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A Discrete Formalism for Reasoning about Action and Change

Brian Knight, Jixin Ma & Taoxin Peng
School of Computing and Mathematical Science, University of Greenwich, Woolwich, London SE18 6PF, UK
Email: b.knight, j.ma, t.peng@greenwich.ac.uk

Abstract

This paper presents a discrete formalism for temporal reasoning about actions and change, which enjoys an explicit representation of time and action/event occurrences. The formalism allows the expression of truth values for given fluents over various times including non-decomposable points/moments and decomposable intervals. Two major problems which beset most existing interval-based theories of action and change, i.e., the so-called *dividing instant problem* and the *intermingling problem*, are absent from this new formalism. The dividing instant problem is overcome by excluding the concepts of ending points of intervals, and the intermingling problem is bypassed by means of characterising the fundamental time structure as a well-ordered discrete set of non-decomposable times (points and moments), from which decomposable intervals are constructed. A comprehensive characterisation about the relationship between the negation of fluents and the negation of involved sentences is formally provided. The formalism provides a flexible expression of temporal relationships between effects and their causal events, including delayed effects of events which remains a problematic question in most existing theories about action and change.

1 Introduction

Modelling the dynamic aspects of the world in terms of representing and reasoning about actions and change is one of the most important problems in the domain of artificial intelligence. Several approaches have been proposed for dealing with this problem over the past half century, including McCarthy and Hayes' framework of situation calculus[17,19], which is probably the most influential formalism regarding this area. Several extensions to the framework have been proposed to add temporal features into the situation calculus, e.g. Gelfond, Lifschitz & Rabinov[9], Miller & Shanahan[20], Pinto & Reiter[21], Schubert[24], in order to enrich the temporal ontology. These formalisms usually associate entities such as situations/states, fluents, and actions with some special time, where time elements are characterised as points and intervals are constructed out of points. However, these approaches have not gone as far as one would like for dealing with temporal issues in representing and reasoning about actions and their effects, and there are still some problematic issues which have not been satisfactorily solved.

Generally speaking, the world persists in a given state until some action is carried out to change it into another state; also, while some actions may be instantaneous, most of them perform over some interval of time. Hence, intervals are needed for expressing the time spans of situations and actions. For instance, in Pinto and Reiter's formalism[21], the time span of a given situation is characterised in terms of its starting time point and ending time point during which no fluents change truth values. However, the approach that characterises intervals as derived structure of points may lead to the so-called *Dividing Instant Problem*[5,26], that is the question of specifying whether time spans of situations are closed or open at their starting/ending points: If all intervals include their ending-points, then adjacent intervals would have ending-points in common. Hence, if two adjacent intervals correspond to states of truth and falsity of a given fluent, there will be a point at which the fluent is both true and false. Similarly, if all intervals don't include their ending-points, there will be points at which the truth or falsity of some fluents are undefined. Another approach is to take point-based intervals as semi-open (e.g., all intervals include their left ending-points, and exclude their right ones) so that they may sit conveniently next to one another. However, on the one hand, since this approach insists that every interval contains only a single ending-point, the choice of which ending-point of intervals should be included/excluded seems arbitrary, and hence unjustifiable and artificial. On the other hand, although the approach may offer a solution to some practical questions, there are some other critical questions which remain problematical (examples are given by Galton[7] and Ma & Knight[16]). The fundamental reason is that in a system where time intervals are all taken as semi-open, it will be difficult to represent time points in an consistent structure so that they can stand between intervals conveniently.

The second question is that in those existing theories about action (event) and change (including that of Pinto and Reiter[21], of Miller and Shanahan[20], and of Allen and Ferguson[1]), which allows intervals (primitive or point-based) as time objects, the negation of a given fluent and the relationship between a negative fluent and sentences which involve the fluent have not been formally addressed. This is in fact a very important issue since we may face the possibility that some fluents might be neither true nor false <u>throughout</u> some specified intervals. Additionally, in a logic where some time intervals are characterised as infinitely decomposable, the so-called *intermingling problem*[8,10,26] may arise, that is, the possibility of indefinitely intermingled time intervals within each of which a fluent f takes both true and false values. This will lead to some difficulties in characterising the relationships between the negation of a fluent and the negation of the corresponding sentence involving that fluent.

Another question is that, in most of the existing systems mentioned above, the effects of actions/events have been represented by the results just after executing actions. Actions with delayed effects have not been successfully dealt with. In fact, temporal relationship between actions and their effects is quite complex and interesting. Gelfond *et al*.[9] proposed an approach using the notation *Duration of Actions* to describe an action with delayed effects. They simply count the delay time in the duration of actions. So the actions have been considered to continue until the results of actions appear. However, this does not seem to capture the common-sense concept about delayed effects of an action, which intuitively means that there is a delay time between the action and its results.

The objective of this paper is to develop a discrete formalism for temporal reasoning about actions and change which enriches the ontology of the situation calculus by providing an explicit representation of time and action/event occurrences. The formalism is presented in section 2. Section 2.1 describes the main features of the underlying time structure which is characterised as a well-ordered discrete set of times (points and moments) with no limit elements. In section 2.2, we firstly introduce the definitely two-valued binary predicate *Holds*(*f, p*) for each pair of a fluent *f* and a prime time *p*. Axioms characterising the closure of the underlying time line are then presented, and the predicate *Holds* is extended to govern all times in the closure including the prime times. The advantage of such a time model is that, on the one hand, since it is not forced to explicitly specify the starting and ending points of time intervals, the *Dividing Instant Problem* is bypassed; on the other hand, since each time in the closure is characterised as an *ordered union* of prime times, the possibility of *intermingling* is definitely excluded, and hence, the relationship between the negation of a fluent and the negation of the sentence involving the fluent can be formally well characterised. A possible state of the world is defined in section 2.3 as a subset of the set of all fluents, while a situation is characterised as a pair of a state *s* and a time *t*, such that over time *t* the world holds in state *s*. Section 2.4 deals with actions and their effects, while an event is formally characterised as a pair of an action *a* and a time *t* such that action *a* performs over time *t*. Then a short discussion about *frame problem* well be given in 2.5. In section 3, we address the problematic issue: expressing the *delay effects* of events. Finally, section 4 concludes the paper.

2. The Formalism

We propose the formalism as a revised version of McCarthy and Hayes' situation calculus[17,19], by extending the ontology to provide an explicit representation of time and action occurrences. The extended framework accommodates three disjoint nonempty sets of symbols, **P**, **F** and **A**, called *prime times*, *propositional fluents* and *actions*, respectively. We shall denote the elements of **P**, **F** and **A** as *p*, *f* and *a* (possibly indexed), and adopt the conventional theories of reals and integers.

2.1 The Time Structure

We assume that the set of the prime times is *totally ordered*. We use *Dur* to denote a function from **P** to $\mathbf{R_0}^+$, the set of non-negative real number, so that *Dur* assigns to each prime time a non-negative real number, called the duration of the time. We shall call *p* a prime interval if $Dur(p) > 0$, otherwise, *p* is called a point. Additionally, we assume that **P** is *similar to* [15] **Z**, the set of integers. That is, there exists a one-to-one mapping between the elements which preserve the order relation.

We shall use *Meets* to denote the immediate predecessor relation over **P**, so that $Meets(p_1, p_2)$ represents that prime time p_1 is the immediate predecessor of prime time p_2. Also, we impose the following axiom:

(E) $\forall p_1,p_2 \in \mathbf{P}(Meets(p_1, p_2) \Rightarrow Dur(p_1) > 0 \vee Dur(p_2)>0)$
that is, no two points can meet each other.

From the property of the similar function, we have:

- **P** is a discrete collection of prime times which is well- ordered by the binary relation *Meets*;
- **P** has no *limit elements* [15];
- The fundamental time structure is *linear*, not branching from any time into either the past or the future;
- The fundamental time structure is *unbounded* in both the past and the future;
- *Circular* times are excluded;

It is important to note that prime intervals and points have no internal structure. In other words the elements of **P** are all non-decomposable, even some of them, i.e., the prime intervals, may have a positive duration. In fact, prime intervals are like Allen and Hayes' *moments* [2] i.e. indivisible intervals. In this paper, we shall use the term moment and "prime interval" interactively.

Based on the fundamental time structure, we define the corresponding closure **T** whose elements are generally called *times*, which are not necessarily non-decomposable moments or points. We shall denote the elements of **T** as *t* (possibly indexed), and use Dur_T to denote, as the extension of *Dur*, the function from **T** to $\mathbf{R_0}^+$, so that Dur_T assigns to each element in **T** a non-negative real number. Correspondingly, we shall call a time *t* an interval if $Dur_T(t) > 0$ (Hence, specially, a moment is an interval), otherwise, *t* is called a point. We also define a binary relation $Meets_T \subseteq \mathbf{T} \times \mathbf{T}$ as the extension of *Meets*, so that $Meets_T(t_1, t_2)$ denotes that time t_1 is one of the immediate predecessors of time t_2. The imposed axioms are:

(T1) $\forall p \in \mathbf{P}(p \in \mathbf{T})$

that is **T** is the extension of **P**;

(T2) $\forall t \in \mathbf{T}(t \in \mathbf{P} \Rightarrow Dur_T(t) = Dur(t))$
that is, Dur_T is the extension of Dur ;

(T3) $\forall t_1,t_2 \in \mathbf{T}(t_1,t_2 \in \mathbf{P} \Rightarrow (Meets(t_1, t_2) \Leftrightarrow Meets_T(t_1, t_2)))$
that is, $Meets_T$ is the extensions of *Meets*;

(T4) $\forall t_1,t_2 \in \mathbf{T}(\exists t_1',t_2' \in \mathbf{T}(Meets_T(t_1',t_1) \wedge Meets_T(t_1',t_2) \wedge Meets_T(t_1,t_2') \wedge Meets_T(t_2,t_2'))$
$\Leftrightarrow t_1 = t_2)$
that is, two times are identical if and only if they have the same immediate predecessor and the same immediate successor.

(T5) $\forall t_1,t_2 \in \mathbf{T}(Meets_T(t_1,t_2) \Rightarrow$
$\exists t \in \mathbf{T} \forall t',t'' \in \mathbf{T}(Meets_T(t',t_1) \wedge Meets_T(t_2,t'') \Rightarrow Meets_T(t',t) \wedge Meets_T(t,t''))$

By axiom (T4) and (T5), for any two adjacent times, t_1 and t_2, we may denote the *adjacent union* of t_1 and t_2 as a new time, $t = t_1 \oplus t_2$, called an interval. N.B. $t_1 \oplus t_2$ always implies that $Meets_T(t_1,t_2)$.

(T6) $\forall t \in \mathbf{T}(\exists p_1,...,p_n \in \mathbf{P}(t = p_1 \oplus ... \oplus p_n))$
that is, each element of **T** is in the form of adjacent union of a sequence of prime times.

(T7) $\forall t_1,t_2 \in \mathbf{T}(Meets_T(t_1,t_2) \Rightarrow Dur_T(t_1 \oplus t_2) = Dur_T(t_1) + Dur_T(t_2))$
where "+" is the conventional arithmetic addition operator. That is, the duration of the combined times $t_1 \oplus t_2$ is identical to the sum of duration of t_1 and duration of t_2.

In what follows, without confusion, we shall simply write Dur_T as *Dur*, and write $Meets_T$ as *Meets*.

2.2 Fluents

We introduce a binary predicate, *Holds*, over $\mathbf{F} \times \mathbf{P}$, so that we substitute the formula $Holds(f, p)$ for each pair of a fluent f and a prime time p, denoting that fluent f holds true with respect to prime time p.

Corresponding to the extension from **P** to its closure **T**, we also extend the predicate, *Holds*, which is primitively defined over $\mathbf{F} \times \mathbf{P}$, to $\mathbf{F} \times \mathbf{T}$, so that we substitute the formula $Holds(f,t)$ for each pair of a fluent f and a time t, denoting that fluent f holds true with respect to time t.

However, unlike the prime times of **P** which are all non-decomposable, an element of **T** may be an interval which can be decomposed into a sequence of sub-intervals/internal-points of itself. Hence, when intervals in **T** are allowed to be arguments of the predicate *Holds*, we will face the possibility that a fluent f might neither hold true nor hold false throughout some interval *t*. That is, it may be the case that fluent *f* holds true with respect to some sub-interval/internal-point of *t* but holds false with respect to some other sub-interval/internal-point of *t*. As pointed out by Shoham[25], Bacchus *et al.*[6] and Allen and Ferguson[1], there are two ways we might interpret the negative sentence $\neg Holds(f,t)$. In the strong interpretation of negation, $\neg Holds(f, t)$ is true if and only if *f* holds false throughout *t*, so neither $Holds(f, t)$ nor $\neg Holds(f, t)$ would be true in the case that fluent *f* holds true with respect to some sub-interval/internal-point of *t* and also holds false with respect to some other sub-interval/internal-point of *t*. So, this strong interpretation of negation does not preserve *Holds* as a two-valued predicate any more. In the weak interpretation, $\neg Holds(f, t)$ is true if and only if it is not the case that *f* holds true throughout *t*, and hence $\neg Holds(f, t)$ is true if *f* changes truth-value over time *t*.

In this paper, we shall take the weak interpretation of negation as the constraint imposed on the *Holds* predicate, since it seems to be the appropriate interpretation for the standard definition of implication and preserves a simple two-valued logic[1]:

(F1) $\forall f \in \mathbf{F} \forall t \in \mathbf{T}(Holds(f, t) \Leftrightarrow \forall t' \in \mathbf{T}(Sub(t', t) \Rightarrow Holds(f, t')))$

where $Sub(t_1,t_2)$ denotes that time t_1 is a part of time t_2, and the binary relation $Sub \subseteq \mathbf{T} \times \mathbf{T}$ is defined as below:

$$\begin{aligned} \forall t_1,t_2 \in \mathbf{T}(Sub(t_1,t_2) \Leftrightarrow & \\ & t_1 = t_2 \\ & \vee \exists t' \in \mathbf{T}(t_1 \oplus t' = t_2) \\ & \vee \exists t' \in \mathbf{T}(t' \oplus t_1 = t_2) \\ & \vee \exists t',t'' \in \mathbf{T}(t' \oplus t_1 \oplus t'' = t_2) \end{aligned}$$

Hence, a fluent *f* holds true with respect to time *t* if and only if it holds true with respect to any sub-interval/internal-point of *t* (including *t* itself), that is *f* holds true throughout *t*.

By (F1), it is straightforward to infer that, for any fluent *f* and any time t_1, t_2:

$$Holds(f, t_1) \wedge Holds(f, t_2) \wedge Meets(t_1, t_2) \Rightarrow Holds(f, t_1 \oplus t_2)$$

that is, if a fluent *f* holds true with respect to two adjacent times, t_1 and t_2, respectively, then *f* holds true with respect to the ordered union time, $t_1 \oplus t_2$.

However, in some cases we do not want to express that a fluent holds true throughout a given time, but only that the fluent holds true sometime during the given time. In other words, we would like to express the knowledge that, for a given fluent *f* and a given time *t*, there exists some sub-interval/internal-point *t'* of *t* such that fluent *f* holds true with respect to *t'*. Hence, we introduce an additional binary predicate, *Holds-in*, over $\mathbf{F} \times \mathbf{T}$:

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(F2) $\forall f \in \mathbf{F} \forall t \in \mathbf{T}(Holds\text{-}in(f, t) \Leftrightarrow \exists t' \in \mathbf{T}(Sub(t', t) \wedge Holds(f, t')))$

By (F1) and (F2), we can easily infer that, for any fluent *f* and any time *t*:

$$Holds(f, t) \Rightarrow Holds\text{-}in(f, t)$$

In this paper, we shall use not(*f*) to represent the negation of fluent *f*, to be kept distinct from ordinary sentence-negation, symbolised by "$\neg$". We say fluent *g* is the negation of fluent *f*, that is not(*f*), if *g* satisfies:

$$\forall t \in \mathbf{T}(Holds(g, t) \Leftrightarrow \forall t' \in \mathbf{T}(Sub(t', t) \Rightarrow \neg Holds(f, t))$$

and

$$\forall t \in \mathbf{T}(\neg Holds(f, t) \Leftrightarrow \exists t' \in \mathbf{T}(Sub(t', t) \wedge Holds(g, t')))$$

that is, not(*f*) holds true throughout time *t* if and only if *f* does not hold true throughout any sub-interval/internal-point of *t*; also, *f* does not hold true throughout time *t* if and only if there exists a sub-element of *t* throughout which not(*f*) holds true. Hence, for any fluent *f* and any time *t*, by definition we have:

$$Holds(\text{not}(f), t) \Leftrightarrow \neg Holds\text{-}in(f, t)$$

and

$$\neg Holds(f, t) \Leftrightarrow Holds\text{-}in(\text{not}(f), t)$$

2.3 States and Situations

We define a possible state of the world as a subset of **F**, the set of all fluents. We can interpret this subset as the set of fluents which are true in that state, all others being false. We shall denote the set of all the possible states, that is the power set of **F**, as **S**. Elements of **S** will be denoted by (possibly indexed) *s*.

For the reason of simplicity, we shall also use *Holds*(*s*, *t*) to denote that *s* is the state (of the world) with respect to time *t*:

$$\text{(S1) } \forall s \in \mathbf{S} \forall t \in \mathbf{T}(Holds(s, t) \Leftrightarrow \forall f \in \mathbf{F}(f \in s \Rightarrow Holds(f, t)) \wedge \mathrm{f} \notin s \Rightarrow Holds(\text{not}(f), t)))$$

that is, *s* is the state with respect to time *t* if and only if every fluent belonging to s holds true with respect to time *t*, and for every fluent not belonging to s, its negation holds true with respect to any part of time *t*.

By (S1) and the previous definitions, it is straightforward to infer that:

$$\forall s_1, s_2 \in \mathbf{S} \forall t \in \mathbf{T}(Holds(s_1, t) \wedge Holds(s_2, t) \Rightarrow s_1 = s_2)$$

That is, with respect to a given time, the state of the world is unique. However, there is nothing to stop a temporally contiguous time having the same state. In order to capture the relationships between states and times, we introduce the concept of situations. A situation is defined as the state of the world associated with a particular time over which the world holds in that state. We shall denote the set of all situations as the binary relation **Sit** $\subseteq$ **S**$\times$**T**, such that:

(S2) $\forall sit\in \mathbf{Sit}\exists s\in \mathbf{S}\exists t\in \mathbf{T}(sit = <s, t> \wedge Holds(s, t))$
(S3) $\forall sit\in \mathbf{Sit}\forall s_1,s_2\in \mathbf{S}\forall t_1,t_2\in \mathbf{T}(sit = <s_1, t_1> \wedge sit = <s_2, t_2>$
$\Rightarrow s_1 = s_2 \wedge t_1 = t_2)$

Hence, the representation of any situation is unique. In what follows, if $sit = <s, t>$ is a situation, we shall call s and t its reference state and reference time, and denote them as *State*(*sit*) and *Time*(*sit*), respectively. In fact, *State* and *Time* can be seen as two functions from **Sit** to **S** and **T**, respectively. By (S2) and (S3), it is easy to see that:

$$\forall sit_1, sit_2\in \mathbf{Sit}(Time(sit_1) = Time(sit_2) \Rightarrow State(sit_1) = State(sit_2))$$

That is, if two situations have the same reference time they must have the same reference state, and hence they are identical.

For reasons of simplicity, in what follows, we may use *Holds*(*f*, *sit*) to denote that fluent *f* is observed as true in situation *sit*, providing that:

(S4) $\forall sit\in \mathbf{Sit}\forall f\in \mathbf{F}(Holds(f, sit) \Leftrightarrow f\in State(sit))$

N.B. For the convenience of expression, in what follows, we shall call situation *sit* a prime situation if its reference time is a prime one.

2.4 Actions and Events

We introduce the binary predicate, *Performs*, over **A**$\times$**T**, so that *Performs*(*a*, *t*) represents that action *a* acts over time *t*.

(A1) $\forall a\in \mathbf{A}\forall t_1,t_2\in \mathbf{T}(Meets(t_1, t_2) \wedge Performs(a, t_1) \wedge Performs(a, t_2)$
$\Leftrightarrow Performs(a, t_1\oplus t_2))$

that is, if an action performs over two adjacent times respectively, then it performs over the ordered union of these two times.

The world holds in one state until an action is performed over some special time to change it into another state. We shall call such a phenomenon an event. Hence, analogously to the form of a situation which are defined as a pair of a state and a time, an event is given in the form of a pair of an action and a time, and we shall denote the set of all events as a binary relation, **E** $\subseteq$ **A**$\times$**T**, such that:

(A2) $\forall a \in \mathbf{A} \forall t \in \mathbf{T}(<a, t> \in \mathbf{E} \Leftrightarrow Performs(a, t))$

Again, analogously to the definitions of the reference state and the reference time of a given situation, for an event in **E**, say $e = <a, t>$, we shall call a and t the *reference action* and the *reference time* of event e, and denote them as $a = Action(e)$ and $t = Time(e)$ respectively. Then the following axiom ensures that the representation of any event is unique:

(A3) $\forall e_1, e_2 \in \mathbf{E}(e_1 = e_2 \Leftrightarrow Action(e_1) = Action(e_2) \wedge Time(e_1) = Time(e_2))$

Whereas the intuition behind the notion of situation is persistence, the intuition behind the notion of event is change. To express knowledge about the result of the occurrence of an event in a given situation, we introduce the ternary function *Result* which maps an event, a situation and a time to a prime situation, so that $Result(e,sit,t)$ intuitively denotes the prime situation immediately after time t, as the result of the occurrence of event e in situation sit (see Fig.1). Here we use the prime situation to ensure that the result situation is unique.

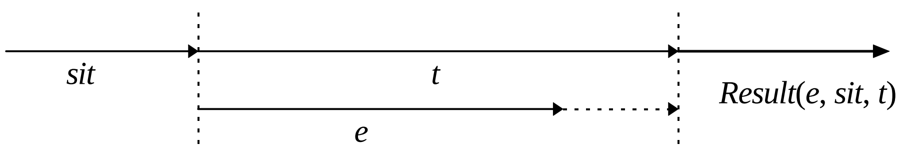

Figure 1

The domain of function *Result*, **Dom**(*Result*), is a subset of **Sit**×**E**×**T**, such that:

(A4) $\forall e \in \mathbf{E} \forall sit \in \mathbf{Sit} \forall t \in \mathbf{T}((e,sit,t) \in \mathbf{Dom}(Result) \Rightarrow$
$Meets(Time(sit), Time(e))$
$\wedge Sub(Time(e), t)$
$\wedge Meets(Time(sit), t)$
$\wedge Meets(t, Time(Result(e,sit,t))))$

This axiom specifies the temporal relationship among situations sit, $Result(e,sit,t)$ and event e.

N.B. In the above, $Result(e,sit,t)$ represents the prime situation immediately after time t, as the result of the occurrence of event e in situation sit. Here, the third argument, t, of function *Result* which stands between $Time(sit)$ and $Time(Result(e,sit,t))$ is needed in order to preserve the uniqueness of the result situation. This approach will allow expression of various cases of temporal relationships between the reference time of an event and the reference time of the result situation, including the case where there is a "delay time" between the occurrence of an event and the result situation (see next section).

2.5 The Frame Problem

Briefly, the frame problem is the need for specifying everything that does not change. Based on the situation calculus, there are mainly two ways for solving this problem: monotonic and nonmonotonic. Schubert[24] and Reiter[22] developed monotonic approaches to this problem based on the idea of "explanation closure". For instance, Reiter provides a solution to the frame problem, using successor state axioms[22]. Each such axiom provides a complete characterisation of a fluent's truth value in the next state *Result*(a, s) in terms of what is true of the state s. Also a number of solutions based on the use of nonmonotonic formalisms have been proposed[4,18]. For instance, Baker proposes a nonmonotonic solution to the frame problem, using the formalism of circumscription. His work is based on the idea that since the abnormality predicate takes a situational argument, "it is important for the meanings of the situations to be held constant across the various models being compared"[4]. The major change suggested by him is to employ a new circumscription policy: to minimise *Ab* while varying *Result* and S_0 (as opposed to varying *Holds*, as is done by Hanks and McDermott[11]). According to the power of the expression of this enriched language together with the preservation of the most appealing characteristics of these existing systems, it is not difficult to extend their solutions within our formalism. In this paper we will not deal with this work.

3. An Illustrating Example

The formalism proposed in this paper is in fact achieved by means of synthesising the quintessence of some representative theories, including that of Allen and Ferguson[1,3], Kowalski and Sergot[12], Sandewall and Ronnquist[23], Lifschitz[13], Gelfond *et al*.[9], Lin and Shoham[14], Miller and Shanahan[20], and Pinto and Reiter[21], etc. Hence, it is not surprising for us to believe that such an extension retains most appealing characteristics of these existing theories, without bearing their corresponding deficiencies. Especially, the new formalism provides a more flexible expression of temporal relationships between effects and the corresponding causal events, and overcomes/bypasses the *dividing instant problem* and *intermingling problem*.

In many cases, the effects of an action take place immediately after the action performed. However, sometimes there may be some time delay between an action and its effects. Consider the following example:

> *25 seconds after a pedestrian starts pressing the button at the crosswalk, the pedestrian crossing light turns to yellow from red, and after another 5 seconds it turns to green.*

In most existing formalisms based on situation calculus and event calculus, this description can not be represented correctly. Gelfond *et al.*[9] proposed an approach using the notation *duration of actions* to describe an action with delayed effects. They simply count the delayed time by the duration of actions. For instance, they use the following formula to represent that after 25 second, the crossing light turns to yellow:

$$Dur(Press) = 1,$$
$$Dur(a) = 24 \Rightarrow Holds(Yellow, Result((Press; a), sit))$$

where the delayed time is represented as action *a* with a duration of 24 seconds, while the action affecting the truth value of traffic light is divided into two actions, *Press* and *a*, which perform successively. However, since an action with some delayed effects actually means that there is a time delay between the action and its effects, Gelfond *et al*'s approach seems unintuitive, and not capable for expressing the knowledge that after another 5 seconds, the light turns to green. In fact, this is due to the difficulty with such an approach in dealing with the persistence of the truth value of fluents over the delayed times involved.

To express this example in the formalism proposed here, we employ the following three fluents for describing the state of the pedestrian crossing light:

RedOn: the light at the crosswalk is red;
YellowOn: the light at the crosswalk is yellow;
GreenOn: the light at the crosswalk is green;

We assume that in any situation there is exactly one of the three fluents that holds true. This assumption can be described as a domain constraint axiom:

(D1)

$$\forall sit((Holds(RedOn,sit)\wedge\neg Holds(YellowOn,sit)\wedge\neg Holds(GreenOn,sit))$$
$$\vee(\neg Holds(RedOn, sit)\wedge Holds(YellowOn, sit)\wedge\neg Holds(GreenOn, sit))$$
$$\vee(\neg Holds(RedOn, sit)\wedge\neg Holds(YellowOn, sit)\wedge Holds(GreenOn, sit)))$$

Let *PressButton* denote the action of pressing the button, and Sit_0 denote the situation in which the red light is on, the yellow and green lights are off:

$$Holds(RedOn, Sit_0)$$

Assuming in situation Sit_0 a pedestrian presses the button, e.g., for 1 second, let $E = <PressButton, T_E>$, where $Dur(T_E) = 1$, then we have

$$Holds(YellowOn, Result(Sit_0, E, T_1))$$

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$Holds(GreenOn, Result(Sit_0, E, T_2))$

where $Dur(T_1) = 25$ and $Dur(T_2) = 30$.

Here we can successfully express the fact that there is a delayed time, say T_{D1}, standing between the reference time of event E and the reference time of the result situation $Result(Sit_0, E, T_1)$, that is:

$Meets(Time(E), T_{D1})$,
$Meets(T_{D1}, Time(Result(Sit_0, E, T_1)))$.

Similarly, we can express that there is a delayed time, say T_{D2}, standing between the reference time of event E and the reference time of the result situation $Result(Sit_0, E, T_2)$, that is:

$Meets(Time(E), T_{D2})$,
$Meets(T_{D2}, Time(Result(Sit_0, E, T_2)))$

Additionally, we can express T_{D2} as: T_{D1}Å$Time(Result(Sit_0, E, T_1))\oplus T_3$, where T_3 is the extension of $Time(Result(Sit_0, E, T_1))$ since $Time(Result(Sit_0, E, T_1))$ is a prime time and its duration may be less than 5 seconds. The above knowledge can be graphically presented as Figure 2.

However, with respect to this expression the frame problem arises. That is, during the delayed time T_{D1}: does the truth value of fluent *RedOn* persist?

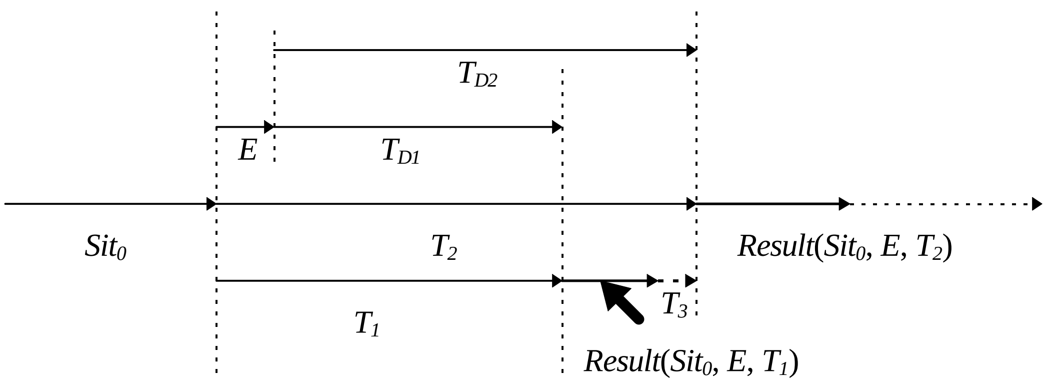

Figure 2

4. Concluding Remarks

In this paper, we have developed a discrete formalism for temporal reasoning about action and change. The main contributions are:

- The formalism proposed here allows a comprehensive characterisation about relationship between the negation of a given fluent and the negation of sentences involving that fluent. Since each time is defined as an ordered union of points/moments that are non-decomposable, the so-called *deviding instant problem* and *intermingling problem* are definitely excluded.
- We formulate some key terms of the extended situation calculus, such as states, situations, actions and events so as to ensure "common-sense" causality. The distinction between states and situations is formally made by means of defining a situation as a pair of a state and a time over which the world holds in the state. In a completely analogous way, the distinction between actions and events is made by defining an event as a pair of an action and a time over which the action performs.
- A flexible temporal relationship between effects and their causes can be expressed, including the case of immediate effects and the case there some delay times between the effects and their causes. It seems that most existing versions of the situation calculus may be subsumed from this new formalism by means of simply specifying $Sub(Time(e), t)$ in (A4) as $Equals(Time(e), t)$.

Hence, while the new formalism retains the most appealing characteristics of these existing systems, it does enjoy a more powerful expressiveness.

Since the fundamental time model is discrete, it will be difficult to model some continuous changes from the theoretical view, for which a dense time model would become necessary[7,16]. However, if the time model is extended to a dense one which accommodates both points/moments and interval, some special axioms may be needed for dealing with the *intermingling problem* for the sake of providing a satisfactory characterisation about the negation of fluents and negation of sentence.

Key Words: Knowledge representation, Temporal reasoning, Artificial intelligence, Actions and change

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