Research Article

Multimodal Network Equilibrium with Stochastic Travel Times

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The private car, unlike public traffic modes (e.g., subway, trolley) running along dedicated track-ways, is invariably subject to various uncertainties resulting in travel time variation. A multimodal network equilibrium model is formulated that explicitly considers stochastic link capacity variability in the road network. The travel time of combined-mode trips is accumulated based on the concept of the mean excess travel time (METT) which is a summation of estimated buffer time and tardy time. The problem is characterized by an equivalent VI (variational inequality) formulation where the mode choice is expressed in a hierarchical logit structure. Specifically, the supernetwork theory and expansion technique are used herein to represent the multimodal transportation network, which completely represents the combined-mode trips as constituting multiple modes within a trip. The method of successive weighted average is adopted for problem solutions. The model and solution method are further applied to study the trip distribution and METT variations caused by the different levels of the road conditions. Results of numerical examples show that travelers prefer to choose the combined travel mode as road capacity decreases. Travelers with different attitudes towards risk are shown to exhibit significant differences when making travel choice decisions.

1. Introduction

Modeling of multimodal travel behavior has attracted a growing amount of research interests as the combined-mode trip becomes common place in metropolis. In contrast to single-mode models, multimode models not only aim to integrate single-mode trips, but also take account of feasible alternative modes within a trip using two or more modes (e.g., park and ride (P&R) trips). Yet not all the combinations are possible in the real network; for example, few travelers would transfer from the private car to bus and then switch back to private car, because the private car is already part of the previous segment.

Many existing multimodal traffic assignment models are formulated with equilibrium constraints. The core issue is to deal with the interaction between mode choice and route choice [1]. The former one is commonly modeled with hierarchical logit structure, which assigns the trips on each reasonable travel mode on the basis of the minimum perceived travel cost. The latter one follows two basic kinds of assumptions including deterministic assumption and stochastic assumption. The deterministic assumption considers that all travelers know the perfect information and choose the best route with the minimum travel cost. Stochastic models deem that traveler's perception is affected by some random errors, and they choose their perceived best routes according to some probabilities. Meanwhile, the generalized travel cost essentially consists of driving cost and transfer cost as the consideration of transfer behavior.

At present, multimodal network equilibrium in normal traffic conditions has been much researched; mathematical programming formulation, variational inequality theory, and fixed-point method have been developed and successfully applied in modeling real networks [2, 3]. Zhou et al. [4] presented two different equations to formulate the combined travel tips problem which incorporated traditional four-step model. Xu and Gao's [5] grouped all travelers in different classes with various travel behavior, like the mode/route selection and transfer points. They formulated this equilibrium model as a fixed-point equation and also

analyzed its existence and uniqueness. Khani et al. [6] divided multiple traffic modes into the two major categories of private and public modes, with transfer constraints. For solving the equilibrium model, they proposed an intermodal path algorithm according to generalized travel cost, including private-side travel cost, public-side travel cost, and transfer cost. Reviews of these models are reported in Lo et al. [7], García and Marín [8], and Li et al. [9].

Much research has been made in recent decade in the formulation, analysis, and computation of stochastic travel time. Given the nongeneralizable sources of uncertainties causing travel time variability [10–12], the route choice criterion under such uncertain variations is not the same as under normal traffic conditions. Uchida and Iida [13] defined the effective travel time as the sum of mean travel time plus a safety margin to model the travel uncertainty. Chan and Lam [14] introduced a concept of path preference index (PI) to quantify the attractiveness of each alternative route. Lo et al. [15] postulated that travelers would make decision based on the travel time budget (TTB), which is a summation of expected travel time and travel time margin. This TTB differs with travel risk attitude. Several studies have shown that the TTB concept is more grounded in reality than effective travel time [11, 16]. Recently, Zhou and Chen [17] and Chen and Zhou [18–20] assumed that travelers would like to choose the route with the minimal mean excess travel times (METT), where METT is the sum of buffer time and tardy time. Buffer time guarantees the travel time reliability and tardy time reflects the unreliability factor beyond the accepted range of travel time. In contrast to TTB, METT contains reliable and unreliable elements together in the travel choice procedure, which is a more conservative measure of risk. The network equilibrium with METT concept is named as mean excess traffic equilibrium (METE). Nie [21] assumed that traveler will select the routes with the minimal budget to assure the probability of on time arrival. Wu and Nie [22] developed a new solution algorithm for the stochastic route choice problem to avoid the route enumeration. Xu et al. [23] extended the METE model to the case with multiclass users, where travelers have different risk-aversion attitudes. Sun and Gao [24] presumed that the travel time has a certain degree of robustness and proposed a robust mean-excess travel time (RMETT) to discuss the route travel behavior.

Network equilibrium models which simultaneously focus on the combined-mode trip and the travel time variability have not received much attention in the literature. Meng et al. [25] studied the travel behavior in a degradable multimodal transportation network. They assumed only that travelers choose the routes which have the minimum expected travel time. Häme and Hakula [26] considered the travel uncertainty in a multimodal network, where the travel decision is described as Markov decision process.

The present approach for formulating network equilibrium model with stochastic travel time can be taken as the development of the model of Chen and Zhou [19] to multimodal network with combined modes. The subway network with overlapping private car network is based on the supernetwork structure. The subway network running in the specific environment is unaffected by traffic uncertainties, while the private car may suffer from various exogenous sources of uncertainties which may cause road capacity variations. The travel time variability is expressed by METT. The user equilibrium is applied under the uncertainty assumption, and the mode choice is modeled with a logit formulation. The equivalent variational inequality model is proposed with a route-based solution algorithm.

The following sections are organized as follows: METT is introduced in Section 2. Section 3 proposes the multimodal traffic network equilibrium model. A heuristic solution algorithm is given in Section 4. Section 5 shows the numerical examples to explain the application. Conclusions and discussions are drawn in Section 6.

2. Multimodal Network

2.1. Notation

$G(N, L)$: a multimodal transportation network,

N: set of nodes,

L: set of links, l is one of the links $l \in L$,

A, X, Y: set of road link, subway link, and the transfer link, $L = A \cup X \cup Y$,

W: set of OD (origin and destination) pairs, w is one of the OD pairs $w \in W$,

q_w: traffic demand of OD pair w,

K_w: set of routes of OD pair w, k is one of the routes $k \in K$,

f_k: flow on route k,

x_l: flow on link l,

δ_{lk}: route-link incidence variable, where $\delta_{lk} = 1$ if route k uses link l and 0 otherwise.

2.2. Supernetwork.
The supernetwork concept has been used extensively to describe the multimodal transportation network since it was first proposed by Sheffi in 1985 [27–29]. Figure 1 shows an example of a small multimodal transportation network, which has 9 nodes and 11 links. The transfer links l_8 and l_9 effectively connect the two different traffic modal layers. The route 10-11 is part of a subway line, which corresponds with the road route 6-7.

3. Traffic Equilibrium Model

3.1. Route Travel Time.
According to Chen and Zhou [19], the travelers cannot make correct estimation of their total travel time as travel time varies. In multimodal transportation network, however, travelers who choose the subway trip can perceive their travel time more accurately as the subway runs to a fixed schedule. Therefore, the travel time of a combined-trip trip which involves at least two traffic modes contains the variable travel time in the uncertain portion of the network,
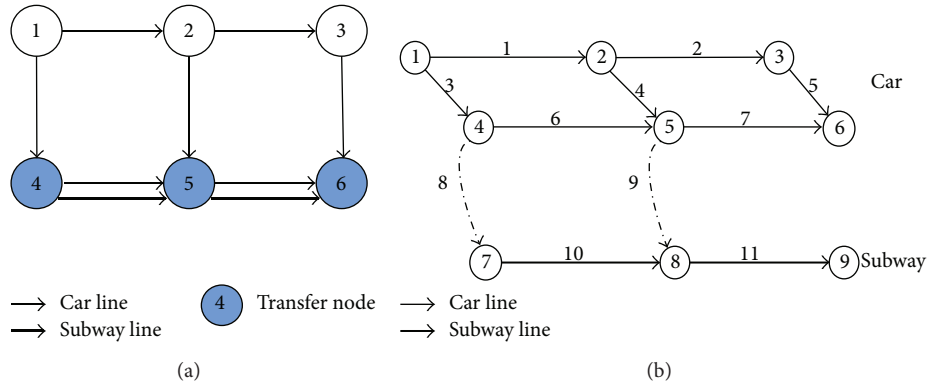
FIGURE 1: Multimodal network.

the certain running time, and the transfer time. The general route travel time should be

$$t_k = \sum_{l \in A} \delta_{lk} t_l + \sum_{l \in X} \delta_{lk} t_l + \sum_{l \in Y} \delta_{lk} t_l, \quad \forall k \in K_w, \ w \in W, \quad (1)$$

where t_k is the travel time of route k and t_l is the travel time of link l. The link travel time is calculated based on the BPR (Bureau of Public Roads) function [30] as

$$t_l(x_l, C_l) = t_l^0 \left[1 + \beta \left(\frac{x_l}{C_l} \right)^n \right], \quad \forall l \in A, \quad (2)$$

where t_l^0 is the free-flow link travel time and C_l is the road capacity on link l and β and n are the nonnegative parameters in BPR function. In conventional analyses, the road capacity is regarded as a constant; under stochastic nature of the road capacity variations or degradations, the capacity of each link C_l is subject to uncertainty and should be a random variable. Therefore, the link travel time t_l is a random variable as well, which is a common assumption in several studies [19, 21, 31–33]. The link travel time is described by a probability distribution. According to (2), its mean $E(t_l)$ and variance $(\sigma(t_l))^2$ can be determined as

$$E(t_l) = E\left(t_l^0\right) + \beta t_l^0 E\left[\left(\frac{x_l}{C_l} \right)^n \right], \quad \forall l \in A,$$

$$(\sigma(t_l))^2 = \left(\sigma\left(t_l^0\right)\right)^2 + \beta^2 \left(t_l^0\right)^2 \left\{ \sigma \left[\left(\frac{x_l}{C_l} \right)^n \right] \right\}^2, \quad \forall l \in A. \quad (3)$$

Assuming that the free-flow link travel time is a constant and the capacity degradation random variable C_l is independent of the amount of traffic on it, $E(1/C_l^n)$ and $[\sigma(1/C_l^n)]^2$ can be expressed as

$$E\left(\frac{1}{C_l^n} \right) = \int_{\theta \bar{c}_l}^{\bar{c}_l} \frac{1}{c_l^n} \cdot \frac{1}{\bar{c}_l - \theta \bar{c}_l} dc_l$$

$$= \frac{1 - \theta_l^{1-n}}{\bar{c}_l^n (1 - \theta_l)(1 - n)}, \quad \forall l \in A,$$

$$\left[\sigma \left(\frac{1}{C_l^n} \right) \right]^2 = \frac{1 - \theta_l^{1-2n}}{\bar{c}_l^{2n}(1 - \theta_l)(1 - 2n)}$$

$$- \left[\frac{1 - \theta_l^{1-n}}{\bar{c}_l^n (1 - \theta_l)(1 - n)} \right]^2, \quad \forall l \in A, \quad (4)$$

where the term $1/(\bar{c}_l - \theta_l \bar{c}_l)$ is the probability density function (PDF) of the uniform distribution with upper bound $\bar{c}_l$ and lower bound $\theta_l \bar{c}_l$ [15]; c_l is the design capacity of C_l; and θ_l is the degradation parameter.

Upon simplification, the mean and variance of the road link travel time are expressed as

$$E(t_l) = t_l^0 + \beta t_l^0 x_l^n \frac{1 - \theta_l^{1-n}}{\bar{c}_l^n (1 - \theta_l)(1 - n)}, \quad \forall l \in A, \quad (5)$$

$$(\sigma(t_l))^2 = \beta^2 \left(t_l^0\right)^2 x_l^{2n} \left\{ \frac{1 - \theta_l^{1-2n}}{\bar{c}_l^{2n}(1 - \theta_l)(1 - 2n)} \right.$$

$$\left. - \left[\frac{1 - \theta_l^{1-n}}{\bar{c}_l^n(1 - \theta_l)(1 - n)} \right]^2 \right\}, \quad \forall l \in A. \quad (6)$$

Assume that the link travel time in subway network is determined by the subway run time, while the transfer travel time includes the walking time and the waiting time; that is,

$$t_l = t_l^{\text{walk}} + t_l^{\text{wait}}, \quad \forall l \in E, \quad (7)$$

where t_l^{walk} and t_l^{wait} are the walking time and waiting time on link l. t_l^{wait} is influenced by the departure frequency as $t_l^{\text{wait}} = 1/2f$, where f is the departure frequency of the objective subway at the transfer node and the average of the waiting time is considered for travelers with different arrival time. t_l^{wait} is influenced by distance between the parking lot and the subway platform, which can be regarded as a deterministic value based on the average walking time for a normal person.

Based on the central-limit theorem, the mean $E(t_k)$ and variance $(\sigma(t_k))^2$ of the route travel time in the multimodal network can be calculated by

$$
\begin{aligned}
E(t_k) = &\sum_{l \in A} \delta_{lk} \left[t_l^0 + \beta t_l^0 x_l^n \frac{1 - \theta_l^{1-n}}{\bar{c}_l^n (1 - \theta_l)(1 - n)} \right] \\
&+ \sum_{l \in X} \delta_{lk} t_l + \sum_{l \in Y} \delta_{lk} t_l, \quad \forall k \in K_w, \ w \in W,
\end{aligned}
$$

$$
\begin{aligned}
(\sigma(t_k))^2 = \sum_{l \in A} \delta_{lk} \beta^2 \left(t_l^0\right)^2 x_l^{2n} &\left\{ \frac{1 - \theta_l^{1-2n}}{\bar{c}_l^{2n}(1 - \theta_l)(1 - 2n)} \right. \\
&\left. - \left[\frac{1 - \theta_l^{1-n}}{\bar{c}_l^n(1 - \theta_l)(1 - n)} \right]^2 \right\},
\end{aligned} \tag{8}
$$

$$
\forall k \in K_w, \quad w \in W.
$$

3.2. Mean Excess Travel Time.
In accordance with Chen and Zhou [19], travelers search a route with consideration of two components: one is the corresponding TTB allowing for on time arrival, the other being the impacts of excessively late arrival. Upon further mathematical operations, the METT of route k can be represented as

$$
B_k = E(t_k) + \frac{\sigma(t_k)}{\sqrt{2\pi}(1 - \alpha)} \exp \left(-\frac{\left(\Phi^{-1}(\alpha)\right)^2}{2} \right), \tag{9}
$$

$$
\forall k \in K_w, \quad w \in W,
$$

where B_k is the METT of route k and $\Phi(\cdot)$ is the standard normal cumulative distribution function (CDF).

3.3. Equilibrium Conditions.
The multimodal traffic network equilibrium condition in the case of METT can be stated as follows: flow f_k on route k is positive if METT on k is equal and minimal METT; all unused routes have an equal or higher METT, which can be expressed as

$$
B_k \begin{cases} = U_w, & \text{if } f_k > 0 \\ \geq U_w, & \text{if } f_k = 0 \end{cases} \tag{10}
$$

$$
\forall k \in K_w, \quad w \in W,
$$

where U_w is the minimal METT under the condition of user equilibrium between the OD pair w.

3.4. VI Formulation.
The multimodal traffic network equilibrium conditions [34] can be expressed as a VI formulation:

$$
\sum_w \sum_k B_k^* (f_k^*) (f_k - f_k^*) \geq 0, \tag{11}
$$

where * represents the variable value at equilibrium. The feasible set Ω is defined by the following equations where the conservation of traffic demand, traffic flow, and nonnegative constraint can be satisfied:

$$
\sum_k f_k = q_w, \quad \forall w \in W, \tag{12}
$$

$$
x_l = \sum_w \sum_k f_k \delta_{lk}, \quad \forall k \in K_w, \ w \in W, \tag{13}
$$

$$
f_k \geq 0, \quad \forall k \in K_w, \ w \in W, \tag{14}
$$

$$
q_w \geq 0, \quad \forall w \in W. \tag{15}
$$

The VI formulation of (11) is equivalent with the multimodal traffic network equilibrium condition (10). Equation (11) is derived on the basis of (10). From (10), one gets

$$
\begin{aligned}
(B_k^* - U_w) f_k^* &= 0, \\
(B_k^* - U_w) &\geq 0.
\end{aligned} \tag{16}
$$

Combining (16) with (14) leads to

$$
\begin{aligned}
(B_k^* - U_w)(f_w - f_k^*) &= (B_k^* - U_w) f_w \\
-(B_k^* - U_w) f_k^* &= (B_k^* - U_w) f_w \geq 0.
\end{aligned} \tag{17}
$$

Adding up (10) for all the routes in all OD pairs leads to

$$
\sum_w \sum_k (B_k^* - U_w) \cdot (f_k - f_k^*) \geq 0. \tag{18}
$$

As $\sum_k f_k = \sum_k f_k^* = q_w$ for an OD pair w, then the term $\sum_w \sum_k U_w(f_k - f_k^*) = 0$. Based on (18) and $\sum_w \sum_k U_w(f_k - f_k^*) = 0$, the multimodal traffic network equilibrium conditions of (11) can thus be deduced from (10).

As (11) is equivalent to (18), one can also derive (10) on the basis of (18). Let all traffic flows on the nonequilibrium routes in all the OD pairs be equal to equilibrium flow; then (17) can be simplified as

$$
(B_k^* - U_w)(f_k - f_k^*) \geq 0, \tag{19}
$$

when $f_k^* > 0$, $(f_k - f_k^*)$ could be positive or negative. When $f_k^* = 0$, then $f_k^* \geq 0$. To prove (10), B_k^* must be greater than or equal to U_w. Derivation of all the routes with the same process shows that (10) is valid. Therefore, (10) is equivalent to (11). The feasible set is close, nonempty, and convex, as it is composed of nonnegative linear formulations. Meanwhile, as the travel demand is bounded, the feasible set is compact. Together with B_k, which is continuous, the VI formulation (11) has at least one solution based on the standard theorem.

TABLE 1: Parameters in car subnetwork.

Link	t_l^0	C_l
(1, 2)	0.6	2000
(2, 3)	0.4	2000
(1, 4)	0.3	1000
(2, 5)	0.3	1000
(3, 6)	0.3	1000
(4, 5)	0.4	2000
(5, 6)	0.2	2000

4. Solution Algorithm

A solution procedure is developed based on MSWA (method of successive weighted average) [35]. The difference between MSWA and traditional MSA (method of successive average) is that the iteration step size of MSWA is not a fixed value but gives more weight to more recent iteration points which will speed up the convergence. Specific steps are as follows.

Step 1 (Initialization). Set the iteration $n = 1$ and the link flow $x_l^{(1)} = 0$; calculate the METT of route $k\{B_k^{i(0)}\}$ based on (9) with the free-flow travel time $\{t_l(0)\}$; assign travel demand $\{q_w\}$ on the network to obtain the original feasible link flow $\{x_a^{i(0)}\}$ based on the all-or-nothing method.

Step 2. Update travel time and calculate $E(t_l)^{(n)}$ and $\sigma(t_l)^{(n)}$.

Step 3. Determine a descent direction. Calculate the METT of route $k\{B_k^{i(n)}\}$ and assign $\{q_w\}$ on the network to obtain the auxiliary link flow $\{y_a^{i(n)}\}$ based on the all-or-nothing method.

Step 4 (Iteration). Let $d = 1$; calculate the link flow by

$$x_l^{(n+1)} = x_l^{(n)} + \chi^{(n)}\left(y_l^{(n)} - x_l^{(n)}\right),$$
$$\chi^{(n)} = \frac{n^d}{1^d + 2^d + 3^d + \cdots + n^d}. \tag{20}$$

Step 5 (Convergence Judgment). Let the merit function G be defined as

$$G = \sqrt{\sum\left(x_l^{(n+1)} - x_l^{(n)}\right)^2 \left(\sum x_l^{(n)}\right)^{-1}}. \tag{21}$$

If $G \le \varepsilon$ (where ε is the convergence criteria), then stop; otherwise, $n = n + 1$, and return to Step 2.

5. Experiments and Analysis

5.1. A Small Network. Figure 1 gives the first experiment network which includes 1 OD pair (1,6). The road network parameters are defined in Table 1, while the parameters for subway links and transfer links are $t_{10} = 0.4$, $t_{11} = 0.2$, $t_{8,walk} = 0.05$, $t_{8,wait} = 0.05$, $t_{9,walk} = 0$, and $t_{9,wait} = 0.05$. Five thousand units travel demand is considered, while other information is $\alpha = 0.9$, $\beta = 0.15$, $n = 4$, and $\varepsilon = 0.001$.

Figure 2 gives the convergence performance of the MSWA-based algorithm at different confidence level. As the
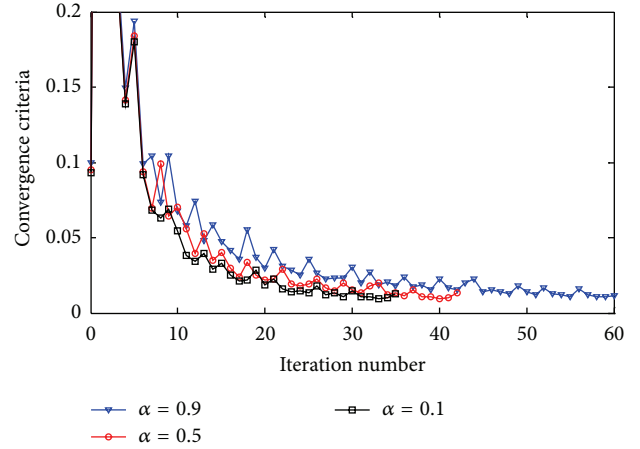

FIGURE 2: Convergence curve of the algorithm in the small network.

iteration time increases, the convergence criteria gradually decrease. When the confidence level $\alpha = 0.1$, 0.5, and 0.9, the MSWA algorithm terminates at iterations 36, 43, and 62, respectively. In contrast, the MSA algorithm needed 75, 92, and 137 times of iteration to achieve convergence under the same conditions of this experiment.

Table 2 shows the traffic assignment results under different road conditions with the three confidence levels $\alpha = 0.1$, 0.5, and 0.9. Four cases of road network are designed including normal (0.95), mild degradation ($\theta = 0.85$), moderate degradation ($\theta = 0.70$), and severe degradation ($\theta = 0.55$). The flow results show that, with the degradation of the road network, the car trip decreases while the P&R trip increases at different levels. It is because degradation increases the travel time in the road network, and travelers prefer P&R trip given the punctuality of subway network.

With the degradation of the road network, the METT for different confidence level increases in varying degrees. As shown in Figure 3, the intensity of METT increases for the case $\alpha = 0.9$ which is much greater than at $\alpha = 0.1$ (2 times in the severe condition), while that for $\alpha = 0.5$ is in between. When travelers have 10% confidence level of on time arrival, they would be much less inclined (<5%) to change their METT; on the contrary, if travelers have 90% confidence level of on time arrival (almost 27%), they would be more likely to adjust their perceived METT according to the network condition.

With the degradation of the road network, the ratio of car trip decreases with three confidence level cases. As shown in Figure 4, the biggest decline is the case when $\alpha = 0.9$ as the travelers perceive the situation better than other cases. When $\alpha = 0.1$, travelers would rather maintain their existing knowledge than explore the new information. The combined-trip becomes more advantageous as the road network becomes more uncertain.

5.2. Sioux Fall Network. To further illustrate the performance of the algorithm, a test is performed on a modified network from the Sioux Fall Network [36] as shown in Figure 5, which has 24 nodes, 76 road links, 24 subway links, and 36 OD pairs.

Table 2: Equilibrium results under different road conditions.

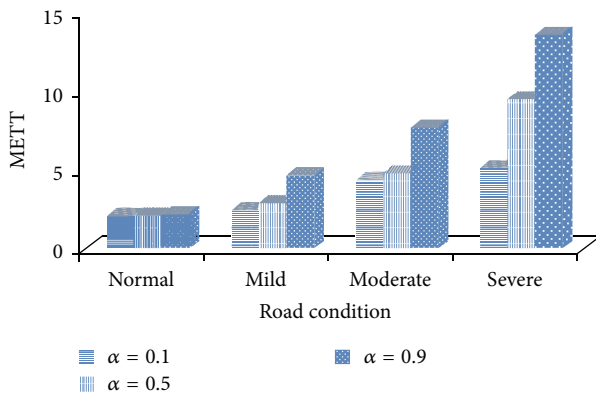| Confidence level	0.1		0.5		0.9	
Traffic mode	Car	P&R	Car	P&R	Car	P&R
Normal network						
Flow	3590	1410	3590	1410	3590	1410
METT	2.13	2.18	2.17	2.19	2.20	2.21
Mild degradation						
Flow	3482	1518	3328	1672	3123	1877
METT	2.47	2.59	2.93	3.02	4.68	4.79
Moderate degradation						
Flow	3428	1572	3061	1939	2800	2200
METT	4.39	4.45	4.78	4.87	7.72	7.93
Severe degradation						
Flow	3231	1769	2796	2204	2334	2667
METT	5.16	5.29	9.49	9.78	13.68	14.07

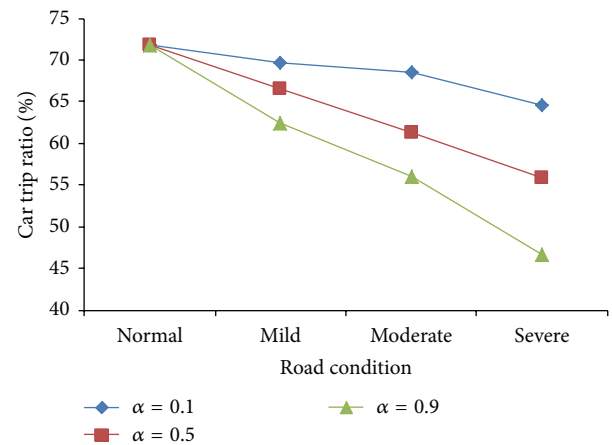Figure 3: METT in different condition with different confidence level.

Figure 4: Car trip ratio in different condition with different confidence level.

The bold lines are the subway lines, and hatched nodes are the origin nodes and destination nodes.

The proposed algorithm terminates after 37 iterations and the CPU (4∗2core, 2.13 GHz, RAM8Gb) time is 8.7 s. When the confidence level $\alpha = 0.5$ in the normal condition, the convergence of the algorithm by the RMSE (root mean square error) is shown in Figure 6. This figure shows that the algorithm can quickly reach the convergence precision, which is effective and exercisable.

Two routes connecting OD pair (1, 20) are also examined in both normal condition and moderate degradation condition, including a car route 13-41-42-43-44-45-46 and a P&R route 13-41-5-6-7-8-9. The route flow evolutions of the two routes during the iteration process are depicted in Figures 7 and 8. From these figures, one can see that in the normal condition the flow of the car route rises quickly while the growth of the P&R route flow is slow. In the moderate degradation, however, the P&R route flow maintains a strong momentum of growth, while the car route flow does not increase at a rapid pace. These results also confirm that the attraction of combined travel mode under moderate road

network degradation is significantly stronger than under normal condition.

6. Conclusions

In this study, the METT model is extended into the multimodal transportation network with consideration of combined modes. In the car transportation network, the travelers cannot make a correct estimate of their total travel time as the travel time variability, while travelers who choose the subway trip can perceive their travel time more accurately as the subway runs to a fixed schedule. Based on the supernetwork theory and extension technique, the combined-mode trips are formulated as an equivalent VI equation which is solved by a MSWA-based algorithm. Experiment results showed the practicality and effectiveness of the developed model and algorithm and also revealed that, with the degradation of road network, travelers prefer to choose the combined trips to decrease their mean excess travel time. The attraction of the combined travel mode will become stronger as the
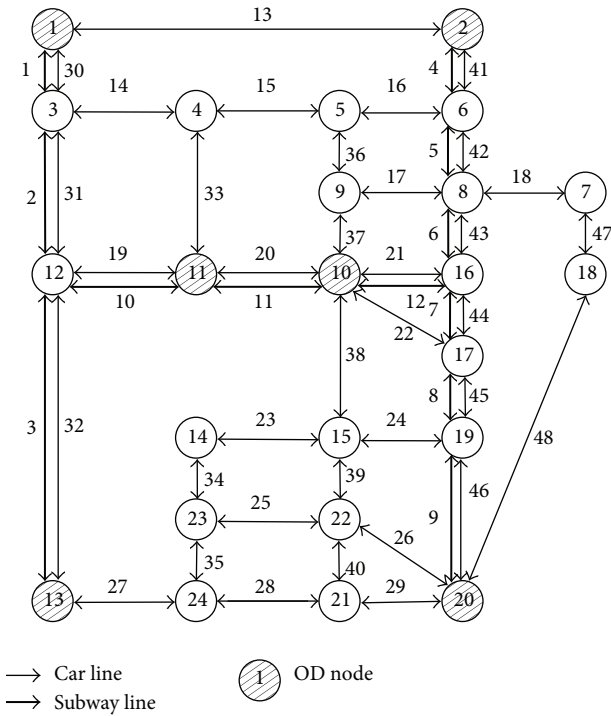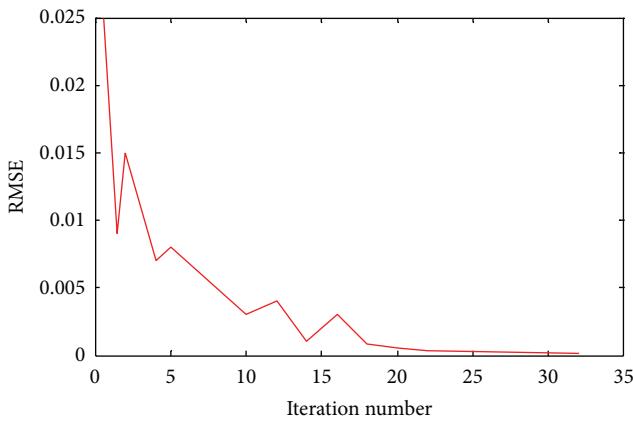
Figure 5: Sioux Fall Network.

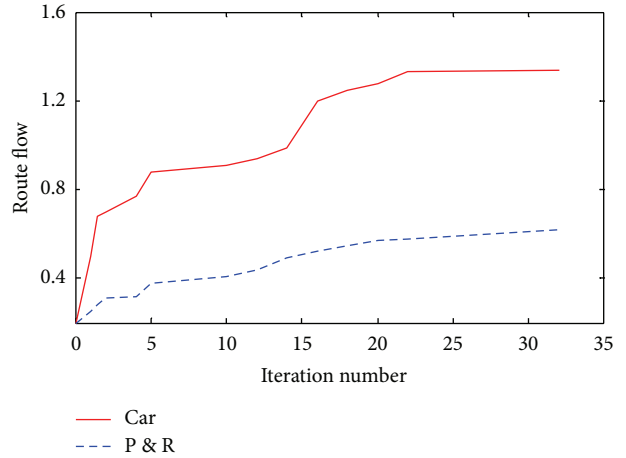Figure 6: Convergence curve of the algorithm in Sioux Fall Network.

Figure 7: Route flow evolution in the normal condition for OD pair (1, 20).

Figure 8: Route flow evolution under moderate degradation for OD pair (1, 20).

degradation of the road network increases. Travelers with different attitudes towards risk varied significantly during travel decisions. Next study should be carried out to apply the proposed model to real-world networks and extend the generalized multimodal transportation systems to the public bus. Also the parameter calibration should be carefully focused in practice.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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