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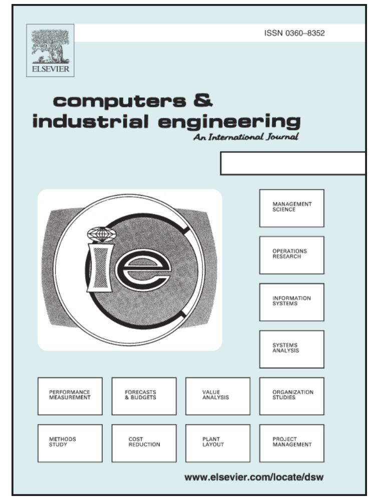

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Reimagining Government Subsidy Policies: Facilitating Echelon Utilization and Sustainable Practices for Retired Battery Systems

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ABSTRACT

The rising number of retired electric vehicle batteries has sparked significant global interest in the recycling and reuse sectors, crucial for achieving a Sustainable Circular Economy and Net-Zero goals. In response, governments are exploring strategic initiatives to enhance the echelon utilization of these batteries, yet current subsidy policies require refinement. This study presents a two-period closed-loop supply chain model involving a battery manufacturer, recycler, power grid company, and government to investigate how subsidies for purchasing reuseable batteries can bolster their utilization for energy storage. The paper analyzes the optimal subsidy amount and assesses stakeholder decisions, profits, and social welfare under scenarios with and without subsidies. The findings reveal that while effective subsidy policies can enhance overall social welfare and support a circular economy, their impact on echelon utilization is influenced by the factors such as battery quality, operating revenue, carbon emission and installation costs. Furthermore, varying installation costs between second-life and new batteries do not necessarily hinder purchase decisions by power companies. This research enriches the discourse on sustainable industrial systems by providing actionable insights for businesses to develop effective strategies in retired battery utilization and guiding policymakers in promoting circular economy initiatives that contribute to industry sustainability and Net-Zero objectives.

Keywords: Retired batteries; Echelon utilization; Subsidy policy design

1 Introduction

Against the backdrop of rising global climate concerns and heightened environmental awareness, electric vehicles (EVs) have gained significant attention as a sustainable mode of transportation. Supported by policies worldwide, EVs are now a strategic priority for achieving sustainable development in the automotive industry (Xing and Yao, 2022). According to International Energy Agency (2024) , global EV sales reached nearly 14 million in 2023, accounting for 18% of total vehicle sales, compared to just 4% in 2020. The IEA forecasts this figure to grow to 17 million by 2024, representing a year-on-year increase of over 20%.

EVs rely on power batteries, a core component driving the expansion of the EV industry(Liu and Wang, 2022). According to SNE Research, the global installed capacity of power batteries reached approximately 705.5 GWh in 2023, growing by 38.6% year-on-year (Electrification Solutions, 2024). However, these batteries have a limited lifespan, typically requiring replacement once their capacity drops below 80% (Chirumalla et al., 2022). Retired batteries contain both hazardous materials (e.g., electrolytes and heavy metals) and valuable metals (e.g., cobalt and nickel) (Liang et al., 2021). Improper disposal risks resource wastage and environmental harm (Dutta et al., 2018). Globally, managing retired batteries and extracting their residual value remain significant challenges.

Key players in the retired battery recycling market include power battery manufacturers (e.g., CATL, LG Energy Solution), EV manufacturers (e.g., Tesla, BMW), and third-party recyclers (e.g., GEM, Accurec). While manufacturers benefit from channel advantages, third-party recyclers often employ advanced technologies, enabling them to capture a significant market share (Zhu et al., 2024). Recycled batteries undergo two main processes: echelon utilization and regeneration (Zhang et al., 2022). Echelon utilization involves repurposing batteries for secondary applications (e.g., grid energy storage or low-speed EVs) after testing and refurbishment. Regeneration entails disassembling batteries with less than 20% capacity to extract valuable metals like cobalt and nickel, facilitating resource circulation (Lei et al., 2024).

Several successful projects highlight the potential of retired batteries in energy storage. For example, 4R Energy in Japan repurposes Nissan Leaf batteries for residential and commercial applications (Nissan, 2021). FreeWire, a U.S.-based company, has developed a mobile EV charging device powered by retired automotive batteries and provided a convenient charging solution(Scooter, 2023). Similarly, Bosch in Germany has integrated retired BMW batteries into a 2MW/2MWh photovoltaic energy storage system (Geng et al., 2022). These initiatives demonstrate the versatility of retired batteries. According to Greenpeace and All-China Environment Federation (2020), global EV batteries are projected to retire at a scale of 463 GWh by 2030. Reusing 80% of these batteries could meet global energy storage needs, with an estimated market value of $15 billion. However, challenges remain, including limited availability of suitable batteries and barriers such as high costs and technical complexities (Gu et al., 2021).

Currently, governments of different countries have formulated various policies for the recycling and reuse of retired batteries (Zhang et al., 2023). For example, the *Regulation Concerning Batteries and Waste Batteries* issued by the European Union stipulates minimum recycling rates and material recovery targets for power batteries, aiming to promote a circular economy by regulating the entire lifecycle of batteries (Council of the European Union, 2023). Germany provides subsidies to enterprises or individuals that deploy batteries for use in charging stations or photovoltaic energy storage systems (Bank aus Verantwortung, 2025). Meanwhile, the Chinese central government has issued the *Administrative Measures for Echelon Utilization of Power Batteries of New Energy Vehicles* to ensure standardized and efficient echelon utilization of retired batteries (Ministry of Industry and Information Technology of the People's Republic of China, 2023). While in this context, Chengdu in China is offering subsidies up to RMB 5 million to qualified battery echelon utilization companies (Chengdu Municipal Bureau of Economic and Information Technology, 2024). For the United States, the federal government supports state and local governments in expanding battery recycling efforts through funding allocations and the establishment of battery recycling points (United States Department of Energy, 2024). The Californian Energy Commission has developed a pilot project to incentivize the secondary use of batteries in order to promote the further spread of renewable energy (California Energy Commission, 2024).

In view of the above, although there are a number of strategic proposals to support the recycling and reuse of retired batteries in various countries, it can be found that there are no explicit subsidy policies being proposed to promote the echelon utilization of retired batteries. As for the development of EVs, government subsidies for the purchase of EVs and subsidies for the charging station installation have well and effectively promoted the EV industry (International Energy Agency, 2023; Yu et al., 2022). Moreover, as the EV market expands, the number of batteries will also increase. When they are retired from EVs, the whole life cycle of the batteries can be extended through echelon utilization, thereby alleviating the challenge of scarce raw materials for batteries and environmental pollution in the production process. This approach not only supports the principles of the circular economy in manufacturing but also contributes to reducing carbon emissions across the supply chain. Given the critical role of purchase subsidies and policies in promoting both the circular economy and the development of EVs, it is essential to evaluate their effectiveness in fostering the recycling and reuse of batteries within closed-loop supply chains. To this end, this study investigates the impact of subsidy policies on promoting the echelon utilization of retired batteries, aiming to provide actionable insights that align with circular economy frameworks and support sustainable industrial practices.

Specifically, in a circumstance where there is a constraint on the quantity of retired batteries, the following research questions will be discussed and answered: (1) Should the government have a purchase subsidy policy in place to promote the echelon utilization? If so, how should it be designed? (2) What is the impact of the purchase subsidy policy on the supply chain members? (3) How do the external factors, such as retired battery return yield, the quality of retired batteries, the installation cost of using batteries, carbon emissions, affect the echelon utilization of retired batteries, and how do they affect the supply chain members?

To answer these questions, this study designs a two-period game model for a closed-loop supply chain (CLSC) of power batteries, respectively considering two scenarios of the government providing and not providing a purchase subsidy. The optimal decisions and profits of supply chain members under the two scenarios are analyzed and compared, complemented by necessary numerical analysis.

The rest of this paper is organized as follows. Section 2 reviews relevant literature and elucidates the innovations of our study. Section 3 provides a detailed description of the model and necessary assumptions. Section 4 analyzes and compares the equilibrium solutions under the two scenarios of no purchase subsidy and purchase subsidy. Section 5 conducts numerical analysis on the equilibrium solutions. Section 6 concludes the paper, further presenting managerial insights and future research directions. All proofs are provided in the Appendix.

2 Research Context

This research is primarily related to three streams of literature: CLSC of power batteries, echelon utilization of retired batteries and government subsidy in CLSC of power batteries. Based on a comprehensive review of existing relevant literature, the current research gap will be identified.

A CLSC is defined as a closed-loop system integrating a forward supply chain and a reverse supply chain, the core of which lies in incorporating the reverse process of product recycling and reuse into the framework of the traditional supply chain, thereby achieving management throughout the entire product lifecycle. Focusing on the field of CLSC of power batteries, rich research results have been produced (Kannan et al., 2010; Mayyas et al., 2019; Zhang et al., 2024). Li and Wei (2025) designed a blockchain-enabled electric vehicle battery recycling system to enhance resilience throughout the EV battery lifecycle.

Scholars have constructed different models grounded in diverse research perspectives, among which the game models considering single or multiple periods have been widely used to study the interest relationships and operational strategies of supply chain members. For example, Liu et al. (2021) modeled a single-period CLSC comprising a battery supplier and an EV manufacturer, considering the uncertain residual capacity of used batteries, and explored the optimal collection and remanufacturing strategies under the supplier-recovery and the manufacturer-recovery models. Zhang and Zhang (2022) built a two-period game model to study the pricing strategy of the manufacturer and the retailer in the CLSC of EV batteries, as well as the financing channel selection of the capital-constrained retailer when undertaking recycling. In the first period, new batteries are produced and sold. In the second period, a portion of the used batteries are recycled for remanufacturing and sold competitively with the new batteries. Gu et al. (2018) developed a three-period model for a CLSC composed of a battery manufacturer and a remanufacturer, and analyzed the optimal manufacturing quantity, remanufacturing quantity, and purchase price. In the first period, all batteries are made from raw materials. In the second period, returned batteries are sorted into low- and high-quality. A proportion of high-quality returns can be reused, while the rest are recycled into materials. The reused batteries are recycled into materials in the third period. In the latter two periods, batteries are made from raw materials and recycled materials.

Unlike ordinary products, power batteries, when retired from EVs, are not scrapped themselves, and can be reused in other fields after recycling, a process known as echelon utilization. Echelon utilization is a crucial link in forming the CLSC of power batteries, and also an important means to realize the residual value of retired power batteries. At present, academic research on echelon utilization has not been fully carried out, with most existing literature primarily concentrating on technical aspects (Abdel-Monem et al., 2017; Gao et al., 2022; Gu et al., 2024; Wang et al., 2023) and environmental impact (Cui et al., 2023; Quan et al., 2022; Wang et al., 2022) of this process. Specifically, Gao et al. (2022) designed a grid-connected photovoltaic-energy storage microgrid by applying retired lithium iron phosphate batteries to the microgrid, and performed experimental studies on its operating performance. The results found that the retired lithium iron phosphate batteries can meet the energy storage requirements of the microgrid. Wang et al. (2023) proposed new available power and energy analysis for battery energy storage systems using active life balancing control techniques, and demonstrated the effectiveness of active life balancing control in extending the life of the battery energy storage systems using retired batteries. Wang et al. (2022) investigated two battery end-of-life management schemes of combining secondary use with recycling and direct recycling, and found that the secondary application of retired batteries in energy storage systems can effectively reduce the net environmental impact of battery life cycle. By evaluating the lifecycle environmental impact of secondary use of lithium-ion batteries in different scenarios, Cui et al. (2023) suggested that reapplying whole packs of retired batteries was better than using only cells or modules due to the environmental loads from diagnosis, disassembly, replacement and testing processes. Recently, echelon utilization has gradually received attention from some scholars in the field of CLSC management for power batteries. Xu et al. (2023) explored the pricing decisions of a new CLSC consisting of a battery manufacturer and an echelon utilization enterprise, considering that the echelon utilization enterprise is responsible for recycling retired batteries and investing in low-carbon innovation. Yan et al. (2024) studied the impact of introducing cascade utilization on the CLSC members of power batteries. The results showed that when the revenue from resource recycling of waste batteries was low, adopting cascade utilization can enhance the corporate profits and consumer welfare.

The CLSC of power batteries involves multiple stakeholders, and government intervention inevitably affects the interest relationships of all parties involved (Guo et al., 2024). Subsidy policy is one of the common means of government intervention. In order to accomplish diverse goals, the government provide different types of subsidies, and the recipients of these subsidies also vary. There is still limited research on government subsidy in CLSC of power batteries. Ding et al. (2020) analyzed and compared the impact of providing collection subsidies to the collecting firm and dismantling subsidies to the dismantling firm, focusing on the optimal collecting strategies under different scenarios and the preferences of the firms and the government regarding these two subsidies. Gu et al. (2021) considered government subsidies to the secondary user, and the relationship between the amount of subsidies and the quality of retired batteries. Chen et al. (2022) examined a supply chain with a battery manufacturer and an EV manufacturer, where the government could choose to offer a subsidy for endurance level or one time quota subsidy to the EV manufacturer for recycling retired batteries. Their research provides the effect of the two forms of recycling subsidies with and without subsidy budget constraints and the corresponding thresholds. Dai et al. (2023) designed a production subsidy for the battery manufacturer and a consumption subsidy for remanufactured battery consumers, and investigated the short-term operation and long-term evolutionary trend of the competitive market between the manufacturer and the remanufacturer under different subsidy policies. Sadrabadi et al. (2024) proposed that the government would pay recycling subsidies to the automotive battery manufacturing factory, the sales representative, or the third-party collection center under three recycling models, and analyzed the pricing strategies involved in the different recycling process of used batteries. Xiao et al. (2024) studied the impact of subsidizing the formal recycler on the recycling supply chain when the formal recycler and the informal recycler compete in recycling retired batteries. The results found that the government subsidy can facilitate an increased flow of retired batteries into the formal recycling channel. Wen et al. (2025) developed a game-theoretic model to investigate the online collection channel and government subsidy strategies, aiming to assess the impact of echelon utilization within the battery collection industry.

Through the above review, it can be found that the current research in the field of CLSC of power batteries mainly focuses on the recycling of retired batteries. The majority of studies on echelon utilization are conducted from a technical perspective, with relatively few from the perspective of supply chain management, that is, there is a lack of thorough exploration on the interest relationships and operational strategies of supply chain members in the echelon utilization stage. Meanwhile, some literature has embarked upon scrutinizing the impact of government subsidy on the CLSC of power batteries, yet the vast majority of these studies concentrate on subsidy for recycling retired batteries. As the echelon utilization of retired batteries has just begun, to the extent of our knowledge, there is scarce relevant research on the design of policies to promote the echelon utilization of retired batteries. Moreover, most of the literature related to government subsidy treats the subsidy merely as an exogenous variable to analyze its impact, without further conclusions on the optimal subsidy.

Compared to the previous research, we this paper emphasizes the following innovations: (1) Drawing from relevant literature, it is reasonable to construct a two-period game model for a CLSC of power batteries in this study. The highlight of this study is that the recycling, both echelon utilization and regeneration utilization of retired batteries are considered, which reflects the reality well; (2) This study fills the research gap by including the government in the game, investigating the government's optimal subsidy policy for the purchase of retired batteries, and discussing its implications; (3) Since the quantity of retired batteries within a certain period is limited, this constraint is considered in the model to ensure the rationality of results.

3 Model Description

Consider a situation in which there is both a battery manufacturer and a battery recycler in the market during a certain period, selling new batteries and echelon utilized batteries to a power grid company (abbreviated as PGC), respectively. After the batteries reach their end of life on EVs, the battery recycler will collect, test and categorize these retired batteries according to their quality. The batteries that comply with the echelon utilization standard will be provided as a potential supply to the PGC for secondary use. The PGC decides how many retired batteries and new batteries to purchase based on the demand, costs and revenue of the batteries. The rest of all the returned batteries that have not been echelon utilized will be regeneration utilized and sold to the battery manufacturer. The battery manufacturer purchases recycled materials and natural raw materials to produce new batteries. During this process, the recycler will set the price for retired batteries first, then the manufacturer will set the price for new batteries, and finally, based on its total demand, the PGC will determine the quantities of both retired and new batteries to purchase.

To improve the secondary utilization rate of retired batteries, governments could adopt various policy intervention, including but not limited to purchase subsidies, tax incentives, mandatory recycling quotas, and carbon trading mechanisms. For the purpose of this study, similar with Wu and Zhang (2025), this study focuses on and simplifies the analysis to a purchase subsidy policy, given its direct impact on the purchasing behavior of the PGC. Specifically, the government's decision on the amount of purchase subsidy is examined as a representative instance of policy intervention (Li et al., 2020; Tang et al., 2019).

This paper considers a two-period CLSC model to study the optimal decisions of the participants in the above situation by focusing on the game among the government, the battery recycler, the battery manufacturer and the PGC. Period 1 involves the manufacturing and selling of power batteries and EVs. The new batteries serve the EV market, where pricing strategies are driven by competitive pressures and production scale considerations. In Period 2, the introduction of the PGC as a user for both new and retired batteries create a differentiated market context. This context requires tailored pricing strategies, as well as the recycling, echelon utilization and regeneration utilization of retired batteries. Additionally, period 2 involves the development of government subsidy policies to support echelon utilization.

In this study, a game theory model is employed to address the problem, specifically through a Stackelberg game approach. The model is developed by solving the decision-making processes over two periods (Period 1 and Period 2) using a backward induction method, enabling the analysis of sequential interactions between the players in the CLSC model. Furthermore, given the constraint on the availability of used batteries, the Karush-Kuhn-Tucker (KKT) conditions are incorporated to address the optimization problem under these limitations. This rigorous methodology ensures that the model effectively captures both the strategic dynamics and the operational constraints.

Figure 1 depicts the interactions of the participants in the two-period game. In addition, this study will solve and analyze the base model in which the government does not provide a purchase subsidy, so as to compare the results before and after providing the subsidy and reveal the implications of the subsidy policy. All notations used throughout the research are listed in Table 1. Besides, this paper uses superscripts N and S to respectively represent the two scenarios of no purchase subsidy and purchase subsidy.

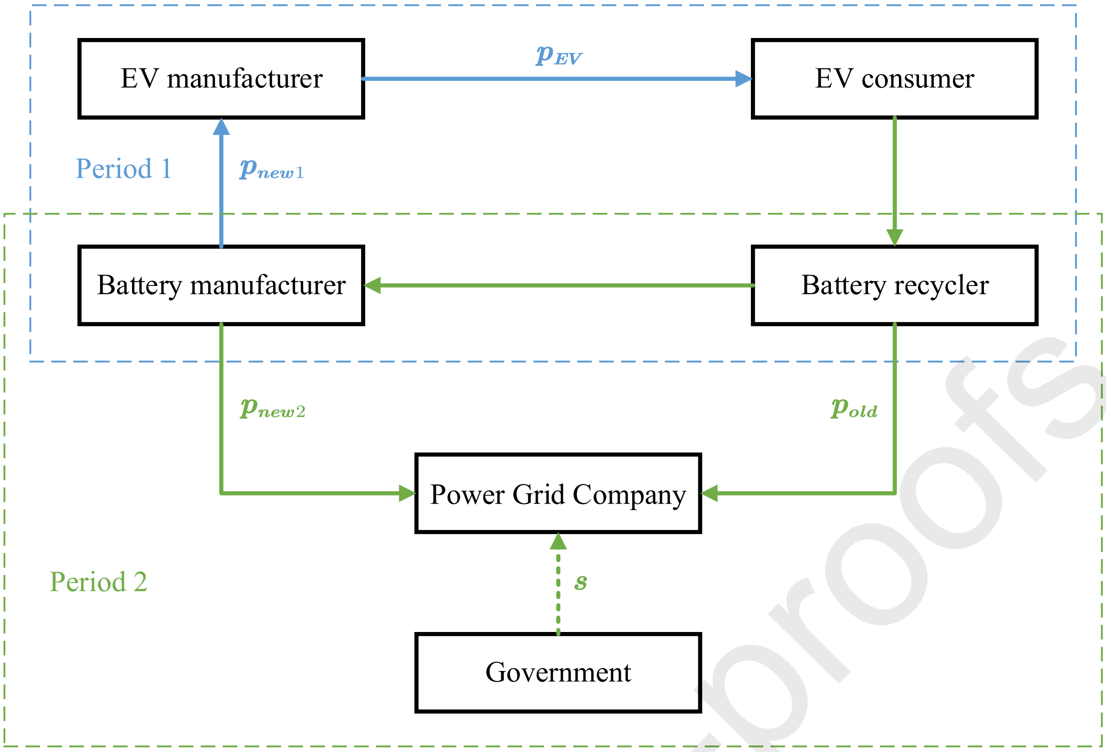


Figure 1 Interactions among the participants in the two-period game

Table 1 Notations

Notations	Descriptions
Parameters	
α	Potential EV market size
β	Price sensitive coefficient
c_{EV}	Manufacturing cost per EV
c_m	Manufacturing cost per new battery
p_n	Natural raw material cost per new battery
c_e	Environmental cost per unit of carbon emissions

e_m	Carbon emission by producing per unit new battery
e_n	Carbon emission by exploiting per unit raw material
D_{PGC}	Total quantity of batteries demanded by PGC
$\bar{q}$	Average quality of reusable batteries
σ^2	Quality variance of reusable batteries
$\hat{q}$	Quality factor of reusable batteries
ξ	Preference sensitivity coefficient to quality fluctuations
V_b	Revenue per new battery for PGC
k_{new}	Installation cost coefficient of new batteries
k_{old}	Installation cost coefficient of reusable batteries
θ	Battery return yield
p_r	Recycled material cost per new battery
φ	Proportion of recycled materials in total disassembled batteries
e_r	Carbon emission by extracting per unit recycled material
c_r	Recycling cost per retired battery
c_d	Disassembling cost per retired battery
η	Proportion of batteries that can be reused in total returned batteries
q_L	Lower limit on the quality of reusable batteries

q_H	Upper limit on the quality of reusable batteries
Variables	
d	Consumer demand for EV
Q_{new1}	Quantity of new batteries sold in period 1
p_{EV}	Selling price per EV
p_{newi}	Selling price per new battery in period i, $i \in \{1, 2\}$
Q_{oldPGC}	Quantity of retired batteries purchased by PGC for reuse
Q_{newPGC}	Quantity of new batteries purchased by PGC
p_{old}	Selling price per retired battery
s	Purchase subsidy per reusable battery
Π_{EV}	Profit for EV manufacturer
Π_{mi}	Profit for battery manufacturer in period i, $i \in \{1, 2\}$
Π_{PGC}	Profit for PGC
Π_r	Profit for battery recycler
SW	Social welfare (i.e., profit for the government)

The research also has some assumptions which are summarized below:

Assumption 1. The PGC is risk-neutral and the quality of the reusable batteries follows a normal distribution with mean $\overline{q}$ and variance σ^2, truncated in $[q_L, q_H]$. $\hat{q} = \overline{q} - \xi\sigma^2$ is defined as the quality factor where ξ is the preference coefficient to describe PGC's sensitivity to quality fluctuations (Newbold et al., 2013; Wen and Siqin, 2020).

Assumption 2. The installation cost coefficient of reusable batteries is greater than that of new batteries, namely $k_{old} > k_{new}$, due to the fact that reusable batteries require additional in-depth testing, classification and refurbishment before secondary use compared to new batteries.

Assumption 3. In the context of ESG considerations, the battery manufacturer is expected to prioritize the use of recycled materials in the production of new batteries.

3.1 Period 1

Consumer demand for EV d. The consumer demand for EV d in period 1 for any given EV selling price p_{EV} takes the following linear form (Yu et al., 2022):

$$d = \alpha - \beta p_{EV}, \quad (1)$$

where α is the potential EV market size and β ($\beta > 0$) is the price sensitive coefficient.

EV manufacturer's decision p_{EV}. Given the selling price of new batteries p_{new1} and the demand function d in (1), the EV manufacturer will decide the EV selling price p_{EV}. In addition to the purchase cost of the new battery, the manufacturing cost of each EV is c_{EV}. Therefore, the EV manufacturer can determine the optimal p_{EV} by maximizing the profit function:

$$\max_{p_{EV}} \Pi_{EV} = (p_{EV} - p_{new1} - c_{EV})d. \quad (2)$$

Battery manufacturer's decision p_{new1}. By noting that the manufacturing cost and raw material cost of each new battery are c_m and p_n, the environmental cost of carbon emissions from producing each new battery and exploiting required raw material is $c_e e_m + c_e e_n$, the quantity of new batteries sold in period 1 is Q_{new1}, which is equal to the quantity of EVs sold, i.e., the consumer demand for EV d given in (1) (Tang et al., 2018), the battery manufacturer can determine the optimal p_{new1} by solving

$$\max_{p_{new1}} \Pi_{m1} = (p_{new1} - c_m - p_n - c_e e_m - c_e e_n)Q_{new1}. \quad (3)$$

Sequence of decisions. The sequence of decisions and resulting events associated with the different participants in period 1 is illustrated in Figure 2. The battery manufacturer decides the selling price per new battery p_{new1} in period 1 first. Next, the EV manufacturer purchases batteries from the battery manufacturer to produce EVs and then decides the selling price per EV p_{EV}. Finally, as the EV selling price p_{EV} is known, the consumer demand for EV d is realized and the quantity of new batteries sold Q_{new1} in period 1 is known.

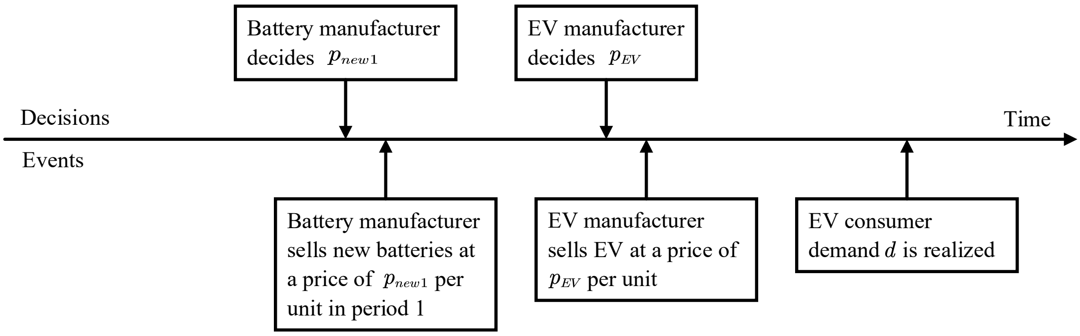


Figure 2 Sequence of decisions in period 1

3.2 Period 2

Power Grid Company's decision Q_{oldPGC}. Given the decisions of the government, the battery recycler and the battery manufacturer (s, p_{old}, p_{new2}), the PGC will decide the quantity of retired batteries to purchase for reuse Q_{oldPGC} that maximizes its profit. Based on the total demand for batteries D_{PGC}, the demanded quantity of new batteries is defined as:

$$Q_{newPGC} = D_{PGC} - Q_{oldPGC}, \tag{4}$$

The study assumes that the quality of each new battery is 1, which can generate a revenue of V_b. The quality of reusable batteries is random and subject to a distribution on $[q_L, q_H]$. In this distribution, the mean of the battery quality is $\bar{q}$ and the variance is σ^2. For all these batteries, only η $(0 < \eta < 1)$ part meets the quality standard and could be considered by the PGC for echelon utilization with the amount $\eta\theta Q_{new1}$. In addition to the cost of purchasing batteries, the PGC also needs to incur the cost of installing them, and this cost exhibits diseconomies of scale. This is because, as the number of batteries increases, the system complexity of battery packs will also increase and it will be more difficult to install and operate them, resulting in the total installation cost increasing exponentially with the number of batteries. To capture the diseconomies of scale of the installation cost, this study shall model this total cost as a quadratic function of the number of batteries, and assume that the installation cost coefficient of new batteries is k_{new} , that of reusable batteries is k_{old}, and $k_{old} > k_{new}$.

Since the quantity of retired batteries provided by the battery recycler is limited, in order to ensure the rationality of decision, the PGC will maximize its profit under this constraint:

$$\max_{Q_{oldPGC}} \Pi_{PGC} = \begin{pmatrix} (Q_{newPGC} + \hat{q} Q_{oldPGC}) V_b - p_{new2} Q_{newPGC} \\ - (p_{old} - s) Q_{oldPGC} - k_{new} Q_{newPGC}^2 - k_{old} Q_{oldPGC}^2 \end{pmatrix},$$

$$\text{subject to } 0 \leq Q_{oldPGC} \leq \eta\theta Q_{new1}, \tag{5}$$

where $\eta\theta Q_{new1}$ is the total number of retired batteries provided by the battery recycler for echelon utilization.

Considering the constraint condition $0 \leq Q_{oldPGC} \leq \eta\theta Q_{new1}$, KKT conditions will be used to solve the PGC's optimal decision problem in Section 4.

Battery manufacturer's decision p_{new2}. The battery manufacturer first uses recycled materials to produce new batteries in period 2. By noting that the recycled material cost of each new battery is p_r, the number of new batteries produced from recycled materials is $\varphi(\theta Q_{new1} - Q_{oldPGC})$, the battery manufacturer can determine the optimal p_{new2} by maximizing the profit function:

$$\max_{p_{new2}} \Pi_{m2} = \begin{pmatrix} (p_{new2} - c_m - c_e e_m) Q_{newPGC} - p_r \varphi(\theta Q_{new1} - Q_{oldPGC}) \\ - (p_n + c_e e_n)(Q_{newPGC} - \varphi(\theta Q_{new1} - Q_{oldPGC})) \end{pmatrix}. \quad (6)$$

Battery recycler's decision p_{old}. Assuming that the proportion of retired batteries is θ in total batteries sold Q_{new1} in period 1, the battery recycler will collect θQ_{new1} retired batteries from customers who purchased EVs in period 1. The total quantity of low-quality retired batteries and unsold high-quality retired batteries is $\theta Q_{new1} - Q_{oldPGC}$. The battery recycler can maximize its profit by deciding the optimal p_{old}:

$$\max_{p_{old}} \Pi_r = \begin{pmatrix} p_{old} Q_{oldPGC} + (p_r - c_e e_r)\varphi(\theta Q_{new1} - Q_{oldPGC}) \\ - c_r \theta Q_{new1} - c_d(\theta Q_{new1} - Q_{oldPGC}) \end{pmatrix}. \quad (7)$$

Government's decision s. The government decides the purchase subsidy of each retired battery s that maximizes social welfare, in order to incentivize the PGC to purchase and reuse retired batteries. By noting that the social welfare consists of the total profits of the PGC, the battery manufacturer and the battery recycler, and deducting the purchase subsidy provided by the government for the PGC, the government can determine the optimal s by solving

$$\max_{s} SW = \Pi_{PGC} + \Pi_{m2} + \Pi_r - sQ_{oldPGC}. \quad (8)$$

Sequence of decisions. Figure 3 depicts the sequence of decisions and resulting events associated with the different participants in period 2. The government will first decide the subsidy s offered to the PGC for purchasing each reusable battery (In the base model, there is no government participation). Next, the battery recycler decides the selling price per battery p_{old}. Upon observing the recycler's decision, the battery manufacturer decides the selling price per new battery p_{new2} in period 2. Finally, given (s, p_{old}, p_{new2}), the PGC decides the quantity of retired batteries purchased Q_{oldPGC}, and then the quantity of new batteries purchased Q_{newPGC} can be determined based on the unmet demand for batteries.

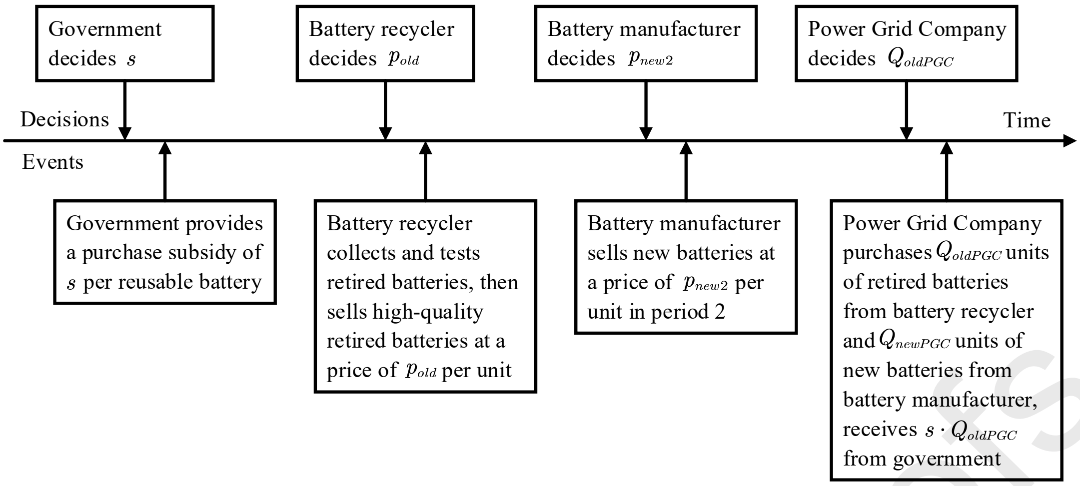


Figure 3 Sequence of decisions in period 2

4 Model Solving and Analysis

In Section 4.1, the paper analyzes the base model in which there is no government participation. In Section 4.2, the base model is extended by considering the situation when the government provides purchase subsidy for the PGC. Specifically, the government will act as the leader of the game in period 2, first deciding the subsidy s of each retired battery purchased by the PGC. In Section 4.3, the paper compares the equilibrium solutions for the two models and analyzes the impact of the optimal subsidy policy.

4.1 No Purchase Subsidy

4.1.1 Period 1

EV manufacturer's decision $\boldsymbol{p_{EV}}$. By substituting the demand function in (1) into the EV manufacturer's profit function in (2), for any given p_{new1}, the EV manufacturer determines the optimal EV selling price p_{EV}^* by solving

$$\underset{p_{EV}}{\text{Max}}\, \Pi_{EV} = (p_{EV} - p_{new1} - c_{EV})(\alpha - \beta p_{EV}). \tag{9}$$

It is obvious that (9) is concave in p_{EV}, and by considering the first-order condition $\frac{\partial \Pi_{EV}}{\partial p_{EV}} = 0$, the EV manufacturer's optimal decision is $p_{EV}^* = \frac{\alpha + \beta(c_{EV} + p_{new1})}{2\beta}$ and the corresponding consumer demand is $d^* = \frac{\alpha - \beta(c_{EV} + p_{new1})}{2}$.

Battery manufacturer's decision $\boldsymbol{p_{new1}}$. Anticipating the EV manufacturer's optimal decision p_{EV}^*, the battery manufacturer determines the optimal selling price of new batteries p_{new1}^* in period 1 by maximizing the profit function:

$$\underset{p_{new1}}{\text{Max}}\,\Pi_{m1} = (p_{new1} - c_m - p_n - c_e e_m - c_e e_n)\frac{\alpha-\beta(c_{EV}+p_{new1})}{2}. \tag{10}$$

As (10) is concave in p_{new1}, when $\frac{\partial \Pi_{m1}}{\partial p_{new1}} = 0$, the battery manufacturer's optimal decision in period 1 is $p_{new1}^* = \frac{\alpha+\beta(c_m+p_n+c_e(e_m+e_n)-c_{EV})}{2\beta}$.

By back-substituting p_{new1}^*, the following proposition can be obtained:

Proposition 1. The optimal selling price of new batteries in period 1 is

$$p_{new1}^* = \frac{\alpha+\beta(c_m+p_n+c_e(e_m+e_n)-c_{EV})}{2\beta}, \tag{11}$$

and the optimal EV selling price is

$$p_{EV}^* = \frac{3\alpha+\beta(c_m+p_n+c_e(e_m+e_n)+c_{EV})}{4\beta}, \tag{12}$$

Additionally, the corresponding consumer demand and the profits of the battery manufacturer and the EV manufacturer are as follows:

$$d^* = \frac{\alpha-\beta(c_m+p_n+c_e(e_m+e_n)+c_{EV})}{4}, \tag{13}$$

$$\Pi_{m1}^* = \frac{(\alpha-\beta(c_m+p_n+c_e(e_m+e_n)+c_{EV}))^2}{8\beta} \tag{14}$$

$$\Pi_{EV}^* = \frac{(\alpha-\beta(c_m+p_n+c_e(e_m+e_n)+c_{EV}))^2}{16\beta}. \tag{15}$$

4.1.2 Period 2

Power Grid Company's decision Q_{oldPGC}. In this model, the government does not provide the purchase subsidy. By substituting Q_{newPGC} given in (4) into the PGC's profit function in (5) and removing the subsidy variable s, the PGC can determine the optimal quantity of retired batteries purchased Q_{oldPGC}^*:

$$\underset{Q_{oldPGC}}{\text{Max}}\,\Pi_{PGC} = \begin{pmatrix} ((D_{PGC} - Q_{oldPGC}) + \hat{q}Q_{oldPGC})V_b - p_{new2}(D_{PGC} - Q_{oldPGC}) \\ - p_{old}Q_{oldPGC} - k_{new}(D_{PGC} - Q_{oldPGC})^2 - k_{old}Q_{oldPGC}^2 \end{pmatrix},$$

$$\text{s.t. } 0 \le Q_{oldPGC} \le \eta\theta Q_{new1}. \tag{16}$$

Now KKT conditions can be used to solve the PGC's optimal decision problem, with the corresponding Lagrange function constructed as follows:

$$\underset{Q_{oldPGC}}{\text{Max}}\ \Pi_{PGC} = \begin{pmatrix} ((D_{PGC} - Q_{oldPGC}) + \hat{q}Q_{oldPGC})V_b - p_{new2}(D_{PGC} - Q_{oldPGC}) \\ - p_{old}Q_{oldPGC} - k_{new}(D_{PGC} - Q_{oldPGC})^2 - k_{old}Q_{oldPGC}^2 \\ + \mu_1(\eta\theta Q_{new1} - Q_{oldPGC}) + \mu_2 Q_{oldPGC} \end{pmatrix}. \quad (17)$$

And then the PGC's optimal decision can be obtained in three different cases, as indicated in Proposition 2.

Proposition 2. (i) If $\frac{2D_{PGC}k_{new}+p_{new2}-p_{old}-V_b(1-\hat{q})}{2(k_{new}+k_{old})} \leq 0$, then the optimal quantity of retired batteries purchased is $Q_{oldPGC}^* = 0$, and the optimal quantity of new batteries purchased is $Q_{newPGC}^* = D_{PGC}$.

(ii) If $0 < \frac{2D_{PGC}k_{new}+p_{new2}-p_{old}-V_b(1-\hat{q})}{2(k_{new}+k_{old})} < \eta\theta Q_{new1}$, then the optimal quantity of retired batteries purchased is $Q_{oldPGC}^* = \frac{2D_{PGC}k_{new}+p_{new2}-p_{old}-V_b(1-\hat{q})}{2(k_{new}+k_{old})}$, and the optimal quantity of new batteries purchased is $Q_{newPGC}^* = \frac{2D_{PGC}k_{old}-p_{new2}+p_{old}+V_b(1-\hat{q})}{2(k_{new}+k_{old})}$.

(iii) If $\frac{2D_{PGC}k_{new}+p_{new2}-p_{old}-V_b(1-\hat{q})}{2(k_{new}+k_{old})} \geq \eta\theta Q_{new1}$, then the optimal quantity of retired batteries purchased is $Q_{oldPGC}^* = \eta\theta Q_{new1}$, and the optimal quantity of new batteries purchased is $Q_{newPGC}^* = D_{PGC} - \eta\theta Q_{new1}$.

It follows from Proposition 2 that both (i) and (iii) are special boundary cases. Therefore, without loss of generality, Proposition 2(ii) is further analyzed in the following corollary:

Corollary 1. (i) $\frac{\partial Q_{oldPGC}^*}{\partial p_{old}} < 0$, $\frac{\partial Q_{newPGC}^*}{\partial p_{old}} > 0$ and (ii) $\frac{\partial Q_{oldPGC}^*}{\partial p_{new}} > 0$, $\frac{\partial Q_{newPGC}^*}{\partial p_{new}} < 0$.

Corollary 1 implies that in the general case, the optimal quantity of retired batteries purchased Q_{oldPGC}^* decreases with the increase of selling price of retired batteries p_{old} and increases with the increase of selling price of new batteries p_{new}. Meanwhile, the optimal quantity of new batteries purchased Q_{newPGC}^* varies inversely.

Battery manufacturer's decision $\boldsymbol{p_{new2}}$. Anticipating the three cases of the PGC's optimal quantities of reusable and new batteries purchased (Q_{oldPGC}^*, Q_{newPGC}^*) as stated in Proposition 2, the battery manufacturer determines the optimal selling price of new batteries p_{new2}^* in period 2 by substituting (Q_{oldPGC}^*, Q_{newPGC}^*) and considering the corresponding condition for each of the three cases.

By using KKT conditions to solve the battery manufacturer's optimal decision problem, the study gets Proposition 3.

Proposition 3. (i) If $\frac{c_m+2D_{PGC}(2k_{new}+k_{old})-p_{old}-V_b(1-\hat{q})+p_n(1-\varphi)+p_r\varphi+c_e(e_m+e_n-e_n\varphi)}{4(k_{new}+k_{old})} \leq 0$, then the optimal selling price of new batteries is $p_{new2}^* = -2D_{PGC}k_{new} + p_{old} + V_b(1-\hat{q})$.

(ii) If $0 < \frac{c_m+2D_{PGC}(2k_{new}+k_{old})-p_{old}-V_b(1-\hat{q})+p_n(1-\varphi)+p_r\varphi+c_e(e_m+e_n-e_n\varphi)}{4(k_{new}+k_{old})} < \eta\theta Q_{new1}$, then the optimal selling price of new batteries is $p^*_{new2} = \frac{1}{2}(c_m + 2D_{PGC}k_{old} + p_{old} + V_b(1-\hat{q}) + p_n(1-\varphi) + p_r\varphi + c_e(e_m + e_n - e_n\varphi))$.

(iii) If $\frac{c_m+2D_{PGC}(2k_{new}+k_{old})-p_{old}-V_b(1-\hat{q})+p_n(1-\varphi)+p_r\varphi+c_e(e_m+e_n-e_n\varphi)}{4(k_{new}+k_{old})} \geq \eta\theta Q_{new1}$, then the optimal selling price of new batteries is $p^*_{new2} = -2D_{PGC}k_{new} + 2\eta\theta Q_{new1}(k_{new} + k_{old}) + p_{old} + V_b(1-\hat{q})$.

Battery recycler's decision p_{old}. Anticipating the battery manufacturer's and the PGC's optimal decisions $(p^*_{new2}, Q^*_{oldPGC})$ above, the battery recycler determines the optimal selling price of reusable batteries p^*_{old} that maximizes its profit by substituting $(p^*_{new2}, Q^*_{oldPGC})$ and considering the updated condition for each of the three cases as shown in Proposition 3.

KKT conditions are also used to solve the battery recycler's optimal decision problem. And then by back-substituting p^*_{old}, the optimal decisions of the battery recycler, the battery manufacturer and the PGC in the three different cases are given by the following proposition.

Proposition 4. (i) If $\frac{c_d+c_m+2D_{PGC}(2k_{new}+k_{old})-V_b(1-\hat{q})+p_n(1-\varphi)+c_e(e_m+e_n-e_n\varphi+e_r\varphi)}{8(k_{new}+k_{old})} \leq 0$, then the three firms' optimal decisions are as follows:

$$p^{N*}_{old} = \begin{pmatrix} c_m + 2D_{PGC}(2k_{new} + k_{old}) - V_b(1-\hat{q}) \\ + p_n(1-\varphi) + p_r\varphi + c_e(e_m + e_n - e_n\varphi) \end{pmatrix}, \quad (18)$$

$$p^{N*}_{new2} = c_m + 2D_{PGC}(k_{new} + k_{old}) + p_n(1-\varphi) + p_r\varphi + c_e(e_m + e_n - e_n\varphi), \quad (19)$$

$$Q^{N*}_{oldPGC} = 0, \quad (20)$$

$$Q^{N*}_{newPGC} = D_{PGC}. \quad (21)$$

Additionally, the corresponding profits of the three firms are as follows:

$$\Pi^{N*}_{r} = \theta Q_{new1}((p_r - c_e e_r)\varphi - c_d - c_r), \quad (22)$$

$$\Pi^{N*}_{m2} = 2D^2_{PGC}(k_{new} + k_{old}) - (c_e e_n + p_n - p_r)(D_{PGC} - \theta Q_{new1})\varphi, \quad (23)$$

$$\Pi^{N*}_{PGC} = D_{PGC}\begin{pmatrix} V_b - c_m - D_{PGC}(3k_{new} + 2k_{old}) \\ - p_n(1-\varphi) - p_r\varphi - c_e(e_m + e_n - e_n\varphi) \end{pmatrix}. \quad (24)$$

(ii) If $0 < \frac{c_d+c_m+2D_{PGC}(2k_{new}+k_{old})-V_b(1-\hat{q})+p_n(1-\varphi)+c_e(e_m+e_n-e_n\varphi+e_r\varphi)}{8(k_{new}+k_{old})} < \eta\theta Q_{new1}$, then the three firms' optimal decisions are as follows:

$$p^{N*}_{old} = \frac{1}{2}\begin{pmatrix} c_m + c_e(e_m + e_n) + 2D_{PGC}(2k_{new} + k_{old}) - V_b(1-\hat{q}) \\ + p_n - c_d - (c_e(e_n + e_r) + p_n - 2p_r)\varphi \end{pmatrix}, \quad (25)$$

$$p_{new2}^{N*} = \frac{1}{4}\begin{pmatrix} 3c_m + 3c_e(e_m + e_n) + 2D_{PGC}(2k_{new} + 3k_{old}) + V_b(1 - \hat{q}) \\ +3p_n - c_d - (3c_e e_n + c_e e_r + 3p_n - 4p_r)\varphi \end{pmatrix}, \tag{26}$$

$$Q_{oldPGC}^{N*} = \frac{c_d + c_m + 2D_{PGC}(2k_{new} + k_{old}) - V_b(1 - \hat{q}) + p_n(1 - \varphi) + c_e(e_m + e_n - e_n\varphi + e_r\varphi)}{8(k_{new} + k_{old})}, \tag{27}$$

$$Q_{newPGC}^{N*} = \frac{-c_d - c_m + 2D_{PGC}(2k_{new} + 3k_{old}) + V_b(1 - \hat{q}) - p_n(1 - \varphi) - c_e(e_m + e_n - e_n\varphi + e_r\varphi)}{8(k_{new} + k_{old})}. \tag{28}$$

Additionally, the corresponding profits of the three firms are Π_r^{N*}, Π_{m2}^{N*} and Π_{PGC}^{N*}, which are described in Appendix (A.1), (A.2) and (A.3).

(iii) If $\frac{c_d + c_m + 2D_{PGC}(2k_{new} + k_{old}) - V_b(1 - \hat{q}) + p_n(1 - \varphi) + c_e(e_m + e_n - e_n\varphi + e_r\varphi)}{8(k_{new} + k_{old})} \geq \eta\theta Q_{new1}$, then the three firms' optimal decisions are as follows:

$$p_{old}^{N*} = \begin{pmatrix} c_m + 2D_{PGC}(2k_{new} + k_{old}) - 4\eta\theta Q_{new1}(k_{new} + k_{old}) \\ -V_b(1 - \hat{q}) + p_n(1 - \varphi) + p_r\varphi + c_e(e_m + e_n - e_n\varphi) \end{pmatrix}, \tag{29}$$

$$p_{new2}^{N*} = \begin{pmatrix} c_m + 2(k_{new} + k_{old})(D_{PGC} - \eta\theta Q_{new1}) \\ + p_n(1 - \varphi) + p_r\varphi + c_e(e_m + e_n - e_n\varphi) \end{pmatrix}, \tag{30}$$

$$Q_{oldPGC}^{N*} = \eta\theta Q_{new1}, \tag{31}$$

$$Q_{newPGC}^{N*} = D_{PGC} - \eta\theta Q_{new1}. \tag{32}$$

Additionally, the corresponding profits of the three firms are as follows:

$$\Pi_r^{N*} = Q_{new1}\theta\begin{pmatrix} \eta\begin{pmatrix} c_m + c_e(e_m + e_n) + 2D_{PGC}(2k_{new} + k_{old}) \\ + p_n - V_b(1 - \hat{q}) - 4(k_{new} + k_{old})\eta\theta Q_{new1} \end{pmatrix} \\ - c_r - c_d(1 - \eta) + (p_r - c_e e_r(1 - \eta) - c_e e_n\eta - p_n\eta)\varphi \end{pmatrix}, \tag{33}$$

$$\Pi_{m2}^{N*} = 2(k_{new} + k_{old})(D_{PGC} - \eta\theta Q_{new1})^2 - (c_e e_n + p_n - p_r)(D_{PGC} - \theta Q_{new1})\varphi, \tag{34}$$

$$\Pi_{PGC}^{N*} = \begin{pmatrix} D_{PGC}(V_b - c_m - p_n) + (2D_{PGC}\eta\theta Q_{new1} + \eta^2\theta^2 Q_{new1}^2)(k_{new} + k_{old}) \\ + D_{PGC}(p_n - p_r)\varphi - D_{PGC}^2(3k_{new} + 2k_{old}) - c_e D_{PGC}(e_m + e_n - e_n\varphi) \end{pmatrix}. \tag{35}$$

The following corollary explains the connotation of the PGC's different optimal decisions in the above three cases from the perspective of the limited quantity of retired batteries:

Corollary 2. (i) When $\frac{c_d + c_m + 2D_{PGC}(2k_{new} + k_{old}) - V_b(1 - \hat{q}) + p_n(1 - \varphi) + c_e(e_m + e_n - e_n\varphi + e_r\varphi)}{8(k_{new} + k_{old})} \leq 0$, it is optimal for the PGC not to purchase retired batteries.

(ii) When $0 < \frac{c_d + c_m + 2D_{PGC}(2k_{new} + k_{old}) - V_b(1 - \hat{q}) + p_n(1 - \varphi) + c_e(e_m + e_n - e_n\varphi + e_r\varphi)}{8(k_{new} + k_{old})} < \eta\theta Q_{new1}$, the quantity of retired batteries that the PGC wants to purchase is less than the quantity of retired batteries provided by the battery recycler, meaning that the supply of retired batteries is sufficient for the

PGC, so the PGC will purchase a portion of the retired batteries provided by the battery recycler for echelon utilization, the optimal quantity of which is $Q_{oldPGC}^{N*} = \frac{c_d+c_m+2D_{PGC}(2k_{new}+k_{old})-V_b(1-\hat{q})+p_n(1-\varphi)+c_e(e_m+e_n-e_n\varphi+e_r\varphi)}{8(k_{new}+k_{old})}$.

(iii) When $\frac{c_d+c_m+2D_{PGC}(2k_{new}+k_{old})-V_b(1-\hat{q})+p_n(1-\varphi)+c_e(e_m+e_n-e_n\varphi+e_r\varphi)}{8(k_{new}+k_{old})} \geq \eta\theta Q_{new1}$, the quantity of retired batteries that the PGC wants to purchase exceeds the quantity of retired batteries provided by the battery recycler, meaning that the supply of retired batteries is insufficient for the PGC, so the PGC can only purchase all the retired batteries provided by the battery recycler for echelon utilization at most.

Without loss of generality, by focusing on Proposition 4(ii) and analyzing the impact of quality factor of reusable batteries $\hat{q}$ on the equilibrium solutions, Corollary 3 is derived as follows.

Corollary 3. (i) $\frac{\partial p_{old}^{N*}}{\partial \hat{q}} > 0$, $\frac{\partial p_{new2}^{N*}}{\partial \hat{q}} < 0$, $\frac{\partial Q_{oldPGC}^{N*}}{\partial \hat{q}} > 0$, $\frac{\partial Q_{newPGC}^{N*}}{\partial \hat{q}} < 0$.

(ii) $\frac{\partial p_{old}^{N*}}{\partial \bar{q}} > 0$, $\frac{\partial p_{new2}^{N*}}{\partial \bar{q}} < 0$, $\frac{\partial Q_{oldPGC}^{N*}}{\partial \bar{q}} > 0$, $\frac{\partial Q_{newPGC}^{N*}}{\partial \bar{q}} < 0$; $\frac{\partial p_{old}^{N*}}{\partial \sigma^2} < 0$, $\frac{\partial p_{new2}^{N*}}{\partial \sigma^2} > 0$, $\frac{\partial Q_{oldPGC}^{N*}}{\partial \sigma^2} < 0$, $\frac{\partial Q_{newPGC}^{N*}}{\partial \sigma^2} > 0$; $\frac{\partial p_{old}^{N*}}{\partial \xi} < 0$, $\frac{\partial p_{new2}^{N*}}{\partial \xi} > 0$, $\frac{\partial Q_{oldPGC}^{N*}}{\partial \xi} < 0$, $\frac{\partial Q_{newPGC}^{N*}}{\partial \xi} > 0$.

Corollary 3(i) shows that the optimal selling price of retired batteries p_{old}^{N*} increases with the increase of $\hat{q}$, and the optimal selling price of new batteries p_{new2}^{N*} decreases with the increase of $\hat{q}$. Also, the optimal quantity of retired batteries purchased Q_{oldPGC}^{N*} increases with the increase of $\hat{q}$, so that the corresponding optimal quantity of new batteries purchased Q_{newPGC}^{N*} decreases with the increase of $\hat{q}$. This is because, the higher the quality of retired batteries, the more motivated the PGC to purchase retired batteries. Consequently, the quantity of retired batteries purchased by the PGC will increase. And the battery recycler will raise the selling price of retired batteries to make more profit. Given the constant total demand for batteries, the quantity of new batteries purchased by the PGC will decrease and the battery manufacturer will reduce the selling price of new batteries to improve the competitiveness of new batteries in the market.

Corollary 3(ii) shows that the optimal selling price and optimal quantity of retired batteries p_{old}^{N*}, Q_{oldPGC}^{N*} increases with the increase of average quality $\bar{q}$; and the optimal selling price and optimal quantity of retired batteries p_{new2}^{N*}, Q_{newPGC}^{N*} decreases with the increase of average quality $\bar{q}$. It can be similarly observed that the variance σ^2 and its sensitivity coefficient ξ are inversely related to the decision variables associated with the old batteries (p_{old}^{N*}, Q_{oldPGC}^{N*}), whereas they are directly related to those associated with the new batteries (p_{new2}^{N*}, Q_{newPGC}^{N*}). In reality, a higher average quality and lower quality variance of retired batteries serve as stronger incentives for the PGC to procure such batteries. Consequently, the procurement volume of retired batteries by the PGC is expected to rise. In response, the battery recycler may increase the selling price in an effort to enhance profitability.

Next, Corollary 4 reveals the impact of installation cost coefficients k_{new} and k_{old} on the equilibrium solutions.

Corollary 4. (i) $\frac{\partial p_{old}^{N*}}{\partial k_{new}} > \frac{\partial p_{new2}^{N*}}{\partial k_{new}} > 0$, $\frac{\partial p_{new2}^{N*}}{\partial k_{old}} > \frac{\partial p_{old}^{N*}}{\partial k_{old}} > 0$.

(ii) The study further denotes $A = \frac{c_d + c_m - V_b(1-\hat{q}) + p_n(1-\varphi) + c_e(e_m + e_n - e_n\varphi + e_r\varphi)}{2D_{PGC}}$ and considers $k_{old} > k_{new} > 0$ so that:

a) If $k_{old} > A$ and $k_{new} > -A$, then $\frac{\partial Q_{oldPGC}^{N*}}{\partial k_{new}} > 0$, $\frac{\partial Q_{newPGC}^{N*}}{\partial k_{new}} < 0$, $\frac{\partial Q_{oldPGC}^{N*}}{\partial k_{old}} < 0$, $\frac{\partial Q_{newPGC}^{N*}}{\partial k_{old}} > 0$.

b) If $k_{old} > A$ and $k_{new} < -A$, then $\frac{\partial Q_{oldPGC}^{N*}}{\partial k_{new}} > 0$, $\frac{\partial Q_{newPGC}^{N*}}{\partial k_{new}} < 0$, $\frac{\partial Q_{oldPGC}^{N*}}{\partial k_{old}} > 0$, $\frac{\partial Q_{newPGC}^{N*}}{\partial k_{old}} < 0$.

c) If $k_{old} < A$ and $k_{new} > -A$, then $\frac{\partial Q_{oldPGC}^{N*}}{\partial k_{new}} < 0$, $\frac{\partial Q_{newPGC}^{N*}}{\partial k_{new}} > 0$, $\frac{\partial Q_{oldPGC}^{N*}}{\partial k_{old}} < 0$, $\frac{\partial Q_{newPGC}^{N*}}{\partial k_{old}} > 0$.

It should be further noted that b) exists only when $A < 0$, but c) exists only when $A > 0$.

Corollary 4(i) indicates that the optimal selling price of retired batteries p_{old}^{N*} and the optimal selling price of new batteries p_{new2}^{N*} are directly proportional to both k_{new} and k_{old}. What's more, the impact of k_{new} on p_{old}^{N*} is greater than that on p_{new2}^{N*}, while the impact of k_{old} on p_{new2}^{N*} is greater than that on p_{old}^{N*}. These results can be explained as follows. The higher installation cost coefficient of new batteries means that the cost of installing the new batteries for the PGC will be higher. As the quantity of retired batteries purchased by the PGC increases, the battery recycler is likely to increase the selling price in an effort to enhance profitability. Meanwhile, fewer new batteries will be sold and the battery manufacturer will also raise the selling price of new batteries to compensate for the resulting loss of profit. And in order to improve the competitiveness of new batteries in the market, the selling price of new batteries will be raised at a slower rate than the selling price of the retired batteries. The impact of the installation cost coefficient of retired batteries is in a similar vein.

Corollary 4(ii) shows the impact of k_{new} and k_{old} on the PGC's optimal decision under different conditions, with counterintuitive results in b) and c) respectively: when $A < 0$ and $k_{new} < -A$, Q_{oldPGC}^{N*} is directly proportional to k_{old}; when $A > 0$ and $k_{old} < A$, Q_{oldPGC}^{N*} is inversely proportional to k_{new}. Obviously, the impact of installation cost under these two conditions is relatively small, thus these results can be interpreted as follows. For the case when k_{new} is small (i.e., $k_{new} < -A$), it can be seen from Corollary 4(i) that, firstly, when k_{old} is also small, the selling prices of both batteries are low and the price gap is small, so the PGC tends to purchase new batteries. Then as k_{old} increases, since p_{new2}^{N*} increases faster than p_{old}^{N*}, the price gap between the two types of batteries will increase. Therefore, the PGC will purchase more retired batteries. On the contrary, for the case when k_{old} is small (i.e., $k_{old} < A$), when k_{new} is also small, the price gap between the two types of batteries is relatively large, so the PGC tends to purchase retired batteries. Then as k_{new} increases, since p_{old}^{N*} increases faster than p_{new2}^{N*}, the price gap between the two types of batteries will decrease. Therefore, the PGC will purchase more new batteries.

Corollary 5. (i) $\frac{\partial p_{new2}^{N*}}{\partial e_m} > \frac{\partial p_{old}^{N*}}{\partial e_m} > 0$, $\frac{\partial p_{new2}^{N*}}{\partial e_n} > \frac{\partial p_{old}^{N*}}{\partial e_n} > 0$, $\frac{\partial p_{old}^{N*}}{\partial e_r} < \frac{\partial p_{new2}^{N*}}{\partial e_r} < 0$.

(ii) $\frac{\partial Q_{oldPGC}^{N*}}{\partial e_m} > 0$, $\frac{\partial Q_{newPGC}^{N*}}{\partial e_m} < 0$, $\frac{\partial Q_{oldPGC}^{N*}}{\partial e_n} > 0$, $\frac{\partial Q_{newPGC}^{N*}}{\partial e_n} < 0$, $\frac{\partial Q_{oldPGC}^{N*}}{\partial e_r} > 0$, $\frac{\partial Q_{newPGC}^{N*}}{\partial e_r} < 0$.

Corollary 5(i) reveals that as e_m or e_n increases, the optimal selling price of reusable batteries p_{old}^{N*} and the optimal selling price of new batteries p_{new2}^{N*} both increase, with p_{new2}^{N*} increasing at a faster rate. Conversely, as e_r increases, p_{old}^{N*} and p_{new2}^{N*} decrease, with p_{old}^{N*} declining at a faster rate. Corollary 5(ii) indicates that the optimal quantity of retired batteries purchased Q_{oldPGC}^{N*} increases with the increase of e_m, e_n or e_r. This is because the higher the carbon emissions by producing new batteries or mining raw materials, the higher the corresponding environmental cost borne by the battery manufacturer. Therefore, the battery manufacturer will raise the selling price of new batteries, which increases the cost for the PGC to purchase new batteries. The PGC will purchase more retired batteries, the battery recycler will raise the selling price of retired batteries to gain more profit. However, in order to preserve the market competitiveness of retired batteries, the increase in the selling price of retired batteries will be smaller than the increase in the selling price of new batteries. On the other hand, higher carbon emissions by extracting recycled materials escalate the corresponding environmental cost for the battery recycler, which means that the battery recycler is more profitable in selling retired batteries rather than recycled materials. Therefore, the battery recycler will set a lower selling price of retired batteries to increase sales while the PGC will purchase less new batteries, and the battery manufacturer will also reduce the selling price of new batteries to compete with the recycler. Meanwhile, the battery manufacturer can only use fewer recycled materials to produce new batteries, which increases the cost of purchasing raw materials, thus attenuating the decrease in the selling price of new batteries.

4.2 Purchase Subsidy

4.2.1 Period 1

In this model, the government will decide the purchase subsidy of each retired battery s in period 2, which does not affect the game in period 1. Therefore, the optimal decisions of the battery manufacturer and the EV manufacturer in period 1 are the same as the ones given in Proposition 1.

4.2.2 Period 2

Power Grid Company's decision Q_{oldPGC}. For any given per-unit purchase subsidy s, the effective purchase price of each retired battery is $(p_{old} - s)$. By substituting Q_{newPGC} given in (4) into the PGC's profit function in (5), the PGC can determine the optimal quantity of retired batteries purchased Q_{oldPGC}^{*}:

$$\underset{Q_{oldPGC}}{\text{Max}}\ \Pi_{PGC} = \begin{pmatrix} ((D_{PGC} - Q_{oldPGC}) + \hat{q}Q_{oldPGC})V_b - p_{new2}(D_{PGC} - Q_{oldPGC}) \\ -(p_{old} - s)Q_{oldPGC} - k_{new}(D_{PGC} - Q_{oldPGC})^2 - k_{old}Q_{oldPGC}^2 \end{pmatrix},$$

$$\text{s.t. } 0 \le Q_{oldPGC} \le \eta\theta Q_{new1}. \tag{36}$$

Analogous to the base model, KKT conditions are used to solve the PGC's optimal decision problem, with the corresponding Lagrange function constructed as follows:

$$\underset{Q_{oldPGC}}{\text{Max}}\ \Pi_{PGC}=\begin{pmatrix}((D_{PGC}-Q_{oldPGC})+\hat{q}Q_{oldPGC})V_b-p_{new2}(D_{PGC}-Q_{oldPGC})\\-(p_{old}-s)Q_{oldPGC}-k_{new}(D_{PGC}-Q_{oldPGC})^2-k_{old}Q_{oldPGC}^2\\+\mu_1(\eta\theta Q_{new1}-Q_{oldPGC})+\mu_2Q_{oldPGC}\end{pmatrix}. \quad (37)$$

And then the PGC's optimal decision can be obtained in three different cases, as indicated in Proposition 5.

Proposition 5. (i) If $\frac{2D_{PGC}k_{new}+p_{new2}-p_{old}-V_b(1-\hat{q})+s}{2(k_{new}+k_{old})}\leq 0$, then the optimal quantity of retired batteries purchased is $Q^*_{oldPGC}=0$, and the optimal quantity of new batteries purchased is $Q^*_{newPGC}=D_{PGC}$.

(ii) If $0<\frac{2D_{PGC}k_{new}+p_{new2}-p_{old}-V_b(1-\hat{q})+s}{2(k_{new}+k_{old})}<\eta\theta Q_{new1}$, then the optimal quantity of retired batteries purchased is $Q^*_{oldPGC}=\frac{2D_{PGC}k_{new}+p_{new2}-p_{old}-V_b(1-\hat{q})+s}{2(k_{new}+k_{old})}$, and the optimal quantity of new batteries purchased is $Q^*_{newPGC}=\frac{2D_{PGC}k_{old}-p_{new2}+p_{old}+V_b(1-\hat{q})-s}{2(k_{new}+k_{old})}$.

(iii) If $\frac{2D_{PGC}k_{new}+p_{new2}-p_{old}-V_b(1-\hat{q})+s}{2(k_{new}+k_{old})}\geq\eta\theta Q_{new1}$, then the optimal quantity of retired batteries purchased is $Q^*_{oldPGC}=\eta\theta Q_{new1}$, and the optimal quantity of new batteries purchased is $Q^*_{newPGC}=D_{PGC}-\eta\theta Q_{new1}$.

From Proposition 5(ii), the impact of the selling prices of retired and new batteries on the PGC's optimal decision is the same as the base model.

Battery manufacturer's decision $\boldsymbol{p_{new2}}$. Anticipating the three cases of the PGC's optimal quantities of retired and new batteries purchased (Q^*_{oldPGC}, Q^*_{newPGC}) as stated in Proposition 5, the battery manufacturer can determine the optimal selling price of new batteries p^*_{new2} in period 2 by substituting (Q^*_{oldPGC}, Q^*_{newPGC}) into its profit function in (6) and considering the corresponding condition for each of the three cases.

By using KKT conditions to solve the battery manufacturer's optimal decision problem, the study gets Proposition 6.

Proposition 6. (i) If $\frac{c_m+2D_{PGC}(2k_{new}+k_{old})-p_{old}-V_b(1-\hat{q})+p_n(1-\varphi)+p_r\varphi+c_e(e_m+e_n-e_n\varphi)+s}{4(k_{new}+k_{old})}\leq 0$, then the optimal selling price of new batteries is $p^*_{new2}=-2D_{PGC}k_{new}+p_{old}+V_b(1-\hat{q})-s$.

(ii) If $0<\frac{c_m+2D_{PGC}(2k_{new}+k_{old})-p_{old}-V_b(1-\hat{q})+p_n(1-\varphi)+p_r\varphi+c_e(e_m+e_n-e_n\varphi)+s}{4(k_{new}+k_{old})}<\eta\theta Q_{new1}$, then the optimal selling price of new batteries is $p^*_{new2}=\frac{1}{2}(c_m+2D_{PGC}k_{old}+p_{old}+V_b(1-\hat{q})+p_n(1-\varphi)+p_r\varphi+c_e(e_m+e_n-e_n\varphi)-s)$.

(iii) If $\frac{c_m+2D_{PGC}(2k_{new}+k_{old})-p_{old}-V_b(1-\hat{q})+p_n(1-\varphi)+p_r\varphi+c_e(e_m+e_n-e_n\varphi)+s}{4(k_{new}+k_{old})} \geq \eta\theta Q_{new1}$, then the optimal selling price of new batteries is $p^*_{new2} = -2D_{PGC}k_{new} + 2\eta\theta Q_{new1}(k_{new}+k_{old}) + p_{old} + V_b(1-\hat{q}) - s$.

Battery recycler's decision p_{old}. Anticipating the battery manufacturer's and the PGC's optimal decisions (p^*_{new2}, Q^*_{oldPGC}) above, the battery recycler can determine the optimal selling price of retired batteries p^*_{old} by substituting (p^*_{new2}, Q^*_{oldPGC}) and considering the updated condition for each of the three cases as shown in Proposition 6.

KKT conditions are also used to solve the battery recycler's optimal decision problem and draw the following proposition.

Proposition 7. (i) If $\frac{c_d+c_m+2D_{PGC}(2k_{new}+k_{old})-V_b(1-\hat{q})+p_n(1-\varphi)+c_e(e_m+e_n-e_n\varphi+e_r\varphi)+s}{8(k_{new}+k_{old})} \leq 0$, then the optimal selling price of new batteries is $p^*_{old} = c_m + 2D_{PGC}(2k_{new}+k_{old}) - V_b(1-\hat{q}) + p_n(1-\varphi) + p_r\varphi + c_e(e_m+e_n-e_n\varphi) + s$.

(ii) If $0 < \frac{c_d+c_m+2D_{PGC}(2k_{new}+k_{old})-V_b(1-\hat{q})+p_n(1-\varphi)+c_e(e_m+e_n-e_n\varphi+e_r\varphi)+s}{8(k_{new}+k_{old})} < \eta\theta Q_{new1}$, then the optimal selling price of new batteries is $p^*_{old} = \frac{1}{2}(c_m + c_e(e_m+e_n) + 2D_{PGC}(2k_{new}+k_{old}) - V_b(1-\hat{q}) + p_n - c_d - (c_e(e_n+e_r) + p_n - 2p_r)\varphi + s)$.

(iii) If $\frac{c_d+c_m+2D_{PGC}(2k_{new}+k_{old})-V_b(1-\hat{q})+p_n(1-\varphi)+c_e(e_m+e_n-e_n\varphi+e_r\varphi)+s}{8(k_{new}+k_{old})} \geq \eta\theta Q_{new1}$, then the optimal selling price of new batteries is $p^*_{old} = c_m + 2D_{PGC}(2k_{new}+k_{old}) - 4\eta\theta Q_{new1}(k_{new}+k_{old}) - V_b(1-\hat{q}) + p_n(1-\varphi) + p_r\varphi + c_e(e_m+e_n-e_n\varphi) + s$.

By further checking the impact of the government subsidy s on the above firms' decisions, the following corollary is derived:

Corollary 6. $\frac{\partial p^*_{old}}{\partial s} > 0, \frac{\partial p^*_{new2}}{\partial s} < 0, \frac{\partial Q^*_{oldPGC}}{\partial s} > 0, \frac{\partial Q^*_{newPGC}}{\partial s} < 0.$

Corollary 6 reveals that the optimal selling price of retired batteries p^*_{old} increases with the increase of purchase subsidy s provided by the government, and the optimal selling price of new batteries p^*_{new2} decreases with the increase of s. Also, the optimal quantity of retired batteries purchased Q^*_{oldPGC} increases with the increase of s, so that the corresponding optimal quantity of new batteries purchased Q^*_{newPGC} decreases with the increase of s. These results can be explained as follows. When the government increases the purchase subsidy, the PGC will be more motivated to purchase retired batteries due to the lower effective purchase price and corresponding higher profit of each retired battery, resulting in an increase in the quantity of retired batteries purchased. Furthermore, the battery recycler will set a higher price for retired batteries to make more profit. As the battery recycler is in a leading position over the PGC in the game, the increase in the selling price of retired batteries allows the recycler to capture a portion of the profit from the government subsidy by squeezing the profit margin of the PGC. On the other hand, the quantity of new batteries purchased by the PGC will decrease because the total demand for batteries is constant. In order to maintain the sales of new batteries, the battery manufacturer will reduce the selling price of new batteries so that the cost of purchasing each new battery will be also lower for the PGC, thereby improving the competitiveness of new batteries in the market.

Government's decision s. Anticipating the three firms' optimal decisions $(p_{old}^*, p_{new2}^*, Q_{oldPGC}^*)$ above, the government can determine the optimal per-unit purchase subsidy s that maximizes the social welfare by substituting $(p_{old}^*, p_{new2}^*, Q_{oldPGC}^*)$ and considering the updated condition for each of the three cases as shown in Proposition 7.

By using KKT conditions to solve the government's optimal decision problem and then back-substituting s^*, the optimal decisions of the government and the three firms in the three different cases are given by the following proposition.

Proposition 8. (i) If $\frac{c_d+c_m+2D_{PGC}k_{new}-V_b(1-\hat{q})+p_n(1-\varphi)+c_e(e_m+e_n-e_n\varphi+e_r\varphi)}{2(k_{new}+k_{old})} \le 0$, then the government and the three firms' optimal decisions are as follows:

$$s^{S*} = \begin{pmatrix} V_b(1-\hat{q}) - 2D_{PGC}(2k_{new}+k_{old}) - c_m - c_d \\ - p_n(1-\varphi) - c_e(e_m+e_n-e_n\varphi+e_r\varphi) \end{pmatrix}, \quad (38)$$

$$p_{old}^{S*} = (p_r - c_e e_r)\varphi - c_d, \quad (39)$$

$$p_{new2}^{S*} = c_m + 2D_{PGC}(k_{new}+k_{old}) + p_n(1-\varphi) + p_r\varphi + c_e(e_m+e_n-e_n\varphi), \quad (40)$$

$$Q_{oldPGC}^{S*} = 0, \quad (41)$$

$$Q_{newPGC}^{S*} = D_{PGC}. \quad (42)$$

Additionally, the corresponding profits of the three firms and the social welfare are as follows:

$$\Pi_r^{S*} = \theta Q_{new1}((p_r - c_e e_r)\varphi - c_d - c_r), \quad (43)$$

$$\Pi_{m2}^{S*} = 2D_{PGC}^2(k_{new}+k_{old}) - (c_e e_n + p_n - p_r)(D_{PGC} - \theta Q_{new1})\varphi, \quad (44)$$

$$\Pi_{PGC}^{S*} = D_{PGC}\begin{pmatrix} V_b - c_m - D_{PGC}(3k_{new}+2k_{old}) \\ - p_n(1-\varphi) - p_r\varphi - c_e(e_m+e_n-e_n\varphi) \end{pmatrix}, \quad (45)$$

$$SW^{S*} = \begin{pmatrix} D_{PGC}(V_b - c_m - D_{PGC}k_{new} - c_e(e_m+e_n) - p_n) \\ +\varphi\theta Q_{new1}(c_e(e_n-e_r)+p_n) - \theta Q_{new1}(c_d+c_r) \end{pmatrix}. \quad (46)$$

(ii) If $0 < \frac{c_d+c_m+2D_{PGC}k_{new}-V_b(1-\hat{q})+p_n(1-\varphi)+c_e(e_m+e_n-e_n\varphi+e_r\varphi)}{2(k_{new}+k_{old})} < \eta\theta Q_{new1}$, then the government and the three firms' optimal decisions are as follows:

$$s^{S*} = \begin{pmatrix} 3c_d + 3c_m + D_{PGC}(4k_{new} - 2k_{old}) - 3V_b(1-\hat{q}) \\ +3p_n(1-\varphi) + 3c_e(e_m + e_n - e_n\varphi + e_r\varphi) \end{pmatrix}, \tag{47}$$

$$p_{old}^{S*} = \begin{pmatrix} 2c_m + 2c_e(e_m + e_n) + 4D_{PGC}k_{new} - 2V_b(1-\hat{q}) \\ +2p_n + c_d - (c_e(2e_n - e_r) + 2p_n - p_r)\varphi \end{pmatrix}, \tag{48}$$

$$p_{new2}^{S*} = 2D_{PGC}k_{old} + V_b(1-\hat{q}) + (p_r - c_e e_r)\varphi - c_d, \tag{49}$$

$$Q_{oldPGC}^{S*} = \frac{c_d + c_m + 2D_{PGC}k_{new} - V_b(1-\hat{q}) + p_n(1-\varphi) + c_e(e_m + e_n - e_n\varphi + e_r\varphi)}{2(k_{new} + k_{old})}, \tag{50}$$

$$Q_{newPGC}^{S*} = \frac{-c_d - c_m + 2D_{PGC}k_{old} + V_b(1-\hat{q}) - p_n(1-\varphi) - c_e(e_m + e_n - e_n\varphi + e_r\varphi)}{2(k_{new} + k_{old})}. \tag{51}$$

Additionally, the corresponding profits of the three firms and the social welfare are Π_r^{S*}, Π_{m2}^{S*}, Π_{PGC}^{S*} and SW^{S*}, which are described in Appendix (A.4), (A.5), (A.6) and (A.7).

(iii) If $\frac{c_d + c_m + 2D_{PGC}k_{new} - V_b(1-\hat{q}) + p_n(1-\varphi) + c_e(e_m + e_n - e_n\varphi + e_r\varphi)}{2(k_{new} + k_{old})} \geq \eta\theta Q_{new1}$, then the government and the three firms' optimal decisions are as follows:

$$s^{S*} = \begin{pmatrix} V_b(1-\hat{q}) + 8\eta\theta Q_{new1}(k_{new} + k_{old}) - 2D_{PGC}(2k_{new} + k_{old}) \\ - c_m - c_d - p_n(1-\varphi) - c_e(e_m + e_n - e_n\varphi + e_r\varphi) \end{pmatrix}, \tag{52}$$

$$p_{old}^{S*} = 4\eta\theta Q_{new1}(k_{new} + k_{old}) + (p_r - c_e e_r)\varphi - c_d, \tag{53}$$

$$p_{new2}^{S*} = \begin{pmatrix} c_m + 2(k_{new} + k_{old})(D_{PGC} - \eta\theta Q_{new1}) \\ + p_n(1-\varphi) + p_r\varphi + c_e(e_m + e_n - e_n\varphi) \end{pmatrix}, \tag{54}$$

$$Q_{oldPGC}^{S*} = \eta\theta Q_{new1}, \tag{55}$$

$$Q_{newPGC}^{S*} = D_{PGC} - \eta\theta Q_{new1}. \tag{56}$$

Additionally, the corresponding profits of the three firms and the social welfare are as follows:

$$\Pi_r^{S*} = \theta Q_{new1}\left(4\eta^2\theta Q_{new1}(k_{new} + k_{old}) - c_r - c_d + (p_r - c_e e_r)\varphi\right), \tag{57}$$

$$\Pi_{m2}^{S*} = 2(k_{new} + k_{old})(D_{PGC} - \eta\theta Q_{new1})^2 - (c_e e_n + p_n - p_r)(D_{PGC} - \theta Q_{new1})\varphi, \tag{58}$$

$$\Pi_{PGC}^{S*} = \begin{pmatrix} D_{PGC}(V_b - c_m - p_n) + \left(2D_{PGC}\eta\theta Q_{new1} + \eta^2\theta^2 Q_{new1}^2\right)(k_{new} + k_{old}) \\ + D_{PGC}(p_n - p_r)\varphi - D_{PGC}^2(3k_{new} + 2k_{old}) - c_e D_{PGC}(e_m + e_n - e_n\varphi) \end{pmatrix}, \tag{59}$$

$$SW^{S*} = \begin{pmatrix} D_{PGC}V_b + \eta\theta Q_{new1}(2D_{PGC}k_{new} + p_n - V_b(1-\hat{q})) \\ -\theta Q_{new1}(c_d + c_r - c_d\eta) - \eta^2\theta^2 Q_{new1}^2(k_{new} + k_{old}) \\ -(c_m + c_e(e_m + e_n))(D_{PGC} - \eta\theta Q_{new1}) - D_{PGC}^2 k_{new} \\ - D_{PGC}p_n + (p_n + c_e(e_n - e_r))\theta Q_{new1}(1-\eta)\varphi \end{pmatrix}. \tag{60}$$

The following corollary explains the connotation of the PGC's different optimal decisions in the above three cases from the perspective of the limited quantity of retired batteries:

Corollary 7. (i) When $\frac{c_d+c_m+2D_{PGC}k_{new}-V_b(1-\hat{q})+p_n(1-\varphi)+c_e(e_m+e_n-e_n\varphi+e_r\varphi)}{2(k_{new}+k_{old})} \leq 0$, it is optimal for the PGC not to purchase retired batteries.

(ii) When $0 < \frac{c_d+c_m+2D_{PGC}k_{new}-V_b(1-\hat{q})+p_n(1-\varphi)+c_e(e_m+e_n-e_n\varphi+e_r\varphi)}{2(k_{new}+k_{old})} < \eta\theta Q_{new1}$, the quantity of retired batteries that the PGC wants to purchase is less than the quantity of retired batteries provided by the battery recycler, meaning that the supply of retired batteries is sufficient for the PGC, so the PGC will purchase a portion of the retired batteries provided by the battery recycler for echelon utilization, the optimal quantity of which is $Q_{oldPGC}^{S*} = \frac{c_d+c_m+2D_{PGC}k_{new}-V_b(1-\hat{q})+p_n(1-\varphi)+c_e(e_m+e_n-e_n\varphi+e_r\varphi)}{2(k_{new}+k_{old})}$.

(iii) When $\frac{c_d+c_m+2D_{PGC}k_{new}-V_b(1-\hat{q})+p_n(1-\varphi)+c_e(e_m+e_n-e_n\varphi+e_r\varphi)}{2(k_{new}+k_{old})} \geq \eta\theta Q_{new1}$, the quantity of retired batteries that the PGC wants to purchase exceeds the quantity of retired batteries provided by the battery recycler, meaning that the supply of retired batteries is insufficient for the PGC, so the PGC can only purchase all the retired batteries provided by the battery recycler for echelon utilization at most.

Based on Corollary 2 and Corollary 7, the PGC's decisions to purchase used batteries for echelon utilization under two scenarios of non-subsidization and subsidization can be summarized in Table 2.

Table 2 Purchase decisions of PGC

	Non-subsidization		Subsidization	
	Q_{oldPGC}^{N*}	Q_{newPGC}^{N*}	Q_{oldPGC}^{S*}	Q_{newPGC}^{S*}
Case (i): not purchasing retired batteries	Eq. (20)	Eq. (21)	Eq. (41)	Eq. (42)
Case (ii): purchasing a portion of retired batteries for reuse	Eq. (27)	Eq. (28)	Eq. (50)	Eq. (51)
Case (iii): purchasing all retired batteries foe reuse	Eq. (31)	Eq. (32)	Eq. (55)	Eq. (56)

Similar to Corollary 4, Corollary 8 shows the impact of quality factor, average quality and quality variance of reusable batteries $\hat{q}$ on the equilibrium solutions given in Proposition 8(ii).

Corollary 8. (i) $\frac{\partial p_{old}^{S*}}{\partial \hat{q}} > 0$, $\frac{\partial p_{new2}^{S*}}{\partial \hat{q}} < 0$, $\frac{\partial Q_{oldPGC}^{S*}}{\partial \hat{q}} > 0$, $\frac{\partial Q_{newPGC}^{S*}}{\partial \hat{q}} < 0$, $\frac{\partial s^{S*}}{\partial \hat{q}} > 0$. (ii) $\frac{\partial p_{old}^{S*}}{\partial \bar{q}} > 0$, $\frac{\partial p_{new2}^{S*}}{\partial \bar{q}} < 0$, $\frac{\partial Q_{oldPGC}^{S*}}{\partial \bar{q}} > 0$, $\frac{\partial Q_{newPGC}^{S*}}{\partial \bar{q}} < 0$, $\frac{\partial s^{S*}}{\partial \bar{q}} > 0$; $\frac{\partial p_{old}^{S*}}{\partial \sigma^2} < 0$, $\frac{\partial p_{new2}^{S*}}{\partial \sigma^2} > 0$, $\frac{\partial Q_{oldPGC}^{S*}}{\partial \sigma^2} < 0$, $\frac{\partial Q_{newPGC}^{S*}}{\partial \sigma^2} > 0$, $\frac{\partial s^{S*}}{\partial \sigma^2} < 0$; $\frac{\partial p_{old}^{S*}}{\partial \xi} < 0$, $\frac{\partial p_{new2}^{S*}}{\partial \xi} > 0$, $\frac{\partial Q_{oldPGC}^{S*}}{\partial \xi} < 0$, $\frac{\partial Q_{newPGC}^{S*}}{\partial \xi} > 0$, $\frac{\partial s^{S*}}{\partial \xi} < 0$.

Corollary 8(i) demonstrates that the impact of $\hat{q}$ on the three firms' optimal decisions is the same as the base model. For the government, the optimal per-unit purchase subsidy s^{S*} increases with the increase of $\hat{q}$. This is because the higher quality of retired batteries will result in the higher selling price of them, the government will provide higher purchase subsidy to enhance the motivation of the PGC to purchase retired batteries.

Corollary 8(ii) demonstrates that the impacts of average quality $\bar{q}$ and variance σ^2 on the three firms' optimal decisions are the same as the base model. Moreover, for the government, the optimal per-unit purchase subsidy s^{S*} increases with the increase of $\bar{q}$ and decrease of σ^2 and ξ. This is mainly because the higher the quality of the retired batteries and the smaller the quality fluctuations, the more motivated the government will be to offer higher purchase subsidies, thereby enhancing the PGC's enthusiasm for purchasing these batteries.

Then, Corollary 9 shows the impact of installation cost coefficients k_{new} and k_{old} on the equilibrium solutions.

Corollary 9. (i) $\frac{\partial p_{old}^{S*}}{\partial k_{new}} > 0$, $\frac{\partial p_{new2}^{S*}}{\partial k_{new}} = 0$, $\frac{\partial p_{old}^{S*}}{\partial k_{old}} = 0$, $\frac{\partial p_{new2}^{S*}}{\partial k_{old}} > 0$; (ii) $\frac{\partial Q_{oldPGC}^{S*}}{\partial k_{new}} > 0$, $\frac{\partial Q_{newPGC}^{S*}}{\partial k_{new}} < 0$, $\frac{\partial Q_{oldPGC}^{S*}}{\partial k_{old}} < 0$, $\frac{\partial Q_{newPGC}^{S*}}{\partial k_{old}} > 0$. (iii) $\frac{\partial s^{S*}}{\partial k_{new}} > 0$, $\frac{\partial s^{S*}}{\partial k_{old}} < 0$.

Corollary 9(i) implies that the optimal selling price of retired batteries p_{old}^{S*} is directly proportional to k_{new}, and the optimal selling price of new batteries p_{new2}^{S*} is directly proportional to k_{old}. The difference from the base model is that p_{old}^{S*} is independent of k_{old}, and p_{new2}^{S*} is independent of k_{new}. This is because, the existence of the optimal per-unit purchase subsidy s^{S*} compensates for p_{old}^{S*} and p_{new2}^{S*}.

Corollary 9(ii) indicates that the optimal quantity of retired batteries purchased Q_{oldPGC}^{S*} increases with the increase of k_{new} and decreases with the increase of k_{old}. Compared to the base model, there are no counterintuitive results. The reason is that as k_{new} or k_{old} changes, the gap between the price of new batteries p_{new2}^{S*} and the effective price of retired batteries $\left(p_{old}^{S*} - s^{S*}\right)$ remains unchanged due to the compensation from the purchase subsidy, so that the PGC' optimal decision is only affected by installation cost.

Corollary 9(iii) demonstrates that the optimal per-unit purchase subsidy s^{S*} increases with the increase of k_{new} and decreases with the increase of k_{old}. This observation is counterintuitive and it can be explained as follows: when k_{new} increases, battery recycler will raise the selling price of retired batteries, the government will provide higher purchase subsidy to incentivize the PGC to purchase retired batteries. On the other hand, when k_{old} increases, the battery manufacturer will raise the selling price of new batteries, which will enhance the motivation of the PGC to purchase retired batteries, the government will reduce the purchase subsidy, thereby saving fiscal expenditure.

Corollary 10. (i) $\frac{\partial p_{old}^{S*}}{\partial e_m} > 0$, $\frac{\partial p_{new2}^{S*}}{\partial e_m} = 0$, $\frac{\partial p_{old}^{S*}}{\partial e_n} > 0$, $\frac{\partial p_{new2}^{S*}}{\partial e_n} = 0$, $\frac{\partial p_{old}^{S*}}{\partial e_r} > 0$, $\frac{\partial p_{new2}^{S*}}{\partial e_r} < 0$;

(ii) $\frac{\partial Q_{oldPGC}^{S*}}{\partial e_m} > 0$, $\frac{\partial Q_{newPGC}^{S*}}{\partial e_m} < 0$, $\frac{\partial Q_{oldPGC}^{S*}}{\partial e_n} > 0$, $\frac{\partial Q_{newPGC}^{S*}}{\partial e_n} < 0$, $\frac{\partial Q_{oldPGC}^{S*}}{\partial e_r} > 0$, $\frac{\partial Q_{newPGC}^{S*}}{\partial e_r} < 0$;

(iii) $\frac{\partial s^{S*}}{\partial e_m} > 0$, $\frac{\partial s^{S*}}{\partial e_n} > 0$, $\frac{\partial s^{S*}}{\partial e_r} > 0$.

Corollary 10(i) shows that the optimal selling price of retired batteries p_{old}^{S*} remains increasing with the increase of e_m or e_n, and the optimal selling price of new batteries p_{new2}^{S*} remains decreasing with the increase of e_r. Due to the compensation from the optimal purchase subsidy s^{S*}, p_{old}^{S*} becomes directly proportional to e_r, and p_{new2}^{S*} becomes independent of e_m and e_n. What's more, as established in Corollary 10(ii), the relationship between the quantities of retired and new batteries purchased by the PGC and the carbon emissions remains unchanged. This can be explained by Corollary 10(iii), as e_m, e_n or e_r increases, the government will raise the purchase subsidy to offset the increase in the selling price of retired batteries, thereby promoting the echelon utilization.

4.3 Model Comparison

Without loss of generality, the paper compares the equilibrium solutions under the two scenarios of no purchase subsidy and purchase subsidy, respectively given in Proposition 4(ii) and Proposition 8(ii).

Power Grid Company's decision and profit. The purpose of the government' purchase subsidy policy is to incentivize the PGC to purchase more retired batteries, thereby promoting the echelon utilization of retired batteries. To examine the effectiveness of government subsidy, the PGC's optimal decisions and corresponding profits under the two different scenarios are compared in the following corollary:

Corollary 11. (i) If $3c_d + 3c_m + D_{PGC}(4k_{new} - 2k_{old}) - 3V_b(1-\hat{q}) + 3p_n(1-\varphi) + 3c_e(e_m + e_n - e_n\varphi + e_r\varphi) > 0$, then $Q_{oldPGC}^{S*} > Q_{oldPGC}^{N*}$; If $3c_d + 3c_m + D_{PGC}(4k_{new} - 2k_{old}) - 3V_b(1-\hat{q}) + 3p_n(1-\varphi) + 3c_e(e_m + e_n - e_n\varphi + e_r\varphi) \le 0$, then $Q_{oldPGC}^{S*} \le Q_{oldPGC}^{N*}$.

(ii) If $3c_d + 3c_m + D_{PGC}(4k_{new} - 2k_{old}) - 3V_b(1-\hat{q}) + 3p_n(1-\varphi) + 3c_e(e_m + e_n - e_n\varphi + e_r\varphi) > 0$, then $\Pi_{PGC}^{S*} > \Pi_{PGC}^{N*}$; If $3c_d + 3c_m + D_{PGC}(4k_{new} - 2k_{old}) - 3V_b(1-\hat{q}) + 3p_n(1-\varphi) + 3c_e(e_m + e_n - e_n\varphi + e_r\varphi) \le 0$, then $\Pi_{PGC}^{S*} \le \Pi_{PGC}^{N*}$.

Corollary 11 reveals that the purchase subsidy policy is not necessarily effective in improving the secondary utilization rate of retired batteries. Only when the certain condition $3c_d + 3c_m + D_{PGC}(4k_{new} - 2k_{old}) - 3V_b(1-\hat{q}) + 3p_n(1-\varphi) + 3c_e(e_m + e_n - e_n\varphi + e_r\varphi) > 0$ is met does the government need to provide the purchase subsidy for the PGC. What's more, under this condition, the purchase subsidy can bring higher profit for the PGC.

Battery manufacturer's decision and profit. By comparing the battery manufacturer' optimal decisions and corresponding profits under the two different scenarios, the following corollary can be obtained:

Corollary 12. (i) If $3c_d + 3c_m + D_{PGC}(4k_{new} - 2k_{old}) - 3V_b(1-\hat{q}) + 3p_n(1-\varphi) + 3c_e(e_m + e_n - e_n\varphi + e_r\varphi) > 0$, then $p_{new2}^{S*} < p_{new2}^{N*}$; If $3c_d + 3c_m + D_{PGC}(4k_{new} - 2k_{old}) - 3V_b(1-\hat{q}) + 3p_n(1-\varphi) + 3c_e(e_m + e_n - e_n\varphi + e_r\varphi) \le 0$, then $p_{new2}^{S*} \ge p_{new2}^{N*}$.

(ii) If $3c_d + 3c_m + D_{PGC}(4k_{new} - 2k_{old}) - 3V_b(1-\hat{q}) + 3p_n(1-\varphi) + 3c_e(e_m + e_n - e_n\varphi + e_r\varphi) > 0$, then $\Pi_{m2}^{S*} < \Pi_{m2}^{N*}$; If $3c_d + 3c_m + D_{PGC}(4k_{new} - 2k_{old}) - 3V_b(1-\hat{q}) + 3p_n(1-\varphi) + 3c_e(e_m + e_n - e_n\varphi + e_r\varphi) \le 0$, then $\Pi_{m2}^{S*} \ge \Pi_{m2}^{N*}$.

Corollary 12 shows that under the certain condition $3c_d + 3c_m + D_{PGC}(4k_{new} - 2k_{old}) - 3V_b(1-\hat{q}) + 3p_n(1-\varphi) + 3c_e(e_m + e_n - e_n\varphi + e_r\varphi) > 0$, the purchase subsidy can bring lower selling price of new batteries and lower profit for the battery manufacturer.

Battery recycler's decision and profit. By comparing the battery recycler' optimal decisions and corresponding profits under the two different scenarios, the following corollary can be drawn:

Corollary 13. (i) If $3c_d + 3c_m + D_{PGC}(4k_{new} - 2k_{old}) - 3V_b(1-\hat{q}) + 3p_n(1-\varphi) + 3c_e(e_m + e_n - e_n\varphi + e_r\varphi) > 0$, then $p_{old}^{S*} > p_{old}^{N*}$; If $3c_d + 3c_m + D_{PGC}(4k_{new} - 2k_{old}) - 3V_b(1-\hat{q}) + 3p_n(1-\varphi) + 3c_e(e_m + e_n - e_n\varphi + e_r\varphi) \le 0$, then $p_{old}^{S*} \le p_{old}^{N*}$.

(ii) If $3c_d + 3c_m + D_{PGC}(4k_{new} - 2k_{old}) - 3V_b(1-\hat{q}) + 3p_n(1-\varphi) + 3c_e(e_m + e_n - e_n\varphi + e_r\varphi) > 0$, then $\Pi_r^{S*} > \Pi_r^{N*}$; If $3c_d + 3c_m + D_{PGC}(4k_{new} - 2k_{old}) - 3V_b(1-\hat{q}) + 3p_n(1-\varphi) + 3c_e(e_m + e_n - e_n\varphi + e_r\varphi) \le 0$, then $\Pi_r^{S*} \le \Pi_r^{N*}$.

Corollary 13 implies that under the certain condition $3c_d + 3c_m + D_{PGC}(4k_{new} - 2k_{old}) - 3V_b(1-\hat{q}) + 3p_n(1-\varphi) + 3c_e(e_m + e_n - e_n\varphi + e_r\varphi) > 0$, the purchase subsidy can bring higher selling price of retired batteries and higher profit for the battery recycler.

Combining the above results, it can be concluded that when government's purchase subsidy policy can incentivize the PGC to purchase more retired batteries, the purchase subsidy is beneficial to both the PGC and the battery recycler, but detrimental to the battery manufacturer. Corollary 14 further explains the distribution of the government subsidy between the PGC and the battery recycler.

Corollary 14. When $3c_d + 3c_m + D_{PGC}(4k_{new} - 2k_{old}) - 3V_b(1-\hat{q}) + 3p_n(1-\varphi) + 3c_e(e_m + e_n - e_n\varphi + e_r\varphi) > 0$, $p_{old}^{S*} - p_{old}^{N*} = p_{old}^{N*} - \left(p_{old}^{S*} - s^{S*}\right) = \frac{1}{2}(3c_d + 3c_m + D_{PGC}(4k_{new} - 2k_{old}) - 3V_b(1-\hat{q}) + 3p_n(1-\varphi) + 3c_e(e_m + e_n - e_n\varphi + e_r\varphi)) = \frac{1}{2}s^{S*} > 0$.

Corollary 14 indicates that under the certain condition $3c_d + 3c_m + D_{PGC}(4k_{new} - 2k_{old}) - 3V_b(1-\hat{q}) + 3p_n(1-\varphi) + 3c_e(e_m + e_n - e_n\varphi + e_r\varphi) > 0$, both the battery recycler and the PGC benefit from the government's purchase subsidy policy. Specifically, the battery recycler can set higher selling price of retired batteries, while the PGC can purchase retired batteries at a lower effective price. Furthermore, the purchase subsidy is distributed equally between the PGC and the battery recycler.

Social Welfare. The study defines the social welfare in the base model as the total profits of the three firms. The social welfare under the two different scenarios is compared in the following corollary:

Corollary 15. $SW^{S*} - SW^{N*} \geq 0$.

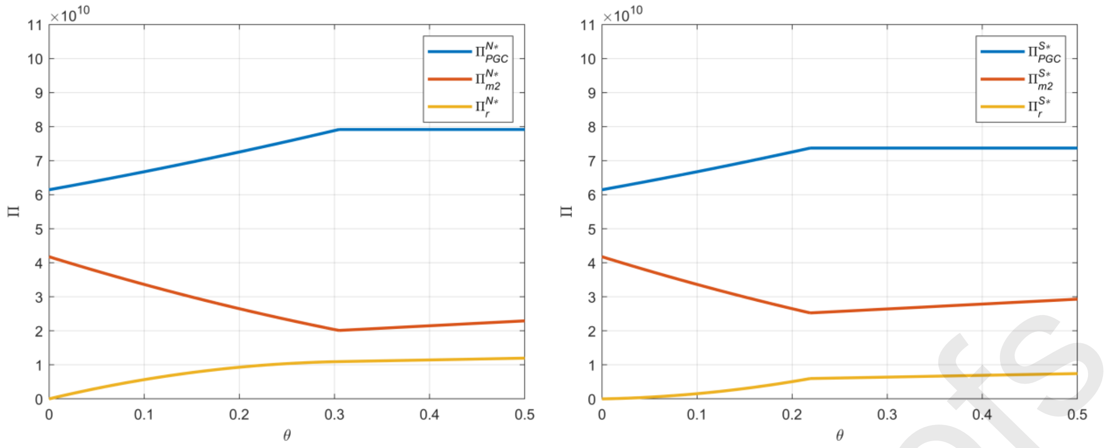
Corollary 15 demonstrates that the social welfare in the scenario where the government provides the subsidy for retired battery echelon utilization is always higher than or equal to the social welfare in the scenario where the government does not provide the subsidy.

5 Numerical Analysis

In Section 4, this paper has analyzed the impact of the average quantity of reusable batteries $\hat{q}$, and the installation cost coefficients k_{new} and k_{old}, the carbon emissions e_m, e_n and e_r on the optimal decisions of the three firms and the government. This section will further discuss the impact of the important parameters on the profits of supply chain members and the PGC's purchase decision cases through numerical analysis. According to the actual situation and combined with the existing literature (Abdel-Monem et al., 2017; Gu et al., 2021; Liu and Wang, 2022; Shao et al., 2018; Xiao et al., 2024; Yu et al., 2022), the model parameters are set as follows: $\alpha = 17000000$, $\beta = 100$, $c_{EV} = 70000$, $c_m = 10000$, $p_n = 40000$, $c_e = 2000$, $e_m = 2$, $e_n = 3$, $e_r = 1$, $D_{PGC} = 600000$, $\hat{q} = 0.8$, $\bar{q} = 0.75$, $\sigma^2 = 0.1$, $\xi = 0.5$, $V_b = 250000$, $k_{new} = 0.03$, $k_{old} = 0.04$, $c_r = 8000$, $c_d = 12000$, $p_r = 30000$, $\varphi = 0.9$, $\theta = 0.3$, $\eta = 0.6$. When discussing the impact of k_{new} and k_{old}, the study ensures that $k_{new} < k_{old}$.

5.1 Impact of parameters on profits of supply chain members

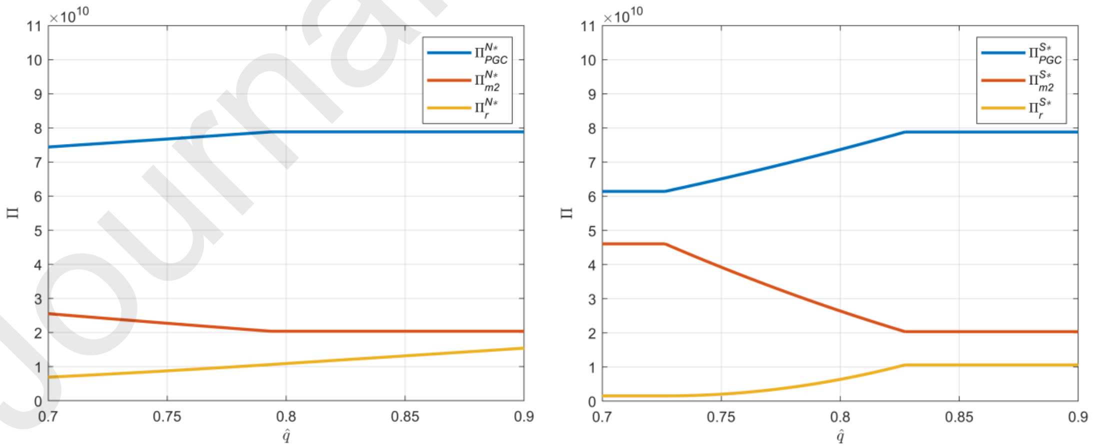
Although the battery return yield θ does not influence the optimal decisions of supply chain members in the general case, it does affect the upper limit of the constraint condition $0 \leq Q_{oldPGC} \leq \eta\theta Q_{new1}$, which represents the maximum quantity of retired batteries that the PGC can purchase. It is necessary to check the impact of θ on the profits of supply chain members. Figure 4 shows that under the two scenarios, as θ increases, the profit for the PGC first increases and then remains constant, the profit for the battery manufacturer first decreases and then increases, and the profit for the battery recycler increases. This is because, as the battery return yield increases from a very low value, the increase in the quantity of retired batteries provided by the battery recycler allows the PGC to purchase more retired batteries to enhance its profit. Simultaneously, the profit for the battery recycler increases, while the profit for the battery manufacturer decreases due to a reduction in the sales of new batteries. When the battery return yield reaches a certain threshold, the quantity of retired batteries purchased by the PGC has already maximized its profit and will not continue to increase. Consequently, the profit for the PGC remains unchanged, while the profits for the manufacturer and the recycler increase due to the increased sales and use of recycled materials.



(a) With no purchase subsidy (b) With purchase subsidy

Figure 4 Impact of θ on profits of supply chain members

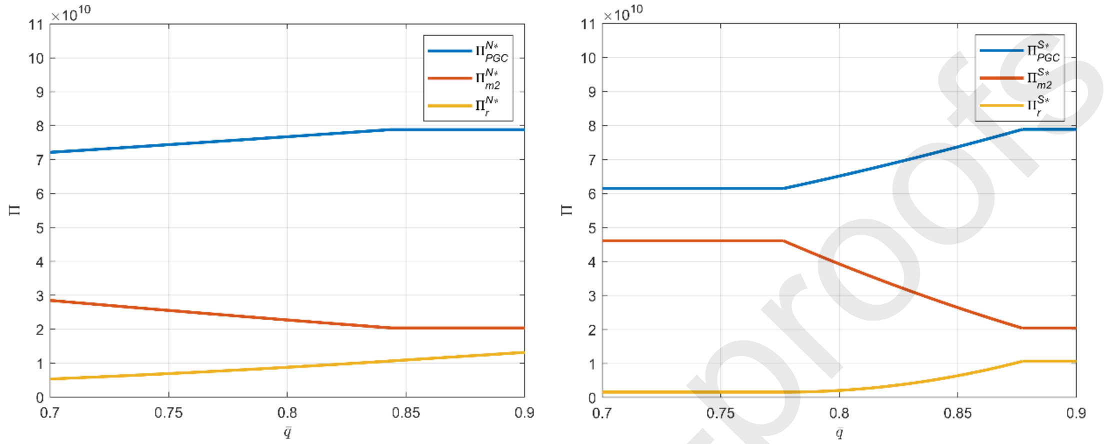
Figure 5 illustrates the impact of $\hat{q}$ on the profits of supply chain members. In both scenarios, as $\hat{q}$ increases, the profits for the PGC and the battery recycler rise, while the profit for the battery manufacturer decreases. This can be explained by the fact that the higher the quality of reusable batteries, the more profitable it is for the PGC to purchase them, so the PGC will purchase more retired batteries and its profit will increase. And the battery recycler will raise the selling price of retired batteries, further boosting its profit. Meanwhile, with a decline in both sales and selling price of new batteries, the battery manufacturer’s profit will diminish.



(a) With no purchase subsidy (b) With purchase subsidy

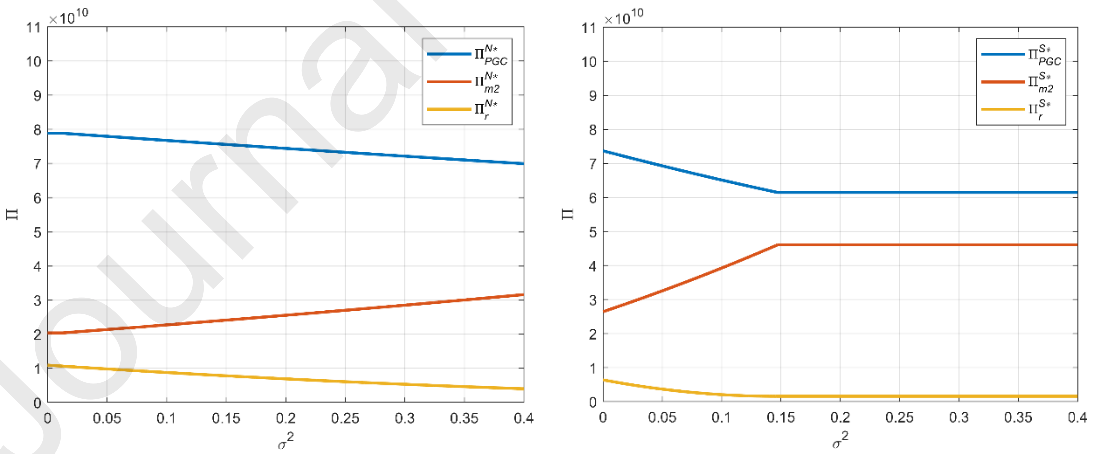
Figure 5 Impact of $\hat{q}$ on profits of supply chain members

Figure 6 and Figure 7 illustrate the impact of $\overline{q}$ and σ^2 on the profits of supply chain members respectively. In both scenarios, as $\overline{q}$ increases, the profits for the PGC and the battery recycler rise, while the profit for the battery manufacturer declines. And the impact of σ^2 is opposite. These results can be explained as followed. The higher the averaged quality of reusable batteries or the lower the variance of quality distribution, the higher the quality factor of reusable batteries that has a positive impact on the echelon utilization. Therefore, the profits for the PGC and the battery recycler will increase. Meanwhile, the battery manufacturer' profit will decrease due to the lower sales of new batteries.



(a) With no purchase subsidy (b) With purchase subsidy

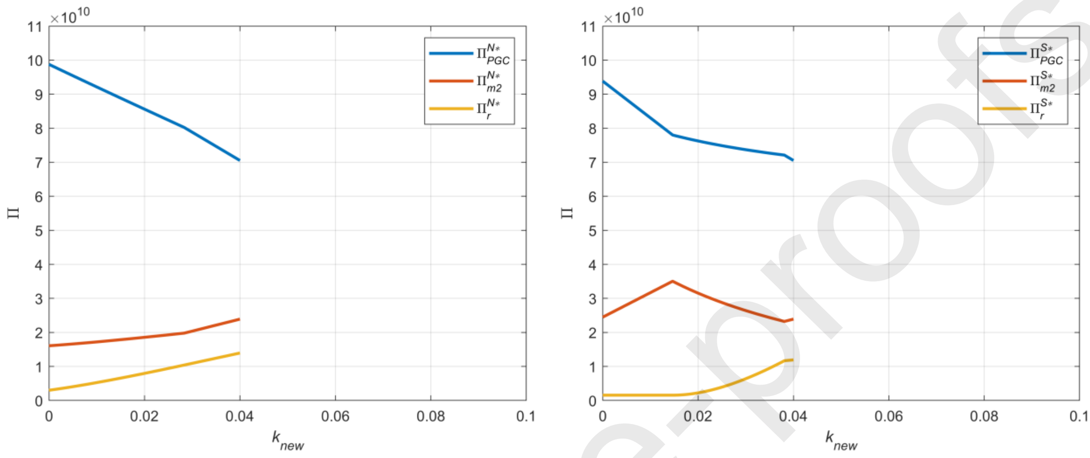
Figure 6 Impact of $\bar{q}$ on profits of supply chain members



(a) With no purchase subsidy (b) With purchase subsidy

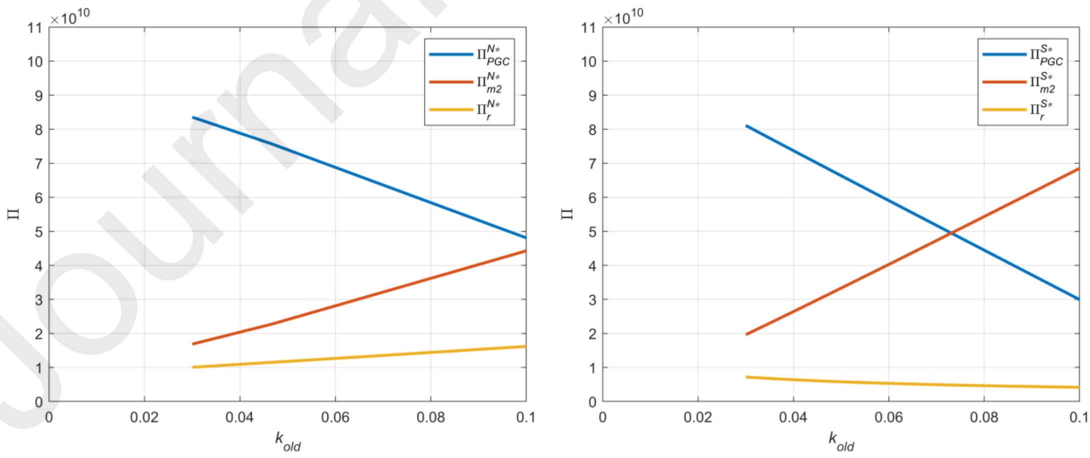
Figure 7 Impact of σ^2 on profits of supply chain members

Figure 8 and Figure 9 describe the impact of k_{new} and k_{old} on the profits of supply chain members respectively. Based on Corollary 4 and Corollary 9, the changes in the profits can be explained as follows. When the government does not provide the subsidy, as k_{new} or k_{old} increases, the profit for the PGC decreases, and the profits for both the battery manufacturer and the battery recycler increase because the selling prices of both types of batteries increase. However, when the government provides the subsidy, with the increase of k_{new}, the selling price of new batteries remains unchanged, and the quantity of new batteries purchased by the PGC decreases, so the profit for the battery manufacturer declines. Similarly, as k_{old} increases, the selling price of retired batteries remains unchanged, and the quantity of retired batteries purchased by the PGC decreases, leading to a decrease in the battery recycler's profit.



(a) With no purchase subsidy (b) With purchase subsidy

Figure 8 Impact of k_{new} on profits of supply chain members

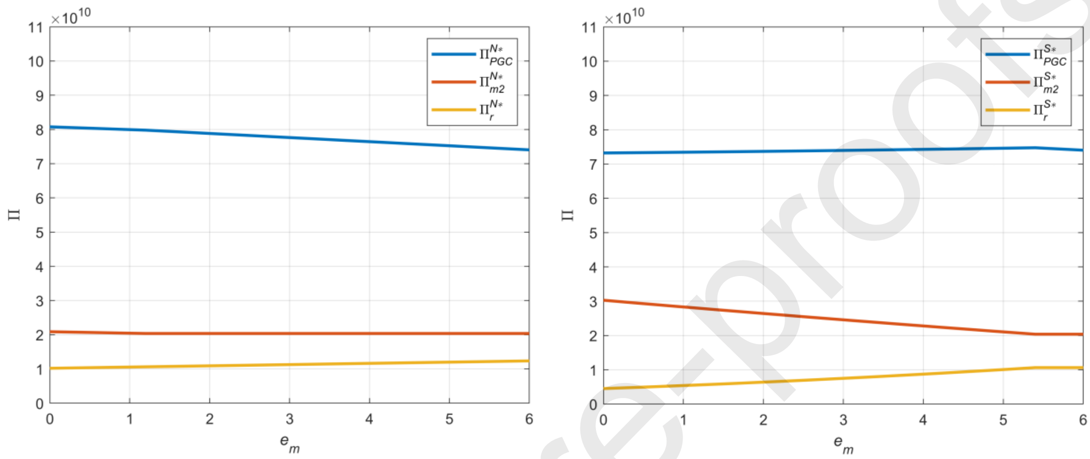


(a) With no purchase subsidy (b) With purchase subsidy

Figure 9 Impact of k_{old} on profits of supply chain members

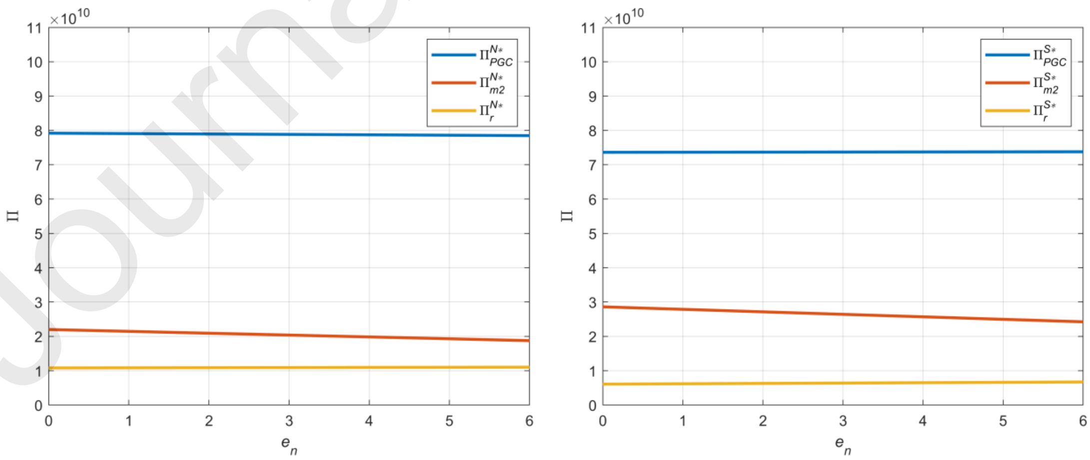
Figure 10, Figure 11 and Figure 12 describe the impact of e_m, e_n and e_r on the profits of supply chain members respectively. When the government does not provide the subsidy, as e_m or e_n

increases, the profits for both the PGC and the battery manufacturer decrease, and the profit for the battery recycler increases. This is because, the selling prices of new and retired batteries increase and the PGC purchases more retired batteries. And as e_r increases, the profit for the PGC increases, and the profits for both the battery manufacturer and the battery recycler decrease because the selling prices of both types of batteries decrease. When the government provides the subsidy, as e_m or e_n increases, the profit for the PGC shifts towards growth. And as e_r increases, the profit for the battery recycler also shifts towards growth. These shifts are attributed to the compensation from the purchase subsidy.



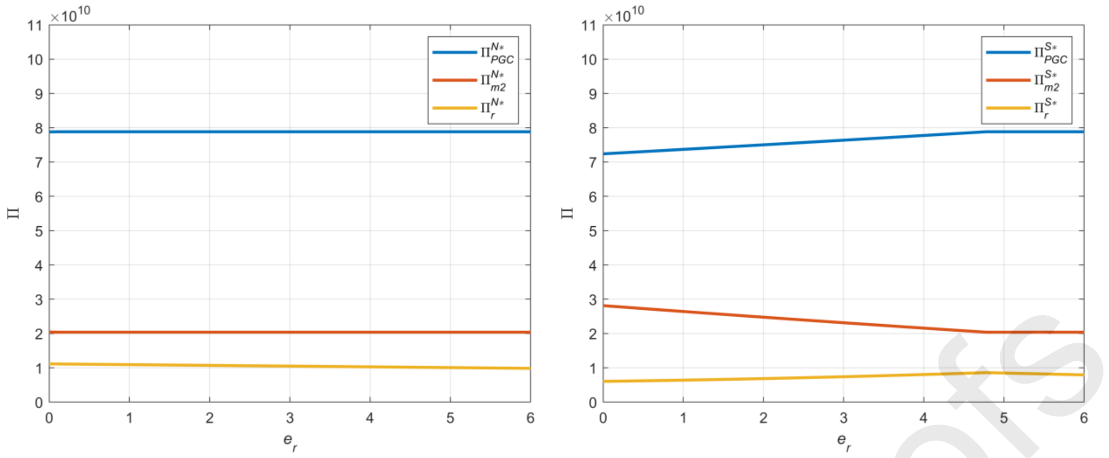
(a) With no purchase subsidy (b) With purchase subsidy

Figure 10 Impact of e_m on profits of supply chain members



(a) With no purchase subsidy (b) With purchase subsidy

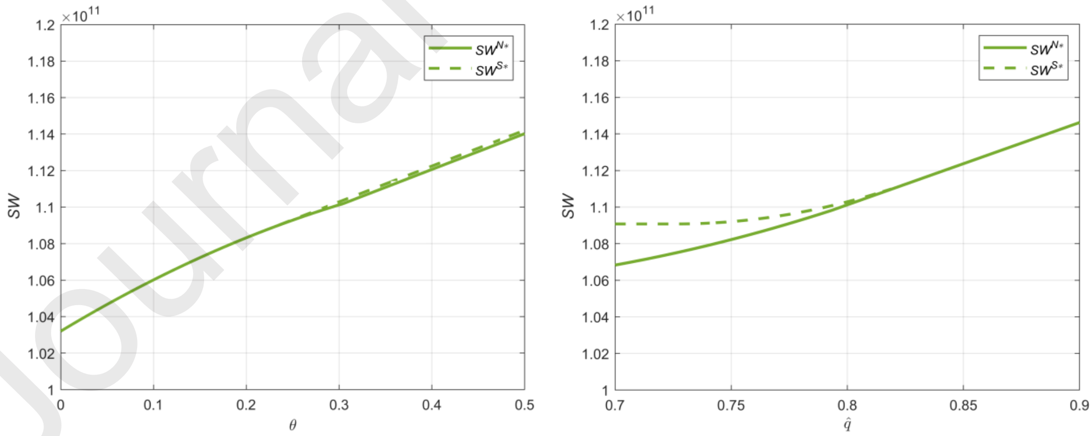
Figure 11 Impact of e_n on profits of supply chain members



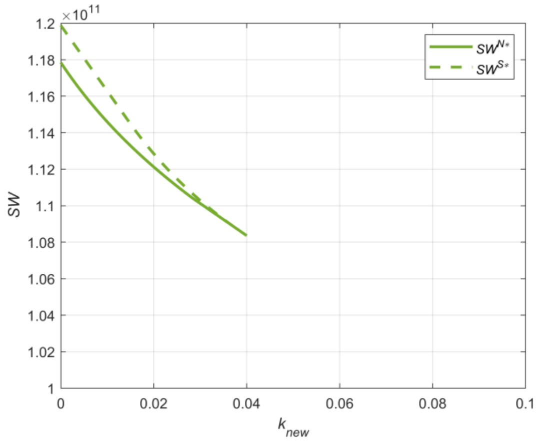
(a) With no purchase subsidy (b) With purchase subsidy

Figure 12 Impact of e_r on profits of supply chain members

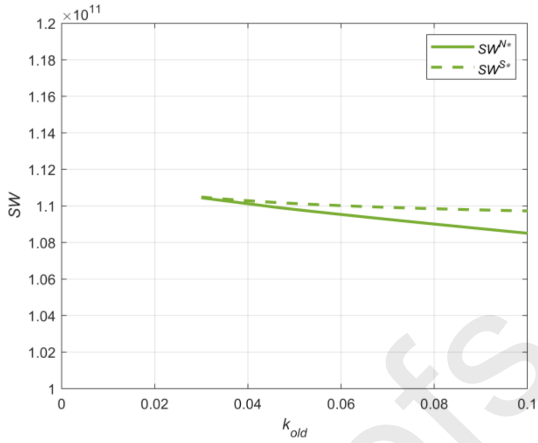
Next, the study further analyzes the impact of the above important parameters on the total social welfare under the two different scenarios. From Figure 13, The impact of battery return yield, average quality and quality factor of reusable batteries on the social welfare is positive, while the variance of quality distribution, installation costs and carbon emissions exert negative influence on the social welfare. Furthermore, it can be observed that the social welfare with purchase subsidy is higher than that with no purchase subsidy, which validates Corollary 15.



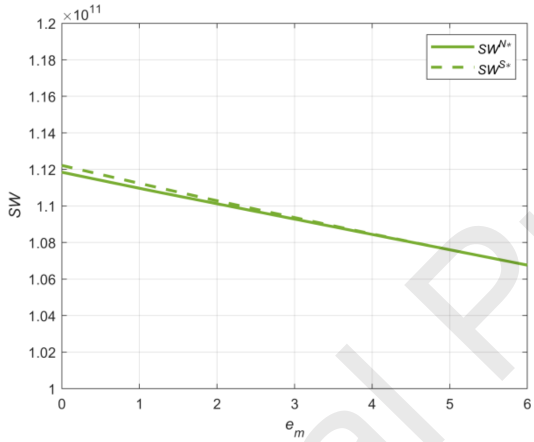
(a) Impact of θ on social welfare (b) Impact of $\hat{q}$ on social welfare



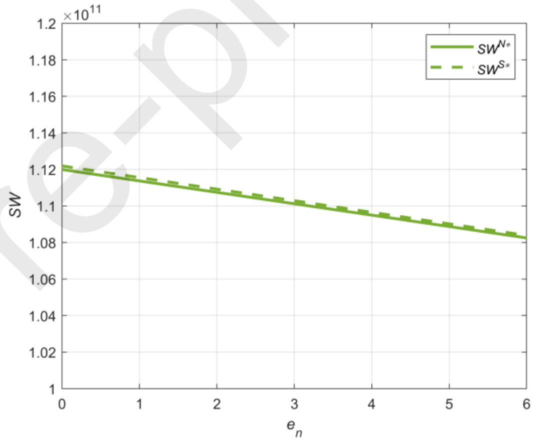
(c) Impact of k_{new} on social welfare



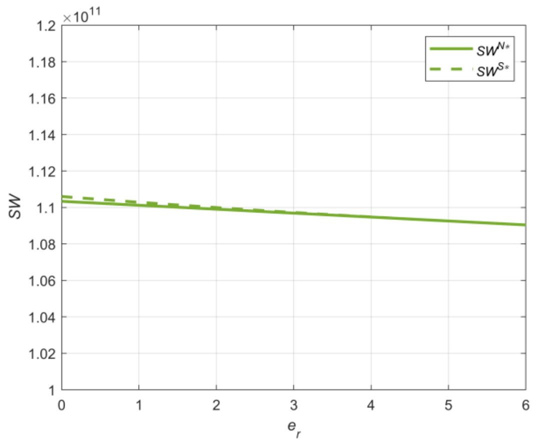
(d) Impact of k_{old} on social welfare



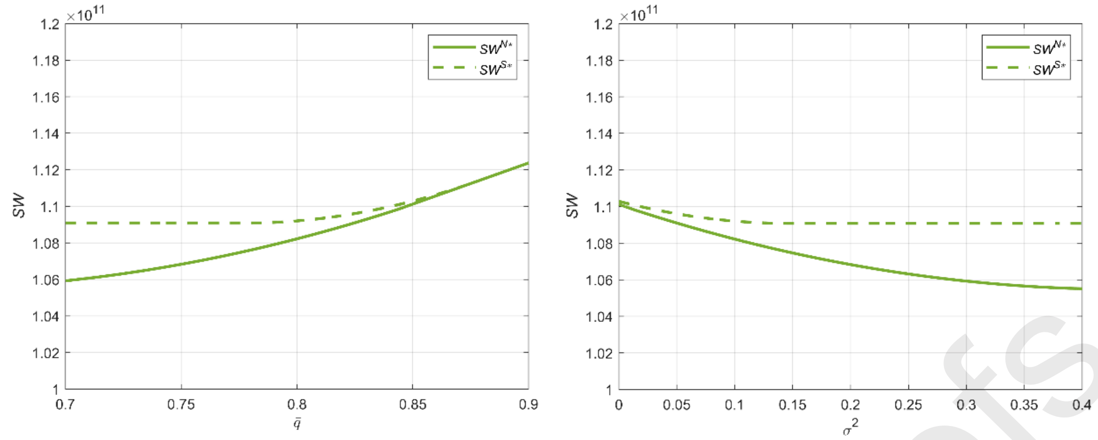
(e) Impact of e_m on social welfare



(f) Impact of e_n on social welfare



(g) Impact of e_r on social welfare

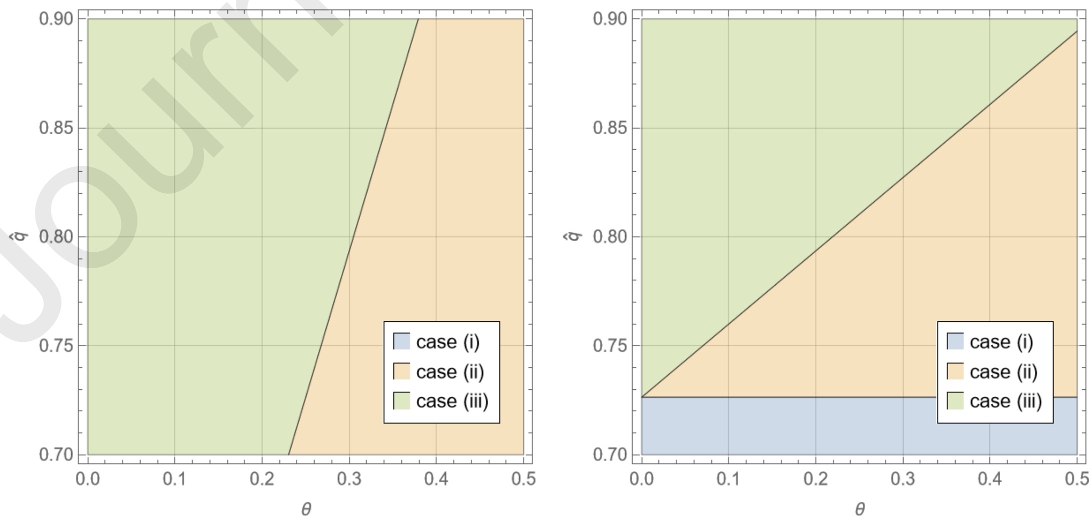


(h) Impact of $\bar{q}$ on social welfare (i) Impact of σ^2 on social welfare

Figure 13 Impact of parameters on social welfare

5.2 Impact of parameters on PGC's purchase decision cases

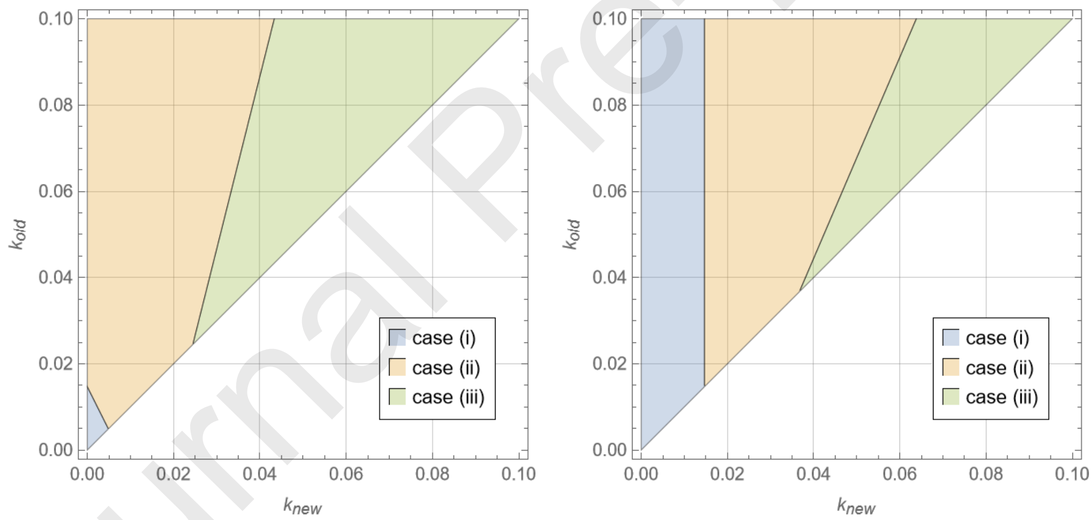
Figure 14 illustrates the impact of θ and $\hat{q}$ on the PGC's purchase decision cases under the two different scenarios. The cases (i), (ii) and (iii) marked in the figure correspond to the three cases of the PGC's purchase decision respectively. Case (i) represents not purchasing retired batteries, case (ii) represents purchasing a portion of the retired batteries provided by the battery recycler for echelon utilization, and case (iii) represents purchasing all the retired batteries. In this example, as θ increases, the PGC shifts from purchasing all the retired batteries to purchasing a portion of the retired batteries provided by the battery recycler. Conversely, as $\hat{q}$ increases, the PGC transitions from purchasing a portion of the retired batteries to purchasing all of them.



(a) With no purchase subsidy (b) With purchase subsidy

Figure 14 Impact of θ and $\hat{q}$ on PGC's purchase decision cases

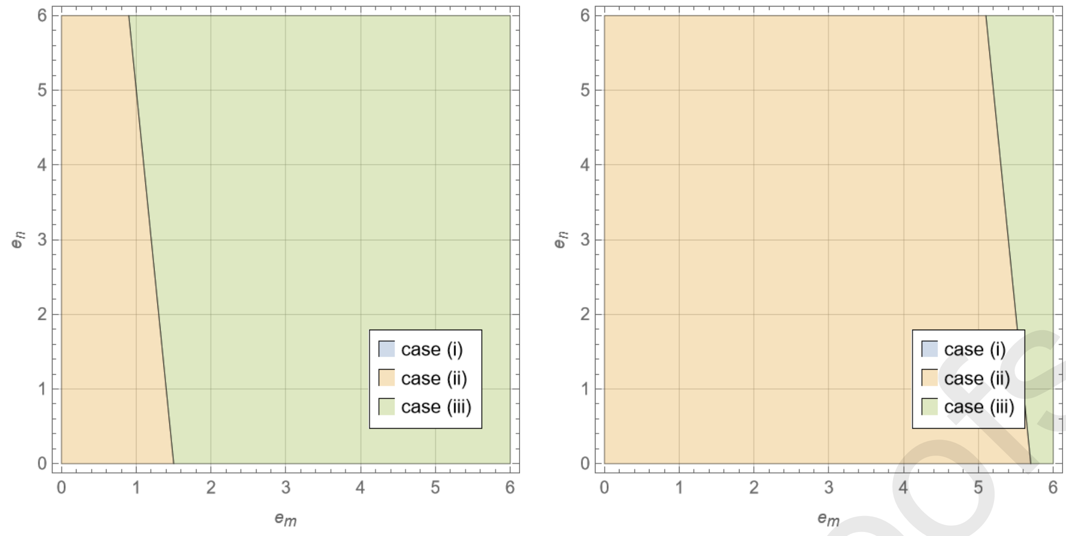
Figure 15 depicts the impact of k_{new} and k_{old} on the PGC's purchase decision cases. Based on Figure 15 (a), the changes in the PGC's purchase decision cases when the government does not provide the subsidy can be analyzed. In this example, when $0 < k_{new} < 0.005$, as k_{old} increases, the PGC shifts from not purchasing retired batteries to purchasing a portion of them. When $0.005 \le k_{new} \le 0.024$, the PGC purchases a portion of the retired batteries. When $0.024 < k_{new} < 0.043$, as k_{old} increases, the PGC shifts from purchasing all the retired batteries to purchasing a portion of them. When $0.043 \le k_{new} < 0.1$, the PGC purchases all the retired batteries. It should be noted that when k_{new} is especially small (i.e., $0 < k_{new} < 0.005$), the result is counterintuitive and essentially similar to the counterintuitive result in Corollary 4(ii). The reason is that the impact of installation cost under this condition is relatively small. As k_{old} increases, the purchase price gap between new and retired batteries also increases. Therefore, the PGC will shift from not purchasing retired batteries to purchasing a portion of them. It can be seen from Figure 15 (b) that under the scenario where the government provides the subsidy, when k_{new} is small, the PGC keeps not purchasing retired batteries, that is, there is no counterintuitive result. The reason for the difference is that as k_{old} increases, the effective purchase price gap between the two types of batteries remains unchanged.



(a) With no purchase subsidy (b) With purchase subsidy

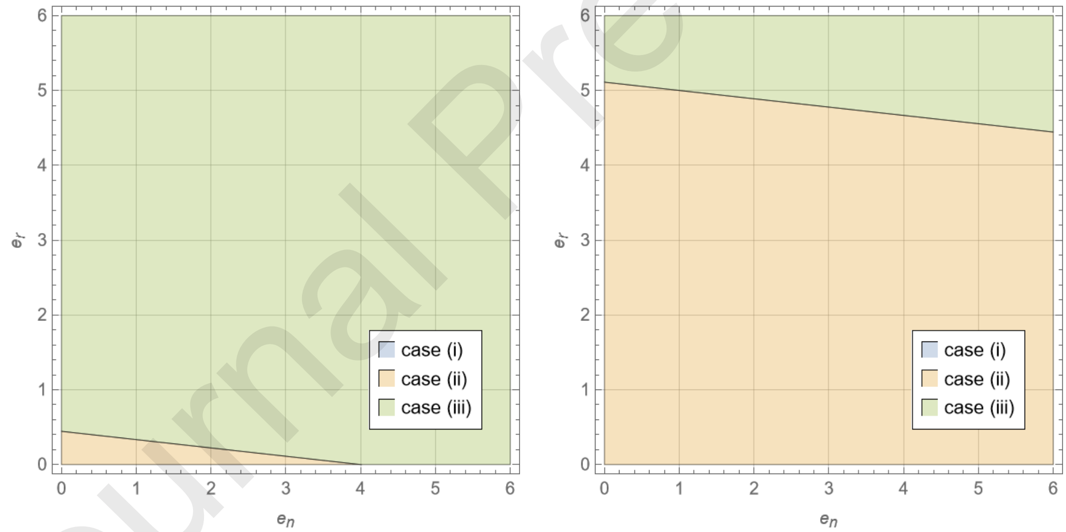
Figure 15 Impact of k_{new} and k_{old} on PGC's purchase decision cases

Figure 16 and Figure 17 describes the impact of e_m, e_n and e_r on the PGC's purchase decision cases. In this example, as e_m, e_n or e_r increases, the PGC shifts from purchasing a portion of the retired batteries provided by the battery recycler to purchasing all of them.



(a) With no purchase subsidy (b) With purchase subsidy

Figure 16 Impact of e_m and e_n on PGC's purchase decision cases



(a) With no purchase subsidy (b) With purchase subsidy

Figure 17 Impact of e_n and e_r on PGC's purchase decision cases

In addition, by comparing the changes in the PGC's purchase decision cases under the two scenarios, it can be found that when the government provides the subsidy, the PGC is more likely to purchase a portion of the retired batteries provided by the battery recycler or not to purchase them, which further validates that the subsidy policy is not necessarily effective in promoting the echelon utilization.

6 Conclusion

In this paper, a two-period game model is designed to study the interest relationships and operational strategies of the CLSC members of power batteries in the echelon utilization stage. Specifically, considering that the government develops a subsidy policy for the purchase of retired batteries, the study explores the government's optimal subsidy and its impact on the decisions and profits of supply chain members, thereby examining the effectiveness of the subsidy policy in promoting the echelon utilization of retired batteries. In addition, the sensitivity analysis of battery return yield θ, quality factors of reusable batteries, carbon emissions, installation cost coefficients of both new and reusable batteries, k_{new} and k_{old}, are incorporated into our study.

The main conclusions are summarized as follows:

(1) When the government provides a purchase subsidy for the PGC, the per-unit subsidy influences the decisions of power battery supply chain members. Higher subsidies encourage the PGC to purchase more retired batteries, but the government balances this with the goal of maximizing social welfare, determining an optimal subsidy level. This optimal level depends on factors like installation costs, carbon emissions, revenue per new battery, and the quality of reusable batteries.

(2) Although purchase subsidies generally enhance social welfare, their effectiveness in promoting echelon utilization depends on parameters like manufacturing costs, demand, installation costs, revenue per battery, and carbon emissions. When subsidies promote reuse, retired battery prices rise while new battery prices fall. The subsidy benefits the PGC and recycler equally but reduces the manufacturer's profit.

(3) Higher carbon emissions from producing new batteries increase environmental costs, raising their prices and encouraging PGCs to buy more retired batteries. To stay competitive, recyclers raise retired battery prices less sharply than new battery prices, preferring to sell reused batteries over recycled materials. Meanwhile, manufacturers lower prices but face constraints from rising raw material costs, creating complex market dynamics.

(4) Reusable batteries are cheaper than new ones but have higher installation costs due to testing, classification, and refurbishment. Without subsidies, changes in installation costs produce counterintuitive effects: as one type's cost rises, the PGC may paradoxically purchase more of that type, driven by shifts in price differences. With subsidies, the price gap remains stable, eliminating these anomalies. Interestingly, the optimal subsidy increases as new battery installation costs rise or reusable battery costs decrease.

The findings above yield key managerial insights for decision-makers in the power battery CLSC. The subsidy policy effectively promotes echelon utilization of retired batteries only under certain conditions. When designing subsidy policies for retired battery purchases, policymakers should consider key factors across the CLSC, determine suitable conditions for subsidies, and set optimal levels accordingly. If subsidies prove ineffective, policymakers can promote the echelon utilization industry through regulations, public awareness campaigns, and other measures. While policymakers focus on social welfare, enterprises in the power battery CLSC aim to maximize their profits. For third-party recyclers, recycling costs directly impact profits. Optimizing processes and upgrading technology can reduce costs and improve efficiency. Similarly, echelon utilization firms should reduce the higher usage costs of retired batteries through improved processing technology. When subsidies are provided, reduced usage costs for reusable batteries will promote echelon utilization. Battery manufacturers can partner with third-party recyclers to collect retired batteries, reducing recycling costs through complementary strengths. Under profit-sharing agreements, manufacturers can also gain a share of the revenue from recycled battery sales. Battery manufacturers can also recover used batteries from echelon utilization firms and extract materials to cut manufacturing costs.

Our research enriches the theoretical understanding of closed-loop supply chains (CLSC) for power batteries, offering valuable insights for policymakers to design effective subsidy policies and guiding enterprises in formulating strategies aligned with circular economy principles. By promoting the echelon utilization of retired batteries, this study addresses critical challenges in the electric vehicle battery manufacturing industry, including resource scarcity, environmental pollution, and inefficiencies in closed-loop systems. Furthermore, it investigates the systemic shift toward a sustainable circular economy for EV batteries, emphasizing ecosystem-level orchestration, stakeholder engagement as key enablers. These contributions enhance the understanding of how systemic changes can support the transition to a circular economy, mitigate climate change, and address the pressing challenges facing industrial systems and closed-loop supply chains.

In the future, this study can be further extended in the following areas. For example, based on the study of a single subsidy policy, comparing different subsidy policies or other intervention means (such as reward-penalty policy, etc.), and examining the effect of combining multiple policies, will yield some interesting findings. Furthermore, future research could delve into the integration of advanced technologies—such as artificial intelligence (AI) for optimizing battery classification accuracy, and blockchain to enhance transparency and traceability across recycling operations. This exploration could also extend to quantifying the comprehensive environmental impacts of battery recycling systems, which includes conducting granular carbon footprint assessments, analyzing emissions associated with dismantling processes, and refining the operational efficiency of closed-loop supply chains. Additionally, adopting advanced uncertainty modelling frameworks—for instance, stochastic optimization and probabilistic analysis—to strengthen the robustness of our analytical approach by explicitly capturing the inherent variability in real-world battery quality parameters.

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Appendix A

The profits of the three firms in Proposition 4(ii) are given below.

$$\Pi_r^{N*} = \begin{pmatrix} \begin{pmatrix} c_m + c_e(e_m + e_n) + 2D_{PGC}(2k_{new} + k_{old}) + p_n \\ - c_d - V_b(1-\hat{q}) - (c_e(e_n + e_r) + p_n - 2p_r)\varphi \end{pmatrix} \\ \begin{pmatrix} c_d + c_m + 2D_{PGC}(2k_{new} + k_{old}) - V_b(1-\hat{q}) \\ + p_n(1-\varphi) + c_e(e_m + e_n - e_n\varphi + e_r\varphi) \end{pmatrix} / (16(k_{new} + k_{old})) \\ + ((p_r - c_e e_r)\varphi - (c_d + c_r))\theta Q_{new1} + (c_d - (p_r - c_e e_r)\varphi) \\ \begin{pmatrix} c_d + c_m + 2D_{PGC}(2k_{new} + k_{old}) - V_b(1-\hat{q}) \\ + p_n(1-\varphi) + c_e(e_m + e_n - e_n\varphi + e_r\varphi) \end{pmatrix} / (8(k_{new} + k_{old})) \end{pmatrix}, \tag{A.1}$$

$$\Pi_{m2}^{N*} = \begin{pmatrix} \begin{pmatrix} c_d + c_m + 2D_{PGC}(2k_{new} + k_{old}) - V_b(1-\hat{q}) + p_n(1-\varphi) \\ -8(k_{new} + k_{old})\theta Q_{new1} + c_e(e_m + e_n - e_n\varphi + e_r\varphi) \end{pmatrix} \\ (p_r - c_e e_n - p_n)\varphi / (8(k_{new} + k_{old})) \\ + \begin{pmatrix} c_d + c_m - 2D_{PGC}(2k_{new} + 3k_{old}) - V_b(1-\hat{q}) \\ -3p_n(1-\varphi) - 4p_r\varphi + c_e(e_m - 3e_n(1-\varphi) + e_r\varphi) \end{pmatrix} \\ \begin{pmatrix} c_d + c_m - 2D_{PGC}(2k_{new} + 3k_{old}) - V_b(1-\hat{q}) \\ + p_n(1-\varphi) + c_e(e_m + e_n - e_n\varphi + e_r\varphi) \end{pmatrix} / (32(k_{new} + k_{old})) \end{pmatrix}, \tag{A.2}$$

$$\Pi_{PGC}^{N*} = \begin{pmatrix} \begin{pmatrix} c_d + c_m + 2D_{PGC}(2k_{new} + k_{old}) - V_b(1-\hat{q}) \\ + p_n(1-\varphi) + c_e(e_m + e_n - e_n\varphi + e_r\varphi) \end{pmatrix} / (64(k_{new} + k_{old})) \\ \begin{pmatrix} c_d + c_m - 2D_{PGC}(6k_{new} - k_{old}) - V_b(1-\hat{q}) \\ + p_n(1-\varphi) + c_e(e_m + e_n - e_n\varphi + e_r\varphi) \end{pmatrix} - D_{PGC}^2 k_{new} \\ + D_{PGC} \begin{pmatrix} c_d - 3c_m - 3c_e(e_m + e_n) - 2D_{PGC}(2k_{new} + 3k_{old}) \\ +3V_b + V_b\hat{q} - 3p_n(1-\varphi) + (c_e(3e_n + e_r) - 4p_r)\varphi \end{pmatrix} / 4 \\ + k_{new} D_{PGC} \begin{pmatrix} c_d + c_m + 2D_{PGC}(2k_{new} + k_{old}) - V_b(1-\hat{q}) \\ + p_n(1-\varphi) + c_e(e_m + e_n - e_n\varphi + e_r\varphi) \end{pmatrix} / 4(k_{new} + k_{old}) \end{pmatrix}, \tag{A.3}$$

The profits of the three firms and the social welfare in Proposition 8(ii) are given below.

$$\Pi_r^{S*} = \begin{pmatrix} \begin{pmatrix} 2(c_m + c_e(e_m + e_n) + 2D_{PGC}k_{new} + p_n - V_b(1-\hat{q})) \\ + c_d - (c_e(2e_n - e_r) + 2p_n - p_r)\varphi \end{pmatrix} \\ \begin{pmatrix} c_d + c_m + 2D_{PGC}k_{new} - V_b(1-\hat{q}) \\ + p_n(1-\varphi) + c_e(e_m + e_n - e_n\varphi + e_r\varphi) \end{pmatrix} / (2(k_{new} + k_{old})) \\ + ((p_r - c_e e_r)\varphi - (c_d + c_r))\theta Q_{new1} + (c_d - (p_r - c_e e_r)\varphi) \\ \begin{pmatrix} c_d + c_m + 2D_{PGC}k_{new} - V_b(1-\hat{q}) \\ + p_n(1-\varphi) + c_e(e_m + e_n - e_n\varphi + e_r\varphi) \end{pmatrix} / (2(k_{new} + k_{old})) \end{pmatrix}, \tag{A.4}$$

$$\Pi_m^{S*} = \begin{pmatrix} \begin{pmatrix} c_d + c_m + 2D_{PGC}k_{new} - V_b(1-\hat{q}) + p_n(1-\varphi) \\ -2(k_{new} + k_{old})\theta Q_{new1} + c_e(e_m + e_n - e_n\varphi + e_r\varphi) \end{pmatrix} \\ (p_r - c_e e_n - p_n)\varphi / (2(k_{new} + k_{old})) \\ + \begin{pmatrix} c_d + c_m - 2D_{PGC}k_{old} - V_b(1-\hat{q}) \\ + p_n(1-\varphi) + c_e(e_m + e_n - e_n\varphi + e_r\varphi) \end{pmatrix} \\ \begin{pmatrix} c_d + c_m - 2D_{PGC}k_{old} - V_b(1-\hat{q}) \\ + p_n - p_r\varphi + c_e(e_m + e_n + e_r\varphi) \end{pmatrix} / (2(k_{new} + k_{old})) \end{pmatrix}, \tag{A.5}$$

$$\Pi_{PGC}^{S*} = \begin{pmatrix} \begin{pmatrix} c_d + c_m - 2D_{PGC}k_{new} - V_b(1-\hat{q}) \\ + p_n(1-\varphi) + c_e(e_m + e_n - e_n\varphi + e_r\varphi) \end{pmatrix} / (4(k_{new} + k_{old})) \\ \begin{pmatrix} c_d + c_m + 2D_{PGC}k_{new} - V_b(1-\hat{q}) \\ + p_n(1-\varphi) + c_e(e_m + e_n - e_n\varphi + e_r\varphi) \end{pmatrix} - D_{PGC}^2 k_{new} \\ + D_{PGC}(c_d - 2D_{PGC}k_{old} + V_b\hat{q} - (p_r - c_e e_r)\varphi) \\ + k_{new} D_{PGC} \begin{pmatrix} c_d + c_m + 2D_{PGC}k_{new} - V_b(1-\hat{q}) \\ + p_n(1-\varphi) + c_e(e_m + e_n - e_n\varphi + e_r\varphi) \end{pmatrix} / (k_{new} + k_{old}) \end{pmatrix}, \tag{A.6}$$

$$SW^{S*}=\begin{pmatrix}\begin{pmatrix}c_d+c_m+2D_{PGC}k_{new}-V_b(1-\hat{q})\\+p_n(1-\varphi)+c_e(e_m+e_n-e_n\varphi+e_r\varphi)\end{pmatrix}^2/(4(k_{new}+k_{old}))\\((c_e(e_n-e_r)+p_n)\varphi-(c_d+c_r))\theta Q_{new1}\\-D_{PGC}(c_m+c_e(e_m+e_n)+D_{PGC}k_{new}+p_n-V_b)\end{pmatrix}, \quad (A.7)$$

Appendix B

Proofs of Proposition 2. From the Lagrange function in (17), $\frac{\partial^2 L(Q_{oldPGC},\mu_1,\mu_2)}{\partial Q_{oldPGC}^2}=-2(k_{new}+k_{old})$ <0, so the Lagrange function is a concave function of Q_{oldPGC}. There are three possible cases for equilibrium solutions:

(i) When $\mu_1=0$, $\mu_2>0$, $\eta\theta Q_{new1}-Q_{oldPGC}>0$, $Q_{oldPGC}=0$, by solving $\frac{\partial L(Q_{oldPGC},\mu_1,\mu_2)}{\partial Q_{oldPGC}}=0$, $Q^*_{oldPGC}=0$ and the corresponding condition $\frac{2D_{PGC}k_{new}+p_{new2}-p_{old}-V_b(1-\hat{q})}{2(k_{new}+k_{old})}\leq 0$ are obtained.

(ii) When $\mu_1=0$, $\mu_2=0$, $\eta\theta Q_{new1}-Q_{oldPGC}>0$, $Q_{oldPGC}>0$, by solving $\frac{\partial L(Q_{oldPGC},\mu_1,\mu_2)}{\partial Q_{oldPGC}}=0$, $Q^*_{oldPGC}=\frac{2D_{PGC}k_{new}+p_{new2}-p_{old}-V_b(1-\hat{q})}{2(k_{new}+k_{old})}$ and the corresponding condition $0<\frac{2D_{PGC}k_{new}+p_{new2}-p_{old}-V_b(1-\hat{q})}{2(k_{new}+k_{old})}<\eta\theta Q_{new1}$ are obtained.

(iii) When $\mu_1>0$, $\mu_2=0$, $\eta\theta Q_{new1}-Q_{oldPGC}=0$, $Q_{oldPGC}>0$, by solving $\frac{\partial L(Q_{oldPGC},\mu_1,\mu_2)}{\partial Q_{oldPGC}}=0$, $Q^*_{oldPGC}=\eta\theta Q_{new1}$ and the corresponding condition $\frac{2D_{PGC}k_{new}+p_{new2}-p_{old}-V_b(1-\hat{q})}{2(k_{new}+k_{old})}\geq\eta\theta Q_{new1}$ are obtained.

All subsequent proofs are similar.

Proofs of Proposition 3. The following Lagrange function is constructed to solve the battery manufacturer's optimal decision problem with no purchase subsidy:

$$\max_{p_{new2}} L=\begin{pmatrix}\begin{pmatrix}p_{new2}-c_m\\-c_e e_m\end{pmatrix}\frac{2D_{PGC}k_{old}-p_{new2}+p_{old}+V_b(1-\hat{q})}{2(k_{new}+k_{old})}-p_r\varphi\left(\theta Q_{new1}-\frac{2D_{PGC}k_{new}+p_{new2}-p_{old}-V_b(1-\hat{q})}{2(k_{new}+k_{old})}\right)\\-(p_n+c_e e_n)\left(\frac{2D_{PGC}k_{old}-p_{new2}+p_{old}+V_b(1-\hat{q})}{2(k_{new}+k_{old})}-\varphi\left(\theta Q_{new1}-\frac{2D_{PGC}k_{new}+p_{new2}-p_{old}-V_b(1-\hat{q})}{2(k_{new}+k_{old})}\right)\right)\\+\mu_3\left(\eta\theta Q_{new1}-\frac{2D_{PGC}k_{new}+p_{new2}-p_{old}-V_b(1-\hat{q})}{2(k_{new}+k_{old})}\right)+\mu_4\frac{2D_{PGC}k_{new}+p_{new2}-p_{old}-V_b(1-\hat{q})}{2(k_{new}+k_{old})}\end{pmatrix}$$

.

By noting that $\frac{\partial^2 L(p_{new2}, \mu_3, \mu_4)}{\partial p_{new2}^2} = -\frac{1}{k_{new}+k_{old}} < 0$, there are three possible cases for equilibrium solutions:

(i) When $\mu_3 = 0$, $\mu_4 > 0$, $\eta\theta Q_{new1} - (2D_{PGC}k_{new} + p_{new2} - p_{old} - V_b(1-\hat{q}))/(2(k_{new}+k_{old})) > 0$, $(2D_{PGC}k_{new} + p_{new2} - p_{old} - V_b(1-\hat{q}))/(2(k_{new}+k_{old})) = 0$, by solving $\frac{\partial L(p_{new2}, \mu_3, \mu_4)}{\partial p_{new2}} = 0$, $p_{new2}^* = -2D_{PGC}k_{new} + p_{old} + V_b(1-\hat{q})$ and the corresponding condition $\frac{(c_m+2D_{PGC}(2k_{new}+k_{old})-p_{old}-V_b(1-\hat{q})+p_n(1-\varphi)+p_r\varphi+c_e(e_m+e_n-e_n\varphi))}{4(k_{new}+k_{old})} \leq 0$ are obtained.

(ii) When $\mu_3 = 0, \mu_4 = 0, \eta\theta Q_{new1} - (2D_{PGC}k_{new} + p_{new2} - p_{old} - V_b(1-\hat{q}))/(2(k_{new}+k_{old})) > 0$, $(2D_{PGC}k_{new} + p_{new2} - p_{old} - V_b(1-\hat{q}))/(2(k_{new}+k_{old})) > 0$, by solving $\frac{\partial L(p_{new2}, \mu_3, \mu_4)}{\partial p_{new2}} = 0$, $p_{new2}^* = \frac{1}{2}(c_m + 2D_{PGC}k_{old} + p_{old} + V_b(1-\hat{q}) + p_n(1-\varphi) + p_r\varphi + c_e(e_m+e_n-e_n\varphi))$ and the corresponding condition $0 < (c_m + 2D_{PGC}(2k_{new}+k_{old}) - p_{old} - V_b(1-\hat{q}) + p_n(1-\varphi) + p_r\varphi + c_e(e_m+e_n-e_n\varphi))/(4(k_{new}+k_{old})) < \eta\theta Q_{new1}$ are obtained.

(iii) When $\mu_3 > 0$, $\mu_4 = 0$, $\eta\theta Q_{new1} - \frac{2D_{PGC}k_{new}+p_{new2}-p_{old}-V_b(1-\hat{q})}{2(k_{new}+k_{old})} = 0$ and $\frac{2D_{PGC}k_{new}+p_{new2}-p_{old}-V_b(1-\hat{q})}{2(k_{new}+k_{old})} > 0$, by solving $\frac{\partial L(p_{new2}, \mu_3, \mu_4)}{\partial p_{new2}} = 0$, $p_{new2}^* = -2D_{PGC}k_{new} + 2\eta\theta Q_{new1}(k_{new}+k_{old}) + p_{old} + V_b(1-\hat{q})$ and the corresponding condition $\frac{c_m+2D_{PGC}(2k_{new}+k_{old})-p_{old}-V_b(1-\hat{q})+p_n(1-\varphi)+p_r\varphi+c_e(e_m+e_n-e_n\varphi)}{4(k_{new}+k_{old})} \geq \eta\theta Q_{new1}$ are obtained.

Proofs of Proposition 4. The following Lagrange function is constructed to solve the battery recycler's optimal decision problem with no purchase subsidy:

$$\max_{p_{old}} L = \left(\begin{array}{c} p_{old}\frac{c_m+2D_{PGC}(2k_{new}+k_{old})-p_{old}-V_b(1-\hat{q})+p_n(1-\varphi)+p_r\varphi+c_e(e_m+e_n-e_n\varphi)}{4(k_{new}+k_{old})} \\ +(p_r - c_e e_r)\varphi\left(\theta Q_{new1} - \frac{c_m+2D_{PGC}(2k_{new}+k_{old})-p_{old}-V_b(1-\hat{q})+p_n(1-\varphi)+p_r\varphi+c_e(e_m+e_n-e_n\varphi)}{4(k_{new}+k_{old})}\right) \\ -c_r\theta Q_{new1} - c_d\left(\theta Q_{new1} - \frac{c_m+2D_{PGC}(2k_{new}+k_{old})-p_{old}-V_b(1-\hat{q})+p_n(1-\varphi)+p_r\varphi+c_e(e_m+e_n-e_n\varphi)}{4(k_{new}+k_{old})}\right) \\ +\mu_5\left(\eta\theta Q_{new1} - \frac{c_m+2D_{PGC}(2k_{new}+k_{old})-p_{old}-V_b(1-\hat{q})+p_n(1-\varphi)+p_r\varphi+c_e(e_m+e_n-e_n\varphi)}{4(k_{new}+k_{old})}\right) \\ +\mu_6\frac{c_m+2D_{PGC}(2k_{new}+k_{old})-p_{old}-V_b(1-\hat{q})+p_n(1-\varphi)+p_r\varphi+c_e(e_m+e_n-e_n\varphi)}{4(k_{new}+k_{old})} \end{array} \right)$$

.

By noting that $\frac{\partial^2 L(p_{old}, \mu_5, \mu_6)}{\partial p_{old}^2} = -\frac{1}{2(k_{new}+k_{old})} < 0$, there are three possible cases for equilibrium solutions:

(i) When $\mu_5 = 0$, $\mu_6 > 0$, $(\eta\theta Q_{new1} - c_m + 2D_{PGC}(2k_{new} + k_{old}) - p_{old} - V_b(1-\hat{q}) + p_n(1-\varphi) + p_r\varphi + c_e(e_m + e_n - e_n\varphi))/(4(k_{new} + k_{old})) > 0$, $(c_m + 2D_{PGC}(2k_{new} + k_{old}) - p_{old} - V_b(1-\hat{q}) + p_n(1-\varphi) + p_r\varphi + c_e(e_m + e_n - e_n\varphi))/(4(k_{new} + k_{old})) = 0$, by solving $\frac{\partial L(p_{old}, \mu_5, \mu_6)}{\partial p_{old}} = 0$, $p_{old}^* = c_m + 2D_{PGC}(2k_{new} + k_{old}) - V_b(1-\hat{q}) + p_n(1-\varphi) + p_r\varphi + c_e(e_m + e_n - e_n\varphi)$ and the corresponding condition $(c_d + c_m + 2D_{PGC}(2k_{new} + k_{old}) - V_b(1-\hat{q}) + p_n(1-\varphi) + c_e(e_m + e_n - e_n\varphi + e_r\varphi))/(8(k_{new} + k_{old})) \leq 0$ are obtained.

(ii) When $\mu_5 = 0$, $\mu_6 = 0$, $\eta\theta Q_{new1} - (c_m + 2D_{PGC}(2k_{new} + k_{old}) - p_{old} - V_b(1-\hat{q}) + p_n(1-\varphi) + p_r\varphi + c_e(e_m + e_n - e_n\varphi))/(4(k_{new} + k_{old})) > 0$, $(c_m + 2D_{PGC}(2k_{new} + k_{old}) - p_{old} - V_b(1-\hat{q}) + p_n(1-\varphi) + p_r\varphi + c_e(e_m + e_n - e_n\varphi))/(4(k_{new} + k_{old})) > 0$, by solving $\frac{\partial L(p_{old}, \mu_5, \mu_6)}{\partial p_{old}} = 0$, $p_{old}^* = \frac{1}{2}(c_m + c_e(e_m + e_n) + 2D_{PGC}(2k_{new} + k_{old}) - V_b(1-\hat{q}) + p_n - c_d - (c_e(e_n + e_r) + p_n - 2p_r)\varphi)$ and the corresponding condition $0 < \frac{c_d + c_m + 2D_{PGC}(2k_{new} + k_{old}) - V_b(1-\hat{q}) + p_n(1-\varphi) + c_e(e_m + e_n - e_n\varphi + e_r\varphi)}{8(k_{new} + k_{old})} < \eta\theta Q_{new1}$ are obtained.

(iii) When $\mu_5 > 0$, $\mu_6 = 0$, $\eta\theta Q_{new1} - (c_m + 2D_{PGC}(2k_{new} + k_{old}) - p_{old} - V_b(1-\hat{q}) + p_n(1-\varphi) + p_r\varphi + c_e(e_m + e_n - e_n\varphi))/(4(k_{new} + k_{old})) = 0$, $(c_m + 2D_{PGC}(2k_{new} + k_{old}) - p_{old} - V_b(1-\hat{q}) + p_n(1-\varphi) + p_r\varphi + c_e(e_m + e_n - e_n\varphi))/(4(k_{new} + k_{old})) > 0$, by solving $\frac{\partial L(p_{old}, \mu_5, \mu_6)}{\partial p_{old}} = 0$, $p_{old}^* = c_m + 2D_{PGC}(2k_{new} + k_{old}) - 4\eta\theta Q_{new1}(k_{new} + k_{old}) - V_b(1-\hat{q}) + p_n(1-\varphi) + p_r\varphi + c_e(e_m + e_n - e_n\varphi)$ and the corresponding condition $\frac{c_d + c_m + 2D_{PGC}(2k_{new} + k_{old}) - V_b(1-\hat{q}) + p_n(1-\varphi) + c_e(e_m + e_n - e_n\varphi + e_r\varphi)}{8(k_{new} + k_{old})} \geq \eta\theta Q_{new1}$ are obtained.

Proofs of Proposition 5. From the Lagrange function in (37), $\frac{\partial^2 L(Q_{oldPGC}, \mu_1, \mu_2)}{\partial Q_{oldPGC}^2} = -2(k_{new} + k_{old}) < 0$, so the Lagrange function is a concave function of Q_{oldPGC}. There are three possible cases for equilibrium solutions:

(i) When $\mu_1 = 0$, $\mu_2 > 0$, $\eta\theta Q_{new1} - Q_{oldPGC} > 0$, $Q_{oldPGC} = 0$, by solving $\frac{\partial L(Q_{oldPGC}, \mu_1, \mu_2)}{\partial Q_{oldPGC}} = 0$, $Q_{oldPGC}^* = 0$ and the corresponding condition $\frac{2D_{PGC}k_{new} + p_{new2} - p_{old} - V_b(1-\hat{q}) + s}{2(k_{new} + k_{old})} \leq 0$ are obtained.

(ii) When $\mu_1 = 0$, $\mu_2 = 0$, $\eta\theta Q_{new1} - Q_{oldPGC} > 0$, $Q_{oldPGC} > 0$, by solving $\frac{\partial L(Q_{oldPGC}, \mu_1, \mu_2)}{\partial Q_{oldPGC}} = 0$,

$Q^*_{oldPGC} = \frac{2D_{PGC}k_{new}+p_{new2}-p_{old}-V_b(1-\hat{q})+s}{2(k_{new}+k_{old})}$ and the corresponding condition $0 < \frac{2D_{PGC}k_{new}+p_{new2}-p_{old}-V_b(1-\hat{q})+s}{2(k_{new}+k_{old})} < \eta\theta Q_{new1}$ are obtained.

(iii) When $\mu_1 > 0$, $\mu_2 = 0$, $\eta\theta Q_{new1} - Q_{oldPGC} = 0$, $Q_{oldPGC} > 0$, by solving $\frac{\partial L(Q_{oldPGC}, \mu_1, \mu_2)}{\partial Q_{oldPGC}} = 0$, $Q^*_{oldPGC} = \eta\theta Q_{new1}$ and the corresponding condition $\frac{2D_{PGC}k_{new}+p_{new2}-p_{old}-V_b(1-\hat{q})+s}{2(k_{new}+k_{old})} \geq \eta\theta Q_{new1}$ are obtained.

Proofs of Proposition 6. The following Lagrange function is constructed to solve the battery manufacturer's optimal decision problem with purchase subsidy:

$$\max_{p_{new2}} L = \left(\begin{array}{c} (p_{new2} - c_m - c_e e_m)\frac{2D_{PGC}k_{old}-p_{new2}+p_{old}+V_b(1-\hat{q})-s}{2(k_{new}+k_{old})} \\ - p_r\varphi\left(\theta Q_{new1} - \frac{2D_{PGC}k_{new}+p_{new2}-p_{old}-V_b(1-\hat{q})+s}{2(k_{new}+k_{old})}\right) \\ -(p_n + c_e e_n)\left(\begin{array}{c} \frac{2D_{PGC}k_{old}-p_{new2}+p_{old}+V_b(1-\hat{q})-s}{2(k_{new}+k_{old})} \\ -\varphi\left(\theta Q_{new1} - \frac{2D_{PGC}k_{new}+p_{new2}-p_{old}-V_b(1-\hat{q})+s}{2(k_{new}+k_{old})}\right) \end{array} \right) \\ + \mu_3\left(\eta\theta Q_{new1} - \frac{2D_{PGC}k_{new}+p_{new2}-p_{old}-V_b(1-\hat{q})+s}{2(k_{new}+k_{old})}\right) \\ + \mu_4\frac{2D_{PGC}k_{new}+p_{new2}-p_{old}-V_b(1-\hat{q})+s}{2(k_{new}+k_{old})} \end{array} \right).$$

By noting that $\frac{\partial^2 L(p_{new2}, \mu_3, \mu_4)}{\partial p^2_{new2}} = -\frac{1}{k_{new}+k_{old}} < 0$, there are three possible cases for equilibrium solutions:

(i) When $\mu_3 = 0$, $\mu_4 > 0$, $\eta\theta Q_{new1} - (2D_{PGC}k_{new} + p_{new2} - p_{old} - V_b(1-\hat{q}) + s)/(2(k_{new} + k_{old})) > 0$, $(2D_{PGC}k_{new} + p_{new2} - p_{old} - V_b(1-\hat{q}) + s)/(2(k_{new} + k_{old})) = 0$, by solving $\frac{\partial L(p_{new2}, \mu_3, \mu_4)}{\partial p_{new2}} = 0$, $p^*_{new2} = -2D_{PGC}k_{new} + p_{old} + V_b(1-\hat{q}) - s$ and the corresponding condition $(c_m + 2D_{PGC}(2k_{new} + k_{old}) - p_{old} - V_b(1-\hat{q}) + p_n(1-\varphi) + p_r\varphi + c_e(e_m + e_n - e_n\varphi) + s)/(4(k_{new} + k_{old})) \leq 0$ are obtained.

(ii) When $\mu_3 = 0$, $\mu_4 = 0$, $\eta\theta Q_{new1} - (2D_{PGC}k_{new} + p_{new2} - p_{old} - V_b(1-\hat{q}) + s)/(2(k_{new} + k_{old})) > 0$, $(2D_{PGC}k_{new} + p_{new2} - p_{old} - V_b(1-\hat{q}) + s)/(2(k_{new} + k_{old})) > 0$, by solving $\frac{\partial L(p_{new2}, \mu_3, \mu_4)}{\partial p_{new2}} = 0$, $p^*_{new2} = \frac{1}{2}(c_m + 2D_{PGC}k_{old} + p_{old} + V_b(1-\hat{q}) + p_n(1-\varphi) + p_r\varphi + c_e(e_m + e_n - e_n\varphi) - s)$ and the corresponding condition $0 < (c_m + 2D_{PGC}(2k_{new} + k_{old}) - p_{old} - V_b(1-\hat{q}) + p_n(1-\varphi) + p_r\varphi + c_e(e_m + e_n - e_n\varphi) + s)/(4(k_{new} + k_{old})) < \eta\theta Q_{new1}$ are obtained.

(iii) When $\mu_3 > 0$, $\mu_4 = 0$, $\eta\theta Q_{new1} - \frac{2D_{PGC}k_{new}+p_{new2}-p_{old}-V_b(1-\hat{q})+s}{2(k_{new}+k_{old})} = 0$, $\frac{2D_{PGC}k_{new}+p_{new2}-p_{old}-V_b(1-\hat{q})+s}{2(k_{new}+k_{old})} > 0$, by solving $\frac{\partial L(p_{new2}, \mu_3, \mu_4)}{\partial p_{new2}} = 0$, $p^*_{new2} = -2D_{PGC}k_{new} + 2\eta\theta Q_{new1}(k_{new}+k_{old}) + p_{old} + V_b(1-\hat{q}) - s$ and the corresponding condition $\frac{c_m+2D_{PGC}(2k_{new}+k_{old})-p_{old}-V_b(1-\hat{q})+p_n(1-\varphi)+p_r\varphi+c_e(e_m+e_n-e_n\varphi)+s}{4(k_{new}+k_{old})} \geq \eta\theta Q_{new1}$ are obtained.

Proofs of Proposition 7. The following Lagrange function is constructed to solve the battery recycler's optimal decision problem with purchase subsidy:

$$\max_{p_{old}} L = \left(\begin{array}{c} p_{old}\frac{(c_m+2D_{PGC}(2k_{new}+k_{old})-p_{old}-V_b(1-\hat{q})+p_n(1-\varphi)+p_r\varphi+c_e(e_m+e_n-e_n\varphi)+s)}{4(k_{new}+k_{old})} \\ +(p_r-c_e e_r)\varphi\left(\theta Q_{new1} - \frac{c_m+2D_{PGC}(2k_{new}+k_{old})-p_{old}-V_b(1-\hat{q})+p_n(1-\varphi)+p_r\varphi+c_e(e_m+e_n-e_n\varphi)+s}{4(k_{new}+k_{old})}\right) \\ -c_r\theta Q_{new1} - c_d\left(\theta Q_{new1} - \frac{c_m+2D_{PGC}(2k_{new}+k_{old})-p_{old}-V_b(1-\hat{q})+p_n(1-\varphi)+p_r\varphi+c_e(e_m+e_n-e_n\varphi)+s}{4(k_{new}+k_{old})}\right) \\ +\mu_5\left(\eta\theta Q_{new1} - \frac{c_m+2D_{PGC}(2k_{new}+k_{old})-p_{old}-V_b(1-\hat{q})+p_n(1-\varphi)+p_r\varphi+c_e(e_m+e_n-e_n\varphi)+s}{4(k_{new}+k_{old})}\right) \\ +\mu_6\frac{c_m+2D_{PGC}(2k_{new}+k_{old})-p_{old}-V_b(1-\hat{q})+p_n(1-\varphi)+p_r\varphi+c_e(e_m+e_n-e_n\varphi)+s}{4(k_{new}+k_{old})} \end{array} \right)$$

.

By noting that $\frac{\partial^2 L(p_{old}, \mu_5, \mu_6)}{\partial p^2_{old}} = -\frac{1}{2(k_{new}+k_{old})} < 0$, there are three possible cases for equilibrium solutions:

(i) When $\mu_5 = 0$, $\mu_6 > 0$, $\eta\theta Q_{new1} - (c_m + 2D_{PGC}(2k_{new}+k_{old}) - p_{old} - V_b(1-\hat{q}) + p_n(1-\varphi) + p_r\varphi + c_e(e_m+e_n-e_n\varphi) + s)/(4(k_{new}+k_{old})) > 0$, $(c_m + 2D_{PGC}(2k_{new}+k_{old}) - p_{old} - V_b(1-\hat{q}) + p_n(1-\varphi) + p_r\varphi + c_e(e_m+e_n-e_n\varphi) + s)/(4(k_{new}+k_{old})) = 0$, by solving $\frac{\partial L(p_{old}, \mu_5, \mu_6)}{\partial p_{old}} = 0$, $p^*_{old} = c_m + 2D_{PGC}(2k_{new}+k_{old}) - V_b(1-\hat{q}) + p_n(1-\varphi) + p_r\varphi + c_e(e_m+e_n-e_n\varphi) + s$ and the corresponding condition $(c_d + c_m + 2D_{PGC}(2k_{new}+k_{old}) - V_b(1-\hat{q}) + p_n(1-\varphi) + c_e(e_m+e_n-e_n\varphi+e_r\varphi) + s)/(8(k_{new}+k_{old})) \leq 0$ are obtained.

(ii) When $\mu_5 = 0$, $\mu_6 = 0$, $\eta\theta Q_{new1} - (c_m + 2D_{PGC}(2k_{new}+k_{old}) - p_{old} - V_b(1-\hat{q}) + p_n(1-\varphi) + p_r\varphi + c_e(e_m+e_n-e_n\varphi) + s)/(4(k_{new}+k_{old})) > 0$, $(c_m + 2D_{PGC}(2k_{new}+k_{old}) - p_{old} - V_b(1-\hat{q}) + p_n(1-\varphi) + p_r\varphi + c_e(e_m+e_n-e_n\varphi) + s)/(4(k_{new}+k_{old})) > 0$, by solving $\frac{\partial L(p_{old}, \mu_5, \mu_6)}{\partial p_{old}} = 0$, $p^*_{old} = \frac{1}{2}(c_m + c_e(e_m+e_n) + 2D_{PGC}(2k_{new}+k_{old}) - V_b(1-\hat{q}) + p_n - c_d - (c_e(e_n+e_r) + p_n - 2p_r)\varphi + s)$ and the corresponding condition $0 < (c_d + c_m + 2D_{PGC}(2k_{new}+k_{old}) - V_b(1-\hat{q}) + p_n(1-\varphi) + c_e(e_m+e_n-e_n\varphi+e_r\varphi) + s)$

$/(8(k_{new}+k_{old})) < \eta\theta Q_{new1}$ are obtained.

(iii) When $\mu_5 > 0$, $\mu_6 = 0$, $\eta\theta Q_{new1} - (c_m + 2D_{PGC}(2k_{new}+k_{old}) - p_{old} - V_b(1-\hat{q}) + p_n(1-\varphi) + p_r\varphi + c_e(e_m+e_n-e_n\varphi) + s)/(4(k_{new}+k_{old})) = 0$, $(c_m + 2D_{PGC}(2k_{new}+k_{old}) - p_{old} - V_b(1-\hat{q}) + p_n(1-\varphi) + p_r\varphi + c_e(e_m+e_n-e_n\varphi) + s)/(4(k_{new}+k_{old})) > 0$, by solving $\frac{\partial L(p_{old},\mu_5,\mu_6)}{\partial p_{old}} = 0$, $p_{old}^* = c_m + 2D_{PGC}(2k_{new}+k_{old}) - 4\eta\theta Q_{new1}(k_{new}+k_{old}) - V_b(1-\hat{q}) + p_n(1-\varphi) + p_r\varphi + c_e(e_m+e_n-e_n\varphi) + s$ and the corresponding condition $\frac{c_d+c_m+2D_{PGC}(2k_{new}+k_{old})-V_b(1-\hat{q})+p_n(1-\varphi)+c_e(e_m+e_n-e_n\varphi+e_r\varphi)+s}{8(k_{new}+k_{old})} \geq \eta\theta Q_{new1}$ are obtained.

Proofs of Proposition 8. The following Lagrange function is constructed to solve the government's optimal decision problem:

$$\max_s L = \left(\begin{array}{c} \left(\frac{-c_d-c_m+2D_{PGC}(2k_{new}+3k_{old})+V_b(1-\hat{q})-p_n(1-\varphi)-c_e(e_m+e_n-e_n\varphi+e_r\varphi)-s}{8(k_{new}+k_{old})} + \hat{q}\frac{c_d+c_m+2D_{PGC}(2k_{new}+k_{old})-V_b(1-\hat{q})+p_n(1-\varphi)+c_e(e_m+e_n-e_n\varphi+e_r\varphi)+s}{8(k_{new}+k_{old})}\right)V_b \\ - k_{new}\left(\frac{-c_d-c_m+2D_{PGC}(2k_{new}+3k_{old})+V_b(1-\hat{q})-p_n(1-\varphi)-c_e(e_m+e_n-e_n\varphi+e_r\varphi)-s}{8(k_{new}+k_{old})}\right)^2 \\ - k_{old}\left(\frac{c_d+c_m+2D_{PGC}(2k_{new}+k_{old})-V_b(1-\hat{q})+p_n(1-\varphi)+c_e(e_m+e_n-e_n\varphi+e_r\varphi)+s}{8(k_{new}+k_{old})}\right)^2 \\ - (c_m+c_e e_m)\frac{-c_d-c_m+2D_{PGC}(2k_{new}+3k_{old})+V_b(1-\hat{q})-p_n(1-\varphi)-c_e(e_m+e_n-e_n\varphi+e_r\varphi)-s}{8(k_{new}+k_{old})} - \\ (p_n+c_e e_n)\left(\frac{-c_d-c_m+2D_{PGC}(2k_{new}+3k_{old})+V_b(1-\hat{q})-p_n(1-\varphi)-c_e(e_m+e_n-e_n\varphi+e_r\varphi)-s}{8(k_{new}+k_{old})} - \varphi\left(\theta Q_{new1} - \frac{c_d+c_m+2D_{PGC}(2k_{new}+k_{old})-V_b(1-\hat{q})+p_n(1-\varphi)+c_e(e_m+e_n-e_n\varphi+e_r\varphi)+s}{8(k_{new}+k_{old})}\right)\right) \\ - c_e e_r\varphi\left(\theta Q_{new1} - \frac{c_d+c_m+2D_{PGC}(2k_{new}+k_{old})-V_b(1-\hat{q})+p_n(1-\varphi)+c_e(e_m+e_n-e_n\varphi+e_r\varphi)+s}{8(k_{new}+k_{old})}\right) \\ - c_r\theta Q_{new1} - c_d\left(\theta Q_{new1} - \frac{c_d+c_m+2D_{PGC}(2k_{new}+k_{old})-V_b(1-\hat{q})+p_n(1-\varphi)+c_e(e_m+e_n-e_n\varphi+e_r\varphi)+s}{8(k_{new}+k_{old})}\right) \\ + \mu_7\left(\eta\theta Q_{new1} - \frac{c_d+c_m+2D_{PGC}(2k_{new}+k_{old})-V_b(1-\hat{q})+p_n(1-\varphi)+c_e(e_m+e_n-e_n\varphi+e_r\varphi)+s}{8(k_{new}+k_{old})}\right) \\ + \mu_8\frac{c_d+c_m+2D_{PGC}(2k_{new}+k_{old})-V_b(1-\hat{q})+p_n(1-\varphi)+c_e(e_m+e_n-e_n\varphi+e_r\varphi)+s}{8(k_{new}+k_{old})} \end{array}\right)$$

.

By noting that $\frac{\partial^2 L(s,\mu_7,\mu_8)}{\partial s^2} = -\frac{1}{32(k_{new}+k_{old})} < 0$, there are three possible cases for equilibrium solutions:

(i) When $\mu_7 = 0$, $\mu_8 > 0$, $\eta\theta Q_{new1} - (c_d + c_m + 2D_{PGC}(2k_{new}+k_{old}) - V_b(1-\hat{q}) + p_n(1-\varphi) + c_e(e_m+e_n-e_n\varphi+e_r\varphi) + s)/(8(k_{new}+k_{old})) > 0$, $(c_d + c_m + 2D_{PGC}(2k_{new}+k_{old}) - V_b(1-\hat{q}) + p_n(1-\varphi) + c_e(e_m+e_n-e_n\varphi+e_r\varphi) + s)/(8(k_{new}+k_{old})) = 0$, by solving $\frac{\partial L(s,\mu_7,\mu_8)}{\partial s} = 0$, $s^* = V_b(1-\hat{q}) - 2D_{PGC}(2k_{new}+k_{old}) - c_m$

$- c_d - p_n(1-\varphi) - c_e(e_m + e_n - e_n\varphi + e_r\varphi)$ and the corresponding condition $(c_d + c_m + 2D_{PGC}k_{new} - V_b(1-\hat{q}) + p_n(1-\varphi) + c_e(e_m + e_n - e_n\varphi + e_r\varphi))/(2(k_{new} + k_{old})) \leq 0$ are obtained.

(ii) When $\mu_7 = 0$, $\mu_8 = 0$, $\eta\theta Q_{new1} - (c_d + c_m + 2D_{PGC}(2k_{new} + k_{old}) - V_b(1-\hat{q}) + p_n(1-\varphi) + c_e(e_m + e_n - e_n\varphi + e_r\varphi) + s)/(8(k_{new} + k_{old})) > 0$, $(c_d + c_m + 2D_{PGC}(2k_{new} + k_{old}) - V_b(1-\hat{q}) + p_n(1-\varphi) + c_e(e_m + e_n - e_n\varphi + e_r\varphi) + s)/(8(k_{new} + k_{old})) > 0$, by solving $\frac{\partial L(s,\mu_7,\mu_8)}{\partial s} = 0$, $s^* = 3c_d + 3c_m + D_{PGC}(4k_{new} - 2k_{old}) - 3V_b(1-\hat{q}) + 3p_n(1-\varphi) + 3c_e(e_m + e_n - e_n\varphi + e_r\varphi)$ and the corresponding condition $0 < (c_d + c_m + 2D_{PGC}k_{new} - V_b(1-\hat{q}) + p_n(1-\varphi) + c_e(e_m + e_n - e_n\varphi + e_r\varphi))/(2(k_{new} + k_{old})) < \eta\theta Q_{new1}$ are obtained.

(iii) When $\mu_7 > 0$, $\mu_8 = 0$, $\eta\theta Q_{new1} - (c_d + c_m + 2D_{PGC}(2k_{new} + k_{old}) - V_b(1-\hat{q}) + p_n(1-\varphi) + c_e(e_m + e_n - e_n\varphi + e_r\varphi) + s)/(8(k_{new} + k_{old})) = 0$, $(c_d + c_m + 2D_{PGC}(2k_{new} + k_{old}) - V_b(1-\hat{q}) + p_n(1-\varphi) + c_e(e_m + e_n - e_n\varphi + e_r\varphi) + s)/(8(k_{new} + k_{old})) > 0$, by solving $\frac{\partial L(s,\mu_7,\mu_8)}{\partial s} = 0$, $s^* = V_b(1-\hat{q}) + 8\eta\theta Q_{new1}(k_{new} + k_{old}) - 2D_{PGC}(2k_{new} + k_{old}) - c_m - c_d - p_n(1-\varphi) - c_e(e_m + e_n - e_n\varphi + e_r\varphi)$ and the corresponding condition $(c_d + c_m + 2D_{PGC}k_{new} - V_b(1-\hat{q}) + p_n(1-\varphi) + c_e(e_m + e_n - e_n\varphi + e_r\varphi))/(2(k_{new} + k_{old})) \geq \eta\theta Q_{new1}$ are obtained.

The highlights and main contributions for the paper *Reimagining Government Subsidy Policies: Facilitating Echelon Utilization and Sustainable Practices for Retired Battery Systems* are shown below:

- A two-period CLSC model for EV battery echelon utilization involving a manufacturer, recycler, power grid company, and government is proposed.
- Optimal decisions for all parties in the CLSC are analyzed to optimize subsidy policies.
- Subsidy policies effectively enhance total social welfare within the supply chain.
- Higher retired battery return yields do guarantee increased profits for recyclers or improved social welfare, with or without subsidies.