

# A causal based method for denoising non-homologous noises in time series manufacturing monitoring data

Changqing Liu<sup>a</sup>, Yingguang Li<sup>a\*</sup>, Jiaqi Hua<sup>a</sup>, Zhiwei Zhao<sup>a</sup>, James Gao<sup>b</sup>

<sup>a</sup>Nanjing University of Aeronautics and Astronautics, Nanjing, 210016, China, \* liyingguang@nuaa.edu.cn

<sup>b</sup>School of Engineering, University of Greenwich, Chatham Maritime, Kent, ME4 4TB, UK

## Abstract :

In manufacturing process monitoring, the obtained data is always affected by multi-noise resources with different statistics features, including non-homologous noises, which adversely affect data analysis, especially the performance of data driven models which are increasingly developed in manufacturing applications. Due to the lack of prior knowledge of the noises, traditional denoising methods based on modeling noise distribution with statistical features have major limitations in denoising non-homologous noises. To address this issue, this paper presents a denoising method for monitoring data with non-homologous noises based on causal inference. The causal relationship between raw data from multi sources, and the noises and monitoring data from sensors is modeled. To remove the influence of noises on monitoring data, the instruments variable is introduced into causal model, which forming new back-door paths between non-homologous noises sources. Then the noise could be denoised according to the prior causal knowledge of the causal model. The method is verified in both simulation and actual machining environment, which lays a data foundation for establishing accurate and stable prediction and control models during manufacturing processes.

**Key words:** Data denoising, Non-homologous noise, Causal-based method, Back-door path.

## 1. Introduction

In Industry 4.0, time-series data is a crucial type of data that is used for analyzing manufacturing processes, predicting failures, and conducting condition monitoring and control. However, recorded time-series signals are often mixed with noises and other types of interferences. Data noise in the manufacturing process refers to the difference between the measurement result of an object and its raw value [1]. It could be divided into noises caused by burst errors, random errors of manufacturing systems, noises caused by external interruptions and noises caused by systematic errors of the measurement system, which is the focused of this paper. It can be divided into noises caused by burst errors, random errors of manufacturing systems, noises caused by external

interruptions and noises caused by systematic errors of measurement systems, which is the focused of this paper. The existence of noises will negatively impact the accuracy and reliability of subsequent analysis. In the aerospace manufacturing industry, the presence of noises can lead to significant losses due to erroneous decision-making or processing based on monitoring data. Therefore, denoising is an important part of data processing in industry 4.0. The denoised data can improve the accuracy of subsequent analysis and diagnosis models, thus improving manufacturing efficiency and saving costs.

During manufacturing, multiple sensors are utilized to monitor the machining processes and collect a plethora of different time-series data from each sensor [2]. Therefore, noise sources are generally reflected in the data collected by related sensors, and noises can be removed by modeling the noise sources. However, there are cases where multi-functional sensors are used to collect data through a single sensor. For example, a single multi-directional vibration sensor is used to measure the vibration of an object in 3 directions (along the  $x$ ,  $y$ , and  $z$  axes). It is difficult to simultaneously model the noises obtained by different components of the sensor in different directions in a single time-series data, making it very challenging to remove the noises from different noise sources, i.e., non-homogeneous noises.

Existing denoising methods can be divided into three categories, i.e., statistics based methods, mechanism mode based methods, and data driven methods. The traditional statistics based data denoising methods require the prior knowledge of noise sources. In the manufacturing process of varying working conditions, the statistical characteristics of noise are not clear and difficult to obtain in advance. Therefore, the denoising methods based on traditional statistics have limitations in dynamic manufacturing conditions. In the traditional denoising methods based on mechanism models, it is difficult to achieve accurate mechanism modeling due to the complexity of machining processes. The mechanism of measurement noise is very complex especially in varying working conditions. Mechanism model based methods can only be carried out with many assumptions, approximations and simplifications, such as continuity hypothesis and linear simplification. Therefore, traditional mechanism model based denoising methods are also difficult to be applied in real manufacturing processes. Data driven methods are being increasingly developed with significant advantages over the traditional methods. However, it is difficult and expensive to obtain sufficient noise labels required, which caused challenge for many data-driven method based on labelled data.

Although there is no prior knowledge for the noise model, in the causal perspective, the noise could be deemed as the unobservable common cause factor in a complex measurement environment, i.e., noise

influencing all monitoring data, which is a type of prior causal knowledge as it could be known through its influence on other monitoring data and used as prior knowledge for the concerned monitoring data. Removing the influence of unobservable common cause factors based on causal inference is an effective way to restore the raw data from monitoring data, i.e., data denoising. Existing half-sibling regression methods based on causal inference use a given causal structure to remove the influence of unobservable common cause factors in a measurement system with the data affected by the same noise source, and realize data denoising in the prediction tasks without the need for modeling noise. However, for the manufacturing processes, these noises have the characteristics of non-homology. The monitoring data is not only affected by the same noise source, but also affected by other different noise sources. For example, the piezoelectric vibration sensor is usually used for measuring vibration signals, where the sensitive crystal components used for measurement in  $x$  and  $y$  directions are different, so the energy conversion process of different crystal components by the external influence of the form and degree are also different, resulting in the different signal noise in  $x$  and  $y$  direction crystal components. Because the non-homology noises cannot be deemed as common cause factors, the existing half-sibling regression method is not suitable for non-homologous noise reduction.

To solve the issue that machining process data is difficult to be denoised due to non-homologous noise, this paper describes a new monitoring data denoising method based on causal inference for machining process with non-homologous noise. By establishing the causal model, the influence of noises is removed by causal intervention based on the new backdoor path on observable working conditions, considering that the change of working conditions in the machining process, e.g., different locations or machining parameters, will cause the change of manufacturing system, producing different noises [3]. The data denoised is modelled by attractor manifold reconstruction residual regression method considering the characteristics of information correlation between the time domain and the space domain of multi-directional time-series monitoring signals in the manufacturing process. The proposed method lays a data foundation for accurate and stable machining process prediction and control using data driven models.

## **2. Related work**

This section reviews and compares previous research and current status of statistics based methods, mechanism mode based methods and data-driven methods, and analyses their problems and limitations.

### **2.1 Statistics based denoising methods**

Statistics-based denoising methods analyse statistical properties of data from different perspectives, such

as in time domain and frequency domain, and use prior knowledge of statistical information of noise, e.g., the distribution of random noise is in the form of Gaussian distribution, the frequency of vibration noise is known and high, before removing its influence. These methods include mean value, wavelet transform [4][5] convolution [6] and other filter methods, like Kalman filter [7] and particle filters [8].

Marchuk et al. [9] studied the estimation of reproduction strategies to compensate the deviation of additional noise components with asymmetric distribution law in situations where signal functions and prior information of noise statistics were very limited. Liu et al. [10] employed the maximum value method to remove the interference signal of machine tools, and denoised monitoring signal using wavelet analysis by comparing the maximum signal in an empty milling segment to the subsequent signal, which can be regarded as the benchmark. Zhang et al. [11] analysed the statistical properties of systematic errors in terms of mean value and covariance, and estimated the corresponding distance and azimuth deviation using the weighted nonlinear least square method. Hu Chen et al. [12] proposed a drivetrain by proposing customized maximal-overlap multiwavelet denoising method, where the strategies of appropriate selections of wavelet base, transform strategy and threshold rule are adopted in the model of wavelet denoising method. It enhances the application of wavelet denoising method in industrial environment. John et al. [13] analyzed the influence of noise in frequency response functions, and proposed two frequency response function de-noising methods by smoothing measured frequency response functions and truncating effects of unmeasured modes in frequency response functions.

Traditional statistical methods can effectively mitigate random noises and system noises to certain extent, given that the statistical properties of these noise sources are priorly known. However, a key limitation of these methods is the requirement for statistical prior knowledge of random noise and system noise (here ‘system’ noise could be the manufacturing system noise, or the measurement system noise, or both), which can be difficult to obtain in varying working conditions. More importantly, these methods do not deal with noises caused by unpredictable interruptions in the environment, or change of manufacturing conditions temporarily. This makes traditional statistical noise denoising methods challenging to implement in the real world scenarios.

## **2.2 Mechanism model based denoising methods**

Mechanism modeling methods realise denoising of data by establishing noise mechanism models based on prior knowledge of the mechanism of noise generation. In the field of measurement, Chen et al. [14] established an angle measurement noise model of the torsional characteristics measurement system with geometric errors,

and denoised the measurement noise. In the field of machining, there exists a kind of error elimination method based on mechanism modeling, which provides some guidance for data noise reduction. Liu et al. [15] analyzed the causes of Abbe offset in CNC machine tools, and used software error compensation based on Abbe principle error model to correct systematic errors. Xiong et al. [16] proposed a closed-loop error compensation method for robot side milling of complex curved surfaces, which aimed to extract the system components of machining errors.

Despite being an effective approach in more established and stable manufacturing situations, the mechanism modeling methods have limitations in accurately reducing measurement noises, particularly when the working conditions are constantly changing in complex and flexible manufacturing systems. The formation mechanism of the measurement noise in complex systems is highly intricate, and the mechanism modeling method can only carry out functions under many assumptions, approximations and simplifications, such as continuity hypothesis and linear simplification of the model. As a result, it is difficult to achieve accurate mechanism modeling and noise reduction in today's complex and flexible manufacturing systems.

### **2.3 Data-driven denoising methods**

Data driven denoising methods are effective for data denoising compared with complex mechanism noise modeling. There are mainly two approaches. One is that the mechanism of the noise model is known, and the data driven method is used to identify the parameters of the model, like the mean value method in a Gaussian noise model. However, due to the need for the known mechanism of the noise model, similar to the mechanism modeling method, it is difficult to apply this method when the prior of noise mechanism is absent in the manufacturing process.

The second approach is a more direct approach in which machine learning models are built between noised data and noise. Xue et al. [17] compensated the measurement error of an industrial robot angle measuring device within the entire perimeter based on the backpropagation neural network model, and obtained the angular displacement of the input and output of the tested reducer during the driving process accurately. Zeng et al. [18] developed a measurement data denoising method based on error similarity analysis. This mathematical model qualitatively analyzed the space similarity of measurement noise and employed simulation and semi-variance graph to conduct quantitative analysis. Jiang et al. [19] proposed a Support Vector Machine method based on improved Particle Swarm Optimization to predict dynamic measurement noise of sensors. The dynamic measurement noise data from two sensors was used as the test data. Yang et al. [20] presented a signal denoising

method based on Generative Adversarial Network. This method had the advantage of strong data-driven model fitting ability. Similarly, Wei et al. [21] proposed denoising framework UKF-CycleGAN for multi-sensor signals based on deep-learning, where the Gaussian fitting distribution of CycleGAN was replaced by Bayesian nonparametric estimation to ensure the quality of denoising data to the greatest extent. Saul et al. [22] proposed a blind denoising method for real time denoising, where noise contrastive estimation was used as a regularization term over the latent space. It achieves blind denoising, because there is no need for clean version of the signals and prior knowledge. However, because data-driven methods rely on mining the correlation between variables, this often leads to modeling errors. Especially for the machining process of variable working conditions, where the data may not conform to the assumption of independent and identical distribution, this may lead to larger modeling errors of data driven methods, and a greater impact on the noise reduction effect of measured data.

To sum up, traditional statistics based and mechanism model-based denoising methods require the prior knowledge about the noise sources to model data noise. However, in the case of complex, variable working conditions in machining processes, this prior knowledge may not be available. It is difficult for data driven denoising methods to obtain sufficient noise labels in the process of varying working conditions. Therefore, the data denoising in the machining process of varying working conditions needs further study.

### 3. Research problem definition

From causal perspective, monitoring data is caused by noise sources and the raw data. Thus, the noises and raw data could be deemed as the cause factors, and the noised monitoring data is the result variable. In causal inference research, obtaining the raw data from noised monitoring data, i.e., data denoising, falls under the category of reconstructing cause variables, which is a common and fundamental issue found in the field of image recognition and other related applications [23]. The reconstruction of the cause variables refers to the derivation of the cause variables through the result variables affected by the cause variables when these causes variables are unobservable. There are three scenarios of cause variable reconstruction, as illustrated in Fig. 1 (a), (b) and (c). Fig. 1 (c) with the reduction of non-homogeneous noise is the focus of this paper, which involves the reconstruction of cause variables with multiple non-homogeneous ( $N^Y$  and  $N^X$ ) and unobservable variables.

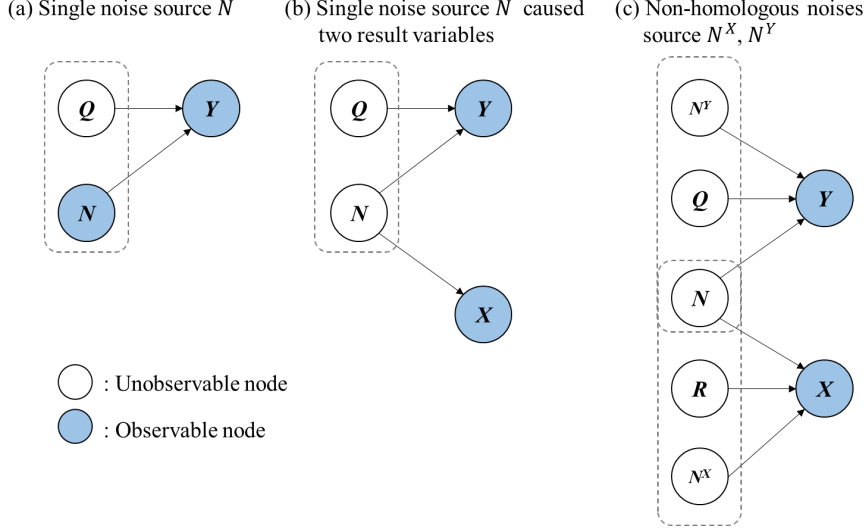


Fig. 1 Three scenarios of causal variable reconstruction

Fig. 1 (a) shows a scenario where two homologous cause variables  $Q$  and  $N$  resulted in a result variable  $Y$ . It is possible to reconstruct the unobservable cause variable  $Q$  using the observable cause variables  $N$  and the result variable  $Y$ . To tackle this, the machine learning method and the causal inference method based on intervention operation are effective means to solve this type of problem[23][24]. In Fig. 1 (b), homologous cause variables  $Q$  and  $N$  resulted in a result variable  $Y$  and both of these cause variables are unobservable. In order to reconstruct cause variable  $Q$ , a new independent and observable result variable  $X$  also affected by cause variable  $N$ , must be considered [25]. Introducing an independent, observable result variable  $X$  also affected by  $N$  is an effective way to solve this problem.

Fig. 1 (c) shows a scenario where homologous cause variables  $Q$ ,  $N$ , and non-homologous  $N^Y$  resulted in a result variable  $Y$ , while cause variable  $N$  together with non-homologous cause variable  $N^X$  resulted in another independent (from  $Y$ ) observable result variable  $X$ . Note that all the cause variables are unobservable, making this a challenging problem in cause variable reconstruction. There is not obvious easy solution to obtain all the unobservable cause variables in this scenario. This scenario is commonly seen in manufacturing process, such as the multi-direction vibration monitoring process, where the monitored data in  $X$  and  $Y$  axes are variables  $X$  and  $Y$ , caused by common noise  $N$ , and non-homologous noises source  $N^X, N^Y$  and raw data  $Q, R$ , respectively. The challenge lies in obtaining information on the unobservable causes, which makes it difficult to remove the associated noise. Therefore, research on this type of cause variable reconstruction problem with multiple unobservable homologous and non-homologous causes is crucial.

#### 4. Overview of research idea and method

Based on the above problem definition, the authors proposed a novel idea and method for denoising monitoring data with both homologous and non-homologous noises using causal knowledge. The proposed method obtains the raw data (i.e., the target data) directly from the noised monitoring data, by modeling the noise sources, which challenges the traditional denoising methods based on prior known noise sources.

For the denoising problem of non-homologous noise data in this paper, an instrument variable is introduced which satisfies the assumption that it is a common cause of non-homologous noise data, which establishes the relationships between non-homologous noises. Based on the knowledge in numerical control (NC) machining, the change of working conditions in the machining process will cause the change of vibration frequency, producing different noises, which means that the working conditions are the causes of different noises, which could be deemed as the common cause factor, this paper introduces machining conditions, like spindle speed and cutting depth, that fulfill these assumptions and deemed as instrument variables to form the backdoor paths, a backdoor path is a path in a causal network that allows for indirect influence between two variables, as the backdoor path between  $X$  and  $Y$  illustrated in Fig. 2 (a).

In this paper, based on the establishment of a structural causal model, some new backdoor paths are introduced through the machining condition. By intervening on the machining condition, an intervention distribution containing non-homologous noise information can be calculated, and the unobserved potential quantity under the condition of the machining condition can be reconstructed. The structural causal model of data denoising is illustrated in Fig. 2. As shown in Fig. 2 (a),  $Q$  and  $R$  are the unobserved target variables to be obtained (i.e., the raw data),  $Y$  and  $X$  are the data measured by two sensors respectively,  $N$  is the homologous noise,  $N^X$  and  $N^Y$  are non-homologous noises. As shown in Fig. 2 (b),  $C$  is the machining condition variable introduced to form backdoor paths. By introducing the machining condition, there are total five backdoor paths between  $Y$  and  $X$ :  $Y \leftarrow N \rightarrow X$ ,  $Y \leftarrow Q \leftarrow C \rightarrow R \rightarrow X$ ,  $Y \leftarrow N^Y \leftarrow C \rightarrow N^X \rightarrow X$ ,  $Y \leftarrow Q \leftarrow C \rightarrow N^X \rightarrow X$ ,  $Y \leftarrow N^Y \leftarrow C \rightarrow R \rightarrow X$ . Based on the backdoor paths, the influence of non-homologous noises ( $N^X$  and  $N^Y$ ) on  $X$  and  $Y$  could be obtained by the information known variable  $Q$ . To reconstruct  $Q$  through  $X$ , all five backdoor paths are considered, which contains the information of noise  $N$ ,  $N^X$  and  $N^Y$ . Therefore, by reconstructing the unobserved potential variables  $R$  and  $Q$  under the machining conditions  $C$ , the non-homologous noise could be eliminated.

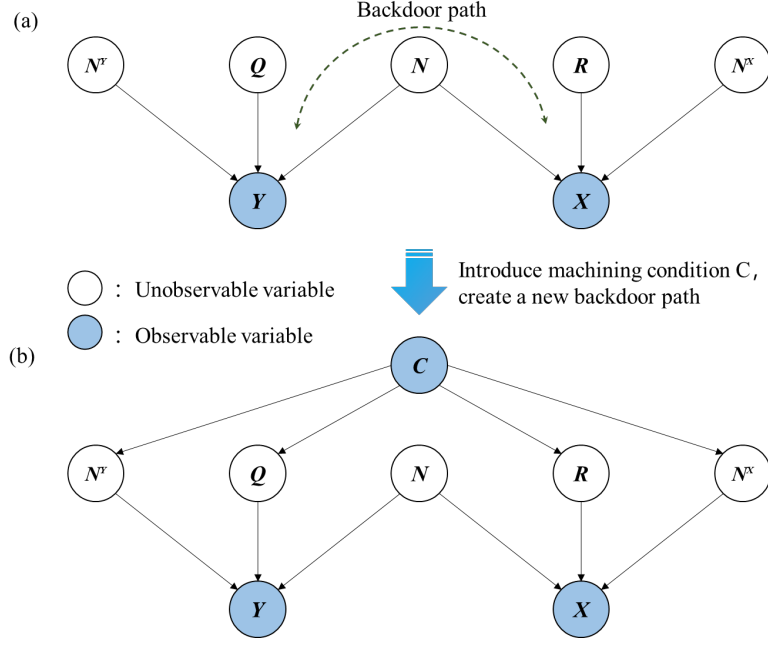


Fig. 2 Structural causal model for data denoising

In this paper, the vibration data reconstruction is used as an example. The raw vibration data generated during the machining process, referred to as  $Q$ , is to be obtained from the monitoring data measured via a vibration sensor installed at the spindle, while the measured noise  $N$  comprises random noise and noise from the NC machining system. Additionally, the monitoring data at each time point is impacted by non-homologous noise, such as different direction components and different channels of the sensor. To address those challenges, this paper introduces components from different directions of the monitoring signal, establishes a new backdoor path through the machining conditions, and establishes a temporal causal model of the monitoring signal of the machining process; Aiming at the characteristics of information correlation between the time domain and space domain of multi-directional monitoring signals in the machining process, the attractor manifold is used to reconstruct residual regression to capture the correlation of multi-directional monitoring signals, so as to reduce the noise in monitoring signals.

## 5. Causal modeling and residual regression reconstruction of monitoring data

### 5.1 Establishment of causal model of monitoring signal based on causal backdoor path

Due to the lack of prior knowledge of noise from sensor components, it is difficult to obtain the raw data  $Q$  which is mixed with noises in the data recorded from the sensor, which is a set of time-series data. It is assumed that monitored data at certain time is influenced by the same noise sources, and non-homologous noises are from different components of the sensor in different directions (in  $X$ ,  $Y$  and  $Z$  axes). Thus, based on the

above causal model, a new backdoor path is introduced using the machining conditions as observable homologous cause variable, to denoise the homologous and non-homologous noises. In this way,  $Q$  can be derived using other known observable causal variables (e.g., obtained from other sensors or other measurement methods) in the same manufacturing system.

Taking the multi-direction vibration measurement using a sensor as an example, if the raw vibration signal in the  $X$ ,  $Y$  and  $Z$  directions are  $R_t$ ,  $Q_t$ ,  $P_t$ , respectively. The data measured by sensor components are  $X_t$ ,  $Y_t$ , and  $Z_t$ , which are the raw data affected by homologous noise  $N_t$  of the sensor, and non-homologous noise of the sensor components in different direction,  $N_t^X$ ,  $N_t^Y$ , and  $N_t^Z$ . To introduce a new backdoor path to construct a new cause graph that meets the independence requirements, the machining conditions, which affect the vibration physical quantity and the noise of vibration signals in different directions, and is observable, are introduced, as illustrated in Fig. 3 (a) and (b).

As shown in Fig. 3 (b),  $C_t$  is the machining condition, specifically referring to feedrate, spindle speed, depth of cut and other known cutting parameters at current moment. The raw vibrations signals  $R_t$ ,  $Q_t$ ,  $P_t$  at moment  $t$  are influenced by the vibration at the immediate previous moment  $t - 1$  and machining conditions, satisfying the causal Markov effect.

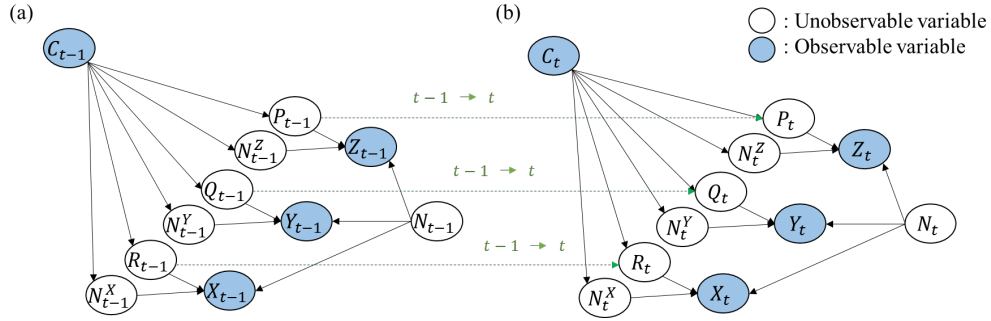


Fig. 3 Time-series causal model for monitoring signals of vibration in the machining process

## 5.2 Residual regression reconstruction of monitored data based on causal model

The temporal causal model in Fig. 3 contains two statistical characteristics that can be used for denoising monitoring data, taking data from  $x$  and  $y$  vibration directions as an example.

- (1) Given measured data  $Y_t$ , measured data  $X_t$  and raw vibration signal  $Q_t$  that are relevant, i.e.  $X_t \not\perp Q_t \mid Y_t$ , once  $Y_t$  is observed,  $X_t$  can contain the information of  $Q_t$ , which means it is feasible to obtain  $Q_t$  using  $X_t$ ;
- (2) Given machining condition  $C_t$ , measured data  $X_t$  and raw vibration signal  $Q_t$  that are independent of each other, i.e.  $X_t \perp Q_t \mid C_t$ , because  $X_t$  does not contain information about  $Q_t$ , so only the influence of non-

homologous noise  $N_t$  and  $N_t^Y$  on  $Y_t$  could be calculated, the influence of  $Q_t$  on  $Y_t$  cannot be calculated.

When  $C_t$  is given,  $Q_t$  and  $R_t$  are independent of each other. For noise  $N_t$ ,  $X_t$  contains the relevant information about the influence of  $N_t$  on  $Y_t$ . It is assuming to predict  $Y_t$  from  $X_t$ , then no information from  $Q_t$  could be extracted, only the information component from  $N_t$  will be extracted. Therefore, trying to predict  $Y_t$  from  $X_t$  is a selective capture tool that can extract the influence of  $N_t$  on  $Y_t$  and then remove it to obtain an estimate of  $Q_t$  to achieve the purpose of reducing homologous noise; For non-homologous noise  $N_t^X$  and  $N_t^Y$ , because there is only one common cause  $C_t$ , the information from non-homologous noise of  $N_t^X$  and  $N_t^Y$  in  $X_t$  and  $Y_t$  is contained in  $C_t$ . Trying to predict  $Y_t$  from  $X_t$  based on  $C_t$  can extract the influence of  $N_t^Y$  on  $Y_t$ , which could be removed and to obtain an idealized estimate of  $Q_t$  to achieve the purpose of reducing non-homologous noise. Aiming to preserve the influence of  $C_t$  on non-homologous noise,  $Q_t$  is reconstructed as the residual of regression of  $Y_t$  by  $X_t$  and  $C_t$ . Even if noise  $N$  contains the temporal structure, it will not affect the reconstruction result:

$$\widehat{Q}_t := Y_t - \mathbb{E}[(Y_t - E[Y_t|C_t])|X_t, C_t] \quad (1)$$

According to literature [25], it can be proved that the reconstruction accuracy of the above reconstruction  $\widehat{Q}_t$  of  $Q_t$ , will not be lower than  $\mathbb{E}[(Y_t - Q_t)^2]$ , i.e.,  $\mathbb{E}[(Y_t - Q_t)^2] \geq \mathbb{E}[(\widehat{Q}_t - Q_t)^2]$ , the proof process is as follow,

$$\begin{aligned} \mathbb{E}[(Y_t - Q_t)^2] &= \mathbb{E}[(Q_t - E[Q_t]) - (Y_t - E[Y_t])]^2 = \mathbb{E}[(Q_t - Y_t - E[Q_t - Y_t])]^2 \quad (2) \\ &\geq \mathbb{E}[(Q_t - Y_t - E[Q_t - Y_t|X_t, C_t])]^2 \\ &= \mathbb{E}[(Q_t - Y_t - E[Q_t|C_t] + E[Y_t|X_t, C_t])]^2 \\ &= \mathbb{E}[(Q_t - (Y_t - E[Y_t|X_t, C_t] + E[Y_t|C_t]))]^2 \\ &= \mathbb{E}[(Q_t - (Y_t - \mathbb{E}[Y_t - E[Y_t|C_t]|X_t, C_t))]^2 = \mathbb{E}[(Q_t - \widehat{Q}_t)]^2 \\ &= \mathbb{E}[(\widehat{Q}_t - Q_t)^2] \end{aligned}$$

On the other hand, it can be proved that the reconstruction accuracy of the above reconstruction  $\widehat{Q}_t$  of  $Q_t$ , will not be lower than  $\mathbb{E}[(\widehat{Q}_t' - Q_t)^2]$  obtained by Half-Sibling Regression method, i.e.,  $\mathbb{E}[(\widehat{Q}_t - Q_t)^2] \leq \mathbb{E}[(\widehat{Q}_t' - Q_t)^2]$ . The proof process is as follow,

$$\begin{aligned}
\mathbb{E}[(\widehat{Q}_t - Q_t)^2] &= \mathbb{E}[(Q_t - \widehat{Q}_t)^2] = \mathbb{E}[(Q_t - (Y_t - \mathbb{E}[Y_t - \mathbb{E}[Y_t|C_t]|X_t, C_t]))^2] \\
&= \mathbb{E}[(Q_t - (Y_t - \mathbb{E}[Y_t|X_t, C_t] + \mathbb{E}[Y_t|C_t]))^2] \\
&= \mathbb{E}[(Q_t - Y_t - \mathbb{E}[Q_t|C_t] + \mathbb{E}[Y_t|X_t, C_t])^2] \\
&= \mathbb{E}[(Q_t - Y_t - \mathbb{E}[Q_t - Y_t|X_t, C_t])^2] \\
&\leq \mathbb{E}[(Q_t - Y_t - \mathbb{E}[Q_t - Y_t|X_t])^2] \\
&= \mathbb{E}[(Q_t - \mathbb{E}[Q_t]) - (Y_t - \mathbb{E}[Y_t|X_t])^2] = \mathbb{E}[(\widehat{Q}_t - Q_t)^2]
\end{aligned} \tag{3}$$

From the causal model in Fig. 3, it could be seen that all other  $X_{t+h}$ ,  $h \neq 0$  may contain more information about  $Y_t$  after predicting  $Y_t$  using  $X_t$ , and the different monitoring signals at different times do not satisfy the independence principle. On the other hand, although the monitoring points in the machining process generate monitoring signals in multi-directions, the multi-directional monitoring signals from the same monitoring point is associated in the time domain and the space domain. It is difficult to capture the relationship, if only regressing  $Y_t$  using  $X_t$ , which affects the performance of the regression model in noise reduction. Therefore, in the monitoring signal denoising process of machining process, the attractors, which contains time-space information in the monitoring signals in different direction, are used in this paper, and the attractor manifold reconstruction is used to model the residual regression model.

In complex systems with deterministic and stochastic components, time-space dynamics can be described as a unique attractor. According to the dynamical system theory, when two time series  $x$  and  $y$  are generated in the same dynamical system, they co-exist in a common attractor, which is the embedded performance of their joint dynamical system. Therefore, each variable contains the state information of other variable [24][25]. However, it is not sufficient to assess the nature of the relationship between variables by sharing a common attractor. This complex information is contained in the local time-space dynamic interactions between the attractors  $M_x$  and  $M_y$  of  $x$  and  $y$ . Thus, the time-space neighborhood in  $M_y$  could be used to predict the time-space neighborhood in  $M_x$ , so as to model the regression model  $\mathbb{E}[Y_t|X_t]$ , to capture the time-space information relationship of multi-direction monitoring signals in machining process.

For a certain deterministic dynamic system such as the sensor monitoring signal in the machining process, it could be represented by an attractor manifold, i.e., the motion trajectory formed by a point in high-dimensional space along with time. In the three-direction vibration monitoring system, the vibration signals in the  $X$ ,  $Y$  and  $Z$  directions belong to the same dynamic system, so the time series  $X_t$ ,  $Y_t$  and  $Z_t$  share a manifold  $M = (X_t, Y_t, Z_t)$ . Thus, the time series of  $X_t$ ,  $Y_t$  and  $Z_t$  constitutes the attractor of the three-dimensional space of the

vibration monitoring signal, that is, the collection or trajectory of the point of the vibration monitoring system space, as shown in Fig. 4. From the perspective of the dynamical system, the time series  $X_t$ ,  $Y_t$  and  $Z_t$  are the one-dimensional representations of the three-dimensional attractor  $M$ . According to Takens' theorem [26], the delay values of these one-dimensional representations can be used to reconstruct the original attractor.

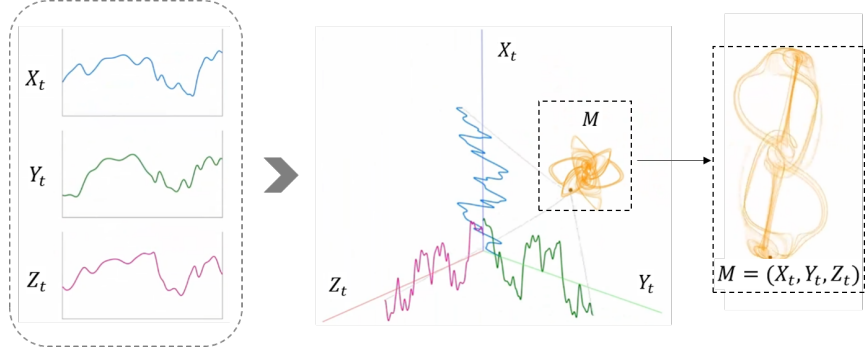


Fig. 4 Attractor flow shape and attractor diagram of three-way vibration monitoring signal

For the attractor of a vibration signal in specific direction, such as the  $X$  direction, the attractor manifold  $M_x = (X_{t-1}, X_t, X_{t+1})$  in the specific direction could be constructed by the vibration signal at the time  $t$  and its adjacent time. In the manifold  $M_x$ , the three-dimensional sequence  $X_{t-1}, X_t, X_{t+1}$  construct the shadow attractor of the vibration monitoring signal in  $X$  direction, as illustrated in Fig. 5 (a). The shadow attractor  $M_x$  maintains the topology of the original attractor  $M$ , which is a one-to-one mapping from  $M$  to  $M_x$ , as illustrated in Fig. 5 (b). In the same way, the attractor manifold  $M_y$  and  $M_z$  could be constructed by the vibration signal time sequence  $Y_t$  and  $Z_t$  in the  $Y$  and  $Z$  direction. Among them, for any time point  $t$ , the points on the manifold  $M$  and the three attractor manifolds  $M_x$ ,  $M_y$  and  $M_z$  are in one-to-one mapping, as shown in Fig. 5 (c).

For the attractor manifold  $M$  of the three-way vibration monitoring signal, the vibration time series in the  $X$  direction is  $X(t) = \{X(1), \dots, X(L)\}$ , where  $L$  is the length of the time series. According to the Takens theorem, the original manifold space can be reconstructed from the delay sequence of  $X(t)$  by selecting the appropriate embedding dimension  $E$  and delay  $\tau$ :

$$M_x = \begin{pmatrix} \underline{x}(1) = \langle X(1), X(1-\tau), \dots, X(1-(E-1)\tau) \rangle \\ \underline{x}(2) = \langle X(2), X(2-\tau), \dots, X(2-(E-1)\tau) \rangle \\ \vdots \\ \underline{x}(L-(E-1)\tau) = \langle X(L), X(L-\tau), \dots, X(L-(E-1)\tau) \rangle \end{pmatrix} \quad (4)$$

$$= (X(t), X(t-\tau), \dots, X(t-(E-1)\tau))$$

Where  $M_x$  is a matrix, the scale is  $E*L$ . In the vibration monitoring signal, take  $E = 3$ , then,

$$M_x = \underline{x}(t) = (X(t), X(t - \tau), X(t - 2\tau)) \quad (5)$$

In the same way,  $M_y$  can be constructed for the vibration monitoring signal in the  $Y$  direction:

$$M_y = \underline{y}(t) = (Y(t), Y(t - \tau), Y(t - 2\tau)) \quad (6)$$

For each point  $\underline{x}(t)$  on  $M_x$ , the distance between any two points is calculated, and the nearest neighbor  $NN\underline{x}(t)$  is extracted from the  $E+1$  neighbor points with the minimum distance. In this paper, the Manhattan distance is used as a distance measurement for the attractor subspace. Then the distance metric matrix  $D_x \in \mathbb{R}^{E-1}$  of  $M_x$  is:

$$D_x = \begin{pmatrix} d(\underline{x}(1), \underline{x}(1)) & \cdots & d(\underline{x}(1), \underline{x}(L - (E - 1)\tau)) \\ \vdots & \ddots & \vdots \\ d(\underline{x}(L - (E - 1)\tau), \underline{x}(1)) & \cdots & d(\underline{x}(L - (E - 1)\tau), \underline{x}(L - (E - 1)\tau)) \end{pmatrix} \quad (7)$$

Where  $d$  is the Manhattan distance  $d(\underline{x}(t_1), \underline{x}(t_2)) = \sqrt{\sum_{i=1}^E (\underline{x}(t_1^i) - \underline{x}(t_2^i))^2}$ . For the point  $\underline{x}(t_1) \in M_x$ , when  $E=3$ , there will be four nearest neighbor points:

$$NN\underline{x}(t_1) = \{s_1, s_2, s_3, s_4\} \quad (8)$$

Therefore, the  $E+1$  nearest neighbors of the vibration monitoring signal time series in the  $X$  direction are:

$$\begin{aligned} NN\underline{x}(t) &= \underset{(E+1)}{\operatorname{argmin}} \left\{ d(\underline{x}(t), \underline{x}(1)), \dots, d(\underline{x}(t), \underline{x}(L - (E - 1)\tau)) \right\} \\ &= \{NN\underline{x}(t_1), NN\underline{x}(t_2), \dots, NN\underline{x}(t_{E+1})\} \end{aligned} \quad (9)$$

Similarly, the distance measure matrix  $D_y$  of  $M_y$  and its nearest neighbor  $NN\underline{y}(t)$  can be constructed.

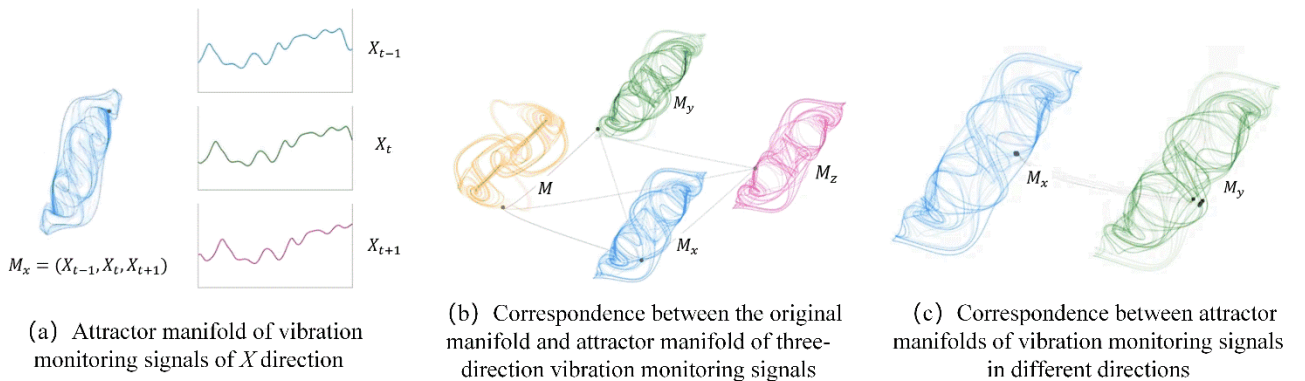


Fig. 5 Attractor flow shape and shadow attractor diagram of triaxial vibration monitoring signal

According to Eq.(1), at the current time  $t$ , the vibration monitoring signal in  $Y$  direction can be

reconstructed by the nearest time-space neighbor of  $M_x$ , and the regression model  $\mathbb{E}[Y_t - E[Y_t|C_t]|NN\underline{x}(t), C_t]$  can also be established. Since the vibration monitoring signal in the  $y$  direction is a one-dimensional vector, and the nearest time-space neighbor of  $M_x$  is a 4-dimensional vector, the fully connected neural network is used to parameterize the regression model to realize the noise reduction of vibration monitoring signal in the  $Y$  direction.

$$\widehat{Q}_t := Y_t - \mathbb{E}[Y_t - E[Y_t|C_t]|NN\underline{x}(t), C_t] \quad (10)$$

## 6. Case analysis and comparison

Due to the difficulty of obtaining raw vibration data in actual environment, both simulation environment and actual environment were applied in this research, where the raw vibration could be simulated in simulation environment and was deemed as the benchmark for comparison between different denoising methods. In the simulation environment, artificial noise was added to the simulated vibration signal, and the proposed denoising method was used to denoise the noised signal. The proposed method was compared with denoising methods based on statistics and half-sibling regression methods.

In addition, the actual cutting experiment of typical aircraft structural parts was designed to verify the proposed method's effectiveness on data driven model for tool wear, where vibration data was the main factor to predict tool wear [26]. The vibration signal data were collected in real-time during cutting process in three directions. The data denoising method was used to reduce the noise of the data. Then, a tool wear prediction model was trained using the denoising data, and its accuracy was compared with that of the half-sibling regression method to verify the effectiveness of the denoising method in improving the prediction model accuracy.

### 6.1 Verification of the proposed method using Simulation data

#### 6.1.1 Details of simulation dataset

A simulation model was used to generate the dataset and validate the proposed method. The simulation model is developed for the milling dynamic, and the milling dynamic equation can be expressed as components in both  $X$  and  $Y$  directions, as shown below:

---


$$\begin{cases} m_x \ddot{x}(t) + c_x \dot{x}(t) + k_x x(t) = F_x(t) \\ m_y \ddot{y}(t) + c_y \dot{y}(t) + k_y y(t) = F_y(t) \end{cases} \quad (11)$$


---

where the velocity in  $X$  and  $Y$  direction is represented by  $\dot{x}(t)$  and  $\dot{y}(t)$ , respectively, while the displacement, which reflects the vibration quantity, is represented by  $x(t)$  and  $y(t)$ , both with initial values of 0.

To generate data in various working conditions, the spindle speed and cutting depth ranges from ranging from 5000rpm to 10000rpm and 0.4mm to 1.8mm, respectively. The speed, acceleration, and displacement in the  $X$  and  $Y$  directions were obtained through numerical integration using the related spindle speed and cutting depth. In total, 1971 simulation data were obtained. Some of the simulated vibration signals are displayed in Fig. 7 (a).

In addition, to simulate real-world scenarios with noise, noise  $N$  conforming to the normal distribution was added to the simulated vibration signal data, where the mean value of noise  $N$  was 0, and the variance ranged from 0.05 to 0.1 to simulate the non-uniform noise of an unknown distribution. Furthermore, noise following different normal distribution variances were applied in the  $X$  and  $Y$  directions to simulate the case of non-homologous noise.

### 6.1.2 Experiment results with traditional denoising method

To illustrate the effectiveness of the proposed denoising method on unknown prior noise, the proposed method was compared with the traditional noise modeling method using statistical characteristics firstly and the average Mean Absolute Error (MAE) is used to evaluate the models denoising performance.

$$MAE = \frac{1}{n} \sum_{i=1}^n |\widehat{Q_{t,de}}^i - \widehat{Q_t}^i| \quad (12)$$

where  $\widehat{Q_{t,de}}^i$  is the denoised signal data,  $\widehat{Q_t}^i$  is the true data without noise.

The comparison results are shown in Fig. 6 (a), where the horizontal axis represents the variance of the noise applied to the vibration signal, and the vertical axis represents the MAE between the signal after noise reduction and the raw signal. The closer the value is to 0, the closer the signal after denoising is to the raw signal, indicating a better noise reduction effect. It can be found that as the distribution of noise  $N$  changes, the traditional noise reduction method can achieve good results only when its noise priori is consistent with the real noise distribution. However, the proposed method does not require noise modeling and can achieve better denoising effect even when the noise distribution changes.

The proposed denoising method presented in this paper was also compared with the half-sibling regression method based on causal reasoning to verify the noise reduction effect on non-homologous noises. The comparison results are shown in Fig. 6 (b). The blue curve and orange curve in Fig. 6 (a) represent the noise removed by the proposed method and the half-sibling regression method, respectively, while Fig. 6 (b) represents the absolute value of the difference between the noise removed by the two methods (MAE is 0.3071).

It can be found that the noise removed by the proposed method is larger than that by the half-sibling regression method.

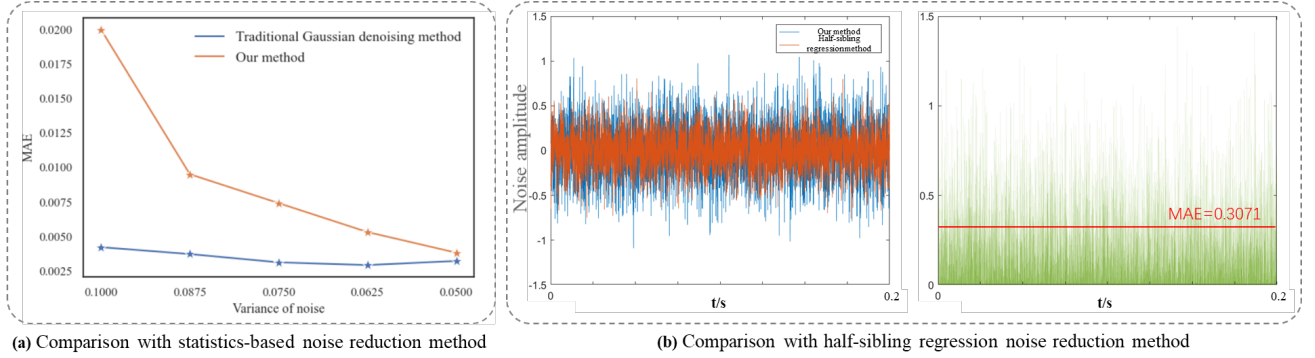


Fig. 6 Noise reduction effect comparison results

### 6.1.3 Experiment results with data-driven denoising method

Furthermore, the proposed method was compared with existing literature using blind denoising method [22]. Except for MAE, the signal-to-noise ratios (SNR) is also used here to evaluate the performance of the two denoising methods.

$$SNR = 10 \lg \frac{P_{signal}}{P_{noise}} = 10 \lg \frac{1}{N} \left( \frac{\sum_{i=1}^N \widehat{Q_{t,de}}^{i,2}}{\sum_{i=1}^N (\widehat{Q_t}^i - \widehat{Q_{t,de}}^i)^2} \right) \quad (13)$$

The simulation vibration clean signal data and data with noise are shown in Fig. 7 (a) and (b), respectively. In this study, the proposed method is compared with blind denoising method, its denoising result is shown in Fig. 7 (c). The method proposed in this paper is used to denoise the simulation vibration signal with noise, and the results are shown in Fig. 7 (d). After denoising, the signal-to-noise ratios (SNR) of the two methods are 3.16 and 18.67, respectively. From the graph and the data of SNR, it can be observed that in terms of the non-homogeneous noise, blind denoising method exhibit significant over-denoising. Conversely, the method proposed in this paper can effectively remove noise while preserving the original signal and maintaining a high SNR. It can be concluded that the proposed model has a better denoising performance for non-homologous noise.

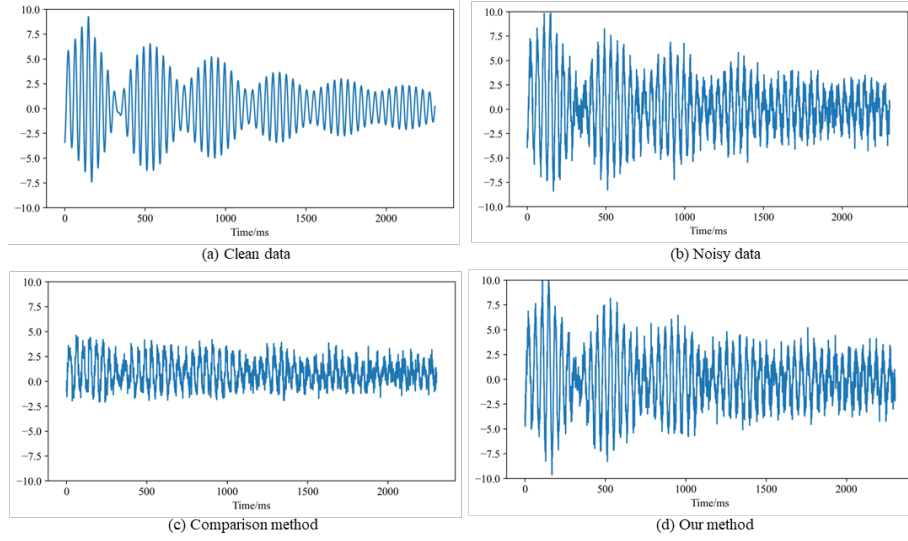


Fig. 7 Denoising results on simulated vibration signals

Furthermore, the spectral analysis of the original signal (Fig. 8 (a)), the noisy signal (Fig. 8 (b)), and the signals after-denoising by the two methods are presented in Fig. 8 (c) and (d). It can be observed from Fig. 8 (c) that the compared blind denoising method introduced numerous additional small peaks within the frequency spectrum, as marked by red circles. In contrast, the method proposed in this paper maintains the integrity of the main signal peaks while effectively restoring the signal frequency values in the intermediate region marked by the red circle. This indicates that the method proposed in this paper can effectively reduce the influence of noise and recover the original signal.

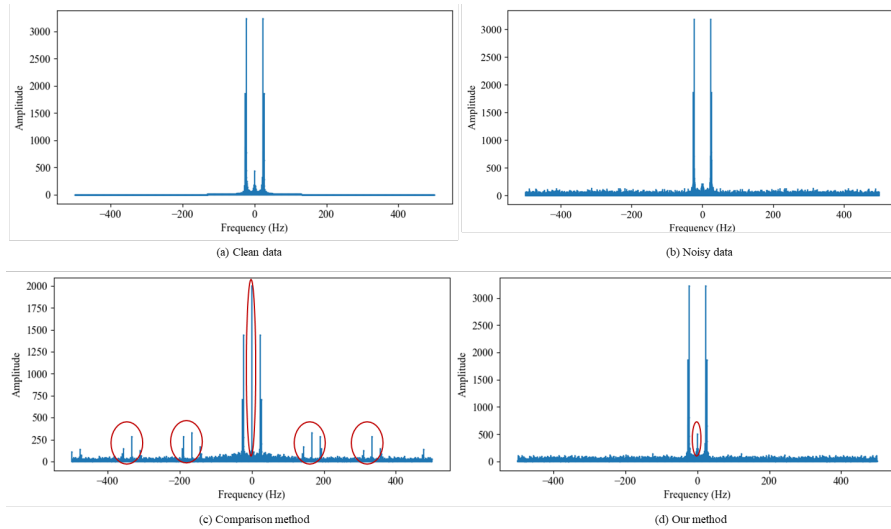


Fig. 8 Frequency domain analysis of denoising results

By comparing the proposed method with existing traditional statistical denoising methods and data-driven denoising methods in a simulation environment, it can be concluded that the proposed method performs better

in removing non-homologous noises in time series manufacturing monitoring data.

## 6.2 Verification of the proposed method using actual processing data

### 6.2.1 Details of experiment environment

In this research, an actual machining experiment was designed to verify the effectiveness of the proposed denoising method in the actual machining process. The experiment was performed on a DMG 80P five-axis machining center using a titanium alloy TC4 workpiece, which is widely used for aircraft parts manufacturing. A solid carbide end milling tool with a diameter of 10mm, and 4 tool teeth was used as the cutting tool. A KSI-108M500 vibration sensor with an acquisition frequency of 8000Hz was used to measure the vibration signals during the cutting process.

The experimental setup is illustrated in Fig. 9. The cutting parameters were designed to meet the requirements of actual parts machining process. The variation cutting conditions were generated by variations in the feed rate per tooth ( $f_z$ : 0.045mm/r-0.055mm/r), spindle speed ( $n$ : 1750r/min-1850r/min), and cutting depth ( $ap$ : 2.5mm-3mm).

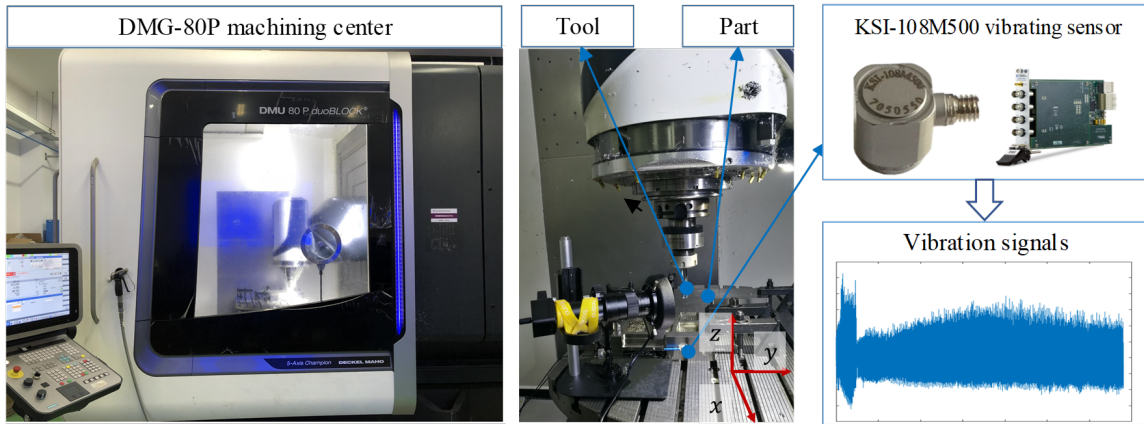


Fig. 9 Denoising experimental scenario

Fig. 10 presents the results of the experimental data collected under different working conditions, represented by groups (a), (b), and (c). Each row of figures displays a group of vibration signals and denoising effect. The horizontal axis represents the sampling point of the vibration signal, and the vertical axis represents the amplitude of the vibration signal. The blue curve in the first column depicts the monitored vibration signal. The proposed denoising method is compared with the half-sib regression denoising method. The green curve in the second column of Fig. 10 shows the denoising effect of the half-sib regression method, while the black curve in the third column represents the noise removed by this method. The denoising effect of the proposed method

is shown by the orange curve in the fourth column of Fig. 10, and the black curve in the fifth column represents the noise removed by the proposed method. It can be found that the proposed method significantly removes the complex non-homologous noise in the machining process comparing with the half-sib regression method, achieving a better denoising effect. Moreover, it effectively retains the trend of vibration signals while reducing noise, thereby laying a strong data foundation for the subsequent machining process state prediction.

In order to illustrate the influence of the attractor manifold reconstruction used in this paper on the denoising effect, and further highlight the advantages of the proposed method over the half-sib regression method, this research designed a regression model precision comparison experiment for predicting  $Y_t$  by  $X_t$  and  $C_t$  on the actual machining vibration signal data. Since the real noise-free signal cannot be obtained under the actual machining condition, the prediction accuracy was used as the evaluation index, the higher the prediction accuracy of the regression model, the more noise information the regression model contains, the more noise information is used, and the higher the reconstruction accuracy of the real signal  $\widehat{Q}_t$ , the better the denoising effect.

This research compared the prediction accuracy (Mean Absolute Error, MAE) of the regression model using the attractor manifold, the regression model without the attractor manifold, and the half-sib regression model under the three working conditions.

$$Error_{MAE} = \frac{1}{n} \sum_{i=1}^n |\widehat{Q_{t,pre}}^i - \widehat{Q}_t^i| \quad (14)$$

where  $\widehat{Q_{t,pre}}^i$  is the reconstructed or predicted signals.

The reconstruction error of the regression model using the attractor manifold was 2.81, the reconstruction error of the regression model without the attractor manifold was 3.20, and the reconstruction error of the half-sib regression model was 3.69. Compared with the half-sib regression method, in this experiment, the proposed method improved the reconstruction accuracy by 23%, and the reconstruction accuracy by 13% compared to the method without using attractor manifold reconstruction. In summary, the regression model constructed using the method proposed in this research demonstrated higher prediction accuracy and better denoising effect.

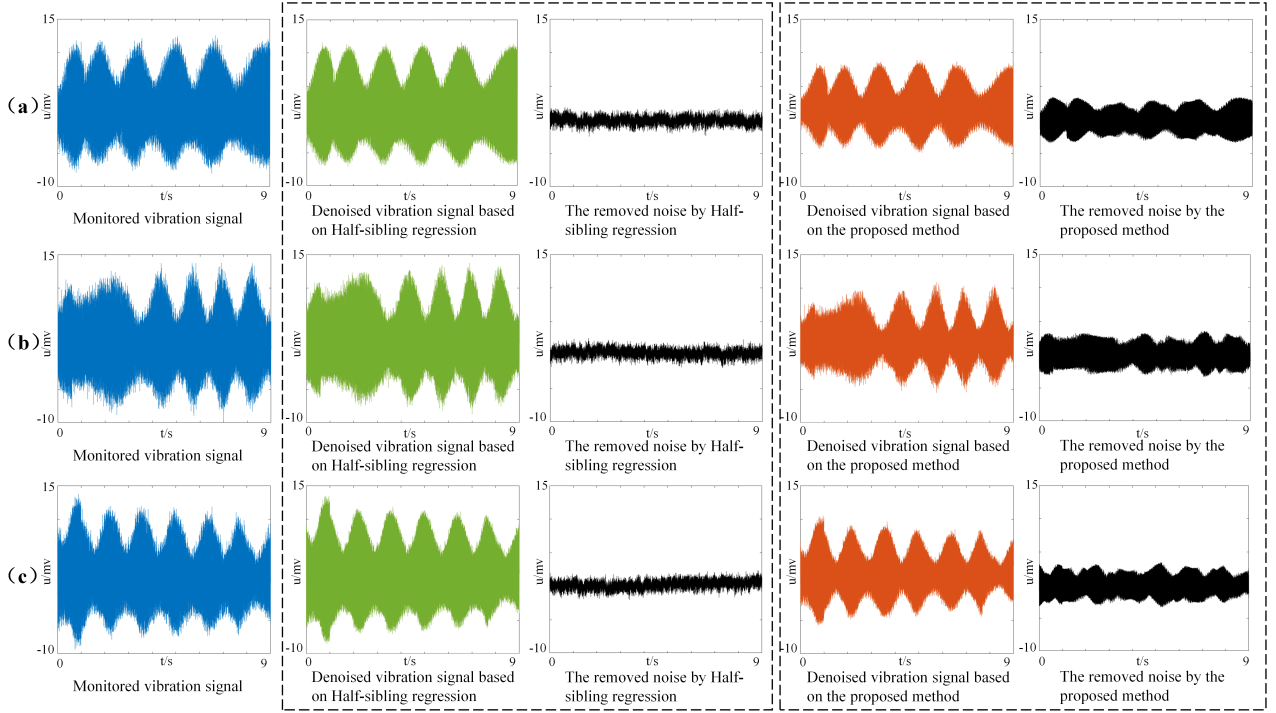


Fig. 10 Comparison of noise reduction results of vibration monitoring signal in machining process

## 7. Conclusions

The reported research investigated the issue of denoising in time-series monitoring data during machining process. The main contributions of this research are outlined below.

- (1) The problem of data denoising was defined from a causal inference perspective, which transformed the problem of data denoising with non-homologous noise into a problem of cause variable reconstruction with multiple unobservable variables. A causal model was established considering raw data, non-homologous noise and monitoring data. New backdoor paths were established by introducing observable machining conditions, which provided a basis for causal intervention.
- (2) In terms of the information correlation between the time domain and the space domain of multi-direction monitoring signals in machining process, the method based on attractor manifold reconstruction residual regression was used to denoise the monitoring data.
- (3) Taking vibration signal data denoising as an example, the proposed method was verified in both simulation environment and actual machining environment. To address the lack of reference in the actual environment, reconstruction accuracy of signal was deemed as the verification index, and the verification results all shown that the proposed method could achieve significant denoising effect.

## Acknowledgments

The reported research was supported by the National Key R&D Programs of China (2021YFB3301302) and the National Natural Science Foundation of China (52175467) led by Changqing Liu, and the National Science Fund of China for Distinguished Young Scholars (51925505) led by Yingguang Li.

## References

- [1] X. Ye, X. Xiao, J. Shi, M. Ling, The new concepts of measurement error theory, *Measurement*. 83 (2016) 96–105. <https://doi.org/10.1016/j.measurement.2016.01.038>.
- [2] X. Zhang, Y. Gao, Z. Guo, W. Zhang, J. Yin, W. Zhao, Physical model-based tool wear and breakage monitoring in milling process, *Mech. Syst. Signal Process.* 184 (2023) 109641. <https://doi.org/10.1016/j.ymssp.2022.109641>.
- [3] J.G. Proakis, D.G. Manolakis, “Digital Signal Processing – Principles, Algorithms & Applications”, Fourth Edition, Pearson Education, Prentice Hall, 2007., Pearson Education India, 2015.
- [4] M. Gao, G. Yu, C. Li, Incipient Gear Fault Detection Using Adaptive Impulsive Wavelet Filter Based on Spectral Negentropy, *Chin J Mech Eng-EN*. (2022) 35. <https://doi.org/10.1186/s10033-022-00678-4>.
- [5] C. Beale, C. Niezrecki, M. Inalpolat, An adaptive wavelet packet denoising algorithm for enhanced active acoustic damage detection from wind turbine blades, *Mech. Syst. Signal Process.* 142 (2020) 106754. <https://doi.org/10.1016/j.ymssp.2020.106754>.
- [6] D. Xue, Linear Convolution Filter to Reduce Computational Complexity Based on Discrete Hirschman Transform, *IEEE Signal Process. Lett.* 26 (2019) 1935–1939. <https://doi.org/10.1109/LSP.2019.2950111>.
- [7] O. Sayadi, M.B. Shamsollahi, ECG denoising and compression using a modified extended Kalman filter structure, *IEEE Trans. Biomed. Eng.* 55 (2008) 2240–2248. <https://doi.org/10.1109/TBME.2008.921150>.
- [8] C. Zhou, Y. Zhang, Particle filter based noise removal method for acoustic emission signals, *Mech. Syst. Signal Process.* 28 (2012) 63–77. <https://doi.org/10.1016/J.YMSSP.2011.08.004>.
- [9] V. Marchuk, S. Makov, A. Minaev, V. Voronin, D.B.T.-E.D.& T.S. Chernyshov, Removing of systematic measurement errors caused by asymmetric distribution law of the noise component, in: 2016 IEEE EastWest Design & Test Symposium (EWDTS), 2016.
- [10] C. Liu, Y. Li, J. Hua, N. Lu, W. Mou, Real-time cutting tool state recognition approach based on

- machining features in NC machining process of complex structural parts, *Int. J. Adv. Manuf. Technol.* 97 (2018) 229–241. <https://doi.org/10.1007/s00170-018-1916-y>.
- [11] Feihu, Zhang, Alois, Knoll, Systematic Error Modeling and Bias Estimation., *Sensors*. (2016).
- [12] J. Chen, Z. Wan, J. Pan, Y. Zi, Y. Wang, B. Chen, H. Sun, J. Yuan, Z. He, Customized maximal-overlap multiwavelet denoising with data-driven group threshold for condition monitoring of rolling mill drivetrain, *Mech. Syst. Signal Process.* 68–69 (2016) 44–67. <https://doi.org/10.1016/j.ymssp.2015.07.022>.
- [13] J.A. Seymour, P. Avitabile, Improved denoising methods for smoothing frequency response functions, *Mech. Syst. Signal Process.* 204 (2023) 110722. <https://doi.org/10.1016/j.ymssp.2023.110722>.
- [14] B. Chen, C. Peng, J. Huang, A New Error Model and Compensation Strategy of Angle Encoder in Torsional Characteristic Measurement System, *Sensors*. 19 (2019).
- [15] H. Liu, H. Xiang, J. Chen, R. Yang, Measurement and compensation of machine tool geometry error based on Abbe principle, *Int. J. Adv. Manuf. Technol.* 98 (2018) 2769–2774. <https://doi.org/10.1007/s00170-018-2471-2>.
- [16] G. Xiong, Z.L. Li, Y. Ding, L.M. Zhu, A closed-loop error compensation method for robotic flank milling, *Robot. Comput. Integr. Manuf.* 63 (2020) 101928. <https://doi.org/10.1016/j.rcim.2019.101928>.
- [17] J. Xue, Z. Qiu, L. Fang, Y. Lu, W. Hu, Angular Measurement of High Precision Reducer for Industrial Robot, *IEEE Trans. Instrum. Meas.* 70 (2021) 1–10. <https://doi.org/10.1109/TIM.2020.3028887>.
- [18] Q. Zeng, S. Li, Z. Deng, X. Huang, An error similarity-based error-compensation method for measurement in the nonuniform temperature field, *Meas. Sci. Technol.* 31 (2020) 085008 (13pp).
- [19] Jiang, Jiang, Jiang, Dingde, Li, Fei, Song, A Sensor Dynamic Measurement Error Prediction Model Based on NAPSO-SVM, 18 (2018) 223. <https://doi.org/10.3390/s18010233>.
- [20] Y. An, H.K. Lam, S.H. Ling, Auto-Denoising for EEG Signals Using Generative Adversarial Network, *Sensors*. 22 (2022) 1750.
- [21] X. Wei, X. Liu, C. Yue, L. Wang, S.Y. Liang, Y. Qin, A multi-sensor signals denoising framework for tool state monitoring based on UKF-CycleGAN, *Mech. Syst. Signal Process.* 200 (2023) 110420. <https://doi.org/10.1016/j.ymssp.2023.110420>.
- [22] S. Langerica, F. Núñez, Contrastive blind denoising autoencoder for real time denoising of industrial IoT sensor data, *Eng. Appl. Artif. Intell.* 120 (2023) 105838.

<https://doi.org/10.1016/j.engappai.2023.105838>.

- [23] B. Scholkopf, D.W. Hogg, D. Wang, D. Foreman-Mackey, D. Janzing, C.J. Simon-Gabriel, J. Peters, Modeling confounding by half-sibling regression, *Proc. Natl. Acad. Sci.* 113 (2016). <https://doi.org/10.1073/pnas.1511656113>.
- [24] D.A. Rand, L.-S. Young, *Dynamical Systems and Turbulence*, Warwick 1980: Proceedings of a Symposium Held at the University of Warwick 1979/80, Springer, 2006.
- [25] E.R. Deyle, G. Sugihara, Generalized Theorems for Nonlinear State Space Reconstruction, *PLoS One*. 6 (2011) e18295. <https://doi.org/10.1371/journal.pone.0018295>.
- [26] B. Qin, Y. Wang, K. Liu, S. Jiang, Q. Luo, A novel online tool condition monitoring method for milling titanium alloy with consideration of tool wear law, *Mech. Syst. Signal Process.* 199 (2023) 110467. <https://doi.org/https://doi.org/10.1016/j.ymssp.2023.110467>.