Abstract: We propose a demand response (DR) solution approach for a real-time pricing model with multi-class users to determine the electricity supply mix. The model aims to address the problem of power consumption overloading in peak hours using the real-time information obtained from the interaction between suppliers and users in a smart grid. The proposed DR algorithm allocates the overloaded demand assigned to a supplier to other electricity suppliers in order to satisfy all users’ demand while the supplier ensures to maximize the utility or reserved demand of users. Furthermore, a priority approach based on different user groups is developed for allocating the extra demand to other suppliers. Numerical experiments have been conducted to analyze the performance of the algorithm and compare the real-time electricity price with the fixed price.

Key words: demand response; classified subscribers; real-time pricing.
1. Introduction

In recent years, renewable energy sources (RES), which are believed to be cleaner and cheaper than traditional fossil power sources, are growing rapidly due to policy stimulations and technological advancements. This development is evidenced by the increasing share of RES in the energy market which was previously dominated by fossil-fueled power. The wider use of RES also changes the traditional supply-driven pricing control to demand-driven pricing control. However, since RES have higher fluctuation in supply quantity and a lower energy density than fossil fuels, it is most likely that users’ electricity consumption demands cannot be satisfied, particularly when the electricity supply only relies on a single RES supplier or a single type of RES. A combination of several RES can offer a more robust solution to the power supply. As a result, many countries or areas such as North America, European Union and China are developing balanced energy supply mix, which allows electricity users to select more than one electricity supplier to meet their demands of electricity rather than solely depending on a single supplier [1].

To select appropriate electricity suppliers, electricity users need an interactive equipment to know the price of each energy supply source and their consumption loads timely. Besides, electricity suppliers also need similar equipment to trace electricity users’ consumption, and then guide users to use electricity appropriately by adjusting the prices. Smart Grid (SG), the newest development of modern power system, can provide the interactive tool required by both electricity users and suppliers. SG is developed based on advanced technology in the power industry, high-speed and bi-directional wireless communication network, advanced sensing measurement technologies, control methods, and decision support system. The interactive equipment in SG is smart meters, which provide a real-time interactive link between electricity users and suppliers. Electricity consumption data obtained from the smart meter can help suppliers analyze, forecast and manage consumption load and further adjust the price of the electricity [2]. Smart meters can also help electricity users to monitor their consumption and their expense, and consequently, make the decision on which supplier they should choose, and when and how much electricity they would like to consume.

Pricing mechanism is the key issue of the electricity market. Fair and reasonable
electricity price can maximize social benefits through the optimal allocation of power resources. It should also allow users to choose different energy suppliers or reasonable power consumption time, which smooth out peak load to fit the intermittent characteristics of distributed generation [3-9].

At present, electricity market pricing strategies mainly include fixed price, time-of-use price [10], ladder-type price, peak-valley price, adaptive price and real-time pricing (RTP). Compared with other pricing methods, RTP is more flexible due to exploiting real-time two-way interactive information between electricity users and suppliers. Real-time communication network and smart meters in SG are preconditions to implement RTP. Through the deployment of smart meters, electricity suppliers use the real-time communication network to obtain electricity consumption data from users and analyze their consumption characteristics and pattern. Based on electricity users’ consumption information, RTP mechanism is designed to guide users to comply with electricity market regulation and shift to more reasonable consumption patterns. In other words, RTP can facilitate the interaction between users and grid load, and eventually balance electricity supply and demand in particular at peak hours.

It should be pointed out that users’ responses should not be neglected during the implementation of the RTP strategy. Users’ responses can represent whether the real-time price can really reflect the real-time change of electricity cost which leads to the optimal allocation of power resources [11-14]. Therefore, the modifications in users’ electricity demand in response to real-time price are considered in order to implement the RTP strategy. In the literature, demand response (DR) is used for this purpose. DR is electricity consumers’ market participation behavior of changing their inherent consumption patterns in response to market prices or incentives based on utilities.

Different types of users have different electricity consumption utilities which leads to different DR to the price of electricity. Setting different real-time prices according to different types of users can guide users’ electricity behavior and realizing peak cutting and valley filling better. to the best of our knowledge, the study of real-time pricing (RTP) according to users’ classification is lack in the smart grid literature.

In this paper, we classify the subscribers as the residential, commercial and industrial
subscribers and build the utility functions based on the concept of microeconomics for the
classified subscribers. Next, the distribution of electricity is considered when all electricity
consumption requirements cannot be supplied by a single supplier. We propose a DR
algorithm, which can allocate the surplus electricity requirements to other suppliers. The DR
method first classifies users to implement the different RTP strategies, then optimizes the
theoretical electricity consumption and finally allocates the overloaded electricity of the
supplier to other suppliers to balance supply and demand under the policy of balanced energy
mix. We conduct simulation experiments to show that the proposed algorithm can reduce the
load on the smart grid to meet the demands of all users while ensures that the supplier can
maximize the utility or satisfy reserved demands of the users based on a priority approach.

The remainder of the paper is structured as follows. Section 2 is a review of related
works to our paper. In Section 3, we describe the RTP model which can give the theoretically
optimal RTP and consumption load. In Section 4, we propose the DR method based on RTP
and reserved electricity consumption requirement of multi-class users in detail. Section 5
provides some numerical results and comparisons. Our concluding comments and final
observations are presented in Section 6.

2. Literature Review

In [15-17], some optimization models were developed to determine the user’s optimal
power consumption and real-time prices based on user’s utility. In [18-20], an RTP strategy
was developed based on game theory. Ma et al. [21] studied the problem of residential load
dispatching in the smart grid from the perspective of cost efficiency. The reliability of SG
requires the balance of supply and demand. DR can match the available supply under RTP.
Tsui et al. [22] proposed that the intelligent home appliances can manage the load
automatically. They also expressed that an optimal DR framework using convex programming
could solve the problem of energy-saving scheduling effectively. Tan et al. [23] developed a
DR management model with renewable distributed generators and plug-in electric vehicles
for re-selling back the generated or stored energy. Wang et al. [24] presented a
consensus-based alternating direction method of multipliers (ADMM) method, which can
solve a dynamic direct current (DC) optimal power flow (DC-OPF) problem of DR in a
distributed manner and adopted three distributed DC-OPF algorithms to discuss different
communication requirement and convergence performance. Darby [25] identified some practical and theoretical problems relating to the potential for residential DR with electric storage heating and applied many of them to heat pumps. Yang et al. [26] proposed an effective DR pricing approach to stimulate different consumers to actively participate in DR. Behboodi et al. [27] used an agent-based method to develop an effective DR control scheme with low complexity for thermostatically controlled loads participating in real-time retail electricity markets under a transactive control paradigm. Nan et al. [28] proposed a DR scheduling model for the residential community, which could reduce the cost of user's electricity consumption, the peak load and peak-valley difference of residential load profile. their model could also provide the decision support of electricity pricing strategies under power market development. Tang et al. [29] developed a direct load control technology for centralized air-conditioning systems to build fast DR for urgent requests of smart grids. Based on the multi-criteria nature of SG problems, Batista et al. [30] presented an exact multi-objective strategy for the optimal demand side management. For managing loads on the demand side, Evora et al. [31] adopted a direct load control (DLC) method based on multi-objective particle swarm optimization (MOPSO) algorithm to build the operation of the appliances with a power restriction. Biscarri et al. [32] proposed a clustering classification algorithm for the automatic classification of electricity customers’ loads, by which new customers can be assigned to a predefined set of clusters in the classification phase. Other DR contributions in SG can be found in [33-42].

3. Real-Time Pricing Model

In this section we propose a real-time pricing model for an SG system with different groups of power users and some power suppliers. In the SG system, smart meters are equipped with real-time data collection and communication functions. Hence the electricity users and suppliers can get access to the information of real-time electricity prices and consumption. Because there exists real-time information exchange between suppliers and users via smart meters, we assume that suppliers can publish real-time prices to different groups of users according to electricity loads and consumption requirements at each time slot $t_k$, and users can accordingly adjust their electricity consumption in real time based on the published electricity prices.
3.1. Multi-class users

Now the planning horizon is divided into $K$ time slots, where $K = |\kappa|$, and $\kappa$ is the set of all time slots. The interaction between users and suppliers would be stronger as the value of $K$ increases since it reflects the frequency of the communication between suppliers and users. In this paper, we consider three groups of users in the smart grid: residential, commercial and industrial. Let $\mathbb{R} = \{1, 2, 3, \ldots, n\}$, $\mathbb{D} = \{1, 2, 3, \ldots, n\}$ and $\mathbb{I} = \{1, 2, 3, \ldots, n\}$ represent the sets of residential, commercial and industrial users, respectively. $x_i^k$, $y_j^k$ and $z_l^k$ represent the actual electricity consumption of the $i$-th residential user, $j$-th commercial user and $l$-th industrial user in the $k$-th time slot respectively, where $i \in \mathbb{R}$, $j \in \mathbb{D}$ and $l \in \mathbb{I}$. To ensure the proper use of electricity, each user group should meet

$$m_{ij}^k \leq x_i^k \leq M_{ij}^k, \quad m_{ij}^k \leq y_j^k \leq M_{ij}^k, \quad m_{ij}^k \leq z_l^k \leq M_{ij}^k,$$

(1)

where $M_{ij}^k$, $M_{ij}^k$, $M_{ij}^k$, $m_{ij}^k$, $m_{ij}^k$ and $m_{ij}^k$ are, respectively, used to represent the total maximum and minimum electricity consumption of residential, commercial and industrial users in the $k$-th time slot.

3.2. Utility functions of different types of users

On the demand side of the SG system, we assume that the electricity usage of each user is independent, and different types of users have different electricity usage characteristics. Samadi et al. show that utility functions can model demand response behaviors of power consumers and refer to the user's satisfaction to consume a certain amount of electricity [16]. For example, as residential users increase their use of electricity, their utility will increase. When electricity consumption reaches a certain level, i.e., electricity demand for everyday life is satisfied, the utility will become saturated and no more utility can be generated. However, the utility of commercial and industrial users has characteristics of continuous increase with power consumption. For describing different electricity usage characteristics, we define the utility functions of different types of users as below:

$$Y(x, a) = \begin{cases} \frac{\alpha}{2a} \frac{x}{\alpha}, & 0 < x \leq \frac{\alpha}{2a} \\ \frac{\alpha}{2a}, & x \geq \frac{\alpha}{2a} \end{cases},$$

(2)

$$N(y, a) = \begin{cases} \beta \log(ay + 1), & y > 0 \\ 0, & y = 0 \end{cases},$$

(3)
where, \( Y(x, \omega), \lambda(y, \omega) \) and \( \Pi(z, \omega) \) denote the utilities of residential, commercial and industrial users at the level of \( x, y \) and \( z \), respectively; and \( \alpha, \omega, \beta \) and \( \mu \) are preset parameters which indicate the user's willingness to participate in DR and reflect the utility increment rate of different users. According to the above defined utility functions, if no electricity is consumed for any \( i \in \mathbb{I}, j \in \mathbb{J} \) and \( l \in \mathbb{L} \), the corresponding utility will be 0.

### 3.3. Electricity supply

Different energy providers may have different cost of generating electricity. In this paper, we consider the average electricity cost of the given energy providers. Following the early studies [15-17], we define electricity suppliers’ cost function \( \Box(L_i) \) as follows.

\[
\Box(L_i) = \Box(L_i^r) + \Box(L_i^c) + \Box(L_i^i)
\]

(5)

with

\[
\Box(L_i^r) = a(L_i^r)^2 + bL_i^r + c, \quad \Box(L_i^c) = a(L_i^c)^2 + bL_i^c + c, \quad \Box(L_i^i) = a(L_i^i)^2 + bL_i^i + c
\]

(6)

where \( a > 0, b \geq 0 \) and \( c \) are the default parameters, \( L_i = (L_i^r, L_i^c, L_i^i) \) and \( L_i^r, L_i^c \) and \( L_i^i \) represent the suppliers’ electricity supply to residential, commercial and industrial users in the \( k \)-th time slot, respectively. Obviously, the cost function \( \Box(L_i) \) is increasing and strictly convex because its first-order derivative is non-negative, and the second-order derivative is positive.

For satisfying the electricity consumption requirement of users at each time slot \( k \), the given supplier should supply the equal electricity to keep the balance, that is,

\[
L_i^g = \sum_{sil} x_i^s + \sum_{sic} y_i^s + \sum_{sil} z_i^s
\]

(7)

where \( L_i^g \) is the power supply provided by the given supplier. However, the energy provider has a maximum and minimum capacity of generating electricity defined as \( L_i^{\text{max}} \) and \( L_i^{\text{min}} \), respectively, at each time slot \( k \), i.e.,

\[
L_i^{\text{min}} \leq L_i^g \leq L_i^{\text{max}}
\]
If the electricity consumption requirement of users exceeds the supplier’s maximum generating capacity, the exceeding part should be satisfied by other suppliers. Meanwhile, if the requirement is less than the supplier’s minimum generating capacity, supplying the electricity by the supplier is not cost effective due to line loss and starting cost. Hence, the required electricity will be transferred to other suppliers who are capable of satisfying this demand. In other words, the electricity consumption requirement of all users at each time slot \( k \) will not be transferred to other suppliers, only if the power supply \( \mathcal{U}_k \) of the power supplier at time slot \( k \) satisfies the condition Eq. (8).

The increasing use of smart electrical appliances gives rise to making electricity reservation in advance by a great number of users. This enables the power suppliers to generate electricity with a plan at every time slot. If the given supplier cannot satisfy the reserved electricity consumption requirement at some time slots, a part of the requirement will be transferred to other suppliers. In this paper, we suppose that the suppliers can learn the electricity consumption requirement of all users by smart meters at next time slots and generate electricity according to the requirement. Moreover, each user will consume the electricity not less than the reserved requirement at every time slot, i.e, the actual electricity consumption requirement will satisfy the following condition:

\[
x^{i}_k \geq v^i_{k}, \quad y^{j}_k \geq v^j_{k}, \quad z^{l}_k \geq m^l_{k}
\]

(9)

where \( v^i_{k}, \; v^j_{k} \) and \( v^l_{k} \) are, respectively, the reserved electricity requirements of the \( i \)-th residential, \( j \)-th commercial and \( l \)-th industrial users at time slot \( k \).

3.4. RTP model of multi-class power users

In this subsection we describe the proposed model. The objective of the RTP model of multi-class power users is to maximize social benefit, which is defined as the difference between the total utility of all users and the cost of suppliers. The RTP model of multi-class power users in the SG system can be formulated as follows.
Model 1 (RTP model of multi-class power users in the smart grid)

\[
\begin{align*}
\max_{\alpha_1, \alpha_2, \alpha_3} & \quad \sum_{i \in I} \gamma(x_i, \alpha_i) + \sum_{j \in J} \lambda(y_j, \alpha_j) + \sum_{k \in K} \Pi(z_k, \alpha_k) - \square(L_k) \\
\text{s.t.} & \quad \sum_{i \in I} x_i^k \leq L_i^k, \quad \sum_{j \in J} y_j^k \leq L_j^k, \quad \sum_{k \in K} z_k^k \leq L_k^k,
\end{align*}
\]

where \( L_i^m \) and \( L_i^m \) are respectively the suppliers’ minimum and maximum supply in the time slot \( k \). These values are calculated as the total minimum and maximum power consumption requirement of all users, respectively, for keeping the balance of supply and demand. That is,

\[
L_i^m = \sum_{j \in J} m_{ij}^k + \sum_{k \in K} m_{ik}^k, \quad L_i^m = \sum_{j \in J} M_{ij}^k + \sum_{k \in K} M_{ik}^k.
\]

This model can directly be solved using some optimization methods such as the interior point method. However, the exact price information cannot be obtained directly. According to [15-17], when problem (10) is converted to the Lagrange dual problem, the optimal Lagrange multiplier is exactly the electricity price in that time slot. Hence, the Lagrange dual problem of the primal problem (10) should be created. First, for each fixed time slot \( k \), the Lagrange function of the primal problem (10) is:

\[
\Psi(X_k, L_k, \lambda) = \sum_{i \in I} \gamma(x_i^k, \alpha_i) + \sum_{j \in J} \lambda(y_j^k, \alpha_j) + \sum_{k \in K} \Pi(z_k^k, \alpha_k) - \square(L_k) \\
- \lambda^k \left( \sum_{i \in I} x_i^k - L_i^k \right) - \lambda^k \left( \sum_{j \in J} y_j^k - L_j^k \right) - \lambda^k \left( \sum_{k \in K} z_k^k - L_k^k \right) \\
+ \sum_{i \in I} \left( \gamma(x_i^k, \alpha_i) - \lambda^k x_i^k \right) + \sum_{j \in J} \left( \lambda(y_j^k, \alpha_j) - \lambda^k y_j^k \right) \\
+ \sum_{k \in K} \left( \Pi(z_k^k, \alpha_k) - \lambda^k z_k^k \right) - \square(L_k).
\]

where \( X_k = (x_i^k, y_j^k, z_k^k) \), and \( \lambda^k = (\lambda_i^k, \lambda_j^k, \lambda_k^k) \) is the Lagrange multiplier of the residential, commercial and industrial users in the time slot \( k \). Considering the separability of the terms in the Lagrange function, the objective function of the dual optimization problem is:

\[
D(\lambda) = \max_{\lambda \in \Lambda} \Psi(X, \lambda) \\
= \sum_{i \in I} R_i(\lambda_i) + \sum_{j \in J} C_j(\lambda_j) + \sum_{k \in K} f_k(\lambda_k) + g_k(\lambda_k) + h_k(\lambda_k),
\]

(13)

where

\[
R_i(\lambda_i) = \max_{\alpha_i \in \mathcal{A}_i} \left[ \gamma(x_i^k, \alpha_i) - \lambda_i x_i^k \right],
\]

(14)

\[
C_j(\lambda_j) = \max_{\alpha_j \in \mathcal{A}_j} \left[ \lambda(y_j^k, \alpha_j) - \lambda_j y_j^k \right],
\]

(15)

\[
f_k(\lambda_k) = \max_{\alpha_k \in \mathcal{A}_k} \left[ \Pi(z_k^k, \alpha_k) - \lambda_k z_k^k \right],
\]

(16)

\[
g_k(\lambda_k) = \max_{\alpha_k \in \mathcal{A}_k} \left[ \Pi(z_k^k, \alpha_k) - \lambda_k z_k^k \right],
\]

(17)

\[
h_k(\lambda_k) = \max_{\alpha_k \in \mathcal{A}_k} \left[ \Pi(z_k^k, \alpha_k) - \lambda_k z_k^k \right],
\]

(18)
Hence, the dual problem would be as follows:

\[
\min_{\lambda^k} \ D(\lambda^{k}) ,
\]

(20)

Note that the first term in \( D(\lambda^{k}) \) in (13) can be decomposed into some separable subproblems (14), (15) and (16) solved by the users and some subproblems (17), (18) and (19) solved by the energy provider, respectively. Thus, for each time slot \( k \), as a medium of exchange for users and suppliers, the electricity price, which is the optimal Lagrange multiplier, is obtained by solving the problem (20). To solve the problem (20) iteratively, the gradient projection iteration method is used as follows:

\[
\lambda^{k}_{x,1,t} = [\lambda^{k}_{x,1} + \gamma t \left( \sum_{i \neq x} \lambda^{k}_{i} y_i - E^{x}(\lambda^{k}_{x}) \right) ] ,
\]

(21)

\[
\lambda^{k}_{y,1,t} = [\lambda^{k}_{y,1} + \gamma t \left( \sum_{i \neq y} \lambda^{k}_{i} y_i - E^{y}(\lambda^{k}_{y}) \right) ] ,
\]

(22)

\[
\lambda^{k}_{z,1,t} = [\lambda^{k}_{z,1} + \gamma t \left( \sum_{i \neq z} \lambda^{k}_{i} y_i - E^{z}(\lambda^{k}_{z}) \right) ] ,
\]

(23)

where \( t \in T \), and \( T \) is the time-series set that the electric power supplier updates \( x^{k}(\lambda^{k}_{x}, \lambda^{k}_{y}, \lambda^{k}_{z}) \) at time slot \( k \). \( x^{k}(\lambda^{k}_{x}, \lambda^{k}_{y}, \lambda^{k}_{z}) \) and \( y^{k}(\lambda^{k}_{y}, \lambda^{k}_{z}) \) are respectively the local optimal values of the problem (14), (15) and (16) via \( x^{k}(\lambda^{k}_{x}, \lambda^{k}_{y}, \lambda^{k}_{z}) \) in (21), (22) and (23). Similarly, \( E^{x}(\lambda^{k}_{x}), E^{y}(\lambda^{k}_{y}) \) and \( E^{z}(\lambda^{k}_{z}) \) are respectively the local optimal values of the problem (17), (18) and (19) via \( x^{k}(\lambda^{k}_{x}, \lambda^{k}_{y}, \lambda^{k}_{z}) \) in (21), (22) and (23). \( \lambda^{k}=(\lambda^{k}_{x}, \lambda^{k}_{y}, \lambda^{k}_{z}) \) is the value of \( \lambda^{k}=(\lambda^{k}_{x}, \lambda^{k}_{y}, \lambda^{k}_{z}) \) in \( t \in T \), \( \gamma=(\gamma_{1}, \gamma_{2}, \gamma_{3}) \) is the step size and \([\delta]=\max\{0, \delta\} \).

4. DR Algorithm of Multi-class Users based on RTP

Based on RTP model discussed above, we propose a DR mechanism for multi-class users to allocate electricity consumption requirement to other suppliers when a single supplier’s supply is insufficient, which is

\[
L_{x}^{\text{min}} < L_{x}.
\]

We assume that a given supplier will first supply the electricity of all users. If the supplier cannot satisfy all the requirements, the surplus requirements will be transferred to other
suppliers. The DR algorithm first calculates the theoretical optimal power consumption $x^*(\lambda^k_i), y^*(\lambda^k_j)$ and $z^*(\lambda^k_l)$ of each group of users by solving Eq. (20) at time slot $k$. On the one hand, we consider the electricity consumption of the users. If the condition Eq. (9) is satisfied, which shows that the actual consumptions of all users are more than their reserved electricity consumption requirement, the optimal power consumption obtained from Eq. (20) will be the final power supply $L_k$ to maximize the total utility of all users in the system, i.e.,

$$L_k = \sum_{i \in I} x^*(\lambda^k_i) + \sum_{j \in J} y^*(\lambda^k_j) + \sum_{l \in L} z^*(\lambda^k_l).$$

If the condition Eq. (9) is not satisfied, the total reserved power consumption requirement of the users will be the final power supply $L_k$ to meet the actual demand of the users, i.e.,

$$L_k = \max\{x^*(\lambda^k_i), y^*(\lambda^k_j), z^*(\lambda^k_l)\}_i + \max\{x^*(\lambda^k_i), y^*(\lambda^k_j), z^*(\lambda^k_l)\}_j + \max\{x^*(\lambda^k_i), y^*(\lambda^k_j), z^*(\lambda^k_l)\}_l.$$

On the other hand, we consider the supply capacity of the given supplier. If $L_k$ satisfies the condition Eq. (8), which means that the supply will not be overloaded, the given supplier can meet the electricity demand of all users at time slot $k$ by supplying the power $L_k$, i.e., $L_k = L_k$. Otherwise, if $L_k$ is lower than the lower bound of the condition Eq. (8), which means that starting the electricity generator to supply the power is not affordable for the supplier, the users’ power request will be directly transferred to other suppliers to supply. If $L_k$ exceeds the upper limit of the condition Eq. (8), that is, the supplier will be overloaded and will not be able to meet the electricity consumption requirement of all users. Hence, a part of consumption request must be transferred to other suppliers to keep the stability of the system. The following is how to transfer the users’ consumption demand to other suppliers.

In order to allocate the users’ electricity consumption demand to other suppliers practically, we will adopt the following allocation priority rule among the different class of users:

Residential users $\triangleright$ Commercial users $\triangleright$ Industrial users \hspace{1cm} (24)

which means that the supplier first gives priority to meeting industrial users’ electricity consumption request, next commercial users’ and lastly residential users’ to keep the stability of industrial production and commercial service.

However, in the same class of users, we will use the following method to transfer the power consumption request of users to other suppliers to minimize the difference between the
optimal and reserved electricity consumption requirements. First, the consumption request of the users with the largest difference between the optimal and reserved electricity consumption request will be transferred to other power suppliers, and then the request with the second largest difference, and so on till the condition Eq. (8) is satisfied for the remaining users' power request.

The DR algorithm can be summarized as follows:

Algorithm 1 (DR algorithm of multi-class users based on RTP in the smart grid):

**Step 0** Initialize \( n_1, n_2, \ldots, n_K, \alpha, \beta, \gamma, \lambda, \mu, \rho \) and \( n_i, n_j, n_k \).

**Step 1** For each \( k \in \kappa \), identify \( \mathbf{v}_i, \mathbf{v}_j, \mathbf{v}_k \), and give \( M^i, M^j, M^k \) and \( T_{\text{min}} \).

**Step 2** Calculate the real-time prices \( \lambda^i, \lambda^j, \lambda^k \) and the optimal power consumptions \( x^i, y^i, z^i \) of each group of the users by solving Eq. (20).

**Step 3** If the condition Eq. (9) is satisfied, the final power supply is

\[
L_k = \sum_{i=1}^{n_k} x^i + \sum_{j=1}^{n_j} y^j + \sum_{l=1}^{n_l} z^l.
\]

Otherwise,

\[
L_k = \sum_{i=1}^{n_k} \max\{x^i, y^i, z^i\} + \sum_{j=1}^{n_j} \max\{y^j, z^j\} + \sum_{l=1}^{n_l} \max\{z^l, x^l\}.
\]

**Step 4** If \( L_k \) satisfies the condition Eq. (8), set \( L_k \) = \( L_k \).

**Step 5** If \( L_k < L_{\text{min}} \), then \( L_k = 0 \) and let \( L_k = L_k \), where \( L_k \) is the power supplied by other suppliers.

**Step 6** Otherwise, if \( \sum_{i=1}^{n_k} z^i > L_{\text{max}} \), Let

\[
L_k = \sum_{i=1}^{n_k} z^i \quad \text{and} \quad L_k = \sum_{i=1}^{n_i} x^i + \sum_{j=1}^{n_j} y^j + \sum_{l=1}^{n_l} z^l.
\]

If \( \sum_{i=1}^{n_i} z^i < L_{\text{max}} < \sum_{j=1}^{n_i} x^i + \sum_{j=1}^{n_j} y^j \), Let

\[
L_k = \sum_{i=1}^{n_i} y^i + \sum_{j=1}^{n_j} x^j \quad \text{and} \quad L_k = \sum_{i=1}^{n_i} x^i + \sum_{j=1}^{n_j} y^j.
\]

If \( \sum_{i=1}^{n_i} x^i + \sum_{j=1}^{n_j} y^j < L_{\text{max}} < \sum_{i=1}^{n_i} x^i + \sum_{j=1}^{n_j} y^j + \sum_{l=1}^{n_l} z^l \), Let

\[
L_k = \sum_{i=1}^{n_i} x^i + \sum_{j=1}^{n_j} y^j + \sum_{l=1}^{n_l} z^l \quad \text{and} \quad L_k = \sum_{i=1}^{n_i} x^i.
\]
where $\mathbb{1}_r$, $\mathbb{1}_c$, and $\mathbb{1}_i$, which are subsets of $\mathbb{1}$, $\mathbb{1}$ and $\mathbb{1}$ respectively, represent the part of the residential, commercial and industrial users whose electricity consumption demands are transferred, respectively.

Note that the users’ optimal electricity consumptions obtained in Step 2 may be less than the reserved electricity consumptions of users. In the actual supply process of electricity, if suppliers cannot meet the users’ reserved electricity demand, other suppliers will be selected to supply the power shortfall. Steps 3-6 give the detailed solution. The whole process of the algorithm for each time slot $k$ is shown in Figure 1.

Fig. 1. DR algorithm of multi-class users based on RTP

5. Numerical analysis

In this section we apply our algorithm for a random sample in order to investigate its efficiency. Matlab 2016 is used to perform numerical simulation of Algorithm 1. We select the following parameters in the simulation.

Suppose that there are $n_r = 100$ residential users, $n_c = 10$ commercial users and $n_i = 2$ industrial users, $K = 24$ represents 24 hours a day. The default parameter $\omega$ is a random number in the interval $[1, 4]$ and remains fixed throughout the algorithm. The constant parameters $[16]$ are set as $a = 0.01$, $b = c = 0$, $\alpha = 0.5$, $\beta = 5$, $\mu = 20$ and $y_2 = y_3 = y_4 = 0.01$, the initial value is $\lambda = 0.4 [16]$.

First, we run Algorithm 1 and find the real-time prices of three types of users at each slot.
as shown in Figure 2. The result shows that commercial and industrial users have insignificant fluctuation in real-time electricity price because they have steady electricity consumption. However, residential users have significant changes in electricity consumption, which indicates that they have a large real-time electricity price fluctuation. As the electricity consumption increases over the day, the electricity price rises, which helps to guide the users to save the electricity.

Fig.2. Real-time electricity prices of different classes of the users
Fig.3. Optimal electricity of real-time and fixed electricity prices

Next, we look at the optimal power consumption $\psi' \left( \lambda_{k} \right), y' \left( \lambda_{k} \right), \psi' \left( \lambda_{k} \right)$ and give their total optimal consumption, which is shown in Figure 3. At the same time, in order to make a comparison, the fixed power price and its corresponding electricity consumption are added. The average value of the fixed price of all users in all time slots is $p = 0.725$. Obviously, the optimal power consumption of the real-time electricity price is between the maximum and minimum supply of the supplier, which can satisfy the condition Eq. (8). Compared with the fixed electricity price, the real-time electricity price can effectively reduce the electricity consumption during the peak hours due to the high efficiency of information transmission. Thus, the load of the electricity grid is reduced.

Then, we conduct a further analysis on the selection of the final suppliers when the condition Eq. (8) is satisfied. Take the residential users as an example. 10 users’ data are selected as shown in Fig. 4. From Fig. 4, the optimal electricity consumption of the users 1, 2, 4, 5, 7 and 9 is less than the reserved power, so the final power supply should be equal to the user’s reserved power consumption requirement. The optimal power consumption of the users 3, 6, 8 and 10 is greater than their reserved power consumption requirement, so their final power supply should be equal to the user's optimal power consumption.
Finally, we analyze the allocation of the electricity when the condition Eq. (8) is not satisfied. Figure 5 shows the supply and demand of the power grid. As shown in Figure 5, the theoretical optimal power consumption is between the minimum and maximum power supply. Although the total grid load computed by the theoretical optimal power consumption meets the condition Eq. (8), it does not meet the reserved requirement of all users. The simulation results show that at time slots 4, 7, 10, 20, 22 and 24, the users’ total electricity consumption requirement exceeds the maximum supply capacity of the supplier, which indicates that the condition Eq. (8) is not satisfied. However, at the other time slots, Eq. (8) will be satisfied if the total consumption requirement is regarded as the final supply of the supplier. Hence, the supplier can meet the reserved electricity requirement of all users except in time slots 4, 7, 10, 20, 22 and 24, although it is not the theoretical optimal power supply.

In time slots 4, 7, 10, 20, 22 and 24, we use the priority rules to meet the reserved electricity requirement of all users. From Step 6 in Algorithm 1, we know that the maximum supply will be the final supply of the supplier. The other power suppliers will make up the shortfall of electricity requirement. The computation result indicates that the final supply of the supplier can meet all industrial and commercial users and a part of the residential users. Hence, we can analyze the priority rule only by the residential users. Fig. 6 gives the electricity requirements of the residential users at the time slots 4, 7, 10, 20, 22 and 24.
In Fig. 6, 0 denotes that the user’s electricity requirement has been transferred, whose electricity will be supplied by other suppliers. For example, at time slot 4, the electricity requirements of the users 47 and 59 will be transferred to other suppliers to supply electricity.

From the above analysis, we can know that compared to fixed price, the DR method based on RTP can meet users’ reserved electricity needs, reduce and stabilize power load and guide users to consume electricity reasonably. Meanwhile, it can also transfer positively the overload electricity requirement to other qualified suppliers according to the priority rule at those time slots, which ensures that the supply network can run properly.

6. Conclusion

The large number of power users in the smart grid causes the complexity of the power supply and demand relationship. Based on the existing literature, a real-time electricity price can be obtained based on the user’s utility. Since different types of users have different power characteristics, the power suppliers usually cannot meet the users’ optimal electricity demand at peak time slots and users’ demand is beyond the supply capacity of a single supplier. To address this issue, we proposed the DR algorithm based on classified users' RTP with the users’ utility in the smart grid. This algorithm can solve the problems of the electricity consumption of the residential, commercial, industrial users with multiple suppliers under the power supply, real-time electricity prices, optimal electricity consumption and the balance between supply and demand. The simulation results show that the DR algorithm based on RTP of multi-class users can reduce the power grid load and meet the reserved electricity consumption of users. The algorithm can also transfer the overloaded electricity demand exceeds the supply capacity of a supplier to other suppliers in peak time slots. This not only ensures the normal supply of the power network and meets all users’ requirement, but also the supplier can maximize the electricity utility or requirement of the key user-groups efficiently.

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References


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