	35.59
Position 6 (middle of wall)	70.7	27.73	8.5	35.53

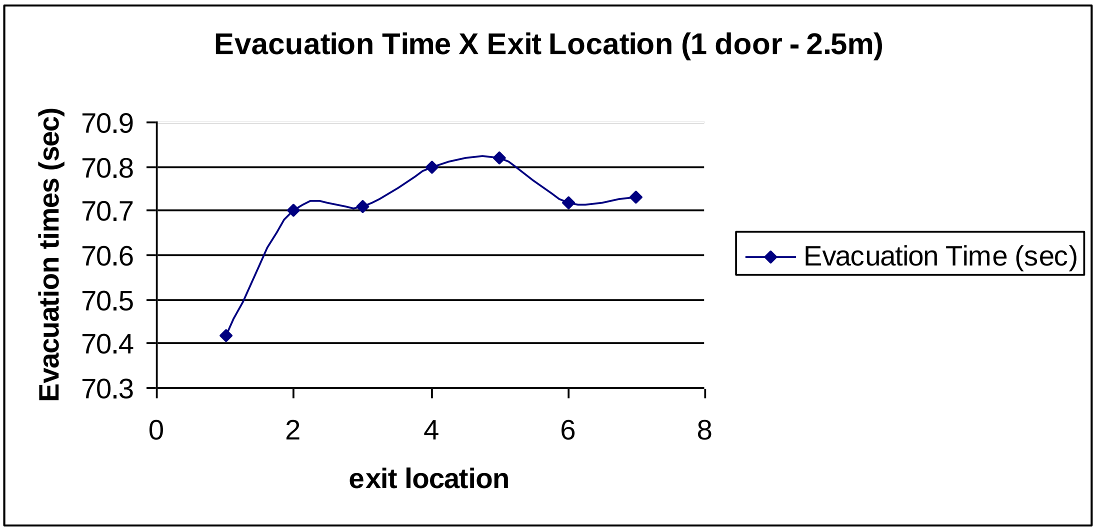


Figure 5.5: Average TET for the 2.5 m exit cases

For this case, the average evacuation time for the exits, which are not in the corner, is 70.7sec and the average evacuation time in the corner is 70.4sec. The difference between the evacuation time for the corner exit and the average evacuation time for the other exits, which are not located in the corner, is now 0.4%.

5.2.4 Discussion on the results

The figure 5.6 presents a graph which shows the normalized curves of "*evacuation times against exit locations"* for all the squared room with one exit cases. Table 5.6 presents all the data for these initial studies.

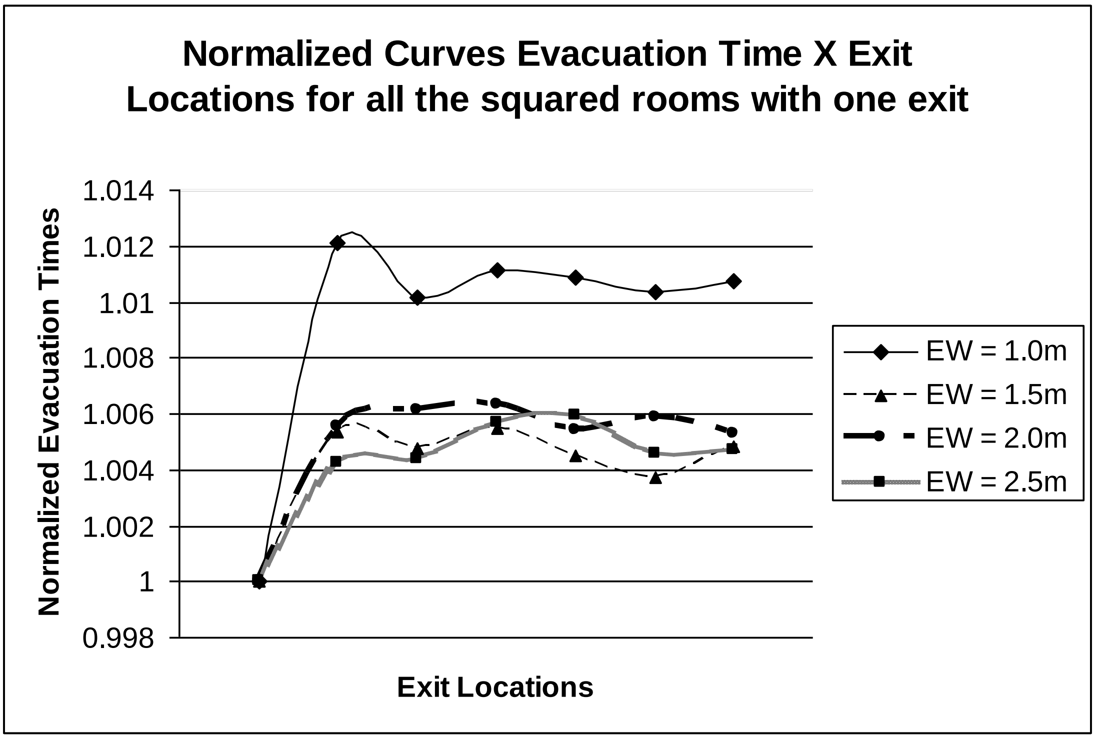


Figure 5.5: Average TET for the 2.5 m exit cases

Table 5.6: Data summary for all squared room with one exit cases

Exit Location (m)	Exit Width (m)	Avr TET (sec)	Avr-CWT (sec)	Avr-Dist (m)	Avr-PET (sec)
0 (in the corner)	1.0	165.4	75.04	8.85	83.11
1.0m from the corner	1.0	167.4	76.56	8.33	84.25
2.0m from the corner	1.0	167.1	76.57	8.1	84.08
3.0m from the corner	1.0	167.2	76.74	8.02	84.19
4.0m from the corner	1.0	167.2	76.77	8.02	84.22
5.0m from the corner	1.0	167.1	76.61	8.17	84.18
in the middle of the wall	1.0	167.2	76.73	8.07	84.21
0 (in the corner)	1.5	113.4	49.06	8.71	57.02
1.0m from the corner	1.5	114.0	49.74	8.23	57.34
2.0m from the corner	1.5	114.0	49.9	8.02	57.35
3.0m from the corner	1.5	114.0	49.95	8.04	57.41
4.0m from the corner	1.5	113.9	49.89	8.09	57.4
5.0m from the corner	1.5	113.8	49.67	8.32	57.35
in the middle of the wall	1.5	113.9	49.79	8.19	57.37
0 (in the corner)	2.0	86.7	35.75	8.62	43.65
1.0m from the corner	2.0	87.2	36.29	8.16	43.84
2.0m from the corner	2.0	87.2	36.44	7.97	43.86
3.0m from the corner	2.0	87.2	36.48	8.01	43.92
4.0m from the corner	2.0	87.2	36.37	8.13	43.9
5.0m from the corner	2.0	87.2	36.18	8.41	43.92
in the middle of the wall	2.0	87.2	36.35	8.13	43.89
0 (in the corner)	2.5	70.4	27.53	8.57	35.39
1.0m from the corner	2.5	70.7	27.96	8.15	35.51
2.0m from the corner	2.5	70.7	28.12	7.98	35.54
3.0m from the corner	2.5	70.8	28.12	8.02	35.58
4.0m from the corner	2.5	70.8	28.02	8.17	35.59
5.0m from the corner	2.5	70.7	27.73	8.5	35.53
in the middle of the wall	2.5	70.7	28.07	8.07	35.56

From the data (i.e., using one single exit), it is possible to see that the lowest value for the evacuation time is obtained when the exit is located in the corner. It seems that, apart from when the exit is initially in the corner, it does not matter where the exit is located, since the differences in evacuation times are negligible.

This shows that when the exit is in the corner, the evacuation time decreases; which implies directly proportional (i.e., the evacuation time is directly proportional to the exit location in this point). As the exit location is increased along the wall, so the evacuation time increases; but the difference is quite insignificant.

Probably, the reason for this is that when the exit is located in the corner the "arc formation" has less impact on the occupants' movement than when the exit is not located in the corner. In other words, in the corner, "the area of the arc" seems to be smaller than when the exit is not located in the corner

In reality, once "the area of the arc" is at its smallest, there is less conflict between the occupants trying to reach the exit; this reduction in conflict means that there is less congestion. Once there is less congestion, movement is less turbulent (i.e., less chaotic); or in others terms is more permanent (i.e., more organized). Once their movement is more organized, the flow rate is better (i.e., higher) which is going to make the evacuation time lower. The figure 5.7 represents this aspect.

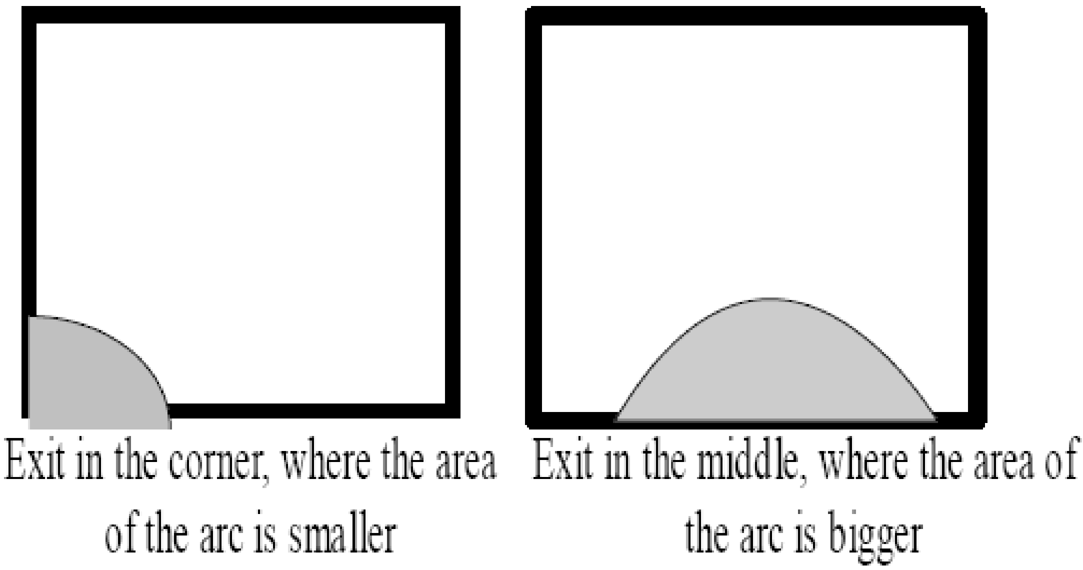


Figure 5.7: Arc formation in the exits

The greater the physical extent and duration that the occupant arch is in contact with the confining walls, the greater the advantage provided by the wall. When the exit is located in the corner, more of the arch comes into contact with the confining walls then if the exit was located away from the corner. Furthermore, occupants immediately surrounding an exit and located at the wall normal to the exit, will experience fewer conflicts or challenges to pass through the exit then occupants immediately surrounding an exit which is located away from the corner. This is not unlike the well known phenomena observed at sports grounds when large crowds attempt to exit through a gate **[40]**. In such situations exits with a barrier positioned perpendicular to the centre of the exit produce higher throughput than exits without the perpendicular barrier. It has been demonstrated that this measure can improve the exit flow rate of the crowd. In reality, when exits are located in the corner, the wall seems to play a similar function to the barrier, restricting the occupants' movement and forcing them to move in the opposite direction and consequently, their movement becomes more organized (this issue is discussed further in the section 5.4.5a).

If the above explanation is correct, then it is expected that investigations will find that as the exit width is increased while keeping the population size fixed, the advantage offered by the corner location should diminish. This is because the wider exit provides greater exit capacity, resulting in more rapid egress. With a more rapid egress the extent and duration of arch contact with the confining walls will diminish. Furthermore, if the population size was decreased while keeping the exit size fixed, it would also be anticipated that the advantage offered by the corner exit would diminish.

These results suggest there is a slight advantage in placing the exit in the corner of the room. Placing the exit in any other location will produce longer egress times and furthermore, outside the corner region, the egress time is not strongly dependent on exit location.

An explanation of these results can be found in the nature of the flow dynamics around the exit. As an instant response time distribution is used in these simulations, a large crowd develops around the exit almost immediately creating a large characteristic arch (see Figure 5.8). Occupant movement within the arch is chaotic with people interacting with all of their neighbours creating many conflicts for space. This produces quite large values for the CWT representing the large amounts of time spent in congestion. However, when the exit is located hard up against the corner of the room, the compartments confining wall allows occupants pressed up against the wall (and those immediately near the wall) to take a more direct path to the exit, reducing the opportunity for conflicts from the "wall side" (see Figure 5.9) while travelling to the exit. The wall effectively provides the occupants with a barrier protecting them from time wasting conflict interactions from one side.

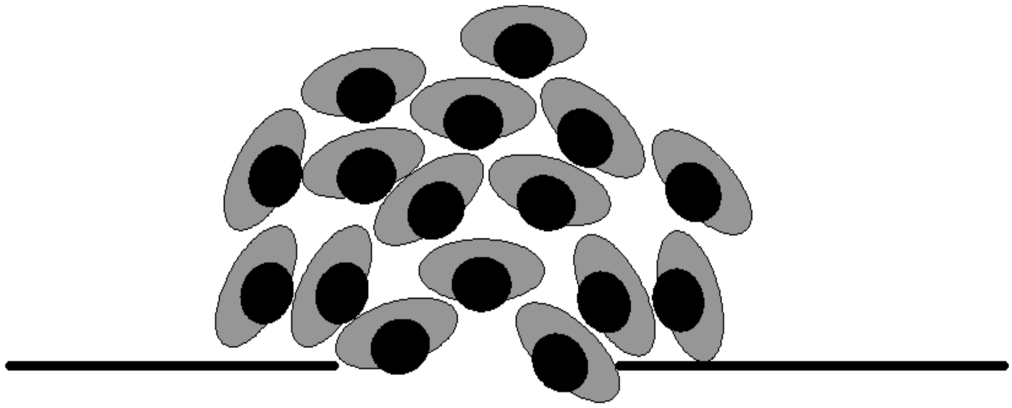


Figure 5.8: Typical occupant arch formed around a congested exit located away from a room corner

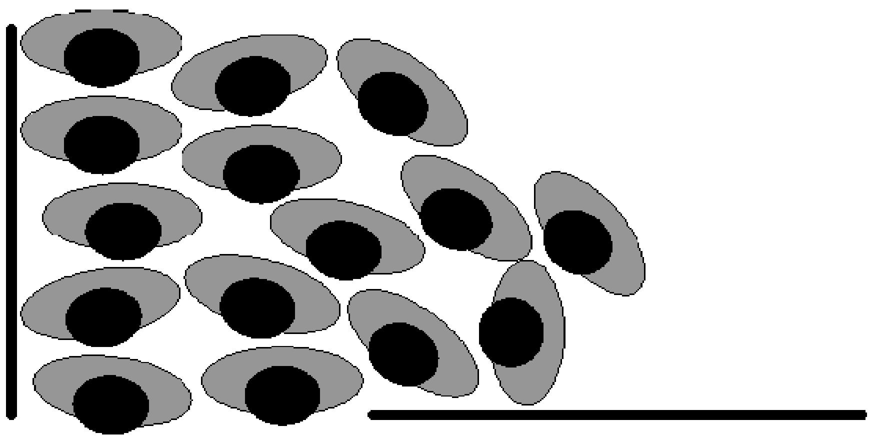


Figure 5.8: Typical arch formation around congested exit located in a room corner

As can be seen in Figure 5.8, this is indeed the case with the exit located in Position 1 producing the smallest AV CWT. However, it can be argued that the AV

CWT is smaller in this case because the duration of the simulation as measured by AV TET is also smaller. A good indication of evacuation efficiency can be obtained by taking the ratio AV CWT/AV PET which is an indication of the fraction of the average PET spent in congestion. The larger the ratio, the more inefficient the evacuation as more of the personal evacuation time is spent in congestion. For the exit located in the corner, the AV CWT/AV PET is 90.25% while the average for all other locations is 91.07%. This supports that view that on average, the evacuation with the corner exit is slightly more efficient than the evacuations with the exit located away from the corner.

This "hypothesis" could be explained, for instance, based on the well-known experiments using some kind of obstacles (i.e., barriers and even columns) in the middle of the exits, which reduces the crowd density and consequently makes the flow rate more efficient. As mentioned before, this aspect is discussed in more detail in the next section of this chapter.

It is also interesting to note that, when the exit width is increased, the location of the exit is not so relevant, as it is possible to see from the values produced for the 2.5m exit. In fact, when the exit width increases, the evacuation time becomes more dependent on the flow rate rather than the exit location. The reason for this is probably that when the exit width is increased, naturally, the flow rate is better and consequently the evacuation time is reduced, by the same explanation given above.

Thus, basically, this relation between the evacuation time and the exit location is summarized below:

$$\left.\begin{array}{l} EL = 0 \Rightarrow ET \downarrow \\ \\ EL \neq 0 \Rightarrow ET \uparrow \end{array}\right\} ELxET$$

When the exit location is at 0 (i.e., which means that the exit is located in the corner), the evacuation time decreases. And when the exit location is different from 0 (i.e., which means that the exit is not located in the corner), the evacuation time increases.

The evacuation time (ET) seems to be directly proportional to the area of the arc (AA):

$$AA \sim ET$$

Based on that, the logical and basic relations are:

$$A\uparrow C\uparrow F\downarrow ET\uparrow$$

The bigger the area of the arc around the exit, so the bigger is the conflict and the lower is the flow rate and consequently higher becomes the evacuation time. It might relevant to mention here that these relations seem to be true when the arc is full of people queuing.

One hypothesis to explain this congestion (i.e., conflict) is that, during an evacuation process, it can be divided into two parts: the conflict during the movement (i.e., when the occupants are moving towards to the exits) and the conflict in the exit. The travel distance (TD), which is directly associated with the enclosure area, lay-out etc., is directly influencing this conflict. It could be possible to observe that the higher is the travel distance; the lowest is the conflict for both cases (during the movement and in the exits). The reason for this can be explained by the fact that when the travel distance is bigger, the arc formation then is decreased, because the occupants take a bigger range of times to present themselves at the congested area. And for this reason, the cumulative time spent by all the occupants in the congestion region, is reduced.

5.4.a The use of Barriers

The use of barriers for improving the flow rate through exits, when the population size is very large, is well known at sports grounds **[42]**. In such situations exits with a barrier positioned perpendicular to the centre of the exit produce higher throughput than exits without the perpendicular barrier.

In order to observe this behaviour, under a different, but similar perspective, a set of simulations were performed. In this set of simulations, the scenarios were based on the same squared room, with a 2.0m wide exit located in the middle of the wall. These simulations were investigated using barriers at the exit, taking into consideration the same conditions for the previous simulations. The scenarios taken into account were:

- 0.5m barrier – a barrier of 0.5m is located in the middle of the exit;
- 1.0m barrier – a barrier of 10m is located in the middle of the exit;
- 2.0m barrier – a barrier of 2.0m is located in the middle of the exit;
- no barrier - the exit is placed with no barrier.

A total of 2,400 simulations were performed.

Therefore, with these 4 scenarios, the objective of these simulations was to investigate how useful it is to place a barrier in the middle of an exit, in terms of the evacuation efficiency, and if there is an advantage of using a barrier, how the relation between its dimensions works

The table 5.7 presents a summary of some of the results.

Table 5.7: Results summary for the use of barriers in the exits

Scenario	Average Evacuation Time (sec)	Average Flow Rate (p/s)
0.5m barrier	167.9	2.36
1.0m barrier	168	2.36
2.0m barrier	169.8	2.36
no barrier	172	2.29

From these results, it is possible to see that the use of barriers in the exits is an advantage, since the flow rates are higher compared to the case in which there is no barrier. In fact, when the barriers are placed at the exit, there is an improvement of 2.96% in the flow rates. This difference can be a significant factor in a real evacuation process, because once the flow rate is higher, people will use the exits more efficiently; which is going to make the evacuation faster. In others words; once the flow rate is improving (i.e., getting higher), the occupants' movement has a tendency to become less “turbulent” and with this, they evacuate in a more organized manner, and consequently, this reduces the time required to evacuate (i.e., less conflicts; less congestion). Figure 5.9 shows this relation between the flow rates and the barriers.

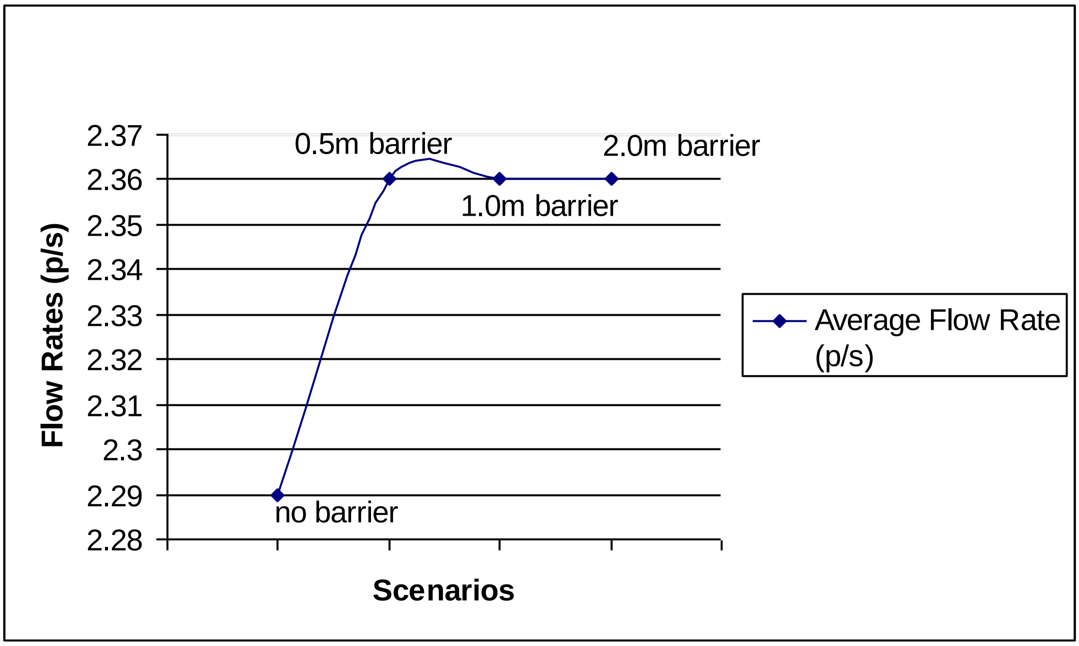


Figure 5.9: Curve Flow Rates “vs” Scenarios

The figure 5.10 shows the relationship between barrier size (including no barrier) and evacuation times.

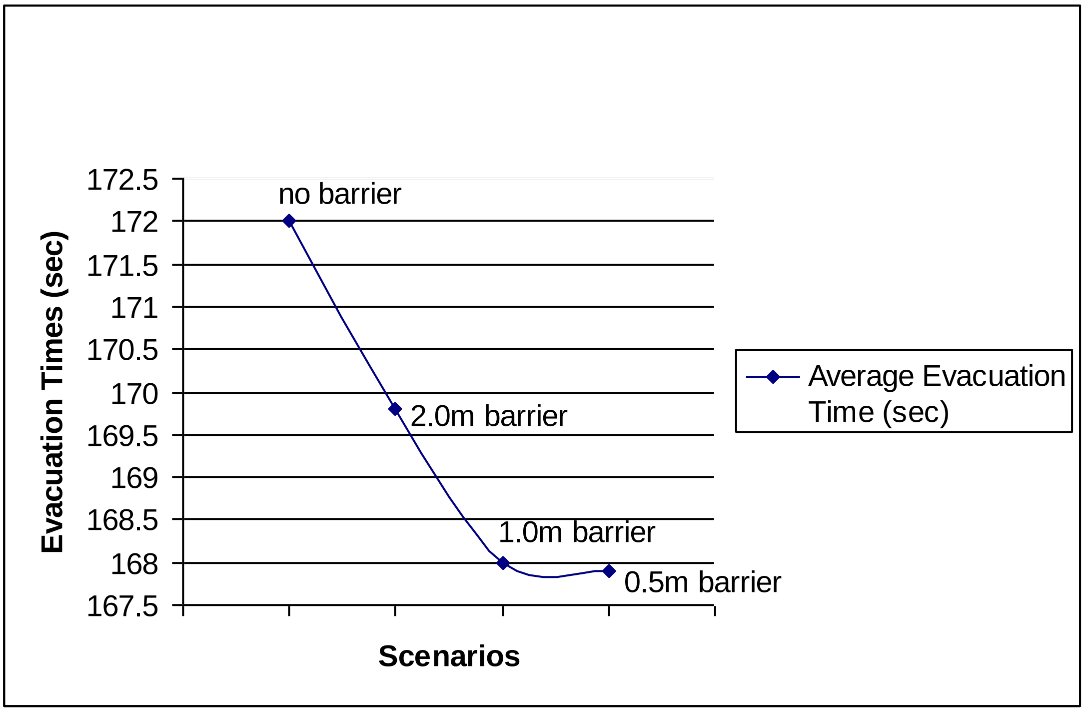


Figure 5.10: Curve Evacuation Times "vs" Scenarios

From this figure, it is also possible to see that the improvement in terms of the evacuation times (i.e., which is also a reflection of the improvement of the flow rates); following a tendency according to the barriers dimensions.

This is also supported by another computational simulation experiment that was run in the Panic software **[42]**.

In this experiment, a similar geometry was also built with 200 people, in which the fire products were also taken into account. Instead of using a barrier in the middle of the exit, a column was placed in front of the exit and it was shown that, with this measure, injuries were avoided. The pillar also had the effect of increasing the flow rate; see table 5.8 and Figure 5.11.

Table 5.8: Results summary for the use of a column place in front of an exit

Type of Simulation	Evacuated safely until 45sec	Injured until 45sec
Without column, possible injuries ignored (Panic / 200 people)	65	-
Without column, injured people don't move (Stampede / 200 people)	44	5
With column, injured people don't move (Column)	72	0

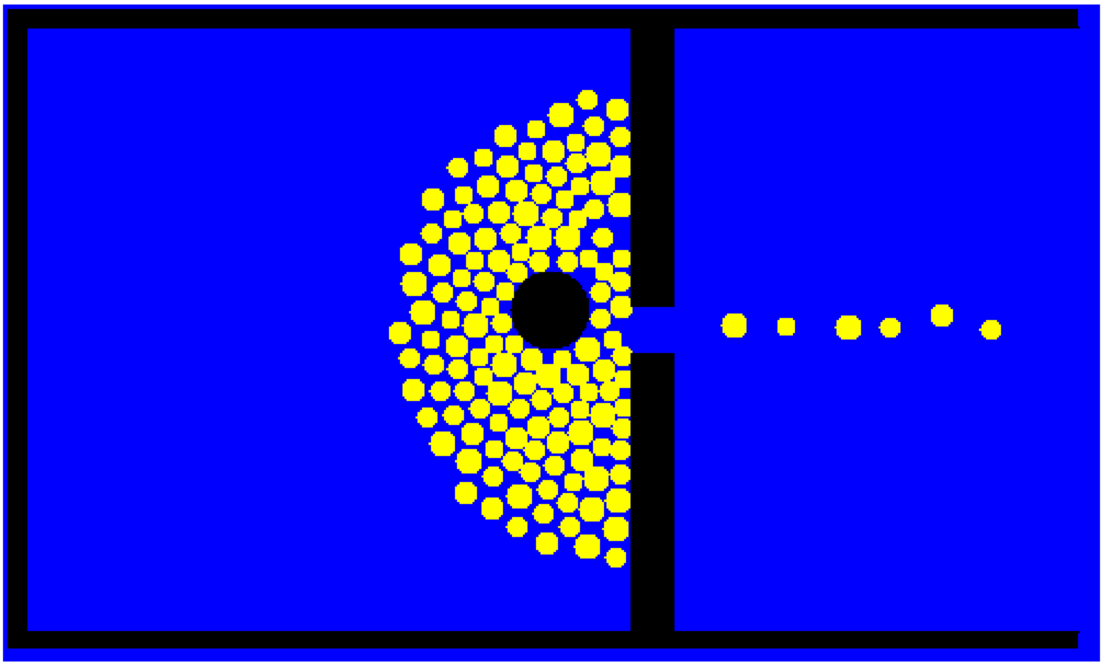


Figure 5.11: The use an obstacle (a column) to improve the flow rate in a single exit room [42]

Therefore, this experiment does support the assumption that having an obstacle near to the exit can improve the flow rate and consequently allow the occupants to evacuate faster and safer. Thus, in terms of evacuation efficiency, it was shown that the use of a barrier, for instance, is an advantage. The question now is: what is the optimal dimension of this barrier? In other words, what is the relationship between the barrier's dimensions and the improvement of the evacuation efficiency?

In order to investigate this question, additional simulations were performed. In total 2,400 simulations were performed, where the barriers of 3.0m; 4.0m; 5.0m and 6.0m were considered. The data (in Table 5.10) shows that, as the barrier becomes longer, so the advantage of having the barrier reduces. In reality, when the barrier is bigger, the evacuation time starts to increase. Probably, the reason for that is based on the fact that when the barrier is bigger, at some point during the evacuation time (i.e., when the number of people starts to decrease; and the formation of the arcs towards of the exit starts to be developed), its size becomes a disadvantage, because it tends to obstruct the occupants' movement. In other words, the barrier's dimensions, in the beginning of the evacuation process, is not a problem, however towards the end of this process, it becomes a critical issue affecting performance, since it becomes, literally a barrier in terms of occupants' movement flexibility.

Table 5.9 presents the evacuation times according to the scenarios at selected stages during the evacuation time.

Table 5.9: Evacuation times for each scenario according to the number of people evacuated

Number of people out	No barrier (sec)	0.5m barrier (sec)	1.0m barrier (sec)	2.0m barrier (sec)	3.0m barrier (sec)	4.0m barrier (sec)	5.0m barrier (sec)	6.0m barrier (sec)
20%	34.7	33.9	32.3	34.5	33.9	34.2	34.9	33.7
40%	69.3	68.7	66.8	66.6	67.0	68.1	68.7	69.1
60%	103.3	102.8	100.3	99.9	101.7	102.5	102.9	104.1
80%	178	174.3	174.7	176.2	176.8	177.3	177.8	178.5
85%	185.8	180.5	182.9	184.9	185.2	185.6	185.9	186.8
90%	194.3	188.5	191.4	193.6	193.9	194.1	194.5	195.3
95%	202.5	198.8	199.8	201.9	202.2	202.2	202.7	203.1
100%	213.4	205.9	207.4	211.7	212.7	212.9	213.1	215.1

From table 5.9, it is clear that having a barrier in the exits is always an advantage than to not having it. And it is also clear to see that, in terms of the barriers' dimensions, it is not an important issue in the early stages of the evacuation. However, at some stage (i.e., when the competitive behaviour amongst the occupants; when their interaction even in physical terms increases; *some people still call this as "panic behaviour"*), for instance, when 80% of the occupants evacuated, the barriers' dimensions start to represent an important factor. The difference observed for each scenario is quite significant. For example, when 90% of the occupants evacuated, the evacuation time for 0.5m barrier scenario is 188.5sec, while for the 2.0m barrier scenario is 193.6sec. This represents a 2.63% difference. The figure 5.12 shows how the barriers' dimensions become a disadvantage in the final stages of the evacuation, when 95% of the occupants are out.

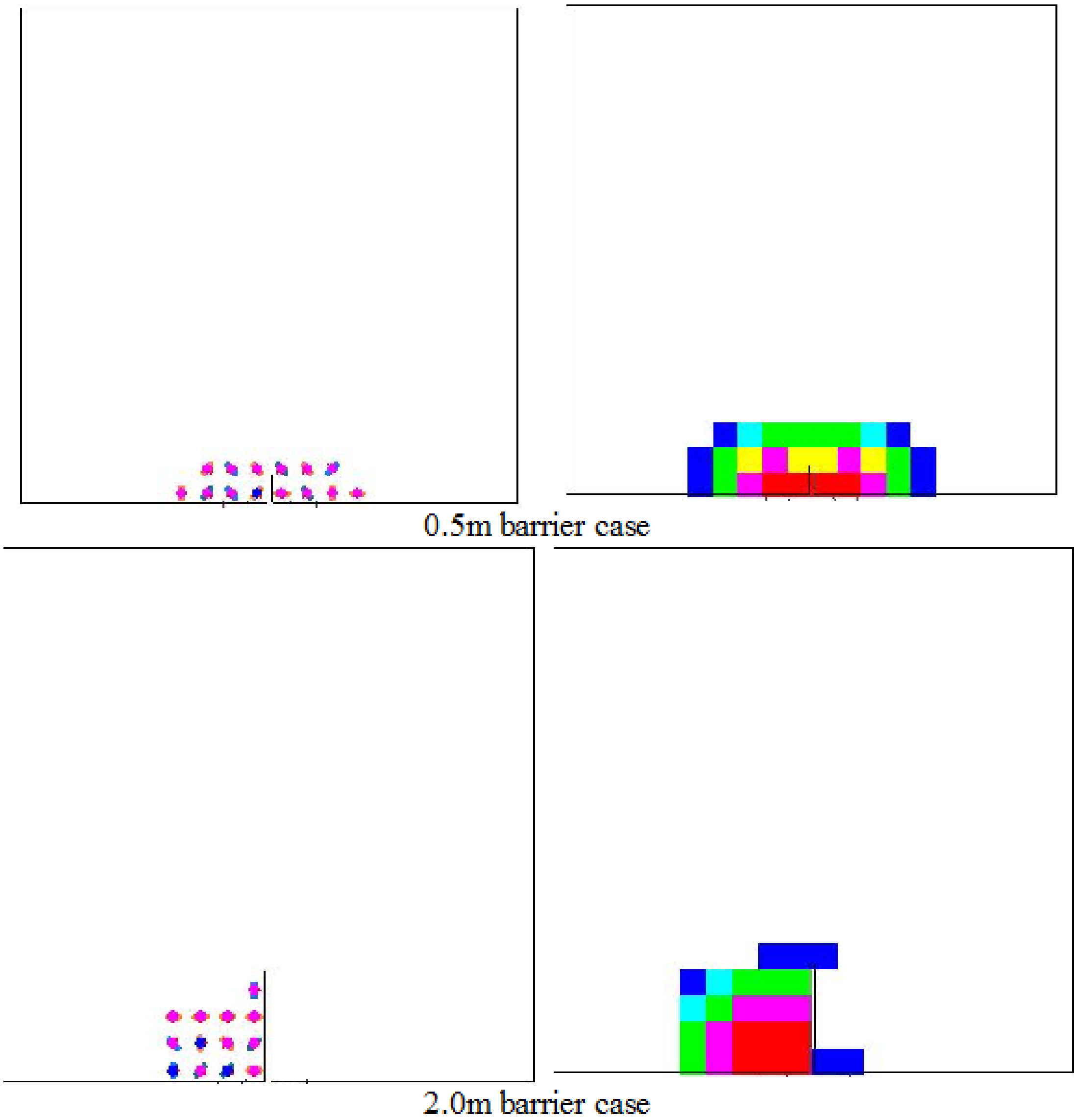


Figure 5.12: The use barrier to improve the flow rate in a single exit

It is clear that the barrier' dimensions, towards the end of the evacuation process, is no longer an advantage; it is actually a disadvantage that has a negative impact on the evacuation time. This can be explained by the fact that at some stage, for instance, for the 2.0m barrier (the biggest barrier), the distribution of people is not completely balanced. So that, if a jam starts to form on one side of the barrier and the people who are in this packed side have no movement flexibility to go to the other side of the barrier. In fact, for the occupants it is preferable to stay in this packed side than to cross over to the other side. This clearly impacts the evacuation efficiency.

Regarding the use of barriers in exits, it could be concluded that having a barrier is better than not having one.

And in terms of having a barrier, it should be attempted its dimensions; because the results suggest that having a shorter barrier is better than having a bigger barrier. For instance, at some stage, when the length of the barrier is bigger than the half of the length of the wall of the room, to have such a long barrier is worse than not to have it. Further comments about this issue are presented in the appendix A of this thesis.

In the next section of this chapter, the same problem with 2 exits is presented and discussed.

5.3 The square room with 2 exits

5.3.1 The problem

The problem to be investigated is similar to the problem from the previous section. However, instead of having a single exit, all the scenarios have two exits. Therefore, the question to be answered now, is: for a simple square room, containing an arbitrarily large population, is there an optimal location for the exits that will minimise the evacuation times? In other words, what is the best relative distance between these two exits for minimising the evacuation times?

This second hypothetical case is mostly the same, with: a simple geometry, a squared room with an area of 100 m^2 and the exits width able to vary from 1.0m to 2.5m. The population of the room consists of 200 people producing a population density of 2 people/m^2. This means that 50% of the area is populated, which gives 50% of freedom for the occupants' movement during the evacuation process.

In the same way that was done for the previous cases using one single exit, the problem with two exits were solved through "brute force method".

For this case with two exits, the problem is a constrained problem, since there two exits. Nevertheless, as mentioned previously, the problem was actually solved as an unconstrained problem. For this, 13 key-positions were defined. Each of the 13 key-positions is associated with a specific coordinate, which represents the positions for the two exits. With this, the constraints were defined "artificially" (i.e., "manually") which avoids having two overlapping exits and enables the problem to be solved as an unconstrained problem.

The details on how the problem was defined and the nature of the design variables, for instance, are further explained in the following sections of this chapter.

5.3.2 The Scenarios

5.3.2a The simulations

The simulations, the evacuation model used and the parameters used for the simulations were discussed in the previous section.

It is important to mention again that, given that each particular scenario is repeated 600 times, an average of the averages can also be determined. These are represented as AV CWT, AV PET and AV Dist.

Unlike the CWT, PET and Dist, the TET is not a personal parameter but represents the total evacuation time for a particular simulation. As each particular scenario is repeated 600 times, the average TET for the scenario can be determined and is represented by AV TET.

Therefore, for the unconstrained problem, as mentioned previously, 13 key-positions were defined; where each of the 13 key-positions is associated with a specific coordinate, which represents the relative positions for both of the two exits. The Figure 5.14 presents these key-positions and their associated coordinate.

According to what was mentioned before, for the two-exit cases, the positions are measured in terms of the relative distance between these two exits along the perimeter of the room. Thus, the distance from the left edge of each exit to the corner is measured similarly to the way that was done for the single exit case. These distances are going to be the design variables for the two exits case. The domains of these design variables are:

$$0 \le D1 \le 39 \qquad\qquad 0 \le D2 \le 39$$

D1 represents the distance from the left edge of exit 1 to the corner

D2 represents the distance from the left edge of exit 2 to the corner

Given that the squared room is 10x10m, these two exits can be located anywhere along its perimeter. Considering also that each exit is a 1m width exit, the domain could be represented by a line (which represents the walls, see figure 5.13), in which the total length is 39m.

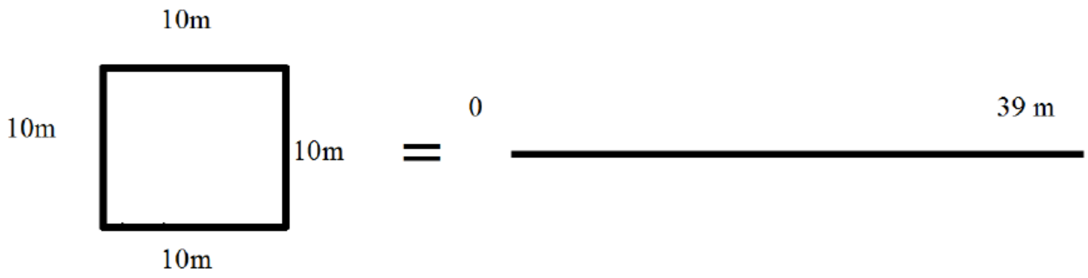
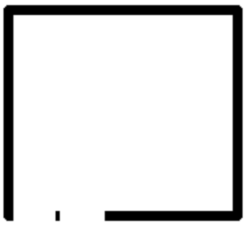


Figure 5.13: Domain for the exit positions for two exits case with 1.0m wide

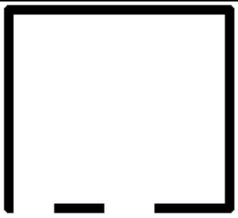
It is important to observe that, given this configuration, the two exits can overlap each other, since they are free to be located anywhere between 0 and 39 along the perimeter line. For this reason, based on this constraint, the problem was solved based on two ways:

– Not taking this constraint into account (i.e., case treated as an unconstrained problem);
– Taking into account this constraint (i.e. a constrained problem).

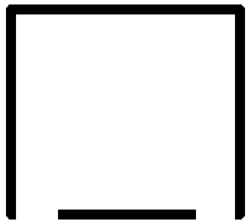
In the first approach, the constraint was not taken into consideration "directly"; however it was considered, since the 13 key-positions were defined so that the constraints were obeyed. I.e. it was assured that the exits would not be overlapping each other.



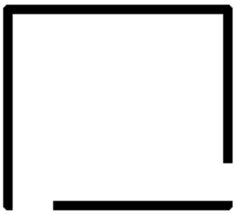
(i) Position 1 : two exits side by side in the corner of wall 1
(D1 = 0 and D2 =1)



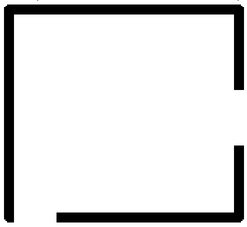
(ii) Position 2 : one exit located in the corner of wall 1 and one located in the middle of wall1
(D1 = 0 and D2 = 4.5)



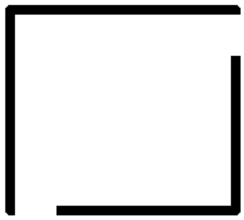
(iii) Position 3 : an exit located in each corner of wall 1
(D1 = 0 and D2 = 9)



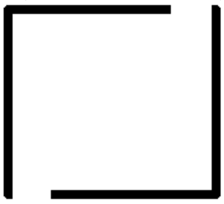
(iv) Position 4 : one exit located in the left corner of wall 1 and the other exit located in the lower corner of wall 2
(D1 = 0 and D2 = 11)



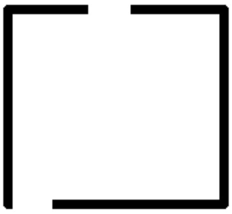
(v) Position 5 : one exit located in the left corner of wall 1 and the other exit located in the middle of wall 2
(D1 = 0 and D2 = 14.5)



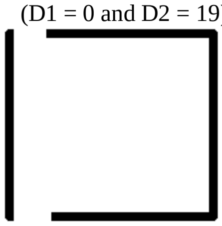
(vi) Position 6 : one exit located in the left corner of wall 1 and the other exit located in the upper corner of wall 2
(D1 = 0 and D2 = 19)



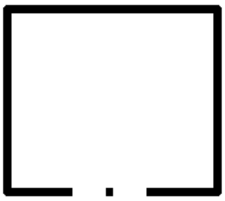
(vii) Position 7 : one exit located in the left corner of wall 1 and the other exit located in the right corner of wall 3
(D1 = 0 and D2 = 21)



(viii) Position 8 : one exit located in the left corner of wall 1 and the other exit located in the middle of wall 3
(D1 = 0 and D2 = 24.5)

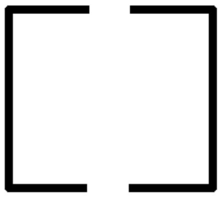


(ix) Position 9 : one exit located in the left corner of wall 1 and the other exit located in the left corner of wall 3
(D1 = 0 and D2 = 29)

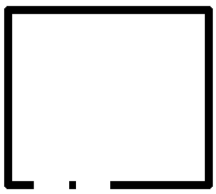


(x) Position 10 : two exits placed side by side located in the middle of wall 1
(D1 = 4 and D2 = 5)

(xi) Position 11 : one exit located in the middle of wall 1 and the other one exit located in the middle of wall 2
(D1 = 4.5 and D2 = 24.5)



(xii) Position 12 : one exit located in the middle of wall 1 and the other exit located in the middle of wall 3
(D1 = 4.5 and D2 = 14.5)



(xiii) Position 13 : the two exits placed side by side located between the corner and middle of wall 1
(D1 = 1.5 and D2 = 2.5)

Figure 5.14: Exit positions for two exits cases

Thus, for the unconstrained problem, the key-positions were manually defined, so as to avoid any overlapping of the two exits.

For the second approach, when the constraint was taken into consideration, an inequality was defined in order to represent this constraint, which prevents the overlapping of the two exits. With this, the objective function i.e., the evacuation time, was subject to this inequality, namely:

$$|D1 - D2| \geq 1$$

In summary, the one single exit case is solved as an unconstrained problem, since there are no constraints. And the two exits case is solved as an unconstrained problem (when the 13 key-positions are defined, avoiding overlapping of the two exits); and also as a constrained problem (when the inequality was defined, to prevent overlaps of the two exits).

It is important to note that the positions 1, 3, 4, 6, 7, 9, 10, 11, 12 and 13 are symmetrical; while positions 2, 5, and 8 are asymmetrical. For the symmetrical positions, naturally, the potential maps (i.e., the areas which "contains" the exits' influence; in others words, the areas in which the exits, potentially, should be used by the occupants, taking into account their location in the environment lay-out) are the same, which means that the potential map for both exits are 50% of the total area of the room. In other words, the potential map of one exit is 50m^2 and the potential map for the other exit is 50m^2, given that the room area is 100m^2. For the asymmetrical positions, these potential maps are different.

This aspect should be mentioned, since this is going to influence the use of the exits and consequently, is going to have an impact on the evacuation times. This is going to be analysed further later in this Chapter. The table 5.10 presents the areas for these asymmetrical positions.

Table 5.10: Potential map areas for the squared room with two exits

Asymmetrical Positions	Potential Map Area for one exit (m^2)	Potential Map Area for another exit (m^2)
2	43.7	56.3
5	44.25	55.75
8	38.5	61.5

5.3.3 Results

A total of 87,000 simulations were performed. The results are discussed in the following sections.

5.4.1 The 1.0m wide exit cases

Detailed results for the cases with two 1.0m wide exits are presented here (see Table 5.11 and Figure 5.15). It is possible to note that, with two 1.0m exits, the compartment empties in approximately 84-98 seconds, depending on exit locations while, with two 2.0 m exits, the compartment empties in approximately 45-49 seconds (as shown in the following sections). As to be expected, it is observed that with two 2.0m exits, the compartment empties in approximately half the time of the compartment that has two 1.0m exits. Also, on comparing these times with the equivalent times for the appropriate single exit cases of scenario 1, it is found that two-exit cases are approximately twice as fast as the equivalent single-exit cases. Again, as expected, the AV CWT for the cases with two 1.0m exits are considerably larger then those for the two 2.0m exit cases, indicating that the occupants of the case with two 1.0m wide exits experienced considerably more congestion than those in the case with two 2.0m wide exits.

Table 5.11: Evacuation data for room with 200 occupants and two 1.0 m exits

Position	AV TET (sec)	AV CWT (sec)	AV Dist (m)	AV PET (sec)	Av Number using Exit 1	Av Number using Exit 2
1	83.8	34.4	8.6	42.2	100	100
2	96.6	36.4	7.4	43.4	85	115
3	87.0	36.0	7.5	43.2	101	100
4	86.5	36.0	7.5	43.1	100	100
5	92.7	37.0	6.1	43.0	90	110
6	86.7	36.0	6.4	42.3	100	101
7	86.6	36.0	6.4	42.3	99	101
8	98.2	38.2	5.5	43.8	83	117
9	86.4	35.5	6.9	42.1	100	100
10	84.4	35.1	8.1	42.6	100	100
11	85.2	35.8	5.9	41.8	98	102
12	86.2	37.0	4.7	41.9	99	101
13	84.1	34.6	8.3	42.4	100	100

When the evacuation times are compared for the different cases with two 1.0 m exits, it is possible to note that there is a complex spread of AV TET ranging from 83.8 sec for Position 1 to 98.2 sec for Position 8, a spread of some 17%. This difference is more significant than the difference found in the single exit cases and suggests that the relative positioning of two exits in a compartment can have a significant impact on the expected egress times.

Examination of Figure 5.15 suggests there is a pattern in the distribution of egress times, with exits located in Positions 1, 10 and 13 producing the shortest evacuation times (83.8 sec, 84.4 sec and 84.1 sec respectively), exits located in Positions 2, 5 and 8 producing the longest egress times (96.6 sec, 92.7 sec and 98.2 sec respectively) and all the other exit configurations producing similarly small evacuation times close to those of those of the shortest egress times.

On examining the exit locations associated with these clusters of exits, it is clear to see that the shortest evacuation times (Positions 1, 10 and 13) are produced when the exits are placed adjacent to each other, whilst the longest evacuation times (Positions 2, 5 and 8) are produced by exits positioned asymmetrically around the compartment and the intermediate evacuation times (Positions 3, 4, 6, 7, 9, 11 and 12) are produced by exits positioned symmetrically or near symmetrically around the compartment. When the exits are presented graphically clustered into configuration groups the differences between the configuration groups becomes apparent (see Figure 5.16).

To explain the difference in performance between the exit configuration groups it is necessary to consider the behaviour of the evacuating population. Recall that within the model, the occupants follow the idealised behaviour of moving towards their nearest exit. If the exits are symmetrically placed around the perimeter of the room, using a distance algorithm to allocate floor area to a particular exit, the room floor area will be divided equally between the two exits, 50% of the floor area being located closer to one exit than the other exit. As the behaviour imposed on the population is to utilise their nearest exit, if the population is randomly positioned within the room, it is reasonable to expect that 50% of the population will go to one exit and 50% will go to the other exit. If the exits are asymmetrically placed around the perimeter of the room, using the same distance algorithm, one exit will be allocated more than 50% of the floor area and will therefore attract more than 50% of the randomly placed population. Thus in the asymmetrical cases there will be an imbalance in the number of people using the exits, thereby prolonging the overall evacuation.

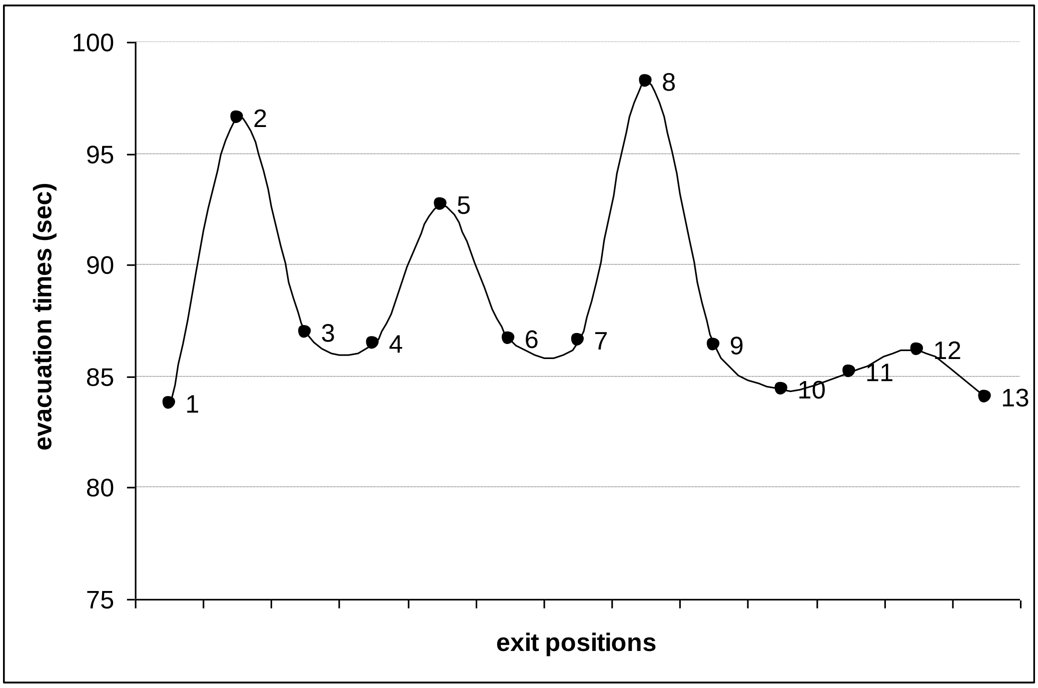


Figure 5.15: Average TET for the 13 cases with two 1.0 m exits

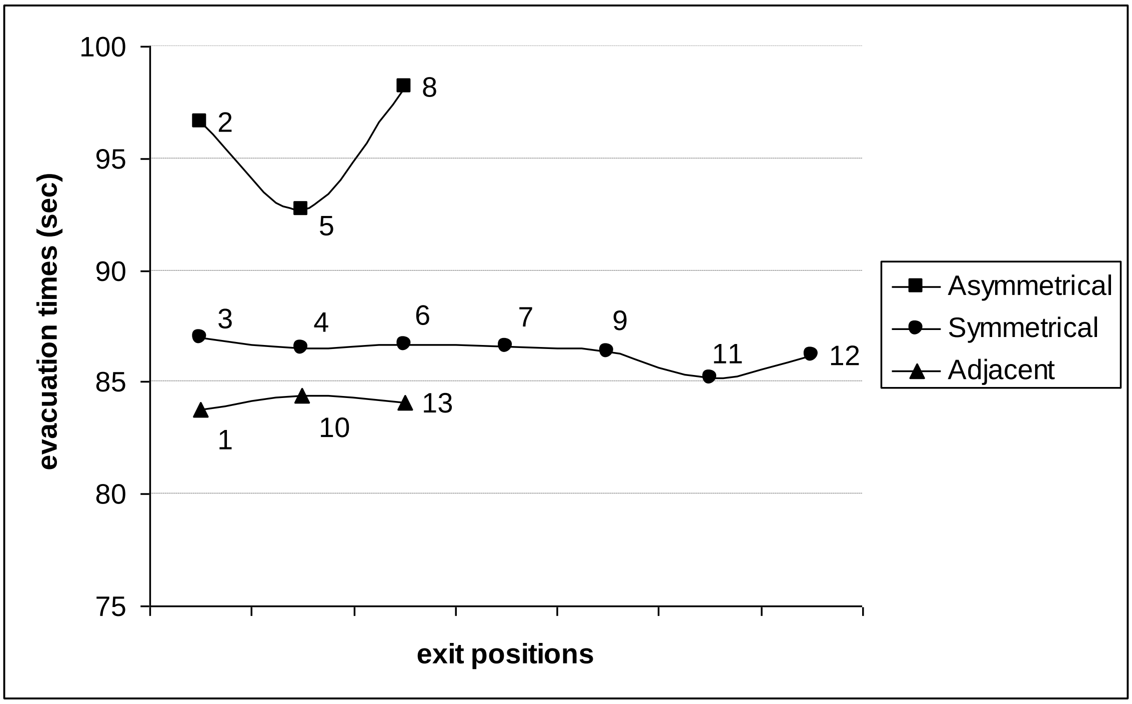


Figure 5.16: Average TET for the 12 cases with two 1.0 m exits clustered in groupings of exit configuration type

The results also suggest that two exits placed side by side produce better evacuation times than two exits distributed around the perimeter of the room. The adjacent exit positions (Positions 1, 10 and 13) produce an average egress time of 84.1 sec while the symmetrically placed exits (Positions 3, 4, 6, 7, 9, 11 and 12) produce an average egress time of 86.4 sec, the latter being some 2.7% longer than the former. Of the adjacent exit locations, the exit pair located in the corner (Position 1) produced the best time for the same reasons as highlighted for the single exit cases.

It is worth noting that in the adjacent exit cases, the two 1.0 m exits were placed side by side, not as a continuous opening of 2.0 m, but as two separate 1.0 m exits with a partition separating the exits. The partition between the exits act as a barrier preventing interaction and hence preventing time wasting conflicts between occupants attempting to utilise the exits. Thus the evacuation time produced by these two adjacent exits is actually better than the evacuation time produced by a single exit of width equal to the sum of the two adjacent exits. This can be seen by comparing the results for the single 2.0 m exit located in Positions 1 (AV TET 86.7 sec) and 6 (AV TET 87.2 sec); (see subsection *5.4.4* of this section, where the 2.0m wide exit cases results are presented and discussed), with the equivalent cases using two adjacent 1.0 m exits, Position 1 (AV TET 83.8 sec) and Position 10 (AV TET 84.4 sec) respectively.

5.4.2 The 1.5m wide exit cases

The Table 5.12 and Figure 5.16 present the results for this case.

Table 5.12: Evacuation data for room with 200 occupants and two 1.5 m exits

Position	AV TET (sec)	AV CWT (sec)	AV Dist (m)	AV PET (sec)	Av Number using Exit 1	Av Number using Exit 2
1	58.3	21.48	8.42	29.22	99.18	100.81
2	62.9	22.35	7.53	29.44	90.53	109.46
3	58.6	22.22	7.65	29.43	100.04	99.95
4	59.9	22.34	7.51	29.45	96.98	103.02
5	62.3	23.39	6.15	29.46	92.24	107.75
6	59.3	22.97	6.3	29.18	99.37	100.62
7	59.6	22.9	6.33	29.12	98.93	101.06
8	65.5	24.25	5.39	29.76	85.75	114.24
9	59.3	22.46	6.77	28.99	99.21	100.78
10	58.5	21.97	8.02	29.42	99.98	100.01
11	59.4	22.88	6.27	29.04	105.38	94.61
12	59.2	24.03	4.57	28.93	99.19	100.8

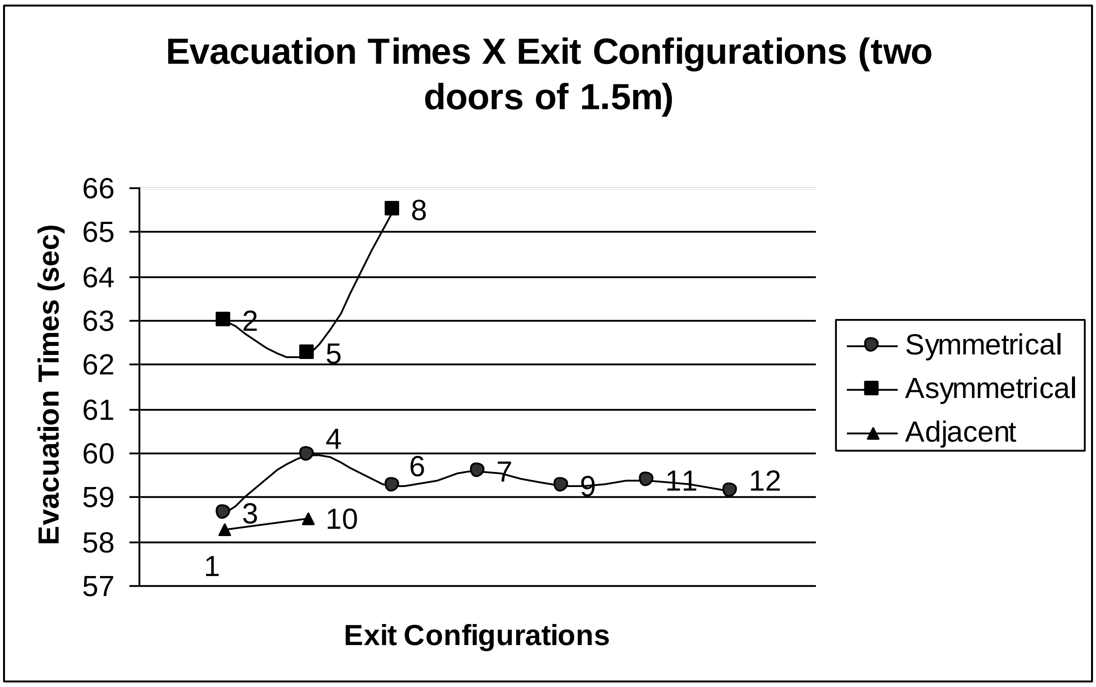


Figure 5.16: Average TET for the 13 cases with two 1.5 m exits clustered in groupings of exit configuration type

It is possible to observe that a similar trend for evacuation efficiency is found for the case with two 1.5m wide exits, as was found for the case with two 1.0m wide exits.

5.4.3 The 2.0m wide exit cases

The Table 5.13 and Figure 5.17 present the results.

It is possible to note that spread in AV TET has decreased from a difference of some 17% for the 1.0 m exits to a difference of 9.3% for the 2.0m exits. It is clear to see that the adjacent exit locations produce an AV TET of 45.2 sec, the symmetric exit locations produce an AV TET of 45.6 sec and the asymmetric exit locations produce an AV TET of 47.7 sec. In particular, the difference between the adjacent and symmetric exit locations has decreased to only 0.9%.

Thus the improved evacuation efficiency, offered by strategically placed exits, decreases as the size of the exits increases while keeping the population size fixed. This is particularly true for the difference between adjacent and symmetrically placed exits.

Table 5.13: Egress data for room with 200 occupants and two 2.0 m exits

Position	**AV TET (sec)**	**AV CWT (sec)**	**AV Dist (m)**	**AV PET (sec)**	**Av Number using Exit 1**	**Av Number using Exit 2**
1	45.2	14.9	8.1	22.4	99	102
2	46.8	15.5	7.5	22.5	94	106
3	45.1	15.5	7.5	22.6	100	100
4	45.9	15.6	7.3	22.6	97	103
5	47.2	16.6	6.0	22.6	94	106
6	45.8	16.4	6.2	22.5	100	100
7	45.7	16.2	6.2	22.4	100	100
8	49.2	17.2	5.3	22.7	88	112
9	45.7	15.9	6.6	22.3	100	101
10	45.1	15.2	7.9	22.6	101	99
11	45.0	16.3	5.9	22.2	97	103
12	45.9	17.5	4.6	22.5	99	101

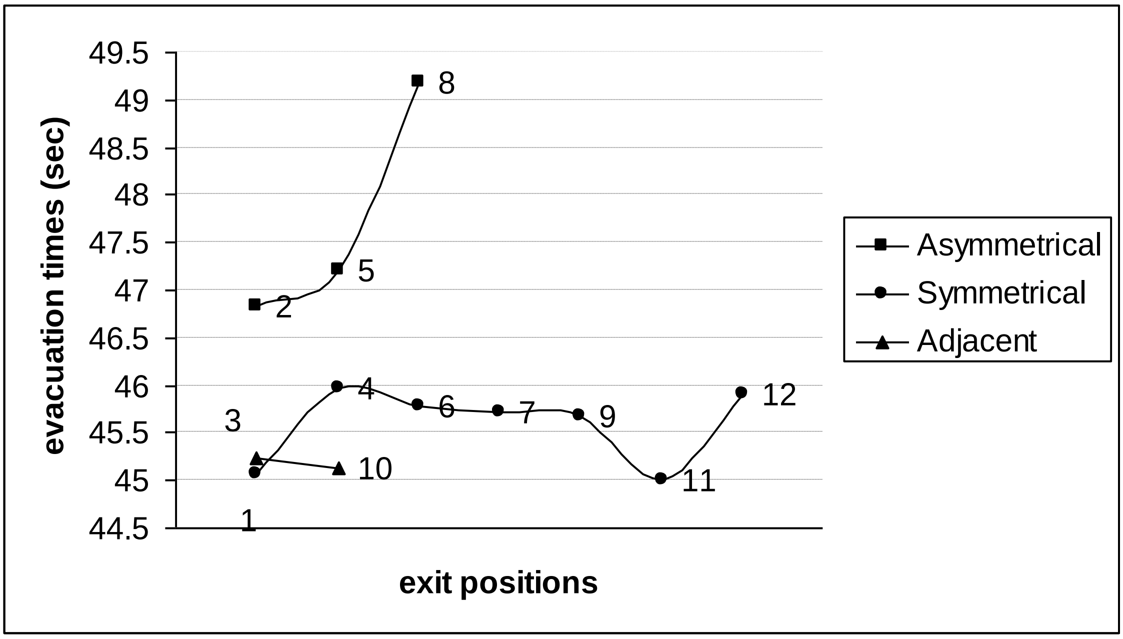


Figure 5.17: Average TET for the 12 cases with two 2.0 m exits clustered in groupings of exit configuration type

From Table 5.13 and Figure 5.17, it is possible to conclude that similar trends of behaviour are observed, as in the previous cases. Figure 5.17 summarizes this behaviour; showing that the best positions to place the two exits are when the exits are side by side. The worst positions are when the exits are located asymmetrically to each other along room perimeter.

5.4.4 The 2.5m wide exit cases

The Table 5.14 and Figure 5.18 present the results.

Table 5.14: Egress data for room with 200 occupants and two 2.5 m exits

Position	AV TET (sec)	AV CWT (sec)	AV Dist (m)	AV PET (sec)	Av Number using Exit 1	Av Number using Exit 2
1	37.5	11.2	7.78	18.47	97.56	102.43
2	38.3	11.38	7.35	18.33	94.02	105.97
3	36.9	11.39	7.42	18.42	100.7	99.29
4	36.9	11.58	7.07	18.36	99.6	100.39
5	37.9	12.52	5.9	18.39	96.61	103.38
6	37.3	12.32	6.02	18.31	98.78	101.21
7	37.3	12.21	6.05	18.23	99.83	100.16
8	39.3	13.03	5.18	18.37	90.18	109.81
9	37.2	11.96	6.29	18.12	99.46	100.53
10	37.4	11.19	7.7	18.4	102.84	97.15
11	37.5	12.36	5.8	18.2	96.29	103.7
12	37.2	13.27	4.51	18.12	99.37	100.62

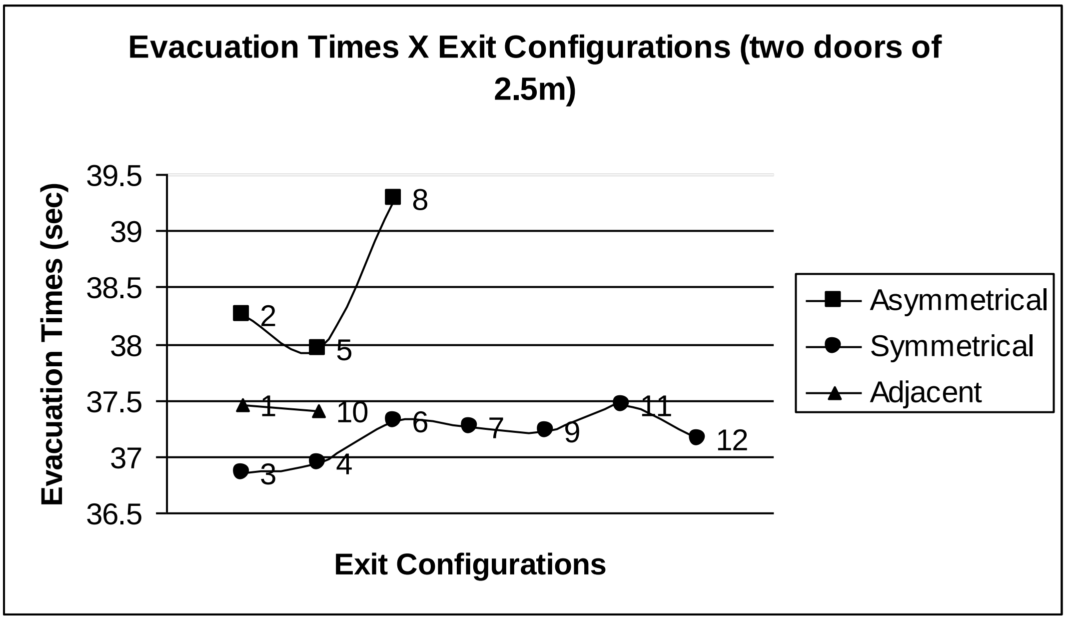


Figure 5.18: Average TET for the 12 cases with two 2.5 m exits clustered in groupings of exit configuration type

5.4.5 Discussion on the results

It is possible to observe, from the results, that the general behaviour trend for all cases is similar. It is clear that, using the widest exits, the impact of where the exits are located becomes less important. This can be explained by the fact that, when the exits are wider, the flow rate becomes the dominant factor for the evacuation time.

It is also apparent that the positions 1, 3, 4, 6, 7, 9, 10, 11, 12 and 13, which have a symmetrical arrangement, give shorter evacuation times than for positions 2, 5, and 8, which are asymmetrical.

Therefore, in terms of having two exits, regarding their locations in the room, the results clearly suggest that it is always better to locate the two exits in symmetrical positions. This lay-out, in terms of occupants' fire safety, provides a more efficient evacuation process, since the exits will be used in a more balanced way by the occupants. (It might be also important to mention that this statement is valid if the assumptions about the population and their responses are reasonable).

In summary, it was revealed that significant advantage can be derived from the strategic positioning of the two exits in a squared room. Analysis suggests that exits placed adjacent to each other produce the minimum evacuation times, exits positioned symmetrically around the perimeter of the room is next best while exits placed asymmetrically around the perimeter of the room produce the longest egress times. Therefore, the scenarios which give the optimal solutions for this problem (i.e., to locate the exits side by side) gives a range of 83.8 – 84.4 seconds. The second best scenarios (i.e., to locate the exits in symmetrical positions in relation to each other) gives a range of 85.2 – 87.0 seconds. And the worst case scenarios (i.e., to locate the exits in asymmetrical positions in relation to each other) gives a range of 92.7 – 98.2 seconds.

5.4 The rectangular room with 2 exits

5.4.1 The problem

The same question is also made for this case study: for a rectangular room with two exits of the same width, what are the best locations to put these two exits for producing minimum evacuation times?

The rectangular room analysed here had the dimensions of 10m x 40m (i.e., $400m^2$) and is populated by 1280 occupants (i.e., which gives a population density of $3.2p/m^2$), see Figure 5.19.

The two exits also have the same dimensions of 1.0m. All the other relevant parameters, such as travel speed, age, response times (RT = 0) etc. were treated the same as used in the evacuation modelling for the previous geometry (i.e., the square room). With this procedure, a fair comparison can then be made between the results. And regarding the simulations, all the scenarios for both geometries were performed 600 times and the occupants' locations were randomised for each run.

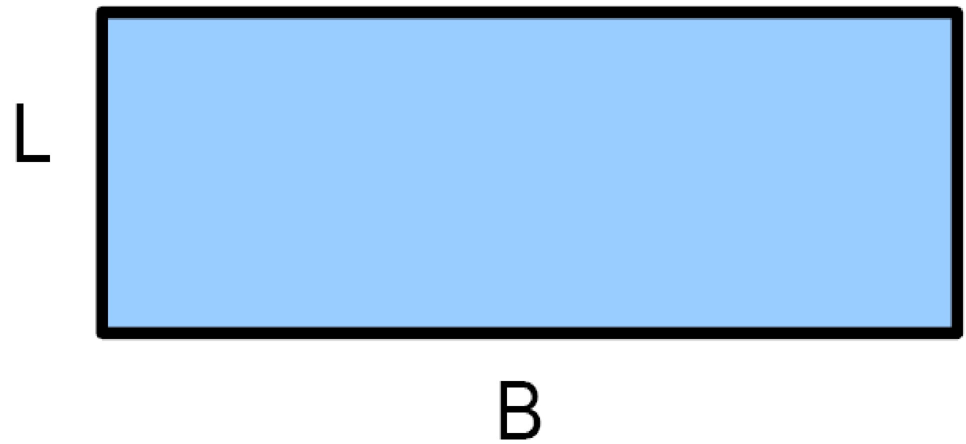


Figure 5.19: Rectangular room

In addition, a parameter of separation "f" is defined. As the name suggests, this parameter defines the separation between the two exits. (It is important to observe that for the rectangular room, the two exits are located in the same wall: the shortest wall L and for the square room it does not matter in which wall the two exits are located, once the walls have the same dimensions). For instance, when *f* is equal to 0 (*f*=0), this means that the two exits will be located side by side in the middle of the wall. As long as the value of f increases, the two exits start to become apart from each other until the maximum value of *f* (*f*=16) is reached. Therefore, when this value is reached, this means that the two exits will be located in the opposite corners from each other in the same wall. The values which the parameter f assumes represent the number of nodes. Furthermore, for *f* = 16 the two exits are separated 16 nodes (i.e., 8m) from each other. These values are also the same for the square room; i.e., *f* = 0 is the minimum value for *f* and *f* = 16 is the maximum value for *f*.

Unlike the square room results, it was observed that for enclosed environments where the relation between the perpendicular walls L and B is asymmetrical (i.e., for instance rectangular room where L is different from B, see Figure 5.19), for *f* =0 and *f* = 16 the evacuation times do not have the lowest values. Surprisingly, they actually have the highest values.

5.4.2 Results

A total of 11,400 simulations were performed for these cases.

The results of these evacuation modelling simulations are presented in the next paragraphs.

The Table 5.15 presents the evacuation times obtained for this case and Figure 5.20 shows the graph Evacuation Times X Values for *f*.

Table 5.15: Values for the rectangular room 10mx40m with 1280 occupants:

Values for *f*	**ET (sec)**
0	546.99
2	538.19
4	537.56
6	538.22
8	538.42
10	538.6
12	538.72
14	537.81
16	546.26

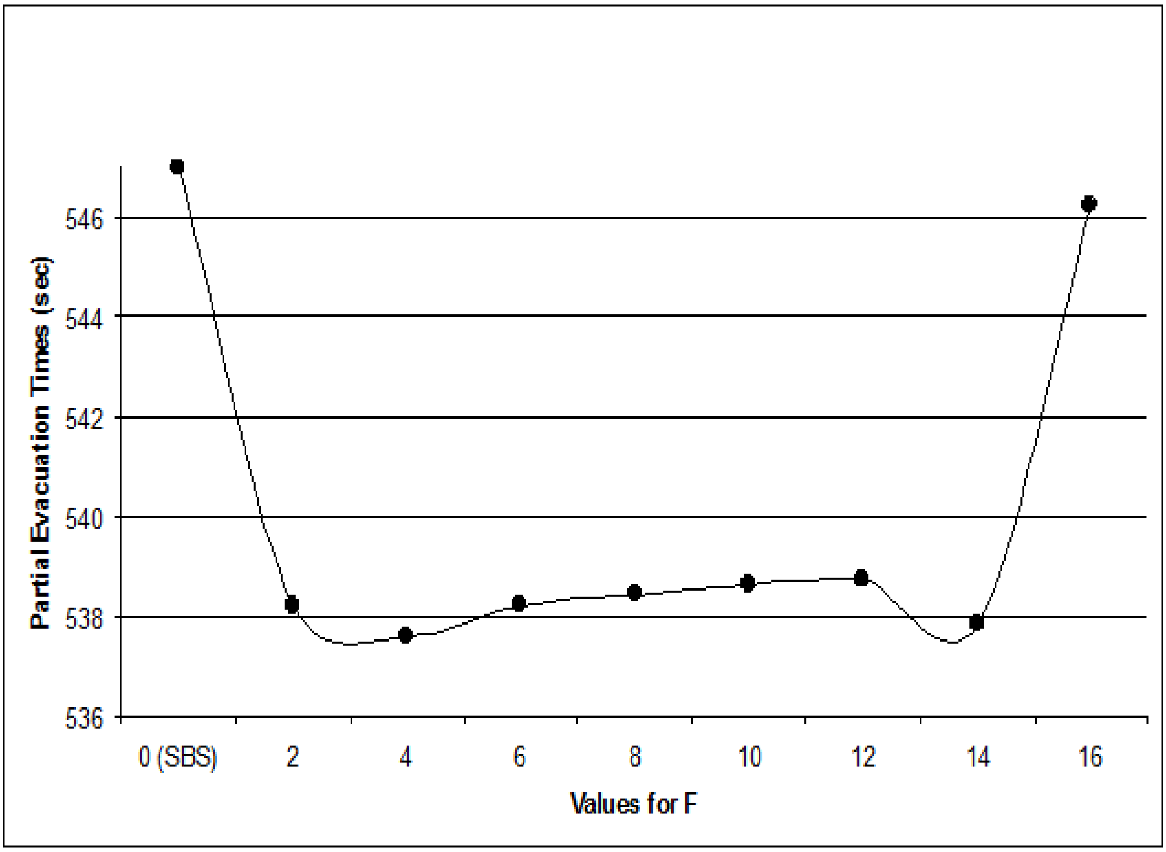


Figure 5.20: Evacuation Times X Values of *f* (for the rectangular room 10m x 40m with 1280 occupants)

Figure 5.20 shows that the trends in behaviour for these cases were very similar. When the exits are located in configurations such as for f = 0 and for f = 16 the evacuation times have the highest values.

The travel distances for all the scenarios for this case, do not seem to influence on the evacuation efficiency, since the results for the cases are all very similar. For instance, the values of the travel distances vary from 27.69m (minimum value) to 29.67m (maximum value).

For the situation where f = 0, despite the exits being side by side, the location seems to be not ideal for this type of enclosure's geometry (i.e., geometries where the perpendicular walls L and B do not have the same dimensions), since the occupants will be travelling to the same point. This generates congestions during the movement escape which impacts the evacuation performance in a negative way, hence reducing the evacuation times, see Figure 5.21.

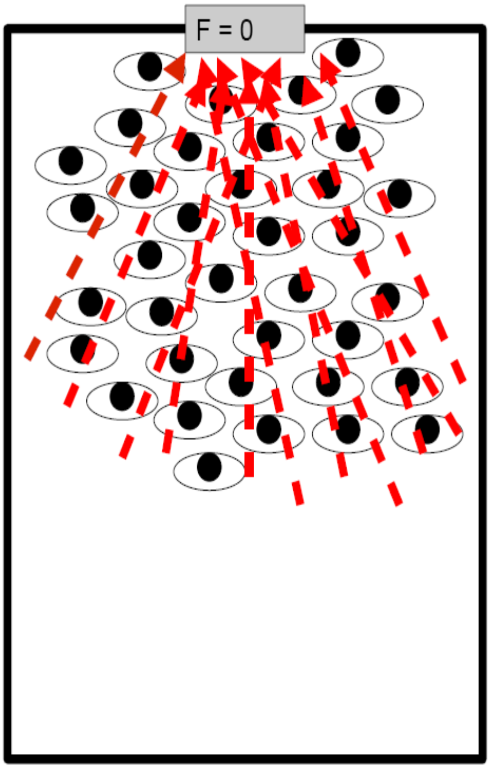


Figure 5.21: Congestions during the escape movement for $f = 0$

The dashed arrows represent the escape movements of the occupants. As it is possible to see, all the arrows are pointing to the same location (the exit) and during the evacuation process; some of these "arrows" will be crossing each other's paths. This is explained by the fact that the occupants have the capability of overtaking each other during the escape movement and/or change the decision of which path they should follow and so on. This will negatively impact the evacuation performance, since this generates more congestion during the escape movement and consequently the flow rate decreases and hence, the evacuation times increase.

Conversely, once the value of f starts to increase, it is clear that the evacuation time decreases substantially. This can be observed by the values of the evacuation times for the intermediary values of f between 0 and 16, as shown in table 9 and figure 7. This can be explained by the fact that, as soon as the exits separate from each other, there will be an improvement on the flow rates of occupants' movement. In fact, it seems that the escape movement becomes “less chaotic”, since the occupants separate into two main groups according to their exit preference. In other words, there will be less congestion during the escape movement and this improves the evacuation efficiency. This phenomenon can be called a “bifurcation effect” and it can be seen for enclosure's geometries where the perpendicular walls L and B are asymmetrical (i.e., they do not have the same dimensions, just like the rectangular room in this study). In summary, the occupants, for these situations, will not need to rush into the same point; therefore there is less competitive behaviour and the movement becomes more coordinated and less chaotic. It seems that for a rectangular room, the parameter f plays a similar function that the barriers played in the square room scenarios.

Nevertheless, it is relevant to mention that this phenomenon is not observed when f assumes the maximum value (i.e., f = 16). In fact, when the exits are located in the opposite corner to each other in the same wall, the evacuation time increases substantially, just like when f = 0. The evacuation movement, through the room, is now very fast because there is less congestion due to the separation of the population between the two target exits, however this introduces greater queuing (and hence increases congestion) close to the exits, which adversely affects the evacuation efficiency.

These conclusions are supported by the values obtained for the cumulative waiting times, CWT, which are shown in table 5.16.

Table 5.16: Values for the CWT for the rectangular room 10mx40m with 1280 occupants:

Values for f	ET (sec)	CWT (sec)
0	546.99	158.76
2	538.19	141.2
4	537.56	141.8
6	538.22	140.23
8	538.42	139.87
10	538.6	142.5
12	538.72	140.45
14	537.81	141.56
16	546.26	157.65

In summary, it seems that, for asymmetrical enclosure's geometry like the rectangular room (i.e., where the perpendicular walls L and B do not have the same dimensions), unlike the square room cases, the time spent towards the exits area – T_2) (i.e., the queuing time at the doors) does not represent such an important factor (for the relationship between the exits locations and the evacuation efficiency) compared to the time spent during the escape movement (T_1), see equation 2 shown previously.

In fact, it is the opposite, the time spent during the movement (T_1) now becomes a more important factor on the relationship between the exits locations and the evacuation efficiency than the time spent towards the exits (T_2); apart from the case where f = 16, where the occupants seem to spend more time in the congestion areas towards the exits.

In order to better understand these results, additional square room scenarios were constructed and analysed. These new scenarios were based on our previous studies. The dimensions were 10m x 10m, but the population now was defined as 320 occupants in order to create the same population density used for the rectangular room (i.e., 3.2p/m^2)..

The new cases used the same parameter for the separation f.

It was observed previously that for enclosed environments, where the relation between the perpendicular walls L and B is symmetrical (i.e., square room where L= B), to have exit(s) located in the corner will produce shorter evacuation times. This was well discussed and validated. This effect can be called as the "corner effect". And as mentioned, this effect seems to be valid for symmetrical geometries as it will be discussed further later on.

It is important to observe that this effect was found for the square room (10m x 10m) with one single exit (and also with two exits located side by side in the corner) and populated by 200 occupants, which had a population density of 2p/m^2. However, this same effect was also found to be true when the population density is increased to 3.2p/m^2 (the value adopted for this new study) for both square and rectangular rooms. (this issue, the *"corner effect"* will not be discussed here on this section, since it is already proved and understood. It might be also relevant to mention that for f = 0, both situations were performed considering the exits located side by side and also using a single big exit. The results have also shown that for f = 0 when the exits are located side by side, there is a small advantage in relation with the other scenario, a single big exit, since the evacuation times were shorter for the previous case. This was also well discussed and understood in the previous studies which looked at the advantage of using barriers in the middle of exits. Therefore, this issue will not be discussed here in this section. Furthermore, every time that $f = 0$ is mentioned in this study, it means the two exits are side by side).

A total of 20,400 simulations were performed for the analysis of the square room. The results of these evacuation modelling simulations are presented in the next paragraphs. Table 5.17 and Figure 5.22 present the results for the square room with 200 occupants and Table 5.18 and Figure 5.23 present the results for the square room with 320 occupants.

Table 5.17: Values for the square room 10mx10m with 200 occupants:

Values for *f*	**ET (sec)**
0	84.4
2	86
4	87.8
6	88.1
8	88
10	87.95
12	87.59
14	87.23
16	87

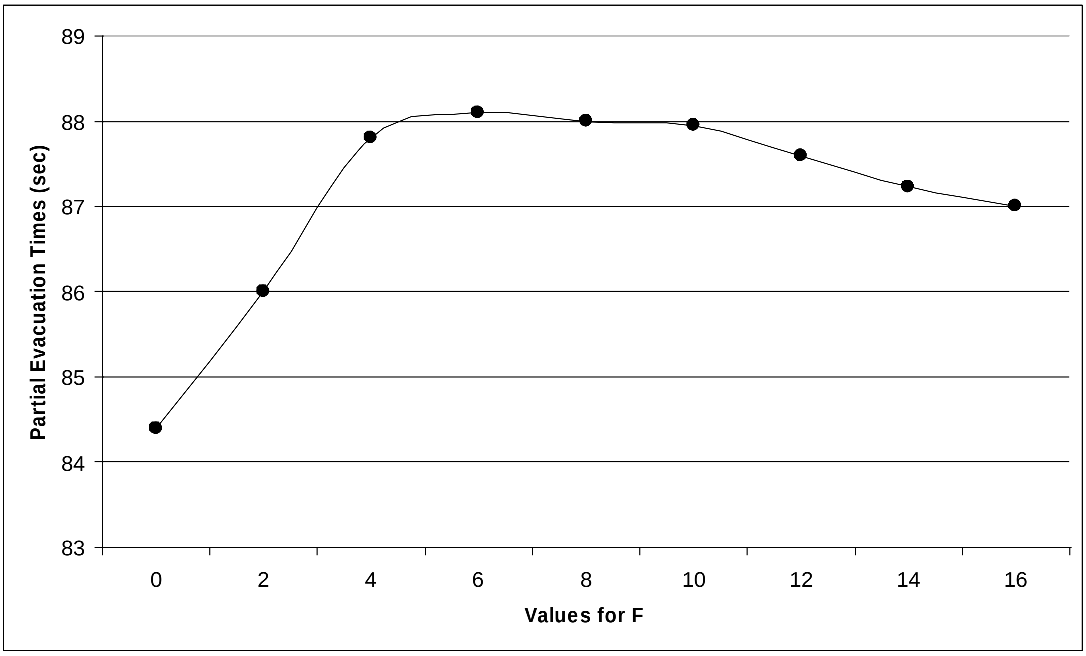


Figure 5.22: Evacuation Times X Values of *f* (for the square room 10m x 10m with 200 occupants)

Table 5.18: Values for the square room 10mx10m with 320 occupants:

Values for F	**ET (sec)**
0	129.7
2	135.8
4	137
6	137.8
8	138.92
10	138.42
12	138.25
14	137.75
16	137.24

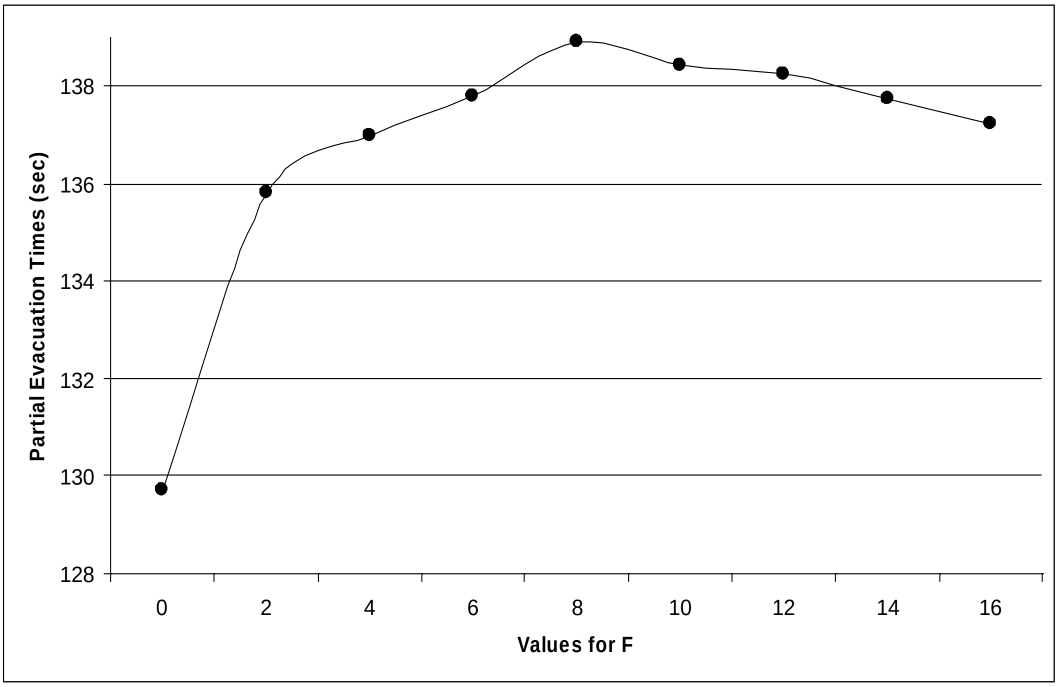


Figure 5.23: Evacuation Times X Values of *f* (for the square room 10m x 10m with 300 occupants)

The figures show that the trends of behaviour for these cases were very similar. When the exits are located in configurations such as for f = 0 and for f = 16 the evacuation times are at their lowest values.

For the situation where f = 0, it is common knowledge that the wider the exit is, so the higher the flow rate will become and consequently the evacuation time will be reduced, as equation 1 shows:

$$ET \sim \frac{1}{EW} \qquad \text{Equation 5.1}$$

where:

ET – Evacuation Time;

EW – Exit Width.

Also, for f = 0, the travel distance is at its shortest value (i.e., 5.44m). Therefore, these two factors combine to make the evacuation time decrease to the lowest value.

And when f = 16, the evacuation times are also reduced. This can be explained by the "corner effect" as previously discussed in our previous studies for the square room with one exit cases as well as the two exits cases.

A scenario where the two exits are located in the opposite corner of each other in the same wall reduces the evacuation times, because there is less conflict between the occupants and therefore, the congestion is reduced, the flow rate is improved and consequently the evacuation time decreases.

Nevertheless, for the intermediary values of *f*, a different behaviour is seen for the evacuation times curve.

When the values of *f* start to increase, it is possible to see that for *f* = 4, *f* = 6 and *f* =8, the values of the evacuation time are proportional to this increase. (For *f* = 2, the evacuation times also increase, but show only a small difference. And in fact, for *f* = 2, it can said that it is still an advantage to have the exits located in this configuration, because the "barrier effect" is still occurring). The evacuation times start to increase gradually showing that for these values of *f* , the evacuation times will not be as good as for *f* = 0. This can be explained by the fact that, as the exits are separated further apart, the congestion between the occupants is now divided between two areas. And since the exits are still close to each other for these cases, these "congestion areas" will impact each other. In other words, the conflict between the occupants is more chaotic, because the congestion area for one exit will be impacted by the congestion area of the other exit and vice-versa, see Figure 5.24.

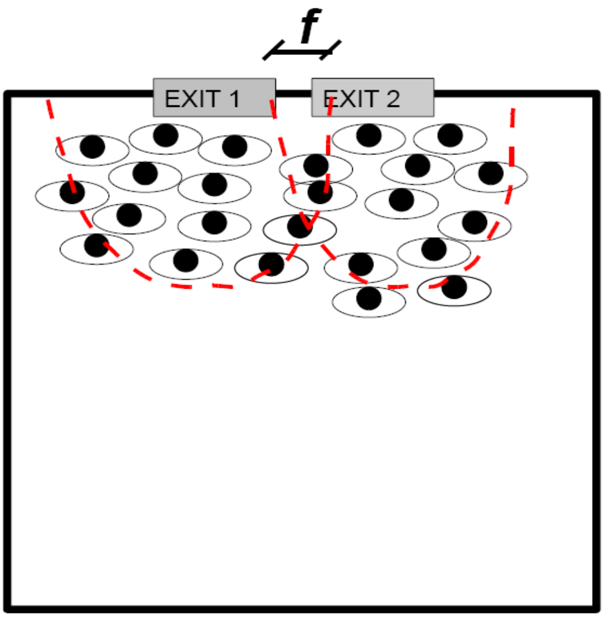


Figure 5.24: Congestion Areas interaction between each other

The dashed lines represent the congestion areas for each exit. It is possible to see that, given the short distance between the two exits defined by the value of *f*, there is a partial overlapping of the two congestion areas. As consequence, there will be conflict between some of the occupants attempting to escape using exit 1 and some of the occupants attempting to escape using exit 2. And in fact, because of the proximity of these exits, some occupants who are in the congestion area of exit 1 might change his decision to use the exit 2 instead and vice-versa. This interaction between the two areas causes more conflict between the occupants and consequently, the flow rate is reduced and as a result, the evacuation time increases.

On the other hand, once the value of *f* continues to increase, there is an improvement on the evacuation performance. This can be seen from $f = 10$, when the evacuation time starts to decrease. In fact, for $f = 10$, $f = 12$, $f = 14$ and until *f* reaches the value of 16, a decrease is observed for the values of the evacuation times.

This can be explained by the fact that, since the values of *f* increase, the two exits start to become more distant from each other; and with this, the congestion areas are also more distant from each other. Furthermore, the conflict between the occupants is reduced, since there will be much less interaction between them. As a consequence of this, the flow rates are improved and the evacuation times are reduced.

In summary, the tables 5.20 and 5.21 present the values of the average cumulative waiting time, CWT.

It is clear to see that the CWT is growing with increasing values of *f*, from $f = 0$ to $f = 8$ until it then starts decreasing as the values of *f*, increase from $f = 10$ to $f = 16$. The CWT represents the time that the occupant spend waiting during the evacuation process and this time is clearly influenced by the time spent near to the exits areas.

Table 5.20: Values CWT for the square room 10mx10m with 200 occupants:

Values for *f*	ET (sec)	CWT (sec)
0	84.4	34.4
2	86	36.4
4	87.8	36.0
6	88.1	36.0
8	88	37.0
10	87.95	36.0
12	87.59	36.0
14	87.23	36.2
16	87	35.5

Table 5.21: Values CWT for the square room 10mx10m with 320 occupants:

Values for *f*	ET (sec)	CWT (sec)
0	129.7	30.47
2	135.8	37.94
4	137	38.24
6	137.8	39.30
8	138.92	40.2
10	138.42	38.36
12	138.25	38.5
14	137.75	38.23
16	137.24	38.21

These logical relations support the idea that the closer the exits are to each other, the closer the congestion areas will be to each other, and so more interaction will take place between the occupants. With increased interaction, there will be more conflicts between the occupants, which generate more congestion, hence reducing the flow rate, increasing the cumulative waiting time and increasing the evacuation time.

Conversely, when the exits become more distant from each other, the opposite behaviour is observed with the result that the evacuation times decrease.

In summary, it seems that for symmetrical enclosure, like the square room (i.e., where the perpendicular walls L and B have the same dimensions), the time spent moving towards the exit areas (T_2), represents a more important factor for the relationship between the exits locations and the evacuation efficiency, than the time spent during the escape movement (T_1), see equation below:

$$ET = T_1 + T_2 \qquad \text{Equation 5.2}$$

where:

ET – Evacuation Time;

T_1 – Time spent during the escape movement;

T_2 – Time spent towards the exits.

Comparing the results obtained from the rectangular room and the square room, it is possible to see that even having the same population density, different behaviour was observed for similar scenarios (i.e., same value for *f*). In the next paragraphs, this issue is discussed with more details.

5.5 Comparison between the results obtained for the square room with two exits and the rectangular room with two exits

For the rectangular room, the results have shown that for $f = 0$ and $f = 16$, the worst evacuation times are obtained. Nevertheless, when *f* assumes values between 0 and 16, the evacuation times are lower than these previously mentioned cases. In fact, for the intermediate values between 0 and 16, very similar evacuation times are produced. This trend of behaviour can be summarized in a general graph as Figure 5.25 presents.

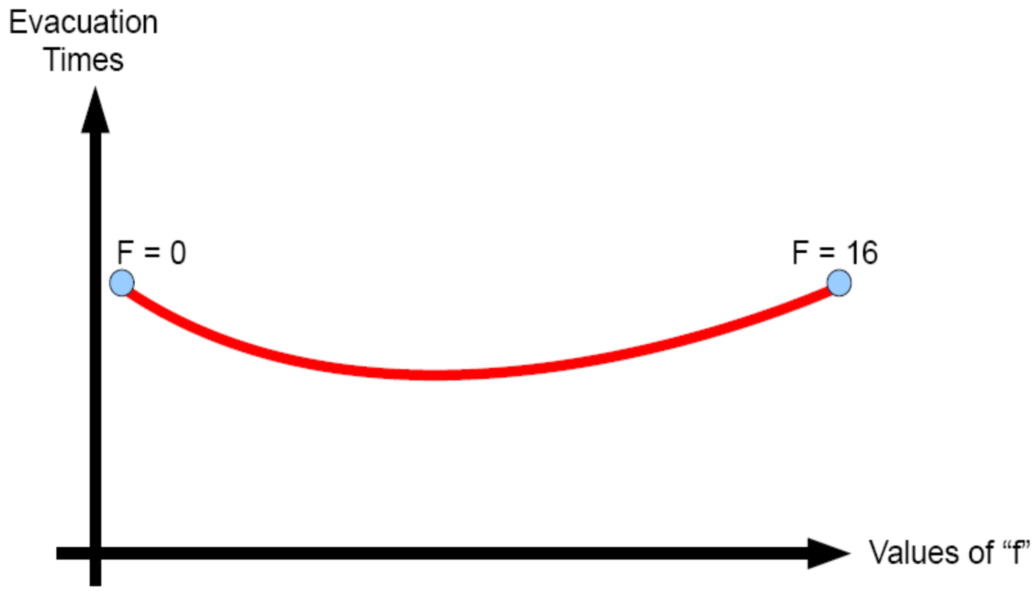


Figure 5.25: Graph showing the trend behaviour of the evacuation times values in relation with the separation parameter F values for the rectangular room

When these results are compared with previous results, such as those produced for the square room (with two exits with the same dimensions, i.e. 1.0m width, and same population density, i.e. 3.2p/m^2, as used for the rectangular room), it is clear to observe that the results are different, see Figure 5.26. This might indicate that, associated with the exits locations, the shape of the room is playing an important rule on the evacuation efficiency.

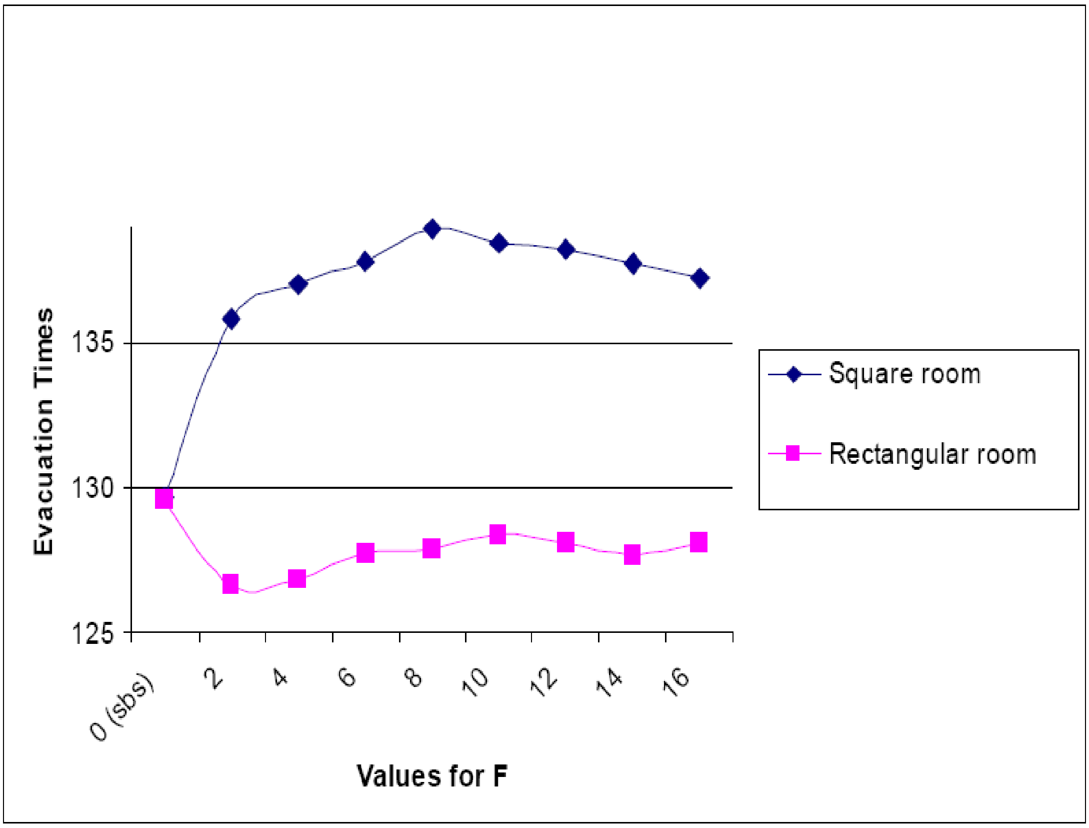


Figure 5.26: Graphs comparing the evacuation times for the square room and for the rectangular room

In fact, as Figure 5.26 shows, for the square room cases, the trend of same graph assumes very different behaviour, as Figure 5.27 shows.

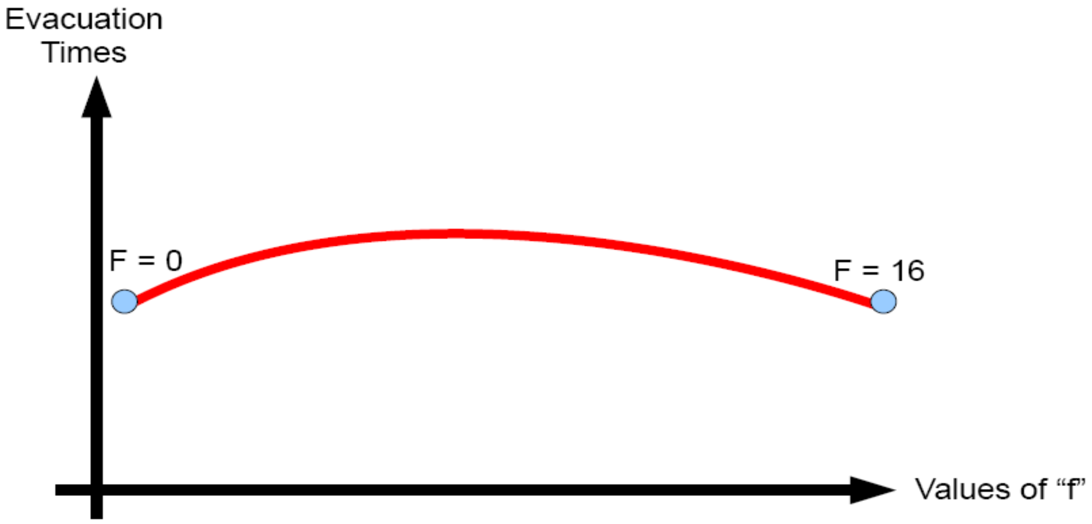


Figure 5.27: Graph showing the trend behaviour of the evacuation times values in relation with the separation parameter F values for the square room

In reality, it seems that the relation between the perpendicular walls L and B within the enclosure's geometry's boundaries does have an impact on the relationship between the exits locations and the evacuation times. For instance, as shown previously, considering the case where the relation between the perpendicular walls is asymmetrical (i.e., rectangular room where L is different from B), for the same values of *f*, the evacuation times are different for the case where the relationship between the perpendicular walls is symmetrical (i.e., square room where L is equal to B).

Therefore, it seems that the shape of the room (i.e., the enclosure's geometry) does have an impact on the evacuation efficiency. This might be explained by the fact that the enclosure's geometry is also associated with key factors which have direct impact on the evacuation efficiency, such as travel distance and population density. Furthermore, it can be said that the enclosure's geometry has an indirect impact on the evacuation efficiency, because it is related, for instance, with the travel distance and the population density.

For this reason, the locations of the exits, along the wall perimeter of any room, do impact the evacuation performance and this impact varies according to the enclosure's geometry. As an example, if an exit, that is located in the corner of a room with a specific shape, generates the lowest evacuation times, this will not necessarily be true for another room with a different shape.

As mentioned previously, both the geometries (square and rectangular rooms) had the same population density, i.e., 3.2p/m^2. For instance, the square room had 10m x 10m as dimensions (i.e., 100m^2) and 320 occupants. And the rectangular room had 15m x 12m as dimensions (i.e., 180m^2) and 576 occupants. These settings allowed a fair comparison to be made between the results in order to check if the geometry of the enclosure had an impact on the relationship between the exits locations and the evacuation efficiency.

In theory, for similar scenarios in which the population density is the same and the lay-out configuration of the exits along the walls is the same (just like the square and rectangular rooms here discussed), the evacuation performance expressed by the evacuation time would be the same or at least similar. Nevertheless, according to what was shown in the previous section, it is clear that the enclosure's geometry does have an impact on this relationship between the exits locations along the wall perimeter and the evacuation efficiency.

The question which now arises is: why does this difference exist?
Therefore, in this section, the possible answers for this question are presented and discussed.

From the results, it was observed that what is an advantage in terms of exits locations for a specific enclosure's geometry is not necessarily true for another different enclosure's geometry. For instance, for the rectangular room when f = 0, the evacuation time assumes the highest value. This starts to change when the value for f increases, making the values of the evacuation times decrease until f = 16, when the evacuation time starts increasing again. While, for the square room, the opposite trend is observed, as the evacuation time assumes the lowest value. When f = 0, the evacuation time assumes the lowest value. And this starts to change when the value of f increases, making the values of the evacuation times, increase until f = 16, when the evacuation time starts decreasing again.

Possible explanations for this might be found in the relationship between the exits locations and the travel distance and also other physical phenomena (i.e., created by the relation occupants-structure) which happen during the evacuation process, such as the 'faster is slower' effect **[41]** and the congestions during the escape movement towards the exits and the congestions near to the exits areas.

With these two case studies, namely the square room and the rectangular room, it becomes clear to conclude that the relationship between exits locations and evacuation efficiency is influenced by the shape of the room (i.e., the enclosure's geometry).

5.6 Concluding Comments

In this chapter, the optimal positioning of exits around the perimeter of a square room was explored, in order to minimise the evacuation times. The evacuation simulations were conducted assuming ideal conditions of zero response times and population behaviour such that occupants would move to their nearest exits. Both assumptions are made to simplify the analysis and to isolate issues associated with exit location.

For this purpose, a squared room with one single exit was initially considered. Firstly, the best solutions were found through data analysis (i.e., using a "brute force method"). The analysis revealed that, for square rooms with a single exit, there is a slight advantage in positioning the exit in the corner of the room. Placing the exit in any other location will produce longer evacuation times and furthermore, if the exit is outside of the corner region, the evacuation time will not be dependent on the exit location. The advantage offered by placing the exit in a room corner diminishes as the size of the exit is increased while keeping the population size constant; or decreasing the population size while keeping the exit width constant.

It is important to note that the results presented here are based only on simulations. However, the findings can be verified based on previous evacuation simulation experimentation as mentioned before. Furthermore, the simplifying assumptions have ignored those factors arising from complex human behaviour and do not take into account any of the constraints imposed by building regulations. However, it is believed that the findings have relevance to practical fire engineering and may assist engineers to optimally position exits even within a constraining regulatory environment.

In this chapter, a square room with two exits was also analyzed. With two exits a much larger range of exit positions are available for consideration. This investigation revealed that significant advantage can be derived from the strategic positioning of the exits. Analysis suggests that exits placed adjacent to each other produce the minimum evacuation times, exits positioned symmetrically around the perimeter of the room is next best; while exits placed asymmetrically around the perimeter of the room produce the longest egress times. Furthermore, adjacent exits located in the corner of the room produce the best egress times and separate but adjacent exits produce better times than a single exit of width equal to the two adjacent exits. The difference between the best and worst configurations is more significant in the case with two exits than in the case with a single exit. The advantage offered by the adjacent exits decreases significantly as the exit size increases or the size of the population serviced by the exit decreases. To a lesser extent, the advantage offered by the symmetrically placed exits over the asymmetrically placed exits also decreases as the exit size increases or the size of the population serviced by the exit decreases.

And finally, a third case study was analysed. This last case was based on a rectangular room with two exits.

The results revealed that the relationship between exit locations and evacuation efficiency is influenced by the shape of the room (i.e., the enclosure's geometry). This conclusion is based on the fact that, for the same exit locations, the trends of evacuation efficiency were completely different for the square room when compared with the rectangular room.

Previous studies, performed during the preparation of some of the case studies for this thesis, where several other evacuation cases were modelled with different shapes (such as circular, triangular, pentagonal, hexagonal and heptagonal) have also shown that the enclosure's geometry seems to have a significant influence on the evacuation performance.

It is expected that the present case studies will help to explain some of these issues. The significance of the findings is such that more research should be directed towards gaining a better understanding of these issues.

This issue is also discussed with more details in the next chapters of this thesis, where more case studies are presented.

In the next chapter, the study cases that were presented in this chapter are solved using the numerical optimisation techniques.

6. Identifying Optimal Solutions to Unconstrained and Constrained Problems Using Numerical Optimisation Techniques

6.1 Introduction

This chapter presents the same problem presented in the previous involving a square room with one and two exits, in which the best location for the exits is found to minimise the evacuation times. In other terms, the optimal location for positioning the exits is determined by finding the minimum evacuation time.

However, now instead of the solutions being found through brute force method; they are found applying the numerical optimisation techniques.

For the scenario with one single exit, the problem is solved as an unconstrained problem. For the scenario with two exits, the problem is solved as both types of problem: unconstrained and constrained.

In the next sections these problems are presented and discussed.

6.2 Problem 1 – Square room with one single exit

The problem to be investigated can be stated very simply as follows: for a room of given size, containing an arbitrarily large population, is there an optimal location for the exit that will minimise the evacuation times?

For simplicity, this first hypothetical case is limited to a simple geometry, namely: a squared room containing one exit. The square room used in the analysis has an area of 100 m^2 and the exit width varies from 1.0m to 2.5m. The population of the room consists of 200 people producing a population density of 2 people/m^2. This means that 50% of the area is populated, which gives 50% of freedom for the occupants' movement during the evacuation process.

For answering the question, evacuation process is analysed under the optimisation theory principles. The exit locations, for the case of 1 single exit, are measured in terms of the distance from the left edge of the exit to the corner of the wall.

For this first case analysed, the problem is an unconstrained problem, because no constraints were considered. The DoE techniques were used for picking up the design points. For all the scenarios considered in this problem, the RSM based on polynomial high order or quadratic smooth were used, once they proved to be appropriate based on the general stepwise regression.

And finally, two types of numerical optimisation techniques were applied: a gradient-based technique and a non gradient-based technique.

It is important to note that the optimal solution is understood as being the global minima region of the objective function (i.e., the evacuation time) in the response surface. However, given that to find precisely the global minima region is not an easy task, if the local minima region near to the global minima region is found, it is assumed that this is also a good result.

The details on how the problem was defined, the nature of the design variables, which DoE technique was used, which RSM was used and which numerical optimisation technique was applied are further explained in the following sections of this chapter. The Table 6.1 brings the summary of this study case.

Table 6.1: Summary of the problem

Type of Problem	DoE Techniques used	RSM used	Numerical Optimisation Techniques used
Unconstrained	– CCD – Central Composite Design – Latin-Hypercube	● Polynomial High Order ● Smooth Quadratic	– Gradient-based technique (Fletcher-Reeves) – Non gradient-based technique (PSO – Particle Swarm Optimisation)

6.2.1 The scenarios

6.2.1a The simulations

The approach adopted involves the use of computer simulation software to simulate the evacuation process for each relevant exit configuration.

The same considerations and assumptions explained in the previous chapter are here adopted. For instance, the simulations were repeated a total of 600 times for each scenario. Thus all the results presented here represent an average over 600 simulations. At the start of each simulation, the starting location of the population was also randomised. This ensured that the population was distributed throughout the confines of the geometry with little bias resulting from population starting position contributing significantly to the overall results.

Given the simplicity of the geometry (i.e., a regular and symmetrical shape – squared room), the domain of the design variable is:

$$0 \leq EP \leq \text{ the middle of the wall}$$

In the next section, the results obtained for this problem are presented and discussed.

6.2.2 Results obtained through the use of numerical optimisation techniques

It was found in previously, through data analysis, that for the single exit case, the best position to locate the exit was in the corner (i.e., EL = 0), which produces an average evacuation time of 165.4 sec. This conclusion was made based on 16,800 simulations.

Now, considering the same case, one single exit, in which it is intended to find the optimal solution through the application of numerical optimisation techniques. For this case, there are two approaches:

- Finding the optimal solution based on these 416,800 simulations;
- Finding the optimal solution based on the design points suggested by the DoE techniques.

All the functions developed from the RSM's are described in the appendix B of this thesis. The numerical approximation method used for all the cases was the General Stepwise Regression. All these problems, as mentioned in the beginning of this chapter, are unconstrained problems.

6.2.2a The 2D analysis

The first analysis is a 2D analysis, i.e., a function with one variable. And the optimisation analysis, in the first investigation will be based on all the data produced from the evacuation modelling simulations; and in the second investigation, it will be based on the design points suggested by the DoE techniques. The statement for this problem for both approaches is:

$$\text{Minimize: } ET = f\,(EL)$$

Where:

ET – the evacuation time;

EL – the exit locations.

The exit location can be placed anywhere from the corner to the middle of the wall, as explained before.

Table 6.2 presents the results from this optimisation analysis.

Table 6.2: Optimal solution for the squared room with one exit based on the 6,600 simulations (2D analysis)

Optimum Solution (0 ; 165.4)	**Numerical Optimisation Techniques**	
Response Surface Models	Fletcher-Reeves	PSO – Particle Swarm Optimisation
Polynomial of 4^{th} order (R^2 = 0.95)	(0;165.4)	(0;165.4)
Polynomial of 5^{th} order (R^2 = 0.99)	(0;165.4)	(0;165.4)
Polynomial of 6^{th} order (R^2 = 0.99)	(0;165.4)	(0;165.4)

It is possible to see clearly that for the two numerical optimisation techniques, Fletcher-Reeves and PSO, and using 3 different types of RSM's, the results are exactly the same for the optimal solution found previously. The coordinate (0; 165.4) indicates the global minima region; where the Exit Location (i.e., exit position) is in the corner (EL = 0) producing an evacuation time (ET) of 165.4 sec. The Figure 6.2 shows this region.

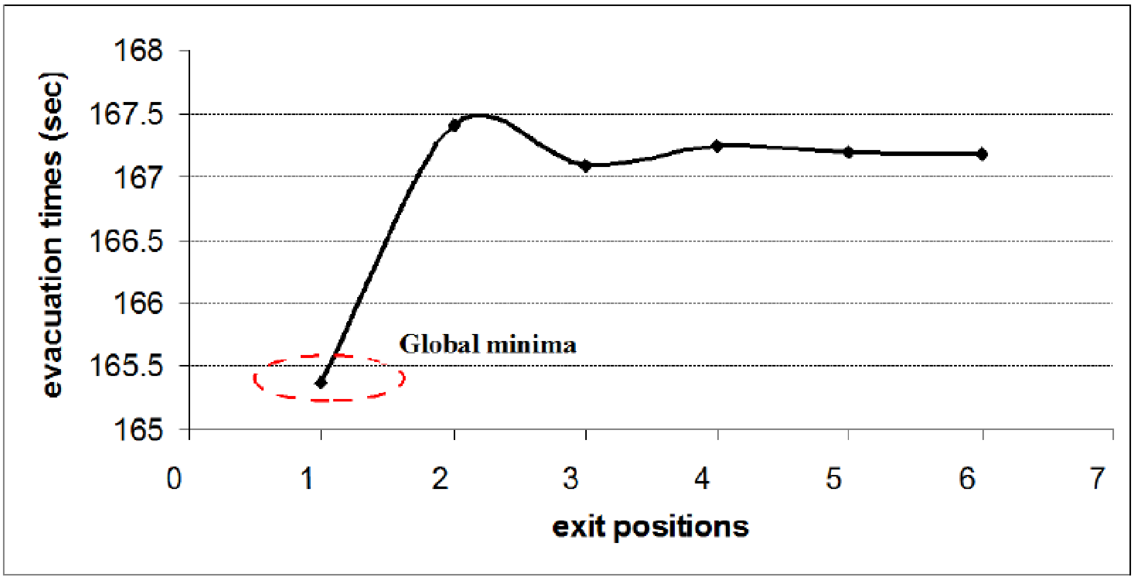


Figure 6.2: Optimal solution for a square room with single exit case

For this case, when using 2 different types of DoE, using one specific RSM and for the two numerical optimisation techniques, it was found the same optimal solution. It is relevant to observe that this solution was found, even using less design points and consequently running less simulations than when all the possible design points were used based on the data obtained from the previous evacuation simulations. Using the first DoE technique, Latin hypercube, 6 design points were used, which implicates in 3,600 simulations. And using the second DoE technique, the CCD, just 3 design points were used, which implicates in 1,800 simulations. In summary, for this case, using just 3 design points, the same solution was found when using 7 design points. Thus, it was avoided to perform 2,400 simulations, saving time. The table 6.3 presents these results.

Table 6.3: Optimal solution for the squared room with one exit based on the design points suggested by the DoE techniques (2D analysis)

Optimum = (0; 165.4) **Response Surface Model** Full-quadratic (R^2 = 0.99)	**Numerical Optimisation Technique**	
DoE Techniques	Fletcher-Reeves	PSO – Particle Swarm Optimisation
Latin Hypercube – 6 points	(0; 165.4)	(0; 165.4)
Central Composite Design (CCD) - 3 points	(0; 165.4)	(0; 165.4)

6.2.2b The 3D analysis

This second analysis is a 3D analysis, i.e., a function with 2 variables. And the optimisation analysis, in the first investigation will be based on all the data produced from the evacuation modelling simulations; and in the second investigation, it will be based on the design points suggested by the DoE techniques. The statement for this problem for both approaches is:

Minimize: ET = f (EL; EW)

Where:

ET – the evacuation time;

EL – the exit locations;

EW - the exit widths.

The exit location can be placed anywhere from the corner to the middle of the wall, as explained before.

The exit width varies from 1.0m to 2.5m.

Table 6.4 presents the results from this optimisation analysis.

It is also relevant to mention that, based on the "brute force method", it was found that the optimal solution would be: (0; 2.5; 70.4). In other words, when the exit is located in the corner (i.e., EL = 0) and with a width of 2.5m, the evacuation time will be 70.4 seconds.

Table 6.4: Optimal solution for the squared room with one exit based on simulations (3D analysis)

Optimum = (0; 2.5; 70.4) Response Surface Model	**Numerical Optimisation Technique**	
Response Surface Model	Fletcher-Reeves	PSO – Particle Swarm Optimisation
Polynomial of higher order (R^2 = 0.99)	(0; 2.48 ; 70.9)	(0; 2.48; 70.9)

From the results, it can be seen that not the global minima region was found. Nevertheless, a good solution was found, because the coordinates also suggested that the best place to locate the exit is in the corner (EL=0); and regarding to the exit width, a close value to the upper value was found (i.e., 2.48m). This will produce an evacuation time of 70.9 seconds which is very close to the optimal solution, 70.4 seconds.

When doing the same analysis, using the same RSM, but using different DoE techniques, similar results were found and they are also satisfactory results. Table 6.5 presents these results and Figure 6.3 shows the response surface built for this 3D analysis.

Table 6.5: Optimal solution for the squared room with one exit based on the design points suggested by the DoE techniques (3D analysis)

Optimum = (0; 2.5; 70.4) **Response Surface Model** Full-quadratic (R^2 = 0.99)	**Numerical Optimisation Technique**	
DoE Techniques	Fletcher-Reeves	PSO – Particle Swarm Optimisation
Central Composite Design (CCD) - 9 points	(0.5; 2.5; 71.42)	(0.5; 2.5; 71.42)
Central Composite Design (CCD) - 9 points	(0; 2.38; 70.18)	(0; 2.38; 70.18)
Latin Hypercube – 6 points	(0.4; 2.5; 70.10)	(0.4; 2.5; 71.57)

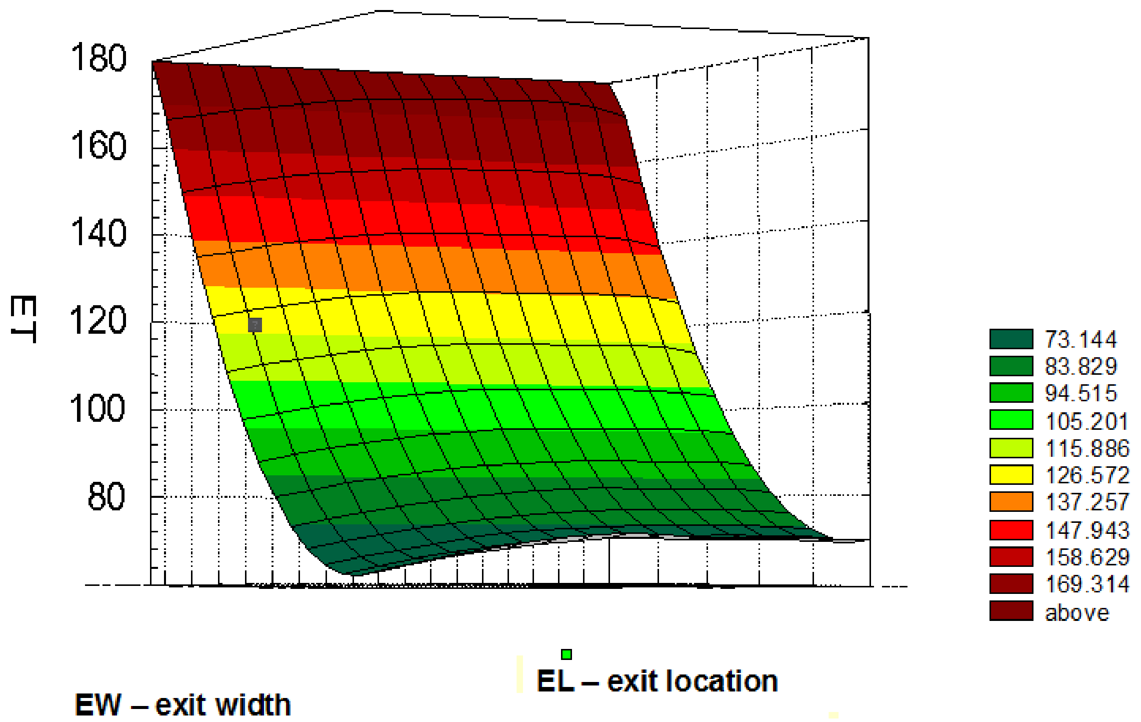


Figure 6.3: Response Surface for the 1 single exit case (3D analysis)

In the next section the same geometry is investigated considering two exits.

6.3 Problem 2 – Square room with two exits

The problem to be investigated is similar to the problem showed in the previous chapter. However, here instead of having a single exit, the scenarios have two exits. Therefore, the question to be answered now is: for a simple square room, containing an arbitrarily large population is there an optimal location for the exits that will minimise the evacuation times? In other terms, what is the best relative distance between these two exits for minimising the evacuation times?

This second hypothetical case is exactly the same: a simple geometry, a squared room. with an area of 100 m^2 and the exits width vary from 1.0m to 2.5m. The population of the room consists of 200 people producing a population density of 2 people/m^2. This means that 50% of the area is populated, which gives 50% of freedom for the occupants' movement during the evacuation process.

In the same way that was done for the case with one single exit, the problem with two exits were solved through two approaches: firstly, based on the analysis from the data obtained from the evacuation simulations and exit; and secondly, based on the use of numerical optimisation techniques.

For this case with two exits, the problem was solved as an unconstrained problem and also as a constrained problem.

Therefore, for the unconstrained problem, 13 key-positions were defined. Each 13 key-positions is associated with a specific coordinate, which represents the positions for the two exits. For the constrained problem, the random DoE technique was used for picking up the design points. For all the scenarios considered in this problem, the RSM based on polynomial high order or quadratic smooth were used, once they proved to be appropriate based on the general stepwise regression.

And finally, two types of numerical optimisation techniques were applied: a gradient-based technique and a non gradient-based technique. The table 6.6 presents this summary.

Table 6.6: Summary of the problem (squared room with two exits)

Type of problem	DoE techniques used	RSM used	Numerical Optimisation Techniques used
Unconstrained	None (13 key-positions defined)	Smooth Quadratic	• Gradient-based technique (Fletcher-Reeves) • Non gradient-based technique (Particle Swarm Optimisation)
Constrained	The random DoE technique	• Polynomial High Order • Smooth Quadratic	• Gradient-based technique (Fletcher-Reeves) • Non gradient-based technique (Particle Swarm Optimisation)

6.3.1 The scenarios

6.3.1a The simulations

The same considerations and assumptions explained in the previous chapter are here adopted.

For the unconstrained problem, as mentioned previously, 13 key-positions were defined; where each 13 key-positions is associated with a specific coordinate, which represents the positions for the two exits. The Figure 6.5 presents these key-positions and their associated coordinate.

And according to what was mentioned before, for the two exits case, the positions are measured in terms of the relative distance between these two exits along the perimeter of the room. Thus, the distance from the left edge of each exit to the corner is measured similarly to the way that was done for the single exit case. These distances are going to be the design variables for the two exits case. The domains of these design variables are:

$$0 \le D1 \le 39 \qquad 0 \le D2 \le 39$$

D1 represents the distance from the left edge of exit 1 to the corner

D2 represents the distance from the left edge of exit 2 to the corner

Given that the squared room is 10x10m, these two exits can be located anywhere along its perimeter. Considering also that each exit is a 1m width exit, the domain could be represented by a line in which the total length is 39m.

It is important to observe that, given this configuration, the two exits can overlap each other, once they are free to be located anywhere between 0 and 39 of this line. For this reason, based on this constraint, the problem was solved based on two ways:

- Not taking into account directly this constraint (i.e.,unconstrained problem);
- Taking into account this constraint (constrained problem).

In the first approach, the constraint was not taken into consideration “directly”; however it was considered, once the 13 key-positions were defined and with this, it was assured that the exits would not overlapping each other.

Thus, for the unconstrained problem, once these key-positions were defined, as it was avoided “artificially” the overlapping of the two exits.

For the second approach, when the constraint was taken into consideration, an inequality was defined in order to represent this constraint, avoiding then the overlapping of the two exits. With this, the objective function i.e., the evacuation time, was subject to this inequality, namely:

$$|D1 - D2| \geq 1$$

In summary, the one single exit case is solved as an unconstrained problem, once there are no constraints. And the two exits case is solved as an unconstrained problem (when the 13 key-positions are defined, avoiding the overlapping of these two exits); and also as a constrained problem (when the inequality was defined, avoiding the overlapping of these two exits).

It is important to note that the positions 1, 3, 4, 6, 7, 9, 10, 11, 12 and 13 are symmetrical; while positions 2, 5, and 8 are asymmetrical. For the symmetrical positions, naturally, the potential maps (i.e., the areas which “contains” the exits' influence; in others words, the areas in which the exits, potentially, should be used by the occupants, taking into account their location in the environment lay-out) are the same, which means that the potential map for both exits are 50% of the total area of the room. In other words, the potential map of one exit is 50m^2 and the potential map for the other exit is 50m^2, given that the room area is 100m^2. For the asymmetrical positions, these potential maps are different.

This aspect should be mentioned, once this is going to influence the use of the exits and consequently is going to have an impact on the evacuation times. This is going to be analysed further later on. The table 6.7 presents the areas for these asymmetrical positions.

Table 6.7: Potential map areas for the squared room with two exits

Asymmetrical Positions	**Potential Map Area for one exit (m^2)**	**Potential Map Area for another exit (m^2)**
2	43.7	56.3
5	44.25	55.75
8	38.5	61.5

6.3.2 Results obtained through the use of numerical optimisation techniques

Now it is intended to solve the same problem using the methodology here proposed in this thesis. According to what was mentioned before, this problem is solved in two manners: firstly, as an unconstrained problem and secondly, as a constrained problem.

In the first approach, DoE technique was not used for picking up the design points. Instead, the pre-defined 13 key-positions were used (see figure 6.2). In fact, the previous data produced was used, and with this data the optimisation methodology was applied in order to find the optimal solution.

In the second approach, the random DoE technique was used for picking up the design points.

Therefore, as mentioned in the beginning of this chapter, all these problems were solved firstly as unconstrained problem and secondly as constrained problem.

All the functions developed from the RSM's are described in the appendix B of this thesis. The numerical approximation method used for all the cases was the General Stepwise Regression.

6.3.2.a The unconstrained problem

The first analysis is an unconstrained problem, taking as reference the 1.0m wide exits.

The statement of the unconstrained problem is:

Minimize: $$ET = f(D1, D2)$$

Where: $$0 \leq D1 \leq 39$$

$$0 \leq D2 \leq 39$$

Where:

ET – the evacuation time;

D1 - the distance from the left edge of exit 1 to the corner;

D2 - the distance from the left edge of exit 2 to the corner.

The exit location can be placed anywhere from the corner to the middle of the wall, as explained before.

Therefore, based on the design points, the RSM used was a smooth quadratic function and the general stepwise regression was used to build this RSM. The response surface is shown in Figures 6.4 and 6.5.

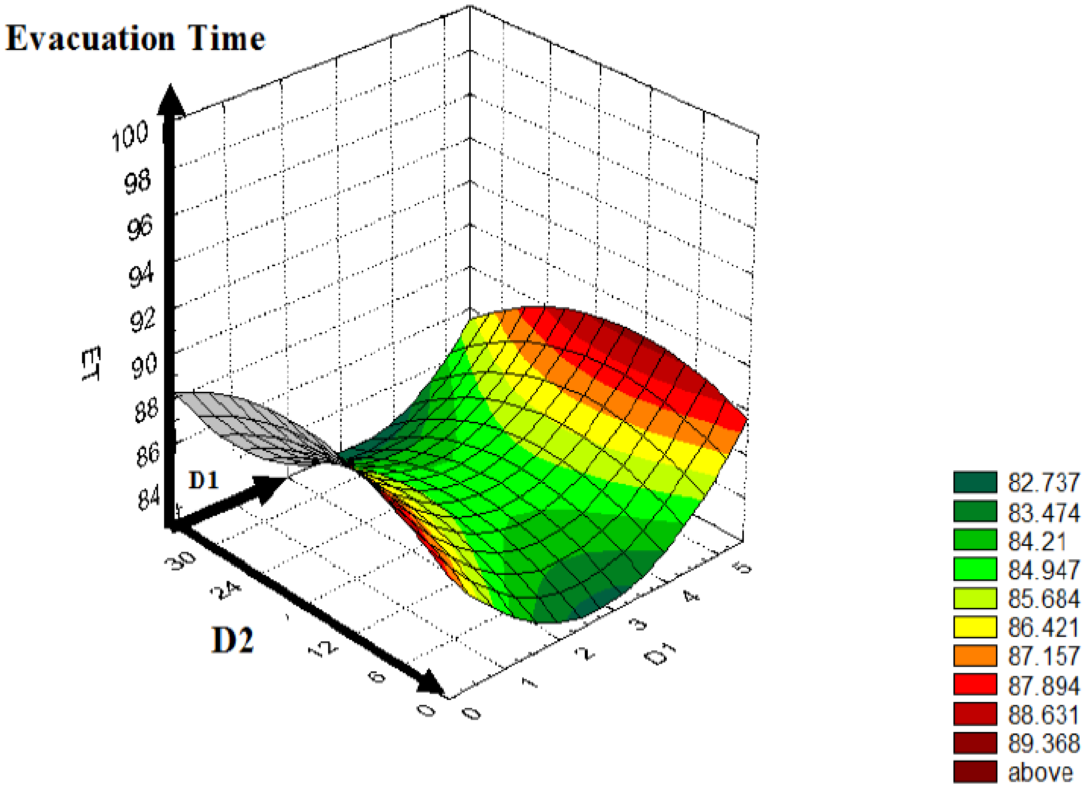


Figure 6.4: Response surface for the squared room with two 1.0m wide exits

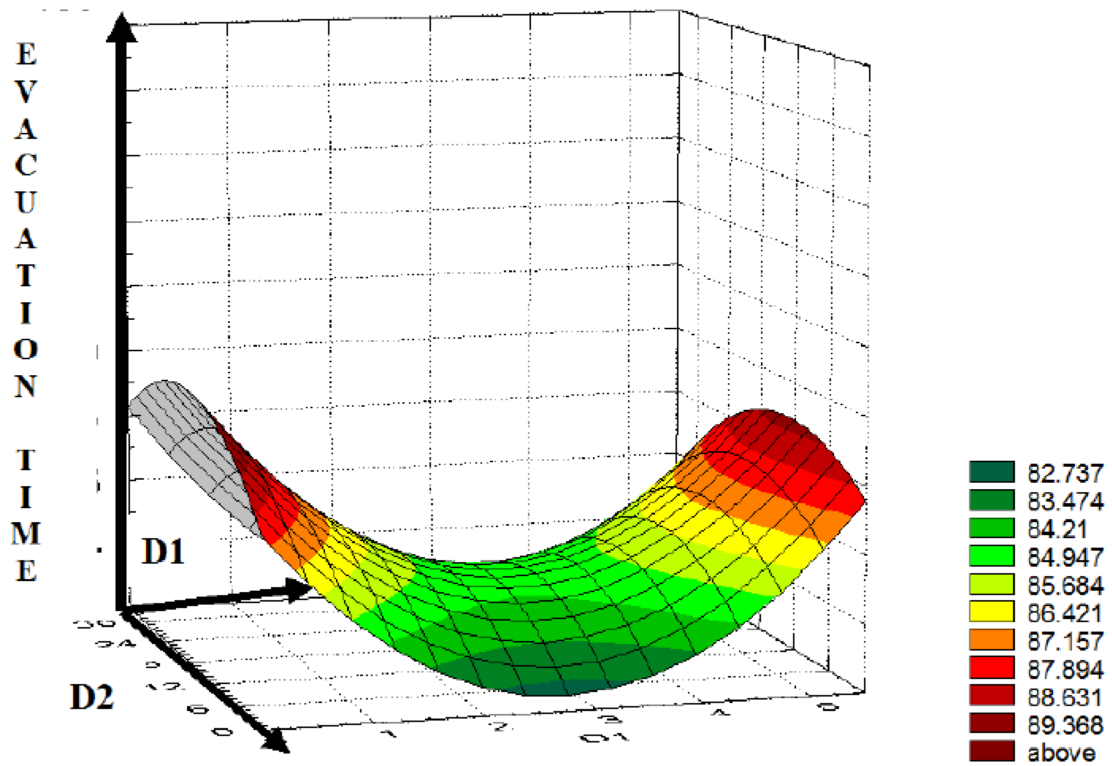


Figure 6.5: Side view of the response surface for the squared room with two 1.0m wide exits

From figures 6.4 and 6.5, it is possible to see the optimal solution region is when the value of the evacuation time is something between 82 and 83 seconds.

The table 6.8 presents the results based on this RSM and the use of the numerical optimisation techniques.

Table 6.8: Optimal solution for the squared room with two 1.0m wide exits based on the established 13 key-positions (unconstrained problem)

Optimum Solution (0 ; 1; 83.8)	**Numerical Optimisation Technique**	
Response Surface Model	Fletcher-Reeves	PSO – Particle Swarm Optimisation
Smooth Quadratic ($R^2 = 0.91$)	(2.70; 1; 82.81)	(2.70; 1; 82.81)

From table 6.8, it is clear to observe that using both numerical optimisation techniques, it was found the same solution: when D1 = 2.70 and D2 = 1, ET = 82.81. In other terms, when the distance from the corner to the left edge of one exit is 2.70m and when the distance from the corner to the left edge of the other exit is 1.0m, the evacuation time is 82.81 seconds. The figure 6.6 shows this scenario.

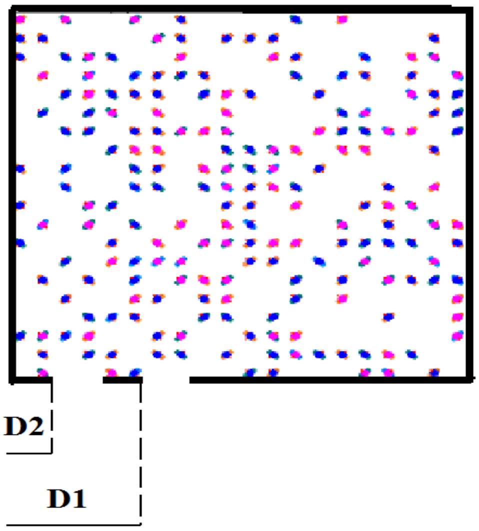


Figure 6.6: Optimal solution for the squared room with two exits using the numerical optimisation techniques

This scenario is something close to the case in which the exits are located side by side. The two exits are also located somewhere between the corner and the middle of the wall. This result seem to be consistent, once it was shown before that if the two exits are located close (i.e., almost side by side) and somewhere between the corner of the wall and in the middle of it, the evacuation time will be decreased.

6.3.2b The constrained problem

Now solving this same problem but taking into account the constraints, the statement becomes:

Minimize: $ET = f(D1, D2)$

Subject: $|D1 - D2| \geq 1$

Where:
$$0 \leq D1 \leq 39$$
$$0 \leq D2 \leq 39$$

For this problem, given its constraint, the design space is then modified. For this reason, as explained in chapter 4 of this thesis, the random DoE technique was used, once the other DoE techniques can just be applied for the whole design space; i.e., not a modified design space.

As the name suggests, the inconvenience of using the random DoE technique is that the design points are picked up randomly. This does not assure that the design points will represent properly the design space, even being a modified design space. In order to increase the likelihood of covering better the design space, it may be useful to increase the number of design points picked up. However, once again, there is no certainty. Given that, it was also investigated, for this case, how many design points should be picked up until get a reasonable result. Here, the optimisation analysis involved more work, given that a total of 87,000 simulations were performed.

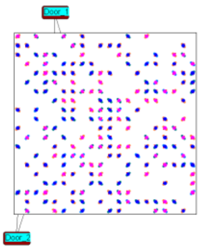
The table 6.9 presents all the results regarding to this case. The general stepwise regression was also used to develop the RSM’s for this problem.

Table 6.9: Optimal solution for the squared room with two 1.0m wide exits based on the design points suggested by the random DoE technique (constrained problem)

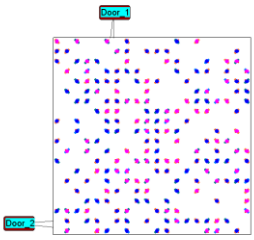
RSM polynomial function	**5 design points**		**10 design points**		**20 design points**	
Numerical Optimisation Technique	**Fletcher-Reeves**	**PSO**	**Fletcher-Reeves**	**PSO**	**Fletcher-Reeves**	**PSO**
Set 1	(27.0; 0 ; 81.77)	(27.0; 0 ; 81.77)	(26.5; 39.0; 81.72)	(26.5; 39.0; 81.72)	(23.0; 39.0; 85.11)	(23.0; 39.0; 85.11)
Set 2	N.S.	N.S.	(0; 39.0; 89.2)	(0; 39.0; 89.2)	(39.0; 17.0; 85.30)	(39.0; 17.0; 85.30)
Set 3	N.S.	N.S.	(39.0; 23.5; 67.93)	(39.0; 23.5; 67.93)	(39.0; 38.0; 89.26)	(1.0; 0; 84.57)
Set 4	N.S.	N.S.	N.S.	N.S.	(39.0; 0; 86.54)	(39.0; 0; 86.54)
Set 5	N.S.	N.S.	N.S.	N.S.	(1.0; 0; 84.37)	(1.0; 0; 84.37)
RSM quadratic smooth	5 design points		10 design points		20 design points	
Numerical Optimisation Technique	Fletcher-Reeves	PSO	Fletcher-Reeves	PSO	Fletcher-Reeves	PSO
Set 1	N.S.	N.S.	(32.5; 0; 76.77)	(32.5; 0; 76.77)	(38.0; 39.0; 83.2)	(38.0; 39.0; 83.2)
Set 2	N.S.	N.S.	(38.0; 39.0; 48.0)	(39.0; 38.0; 48.0)	(0; 1; 86.43)	(39.0; 24.0; 85.81)
Set 3	N.S.	N.S.	(39.0; 27.0; 65.60)	(39.0; 27.0; 65.60)	(1.0 ; 0; 81.64)	(1.0 ; 0; 81.64)
Set 4	N.S.	N.S.	N.S.	N.S.	(37.5; 39.0; 88.32)	(37.5; 39.0; 88.32)
Set 5	N.S.	N.S.	N.S.	N.S.	(0; 1.25; 85.3)	(0; 1.25; 85.3)

N.S. - No Solution found

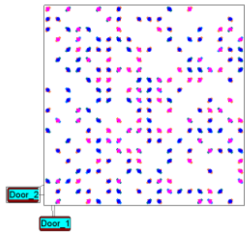
The figures 6.7 and 6.8 present the scenarios which represent these results from table 6.9.



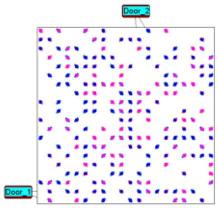
Set 1 (5 design points)
Fletcher-Reeves and PSO
(D1 = 0 and D2 = 27)



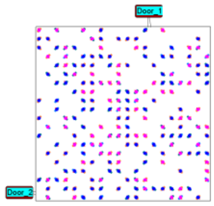
Set 1 (10 design points)
Fletcher-Reeves and PSO
(D1 = 26.5 and D2 = 39)



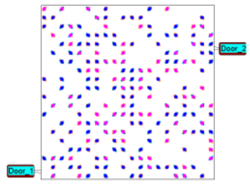
Set 2 (10 design points)
Fletcher-Reeves and PSO
(D1 = 0 and D2 = 39)



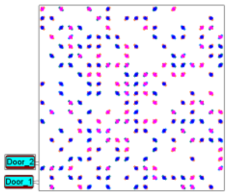
Set 3 (10 design points)
Fletcher-Reeves and PSO
(D1 = 39 and D2 = 23.5)



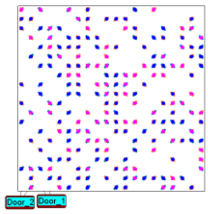
Set 1 (20 design points)
Fletcher-Reeves and PSO
(D1 = 23 and D2 = 39)



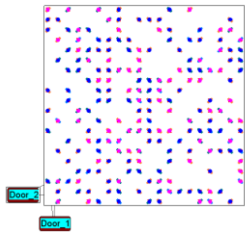
Set 2 (20 design points)
Fletcher-Reeves and PSO
(D1 = 39 and D2 = 17)



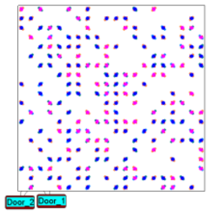
Set 3 (20 design points)
Fletcher-Reeves
(D1 = 39 and D2 = 38)



Set 3 (20 design points)
PSO
(D1 = 1 and D2 = 0)

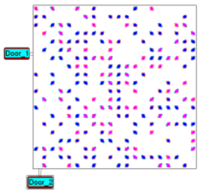


Set 4 (20 design points)
Fletcher-Reeves and PSO
(D1 = 39 and D2 = 0)

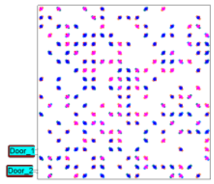


Set 5 (20 design points)
Fletcher-Reeves and PSO
(D1 = 1 and D2 = 0)

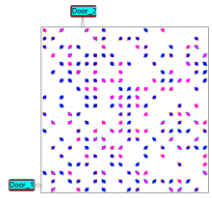
Figure 6.7: Exit positions for two exits cases found using the numerical optimisation techniques (RSM - polynomial function)



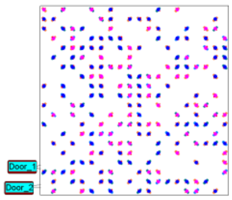
Set 1 (10 design points)
Fletcher-Reeves and PSO
(D1 = 32.5 and D2 = 0)



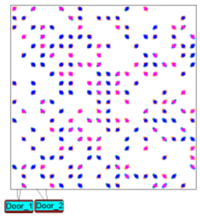
Set 2 (10 design points)
Fletcher-Reeves and PSO
(D1 = 38 and D2 = 39)



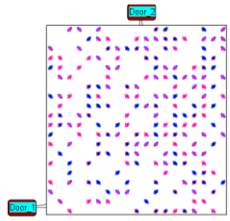
Set 3 (10 design points)
Fletcher-Reeves and PSO
(D1 = 39 and D2 = 27)



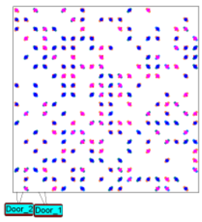
Set 1 (20 design points)
Fletcher-Reeves and PSO
(D1 = 38 and D2 = 39)



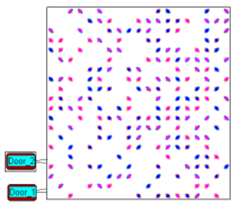
Set 2 (20 design points)
Fletcher-Reeves
(D1 = 0 and D2 = 1)



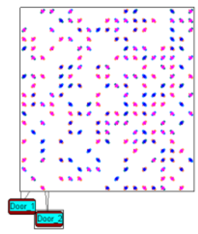
Set 2 (20 design points)
PSO
(D1 = 39 and D2 = 24)



Set 3 (20 design points)
Fletcher-Reeves and PSO
(D1 = 1 and D2 = 0)



Set 4 (20 design points)
Fletcher-Reeves and PSO
(D1 = 37.5 and D2 = 39)



Set 5 (20 design points)
Fletcher-Reeves and PSO
(D1 = 0 and D2 = 1.25)

Figure 6.8: Exit positions for two exits cases found using the numerical optimisation techniques (RSM - quadratic smooth)

It is possible to observe from these results that, for 5 design points, the results were inconsistent. For instance, for sets 2 and 3, it was not possible to complete the optimisation analysis, once the results obtained were completely wrong, once the evacuation time was negative.

For both RMS's, the 3 sets of 10 design points provided some better results than using just 5 design points. However, they were still inconsistent.

And for both RMS's, the 5 sets of 20 design points provided the best solutions. In other terms, these results were the closest to the actual optimal solution (found in the data analysis) for the two exits cases, which is having the two exits located side by side in the corner. For example, using a polynomial function as the RSM, and for both numerical optimisation techniques, namely Fletcher-Reeves and PSO, sets 3, 4 and 5 give robust results in the sense that they suggest that the optimal solution is to have the two exits located side by side and in the corner. Using now the quadratic smooth, which provided a better curve fitting than the polynomial, all the 5 sets of 20 design points were almost consistent. This can be said, because all the most the coordinates are logical, in a sense that the exits are assuming values which mean that they need to be located side by side in the corner and/or at least one exit should be close to the corner and the other one closer this exit (which gives the other second configuration, the asymmetrical positions for these two exits in relation to each other). And besides that, the evacuation times for all cases are consistent.

6.6 Concluding Comments

In this chapter, the optimal positioning of exits around the perimeter of a square room in order to minimise the evacuation times were explored. The evacuation simulations were conducted assuming ideal conditions of zero response times and population behaviour such that occupants would move to their nearest exits. Both assumptions are made to simplify the analysis and to isolate issues associated with exit location.

This chapter presented and discussed a systematic methodology to efficiently optimise evacuation safety aspects of structural designs. Such an approach will be of particular interest to practical fire engineers as it allows the fire engineer rapidly and efficiently optimise their design.

For this purpose, it was considered the previous square room with one single exit and two exits.

For the first problem, the square room with one single exit, the solution was found applying the concept of combining the use of numerical optimisation techniques with evacuation simulation modelling. With this, it was avoided to perform all the 46,200 simulations which were done before. And besides that, the results obtained have shown to be satisfactory, i.e., global minima and local minima closest to the global minima region were found. For all the cases, it was used a gradient-based algorithm (i.e., the Fletcher-Reeves numerical optimisation technique) and non gradient-based algorithm (i.e., the Particle Swarm Optimisation numerical optimisation technique) to find the optimal solution.

For this single exit case, it was defined the problem as being unconstrained. Two different DoE techniques were used. It was found exactly the same optimal solution that was found before, but now using just 3 design points proposed by the Central Composite Design DoE technique; which implicated in 1,800 simulations. With this, it was avoided to perform 2,400 simulations.

For all the cases, the response surface model based on polynomial high order or quadratic smooth seem to be appropriate. The general stepwise regression was the multivariable regression analysis technique here selected to build these response surface models and it worked well, once the coefficients R^2 were higher than 0.90.

The analysis revealed that this methodology seems to be a very powerful tool for evacuation modelling analysis. This systematic methodology to efficiently optimise evacuation safety aspects of structural designs will be extended to more complex designs.

For the second problem, the square room with two exits, was also investigated applying the concept of combining the use of numerical optimisation techniques with evacuation simulation modelling. With this, it was avoided to perform all the 87,000 simulations which were done before. And besides that, the results obtained have shown to be satisfactory, i.e., global minima and local minima closest to the global minima region were found. For all the cases, we have used a gradient-based algorithm (i.e., the Fletcher-Reeves numerical optimisation technique) and non gradient-based algorithm (i.e., the Particle Swarm Optimisation numerical optimisation technique) to find the optimal solution.

The problem was defined as being unconstrained in the first analysis and then constrained in the second analysis.

When the problem was defined as being unconstrained, no DoE technique was used. Instead, 13 key-positions already defined in the previous work was used. The results obtained in this analysis were very close to the global minima solution and it was satisfactory, once the evacuation time was minimized and its value was close to the optimal solution.

When the problem was then defined as being constrained, the random DoE technique was to pick up the design points. For this analysis, 87,000 simulations were performed, once it was intended to check the effectiveness of the random DoE technique. It was found that just with 20 design points we started to find good results; values closer to the global minima.

For all the cases, the response surface model based on polynomial high order or quadratic smooth seem to be appropriate. The general stepwise regression was the multi-variable regression analysis technique here selected to build these response surface models and it worked well, once the coefficients R^2 were higher than 0.90.

The analysis revealed that this methodology seems to be a very powerful tool for evacuation modelling analysis. This systematic methodology to efficiently optimise evacuation safety aspects of structural designs will be extended to more complex designs.

In the next chapter, a more complex study case is investigated using this methodological approach: the use of numerical optimisation techniques and associated concepts.

7. Case Study 1: L-Shaped Room

7.1 Introduction

This chapter presents a case study in which composes the set of the additional more complex scenarios of this thesis.

The case study presented in this chapter is a constrained problem, in where the best location for two exits is found for a “L-shaped” room. This case is similar to the square room with two exits, however, the geometry analysed is more complex given its shape which enables more exit locations combinations. Besides that, based on the increase of exit locations combinations, more small evacuation times will probably be found. In optimisation terms, more local minima regions will probably be found and with that, the response surface will be more challenging for the optimisation methodology.

Clearly, the problem has 2 design variables and it is a constrained problem.

Firstly, the problem is solved using brute force method. For the brute force method, 22 scenarios were analysed. The global minima region was found based on these 22 scenarios.

Secondly, the problem is solved using the numerical optimisation techniques taking into consideration the response surface built based on the 22 scenarios used during the brute force method.

And finally, the problem is solved using the numerical optimisation techniques taking into account the response surfaces built based on the design points suggested by the random DoE technique.

All the data used was obtained from evacuation simulations.

In the next section, the problem is discussed with further details.

7.2 The Problem

The problem to be investigated can be stated very simply as it was the square room: for a room of given size, containing an arbitrarily large population, is there an optimal location for two exits that will minimise the evacuation times?

The same considerations made for the squared room cases are here made as well. The L-shaped room has an area of 108 m^2 and there are two exits of 1.0m width each, see Figure 7.1. The population of the room consists of 216 people producing a population density of 2 people/m^2. This means that 50% of the area is populated, which gives 50% of freedom for the occupants' movement during the evacuation process.

The exit locations are measured in terms of the distance from the left edge of the exit to the corner of the wall.

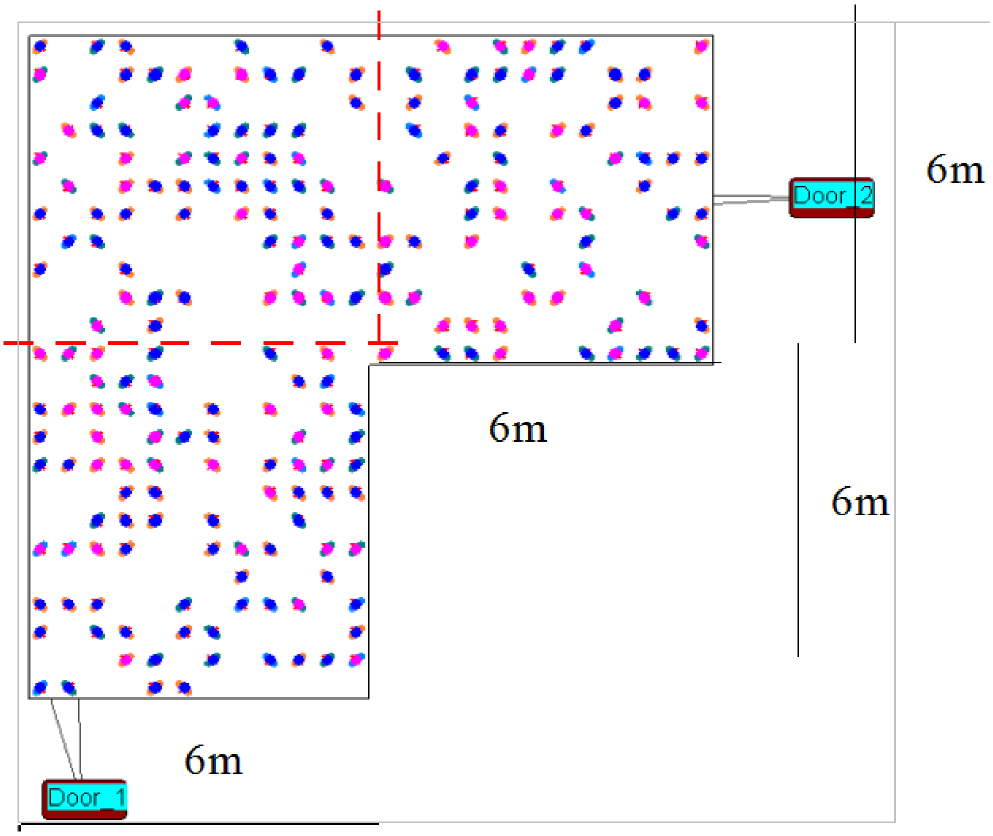


Figure 7.1: L-shaped room

Before solving this problem using the numerical optimisation techniques and the associated concepts (i.e., the methodology presented here in this study), several sets of simulations were performed in order to demonstrate the impact of the exits locations in such type of geometry (i.e., L shaped room). And in addition, the impact of the relationship between the exits locations and the population density on the evacuation efficiency was also investigated.

With this information, it was possible to identify the optimal solution and then investigate shelter the optimisation method could determine the optimal solution.

Therefore, in the early stages of the solution process for this problem, several exit locations scenarios were investigated in order to find the optimal solution. This process did not involve the use of the optimisation methodology. Instead, the brute force method took place.

7.3. Analysis of exit locations impact on the evacuation efficiency for the L-Shaped room

For this initial analysis, the L-shaped room was populated by 216 occupants, producing a population density of 2p/m^2, as mentioned previously. And also, the same geometry was populated by 116 occupants, giving a population density of approximately of 1.07p/m^2 (reducing the population density to almost half of the initial value). With this procedure, it was intended to investigate also the impact of the population density on the relationship between the exits locations and the evacuation efficiency.

In reality, the optimal solution is here found based on brute force method.

The scenarios investigated were similar to the scenarios investigated for the square room with two exits. In other words, key-positions were selected in order to cover the strategic exit locations along the walls which could impact significantly the evacuation performance, see Figure 7.2.

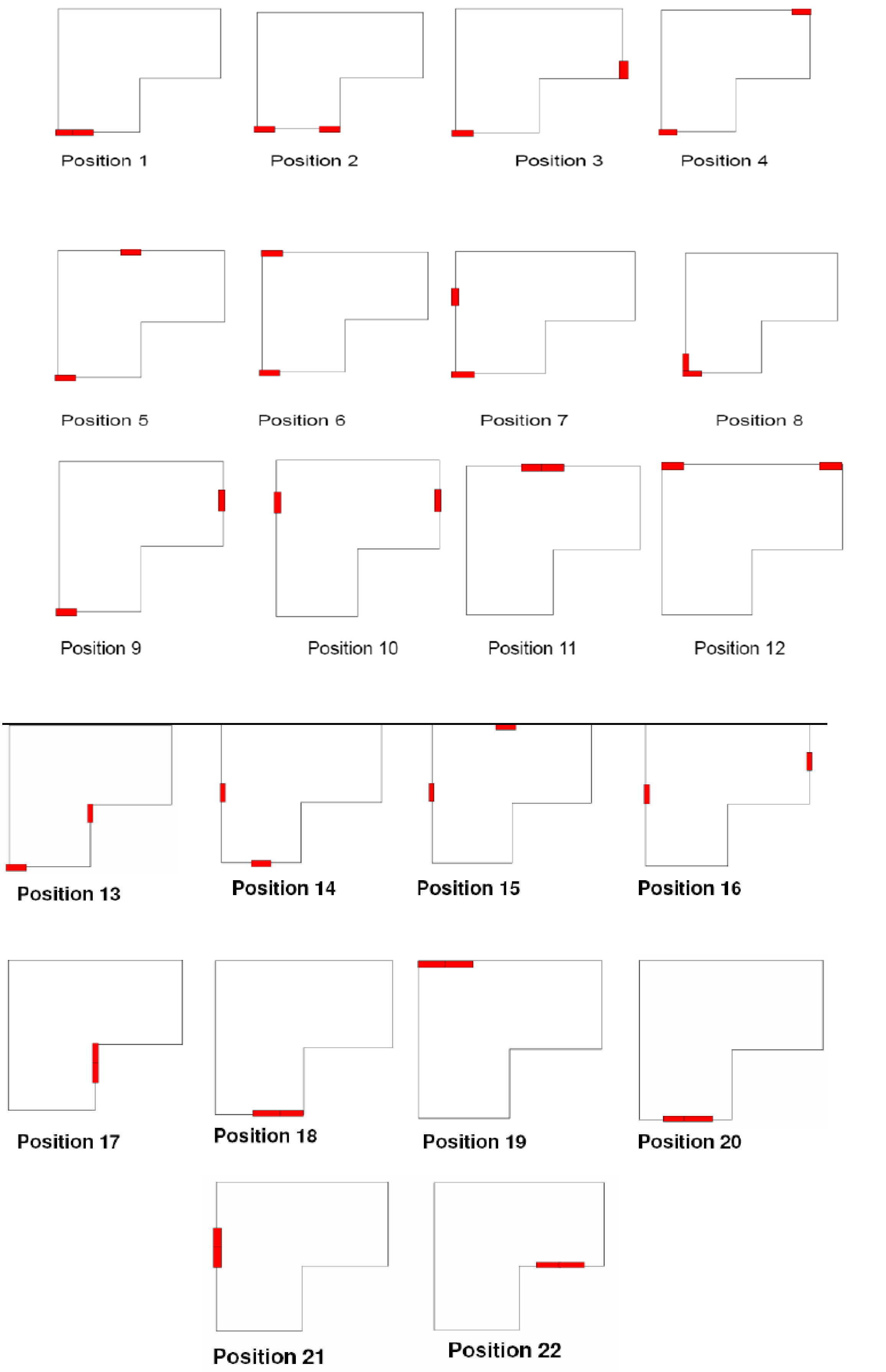


Figure 7.2: Key-positions for the exits in the L-shaped room

In figure 7.2, it is possible to see that 22 key-positions were defined. The red rectangular represent the positions for the exits.

To investigate the impact of population density only the first 12 scenarios will be considered. This is to reduce the number of simulations that need to be performed. In total, 7,200 simulations were performed for this first analysis in order to check if the exits locations, as they had for the square and rectangular rooms presented in chapters 5 and 6, do impact the evacuation efficiency.

The same parameters and assumptions used for the previous evacuation modelling analysis were used here as well.

The results have shown that the best locations for the two exits are when they are located side by side in the corner. For instance, the lowest evacuation times were found for positions 8 and positions 1: 80 sec and 80.2 sec respectively. Position 4 also provided a good result, since the two exits located in a diagonal distance, which represents a symmetrical relative distance between the two exits. Table 7.1 presents the summary of these results.

Table 7.1: Results through data analysis for L-shaped room with two exits populated by 216 occupants

Positions	AV TET (sec)	AV CWT (sec)	AV Dist (m)	AV PET (sec)
1	80.2	32.03	11.88	40.37
2	92.92	35.97	11.30	46.83
3	95.42	38.54	6.95	45.76
4	93.42	37.80	7.25	45.29
5	128.42	47.40	5.76	53.66
6	123.92	43.79	8.18	52.08
7	138.09	49.28	8.10	57.50
8	80	31.2	11.81	40.21
9	96.75	39.50	7.05	46.82
10	110.75	42.08	5.76	48.33
11	94.98	40.40	7.92	48.45
12	110.92	40.44	7.63	48.25

It is clear to see that the exits locations do impact the evacuation efficiency, since the evacuation times are varying according to the different scenarios. The results have shown that the best locations are those in where the two exits are located side by side in the corner. For instance, positions 1 and 8 give the lowest evacuation times: 80.2sec and 80sec respectively. And besides that, the second best locations are those in where the relative distance between the two exits is symmetrical or close to symmetrical. This can be seen for positions 3 and 4 for example, which provided evacuation times of 95.42sec and 93.42sec respectively.

Regarding the results produced based on the same L-shaped room, but populated by 116 occupants, the same 12 key-positions were used as well as the same parameters and assumptions in the evacuation modelling analysis. In total, 7,200 simulations were performed and the results are presented in table 7.2.

Table 7.2: Results through data analysis for L-shaped room with two exits populated by 116 occupants

Positions	AV TET (sec)	AV CWT (sec)	AV Dist (m)	AV PET (sec)
1	50.66	14.79	11.33	25.65
2	51.59	15.34	11.01	25.92
3	52.58	18.15	6.20	24.77
4	53.92	18.87	6.65	25.87
5	67.91	22.73	5.65	28.91
6	65.25	20.17	8.07	28.34
7	82.92	28.95	7.77	36.86
8	50.41	14.01	11.45	24.95
9	53.42	18.78	6.34	25.53
10	59.59	20.86	5.04	26.50
11	54.92	18.05	7.32	25.58
12	59.42	19.35	6.91	26.54

It is also clear to observe that even reducing the population density, the exits locations does still impact the evacuation efficiency. It is evident that reducing the number of occupants, the difference between the evacuation times for the different scenarios decreases. In other terms, for low population density, the exit locations impact the evacuation times, but to a less extent due to smaller number of people.

Regarding the curve “evacuation times *versus* exits positions” shows that for both scenarios (i.e., L-shaped room with 216 occupants and L-shaped room with 116 occupants) the trend behaviour is very similar, see figure 7.3. In reality, the optimal locations are the same found when the room was occupied by 216 people: which are positions 1 and 8. And the worst evacuation time was found for the same position found for the higher populated case: position 7.

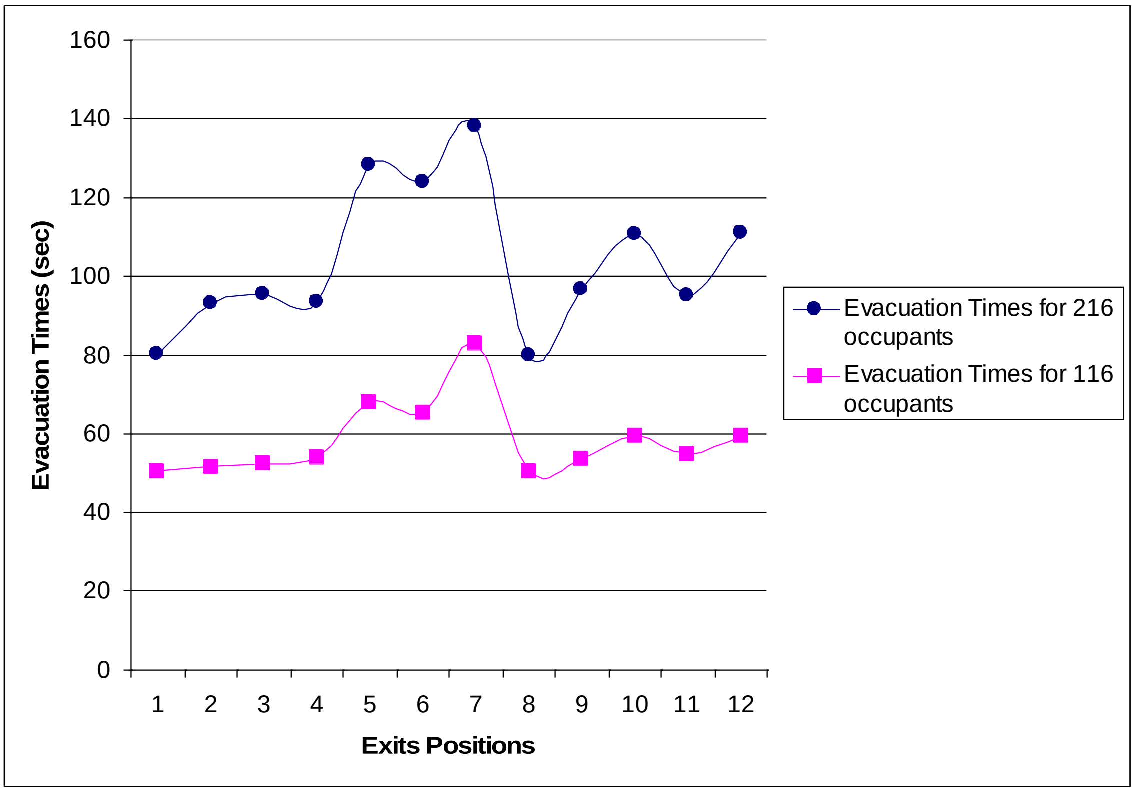


Figure 7.3: Comparison between the evacuation times obtained for 216 people and 116 people for the exits in the L-shaped room

As mentioned previously, from figure 7.3, it becomes clear that for both cases, the exits locations do impact the evacuation efficiency. It is also possible to conclude that the population density influences the evacuation efficiency, since the difference between the evacuation times according to the exits locations decreased. However, despite this, this impact was not significant in terms of changing the relationship between exits locations and evacuation efficiency. It is interesting that even reducing the population density to almost 50%, the exits locations are still impacting the evacuation performance.

In fact, comparing the results obtained with 116 occupants with the results obtained with 216 occupants, it is expected that if the population density is reduced to a value more than 50%, possibly this trend behaviour would change, since there would be much less congestions during the movement escape and towards the exits areas. And with that, the trend behaviour of the evacuation times in relation with the exits locations should be less non-linear.

Nevertheless, it is not the objective of this study to find the "critical population density" which changes the relationship between exits locations and evacuation efficiency, since this study is focusing on assembly structures, where the environments are high dense populated.

In summary, based on these previous analyses, like the previous cases where simple geometries were analysed (i.e., square and rectangular rooms), it was possible to see that the exits locations do impact the evacuation efficiency in more complex geometries such as this L-shaped room.

In fact, as mentioned previously, it was found from this initial analysis that the best locations to reduce the evacuation times will be when the exits are located side by side in the corners areas. And in fact, performing all 22 scenarios, see Figure 7.2, it was possible to analyse in order to demonstrate that the best locations were certainly found for the L-Shaped room. This was done taking into consideration the 216 occupants, since this is the final total number of occupants which will be analysed for this case study. The table 7.3 presents all of this data.

Table 7.3: Complete results for the L-shaped room with 216 occupants and two 1.0m exits

Position	AV TET (sec)	AV CWT (sec)	SIM. AV Dist (m)	GEOM. AV Dist (m)	AV PET (sec)	Max. Travel Distance (m)	Av Number using Exit 1	Av Number using Exit 2
1	80.2	32.03	11.88	12.53	40.37	28.23	108	108
2	92.92	35.97	11.30	11.48	46.83	30.11	107	109
3	95.42	38.54	6.95	7.84	45.76	21.61	110	106
4	93.42	37.80	7.25	7.85	45.29	20.57	108	108
5	128.42	47.40	5.76	5.84	53.66	13.74	120	96
6	123.92	43.79	8.18	8.99	52.08	30.93	138	78
7	138.09	49.28	8.10	9.76	57.50	26.43	125	91
8	80	31.2	11.81	13.32	40.21	29.31	108	108
9	96.75	39.50	7.05	7.39	46.82	22.52	109	107
10	110.75	42.08	5.76	7.22	48.33	22.11	115	101
11	94.98	40.40	7.92	8.48	48.45	25.52	107	109
12	110.92	40.44	7.63	8.58	48.25	30.56	145	71
13	160.08	66.09	6.25	7.35	72.26	16.36	28	188
14	118.58	42.10	7.59	8.32	49.28	21.19	77	139
15	92.92	41.03	5.36	5.61	46.52	12.16	109	107
16	116.92	45.14	5.32	5.57	50.61	12.57	141	75
17	94.27	40.15	7.18	7.61	46.99	17.36	112	104
18	80.45	31.89	11.88	12.98	40.2	29.0	108	108
19	80.5	32.60	11.92	12.39	40.80	28.6	108	108
20	96.09	37.79	11.34	11.74	47.76	25.40	109	107
21	94.49	40.15	7.87	8.32	47.48	25.99	107	109
22	93.48	38.47	9.27	10.09	46.93	22.23	108	108

It is important to observe that the Average Travel Distances were now categorized into two types: Simulation Average Travel Distance (SIM AV Dist.) and Geometry Average Travel Distance (GEOM AV Dist.). The first index was calculated based on the values calculated during the simulation (i.e., taking into account the 216 occupants). And the second index was calculated populating the whole geometry with the maximum number of occupants (i.e., for the L-shaped room: 432 occupants). In summary, the geometry average travel distance represents the “real” travel distance for each scenario and this is why there is small difference between these values and the values obtained from the actual simulations.

Another new parameter inserted on this table was the Maximum Travel Distances as well as the last two columns which show the total number of occupants using each exit for each scenario. These last two columns present an useful information which enables to verify which scenarios amongst these 22 scenarios are symmetrical or asymmetrical in terms of the relative distance between the two exits.

It is interesting to observe that those scenarios which provided the lowest evacuation times, where those in where the exits are located side by side (for a basic physical law: the wider the aperture is, the higher will be the flow rate). And in reality, those scenarios in where the exits were located side by side and in the corner areas provided lower evacuation times. Those scenarios (positions 3, 4 and 9) in where the relative distance between the two exits is symmetrical or close to symmetrical generated also lower evacuation times. And the scenarios which provided the highest evacuation times were those in where the relative distance between the exits is asymmetrical, namely: 5, 6, 7,10, 12, 13, 14 and 16. It is important to mention that position 2 is also asymmetrical, nevertheless the evacuation time generated by this scenario was low (i.e., 92.92sec). For position 2, the two exits are located in the corner regions and this, as it was discussed previously in chapters 5 and 6, provide an advantage since the conflict between the occupants is reduced.

And regarding the position 12, which is similar to position 2, since the two exits are located in the corners. Therefore, it would be expected that this should provide the same advantage of the corner effect. However, for position 12, the evacuation time is high. The possible explanation for that is found in the unbalance in the exits use. In fact, given that the distance between the two exits in position 12 is much bigger than in position 2, the occupants once they are close to one exit, it would be more difficult to go to the other exit. And for position 2, this is not true.

This similar trend behaviour was observed for the square room with two exits, see Figure 7.4.

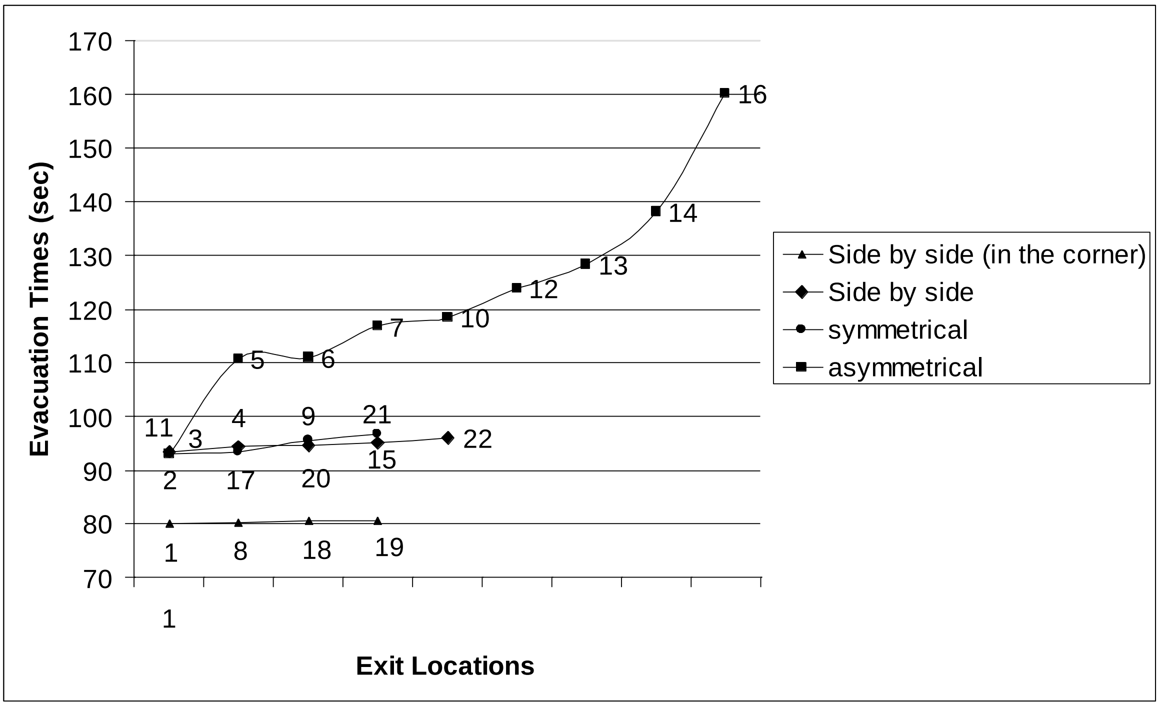


Figure 7.4: Comparison between the evacuation times obtained for 216 people and 116 people for the exits in the L-shaped room

The Table 7.4 presents these values.

Table 7.4: Complete results for the L-shaped room with 216 occupants and two 1.0m exits

Position	Type	AV TET (sec)
1	Side By Side (in the corner)	80.2
8	Side By Side (in the corner)	80
18	Side By Side (in the corner)	80.45
19	Side By Side (in the corner)	80.5
11	Side By Side	94.98
17	Side By Side	94.27
20	Side By Side	96.09
21	Side By Side	94.49
22	Side By Side	93.48
3	Symmetrical	95.42
4	Symmetrical	93.42
9	Symmetrical	96.75
15	Symmetrical	92.92
2	Asymmetrical	92.92
5	Asymmetrical	128.42
6	Asymmetrical	123.92
7	Asymmetrical	138.09
10	Asymmetrical	110.75
12	Asymmetrical	110.92
13	Asymmetrical	160.08
14	Asymmetrical	118.58
16	Asymmetrical	116.92

It is also interesting to observe that for scenarios 6 and 12, the evacuation times are high. From the data, it is possible to see the reason for that. It is based on is based on the fact that they have the biggest Maximum Travel Distances and with that, the Cumulative Waiting Time (CWT) is long as well, which generated high Personal Evacuation Times (PET). In this case, the travel distance played a major factor.

Therefore, in summary, from these 22 scenarios, just 2 scenarios (6 and 12) did follow the "common sense" in defining that the travel distance will be the major factor for defining what the RSET will be for enclosures.

In reality, from the data, it is possible to observe that those scenarios in where the two exits were located side by side and also in where the relative distance between the two exits is symmetrical provide the lowest evacuation times. In the other hand, for those scenarios, in where the relative distance between the two exits is asymmetrical provide the highest evacuation times. This "law" was also observed for the square room with two exits.

It is also clear to conclude that the travel distance is not the major factor for evaluating the evacuation efficiency in those scenarios. For instance, for position 7 which has a lower value for the travel distance when compared with position 2, generates a higher value for the evacuation time than position 2.

In summary, when evaluating the evacuation efficiency in enclosures, each scenario will be a particular scenario. Therefore, the major factor influencing on the evacuation efficiency for a specific case will not be necessarily the same one for another different case. The travel distance can be or can not be the major factor impacting the evacuation performance. The exits locations combined with the shape of the room do also play an important rule on it.

Additionally, it is correct to say that exits locations will define the travel distance; nevertheless, when more than one exit is placed within the enclosure, the relative distance between the exits will then also impact the evacuation performance.

It might be relevant to mention, once again here, that these findings are potentially useful for practical fire safety engineering problems for enclosures. In fact, the fire safety codes which follow a performance-based approach should consider that the travel distance should not be taken as the only major factor to address the occupants' safety within an enclosure (i.e., when preparing the fire design). Furthermore, depending on the scenario, as mentioned before (which is itself a simple/complex system) the travel distance might be or might not be the major factor influencing the evacuation efficiency.

In summary, fire safety engineers/designers should have on their minds that combined with that the exits locations do play a major rule as well.

Clearly, the travel distance is directly associated with the exit location; however, depending on where the exit is located, even when the travel distance is long, this does not mean necessarily that the exit location is badly located in terms of evacuation efficiency. This is explained by the fact that sometimes the time spent towards the exits areas become bigger than the time spent during the escape movement. And for some other cases, the combination of these two times is also affecting the evacuation.

Therefore, in the next sections of this chapter, this optimal solution will be explored through the use of numerical optimisation techniques. It is expected that the same solution will be found, i.e., the global minima region, using much less scenarios.

7.4. Finding the optimal solution using optimisation techniques applied to the response surface built based on the 22 scenarios used for the brute force method

In the previous section, the optimal solution for the L-shaped room was found using brute force method. In this section, the optimal solution is found using the numerical optimisation techniques.

Here, the numerical optimisation techniques are applied taking into consideration the response surface built, see Figure 7.5, based on the 22 scenarios used for the brute force method, see Figure 7.2. Therefore, DoE techniques were not used.

Clearly, for this approach, the problem was solved as being unconstrained, since the design points, the 22 scenarios, were defined artificially.

The response surface model was based on a radial basis function: multiquadratic function, since the behaviour of the surface presented high non-linear characteristics.

Two types of numerical optimisation techniques were applied: a gradient-based technique and a non gradient-based technique.

It is important to note that the optimal solution is understood as being the global minima region of the objective function (i.e., the evacuation time) in the response surface. However, given that to find precisely the global minima region is not an easy task, if the local minima region near to the global minima region is found, it is assumed that this is also a good result.

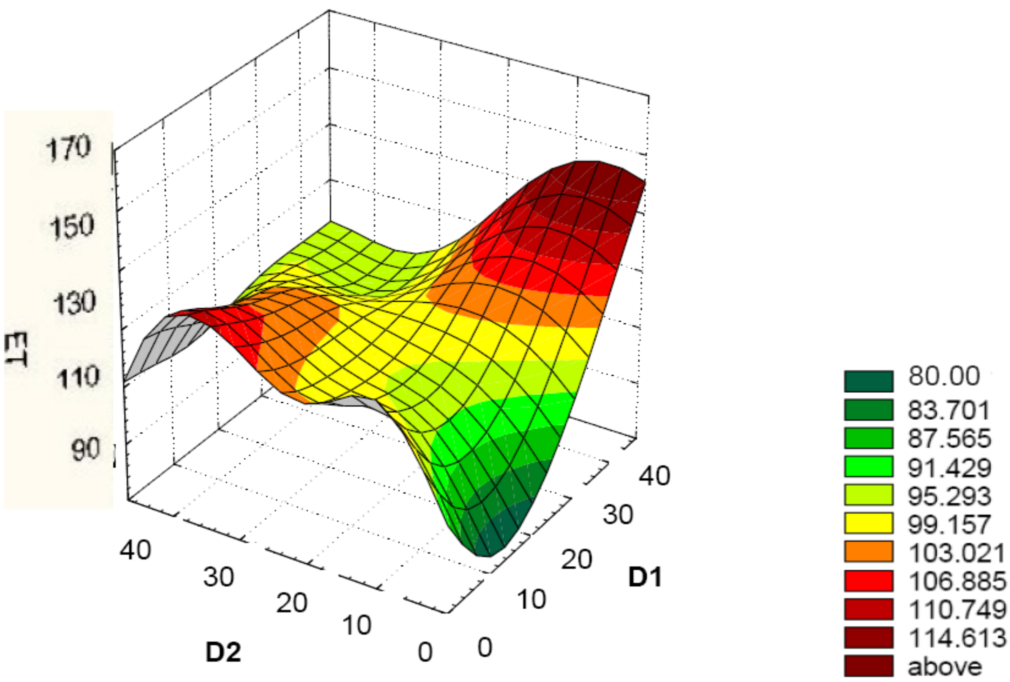


Figure 7.5: Response surface for the L-shaped room with two 1.0m exits based on all 22 design points analysed using "brute force method"

The table 7.5 presents the results.

Table 7.5: Optimal solution for the L-shaped room with two 1.0m wide exits based on the 22 design points used for the brute force method (unconstrained problem)

RSM High Oder Polynomial ($R^2 = 0.97$)	**Numerical Optimisation Techniques**	
22 design points (the same scenarios used for the brute force method)	**Fletcher-Reeves**	**PSO**
	(47; 0; 80)	(47; 0; 80)

It is possible to see that using both numerical optimisation techniques, the optimal solution is found: position 8.

This shows that both techniques are robust, because the response surface model has a high non-linear complex shape. In other terms, the design space has more than one local minima regions and the techniques were able to find the global minima region.

The Figure 7.6 shows the best region where to put two exits of 1.0m each in a L-shaped room to produce minimum evacuation times.

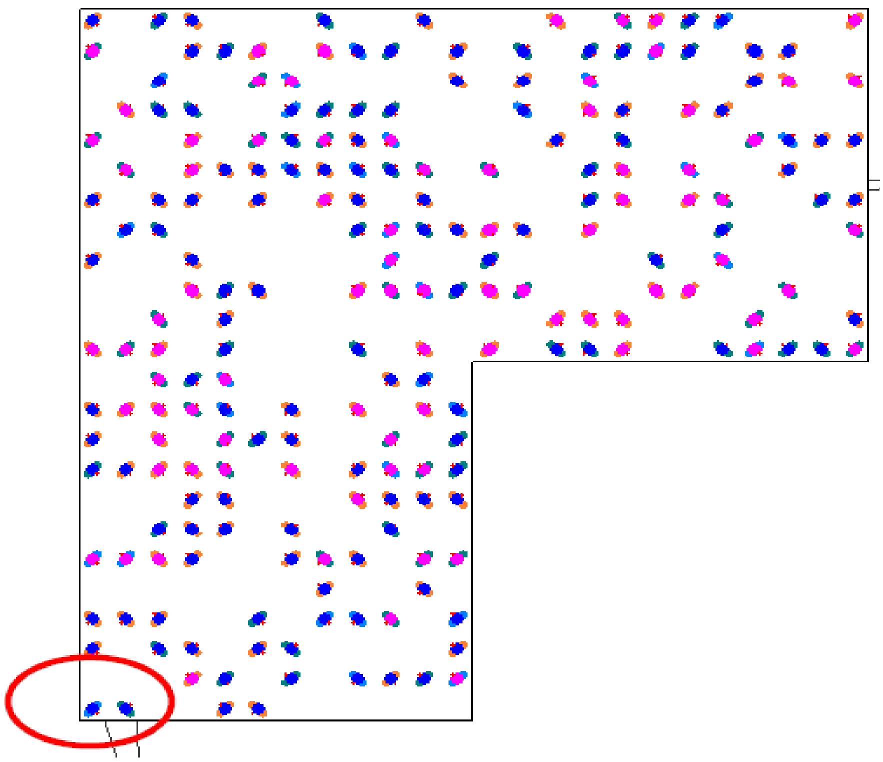


Figure 7.6: Best (optimal) location to put 2 exits of 1.0m each in a L-shaped room

In reality, when the two exits are located somewhere in the region shown by the red ellipse, good results are produced (i.e., short evacuation times).

This seems to be a particular case of the squared room with 1 single exit, where the best place to locate the exit was in the corner, once less congestion was caused. Here again, for this more complex case, the same condition is observed.

Therefore, it is assumed that coordinates close to this region are also good results.

In the next section, the same problem is solved, but now using DoE techniques to pick up the design points.

7.5 The Final Analysis using Classical Optimisation Theory

7.5.1. The Problem

For this study case analysed, the problem is a constrained problem. In other terms, the two exits can be located anywhere along the room perimeter; for this reason constraints must be considered in order to avoid these two exits overlapping each other. The random DoE technique was used for picking up the design points. For all the scenarios considered in this problem, the RSM based on multiple regression function was used, once it proved to be appropriate based on the general stepwise regression.

And finally, two types of numerical optimisation techniques were applied: a gradient-based technique and a non gradient-based technique.

It is important to note that the optimal solution is understood as being the global minima region of the objective function (i.e., the evacuation time) in the response surface. However, given that to find precisely the global minima region is not an easy task, if the local minima region near to the global minima region is found, it is assumed that this is also a good result. The table 7.6 presents this summary.

Table 7.6: Summary of the problem (L-shaped room with two exits)

Type of problem	DoE techniques used	RSM used	Numerical Optimisation Techniques used
Constrained	The random DoE technique	Multiple Regression Function	• Gradient-based technique (Fletcher-Reeves) • Non gradient-based technique (Particle Swarm Optimisation)

In the following sections of this chapter, the details on how the problem was solved and the results are presented and discussed.

7.6 The scenarios

7.6.1 The simulations

The simulations, the evacuation model used and the parameters used for the simulations here for this case are the same which were presented and discussed for the squared room cases in the previous chapters 5 and 6.

Nevertheless, it is important to mention again that, given that each particular scenario is repeated 600 times, an average of the averages can also be determined. These are represented as AV CWT, AV PET and AV Dist.

Unlike the CWT, PET and Dist, the TET is not a personal parameter but represents the total evacuation time for a particular simulation. As each particular scenario is repeated 600 times, the average TET for the scenario can be determined and is represented by AV TET.

For each simulation, the location of each occupant was randomized.

And according to what was mentioned before for the squared room with two exits, the positions are measured in terms of the relative distance between these two exits along the perimeter of the room. Thus, the distance from the left edge of each exit to the corner is measured similarly to the way that was done for the single exit case. These distances are going to be the design variables for the two exits case. The domains of these design variables are:

$$0 < D1 < 47 \qquad 0 < D2 < 47$$

D1 represents the distance from the left edge of exit 1 to the corner

D2 represents the distance from the left edge of exit 2 to the corner

Given that the L-shaped room configuration, see Figure 7.1, these two exits can be located anywhere along its perimeter. Considering also that each exit is a 1m width exit, the domain could be represented by a line, in which the total length is 47m, see Figure 7.7.

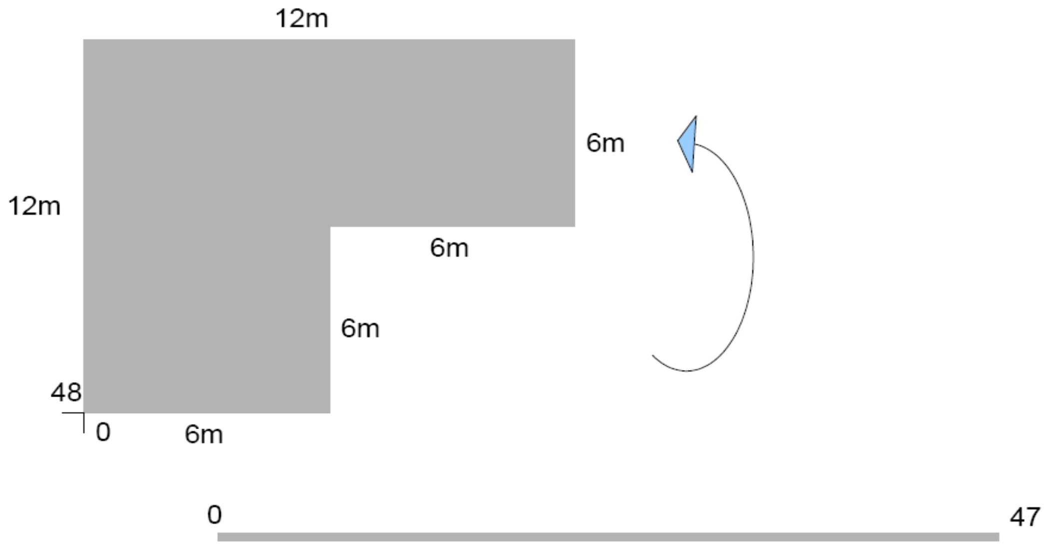


Figure 7.7: Domain for the exit positions for L-shaped room with two exits case with 1.0m wide

It is important to observe that, given this configuration, the two exits can overlap each other, once they are free to be located anywhere between 0 and 47 of this line. For this reason, based on this constraint, the problem was solved as a constrained problem as and the constraint was represented by the following inequality:

$$|D1 - D2| \geq 1$$

With this consideration, it was possible then to avoid the overlapping of the two exits. Therefore, the problem was can be stated as follows:

Minimize: $ET = f(D1, D2)$

Subject to: $|D1 - D2| \geq 1$

Where: $0 < D1 < 47$

$0 < D2 < 47$

Where:

ET – the evacuation time;

D1 - the distance from the left edge of exit 1 to the corner;

D2 - the distance from the left edge of exit 2 to the corner.

7.7 Results

According to what was mentioned previously, the design points used for the simulations of the study case were based on the random DoE technique. For the reason explained before in chapter 6, 3 sets of design points were defined, namely: a) 3 sets of 4 design points; b) 3 sets of 6 design points; c) 3 sets of 10 design points and d) 3 sets of 20 design points.

For design point (which represents a scenario to be analysed through evacuation modelling), 600 simulations were performed. Therefore, a total of 72,000 simulations were performed. (Additional 28,800 simulations were also performed previously in order to find the best L-shaped room area for this analysis).

In this study, the STATISTICA computational package was used for doing the numerical approximations when developing the RSM. And the numerical optimisation techniques were computed using the computational package VisualDOC.

The results for this problem are presented in the following paragraphs.

7.7.1 The 3 sets of 4 design points

For these sets of 4 design points, just the second set produced reasonable design points. In other words, just the second set generated design points able to develop a response surface.

The first and the third sets of 4 design points did not produce design points able to develop a response surface which could represent reasonably the design space. For this reason, using these two sets, it was not possible to find the optimal solution.

The figures 7.8, 7.9 and 7.10 present the design points for sets 1, 2 and 3 respectively. And Figure 7.11 presents the response surface built based on the 4 design points of the second set.

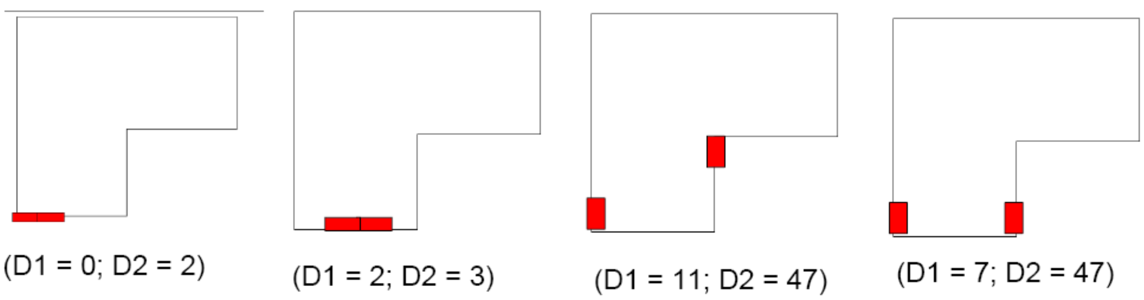


Figure 7.8: Scenarios based on the design points picked up by the random DoE technique (based on the 1st set of 4 design points using the random DoE technique)

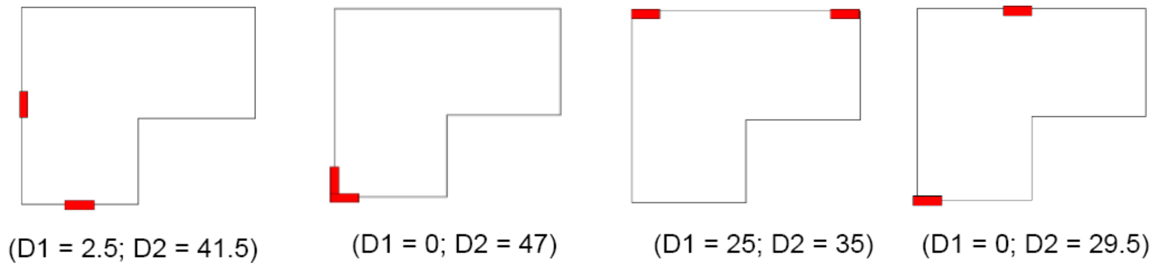


Figure 7.9: Scenarios based on the design points picked up by the random DoE technique (based on the 2nd set of 4 design points using the random DoE technique)

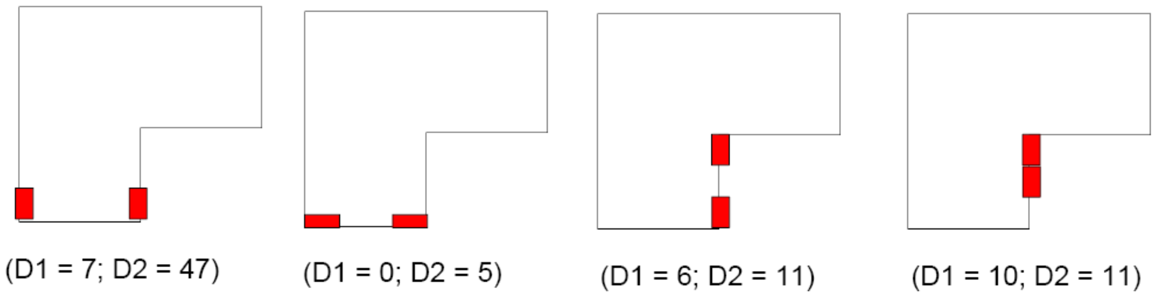


Figure 7.10: Scenarios based on the design points picked up by the random DoE technique (based on the 3rd set of 4 design points using the random DoE technique)

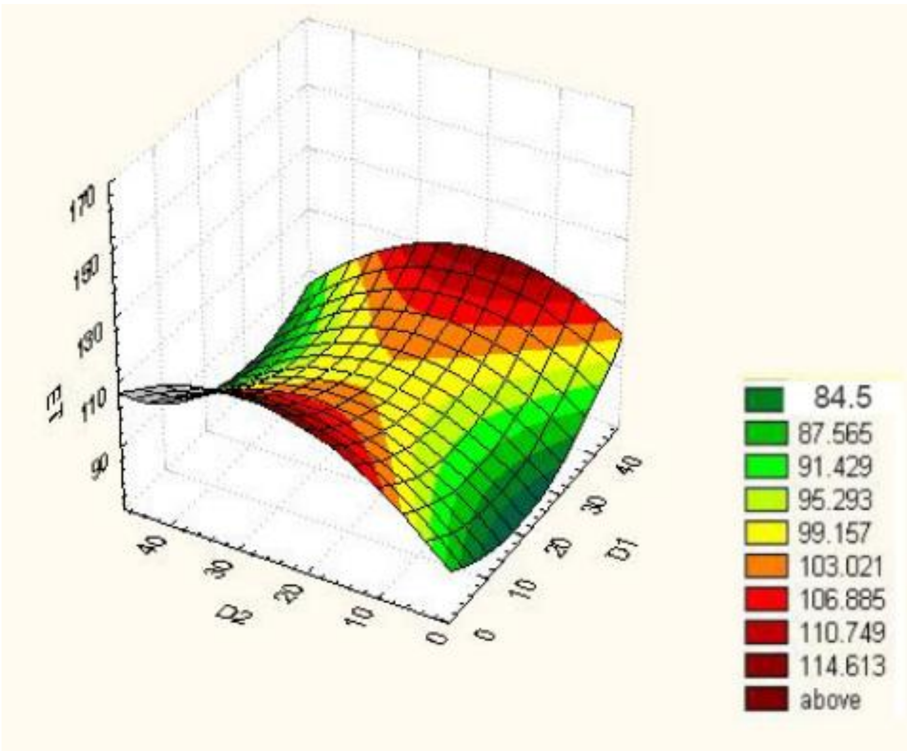


Figure 7.11: Response surface for the L-shaped room with two 1.0m exits (based on 2nd set of 4 design points using the random DoE technique)

The optimal solution was found for the coordinate (47; 46; 84.5). This means that when D1 = 47 and D2 = 46, the evacuation time would be 84.5 sec.

7.7.2 The 3 sets of 6 design points

For these sets of 6 design points, the first set did not produce reasonable design points. Just the second and the third sets generated design points able to develop a response surface.

The figures 7.12, 7.13 and 7.14 present the design points for sets 1, 2 and 3 respectively. And Figures 7.15 and 7.16 present the response surface built based on the 6 design points of the second and third sets.

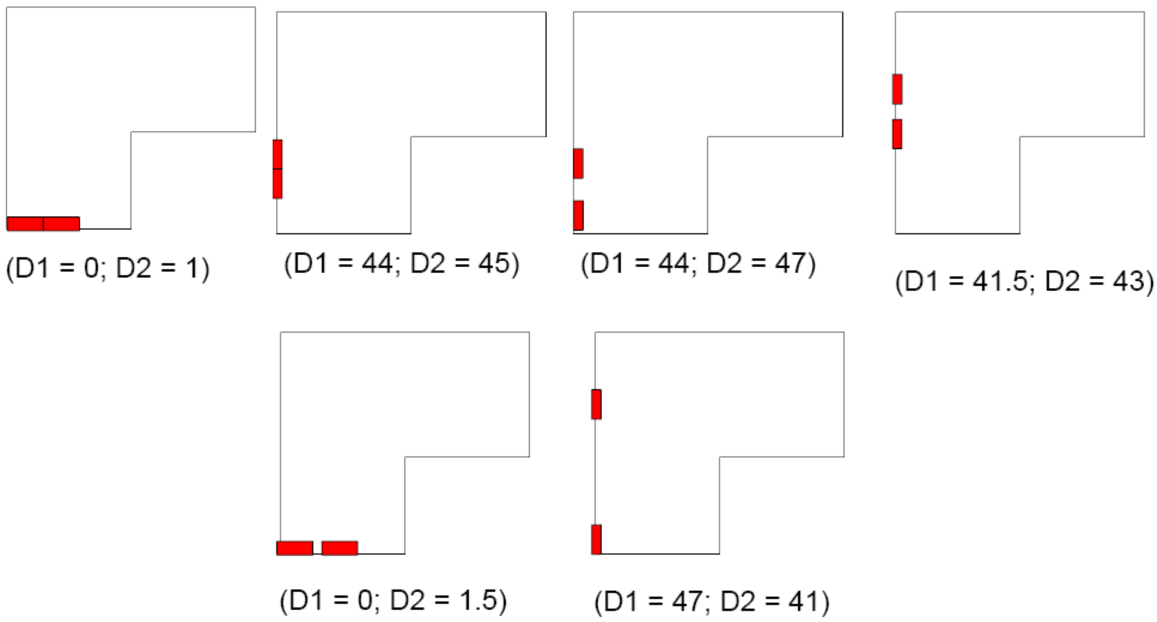


Figure 7.12: Scenarios based on the design points picked up by the random DoE technique (1st set of 6 design points)

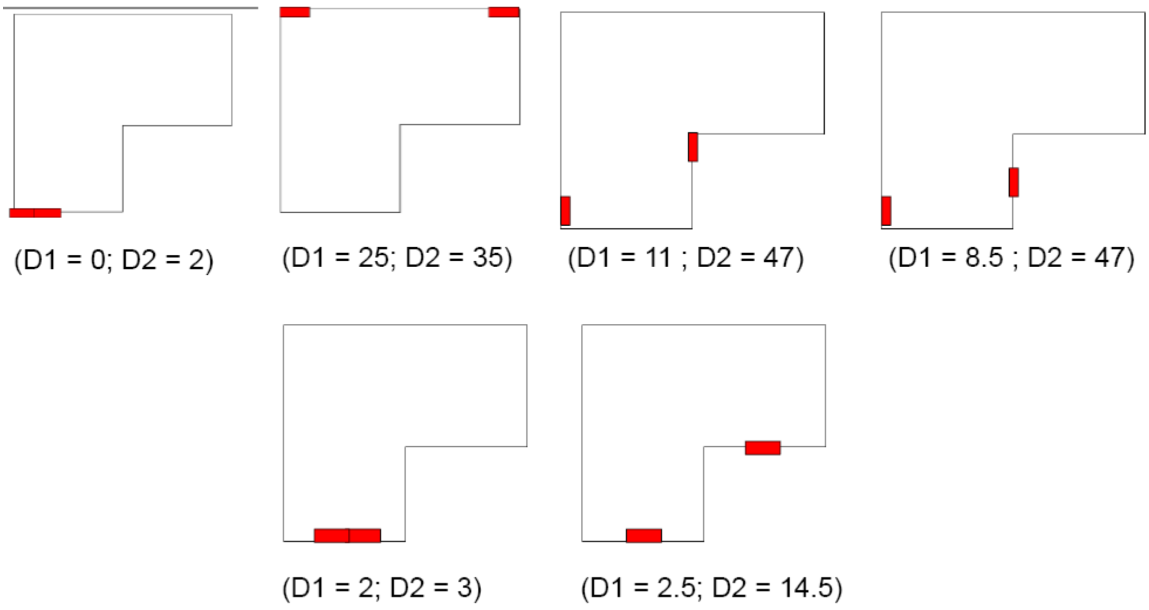


Figure 7.13: Scenarios based on the design points picked up by the random DoE technique (2nd set of 6 design points)

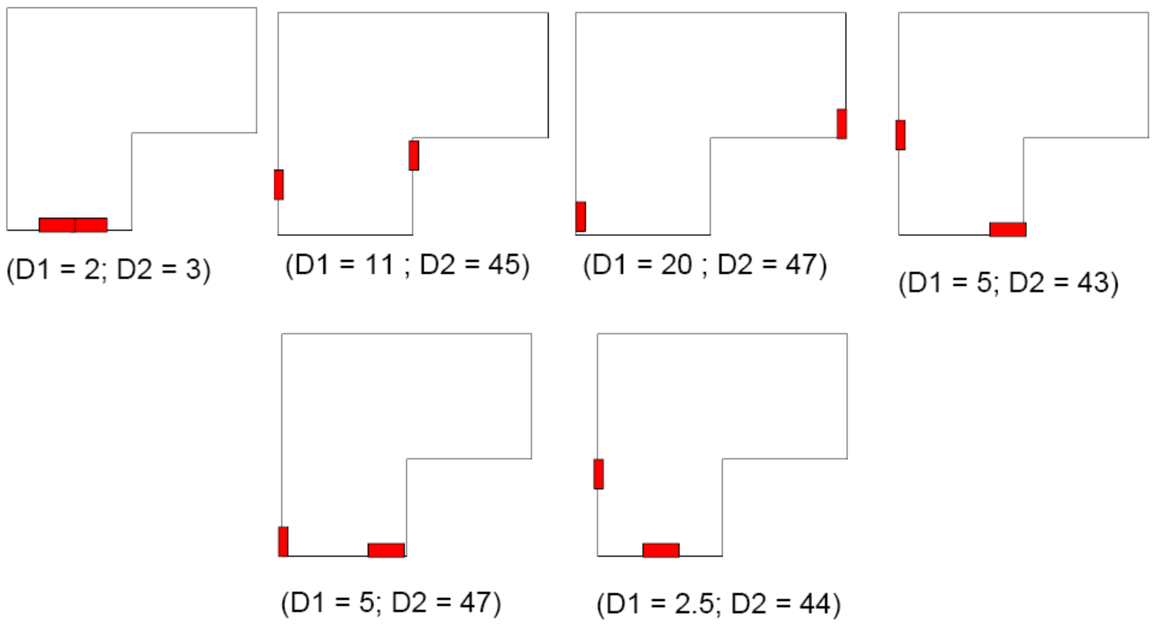


Figure 7.14: Scenarios based on the design points picked up by the random DoE technique (3rd set of 6 design points)

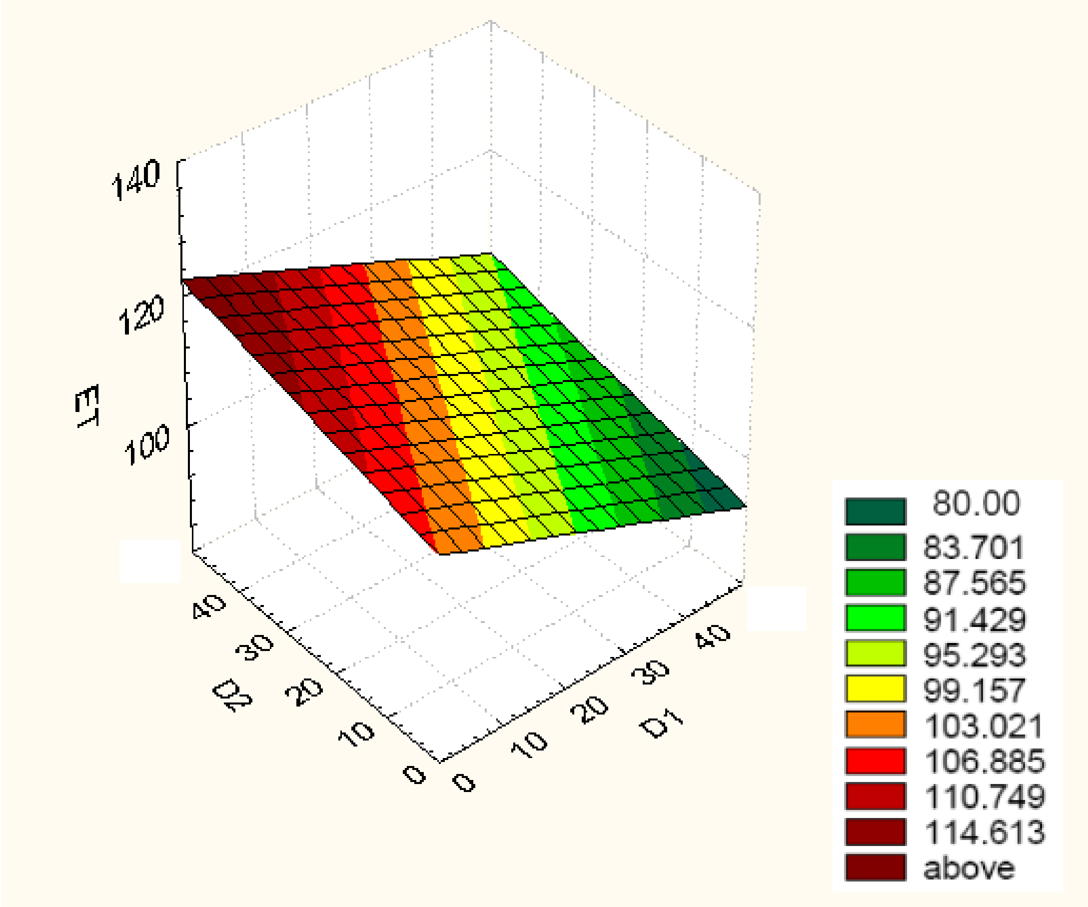


Figure 7.15: Response surface for the L-shaped room with two 1.0m exits (based on the 2nd set of 6 design points using the random DoE technique)

This is a substantial error and from Figure 7.15 it can be possible to see why. When these two response surfaces, the one built based on the 22 design points, see Figure 7.5, and the one shown in Figure 7.15, are compared, visually it is possible to see that they are completely different. Nevertheless, the response surface built based on the 6 design points shown in Figure 7.13, was capable to pass through regions close to the global minima region. And for this reason, it found the coordinate (47; 0; 80.45). This means that when D1=47 and D2 = 0, the evacuation time produced is 80.45 sec.

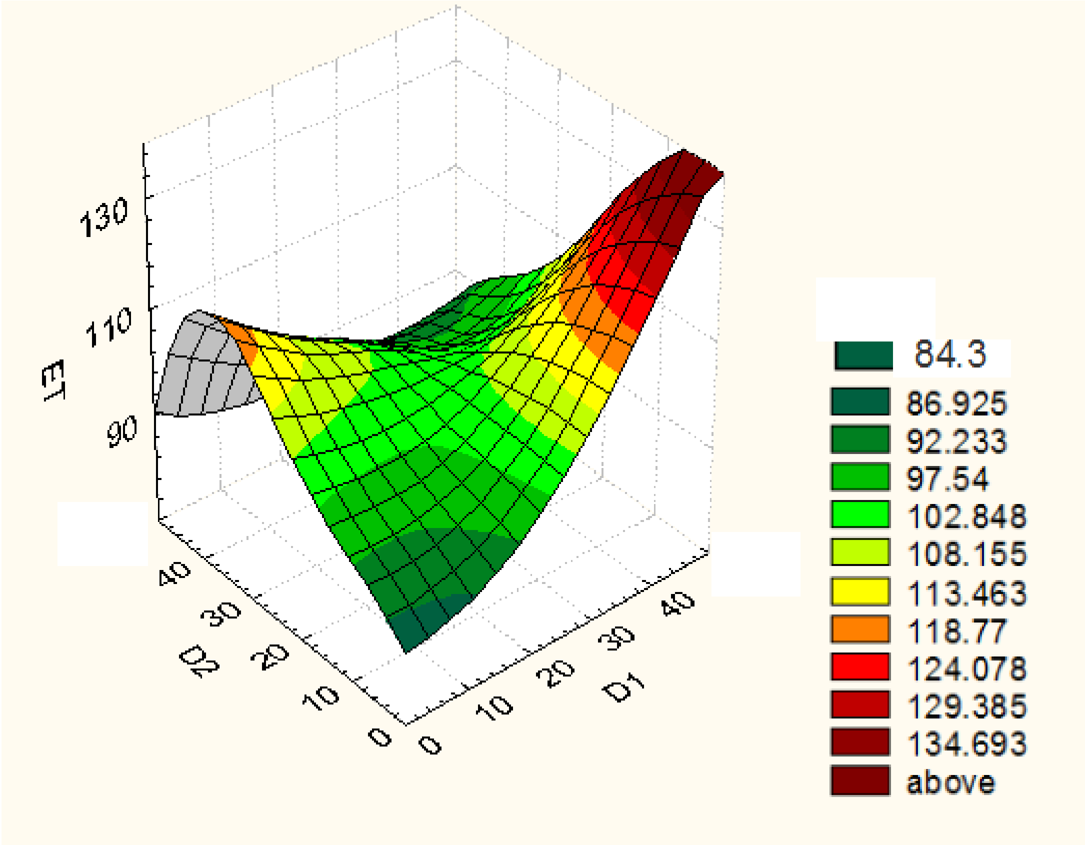


Figure 7.16: Response surface for the L-shaped room with two 1.0m exits (based on the 3rd set of 6 design points using the random DoE technique)

The optimisation analysis has found the coordinate (0; 0; 84.15sec). This means that when D1 = 0 and D2 = 0, the evacuation time, according to this RSM, would be 84.15sec.

7.7.3 The 3 sets of 10 design points

For these sets of 10 design points, all of them produced reasonable design points. Therefore, it was possible to build response surface based on these design points and finally to complete the optimisation analysis.

The design points generated by sets 1 and 2 were very similar and for this reason, the response surfaces were also similar.

The figures 7.17, 7.18 and 7.19 present the design points for sets 1, 2 and 3 respectively. And Figures 7.20, 7.21 and 7.22 present the response surface built based on the 10 design points of the first, second and third sets respectively.

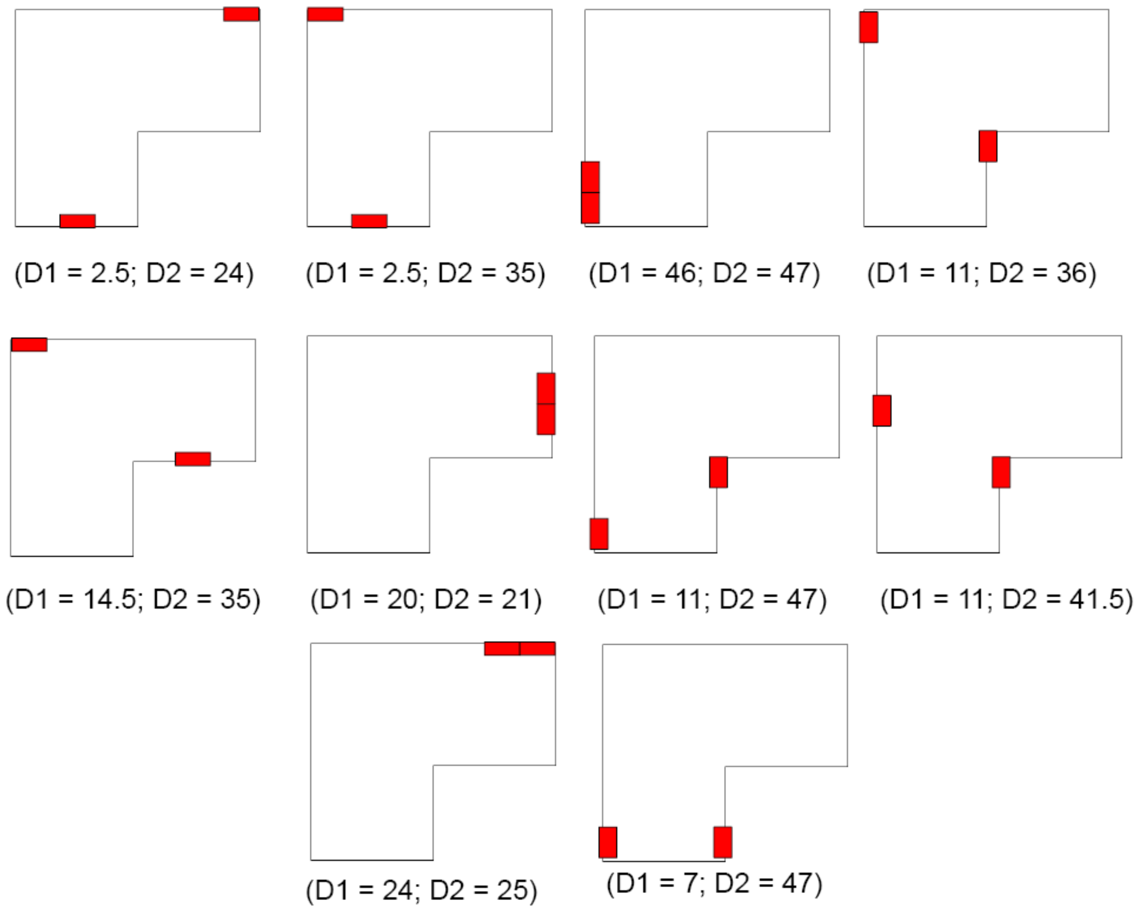


Figure 7.17: Scenarios based on the design points picked up by the random DoE technique (1st set of 10 design points)

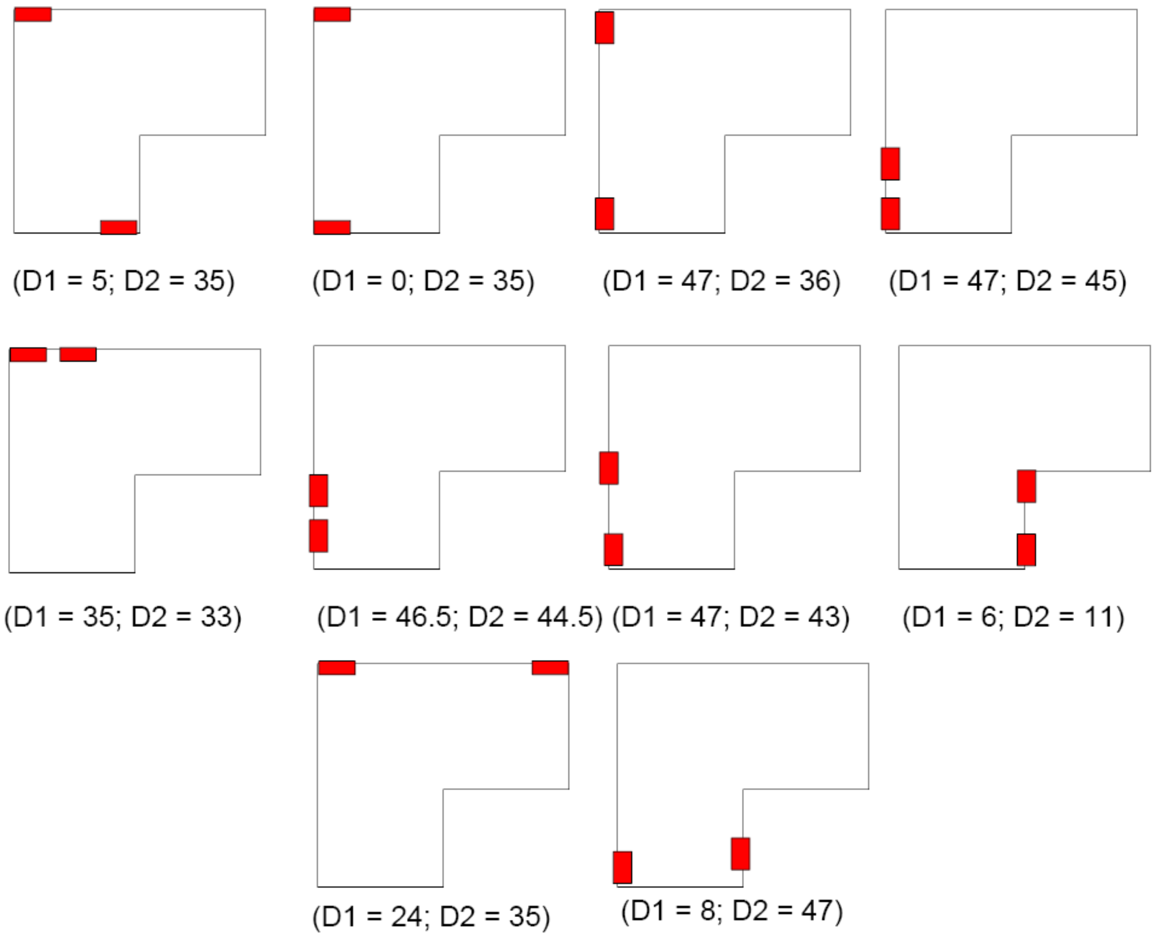


Figure 7.18: Scenarios based on the design points picked up by the random DoE technique (2nd set of 10 design points)

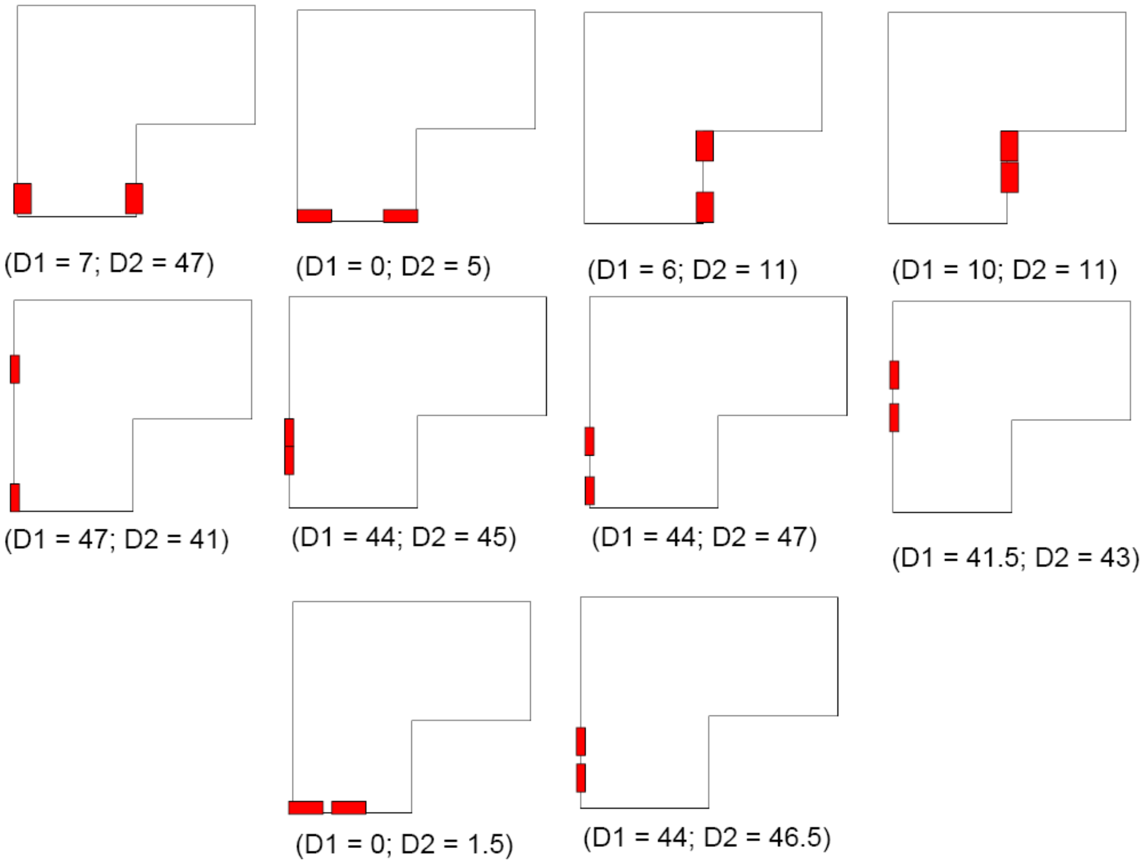


Figure 7.19: Scenarios based on the design points picked up by the random DoE technique (3rd set of 10 design points)

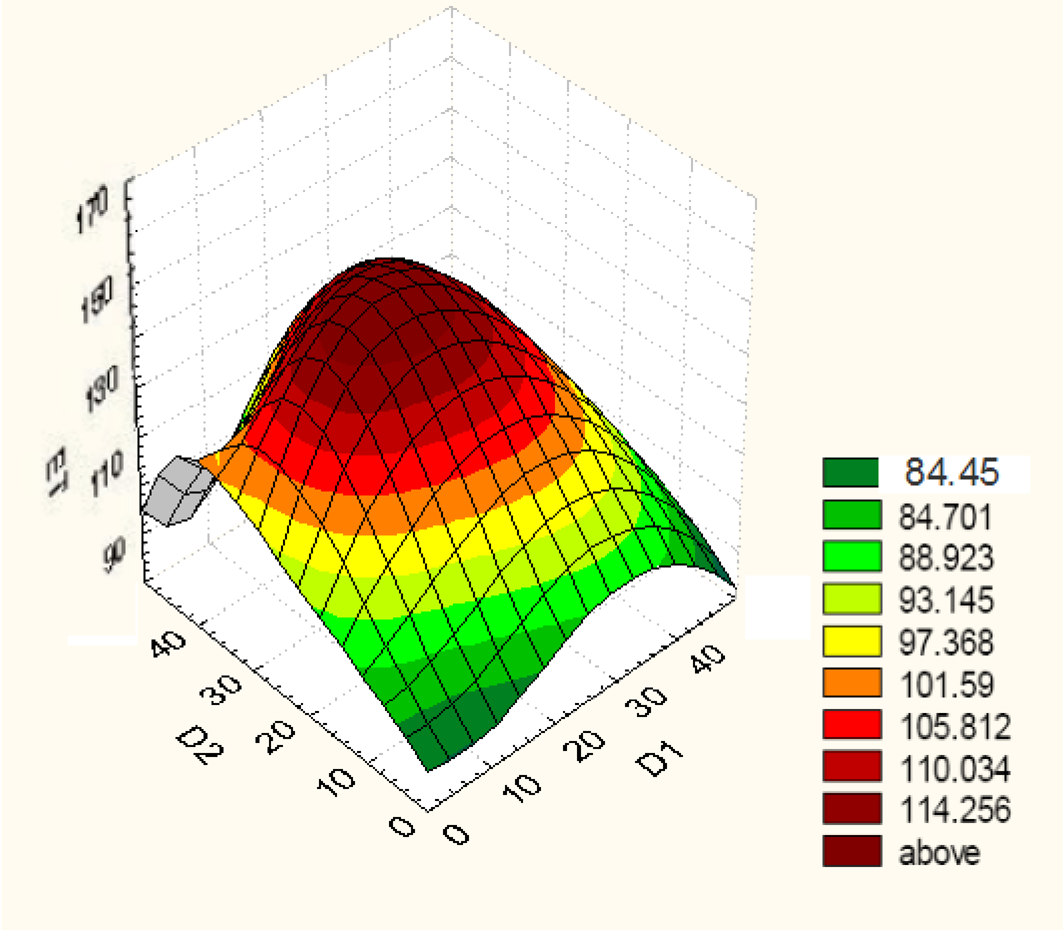


Figure 7.20: Response surface for the L-shaped room with two 1.0m exits (based on the 1st set of 10 design points using the random DoE technique)

The optimisation analysis has found the coordinate (47; 0; 84.56). This means that when D1= 47 and D2 = 0, the evacuation time, according to this RSM, would be 84.56sec.

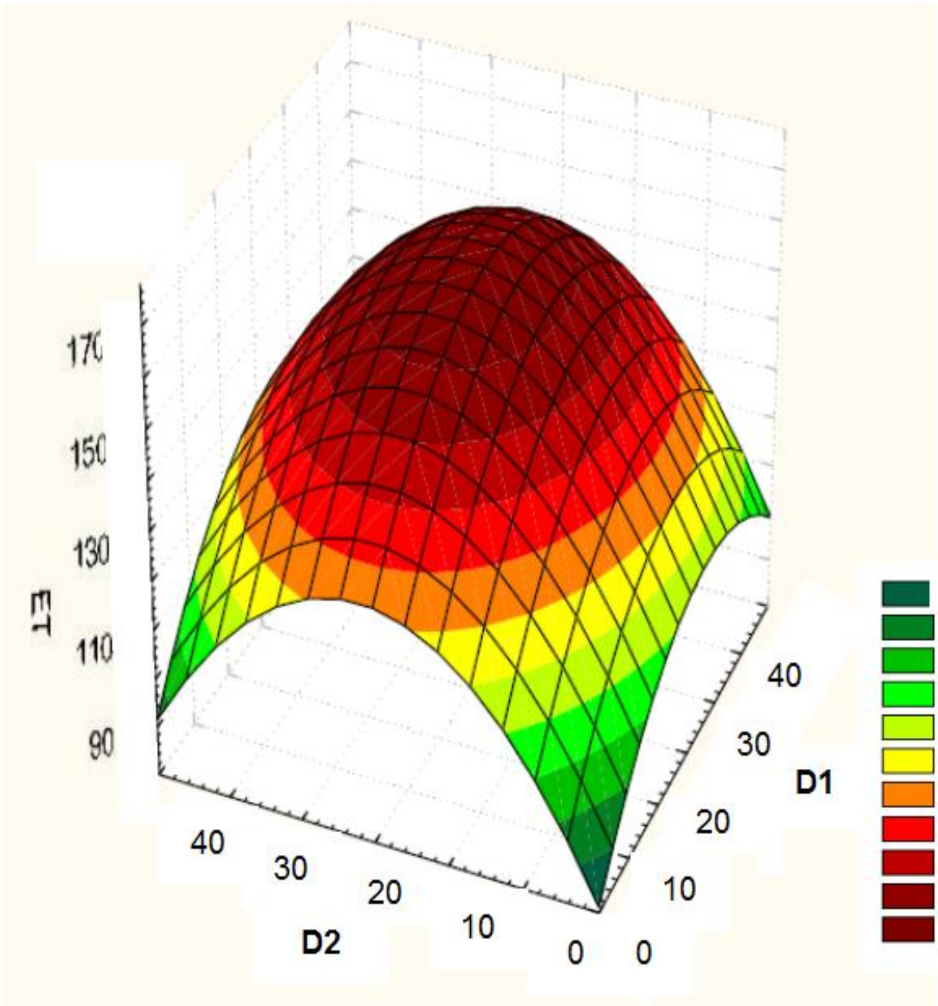


Figure 7.21: Response surface for the L-shaped room with two 1.0m exits (based on the 2nd set of 10 design points using the random DoE technique)

The optimisation analysis has found the coordinate (0; 0; 80.2). This means that when D1=0 and D2 = 0, the evacuation time, according to this RSM, would be 80.2sec.

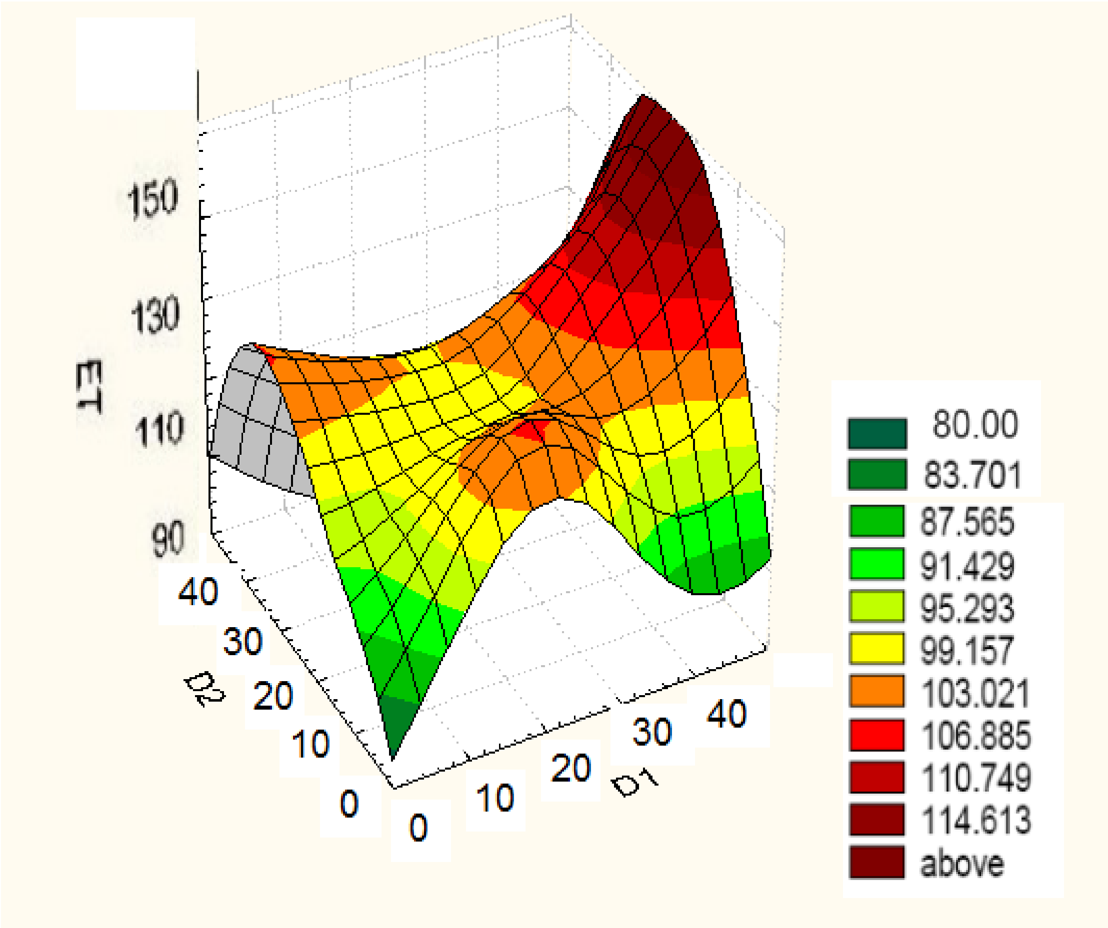


Figure 7.22: Response surface for the L-shaped room with two 1.0m exits (based on the 3rd set of 10 design points using the random DoE technique)

The optimisation analysis has found the coordinate (0; 0 ; 80.5). This means that when D1=0 and D2 = 0, the evacuation time, according to this RSM, would be 80.5sec.

7.7.4 The 3 sets of 20 design points

Similarly to the previous sets of 10 design points, for these final sets of 20 design points, all of these points produced reasonable design points. Furthermore, it was also possible to build response surface based on these design points and finally to complete the optimisation analysis.

The design points generated by these 3 sets of 20 different design points were very similar and for this reason, the response surfaces were also similar.

The figures 7.23, 7.24 and 7.25 present the design points for sets 1, 2 and 3 respectively. And Figures 7.26, 7.27 and 7.28 present the response surface built based on the 20 design points of the first, second and third sets respectively.

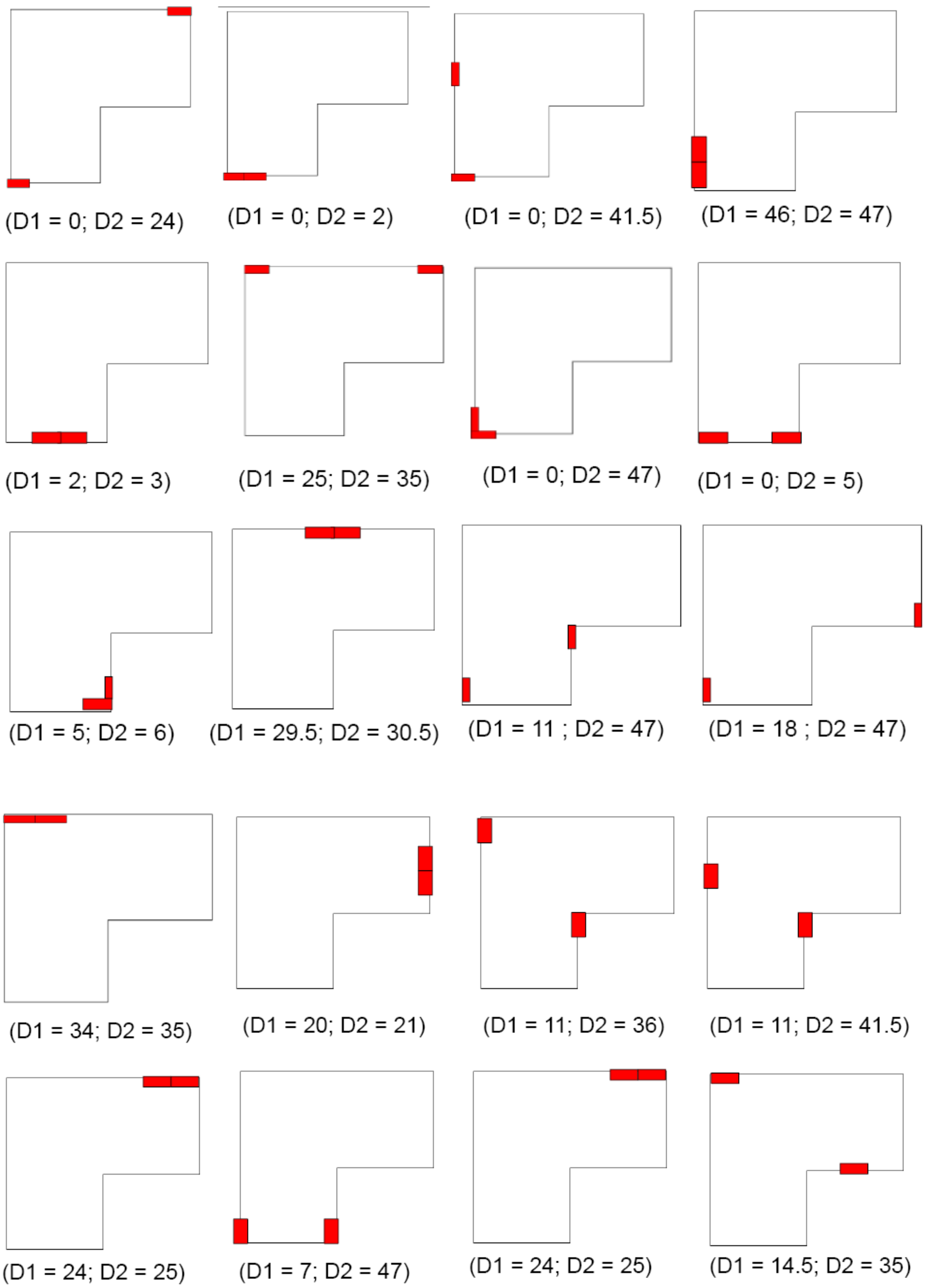


Figure 7.23: Scenarios based on the 1st set of 20 design points picked up by random DoE technique

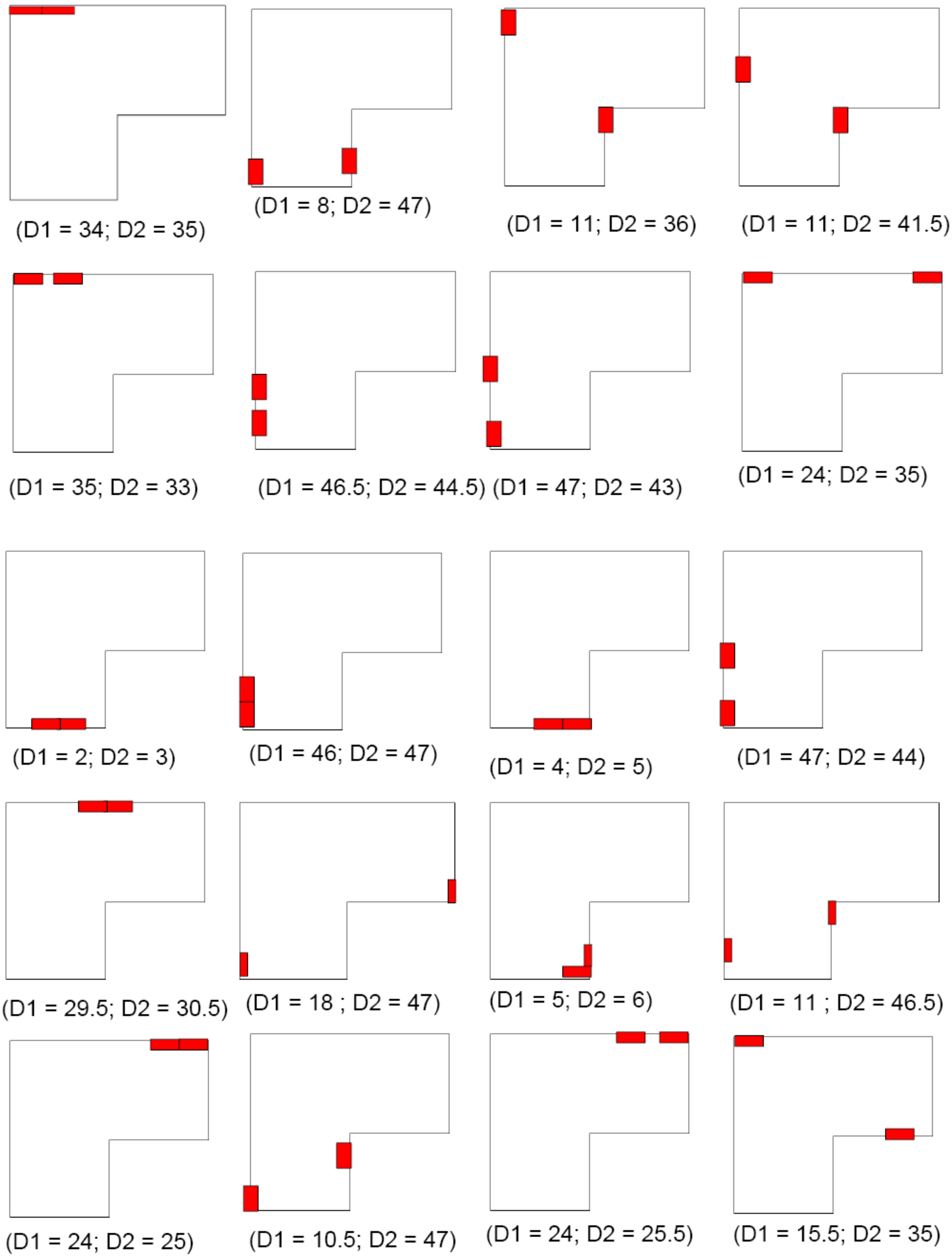


Figure 7.24: Scenarios based on the 2nd set of 20 design points picked up by random DoE technique

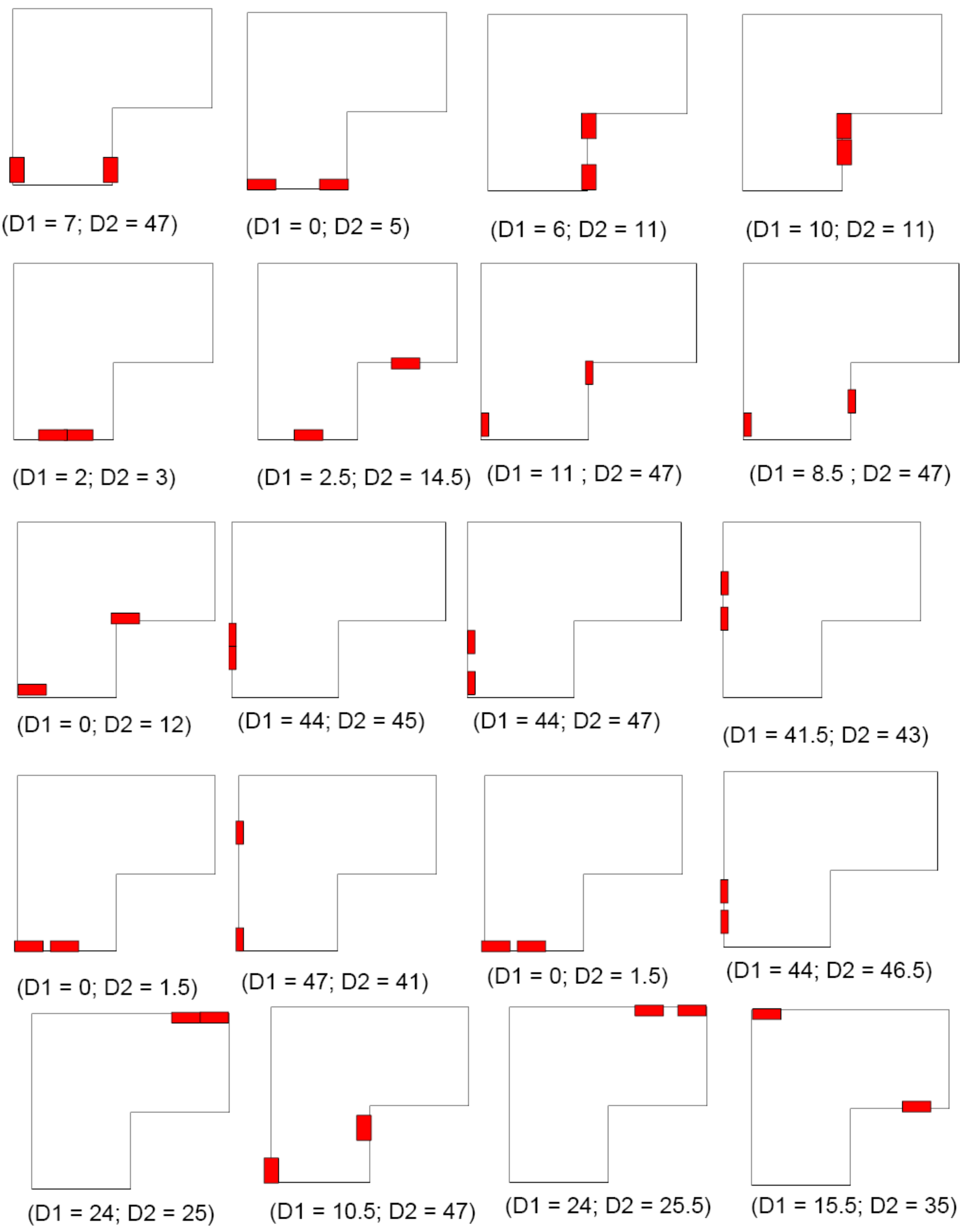


Figure 7.25: Scenarios based on the 3rd set of 20 design points picked up by random DoE technique

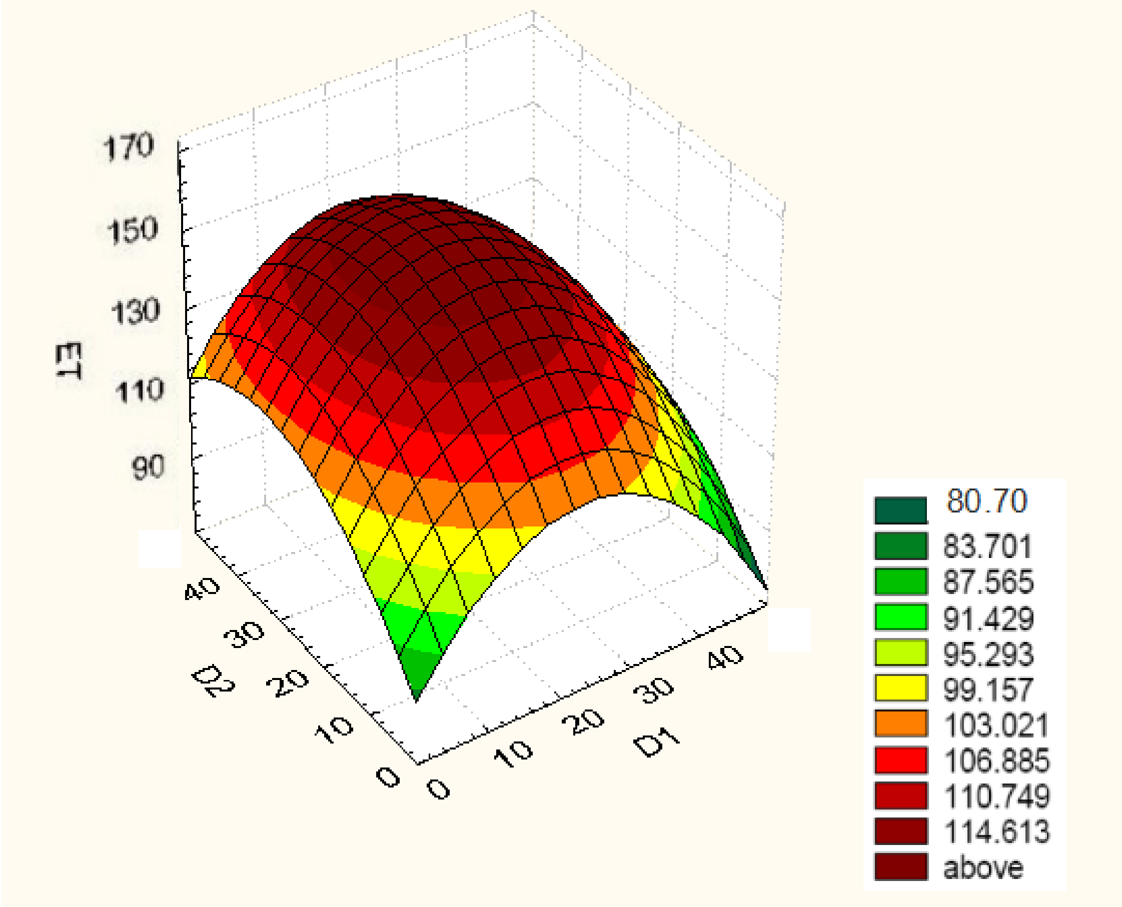


Figure 7.26: Response surface for the L-shaped room with two 1.0m exits (based on the 1st set of 20 design points using the random DoE technique)

The optimisation analysis has found the coordinate (47; 0; 80.95). This means that when D1= 47 and D2 = 0, the evacuation time, according to this RSM, would be 80.95sec.

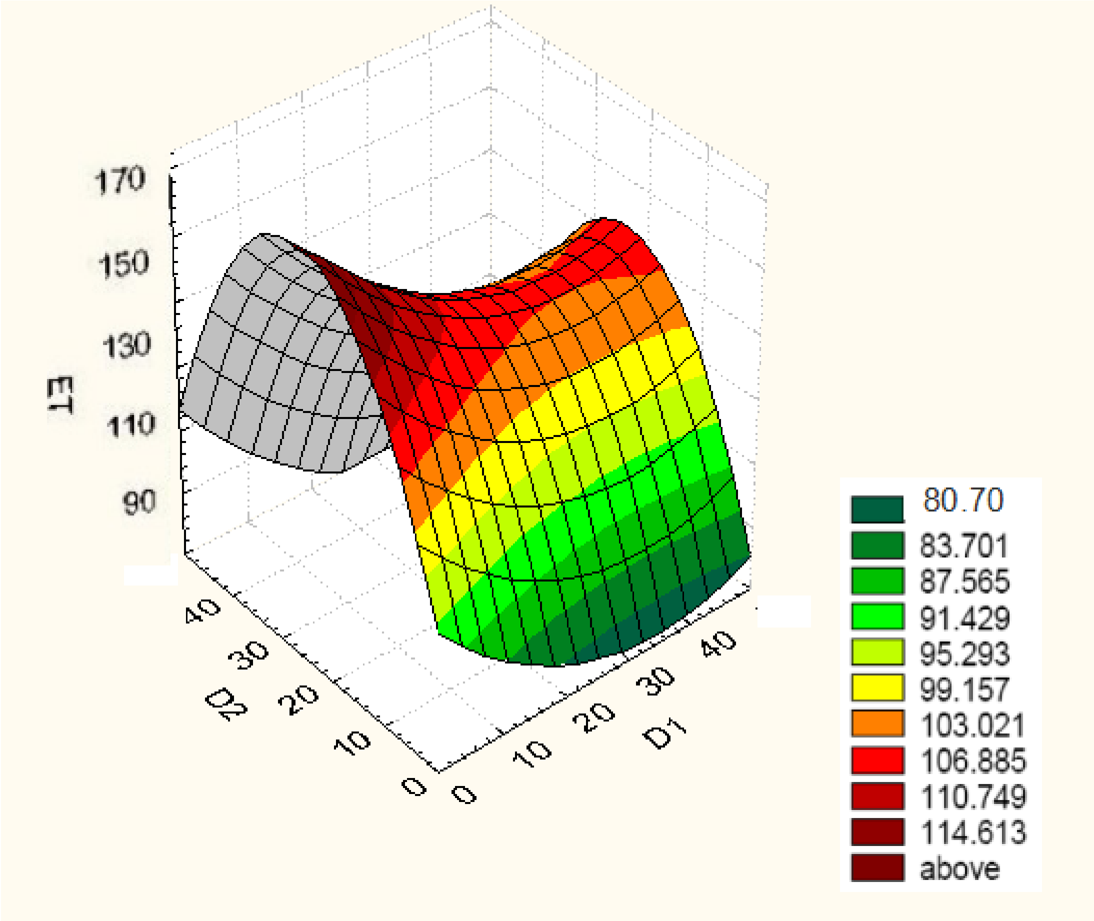


Figure 7.27: Response surface for the L-shaped room with two 1.0m exits (based on the 2nd set of 20 design points using the random DoE technique)

The optimisation analysis has found the coordinate (47; 0; 80.02). This means that when D1= 47 and D2 = 0, the evacuation time, according to this RSM, would be 80.02sec.

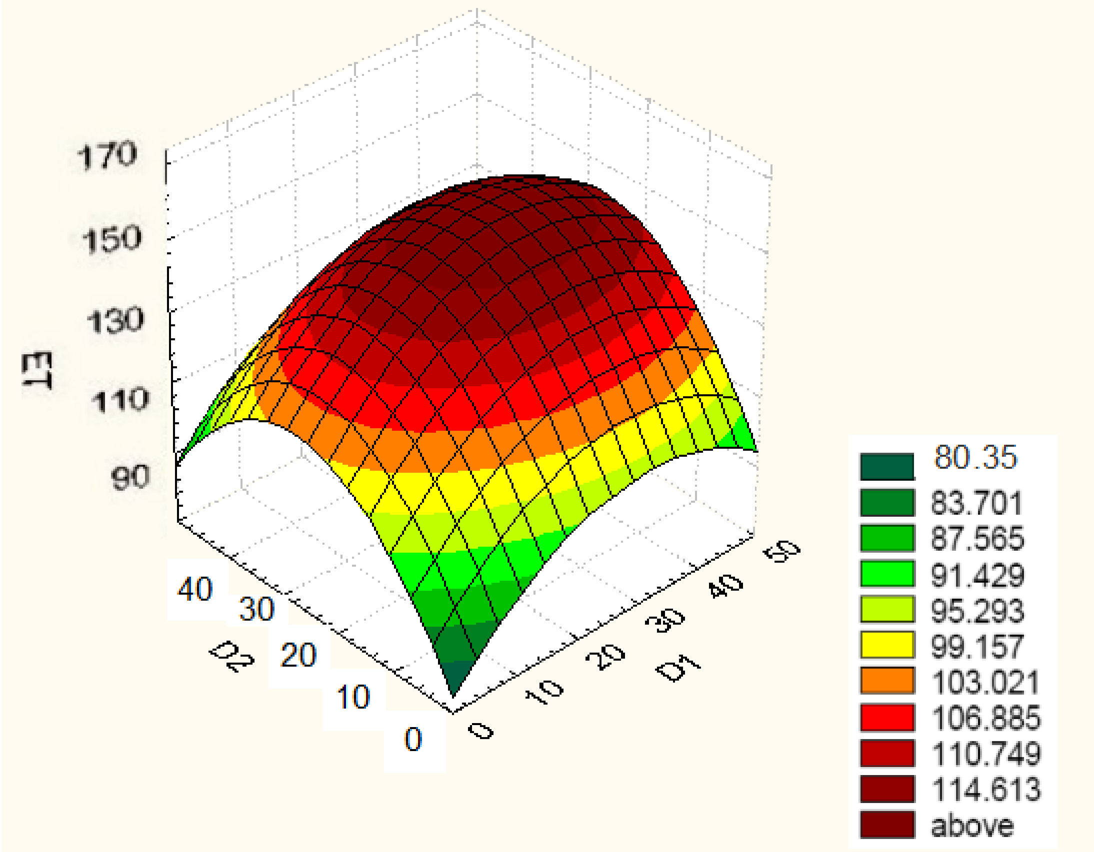


Figure 7.28: Response surface for the L-shaped room with two 1.0m exits (based on the 3rd set of 20 design points using the random DoE technique)

The optimisation analysis has found the coordinate (0; 0; 80. 4). This means that when D1=0 and D2 = 0, the evacuation time, according to this RSM, would be 80.4sec.

The summary of all of these results are presented in Table 7.7.

Table 7.7: Optimal solution for the L-shaped room with two 1.0m wide exits based on the design points suggested by the random DoE technique (constrained problem)

	SET 1		SET2		SET3	
Numerical Optimisation Technique	**Fletcher-Reeves**	**PSO**	**Fletcher-Reeves**	**PSO**	**Fletcher-Reeves**	**PSO**
4 design points (random DoE technique)	N.S.	N.S.	(47; 46;84.5)	(47; 46;84.5)	N.S.	N.S.
6 design points (random DoE technique)	N.S.	N.S.	(47; 0;80.45)	(47; 0;80.45)	(0; 0; 84.15)	(0; 0; 84.15)
10 design points (random DoE technique)	(47; 0; 84.56)	(47; 0; 84.56)	(0; 0; 80.2)	(0; 0 ;80.2)	(0 ; 0 ; 80.5)	(0; 0; 80.5)
20 design points (random DoE technique)	(47; 0; 80.95)	(47; 0; 80.95)	(47; 0; 80.02)	(47; 0; 80.02)	(0; 0;80. 4)	(0; 0;80.4)

N.S. - No Solution found.

It is possible to observe from these results that, for 4 design points, the results were inconsistent for sets 1 and 3, since negative results were found. However for set 2 the results were reasonable

For 6 design points, the results obtained in the 3 sets were more consistent than for 4 design points, apart from set 1, where negative values were found.

For 10 design points, the results were consistent for all the three sets, but the values obtained had some difference when compared with the results for 20 design points.

For 20 design points, the results were almost consistent. And in fact, a local region very close to the global minima region was found for set 2.

In summary, most of these coordinates were close to the global minima which is (47;0; 80). There were some inconsistencies as it was expected given the limitations of the random DoE technique when selecting the design points, as mentioned in the previous chapters (see chapter 3 and chapter 6).

The best RMS’s, for all sets was the high order polynomial function.

Clearly it has shown that the error on the RSM decreased at the moment that the number of design points increased.

Once again, for the two numerical optimisation techniques, a gradient-based technique, Fletcher-Reeves, and a non gradient-based technique, PSO, the results were consistent.

As it was shown in table 7.5, the results were very similar. The scenarios where the coordinates were (D1=46;D1=47); (D1=0;D2=47); (D1=0; D2=1) and variations of that were found. The Figure 7.29 presents these scenarios which the design points picked up by the random DoE technique suggested.

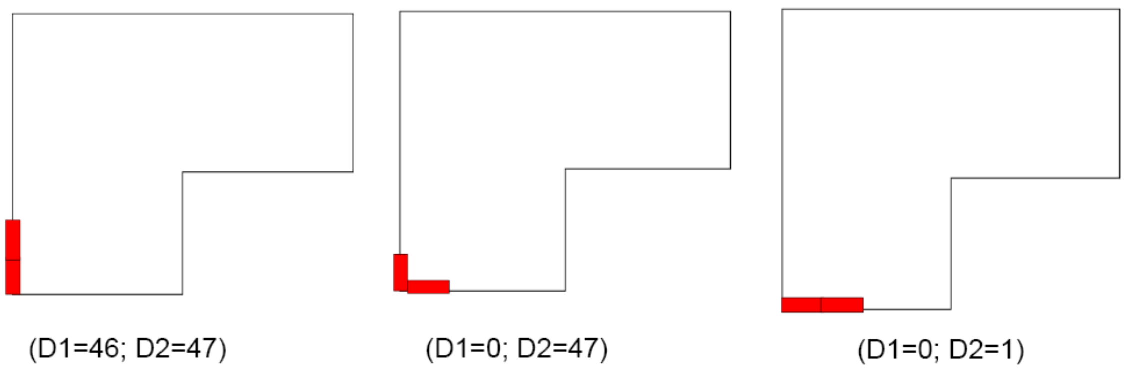


Figure 7.29: Scenarios based on the design points picked up by the random DoE technique

The coordinate where D1=46 and D2=47 was picked up by the following sets: set 2 using 4 design points; sets 1 and 2 using 10 design points and set 3 using 30 design points.

The coordinate where D1=0 and D2=47 was picked up by the following sets: sets 2 and 3 using 6 design points; set 3 using 10 design points and set 1 using 20 design points.

And the coordinate where D1=0 and D2=1 was picked up by set 2 using 20 design points.

7.8 Concluding Comments

In this chapter, the optimal positioning of exits around the perimeter of an L-shaped room and compartmented room in order to minimise evacuation times were explored. The evacuation simulations were conducted assuming ideal conditions of zero response times and population behaviour such that occupants would move to their nearest exits. Both assumptions are made to simplify the analysis and to isolate issues associated with exit location.

This investigation revealed also that significant advantage can be derived from the strategic positioning of the exits.

The solutions were found through the use of numerical optimisation techniques.

The analysis revealed that this systematic methodology to efficiently optimise evacuation safety aspects of structural designs can be applied to more complex designs than just a squared room.

In the next chapter, a second case study with more complex scenario is presented and solved.

8. More Complex Geometries: Case Study 2 – Multi-compartment room

8.1 Introduction

This chapter presents a second case study using more complex geometry than analysed in the previous chapter.

This case study is based on a multi-compartment room. The rooms have the same overall square floor plan as presented previously in chapters 5 and 6, with the same area and the same number of people. However, in this case, there is a smaller room that occupies one corner of the large square area (i.e., an inner room condition), see Figure 8.1. For this reason, despite being a similar geometry to the square room shown previously in chapters 5 and 6, an evacuation event in this particular situation of a multi-compartment room will present different behaviour trends as it will be shown and discussed later in this Thesis.

For this case study, two scenarios were analysed: scenario (1) has one exit from the smaller room leading to the larger room and one exit from the larger room leading to the exterior; and scenario (2) has one exit from the smaller room as in scenario 1 and two exits from the larger room to the exterior. As previously, the problem to be addressed is the determination of the exits locations which produce minimum evacuation times. Scenario 1 is solved as an unconstrained problem and scenario 2 as a constrained problem.

For scenario 1, there are 2 design variables, i.e., the position of the exit located in the smaller room and the position of the exit located in the larger room. As each exit has its own domain; i.e., one exit can be located anywhere along the smaller room perimeter; while the other exit can be located anywhere along the larger room perimeter, there are no constraints. As consequence of this, there is no chance of these exits overlapping each other.

In the second scenario, the problem is solved as a constrained problem. There are three design variables, i.e., the exit located in the smaller room and the other two exits located in the bigger room. Therefore, there are constraints in this problem, because the two exits located in the bigger room do have the same domain, and there is a risk of them overlapping. In other words, a constraint has to be defined, because the two exits in the bigger room perimeter cannot be overlapping.

The solution is found by applying numerical optimisation techniques. The data that was used for the optimisation study was obtained from evacuation simulations.

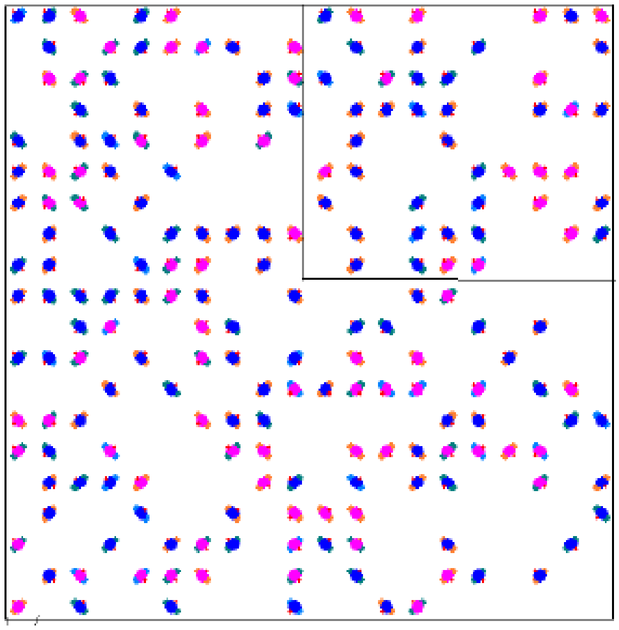


Figure 8.1: Multi-compartment room

8.2 The Problem

The problem to be investigated is simply stated as follows: for a room of given size, containing an arbitrarily large population, is there an optimal location for the exits that will minimise the evacuation times?

The same considerations that were made for the square room cases are also made for this study case. The “compartmented room” has an area of 100 m^2. The population of the room consists of 200 people, giving a population density of 2 people/m^2. This means that 50% of the area is populated, which gives 50% of freedom for the occupants’ movement during the evacuation process. It is important to note that 150 people are located in the bigger room and 50 people are located in the smaller room. This is a proportion that is consistent with the areas of the two rooms.

The exit locations are measured in terms of the distance from the left edge of the exit to the corner of the wall in an anti-clockwise direction, in the same way as the locations were defined for the other study cases.

Nevertheless, before solving this problem using the numerical optimisation techniques and the associated concepts (i.e., the methodology presented here in this study), several sets of simulations were performed in order to demonstrate the impact of the exits locations on multiple connected compartments.

With this information, it was possible to determine what the optimal solution would be, and then to apply the classical optimisation theory methodology to ascertain if it was able to find this solution.

Therefore, in the early stages of the solving process of this problem, several scenarios (i.e., with different exits locations) were investigated, in order to find the optimal solution. This process did not involve the use of the optimisation methodology. Instead, a data analysis based process was performed.

8.3. Investigating the impact of the exit locations on the evacuation efficiency for complex geometries (Multiple Connected Compartments) using brute force approach

8.3.1 Introduction

In this section, evacuation simulations were performed for investigating the impact of the exit locations on the evacuation efficiency for multiple connected compartments. These analyses were based on the brute force method.

For this investigation, two scenarios were analyzed, namely: scenario 1 and scenario 2. These scenarios are presented and discussed in the next sub-section.

8.3.2 Description of the Scenarios

Scenario 1 has one exit for the smaller room and one exit for the bigger room; while scenario 2 has one exit for the smaller room and two exits for the bigger room. As in previous cases, to determine the solutions by “informed brute force”, a number of key-positions were selected in order to cover the strategic exit layout positioning along the walls which could impact significantly the evacuation performance, see Figures 8.2 and 8.3.

For scenario 1, 36 key-positions were identified (see Figure 8.2); and for scenario 2, 40 key-positions were identified (see Figure 8.3).

For scenario 1, the key-positions were determined by keeping one exit located fixed and moving the other exit around the perimeter to key-positions.

And for scenario 2, given its complexity (and also the previous knowledge gained from the square room problems), the key-positions were determined combining possible different strategic positions for all the three exits (i.e., the small compartment exit and the other two larger compartment exits).

For scenario 1, the locations can be described as follows:

Positions 1 to 8: small compartment exit fixed in the corner; larger compartment exit moved from adjacent positions to key-locations on the four other walls.

Positions 9 to 12: larger compartment exit fixed in the corner; small compartment exit moved from adjacent positions to key-location on the the four walls.

Positions 13 to 20: small compartment exit fixed in the middle of the interior wall; larger compartment exit moved from adjacent positions for key-locations on the four other walls.

Positions 21 to 28: small compartment exit fixed in the other end corner; larger compartment exit moved from adjacent positions for key-locations on the four other walls.

Positions 29 to 36: small compartment exit fixed in the middle of the other interior wall; larger compartment exit moved from adjacent positions for key-locations on the four other walls.

For scenario 2, the locations were set up similarly to the way the locations for scenario 1 were set up. And taking into account the previous findings, most scenarios were based on the two larger compartment exits being side by side.

In this way, the majority of expected key-exit locations were covered for both scenarios 1 and 2 (see Figures 8.2 and 8.3).

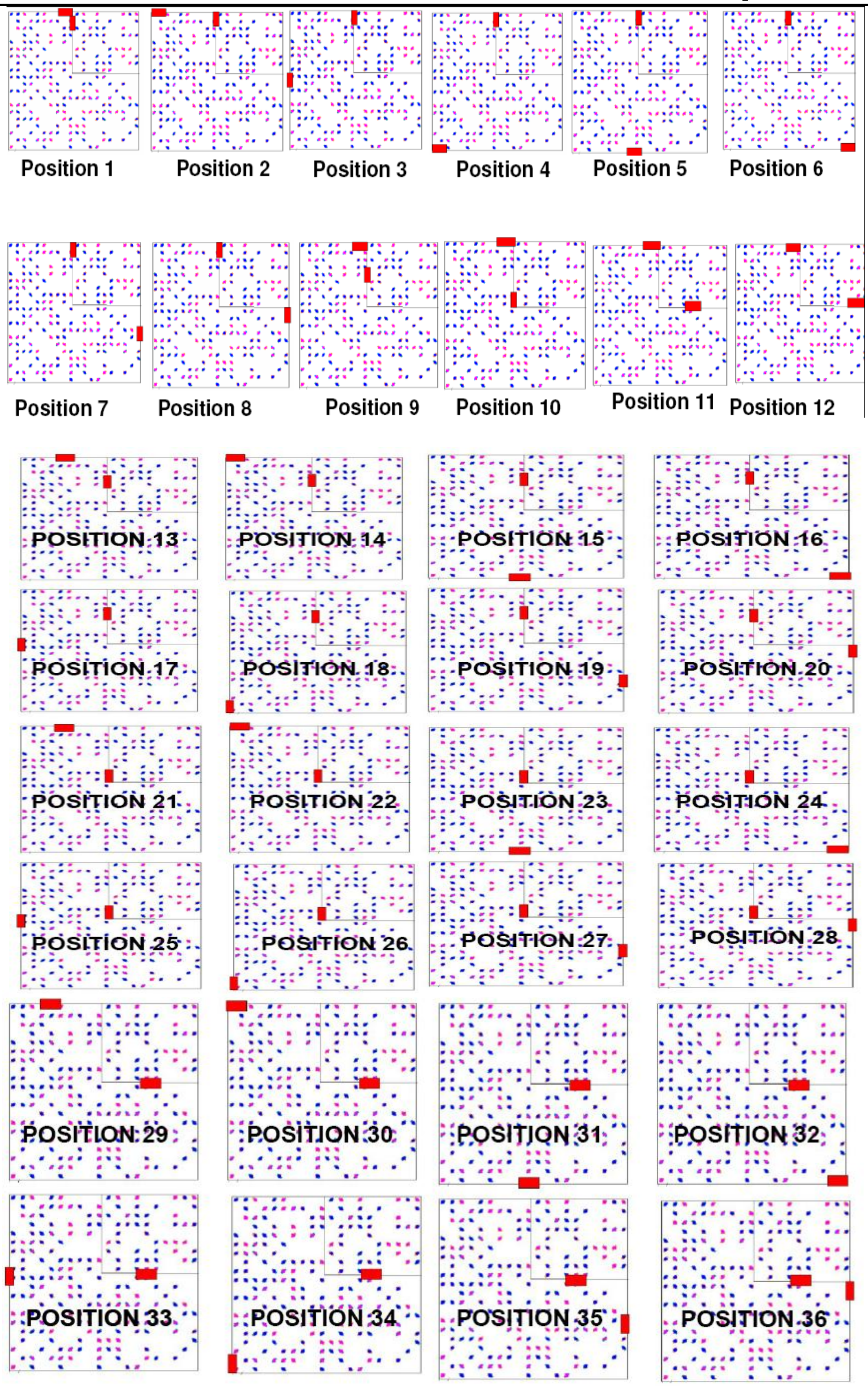


Figure 8.2: The 36 Key-positions for the exits in the Multiple Connected Compartments, scenario 1

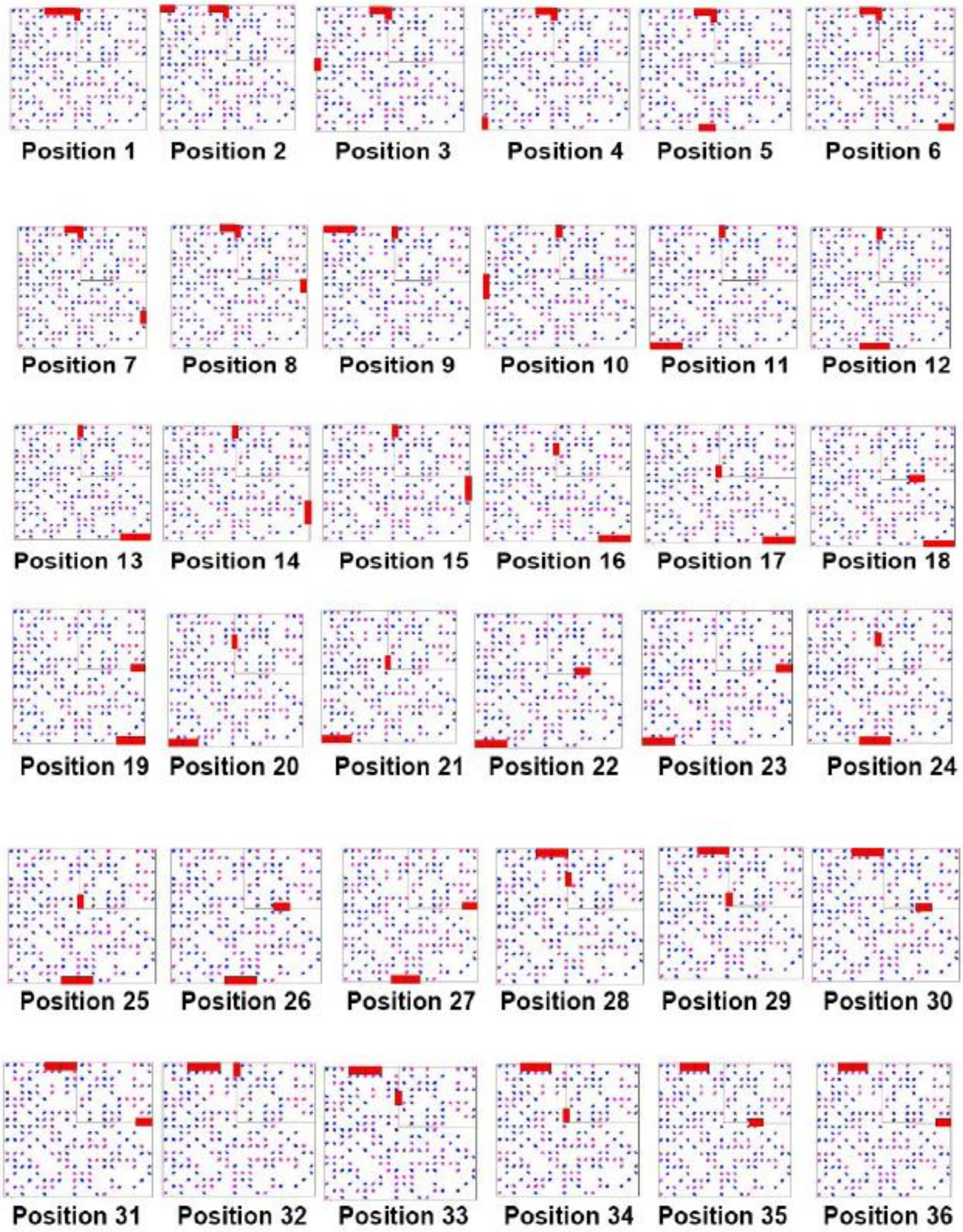


Figure 8.3: The 36 Key-positions for the exits in the Multiple Connected Compartments, scenario 2

In both figures 8.2 and 8.3, it is possible to see the key-positions, where the rectangles represent the exit positions.

Regards the scenario 2, it is clear that there are many other possible combinations than these key-positions investigated; nevertheless, it is already known that two exits side by side will generate lower evacuation times, because the flow rate increases. For this reason, most of the key-positions are based on the two exits located side by side the larger wall.

In the next section, the results for both scenarios are presented and discussed.

8.3.3 Results and Discussion

In total, 46,200 simulations were performed: 24,600 for scenario 1) and 21,600 for scenario 2). These simulations were performed in order to check if the exit(s) locations do have an impact on the evacuation efficiency, as they did for the square and rectangular rooms presented in chapters 5 and 6 as well the L-shaped room presented in chapter 7 of this thesis.

The same parameters and assumptions that were used for the previous evacuation modelling analysis were also used for this study.

It is clear to see, from the results obtained for this first set of simulations, that the exit locations do represent a major influencing factor on the evacuation efficiency as it has been shown in the previous chapters. This can be said because the exit locations will determine the travel distance and therefore, they will impact directly and/or indirectly the evacuation efficiency. According to what was already discussed along this Thesis, the evacuation efficiency can be "measured" by the evacuation time. And the evacuation time, restricting only to the physical aspects of the occupants' movement, can be seen as a function of two main variables, namely:

- The time spent during the actual movement within the enclosure towards the exit(s) (Tsm);
- The time spent around the exit(s) areas (Tsae).

Mathematically speaking:

$$\boldsymbol{ET = Tsm + Tsae} \qquad \text{Equation 8.1}$$

Furthermore, intuitively, it is possible to start to understand why and how the exits locations do have an effect on the evacuation efficiency. This can be said, because, as mentioned previously, the positioning of exits will potentially determine the travel distance(s) and consequently will impact both sub-times: ***Tsm*** and ***Tsae***. (It is important to mention, once again, that the population density is also influencing indirectly this relationship between exit(s) locations and travel distance).

In summary, the exit(s) locations will impact the Cumulative Waiting Time (CWT), which will be observed:

a) During the movement;
b) Around the exit(s) areas;
c) Or both combined (i.e., during the movement + around the exit(s) areas).

Based on that, it was possible to observe, through the results, that very likely these three mentioned situations influencing the evacuation times.

In the next paragraphs the results for both scenarios, 1 and 2, are presented and discussed.

8.3.3.1 Scenario 1)

As mentioned before, in total, 24,600 simulations were performed for this scenario.

The results have shown that position 1 produces the minimum value for the evacuation time: 164.75sec; and position 6 produces the worst value for the evacuation time: 176.59 sec. Table 8.1 presents the summary of these results.

Table 8.1: Complete results for the Multiple Connected Compartments, scenario 1 - one exit for the smaller room and one exit for the bigger room

Position	AV TET (sec)	AV CWT (sec)	SIM. AV Dist (m)	GEOM. AV Dist (m)	AV PET (sec)	Max. Travel Distance (m)
1	164.75	75.38	8.49	8.89	83.93	25.52
2	168.42	74.49	10.07	10.47	84.36	25.99
3	167.75	75.31	8.69	9.10	84.03	26.52
4	166.75	73.04	11.11	11.67	83.75	30.64
5	166.92	73.27	10.33	10.76	83.32	29.85
6	176.59	75.81	13.05	14.09	88.13	34.97
7	167.22	72.10	12.59	13.31	84.03	34.88
8	167.59	71.06	12.73	13.32	83.11	32.80
9	168.10	73.85	8.91	9.28	82.74	21.31
10	170.09	76.31	9.82	10.03	85.94	23.93
11	166.76	72.83	11.29	11.87	83.69	26.56
12	166.18	71.03	12.34	12.85	82.77	32.21
13	170.59	76.92	9.20	9.89	86.06	24.78
14	165.59	73.21	10.10	11.20	84.09	25.67
15	168.09	76.26	8.21	8.75	84.58	25.76
16	167.42	74.57	10.65	11.67	84.89	28.45
17	165.03	72.82	9.58	10.01	85.26	25.38
18	174.09	75.55	12.08	12.89	87.07	32.21
19	165.59	71.54	11.85	12.05	84.86	25.99
20	165.92	71.58	11.64	12.45	84.74	26.04
21	165.42	73.31	9.83	10.47	84.95	26.48
22	171.40	76.33	10.16	11.34	86.26	27.88
23	165.42	74.19	7.95	8.21	85.99	26.40
24	168.25	74.37	9.96	10.53	84.29	29.73
25	167.85	75.52	9.77	8.96	85.34	28.47
26	174.42	77.28	11.44	11.97	88.26	32.45
27	165.92	74.50	11.03	11.85	83.99	27.46
28	165.42	74.18	11.05	11.78	84.01	26.99
29	170.09	73.37	11.65	12.34	84.53	28.76
30	165.92	74.02	11.81	12.65	84.99	26.09
31	168.75	75.43	9.28	10.45	84.62	27.32
32	165.75	74.60	10.08	11.01	85.10	26.43
33	165.02	74.67	9.56	10.78	84.67	26.78
34	168.75	75.05	10.04	11.12	84.89	25.09
35	166.75	74.58	9.30	9.97	84.79	25.98
36	166.09	74.33	9.02	10.23	84.30	25.02

These case study results show some considerable differences in behaviour from the previous cases (that used simpler rooms). Probably the reason for that is based on the complexity involved in these multi-compartment room cases, which is much higher when compared with the square rooms previously investigated. Nevertheless, in some cases, what would be expected seemed to be produced.

For instance, for positions such as 3, 4, 5, 11, 17, 35, 36 and others where the travel distances were small, the evacuation times produced for these positions were also low. In the other hand, for positions such as 6, 18 and 26 where the travel distances were big, the evacuation times produced for these positions were also the highest.

Regards position 1, which has produced the minimum value for the evacuation time (164.75sec), besides having small travel distance, the fact that the two exits were almost located “side by side” (just like a wider exit), allowed an increase of the occupants’ flow rate and consequently a reduction of the evacuation time. This result would be expected. (This is already known and was extensively discussed in chapters 5 and 6).

For position 6, which has produced the worst value for the evacuation time (176.59 sec), this would also be expected since it has the largest travel distance. In this other case, it is likely that the cumulative waiting time was defined by the escape movement (i.e., the time spent during the movement within the enclosure towards the exit(s): Tsm)

But differently from these cases, some other cases, as mentioned before, have presented “unexpected” results; which might be consequence of the complexity involved within inner rooms’ situations.

For example, position 10 has the second smallest travel distance when compared to all other positions. Based on this fact, in theory, position 10 should produce one of the shortest evacuation times; nevertheless, it produces one of the longest evacuation times. This can be explained by the fact that the exits are not well located, since they generate, for this position, the largest cumulative waiting time (CWT). In this particular case, the CWT was probably defined by the time spent around the exit(s) areas. So, in this case, the time spent around the exit(s) areas (Tsae) was the determinant factor for defining the evacuation efficiency, rather than the time spent during the actual movement within the enclosure towards the exit(s) (Tsm).

Another interesting scenario to observe is position 4. This scenario gives a high travel distance; however, the evacuation time has a reasonable value. This might be explained by the fact that the two exits are well located in the corners.

In reality, the point which should be emphasised here is that: for some scenarios, the travel distance will be an important issue to be addressed; and therefore, it should be taken as the main reference when assessing the evacuation efficiency. Nevertheless, in some other scenarios, the travel distance will not be the main factor impacting the evacuation efficiency; and, for this reason, alternative factors, such as the exit(s) locations, should be considered when assessing the evacuation efficiency. In summary, for some situations, to have long travel distances would be better than to have short travel distances. And this is especially true for complex geometries in where the '*pos and cons*' of the length of travel distances needs to be analysed carefully.

Clearly, it is also correct to say that this impact varies according to a set of existent variables, such as: enclosure's geometry; internal lay-out; floor area; population density etc.

In the following section, the results for scenario 2 are presented and discussed.

8.3.3.2 Scenario 2)

As mentioned before, in total, 21,600 simulations were performed for this scenario.

The results for this second scenario, i.e., two exits in the bigger room, have also demonstrated the impact of the exit locations.

In summary, they have shown that position 34 produces the minimum value for the evacuation time: 83.92sec; and position 6 produces the worst value for the evacuation time: 106.09 sec. Table 8.2 presents the summary of these results.

Table 8.2: Complete results for the Multiple Connected Compartments, scenario 2) - one exit for the smaller room and two exits for the bigger room

Pos	AV TET (sec)	AV CWT (sec)	SIM. AV Dist (m)	GEOM. AV Dist (m)	AV PET (sec)	Max. Travel Distance (m)	Av Number using Exit 1	Av Number using Exit 2
1	87.59	33.56	8.39	8.63	44.02	24.11	98	102
2	93.59	36.67	8.07	8.57	42.87	25.73	114	86
3	90.92	35.96	6.37	7.25	43.75	22.69	92	108
4	94.25	37.15	6.34	6.89	43.91	22.64	112	88
5	96.09	37.37	4.81	5.05	42.86	24.19	103	97
6	106.09	40.19	5.80	6.19	46.50	18.07	127	73
7	105.55	38.94	5.66	5.90	45.13	18.31	124	76
8	104.90	35.02	5.82	6.60	39.98	26.23	125	75
9	87.25	34.19	9.77	10.11	43.79	22.16	98	102
10	85.36	34.29	8.48	8.68	42.82	22.86	99	101
11	87.59	33.65	10.64	11.30	43.96	30.56	99	101
12	87.38	34.01	10.00	10.57	43.79	27.68	100	100
13	87.25	31.87	12.69	13.35	43.89	38.97	98	102
14	86.42	31.34	12.34	12.96	43.06	32.64	96	104
15	87.92	31.08	12.75	13.29	43.15	29.76	99	101
16	88.59	33.18	11.69	12.31	44.36	30.23	100	100
17	88.42	33.75	11.04	11.78	44.40	26.31	101	99
18	88.34	34.07	9.83	10.57	43.71	25.40	98	102
19	89.75	35.45	9.84	10.45	45.12	23.57	98	102
20	87.92	34.37	10.33	10.61	44.43	24.93	101	99
21	88.09	35.03	9.88	10.02	44.72	22.99	100	100
22	87.75	34.57	10.03	10.39	44.38	27.31	98	102
23	90.59	34.53	11.01	11.32	45.17	28.64	102	98
24	87.59	35.09	9.36	9.48	44.34	23.69	99	101
25	87.57	35.44	9.35	9.98	44.68	23.11	101	99
26	87.55	36.07	8.16	8.43	44.33	21.23	103	97
27	89.59	36.38	8.59	9.00	45.00	20.81	98	102
28	85.59	34.07	8.91	9.22	42.95	22.99	99	101
29	86.59	35.08	9.58	9.93	44.51	28.02	98	102
30	85.62	33.20	10.67	11.08	43.53	23.99	100	100
31	87.75	32.78	11.79	12.34	44.05	30.76	101	99
32	88.25	34.73	8.73	9.07	43.47	22.61	101	99
33	86.09	34.04	8.97	9.27	42.98	22.28	100	100
34	83.92	32.03	9.71	10.12	41.98	22.78	99	101
35	85.03	32.15	10.72	11.20	42.54	22.86	100	100
36	86.92	32.40	11.81	12.54	43.69	33.64	100	100

From the results, it is possible to see that positions 6 and 7, which have the smallest travel distance, produced the largest evacuation times: 106.09sec and 105.55sec respectively. These positions, in theory, are good positions to locate the exits, however, their cumulative waiting times have shown that some time was spent in movement towards the exit areas. This can be said, because the exit of the smaller room is too close to one of the exits of the bigger room and this proximity seems to give a disadvantage, since it generates additional congestion. A similar behaviour was also found for position 8 which produced an evacuation time of 104.90 sec. In reality, for positions 2, 3, 4 and 5 this effect can be seen with progressively increasing severity.

In the other hand, position 1 produces a good result, since the two exits of the bigger room are located side by side; however as the two exits are separated (with one of the exits remaining close to the exit of the smaller room) a significant increase is observed in the evacuation time. Overall, the results have shown that the best combination of locations for the two exits is when they are located side by side in the corner. For instance, the lowest evacuation time was found for position 34 with an evacuation time of 83.92sec. And in fact, the lowest evacuation times were found when the two exits are located side by side. In general, it can be said again that, as mentioned previously, that the positions of the exits do represent a major factor on the evacuation efficiency.

In the next section, these two scenarios, scenario 1, with one exit in the smaller room and one exit in the bigger room, and scenario 2, with one exit in the smaller room and two exits in the bigger room, are solved using the methodology proposed in this thesis.

8.4 The Final Analysis using Classical Optimisation Theory

8.4.1. The Problem

The problem that is to be analysed is that same case study already described in this chapter. To recap, this case has the first scenario that is solved as an unconstrained problem and the second scenario is solved as a constrained problem.

For the unconstrained problem, two different DoE techniques were used for the selection of the design points, namely: the CCD (Composite Central Design) and the Latin-hypercube.

For the constrained problem the random DoE technique was used to select the design points.

For the unconstrained problem, two RSM's were used: a high polynomial order and a multiple regression function. These RSM's proved to be appropriate and were based on the general stepwise regression.

For the constrained problem, the RSM used was the multiple regression function, since other RMS's did not provide accurate results. This RSM was also developed based on the general stepwise regression.

Finally, two types of numerical optimisation techniques were applied: a gradient-based technique (Fletcher-Reeves) and a non gradient-based technique (PSO – Particle Swarm Optimisation).

As considered for the other study cases, the optimal solution is understood to be the global minima region of the objective function (i.e., the evacuation time) in the response surface. However, given that to find precisely the global minima region is not an easy task, if the local minima region near to the global minima region is found, it is assumed that this is also a good result. The table 8.3 presents this summary.

Table 8.3: Summary of the problem (compartmented room)

Type of problem	**DoE techniques used**	**RSM used**	**Numerical Optimisation Techniques used**
Unconstrained	• CCD (Central Composite Design • LatinHypercube	• Polynomial • Multiple Regression Function	• Gradient-based technique (Fletcher-Reeves) • Non gradient-based technique (Particle Swarm Optimisation)
Constrained	The random DoE technique	Multiple Regression Function	• Gradient-based technique (Fletcher-Reeves) • Non gradient-based technique (Particle Swarm Optimisation)

The following sections of this chapter detail how the problem was solved and also present and discuss the results.

8.5 The scenarios

8.5.1 The simulations

The simulations, the evacuation model used and the parameters used for the simulations here for this case are the same as those which were presented and discussed for the

For each simulation, the initial location of each occupant was randomized.

And according to what was mentioned before for the square room scenario, with two exits, the positions are measured in terms of the relative distance between these two exits along the perimeter of the room. Thus, the distance from the left edge of each exit to the corner is measured similarly to the way that it was measured for the single exit case. These distances are taken to be the design variables for the two exits case. The domains of these design variables are:

$$0 < D1 < 29 \qquad 0 < D2 < 9 \qquad 0 < D3 < 29$$

D1 represents the distance from the left edge of exit 1 to the corner

D2 represents the distance from the left edge of exit 2 to the corner

D3 represents the distance from the left edge of exit 3 to the corner

It should be noted that exit 1 is the exit located in the "bigger" room wall perimeter. Exit 2 is the exit located in the "smaller" room wall perimeter. And exit 3 (which is considered only for the constrained problem) is the additional exit that is located in the "bigger" room wall perimeter.

Given that the room configuration is compartmented, see Figure 8.6, the domains of exit 1 and exit 2 are not the same; these two exits can be located anywhere along its perimeter. However, when considering the scenario where the exit 3 is included, exit 1 and exit 3 have the same domain; therefore, these two exits cannot be located such that they overlap.

Considering also that each exit is a 1m width exit, the domain of exit 1 and 3 could be represented by a line in which the total length is 29m; and for exit 2, a line in which the total length is 9m, see Figure 8.4.

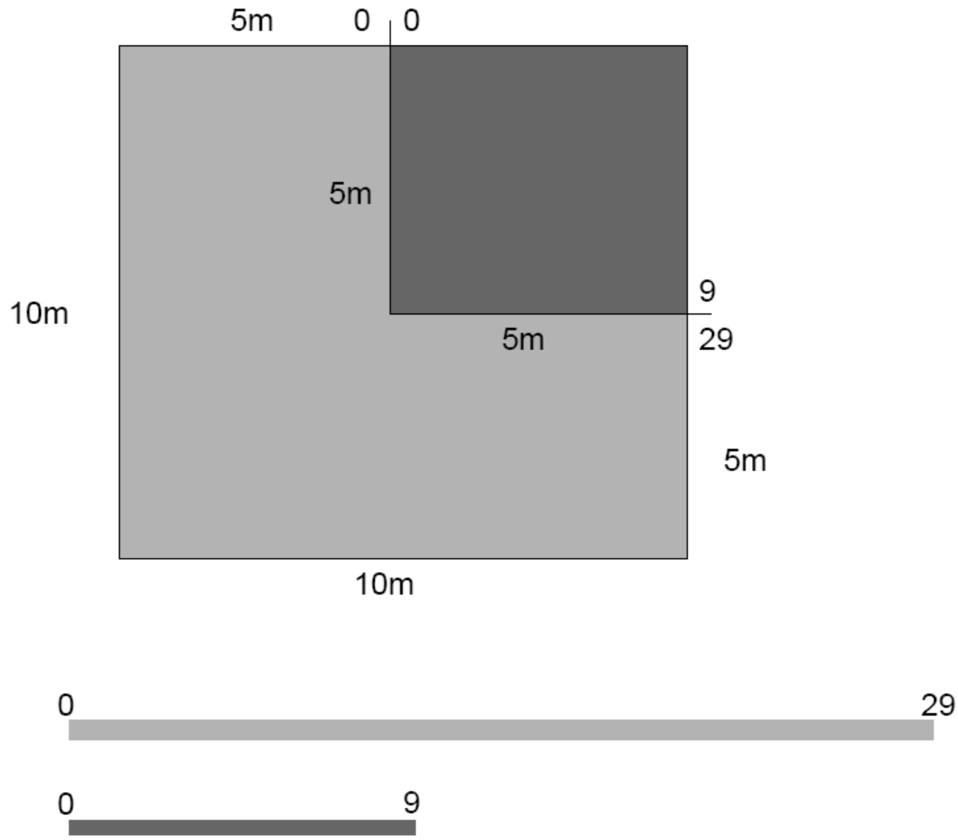


Figure 8.4: Domains for the exits positions for the compartmented room

It is important to note that, given this configuration, the two exits, exit 1 and exit 3, can overlap each other, since they are free to be located anywhere between 0 and 29, along the perimeter line. For this reason, based on this constraint, the problem was solved as a constrained problem, with the constraint represented by the following inequality:

$$|D1 - D3| > 1$$

8.6 Results

For the unconstrained problem, as mentioned previously, two DoE techniques were used for selecting the design points. Using the CCD, a set of 9 design points was used. Using the Latin-Hypercube, also two sets were defined; the first set used 3 design points and the second set used 5 design points. For each design point, 600 simulations were performed. Therefore, a total of 14,400 simulations were performed.

For the constrained problem, the random DoE technique was used for selecting the design points. For this, 3 sets were defined. The first set used 5 design points; the second set used 10 design points and the third set used 15 design points. As usual, for each design point, 600 simulations were performed. Therefore, a total of 18,000 simulations were performed.

Similarly to the previous analyses presented in the previous chapters, the computational package MATLAB was used to compute the numerical approximations used to develop the RSM, whilst the numerical optimisation techniques were computed using VisualDOC.

The results for this problem are presented and discussed in the following sections of this chapter.

8.6.1 The unconstrained problem (scenario 1)

The statement of the unconstrained problem is:

Minimize: $ET = f(D1, D2)$

Where:

$0 < D1 < 29$

$0 < D2 < 9$

Where:

ET – the evacuation time;

D1 - the distance from the left edge of exit 1 to the corner;

D2 - the distance from the left edge of exit 2 to the corner;

The results for this problem are presented in tables 8.4 and 8.5.

Table 8.4: Optimal solution for the compartmented room (unconstrained problem)

Response Surface Model: High Polynomial Order (R^2 = 0.91)	**Numerical Optimisation Technique**	
DoE Technique	Fletcher-Reeves	PSO – Particle Swarm Optimisation
Latin Hypercube – 3 design points	N.S.	N.S.
Latin Hypercube – 6 design points	(0; 9; 166.18)	(0; 9; 166.18)
CCD – 9 design points	(0; 0; 164.75)	(0; 0; 164.75)

N.S. - No Solution found.

Table 8.5: Optimal solution for the compartmented room (unconstrained problem)

Response Surface Model: Multiple Regression Function (R^2 = 0.99)	**Numerical Optimisation Technique**	
DoE Technique	Fletcher-Reeves	PSO – Particle Swarm Optimisation
Latin Hypercube – 3 design points	(0; 9; 166.18)	(0; 9; 166.18)
Latin Hypercube – 6 design points	(0; 0; 164.75)	(0; 0; 164.75)
CCD – 9 design points	(0; 0; 164.75)	(0; 0; 164.75)

The results have shown to be consistent for all the cases that were investigated. Using the multiple regression function, as the RSM, was shown to produce more accurate results than using the high polynomial order.

From the results, it is possible to conclude that the best locations for the two exits are almost “side by side” and in the corner, as shown in Figure 8.5. These results can be seen as a particular case of the previous study for the square room with one exit (see chapter 5), where having an exit in the corner produced lower evacuation times than having an exit anywhere else (except in the corner).

Additionally, to have the two exits located far away from each other, also produces low evacuation times. This can be seen as a special case of the previous study (see chapter 6) for the squared room with two exits; where having two exits located in a symmetrical distance produces good results in terms of minimising the evacuation times.

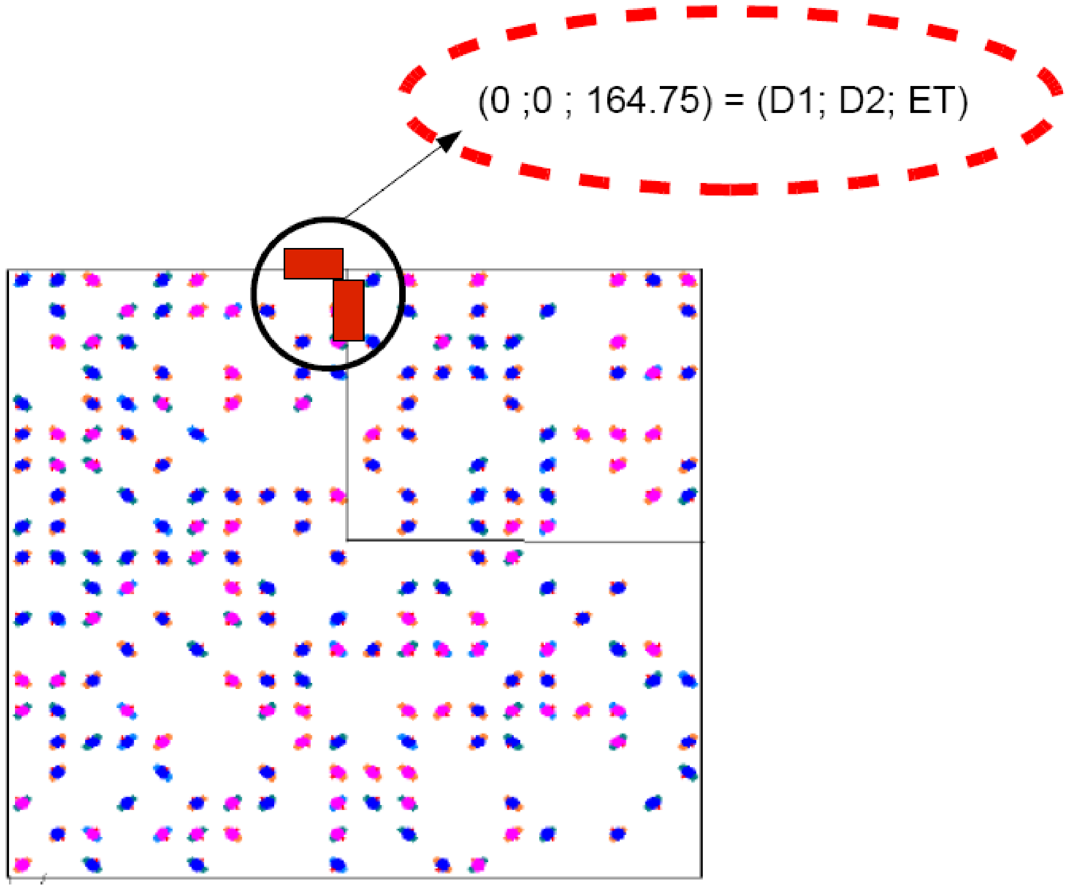


Figure 8.5: Optimal exits locations for the compartmented room (unconstrained problem)

8.6.2 The constrained problem (scenario 2)

This scenario is almost the same problem as scenario 1 except that it has a third exit 3 and consequently there is a constraint on the exit locations. The statement of the problem is given as:

Minimize:

$$ET = f(D1,D2,D3)$$

Subject:

$$|D1 - D3| > 1$$

Where:

$$0 < D1 < 29$$

$$0 < D2 < 9$$

$$0 < D3 < 29$$

Where:

ET – the evacuation time;

D1 - the distance from the left edge of exit 1 to the corner;

D2 - the distance from the left edge of exit 2 to the corner;

D3 - the distance from the left edge of exit 3 to the corner.

The results for this problem are presented in table 8.6.

Table 8.6: Optimal solution for the compartmented room (constrained problem)

Response Surface Model: Multiple Regression Function (R^2 = 0.99)	**Numerical Optimisation Technique**	
DoE Technique	Fletcher-Reeves	PSO – Particle Swarm Optimisation
Random (5 design points)	(1.5 ;7; 2.5 ;85.03)	(1.5 ;7; 2.5 ;85.03)
Random (10 design points)	(1.5 ;4; 2.5;83.92)	(1.5 ;4; 2.5;83.92)
Random (15 design points)	(1.5 ;4; 2.5;83.92)	(1.5 ;4; 2.5;83.92)

From the results, it is clear to see that, in relation to exits 1 and 3 (i.e. those exits which are located in the “bigger room” wall perimeter), having both these exits side by side, produces the lowest evacuation time, see Figure 8.6.

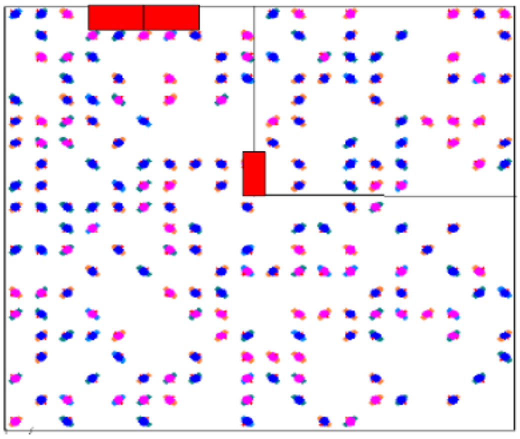


Figure 8.6: Optimal exits locations for the compartmented room (constrained problem), where the coordinate is (D1;D2;D3;ET = 1.5 ; 4; 2.5; 83.92)

Using buildingEXODUS, taking into consideration D1 = 1.5 and D2 = 4, the evacuation time produced is 84.01 seconds. This supports the result obtained through the methodology proposed in this thesis.

8.6 Concluding Comments

In this chapter, a study case was explored to investigate the optimal positioning of exits around the perimeter of compartmented room (i.e., "multiple connected compartments") in order to minimise the evacuation times. The evacuation simulations were conducted assuming ideal conditions of zero response times and population behaviour such that occupants would move to their nearest exits. Both assumptions are made to simplify the analysis and to isolate issues associated with exit location.

This investigation also revealed that a significant advantage can be derived from the strategic positioning of the exits.

For this case study, two scenarios were analysed: scenario 1 had one exit for the smaller room and one exit for the bigger room; whilst scenario 2 had one exit for the smaller room and two exits for the bigger room. Therefore, this problem was solved as an unconstrained problem for the scenario 1) and as a constrained problem for the scenario 2).

The solutions were initially found through "brute force method" and then through the use of numerical optimisation techniques.

For the first scenario, it was found that the lowest evacuation times are found when the exits are close to each other (producing a similar effect as if they were located side by side) and when the two exits are located in symmetrical locations.

For the second scenario, it was found that the lowest evacuation times occur when the two exits of the bigger room are located side by side.

For both cases, it was observed that the exit locations do play an important role in determining the evacuation efficiency.

The analysis using the optimisation techniques has revealed again that this systematic methodology to efficiently optimise evacuation safety aspects of structural designs can be applied to more complex designs than to a simple square room.

In the next chapter, a second set of study cases with more complex scenarios are presented and solved.

9. More Complex Geometries: Case Study 3 – Multiple Exits of Varying Sizes

9.1 Introduction

The problem investigated in this chapter is complex as it involves multiple exits of varying size and flow capacity with an uniform array of obstacles restricting flow in parts of compartment. The objective of the analysis is to determine the location of the exits required to produce minimum evacuation times.

In concept, the problem is similar to determining the optimal location of exits in an aircraft style geometry. The problem consists of a rectangular geometry with an uniform array of seating, a central aisle and three exits (of varying size) located in each of the long side of the compartment.

It is important to note that these simulations are not intended to model aircraft certification egress times **[120]**; but is intended to model an evacuation situation in which the occupants attempt to use their nearest exit. It has been argued that this is a more realistic accident scenario than to standard certification trial based as data derived from real accident investigation **[120]**.

It is also important to note that the buildingEXODUS software is used to perform evacuation simulation in this chapter and not airEXODUS. Thus the comparison with aircraft is intended to be only approximate as typical aircraft evacuation behaviours such as seat climbing are not included and the exit flow rates only approximate true ground in real aircraft. The problem is however relevant to aircraft design as the location of aircraft exits is currently dictated by aircraft structural and weight considerations, rather than on optimizing evacuation considerations.

The geometry under investigation is 25.5m long and 3.5m wide; contains 156 people and the exits have flow rates which vary from 0.77p/s to 1.33p/s. This lay-out approximates that of a B 737.

Unlike in previous chapters, a brute force solution to this problem is not attempted due to the complexity of the problem. Suggested ideal solutions are derived from a logical study of the problem.

9.2 The Problem

The problem to be investigated can be stated very simply as follows: for a rectangular shaped compartment with discrete space of given size with an uniform array of obstacles and a central aisle, containing an arbitrarily large population, is there an optimal location for six exits that will minimise the evacuation time?

The rectangular shaped room of dimension 25.5m x 3.5m has an effective area of 53.25 m^2 with 6 exits of widths of 0.5m, see Figure 9.1. The population of the room consists of 156 people producing a population density of approximately 2.93people/m^2 (i.e., every set is initially occupied by an agent) In this geometry, due to the presence of obstacles (seats), the occupants' movement is restricted. It is also intended to consider the impact that the lay-out has on the evacuation efficiency. The configuration to be examined is similar to that of a narrow body, single aisle aircraft.

The problem to be addressed is concerned with determining the optimal location of exits in this geometry to minimise evacuation time.

In actual narrow body aircraft, the typical exit layout consists of three exit pairs; one exit pair located in the extreme front (two exits), one located over the wing, near to the centre, and the third pair located in the extreme rear, see Figure 9.1.

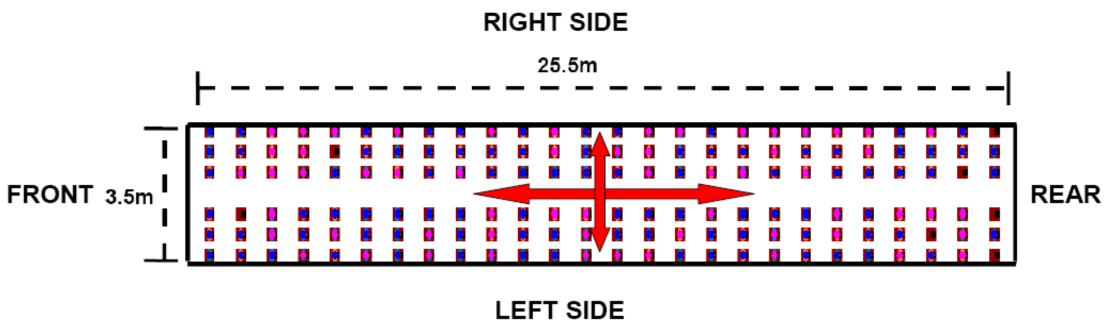


Figure 9.1: Rectangular shaped room with multiple exits (arrows represent the movement options)

In Figure 9.1, the nodes represent seat rows where people are seated. People cannot move forward or backward through the seats, but can move along the seat row to the central aisle.

In aircraft, the front are rear exit pairs are similar in size and are referred to as Type I exits and the overwing exits are smaller and referred to as Type III exit.

Within our problem, we will identify the Type I exit as “large” and the Type III as “small” exits.

In addition, we will set the flow rate of the large exit to 1.33p/s and the small exit as 0.77p/s. While these are not the flow rates achieved by the exits in reality, they will be sufficient for our analysis and reflect the actual competitive difference between the exits.

Thus in our problem, there are four large and two small exits. Movement in the aircraft is restricted by the seats and generally occurs along the central aisle.

The problem we will address concerns identifying the optimal location of the exits subject to various constraints.

Using a conventional aircraft exit layout, assuming all exits were of “large” type, the configuration with 156 people would achieve an average evacuation time of 76.8sec determined using the buildingEXODUS software. If we replace the two central large exits with small exits, the evacuation time becomes 104.5sec.

The question addressed in this chapter is, using the optimisation techniques within the methodology developed in this thesis, can we determine what the optimal exit locations is for this “aircraft style” geometry?

To address this problem, four scenarios are considered; each with increasing complexity.

9.2.1 The scenarios

The scenarios which are investigated in this chapter are as follows:

9.2.1.1 Scenario 1: identical exits, in pairs, three on each side

This first scenario in summary can be described as follows:

- three exits on each side;
- left and right exits are positioned in the same locations (as pairs);
- all six exits are assumed to be identical and of the "large" type.

Therefore, scenario 1 assumed that the three exits in left side would have the same relative locations along the edge that the three exits in the right side would have. For this reason, this problem is a three variable problem. All of the six exits have the same maximum flow rate of 1.33 occupants/meter/second (occ/m/s).

9.2.1.2 Scenario 2: four large, two small, in pairs, three on each side

This second scenario in summary can be described as follows:

- three exits on each side;
- left and right exits are positioned in the same location (as pairs);
- four exits are of the "large" type and two exits are of the "small" type.

Therefore, scenario 2 assumed that the three exits in left side would have the same relative locations along the edge that the three exits in the right side would have. For this reason, this problem, like scenario 1, is a three variables problem. Nevertheless, not all of the six exits have the same maximum flow rate. Two exits in side 1 have a maximum flow rate of 1.33 occ/m/s and one exit in side 1 has the maximum flow rate of 0.77 occ/m/s.

9.2.1.3 Scenario 3: identical exits, three on each side

This third scenario in summary can be described as follows:

- three exits on each side;
- left and right exits are not necessarily at the same location;
- all six exits are assumed to be identical and of the "large" type.

Therefore, scenario 3 assumed that the three exits in left side would not have the same relative locations along the edge that the three exits in the right side would have. For this reason, this problem, differently from scenarios 1 and 2, is a six variables problem. And all of the six exits have the same maximum flow rate of 1.33 occ/m/s.

9.2.1.4 Scenario 4: four large, two small, three on each side

This final scenario in summary can be described as follows:

- three exits on each side;
- left and right exits are not necessarily at the same locations;
- four exits are of the “large” type and two exits are of the “small” type.

Therefore, scenario 4 assumed that the three exits in side 1 would not have the same relative locations along the edge that the three exits in the side 2 would have and vice-versa. For this reason, this problem, similarly from scenario 3, is a six variables problem. Nevertheless, not all of the six exits have the same maximum flow rate. Two exits in side 1 have a maximum flow rate of 1.33 occ/m/s and one exit in side 1 has the maximum flow rate of 0.77 occ/m/s. In summary, four of the exits have a flow rate of 1.33 occ/m/s, HMSO (Her Majesty's Stationery Office). The other two exits have a flow rate of 0.77 occ/m/s.

9.3 The Simulations

Regarding the simulations, the evacuation model used and the parameters used for the simulations of this case, are the same as those presented and discussed for the previous case studies shown in chapters 5, 6, 7 and 8.

However, it is important to note again that, given that each particular scenario is repeated 600 times and for each simulation, the location of each occupant was swapped. In other terms, for each simulation, the locations of each occupant were swapped.

The exits' locations are measured in terms of the distance from the front edge of the exit to the corner of the wall. Three exits will be located in the right side and the other three exits will be located in the left side.

As mentioned previously, before starting the optimisation methodology analysis, some previous simulations were performed in order to estimate the optimal solutions. These simulations were based on informed judgement. The analysis of these results suggested which configurations for the exits would generate the lowest evacuation times.

These results were estimated based on logical analysis, are presented in the next section.

9.4 Identifying Optimal Solutions through Logical Analysis

As the possible number of configurations for this problem is very large, the brute force approach used in previous chapters would require several cases to be run. So in this case, informed expert opinion is used to suggest optimal exit configuration for each scenario. Each "suggested" optimal case is then run using buildingEXODUS and compared with the standard times identified in section 9.2.

9.4.1 Scenario 1: identical exits, in pairs, three on each side

The lowest evacuation time will occur when all the exits "finish" at the same time. As all the exits have the same flow rate, this will occur when equal number of people use each exit. Given that the agents, in this problem, will go to their nearest exit, the problem becomes a geometrical one of positioning the exits. (It is also relevant to mention that they could randomly go in either direction, but for our purpose, to find ideal split of paths plus hence exits locations, we will assume that 50% go one way and 50% go the other way).

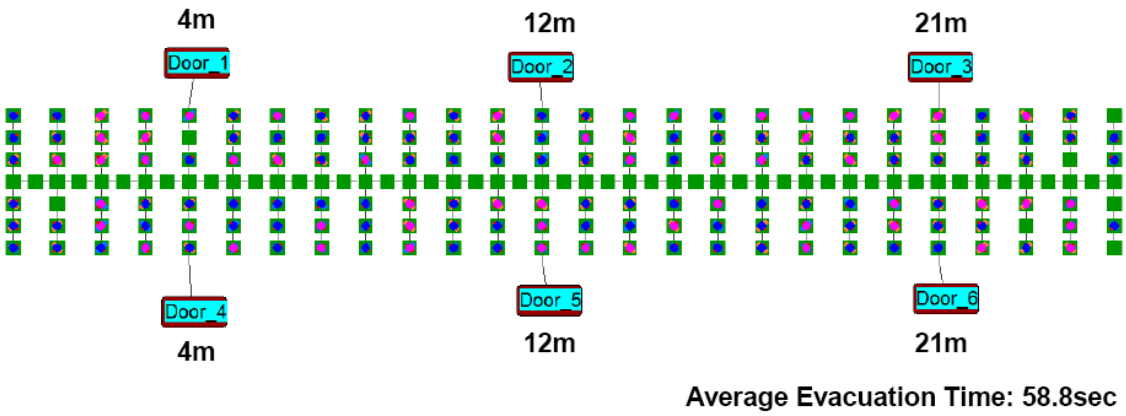


Figure 9.2: The expected optimal solution for the exits locations for the rectangular shaped room with multiple exits

To achieve this, the 2 exits on the left are located 4m from the left corner; the 2 exits in the middle are located 12m from the left corner; and the 2 exits in the right end are located 21m from the left corner. Therefore, it is possible to see that these exits are well distributed along the walls. The evacuation time produced using this configuration was 59.8sec (i.e., this is the average value based on 600 simulations). It is important to observe that, all the exits, for this analysis, had the same maximum flow rate of 1.33 occ/sec; and all exits on side 1 had the same locations as the exits on side 2. This time is considerably less than the time achieved for the standard layout 76.8sec (see section 9.2).

This demonstrates that the evacuation time is shorter using this configuration for the exit layout rather than using the conventional aircraft one. In fact, with this new configuration, the evacuation time was 17sec quicker which means a reduction of 22% in the evacuation time.

In reality, the evacuation time efficiency was improved for this case, because differently from the conventional aircraft case, as mentioned before, the exits were better distributed (i.e., well spread along the walls). For this reason, the likelihood of having one exit being more used than another was considerably reduced and consequently, the congestions were avoided. Therefore, a decrease of the evacuation time value was achieved.

It is argued that this is the "optimal" exit configuration using these constraints.

Here, once again, the impact of the exits locations on the evacuation efficiency have shown to be an important issue to be considered when addressing occupants' safety in enclosures. It is also relevant to observe that even when the exits have same flow rates, the exits locations do still play a major rule.

In the next section, scenario 2 (in which the exits do not have the same flow rates) is presented and investigated.

9.4.2 Scenario 2: four large, two small, in pairs, three on each side

Similarly to the previous scenario, the lowest evacuation time for this particular scenario will also be obtained when the exits are well distributed along the walls. In fact, this distribution is based on the two exits with flow rate of 0.77 occ/m/s located in the middle of the walls (marked with the red ellipses in the drawing) and the other four exits with flow rate of 1.33 occupants/meter/second (occ/m/s) located well apart from themselves as shown in Figure 9.3. It is also important to observe that, like scenario 1, all the exits on right side dad the same locations as the exits on the left side.

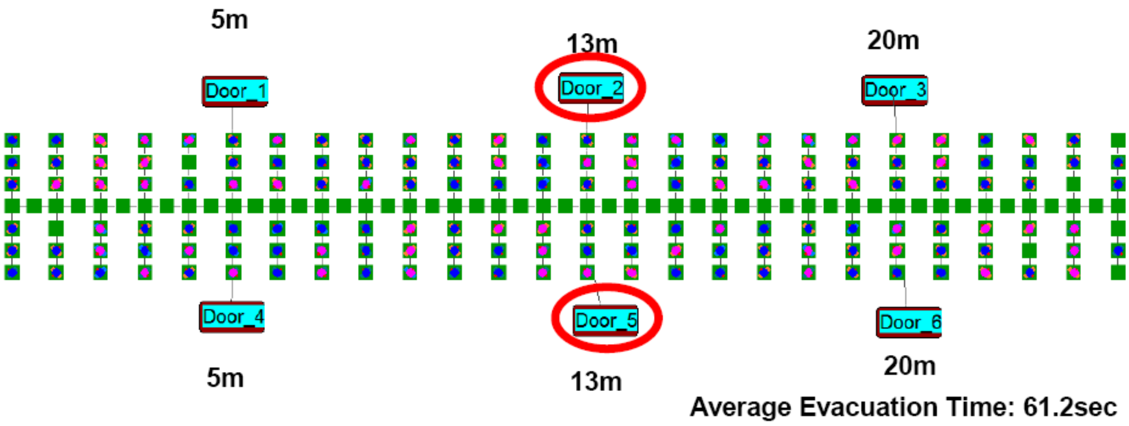


Figure 9.3: The expected optimal solution for the exits locations for the rectangular shaped room with multiple exits (scenario 2)

The exact locations for these exits are: the two exits on the left (with flow rate of 1.33 occ/m/s) located 5m from the left corner; the two exits in the middle (with flow rate of 0.77 occ/m/s) located 13m from the left corner and the two exits in the right end (with flow rate of 1.33occ/m/s) located 20m from the left corner. In other words, the coordinate for this solution is (5;**13**;20;5;**13**;20), which produced an evacuation time of 61.2sec. This evacuation time is a result of an average of 600 simulations.

This optimal solution was found based that for wider exits should capture similarly number of occupants and not be influenced by the other two smaller exits. This is an important issue to be considered, because differently from scenario 1, where all exits had the same flow rate, in this scenario, two exits have smaller flow rates. For this reason, the ideal exit locations configuration would be the one in which these two smaller exits would not affect the catchment areas for the rest of the four bigger exits, as shown in Figure 9.3.

It is also important to observe that in a conventional aircraft exit layout, for this same scenario (i.e., four exits are of “large” type and two exits are of “small” type and the configuration with 156 people), as showed previously, the average evacuation time was 104.5sec. And now with these new exits locations, the evacuation time produced was 61.2sec. This demonstrates the evacuation time is shorter using this configuration for the exit layout rather than using the conventional aircraft one. In fact, with this new configuration, the evacuation time was 43.3sec quicker which means a reduction of 41.46% in the evacuation time.

This achievement, based on a real situation (i.e., aircraft geometry in where the exits do not have same flow rates) has shown, once again, how the exits locations do impact substantially the evacuation efficiency.

9.4.3 Scenario 3: identical exits, three on each side

For this scenario, all the exits had the same flow rates (i.e., 1.33 occ/m/s) and the exits in the right side are not necessarily at the same location of those in the left side. The lowest evacuation time is obtained when these exits are well distributed in alternate locations as shown in Figure 9.4

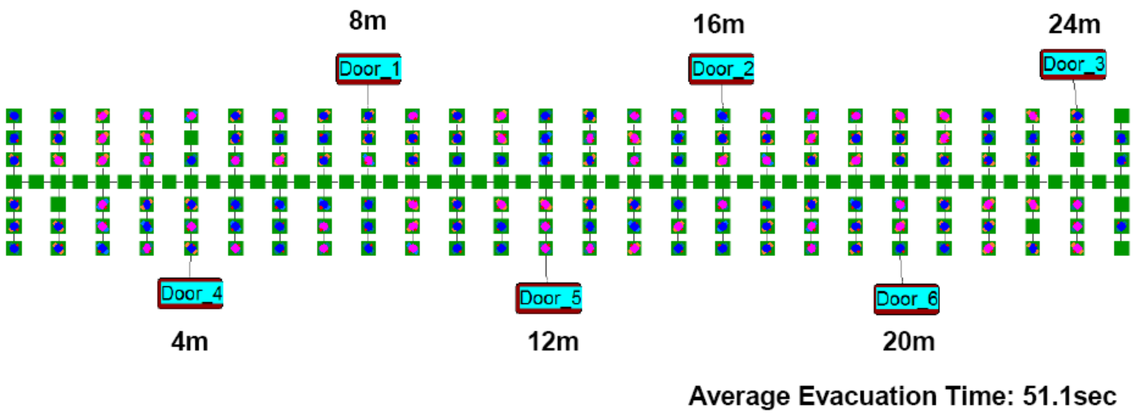


Figure 9.4: The expected optimal solution for the exits locations for the rectangular shaped room with multiple exits (scenario 3)

As shown in Figure 9.4, this optimal solution was found based that all these six exits should be located well distributed along the walls on alternate positions. With this, all of these exits have equal balance in terms of occupants' usage. In other terms, these locations attempt to avoid these exits influence themselves in terms of potential usage.

To achieve this, the 2 exits on the left are located 8m from the left corner (in the right side) and 4m from the left corner (in the left side); the 2 exits in the middle are located 16m from the left corner (in the right side) and 12m from the left corner (in the left side) and the 2 exits in the right end are located 24m from the left corner (in the right side) and 20m from the left corner (in the left side). Therefore, it is possible to see that these exits are well distributed along the walls.

In summary, the coordinate for the exits locations is: (8 16; 24; 4; 12; 20). With these locations, the evacuation time obtained was 51.1sec (the average value for 600 simulations).

It is important to observe that there is no exit located in the corner and they are all alternatively distributed along the opposite walls. In other words, there are no exits facing each other. With this, there is much less chance of conflicts between the occupants towards the exits areas and consequently making the evacuation more efficient.

Clearly, there would be always some kind of conflict amongst the occupants; however, with this location, it seems that this issue was minimized, allowing a good usage of each exits' catchment areas. As consequence of that, the evacuation efficiency was improved and this is why this particular exit locations configuration has produced the lowest evacuation time.

9.4.4 Scenario 4: four large, two small, three on each side

For this final scenario, similarly to the previous one, the exits in the right side are not necessarily at the same location of those in the left side. But in the other hand, these exits had different flow rates: four exits with 1.33 occ/m/s and two exits with 0.77occ/m/s. The lowest evacuation time is obtained when the two exits with flow rate of 0.77 occ/m/s are located in the opposite corners (marked with the red ellipses in the drawing) in a diagonal position; and the other four exits with flow rate of 1.33 occ/m/s located well apart from themselves in alternate positions as shown in Figure 9.5.

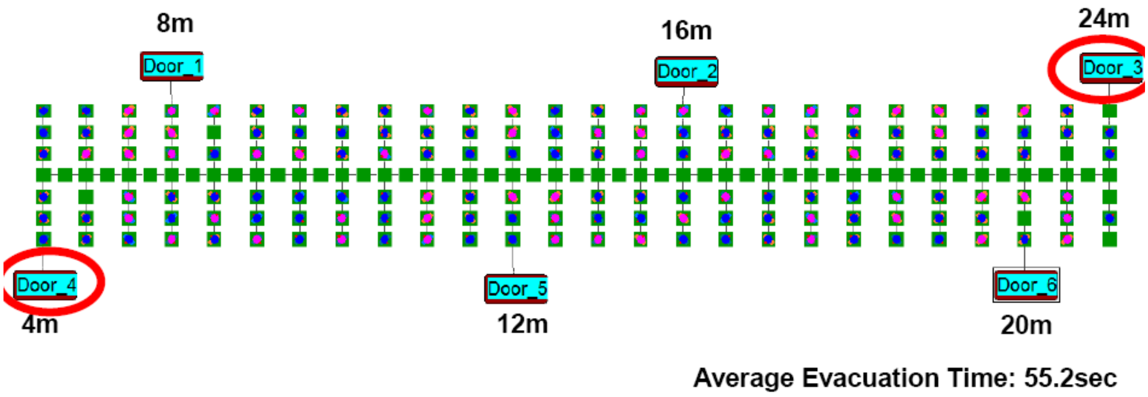


Figure 9.5: The expected optimal solution for the exits locations for the rectangular shaped room with multiple exits (scenario 4)

This configuration is similar to the one found for the previous case; where the exits were also well and alternatively distributed along the walls. In other words, these exits are located in positions in which they can capture the maximum number of occupants, without influencing much the potential users of the other exits. Besides that, the two exits with smaller flow rates were "isolated" in the corners. With this configuration, it was possible to enable these two smaller exits not to capture potential users for the bigger exits.

The coordinate for exits locations for this solution is: (**3**; 15; 25; 0; 11; **23**) and this gave an evacuation time of 55.2ssec. This value was also based on an average of 600 simulations.

In other terms, the coordinate was based on: the 2 exits on the left are located 3m from the left corner (in the right side and with flow rate of 1.33 occ/m/s) and in the corner (in the left side and with flow rate of 0.33 occ/ms/); the 2 exits in the middle are located 15m from the left corner (in the right side) and 11m from the left corner (in the left side) and both with a flow rate of 1.33 occ/m/s; and the 2 exits in the right end are located 25m from the left corner (in the right side and with a flow rate of 1.33occ/m/s) and 23m from the left corner (in the left side and with a flow rate of 0.77 occ/m/s). Therefore, it is possible to see that these exits are well distributed along the walls.

In the next section, the optimal solutions for these four scenarios are found through the use of the optimisation methodology developed in this thesis.

9.5 Determining optimal exit location using the Optimisation Methodology

The four scenarios presented and discussed previously are investigated now using the optimisation methodology proposed in this Thesis.

For all these four scenarios, the random DoE technique was used for picking up the design points. And the RSM based on high order polynomial function was used, once it proved to be appropriate based on the general stepwise regression.

Finally, two types of numerical optimisation techniques were applied: a gradient-based technique (i.e., Fletcher-Reeves) and a non gradient-based technique (PSO: Particle Swarm Optimisation).

In this study, the STATISTICA computational package was used to perform the numerical approximations when developing the RSM. The numerical optimisation techniques were computed using the VisualDOC software.

For this section, a total of 1,000,800 simulations were performed for all scenarios; in which:

- 439,200 simulations for scenario 1;
- 187,200 simulations for scenario 2;
- 187,200 simulations for scenario 3;
- 187,200 simulations for scenario 4.

9.5.1 Scenario 1: solution using the optimisation methodology

9.5.1.1 Problem statement

This scenario involves identical exits, in pairs, three on each side.

Therefore, scenario 1 assumed that the three exits in side 1 would have the same relative locations along the edge that the three exits in the side 2 would have and vice-versa. For this reason, this problem is a three variables problem. And all of the six exits have the same maximum flow rate of 1.33 occ/m/s.

This first consideration, as mentioned before, assumed that the three exits in side 1 would have the same relative locations that the three exits in the side 2 would have and vice-versa.

From Figure 9.1, it is possible to define the domains of each design variable:

$0 < D1 < 25 \qquad 0 < D2 < 25 \qquad 0 < D3 < 25$

$0 < D4 < 25 \qquad 0 < D5 < 25 \qquad 0 < D6 < 25$

Where:

D1 represents the distance from the left edge of exit 1 to the corner

D2 represents the distance from the left edge of exit 2 to the corner

D3 represents the distance from the left edge of exit 3 to the corner

D4 represents the distance from the left edge of exit 4 to the corner

D5 represents the distance from the left edge of exit 5 to the corner

D6 represents the distance from the left edge of exit 6 to the corner

For reasons of consistency, it is important to mention that exits 1, 2 and 3 are located in side 1 and the exits 4, 5 and 6 are located in side 2.

Once the locations for exits 4, 5 and 6 will be the same for the exits 1, 2 and 3, respectively; this problem can be solved as a 3 variables problem:

Minimize: $ET = f\,(D1,D2,D3)$

Where: $0 < D1 < 25 \qquad 0 < D2 < 25 \qquad 0 < D3 < 25$

And given that these exits should not overlap each other. Clearly, the problem is a constrained problem, therefore, two constraints were considered:

$$|D1 - D2| > 0.5$$

$$|D2 - D3| > 0.5$$

9.5.1.2 Problem methodology

For this study case, the random DoE technique was used for selecting the design points. For this, three different sets of different design points consisting of: 4, 10, 15, 20, 25 and 30 were defined.

For scenario 1, two types of response surface models were used, namely: the high-order polynomial and the *multiquadratic* function. For this reason, a total of 439,200 simulations were performed.

For the other scenarios, just the *multiquadratic* function was used, therefore, for each one of these scenarios, 187,200 simulations were performed.

Furthermore, as mentioned previously, given that for each design point, 600 simulations were performed; a total of 1,000,800 simulations were performed for all scenarios.

9.5.1.3 Results

As mentioned before, the random DoE technique was used for selecting the design points. The first response surface model used was polynomial function, with 4, 10, 15, 20, 25 and 30 design points. For each case, three sets of different design points were investigated. The second response surface model used was the *multiquadratic* function, with 20, 25 and 30 design points.

The best configurations for all the three sets are presented in Figure 9.6. Presented in Figure 9.7 is one set of 30 design points used in the analysis. These design points represent SET 1. They are presented to show the nature of the randomly selected points used in the analysis.

Considering these 30 design points for SET 1 as shown in Figure 9.7, it is possible to note that some of the 30 cases had both the forward and rear exit pairs located away from the ends of the compartment. This is similar to the trend required by the “optimal” solution. Thus some of the randomly selected design points were appropriately steering the response surface in the correct direction. This would appear to be a reasonable selection of points.

Table 9.1 Results for Case Study 3: Scenario 1

SET 1			SET 2			SET 3		
RSM Poly.	**Numerical Optimisation Techniques**		**RSM Poly.**	**Numerical Optimisation Techniques**		**RSM Poly.**	**Numerical Optimisation Technique**	
Design Points	Fletcher-Reeves	PSO	Design Points	Fletcher-Reeves	PSO	Design Points	Fletcher-Reeves	PSO
4	N.S.	N.S.	4	N.S.	N.S.	4	N.S.	N.S.
6	N.S.	N.S.	6	N.S.	N.S.	6	N.S.	N.S.
10	N.S.	N.S.	10	N.S.	N.S.	10	N.S.	N.S.
15	N.S.	N.S.	15	N.S.	N.S.	15	N.S.	N.S.
20	(3;23;25; 3; 23; 25; 87.5)	(3;23;25; 3; 23; 25; 87.5)	20	(5;18;22; 5; 18; 22; 84.7)	(5;18;22; 5; 18; 22; 84.7)	20	(0; 10; 18; 0; 10; 18; 74.9)	(0; 10; 18; 0; 10; 18; 74.9)
25	(6;20;25; 6; 20; 25; 87.9)	(6;20;25; 6; 20; 25; 87.9)	25	(5;22;25; 5; 22; 25; 88)	(5;22;25; 5; 22; 25; 88)	25	(0; 8; 24; 0; 8; 24;74.6)	(0; 8; 24; 0; 8; 24;74.6)
30	(0;15;25;0;15;25; 73.7)	(0;15;25;0;15;25; 73.7)	30	(0;12;23;0;12;23; 74.3)	(0;12;23;0;12;23; 74.3)	30	(1;8; 25;1;8;25; 75.3)	(1;8; 25;1;8;25; 75.3)

N.S. - No Solution found

Figure 9.6 presents the optimal exits location configurations for these 3 sets.

The results are presented in Table 9.1. as can be seen, at least 20 design points were required to produce potential candidate solutions. And both numerical optimisation techniques, the Fletcher-Reeves and the PSO, produced the same results which show their robustness.

However, the results for the 20, 25 and 30 design points produce solutions which are considerably different to the optimal configuration found in section 9.2. Here, we will discuss the solutions produced by the 30 design points' cases, but similar arguments applying to the 20 and 25 design points' cases.

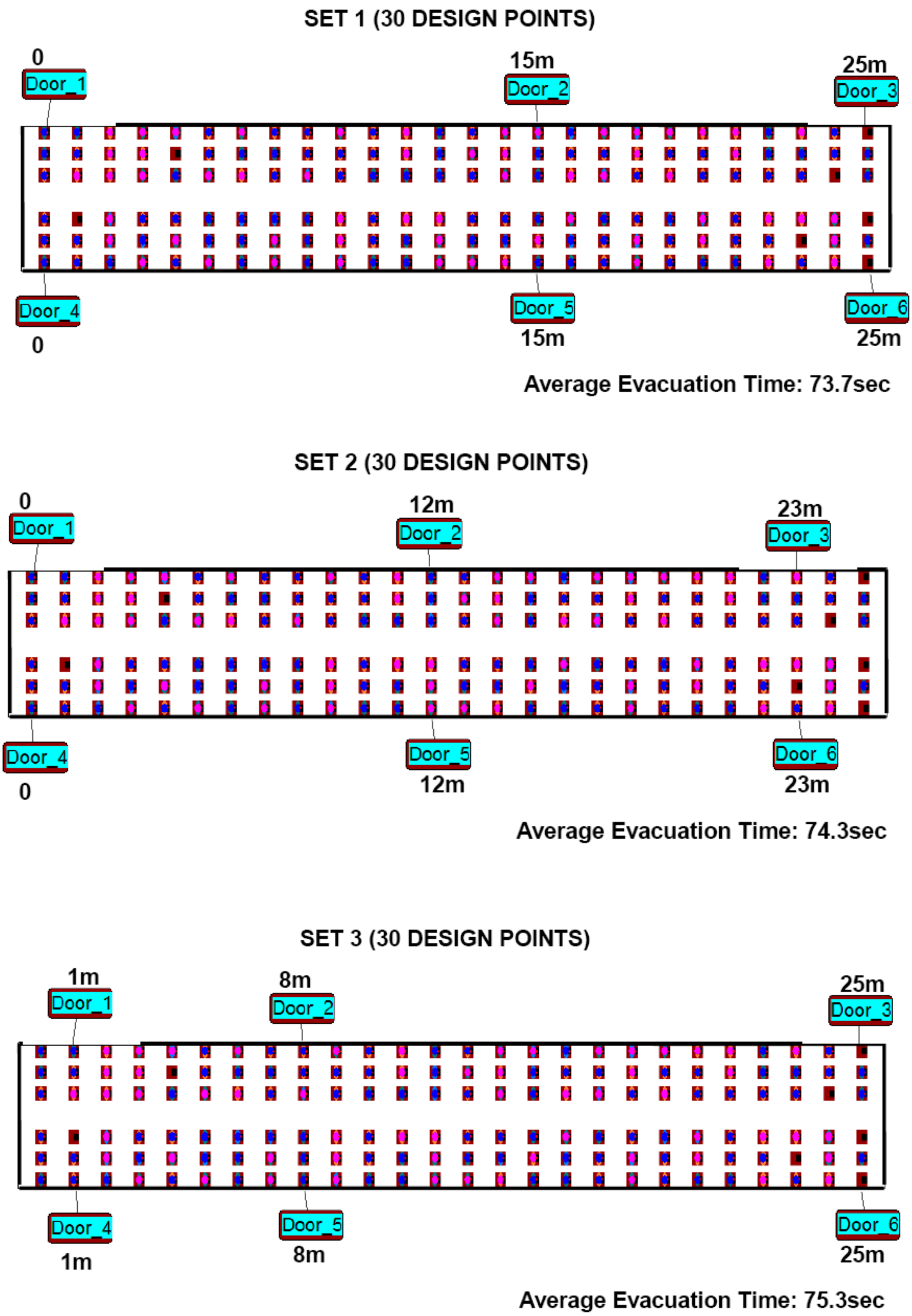


Figure 9.6: The 3 best configurations for all the three sets for Scenario 1

Presented in Figure 9.6 are the three solutions derived from the 30 design points for each the three sets of design points. All these best solutions show that the exits should be well distributed for producing lower evacuation times. However, from the ideal solution found section 9.2, it is clear that none of these cases result in the optimal distribution of exits.

Furthermore, the configurations were run in buildingEXODUS to determine the evacuation times they would produce in order to determine whether the predicted times were actually close to the simulated times.

Each configuration was tested and run 600 times, which generated a total of 1,800 simulations. For SET 1, the average evacuation time generated for the buildingEXODUS simulation was 75.8sec. This compares with a predicted 73.7sec produced by the optimisation technique. Clearly there is a difference between the actual simulation for this configuration and the time produced by the optimisation technique. However, this difference is very small: 2.77%.

For SET 2 and SET 3, we found similar difference between the simulated times produced for each of the "optimal" candidate solutions and the times generated by the optimisation method. This suggests that the times found by the optimisation method for this case are representative of the actual times produced by the simulation model for the specific cases.

It is relevant to mention that not simply the predicted evacuation time is important, but also the actual exit configuration. While the time produced by the predicted optimal solution (75.8sec) is only 25% greater than the actual optimal time (59.8sec), the predicted configuration is very different to the correct configuration. The most important physical array is that the forward and rear exits are still located in the extremities of the geometry. Whereas the actual optimal solution has the exits located some distant away from the extreme ends. The predicted solutions are thus significantly physically different to the actual solutions.

The failure of the optimisation method to determine the correct exit configuration may be the consequence of three reasons: the design points were not well located in the design space and/or the response surface model was not capable of producing a reasonable representation of the response surface, or there may not be sufficient design points.

As mentioned before, for all these three sets, all the design points were picked up randomly and also they were different. In other terms, for all these sets, the design points were not repeated. It is also important to mention that the actual solution configuration was not picked up amongst the considered design points. For instance, Figure 9.7 shows the different 30 design points for SET 1 and it is clear that the optimal configuration is not amongst the design points.

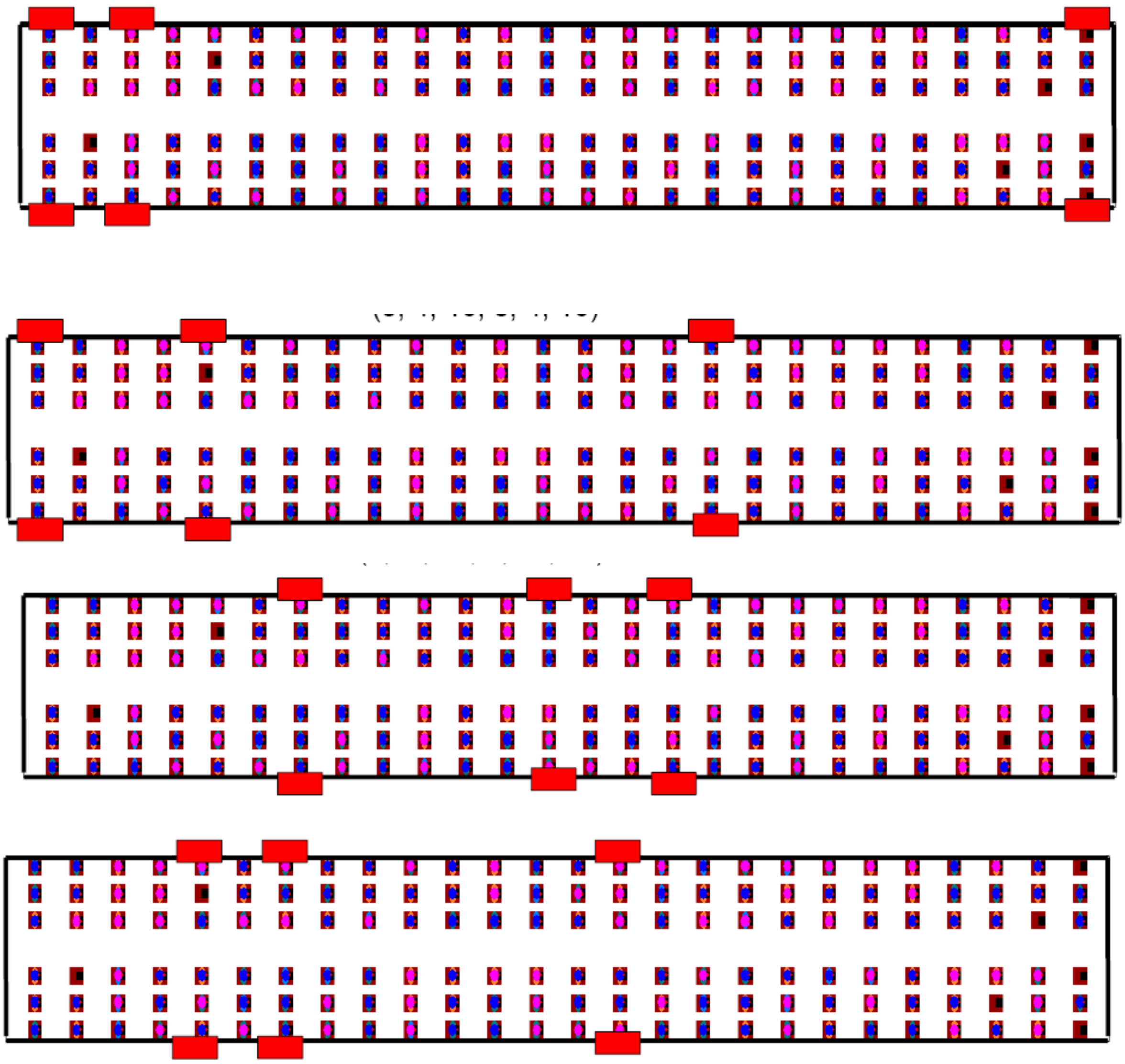


Figure 9.7a: The 30 different design points of SET 1

Figure 9.7b: The 30 different design points of SET 1

Figure 9.7c: The 30 different design points of SET 1

Figure 9.7d: The 30 different design points of SET 1

Figure 9.7e: The 30 different design points of SET 1

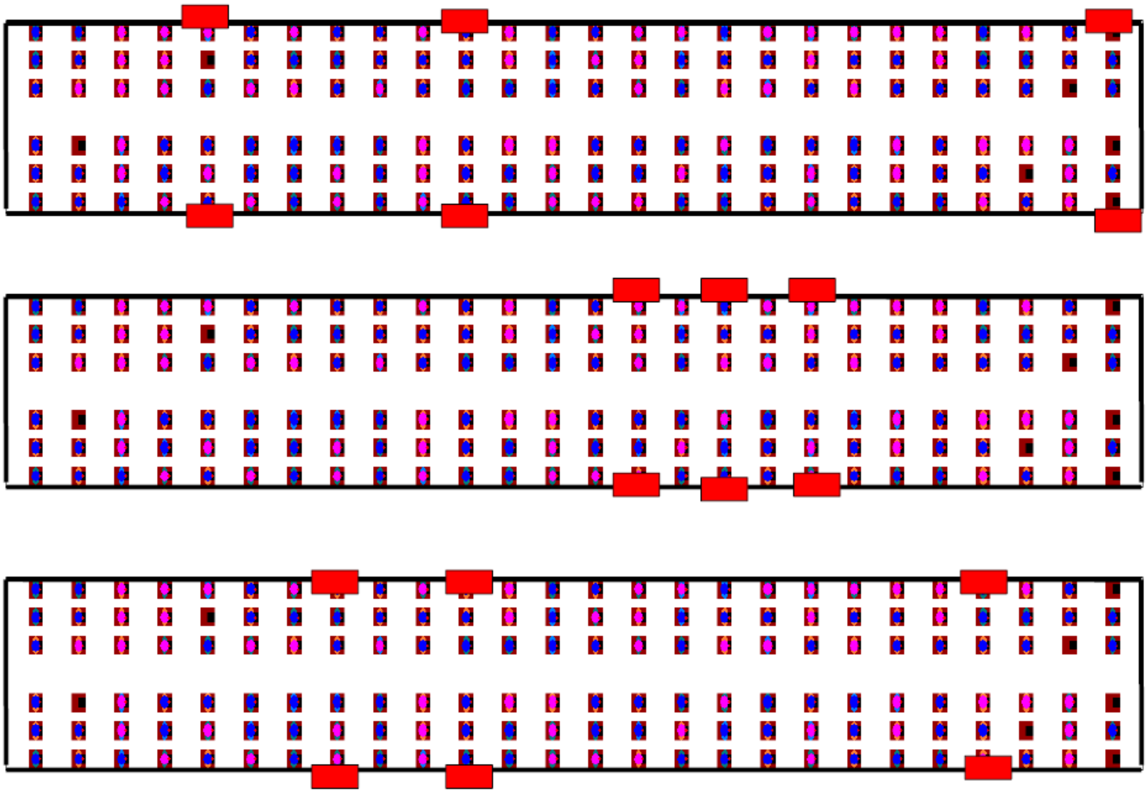


Figure 9.7f: The 30 different design points of SET 1

As we have investigated three different random sets of design points, the poor results are unlikely to be due to the nature of the design points.

Also, we have investigated up to 30 design points, so if the techniques require more design points, the methodology is less likely to be attractive to practical engineers due to the large amount of efforts that would be required to use the technique.

Thus, we will investigate the nature of the RSM on the accuracy of the produced results.

To investigate this, an additional optimisation analysis is performed. In this analysis, the response surface model used is based on the *multiquadratic* function. The same sets of design points used in the previous analysis are again used here. In total, 198,000 additional simulations were performed for this analysis. The results are shown in Table 9.2.

Table 9.2 Results for Scenario 1 (changing the RSM)

SET 1			SET 2		SET 3	
RSM Multiquadratic	**Numerical Optimisation Techniques**		**Numerical Optimisation Techniques**		**Numerical Optimisation Techniques**	
Number of Design Points	Fletcher-Reeves	PSO	Fletcher-Reeves	PSO	Fletcher-Reeves	PSO
20	(18;20;25; 18; 20; 25; 101)	(18;20;25; 18; 20; 25; 101)	(16;21;25; 16; 21; 25; 102.8)	(16;21;25; 16; 21; 25; 102.8)	(0;10;25;0;10;25;68.9)	(0;10;25;0;10;25;68.9)
25	(6; 20;23; 6; 20; 23; 86.8)	(6; 20;23; 6; 20; 23; 86.8)	(7; 15; 19; 7; 15; 19; 74.9)	(7; 15; 19; 7; 15; 19; 74.9)	(5; 15; 20; 5; 15; 20; 67.9)	(5; 15; 20; 5; 15; 20; 67.9)
30	(5; 15; 21; 5; 15; 21; 67.6)	(5; 15; 21; 5; 15; 21; 67.6)	(3; 12; 20; 3; 12; 20; 68.8)	(3; 12; 20; 3; 12; 20; 68.8)	(4; 12; 18; 4; 12; 18; 68.5)	(4; 12; 18; 4; 12; 18; 68.5)

Figure 9.8 presents the best exits location configurations found for these three sets, now using a different RSM.

As with the high-order polynomial RSM, the multi-quadratic RSM using less than 20 design points fails to produce a potential candidate solution. However, with 20 or more design points, potential solutions can be found. Furthermore, with 30 design points, a potentially viable optimal solution is found (see Figure 9.8). In this case, as in the optimal solution, and unlike the cases using the high-order polynomial RSM, the end exit pairs are located away from the end walls of the compartment, and the middle exits are located towards the middle of the geometry.

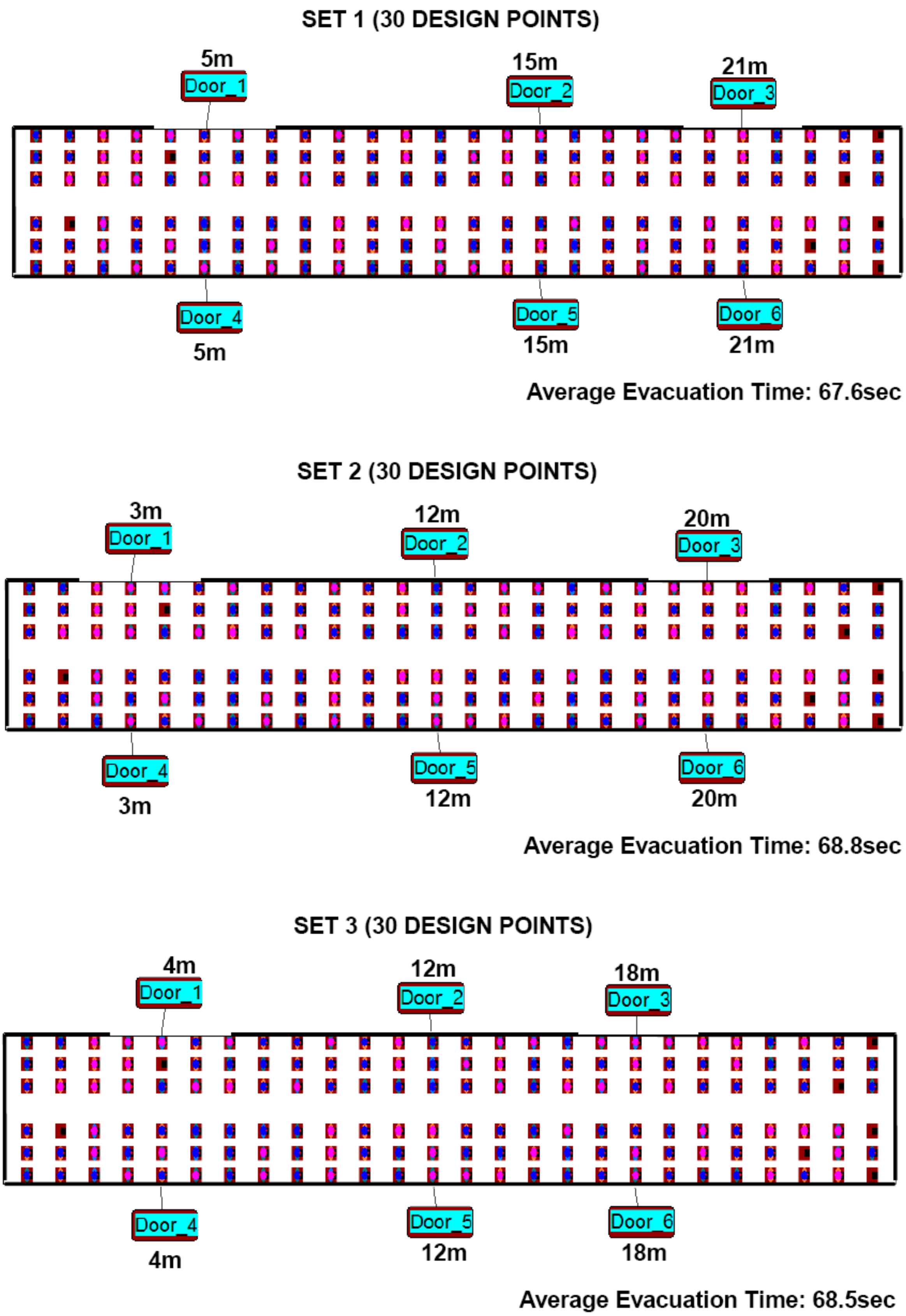


Figure 9.8: The 3 best configurations for all the three sets for Scenario 1 (using an improved RSM)

From Figure 9.8, it is clear to observe that the result obtained based on this additional analyses taken the same design points used in the initial sets (i.e., sets 1, 2 and 3), but changing the RSM from a high polynomial function order to a *multiquadratic* function, improved the results. The solutions obtained using the *multiquadratic* function as the RSM while much improved does not produce the exact solutions.

Nevertheless, it achieved an improvement, since lower evacuation times were obtained when compared with the values found when the High order polynomial was used as the RSM. This can be explained by the fact that the exits are better distributed along the walls. Therefore, in reality, the results are better not only in terms of time, but most important, the exit locations are more closely in agreement with those in the optimal solutions.

In order to check the consistency of this result, additional evacuation simulations were performed based on these best solutions. The simulations were performed 600 times. The evacuation times produced based on the average of these 600 simulations were: for SET 1 was 68.9sec; for SET 2 was 69.5sec and for SET 3 was 70.1sec. These are small differences when compared with values obtained from the optimisation analyses; therefore the results can be considered consistent.

Based on that, it is possible to conclude that the *multiquadratic* function seems to be a more appropriate RSM for these types of analyses. This is further confirmed by comparing the regression coefficients for the *multiquadratic* function with that of the high-order polynomial function. The regression coefficient for the multiquadratic function is 0.97 while for the high-order polynomial was 0.95. This means that the error when approximating the curve to the actual design points along the design space is smaller when using the *multiquadratic* function than when using the high-order polynomial function.

For this reason, the *multiquadratic* function is used as the RSM for the other scenarios. And in fact, this type of radial-basis function, as discussed in chapter 3, seems to be more suitable for complex optimisation problems where the number of design variables becomes an important issue to be addressed.

It was also found that for these analyses, the design points are also an important issue, since resealable good results just started to be found when the number of design points was increased. Nevertheless, based on the fact that different sets of different design points were tested here, the major impact seems to be regards the RSM rather than the design points. This is said, because for this complex study given its number of design variables, it is expected that a larger amount of design points would be required for obtaining reasonable results.

It seems that from 20 design points using the *multiquadratic* function, good results (i.e., local minima regions close to the global minimum region) can be achieved.

In the next section, scenario 2 is investigated using the optimisation methodology.

9.5.2 Scenario 2: solution using the optimisation methodology

9.5.2.1 Problem statement

This scenario involves four large, two small, in pairs, three on each side.

Therefore, scenario 2 assumed that the three exits in side 1 would have the same relative locations along the edge that the three exits in the side 2 would have and vice-versa. For this reason, this problem, like scenario 1, is a 3 variables problem. Nevertheless, not all of the 6 exits have the same maximum flow rate. Two exits in side 1 have a maximum flow rate of 1.33 occupants/meter/second (occ/m/s) and one exit in side 1 has the maximum flow rate of 0.77 occupants/meter/second (occ/m/s). In summary, four of the exits have a flow rate of 1.33 occupants/meter/second (occ/m/s), HMSO. The other two exits have a flow rate of 0.77 occupants/meter/second (occ/m/s).

The problem here is defined similarly to the way the problem in scenario 1 was defined. From Figure 9.1, it is possible to define the domains of each design variable:

$0 < D1 < 25$ $0 < D2 < 25$ $0 < D3 < 25$

$0 < D4 < 25$ $0 < D5 < 25$ $0 < D6 < 25$

Where:

D1 represents the distance from the left edge of exit 1 to the corner

D2 represents the distance from the left edge of exit 2 to the corner

D3 represents the distance from the left edge of exit 3 to the corner

D4 represents the distance from the left edge of exit 4 to the corner

D5 represents the distance from the left edge of exit 5 to the corner

D6 represents the distance from the left edge of exit 6 to the corner

For reasons of consistency, it is important to mention that exits 1, 2 and 3 are located in side 1 and the exits 4, 5 and 6 are located in side 2.

Once the locations for exits 4, 5 and 6 will be the same for the exits 1, 2 and 3, respectively; this problem can be solved as a 3 variables problem:

Minimize: $ET = f\,(D1,D2,D3)$

Where: $0 < D1 < 25$ $0 < D2 < 25$ $0 < D3 < 25$

And given that these exits should not overlap each other. Clearly, the problem is a constrained problem, therefore, two constraints were considered:

$$|D1 - D2| > 0.5$$

$$|D2 - D3| > 0.5$$

9.5.2.2 Problem methodology

Similarly to what was done for scenario 1, for this study case, the random DoE technique was used for picking up the design points. Again, 3 sets of different design points were defined: 4 design points, 10 design points, 15 design points, 20 design points, 25 design points, 30 design points and 35 design points. Just from 15 design points and above 15 design points, some consistent results were found.

In total, given that for each design point, 600 simulations were performed; which required a total of 187,200 simulations to be performed.

9.5.2.3 Results

Similarly to the previous case study, the random DoE technique was used for selecting the design points. The response surface model used was the *Multiquadratic* Function for the reasons explained previously. For this scenario, 3 sets of different design points were defined: 4, 6, 10, 15, 20, 25, 30 design points and 35 design points. The results are presented in Table 9.3; as can be seen, at least 20 design points were required to produce potential candidate solutions. Both numerical optimisation techniques, the Fletcher-Reeves and the PSO, produced the same results which show their robustness.

Table 9.3 Results for Case Study 3: Scenario 2

SET 1		
Response Surface Model *Multiquadratic* Function	**Numerical Optimisation Technique**	
DoE - Random DoE technique	Fletcher-Reeves	PSO – Particle Swarm Optimisation
4 design points	N.S.	N.S.
6 design points	N.S.	N.S.
10 design points	N.S.	N.S.
15 design points	N.S.	N.S.
20 design points	(1; **3**; 25; 1; **3**; 25; 73.5)	(1; **3**; 25; 1; **3**; 25; 73.5)
25 design points	(**0**; 7; 25.5; **0**; 7; 25.5; 67.9)	(**0**; 7; 25.5; **0**; 7; 25.5; 67.9)
30 design points	(4;**12**;20;4;**12**;20;66.5)	(4;**12**;20;4;**12**;20;66.5)
35 design points	(3;**10**;22;3;**10**;22;66.1)	(3;**10**;22;3;**10**;22;66.1)
SET 2		
Response Surface Model *Multiquadratic* Function	**Numerical Optimisation Technique**	
DoE - Random DoE technique	Fletcher-Reeves	PSO – Particle Swarm Optimisation
4 design points	N.S.	N.S.
6 design points	N.S.	N.S.
10 design points	N.S.	N.S.
15 design points	N.S.	N.S.
20 design points	(**16;**19;25;**16**; 19; 25; 70.8)	(**16;**19;25;**16**; 19; 25; 70.8)
25 design points	(0; **1**; 25; 0; **1**; 25;68.9)	(0; **1**; 25; 0; **1**; 25;68.9)
30 design points	(**0**; 2; 23; **0**; 2; 23; 68.6)	(**0**; 2; 23; **0**; 2; 23; 68.6)
35 design points	(4;**11**;24;4;**11**;24;67)	(4;**11**;24;4;**11**;24;67)
SET 3		
Response Surface Model *Multiquadratic* Function	**Numerical Optimisation Technique**	
DoE - Random DoE technique	Fletcher-Reeves	PSO – Particle Swarm Optimisation
4 design points	N.S.	N.S.
6 design points	N.S.	N.S.
10 design points	N.S.	N.S.
15 design points	N.S.	N.S.
20 design points	(6; 15; **25**; 6; 15; **25** ;69.9)	(6; 15; **25**; 6; 15; **25** ;69.9)
25 design points	(0; 20; **25**;0; 20; **25**;64.7)	(0; 20; **25**;0; 20; **25**;64.7)

30 design points	(6;**13**;19;6;**13**;19;62.9)	(6;**13**;19;6;**13**;19;62.9)
35 design points	(5;**10**;20;5;**10**;20;62.5)	(5;**10**;20;5;**10**;20;62.5)

From the results, it was also possible to see that the best type of scenario is when the exits are well distributed.

Figure 9.9 presents the optimal exits location configurations for these 3 sets.

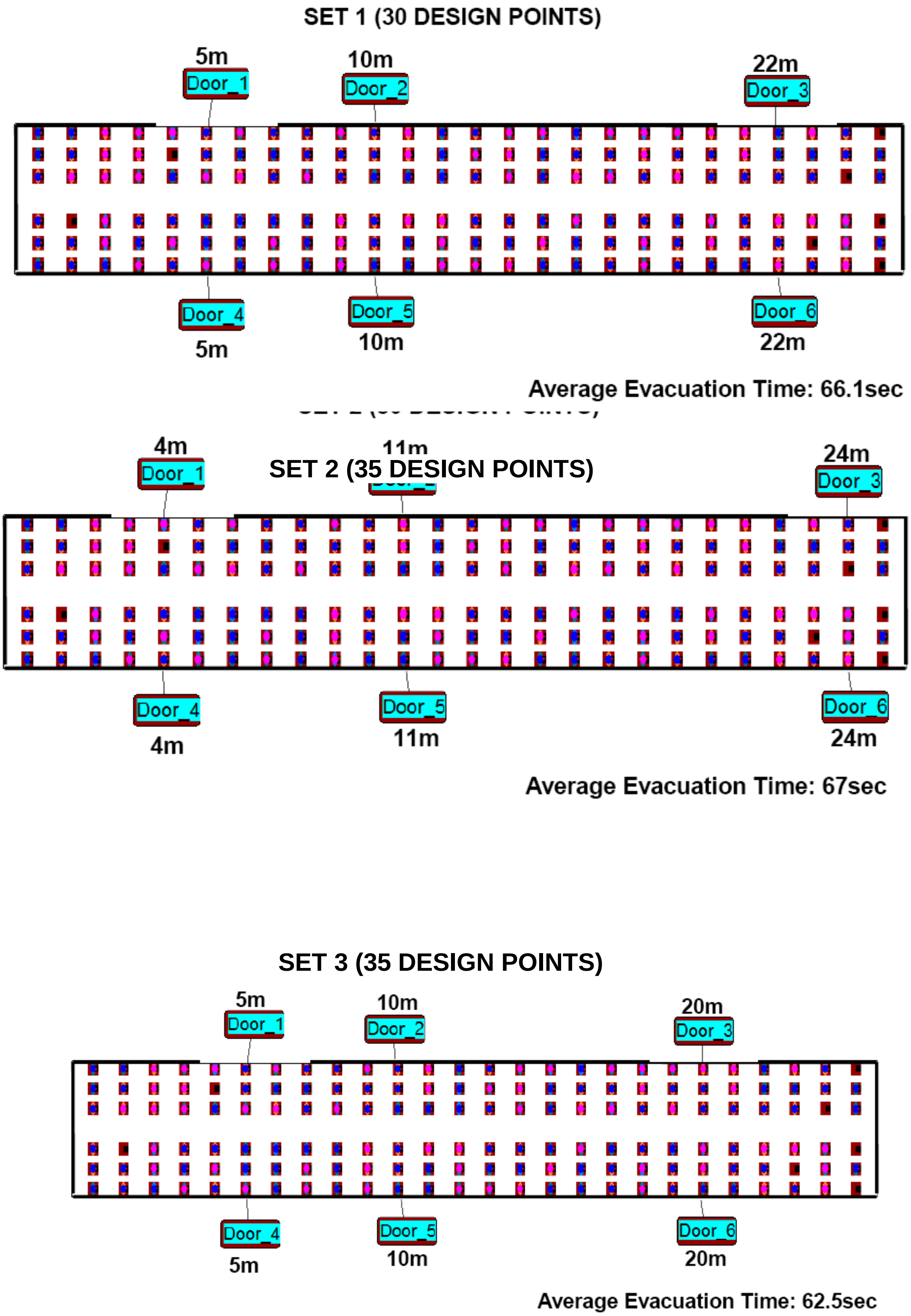


Figure 9.9: The 3 best configurations for all the three sets for Scenario 2

As it is possible to see from Figure 9.9, similarly to scenario 1, apart from SET 2, all the best solutions for these 3 sets of design points show that the exits should be well distributed for producing lower evacuation times.

All the exits with flow rate of 0.77 occ/m/s are located in the middle of the walls.

In order also to check the consistency of these results, additional evacuation simulations were performed based on these best solutions. Each simulation was performed 600 times, which generated a total of 1,800 simulations. The results have shown to be consistent.

For SET 1, the evacuation time produced was 67.6sec; while the actual result from the optimisation analysis was 66.1sec. For SET 2, the evacuation time produced was 68.1sec; while the actual result from the optimisation analysis was 67sec. And finally, for SET 3, the evacuation time produced was 64.5sec; while the actual result from the optimisation analysis was 62.5sec.

It is important to mention that all the sets using 20 and 25 design points, the data did not produce reasonable good results. The actual evacuation times were not much higher than the best results; nevertheless the exit locations were not good.

It is also important to mention that it was not possible to find a lower evacuation time than the one found through logical analysis (61.2sec). Nevertheless, the actual physical locations of the exits are also consistent and considered to be optimal in terms of practical design.

In summary, these results are showing that the optimisation techniques when properly used can find very good results indeed. And in fact, these results are not only "good" numerically speaking (since the values were close to the optimal value found previously); but also importantly, the results produced using these techniques were also good in terms of design. In other words, the exits locations configurations were consistent to what has been shown previously in section 9.4.2, in where the optimal solution was also found when the exits were well distributed along the walls; and besides that the smaller exits were also located in the middle of the walls.

In the next section, scenario 3 is investigated using the optimisation methodology developed in this research.

9.5.3 Scenario 3: solution using the optimisation methodology

9.5.3.1 Problem statement

This scenario involves identical exits, not in pairs, three on each side.

Therefore, scenario 3 assumed that the three exits in side 1 would not have the same relative locations along the edge that the three exits in the side 2 would have and vice-versa. For this reason, this problem, differently from scenarios 1 and 2, is a 6 variables problem. And all of the 6 exits have the same maximum flow rate of 1.33 occ/m/s.

Once the locations for exits 4, 5 and 6 will not be necessarily the same for the exits 1,2 and 3, respectively; this problem has to be solved as a 6 variables problem:

Minimize: ET = *f* (D1,D2,D3,D4,D5,D6)

Where: $0 < D1 < 25$ $0 < D2 < 25$ $0 < D3 < 25$

$0 < D4 < 25$ $0 < D5 < 25$ $0 < D6 < 25$

And given that these exits should not overlapping each other (on the same edge), four constraints were considered:

$$|D1 - D2| > 0.5$$

$$|D2 - D3| > 0.5$$

$$|D4 - D5| > 0.5$$

$$|D5 - D6| > 0.5$$

Clearly, the problem is a constrained problem, since the exits should not overlap each other, in both sides of the geometry.

9.5.3.2 Problem methodology

For this study case, the random DoE technique was used to select the design points. In a similar way to the previous consideration of this problem, a number of different 3 sets of design points were defined: 7 design points, 10 design points, 15 design points, 20 design points, 25 design points, 30 design points and 35 design points. In these investigations, consistent results were only found using 20 design points or more.

In total, given that for each design point, 600 simulations were performed, a total of 187,200 simulations were required for all of the design points.

9.5.3.3 Results

Similarly to the previous case studies, the random DoE technique was used for picking up the design points. The response surface model used was the *Multquadratic* Function for the reasons explained previously. For this scenario, 3 sets of different design points were defined: 7 design points, 10 design points, 15 design points, 20 design points, 25 design points, 30 design points and 35 design points. Both numerical optimisation techniques, the Fletcher-Reeves and the PSO, produced the same results which show their robustness.

The table 9.4 presents the summary of the results.

Table 9.4 Results for Case Study 3: Scenario 3

SET 1		
Response Surface Model *Multiquadratic* Function	**Numerical Optimisation Technique**	
DoE - Random DoE technique	Fletcher-Reeves	PSO – Particle Swarm Optimisation
7 design points	N.S.	N.S.
10 design points	N.S.	N.S.
15 design points	N.S.	N.S.
20 design points	N.S.	N.S.
25 design points	(0; 11; 25; 4; 12; 23; 56.8)	(0; 11; 25; 4; 12; 23; 56.8)
30 design points	(7; 11; 25; 4; 18; 23; 54.9)	(7; 11; 25; 4; 18; 23; 54.9)
35 design points	(3; 12; 25; 1; 7; 19; 53.8)	(3; 12; 25; 1; 7; 19; 53.8)
SET 2		
Response Surface Model *Multiquadratic* Function	**Numerical Optimisation Technique**	
DoE - Random DoE technique	Fletcher-Reeves	PSO – Particle Swarm Optimisation
7 design points	N.S.	N.S.
10 design points	N.S.	N.S.
15 design points	N.S.	N.S.
20 design points	N.S.	N.S.
25 design points	(0; 15; 25; 5; 10; 25; 62.9)	(0; 15; 25; 5; 10; 25; 62.9)
30 design points	(5;10; 25; 7; 19; 22; 55.9)	(5;10; 25; 7; 19; 22; 55.9)
35 design points	(4; 10; 24; 0; 7; 17; 54.7)	(4; 10; 24; 0; 7; 17; 54.7)
SET 3		
Response Surface Model *Multiquadratic* Function	**Numerical Optimisation Technique**	
DoE - Random DoE technique	Fletcher-Reeves	PSO – Particle Swarm Optimisation
7 design points	N.S.	N.S.
10 design points	N.S.	N.S.
15 design points	N.S.	N.S.
20 design points	(0; 17; 20; 5; 12; 25; 64.5)	(0; 17; 20; 5; 12; 25; 64.5)
25 design points	(0; 20; 25; 10; 15; 19; 68.7)	(0; 20; 25; 10; 15; 19; 68.7)
30 design points	(4; 15; 25; 0; 9; 21; 52.5)	(4; 15; 25; 0; 9; 21; 52.5)
35 design points	(0; 8; 22; 3; 14; 25; 52.1)	(0; 8; 22; 3; 14; 25; 52.1)

N.S. - No Solution found.

From the results, it was also possible to see that, similarly to the previous scenarios, the best type of scenario is when the exits are well distributed. It is interesting to observe that the best result amongst these three sets for scenario 3, is obtained for SET 3, where the exits are much better distributed along the walls when compared with SETS 1 and 2, see Figure 9.10.

Figure 9.10 presents the optimal exits location configurations for these 3 sets.

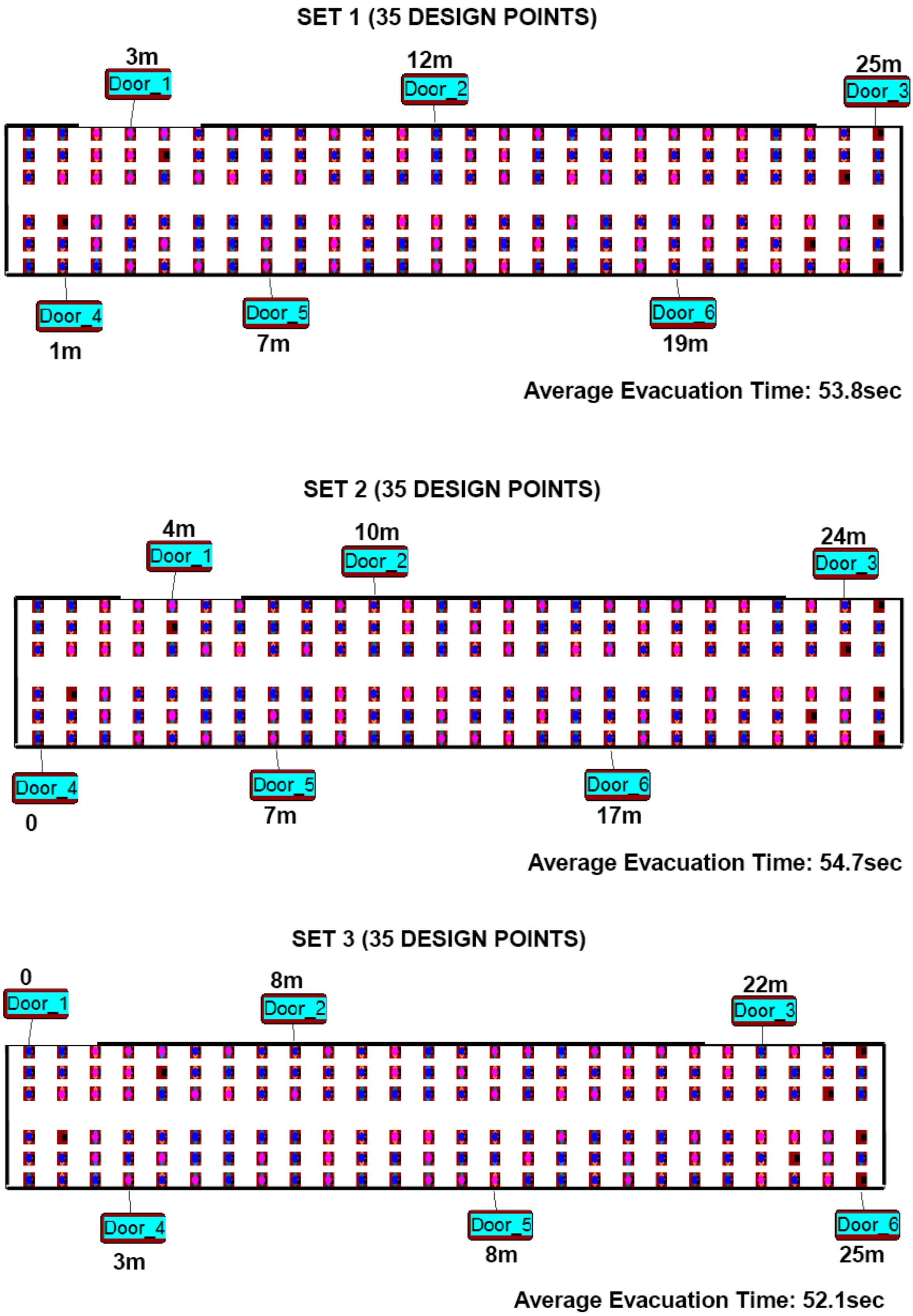


Figure 9.10: The best configurations for all the three sets for Scenario 3

Here again, in order also to check the consistency of these results, additional evacuation simulations were performed based on these best solutions. Each simulation was performed 600 times, which generated a total of 1,800 simulations. The results have shown to be consistent.

For SET 1, the evacuation time produced was 54.2sec; while the actual result from the optimisation analysis was 53.8sec. For SET 2, the evacuation time produced was 55.7sec; while the actual result from the optimisation analysis was 54.5sec. And finally, for SET 3, the evacuation time produced was 53.4sec; while the actual result from the optimisation analysis was 52.1sec.

It is important to mention that for this scenario, the optimal solution based on the logical analysis predicted an evacuation time of 51.1sec. For this reason, using the Optimisation Methodology for this particular scenario, the optimal solution (or what it is believed to be the optimal solution) was not found. Nevertheless, it can be considered that good results were found, since the exits locations are well distributed similarly to the optimal solution; and as it was mentioned before, this is also one of the objectives of this study: not only to find the actual minimum evacuation time, but the optimal design.

Based on the trend shown in the results, clearly, it would be expected that, very likely, if the number of design points is increased, better results would be found (i.e., much closer to the actual optimal result found through the logical analysis).

In the next section, the last scenario is investigated using the optimisation methodology.

9.5.4 Scenario 4: solution using the optimisation methodology

9.5.4.1 Problem statement

This scenario in summary can be described as follows: 3 exits on each side; left and right exits are not necessarily at the same location; not all exits have the same maximum flow rate.

Therefore, scenario 4 assumed that the three exits in side 1 would not have the same relative locations along the edge that the three exits in the side 2 would have and vice-versa. For this reason, this problem, similarly from scenario 3, is a 6 variables problem. Nevertheless, not all of the 6 exits have the same maximum flow rate. Two exits in side 1 have a maximum flow rate of 1.33 occupants/meter/second (occ/m/s) and one exit in side 1 has the maximum flow rate of 0.77 occupants/meter/second (occ/m/s). In summary, four of the exits have a flow rate of 1.33 occupants/meter/second (occ/m/s), HMSO. The other two exits have a flow rate of 0.77 occupants/meter/second (occ/m/s).

Similarly to scenario 3, once the locations for exits 4,5 and 6 will not be necessarily the same for the exits 1,2 and 3, respectively; this problem has to be solved as a 6 variables problem:

Minimize: ET = f (D1,D2,D3,D4,D5,D6)

Where: $0 < D1 < 25$ $0 < D2 < 25$ $0 < D3 < 25$

$0 < D4 < 25$ $0 < D5 < 25$ $0 < D6 < 25$

And given that these exits should not overlapping each other (on the same edge), four constraints were considered:

$$|D1 - D2| > 0.5$$

$$|D2 - D3| > 0.5$$

$$|D4 - D5| > 0.5$$

$$|D5 - D6| > 0.5$$

9.5.4.2 Problem methodology

Similarly to the previous scenario, the problem here is also a constrained problem, since the exits should not overlap each other, in both sides of the geometry.

For this study case, the random DoE technique was used to select the design points. And 3 sets of different design points were defined: 7 design points, 10 design points, 15 design points, 20 design points, 25 design points, 30 design points and 35 design points. In these investigations, consistent results were only found using 20 design points or more.

In total, given that for each design point, 600 simulations were performed, a total of 187,200 simulations were required for all of the design points.

9.5.4.3 Results

And finally, also similarly to the previous case studies, the random DoE technique was used for picking up the design points. The response surface model used was the *Multquadratic* Function for the reasons explained previously. For this scenario, 3 sets of different design points were defined: 7 design points, 10 design points, 15 design points, 20 design points, 25 design points, 30 design points and 35 design points. Both numerical optimisation techniques, the Fletcher-Reeves and the PSO, produced the same results which show their robustness. The table 9.5 presents this summary.

Table 9.5 Results for Case Study 3: Scenario 4

SET 1		
Response Surface Model *Multiquadratic* Function	**Numerical Optimisation Technique**	
DoE - Random DoE technique	Fletcher-Reeves	PSO – Particle Swarm Optimisation
7 design points	N.S.	N.S.
10 design points	N.S.	N.S.
15 design points	N.S.	N.S.
20 design points	(**0**;17; 25; 9; 17; **25**; 58.2)	(**0**;17; 25; 9; 17; **25**; 58.2)
25 design points	(**5**; 17; 25; 9; 17; **25**; 58.8)	(**5**; 17; 25; 9; 17; **25**; 58.8)
30 design points	(**0**; 12; 22; 5; 18; **25**; 56)	(**0**; 12; 22; 5; 18; **25**; 56)
35 design points	(**0**; 9; 18; 4; 14; **25**; 55.8)	(**0**; 9; 18; 4; 14; **25**; 55.8)
SET 2		
Response Surface Model *Multiquadratic* Function	**Numerical Optimisation Technique**	
DoE - Random DoE technique	Fletcher-Reeves	PSO – Particle Swarm Optimisation
7 design points	N.S.	N.S.
10 design points	N.S.	N.S.
15 design points	N.S.	N.S.
20 design points	N.S.	N.S.
25 design points	(0; **19**; 25; 7; 15; 20; 100.7)	(0; **19**; 25; **7**; 15; 20; 100.7)
30 design points	(**0**; 12; 13; 6; 20; **25**; 68.4)	(**0**; 12; 13; 6; 20; **25**; 68.4)
35 design points	(**3**; 12; 25; 0; 7; **19**; 56.1)	(**3**; 12; 25; 0; 7; **19**; 56.1)
SET 3		
Response Surface Model *Multiquadratic* Function	**Numerical Optimisation Technique**	
DoE - Random DoE technique	Fletcher-Reeves	PSO – Particle Swarm Optimisation
7 design points	N.S.	N.S.
10 design points	N.S.	N.S.
15 design points	N.S.	N.S.
20 design points	(**0**;15; 25; 10; 20; **23**; 101.3)	(**0**;15; 25; 10; 20; **23**; 101.3)
25 design points	(**5**; 15; 25; 0; **10**; 20; 59.1)	(**5**; 15; 25; 0; **10;** 20; 59.1)
30 design points	(**4**; 15; 25; 0; 9; **21**; 54)	(**4**; 15; 25; 0; 9; **21**; 54)
35 design points	(4; 14; **25**;**0**; 10; 18; 53.8)	(4; 20; **25**;**0**; 10; 18; 53.8)

N.S. - No Solution found.

Once again, from the results, it is possible to see that, in all scenarios, the exits are well distributed (see Figure 9.11). The best scenarios were obtained using 30 design points. It is also important to mention that these optimal scenarios were obtained when the exits with lower flow rate were located in the corners of the walls.

Figure 9.11 presents the optimal exits location configurations for these 3 sets.

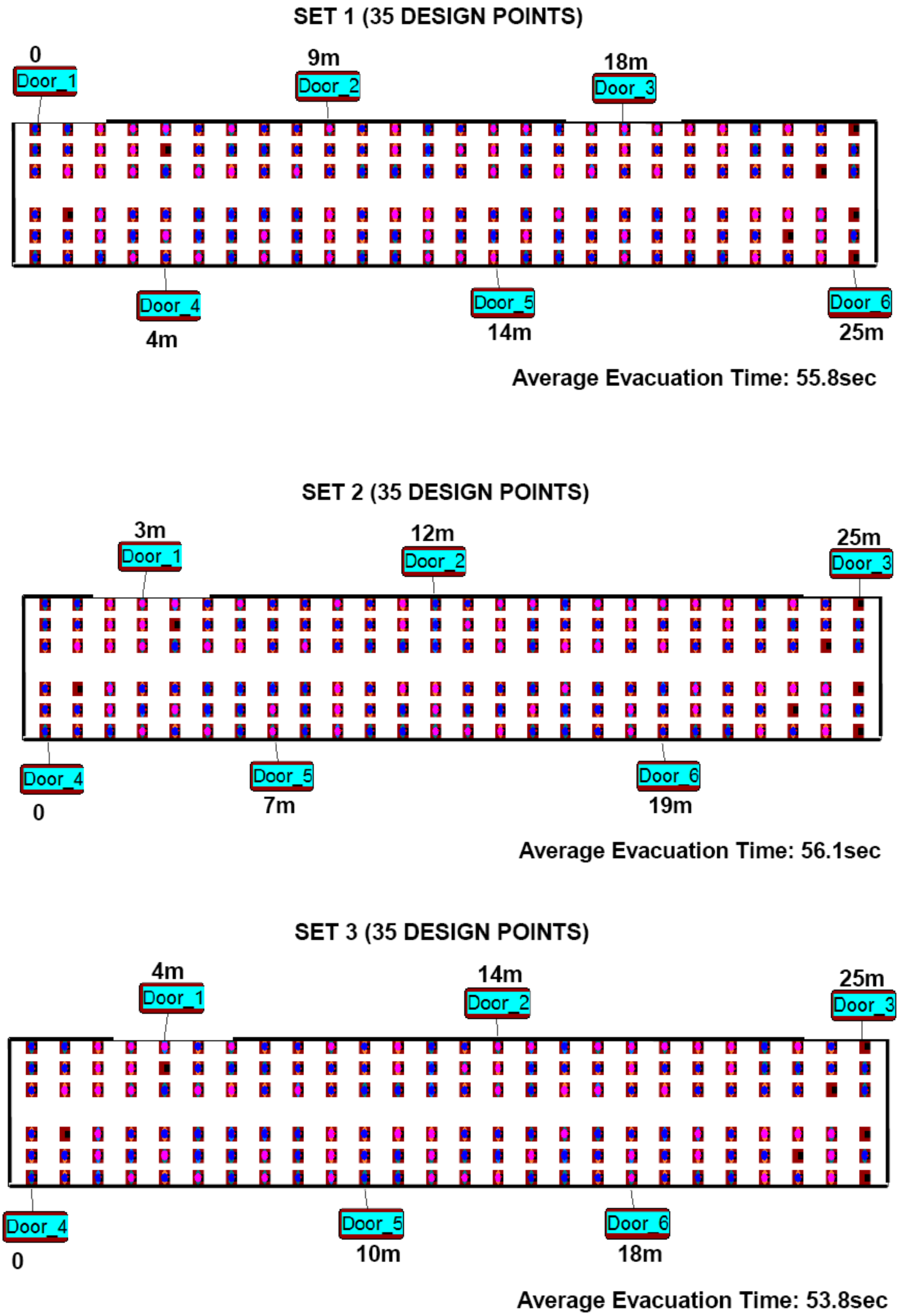


Figure 9.11: The best configurations for all the three sets for Scenario 4

And finally, in order also to check the consistency of these results, additional evacuation simulations were performed based on these best solutions. Each simulation was performed 600 times, which generated a total of 1,800 simulations. The results have shown to be consistent.

For SET 1, the evacuation time produced was 56.1sec; while the actual result from the optimisation analysis was 55.8sec. For SET 2, the evacuation time produced was 58.8sec; while the actual result from the optimisation analysis was 56.1sec. And finally, for SET 3, the evacuation time produced was 54.2sec; while the actual result from the optimisation analysis was 53.8sec.

It is also important to mention that it was not possible to find a lower evacuation time than the one found through logical analysis. In reality, previously, it was found that the best configuration provided an evacuation time of 55.2sec. And now through the use of the Optimisation Methodology, it was found that the best configuration gives an evacuation time of 54.2sec. Despite this, it can be considered that the results are good, given the small difference between the actual optimal solution and the solutions found using the techniques. Besides that, the actual physical locations of the exits are also consistent and have shown that the lowest evacuation times are produced when the small type exits are located in the corners.

9.5 Concluding Comments

In this chapter, the final case study was investigated in where the optimal positioning of exits around the perimeter of rectangular shaped room (with discrete spaces) in order to minimise evacuation times were explored. For this particular geometry, there were multiple exits of varying sizes.

The evacuation simulations were conducted assuming ideal conditions of zero response times and population behaviour such that occupants would move to their nearest exits. Both assumptions were made to simplify the analysis and to isolate issues associated with exit location.

For this geometry, 6 exits were available and they did not have the same flow rate. The results for this problem were found through the use of numerical optimisation techniques.

This investigation revealed also that significant advantage can be derived from the strategic positioning of the exits. And besides that, the way the space is presented, discrete instead of continuum, plays also an important rule on the evacuation efficiency.

The problem was solved taking into account two considerations. In the first one, the three exits located in one wall would have the same locations of the three other exits located in the opposite wall. And in the second consideration, these exits would not have necessarily the same locations as the exits located in the opposite wall.

Therefore, for consideration 1, the problem was solved as a 3 variables problem and two constraints were taken into account. While for consideration 2, the problem was solved as a 6 variables problem and five constraints were taken into account.

Several design points were analyzed for both scenarios, and given the problem nature, i.e., constrained problems, the random DoE technique was used for picking up the design points.

As mentioned in the previous chapters and based on the results, it is clear that for complex cases, with more than 3 design variables for instance, the random DoE technique is not suitable, since consistent results are often just found from a large amount of design points.

It was possible to conclude that the *multiquadratic* function seems to be a more appropriate RSM for these types of analyses. This might be explained by the fact that the regression coefficient is better and therefore, the error when approximating the curve to the actual design points along the design space is smaller when using the *multiquadratic* function than when using the high-order polynomial function.

For this reason, the *multiquadratic* function is used for the other scenarios. And in fact, this type of radial-basis function, as discussed in chapter 3, seems to be more suitable for complex optimisation problems where the number of design variables becomes an important issue to be addressed.

It was also found that for these analyses, the design points are also an important issue, since reasonable good results just started to be found when the number of design points was increased.

It seems that from 20 design points using the *multiquadratic* function, good results (i.e., local minima regions close to the global minimum region) can be achieved.

In the next chapter, this issue is discussed with more details and a DoE method is proposed to bring some light to this issue.

10. Introduction

In this chapter a generic DoE method: the “modified CCD DoE method” is presented. This method is a hybrid DoE method which can be applied for both unconstrained and constrained problems. The Modified CCD Method was developed through data analyses based on the previous studies presented in this thesis, which included evacuation simulations of a variety of different types of scenario. This new method has shown itself to be more robust when compared to the random DoE technique.

From the previous case studies, it became clear that, while the Random DoE technique may be viable to address relatively simple problems; but even for simple problems, the Random DoE technique might not be suitable. The reason for that is based on the fact that the design points selected by this technique can be concentrated within a particular area within the design space which could be far from the optimal solution region. In addition to that, the DoE technique clearly is not viable for assessing more complex situations involving many exits, exits of varying size and complex shaped compartments.

Therefore, in the following paragraphs of this chapter, the modified CCD method (MCCD method) is presented and discussed. This method is also applied to all of the previous case studies, which have been analysed in this thesis, to demonstrate the practical use of the method.

10.1 Review on the general problem

As mentioned previously, the problems investigated in this chapter had the same key question: for an arbitrarily complex structure, containing an arbitrarily large population; is there an optimal location for the exits that will minimise egress times?

In the previous chapters, the optimal positioning of exits around the perimeter of different enclosures was explored in order to minimise egress times. It was found that the exit position and, in case of two exits, the distribution of exits around the perimeter, does have an impact on the egress time.

In order to solve this problem, this thesis has presented and discussed a systematic methodology which combines the use of numerical optimisation techniques and evacuation simulation analysis, see Figure 10.1. This methodology has been found to provide the best solution in terms of giving an optimal or near optimal solution (i.e., determination of the best exit position to minimise the egress time).

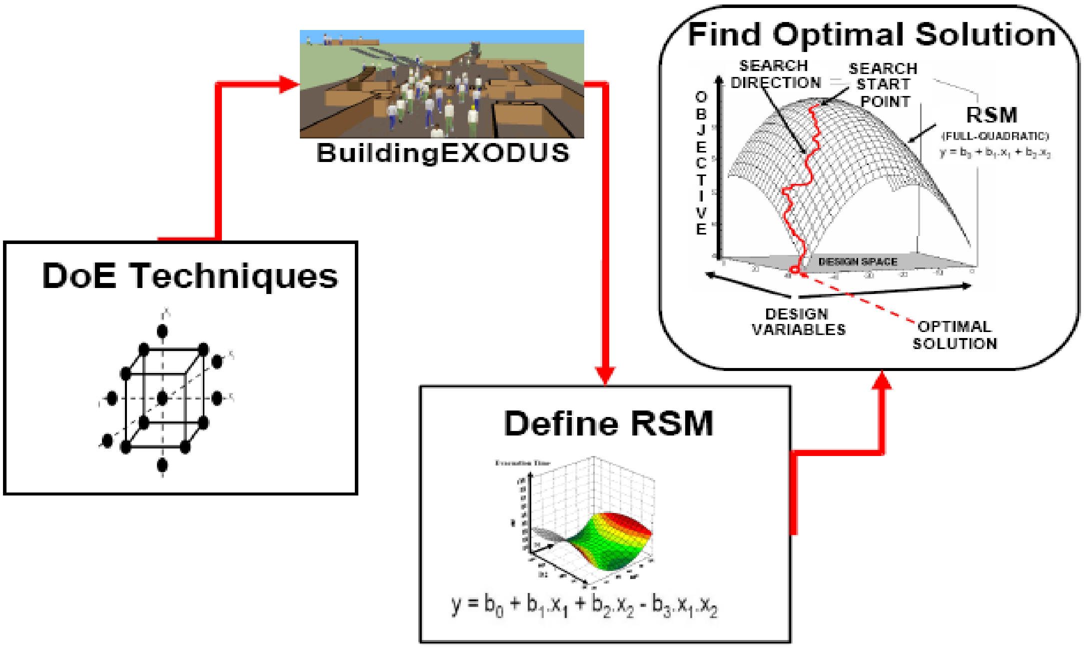


Figure 10.1: The proposed optimisation methodology

For all of the case studies analysed, in which the geometry had two or more exits, the optimisation problem was solved as constrained. For this, the random DoE technique was used to select the design points.

It became clear that the random DoE technique has shown to be not suitable for more complex cases, where the time spent performing the whole analysis becomes an issue that needed to be addressed. Therefore, an improvement on how these design points are selected along the design space was needed; especially for constrained problems where the design space is modified.

For this reason, this chapter presents a DoE method, the *Modified CCD Method*, which picks up design points strategically "well located" along the design space. This is an important issue to be addressed, especially for constrained problems, as mentioned previously.

As discussed previously in this thesis, there are few DoE techniques available; nevertheless, their use is limited to unconstrained problems, where the design space is regular. For constrained problems, where the design space is modified, and consequently irregular, the standard DoE techniques, such as the CCD and Factorial designs, will not be able to be applied. The main reason for that is due the fact that these conventional DoE techniques have their algorithms based on combinations which rely on the whole design space. In other words, if the design space is modified, these

DoE techniques would not be able to select the new design points which are located in the new boundaries imposed by the new design space (i.e., modified design space). For instance, the following example can illustrate what was said. Assuming a function *f* dependent on two design variables X and Y; and that both design variables have the same domain: -1 as the lower value and +1 as the upper value. Therefore, the problem can be mathematically stated as it follows:

Objective function: $f\,(X; Y)$

Where: $-1 < X < 1$

$-1 < Y < 1$

Based on that, the design space would be:

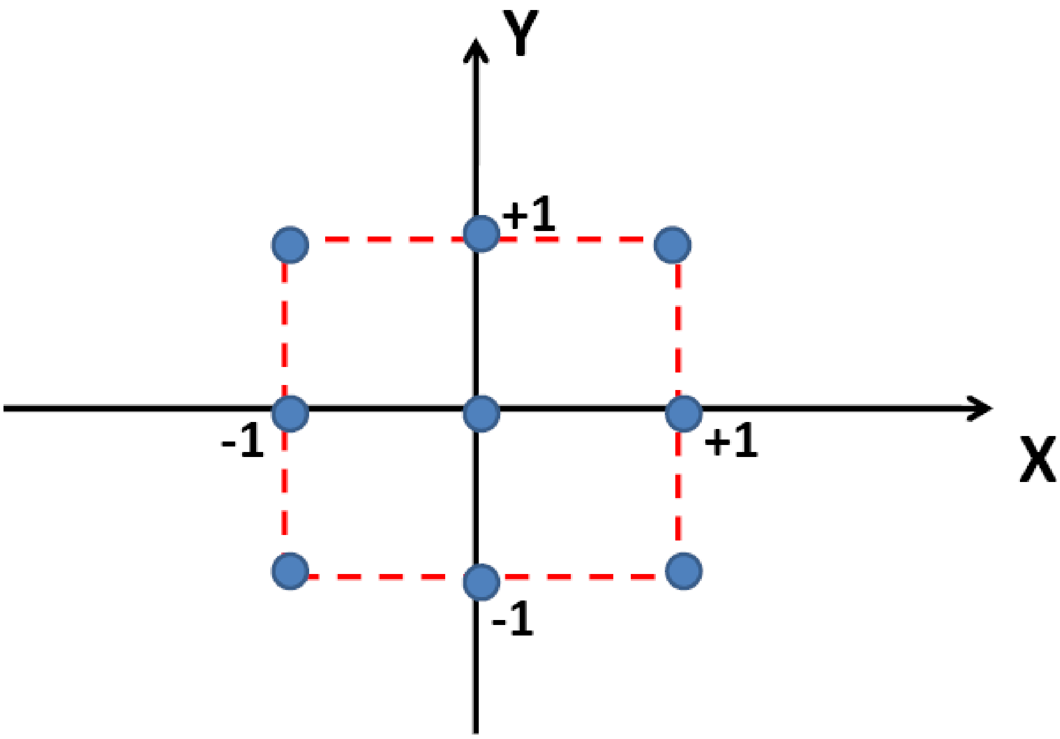


Figure 10.2: Example of an unconstrained design space for two design variables

For defining the design points, using conventional DoE techniques, such as the CCD technique, it would be straight forward: the 9 points indicated in the design space would be selected.

However, for the same problem, but considering the following constraints, namely:

$$-1.5 < X + Y$$

$$1 > X + Y$$

The design space would be different, since the domains of the design variables are now limited, as Figure 10.3 illustrates this condition:

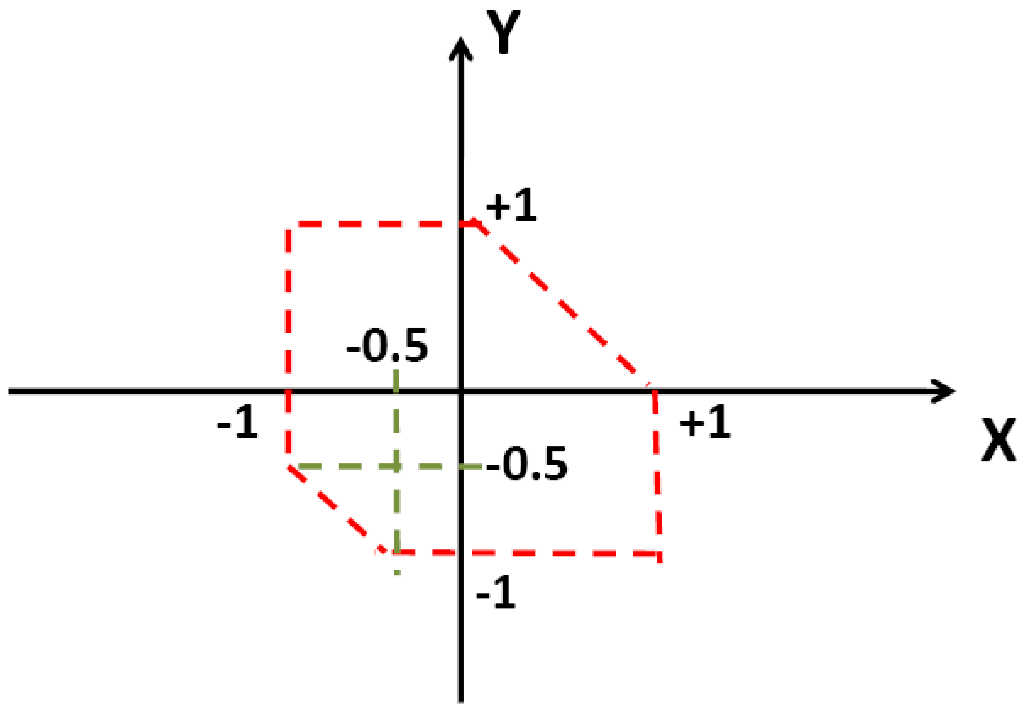


Figure 10.3: Example of a constrained design space for two design variables

It is clear to see, even with this simple example, that using the CCD technique, it would not be possible to select the design points for this modified design space. For instance, the original axial points are not the same and the algorithm of the CCD technique does not cover different combinations between the design variables' values other than those related with their upper and lower values. In other words, the design points selected by the CCD technique are based on the upper and lower values of the design points only. For this reason, in the constrained example shown before, the design points selected by the CCD technique would not cover the design space (i.e., represent it realistically) as Figure 10.4 shows.

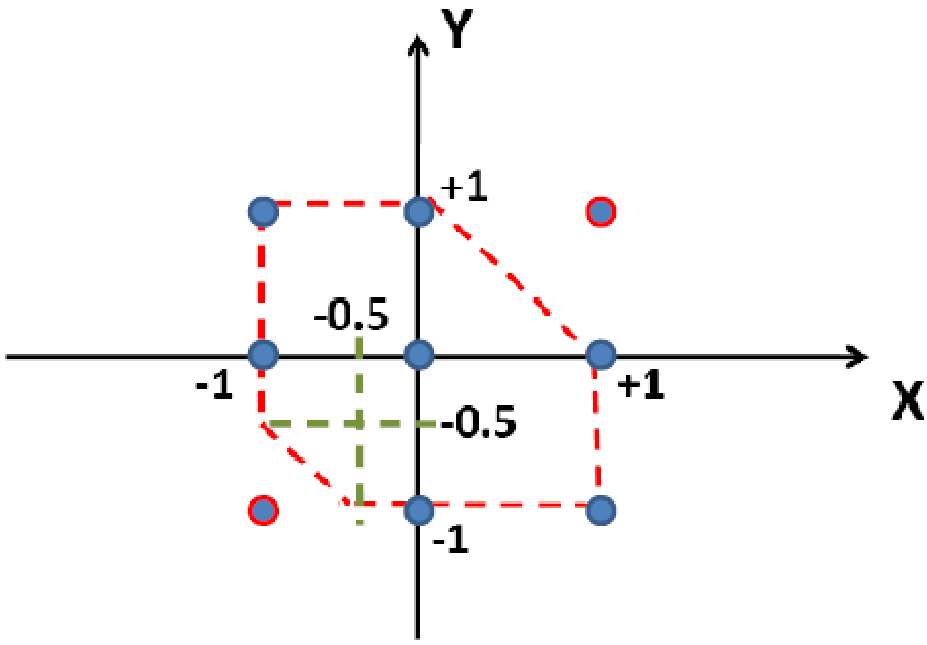


Figure 10.4: Design points generated using the CCD technique for a constrained design space for two design variables

The major issue in not covering properly the design space is when solving the problem (i.e., applying the RSM and then to use the numerical optimisation techniques for finding the optimal solution(s)). For example, the optimal solution might not be found, since this solution can be within the region where the set of design points did not cover.

As discussed in Chapter 3 of this thesis, an alternative to conventional DoE techniques for addressing this issue is the Random DoE technique. However, this technique does also have its limitations.

In the next section, the use and limitations of the random DoE technique are discussed.

10.2.1 The Random DoE Technique: applications and limitations

There are situations where the use of conventional DoE techniques would not be applicable and therefore the use of the random DoE technique would be required. A typical example of a situation where the random DoE technique may be appropriate is when the design region (i.e., experimental region or design space) is irregular; in other terms, when the design space is modified. And this is very common when the problem is constrained.

For instance: if a designer is investigating the properties of a particular adhesive. The adhesive is applied to two parts and then cured at an elevated temperature. Over the ranges of these two design variables, taken as -1 to +1 on the usual coded variable scale, the designer knows that if too little adhesive is applied and the cure temperature is too low, the parts will not bond satisfactorily. In terms of the coded variables, this leads to a constraint on the design variables:

$$-1.5 < x_1 + x_2$$

Where x_1 is the application amount of the adhesive and x_2 is the temperature. Therefore, if the temperature is too high and too much adhesive is applied, the parts will either be damaged by heat stress or an inadequate bond will result. Furthermore, there is another constraint on the design variable levels:

$$x_1 + x_2 < 1$$

The Figure 10.4, presented previously, shows the design region, as a result from applying these constraints.

Therefore, for this type of situation, where the design space is modified (once the levels of freedom are restricted), alternative techniques must be applied. For situations of this kind, the complexity involved requires other alternative “DoE techniques”, such as, so called “computer-generated designs”. This type of DoE techniques is also more commonly known as random DoE technique or simply random design. As the name suggests, the technique chooses design points randomly throughout the design space. In this technique, no statistical criterion is used for the selection of design points.

The values of the design variables are calculated by mapping the result of a random number generator in the range defined by the minimum and maximum values of the design variables. For instance, the random number generator used to construct the random design can make use of a uniform statistical distribution to create new random points. This means that every design point in the design space has an equal probability of being chosen as the next random point. The Monte Carlo technique is often used to generate the design points for this purpose.

Nevertheless, the use of the random DoE technique has its limitations. The design points picked up by the random DoE technique, based on its nature, very often do not properly cover the whole design space. For example, taking the problem shown in Figure 10.4, and using the Random DoE technique for generating a set of 5 design points and a set of 10 design points, it is possible to see, through Figure 10.5, that these design points are not well located within the modified design space.

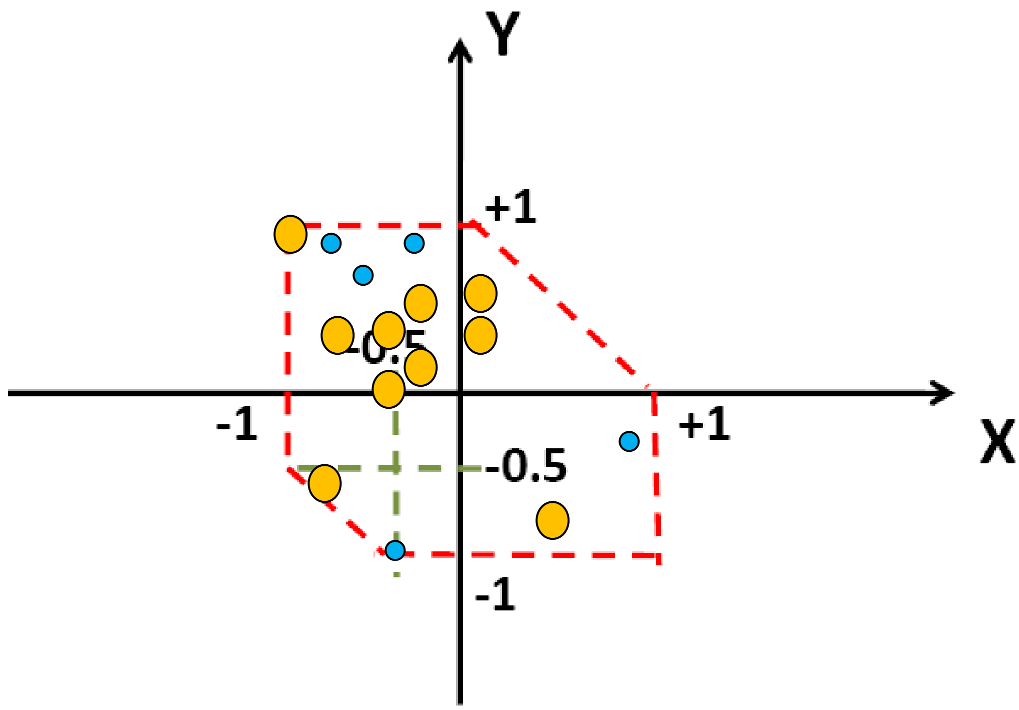


Figure 10.5: Locations of two different sets of designs points (one set with 5 and another one with 10 different design points) using the Random DoE

The smaller circles are those 5 design points selected in the first set; and those bigger 10 design points were selected in the second set. It is clear to see that these design points do not cover properly the space. This can be a problematic issue, because in many cases, the optimal solution region might not be captured by these points.

In theory, as the number of design points increases, the coverage gets more complete. Nevertheless, as mentioned previously, in practical terms, this is not true, since the location of the design points within the design space is more crucial). And for this reason, it is common that this type of DoE technique requires a large amount of design points. Even increasing the number of design points, this does not ensure satisfactory coverage of the design space.

Based on these issues, it became clear that the random DoE technique was shown to be not particularly suitable for more complex cases, when the time spent during the whole analysis becomes a limiting factor. Therefore, an improvement is needed for the selection of design points from the design space; especially for constrained problems where the design space is modified.

In the next section, the new DoE method proposed in this study, is presented and discussed.

10.3 The Hybrid DoE Method: The Modified CCD (MCCD) Method

The *Modified CCD Method*, or simply the MCCD Method, is a generic DoE method which can be applied for both types of optimisation problem: i.e. unconstrained and constrained. The method is a hybrid DoE method which combines principles found in the Central Composite Design, CCD, with concepts found in the combinatorial analysis field in pure mathematics. Given its simplicity, flexibility and nature, the MCCD method is a method and not a technique itself. In fact, it defines a set of procedures to be used to accomplish specific task(s): (i) selecting the design points within the design space and (ii) reduce the number of design points selected for finding the best solution and/or a good solution.

This method was developed through data analyses in a trial and error approach. The data used was based on our previous studies which included evacuation simulations of several types of scenarios; which varied in complexity and nature. The method has shown to be more robust than the random DoE technique. For instance, using the proposed DoE method for one of the problems presented in this thesis, we have found the global minima using only 3 design points instead of the 20 design points suggested for the random DoE technique. Using the MCCD method, it was possible to avoid having to perform additional 85,200 simulations, which considerably reduced the time needed for the simulations. (It might be also important to mention that the simulations, especially for fire modelling, are massively more computationally expensive and hence minimising the required numbers of simulations is critical). In addition to that, although for some cases when the results obtained from the design points selected using the MCCD method, were not the best solution (or what was expected to be the best solution) in terms of numerical values; the final output (i.e., how the exits were located along the walls) was well distributed. Furthermore, the method seems to be robust, since complex scenarios were also investigated with satisfactory results as they will be shown later in this chapter.

From basic concepts in regression analysis, it is known that, when developing the response surface, the minimum number of design points must be at least equal to the number of design variables found in the problem. The more design points that are analysed, so the response surface modelling will improve. Nevertheless, this is only true when the design points are well located in the design space.

Therefore, in theory the accuracy of the response surface modelling is directly proportional to the number of design points. However, based on previous findings and as discussed previously, this thesis supports the hypothesis that in practice the accuracy of the response surface modelling is not necessarily directly proportional to the number of design points. In reality, this accuracy is dependent on the locations of the design points along the design space rather than the quantity of design points.

Many of the conventional DoE techniques, such as the CCD and the Factorial Designs, which are well known and robust DoE techniques, apply algorithms which significantly increase the number of design points according to the number of design variables. For this reason, when the problem involves a small number of design variables, such techniques are highly recommended; however these techniques are not recommended as the number of design variables increases. This becomes an even bigger issue when the design points are collected through laboratory experiments and/or through some computational simulations, such as Computational Fluid Dynamics, CFD, since to perform these experiments is expensive and time consuming. Table 10.1 presents a comparison between the factorial designs and the CCD in terms of design points produced using them.

Another issue, which is very important when using a DoE technique, is the location of the design points along the design space, as mentioned previously. The CCD, for instance, has a structure which allows core design points to be selected. For instance, a central point and axial points along the design space are recommended for this technique. However, as table 10.1 shows, this technique is not feasible in cases in which there are more than three design variables involved. Other well known DoE techniques, such as the Latin-Hypercube and the Taguchi Design, for instance, have their limitations in terms of the selection of “good design points”. For example, the Latin-Hypercube and the Taguchi Design, when the number of design variables involved in the problem exceed three design variables, the techniques start to select repeated design points and/or very similar design points (i.e., design points which are very close to each other in the design space). It should also be remembered that, all of these conventional DoE techniques are restricted to unconstrained problems, which limits their usage, and consequently their flexibility.

Table 10.1: Design matrix for fractional factorial designs and CCD

Number of Design Variables	Full Factorial Design	Three-Level Factorial Design	Central Composite Design
n	**2^n**	**3^n**	**$1+2n+2^n$**
2	4	9	9
3	8	27	15
4	16	81	25
5	32	243	43
6	64	729	77
7	128	2187	143
8	256	6561	273

From Table 10.1 it becomes clear that, despite the fact that these DoE techniques produce "good design points" (i.e., well located in the design space), it becomes impractical to perform experiments based on these DoE techniques. The reason for that is based on the fact that their algorithms have exponential formulations.

Also, like other DoE techniques, their applications are limited to unconstrained problems where the domains of the design variables are not restricted to any conditions.

Following on from these considerations, the ideal DoE technique/method would be the one which covers reasonably the following features, namely:

- produce "good design points", which cover the most important combinations between the design variables (i.e., well located design points along the design space);
- the number of design points that are required for the technique does not increase substantially as the number of design variables increases;

– has flexibility in terms of where the design points are located in the design space, for both cases: i.e. unconstrained and constrained problems (i.e., when the problem is constrained, as mentioned before, the original design space becomes a modified design space and the domain for the design variables' values are then restricted to conditions. The conventional DoE techniques do not work in such types of problems).

These three principles are found in the MCCD method proposed here in this chapter. According to what was mentioned previously in this section, the MCCD method consists in a hybrid DoE method which applies principles found in the CCD. For instance, the MCCD method also uses design points which are "similar" to the factorial design points. The figure 10.6 illustrates this.

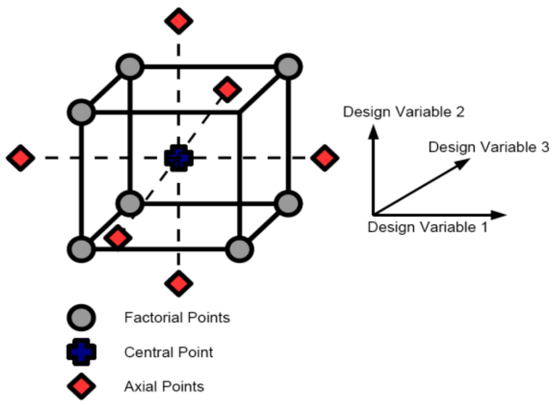


Figure 10.6: Example of a CCD with three design variables

From figure 10.6, it is possible to see that the factorial points are able to cover the whole design space within any *n*-dimension, since they are located on its boundaries. However, this is true for unconstrained problems; and what if the problem becomes constrained?

Consider the example: a function *f* of 2 design variables X and Y. For both design variables, the lower limit is -1 and the upper limit is 1. The design matrix is represented in Table 10.2. The geometric illustration for this case is shown in Figure 10.7. The mathematical statement for this problem is shown below:

Objective Function: $OF = f\,(X; Y)$

where: $-1 < X < 1$

$-1 < Y < 1$

Table 10.2: Design matrix for Factorial design in two design variables (using the ideal number of design points)

Design Point	**X**	**Y**
Design point 1	1	1
Design point 2	1	-1
Design point 3	-1	1
Design point 4	-1	-1

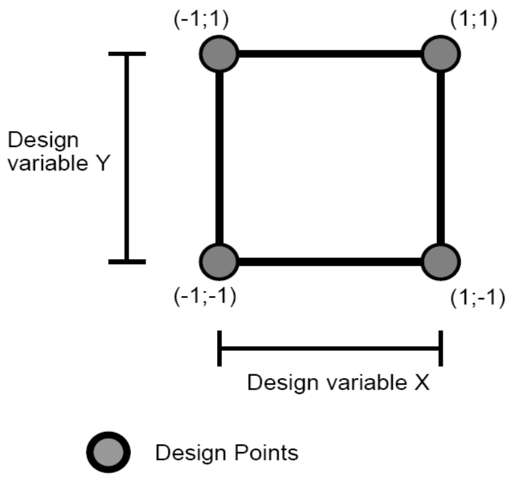


Figure 10.7: Example of a MCCD with two design variables

Taking the same example, but now with three design variables, where the third design variable, Z, has also the same upper and lower limits. The design matrix is shown in Table 10.3 and the geometric representation is shown in Figure 10.8.

Table 10.3: Design matrix for the Factorial design in three variables (using the minimum number of design points)

Design Point	**X**	**Y**	**Z**
Design point 1	1	1	-1
Design point 2	1	-1	1
Design point 3	-1	1	1
Design point 4	-1	-1	-1

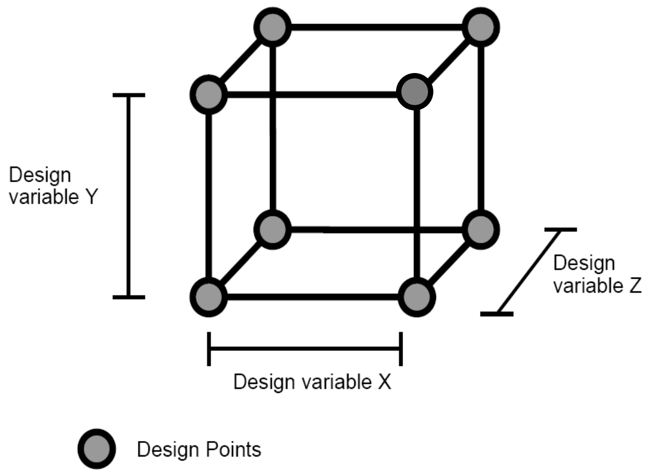


Figure 10.8: Example of a MCCD method three design variables

These examples just illustrate simple cases in where there are no constraints. However, the same principle can be applied for more complex cases; i.e., those with constraints and more design variables.

In the next sub-section, a brief review of the CCD Technique is presented (i.e., how it works) and how the MCCD method was developed.

10.3.1 The CCD Technique and the MCCD Method

In this sub-section, the way how CCD Technique works is briefly presented and how the MCCD method was developed and consequently how it works.

The two fundamental rules when using any DoE technique (or method, if it is the case) are:

a) to know how many design points will be used;

b) to know where these design points will be located.

In the next paragraphs, these rules will be explained for both: CCD technique and MCCD method.

CCD Technique

As discussed in Chapter 3 of this thesis, the formula used to define the number of design points for the CCD is $1+2n+2^n$ (where n is the number of design variables). The Table 10.1, shown previously, presents the number of design points which would be required if the DoE technique is applied according to the different number of design variables.

Regarding to the locations of the design points, the CCD technique will define them according to the "type of design points". In reality, the CCD consists of three different types of design points, namely: factorial points; central point and axial points.

The factorial points are the same points described in the factorial designs; either from a full factorial design or from a fractional factorial design.

The central point, as the name suggests, is a single point at the centre of the design space.

The axial points, as the name suggests, are the points located on the axes of the coordinate system and located symmetrically with respect to the central point.

Therefore, as a simplified example, considering an objective function *f*, dependent on two design variables (X and Y) with the following domains:

$$X_1 < X < X_2$$

$$Y_1 < Y < Y_2$$

The design space for this problem is shown in Figure 10.9.

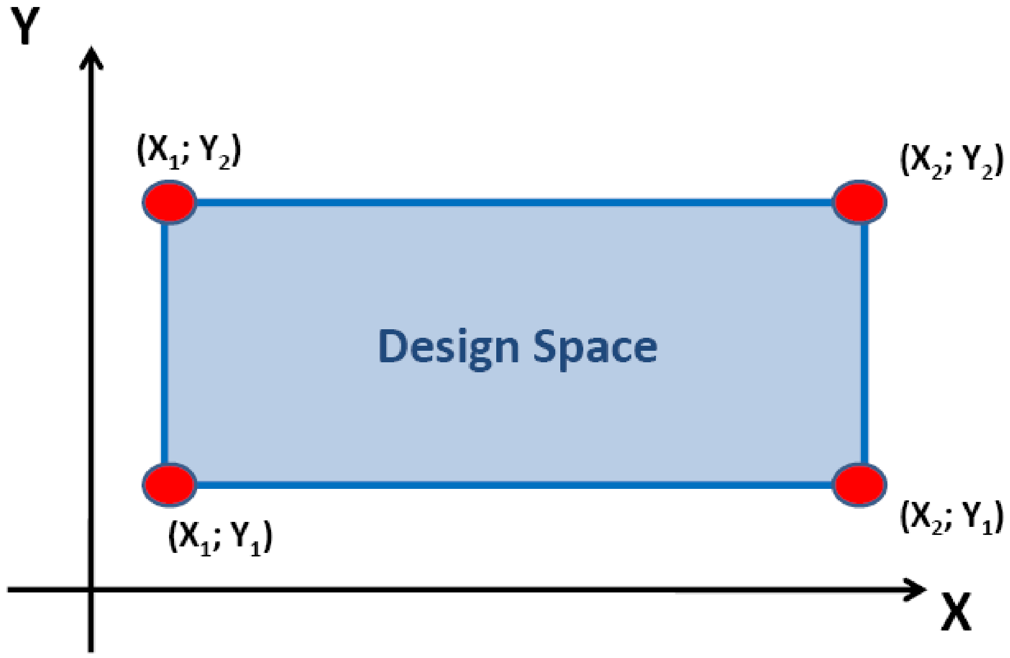


Figure 10.9: Representation of a generic design space for two design variables

The total number of design points according to CCD technique, for 2 design variables produces 9 design points:

$$1+2n+2^n = 1 + 2.(2) + 2^2 = 1 + 4 + 4 = 9 \text{ design points}$$

The locations of these points are:

FACTORIAL POINTS (FP): the general formula for defining the number of factorial points is 2^n (where n is the number of design variables). Since the number of design variables is 2, therefore, the total number of factorial points is 4. The factorial points will be the design points located in the extremes of the design space. They are already shown in Figure 10.9. The formulation for defining their locations can be described as follows:

- design point 1 (extreme 1): lower values for design variables X and Y: $(X_1; Y_1)$;
- design point 2 (extreme 2): upper value for design variable X and lower value for design variable Y: $(X_2; Y_1)$;
- design point 3 (extreme 3): upper value for design variable Y and lower value for design variable X: $(X_1; Y_2)$;
- design variable 4 (extreme 4): upper values for design variables X and Y: $(X_2; Y_2)$.

This logical sequence is adopted for problems involving "n" design variables; i.e., the factorial points will be located always in the extremes of the design space. It is important to observe, that the factorial points are only located in the "design space" and for this reason the CCD technique can only work for unconstrained problems.

CENTRAL POINT (CP): the central point will be the design point located at the centre of the design space. For this reason, the formulation used to define its location is:

- design point 5: Midpoint in X: $(X_1 + X_2)/2$; Midpoint in Y: $(Y_1 + Y_2)/2$

AXIAL POINTS (AP): the axial points will be the design points located on the axes of the coordinate system and located symmetrically with respect to the central point. Therefore, the general formulation for defining how many axial points will be necessary is 2n. For this reason, the total number of axial points for this will be 2n = 2.2 = 4. Their locations can be defined by the generic formulations as described bellow:

- design point 6: lower value of X and midpoint of Y: (X1; $(Y_1 + Y_2)/2$);
- design point 7: upper value of X and midpoint of Y: (X2; $(Y_1 + Y_2)/2$);
- design point 8: midpoint of X and lower value of Y: ($(X_1 + X_2)/2$; Y_1);
- design point 9: midpoint of X and upper value of Y: ($(X_1 + X_2)/2$; Y_2);

This logical sequence is adopted for problems involving "n" design variables; i.e., the axial points will be located always in the exact axis of the design space. For this reason, when the design space is modified, these axial points are not used and therefore, also for this reason, the CCD technique can only work for unconstrained problems (where the design spaces are unmodified).

These design points are indicated in Figure 10.10.

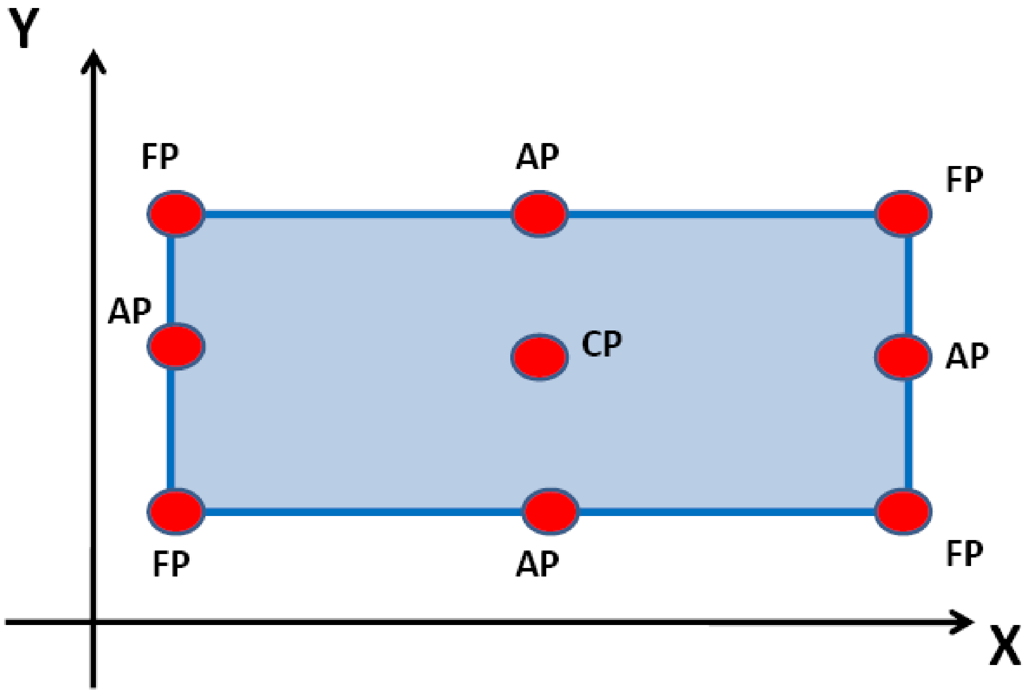


Figure 10.10: Location of the design points using the CCD technique for a design space for two design variables

MCCD Method

In the previous sub-section, the basic principles to be adopted when using the CCD technique were explained. Now, the basic principles to be adopted when using the MCCD method are explained. For didactic reasons, the same hypothetical problem with two design variables is presented:

Objective function: f (X; Y);

Where: $X_1 < X < X_2$

$Y_1 < Y < Y_2$

Nevertheless, now the problem is constrained. Furthermore, Y will be subjected to a constraint C. For this reason, the design space will be modified. Figure 10.11 illustrates how the design space shown in Figure 10.10 would be modified if the constraint C is inserted into the problem context.

Assuming that the constraint C is a constant value (Y/2) and that the condition is: Y < C

The design space is represented in Figure 10.11.

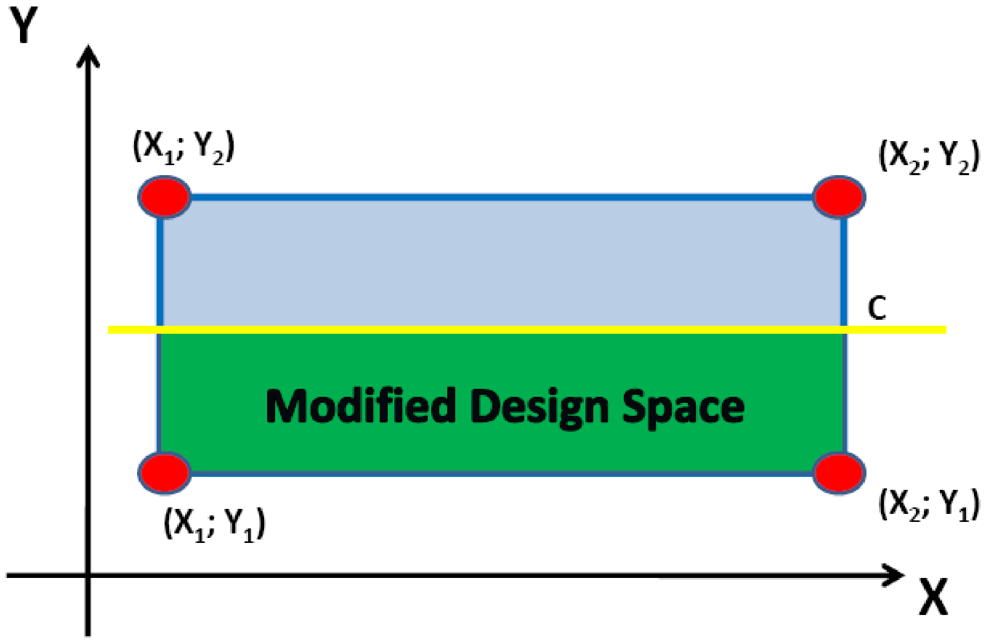


Figure 10.11: Representation of a generic modified design space due to a constraint (for two design variables)

Clearly, even for this simple example, the CCD technique would not be able to pick up the design points; since the design space is constrained (this issue was already discussed previously in this chapter).

As mentioned before and as the name suggests, the 'modified' CCD method follows some basic principles from the CCD. These basic principles are regarding the conception of the design points. The MCCD does also classify the design points into three main types of similar nature (not the same nature) of the CCD and they are:

- The central point: as the name suggests, is the point at the centre of the "modified" design space;
- The axial points, as the name suggests, are the points located on the axes of the "modified" design space (and not necessarily of the actual coordinate system defined by the original design space) and might be or not located symmetrically with respect to the central point;
- The "quasi-factorial points" are not the same points described in the factorial designs. This is the biggest difference between the MCCD and CCD. In fact, the quasi-factorial points are not factorial in its pure essence and are always located in the extremes of the modified design space and not of the actual design space.

Therefore, the design points which would be selected using the MCCD for the example are presented in Figure 10.12.

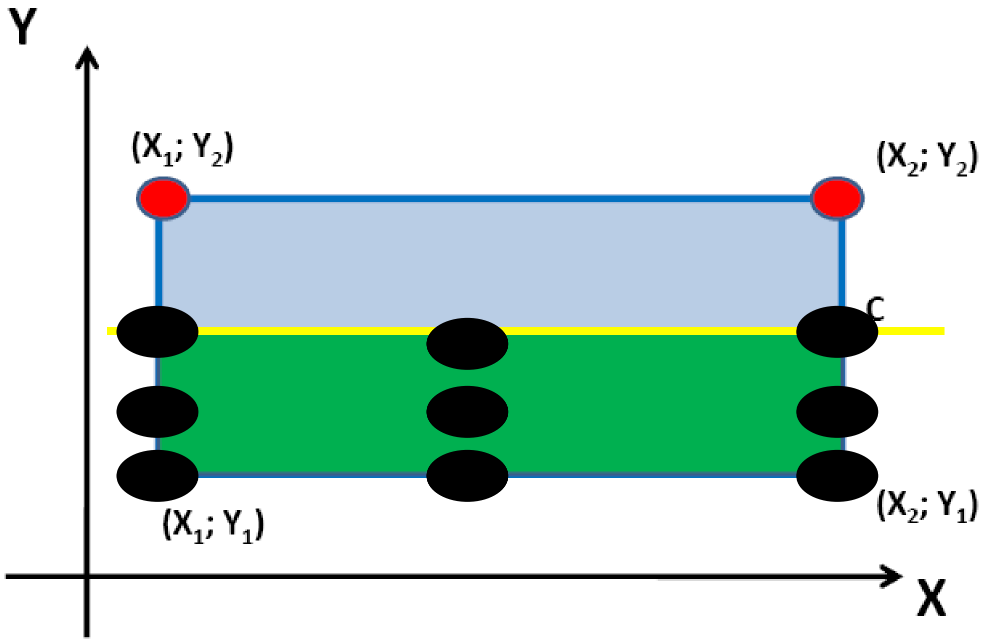


Figure 10.12: Locations of the design points using the MCCD method for a generic modified design space due to a constraint (for two design variables)

The black ellipses represent the design points (i.e., the quasi-factorial points; the axial points and the central point) selected by the MCCD method.

It is possible to see the main differences between the CCD technique and the MCCD method. One of the core-features of the MCCD method is to allow the design points' locations to be readjusted according to the modified design space geometry.

It is also relevant to mention, once again, that despite the MCCD had being developed based on some principles of the CCD technique, it was also developed based on data analyses of several (in fact, millions) of simulations. Therefore, the MCCD is essentially based on the observation of computational experiment results, i.e., trial and error approach.

Before starting to present some examples of how to use the MCCD method, it is also relevant to introduce some other new concepts which the MCCD is based on:

- "region of interest": is the geometric region defined by lower and upper limits or combinations of the different levels of the design variables' domains within the real domain or modified domain that are of interest to the experimenter. The region of interest can coincide or not with the modified design space. This will also depend on the level of knowledge that the experimenter has about its problem. In some cases, the experimenter already knows which specific region within the modified design space is feasible or not for exploring. The concept of "region of interest" does provide even a bigger flexibility to readjust the modified design space into a smaller region within it. This principle follows, in somehow, the principle of the Fractals, (FALCONER, K. (1997)).

Furthermore, in the next paragraphs, some examples will be presented, showing how the MCCD method would be applied in its principle. The design points suggested in the next paragraphs are results of the following principles/steps of the MCCD, namely:

1) *define your design variables domains (modified or not), considering the existence or not of constraints;*
2) *if your problem is 2D or 3D in nature, draw the modified design space and/or the region of interest within it, if it is the case. (In case, the problem is greater than 3D, clearly, to draw the modified design space and/or the region of interest would not be possible);*
3) *start selecting your design points. (In the CCD, the design points are started to be selected with the factorial points and then the centre point and finally the axial points). Using the MCCD method, start with the "quasi-factorial points" which will be the extremes of your modified design space or your region of interest; then define the axial points and if needed define the centre point (which is not necessarily related with the axial points and this is why the axial points are selected before the centre point; differently from the CCD technique).*

These are the basic steps when using the MCCD method. As it is possible to see, the main design points when using the MCCD method are the "quasi-factorial points"; the second most important points are the axial points and the centre point.

For instance, given a response θ, which is a function of two design variables, α and β, these design variables will have their lower and upper limits in which their values will vary. Based on that, the design space in then defined, in where the combination between the values of both design variables will correspond to a unique value for the response, see Figure 10.13.

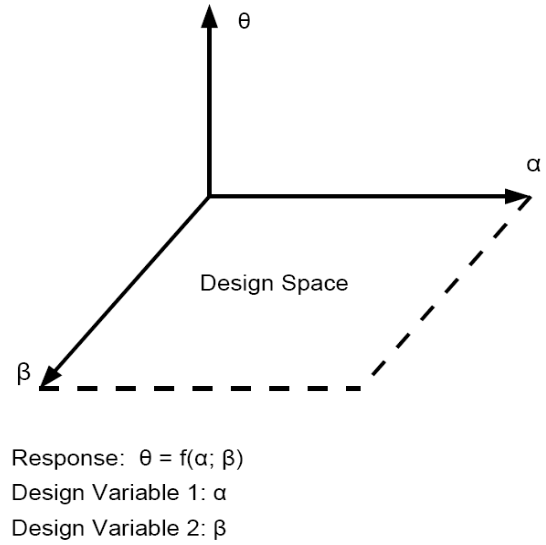


Figure 10.13: Hypothetical design space for two design variables

Based on this statement, and considering the existence of constraints, many possible problems can be extended from there. For example, some possible situations are as follows:

- two design variables with the same nature and same domain;
- two design variables with the same nature and different domains;
- two design variables with different nature and different domains.

There are many other possible situations which can be increased in complexity as many design variables as required for the chosen application area.

And regarding the constraints, there are also many possible scenarios, which will be impacted by the number of constraints, their nature (i.e., if it is inequality, a function etc.) and their relationship with the design variables and the response.

It becomes clear that to enumerate all the possible scenarios is almost an impossible task. Nevertheless, theoretically speaking, the basic principle proposed by the MCCD method can be extended to many different possible scenarios.

Furthermore, assuming this principle of the MCCD method, the formulations for some constrained optimisation problems are presented in the following paragraphs.

Firstly, considering the hypothetical problem shown in Figure 10.3, in which a response, F, is a function of two design variables: X and Y. These design variables will have their lower and upper limits between which their values will vary. The general problem can be stated as follows:

Optimize (minimize or maximize) F,

Where: $F = f\,(X; Y)$,

$-1 < X < 1$

$-1 < Y < 1$

Subjected to the following constraints:

$$-1.5 < X + Y$$

$$1 > X + Y$$

Based on that, applying the MCCD method, the first step was defined through the mathematical statement shown above. The second step of the MCCD method can be shown in Figure 10.14:

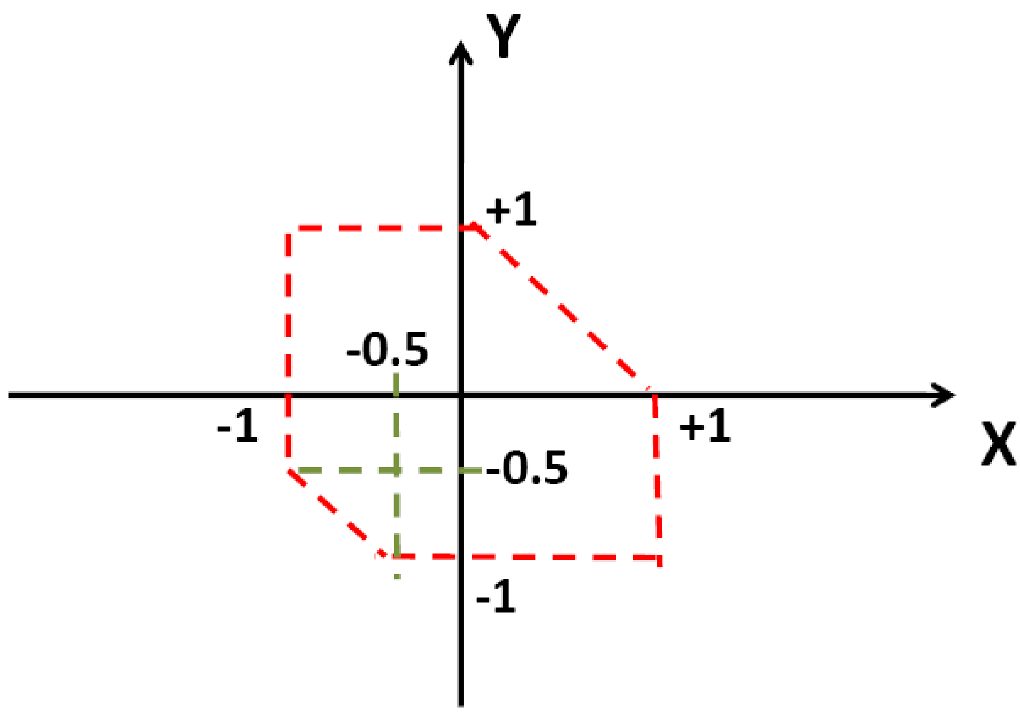


Figure 10.14: Representation of a modified design space for a hypothetical constrained problem

And finally, the design points to be selected presented in Figure 10.15.

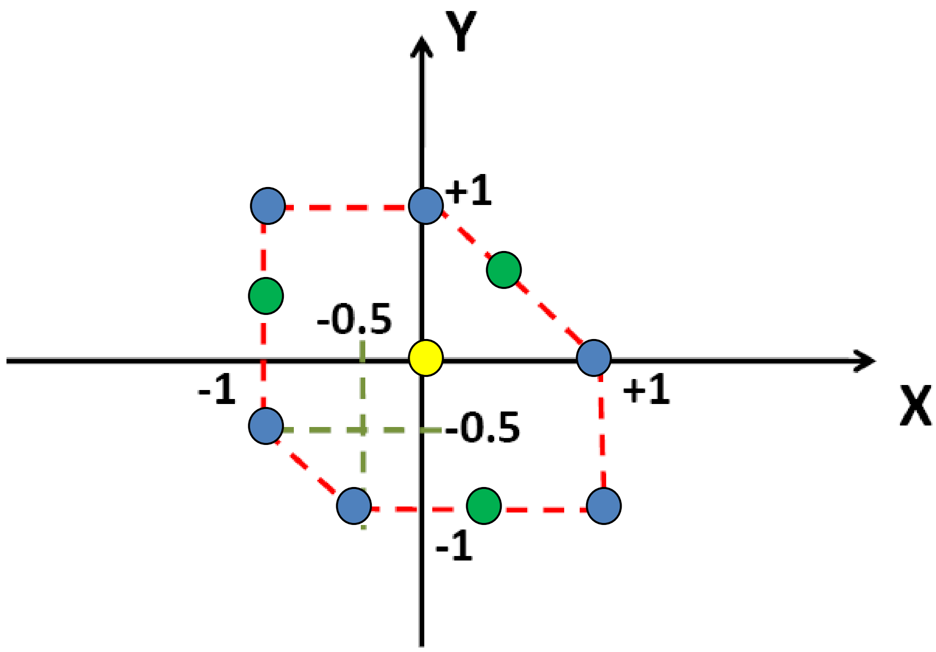


Figure 10.15: Location s of the design points using the MCCD method for a modified design space for a hypothetical constrained problem

As it is possible to see, there would be necessary 6 quasi-factorial points to cover the modified design space. The quasi-factorial points are very important design points, because they can represent the whole modified design space, covering its boundaries. In theory, there would be necessary 6 axial points. However, as explained before, these design points are not always needed and especially for 2D and 3D problems where it is possible to draw a sketch of the design space (and/or modified design space and/or region of interest). Here, it is possible to see that just 3 axial points would be worth to be investigated (the green circles), since they would help to cover better the whole space. The other 3 axial points would not be essential, since they would be too close to the other quasi-factorial points. And the central point is represented by the yellow circle.

It can be said, based on this and other examples shown in this chapter that, in principle, the number of axial points will be the same as the number of quasi-factorial points. It can be also said that, for those cases in which it is possible to draw a sketch of the design space (which can also be a modified space and/or a region of interest), 2D and 3D problems, the designer will have the advantage of excluding those design points which are not “essential” for covering the design space. (It is also important to note that, for those problems in which the designer has an idea of where to investigate, some design points can be also excluded; this will depend on the designer’s knowledge of its problem, as discussed previously in this chapter). For those problems which are greater than 3D in nature and/or the designer do not know anything about the design space, it would be recommended to investigate all the design points suggested by the MCCD method.

Differently from the CCD, in which the number of design points can be estimated through a simple formula which will be dependent on the number of design variables; the MCCD method does not have a specific formula. In fact, the main difference between the CCD and the MCCD is that the CCD will give a fixed total number of design points and fixed locations for these points. This information will be dependent on the number of design variables as well as the design variables domains. In the other hand, the MCCD will suggest the locations of the design points which will be based mainly on: a) the constraint(s); b) the design variables’ domains and c) the number of design variables. In other words, the most important information for defining the design points when using the MCCD method is the constraints plus the design variable(s) domains rather than the number of design variables. This is why the quasi-factorial points are extremely important for the MCCD; indeed, the quasi-factorial points define the main feature of the MCCD: adaptability and flexibility in terms of defining the design points within the modified design space. In other words, using the MCCD, there is no “pre-fixed” locations for the design points, since these locations will be varying according to the nature of the constraints.

Previously in this chapter, a simple definition of the design points used by the MCCD was given. Now, a more specific discussion on these design points is presented as follows:

QUASI-FACTORIAL POINTS (QFP): the quasi-factorial points can be considered the most important design points when using the MCCD method. This is said, because they will cover the actual modified design space (and/or the region of interest), since they are based on the constraints and their relations with the design variables’ domains. For this reason, the QFP will represent the actual boundaries of the “new design space” and not the original design space. In other words, the QFP will be the extreme points of the new design space. As mentioned before, the QFP are responsible for the MCCD's flexibility in selecting the design points which will be adapted to the new design space.

AXIAL POINTS (AP): the axial points are those located between the QFP. Differently from the CCD, when using the MCCD, the AP will not always be necessary as shown in Figure 10.15. This can be also seen as advantage when using the MCCD; especially for 2D and 3D problems.

CENTRAL POINT (CP): the central point (which in fact, can be more than one, depending on the configuration of the modified design space; since there might be cases, in which the constraints might subdivide the original design space into two or more regions) is defined based on the actual modified design space. For this reason, the CP is defined after the QFP are defined. The CP is/are not always necessary neither.

As mentioned before, there are many possible scenarios; and therefore, it would be out of scope to list all of them and/or attempt to consider a large amount of scenarios. Therefore, the creation of a general formula would just work for the actual problem itself (or other problems which have the same nature: same number of design variables; same type of design variables and same types of constraints; allowing just different values for the domains and the eventual numbers associated with the constraints). Therefore, the main principle of the MCCD has been given through the definition of the types of design points used by the MCCD and the basic steps when solving the problem.

In summary, the basic principle shown in these cases can be extended to many different possible scenarios. Each case will be a particular case in which the flexibility of this method can be applied. (In reality, as mentioned previously, combinatorial principles can be integrated with this method, similarly to combinatorial topology commonly used for mesh generation **[121]**. For instance, an optimisation problem with 6 design variables with 3 constraints was solved using the MCCD Method and the method produced acceptable design points, since the problem was solved satisfactorily.

In the next section, the results for some of the previous case studies, but now using the MCCD method, are presented and discussed.

10.4 The use of the MCCD Method

In this section, the MCCD Method is applied for some of the previous case studies, namely: square room with two exits, L-Shaped room with two exits, multiple connected compartments (scenario 2: one exit for the smaller room and two exits for the bigger room) and the geometry with multiple exits of varying sizes.

The results are presented and discussed in the next paragraphs.

10.4.1 The square room with two exits

In Chapter 5, it was revealed that significant advantage can be derived from the strategic positioning of the two exits in a squared room. Analysis suggests that exits placed adjacent to each other produce the minimum egress times, exits positioned symmetrically around the perimeter of the room is next best while exits placed asymmetrically around the perimeter of the room produce the longest egress times. Therefore, the scenarios which give the optimal solutions for this problem (i.e., to locate the exits side by side) gives a range of 83.8 – 84.4 seconds. The second best scenarios (i.e., to locate the exits in symmetrical positions in relation to each other) gives a range of 85.2 – 87.0 seconds. The worst case scenarios (i.e., with exits located in asymmetrical positions in relation to each other) gave a range egress times of 92.7 – 98.2 seconds. This conclusion was made based on 7,800 simulations.

In Chapter 6, it was shown that a methodological approach which involves optimisation theory and its’ associated concepts, which included the use of DoE techniques, can satisfactorily solve the problem. For this problem, given its constraint, the design space is then modified. For this reason, it was necessary to use the random DoE technique, once the other DoE techniques can just be applied for the whole design space; i.e., not a modified design space.

As the name suggests, the inconvenience of using the random DoE technique is that the design points are selected randomly. This does not assure that the design points will properly represent the whole of the design space, even if it is a modified design space. These considerations are similar to a gambling game. In order to increase the likelihood of having better coverage of the design space, it may be necessary to increase the number of design points selected. However, once again, this improvement cannot be assured. For this case, it was investigated how many design points needed to be selected before a reasonable result was obtained. It was found that the optimisation analysis involved more work, since a total of 87,000 simulations were performed.

Conversely, using the MCCD method for defining the design point selections, only three design points were used, which required a total of only 1,800 simulations. (It is important to mention that the problem here solved is classified as case 1), see table 10.5). With this 85,200 simulations were avoided. And besides that, the results obtained, using the MG method, were found to be satisfactory as shown in Table 10.8. In fact, the exact global solution was found. The problem was solved using the gradient-based algorithm (i.e., the Fletcher-Reeves numerical optimisation technique) and the non gradient-based algorithm (i.e., the Particle Swarm Optimisation numerical optimisation technique).

In order to check the consistency of the MCCD method, two response surface models were used: one based on the polynomial high order function and the other one based on the *multiquadratic* function. The general stepwise regression was the multivariable regression analysis technique here selected to build these response surface models and it worked well, since the coefficients R^2 were higher than 0.90. In other words, both response surface models seemed to be appropriate, since the results were the same. This seemed to be consequence of the good locations of the design points within the design space.

In summary, the problem was:

Minimize: $ET = f(D1, D2)$

Subject: $|D1 - D2| \geq 1$

Where:
$$0 \leq D1 \leq 39$$
$$0 \leq D2 \leq 39$$

Where:

ET – the evacuation time;

D1 - the distance from the left edge of exit 1 to the corner;

D2 - the distance from the left edge of exit 2 to the corner.

Based on this, the modified design space can be drawn. Figure 10.16 presents it.

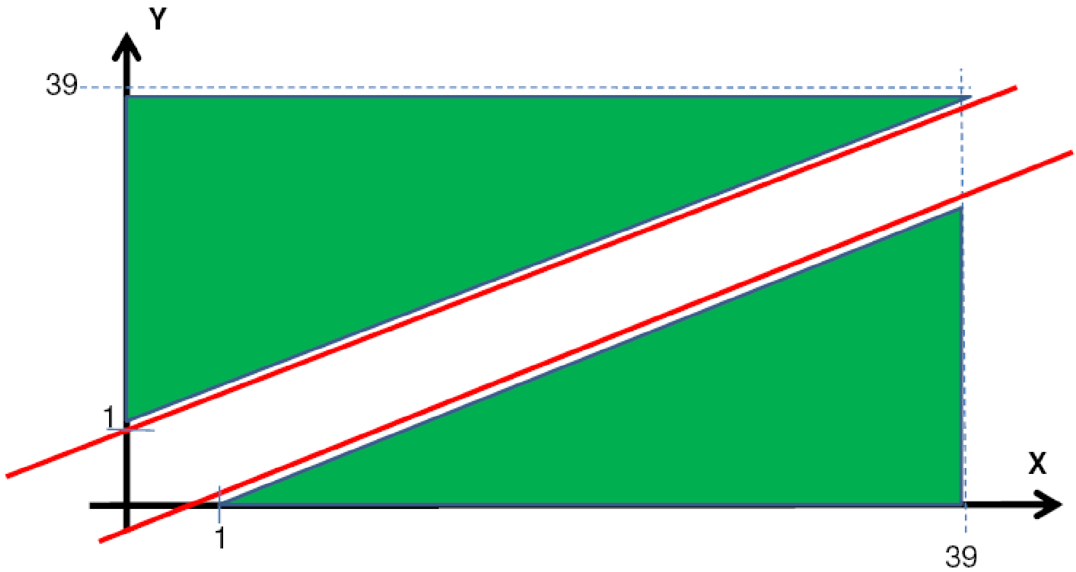


Figure 10.16: Representation of the modified design space

From Figure 10.17, it is possible to see that the original design space was divided into two regions (this type of situation is possible as mentioned previously in this chapter; and for this reason, in cases like this one, there will be more than one central point).

Following the steps proposed when using the MCCD method, the design points which would be selected are shown in Figure 10.17.

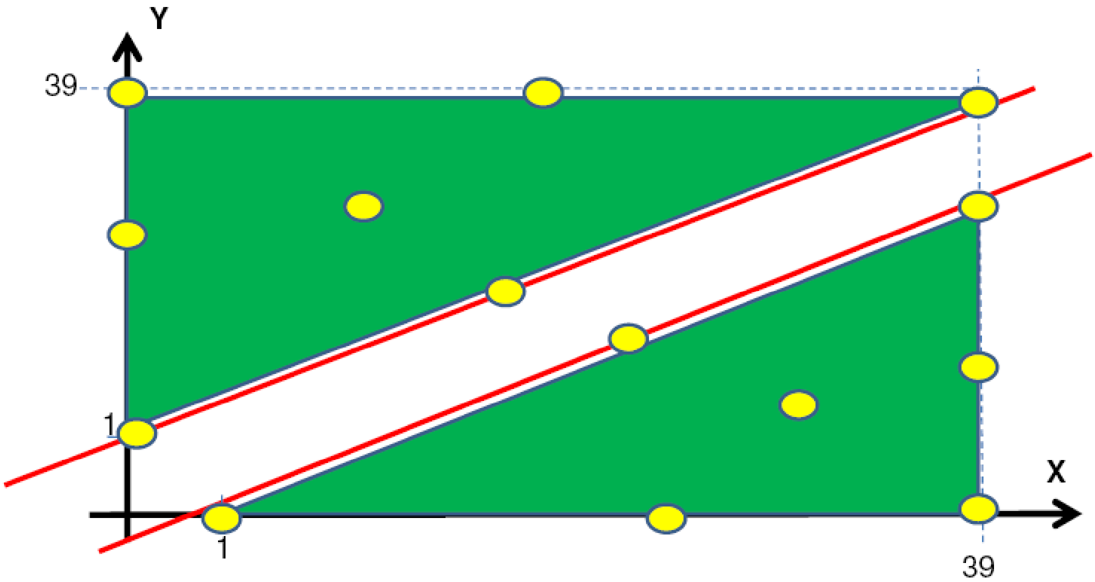


Figure 10.17: Locations of the design points using the MCCD method

There would be in total 14 design points which are represented by the yellow circles.

It is also important to note that, for this particular problem, in which the two regions within the modified design space are symmetrical, only one of these regions would be necessary to be investigated. In additional to that, since both design variables have the same nature as well as the same domain (and also the same constraint associated to each other), it would be possible to exclude some design points. Therefore, the design points which would be worthily to investigate (i.e., those which potentially cover satisfactorily the modified design space) are shown in Figure 10.18.

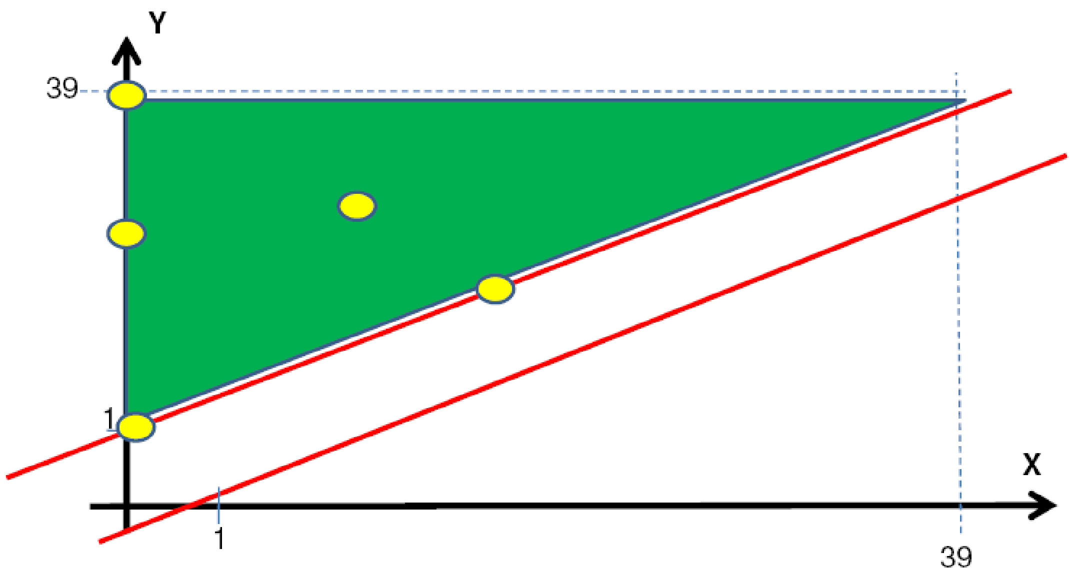


Figure 10.18: Locations of the "selected" design points using the MCCD method

Thus, instead of having the original 14 design points, only 5 design points would be investigated. Numerically speaking, these design points selected are:

Design Point 1 (quasi-factorial point): (0; 39)

Design Point 2 (axial point): (0; 19.5)

Design Point 3 (quasi-factorial point): (0; 1)

Design Point 4 (axial point): (20; 18)

Design Point 5 (central point): (11; 25)

Figure 10.19 presents the design points (associated with the respective scenarios) selected by the MCCD DoE method for this problem.

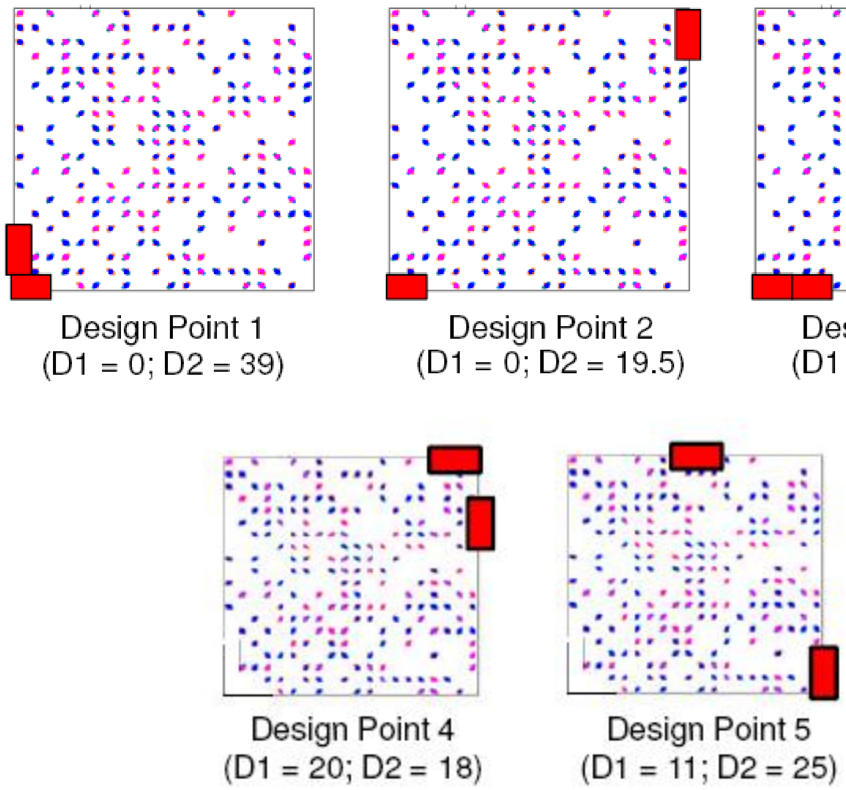


Figure 10.19: The 5 Design Points (and the associated scenarios) selected by the MCCD Method for the square room problem

The results are presented in table 10.4.

Table 10.4: Optimal solution for the squared room with two 1.0m wide exits

Optimum Solution (0 ; 1; 83.8)	**Numerical Optimisation Technique**	
Using the High Polynomial Function as the RSM	Fletcher-Reeves	PSO – Particle Swarm Optimisation
The MCCD Method (5 design points)	(0;1;83.8)	(0;1;83.8)
Using the *Multiquadratic* Function as the RSM	Fletcher-Reeves	PSO – Particle Swarm Optimisation
The MCCD Method (5 design points)	(0;1;83.8)	(0;1;83.8)

It is interesting to observe that the three design points that were selected by the MCCD Method, for this problem, were well located design points along the design space, once the minima region was picked up. In fact, the MCCD method has picked up the best solution as one of the design points; and probably, for this reason, the exact global minima region was found.

As mentioned before, the solution of the problem using the proposed DoE method in this thesis, has given the global minima solution as: (0;1;83.8) which means that one of the exits is located in the corner and the other exit is located besides the first exit producing a total evacuation time of 83.8 sec. In fact, we have found the global minima, i.e. the same results which we found using the "brute force" method. When the random technique was used, the global minima region was not found, but the local minima region was found and was close to the global minima.

Nevertheless, since this first problem is reasonably simple, i.e. it is based on a simple geometry (square room), more complex problems were also solved using the MCCD Method. These are presented in the following sections. However, before starting to show the application of the MCCD method to the other problems, it would be worth to investigate further this first problem. Considering the same square room with the same considerations (i.e., population density, exits' widths, simulation parameters etc), but now the floor area would not be completely free as Figure 10.20 shows.

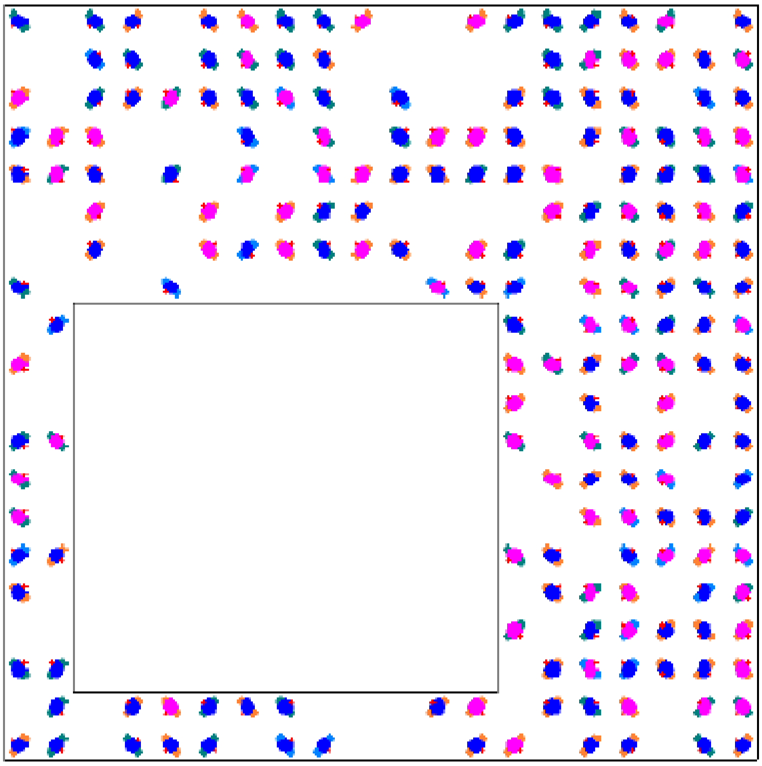


Figure 10.20: Square room taking into account the lay-out within the enclosure

As it is possible to see in Figure 10.20, the occupants will not have the same type of freedom in terms of movement as they had in the original square room. Clearly, there is an "empty space" (represented by the white square area) within the floor are, which represents the lay-out of the enclosure. In other words, the occupants will not be able to walk through the empty space.

The problem to be solved would be exactly the same as previously stated: to find the best locations for two exits with the same widths (i.e., 1m) for minimizing the evacuation times. This will also include the same mathematical formulation; the same modified design space; the same design points used etc. The design points are shown in Figure 10.21.

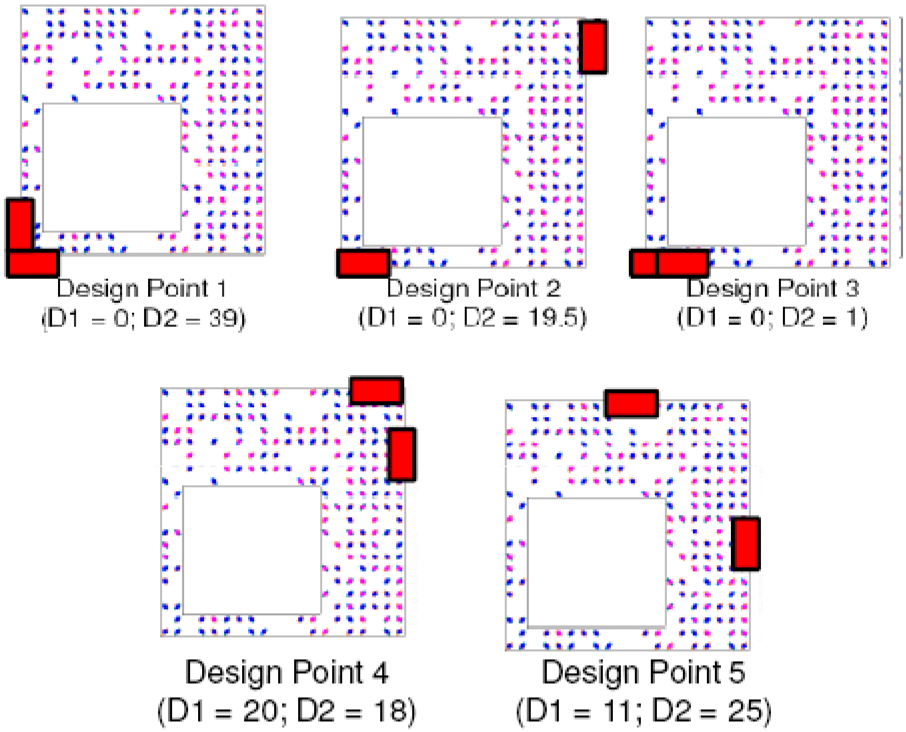


Figure 10.21: The 5 Design Points (and the associated scenarios) selected by the MCCD Method for the additional square room problem (taking into account the an arbitrary lay-out in the room)

It is possible to anticipate that, despite the problem being exactly the same mathematically speaking; the way the design variables will impact the objective function (i.e., the evacuation time) will certainly be different. As consequence of that, the results should be different from the results obtained from the previous square room. For instance, it is possible to see that design point 3 will not produce a good evacuation time, since people will be "squeezed" in a very narrow area for reaching the exits.

For both Numerical Optimisation techniques (i.e., Fletcher-Reeves and the PSO) and using the *Multiquadratic* function as the RSM, the best solution found was: (10; 20; 84.56). This means that for one exit located at 10 meters from the corner and the other one, 20 meters from the corner, it will produce an evacuation time of 84.56 seconds. In This result is shown in Figure 10.22; and in order to check the consistence of this result, 600 simulations based on this configuration was performed, using the same procedures and assumptions adopted in all the cases investigated along this thesis.

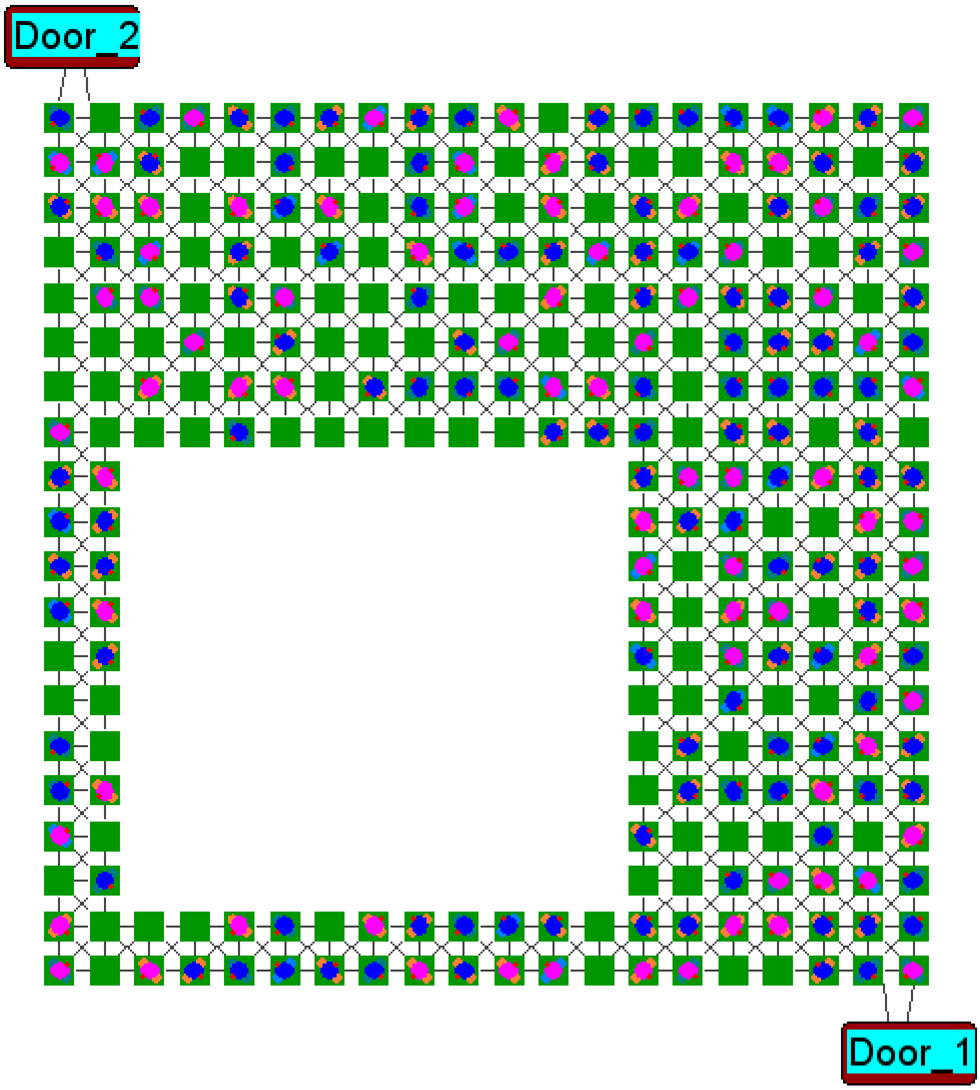


Figure 10.22: Optimal solution for the additional square room problem (taking into account the arbitrary lay-out in the room)

The average evacuation time based on the 600 simulations using buildingEXODUS is 85.14 seconds. Furthermore, it can be concluded that the result seems to be robust and the MCCD method worked for this different scenario, in which the optimal solution was not known. This is a positive outcome of the use of the MCCD method proposed in this thesis.

In the next sections, the other problems investigated in this thesis are solved using the MCCD method.

10.4.2 The L-Shaped room with two exits

The solution for this problem is already known based as shown in Chapter 7. In fact, the solution was found in two different ways: using "brute force method" and using the numerical optimisation techniques and associated concepts. For both methods, the global minima solution and local minima regions close to the global minima region were found. The best solution, i.e., the optimal locations for the two exits to produce the lowest evacuation time, was found for D1 = 47 and D2 = 0, which produced an evacuation time of 80 seconds. Clearly, those cases with coordinates similar to this also provided similar results.

Nevertheless, these solutions were found based on several investigations which required a large amount of simulations. For example, using the brute force method, the optimal solution was found using 34 separate scenarios, which required a total of 20,400 simulations.

The case was then analysed using numerical optimisation techniques, based on the random DoE technique (due to the fact that the problem is constrained). For the same reasons as were explained previously (i.e., the limitations of this type of DoE technique) three sets of design points were defined. These sets were: three sets of four different design points; three sets of six different design points; three sets of 10 different design points and three sets of 20 different design points. This required a total of 72,000 simulations.

Now numerical optimisation techniques, using the MCCD method, are used to solve the same problem. It was found that an optimal solution was also found, but using just three design points, which only required 1,800 simulations when compared to the 92,400 simulations that were needed previously. It is also important to mention that this problem, like the previous problem, is classified as case 1), see table 10.5.

Similarly to the previous problem, the square room with two exits, the solution was found using the gradient-based algorithm (i.e., the Fletcher-Reeves numerical optimisation technique) and the non gradient-based algorithm (i.e., the Particle Swarm Optimisation numerical optimisation technique). The response surface model based on polynomial high order seemed to be appropriate. The general stepwise regression was the multivariable regression analysis technique was selected to build these response surface models and it worked well, since the coefficients R^2 were higher than 0.90.

The Table 10.5 presents the results and Figure 10.23 presents the design points (associated with the respective scenarios) selected by the MCCD DoE method for this problem.

Similarly to what was done for the previous case, in order to check the consistency of the MCCD method, two response surface models were used: one based on the polynomial high order function and the other one based on the multiple regression function. The general stepwise regression was the multivariable regression analysis technique here selected to build these response surface models and it worked well, since the coefficients R^2 were higher than 0.90. Here again, both response surface models seemed to be appropriate, since the results were the same. The reason for that might be explained by the fact that the MCCD method has picked up well located design points within the design space.

In summary, the problem was:

Minimize: $ET = f(D1, D2)$

Subject to: $|D1 - D2| \geq 1$

Where: $0 < D1 < 47$

$0 < D2 < 47$

Where:

ET – the evacuation time;

D1 - the distance from the left edge of exit 1 to the corner;

D2 - the distance from the left edge of exit 2 to the corner.

The modified design space would be similar to the one shown for the previous problem; in fact the only difference would be the numerical values.

Therefore, for the same reasons explained previously, only one region of the modified design space would be considered; and furthermore, this would allow reducing the number of design points to be investigated. In addition to that, we could attempt to not consider the central point and one of the axial points, for reducing even more the number of design points to be considered. This will reduce immensely the number of simulations to be performed and the coverage of the modified design space, in theory, will not be compromised. In case, the results to be found are not those to be expected (i.e., the best solution or a close solution to the best solution); then the other axial point and/or the central point could be considered for a further investigation.

The design points to be considered for this analysis are:

Design Point 1 (quasi-factorial point): (0; 47)

Design Point 2 (axial point): (0; 23.5)

Design Point 3 (quasi-factorial point): (0; 1)

The Figure 10.23 presents the design points (associated with the respective scenarios) selected by the MCCD DoE method for this problem.

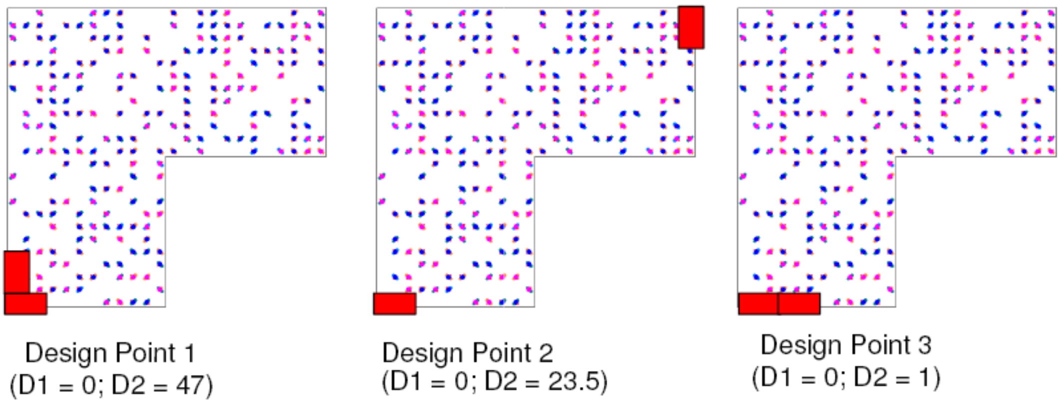


Figure 10.23: The 3 Design Points (and the associated scenarios) selected by the MCCD Method for the L-shaped room problem

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The results are presented in table 10.5.

Table 10.5: Optimal solution for the L-shaped room with two 1.0m wide exits

Optimum Solution (47 ; 0; 80)	**Numerical Optimisation Technique**	
Using Polynomial Function as the RSM	Fletcher-Reeves	PSO – Particle Swarm Optimisation
The MCCD DoE Method (3 design points)	(0; 47; 80)	(0; 47; 80)
Using the *Multiquadratic* Function as the RSM	Fletcher-Reeves	PSO – Particle Swarm Optimisation
The MCCD DoE Method (3 design points)	(0; 47; 80)	(0; 47; 80)

As it was possible to see, the MCCD Method for this more complex problem has also generated strategically well located design points. Similarly to the previous case, the MCCD method has picked up the best solution as one of the design points. Probably, for this reason, the exact global minima region was found.

10.4.3 The multiple connected compartments

In Chapter 8 of this thesis, this case study was investigated. Here, scenario 2, which is a constrained problem (i.e., in which one exit is located in the smaller room and two exits are located in the bigger room) is solved using the MCCD Method.

Previously for this scenario, the solution was found through the use of a “brute force method” and also through the use of numerical optimisation techniques.

For the first approach, 36 different scenarios were analysed, which required 21,600 simulations.

For the second approach, 18,000 simulations were performed. The random DoE technique was used to select the design points (i.e., which define each scenario to be run using the evacuation simulation model).

As it was observed for the previous case studies, the locations of the exits do have a significant impact on the evacuation efficiency. The results showed that the lowest evacuation times were found when the two exits were located side by side.

When this same problem is solved, using the MCCD Method, it required only 4 design points. In reality, the same global minima region was not found at this time; nevertheless, another optimal solution was found. It is also important to mention that this problem, like the previous problem, is classified as case 2), see table 10.6.

Similarly to the previous problems, the solution was found using the gradient-based algorithm (i.e., the Fletcher-Reeves numerical optimisation technique) and the non gradient-based algorithm (i.e., the Particle Swarm Optimisation numerical optimisation technique).

Once again, both response surface models, namely the polynomial high order and the *multiquadratic* function, were used for testing the consistency of the MCCD method. (The general stepwise regression was the multivariable regression analysis technique here selected to build these response surface models and it worked well, because the coefficients R^2 were higher than 0.90).

In summary the problem was stated as it follows:

Minimize:

$$ET = f\,(D1, D2, D3)$$

Subject:

$$|D1 - D3| > 1$$

Where:

$$0 < D1 < 29$$

$$0 < D2 < 9$$

$$0 < D3 < 29$$

Where:

ET – the evacuation time;

D1 - the distance from the left edge of exit 1 to the corner;

D2 - the distance from the left edge of exit 2 to the corner;

D3 - the distance from the left edge of exit 3 to the corner.

And the design points selected were:

Design Point 1 (quasi-factorial point): (0; 0; 29)

Design Point 2 (quasi-factorial point): (0; 0; 14.5)

Design Point 3 (quasi-factorial point): (0; 9; 1)

Design Point 4 (axial point): (14.5; 4.5; 15.5)

The Figure 10.24 presents the design points (associated with the respective scenarios) selected by the MCCD DoE method for this problem.

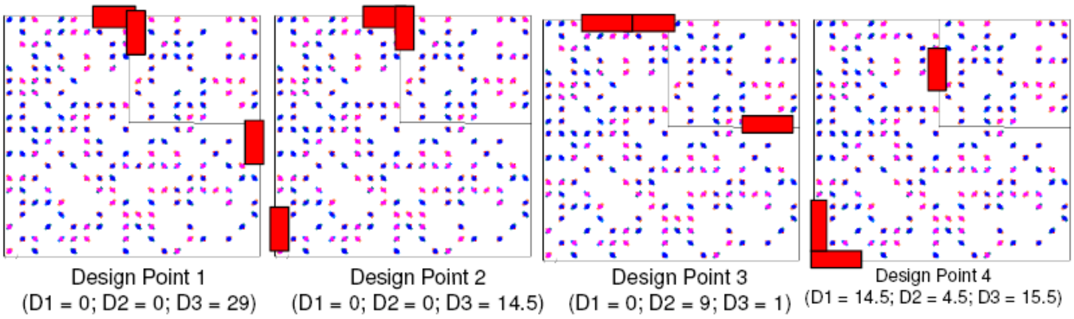


Figure 10.24: The 4 Design Points (and the associated scenarios) selected by the MCCD Method for the multiple connected compartments problem

The Table 10.6 presents the results.

Table 10.6: Optimal solution for the multiple connected compartments (with one exit of 1.0m in the smaller room and two exits of 1.0m each in the bigger room)

Optimum Solution (1.5 ;4; 2.5;83.92)	**Numerical Optimisation Technique**	
Using Polynomial Function as the RSM	Fletcher-Reeves	PSO – Particle Swarm Optimisation
The MCCD Method (4 design points)	(4,5 ;0 ; 5,5 ;86.95)	(4,5 ;0 ; 5,5 ;86.95)
Using the *Multiquadratic* Function as the RSM	Fletcher-Reeves	PSO – Particle Swarm Optimisation
The MCCD DoE Method (4 design points)	(4,5 ;0 ; 5,5 ;86.95)	(4,5 ;0 ; 5,5 ;86.95)

From Table 10.6 and Figure 10.24, it is possible to see that the exact optimal solution found previously was not found this time. The MCCD method has not picked up the solution as one of the design points, differently from what happened in the two previous cases. However, another optimal solution was found as well, where the two exits in the bigger room are also located side by side. The evacuation time produced based on this scenario is 86.95sec which can be considered a good result based on the previous results using the brute force method and numerical optimisation techniques (using the Random DoE Technique). At this time, instead of performing 21,600 simulations (based on brute force method) or 18,000 simulations (using numerical optimisation techniques based on the design points picked up by the Random DoE technique), only 2,400 simulations were performed. And the results can be considered satisfactory, since a local minima region close to the global minima region was found. This means that the MCCD method was able to pick up four strategically well located design points within the whole design space, see Figure 10.25.

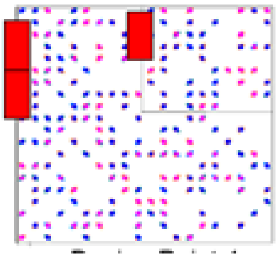


Figure 10.25: The optimal solution found using the MCCD Method for the multiple connected compartments problem (where D1 = 4.5; D2 = 0 and D3 = 5.5, which produced an evacuation time of 86.5sec)

Previously, as mentioned before, a total of 39,600 simulations (*i.e., 21,600 simulations based on brute force method plus 18,000 simulations using numerical optimisation techniques based on the design points picked up by the Random DoE technique*) were performed to find the best solution, where the best evacuation time was 83.92 seconds. Now using the MCCD, only 2,400 simulations were performed and the evacuation time was 86.5 seconds. This gives a small difference of 2.58 seconds. And in addition to that, the actual exits' locations were also good when comparing with the best solution, shown in Figure 10.26. This shows that the use of the MCCD method was satisfactory.

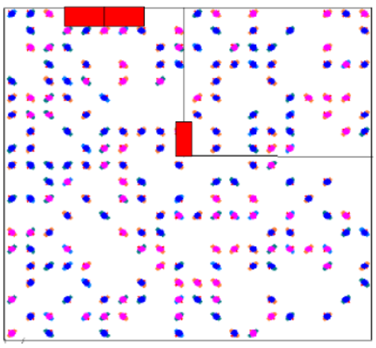


Figure 10.26: Optimal exits locations for the compartmented room (constrained problem), where the coordinate is (D1;D2;D3;ET = 1.5 ; 4; 2.5; 83.92)

It is also important to observe that additional evacuation simulations were performed for checking the consistency of this result. These simulations were based on the best scenario found using the MCCD method for picking up the design points. A total of 600 simulations were run following the same procedures which have been adopted in this study described in the previous chapters of this Thesis. The average evacuation time for these 600 simulations was 85.9sec. The difference between what was found 86.5sec and the actual result from the simulations, 85.9sec, is insignificant. Furthermore the result is consistent, showing the robustness of the MCCD method.

10.4.4 Geometry with multiple exits of varying sizes

In the previous chapter of this thesis, Chapter 9, this case study was discussed. Therefore, the solution for it has already been presented. Here, the scenarios 1 and 2, which are also constrained problems, are solved using the MCCD Method.

The solutions found for these scenarios were based on the use of numerical optimisation techniques as well as through logical analysis. When applying the numerical optimisation techniques, the design points were selected using the random DoE technique.

For the random DoE technique, various sets of different design points were defined. This required a large amount of evacuation simulations to be performed. For instance:

- scenario 1: 439,200 simulations;
- scenario 2; 208,200 simulations.

In summary, a total of 647,400 simulations were performed for these two scenarios. This number is a result of the following consideration:

For each scenario, three sets of different design points were investigated. For instance:

- scenario 1: three sets of 4, 10, 15, 20, 25, and 30 different design points, which meant that 312 different design points were investigated;
- scenario 2: three sets of 4, 10, 15, 20, 25, 30 and 35 different design points, which meant that 417 design points were investigated.

Furthermore, in total, 729 design points were used during the numerical optimisation analyses for find the best solutions for these two scenarios.

It is clear to conclude that for real engineering problems, where time is a key-issue, this amount of design points is not feasible to be investigated. In practical terms, engineers/designers and/or modellers/experimenters would not be able to investigate such large number of design points. This becomes more critical when the data analyses require more complex computational modelling simulations, such as CFD modelling and/or laboratorial tests, which are expensive to be performed.

Now the same problems based on the mentioned scenarios, are solved using the MCCD method. And instead of 729 different design points used from the random DoE technique, only 8 different design points were used (i.e., 4 design points for scenarios 1 and 2).

This considerably reduced the number of required simulations: from 647,400 simulations to 4,800 simulations (i.e., 2,400 simulations for scenarios 1 and 2). Also, the results found using the MCCD method were satisfactory as they are shown in the next paragraphs of this section.

The solutions were found in a similar way to the previous problems, i.e. using the gradient-based algorithm (i.e., the Fletcher-Reeves numerical optimisation technique) and the non gradient-based algorithm (i.e., the Particle Swarm Optimisation numerical optimisation technique). The response surface models were based on the polynomial high order as well as the *multiquadratic* function, for the same reasons explained in the previous sections of this Chapter. It is important to mention that satisfactory results were obtained for both response surface models. This shows, once again, that the MCCD seemed to be appropriate to be used for more complex scenarios as well (i.e., with more than three design variables).

The general stepwise regression was the multivariable regression analysis technique here selected to build these response surface models and it worked well, since the coefficients R^2 were higher than 0.90.

In the next paragraphs the results are presented and discussed.

10.4.4.1 Scenario 1: identical exits, in pairs, three on each side

For this scenario, as mentioned before, three sets of 4, 10, 15, 20, 25, and 30 different design points were investigated. And considering also that for this particular scenario, the most appropriate response surface model was also investigated, a total of 439,200 simulations were required.

Given the nature of this problem, the general statement for it is:

Minimize: ET = *f* (D1, D2, D3)

Where: $0 < D1 < 25$ $0 < D2 < 25$ $0 < D3 < 25$

And given that these exits should not overlap each other. Clearly, the problem is a constrained problem, therefore, two constraints were considered:

$$|D1 - D2| > 0.5$$

$$|D2 - D3| > 0.5$$

ET – represents the evacuation time

D1 represents the distance from the left edge of exit 1 to the corner

D2 represents the distance from the left edge of exit 2 to the corner

D3 represents the distance from the left edge of exit 3 to the corner

It is also important to remind that, D4, D5 and D6 are not included as design variables for this problem. The reason for that is because this problem is a three variables problem, since it was assumed that the three exits in side 1 would have the same relative locations along the edge that the three exits in the side 2 would have and vice-versa. Therefore:

D4 represents the distance from the left edge of exit 4 to the corner

D5 represents the distance from the left edge of exit 5 to the corner

D6 represents the distance from the left edge of exit 6 to the corner

And the design points selected were:

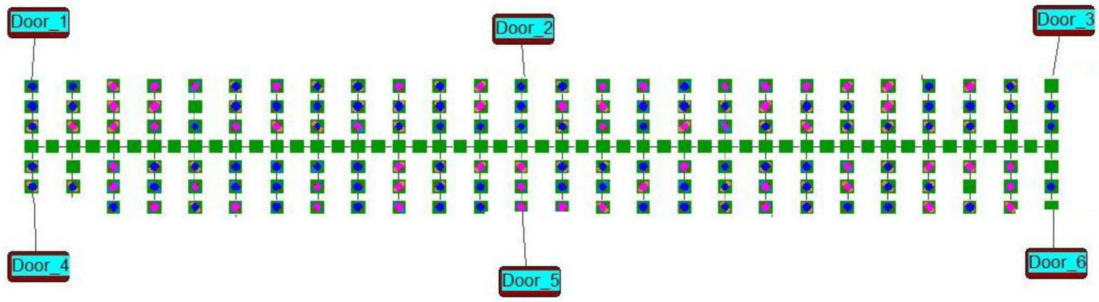
Design Point 1 (quasi-factorial point): (0; 12.5; 25)

Design Point 2 (quasi-factorial point): (0; 25; 0.5)

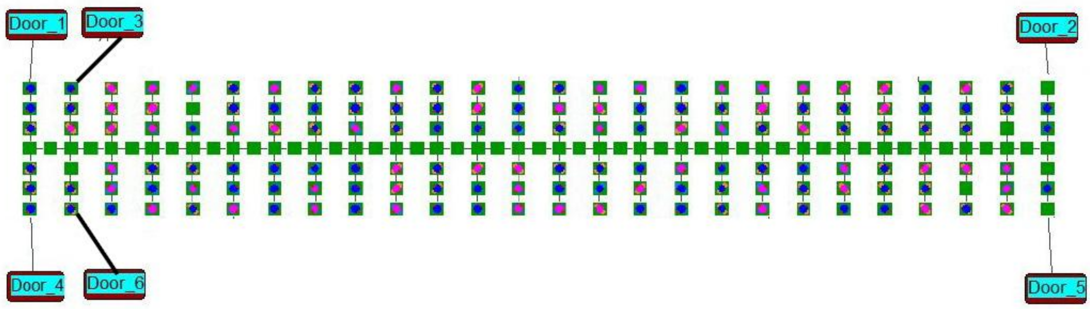
Design Point 3 (quasi-factorial point): (0; 0.5; 24.5)

Design Point 4 (axial point): (12.5; 13; 25)

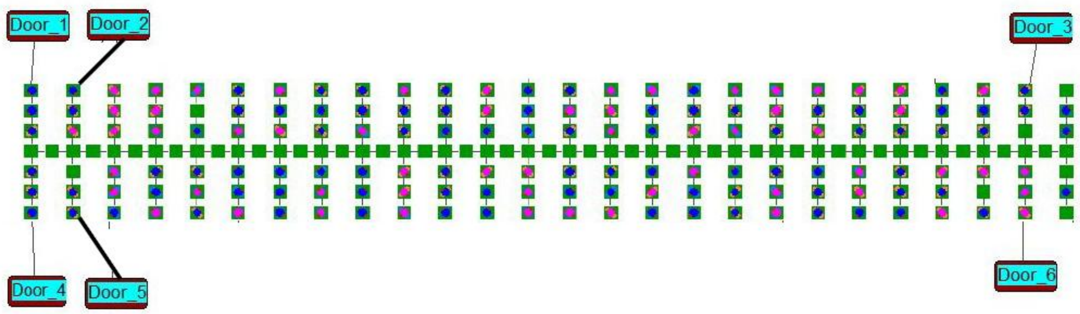
The Figure 10.27 presents the design points (associated with the respective scenarios) selected by the MCCD DoE method for this problem.



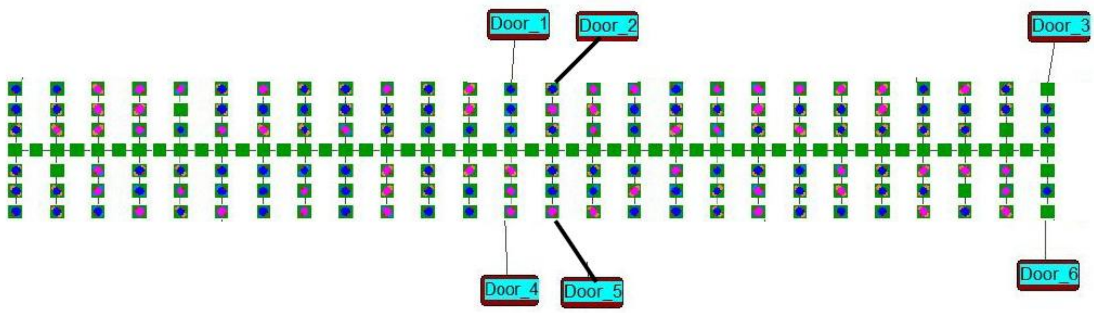
Design Point 1 (0; 12.5; 25; 0; 12.5; 25)



Design Point 2 (0; 25; 0.5; 0; 25; 0.5)



Design Point 3 (0; 0.5; 24.5; 0; 0.5; 24.5)



Design Point 4 (0; 0.5; 25; 0; 0.5; 25)

Figure 10.27: The 4 Design Points (and the associated scenarios) selected by the MCCD Method for the geometry with multiple exits of varying sizes (scenario 1)

The Table 10.7 presents the results.

Table 10.7: Optimal solution for the geometry with multiple exits of varying sizes (scenario 1)

Optimum Solution (4 ;12; 21; 58.8)	**Numerical Optimisation Technique**	
Using Polynomial Function as the RSM	Fletcher-Reeves	PSO – Particle Swarm Optimisation
The MCCD DoE Method (4 design points)	(0 ; 15; 23; 69.58)	(0 ; 15; 23; 69.58)
Using the *Multiquadratic* Function as the RSM	Fletcher-Reeves	PSO – Particle Swarm Optimisation
The MCCD DoE Method (4 design points)	(3 ;13 ; 22; 62.3)	(3 ;13 ; 22; 62.3)

From Table 10.7 and Figure 10.27, it is possible to see that the exact optimal solution found previously was not found this time. However, another good solution was found as well using the *multiquadratic* function as the RSM. The result was close to the assumed global minima region. For instance, the evacuation time produced based on this scenario using logical analysis was 58.8sec.. At this time, using the MCCD method as the DoE technique, the evacuation time found was 62.3sec. (It is also important to note that this result was better than the result found using the Numerical Optimisation methodology, 67.6sec, see section 9.5.1 of the previous Chapter and Figure 10.17 of this Chapter). This is a good result indeed, not only in terms of the numerical small difference, but especially due to the exits locations, which were also well distributed, see Figures 10.28, 10.29 and 10.30.

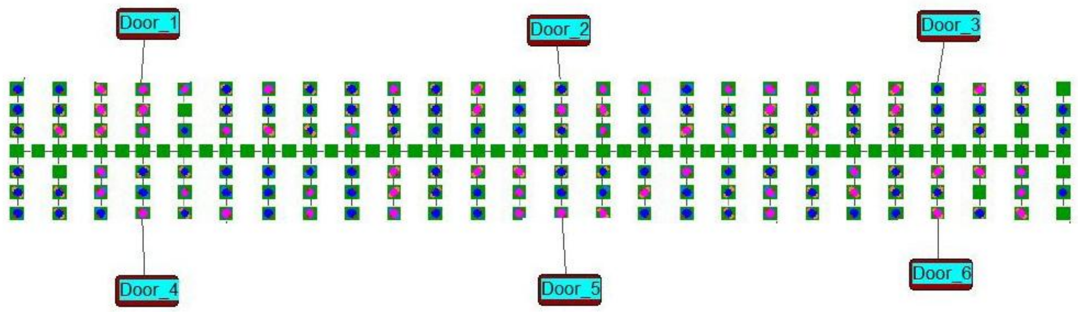


Figure 10.28: The optimal solution found using the MCCD Method for the Geometry with multiple exits of varying sizes, Scenario 1 - *identical exits, in pairs, three on each side* (where D1 = 3; D2 = 13; D3 = 22; D4 = 3; D5 = 13 and D6 = 22, which produced an evacuation time of 62.3sec)

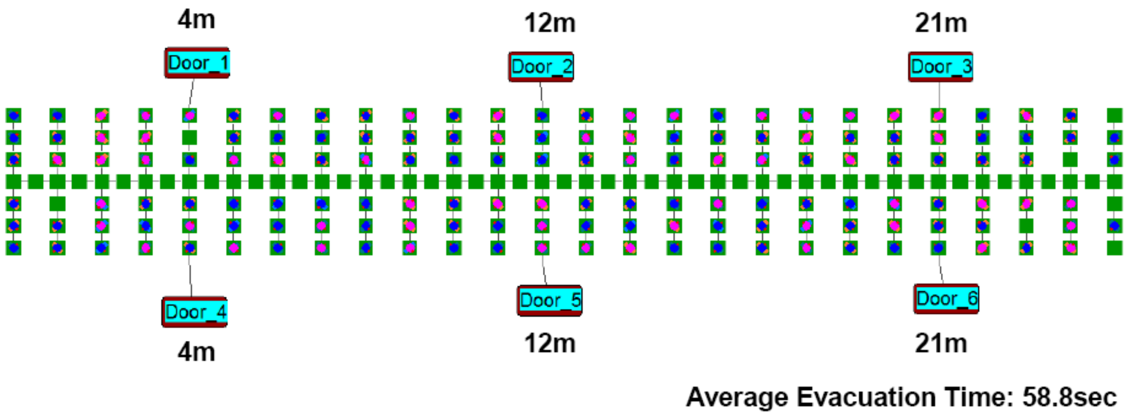


Figure 10.29: The optimal solution found using logical analysis (where D1 = 4; D2 = 12; D3 = 21; D4 = 4; D5 = 12 and D6 = 21, which produced an evacuation time of 58.8sec)

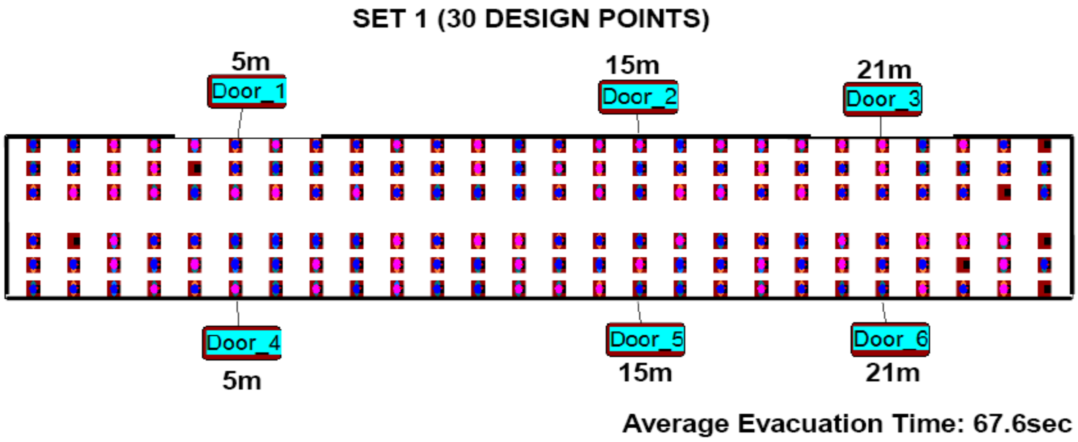


Figure 10.30: The optimal solution found using the Random DoE technique (where D1 = 5; D2 = 15; D3 = 21; D4 = 5; D5 = 15 and D6 = 21, which produced an evacuation time of 67.6sec)

It is also important to observe that additional evacuation simulations were performed for checking the consistency of this result. These simulations were based on the best scenario found using the MCCD method for picking up the design points. A total of 600 simulations were run following the same procedures which have been adopted in this study described in the previous chapters of this Thesis. The average evacuation time for these 600 simulations was 63.5sec. The difference between what was found using the Numerical Optimisation techniques, 62.3sec, and the actual result from the simulations, 63.5sec, is not relevant. Furthermore the result is consistent, showing the robustness of the MCCD method for this more complex problem.

It can be concluded that the MCCD method worked very satisfactorily for this case. This is a good achievement, because this is a complex case and specially, because the design points selected using the MCCD did not pick up the actual best solution. In addition to that, the results found using the MCCD method were better than the results obtained using the Random DoE technique; and using much less points and consequently performing much less simulations as mentioned previously.

10.4.4.2 Scenario 2: four large, two small, in pairs, three on each side

For this scenario, as mentioned before, three sets of 4, 10, 15, 20, 25, 30 and 35 different design points were investigated. A total of 208,200 simulations were required.

Given the similar nature to the previous study case (in which it was assumed that the three exits in side 1 would have the same relative locations along the edge that the three exits in the side 2 would have and vice-versa), the general statement for it is:

Minimize: ET = f (D1, D2, D3)

Where: $0 < D1 < 25$ $\quad$ $0 < D2 < 25$ $\quad$ $0 < D3 < 25$

And given that these exits should not overlap each other. Clearly, the problem is a constrained problem, therefore, two constraints were considered:

$$|D1 - D2| > 0.5$$

$$|D2 - D3| > 0.5$$

ET – represents the evacuation time

D1 represents the distance from the left edge of exit 1 to the corner

D2 represents the distance from the left edge of exit 2 to the corner

D3 represents the distance from the left edge of exit 3 to the corner

As mentioned in the previous case, the design variables D4, D5 and D6 were not included for this problem neither.

The design points selected were the same ones selected for the previous problem, namely:

Design Point 1 (quasi-factorial point): (0; 12.5; 25)

Design Point 2 (quasi-factorial point): (0; 25; 0.5)

Design Point 3 (quasi-factorial point): (0; 0.5; 24.5)

Design Point 4 (axial point): (12.5; 13; 25)

The Figure 10.31 presents the design points (associated with the respective scenarios) selected by the MCCD DoE method for this problem.

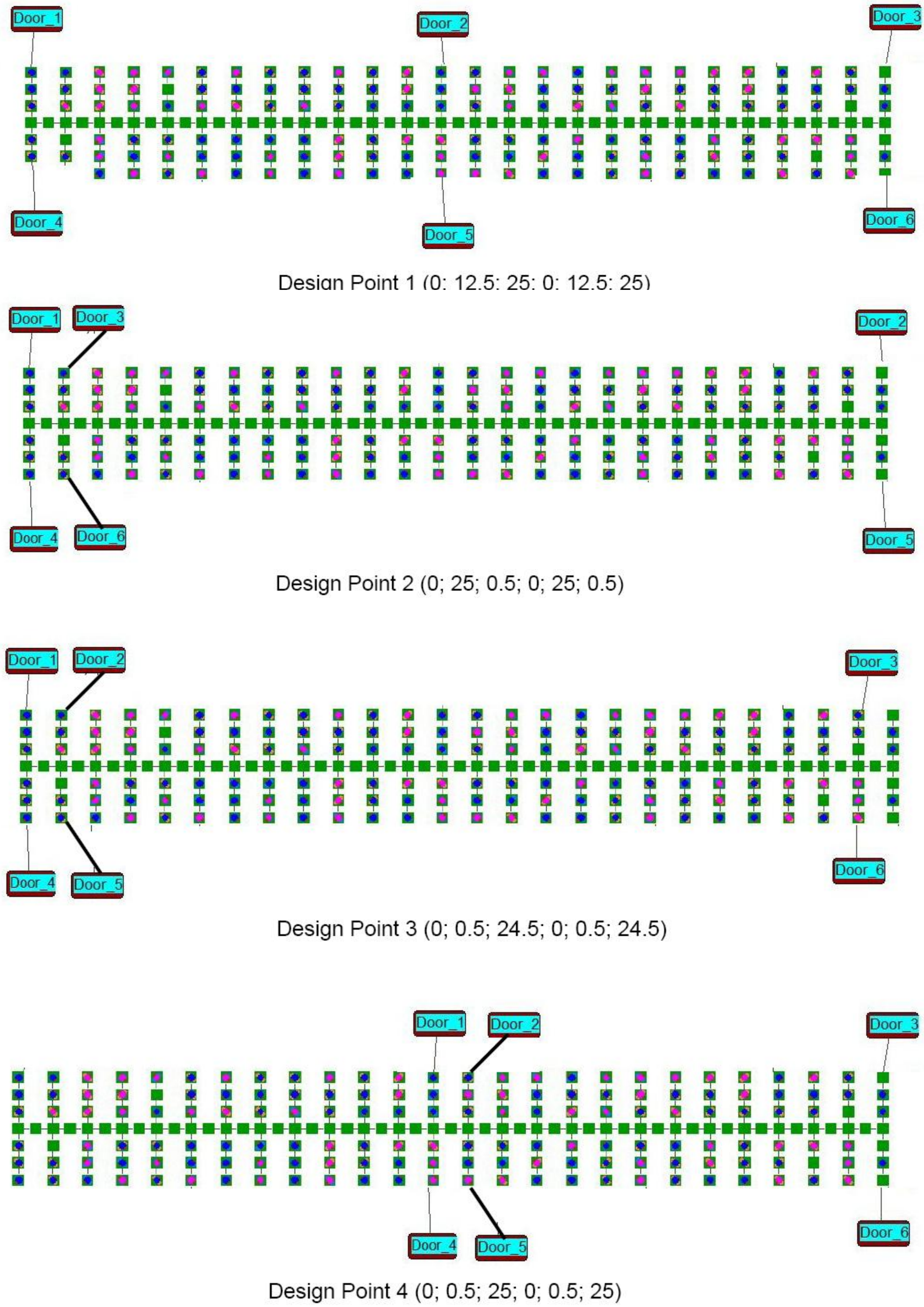


Figure 10.31: The 4 Design Points (and the associated scenarios) selected by the MCCD Method for the geometry with multiple exits of varying sizes (scenario 2)

The Table 10.8 presents the results.

Table 10.8: Optimal solution for the geometry with multiple exits of varying sizes (scenario 2)

Optimum Solution (5 ;13; 20; 61.2)	**Numerical Optimisation Technique**	
Using Polynomial Function as the RSM	Fletcher-Reeves	PSO – Particle Swarm Optimisation
The MCCD DoE Method (4 design points)	(2 ; 18; 25; 72.7)	(2 ; 18; 25; 72.7)
Using the *Multiquadratic* Function as the RSM	Fletcher-Reeves	PSO – Particle Swarm Optimisation
The MCCD DoE Method (4 design points)	(4 ;12 ; 23; 64.8)	(4 ;12 ; 23; 64.8)

From Table 10.8 and Figure 10.31, it is possible to see that the exact optimal solution found previously was not found this time. Nevertheless, similarly to the previous case, another good solution was found as well. Indeed, the result was close to the assumed global minima region found through logical analysis in Chapter 9. For example, the evacuation time produced based on this scenario using the logical analysis was 61.2seconds, see Figure 10.34. At this time, using the MCCD method as the DoE technique, the evacuation time found was 64.8sec. (It is also relevant to observe that this result was not better than the result found using the Numerical Optimisation Methodology, which was 62.5sec, see Section 9.5.2 from the previous Chapter. However, the difference between this result and the result found using the MCCD method for picking up the design point is not significant. And in addition to that, the actual exits distribution along the walls seemed to be better distributed). This is a good result indeed, not only in terms of the numerical small difference, but especially due to the exits locations, which were also well distributed, and having the small exits located in the middle; which are indicated with the ellipses in Figure 10.32.

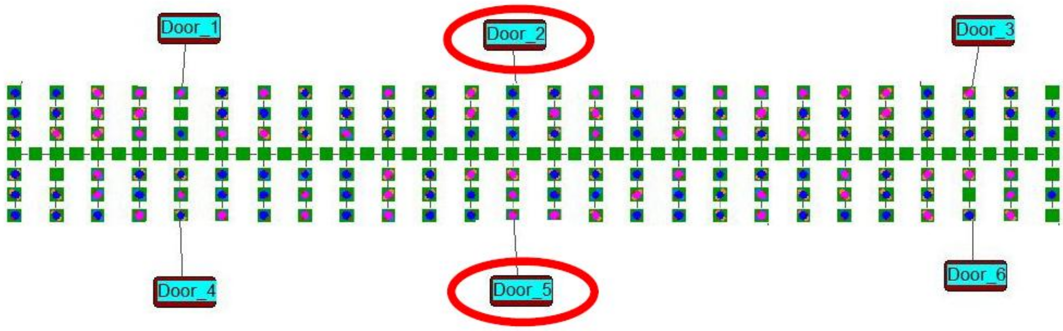


Figure 10.33: The optimal solution found using the MCCD Method for the Geometry with multiple exits of varying sizes, Scenario 2 - *identical exits, in pairs, three on each side* (where D1 = 4; D2 = 12; D3 = 23; D4 = 4; D5 = 12 and D6 = 23, which produced an evacuation time of 64.8sec)

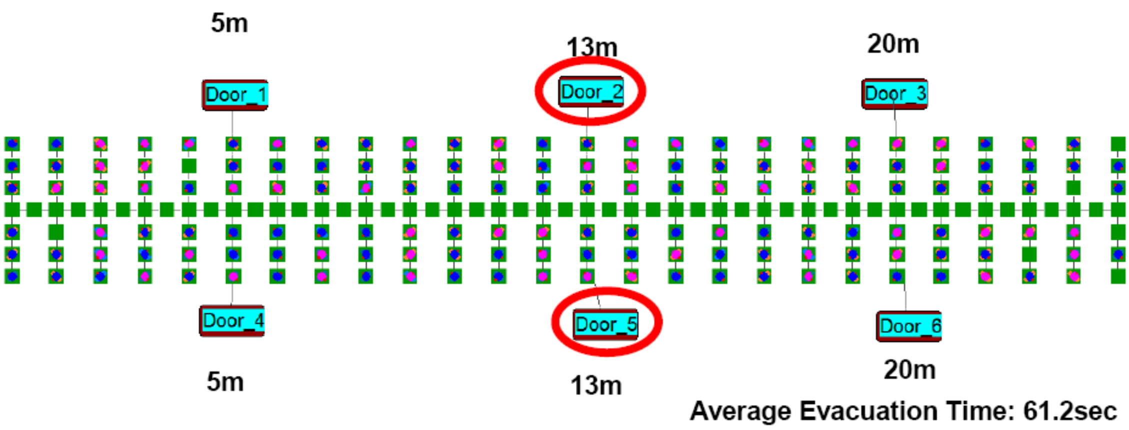


Figure 10.34: The expected optimal solution for the exits locations for the rectangular shaped room with multiple exits (scenario 2) using logical analysis

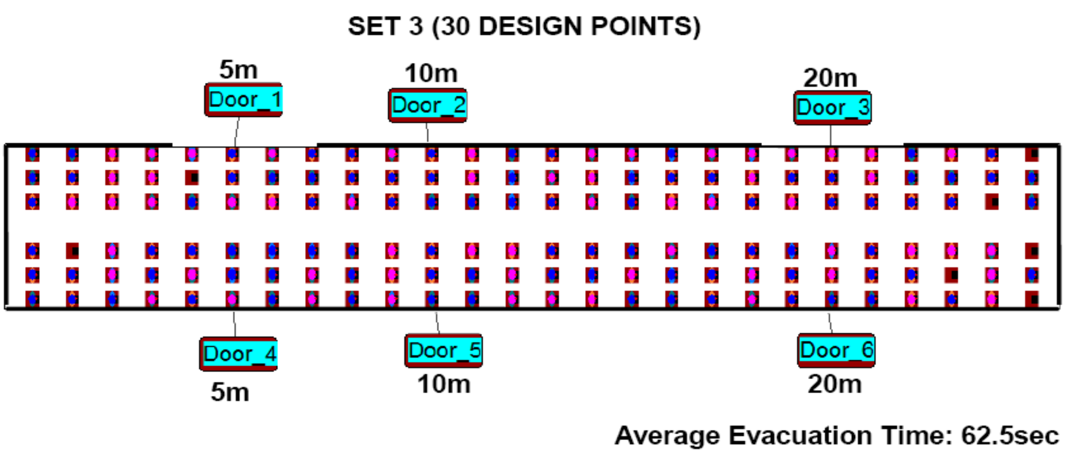


Figure 10.35: The best solution found using the Random DoE technique

Here similarly to what was done previously, additional evacuation simulations were performed for checking the consistency of this result. These simulations were based on the best scenario found using the MCCD method for picking up the design points. A total of 600 simulations were run following the same procedures which have been adopted in this study described in the previous chapters of this Thesis. The average evacuation time for these 600 simulations was 65.6sec. The difference between what was found using the Numerical Optimisation techniques, 64.8sec, and the actual result from the simulations, 65.6sec, is very small. For this reason, the result is consistent, proving that the MCCD method can be applied for complex scenarios with an acceptable degree of accuracy.

10.5 Concluding Comments

In this chapter, the MCCD method (i.e., *Modified CCD method*) was presented. The objective of this method is to reduce the effort involved in determining the design points for solving constrained optimisation problems. The MCCD method is a hybrid and flexible DoE method based on principles found in the CCD and combinatorial analysis. The MCCD method allows the user to find the optimal solution using much less design points than would be required by conventional DoE techniques.

Here, the MCCD method is applied for some of the previous case studies, namely: the square room with two exits, the L-Shaped room with two exits, the multiple connected compartments (scenario 2: one exit for the smaller room and two exits for the bigger room) and the geometry with multiple exits of varying sizes, for scenarios 1 and 2. All of these problems were based on evacuation simulation analyses in which the best locations for two exits, or more, were found to generate minimum evacuation times.

The MCCD method has proved to produce “good design points” (i.e., well located design points along the design space). Also important is the fact that the number of design points do not increase substantially according to the number of design variables. Finally, the MCCD has shown to be potentially flexible in terms of where the design points are located in the design space.

Some more complex cases were explored using the MCCD method, in which the number of design variables is increased as well as the number of constraints. The results have also shown to be satisfactory, supporting the use of the MCCD method as an alternative DoE method for more complex constrained problems.

It is expected that the MCCD method should be used on even more complex cases and for a different range of applications. Nevertheless, it is suggested also that a more accurate matrix needs to be developed for that; which will allow more flexibility for the method. This is why the method could not be used for scenarios 3 and 4 for the geometry with multiple exits of varying sizes. Nevertheless, the basis of the method is built.

The next chapter has some considerations about this issue and the general application of DoE techniques, RSM and numerical optimisation techniques applied to evacuation modelling analyses are presented.

11. Guidelines and Recommendations on the Use of Numerical Optimisation Techniques for Evacuation Modelling Analyses

This chapter presents some guidelines and recommendations on the use of the methodology developed in this study.

Some of these issues were discussed with further details in Chapters 3 and 4 and also along this thesis; however, they will be summarized here in a systematic way.

11.1. The nature of the variables

It is a fundamental condition within the optimisation theory that the design variables must have the same nature (i.e., continuum or discrete).

In this study, the design variable analysed was the positioning of exit(s). This design variable was considered to be continuum as its values, theoretically, had the freedom to vary from any value between a lower value and a upper value. Nevertheless, in case that if it was just possible to represent this design value as being a discrete variable instead of being a continuum variable, another more appropriate numerical optimisation technique should be used, such as Genetic Algorithm (GA). In reality, there are few practical and real problems in where this would be found. For instance, if there is/are column(s) in the wall; clearly the design variable values would not vary its values freely.

Another crucial issue that should be reinforced is the statement of the dependent variable(s) and the design variable(s) and their relationships.

For instance, as the name clearly suggests, the dependent variable(s) is(are) dependent on other variables (i.e., the design variables = independent variables). The dependent variable is the objective function (i.e., the response). This, in a first analysis might seem to be something straight forward to be defined. Nevertheless, it is not and if its relations with the design variables are not defined properly, the whole methodology will be inefficient, because it will be based on a false statement which will not reflect the real problem. This is also extended to the design variables, which must be independent from each other, including also from the dependent variable.

In this study, the evacuation time was taken as the dependent variable, because its values depend on the values of the positioning of exits. And besides that, the evacuation time does represent numerically the evacuation efficiency.

It might be also important to mention that in a hypothetical and unlikely case that the evacuation time would not be able to measured and/or the fire safety engineer is interested in another variable to be the dependent variable, the flow rate as well as the congestion could be taken as the dependent variable. These two variables are also dependent on the positioning of the exits and they do also serve as numerical indexes of the evacuation efficiency.

11.2. The values of the variables

In all the case studies investigated in this research, apart from the case study analysed in Chapter 8, the design variables had the same domain. In other words, the design variables values had the same lower value and upper value limits. All these values for all case studies, with no exception, were real and positive values. Nevertheless, if any of these problems involved negative values, the numerical optimisation techniques used in this study, the Fletcher-Reeves and the PSO methods, would still be suitable for the optimisation analyses.

11.3. The constraints

The constraints used for all the case studies investigated in this research were inequalities. However, even if the constraints were functions and/or composed functions, the numerical optimisation techniques used in this study would still be suitable.

11.4. The DoE techniques

The use of Design of Experiments techniques within the methodological approach has shown that for unconstrained problems, the best techniques are the Central Composite Design (CCD) and the Latin-Hypercube. These DoE techniques proved to cover the design space better than other DoE techniques, such as the Taguchi technique, despite of this one being popular.

For constrained problems, the use of random DoE technique has shown to be not consistent and not feasible for more complex problems which involve more variables and expensive computational analyses.

In the other hand, the “modified CCD” developed in this study seems to be a potential aid when solving constrained problems.

11.5. The Response Surface Models

Evacuation processes, even when they take place in simple scenarios like a square room, seem to be in nature a very complex, dynamic and aperiodic system. And besides that, through the analyses performed in this study, evacuation processes also have shown to be highly dependent on the initial conditions and do present non-linear trend behaviours along the time. In summary, evacuation processes present the following characteristics: non-linearity; dynamical behaviour; non-periodicity; deterministic; multi variable dependent and highly sensitive to the initial conditions. Based on that, functions such as the full-quadratic, cubic, high-order polynomial are recommended to be the response surface model. And in some cases for more complex problems involving more than 4 design variables and more than 2 constraints, it seems that radial-basis functions, like the *multiquadratic* are the most appropriate response surface model.

11.6. The Numerical Optimisation Techniques

The Fletcher-Reeves method and the PSO have produced satisfactorily results, because the global minima region and a local minima region closest to the global minima region were found. Nevertheless, it is recommended for more complex cases that the Fletcher-Reeves should be applied, since it is a gradient-based method and for this reason its algorithm works more consistently in cases where the complexity involves non-linearity and high dynamical behaviours are observed. In case that the Fletcher-Reeves method is not possible to be applied, it is strongly recommended that another gradient-based method should be applied.

11.7. The nature of the problem

The case studies investigated in this thesis were based on a single objective function. Nevertheless, there might be cases where more than one objective function would be taken into account (i.e., multi-objective optimisation problems); which is out of scope of this study. For these types of problems, the Fletcher-Reeves method would be recommended or other robust gradient-based method.

11.8 The data

The data generated for this study was produced through an evacuation simulation model and the scenarios were based on enclosures where the built areas were not bigger than $300m^2$. For this reason, several simulations could be done, because they were not time consuming in terms computational performance. Therefore, for scenarios where the built area is smaller or equal to $300m^2$, it is recommended that, at least, 600 simulations should be performed, given that the likelihood of bias on the values decreases considerably from 600 simulations.

In the next Chapter, the conclusions of this study are presented as well as some ideas are proposed for future work.

12. Conclusion

This chapter concludes the thesis and summarises the major findings. Recommendations for future research on the methodology developed in this research and also on the area of evacuation modelling analysis studies are given.

12.1 Conclusions: the implication of the work

This study is aimed to develop a numerical methodology to allow the efficient optimisation of fire safety aspects of structural designs; which is the main objective of this study. And besides that, this study is also aimed to understand how some core variables impact the evacuation efficiency; this is the secondary objective of this study. In order to achieve these goals, this study has investigated a number of a variety of practical problems. These problems are represented by four case studies which vary from complexity to the nature of the variables. These case studies involved both types of problems found within the classical optimisation field, namely: unconstrained and constrained.

Therefore, regarding the principal objective, the methodology developed and demonstrated in this Ph.D. thesis included the use of an evacuation model, the buildingEXODUS simulation tool, followed by the application of optimisation techniques (both gradient and non-gradient based numerical optimisation techniques) as well as different types of DoE techniques and Response Surface Models.

The results obtained have shown to be satisfactory, i.e., the evacuation times were minimized considering the exits' locations. The analysis revealed that this methodology seems to be a very powerful tool for evacuation modelling analysis, improving the designs according to a number of different variables.

Furthermore, in relation with the methodological approach developed in this study, some findings and contributions can be mentioned:

- The methodology has proved to work satisfactorily, given that the global minima region and local minima region close to the global minima region were found during the analyses;
- The methodology has shown to be efficient, since it saved time during the analyses, because in all the cases, the number of simulations were and/or could be reduced significantly;
- For problems with design variables with the same nature and same domains, the Fletcher-Reeves and Particle Swarm Optimisation methods work accurately;

- For problems with two or three design variables, the high-order polynomial function works appropriately as the Response Surface Model;
- For problems with six design variables and with more than two constraints, the high-order polynomial function does not work consistently in all cases;
- For problems with six design variables and with more than two constraints, the *multiquadratic* function seemed to work robustly;
- The Central Composite Design (CCD) and the Latin-Hypercube DoE techniques have shown to work satisfactorily;
- For constrained problems, a hybrid DoE method which has been called here in this research as the “Modified CCD” has shown to be more efficient than the random DoE technique.

And regarding the secondary objective of this study, some findings can be mentioned:

- The positioning of exits around the perimeter of enclosures does impact the evacuation efficiency;
- The analyses revealed that for square rooms with a single exit, there is a slight advantage in positioning the exit in the corner of the room;
- The impact of the positioning of exits, along the wall perimeter of any room, varies according to the enclosure's geometry;
- For square rooms with two exits, it was found that exits positioned symmetrically around the perimeter of the room produce shorter evacuation times than when the exits are placed asymmetrically around the perimeter of the room;
- For more complex geometries with two exits, like an L-shaped room, the advantage of the symmetrical relative distance between two exits is still observed;
- The exits locations within enclosures do play an important rule on the evacuation efficiency and therefore, the maximum travel distance should not be the only factor to be considered within the fire safety codes;
- It was shown that short travel distances will not necessarily generate short evacuation times;

- It was shown that the queuing time spent near to the exits can be substantially more effective than the time spent during the escape movement towards the exits;
- For scenarios similar to aircraft geometries, the lowest evacuation times were found when the exits were well distributed along the walls;
- Evacuation processes, even when they take place in simple scenarios like a square room, seem to be in nature a very complex, dynamic and aperiodic system;
- Evacuation processes also have shown to be highly dependent on the initial conditions and do present non-linear trend behaviours along the time.

This study revealed that the classical concepts and numerical methods found in the optimisation theory and applied statistics can be successfully applied to provide fire safety in enclosures more efficiently.

It was also shown through this study that the evacuation efficiency is dependent on many variables (such as the exits positions within the enclosure's boundaries) and/or the combinations of these variables.

In summary, this integrated approach is intended to help fire safety engineers and designers to develop optimal designs (i.e., safe designs) in a optimized manner. In reality, this was the motivation of this study: to introduce numerical optimisation techniques and associated concepts, well known within the operational research field, as an approach for a more efficient and systematic procedure when developing and/or improving fire safety designs.

12.2 Future work

This research intends to be a beginning of a new approach within the Fire Safety Engineering field and more specifically within the Computational Fire Engineering modelling community.

This systematic methodology to efficiently optimise evacuation safety aspects of structural designs should be also extended to more complex designs, such as larger enclosures and open spaces.

The study is focused on evacuation modelling; nevertheless the methodology developed here could equally be applied to other fields, such as safety engineering, fire modelling, business management etc.

For instance, in the safety engineering field, there is a wide range of real problems which this methodology can be helpful and useful:

- The modelling of crowds;
- Safety management within off-shore platforms.

In the fire modelling, there are practical problems which this methodology can also bring some light, such as:

- The sizing and positioning of smoke extraction vents;
- The modelling of cable fires;
- The optimal design of sprinklers to better define the gaps designs to generate more efficient water mist to extinguish the fire.

And in the business management field, evacuation models can be used combined with this methodology to maximize the use of floors within a specific area of shopping centre for example.

Besides these recommendations, there are other few, which may be of interest of new researchers:

- In Chapter 2, human behaviour in emergency situations was discussed briefly. In the early stages of this Ph.D. study, some time was spent on this subject. Some discussions on what it has started to be called as "fire safety culture" were initiated which resulted in three publications **[1,2,280]**. Therefore, it is expected that further studies should be conducted to analyse if the "fire safety culture" does impact human behaviour during fires and if so, how significant this influence is. (In fact, a recent new project developed based on this issue has been funded to the tune of 2 million Euros by the

European Union **[281]**. This research project will compare how people behave when fleeing from emergencies in seven different countries of the old continent of Europe. These kinds of studies are potentially helpful in order to bring some light on this topic of fire safety culture);

- It is intended that further research showing more practical applications of the CFE models within the FSE field should be developed. For instance, the use of these models combined with Multicriteria Decision-Making models and also with the methodology developed in this study for performing fire risks assessments, implementing fire risks management and/or improving fire safety procedures should be investigated in depth **[1]**;
- Based on the fact that evacuation processes present the following features: non-linear; dynamic; non-periodic; deterministic; multi variable dependent and highly sensitive to the initial conditions, theoretically they are chaotic systems, since all these principles are observed in the chaos theory. Therefore, the understanding of evacuation processes in enclosures as being chaotic systems might be helpful in terms of modelling, not only regarding the physical aspects (i.e., relationships: occupant-structure, occupant-occupant and fire-occupant) but also the psychological aspects. *A prior*, the “Lotka-Volterra” prey-predator equation could be used as some studies have shown for modelling open fires (i.e., fires in forests) and the interaction between cold air mass and the fire plume **[282]**.

And finally due the limitations of resources and time, which have been the major constraints, the recommendations below may further improve this study:

- This study has shown that the geometry's shape seems to impact the evacuation efficiency. For this reason, some further research on this issue might be relevant in this field;
- Some real scale experiments with low costs could be conducted for analysing some findings in terms of the positioning of exits impact on the evacuation efficiency;
- Some small scale experiments, using ants for instance, could be conducted for analysing some findings in terms of the positioning of exits impact on the evacuation efficiency;

- Development of a more flexible hybrid DoE method for constrained problems and/or improvement of the modified CCD method proposed in this study.

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148. TAVARES, R.M., *Analysis of the interaction between the ventilation factor and the fire plume in enclosures fires under the Chaos Theory perspective*, Master's Degree Dissertation, UFPE – Universidade Federal de Pernambuco (Federal University of Pernambuco), Pernambuco, Brazil, 2003.

APENDIX A: LIST OF PUBLICATIONS

Evacuation Modeling Analysis within the Operational Research Context: a Combined Approach for Improving Enclosure Designs

Publication Type: Journal Article

Authors: Rodrigo Machado Tavares; Edwin R. Galea

Source: Building and Environment, Elsevier Ltd., Volume 44, Issue 5, p.1005-1016 (2009)

The use of a mathematical multicriteria decision-making model for selecting the fire origin room

Publication Type: Journal Article

Authors: Rodrigo Machado Tavares; João Marcelo Lopes Tavares; Sara L. Parry-Jones

Source: Building and Environment, Elsevier Ltd., Volume 43, Issue 12, p.2090-2100 (2008)

Numerical Optimisation Techniques Applied to Evacuation Analysis

Publication Type: Conference Paper

Authors: Rodrigo Machado Tavares; Edwin R. Galea

Source: PED 2008 - 4th International Conference on Pedestrian Dynamics, Springer Berlin Heidelberg, Wuppertal, germany , p.555-561 (2008)

Evacuation Processes Versus Evacuation Models: "Quo Vadimus"?

Publication Type: Journal Article

Authors: Rodrigo Machado Tavares

Source: Fire Technology, Springer Netherlands, Volume 45, Issue 4, p.419-430 (2009)

An Analysis of the Fire Safety Codes in Brazil: Is the Performance-Based Approach the Best Practice

Publication Type: Journal Article

Authors: Rodrigo Machado Tavares

Source: Fire Safety Journal, Elsevier Ltd., Volume 44, Issue 5, p.749-755 (2009)

Collection and Analysis of Pre-Evacuation Time Data Collected from Evacuation Trials Conducted in Library Facilities in Brazil

Publication Type: Journal Article

Authors: Rodrigo Machado Tavares; S. Gwynne; E. R. Galea

Source: Journal of Applied Fire Science, Baywood Publishing Company, Inc., Volume 15, Issue 1, p.23-40 (2006)