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Solving the Black–Scholes European options model using the reduced differential transform method with powered modified log-payoff function

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ABSTRACT

This study introduces a novel analytical method for the Black–Scholes European options model, employing modified log-payoff functions raised to a power. The main motivation for this study stems from the need to develop more efficient and accurate analytical techniques for option pricing, particularly under the assumptions of the Black–Scholes framework. The proposed method utilizes the reduced differential transform method (RDTM), which provides a straightforward, flexible, and precise approach for solving the model. A significant contribution of this work is the ability to swiftly obtain explicit solutions with reduced computational time compared to traditional methods. The research also demonstrates that the sensitivities of the European call and put option prices, commonly known as the "Greeks," can be effectively captured using this approach. Importantly, this model operates under the assumption that assets follow geometric Brownian motion and do not yield dividends. The findings from this study highlight the potential of RDTM as a powerful tool in the realm of financial mathematics, offering substantial improvements in the computational efficiency and accuracy of option pricing models.

1. Introduction

The Black–Scholes model, a seminal option pricing framework introduced by Black and Scholes[1], has garnered widespread adoption across various financial market domains since its inception. Shinde and Takale[2] have delineated several practical applications of this model, showcasing its versatility and efficacy. Fadugba et al[3] proposed a novel framework leveraging the integral transform of Mellin type (MT) for the direct evaluation of the Black–Scholes–Merton model (BSMM) for a European put option on dividend yield, incorporating a modified log-payoff function within the context of geometric Brownian motion. Building upon this foundation, Fadugba et al[4] further advanced the methodology of MT to obtain an analytic solution of BSMM for a European power put option, incorporating dividend yield and a modified-log-power payoff function under geometric Brownian motion. Meanwhile, Bossu et al[5] investigated the replication challenge of a target European option within a static portfolio framework, comprising cash, the underlying asset, and a selection of replicant European options. Sobamowo[6] devised a partial Taylor series expansion method to solve the Black–Scholes differential equation for log-payoff functions, offering an alternative perspective on option pricing methodologies. Ouafoudi and Gao[7] pioneered the application of a modified homotopy perturbation method combined with the Sumudu transform method to address fractional Black–Scholes models. Concurrently, Edeki et al[8] presented an analytical solution for a Black–Scholes

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model with non integer flavor using a hybrid approach of fractional complex modified differential transform method. Tour et al[9] delved into European and double barrier options within a time-fractional framework of the Black–Scholes equation, expanding the scope of option pricing methodologies. Vijayan and Manimaran[10] explored the solution of fractional Black–Scholes equations through a semi-analytical method known as homotopy analysis Shehu transform method, offering insights into alternative numerical techniques. Further contributions include the work of Durojaye and Kazeem[11], who discussed an approximate solution for the pricing model of European call options via a semi-analytical approach, and Fadugba and Edogbanya[12], who considered two semi-analytical methods for solving the time-fractional Black–Scholes equation. The following literature provide a broad understanding of both foundational concepts and advanced modifications in option pricing models; Schaefer[13],Hull[14],Merton[15], Wilmott[16],Williams[17],Chen et al[18],Stiglitz and Weiss[19],Glosten and Milgrom[20],Luenberger[21]. Ghevariya and Sharma[22] presented an innovative analytical solution for pricing European options by incorporating log-power payoff functions into the Black–Scholes framework. This method produced clear pricing formulas and improved comprehension of how adjustments to the payoff function impact option values. Li and Wu[23] explored Monte Carlo simulation for pricing European options with log-power payoffs and found it effective for estimating prices. Their results, which matched analytical solutions, underscored the reliability and adaptability of Monte Carlo methods for complex payoff structures. For more details on the use of log-payoff functions, see Singh and Kumar[24],Brown and Green[25],Morales and Sanchez[26],Lee and Smith[27], just to mention a few. In this study, we extend the application of the semi-analytical Reduced Differential Transform Method (RDTM) to the Black–Scholes option model under a powered modified log-payoff function. RDTM was chosen for its simplicity, efficiency, and remarkable precision. Its flexibility and ability to significantly reduce computational time further underscore its superiority over other analytical or numerical methods. Assuming geometric Brownian motion (GBM) simplifies the model by presuming constant volatility and a log-normal distribution, but it may not capture real-world market behaviors like stochastic volatility or price jumps. Consequently, while GBM provides a useful framework, its applicability in real-world scenarios may be limited, necessitating adjustments or supplementary models for accurate pricing and risk assessment; see Nepusz et al[28]. Section 2 outlines the methodology to be applied, leading to the presentation of two theorems in Section 3. Section 4 captures the sensitivity analysis equations of European call and put option prices under powered modified log-payoff function (PMLPF). Section 5 presents illustrative examples, results and discussion. Finally, in Section 6, we offer concluding remarks and suggest avenues for future research.

2. Reduced differential transform method

In the specific domain of interest concerning time (t) and space (x), assuming the function $\beta(x,t)$ possesses analytical properties and maintains continuous differentiability, the Reduced Differential Transform (RDT) can be elucidated as follows Keskin and Oturanc[29,30]

$$\beta_k^*(x) = \frac{1}{k!}\left[\frac{\partial^k \beta(x,t)}{\partial t^k}\right]_{t=0}, \tag{1}$$

Alternatively, the inversion of the Reduced Differential Transform (RDT) of $\beta_k^*(x)$ is defined as follows Keskin and Oturanc[29]:

$$\beta(x,t) = \sum_{k=0}^{\infty} \beta_k^*(x) t^k. \tag{2}$$

The subsequent outcomes highlight certain characteristics of the Reduced Differential Transform Method (RDTM) pertinent to the current investigation.[29]

(i) If $\beta(x,t) = c(x,t) \pm d(x,t)$, then $\beta_k^*(x) = c_k^*(x) \pm d_k^*(x)$.
(ii) If $\beta(x,t) = q b(x,t)$, then $\beta_k^*(x) = q b_k^*(x)$.
(iii) If $\beta(x,t) = \frac{\partial b(x,t)}{\partial x}$, then $\beta_k^*(x) = \frac{\partial b_k^*(x)}{\partial x}$.
(iv) If $\beta(x,t) = \frac{\partial^r b(x,t)}{\partial t^r}$, then $\beta_k^*(x) = \frac{(k+r)! b_{k+r}^*(x)}{k!}$.

3. Exact solution of Black–Scholes European call and put options equations (BSECPOEs) using (RDTM)

This section presents the exact solution of BSECPOEs on powered modified log-payoff functions via RDTM in the following results.

Theorem 1. *Consider the BSECOE of the form*

$$\begin{aligned}
&\psi_t + rS\psi_S + \frac{\sigma^2 S^2}{2}\psi_{SS} = r\psi,\\
&\psi(S,t) \to \infty, S \to \infty, \text{ for fixed } t,\\
&\psi(0,t) = 0, \forall\, t,\\
&\psi(S,T) = \max\{S^l \ln(SK^{-1}), 0\}, l \in \mathbb{Z}^+, \ \forall\, S,
\end{aligned} \tag{3}$$

where $l, \psi, t, r, S, \sigma, T$ are the power of the modified log-payoff function, European call option price, current time, risk-neutral interest rate, underlying asset price, volatility and time to maturity, respectively. The exact solution of BSECOE on powered modified log-payoff function via reduced differential transform is given by

$$\psi(S,t) = S^l e^{(l-1)(r+0.5l\sigma^2)(T-t)}\left[\ln\left(\frac{S}{K}\right) + (r + 0.5(2l-1)\sigma^2)(T-t)\right], \quad S \geq K \tag{4}$$

Proof. By means of variables transformation

$$S = Ke^h, \tau = 0.5\sigma^2(T-t), \psi(S,t) = K\eta(h,\tau) \tag{5}$$

Eq. (3) becomes

$$\eta_\tau - \eta_{hh} - (y-1)\eta_h + y\eta = 0, \eta(h,0) = \max\{K^{l-1} h e^{lh}, 0\} \tag{6}$$

with

$$y = \frac{2r}{\sigma^2}$$

Applying the reduced differential transform on (6), yields

$$(n+1)\eta_{n+1}^* = \eta_{n\,hh}^* + (y-1)\eta_{n\,h}^* - y\eta_n^*, \eta^*(h,0) = \max\{K^{l-1} h e^{lh}, 0\}, n \in Z^+ \tag{7}$$

For $n = 0$, (7) becomes

$$\begin{aligned}
\eta_1^* &= \eta_{0\,hh}^* + (y-1)\eta_{0\,h}^* - y\eta_0^*\\
&= \max\left\{K^{l-1} e^{lh}(Bh + y + 2l - 1)\right\}
\end{aligned} \tag{8}$$

Thus, (8) becomes

$$\eta_1^* = \begin{cases} K^{l-1} e^{lh}(Bh + y + 2l - 1), & h > 0\\ 0, & h < 0 \end{cases} \tag{9}$$

with

$$B = l^2 + l(y-1) - y$$

Similarly, for $n = 1, 2, 3, 4$, one gets

$$\eta_2^* = \begin{cases} \frac{K^{l-1}e^{lh}}{2!}[B^2 h + 2B(y + 2l - 1)], & h > 0\\ 0, & h < 0 \end{cases} \tag{10}$$

$$\eta_3^* = \begin{cases} \frac{K^{l-1}e^{lh}}{3!}[B^3 h + 3B^2(y + 2l - 1)], & h > 0\\ 0, & h < 0 \end{cases} \tag{11}$$

$$\eta_4^* = \begin{cases} \frac{K^{l-1}e^{lh}}{4!}[B^4 h + 4B^3(y + 2l - 1)], & h > 0\\ 0, & h < 0 \end{cases} \tag{12}$$

$$\eta_5^* = \begin{cases} \frac{K^{l-1}e^{lh}}{5!}[B^5 h + 5B^4(y + 2l - 1)], & h > 0\\ 0, & h < 0 \end{cases} \tag{13}$$

In general, one obtains

$$\eta_j^* = \begin{cases} \frac{K^{l-1}e^{lh}}{j!}[B^j h + jB^{j-1}(y+2l-1)], & h>0 \\ 0, & h<0 \end{cases} \tag{14}$$

By means of the inversion formula of the reduced differential transform method, one obtains

$$\begin{aligned}\eta(h,\tau) &= \lim_{j\to\infty}\sum_{m=0}^{j}\eta_m^*\tau^m \\ &= \eta_0^* + \eta_1^*\tau + \eta_2^*\tau^2 + \eta_3^*\tau^3 + \cdots \\ &= \begin{cases} K^{l-1}e^{(lh+B\tau)}[h+(y+2l-1)\tau], & h>0 \\ 0, & h<0 \end{cases}\end{aligned} \tag{15}$$

Substituting (5) into (15), (4) follows.

Theorem 2. *Consider the BSEPOE of the form*

$$\begin{aligned}&\phi_t + rS\phi_S + \frac{\sigma^2 S^2}{2}\phi_{SS} = r\phi, \\ &\phi(S,t)\to 0, S\to\infty, \text{for fixed } t, \\ &\phi(0,t) = 0, \forall\, t, \\ &\phi(S,T) = \max\{S^l\ln(KS^{-1}),0\}, l\in\mathbb{Z}^+,\ \forall\ S,\end{aligned} \tag{16}$$

where l,ϕ,t,r,S,σ,T are the power of the modified log-payoff function, European put option price, current time, risk-neutral interest rate, underlying asset price, volatility and time to maturity, respectively. The exact solution of BSEPOE for powered modified log-payoff via reduced differential transform is given by

$$\phi(S,t) = S^l e^{(l-1)(r+0.5l\sigma^2)(T-t)}\left[\ln\left(\frac{K}{S}\right) - (r+0.5(2l-1)\sigma^2)(T-t)\right], \quad S\le K \tag{17}$$

Proof. By means of variables transformation

$$S = Ke^h, \tau = 0.5\sigma^2(T-t), \phi(S,t) = K\zeta(h,\tau) \tag{18}$$

Eq. (16) becomes

$$\zeta_\tau - \zeta_{hh} - (y-1)\zeta_h + y\zeta = 0, \zeta(h,0) = \max\{-K^{l-1}he^{lh},0\} \tag{19}$$

with

$$y = \frac{2r}{\sigma^2}$$

Applying the reduced differential transform on (19), yields

$$(n+1){\zeta^*}_{n+1} = {\zeta_n}^*{}_{hh} + (y-1)\zeta^*_{n\,h} - y\zeta^*_n, \zeta^*(h,0) = \max\{-K^{l-1}he^{lh},0\}, \quad n\in Z^+ \tag{20}$$

Following the same procedures as above and by means of the inversion formula of the reduced differential transform method, one obtains

$$\begin{aligned}\zeta(h,\tau) &= \lim_{j\to\infty}\sum_{m=0}^{j}\zeta_m^*\tau^m \\ &= \zeta_0^* + \zeta_1^*\tau + \zeta_2^*\tau^2 + \zeta_3^*\tau^3 + \cdots \\ &= \begin{cases} K^{l-1}e^{(lh+B\tau)}[-h-(y+2l-1)\tau], & h>0 \\ 0, & h<0 \end{cases}\end{aligned} \tag{21}$$

with $B = l^2 + l(y-1) - y$. Substituting (18) into (21), (17) follows.

Remark 1.

- For a special, setting $l=1$, (4) and (17) become the closed form formulas for European call and put options on modified log-payoff function via RDTM

$$\psi(S,t) = S\left[\ln\left(\frac{S}{K}\right) + (r+0.5\sigma^2)\tau\right], \tau = T-t, S\ge K \tag{22}$$

and

$$\phi(S,t) = S\left[\ln\left(\frac{K}{S}\right) - (r+0.5\sigma^2)\tau\right], \tau = T-t, S\le K \tag{23}$$

respectively.

- The analytical values (AV) for European call and put options on modified log-payoff function are given by Dedania and Ghevariya[31]

$$c_{cs}(S,t) = S\left\{\sigma\sqrt{\tau}\rho(d^*) + \left[\ln\left(\frac{S}{K}\right) + (r+0.5\sigma^2)\tau\right]\mathcal{N}(d^*)\right\} \tag{24}$$

and

$$p_{ps}(S,t) = S\left\{\sigma\sqrt{\tau}\rho(d^*) - \left[\ln\left(\frac{S}{K}\right) + (r+0.5\sigma^2)\tau\right]\mathcal{N}(-d^*)\right\} \tag{25}$$

respectively, where

$$\begin{aligned}&d^* = \frac{\ln\left(\frac{S}{K}\right) + ((r+0.5)\sigma^2)\tau}{\sigma\sqrt{\tau}}, \rho(d^*) = \frac{1}{\sqrt{2\pi}}e^{-0.5d^2}, \\ &\mathcal{N}(z) = \int_{-\infty}^{z}\rho(z)dz, \mathcal{N}(z)+\mathcal{N}(-z) = 1, \tau = T-t.\end{aligned} \tag{26}$$

- Eqs. (4) and (17) are especially helpful in markets with non-standard or extreme payoff characteristics because they can better capture the empirical characteristics of some financial instruments and market behaviors by raising the payoff function to a power. These characteristics are frequently not well-represented by standard log-payoff functions.

4. Sensitivity analysis equations of European call and put option prices under MLPPF

Sensitivity analysis entails examining the individual effects of variations in each parameter on the option value, while holding other parameters constant. It aids in gauging the significance of each input parameter and the potential impact of their changes on the option value.

4.1. Sensitivity analysis equations of European call option price

The partial derivatives of the European call option price (4) with respect to S,K,r,σ,τ,l for $S\ge K$ are obtained, respectively as are obtained as

$$\begin{aligned}\frac{\partial\psi}{\partial S} =& lS^{l-1}e^{(l-1)(r+0.5l\sigma^2)(T-t)}\left[\ln\left(\frac{S}{K}\right) + (r+0.5(2l-1)\sigma^2)\tau\right] \\ &+S^l e^{(l-1)(r+0.5l\sigma^2)\tau}\left[\frac{1}{S} + (r+0.5(2l-1)\sigma^2)\tau\right]\end{aligned} \tag{27}$$

$$\frac{\partial\psi}{\partial K} = -S^l e^{(l-1)(r+0.5l\sigma^2)\tau}\left[\frac{1}{K}\right] \tag{28}$$

$$\frac{\partial\psi}{\partial r} = S^l e^{(l-1)(r+0.5l\sigma^2)\tau}\left[\tau^2(l-1) + \tau(2l-1)\sigma^2\right] \tag{29}$$

$$\frac{\partial\psi}{\partial\sigma} = S^l e^{(l-1)(r+0.5l\sigma^2)\tau}\left[\tau(l-1)l\sigma + \tau(2l-1)\sigma\right] \tag{30}$$

$$\begin{aligned}\frac{\partial\psi}{\partial\tau} =& S^l e^{(l-1)(r+0.5l\sigma^2)\tau}\Big[(l-1)(r+0.5l\sigma^2) \\ &+ \ln\left(\frac{S}{K}\right) + (r+0.5(2l-1)\sigma^2)\tau\Big]\end{aligned} \tag{31}$$

$$\frac{\partial\psi}{\partial l} = S^l e^{(l-1)(r+0.5l\sigma^2)\tau}\left[\ln\left(\frac{S}{K}\right) + (r+0.5(2l-1)\sigma^2)\tau + \tau(r+0.5\sigma^2)\right] \tag{32}$$

Table 1
The European call option prices for various values of $S = 60, 70, 80, 90, 100$ via RDTM and AV under PMLPF with $l = 1$.

S	RDTM, $\psi(S,t)$	AV, $c_{cs}(S,t)$	AE, $\lvert\psi(S,t) - c_{cs}(S,t)\rvert$
60	6.2707	6.7291	0.4584
70	18.1063	18.1170	0.0107
80	31.3755	31.3756	0.0001
90	45.8979	45.8979	0.0000
100	61.5337	61.5337	0.0000

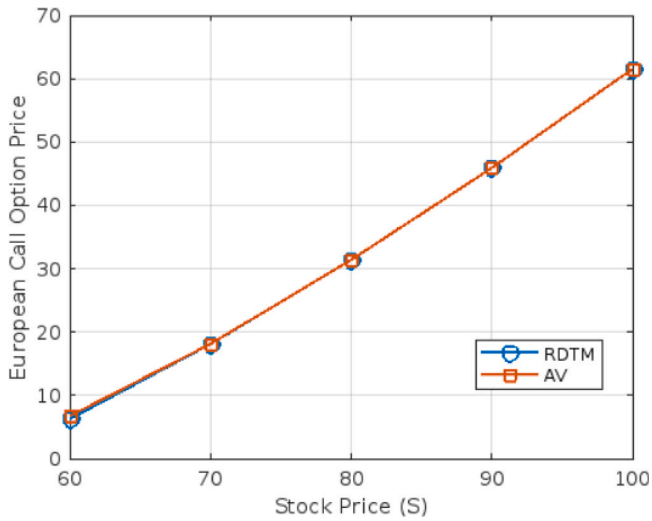

Fig. 1. RDTM versus AV using Table 1.

4.2. Sensitivity analysis equations of European put option price

The partial derivatives of the European put option price (17) with respect to S, K, r, σ, τ, l for $S \leq K$ are obtained, respectively as

$$\frac{\partial \phi}{\partial S} = S^{l-1} e^{(l-1)(r+0.5l\sigma^2)\tau} \left[\ln\left(\frac{K}{S}\right) - (r + 0.5(2l-1)\sigma^2)\tau - \frac{l}{S} \right] \tag{33}$$

$$\frac{\partial \phi}{\partial K} = -S^{l-1} e^{(l-1)(r+0.5l\sigma^2)\tau} \left[\ln\left(\frac{K}{S}\right) - (r + 0.5(2l-1)\sigma^2)\tau \right] \tag{34}$$

$$\frac{\partial \phi}{\partial r} = S^{l} e^{(l-1)(r+0.5l\sigma^2)\tau} (-\tau) \tag{35}$$

$$\frac{\partial \phi}{\partial \sigma} = S^{l} e^{(l-1)(r+0.5l\sigma^2)\tau} \left[-\tau(2l-1) + \ln\left(\frac{K}{S}\right)\sigma\tau \right] \tag{36}$$

$$\frac{\partial \phi}{\partial \tau} = S^{l}(l-1) e^{(l-1)(r+0.5l\sigma^2)\tau} \left[\ln\left(\frac{K}{S}\right) - (r + 0.5(2l-1)\sigma^2)\tau \right] \tag{37}$$

$$\begin{aligned} \frac{\partial \phi}{\partial l} =& S^{l} \ln(S) e^{(l-1)(r+0.5l\sigma^2)\tau} \left[\ln\left(\frac{K}{S}\right) - (r + 0.5(2l-1)\sigma^2)\tau \right] \\ &+ S^{l} e^{(l-1)(r+0.5l\sigma^2)\tau} \left[\frac{(r+0.5\sigma^2)\tau}{l} - \tau \right] \end{aligned} \tag{38}$$

Remark 2. Two key findings emerged from the sensitivity analysis of European option models on powered modified log-payoff are:

- The model's sensitivity to variations in the payoff structure is first shown by the way that changing the power parameter in the modified log-payoff function dramatically affects option pricing.
- The finding also showed that option prices are quite sensitive to changes in volatility and time to maturity, highlighting the importance of these variables in risk management and pricing techniques.

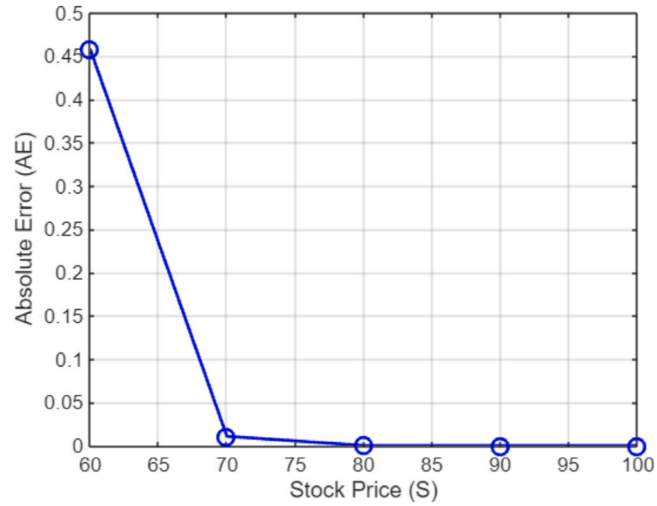

Fig. 2. Absolute error via RDTM using Table 1.

5. Illustrative examples and discussion of results

5.1. Illustrative examples

Example 1. Examining the valuation of European call options utilizing a powered modified log-payoff function, the parameters considered are $l = 1, K = 55, r = 5\%, \sigma = 20\%, t = 0, T = 25\%$. The European call option prices for various values of $S = 60, 70, 80, 90, 100$ via RDTM (22) and AV (24) and the absolute error (AE) are listed in Table 1. The effect of the risk-free interest rate (r) on the prices of European call option obtained via RDTM and AV is displayed in Table 2.

Example 2. Examining the valuation of European put options utilizing a powered modified log-payoff function, the parameters considered are $l = 1, K = 120, r = 5\%, \sigma = 20\%, t = 0, T = 25\%$. The European put option prices for various values of $S = 60, 70, 80, 90, 100$ via RDTM (23) and AV (25) and the absolute error (AE) are listed in Table 3. The effect of the volatility on the prices of European put option obtained via RDTM and AV is displayed in Table 4.

5.2. Results and discussion

The plot in Fig. 1 compares the results of the European call option prices obtained through the RDTM method against AV, as shown in Table 1. From Fig. 1, it is observed that for most values of S, the prices obtained from both methods are very close, with the absolute error being minimal. This indicates that the RDTM method is consistent with the AV in pricing European call options for the given parameters. However, from Fig. 2 there is a slightly higher absolute error observed for smaller values of S compared to larger values. This discrepancy is due to the complexity of the pricing model for options that are in the money. Table 2 displays how changes in r affect the prices of European call options, comparing results obtained from both RDTM and AV. It is observed from Figs. 3 and 4 that options prices are significantly influenced by changes in prevailing risk-free interest rate (r), leading to an impact on call option premium. In other words, call options tend to benefit from rising risk-free interest rate. In Fig. 5, it is evident that the European call prices acquired through RDTM align closely with those derived using the AV method across various combinations of r and S. The plot in Fig. 6 compares the results of the European put option prices obtained through the RDTM method against AV, as shown in Table 3. From Fig. 6, it is observed that for most values of S, the prices obtained from both methods are very close, with the absolute error being minimal. This indicates that the RDTM method is consistent

Table 2
The European put option prices for various values of $S = 60, 70, 80$ and $\sigma = 0.1, 0.2, 0.3$ via RDTM and AV under PMLPF with $l = 1$.

S	r	RDTM, $\psi(S,t)$	AV $c_{cs}(S,t)$
60	0.01	6.2707	6.7291
	0.02	18.1063	18.1170
	0.03	31.3755	31.3756
	0.04	45.8979	45.8979
	0.05	61.5337	61.5337
70	0.01	7.1234	7.5793
	0.02	19.2098	19.2258
	0.03	32.7241	32.7299
	0.04	47.5012	47.5102
	0.05	63.3902	63.3945
80	0.01	8.0567	8.522
	0.02	20.4032	20.4267
	0.03	34.1745	34.1866
	0.04	49.2155	49.2346
	0.05	65.3696	65.3778
90	0.01	9.0801	9.5587
	0.02	21.6876	21.7235
	0.03	35.7270	35.7481
	0.04	51.0423	51.0751
	0.05	67.4723	67.4907
100	0.01	10.1945	10.6910
	0.02	23.0631	23.1199
	0.03	37.3814	37.4168
	0.04	52.9787	53.0361
	0.05	69.6988	69.7418

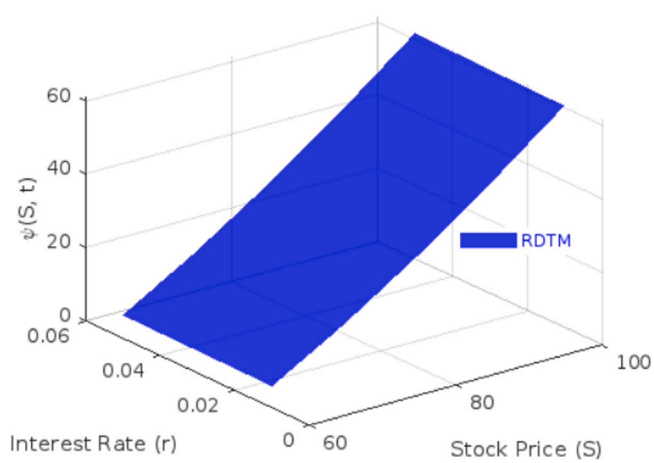

Fig. 3. The plot of the effect of varying r and S on the prices of a European call option via RDTM under PMLPF with $l = 1$ using Table 2.

with the AV in pricing European put options for the given parameters. However, from Fig. 7 there is a slightly higher absolute error observed for larger values of S compared to smaller values. This discrepancy is due to the complexity of the pricing model for put options that are in the money. Table 3 presents the influence of σ on European put option prices, showcasing the results obtained from both RDTM and AV. Figs. 8 and 9 demonstrate that heightened volatility correlates with a stock price recovery, diminishing the payoff and consequently reducing the price of a deeply in-the-money European put option. In other words, RDTM maintains reliable performance under varying market conditions, including high volatility and financial crises, by adapting effectively to market dynamics. The results depicted in Fig. 10 reveal the coherence between European put prices computed using RDTM and those derived through the AV method across a range of σ and S.

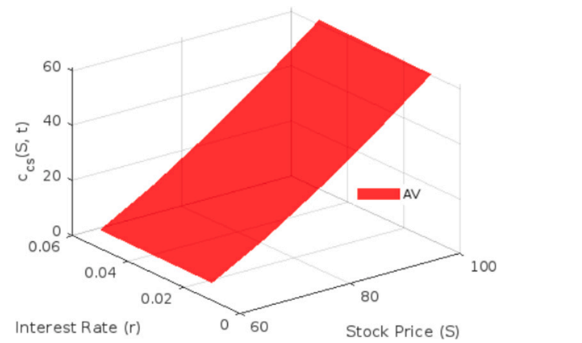

Fig. 4. The plot of the effect of varying r and S on the prices of a European call option via AV under PMLPF with $l = 1$ using Table 2.

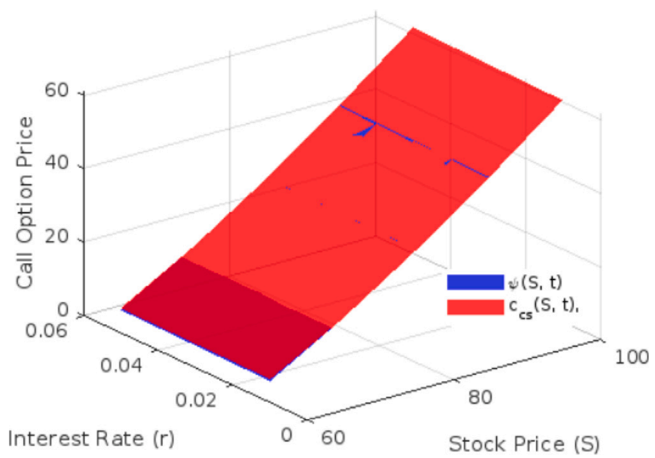

Fig. 5. RDTM versus AV by varying r and S on the prices of a European put option under PMLPF with $l = 1$ using Table 2.

Table 3
The European put option prices for various values of $S = 60, 70, 80, 90, 100$ via RDTM and AV under PMLPF with $l = 1$.

S	RDTM, $\phi(S,t)$	AV, $p_{ps}(S,t)$	AE, $\lvert\phi(S,t) - p_{ps}(S,t)\rvert$
60	40.5388	40.5388	0.0000
70	36.5048	36.5048	0.0000
80	31.0372	31.0373	0.0001
90	24.3164	24.3259	0.0095
100	16.4822	16.6894	0.2072

Table 4
The European put option prices for various values of $S = 60, 70, 80$ and $\sigma = 0.1, 0.2, 0.3$ via RDTM and AV under PMLPF with $l = 1$.

S	σ	RDTM, $\phi(S,t)$	AV $p_{ps}(S,t)$
60	0.1	40.7638	40.7638
	0.2	40.5388	40.5388
	0.3	40.1638	40.1638
70	0.1	36.7673	36.7673
	0.2	36.5048	36.5048
	0.3	36.0673	36.0680
80	0.1	31.3372	31.3372
	0.2	31.0372	31.0373
	0.3	30.5372	30.5581

6. Concluding remarks and future works

A European-style option model with dividend-free in the form of a partial differential equation has been examined, which is affected by the major parameters: S, σ, r, K, T. The principle of RDTM is used to solve the general European style Black–Scholes option model with

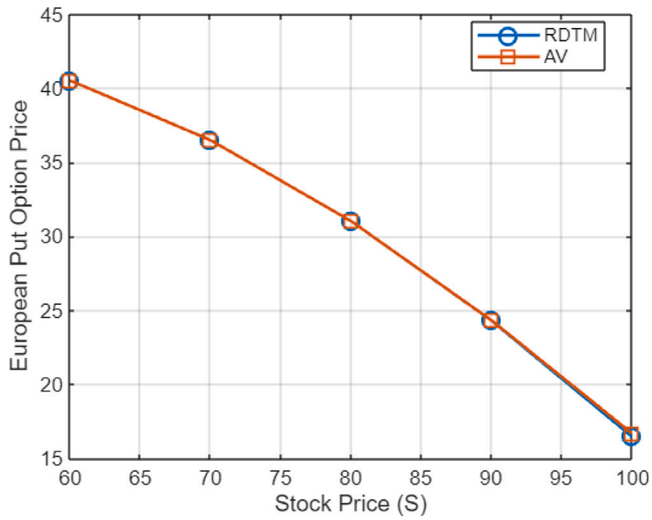

Fig. 6. RDTM versus AV using Table 3.

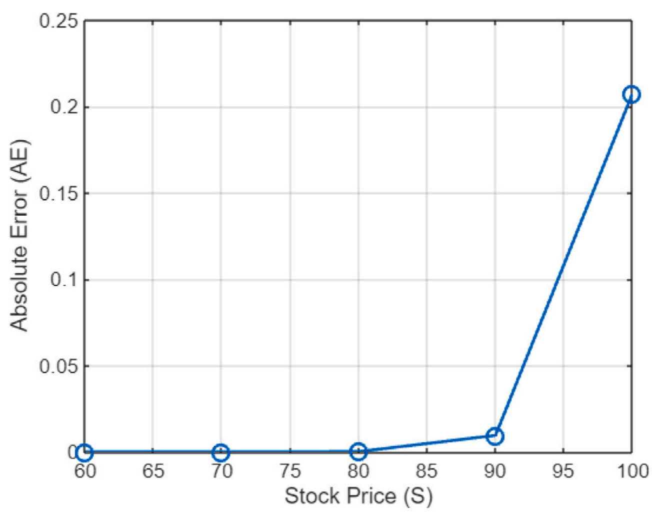

Fig. 7. Absolute error via RDTM using Table 3.

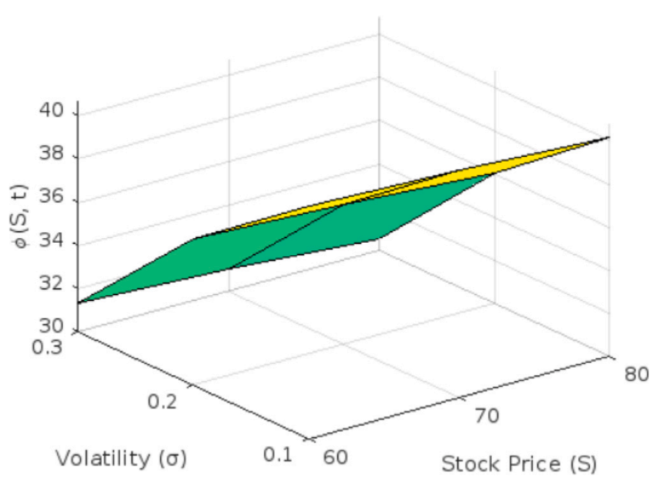

Fig. 8. The plot of the effect of varying σ and S on the prices of a European put option via RDTM under PMLPF with $l = 1$ using Table 4.

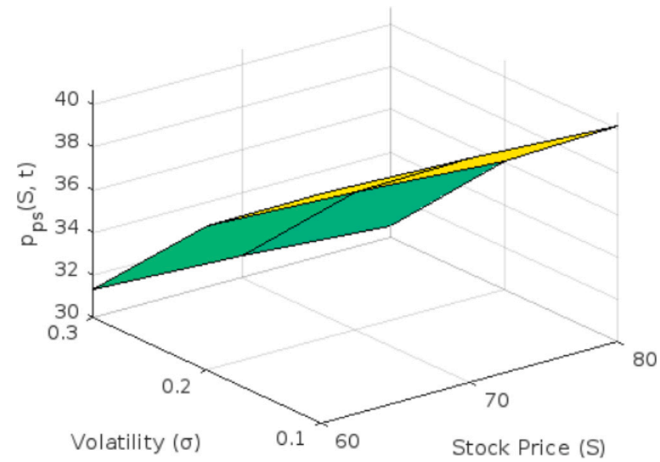

Fig. 9. The plot of the effect of varying σ and S on the prices of a European put option via AV under PMLPF with $l = 1$ using Table 4.

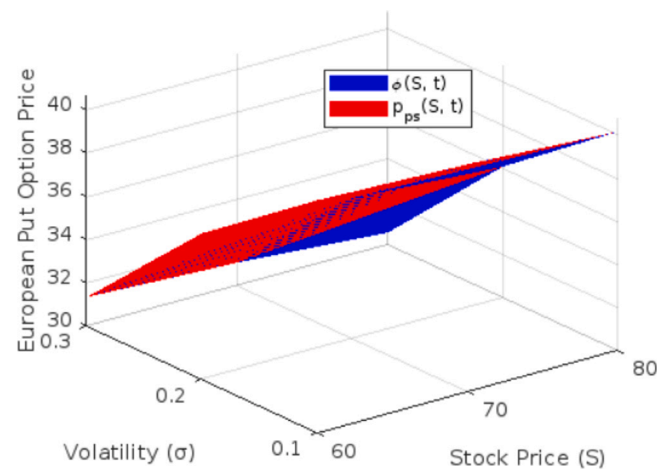

Fig. 10. RDTM versus AV by varying σ and S on the prices of a European put option under PMLPF with $l = 1$ using Table 4.

constant volatility, where the option's return is modeled as a powered modified log-payoff. The approach is theoretically simple and user-friendly. The results obtained agree well with their previous execution iterations. While providing more accuracy, RDTM is easier to apply. In the applied sciences and engineering, it can also be used as a alternative approach for other financial models and related models when nonlinearity appears difficult to handle. The RDTM is shown to be convergent and produces the necessary solutions in an explicit form. The derived closed-form formulas for European call and put options with powered modified log-payoff using RDTM for $l = 1$, as shown in Eqs. (22) and (23) respectively, are consistent with the outcomes documented in (24) and (25), respectively.[3,4,8,11,12,31,32] The inclusion of powered modified log-payoff into the European Black–Scholes option framework is a significant advancement in the financial market. By establishing option pricing, stakeholders gain a mathematical instrument for accurate appraisal. This inclusion paves the way for the creation of novel financial products based on options pricing concepts. It is important to note that the plots presented in Figs. 1, 5, 6, and 10 indicate that the RDTM approach offers precise pricing for European call and put options over various values of r, σ, and S, demonstrating good agreement with the analytical valuation method. It is noteworthy to say that the RDTM-based solutions exhibit strong stability, maintaining accuracy across various initial conditions and parameter changes. Additionally, the method shows reliable convergence, with solutions quickly approaching the true value, ensuring dependable results for the Black–Scholes model with modified powered log-payoff functions. RDTM's key limitations involve its reliance on specific initial and boundary conditions, which may limit its use in more complex scenarios, and its struggles with discontinuities in payoff functions. Future research will aim to broaden the method's applicability by integrating hybrid and adaptive techniques, while also focusing on the stability of option sensitivities, exploring new applications, and advancing numerical solutions for multi-dimensional cases.

CRediT authorship contribution statement

S.E. Fadugba: Supervision, Conceptualization (lead), Methodology, Writing the original - draft. **A.M. Udoye:** Conceptualization (support), Methodology. **S.C. Zelibe:** Project administration. **S.O. Edeki:** Validation (support). **C. Achudume:** Validation (support). **A.A. Adeyanju:** Writing - Review & Editing. **O. Makinde:** Writing - Review & Editing. **P.A. Bankole:** Validation (support), Writing - Review & Editing. **M.C. Kekana:** Validation (support) & Software.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Data will be made available on request.

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