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Financial stress evaluation: a complexity science approach

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Abstract

Financial markets typically exhibit dynamically complex properties as they continuously interact with economic and environmental factors. For example, the efficient market hypothesis suggests a considerable difference in the structural complexity of security prices between "normal" (stable markets) and "abnormal" (financial crises) situations. Considering the analogy between market undulation of price time series and physiological stress of bio-signals, we investigate whether physical stress indices in bio-systems can be adopted and modified to measure "standard stress" in financial markets. We employ structural complexity analysis based on univariate and multivariate sample entropy variants to estimate the overall stress level of financial markets and the performance of individual financial indices. Furthermore, we propose a novel graphical framework to determine the sensitivity of individual assets and stock markets to financial crises. Catastrophe theory and entropy-based stress evaluations are used to ascertain the unique performance of each index or individual stock in response to different types of crisis. Four major indices and four individual equities with gold prices are considered over 31 years, from 1991 to 2021. Results show the feasibility of measuring financial stress and reveal the relationship between structural complexity among economic indices and within each price time series.

Keywords: Multivariate multiscale entropy, Dynamics in financial systems, Determinism, Recurrence plot analysis, Catastrophe theory, Arousal–performance plot

Introduction

The characteristics of stock markets' structural complexity have long been investigated as efficient indicators of financial health and economic stability. Techniques such as noise reduction and permutation entropy have been instrumental in revealing deterministic patterns within financial systems, distinguishing chaotic behaviors from random fluctuations. These methods enhance the understanding of market dynamics by identifying underlying structural patterns, offering valuable insights for theoretical and practical applications (Soofi and Cao 2002; Zanin 2008). The cornerstone of modern financial theory, the EMH (EMH), states that the underlying value of an asset incorporates all the available information to ensure that the stock always trades at a fair value (Fama 1970). Hence, in "normal" situations, certain events barely influence markets, which respond evenly to continuous stimuli of economic change. In the context of complexity science, security prices in normal situations exhibit high randomness and uncertainty, with their historical values exhibiting low predictability. Conversely, when an economic crisis occurs, irrational fluctuations (e.g., panic buying or selling) can influence markets, showing a low degree of randomness and high determinism (García et al. 2016; Hemakom et al. 2016).

Analogous to the well-known automatic fight-or-flight response in physiological stress studies in human cognitive science (McCarty 2016), financial stress can also be considered a deviation from the normal functioning of financial markets (Hakkio and Keeton 2009). Considering the similarity between the complexity-loss theory (CLT) in human body functions (Lipsitz and Goldberger 1992) and the implications of EMH in financial markets, the concept of sympatho-vagal balance in bio-systems can also be used to describe the acceleration–stabilization type of behavior in financial systems (Malik et al. 1996; von Rosenberg et al. 2017). In human-centered sciences, the sympatho-vagal balance refers to the joint influence of the sympathetic nervous system (SNS) and parasympathetic nervous system (PNS) (Malik et al. 1996), which accelerate and decelerate body functions, respectively. Regarding the analysis of financial systems, supply and demand drive and sustain the acceleration–stabilization behavior, resulting in market expansions and recessions (García et al. 2016).

In terms of practical implications, leveraging biosystem concepts to measure financial stress offers a novel perspective in risk assessment and management. Using entropy-based measures, the analogy between physiological and financial stress provides a systematic and non-parametric approach to gauge market stability. This framework allows for the identification of "standard stress" in financial systems, which can influence risk assessment and mitigation strategies via early detection of systemic risks. Moreover, regulators and policymakers can adopt additional dynamic risk management strategies using entropy-based insights for continuous market monitoring. Such metrics allow for a more nuanced understanding of complex market behaviors, particularly under interconnected economic and environmental factors. Furthermore, incorporating entropy-based metrics such as stress testing and early warning systems into policy decisions can enhance the resilience of financial markets by detecting stress levels before a crisis escalates. This study contributes to policy-making by offering new tools for real-time monitoring and intervention, potentially enabling regulators to adjust capital requirements or implement trading halts to mitigate market volatility.

As stated in EMH, the asset price contains all available information. In addition, most financial studies have been implemented based on the return time series, the difference between the two consecutive prices, $x(t+1)$ and $x(t)$, to obtain the dynamics of price change. In this work, we first apply a moving average (MA) filter to the detrended data to maintain the maximal information of the price time series in the signal. Given the potential of nonlinear methods to reveal financial stress, we employ complexity science, that is, structural complexity analysis based on historical data, to predict the occurrence of an "abnormal" situation, that is, a financial crash. Such a sudden change in the behavior of the financial system results from the smooth changes that jointly arise from economic and non-economic factors. This phenomenon was conceptualized by Ren'e Thom, who termed it catastrophe theory (Thom 1972). Although broadly applicable to dynamical systems, catastrophe theory has seen limited use in economics despite its potential to model discontinuities such as stock market crashes. However, applications of stochastic cusp models and financial brittleness functions have demonstrated their ability to explain sudden market transitions, such as the 1987 crash, where internal factors like sentiment play a pivotal role. These models underscore the importance of understanding bifurcations within financial systems to better anticipate systemic risks and design effective macroprudential policies (Wesselbaum 2017; Birău 2013; Yang et al. 2009; Baruník and Vosvrda 2009). Therefore, we demonstrate that financial stress measurement serves as an insightful and practical method for quantitative analysis. To this end, we considered 31 years of historical data from January 1991 to December 2021, with one data point per weekday. The main investigation was conducted over four stock indices (Dow Jones Industrial Average, NASDAQ Composite, Russell 2000, and Standard & Poor's 500) and four individual equities from different industries (Apple Inc., Microsoft, McDonald's, and American International Group). The catastrophe plots exhibit the unique performance of each index or individual stock in response to different crises in the last several decades.

The main contribution of this study is the application of the catastrophe theory combined with entropy measures derived from the Mod-MSE/MMSE to produce catastrophe plots for visualizing systemic financial stress. This method offers a unique descriptive framework for understanding the evolution of financial crises, distinguishing it from traditional financial stress indices, which mainly reflect the unfolding of coincident stress. In addition, this study extends the analysis timeline to include the COVID-19 crisis, providing a comprehensive comparison of this unprecedented event with earlier crises, such as the 2008 financial crisis. The visualization of these different types of crises within the same analytical framework allows for a more in-depth understanding of how various systemic disruptions manifest in financial markets. This extension offers a better grasp of the pandemic's impact and the broader historical context of financial stress. The catastrophe plots offer an innovative way to represent the complex build-up of stress in the financial system, making this framework a valuable tool for researchers and policymakers who aim to study and compare crises over time.

The remainder of this paper is organized as follows. Section Literature review provides a literature review of entropy-based methods and financial stress indices. Section Algorithm and methods presents the details of the nonlinear algorithms and methodologies. Section Data overview and methods presents a summary of the data and methods to provide an overview of the asset price time series and analytical flow. Section Results and analysis illustrates the results of the measures and analyzes the nonlinear properties of financial markets. Section Catastrophe plots based on entropy proposes the framework of the catastrophe theory in financial investigations and provides initial analyses based on the catastrophe theory. Finally, the last section presents the conclusions.

Literature review

Recent advances in entropy-based financial analysis have strengthened the application of complexity methodologies to market dynamics. For example, entropy-based analysis has been employed to examine the efficiency of financial markets during the COVID-19 pandemic (Wang and Wang 2021), revealing significant structural changes in market complexity during this unprecedented crisis. Similarly, research on information diffusion and entropy-based network dynamics in equity markets (Bekiros et al. 2017) has elucidated how entropy measures can capture the complex interrelationships between market participants during periods of stress.

Entropy-based measures have been effectively employed to reveal complex features in the nonlinear domain based on these recent developments. Approximate entropy and multiscale entropy extend traditional approaches by capturing the irregularities and synchrony within financial datasets. This case offers a robust framework to understand system stability and diversification. These advancements demonstrate the versatility of entropy-based methods in quantifying financial stress and discovering dynamics across variables (Pincus 2008; Ahmed et al. 2017; Pincus and Kalman 2004; Wu et al. 1996). The nonlinear methods used in the following analyses include entropy-based univariate or multivariate algorithms. Examples include the recurrence quantification analysis (RQA) (Marwan et al. 2007) and assessment of the latent index of stress using instantaneous amplitude (iA-ALIS) methods (von Rosenberg et al. 2017). Traditional entropy methods aimed at signal irregularity quantification have been applied in financial investigations owing to their model-independent and prejudice-free analysis, including approximate entropy (Pincus 2008; Pincus and Kalman 2004), sample entropy (Wu et al. 2018), and permutation entropy (Yin and Shang 2016; Zhang and Shang 2019). Advancements in multiscale entropy methodologies have provided deeper insights into the complexity of financial time series, a crucial factor in nonlinear system dynamics. For instance, composite multiscale entropy and refined permutation entropy methods overcome the limitations of traditional approaches, enabling the analysis of short-term data with high precision. These techniques reveal how financial systems vary structurally across multiple temporal scales, making them vital tools for understanding market behavior during periods of stress (Yin and Shang 2015; Zhang and Shang 2018; Niu and Wang 2015). In addition to univariate entropy analysis, enhanced multivariate methodologies have been employed, such as multivariate multiscale sample entropy (Er and Mandic 2013). RQA is another popular nonlinear measurement method for quantifying determinism via predictability, which has been widely considered in physiological studies (Yang 2010; Cao et al. 2004). Stock market volatility reflects the degree of uncertainty in asset price fluctuations, a well-established concept linked to financial stress. Intrinsic multiscale analysis methods, such as multiscale sample entropy and stress metrics, have been applied to reveal nonlinear relationships between market dynamics and structural indicators, particularly during crises such as the subprime mortgage crisis. These methods highlight the complexity-loss hypothesis in financial stress contexts, suggesting that periods of heightened volatility correspond to significant structural changes (Pincus 2008; Hemakom et al. 2016; Pincus and Kalman 2004). Closely relevant to the volatility measure, the high degree of determinism (DET) given by RQA indicates low volatility and high predictability (Strozzi et al. 2008; Ruiz et al. 2012). assessment of the latent index of stress (ALIS) was introduced to detect financial crises primarily in stock markets by examining the power in low- and high-frequency bands of dynamical prices (García et al. 2016; Hemakom et al. 2016). Expanding on the ALIS, an assessment based on the instantaneous amplitude, iA-ALIS, was proposed as a highly reliable indicator of financial stress (von Rosenberg et al. 2017).

Although entropy-based methodologies provide powerful analytical tools, their relative advantages and limitations must be considered within the broader context of financial stress measurement approaches. Quantitative methods, such as the aforementioned algorithms, offer the advantage of precise stress testing. The reason is that they can systematically analyze financial data using numerical indicators, allowing for further statistical analysis and visualization. These methods can effectively measure the impact of known factors, providing a clear, data-driven picture of financial stress. However, quantitative analysis has limitations, particularly in accounting for new or unprecedented stress events not present in historical data. For example, supply chain disruptions post-COVID-19 may be readily captured in qualitative assessments because of their evident impact on the economy. However, they could be missed or underestimated in purely quantitative models if these events are not included in the dataset (McKibbin and Fernando 2021). Qualitative analysis in finance has the advantage of capturing the influence of policy regimes and contextual factors that may not be reflected in historical data (Gagnon et al. 2011). This approach allows for a deeper understanding of financial stress by considering external influences, such as government policies or global events, and by visualizing historical events' patterns. However, the timing of qualitative analysis can be challenging because predicting and quantifying exogenous factors, such as sudden policy changes or unexpected global events, is difficult, making it harder to incorporate them into a precise model (Baker et al. 2016).

A comparative analysis with established financial stress indices, such as the St. Louis Fed Financial Stress Index (STLFSI), the Kansas City Financial Stress Index (KCFSI), and the Office of Financial Research Financial Stress Index (OFR FSI), reveals several distinct advantages of our entropy-based framework to contextualize our methodological approach. The STLFSI and KCFSI are constructed using financial indicators normalized based on their standard deviations from historical means, an approach that inherently assumes a Gaussian distribution of the underlying data (Manamperi 2015; Hakkio and Keeton 2009). This reliance on a Gaussian framework is limiting, particularly during periods of significant financial market stress when the assumption of normal distribution is often violated. By contrast, the OFR FSI incorporates a broad array of 33 financial market variables categorized across five segments (i.e., credit, equity valuation, funding, safe assets, and volatility) to offer a more comprehensive understanding of stress across global financial markets (Monin 2019). However, despite its breadth, the OFR FSI relies on the historical weighting of these variables, potentially overlooking the real-time shifts in market dynamics during crises.

Moreover, our method falls between index-specific and composite categories. The method is composite, as is the case with STLFSI, KCFSI, and OFR FSI, in the sense that it simultaneously analyzes several individual market indices while being specific enough to examine the balance or imbalance aspect of markets. Our approach offers a more holistic understanding of the macroeconomic environment and interconnected financial risks. In addition, our approach simultaneously identifies early signals of market stress and asset mispricing, leading to better-informed decisions in financial analysis and risk management. Our method avoids these limitations by not depending on the standard deviation of the data, eliminating the need to assume a Gaussian distribution. This approach allows us to more effectively capture nonlinearities and tail risks that frequently characterize periods of heightened financial stress. Our method offers a more robust and flexible measure of financial stress by focusing on the direct impact of large-scale asset purchases and considering systemic risks that arise from the interconnectedness of financial institutions. Particularly, it has the ability to capture and forecast systemic risks in a more timely and adaptable manner than the STLFSI, KCFSI, and OFR FSI, providing policymakers with a more relevant tool for crisis detection and financial stability monitoring.

Algorithm and methods

The algorithms used in this study are introduced and described in this section.

Modified univariate multiscale sample entropy & modified multivariate multiscale sample entropy

Moving average filter

To obtain the scaled and detrended signal, $y^{(\tau)}(j)$, a moving average (MA) filter, is first used to remove the local trend, $s^{(\tau)}(j)$, from the original time series, $\{x(i)\}_{i=1}^{N}$, as follows:

$$s^{(\tau)}(j) = \frac{1}{\tau} \sum_{i=j-\tau/2-1}^{j+\tau/2-1} x(i), \quad 1 \leq j \leq N - \tau + 1. \tag{1}$$

$$y^{(\tau)}(j) = x(i) - s^{(\tau)}(j) \tag{2}$$

where τ is a predefined scale factor. Unlike the traditional coarse-graining process (Costa et al. 2002), the scaling given by the MA filter can maintain the original signal length and better fit the intrinsic properties of the applied data.

Multivariate sample entropy

Sample entropy is a standard approach for evaluating the degree of time series irregularity and randomness based on their temporal dynamics. It has been widely applied in real-world complex systems (Richman and Moorman 2000; Humeau-Heurtier 2015). Sample entropy is built based on the probability of similarity between the embedding (delay) vectors, where high similarity demonstrates high predictability at multiple scales. To this end, we applied the MA filter to implement the scaling process for each time series, which was termed the modified univariate multiscale sample entropy (Mod-MSE) (Wu et al. 2013). Owing to the intrinsic properties of the financial data, the predefined scale factor is set as 1 week, $\tau = 5$. The Mod-MSE was applied to every index and stock individually to evaluate the stress of each index and equity, reflecting their response to the external environment. However, no individual stock or single index can sufficiently represent the performance of the entire financial market. Therefore, the enhanced multivariate entropy is employed to estimate the overall stress of the financial market. The multivariate entropy method accounts for the cross-channel dependencies in the multivariate data by constructing composite delay vectors (CDV), $\mathbf{X}_M(i)$, derived from the original p-channel signal, $\{x_{k,i}\}_{i=1}^{N}$, $1 \leq k \leq p$, in the form

$$\begin{aligned}\mathbf{X}_M(i) = [&x_{1,i}, x_{1,i+l_1}, \ldots, x_{1,i+(m_1-1)l_1}, \\ &x_{2,i}, x_{2,i+l_2}, \ldots, x_{2,i+(m_2-1)l_2}, \\ &\vdots \\ &x_{p,i}, x_{p,i+l_p}, \ldots, x_{p,i+(m_p-1)l_p},]\end{aligned} \tag{3}$$

where m_k and l_k denote the embedding dimension and time delay set to k^{th} channel, respectively.

Based on a combination of the MA filter (as a scaling process) with multivariate sample entropy, the Modified MMSE (Mod-MMSE) was employed across multiple channels as a scaling process) with multivariate sample entropy. The details of Mod-MSE and Mod-MMSE are given in Algorithm 1.

Algorithm 1. Modified Univariate Multiscale Sample Entropy (Mod-MSE) & Modified Multivariate Multiscale Sample Entropy (Mod-MMSE)

Given a multivariate data set with P channels $\{x_{k,i}\}_{i=1}^{N}$, $1 \le k \le p$, of length N, or a univariate data set with $P = 1$.

1) Standardize the original datasets by subtracting the mean and dividing by the standard deviation for each channel.

2) Scale the normalized datasets, $\{y_{k,i}^{(\tau)}\}_{j=1}^{N-\tau+1}$, for each channel following on (1) and (2).

3) Form the composite delay matrix, $\mathbf{Y}_M(i)$, according to the embedding dimension, M, and the time delay, L, as shown in Eq. (3).

4) Compute the distance between all pairwise composite delay vectors, $Y_M(i)$ and $Y_M(j)$, based on the Chebyshev distance, as $d_M(i,j) = max\{Y_M(i+k) - Y_M(j+k)|| \, i \neq j\}$. The number of matching patterns, $B_M(i)$, is defined as similar pairs of delay vectors that satisfy the criterion $d_M(i,j) \le r$.

5) Compute the estimated local probability of $B_M(i)$ by $C_M(i) = \frac{B_M(i)}{N-n-1}$, where $n = max(M) * max(L)$, and the estimated global probability is $\Phi_M = \frac{\sum_{i=1}^{N-n} C_M(i)}{N-n}$.

6) Repeat steps 1–5 with an increased embedding dimension, $M^* = M + 1$, and obtain the updated global probability, denoted as $\Phi_{M^*} = \frac{\sum_{i=1}^{N-n} C_{M^*}(i)}{N-n}$, $n = max(M^*) * max(L)$.

7) The modified univariate or multivariate multiscale sample entropy is defined as follows:
$Mod\text{-}\,MMSE(m, l, r, N) = -\ln\left[\frac{\Phi_{M^*}}{\Phi_M}\right]$.

Our selection of entropy-based measures for financial stress quantification is driven by several methodological considerations that are directly aligned with our research objectives (García et al. 2016; Yin and Shang 2015; Richman and Moorman 2000; Wu et al. 2018). Unlike traditional volatility measures that assume specific distributions, entropy-based approaches can detect structural complexity changes in nonstationary financial time series without distributional assumptions, which is a crucial advantage when analyzing market behavior across diverse economic regimes.

Empirical testing across multiple financial datasets showed that $m = 2$ provides an optimal discrimination between normal and crisis periods while minimizing computational complexity (García et al. 2016; Richman and Moorman 2000). Hence, the embedding dimension ($m = 2$) was selected. Higher dimensions increased the computational demands without significantly improving the discrimination power. The delay factor ($l = 1$) was chosen to reflect immediate dependencies in daily price movements, which our preliminary analysis showed was most effective for detecting rapid complexity changes during crisis onset.

The scale factor ($\tau = 5$) corresponds to a 1-week trading period, which empirical testing revealed to be optimal for capturing market transitions while filtering daily noise.

Smaller scales showed excessive sensitivity to random fluctuations, whereas larger ones dampened important short-term changes in complexity during crises. This parameter combination maximizes the method's sensitivity to genuine complexity loss during stress periods while maintaining signal fidelity, directly supporting our research objective of distinguishing between normal market fluctuations and crisis-induced structural changes.

Recurrence quantification analysis

The recurrence plot (RP) is a traditional methodology for identifying hidden correlations in multidimensional spaces, without the limitation of data stationarity and size restriction (Webber and Zbilut 1994; Eckmann et al. 1995). The univariate time series, $\{x(i)\}_{i=1}^{N}$, is reconstructed into a phase space according to the optimal embedding dimension, m, and time delay, l, using Takens' embedding theorem (Takens 1981), as follows:

$$x_m(i) = [x(i), x(i+l), \ldots, x(i+(m-1)l)] \tag{4}$$

The optimal combination of the embedding dimension and the time delay can be selected via different methods, such as false nearest neighbors for embedding dimension (Cao 1997) and minimum mutual information for delay factor (Fraser and Swinney 1986). In this study, we choose the joint selection of the optimal embedding dimension and time delay using the differential entropy-based method introduced in Gautama et al. (2003).

Algorithm 2. Recurrence Quantification Analysis (RQA)

Given a univariate data set $\{x(i)\}_{i=1}^{N}$ of length N.

1) Construct the delay vectors (DVs), x_m, according to Takens' embedding theorem, as in Eq. (4).

2) Generate the RP matrix, composed of the pairwise Euclidean distances between delay vectors (DVs), as $RP(i,j) = \Theta(\varepsilon - ||x_m(i) - x_m(j)||$, $i,j = 1, \ldots, N-n-1$, $i \neq j$, where $||\cdot||$ designates the Euclidean distance; $\Theta(\cdot)$ refers to the Heaviside function; ε denotes the threshold when defining the similarity between DVs, set as 60% of the mean Euclidean distance of the DVs; and $n = (m-1) * l$.

3) The DET can be calculated as the percentage of recurrence points that form diagonal lines in the RP matrix, that is, $DET = \frac{\sum_{j=j_{min}}^{N-n-1} j \cdot P(j)}{\sum_{j=1}^{N-n-1} j \cdot P(j)}$ where $P(j)$ is the number of diagonal lines of length j and j_{min} is the minimum number of points to be considered as a diagonal line, set as $j_{min} = 2$.

The outcome of RP is a matrix summarizing the distance between the delay vectors (DVs). Given a threshold, ε, the RP matrix can be plotted as a gray image. Every element in the matrix is converted into a pixel color based on the relation between ε and the distances between DVs. Several probabilistic measures can be implemented according to the RP matrix, such as DET and laminarity (LAM). Both measures can indicate the inverse of the volatility, where DET is the percentage of recurrent points forming diagonal line structures and LAM is the percentage forming vertical lines (Strozzi et al. 2008). Volatility is an important property in financial markets that reflects implicit risk and is generally referred to as the degree of uncertainty about the future price (Pincus and Kalman 2004); hence, it also reflects the degree of predictability. Here, we employed the DET index in RQA, where the length of a diagonal line in RP reflects the number of consecutive recurrent states. Considering the inverse relation of DET and volatility, we expect that high determinism refers to the high predictability of future prices, representing the "abnormal" situation in the financial market (Schreiber 1999). Therefore, the change in DET is positively related to the stress level of the estimated index. Algorithm 2 outlines the process of DET calculation.

Assessment of latent index of stress with instantaneous amplitude (iA-ALIS)

ALIS was proposed to quantify the "stress level of a financial organism" in Hemakom et al. (2016). The original ALIS used the low- and high-frequency band power of the detrended price after the MA filter. The low-frequency band is considered to occupy the frequency band below 0.0042 Hz ($=\frac{1}{240}$) with a time window of one year by considering the intrinsic properties of financial data. By contrast, the high-frequency band is set between 0.0167 Hz ($=\frac{1}{60}$) and 0.2 Hz ($=\frac{1}{5}$), corresponding to 2 months and 5 days, respectively. Recently, ALIS has been enhanced with correct signal power estimation by employing instantaneous amplitude via the Hilbert transform, as discussed in von Rosenberg et al. (2017). Therefore, we employed the ALIS with Instantaneous Amplitude, iA-ALIS, based on the detrended financial time series. The higher the iA-ALIS index, the more stressful the stock, with the threshold between stressed and normal derived based on the median value. The details of iA-ALIS are summarized in Algorithm 3.

Algorithm 3. Assessment of Latent Index of Stress with Instantaneous Amplitude (iA-ALIS)

Given a univariate data set $\{x(i)\}_{i=1}^{N}$ of length *N*.

1) Remove the trend of the data by a MA filter with a window of 1 year, and obtain the detrended data, $\{z(i)\}_{i=1}^{N}$.

2) Bandpass-filter the detrended data, *z*(*i*), into the low-frequency Band and high-frequency bands.

3) Apply the Hilbert transform to low-frequency and high-frequency, and obtain the instantaneous amplitude based on the analytic signals given by the Hilbert transform at every time point, denoted as iA_{LF} and iA_{HF}.

4) Take 4 years as the window length and 1 month as an increment. In every time window, the 20% largest and smallest values are excluded to remove the outliers and then calculate the mean *iA* for every window, denoted as *LF*(*d*) and *HF*(*d*), where *d* refers to a month.

5) Normalize the *LF*(*d*) and *HF*(*d*) series by subtracting the mean and dividing by the standard deviation to alleviate the scaling problem.

6) Remove the offset of *LF*(*d*) and *HF*(*d*).

7) The ALIS is given by $ALIS(d) = LF(d) + HF(d)$

8) The median value in the *ALIS*(*d*) is a market stress threshold.

Data overview and methods

We applied several methodologies to four groups of indices or stocks over the last 31 years, spanning the period between 1991.01.01 and 2021.12.31:

- Stock market index
  - Dow Jones Industrial Average (DJIA/DOW): 30 large companies;
  - NASDAQ Composite (NAS): Mid- and large-caps;
  - Russell 2000 (RUS): Smaller companies;
  - Standard & Poor's 500 (SNP): 500 large companies.
- Price of the metal
  - Gold (Au);
  - Silver (Ag);
  - Copper (Cu);
  - Platinum (Pt).
- Equity
  - Apple Inc.: a large technology company;
  - Microsoft: a large technology company;
  - McDonald's: Fast food company;
- Currency
  - EUR-GBP;
  - GBP-JPY;
  - GBP-USD;

- American International Group (AIG): Insurance company.	- USD-JPY.

Figure 1 provides an overall view and exhibits the original price in the upper panels and the detrended price in the bottom panels of the six indices or stocks. The detrended price in blue in each figure was produced by a MA filter at scale = 5, where the main signals involved in the following analyses were the detrended dynamical time series. The price becomes more dynamic when a crisis arises, as can be observed in the detrended signals with the removal of the local mean. In Fig. 1, the SNP and DJIA/DOW are collections of large companies in the US market. The NASDAQ and Apple Inc. illustrate the performance of the technology industry, where Apple Inc. is the largest stock in the NAS index. Furthermore, the bottom panels in Fig. 1 represent the food industry (McDonald's) and real estate (AIG Insurance), respectively.

In the past 30 years, seven consecutive periods of different natures were identified based on the world economies (Ofek and Richardson 2003; Demyanyk and Van Hemert 2011; Alabdullah et al. 2020; Baker et al. 2020) and are marked at the top of each figure:

1) Economic boom/Dot-com bubble: 1997.01.01 to 1999.12.31.
2) Internet bubble burst (Crisis-1): 2000.01.01 to 2003.12.31.
3) Economic recovery: 2004.01.01 to 2007.12.31.
4) Subprime mortgage crisis (Crisis-2): 2008.01.01 to 2011.12.31.
5) Post-global financial crisis (GFC) recovery: 2012.01.01 to 2014.12.31.
6) Bull run: 2015.01.01 to 2019.12.31.
7) COVID pandemic (Crisis-3): 2020.01.01 to 2021.12.31.

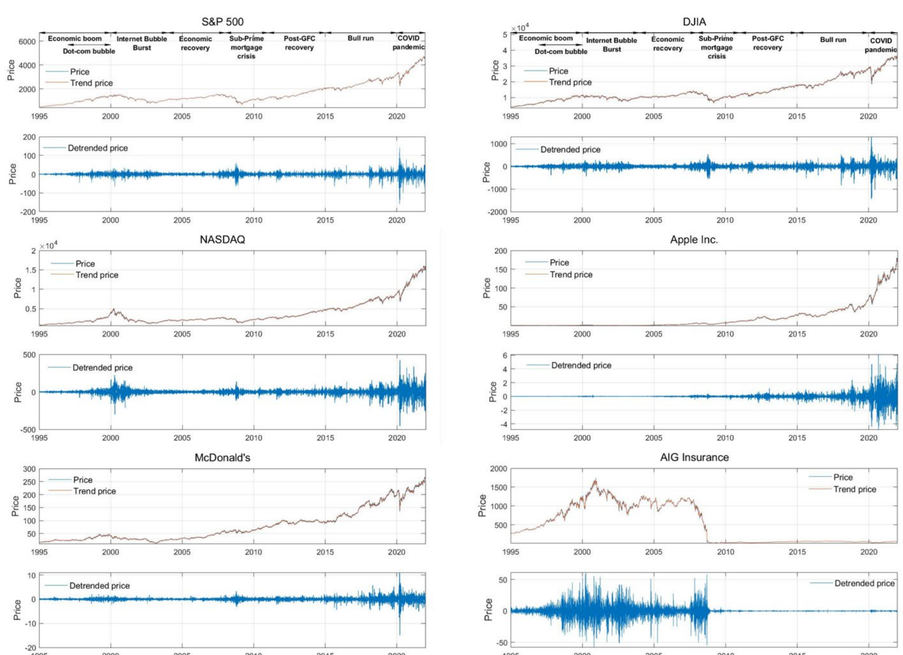

Fig. 1 Exemplary price time series and their detrended dynamical signals over 1995-2021

The selected financial episodes span over 30 years and reflect various economic conditions that have influenced global markets. The Dot-com bubble (1997–1999) was a period of rapid growth in the technology sector. This development was driven by speculative investments in Internet-based companies. This phenomenon was followed by the Internet bubble burst (2000–2003), during which many overvalued tech stocks collapsed. The economic recovery (2004–2007) marked a period of stable growth driven by rising housing prices and increased credit availability. However, this event was followed by the subprime mortgage crisis (2008–2011), which triggered the global financial crisis, leading to severe disruptions in global financial markets. The Post-GFC recovery (2012–2014) was characterized by gradual stabilization and financial reforms to mitigate systemic risks. This period was followed by a bull run (2015–2019), driven by strong corporate earnings and low-interest-rate environments. Finally, the COVID-19 pandemic (2020–2021) caused unprecedented global economic disruptions, leading to sharp market declines and heightened volatility across all sectors. These varied episodes enable a comprehensive assessment of the robustness and sensitivity of the proposed indicators across different types of financial stress events. Different crises show varying influences on different industries, which is reflected in the following complexity analysis. Apart from the impact of the crisis, the trend of prices generally increases as time goes by, except for the AIG insurance, which was severely attacked by the subprime mortgage crisis around 2008.

The length of 1 year contains 261 points owing to the properties of the price time series (one data point per weekday). The analyses for the univariate time series include Mod-MSE, RQA, and iA-ALIS. The multivariate time series analysis was generated using Mod-MMSE. Furthermore, from the Mod-MSE and Mod-MMSE analyses, the catastrophe analyses examine the performance of individual price series and the overall US financial market. Figure 2 shows the analysis framework using the algorithms presented in Section Algorithm and methods.

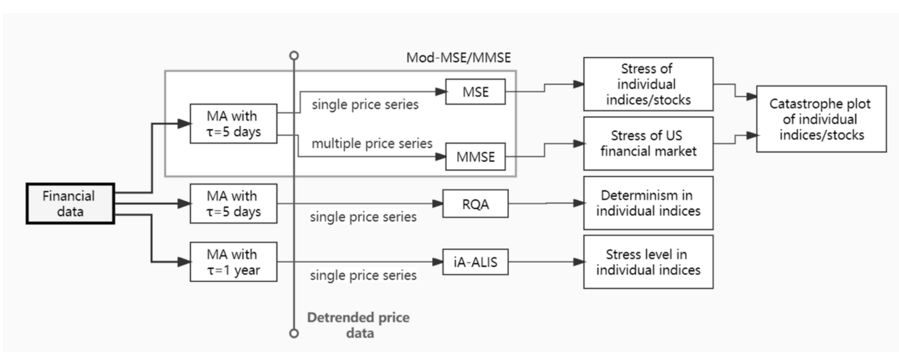

Fig. 2 The employed analysis framework illustrating the data processing flow. First, financial data are detrended using Moving Average (MA) filters at different time scales. The detrended data then undergoes three parallel analytical processes: (1) Modified Multiscale Sample Entropy (Mod-MSE/MMSE) for both individual assets and the entire market, (2) Recurrence Quantification Analysis (RQA) to assess determinism in individual indices, and (3) Assessment of Latent Index of Stress (iA-ALIS) to evaluate stress levels. The Mod-MSE and MMSE results were then used to construct Catastrophe plots that visualized the relationship between market-wide stress and individual asset performance

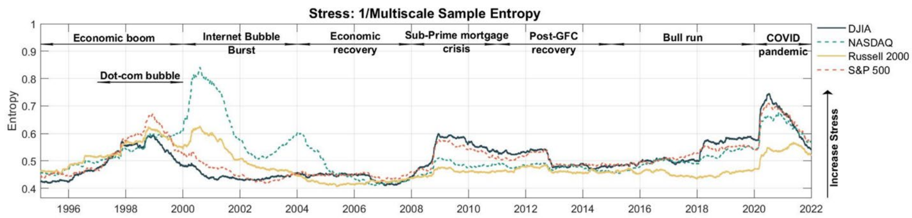

Fig. 3 Financial stress of four indices (DJIA, NASDAQ, Russell 2000 and S&P 500) estimated by Mod-MSE over 1995-2022

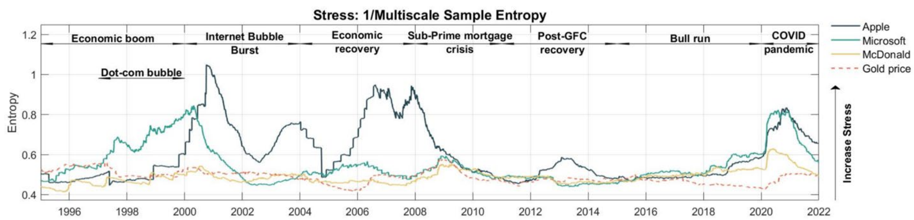

Fig. 4 Financial stress of four leading equities in their sector (Apple Inc., Microsoft, McDonald's and Gold price), estimated by Mod-MSE over 1995-2022

Results and analysis

Modified univariate multiscale sample entropy (Mod-MSE) and modified multivariate multiscale sample entropy (Mod-MMSE)

The different settings of the predefined scale factors, τ, have been discussed in Hemakom et al. (2016), where the various values exhibited the same trend. Here, the short-term MA filter with $\tau = 5$ was selected as it provided the most distinct tracking of the financial stress evolution. The analysis window was set to $N = 1044$ (261 points$\times$4 years) with a 1-day increment. Therefore, considering data from 1991, the complexity plot started from 1995, whereby each entropy value was calculated based on the historical price in the past 4 years. The default parameters of Mod-MSE/Mod-MMSE were set to the embedding dimension $m = 2$ and the delay factor $l = 1$.

Figures 3 and 4 show the complexity of the four indices and four equities considered via Mod-MSE, respectively. The figures show the reciprocal of Mod-MSE, representing the level of stress according to the complexity-loss hypothesis. The increase of Mod-MSE indicated higher randomness referring to a normal period and a decrease in stress when the dynamics of price were balanced and influenced by multiple factors. The complexity plots in Fig. 3 reveal significant insights into market dynamics during different economic periods. During normal periods (economic recovery, post-GFC recovery, bull run), higher MSE values (i.e., lower stress) indicate greater randomness in price movements, which aligns with the efficient market hypothesis where prices incorporate all available information. A sharp decrease in entropy (i.e., increase in stress) during crisis periods (2000–2003, 2008–2011, 2020–2021) quantifies the systematic loss of complexity that characterizes financial stress.

Notably, NASDAQ (green line) exhibits the most dramatic response to the Internet bubble burst, showing a peak value in 2000, followed by a steep decline in 2002, indicating a stress reduction. This event significantly exceeds its response to the subprime crisis, numerically confirming the technology sector's greater vulnerability to the earlier crisis. Conversely, Russell 2000 (yellow line) showed a more gradual stress response but maintained elevated stress levels for longer periods, particularly during the recovery phases. For instance, although S&P 500 and DJIA returned to lower stress levels within approximately 12–18 months after the 2008 crisis, the Russell 2000 maintained higher stress levels until mid-2011, providing quantitative evidence of the diminished resilience of small-cap companies to system-wide shocks.

Figure 4 depicts the distinct stress profiles of individual equities across different market conditions. Gold prices (dashed line) demonstrate remarkable stability, maintaining stress values between 0.4 and 0.5 throughout most of the 27 years, with minimal response even during the COVID-19 pandemic. This quantitatively supports gold's status as a safe-haven asset. Apple Inc. (black line) shows pronounced volatility in the company's stress levels, with dramatic peaks reaching above 1.0 during the early 2000 s and around 2008, indicating periods of extreme structural change in the company's price dynamics. These peaks align with major product innovations and business model transitions, indicating that internal corporate developments can generate complexity patterns comparable to market-wide crises. McDonald's (yellow line) exhibits a more similar stress profile to gold than to technology stocks, reflecting its status as a consumer staples company with relatively stable demand patterns regardless of broader economic conditions.

As shown in Figs. 3 and 4, entropy analyses based on individual indices or stocks can assess the internal stress from the signal dynamics. Although S&P 500 and DJIA are generally considered the leading indices, single-channel analysis is suboptimal for evaluating the overall performance of the US financial market. To this end, we applied the multivariate Mod-MSE with the four representative indices in Fig. 3 as multichannel data. The Mod-MSE of the leading index (S&P 500) is jointly plotted with Mod-MMSE in Fig. 5 to visualize the advantages of multivariate analysis. Figure 5 reveals critical insights about system-wide stress as measured by multivariate entropy compared with individual index stress. The multivariate measure (black line) shows dramatically higher stress levels during crisis periods than the univariate S&P 500 measure (dashed red line). This quantitative difference demonstrates that the loss of system-wide complexity can be significantly greater than that observed in any single index.

The temporal evolution of stress also differs between measures. The multivariate stress began increasing approximately 4–6 months before significant changes appear in the S&P 500 stress during the Dot-com bubble (late 1998) and the subprime crisis

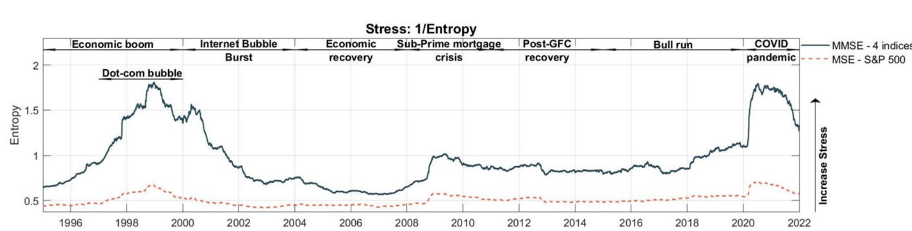

Fig. 5 Financial stress of US market estimated by Mod-MSE and Mod-MMSE over 1995-2022

(mid-2007), providing empirical evidence for the multivariate analysis's early warning capability. Additionally, the rate of stress increase differed substantially, with multivariate stress increasing at approximately twice the rate of univariate stress during crisis onset. These differences underscore the importance of considering market-wide complexity changes rather than relying solely on individual index analysis when assessing financial system stability.

Recurrence quantification analysis

Next, we applied the RQA to yield the DET of each index, which is another quantitative way to evaluate the stress level of the financial market. Recall that the EMH indicates that stock prices behave in a random (uncertain) way during normal financial regimes (Strozzi et al. 2008). The lower the determinism, the more stochastic components the system contains, referring to a normal situation in line with the high randomness in the entropy analysis. Therefore, the DET metric is consistent with the stress level of the index or stock; that is, we expect low determinism in normal situations and high determinism during financial crises.

In the analysis, the predefined scale factor, τ, was set to 5 days and the window for RQA to 4 years. The optimal combination of the embedding dimension, m, and the delay parameter, l, was jointly selected using the DE-based method, proposed in Gautama et al. (2003).

The determinism values in Fig. 6 provide complementary insights to the entropy analysis. Entropy measures randomness, whereas DET quantifies predictability in the time series. During crisis periods, all indices show elevated DET values, with peaks during the COVID-19 pandemic, compared with values during stable periods such as 2014–2016. This dramatic contrast numerically demonstrates how market behavior shifts from being unpredictable (aligned with efficient market theory) to being highly deterministic during periods of stress.

The indices exhibit distinct DET signatures: NASDAQ (green line) shows a peak DET of approximately 0.6 during the Internet bubble, but only reaches approximately 0.3 during the subprime crisis, reinforcing our entropy-based findings about sector-specific vulnerability. The delayed response pattern of Russell 2000 is particularly evident in the DET analysis, where its determinism increases approximately 3–4 months after the other indices during each crisis onset and remains elevated for an average of 5–7 months longer during recovery periods. This quantifies the distinct

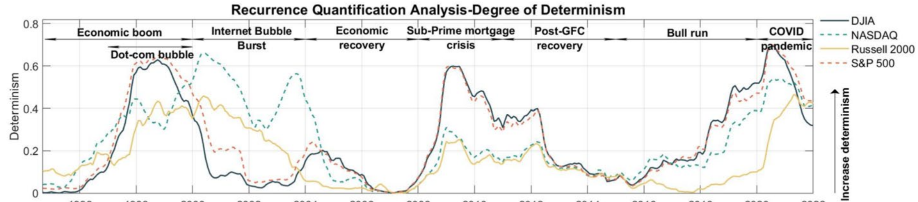

Fig. 6 Degree of determinism of four indices (DJIA, NASDAQ, Russell 2000 and S&P 500), estimated by RQA over 1995-2022

stress response characteristics of small-cap markets, potentially reflecting their lower liquidity and information flow compared with large-cap indices.

Assessment of latent index of stress with instantaneous amplitude (iA-ALIS)

The third methodology used to estimate the stress level of financial indices or stocks is the iA-ALIS (von Rosenberg et al. 2017). The ALIS was originally proposed to examine the "economic organism" through the complexity-loss hypothesis, where a high stress level is indicated by the high value of ALIS (Hemakom et al. 2016). Here, we applied the enhanced iA-ALIS, whereby when estimating the "power" of the low-frequency band and high-frequency band, the instantaneous amplitude is used in place of the absolute power. Following (Hemakom et al. 2016), a 4-year sliding window was employed with a 1-day increment, and the detrended data were given by MA filter on a 1-year scale.

Figure 7 shows the stress levels of every index considered (DJIA, NASDAQ, Russell 2000, and S&P 500) given by iA-ALIS. The black dashed line at the bottom represents the threshold, which is the indices' median value over time. Notably, the iA-ALIS exhibited substantially high levels during the Internet bubble burst and the subprime mortgage crisis. However, considering the impact of the COVID-19 pandemic, the dramatically high stress caused by the pandemic among all industries rendered the measures of the previous two crises less significant when visualizing. During the Internet bubble burst, the NASDAQ demonstrated a higher stress level than the other three indices, as expected, which is supported by the Mod-MSE and RQA analyses.

Moreover, the problem with iA-ALIS is observed, whereby the highly dynamic changes in daily price were difficult to distinguish. With the same resolution, Mod-MSE in Fig. 3 and RQA in Fig. 6 were able to give more details of the stress evolution, whereas the iA-ALIS measure, as shown in Fig. 7, smoothly evaluated the stress levels. Although the periods of crisis could be marked by iA-ALIS in 2000, 2008, and 2020, limited information can be observed from iA-ALIS for further analysis.

Catastrophe plots based on entropy

The complexity theory indicates that all aspects of complex problems have the characteristics of catastrophes (Yang et al. 2009). Catastrophe theory is a branch of applied mathematics that was developed by Rene Thom in the late 1960 s (Thom 1972), whereby the basic idea of catastrophe theory aims to explain the breakdown of relationships in variables of a dynamic system (Stewart and Peregoy 1983). More specifically, this theory can potentially describe the ability of a smooth change in system parameters to generate

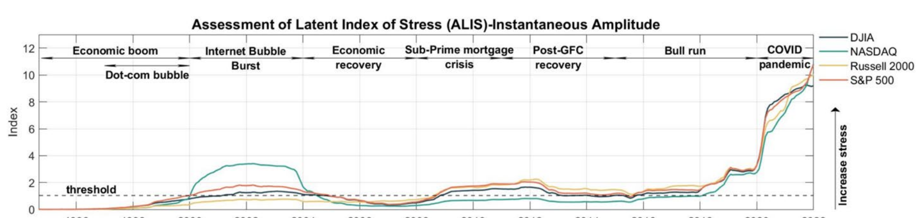

Fig. 7 Financial stress of four indices (DJIA, NASDAQ, Russell 2000 and S&P 500) estimated by iA-ALIS over 1995-2022

catastrophic behaviors (i.e., abrupt, discontinuous, sudden change) in a dependent variable, termed critical points (Stewart and Peregoy 1983; Wesselbaum 2017; Birău 2013; Baruník and Vosvrda 2009). Catastrophe theory can describe all aspects of natural phenomena because of their complex properties; it embodies a theory of great generality that is perceived as a state of mind (Zeeman 1979; Birău 2013).

In the realm of physiology, Hardy and Fazey (1987) stated that physiological arousal is related to performance in an inverted-U hypothesis when the athlete is not worried or has low cognitive anxiety. If cognitive anxiety is high, the increases in arousal pass a point of optimal arousal and a rapid decline in performance occurs (McNally 2002). Catastrophe theory has also been involved in brain modeling (Zeeman 1973); however, the catastrophe model remains in its conceptual framework state without computational analysis (McNally 2002). Catastrophe theory represents a unique hypothesis made up of different mathematical structures in financial applications, in contrast to the EMH (Birău 2013). Indeed, catastrophe theory has been tentatively employed to explain discontinuous jumps in bank investment (Wesselbaum 2017). Studies have shown that it could better explain the stock exchange crash than other models (Baruník and Vosvrda 2009). However, few studies have used catastrophe theory in economics, with most relying on qualitative descriptions rather than quantitative applications (Wesselbaum 2017; Baruník and Vosvrda 2009). Considering the high complexity and unpredictability of the stock market and its chaotic and uncertain behaviors, catastrophe theory can explain the occurrence of financial events (Birău 2013). To this end, we propose a practical framework for the application of catastrophe theory based on entropy-based complexity estimation.

Arousal–performance index plot

Considering the similarity between the financial and physiological systems, an analogy can be drawn between the catastrophe theory applied to athlete performance and index or stock performance. The catastrophe plot in physiological systems reflects the relationship between physiological arousal (anxiety) and performance. Accordingly, in the financial market, we observe the performance of individual indices or stocks evaluated by Mod-MSE; that is, the higher the stress level given by lower Mod-MSE, the lower the performance. Although the arousal in the physiological system is an internal trigger that determines performance, we modeled the dependent factor (arousal) as the external stress imposed by the external environment on the whole market and quantified by the Mod-MMSE because of the intrinsic property of a financial system. Therefore, the financial market catastrophe framework describes the relationship between the level of overall market anxiety (external stress) and the performance of individual indices or stocks (internal stress).

Two indices (i.e., S& P 500 and NASDAQ), three individual stocks in different industries (i.e., Apple Inc., AIG, and McDonald's), and the price of gold (AU) were selected to demonstrate the arousal–performance plot, as shown in Fig. 8. Good performance is reflected in low internal stress, indicated as high sample entropy, whereas strong arousal or stimulus is designated as high external stress, given by the reciprocal of multiscale multivariate sample entropy, as shown in Fig. 5. The line plots in Fig. 8 depict a relationship between the performance of each index or individual stock and external stimuli

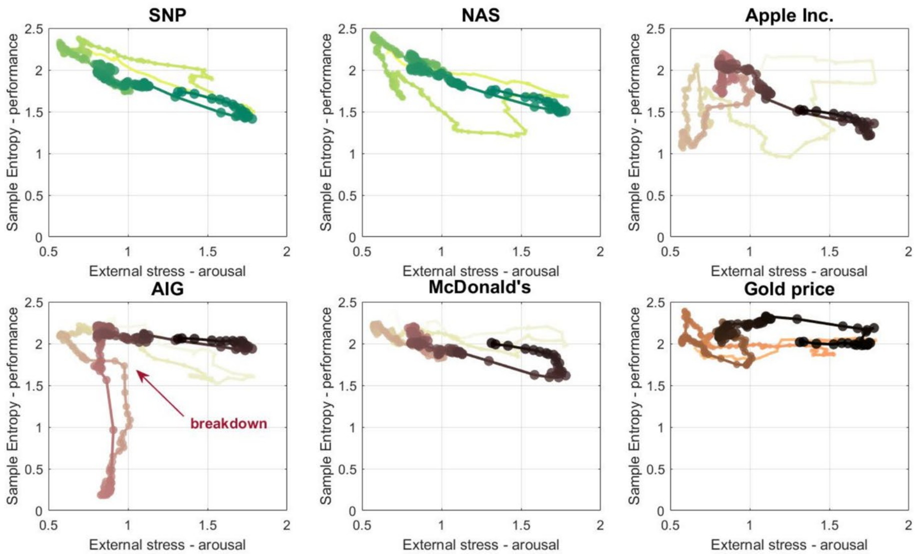

Fig. 8 Evaluation of Catastrophe Plots of 6 indices (S&P 500, NASDAQ, Apple Inc., AIG, McDonald's and gold price) over the period 1995-2022. The plots are color-coded, starting with the lightest shade (early years) and ending with the darkest shade (recent years). The y-axis represents the individual asset performance measured by Sample Entropy, where higher values indicate better performance (lower stress). The x-axis shows the external market stress (arousal) measured by the MMSE reciprocal, where higher values indicate increased market-wide stress. The slope of each curve reflects the asset's sensitivity to market-wide stress - steeper negative slopes indicate greater vulnerability, whereas flatter trajectories demonstrate resilience. Note the dramatic breakdown point in AIG's plot, which represents a catastrophic performance collapse during the 2008 financial crisis

during the entire 27 years (1995–2022), where light colors denote early years and dark colors represent late years; all the plots were adjusted to the same axis scale.

The catastrophe plots of the two indices (i.e., S&P 500 and NASDAQ) are shown in the first two panels in Fig. 8. As a collection of large companies, S&P 500 and NASDAQ exhibit a similar tendency of performance increase as external stress decreases. Observe that the curve of S&P 500 tends toward more of a linear relationship compared with the curve of the NASDAQ, showing the higher predictability or regularity of S&P 500. The larger slope of the NASDAQ curve in light green indicates that the technology market was under higher stress (with suboptimal performance) in the early years than in recent years.

Next, the curve of the most significant stock in NASDAQ, Apple Inc., is in line with the change of NASDAQ in recent years, as shown in dark colors. The reason is the leading role of Apple Inc. in the technology market in the United States. As for AIG, the only individual stock that has experienced a catastrophic change among the given indices is plotted in the first graph of the second row in Fig. 8. In the early and late years, AIG exhibits a relatively stable response to the overall external stress. The sharp drop in the middle years (during the subprime mortgage crisis) reflects the sharp increase in the internal stress of the individual stock in response to a small change in external stress. Therefore, AIG equity exhibited two critical points in the catastrophe plot: (i) the lowest performance or highest stress level that the stock could sustain; and (ii) the highest performance point that could bring the equity back to a normal state. Note that the recovery point is higher than the breakdown point. In terms of highly robust stocks, such as

McDonald's and the gold price, both exhibit a relatively flat relationship between arousal and performance. The flat curves show the highly stable performance of the equity or index under large changes in external stress. McDonald's stock indicates a decrease in stability in recent years compared with the early years, as evidenced by the more inclined tendency, whereas the price of gold maintains a high performance without apparent influence from the stress caused by environmental change.

Arousal–performance plot of crisis index

Next, we extracted the three specific 2-year periods of crisis from the previous catastrophe plot to discuss the response of each index or individual stock to different crises. These are the Internet bubble burst between 2002 and 2002 in green, the subprime mortgage crisis between 2008 and 2010 in brown, and the COVID-19 pandemic between 2020 and 2022 in red. We used the gradient color to indicate the direction of each segment, where the lightest color refers to the start of the selected time. The selected catastrophe plots are illustrated in Fig. 9.

According to the catastrophe theory, with a fixed increase in the external stress (arousal), a good response should be shown as a small decrease or no decrease in sample entropy (performance). Therefore, the slope of the lines could quantitatively reflect the overall performance of the index in response to different crises. Three indices and three individual stocks are examined in Fig. 9, with the same axis scale. The index in the first

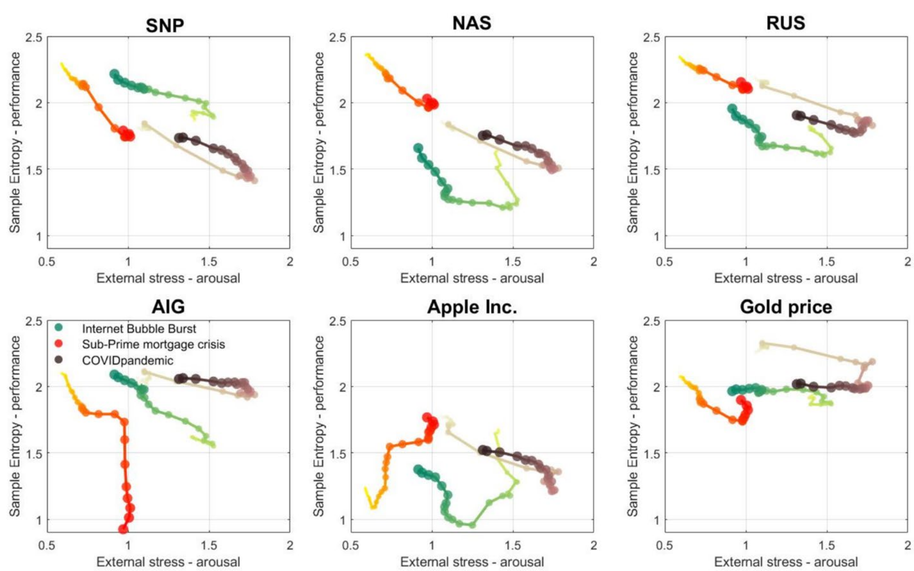

Fig. 9 Catastrophe Plots of 6 indices (S&P 500, NASDAQ, Russell 2000, AIG, Apple Inc., and gold price) during three crises (Internet Bubble Burst in green, Sub-Prime mortgage crisis in red, and COVID pandemic in dark gray). Each segment is color-coded, starting from the lightest shade and ending in the darkest shade for 2 years. The y-axis shows Sample Entropy (performance), where higher values indicate better performance, and the x-axis represents the external market stress (arousal). These plots reveal distinct crisis response signatures: S&P 500 shows varying degrees of resilience across crises, with the steepest decline during the subprime crisis; the NASDAQ demonstrates extreme vulnerability during the Internet Bubble; AIG exhibits a catastrophic vertical trajectory during the subprime crisis; Apple shows positive, countercyclical behavior during the Sub-Prime crisis with a positive slope; and the gold price maintains relatively flat trajectories across all crises, confirming its safe-haven status. The varying slopes across assets and crises illustrate how different market segments exhibit unique sensitivity patterns to different types of financial stress

panel indicated that S&P 500 showed stable responses to all three crises with different degrees of robustness. Among the selected segments, the subprime mortgage crisis had the largest impact on the S&P 500 index (in red) with a sharp decrease in performance. This difference indicates that the subprime crisis was more impactful on broader market indices than the Internet bubble.

NASDAQ exhibits distinctly different patterns across the three crises, with the Internet bubble burst demonstrating a significantly steep negative slope. This case confirms that technology-dominated indices suffered a greater stress impact during the tech bubble burst than during other crises. Russell 2000 displays more horizontal trajectories during crisis recovery phases, particularly after the Internet bubble burst, where its slope flattens as the crisis evolves. Thus, smaller companies employed more rapid adaptation strategies during that crisis compared with during the COVID-19 pandemic, where the slope remains consistently steep throughout.

In terms of the equities in the second row in Fig. 9, in line with the plot in Fig. 8, the catastrophe plot of the insurance company, AIG, exhibits a dramatic breaking point during the subprime mortgage crisis, with an almost vertical performance collapse as external stress increased marginally. This phenomenon represents a catastrophic phase transition in which performance drops—a reduction triggered by just a small increase in the external stress. During the COVID-19 pandemic, AIG reverted back to a stable state, showing a modest negative slope compared with other indices. As for Apple Inc. stock, the nondecreasing tendency during the subprime mortgage crisis (in red) demonstrated the limited influence of specific stimuli, with a slightly positive slope indicating performance resilience or even improvement despite increasing market stress. The most significant crisis for Apple Inc. was the Internet bubble burst (in green), as expected, with a steep downward trajectory representing greater performance sensitivity than that during the COVID-19 pandemic. The gold price is given in the last panel. As the most stable index, the gold price has been at a relatively high level throughout the three crises, with remarkably flat trajectories. This case demonstrates its safe-haven properties with less sensitivity to market stress than typical equity indices.

Arousal–performance plot of crisis

Finally, we selected the arousal–performance plots of five indices or individual stocks in each of the crises to compare the performance of indices or individual stocks in each of the crises. Figure 10 provides a crisis-centric view that enables a direct comparison of how different assets respond to the same type of crisis. The left panel shows the performance during the Internet bubble burst between 2000 and 2002. The two most stable indices are the S&P 500 and gold prices (in blue and red, respectively) with flattening tendencies and shallow slopes. These contrast sharply with technology-focused assets, where NASDAQ and Apple Inc. (in brown and gray) exhibit much steeper trajectories. This performance gap is significant—Apple's entropy values drop sharply over the same stress range where S&P 500 decreases moderately.

The subprime mortgage crisis (middle panel) presents a markedly different pattern across assets. The AIG (in green) exhibits the most dramatic stress response, with a near-vertical trajectory representing a catastrophic performance collapse as the stress approaches 1.0. This visualization visualizes the company's entropy reduction occurring

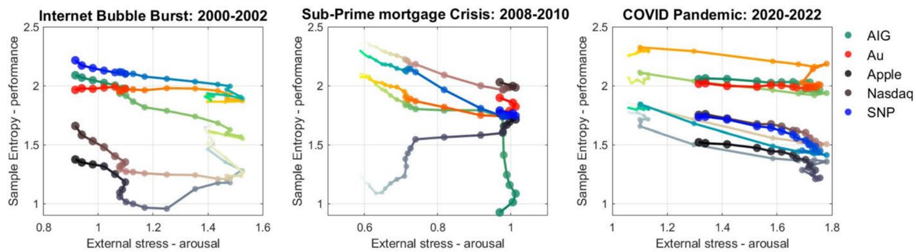

Fig. 10 Catastrophe Plots of three 2-year crises periods (Internet Bubble Burst 2000-2002, Sub-Prime mortgage crisis 2008-2010, and COVID pandemic 2020-2022) for selected 5 indices (S&P 500, NASDAQ, Apple Inc., AIG and gold price). Each segment is color-coded, starting from the lightest shade (beginning of crisis period) and ending in the darkest shade (end of crisis period). Comparing these plots reveals how different assets respond to specific types of crises: technology assets (NASDAQ, Apple) show steeper declines in performance during the Internet Bubble, AIG exhibits a catastrophic vertical drop during the Sub-Prime crisis, and the COVID-19 pandemic produced more uniform stress responses across all assets except gold. The different x-axis ranges across the panels highlight the varying levels of system-wide stress experienced during each crisis

over a minimal stress increase of a small unit—a quintessential example of catastrophe behavior where small parameter changes trigger system-wide reorganization. The remaining stocks showed similar moderate slopes, except for Apple Inc., which exhibited a counter-intuitive positive slope, indicating that performance increases with external stress. This unique positive correlation indicates that the company's fundamental business model operated independently from, or even benefited from, the factors driving the housing and financial crisis.

The COVID-19 pandemic (right panel) presents the most uniform stress response pattern across the entire 31-year study period. Except for gold, which maintains its characteristically flat profile, all indices or stocks display remarkably similar trajectories with negative slopes clustering within a narrow range. This unprecedented uniformity is further evidenced by the parallel nature of the trajectories and the similar magnitudes of ER across all equities. The stress range is also distinctive, spanning from 1 to 1.8, the widest range observed among all three crises. These patterns confirm that the pandemic represented a truly systemic shock affecting diverse sectors through similar mechanisms. On the contrary, previous crises displayed more heterogeneous impacts across different market segments. The highly aligned tendencies of all indices emphasize the uniquely pervasive influence of the pandemic across all industries in the US financial market.

Connecting omplexity measures to catastrophe theory framework

As presented in Section Results and analysis, the entropy-based complexity measures provide the quantitative foundation for our catastrophe theory framework. This connection operates through direct mappings between our analytical approaches: Mod-MSE values quantify individual asset performance (y-axis in catastrophe plots), whereas the reciprocal of Mod-MMSE represents system-wide stress (x-axis). These mappings create a coherent methodological bridge that transforms the complexity analysis into visual representations of the stress–performance relationships.

The alignment between our entropy findings and catastrophe plots can be shown through specific examples. The AIG's breaking point in Fig. 9 corresponds precisely to its period of maximum entropy reduction identified in our univariate analysis. Similarly, NASDAQ's steeper trajectory during the Internet bubble compared with other crises directly reflects its larger entropy reduction during that period, as shown in Fig. 3.

Our catastrophe theory framework is in line with and extends recent findings in financial complexity research. The differential response patterns identified across market sectors support findings on sector-specific complexity dynamics during crises, where significant heterogeneity in entropy measures was observed across different economic sectors (Caraiani 2014). The breaking points observed in our catastrophe plots empirically support the Dragon–King theory of financial crashes. This theory proposes that certain extreme events follow different generating mechanisms than normal market fluctuations (Sornette 2009).

Furthermore, the disproportionate complexity loss in specific assets during crises found in this study correspond with the power law analyses of market phase transitions in Yalamova and McKelvey (2011). Their identification of critical transition points in financial time series using power law distributions aligns with our catastrophe theory approach, although our entropy-based methodology provides a more direct quantification of changes in structural complexity.

The RQA results in Section Results and analysis provide complementary evidence supporting the proposed catastrophe framework. Assets exhibiting high determinism (DET) values during crises, such as the extended recovery periods of the Russell 2000 in Fig. 6, manifest as distinctive trajectory patterns in the catastrophe plots. This notion aligns with the findings that recurrence quantification metrics can effectively identify regime shifts in financial markets (Addo et al. 2013).

The practical implications of our findings extend beyond theoretical validation. Battiston et al. (2016) demonstrated that early warning signals derived from complexity measures can enhance regulatory stress testing frameworks. Our catastrophe theory approach has similar potential for identifying system vulnerabilities before they manifest as full-blown crises, particularly by identifying assets that display steep performance–stress slopes in their catastrophe plots.

By establishing these connections, we create a comprehensive analytical chain from entropy calculation to determinism quantification to catastrophe visualization. This integrated approach facilitates a more nuanced interpretation of financial stress dynamics than would have been possible through any single methodology in isolation. This result supports the assertion that MFMs are essential for capturing the complex dynamics of financial markets (Mandelbrot 1999).

Conclusion

We estimated financial stress from a nonlinear dynamics viewpoint and examined the significance of structural complexity features in financial analysis. This was achieved for four stock market indices and four individual equities from 1991 to 2021. Financial stress was estimated based on Mod-MSE in univariate and multivariate cases. In addition, the univariate RQA and iA-ALIS methods give the DET and stress change, in line with the entropy-based analysis. All three nonlinear approaches have demonstrated their ability to quantify financial stress, where multivariate entropy is the most information-rich and physically meaningful.

Furthermore, a novel framework based on the catastrophe theory has been proposed, where arousal–performance plots have been employed to visualize the response of each financial index or stock. We have adopted Mod-MMSE of four major indices as a metric of external arousal and Mod-MSE of each index or individual stock as a performance metric. The analysis demonstrates that the same crisis triggers different performance changes in various industries and that the same index or equity exhibits various robustness to different types of crises (the Internet bubble burst, subprime mortgage crisis, and COVID-19 pandemic crisis). Finally, the performance of index or individual stock has been qualitatively and quantitatively explored through catastrophe plots.

Although the current analysis concludes in 2021, the timeline gap arises from the extended revision process. We refer to recent studies that explored financial market dynamics from 2022 to 2024 to contextualize our findings. This period has been characterized by several significant developments, including post-COVID-19 inflationary pressures, central bank monetary tightening, geopolitical instability, and notable financial sector stress events, such as the collapse of the Silicon Valley Bank in March 2023. These events continue to test market resilience and demonstrate the relevance of financial stress measurement frameworks. Recent studies have examined these phenomena through analyses of stock market volatility following the COVID-19 pandemic (Khan et al. 2024), gold's performance as a safe-haven asset in turbulent markets (Dammak et al. 2024), and advancements in financial stress measurement (Chavleishvili and Kremer 2023). The entropy-based measures developed in this study could be particularly valuable for analyzing how these recent shocks propagate through different market segments, particularly as digitalization and fintech innovations continue to transform financial market structures and transmission mechanisms. In addition, recent developments in fintech highlight the potential for artificial intelligence algorithms and cloud computing technologies to optimize financial services and enhance stress quantification frameworks (Lăzăroiu et al. 2023). Customer-centric approaches in fintech further emphasize the importance of innovative service delivery and customer loyalty in financial stability (Barbu et al. 2021), whereas Internet of Things–based big data management demonstrates the transformative potential of automated, data-driven processes for financial stress measurement, and operational efficiency (Andronie et al. 2023). These advancements confirm the ongoing relevance of the methods and conclusions presented in this study. Moreover, such advancements point toward future opportunities for extending this framework with updated datasets to assess stress under post-pandemic conditions and in digitally transformed financial environments.

However, this study has limitations that must be acknowledged. While our framework provides a novel methodology, its scope is limited to the dataset used, which ends in 2021; moreover, the study focuses on specific stock indices and equities. Expanding the dataset and applying the framework to a broader range of financial markets and asset classes could validate and enhance its generalizability. Furthermore, incorporating real-time data and leveraging advances in artificial intelligence and machine learning could improve predictive accuracy and scalability. Furthermore, future studies should address the challenges presented by post-pandemic market dynamics, including geopolitical uncertainties and rapid technological changes in the financial landscape.

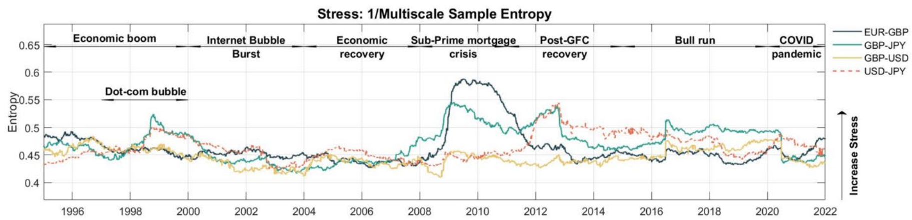

Fig. 11 Financial stress of four currencies (EUR-GBP, GBP-JPY, GBP-USD and USD-JPY) estimated by Mod-MSE over 1995-2022

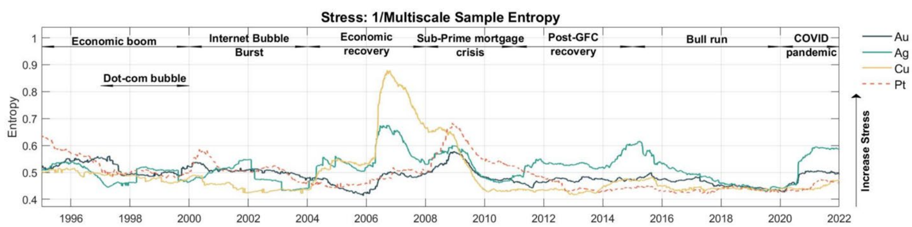

Fig. 12 Financial stress of four metal prices (Gold-Au, Silver-Ag, Copper-Cu and Platinum-Pt) estimated by Mod-MSE over 1995-2022

Appendix A Univariate multiscale sample entrnameopy on currency and metal price

We applied univariate analysis via modified multiscale sample entropy on currency indices and metal price time series. Several factors can influence the value of a currency, including the government's economic policies and its national central bank (García et al. 2016). Furthermore, national central banks are closely associated with their metallic reserves (Hawtrey 2012).

The four currency indices are EUR-GBP, GBP-JPY, GBP-USD, and USD-JPY. The reciprocal of Mod-MSE is plotted in Fig. 11 in line with the complexity-loss theory. The Mod-MSE stress level showed that the USD-JPY (in red) was at a high stressna level before the end of the SPM crisis. The GBP-JPY (in green) and EUR-GBP (in black) indices exhibited higher stress levels in subprime mortgages than those in the Internet bubble burst. Generally, the COVID-19 pandemic impacted the forex market less than the previous two global crises.

Figure 12 presents the stress levels of four mental prices: gold (Au), silver (Ag), copper (Cu), and platinum (Pt). In general, the Internet bubble burst influenced metal prices less than other stocks given in Section Results and analysis. The gold price (in black) and platinum price (in red) are at a low-stress level over time, as expected. Copper price has shown the most sensitivity during the subprime mortgage crisis, whereas silver received more impact from the COVID-19 pandemic.

Authors' contributions

Conceptualization: Hongjian Xiao, and Danilo P. Mandic.; Methodology, data curation and investigation: Hongjian Xiao, Yao Lei Xu and Danilo P. Mandic.; Software, formal analysis, validation, resources, visualization, writing---original draft preparation: Hongjian Xiao.; Project administration, supervision, writing---review and editing: Ana Cukic, Anthony G. Constantinides and Danilo P. Mandic. All authors have read and agreed to the published version of the manuscript.

Data availability

All the price data were downloaded from Bloomberg which are publicly available.

Declaration

Competing interests

No conflict of interest.

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