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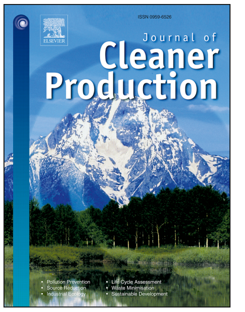

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Developing pricing strategy to optimise total profits in an electric vehicle battery closed loop supply chain

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Abstract

This paper studies a three-period electric vehicle battery recycle and reuse closed-loop supply chain consisting of a battery manufacturer and a remanufacturer. Differing from other products and existing research, used electric vehicle batteries can be instantly reused for other purposes before recycling, such as energy storage. In order to optimize total profits in the whole supply chain in different batteries period of use, this paper develops the optimal pricing strategy between manufacturer and remanufacturer, discusses the relationships between return yield, sorting rate, recycling rate in order to optimize total profit in different period. The result suggests that, comparing with new battery manufacturing, battery recycling and reusing would contribute to reduce raw material consumption hence reduce environmental impact, but may not gain financial benefits. It also notes that although the close-loop supply chain is nonlinearly complicated, some relationships between parameters can be treated as linear or quadratic. The results of this research will help practitioners to better understand the entire closed-loop supply chain in order to enhance its collaboration.

Keywords: Closed-loop supply chain, Electric vehicle battery, Recycle, Reuse, Profit

1. Introduction

Currently, Electric Vehicles (EVs) considered as one of the future development directions for the automotive industry. According to International Energy Agency (2016), from 2005 to 2010 the number of EV sales worldwide, which includes both battery EV and Plug-in Hybrid EV (PHEV), has increased from 1,670 to 12,480. By 2015, EV sales reached 1,256,900 which is almost 752 times than 10 years ago.

One of the most important parts on EV is the battery. Here are two main reasons: Firstly, approximately 50% of the cost of an EV is attributed to the battery (Lih et al., 2012). And secondly, unlike gasoline vehicles (GVs) that have a short refuelling time (5 minutes), the EV charging time is long. A typical EV model (Nissan LEAF 40kWh) takes 8 hours to charge from empty with a 6kW home charging point or 40 minutes super charge from empty to 80% capacity of electricity (Nissan, 2018). Nevertheless, an EV cannot use the original battery until its end of life. Normally, due to performance and safety concerns, the EV battery has to be removed when its capacity falls to 70～80% (McIntire-Strasburg, 2015). Moreover, with the increasing popularity of EVs, more and more batteries will need to be replaced. Discarding these batteries would constitute bad environmental practice, with more far reaching long term effects. Used batteries must be recycled or reused rather than discarded (Yu et al., 2013).

In many countries, similar to normal batteries, it is not allowed to put used automotive batteries through to landfill or incineration. Instead, various EV battery collection and recycle schemes have been set up. For instance, in North America, Tesla, working with Kinsbursky Brothers, recycles about 60% of its battery packs; in Europe, Tesla started working with Umicore on recycling (Kelty, 2011); Nissan and Volkswagen require their EV customers to return used batteries to licensed points or local authority battery collection schemes (Nissan, 2015; Volkswagen, 2016).

In addition, some organisations have already noticed the reuse of EV batteries when the EV industry just started. In the early 2010, the US National Renewable Energy Laboratory has undertaken a project on EV battery reuse (Newbauer and Pesaran, 2010). The report of the project showed that secondary use of EV batteries is necessary and the recycled EV batteries can be reused in the following ways: (a) grid-based stationary use, such as energy time shifting, renewable capacity firming; (b) off-grid stationary use, for instance, as backup power and remote installations (see Heymans et al. (2014) as well); and (c) mobile, for example, as commercial idle management or public transportation. These applications for second use of EV batteries would significantly increase the total lifetime value, both economically and environmentally. It is also gratifying that, currently, more and more EV manufacturers are considering the secondary use of EV batteries: BMW and Nissan are expected to reuse returned batteries as home energy storage (Ayre, 2016; Dalton, 2016). Chevrolet has set up an energy storage station using old EV batteries at the General Motors facility in Michigan (Voelcker, 2016). In a summary, as can be found, recycling of EV battery has been widely accepted and operated by the EV companies. The companies are also aware of the potential value of secondary use of EV batteries. However, the effect of collaboration between reusing and recycling of EV batteries is lack of employment.

Accordingly, this work develops an EV battery closed-loop supply chain (CLSC) model and investigates the pricing strategy to optimise the total profits of the supply chain. In detail, this research attempts to answer the following research questions: (1)What are the relationships between relevant parameters and profit for EV battery manufacturer and remanufacturer? (2) How to balance the accuracy and complexity of the result. In other words, how to simplify the relationship to the level that general practitioners can understand when making a decision.

The rest of the paper is organised as follows. The next section reviews some relevant papers. Section 3 describes the model and derives the optimal quantity, the optimal purchase price and the maximised profit for manufacturer and remanufacturer, respectively. Section 4 analyses the relationship between each parameter and the optimal profit in both period 2 and 3. Section 5 conducts some numerical experiments to express the findings graphically. Section 6 concludes and discusses the limitation of the research.

2. Literature review

There is not much literature relating to EV battery recycle and reuse. Richa et al. (2014) forecast the value and quantity of EV battery waste and then stated as a suggestion that, in order to increase the economic efficiency, the EV end-of-life battery management system must include an increase in reuse avenues before recycle or disposal. And Lih et al. (2012) discussed the technology challenges, cost issues and business model for the EV battery secondary use applications. The results show that, second use of EV battery is a perfect win-win deal which will probably create long-term and stable profits. The research also estimates that, the profit rate could reach around 35% in the 15 service years of a 10kWh Li-ion battery pack. Neubauer and Pesaran (2011) estimated the impact of EV battery second use on the initial cost of PHEV/EV batteries to automotive consumers and explored the potential applications for grid-based energy storage. Although second use of battery is not expected to significantly affect today’s PHEV/EV prices, it has the potential to become a common component in the future EV battery life cycles and to transform markets in need of cost-effective energy storage. Some research also studied the reuse of EV batteries with focusing in the aspect of technology. For example, Tong et al. (2017) proposed a solar energy time shifting and demand side management system for secondary use of EV batteries with objectives to maximize economic benefits, minimize grid energy consumption, or a balance of both. In terms of energy storage, Patten et al. (2011) suggested a wind energy storage system to increase energy capacity factor, improve utilization, and make more efficient use of EV batteries prior to recycling.

As can be seen, there is few research examining how reused and recycled EV batteries affect the operational performance and profit of a CLSC jointly. In other words, from the first use on the EV to reuse for other purposes and then entering the recycle or remanufacture process, the EV battery CLSC is considered as a multi-period CLSC, which is also supported by Yu et al. (2013). As a matter of fact, there are large volumes of papers studying the CLSC from the multi-period perspective. For instance, Majumder and Groenevelt (2001) studied a two-period competition model between an original equipment manufacturer (OEM) and a local remanufacturer by fixing the total cost for dealing with the returned items. The model developed by Mitra and Webster (2008) analysed the regulation of remanufacturing activities. Moreover, Ferrer and Swaminathan (2010) analysed the (re)manufacturer monopoly environment from a multi-period planning horizon, and develop a strategy in order to optimise the profit for the firm. However, these studies did not take into consideration about the process of secondary use of product. This means that, existing research cannot reflect the features of EV used battery CLSC from return, reuse to recycle.

Therefore, the literature review suggests that, there is little research studying the EV battery combined with recycling and second use processes. On another aspect, existing CLSC models are not able to reflect the practices of used EV battery reuse and recycling and the characteristics of such CLSC; that is, unlike normal goods, EV batteries cannot be reused for their original purpose when it degrades down to two thirds of full capacity, which significantly complicates CLSC operations. Moreover, majority of the mathematical models and their results from the relevant literature appear to be too complicated for general practitioners to understand, e.g., Cai et al. (2014) and Bulmus et al. (2014), which significantly limits the application and implication of these research outcomes. Hence, this study aims to fill the CLSC research gap in EV battery reusing and recycling and to help managers better understand the CLSC. The objective of this paper is to design a model to describe a three-period EV battery CLSC, then explores the interrelationship between EV battery manufacturer and remanufacturer, explains the reasons why recycling is still underdeveloped, and how profit can be increased by using right pricing strategy.

Similar with Bulmus et al. (2014), we involve EV battery manufacturer, who produces new EV batteries. Then we enrol more participants in the second period: the EV battery manufacturer (same as period 1), and the remanufacturer, who collects used batteries and sorts them into high-quality and low-quality returns (Cai et al., 2014). Then, reusable batteries will be selected from high-quality returns, i.e. second market for reusing battery. To reflect the specific feature of used battery, the author proposes a third period where reused batteries are collected for recycling. In this study, we aim to optimise the total profit in the entire Supply Chain (SC) by taking into consideration the return rate, sorting rate, processing cost and recycling rate.

3. Model description

We consider a three-period model to describe an EV battery manufacturing/ remanufacturing system as shown in Fig. 1. Initially, the demand for EV battery raw material is based on the quantity of required EV batteries. Furthermore, the demand for EV batteries depends on the EV market size.

Fig. 1: A three-period model in manufacturing/remanufacturing system

• In period 1, all EV batteries are made from raw materials. Battery manufacturing quantity is based on EV demand.

• In period 2, batteries are made from raw materials, and high- and low-quality returns. First, a proportion of θ of used EV batteries is returned. We categorise returns into two classes (Cai et al., 2014; Gaines and Singh, 1995): a proportion (α) of high-quality returns and a proportion $(1-\alpha)$ of low-quality returns. Then, high-quality returns are sorted again: a proportion of β will be reused, meanwhile $(1-\beta)$ of them will be recycled directly. Because of depletion in the battery recycling process, we set λ_l and λ_h as the remanufacturing rate for low- and high-quality returns. This indicates that λ_l (or λ_h) of low- (or high-) quality returns can be recycled to materials.

• In period 3, batteries are made from raw materials, high- and low-quality returns and reused batteries. Those reused batteries reach their end of life and will be recycled as well. The recycling rate for reused batteries is λ_u. The other returns will be recycled as indicated in period 2.

The notations are listed in Table 1.

Table 1: Notations

Input parameters

θ	Battery return yield	α	High-quality used battery sorting rate
β	Reusable battery sorting rate	c_{ntr}	Cost for new EV battery (including material cost and manufacturing cost)
c_h	Remanufacturing cost for high-quality returns	c_l	Remanufacturing cost for low-quality returns
c_u	Remanufacturing cost for reused battery	λ_h	High quality returns recycling rate ($0<\lambda_h<1$)
λ_l	Low quality returns recycling rate ($0<\lambda_l<1$)	λ_u	Reused EV battery recycling rate ($0<\lambda_u<1$)
M_{EV}	EV market size		
Decision variables			
$p_{EVi};i=1,2,3$	EV price in period i	$q_{EVi};i=1,2,3$	EV demand in period i
$p_i;i=1,2,3$	Battery price in period i	$q_{in};i=1,2,3$	Battery quantity made from raw material in period i
$q_{il};i=2,3$	Quantity of batteries re-manufactured from low-quality returns in period i	$q_{ih};i=2,3$	Quantity of batteries re-manufactured from high-quality returns in period i
q_{3u}	Quantity of batteries re-manufactured from reused batteries in period 3	$s_{il};i=2,3$	Price of purchasing low-quality returns in period i
$s_{ih};i=2,3$	Price of purchasing high-quality returns in period i	s_{3u}	Price of purchasing reused batteries in period 3
Objective variables			
$\Pi_i;i=1,2,3$	Total profit in period i		
Intermediate variables			
v	Customer's willingness to pay for the EV	H	Coefficient between battery material quantity and EV sold quantity: $q_i = Hq_{EVi},(H>0)$
δ_m	Coefficient between EV sale price and the value of battery on EV: $p_i=\delta_m p_{EVi},(0<\delta_m<1)$	k	For simplification, suppose $k=\delta_m/\left(HM_{EV}\right)$
$\Pi_{in};i=1,2,3$	Profit from new battery manufacturer in period i	$\Pi_{il};i=2,3$	Profit from low-quality returns re-manufacturer in period i
$\Pi_{ih};i=2,3$	Profit from high-quality returns remanufacturer in period i	Π_{3u}	Profit for reused battery returns remanufacturer in period 3

3.1. Nash equilibrium in period 1

The EV market size is defined as M_{EV}. Similar with Ferguson and Toktay (2006) and Debo et al. (2005), both EV price p_{EV1} and customer's willingness v are uniformly distributed between 0 and 1 (i.e., $v \in [0,1]$ and $p_{EV1} \in [0,1]$). By adopting same utility-based approach as Bulmus et al. (2014), customers utility of buying EV is $\left(v - p_{EV1}\right)$. Therefore, the quantity of EV which is sold in this period becomes

$$q_{EV1} = M_{EV}\left(1 - p_{EV1}\right) \quad (1)$$

The demand for battery material is based on the demand quantity of EVs, that is $q_1 = Hq_{EV1}\,(H > 0)$. And the EV battery price accounts for δ_m of the total EV price ($p_1 = \delta_m p_{EV1}$). In period 1, all EV batteries are made from the raw materials, that is $q_{1n} = q_1$. Hence, through substituting q_1 and p_1 into Eq. 1, we have

$$q_{1n} = HM_{EV}\left(1 - p_1 / \delta_m\right) \qquad (2)$$

Let $k = \delta_m / \left(HM_{EV}\right)$, and through formula transformation, the battery price in period 1 is

$$p_1 = \delta_m - kq_{1n} \quad (3)$$

Battery manufacturer's profit is the sale price minus both the new EV battery cost (including both raw material cost and manufacturing cost), then multiply by the quantity of sold. Through substituting Eq. 3, the profit can be expressed as

$$\Pi_1 = \Pi_{1n} = \left(p_1 - c_{ntr}\right)q_{1n} = \left(\delta_m - kq_{1n} - c_{ntr}\right)q_{1n} \qquad (4)$$

3.2. Nash equilibrium in period 2

Similar to period 1, the entire demand for EV depends on market size and EV price in period 2:

$$q_{EV2} = M_{EV}\left(1 - p_{EV2}\right) \quad (5)$$

With $q_2 = Hq_{EV2}$ and $p_2 = \delta_m p_{EV2}$, the quantity of EV batteries required in this period is

$$q_2 = HM_{EV}\left(1 - p_{EV2} / \delta_m\right) \qquad (6)$$

Let $k = \delta_m / \left(HM_{EV}\right)$, then we can derive the EV battery cost function by inversing Eq. 6:

$$p_2 = \delta_m - kq_2 \quad (7)$$

In this period, θ of batteries will be returned. These returned batteries will be sorted into three classes: reusable returns, high-quality returns and low-quality returns. As shown in Fig. 1, those reusable returns will be reused to other places, for example, as energy storage. Both high- and low-quality returns will be recycled into battery materials directly. Therefore, in period 2, the battery materials come from three sources: raw natural materials, and material recycled from both high-quality returns and low-quality returns. The amount of raw natural materials required amounts to the material requirement for making a battery minus the quantity of materials recycled from the returned batteries:

$$q_{2n} = q_2 - q_{2l} - q_{2h} \qquad (8)$$

We can derive the inverse the demand function Eq. 8 by substituting Eq. 7 as follows:

$$p_2 = \delta_m - k\left(q_{2n} + q_{2l} + q_{2h}\right) \quad (9)$$

The total return at period 2 is the return rate θ multiplied by the quantity of battery material in the previous period, i.e. $H\theta q_{EV1}$. As mentioned, $(1-\alpha)$ of them are classified as low-quality returns. For the other returns, β of them are high-quality returns which will be recycled, while $(1-\beta)$ of them will be sorted as reusable returns. The demands for high-quality returned batteries and low-quality returned batteries are respectively:

$$\begin{cases} q_{2h} = H\theta q_{EV1}\alpha(1-\beta)(1-s_{2h}) \\ q_{2l} = H\theta q_{EV1}(1-\alpha)(1-s_{2l}) \end{cases} \quad (10)$$

The quantity of materials made from different categories of returns are the quantity of returns multiplied by the returned batteries recycling rate, λ_h and λ_l. The profit for new battery manufacturer and the low-quality battery remanufacturer is defined as battery sale revenue minus recycling cost and returned battery purchase cost.

The profit functions for the new battery manufacturer (Π_{2n}), low-quality and high-quality battery remanufacturer in period 2, i.e. Π_{2l} and Π_{2h}, are

$$\begin{cases} \Pi_{2n} = \left(p_2 - c_{ntr}\right)q_{2n} \\ \Pi_{2l} = \left(\lambda_l p_2 - c_l - s_{2l}\right)q_{2l} \\ \Pi_{2h} = \left(\lambda_h p_2 - c_h - s_{2h}\right)q_{2h} \end{cases} \quad (11)$$

In summary, the total profit in period 2 could be

$$\Pi_2 = \Pi_{2n} + \Pi_{2l} + \Pi_{2h} \quad (12)$$

The existence and uniqueness of Nash equilibrium in period 2 can be proved (see Appendix A) The optimal total profit is achieved by using first-order condition, that is $\frac{\partial \Pi_2}{\partial q_{2n}} = \frac{\partial \Pi_2}{\partial s_{2h}} = \frac{\partial \Pi_2}{\partial s_{2l}} = 0$.

Because of the length limit of paper, the optimal values q^*_{2n}, s^*_{2l}, s^*_{2h}, q^*_{2h}, q^*_{2l} are shown in detail from Eq. A.43 to Eq. A.47 in Appendix A as well.

3.3. Nash equilibrium in period 3

Similarly, the EV quantity in period 3 is

$$q_{EV3} = M_{EV}\left(1 - p_{EV3}\right) \quad (13)$$

With $q_3 = Aq_{EV3}$ and $p_3 = \delta_m p_{EV3}$, the total demand for EV batteries in this period is

$$q_3 = HM_{EV}\left(1 - p_3/\delta_m\right) \quad (14)$$

Let $k = \delta_m / \left(HM_{EV}\right)$, then we can achieve the price function by deriving from Eq. 14:

$$p_3 = \delta_m - kq_3 \quad (15)$$

In this period, the battery material consists of raw natural materials, high and low-quality returns and end-of-life reused battery returns. The demand for batteries made from raw natural materials is total market demand minus all EV batteries made from returns:

$$q_{3n} = q_3 - q_{3l} - q_{3h} - q_{3u} \qquad (16)$$

And the price

$$p_3 = \delta_m - k\left(q_{3n} + q_{3l} + q_{3h} + q_{3u}\right) \qquad (17)$$

The return quantity in period 3 is new batteries manufactured in period 2 multiplied by the return rate, i.e. $H\theta q_{EV2}$. In this period, all returns in the three categories (low-quality, high-quality and reused returns) will be recycled with the quantity:

$$\begin{cases} q_{3l} = H\theta q_{EV2}(1-\alpha)(1-s_{3l}) \\ q_{3h} = H\theta q_{EV2}\alpha(1-\beta)(1-s_{3h}) \\ q_{3u} = H\theta q_{EV2}\alpha\beta(1-s_{3u}) \end{cases} \qquad (18)$$

The entire profit for the new product manufacturer is new EV battery demand multiplied by each new EV battery's profit that can be earned in manufacturing. The profits for batteries made from recycled or reused returns are the revenues minus all the costs. By supposing Eq. B.48, the profits are:

$$\begin{cases} \Pi_{3n} = \left(p_3 - c_{ntr}\right)q_{3n} \\ \Pi_{3l} = \lambda_l q_{3l} p_3 - \left(c_l + s_{3l}\right)q_{3l} \\ \Pi_{3h} = \lambda_h q_{3h} p_3 - \left(c_h + s_{3h}\right)q_{3h} \\ \Pi_{3u} = \lambda_u q_{3u} p_3 - \left(c_u + s_{3u}\right)q_{3u} \end{cases} \qquad (19)$$

The entire profit is a sum profit for manufacturer/ remanufacturers:

$$\Pi_3 = \Pi_{3n} + \Pi_{3l} + \Pi_{3h} + \Pi_{3u} \qquad (20)$$

The existence and uniqueness of Nash equilibrium in this period can be found in Appendix B. By using first-order condition to acquire the optimal profit for each agent, the optimal values q_{3n}^*, s_{3l}^*, s_{3h}^*, s_{3u}^*, q_{3l}^*, q_{3h}^*, q_{3u}^* in period 3 are expressed from Eq. B.53 to Eq. B.59 in detail in Appendix B.

4. Discussion

In this section, we will analyse the relationships between the parameters (i.e., θ, α, β, λ_l, λ_h, λ_u, c_{ntr}, c_l, c_h, c_u) and the total profit in both period 2 and 3. Two definitions and lemmas are first presented as preparation:

Definition 1. According to Fraden (2004) and Cooper (1970), **linearity** is defined as a ratio of maximum deviation between the practical curve and fitted straight line with full scale output, that is $\eta = max(\Delta Y)/Y{*}\%100$.

Fig. 2: Example for linearity

Fig. 2 is the schematic diagram for linearity. Through linear regression, the optimal fitted line can be solved. For simplification, we draw a simple fitted line just connecting two points on the curve which a cross point between $x=0$ and the original curve and a cross point between $x=1$ and the original curve. It is easy to prove that the optimal linearity of the curve is always less than or equal to linearity for the simple fitted line. If the linearity $\eta = 0$, the function is completely linear. In this research, we assert that the curve is approximately linear if the linearity of the simple fitted line is $\pm 10\%$.

Definition 2. According to the algorithms (Sedgewick, 1988), **grid search** is an ergodic global

searching method used to find the target value.
In this research, grid search is used to find the maximum and minimum linearity by traversal through searching the value of all possible parameter based on the resolution.

Lemma 1. Function with format

$$f(x)=\frac{N_0+N_1x+N_2x^2}{D_0+D_1x} \qquad (21)$$

If $(N_0+N_1x)\gg N_2x^2$ holds, then $f(x)$ approaches

$$f(x)\approx\frac{N_0+N_1x}{D_0+D_1x} \qquad (22)$$

Lemma 2. Given $q_{EV}<M_{EV}$ and $0<\delta_m<1$, $0<Hkq_{EV}<1$ holds.

Proof: Substituting $k=\delta_m/(HM_{EV})$ into Hkq_{EV}, we get $Hkq_{EV}=\frac{q_{EV}}{M_{EV}}\delta_m$. As the quanitity of EV is less than the market size ($q_{EV}<M_{EV}$) and the battery price is less than the price of entire EV ($0<\delta_m<1$), we have $0<\frac{q_{EV}}{M_{EV}}\delta_m<1$. Hence, $0<Hkq_{EV}<1$.

Based on proposed definations and lemmas, the monotonicity and linearity are discussed and analysed. Due to the length limitations of research paper, all proofs between paramters in period 2 and 3 are shown in appendix C and D respectively. In the appendix, we discuss the relationship between parameters (return yields, recycling rates and remanufacturing costs) and total optimal profit in both period 2 and 3, especially the range of first-order derivative and range of linearity. Although the profit functions are too complex, they can be simplified for approximation. The character of first-order derivative represents the monotonicity. The range of linearity indicates the possibility that a relationship is supposed to be linear. With all proofs in appendix C and D, to summarise, all relationships can be encapsulated in Table 2. We use "L" to express the linear relationship; and "Q" to show the quadratic relation; then "$\uparrow/\downarrow$" represent the positive/negative correlation; while "N/A" shows that the relationship is inapplicable.

Table 2: Summary for theorems

	θ	α	β	λ_l	λ_h	λ_u	c_{ntr}	c_l	c_h	c_u
Π_2^*	$\downarrow$& L	L	$\uparrow$& L	$\uparrow$	$\uparrow$	N/A	Q	$\downarrow$&Q	$\downarrow$&Q	N/A
Π_3^*	$\downarrow$& L	L	L	$\uparrow$	$\uparrow$	$\uparrow$	Q	$\downarrow$&Q	$\downarrow$&Q	$\downarrow$&Q

Therefore, we reach six observations, as shown in the propositions below:

Proposition 1. The relationship between θ, α, β and total optimal profit in period 2/period 3 (i.e., Π_2/Π_3) can be treated as linear, based on $\pm10\%$ linearity limit condition.

Proposition 2. The relationship between all the cost (c_{ntr}, c_l, c_h, c_u) and optimal profit Π_2, Π_3 is quadratic.

Proposition 3. β has a positive correlation with Π_2.

Proposition 4. All recycling rates λ are positively correlated with both Π_2 and Π_3.

Proposition 5. The relationships between θ, c_l, c_h, c_u and Π_2, Π_3 are negative.

Proposition 6. Other relationships depend on different situations and initial values.

Based on the propositions, we consider the following management insight discussions:

• The higher the EV battery return yield θ, the lower the total optimal profit in both period 2 and 3. This indicates that returns are not able to generate more profit for the SC. This finding is similar with Tierney (1996) who argued that recycling may not be a worth activity from economic point of view. However, it is appreciated that increased recycling is able to reduce consumption of new products and materials (Thomas, 2003). Meanwhile, this finding explains why EV battery recycling have not been adopted widely.

• The relationship between reusable battery return yield β and optimal profit in periods 2 and 3 is different. It has a positive linear relationship with the optimal profit in period 2. This means the more reusable EV batteries the higher profit in period 2. But in period 3, which is more complex than period 2, the trend of relationship depends on the initial value of parameters. Therefore, in order to increase the overall profit, EV battery should not be used till the end of its life cycle. Instead, it should go to 2nd stage of reuse when it reaches 60-70% of full capacity.

• Recycling rates for different quality of returns λ_l, λ_h and λ_u have a positive relationship with optimal profit. This is because the higher quality of returns, the less effort (hence lower cost) needed in the recycling process, resulting in higher recycling efficiency. As an example, Gaines (2014) looks ahead at how to improve the recycling efficiency technically.

• According to King et al. (2006), remanufacturing could be the best solution to deal with the returns. And furthermore, in EV battery CLSC, the higher costs of recycling operations (i.e. c_l, c_h and c_u) the lower optimal profit in both period 2 and 3. Producing new EV battery is more profitable than recycling used batteries. Therefore, how to reduce the recycling cost in the entire CLSC is a considerable problem. There are some solutions, for example, to develop better technologies (Hutchinson, 2008) or export recycling operations to the countries with lower processing costs (Geyer and Blass, 2010).

• Although relationships between parameters and entire profit are complicated, our research finds that they actually can be simplified as linear or quadric. For instance, relationships between θ, α, β and Π_2, Π_3 can be considered a linear relationship and relationships betwee c_{ntr}, c_l, c_h, c_u and Π_2, Π_3 can be treated as quadric. This simplification would be considerably helpful to managers when analysing the SC and making decisions.

5. Numerical experiments

The previous section analysed and discussed the relationships among parameters. This section will have some numerical experiments as the implementation of the model. Section 5.1 proposes a numerical example as a case to explain how the model can be used in industry. And section 5.2 describes the use of methodology in practice and verify the relationships as shown in Table 2.

5.1. Numerical example

According to International Energy Agency (2017), EV market size is predicted to 18,000,000 in 2020, i.e. $M_{EV} = 18,000,000$. And EV battery price accounts for around 30% of electric car price i.e. $\delta_m = 0.3$. According to Fred Lambert (2017) and Mark (2014), taking Tesla Model as an example, the whole value for each EV battery is £11700. According to Binks (2016) and Will Date (2015), it averagely costs £860 to process a used battery. We assume the low-quality used battery recycling processing cost is £950, high-quality is £850 and reusable battery is £800. To normalize the cost into the same scale, without losing generality, set $c_{ntr} = 0.2$ as benchmarking, other costs against the benchmark $c_l = 0.2*950/11700 = 0.016$, $c_h = 0.2*850/11700 = 0.015$,

$c_u = 0.2*800/11700 = 0.014$. We can come up with a numerical example to demonstrate the model. All input parameters of numerical example are summarised in Table 3:

Table 3: Numerical example parameters

$M_{EV} = 18000000$	$H = 4$	$\delta_m = 0.4$	$\theta = 0.3$	$\alpha = 0.8$
$\beta = 0.7$	$c_{ntr} = 0.2$	$c_h = 0.015$	$c_l = 0.016$	$c_u = 0.014$
$\lambda_l = 0.8$	$\lambda_h = 0.9$	$\lambda_u = 0.85$		

In **period 1**, with equations from Eq. 1 to Eq. 4, the optimal quantity for battery raw materials (for period 1, this is also the optimal total quantity) is $q_1 = q_{1n} = 1.8\times10^7$. The optimal sale price is $p_1 = 17550$ and the optimal profit in this stage is $\Pi_1 = \Pi_{1n} = 1.8\times10^6$. Moreover, the optimal EV sale quantity is $q_{EV1} = 4.5\times10^6$.

While in **period 2** we substitute q^*_{EV1} as the initial input parameter for this period. By applying Eq. A.43 to Eq. A.47, the optimal values are as follows in Table 4.

Table 4: Optimal values in period 2

$q^*_{2n} = 1.70\times10^7$	$q^*_{2l} = 4.73\times10^5$	$q^*_{2h} = 5.48\times10^5$	$q^*_2 = 1.81\times10^7$
$p^*_2 = 17526$	$\Pi^*_2 = 1.36\times10^6$	$q^*_{EV2} = 4.52\times10^6$	

And in **period 3**, we substitute q^*_{EV2} as initial EV quantity in this period. By applying Eq. B.49 to Eq. B.59, the optimal values in this period are as shown in Table 5.

Table 5: Optimal values in period 3

$q^*_{3n} = 1.58\times10^7$	$q^*_{3l} = 4.75\times10^5$	$q^*_{3h} = 5.50\times10^5$	$q^*_{3u} = 1.30\times10^6$
$q^*_3 = 1.82\times10^7$	$p^*_3 = 20475$	$\Pi^*_3 = 799660$	$q^*_{EV3} = 4.54\times10^6$

In a summary, in this numerical example, the optimal manufacturing yield and the optimal price in each of periods can be derived. Also, in this example, as can be seen, from period 1 to period 3 quantity of batteries made by the raw material is decreasing and the total profit is decreasing as well. This indicates that, with the increasing returned EV batteries, battery production is less dependent on natural resources, but in the meantime, the total profit is reduced because of the cost of remanufacturing.

5.2. Analysis

Based on the initial numerical input in Table 3, this subsection shows the relationship between parameters and total profits by with figures.

5.2.1. Period 2

• Relationship between θ, α, β and total profit in period 2

Fig. 3: θ, α, β vs total profit in period 2

The function between battery return yield θ, high-quality used battery sorting rate α, reusable battery sorting rate β and the optimal total profit Π^*_2, as shown in appendix (see Eq. C.60, Eq. C.67, Eq. C.74), are expressed as the original curves in Fig. 3. And the fitted line with '*' are drawn through linear regression. In addition, three functions below express the profit functions and fitted lines.

$$\Pi_2^*(\theta) = -1376160\theta + \frac{1.95883\times10^{13}}{\theta+3846.15} - 5.09116\times10^{9} \approx -1.46\times10^{6}\theta + 1.80\times10^{6} \quad (23)$$

$$\Pi_2^*(\alpha) = \frac{1.06\times10^{13}}{3604.68-\alpha} - 72527.4\alpha - 2.95\times10^{9} \approx 7.46\times10^{5}\alpha + 7.64\times10^{5} \quad (24)$$

$$\Pi_2^*(\beta) = \frac{2.1422\times10^{14}}{16668.7-\beta} - 1.29\times10^{10} \approx 7.71\times10^{5}\beta + 8.22\times10^{5} \quad (25)$$

• Relationships between λ_l, λ_h and total profit in period 2

The figure and equations below show functions λ_l and λ_h with optimal profit in period 2 (As shown in Eq. C.80 and Eq. C.88 in the appendix), as well as the fitted line by linear regression.

Fig. 4: λ_l, λ_h vs total profit in period 2

$$\Pi_2^*(\lambda_l) = \frac{8.81\times10^{7}\lambda_l + 1.06\times10^{10}}{\lambda_l(\lambda_l-2)+667.68} - 1.46\times10^{7} \approx 1.57\times10^{5}\lambda_l + 1.24\times10^{6} \quad (26)$$

$$\Pi_2^*(\lambda_h) = \frac{8.79\times10^{7}\lambda_h + 8.78\times10^{9}}{\lambda_h(\lambda_h-2)+556.59} - 1.46\times10^{7} \approx 1.88\times10^{5}\lambda_h + 1.20\times10^{6} \quad (27)$$

• Relationship between c_l, c_h and total profit in period 2

Through Eq. C.102 and Eq. C.108, we can describe the quadratic relationships using the figures and expressions below:

Fig. 5: c_l, c_h vs total profit in period 2

$$\Pi_2^*(c_l) = -0.27\times10^{6}c_l^2 - 0.46\times10^{6}c_l + 1.37\times10^{5} \quad (28)$$

$$\Pi_2^*(c_h) = -0.32\times10^{6}c_h^2 - 0.54\times10^{6}c_h + 1.37\times10^{6} \quad (29)$$

With the concept of linearity, in period 2, the linearities are summarised as $\eta\left(\Pi_2^*(\theta)\right) = 0.67\%$, $\eta\left(\Pi_2^*(\alpha)\right) = 0.71\%$, $\eta\left(\Pi_2^*(\beta)\right) = 0.87\%$, $\eta\left(\Pi_2^*(\lambda_l)\right) = 2.95\%$, $\eta\left(\Pi_2^*(\lambda_h)\right) = 1.37\%$. This means, although the relationships are complex, they can be treated as linear. And policy makers will be easier to analyse the profit. In this specific case, in order to increase the revenue, the rate of high-quality used battery and reusable battery should be increased and the waste in remanufacturing processes should be decreased. For example, encourage EV users to use batteries carefully and test them in time so more surplus value could be remained at reusable level; while the remanufacturer should improve the techniques in recycling and remanufacturing. What is more, the cost for the remanufacturing has quadratic relationship with the total profit Π_2^*. And the higher cost, the less total optimal profit.

5.2.2. Period 3

All relationships and description functions for period 3 are given below.

• Relationships between θ, α, β and total profit in period 3

The functions between Battery yield θ, high-quality used battery sorting rate α, reusable battery sorting rate β and optimal profit in period 3 (Π_3^*) are shown in Eq. D.114, Eq. D.120, Eq. D.127.

Fig. 6: α, β, θ vs total profit in period 3

Fig. 6 shows the functions and the fitted lines. The mathematical expressions are shown below.

$$\Pi_3^*(\theta)=\frac{\begin{pmatrix}3.13\times10^{-13}\theta^6-1.54\times10^7\theta^5+1.89\times10^{26}\theta^4\\+9.68\times10^{31}\theta^3+1.43\times10^{37}\theta^2+5.02\times10^{41}\theta-2.72\times10^{41}\end{pmatrix}}{\theta^4-2.46\times10^{19}\theta^3-3.47\times10^{26}\theta^2-8.70\times10^{31}\theta-1.51\times10^{35}}$$
$$\approx-3.36\times10^6\theta+1.81\times10^6 \quad (30)$$

$$\Pi_3^*(\alpha)=\frac{\begin{pmatrix}-1.66\times10^{-14}\alpha^6-1.47\times10^7\alpha^5-3.24\times10^{27}\alpha^4\\+2.02\times10^{33}\alpha^3-3.15\times10^{38}\alpha^2-5.53\times10^{41}\alpha-8.59\times10^{42}\end{pmatrix}}{\alpha^4+4.42\times10^{20}\alpha^3-5.87\times10^{27}\alpha^2+1.79\times10^{33}\alpha-1.12\times10^{37}}$$
$$\approx0.61\times10^5\alpha+7.51\times10^5 \quad (31)$$

$$\Pi_3^*(\beta)=\frac{\begin{pmatrix}2.40\times10^{-14}\beta^6+2.47\times10^7\beta^5+6.33\times10^{27}\beta^4-8.89\times10^{33}\beta^3\\+3.13\times10^{39}\beta^2-7.39\times10^{42}\beta+2.37\times10^{44}\end{pmatrix}}{\left(\beta^4+5.14\times10^{20}\beta^3-3.12\times10^{28}\beta^2+2.16\times10^{34}\beta+2.88\times10^{38}\right)}$$
$$\approx-0.21\times10^5\beta+8.14\times10^5 \quad (32)$$

• Relationships between λ_l, λ_h, λ_u and total profit in period 3

The figure below shows the relationships between high-quality, low-quality, reused EV recycling rate (λ_l, λ_h, λ_u) and optimal profit in period 3 (Π_3^*). The functions are described in Eq. D.134, Eq. D.142, Eq. D.150. The linear regressed lines are also shown in the figure.

Fig. 7: λ_l, λ_h, λ_u vs total profit in period 3

$$\Pi_3^*(\lambda_l)=\frac{\begin{pmatrix}2.32\times10^8\lambda_l^6-1.73\times10^7\lambda_l^5-4.66\times10^{13}\lambda_l^4+5.28\times10^{14}\lambda_l^3\\-2.18\times10^{19}\lambda_l^2+1.70\times10^{20}\lambda_l+6.50\times10^{20}\end{pmatrix}}{\begin{pmatrix}\lambda_l^6-8.04\lambda_l^5+3\times10^6\lambda_l^4-1.69\times10^7\lambda_l^3+1.44\times10^{12}\lambda_l^2\\-2.88\times10^{12}\lambda_l+9.62\times10^{14}\end{pmatrix}}$$
$$\approx1.66\times10^5\lambda_l+6.64\times10^5 \quad (33)$$

$$\Pi_3^*(\lambda_h)=\frac{\begin{pmatrix}2.30\times10^8\lambda_h^5-1.71\times10^7\lambda_h^6-3.87\times10^{13}\lambda_h^4\\+4.38\times10^{14}\lambda_h^3-1.51\times10^{19}\lambda_h^2+1.18\times10^{20}\lambda_h+3.53\times10^{20}\end{pmatrix}}{\begin{pmatrix}\lambda_h^6-8.03\lambda_h^5+2.50\times10^6\lambda_h^4-1.41\times10^7\lambda_h^3+1\times10^{12}\lambda_h^2\\-2\times10^{12}\lambda_h+5.57\times10^{14}\end{pmatrix}}$$
$$\approx2.11\times10^5\lambda_h+6.13\times10^5 \quad (34)$$

$$\Pi_3^*(\lambda_u)=\frac{8.78\times10^7\lambda_u+3.64\times10^9}{\lambda_u(\lambda_u-2)+239.11}-1.48\times10^7$$
$$\approx4.37\times10^5\lambda_u+4.32\times10^5 \quad (35)$$

• Relationships between c_l, c_h, c_u and total profit in period 3

Fig. 8: c_h, c_l, c_u vs total profit in period 3

$$\Pi_3^*(c_l)=-2.74\times10^5c_l^2-4.69\times10^5c_l+8.07\times10^5 \quad (36)$$

$$\Pi_3^*\left(c_h\right) = -3.27\times 10^5 c_h^2 - 5.41\times 10^5 c_h + 8.08\times 10^5 \qquad (37)$$

$$\Pi_3^*\left(c_u\right) = -0.76\times 10^6 c_u^2 - 1.28\times 10^6 c_u + 0.82\times 10^6 \qquad (38)$$

Similar with period 2, in period 3, the linearities are $\eta\left(\Pi_3^*\left(\theta\right)\right) = 0.58\%$, $\eta\left(\Pi_3^*\left(\alpha\right)\right) = 1.26\%$, $\eta\left(\Pi_3^*\left(\beta\right)\right) = 0.83\%$, $\eta\left(\Pi_3^*\left(\lambda_l\right)\right) = 1.61\%$, $\eta\left(\Pi_3^*\left(\lambda_h\right)\right) = 1.87\%$, $\eta\left(\Pi_3^*\left(\lambda_u\right)\right) = 0.72\%$. Therefore, these relationships can be treated as linear as well. In this period, the more reusable batteries, the less total profit. It means that, although reusable batteries are benefit to the environmental sustainability, it may reduce the total profit in period 3. The remanufacturing costs for different quality returns are quadric with the total profit in period 3.

To conclude, in this numerical analysis, through observing formulas from Eq. 23 to Eq. 38, all equations revalidate the propositions in section 4. Moreover, Fig. 3 to Fig. 8 show the relationships between independent variables and dependent variables, Π_2 and Π_3. Although profit functions themselves are complex and non-linear, as can be seen from Fig. 3 to Fig. Fig. 6, in this given case, all sorting rates (θ, α, β) and all recycling rate (λ_l, λ_h, λ_u ,) can still be regressed with a straight line. All the cost has quadratic relation with total profit.

6. Conclusion

In this paper, we proposed a three period EV battery closed-loop supply chain to describe the return, reuse and recycle remanufacturing process. Differing from other product, EV battery should be disassembled from the vehicle when its capacity falls to 70%～80%. But it can be reused for other purposes. In period 1, all batteries are made from raw materials. In period 2, some used batteries are returned and they are sorted to high-quality and low-quality used batteries while in this period, some batteries are selected for reusing. And in period 3, after reusing, used battery has to be recycled. The Nash equilibrium between (re)manufacturers in period 2 and period 3 are and the optimal (re)manufacturing quantities and acquisition prices are derived. We then analysed the relationships between used battery return yield θ, high-quality sorting rate of used battery α, reusable sorting rate β, recycling rate (λ_l, λ_h and λ_u), (re-)manufacturing costs (c_l, c_h and c_u) and total optimal profit in both period 2 and period 3 (Π_2 and Π_3). As complexity of the CLSC model, these relationships are difficult to describe. Through the normalization process, costs and prices are distributed into [0,1], by using the grid search method, portfolio of all parameter values are traversed.

In comparing with the existing models, such as Neubauer and Pesaran (2011) and Tong et al. (2017), it can be found that EV battery CLSC is still a relatively new topic. Firstly, current research on EV batteries is more focused on technology. Secondly, the existing CLSC models, such as Bulmus et al. (2014) and Cai et al. (2014), are not able to describe the features of EV battery, e.g., the combined features of reusing and recycling. We envision that, the model developed in this paper fills the research gap in the EV battery CLSC and EV industry.

Through discussion, the findings can be summarised as: (1) The sorting rate has linear relationship with optimal profit. This conclusion simplifies the difficulties of enterprise managers to analyse the EV battery supply chain; (2) In the EV battery CLSC, the more return batteries, the less profit. This finding also explains why recycling is not widely accepted by manufacturers even though more returns will reduce the consumption of natural resources. Therefore, government should try to take some incentives to increase the economic benefits of recycling; (3) The more reusable batteries, the more profit in period 2. So, reusable returns are encouraged. Hence, EV service providers should encourage customers to use the battery carefully; (4) The higher recycling rate and the lower recycling processing cost, the higher profit. This requires increasing efficiency and technological innovation in recycling operations; (5) The relationship between high-quality return yield and profit is uncertain but it is still encouraged in order to increase the sorting rate for the reusable batteries. Therefore, to conclude, this research develops a unique EV battery CLSC model which reflects the special characteristics of EV battery. All these findings have answered the research questions and will help

EV battery manufacturers and remanufacturers make better decision in cooperation. And to sustain recycling business, governments support is vitally important to keep the business going.

7. Limitation and future research

This research has proposed a CLSC model to illustrate the whole life cycle of EV batteries. It takes more consideration on economics while the environmental influence of EV batteries is less discussed. Therefore, future research will try to quantify the environmental impact in the CLSC. Moreover, how to determine the balance between the economy and the environment in EV battery CLSC is a future research direction as well.

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Supplementary Material

Appendix A. Optimal values calculation and proof for period 2

By substituting Eq. 9, and supposing $A_{2h} = H\theta q_{_{EV1}}\alpha\left(1-\beta\right)$, $A_{2l} = H\theta q_{_{EV1}}\left(1-\alpha\right)$ and also $A_2 = A_{2h} + A_{2l}$, the profit functions for the new battery manufacturer (Π_{2n}), low-quality and high-quality battery remanufacturer in period 2, i.e. Π_{2l} and Π_{2h}, are

$$\begin{cases} \Pi_{2n} = \left(p_{_2} - c_{ntr}\right)q_{_{2n}} \\ \quad = q_{2n}\left(-k\left(A_2 + q_{2n} - A_{2h}s_{2h} - A_{2l}s_{2l}\right) - c_{ntr} + \delta_m\right) \\ \Pi_{2l} = \left(\lambda_l p_{_2} - c_{_l} - s_{_{2l}}\right)q_{_{2l}} \\ \quad = A_{2l}\left(1-s_{2l}\right)\left(\lambda_l\left(\delta_m - k\left(A_2 + q_{2n} - A_{2h}s_{2h} - A_{2l}s_{2l}\right)\right) - c_{_l} - s_{2l}\right) \\ \Pi_{2h} = \left(\lambda_h p_{_2} - c_{_h} - s_{_{2h}}\right)q_{_{2h}} \\ \quad = A_{2h}\left(1-s_{2h}\right)\left(\lambda_h\left(\delta_m - k\left(A_2 + q_{2n} - A_{2h}s_{2h} - A_{2l}s_{2l}\right)\right) - c_{_h} - s_{2h}\right) \end{cases} \tag{A.1}$$

In summary, the total profit in period 2 could be

$$\Pi_2 = \Pi_{2n} + \Pi_{2l} + \Pi_{2h}$$
$$= \begin{pmatrix} q_{2n}\left(\delta_m - k\left(-A_{2h}s_{2h} - A_{2l}s_{2l} + A_2 + q_{2n}\right) - c_{ntr}\right) \\ +A_{2h}\left(1-s_{2h}\right)\left(\lambda_h\left(\delta_m - k\left(-A_{2h}s_{2h} - A_{2l}s_{2l} + A_2 + q_{2n}\right)\right) - c_{_h} - s_{2h}\right) \\ +A_{2l}\left(1-s_{2l}\right)\left(\lambda_l\left(\delta_m - k\left(-A_{2h}s_{2h} - A_{2l}s_{2l} + A_2 + q_{2n}\right)\right) - c_{_l} - s_{2l}\right) \end{pmatrix} \tag{A.2}$$

The Hessian matrix for Π_2 (Eq. A.2) is

$$H\left(\Pi_2\right) = \begin{bmatrix} 0 & kA_{2h} & kA_{2l} \\ kA_{2h} & -2A_{2h}\left(kA_{2h}\lambda_h - 1\right) & -kA_{2h}A_{2l}\left(\lambda_h + \lambda_l\right) \\ kA_{2l} & -kA_{2h}A_{2l}\left(\lambda_h + \lambda_l\right) & -2A_{2l}\left(kA_{2l}\lambda_l - 1\right) \end{bmatrix} \tag{A.3}$$

We have $-2A_{2h}\left(kA_{2h}\lambda_h - 1\right) > 0$ and $-2A_{2l}\left(kA_{2l}\lambda_l - 1\right) > 0$. Therefore, $H\left(\Pi_2\right)$ is positive-semidefinite and the existence and uniqueness Nash equilibrium can be proved. The optimal total profit will be achieved by using first-order condition, that is $\frac{\partial \Pi_2}{\partial q_{_{2n}}} = \frac{\partial \Pi_2}{\partial s_{_{2h}}} = \frac{\partial \Pi_2}{\partial s_{_{2l}}} = 0$:

$$\begin{cases} \frac{\partial \Pi_2}{\partial q_{2n}} = \begin{pmatrix} -k\left(-s_{2h}^* A_{2h} - s_{2l}^* A_{2l} + A_2 + q_{2n}^*\right) - k\left(1-s_{2h}^*\right)A_{2h}\lambda_h \\ -k\left(1-s_{2l}^*\right)A_{2l}\lambda_l - c_{ntr} - kq_{2n}^* + \delta_m \end{pmatrix} = 0 \\ \frac{\partial \Pi_2}{\partial s_{2h}} = \begin{pmatrix} -A_{2l}\left(\lambda_l\left(\delta_m - k\left(-s_{_{2h}}^* A_{2h} - s_{_{2l}}^* A_{2l} + A_2 + q_{2n}^*\right)\right) - c_{_l} - s_{2l}^*\right) \\ +k\left(1-s_{2h}^*\right)A_{2h}A_{2l}\lambda_h + kq_{_{2n}}^* A_{2l} + \left(1-s_{2l}^*\right)A_{2l}\left(kA_{2l}\lambda_l - 1\right) \end{pmatrix} = 0 \\ \frac{\partial \Pi_2}{\partial s_{2l}} = \begin{pmatrix} -A_{2h}\left(\lambda_h\left(\delta_m - k\left(-s_{2h}^* A_{2h} - s_{2l}^* A_{2l} + A_2 + q_{_{2n}}^*\right)\right) - c_{_h} - s_{2h}^*\right) \\ +k\left(1-s_{2l}^*\right)A_{2h}A_{2l}\lambda_l + kq_{2n}^* A_{2h} + \left(1-s_{2h}^*\right)A_{2h}\left(kA_{2h}\lambda_h - 1\right) \end{pmatrix} = 0 \end{cases} \tag{A.4}$$

The expressions for optimal values are

$$q^*_{2n} = \frac{\begin{pmatrix} k(A_{2l}\lambda_l^2(kA_{2h}(-c_h + c_{ntr} + 1) + 2(kA_{2l} + \delta_m)) \\ +A_{2l}\lambda_l(kA_{2h}(\lambda_h(c_h + c_l - 2c_{ntr} + 2) + c_h - c_l - 2) \\ -2(kA_{2l} + c_l - 2c_{ntr} + \delta_m + 1)) \\ -c_h + c_l - 2) + 2kA_{2h}(\lambda_h - 1) - 2(c_h - 2c_{ntr} + \delta_m) + 2\lambda_h\delta_m) \\ -2(A_{2h}(c_h + \lambda_h - 1) + A_{2l}(c_l - 1)) - 2A_2(kA_{2h}(\lambda_h - 1)\lambda_h \\ +kA_{2l}(\lambda_l - 1)\lambda_l + 2)) \end{pmatrix}}{2k(kA_{2h}(\lambda_h - 1)^2 + kA_{2l}(\lambda_l - 1)^2 + 4)} \quad \text{(A.5)}$$

$$s^*_{2l} = \frac{\begin{pmatrix} kA_{2h}(-c_l(\lambda_h - 1)^2 + c_h(\lambda_h - 1)(\lambda_l - 1) + \lambda_l(c_{ntr}(-\lambda_h) \\ +c_{ntr} + \lambda_h + 1) + \lambda_h(c_{ntr}(\lambda_h - 1) + \lambda_h - 3)) \\ +c_{ntr}(\lambda_l + 1) - 2c_l + (\lambda_l - 1)\delta_m + 2) \end{pmatrix}}{2(kA_{2h}(\lambda_h - 1)^2 + kA_{2l}(\lambda_l - 1)^2 + 4)} \quad \text{(A.6)}$$

$$s^*_{2h} = \frac{\begin{pmatrix} kA_{2l}(-c_h(\lambda_l - 1)^2 + c_l(\lambda_h - 1)(\lambda_l - 1) - \lambda_l((c_{ntr} - 1)\lambda_h + c_{ntr} + 3) \\ +(c_{ntr} + 1)\lambda_h + (c_{ntr} + 1)\lambda_l^2) \\ -2A_2k(\lambda_h - 1) - 4c_h + 2c_{ntr} - 2\delta_m + 4 \end{pmatrix}}{2(kA_{2h}(\lambda_h - 1)^2 + kA_{2l}(\lambda_l - 1)^2 + 4)} \quad \text{(A.7)}$$

$$q^*_{2h} = A_{2h}\left(1 - s^*_{2h}\right) \quad \text{(A.8)}$$

$$q^*_{2l} = A_{2l}\left(1 - s^*_{2l}\right) \quad \text{(A.9)}$$

Appendix B. Optimal values calculation and proof for period 3

For simplification, let $A_{3l} = H\theta q_{EV2}(1-\alpha)$, $A_{3h} = H\theta q_{EV2}\alpha(1-\beta)$ and $A_{3u} = H\theta q_{EV2}\alpha\beta$, and $A_3 = A_{3l} + A_{3h} + A_{3u}$, then Eq. 18 can be rewritten as:

$$\begin{cases} q_{3l} = A_{3l}(1 - s_{3l}) \\ q_{3h} = A_{3h}(1 - s_{3h}) \\ q_{3u} = A_{3u}(1 - s_{3u}) \end{cases} \quad \text{(B.1)}$$

The profits for batteries made from recycled or reused returns are the revenues minus all the costs. By supposing Eq. B.1, the profits are:

$$\begin{cases}\Pi_{3n} = \left(p_3 - c_{ntr}\right) q_{3n} \\ \quad = \left(\delta_m - k\left(q_{3n} + A_3 - A_{3l}s_{3l} - A_{3h}s_{3h} - A_{3u}s_{3u}\right) - c_{ntr}\right) q_{3n} \\ \Pi_{3l} = \lambda_l q_{3l} p_3 - \left(c_l + s_{3l}\right) q_{3l} \\ \quad = \begin{pmatrix} \lambda_l A_{3l}\left(1 - s_{3l}\right)\left(\delta_m - k\left(q_{3n} + A_3 - A_{3l}s_{3l} - A_{3h}s_{3h} - A_{3u}s_{3u}\right)\right) \\ -\left(c_l + s_{3l}\right) A_{3l}\left(1 - s_{3l}\right) \end{pmatrix} \\ \Pi_{3h} = \lambda_h q_{3h} p_3 - \left(c_h + s_{3h}\right) q_{3h} \\ \quad = \begin{pmatrix} \lambda_h A_{3h}\left(1 - s_{3h}\right)\left(\delta_m - k\left(q_{3n} + A_3 - A_{3l}s_{3l} - A_{3h}s_{3h} - A_{3u}s_{3u}\right)\right) \\ -\left(c_h + s_{3h}\right) A_{3h}\left(1 - s_{3h}\right) \end{pmatrix} \\ \Pi_{3u} = \lambda_u q_{3u} p_3 - \left(c_u + s_{3u}\right) q_{3u} \\ \quad = \begin{pmatrix} \lambda_u A_{3u}\left(1 - s_{3u}\right)\left(\delta_m - k\left(q_{3n} + A_3 - A_{3l}s_{3l} - A_{3h}s_{3h} - A_{3u}s_{3u}\right)\right) \\ -\left(c_u + s_{3u}\right) A_{3u}\left(1 - s_{3u}\right) \end{pmatrix} \end{cases} \tag{B.2}$$

The entire profit is a sum profit for manufacturer/ remanufacturers:

$$\Pi_3 = \Pi_{3n} + \Pi_{3l} + \Pi_{3h} + \Pi_{3u}$$

$$= \begin{pmatrix} q_{3n}\left(k\left(A_{3h}s_{3h} + A_{3l}s_{3l} + A_{3u}s_{3u} - A_3 - q_{3n}\right) - c_{ntr} + \delta_m\right) \\ + A_{3h}\left(s_{3h} - 1\right)\left(c_h + s_{3h}\right) + A_{3l}\left(s_{3l} - 1\right)\left(c_l + s_{3l}\right) \\ + A_{3u}\left(s_{3u} - 1\right)\left(c_u + s_{3u}\right) \\ - A_{3h}\lambda_h\left(s_{3h} - 1\right)\left(k\left(A_{3h}s_{3h} + A_{3l}s_{3l} + A_{3u}s_{3u} - A_3 - q_{3n}\right) + \delta_m\right) \\ - A_{3l}\lambda_l\left(s_{3l} - 1\right)\left(k\left(A_{3h}s_{3h} + A_{3l}s_{3l} + A_{3u}s_{3u} - A_3 - q_{3n}\right) + \delta_m\right) \\ - A_{3u}\left(s_{3u} - 1\right)\lambda_u\left(k\left(A_{3h}s_{3h} + A_{3l}s_{3l} + A_{3u}s_{3u} - A_3 - q_{3n}\right) + \delta_m\right) \end{pmatrix} \tag{B.3}$$

In period 3, much like period 2, the Hessian matrix for Π_3, i.e., Eq. B.3, is

$$H\left(\Pi_3\right) =$$
$$\begin{bmatrix} 0 & kA_{3h}\left(1 + \lambda_h\right) & kA_{3l}\left(1 + \lambda_l\right) & kA_{3u}\left(1 + \lambda_u\right) \\ kA_{3h}\left(1 + \lambda_h\right) & -2A_{3h}\left(kA_{3h}\lambda_h - 1\right) & -kA_{3h}A_{3l}\left(\lambda_h + \lambda_l\right) & -kA_{3h}A_{3u}\left(\lambda_h + \lambda_u\right) \\ kA_{3l}\left(1 + \lambda_l\right) & -kA_{3h}A_{3l}\left(\lambda_h + \lambda_l\right) & -2A_{3l}\left(kA_{3l}\lambda_l - 1\right) & -kA_{3l}A_{3u}\left(\lambda_l + \lambda_u\right) \\ kA_{3u}\left(1 + \lambda_u\right) & -kA_{3h}A_{3u}\left(\lambda_h + \lambda_u\right) & -kA_{3l}A_{3u}\left(\lambda_l + \lambda_u\right) & -2A_{3u}\left(kA_{3u}\lambda_u - 1\right) \end{bmatrix} \tag{B.4}$$

We have $-2A_{3h}\left(kA_{3h}\lambda_h - 1\right) > 0$, $-2A_{3l}\left(kA_{3l}\lambda_l - 1\right) > 0$ and $-2A_{3u}\left(kA_{3u}\lambda_u - 1\right) > 0$. Therefore $H\left(\Pi_3\right)$ is positive-semidefinite and we have existence and uniqueness of Nash equilibrium in this period. By using first-order condition to acquire the optimal profit for each agent:

$$\begin{cases}\left(\begin{array}{l} k\left(s^*_{3h}A_{3h}+s^*_{3l}A_{3l}+s^*_{3u}A_{3u}-A_3-q^*_{3n}\right)+k\left(s^*_{3h}-1\right)A_{3h}\lambda_h \\ +ks^*_{3l}-1)A_{3l}\lambda_l+k(s^*_{3u}-1)A_{3u}\lambda_u-c_{ntr}-kq^*_{3n}+\delta_m \end{array}\right)=0 \\ \left(\begin{array}{l} A_{3l}\left(c_l+s^*_{3l}\right)-A_{3l}\lambda_l\left(k\left(s^*_{3h}A_{3h}+s^*_{3l}A_{3l}+s^*_{3u}A_{3u}-A_3-q^*_{3n}\right)+\delta_m\right)+kq^*_{3n}A_{3l} \\ -k\left(s^*_{3h}-1\right)A_{3h}A_{3l}\lambda_h-k\left(s^*_{3l}-1\right)A_{3l}^2\lambda_l-k\left(s^*_{3u}-1\right)A_{3l}A_{3u}\lambda_u+\left(s^*_{3l}-1\right)A_{3l} \end{array}\right)=0 \\ \left(\begin{array}{l} A_{3h}\left(c_h+s^*_{3h}\right)-A_{3h}\lambda_h\left(k\left(s^*_{3h}A_{3h}+s^*_{3l}A_{3l}+s^*_{3u}A_{3u}-A_3-q^*_{3n}\right)+\delta_m\right)+kq^*_{3n}A_{3h} \\ -k\left(s^*_{3l}-1\right)A_{3h}A_{3l}\lambda_l-k\left(s^*_{3h}-1\right)A_{3h}^2\lambda_h-k\left(s^*_{3u}-1\right)A_{3h}A_{3u}\lambda_u+\left(s^*_{3h}-1\right)A_{3h} \end{array}\right)=0 \\ \left(\begin{array}{l} A_{3u}\left(c_u+s^*_{3u}\right)-A_{3u}\lambda_u\left(k\left(s^*_{3h}A_{3h}+s^*_{3l}A_{3l}+s^*_{3u}A_{3u}-A_3-q^*_{3n}\right)+\delta_m\right)+kq^*_{3n}A_{3u} \\ -k\left(s^*_{3h}-1\right)A_{3h}A_{3u}\lambda_h-k\left(s^*_{3l}-1\right)A_{3l}A_{3u}\lambda_l-k\left(s^*_{3u}-1\right)A_{3u}^2\lambda_u+\left(s^*_{3u}-1\right)A_{3u} \end{array}\right)=0 \end{cases} \quad \text{(B.5)}$$

Through solving the above equations, the optimal values in period 3 are

$$q^*_{3n}=\frac{\left(\begin{array}{l} k(A_{3h}(kA_{3l}(\lambda_h^2(-c_l+c_{ntr}+1)+\lambda_h(\lambda_l(c_h+c_l-2c_{ntr}+2)-c_h+c_l-2) \\ +\lambda_l\left(\lambda_l\left(-c_h+c_{ntr}+1\right)+c_h-c_l-2\right))+kA_{3u}(\lambda_u^2\left(-c_h+c_{ntr}+1\right) \\ +\lambda_u(\lambda_h(c_h-2c_{ntr}+c_u+2)+c_h-c_u-2)+\lambda_h(\lambda_h(c_{ntr}-c_u+1) \\ -c_h+c_u-2))+2\lambda_h\left(-c_h+2c_{ntr}+\left(\lambda_h-1\right)\delta_m\right))-2(A_{3h}\left(c_h+\lambda_h-1\right) \\ +A_{3l}\left(c_l-1\right)+A_{3u}\left(c_u-1\right))+A_{3u}\lambda_u^2\left(kA_{3l}\left(-c_l+c_{ntr}+1\right)+2\left(kA_{3u}+\delta_m\right)\right) \\ +A_{3l}\lambda_l(kA_{3u}(\lambda_l(c_{ntr}-c_u+1)-c_l+c_u-2)+2kA_{3l}(\lambda_l-1) \\ -2\left(c_l-2c_{ntr}+\delta_m+1\right)+2\lambda_l\delta_m)+A_{3u}\lambda_u(kA_{3l}(\lambda_l\left(c_l-2c_{ntr}+c_u+2\right) \\ +c_l-c_u-2)-2\left(kA_{3u}-2c_{ntr}+c_u+\delta_m+1\right))+2kA_{3h}^2\left(\lambda_h-1\right)\lambda_h \\ -2A_3\left(kA_{3h}\left(\lambda_h-1\right)\lambda_h+kA_{3l}\left(\lambda_l-1\right)\lambda_l+kA_{3u}\left(\lambda_u-1\right)\lambda_u+2\right)) \\ -4c_{ntr}+4\delta_m \end{array}\right)}{2k(kA_{3h}(\lambda_h-1)^2+kA_{3l}(\lambda_l-1)^2+kA_{3u}(\lambda_u-1)^2+4)}$$

(B.6)

$$s^*_{3l}=\frac{\left(\begin{array}{l} kA_{3h}(-c_l(\lambda_h-1)^2+c_h(\lambda_h-1)(\lambda_l-1)+\lambda_l(c_{ntr}(-\lambda_h)+c_{ntr}+\lambda_h+1) \\ +\lambda_h\left(c_{ntr}\left(\lambda_h-1\right)+\lambda_h-3\right))+kA_{3u}c_{ntr}\lambda_l+kA_{3u}\lambda_u^2\left(-c_l+c_{ntr}+1\right) \\ +kA_{3u}\lambda_u\left(-c_{ntr}\left(\lambda_l+1\right)+c_u\left(\lambda_l-1\right)+2c_l+\lambda_l-3\right)-kA_{3u}c_u\lambda_l-kA_{3u}c_l \\ +kA_{3u}c_u+2kA_{3l}\lambda_l^2-2kA_{3l}\lambda_l-2A_3k\left(\lambda_l-1\right)+kA_{3u}\lambda_l+2c_{ntr}\lambda_l-4c_l \\ +2c_{ntr}+2\lambda_l\delta_m-2\delta_m+4 \end{array}\right)}{2(kA_{3h}(\lambda_h-1)^2+kA_{3l}(\lambda_l-1)^2+kA_{3u}(\lambda_u-1)^2+4)} \quad \text{(B.7)}$$

$$s^*_{3h} = \frac{\begin{pmatrix} kA_{3l}(-c_h(\lambda_l-1)^2 + c_l(\lambda_h-1)(\lambda_l-1) - \lambda_l((c_{ntr}-1)\lambda_h + c_{ntr}+3) \\ +(c_{ntr}+1)\lambda_h + (c_{ntr}+1)\lambda_l^2) + kA_{3u}c_{ntr}\lambda_h + kA_{3u}\lambda_u^2(-c_h + c_{ntr}+1) \\ +kA_{3u}\lambda_u(-c_{ntr}(\lambda_h+1) + c_u(\lambda_h-1) + 2c_h + \lambda_h - 3) - kA_{3u}c_u\lambda_h \\ -kA_{3u}c_h + kA_{3u}c_u + 2kA_{3h}\lambda_h^2 - 2kA_{3h}\lambda_h - 2A_3k(\lambda_h-1) + kA_{3u}\lambda_h \\ +2c_{ntr}\lambda_h - 4c_h + 2c_{ntr} + 2\lambda_h\delta_m - 2\delta_m + 4 \end{pmatrix}}{2(kA_{3h}(\lambda_h-1)^2 + kA_{3l}(\lambda_l-1)^2 + kA_{3u}(\lambda_u-1)^2+4)} \quad \text{(B.8)}$$

$$s^*_{3u} = \frac{\begin{pmatrix} \lambda_u(k(A_{3h}(\lambda_h(c_h - c_{ntr}+1) - c_h + c_{ntr}+1) + A_{3l}(\lambda_l(c_l - c_{ntr}+1) - c_l \\ +c_{ntr}+1) - 2A_{3u} - 2A_3) + 2(c_{ntr}+\delta_m)) + k(A_{3h}(\lambda_h^2(c_{ntr}-c_u+1) \\ -\lambda_h(c_h + c_{ntr} - 2c_u + 3) + c_h - c_u) + A_{3l}(\lambda_l^2(c_{ntr}-c_u+1) \\ -\lambda_l(c_l + c_{ntr} - 2c_u + 3) + c_l - c_u) + 2A_3) + 2kA_{3u}\lambda_u^2 \\ +2(c_{ntr} - 2c_u - \delta_m + 2) \end{pmatrix}}{2(kA_{3h}(\lambda_h-1)^2 + kA_{3l}(\lambda_l-1)^2 + kA_{3u}(\lambda_u-1)^2+4)} \quad \text{(B.9)}$$

$$q^*_{3l} = A_{3l}\left(1 - s^*_{3l}\right) \quad \text{(B.10)}$$

$$q^*_{3h} = A_{3h}\left(1 - s^*_{3h}\right) \quad \text{(B.11)}$$

$$q^*_{3u} = A_{3u}\left(1 - s^*_{3u}\right) \quad \text{(B.12)}$$

Appendix C. Proof for relationships between parameters and total optimal profits in Period 2

Relationship between θ and Π_2

The total optimal profit in period 2 can be rewritten as

$$\Pi_2(\theta) = \frac{N_{10} + N_{11}\theta + N_{12}\theta^2}{4k(-4 + D_{11}\theta)} \quad \text{(C.1)}$$

$$\begin{cases} 0 \le N_{10} \le 0 \\ 1.23\times 10^{-7} \le N_{11} \le 24 \\ 0 \le N_{12} \le 0 \\ -1 \le D_{11} \le -2.86\times 10^{-7} \end{cases} \quad \text{(C.2)}$$

Expressions of N_{10}, N_{11}, N_{12}, and D_{11} are shown from Eq. C.4 to Eq. C.7 in Appendix. Based on the range above, we have

(i) Because of $N_{12} \approx 0$, $N_{10} \approx 0$ and Lemma 1, $\Pi_2(\theta) = \dfrac{N_{11}\alpha}{4k(-4 + D_{11}\alpha)}$

(ii) By using grid search, the range of first order of Π_2 on θ is $-6/(4k) \le \Pi_2'(\theta) \le -8.53/(4k)\times 10^{-8}$, also $\Pi_2'(\theta) < 0$. Therefore, the relationship is decremented.

(iii) For simplification, we assume the fitted line crosses (a_1, b_1) and (a_2, b_2), where $a_1 = 0$, $b_1 = \Pi_2(a_1)$, $a_2 = 1$, $b_2 = \Pi_2(a_2)$. And the fitted straight line is

$$\Pi_2(\alpha)=\frac{b_2-b_1}{a_2-a_1}\alpha+\left(b_2-\frac{b_2-b_1}{a_2-a_1}a_2\right) \quad \text{(C.3)}$$

Through using gird search and based on the assumption above, the linearity range is $-5.57\% \le \eta_{2\theta} \le 0$. So, θ and Π_2 can be treated as linear relationship.

Therefore, the relationship between $\Pi_2(\theta)$ and θ is negative linear.

$$N_{10}=-4(c_{ntr}-\delta_m)^2 \quad \text{(C.4)}$$

$$N_{11}=\begin{pmatrix} 4(Hkq_{EV1})(-\alpha\beta+(\alpha-\alpha\beta)c_h^2+\alpha(\beta-1)c_h(c_{ntr}(\lambda_h+1) \\ +(\lambda_h-1)\delta_m-2)+\alpha(c_{ntr}\left(\beta+(\beta-1)\lambda_h\left(-\lambda_h\delta_m+\delta_m+1\right)\right) \\ -(\beta-1)c_{ntr}^2\lambda_h+\delta_m\left((\beta-1)\lambda_h-\beta\right))-(\alpha-1)c_l^2-(\alpha-1)c_{ntr} \\ \cdot\lambda_l^2\delta_m-(\alpha-1)\lambda_l\left(\left(c_{ntr}-1\right)c_{ntr}-\left(c_{ntr}+1\right)\delta_m\right)+(\alpha-1)c_l \\ \cdot\left(c_{ntr}\left(\lambda_l+1\right)+\left(\lambda_l-1\right)\delta_m-2\right)-c_{ntr}+\delta_m+1) \end{pmatrix} \quad \text{(C.5)}$$

$$N_{12}=(\alpha-1)\alpha(\beta-1)(Hkq_{EV1})^2\left(-c_l(\lambda_h-1)+c_h(\lambda_l-1)+(c_{ntr}-1)(\lambda_h-\lambda_l)\right)^2 \quad \text{(C.6)}$$

$$D_{11}=(Hkq_{EV1})\left(\alpha\beta+\alpha(\beta-1)(\lambda_h-2)\lambda_h+(\alpha-1)\lambda_l^2-2(\alpha-1)\lambda_l-1\right) \quad \text{(C.7)}$$

Relationship between α and Π_2

$$\Pi_2(\alpha)=\frac{N_{20}+N_{21}\alpha+N_{22}\alpha^2}{4k(D_{20}+D_{21}\alpha)} \quad \text{(C.8)}$$

$$\begin{cases} -4\le N_{20}\le 20 \\ -24\le N_{21}\le 21 \\ 0\le N_{22}\le -2.27\times10^{-33} \\ -5\le D_{20}\le -4 \\ -1\le D_{21}\le 1 \end{cases} \quad \text{(C.9)}$$

We find The first derivative $-5.25/(4k)\le \Pi_2'(\alpha)\le 6.25/(4k)$ and the linearity is $-5.57\%\le \eta_{2\alpha}\le 5.57\%$. Therefore, the relationship between α and Π_2 can be treated as linear.

$$N_{20}=\begin{pmatrix} 4(\theta(Hkq_{EV1})c_l^2-\theta(Hkq_{EV1})c_l(c_{ntr}(\lambda_l+1)+(\lambda_l-1)\delta_m-2) \\ +c_{ntr}\left(\delta_m\left(\theta(Hkq_{EV1})(\lambda_l-1)\lambda_l+2\right)-\theta(Hkq_{EV1})(\lambda_l+1)\right) \\ +c_{ntr}^2(\theta(Hkq_{EV1})\lambda_l-1)+\theta(Hkq_{EV1})-\delta_m(\theta(Hkq_{EV1})(\lambda_l-1)+\delta_m)) \end{pmatrix} \quad \text{(C.10)}$$

$$N_{21}=\begin{pmatrix}\theta(Hkq_{EV1})(-4\beta-(\beta-1)c_h^2(\theta(Hkq_{EV1})+\theta(Hkq_{EV1})(\lambda_l-2)\lambda_l+4)\\+c_l^2\left(-(\beta-1)\theta\left(Hkq_{EV1}\right)\left(\lambda_h-2\right)\lambda_h-\beta\theta\left(Hkq_{EV1}\right)+\theta\left(Hkq_{EV1}\right)-4\right)\\+2(\beta-1)c_h(\theta(Hkq_{EV1})c_l(\lambda_h-1)(\lambda_l-1)+\lambda_h(c_{ntr}(\theta(Hkq_{EV1})\\-\theta(Hkq_{EV1})\lambda_l+2)+\theta(Hkq_{EV1})\left(\lambda_l-1\right)+2\delta_m)+\theta(Hkq_{EV1})\left(c_{ntr}-1\right)\\\cdot\left(\lambda_l-1\right)\lambda_l+2\left(c_{ntr}-\delta_m-2\right))+2\lambda_l((\beta-1)\theta(Hkq_{EV1})\left(c_{ntr}-1\right)^2\lambda_h\\+2\left(c_{ntr}\left(-c_{ntr}+\delta_m+1\right)+\delta_m\right))+2c_l(c_{ntr}((\beta-1)\theta(Hkq_{EV1})\left(\lambda_h-1\right)\\\cdot\left(\lambda_h-\lambda_l\right)+2\left(\lambda_l+1\right))+(\beta-1)\theta(-(Hkq_{EV1}))\left(\lambda_h-1\right)\left(\lambda_h-\lambda_l\right)\\+2\left(\lambda_l-1\right)\delta_m-4)-\beta\theta(Hkq_{EV1})c_{ntr}^2\lambda_h^2+2\beta\theta(Hkq_{EV1})c_{ntr}\lambda_h^2\\+\theta\left(Hkq_{EV1}\right)c_{ntr}^2\lambda_h^2-2\theta\left(Hkq_{EV1}\right)c_{ntr}\lambda_h^2-4\beta c_{ntr}\lambda_h^2\delta_m+4\beta c_{ntr}\lambda_h\delta_m\\+4c_{ntr}\lambda_h^2\delta_m-4c_{ntr}\lambda_h\delta_m-4\beta c_{ntr}^2\lambda_h+4\beta c_{ntr}\lambda_h+4c_{ntr}^2\lambda_h-4c_{ntr}\lambda_h\\+\lambda_l^2((\beta-1)\theta(-(Hkq_{EV1}))(c_{ntr}-1)^2-4c_{ntr}\delta_m)+4\beta c_{ntr}\\-\beta\theta(Hkq_{EV1})\lambda_h^2+\theta(Hkq_{EV1})\lambda_h^2+4\beta\lambda_h\delta_m-4\lambda_h\delta_m-4\beta\delta_m)\end{pmatrix}\quad\text{(C.11)}$$

$$N_{22}=(\beta-1)\theta^2(Hkq_{EV1})^2(-c_l(\lambda_h-1)+c_h(\lambda_l-1)+(c_{ntr}-1)(\lambda_h-\lambda_l))^2\quad\text{(C.12)}$$

$$D_{20}=\theta\left(-\left(Hkq_{EV1}\right)\right)-\theta\left(Hkq_{EV1}\right)\left(\lambda_l-2\right)\lambda_l-4\quad\text{(C.13)}$$

$$D_{21}=\theta\left(Hkq_{EV1}\right)\left(\beta+\left(\beta-1\right)\left(\lambda_h-2\right)\lambda_h+\left(\lambda_l-2\right)\lambda_l\right)\quad\text{(C.14)}$$

Relationship between β and Π_2

$$\Pi_2\left(\beta\right)=\frac{N_{30}+N_{31}\beta}{4k\left(D_{30}+D_{31}\beta\right)}\quad\text{(C.15)}$$

$$\begin{cases}-4\le & N_{30}\le 20\\-24\le & N_{31}\le -1.64\times10^{-7}\\-5\le & D_{30}\le -4\\3.29\times10^{-7}\le & D_{31}\le 1\end{cases}\quad\text{(C.16)}$$

We find The first derivative $9.22\times10^{-8}/\left(4k\right)\le\Pi_2'\left(\beta\right)\le 6.25/\left(4k\right)$ and the linearity is $7.44\times10^{-10}\le\eta_{2\beta}\le 5.57\%$. Therefore, the relationship between β and Π_2 can be treated as positive

$$N_{30}=\begin{pmatrix} -\alpha^2\theta^2\left(Hkq_{EV1}\right)^2 c_{ntr}^2\lambda_h^2+2\alpha^2\theta^2\left(Hkq_{EV1}\right)^2 c_{ntr}\lambda_h^2+\alpha\theta^2\left(Hkq_{EV1}\right)^2 \\ \cdot c_{ntr}^2\lambda_h^2-2\alpha\theta^2(Hkq_{EV1})^2 c_{ntr}\lambda_h^2+\alpha\theta(Hkq_{EV1})c_h^2((Hkq_{EV1})(\theta-\alpha\theta) \\ -(\alpha-1)\theta(Hkq_{EV1})(\lambda_l-2)\lambda_l+4)-\left(\alpha-1\right)\theta\left(Hkq_{EV1}\right)c_l^2 \\ \cdot(\alpha\theta(Hkq_{EV1})(\lambda_h-2)\lambda_h+\alpha\theta(Hkq_{EV1})+4)+2\alpha\theta(Hkq_{EV1})c_h \\ \cdot((\alpha-1)\theta(Hkq_{EV1})c_l(\lambda_h-1)(\lambda_l-1)+\lambda_h(c_{ntr}((\alpha-1)\theta\left(Hkq_{EV1}\right) \\ -(\alpha-1)\theta(Hkq_{EV1})\lambda_l-2)+(\alpha-1)\theta(Hkq_{EV1})(\lambda_l-1)-2\delta_m) \\ +(\alpha-1)\theta(Hkq_{EV1})(c_{ntr}-1)(\lambda_l-1)\lambda_l-2c_{ntr}+2\delta_m+4)+2\left(\alpha-1\right) \\ \cdot\theta(Hkq_{EV1})\lambda_l(\alpha\theta(Hkq_{EV1})(c_{ntr}-1)^2\lambda_h+2(c_{ntr}(-c_{ntr}+\delta_m+1)+\delta_m)) \\ +2(\alpha-1)\theta(Hkq_{EV1})c_l(c_{ntr}(\alpha\theta(Hkq_{EV1})(\lambda_h-1)(\lambda_h-\lambda_l)+2(\lambda_l+1)) \\ +\alpha\theta(-(Hkq_{EV1}))(\lambda_h-1)(\lambda_h-\lambda_l)+2(\lambda_l-1)\delta_m-4)+4\alpha\theta\left(Hkq_{EV1}c_{ntr}\right) \\ \cdot\lambda_h^2\delta_m-4\alpha\theta(Hkq_{EV1})c_{ntr}\lambda_h\delta_m+4\alpha\theta(Hkq_{EV1})c_{ntr}^2\lambda_h-4\alpha\theta(Hkq_{EV1})c_{ntr}\lambda_h \\ -(\alpha-1)\theta(Hkq_{EV1})\lambda_l^2(\alpha\theta(Hkq_{EV1})(c_{ntr}-1)^2+4c_{ntr}\delta_m)-4\theta(Hkq_{EV1})c_{ntr} \\ +8c_{ntr}\delta_m-4c_{ntr}^2-\alpha^2\theta^2\left(Hkq_{EV1}\right)^2\lambda_h^2+\alpha\theta^2\left(Hkq_{EV1}\right)^2\lambda_h^2-4\alpha\theta\left(Hkq_{EV1}\right)\lambda_h\delta_m \\ +4\theta\left(Hkq_{EV1}\right)+4\theta\left(Hkq_{EV1}\right)\delta_m-4\delta_m^2 \end{pmatrix}$$

linear.

(C.17)

$$N_{31}=\begin{pmatrix} \alpha\theta\left(Hkq_{EV1}\right)(c_h^2((\alpha-1)\theta(Hkq_{EV1})+(\alpha-1)\theta(Hkq_{EV1})(\lambda_l-2)\lambda_l-4) \\ +(\alpha-1)\theta(Hkq_{EV1})c_l^2(\lambda_h-1)^2+2c_h(-(\alpha-1)\theta(Hkq_{EV1})c_l(\lambda_h-1) \\ \cdot(\lambda_l-1)+\lambda_h((\alpha-1)\theta(Hkq_{EV1})(c_{ntr}-1)\lambda_l+c_{ntr}((Hkq_{EV1})(\theta-\alpha\theta)+2) \\ +(\alpha-1)\theta(Hkq_{EV1})+2\delta_m)-\left(\alpha-1\right)\theta\left(Hkq_{EV1}\right)\left(c_{ntr}-1\right)\left(\lambda_l-1\right)\lambda_l \\ +2(c_{ntr}-\delta_m-2))-2\left(\alpha-1\right)\theta\left(Hkq_{EV1}\right)c_l\left(c_{ntr}-1\right)\left(\lambda_h-1\right)\left(\lambda_h-\lambda_l\right) \\ -2(\alpha-1)\theta(Hkq_{EV1})(c_{ntr}-1)^2\lambda_h\lambda_l+\lambda_h(\lambda_h((\alpha-1)\theta(Hkq_{EV1})(c_{ntr}-1)^2 \\ -4c_{ntr}\delta_m)+4(c_{ntr}(-c_{ntr}+\delta_m+1)+\delta_m))+\left(\alpha-1\right)\theta(Hkq_{EV1})(c_{ntr}-1)^2\lambda_l^2 \\ +4(c_{ntr}-\delta_m-1)) \end{pmatrix} \quad \text{(C.18)}$$

$$D_{30}=\theta\left(Hkq_{EV1}\right)\left(-\alpha\lambda_h^2+2\alpha\lambda_h+\left(\alpha-1\right)\left(\lambda_l-2\right)\lambda_l\right)+\theta\left(-\left(Hkq_{EV1}\right)\right)-4 \quad \text{(C.19)}$$

$$D_{31}=\alpha\theta(Hkq_{EV1})(\lambda_h-1)^2 \quad \text{(C.20)}$$

Relationship between λ_l and Π_2

$$\Pi_2\left(\lambda_l\right)=\frac{N_{40}+N_{41}\lambda_l+N_{42}\lambda_l^2}{4k\left(D_{40}+D_{41}\lambda_l+D_{42}\lambda_l^2\right)} \quad \text{(C.21)}$$

$$\begin{cases} -4 \le & N_{40} \le 20 \\ -16 \le & N_{41} \le -5.69\times10^{-8} \\ 6.11\times10^{-8} \le & N_{42} \le 4 \\ -5 \le & D_{40} \le -4 \\ 8.69\times10^{-8} \le & D_{41} \le 2 \\ -1 \le & D_{42} \le -9.08\times10^{-8} \end{cases} \quad \text{(C.22)}$$

We find The first derivative $8.68\times10^{-8}/(4k) \le \Pi_2'(\lambda_l) \le 2.33/(4k)$ and the linearity is $0\% \le \eta_{2\lambda_l} \le 24.72\%$ .Therefore, the relationship between λ_l and Π_2 can be treated as positive.

$$N_{40} = \begin{pmatrix} \alpha(\beta-1)\theta(Hkq_{EV1})c_h^2\left((\alpha-1)\theta(Hkq_{EV1})-4\right)+(\alpha-1)\theta(Hkq_{EV1})c_l^2 \\ \cdot(\alpha(\beta-1)\theta(Hkq_{EV1})(\lambda_h-2)\lambda_h+\alpha(\beta-1)\theta(Hkq_{EV1})-4) \\ +2\alpha(\beta-1)\theta(Hkq_{EV1})c_h((\alpha-1)\theta(Hkq_{EV1})c_l(\lambda_h-1) \\ +c_{ntr}\left(\lambda_h\left(-\alpha\theta(Hkq_{EV1})+\theta(Hkq_{EV1})+2\right)+2\right)+(\alpha-1)\theta(Hkq_{EV1})\lambda_h \\ +2(\lambda_h-1)\delta_m-4)+2(\alpha-1)\theta(Hkq_{EV1})c_l\left(c_{ntr}(2-\alpha(\beta-1)\theta\right. \\ \cdot(Hkq_{EV1})(\lambda_h-1)\lambda_h)+\alpha(\beta-1)\theta(Hkq_{EV1})(\lambda_h-1)\lambda_h-2\delta_m-4) \\ +\alpha(\beta-1)\theta(Hkq_{EV1})\lambda_h^2((\alpha-1)\theta(Hkq_{EV1})(c_{ntr}-1)^2-4c_{ntr}\delta_m) \\ -4\alpha(\beta-1)\theta(Hkq_{EV1})\lambda_h\left((c_{ntr}-1)c_{ntr}-(c_{ntr}+1)\delta_m\right) \\ +4(c_{ntr}-\delta_m)\left(-c_{ntr}+\theta(Hkq_{EV1})(\alpha\beta-1)+\delta_m\right)-4\theta(Hkq_{EV1})(\alpha\beta-1) \end{pmatrix} \quad \text{(C.23)}$$

$$N_{41} = \begin{pmatrix} 2(\alpha-1)\theta(Hkq_{EV1})(\alpha(\beta-1)\theta(-(Hkq_{EV1}))c_h^2+c_l(\alpha(\beta-1)\theta \\ \cdot(Hkq_{EV1})(c_{ntr}-1)\lambda_h+c_{ntr}(2-\alpha(\beta-1)\theta(Hkq_{EV1})) \\ +\alpha(\beta-1)\theta(Hkq_{EV1})+2\delta_m)-\alpha(\beta-1)\theta(Hkq_{EV1})c_h\left(c_l(\lambda_h-1)\right. \\ -(c_{ntr}-1)(\lambda_h+1))-\alpha(\beta-1)\theta(Hkq_{EV1})(c_{ntr}-1)^2\lambda_h \\ +2(c_{ntr}(-c_{ntr}+\delta_m+1)+\delta_m)) \end{pmatrix} \quad \text{(C.24)}$$

$$N_{42} = (\alpha-1)\theta(Hkq_{EV1})(\alpha(\beta-1)\theta(Hkq_{EV1})(c_h-c_{ntr}+1)^2-4c_{ntr}\delta_m) \quad \text{(C.25)}$$

$$D_{40} = \alpha(\beta-1)\theta(Hkq_{EV1})(\lambda_h-2)\lambda_h+\theta(Hkq_{EV1})(\alpha\beta-1)-4 \quad \text{(C.26)}$$

$$D_{41} = -2(\alpha-1)\theta(Hkq_{EV1}) \quad \text{(C.27)}$$

$$D_{42} = (\alpha-1)\theta(Hkq_{EV1}) \quad \text{(C.28)}$$

Relationship between λ_h and Π_2

$$\Pi_2(\lambda_h) = \frac{N_{50}+N_{51}\lambda_h+N_{52}\lambda_h^2}{4k\left(D_{50}+D_{51}\lambda_h+D_{52}\lambda_h^2\right)} \quad \text{(C.29)}$$

$$\begin{cases} -4 \le & N_{50} \le 20 \\ -16 \le & N_{51} \le -7.66\times10^{-8} \\ 6.92\times10^{-8} \le & N_{52} \le 4 \\ -5 \le & D_{50} \le -4 \\ 9.10\times10^{-8} \le & D_{51} \le 2 \\ -1 \le & D_{52} \le -8.87\times10^{-8} \end{cases} \quad \text{(C.30)}$$

We find The first derivative $9.04\times10^{-8}/(4k) \le \Pi_2'(\lambda_h) \le 2.33/(4k)$ and the linearity is $-24.25\% \le \eta_{2\lambda_h} \le 24.25\%$. Therefore, the relationship between λ_h and Π_2 can be treated as positive.

$$N_{50} = \left(\begin{array}{l} \alpha(\beta-1)\theta(Hkq_{EV1})c_h^2((\alpha-1)\theta(Hkq_{EV1})+(\alpha-1)\theta(Hkq_{EV1}) \\ \cdot(\lambda_l-2)\lambda_l-4)+2\alpha(\beta-1)\theta(Hkq_{EV1})c_h((\alpha-1)\theta(Hkq_{EV1}) \\ \cdot(\lambda_l-1)(c_l-(c_{ntr}-1)\lambda_l)+2(c_{ntr}-\delta_m-2))+(\alpha-1)\theta(Hkq_{EV1})c_l^2 \\ \cdot(\alpha(\beta-1)\theta(Hkq_{EV1})-4)+(\alpha-1)\theta(Hkq_{EV1})\lambda_l^2(\alpha(\beta-1)\theta(Hkq_{EV1}) \\ \cdot(c_{ntr}-1)^2-4c_{ntr}\delta_m)+2(\alpha-1)\theta(Hkq_{EV1})c_l(c_{ntr}(\lambda_l(2-\alpha(\beta-1) \\ \cdot\theta(Hkq_{EV1}))+2)+\alpha(\beta-1)\theta(Hkq_{EV1})\lambda_l+2(\lambda_l-1)\delta_m-4) \\ -4(\alpha-1)\theta(Hkq_{EV1})\lambda_l((c_{ntr}-1)c_{ntr}-(c_{ntr}+1)\delta_m)+4(c_{ntr}-\delta_m) \\ \cdot(-c_{ntr}+\theta(Hkq_{EV1})(\alpha\beta-1)+\delta_m)-4\theta(Hkq_{EV1})(\alpha\beta-1) \end{array} \right) \quad \text{(C.31)}$$

$$N_{51} = \left(\begin{array}{c} 2\alpha(\beta-1)\theta(Hkq_{EV1})(c_h((\alpha-1)\theta(Hkq_{EV1})(\lambda_l(-c_l+c_{ntr}-1) \\ +c_l-c_{ntr})+2c_{ntr}+(\alpha-1)\theta(Hkq_{EV1})+2\delta_m)+(\alpha-1)\theta \\ \cdot(-(Hkq_{EV1}))c_l^2+(\alpha-1)\theta(Hkq_{EV1})c_l(c_{ntr}-1)(\lambda_l+1)-(\alpha-1) \\ \cdot\theta(Hkq_{EV1})(c_{ntr}-1)^2\lambda_l+2(c_{ntr}(-c_{ntr}+\delta_m+1)+\delta_m)) \end{array} \right) \quad \text{(C.32)}$$

$$N_{52} = \alpha(\beta-1)\theta(Hkq_{EV1})((\alpha-1)\theta(Hkq_{EV1})(c_l-c_{ntr}+1)^2-4c_{ntr}\delta_m) \quad \text{(C.33)}$$

$$D_{50} = \theta(Hkq_{EV1})(\alpha\beta-1)+(\alpha-1)\theta(Hkq_{EV1})\lambda_l^2-2(\alpha-1)\theta(Hkq_{EV1})\lambda_l-4 \quad \text{(C.34)}$$

$$D_{51} = -2\alpha(\beta-1)\theta(Hkq_{EV1}) \quad \text{(C.35)}$$

$$D_{52} = \alpha(\beta-1)\theta(Hkq_{EV1}) \quad \text{(C.35)}$$

Relationship between c_{ntr} and Π_2

The equation below shows the relationship between c_{ntr} and Π_2. Expressions from N_{60} to D_{60} are presented from Eq. C.39 to Eq. C.42 in appendix as well.

$$\Pi_2(c_{ntr}) = \frac{N_{60}+N_{61}c_{ntr}+N_{62}c_{ntr}^2}{4kD_{60}} \quad \text{(C.37)}$$

By using gird searching method again, the ranges for each character are

$$\begin{cases} -4 \le & N_{60} \le 20 \\ -16 \le & N_{61} \le 8 \\ -4 \le & N_{62} \le -2.18\times 10^{-7} \\ -5 \le & D_{60} \le -4 \end{cases} \quad \text{(C38)}$$

We then find that the first derivative $-2/(4k) \le \Pi_2'(c_{ntr}) \le 4/(4k)$ and the linearity is $-6.25\times 10^{7} \le \eta_{2c_{ntr}} \le 1.15\times 10^{8}$. Back to Eq. C.37 again, the relationship can be quadratic.

$$N_{60} = \begin{pmatrix} \alpha(\beta-1)\theta(Hkq_{EV1})c_h^2((\alpha-1)\theta(Hkq_{EV1})+(\alpha-1)\theta(Hkq_{EV1}) \\ \cdot(\lambda_l-2)\lambda_l-4)+(\alpha-1)\theta(Hkq_{EV1})c_l^2(\alpha(\beta-1)\theta(Hkq_{EV1})(\lambda_h-2)\lambda_h \\ +\alpha(\beta-1)\theta(Hkq_{EV1})-4)+2\alpha(\beta-1)\theta(Hkq_{EV1})c_h(-(\alpha-1)\theta \\ \cdot(Hkq_{EV1})c_l(\lambda_h-1)(\lambda_l-1)-(\alpha-1)\theta(Hkq_{EV1})(\lambda_l-1)(\lambda_h-\lambda_l) \\ +2(\lambda_h-1)\delta_m-4)+2(\alpha-1)\theta(Hkq_{EV1})c_l(\alpha(\beta-1)\theta(Hkq_{EV1})(\lambda_h-1) \\ \cdot(\lambda_h-\lambda_l)+2(\lambda_l-1)\delta_m-4)+\alpha^2\beta\theta^2(Hkq_{EV1})^2\lambda_h^2-\alpha^2\theta^2(Hkq_{EV1})^2\lambda_h^2 \\ -\alpha\beta\theta^2(Hkq_{EV1})^2\lambda_h^2+\alpha\theta^2(Hkq_{EV1})^2\lambda_h^2+2(\alpha-1)\theta(Hkq_{EV1})\lambda_l \\ \cdot(2\delta_m-\alpha(\beta-1)\theta(Hkq_{EV1})\lambda_h)+4\alpha\beta\theta(Hkq_{EV1})\lambda_h\delta_m-4\alpha\theta(Hkq_{EV1}) \\ \cdot\lambda_h\delta_m+(\alpha-1)\alpha(\beta-1)\theta^2(Hkq_{EV1})^2\lambda_l^2-4\alpha\beta\theta(Hkq_{EV1})+4\theta(Hkq_{EV1}) \\ -4\alpha\beta\theta(Hkq_{EV1})\delta_m+4\theta(Hkq_{EV1})\delta_m-4\delta_m^2 \end{pmatrix}$$

(C.39)

$$N_{61} = \begin{pmatrix} 2(\theta(Hkq_{EV1})((\alpha-1)c_l(2(\lambda_l+1)-\alpha(\beta-1)\theta(Hkq_{EV1})(\lambda_h-1) \\ \cdot(\lambda_h-\lambda_l))+\alpha(\beta-1)c_h(\lambda_h(-\alpha\theta(Hkq_{EV1})+\theta(Hkq_{EV1})+(\alpha-1)\theta \\ \cdot(Hkq_{EV1})\lambda_l+2)+(\alpha-1)\theta(-(Hkq_{EV1}))(\lambda_l-1)\lambda_l+2)+2(\alpha-1) \\ \cdot\lambda_l(\alpha(\beta-1)\theta(Hkq_{EV1})\lambda_h+\delta_m+1)-\alpha(\beta-1)\lambda_h((\alpha-1)\theta \\ \cdot(Hkq_{EV1})\lambda_h+2(\lambda_h-1)\delta_m-2)-(\alpha-1)\lambda_l^2(\alpha(\beta-1)\theta(Hkq_{EV1}) \\ +2\delta_m))+2\theta(Hkq_{EV1})(\alpha\beta-1)+4\delta_m) \end{pmatrix} \quad \text{(C.40)}$$

$$N_{62} = \begin{pmatrix} \theta(Hkq_{EV1})((\alpha-1)\alpha(\beta-1)\theta(Hkq_{EV1})\lambda_h^2-2\alpha(\beta-1)\lambda_h \\ \cdot((\alpha-1)\theta(Hkq_{EV1})\lambda_l+2)+(\alpha-1)\lambda_l(\alpha(\beta-1)\theta(Hkq_{EV1})\lambda_l-4)) \end{pmatrix} \quad \text{(C.41)}$$

$$D_{60} = \theta(Hkq_{EV1})\left(\alpha(\beta-1)(\lambda_h-2)\lambda_h+(\alpha-1)\lambda_l^2-2(\alpha-1)\lambda_l\right)+\theta(Hkq_{EV1})(\alpha\beta-1)-4$$
(C.42)

Relationship between c_l and Π_2

$$\Pi_2(c_l) = \frac{N_{70}+N_{71}c_l+N_{72}c_l^2}{4kD_{70}} \quad \text{(C.43)}$$

$$\begin{cases} -4 \le N_{70} \le 20 \\ 9.18\times 10^{-8} \le N_{71} \le 12 \\ 8.24\times 10^{-8} \le N_{72} \le 4 \\ -5 \le D_{70} \le -4 \end{cases} \quad \text{(C.44)}$$

We find The first derivative $-4.24/(4k) \le \Pi_2'(c_l) \le -9.06\times10^{-8}/(4k)$ and the linearity is $-25\% \le \eta_{2c_l} \le -6.29\%$.Therefore, the relationship between c_l and Π_2 has negative relationship.

$$N_{70} = \begin{pmatrix} \alpha(\beta-1)\theta(Hkq_{EV1})c_h^2((\alpha-1)\theta(Hkq_{EV1})+(\alpha-1)\theta(Hkq_{EV1})(\lambda_l-2) \\ \cdot\lambda_l-4)+2\alpha(\beta-1)\theta(Hkq_{EV1})c_h(c_{ntr}(\lambda_h(-\alpha\theta(Hkq_{EV1})+\theta(Hkq_{EV1}) \\ +(\alpha-1)\theta(Hkq_{EV1})\lambda_l+2)+(\alpha-1)\theta(-(Hkq_{EV1}))(\lambda_l-1)\lambda_l+2) \\ -(\alpha-1)\theta(Hkq_{EV1})(\lambda_l-1)(\lambda_h-\lambda_l)+2(\lambda_h-1)\delta_m-4) \\ +2c_{ntr}(2\theta(Hkq_{EV1})(\alpha\beta+\alpha(\beta-1)\lambda_h-1)+(\alpha-1)\theta(Hkq_{EV1}) \\ \cdot(2\lambda_l-\alpha(\beta-1)\theta(Hkq_{EV1})(\lambda_h-\lambda_l)^2)+2\delta_m(\theta(Hkq_{EV1}) \\ \cdot\left(-\alpha(\beta-1)(\lambda_h-1)\lambda_h-(\alpha-1)\lambda_l^2+(\alpha-1)\lambda_l\right)+2))+c_{ntr}^2 \\ \cdot(\theta(Hkq_{EV1})((\alpha-1)\alpha(\beta-1)\theta(Hkq_{EV1})\lambda_h^2-2\alpha(\beta-1)\lambda_h \\ \cdot((\alpha-1)\theta(Hkq_{EV1})\lambda_l+2)+(\alpha-1)\lambda_l(\alpha(\beta-1)\theta(Hkq_{EV1})\lambda_l-4))-4) \\ +\alpha^2\beta\theta^2(Hkq_{EV1})^2\lambda_h^2-\alpha^2\theta^2(Hkq_{EV1})^2\lambda_h^2-\alpha\beta\theta^2(Hkq_{EV1})^2\lambda_h^2 \\ +\alpha\theta^2(Hkq_{EV1})^2\lambda_h^2+2(\alpha-1)\theta(Hkq_{EV1})\lambda_l(2\delta_m-\alpha(\beta-1)\theta \\ \cdot(Hkq_{EV1})\lambda_h)+4\alpha\beta\theta(Hkq_{EV1})\lambda_h\delta_m-4\alpha\theta(Hkq_{EV1})\lambda_h\delta_m \\ +(\alpha-1)\alpha(\beta-1)\theta^2(Hkq_{EV1})^2\lambda_l^2-4\alpha\beta\theta(Hkq_{EV1})+4\theta(Hkq_{EV1}) \\ -4\alpha\beta\theta(Hkq_{EV1})\delta_m+4\theta(Hkq_{EV1})\delta_m-4\delta_m^2 \end{pmatrix} \quad (C.45)$$

$$N_{71} = \begin{pmatrix} 2(\alpha-1)\theta(Hkq_{EV1})(-\alpha(\beta-1)\theta(Hkq_{EV1})c_h(\lambda_h-1)(\lambda_l-1) \\ -c_{ntr}\left(\alpha(\beta-1)\theta(Hkq_{EV1})(\lambda_h-1)(\lambda_h-\lambda_l)-2(\lambda_l+1)\right) \\ +\alpha(\beta-1)\theta(Hkq_{EV1})(\lambda_h-1)(\lambda_h-\lambda_l)+2(\lambda_l-1)\delta_m-4) \end{pmatrix} \quad (C.46)$$

$$N_{72} = (\alpha-1)\theta(Hkq_{EV1})\left(\alpha(\beta-1)\theta(Hkq_{EV1})(\lambda_h-2)\lambda_h+\alpha(\beta-1)\theta(Hkq_{EV1})-4\right) \quad (C.47)$$

$$D_{70} = \theta(Hkq_{EV1})\left(\alpha(\beta-1)(\lambda_h-2)\lambda_h+(\alpha-1)\lambda_l^2-2(\alpha-1)\lambda_l\right)+\theta(Hkq_{EV1})(\alpha\beta-1)-4$$
(C.48)

Relationship between c_h and Π_2

$$\Pi_2(c_h) = \frac{N_{80}+N_{81}c_h+N_{82}c_h^2}{4kD_{80}} \quad (C.49)$$

$$\begin{cases} -4 \le N_{80} \le 20 \\ 1.56\times10^{-7} \le N_{81} \le 12 \\ 6.66\times10^{-8} \le N_{82} \le 4 \\ -5 \le D_{80} \le -4 \end{cases} \quad (C.50)$$

We find The first derivative $-4.24/(4k) \le \Pi_2'(c_l) \le -9.10\times10^{-8}/(4k)$ and the linearity is $-24.91\% \le \eta_{2c_h} \le -6.25\%$.Therefore, the relationship between c_h and Π_2 has negative relationship.

$$N_{80}=\begin{pmatrix}(\alpha-1)\theta(Hkq_{EV1})c_l^2(\alpha(\beta-1)\theta(Hkq_{EV1})(\lambda_h-2)\lambda_h+\alpha(\beta-1)\theta\\ \cdot(Hkq_{EV1})-4)+2(\alpha-1)\theta(Hkq_{EV1})c_l(c_{ntr}(2(\lambda_l+1)-\alpha(\beta-1)\\ \cdot\theta(Hkq_{EV1})(\lambda_h-1)(\lambda_h-\lambda_l))+\alpha(\beta-1)\theta(Hkq_{EV1})(\lambda_h-1)(\lambda_h-\lambda_l)\\ +2(\lambda_l-1)\delta_m-4)+2c_{ntr}(2\theta(Hkq_{EV1})(\alpha\beta+\alpha(\beta-1)\lambda_h-1)\\ +(\alpha-1)\theta(Hkq_{EV1})\left(2\lambda_l-\alpha(\beta-1)\theta(Hkq_{EV1})(\lambda_h-\lambda_l)^2\right)\\ +2\delta_m\left(\theta(Hkq_{EV1})\left(-\alpha(\beta-1)(\lambda_h-1)\lambda_h-(\alpha-1)\lambda_l^2+(\alpha-1)\lambda_l\right)+2\right))\\ +c_{ntr}^2(\theta(Hkq_{EV1})((\alpha-1)\alpha(\beta-1)\theta(Hkq_{EV1})\lambda_h^2-2\alpha(\beta-1)\lambda_h\\ \cdot((\alpha-1)\theta(Hkq_{EV1})\lambda_l+2)+(\alpha-1)\lambda_l(\alpha(\beta-1)\theta(Hkq_{EV1})\lambda_l-4))-4)\\ +\alpha^2\beta\theta^2(Hkq_{EV1})^2\lambda_h^2-\alpha^2\theta^2(Hkq_{EV1})^2\lambda_h^2-\alpha\beta\theta^2(Hkq_{EV1})^2\lambda_h^2\\ +\alpha\theta^2(Hkq_{EV1})^2\lambda_h^2+2(\alpha-1)\theta(Hkq_{EV1})\lambda_l\left(2\delta_m-\alpha(\beta-1)\theta(Hkq_{EV1})\lambda_h\right)\\ +4\alpha\beta\theta(Hkq_{EV1})\lambda_h\delta_m-4\alpha\theta(Hkq_{EV1})\lambda_h\delta_m+(\alpha-1)\alpha(\beta-1)\theta^2\\ \cdot(Hkq_{EV1})^2\lambda_l^2-4\alpha\beta\theta(Hkq_{EV1})+4\theta(Hkq_{EV1})-4\alpha\beta\theta(Hkq_{EV1})\delta_m\\ +4\theta(Hkq_{EV1})\delta_m-4\delta_m^2\end{pmatrix}$$

(C.51)

$$N_{81}=\begin{pmatrix}-2\alpha(\beta-1)\theta(Hkq_{EV1})((\alpha-1)\theta(Hkq_{EV1})c_l(\lambda_h-1)(\lambda_l-1)\\ +\lambda_h(c_{ntr}((\alpha-1)\theta(Hkq_{EV1})-(\alpha-1)\theta(Hkq_{EV1})\lambda_l-2)\\ +(\alpha-1)\theta(Hkq_{EV1})(\lambda_l-1)-2\delta_m)+(\alpha-1)\theta(Hkq_{EV1})(c_{ntr}-1)\\ \cdot(\lambda_l-1)\lambda_l-2c_{ntr}+2\delta_m+4)\end{pmatrix} \quad \text{(C.52)}$$

$$N_{82}=\alpha(\beta-1)\theta(Hkq_{EV1})((\alpha-1)\theta(Hkq_{EV1})+(\alpha-1)\theta(Hkq_{EV1})(\lambda_l-2)\lambda_l-4) \quad \text{(C.53)}$$

$$D_{80}=\theta(Hkq_{EV1})\left(\alpha(\beta-1)(\lambda_h-2)\lambda_h+(\alpha-1)\lambda_l^2-2(\alpha-1)\lambda_l\right)+\theta(Hkq_{EV1})(\alpha\beta-1)-4$$

(C.54)

Appendix D. Proof for relationships between parameters and total optimal profits in Period 3

Relationship between θ and Π_3

$$\Pi_3(\theta)=\frac{N_{90}+N_{91}\theta+N_{92}\theta^2}{4k(-4+D_{91}\theta)} \quad \text{(D.1)}$$

$$\begin{cases}0\le N_{90}\le 0\\ 3.80\times10^{-8}\le N_{91}\le 24\\ -1.83\times10^{-16}\le N_{92}\le 7.01\times10^{-17}\\ -1\le D_{91}\le -1.35\times10^{-7}\end{cases} \quad \text{(D.2)}$$

We find The first derivative $-6/(4k) \le \Pi_3'(\theta) \le -8.06\times 10^{-8}$ and the linearity is $-5.57\% \le \eta_{3\theta} \le -1.86\times 10^{-13}$. Therefore, the relationship between θ and Π_3 can be treated as negative linear.

$$N_{90} = -4(c_{ntr} - \delta_m)^2 \qquad \text{(D.3)}$$

$$N_{91} = \left(\begin{array}{l} 4(Hkq_{EV2})((\alpha-\alpha\beta)c_h^2 + \alpha(\beta-1)c_h(c_{ntr}(\lambda_h+1) + (\lambda_h-1)\delta_m - 2) \\ -\alpha\beta c_{ntr}\lambda_h^2\delta_m + \alpha\beta c_{ntr}\lambda_h\delta_m + \alpha c_{ntr}\lambda_h^2\delta_m - \alpha c_{ntr}\lambda_h\delta_m - \alpha\beta c_{ntr}^2\lambda_h \\ +\alpha\beta c_{ntr}\lambda_h + \alpha c_{ntr}^2\lambda_h - \alpha c_{ntr}\lambda_h - (\alpha-1)c_l^2 - \alpha c_{ntr}\lambda_l^2\delta_m + \alpha c_{ntr}\lambda_l\delta_m \\ +(\alpha-1)c_l\left(c_{ntr}(\lambda_l+1) + (\lambda_l-1)\delta_m - 2\right) + c_{ntr}\lambda_l^2\delta_m - c_{ntr}\lambda_l\delta_m \\ -\alpha c_{ntr}^2\lambda_l + \alpha c_{ntr}\lambda_l + c_{ntr}^2\lambda_l - c_{ntr}\lambda_l + \alpha\beta c_{ntr}\delta_m\lambda_u^2 \\ +\alpha\beta\lambda_u\left(-c_{ntr}(c_u+\delta_m+1) - (c_u+1)\delta_m + c_{ntr}^2\right) + \alpha\beta c_u\delta_m - \alpha\beta c_{ntr}c_u \\ -c_{ntr} + \alpha\beta c_u^2 + 2\alpha\beta c_u + \alpha\beta\lambda_h\delta_m - \alpha\lambda_h\delta_m + \alpha\lambda_l\delta_m - \lambda_l\delta_m + \delta_m + 1) \end{array}\right) \qquad \text{(D.4)}$$

$$N_{92} = \left(\begin{array}{l} (Hkq_{EV2})^2\alpha(-\alpha c_u^2\beta^2 - \alpha c_{ntr}^2\lambda_h^2\beta^2 - \alpha c_u^2\lambda_h^2\beta^2 - \alpha\lambda_h^2\beta^2 + 2\alpha c_{ntr}\lambda_h^2\beta^2 \\ -2\alpha c_u\lambda_h^2\beta^2 + 2\alpha c_{ntr}c_u\lambda_h^2\beta^2 + 2\alpha c_u^2\lambda_h\beta^2 + 2\alpha c_u\lambda_h\beta^2 - 2\alpha c_{ntr}c_u\lambda_h\beta^2 \\ +c_u^2\beta + 2\alpha c_{ntr}^2\lambda_h^2\beta - c_{ntr}^2\lambda_h^2\beta + \alpha c_u^2\lambda_h^2\beta + 2\alpha\lambda_h^2\beta - 4\alpha c_{ntr}\lambda_h^2\beta \\ +2c_{ntr}\lambda_h^2\beta + 2\alpha c_u\lambda_h^2\beta - 2\alpha c_{ntr}c_u\lambda_h^2\beta - \lambda_h^2\beta - \alpha c_u^2\lambda_l^2\beta + c_u^2\lambda_l^2\beta \\ -2\alpha c_u\lambda_l^2\beta + 2\alpha c_{ntr}c_u\lambda_l^2\beta - 2c_{ntr}c_u\lambda_l^2\beta + 2c_u\lambda_l^2\beta - (\alpha\beta-1) \\ \cdot(c_{ntr}-1)^2\lambda_u^2\beta - 2\alpha c_u^2\lambda_h\beta - 2\alpha c_u\lambda_h\beta + 2\alpha c_{ntr}c_u\lambda_h\beta + 2\alpha c_u^2\lambda_l\beta \\ -2c_u^2\lambda_l\beta + 2\alpha c_u\lambda_l\beta - 2\alpha c_{ntr}c_u\lambda_l\beta + 2c_{ntr}c_u\lambda_l\beta - 2c_u\lambda_l\beta - 2\alpha c_{ntr}^2\lambda_h\lambda_l\beta \\ +2c_{ntr}^2\lambda_h\lambda_l\beta - 2\alpha\lambda_h\lambda_l\beta + 4\alpha c_{ntr}\lambda_h\lambda_l\beta - 4c_{ntr}\lambda_h\lambda_l\beta + 2\lambda_h\lambda_l\beta \\ +2(c_{ntr}-1)((c_{ntr}-1)(\alpha(\beta-1)\lambda_h + (\alpha-1)\lambda_l) + c_u(\alpha\beta + \lambda_l \\ -\alpha((\beta-1)\lambda_h + \lambda_l) - 1))\lambda_u\beta - \alpha c_{ntr}^2\lambda_h^2 + c_{ntr}^2\lambda_h^2 - \alpha\lambda_h^2 + 2\alpha c_{ntr}\lambda_h^2 \\ -2c_{ntr}\lambda_h^2 + \lambda_h^2 - \alpha c_{ntr}^2\lambda_l^2 + c_{ntr}^2\lambda_l^2 - \alpha\lambda_l^2 + 2\alpha c_{ntr}\lambda_l^2 - 2c_{ntr}\lambda_l^2 + \lambda_l^2 \\ +2\alpha c_{ntr}^2\lambda_h\lambda_l - 2c_{ntr}^2\lambda_h\lambda_l + 2\alpha\lambda_h\lambda_l - 4\alpha c_{ntr}\lambda_h\lambda_l + 4c_{ntr}\lambda_h\lambda_l - 2\lambda_h\lambda_l \\ -(\beta-1)c_h^2\left(\alpha\beta\lambda_u^2 - 2\alpha\beta\lambda_u + \alpha(\beta-1) - (\alpha-1)(\lambda_l-2)\lambda_l + 1\right) \\ +(\alpha-1)c_l^2\left(-\beta\lambda_u^2 + 2\beta\lambda_u + (\beta-1)(\lambda_h-2)\lambda_h - 1\right) + 2(\alpha-1)c_l \\ \cdot(\beta c_u(\lambda_l-1)(\lambda_u-1) - (c_{ntr}-1)(-\beta\lambda_u^2 + \beta(\lambda_l+1)\lambda_u + (\beta-1)\lambda_h \\ \cdot(\lambda_h - \lambda_l - 1) - \lambda_l)) + 2(\beta-1)c_h(-(\alpha-1)c_l(\lambda_h-1)(\lambda_l-1) + \alpha\beta c_u \\ \cdot(\lambda_h-1)(\lambda_u-1) - (c_{ntr}-1)(-\alpha\beta\lambda_u^2 + \alpha\beta\lambda_u + (\alpha-1)(\lambda_l-1)\lambda_l \\ +\lambda_h(-\beta\alpha - \lambda_l\alpha + \beta\lambda_u\alpha + \alpha + \lambda_l - 1)))) \end{array}\right) \qquad \text{(D.5)}$$

$$D_{91} = \left(\begin{array}{c} (Hkq_{EV2})(\alpha(\beta-1)\lambda_h^2 - 2\alpha(\beta-1)\lambda_h + (\alpha-1)(\lambda_l-2)\lambda_l \\ -\alpha\beta\lambda_u^2 + 2\alpha\beta\lambda_u) - (Hkq_{EV2}) \end{array}\right) \qquad \text{(D.6)}$$

Relationship between α and Π_3

$$\Pi_3(\alpha)=\frac{N_{100}+N_{101}\alpha+N_{102}\alpha^2}{4k\left(D_{100}+D_{101}\alpha\right)} \qquad \text{(D.7)}$$

$$\begin{cases} -4 \le N_{100} \le 20 \\ -19.2 \le N_{101} \le 21 \\ -3.77\times10^{-16} \le N_{102} \le 1.42\times10^{-16} \\ -5 \le D_{100} \le -4 \\ -1 \le D_{101} \le 1 \end{cases} \qquad \text{(D.8)}$$

We find The first derivative $-5.25/\left(4k\right) \le \Pi_3'\left(\alpha\right) \le 4.76/\left(4k\right)$ and the linearity is $-5.41\% \le \eta_{3\alpha} \le 5.46\%$. Therefore, the relationship between α and Π_3 can be treated as linear.

$$N_{100}=\begin{pmatrix} 4(\theta(Hkq_{EV2})c_l^2-\theta(Hkq_{EV2})c_l(c_{ntr}(\lambda_l+1)+(\lambda_l-1)\delta_m-2) \\ +c_{ntr}\left(\delta_m\left(\theta\left(Hkq_{EV2}\right)\left(\lambda_l-1\right)\lambda_l+2\right)-\theta\left(Hkq_{EV2}\right)\left(\lambda_l+1\right)\right) \\ +c_{ntr}^2(\theta(Hkq_{EV2})\lambda_l-1)+\theta(Hkq_{EV2})-\delta_m(\theta(Hkq_{EV2})(\lambda_l-1) \\ +\delta_m)) \end{pmatrix} \qquad \text{(D.9)}$$

$$N_{101} = \left(\begin{array}{l} (Hkq_{EV2})\theta(-(\beta-1)((Hkq_{EV2})\theta+(Hkq_{EV2})(\lambda_l-2)\lambda_l\theta+4)c_h^2 \\ +2(\beta-1)(2\left(c_{ntr}-\delta_m-2\right)+(Hkq_{EV2})\theta c_l\left(\lambda_h-1\right)\left(\lambda_l-1\right) \\ +(Hkq_{EV2})\theta\left(c_{ntr}-1\right)\left(\lambda_l-1\right)\lambda_l+\lambda_h(2\delta_m+(Hkq_{EV2})\theta\left(\lambda_l-1\right) \\ +c_{ntr}\left((Hkq_{EV2})\theta-(Hkq_{EV2})\lambda_l\theta+2\right)))c_h+4\beta c_u^2+(Hkq_{EV2})\beta\theta c_u^2 \\ +(Hkq_{EV2})\theta c_{ntr}^2\lambda_h^2-(Hkq_{EV2})\beta\theta c_{ntr}^2\lambda_h^2+(Hkq_{EV2})\theta\lambda_h^2 \\ -\left(Hkq_{EV2}\right)\beta\theta\lambda_h^2-2\left(Hkq_{EV2}\right)\theta c_{ntr}\lambda_h^2+2\left(Hkq_{EV2}\right)\beta\theta c_{ntr}\lambda_h^2 \\ -4\beta c_{ntr}\delta_m\lambda_h^2+4c_{ntr}\delta_m\lambda_h^2+\left(Hkq_{EV2}\right)\theta c_{ntr}^2\lambda_l^2+\left(Hkq_{EV2}\right)\beta\theta c_u^2\lambda_l^2 \\ +\left(Hkq_{EV2}\right)\theta\lambda_l^2-2\left(Hkq_{EV2}\right)\theta c_{ntr}\lambda_l^2+2\left(Hkq_{EV2}\right)\beta\theta c_u\lambda_l^2 \\ -2(Hkq_{EV2})\beta\theta c_{ntr}c_u\lambda_l^2-4c_{ntr}\delta_m\lambda_l^2+\beta((Hkq_{EV2})\theta(c_{ntr}-1)^2 \\ +4c_{ntr}\delta_m)\lambda_u^2+8\beta c_u-4\beta c_{ntr}c_u+4\beta c_u\delta_m-4\beta c_{ntr}^2\lambda_h+4c_{ntr}^2\lambda_h \\ +4\beta c_{ntr}\lambda_h-4c_{ntr}\lambda_h+4\beta\delta_m\lambda_h+4\beta c_{ntr}\delta_m\lambda_h-4c_{ntr}\delta_m\lambda_h \\ -4\delta_m\lambda_h-4c_{ntr}^2\lambda_l-2\left(Hkq_{EV2}\right)\beta\theta c_u^2\lambda_l+4c_{ntr}\lambda_l-2\left(Hkq_{EV2}\right) \\ \cdot\beta\theta c_u\lambda_l+2(Hkq_{EV2})\beta\theta c_{ntr}c_u\lambda_l+4c_{ntr}\delta_m\lambda_l+4\delta_m\lambda_l-2(Hkq_{EV2}) \\ \cdot\theta c_{ntr}^2\lambda_h\lambda_l+2(Hkq_{EV2})\beta\theta c_{ntr}^2\lambda_h\lambda_l-2(Hkq_{EV2})\theta\lambda_h\lambda_l+2(Hkq_{EV2}) \\ \cdot\beta\theta\lambda_h\lambda_l+4(Hkq_{EV2})\theta c_{ntr}\lambda_h\lambda_l-4(Hkq_{EV2})\beta\theta c_{ntr}\lambda_h\lambda_l \\ -2\beta(((Hkq_{EV2})\theta\lambda_l-2)c_{ntr}^2+(((Hkq_{EV2})\theta+2)c_u+2(\delta_m+1) \\ -(Hkq_{EV2})\theta\left(c_u+2\right)\lambda_l)c_{ntr}+2\delta_m+c_u\left(2\delta_m+(Hkq_{EV2})\theta\left(\lambda_l-1\right)\right) \\ +(Hkq_{EV2})\theta\lambda_l)\lambda_u+c_l^2((Hkq_{EV2})\theta+(Hkq_{EV2})(\beta\lambda_u^2-2\beta\lambda_u-(\beta-1) \\ \cdot\left(\lambda_h-2\right)\lambda_h)\theta-4)+2c_l((Hkq_{EV2})\beta\theta\lambda_u^2-(Hkq_{EV2})\beta\theta\left(\lambda_l+1\right)\lambda_u \\ -2\delta_m+2\delta_m\lambda_l-\left(Hkq_{EV2}\right)\theta\lambda_h\lambda_l+\left(Hkq_{EV2}\right)\beta\theta\lambda_h\lambda_l+\left(Hkq_{EV2}\right)\theta \\ \cdot(\lambda_l-(\beta-1)(\lambda_h-1)\lambda_h)-(Hkq_{EV2})\beta\theta c_u(\lambda_l-1)(\lambda_u-1) \\ +c_{ntr}(2(\lambda_l+1)+(Hkq_{EV2})\theta(-\beta\lambda_u^2+\beta(\lambda_l+1)\lambda_u+(\beta-1)\lambda_h \\ \cdot(\lambda_h-\lambda_l-1)-\lambda_l))-4)) \end{array} \right)$$

(D.10)

$$N_{102}=\left(\begin{array}{l}\theta^2(Hkq_{EV2})^2(-2\beta c_{ntr}^2\lambda_h\lambda_l+4\beta c_{ntr}\lambda_h\lambda_l+2c_{ntr}^2\lambda_h\lambda_l-4c_{ntr}\lambda_h\lambda_l\\+2\beta(c_{ntr}-1)\lambda_u\left((c_{ntr}-1)\left((\beta-1)\lambda_h+\lambda_l\right)+c_u\left(\beta-(\beta-1)\lambda_h-\lambda_l\right)\right)\\+2c_l(\beta c_u(\lambda_l-1)(\lambda_u-1)-(c_{ntr}-1)((\beta-1)\lambda_h(\lambda_h-\lambda_l-1)-\lambda_l\\+\beta(\lambda_l+1)\lambda_u-\beta\lambda_u^2))+2(\beta-1)c_h(-c_l(\lambda_h-1)(\lambda_l-1)-(c_{ntr}-1)\\\cdot(\lambda_h(-\beta-\lambda_l+\beta\lambda_u+1)+(\lambda_l-1)\lambda_l-\beta(\lambda_u-1)\lambda_u)+\beta c_u(\lambda_h-1)\\\cdot(\lambda_u-1))-(\beta-1)c_h^2\left(\beta-(\lambda_l-2)\lambda_l+\beta(\lambda_u-2)\lambda_u-1\right)\\+c_l^2\left((\beta-1)(\lambda_h-2)\lambda_h-\beta\lambda_u^2+2\beta\lambda_u-1\right)+\beta^2\left(-c_{ntr}^2\right)\lambda_h^2\\+2\beta^2c_{ntr}\lambda_h^2+2\beta c_{ntr}^2\lambda_h^2-4\beta c_{ntr}\lambda_h^2-c_{ntr}^2\lambda_h^2+2c_{ntr}\lambda_h^2+2\beta^2c_{ntr}c_u\lambda_h^2\\-2\beta^2c_{ntr}c_u\lambda_h-2\beta c_{ntr}c_u\lambda_h^2+2\beta c_{ntr}c_u\lambda_h-\beta^2c_u^2\lambda_h^2-2\beta^2c_u\lambda_h^2\\+2\beta^2c_u^2\lambda_h+2\beta^2c_u\lambda_h+\beta c_u^2\lambda_h^2+2\beta c_u\lambda_h^2-2\beta c_u^2\lambda_h-2\beta c_u\lambda_h-c_{ntr}^2\lambda_l^2\\+2c_{ntr}\lambda_l^2+2\beta c_{ntr}c_u\lambda_l^2-2\beta c_{ntr}c_u\lambda_l-\beta c_u^2\lambda_l^2-2\beta c_u\lambda_l^2+2\beta c_u^2\lambda_l\\+2\beta c_u\lambda_l-\beta^2(c_{ntr}-1)^2\lambda_u^2-\beta^2c_u^2-\beta^2\lambda_h^2+2\beta\lambda_h^2-\lambda_h^2-2\beta\lambda_h\lambda_l\\+2\lambda_h\lambda_l-\lambda_l^2)\end{array}\right) \quad \text{(D.11)}$$

$$D_{100}=\theta\left(-(Hkq_{EV2})\right)-\theta\left(Hkq_{EV2}\right)(\lambda_l-2)\lambda_l-4 \qquad \text{(D.12)}$$

$$D_{101}=\theta\left(Hkq_{EV2}\right)\left((\beta-1)(\lambda_h-2)\lambda_h+(\lambda_l-2)\lambda_l-\beta\lambda_u^2+2\beta\lambda_u\right) \qquad \text{(D.13)}$$

Relationship between β and Π_3

$$\Pi_3(\beta)=\frac{N_{110}+N_{111}\beta+N_{112}\beta^2}{4k\left(D_{110}+D_{111}\beta\right)} \qquad \text{(D.14)}$$

$$\begin{cases}-4\le N_{110}\le 20\\-19.2\le N_{111}\le 21\\0\le N_{112}\le -1.10\times10^{-31}\\-5\le D_{110}\le -4\\-1\le D_{111}\le 1\end{cases} \qquad \text{(D.15)}$$

We find The first derivative $-5.25/(4k) \le \Pi_3'(\beta) \le 4.76/(4k)$ and the linearity is $-5.57\% \le \eta_{3\beta} \le 5.31\%$. Therefore, the relationship between β and Π_3 can be treated as linear.

$$N_{110} = \begin{pmatrix} -\alpha^2\theta^2 (Hkq_{EV2})^2 c_{ntr}^2 \lambda_h^2 + 2\alpha^2\theta^2 (Hkq_{EV2})^2 c_{ntr}\lambda_h^2 + \alpha\theta^2 (Hkq_{EV2})^2 \\ \cdot c_{ntr}^2 \lambda_h^2 - 2\alpha\theta^2 (Hkq_{EV2})^2 c_{ntr}\lambda_h^2 + \alpha\theta(Hkq_{EV2})c_h^2((Hkq_{EV2})(\theta-\alpha\theta) \\ -(\alpha-1)\theta(Hkq_{EV2})(\lambda_l-2)\lambda_l+4) - (\alpha-1)\theta(Hkq_{EV2})c_l^2 \\ \cdot(\alpha\theta(Hkq_{EV2})(\lambda_h-2)\lambda_h + \alpha\theta(Hkq_{EV2})+4) + 2\alpha\theta(Hkq_{EV2})c_h \\ \cdot((\alpha-1)\theta(Hkq_{EV2})c_l(\lambda_h-1)(\lambda_l-1) + \lambda_h(c_{ntr}((\alpha-1)\theta(Hkq_{EV2}) \\ -(\alpha-1)\theta(Hkq_{EV2})\lambda_l-2) + (\alpha-1)\theta(Hkq_{EV2})(\lambda_l-1) - 2\delta_m) \\ +(\alpha-1)\theta(Hkq_{EV2})(c_{ntr}-1)(\lambda_l-1)\lambda_l - 2c_{ntr} + 2\delta_m + 4) \\ +2(\alpha-1)\theta(Hkq_{EV2})\lambda_l(\alpha\theta(Hkq_{EV2})(c_{ntr}-1)^2\lambda_h \\ +2(c_{ntr}(-c_{ntr}+\delta_m+1)+\delta_m)) + 2(\alpha-1)\theta(Hkq_{EV2})c_l \\ \cdot(c_{ntr}(\alpha\theta(Hkq_{EV2})(\lambda_h-1)(\lambda_h-\lambda_l)+2(\lambda_l+1)) \\ +\alpha\theta(-(Hkq_{EV2}))(\lambda_h-1)(\lambda_h-\lambda_l)+2(\lambda_l-1)\delta_m-4) \\ +4\alpha\theta(Hkq_{EV2})c_{ntr}\lambda_h^2\delta_m - 4\alpha\theta(Hkq_{EV2})c_{ntr}\lambda_h\delta_m \\ +4\alpha\theta(Hkq_{EV2})c_{ntr}^2\lambda_h - 4\alpha\theta(Hkq_{EV2})c_{ntr}\lambda_h - (\alpha-1)\theta(Hkq_{EV2}) \\ \cdot\lambda_l^2(\alpha\theta(Hkq_{EV2})(c_{ntr}-1)^2 + 4c_{ntr}\delta_m) - 4\theta(Hkq_{EV2})c_{ntr} \\ +8c_{ntr}\delta_m - 4c_{ntr}^2 - \alpha^2\theta^2(Hkq_{EV2})^2\lambda_h^2 + \alpha\theta^2(Hkq_{EV2})^2\lambda_h^2 \\ -4\alpha\theta(Hkq_{EV2})\lambda_h\delta_m + 4\theta(Hkq_{EV2}) + 4\theta(Hkq_{EV2})\delta_m - 4\delta_m^2 \end{pmatrix} \tag{D.16}$$

$$N_{111}=\left(\begin{array}{l}\alpha\theta(Hkq_{EV2})(-2c_h((\alpha-1)\theta(Hkq_{EV2})c_l(\lambda_h-1)(\lambda_l-1)\\ -c_{ntr}(\lambda_h(-2\alpha\theta(Hkq_{EV2})+\theta(Hkq_{EV2})+\theta(Hkq_{EV2})((\alpha-1)\lambda_l\\ +\alpha\lambda_u)+2)+\theta(Hkq_{EV2})(-(\alpha-1)(\lambda_l-1)\lambda_l-\alpha\lambda_u^2+\alpha\lambda_u)+2)\\ -\alpha\theta\left(Hkq_{EV2}\right)c_u\lambda_h+\alpha\theta\left(Hkq_{EV2}\right)\lambda_u\left(c_u\left(\lambda_h-1\right)+\lambda_h+1\right)\\ +\alpha\theta\left(Hkq_{EV2}\right)c_u+\left(1-2\alpha\right)\theta\left(Hkq_{EV2}\right)\lambda_h+\alpha\theta\left(Hkq_{EV2}\right)\lambda_h\lambda_l\\ -\theta\left(Hkq_{EV2}\right)\lambda_h\lambda_l-2\lambda_h\delta_m-\alpha\theta\left(Hkq_{EV2}\right)\lambda_l^2+\alpha\theta\left(Hkq_{EV2}\right)\lambda_l\\ +\theta(Hkq_{EV2})\lambda_l^2-\theta(Hkq_{EV2})\lambda_l-\alpha\theta(Hkq_{EV2})\lambda_u^2+2\delta_m+4)\\ +2c_u(\alpha\theta(-(Hkq_{EV2}))(c_{ntr}-1)\lambda_h^2+\alpha\theta(Hkq_{EV2})(c_{ntr}-1)\lambda_h\\ \cdot(\lambda_u+1)-\lambda_u((\alpha-1)\theta(Hkq_{EV2})(c_{ntr}-1)\lambda_l+c_{ntr}(\theta(Hkq_{EV2})+2)\\ +\theta(-(Hkq_{EV2}))+2\delta_m)+(\alpha-1)\theta(Hkq_{EV2})\left(c_{ntr}-1\right)\left(\lambda_l-1\right)\lambda_l\\ +(\alpha-1)\theta(Hkq_{EV2})c_l(\lambda_l-1)(\lambda_u-1)-2c_{ntr}+2\delta_m+4)\\ \cdot(2c_{ntr}(\theta(Hkq_{EV2})((1-2\alpha)\lambda_h+2(\alpha-1)\lambda_l+\lambda_u)-2\delta_m\\ \cdot\left(\lambda_h+\lambda_u-1\right)+2)-2(\alpha-1)\theta(Hkq_{EV2})c_l\left(c_{ntr}-1\right)\left(\lambda_h-\lambda_l+\lambda_u-1\right)\\ +c_{ntr}^2\left(\left(2\alpha-1\right)\theta\left(Hkq_{EV2}\right)\lambda_h-\theta\left(Hkq_{EV2}\right)\left(2\left(\alpha-1\right)\lambda_l+\lambda_u\right)-4\right)\\ +(\alpha-1)\theta(Hkq_{EV2})c_l^2(\lambda_h+\lambda_u-2)+\theta(Hkq_{EV2})((2\alpha-1)\lambda_h\\ -2(\alpha-1)\lambda_l-\lambda_u)+4\delta_m)+c_h^2((2\alpha-1)\theta(Hkq_{EV2})+\theta(Hkq_{EV2})\\ \cdot\left((\alpha-1)\left(\lambda_l-2\right)\lambda_l+\alpha\lambda_u^2-2\alpha\lambda_u\right)-4)+c_u^2(\theta(Hkq_{EV2})\\ \cdot(\alpha\lambda_h^2-2\alpha\lambda_h-(\alpha-1)(\lambda_l-2)\lambda_l)+\theta(Hkq_{EV2})+4))\end{array}\right) \quad \text{(D.17)}$$

$$N_{112}=\alpha^2\theta^2(-(Hkq_{EV2})^2)((c_{ntr}-1)(\lambda_h-\lambda_u)-c_u(\lambda_h-1)+c_h(\lambda_u-1))^2 \quad \text{(D.18)}$$

$$D_{110}=\begin{pmatrix}\alpha\theta\left(-\left(Hkq_{EV2}\right)\right)\lambda_h^2+2\alpha\theta\left(Hkq_{EV2}\right)\lambda_h-\theta\left(Hkq_{EV2}\right)\\ +\left(\alpha-1\right)\theta\left(Hkq_{EV2}\right)\left(\lambda_l-2\right)\lambda_l-4\end{pmatrix} \quad \text{(D.19)}$$

$$D_{111}=\alpha\theta\left(Hkq_{EV2}\right)\lambda_h^2-2\alpha\theta\left(Hkq_{EV2}\right)\lambda_h-\alpha\theta\left(Hkq_{EV2}\right)\lambda_u^2+2\alpha\theta\left(Hkq_{EV2}\right)\lambda_u \quad \text{(D.20)}$$

Relationship between λ_l and Π_3

$$\Pi_3\left(\lambda_l\right)=\frac{N_{120}+N_{121}\lambda_l+N_{122}\lambda_l^2}{4k\left(D_{120}+D_{121}\lambda_l+D_{122}\lambda_l^2\right)} \quad \text{(D.21)}$$

$$\begin{cases}-4\le N_{120}\le 20\\ -16\le N_{121}\le -9.09\times10^{-8}\\ 9.10\times10^{-8}\le N_{122}\le 4\\ -5\le D_{120}\le -4\\ 9.03\times10^{-8}\le D_{121}\le 2\\ -1\le D_{122}\le -9.28\times10^{-8}\end{cases} \quad \text{(D.22)}$$

We find The first derivative $9.08\times10^{-8}/(4k)\le\Pi_3'(\lambda_l)\le2.33/(4k)$ and the linearity is $0.05\%\le\eta_{3\lambda_l}\le24.99\%$. Therefore, the relationship between λ_l and Π_3 can be treated as positive linear.

$$N_{120}=\left(\begin{array}{l}
-(Hkq_{EV2})^2\alpha^2\beta^2c_u^2\theta^2+(Hkq_{EV2})^2\alpha\beta c_u^2\theta^2-(Hkq_{EV2})^2\alpha^2\lambda_h^2\theta^2\\
-(Hkq_{EV2})^2\alpha^2\beta^2\lambda_h^2\theta^2-(Hkq_{EV2})^2\alpha^2c_{ntr}^2\lambda_h^2\theta^2-(Hkq_{EV2})^2\alpha^2\beta^2\\
\cdot c_{ntr}^2\lambda_h^2\theta^2+(Hkq_{EV2})^2\alpha c_{ntr}^2\lambda_h^2\theta^2+2(Hkq_{EV2})^2\alpha^2\beta c_{ntr}^2\lambda_h^2\theta^2\\
-(Hkq_{EV2})^2\alpha\beta c_{ntr}^2\lambda_h^2\theta^2-(Hkq_{EV2})^2\alpha^2\beta^2c_u^2\lambda_h^2\theta^2+(Hkq_{EV2})^2\alpha^2\beta\\
\cdot c_u^2\lambda_h^2\theta^2+(Hkq_{EV2})^2\alpha\lambda_h^2\theta^2+2(Hkq_{EV2})^2\alpha^2\beta\lambda_h^2\theta^2-(Hkq_{EV2})^2\alpha\beta\lambda_h^2\theta^2\\
+2(Hkq_{EV2})^2\alpha^2c_{ntr}\lambda_h^2\theta^2+2(Hkq_{EV2})^2\alpha^2\beta^2c_{ntr}\lambda_h^2\theta^2-2(Hkq_{EV2})^2\alpha\\
\cdot c_{ntr}\lambda_h^2\theta^2-4(Hkq_{EV2})^2\alpha^2\beta c_{ntr}\lambda_h^2\theta^2+2(Hkq_{EV2})^2\alpha\beta c_{ntr}\lambda_h^2\theta^2\\
-2(Hkq_{EV2})^2\alpha^2\beta^2c_u\lambda_h^2\theta^2+2(Hkq_{EV2})^2\alpha^2\beta c_u\lambda_h^2\theta^2+2(Hkq_{EV2})^2\alpha^2\\
\cdot\beta^2c_{ntr}c_u\lambda_h^2\theta^2-2(Hkq_{EV2})^2\alpha^2\beta c_{ntr}c_u\lambda_h^2\theta^2+2(Hkq_{EV2})^2\alpha^2\beta^2c_u^2\lambda_h\theta^2\\
-2(Hkq_{EV2})^2\alpha^2\beta c_u^2\lambda_h\theta^2+2(Hkq_{EV2})^2\alpha^2\beta^2c_u\lambda_h\theta^2-2(Hkq_{EV2})^2\alpha^2\beta\\
\cdot c_u\lambda_h\theta^2-2(Hkq_{EV2})^2\alpha^2\beta^2c_{ntr}c_u\lambda_h\theta^2+2(Hkq_{EV2})^2\alpha^2\beta c_{ntr}c_u\lambda_h\theta^2\\
+4(Hkq_{EV2})\alpha\beta c_u^2\theta+4(Hkq_{EV2})\alpha c_{ntr}\delta_m\lambda_h^2\theta-4(Hkq_{EV2})\alpha\beta c_{ntr}\delta_m\lambda_h^2\theta\\
+(Hkq_{EV2})\alpha\beta(4c_{ntr}\delta_m-(Hkq_{EV2})(\alpha\beta-1)\theta(c_{ntr}-1)^2)\lambda_u^2\theta+4(Hkq_{EV2})\theta\\
-4(Hkq_{EV2})c_{ntr}\theta+8(Hkq_{EV2})\alpha\beta c_u\theta-4(Hkq_{EV2})\alpha\beta c_{ntr}c_u\theta+4(Hkq_{EV2})\\
\cdot\delta_m\theta+4(Hkq_{EV2})\alpha\beta c_u\delta_m\theta+4(Hkq_{EV2})\alpha c_{ntr}^2\lambda_h\theta-4(Hkq_{EV2})\alpha\beta c_{ntr}^2\lambda_h\theta\\
-4(Hkq_{EV2})\alpha c_{ntr}\lambda_h\theta+4(Hkq_{EV2})\alpha\beta c_{ntr}\lambda_h\theta-4(Hkq_{EV2})\alpha\delta_m\lambda_h\theta\\
+4(Hkq_{EV2})\alpha\beta\delta_m\lambda_h\theta-4(Hkq_{EV2})\alpha c_{ntr}\delta_m\lambda_h\theta+4(Hkq_{EV2})\alpha\beta c_{ntr}\delta_m\lambda_h\theta\\
+2(Hkq_{EV2})\alpha\beta(((Hkq_{EV2})\alpha(\beta-1)\theta\lambda_h+2)c_{ntr}^2+(((Hkq_{EV2})(\alpha\beta-1)\theta\\
-2)c_u-2(\delta_m+1)-(Hkq_{EV2})\alpha(\beta-1)\theta(c_u+2)\lambda_h)c_{ntr}-2\delta_m\\
+c_u((Hkq_{EV2})(1-\alpha\beta)\theta-2\delta_m)+(Hkq_{EV2})\alpha(\beta-1)\theta(c_u+1)\lambda_h)\\
-(Hkq_{EV2})\alpha(\beta-1)c_h^2((Hkq_{EV2})(\alpha(\beta-1)+1)\theta+(Hkq_{EV2})\alpha\beta(\lambda_u-2)\lambda_u\\
+4)\theta-2(Hkq_{EV2})\alpha(\beta-1)c_h(-(Hkq_{EV2})\alpha\beta\theta(c_{ntr}-1)\lambda_u^2+(Hkq_{EV2})\alpha\beta\theta\\
\cdot(c_{ntr}-1)(\lambda_h+1)\lambda_u-2c_{ntr}+2\delta_m-(Hkq_{EV2})(\alpha-1)\theta c_l(\lambda_h-1)\\
-((Hkq_{EV2})(-\beta\alpha+\alpha-1)\theta+((Hkq_{EV2})(\alpha(\beta-1)+1)\theta+2)c_{ntr}+2\delta_m)\lambda_h\\
-(Hkq_{EV2})\alpha\beta\theta c_u(\lambda_h-1)(\lambda_u-1)+4)\theta+(Hkq_{EV2})(\alpha-1)c_l^2(-(Hkq_{EV2})\alpha\theta\\
+(Hkq_{EV2})\alpha(-\beta\lambda_u^2+2\beta\lambda_u+(\beta-1)(\lambda_h-2)\lambda_h)\theta-4)\theta-2(Hkq_{EV2})(\alpha-1)\\
\cdot c_l(2(\delta_m+2)+(Hkq_{EV2})\alpha\theta(\beta\lambda_u^2-\beta\lambda_u-(\beta-1)(\lambda_h-1)\lambda_h+\beta c_u(\lambda_u-1))\\
+c_{ntr}((Hkq_{EV2})\alpha\theta(-\beta\lambda_u^2+\beta\lambda_u+(\beta-1)(\lambda_h-1)\lambda_h)-2))\theta-4c_{ntr}^2\\
-4\delta_m^2+8c_{ntr}\delta_m
\end{array}\right)\quad\text{(D.23)}$$

$$N_{121}=\begin{pmatrix}-2(\alpha-1)\theta(Hkq_{EV2})(\alpha(\beta-1)\theta(Hkq_{EV2})c_h^2+c_l(c_{ntr}(\alpha\theta(Hkq_{EV2})\\ \cdot\left(-\beta\lambda_h+\lambda_h+\beta\lambda_u\right)+\alpha\theta(-(Hkq_{EV2}))-2)-\alpha\theta(Hkq_{EV2})\\ \cdot\left(-\beta\left(c_u+\lambda_h\right)+\beta\left(c_u+1\right)\lambda_u+\lambda_h\right)+\alpha\theta(Hkq_{EV2})-2\delta_m)\\ +\alpha(\beta-1)\theta(Hkq_{EV2})c_h\left(c_l\left(\lambda_h-1\right)-\left(c_{ntr}-1\right)\left(\lambda_h+1\right)\right)\\ +\alpha\beta\theta\left(Hkq_{EV2}\right)c_{ntr}^2\lambda_h-2\alpha\beta\theta\left(Hkq_{EV2}\right)c_{ntr}\lambda_h-\alpha\theta\left(Hkq_{EV2}\right)c_{ntr}^2\lambda_h\\ +2\alpha\theta\left(Hkq_{EV2}\right)c_{ntr}\lambda_h-\alpha\beta\theta\left(Hkq_{EV2}\right)\left(c_{ntr}-1\right)\lambda_u\left(c_{ntr}-c_u-1\right)\\ +\alpha\beta\theta\left(Hkq_{EV2}\right)c_{ntr}c_u-\alpha\beta\theta\left(Hkq_{EV2}\right)c_u^2-\alpha\beta\theta\left(Hkq_{EV2}\right)c_u-2c_{ntr}\delta_m\\ -2(c_{ntr}+\delta_m)+2c_{ntr}^2+\alpha\beta\theta(Hkq_{EV2})\lambda_h-\alpha\theta(Hkq_{EV2})\lambda_h)\end{pmatrix} \quad \text{(D.24)}$$

$$N_{122}=\begin{pmatrix}(\alpha-1)\theta(Hkq_{EV2})(\alpha(\beta-1)\theta(Hkq_{EV2})c_h^2-2\alpha(\beta-1)\theta(Hkq_{EV2})c_h\\ \cdot(c_{ntr}-1)-\alpha\theta(Hkq_{EV2})(-2\beta(c_{ntr}-1)c_u+(c_{ntr}-1)^2+\beta c_u^2)\\ -4c_{ntr}\delta_m)\end{pmatrix} \quad \text{(D.25)}$$

$$D_{120}=\alpha\theta\left(Hkq_{EV2}\right)\left(\left(\beta-1\right)\left(\lambda_h-2\right)\lambda_h-\beta\lambda_u^2+2\beta\lambda_u\right)+\theta\left(-\left(Hkq_{EV2}\right)\right)-4 \quad \text{(D.26)}$$

$$D_{121}=-2\left(\alpha-1\right)\theta\left(Hkq_{EV2}\right) \quad \text{(D.27)}$$

$$D_{122}=\left(\alpha-1\right)\theta\left(Hkq_{EV2}\right) \quad \text{(D.28)}$$

Relationship between λ_h and Π_3

$$\Pi_3\left(\lambda_h\right)=\frac{N_{130}+N_{131}\lambda_h+N_{132}\lambda_h^2}{4k\left(D_{130}+D_{131}\lambda_h+D_{132}\lambda_h^2\right)} \quad \text{(D.29)}$$

$$\begin{cases}-4\le N_{130}\le 20\\ -16\le N_{131}\le -9.09\times10^{-8}\\ 5.20\times10^{-8}\le N_{132}\le 4\\ -5\le D_{130}\le -4\\ 1.49\times10^{-7}\le D_{131}\le 2\\ -1\le D_{132}\le -9.29\times10^{-8}\end{cases} \quad \text{(D.30)}$$

We find The first derivative $9.08\times10^{-8}/\left(4k\right)\le \Pi_3'\left(\lambda_h\right)\le 2.33/\left(4k\right)$ and the linearity is $-31.14\%\le\eta_{3\lambda_h}\le 25.04\%$. Therefore, the relationship between λ_h and Π_3 has positive relationship.

$$N_{130}=\left(\begin{array}{l}
(Hkq_{EV2})^2\alpha(-(\alpha(\beta-1)+1)(\beta-1)c_h^2-(\beta-1)(\alpha\beta\lambda_u^2-2\alpha\beta\lambda_u\\
-(\alpha-1)\left(\lambda_l-2\right)\lambda_l)c_h^2+2(\beta-1)((\alpha-1)c_l\left(\lambda_l-1\right)-\alpha\beta c_u\left(\lambda_u-1\right)\\
-\left(c_{ntr}-1\right)\left(-\alpha\beta\lambda_u^2+\alpha\beta\lambda_u+(\alpha-1)\left(\lambda_l-1\right)\lambda_l\right))c_h-(\alpha-1)c_l^2\\
-\alpha\beta^2c_u^2+\beta c_u^2-\alpha c_{ntr}^2\lambda_l^2+c_{ntr}^2\lambda_l^2-\alpha\beta c_u^2\lambda_l^2+\beta c_u^2\lambda_l^2-\alpha\lambda_l^2\\
+2\alpha c_{ntr}\lambda_l^2-2c_{ntr}\lambda_l^2-2\alpha\beta c_u\lambda_l^2+2\beta c_u\lambda_l^2+2\alpha\beta c_{ntr}c_u\lambda_l^2\\
-2\beta c_{ntr}c_u\lambda_l^2+\lambda_l^2-\beta\left(\alpha\beta-1\right)\left(c_{ntr}-1\right)^2\lambda_u^2-2\left(\alpha-1\right)\beta c_l\lambda_u^2\\
+2\alpha\beta c_u^2\lambda_l-2\beta c_u^2\lambda_l-2\left(\alpha-1\right)c_l\lambda_l+2\left(\alpha-1\right)c_lc_{ntr}\lambda_l+2\alpha\beta c_u\lambda_l\\
-2\beta c_u\lambda_l-2\alpha\beta c_{ntr}c_u\lambda_l+2\beta c_{ntr}c_u\lambda_l+2\left(\alpha-1\right)\beta c_lc_u\left(\lambda_l-1\right)\\
\cdot(\lambda_u-1)-2\beta(\alpha\beta-1)c_u\lambda_u+2\beta(\alpha\beta-1)c_{ntr}c_u\lambda_u+2(\alpha-1)\beta c_{ntr}^2\lambda_l\lambda_u\\
-2\left(\alpha-1\right)\beta c_lc_{ntr}\lambda_l\lambda_u+2\left(\alpha-1\right)\beta\left(c_u+1\right)\lambda_l\lambda_u-2\left(\alpha-1\right)\beta c_{ntr}\left(c_u+2\right)\\
\cdot\lambda_l\lambda_u+2(\alpha-1)\beta c_l(\lambda_l+1)\lambda_u-(\alpha-1)\beta c_l^2(\lambda_u-2)\lambda_u+2(\alpha-1)\beta c_l\\
\cdot c_{ntr}\left(\lambda_u-1\right)\lambda_u)\theta^2+4(Hkq_{EV2})((\alpha-\alpha\beta)c_h^2+\alpha(\beta-1)\left(c_{ntr}-\delta_m-2\right)c_h\\
-\left(\alpha-1\right)c_l^2+\alpha\beta c_u^2-\alpha c_{ntr}\delta_m\lambda_l^2+c_{ntr}\delta_m\lambda_l^2+\alpha\beta c_{ntr}\delta_m\lambda_u^2-c_{ntr}+2\alpha\beta c_u\\
-\alpha\beta c_{ntr}c_u+\alpha\beta c_u\delta_m+\delta_m-\alpha c_{ntr}^2\lambda_l+c_{ntr}^2\lambda_l+\alpha c_{ntr}\lambda_l-c_{ntr}\lambda_l+\alpha\delta_m\lambda_l\\
+\alpha c_{ntr}\delta_m\lambda_l-c_{ntr}\delta_m\lambda_l-\delta_m\lambda_l+\left(\alpha-1\right)c_l\left(\delta_m\left(\lambda_l-1\right)+c_{ntr}\left(\lambda_l+1\right)-2\right)\\
+\alpha\beta c_{ntr}^2\lambda_u-\alpha\beta c_{ntr}\lambda_u-\alpha\beta c_{ntr}c_u\lambda_u-\alpha\beta\delta_m\lambda_u\\
-\alpha\beta c_{ntr}\delta_m\lambda_u-\alpha\beta c_u\delta_m\lambda_u+1)
\end{array}\right)$$

(D.31)

$$N_{131}=\left(\begin{array}{l}
-2\alpha(\beta-1)\theta(Hkq_{EV2})(c_h(\theta(Hkq_{EV2})((\alpha-1)c_l(\lambda_l-1)+c_{ntr}\\
\cdot(\alpha(-\beta)+\alpha-\alpha\lambda_l+\lambda_l+\alpha\beta\lambda_u-1)+\alpha(-\beta(c_u+1)\lambda_u+\beta c_u+\lambda_l)\\
-\lambda_l)-2c_{ntr}+\theta(Hkq_{EV2})(\alpha(\beta-1)+1)-2\delta_m)+(\alpha-1)\theta(Hkq_{EV2})c_l^2\\
-\left(\alpha-1\right)\theta\left(Hkq_{EV2}\right)c_l\left(c_{ntr}-1\right)\left(\lambda_l+1\right)+\alpha\theta\left(Hkq_{EV2}\right)c_{ntr}^2\lambda_l-2\alpha\theta\\
\cdot(Hkq_{EV2})c_{ntr}\lambda_l-\theta(Hkq_{EV2})c_{ntr}^2\lambda_l+2\theta(Hkq_{EV2})c_{ntr}\lambda_l-\alpha\beta\theta\\
\cdot(Hkq_{EV2})(c_{ntr}-1)\lambda_u(c_{ntr}-c_u-1)+\alpha\beta\theta(Hkq_{EV2})c_{ntr}c_u\\
-\alpha\beta\theta\left(Hkq_{EV2}\right)c_u^2-\alpha\beta\theta\left(Hkq_{EV2}\right)c_u-2c_{ntr}\delta_m-2\left(c_{ntr}+\delta_m\right)\\
+2c_{ntr}^2+\alpha\theta(Hkq_{EV2})\lambda_l-\theta(Hkq_{EV2})\lambda_l)
\end{array}\right) \quad \text{(D.32)}$$

$$N_{132}=\left(\begin{array}{c}
\alpha(\beta-1)\theta(-(Hkq_{EV2}))((\alpha-1)\theta(-(Hkq_{EV2}))c_l^2+2(\alpha-1)\theta\\
\cdot(Hkq_{EV2})c_l(c_{ntr}-1)+\theta(Hkq_{EV2})((\alpha(\beta-1)+1)(c_{ntr}-1)^2\\
-2\alpha\beta(c_{ntr}-1)c_u+\alpha\beta c_u^2)+4c_{ntr}\delta_m)
\end{array}\right) \quad \text{(D.33)}$$

$$D_{130}=\theta\left(-\left(Hkq_{EV2}\right)\right)+\theta\left(Hkq_{EV2}\right)\left(\left(\alpha-1\right)\left(\lambda_l-2\right)\lambda_l-\alpha\beta\lambda_u^2+2\alpha\beta\lambda_u\right)-4 \quad \text{(D.34)}$$

$$D_{131}=-2\alpha\left(\beta-1\right)\theta\left(Hkq_{EV2}\right) \quad \text{(D.35)}$$

$$D_{132}=\alpha\left(\beta-1\right)\theta\left(Hkq_{EV2}\right) \quad \text{(D.36)}$$

Relationship between λ_u and Π_3

$$\Pi_3\left(\lambda_u\right)=\frac{N_{140}+N_{141}\lambda_u+N_{142}\lambda_u^2}{4k\left(D_{140}+D_{141}\lambda_u+D_{142}\lambda_u^2\right)} \quad \text{(D.37)}$$

$$\begin{cases} -4 \le N_{140} \le 20 \\ -16 \le N_{141} \le -9.18\times10^{-8} \\ -9.09\times10^{-8} \le N_{142} \le 4 \\ -5 \le D_{140} \le -4 \\ -9.17\times10^{-8} \le D_{141} \le 2 \\ -1 \le D_{142} \le -9.10\times10^{-8} \end{cases} \quad \text{(D.38)}$$

We find The first derivative $6.01\times10^{-8} \le \Pi_3{}'\left(\lambda_u\right) \le 2.33/\left(4k\right)$ and the linearity is $0.02\% \le \eta_{3\lambda_u} \le 24.97\%$. Therefore, the relationship between λ_u and Π_3 cannot be treated as positive linear.

$$N_{140}=\left(\begin{array}{l} (Hkq_{EV2})^2\alpha(-\alpha c_u^2\beta^2-\alpha c_{ntr}^2\lambda_h^2\beta^2-\alpha c_u^2\lambda_h^2\beta^2-\alpha\lambda_h^2\beta^2+2\alpha c_{ntr}\lambda_h^2\beta^2 \\ -2\alpha c_u\lambda_h^2\beta^2+2\alpha c_{ntr}c_u\lambda_h^2\beta^2+2\alpha c_u^2\lambda_h\beta^2+2\alpha c_u\lambda_h\beta^2-2\alpha c_{ntr}c_u\lambda_h\beta^2 \\ +c_u^2\beta+2\alpha c_{ntr}^2\lambda_h^2\beta-c_{ntr}^2\lambda_h^2\beta+\alpha c_u^2\lambda_h^2\beta+2\alpha\lambda_h^2\beta-4\alpha c_{ntr}\lambda_h^2\beta \\ +2c_{ntr}\lambda_h^2\beta+2\alpha c_u\lambda_h^2\beta-2\alpha c_{ntr}c_u\lambda_h^2\beta-\lambda_h^2\beta-2\alpha c_u^2\lambda_h\beta-2\alpha c_u\lambda_h\beta \\ +2\alpha c_{ntr}c_u\lambda_h\beta-\alpha c_{ntr}^2\lambda_h^2+c_{ntr}^2\lambda_h^2-\alpha\lambda_h^2+2\alpha c_{ntr}\lambda_h^2-2c_{ntr}\lambda_h^2+\lambda_h^2 \\ -(\alpha-1)((c_{ntr}-1)^2-2\beta c_u(c_{ntr}-1)+\beta c_u^2)\lambda_l^2+(\alpha-1)c_l^2 \\ \cdot((\beta-1)(\lambda_h-2)\lambda_h-1)+2(\alpha-1)(\beta c_u(-c_{ntr}+c_u+1)-(\beta-1) \\ \cdot\left(c_{ntr}-1\right)^2\lambda_h)\lambda_l-(\beta-1)c_h^2\left(\alpha(\beta-1)-(\alpha-1)\left(\lambda_l-2\right)\lambda_l+1\right) \\ +2\left(\alpha-1\right)c_l\left(\left(c_{ntr}-1\right)\left(\lambda_l-\left(\beta-1\right)\lambda_h\left(\lambda_h-\lambda_l-1\right)\right)+c_u\left(\beta-\beta\lambda_l\right)\right) \\ +2(\beta-1)c_h(-\alpha\beta c_u(\lambda_h-1)-(\alpha-1)c_l(\lambda_l-1)(\lambda_h-1)+(c_{ntr}-1) \\ \cdot\left(\lambda_h\left(\alpha(\beta-1)+(\alpha-1)\lambda_l+1\right)-(\alpha-1)\left(\lambda_l-1\right)\lambda_l\right)))\theta^2+4(Hkq_{EV2}) \\ \cdot((\alpha-\alpha\beta)c_h^2+\alpha(\beta-1)(\delta_m(\lambda_h-1)+c_{ntr}(\lambda_h+1)-2)c_h \\ -(\alpha-1)c_l^2-(\alpha-1)c_{ntr}\delta_m\lambda_l^2-c_{ntr}+\delta_m+\alpha(\beta c_u^2+\beta(-c_{ntr}+\delta_m+2)c_u \\ +(\beta-1)\lambda_h\left(\delta_m+c_{ntr}\left(-c_{ntr}+\delta_m-\delta_m\lambda_h+1\right)\right))-(\alpha-1)(\left(c_{ntr}-1\right)c_{ntr} \\ -(c_{ntr}+1)\delta_m)\lambda_l+(\alpha-1)c_l(\delta_m(\lambda_l-1)+c_{ntr}(\lambda_l+1)-2)+1) \\ -4(c_{ntr}-\delta_m)^2 \end{array}\right)$$

(D.39)

$$N_{141}=\left(\begin{array}{c} 2\alpha\beta\theta^2(Hkq_{EV2})^2(\alpha(\beta-1)c_h^2+(c_{ntr}-1)((c_{ntr}-1)(\alpha(\beta-1)\lambda_h \\ +(\alpha-1)\lambda_l)+c_u\left(\alpha\beta-\alpha\left((\beta-1)\lambda_h+\lambda_l\right)+\lambda_l-1\right))-\alpha(\beta-1)c_h \\ \cdot\left(\lambda_h\left(c_{ntr}-c_u-1\right)+c_{ntr}+c_u-1\right)+(\alpha-1)c_l^2-(\alpha-1)c_l \\ \cdot\left(\lambda_l\left(c_{ntr}-c_u-1\right)+c_{ntr}+c_u-1\right))+2\alpha\beta\theta(Hkq_{EV2}) \\ \cdot\left(2c_{ntr}\left(c_{ntr}-c_u-1\right)-2\delta_m\left(c_{ntr}+c_u+1\right)\right) \end{array}\right) \quad \text{(D.40)}$$

$$N_{142}=\begin{pmatrix}\alpha\beta\theta(Hkq_{EV2})(\alpha(\beta-1)\theta(-(Hkq_{EV2}))c_h^2+2\alpha(\beta-1)\theta(Hkq_{EV2})c_h\\ \cdot(c_{ntr}-1)+\theta(Hkq_{EV2})(-(\alpha-1)c_l^2+2(\alpha-1)c_l(c_{ntr}-1)\\ -(\alpha\beta-1)(c_{ntr}-1)^2)+4c_{ntr}\delta_m)\end{pmatrix} \qquad \text{(D.41)}$$

$$D_{140}=\theta\left(Hkq_{EV2}\right)\left(\alpha\left(\beta-1\right)\left(\lambda_h-2\right)\lambda_h+\left(\alpha-1\right)\lambda_l^2-2\left(\alpha-1\right)\lambda_l\right)+\theta\left(-\left(Hkq_{EV2}\right)\right)-4$$
(D.42)

$$D_{141}=2\alpha\beta\theta\left(Hkq_{EV2}\right) \quad \text{(D.43)}$$

$$D_{142}=\alpha\beta\theta\left(-\left(Hkq_{EV2}\right)\right) \qquad \text{(D.44)}$$

Relationship between c_{ntr} and Π_3

$$\Pi_3\left(c_{ntr}\right)=\frac{N_{150}+N_{151}c_{ntr}+N_{152}c_{ntr}^2}{4kD_{150}} \qquad \text{(D.45)}$$

$$\begin{cases}-4\le N_{150}\le 20\\ -16\le N_{151}\le 8\\ -4\le N_{152}\le -3.34\times10^{-7}\\ -5\le D_{150}\le -4\end{cases} \qquad \text{(D.46)}$$

We find The first derivative $-2/\left(4k\right)\le \Pi_3'\left(c_{ntr}\right)\le 4/\left(4k\right)$ and the linearity is $-1.24\times10^7\le\eta_{3c_{ntr}}\le 1.82\times10^7$. Therefore, the relationship between c_n and Π_3 cannot be treated as linear.

$$N_{150}=\begin{pmatrix}(Hkq_{EV2})^2\alpha(-\alpha c_u^2\beta^2-\alpha c_u^2\lambda_h^2\beta^2-\alpha\lambda_h^2\beta^2-2\alpha c_u\lambda_h^2\beta^2+2\alpha c_u^2\lambda_h\beta^2\\ +2\alpha c_u\lambda_h\beta^2+c_u^2\beta+\alpha c_u^2\lambda_h^2\beta+2\alpha\lambda_h^2\beta+2\alpha c_u\lambda_h^2\beta-\lambda_h^2\beta-\alpha c_u^2\lambda_l^2\beta\\ +c_u^2\lambda_l^2\beta-2\alpha c_u\lambda_l^2\beta+2c_u\lambda_l^2\beta+\left(1-\alpha\beta\right)\lambda_u^2\beta-2\alpha c_u^2\lambda_h\beta-2\alpha c_u\lambda_h\beta\\ +2\alpha c_u^2\lambda_l\beta-2c_u^2\lambda_l\beta+2\alpha c_u\lambda_l\beta-2c_u\lambda_l\beta-2\alpha\lambda_h\lambda_l\beta+2\lambda_h\lambda_l\beta\\ +2\left(\alpha\left(\beta-1\right)\lambda_h+\left(\alpha-1\right)\lambda_l+c_u\left(-\alpha\beta+\alpha\left(\beta-1\right)\lambda_h+\left(\alpha-1\right)\lambda_l+1\right)\right)\\ \cdot\lambda_u\beta-\alpha\lambda_h^2+\lambda_h^2-\alpha\lambda_l^2+\lambda_l^2+2\alpha\lambda_h\lambda_l-2\lambda_h\lambda_l-(\beta-1)c_h^2(\alpha\beta\lambda_u^2\\ -2\alpha\beta\lambda_u+\alpha(\beta-1)-(\alpha-1)\left(\lambda_l-2\right)\lambda_l+1)+(\alpha-1)c_l^2(-\beta\lambda_u^2+2\beta\lambda_u\\ +(\beta-1)\left(\lambda_h-2\right)\lambda_h-1)+2(\alpha-1)c_l(-\beta\lambda_u^2+\beta\left(\lambda_l+1\right)\lambda_u+(\beta-1)\lambda_h\\ \cdot\left(\lambda_h-\lambda_l-1\right)-\lambda_l+\beta c_u\left(\lambda_l-1\right)\left(\lambda_u-1\right))+2(\beta-1)c_h(\alpha\lambda_l^2-\lambda_l^2-\alpha\lambda_l\\ -\alpha\lambda_h\lambda_l+\lambda_h\lambda_l+\lambda_l-\alpha\beta\lambda_u^2+\alpha\lambda_h-\alpha\beta\lambda_h-\lambda_h-(\alpha-1)c_l\left(\lambda_h-1\right)\\ \cdot\left(\lambda_l-1\right)+\alpha\beta c_u\left(\lambda_h-1\right)\left(\lambda_u-1\right)+\alpha\beta\left(\lambda_h+1\right)\lambda_u))\theta^2+4(Hkq_{EV2})\\ \cdot((\alpha-\alpha\beta)c_h^2+\alpha(\beta-1)(\delta_m(\lambda_h-1)-2)c_h-(\alpha-1)c_l^2+\alpha\beta c_u^2+2\alpha\beta c_u\\ +\alpha\beta c_u\delta_m+\delta_m-\alpha\delta_m\lambda_h+\alpha\beta\delta_m\lambda_h+\left(\alpha-1\right)c_l\left(\delta_m\left(\lambda_l-1\right)-2\right)+\alpha\delta_m\lambda_l\\ -\delta_m\lambda_l-\alpha\beta(c_u+1)\delta_m\lambda_u+1)\end{pmatrix}$$

(D.47)

$$N_{151}=\begin{pmatrix} 2\alpha\theta^2(Hkq_{EV2})^2(\beta\lambda_u(c_u(\alpha\beta-\alpha((\beta-1)\lambda_h+\lambda_l)+\lambda_l-1) \\ -2\alpha(\beta-1)\lambda_h-2(\alpha-1)\lambda_l)-(\alpha-1)c_l((\beta-1)\lambda_h\left(\lambda_h-\lambda_l-1\right) \\ -\lambda_l+\beta\left(\lambda_l+1\right)\lambda_u-\beta\lambda_u^2)-(\beta-1)c_h(\lambda_h(\alpha(-\beta)+\alpha-\alpha\lambda_l+\lambda_l \\ +\alpha\beta\lambda_u-1)+(\alpha-1)\left(\lambda_l-1\right)\lambda_l-\alpha\beta\lambda_u^2+\alpha\beta\lambda_u)+\alpha\beta^2c_u\lambda_h^2 \\ -\alpha\beta^2c_u\lambda_h-\alpha\beta c_u\lambda_h^2+\alpha\beta c_u\lambda_h+\alpha\beta c_u\lambda_l^2-\alpha\beta c_u\lambda_l-\beta c_u\lambda_l^2+\beta c_u\lambda_l \\ +\alpha\beta^2\lambda_h^2-2\alpha\beta\lambda_h^2+\alpha\lambda_h^2+\beta\lambda_h^2-\lambda_h^2+2\alpha\beta\lambda_h\lambda_l-2\alpha\lambda_h\lambda_l-2\beta\lambda_h\lambda_l \\ +2\lambda_h\lambda_l+\alpha\lambda_l^2-\lambda_l^2+\beta(\alpha\beta-1)\lambda_u^2)+4\theta(Hkq_{EV2})(\alpha(\beta-1)c_h\left(\lambda_h+1\right) \\ +(\alpha-1)c_l\left(\lambda_l+1\right)-\alpha\beta\lambda_u\left(c_u+\delta_m+1\right)-\alpha\beta c_u+\alpha\beta\lambda_h-\alpha\lambda_h \\ -\alpha\beta\lambda_h^2\delta_m+\alpha\beta\lambda_h\delta_m+\alpha\lambda_h^2\delta_m-\alpha\lambda_h\delta_m+\alpha\lambda_l-\lambda_l-\alpha\lambda_l^2\delta_m+\alpha\lambda_l\delta_m \\ +\lambda_l^2\delta_m-\lambda_l\delta_m+\alpha\beta\delta_m\lambda_u^2-1) \end{pmatrix} \quad \text{(D.48)}$$

$$N_{152}=\begin{pmatrix} \theta(-(Hkq_{EV2}))(\alpha(\beta-1)\theta(Hkq_{EV2})(\alpha(\beta-1)+1)\lambda_h^2-2\alpha(\beta-1)\lambda_h \\ \cdot((\alpha-1)\theta(-(Hkq_{EV2}))\lambda_l+\alpha\beta\theta(Hkq_{EV2})\lambda_u-2)+(\alpha-1)\lambda_l \\ \cdot(\alpha\theta(Hkq_{EV2})\lambda_l+4)-2\alpha\beta\lambda_u((\alpha-1)\theta(Hkq_{EV2})\lambda_l+2) \\ +\alpha\beta\theta(Hkq_{EV2})(\alpha\beta-1)\lambda_u^2)-4 \end{pmatrix} \quad \text{(D.49)}$$

$$D_{150}=\begin{pmatrix} \theta(Hkq_{EV2})(\alpha(\beta-1)\lambda_h^2-2\alpha(\beta-1)\lambda_h+(\alpha-1)(\lambda_l-2)\lambda_l \\ -\alpha\beta\lambda_u^2+2\alpha\beta\lambda_u)+\theta(-(Hkq_{EV2}))-4 \end{pmatrix} \quad \text{(D.50)}$$

Relationship between c_l and Π_3

$$\Pi_3\left(c_l\right)=\frac{N_{160}+N_{161}c_l+N_{162}c_l^2}{4kD_{160}} \quad \text{(D.51)}$$

$$\begin{cases} -4\le N_{160}\le 20 \\ 9.08\times10^{-8}\le N_{161}\le 12 \\ 8.28\times10^{-8}\le N_{162}\le 4 \\ -5\le D_{160}\le -4 \end{cases} \quad \text{(D.52)}$$

We find The first derivative $-4.23/\left(4k\right)\le\Pi_3'\left(c_l\right)\le-9.13\times10^{-8}/\left(4k\right)$ and the linearity is $-24.99\%\le\eta_{3c_l}\le-6.25\%$. Therefore, the relationship between c_l and Π_3 cannot be treated as negative linear.

$$N_{160}=\begin{pmatrix}\alpha\theta^2(Hkq_{EV2})^2(2(\beta-1)c_h(\alpha\beta c_u(\lambda_h-1)(\lambda_u-1)-(c_{ntr}-1)\\ \cdot(\lambda_h(\alpha(-\beta)+\alpha-\alpha\lambda_l+\lambda_l+\alpha\beta\lambda_u-1)+(\alpha-1)(\lambda_l-1)\lambda_l-\alpha\beta\lambda_u^2\\ +\alpha\beta\lambda_u t))+2\beta\left(c_{ntr}-1\right)c_u(\alpha(\beta-1)\lambda_h^2-\alpha(\beta-1)\lambda_h\left(\lambda_u+1\right)+(\alpha-1)\\ \cdot\left(\lambda_l-1\right)\lambda_l+\lambda_u\left(\alpha\beta-\alpha\lambda_l+\lambda_l-1\right))-\left(c_{ntr}-1\right)^2((\beta-1)(\alpha(\beta-1)\\ +1)\lambda_h^2-2(\beta-1)\lambda_h\left(-\alpha\lambda_l+\lambda_l+\alpha\beta\lambda_u\right)+(\alpha-1)\lambda_l^2-2(\alpha-1)\beta\lambda_l\lambda_u\\ +\beta(\alpha\beta-1)\lambda_u^2)-(\beta-1)c_h^2(\alpha(\beta-1)-(\alpha-1)\left(\lambda_l-2\right)\lambda_l+\alpha\beta\lambda_u^2\\ -2\alpha\beta\lambda_u+1)+\beta c_u^2(-\alpha\beta+(\alpha-\alpha\beta)\lambda_h^2+2\alpha(\beta-1)\lambda_h-(\alpha-1)\left(\lambda_l-2\right)\\ \cdot\lambda_l+1))+4\theta(Hkq_{EV2})((\alpha-\alpha\beta)c_h^2-c_{ntr}(\alpha\beta c_u\left(\lambda_u+1\right)+\alpha(\beta-1)\lambda_h^2\delta_m\\ -\alpha(\beta-1)\lambda_h\left(\delta_m+1\right)+(\alpha-1)\lambda_l\left(\left(\lambda_l-1\right)\delta_m-1\right)-\alpha\beta\delta_m\lambda_u^2\\ +\alpha\beta\left(\delta_m+1\right)\lambda_u+1)+c_{ntr}^2\left((\alpha-\alpha\beta)\lambda_h-(\alpha-1)\lambda_l+\alpha\beta\lambda_u\right)+\alpha(\beta-1)\\ \cdot c_h(c_{ntr}(\lambda_h+1)+(\lambda_h-1)\delta_m-2)-\alpha\beta c_u\delta_m\lambda_u+\alpha\beta c_u\delta_m+\alpha\beta c_u^2\\ +2\alpha\beta c_u+\alpha\beta\lambda_h\delta_m-\alpha\lambda_h\delta_m+\alpha\lambda_l\delta_m-\lambda_l\delta_m+\delta_m-\alpha\beta\delta_m\lambda_u+1)\\ -4(c_{ntr}-\delta_m)^2\end{pmatrix} \quad \text{(D.53)}$$

$$N_{161}=\begin{pmatrix}2(\alpha-1)\alpha\theta^2(Hkq_{EV2})^2(-(\beta-1)c_h(\lambda_h-1)(\lambda_l-1)-(c_{ntr}-1)\\ \cdot((\beta-1)\lambda_h(\lambda_h-\lambda_l-1)-\lambda_l+\beta(\lambda_l+1)\lambda_u-\beta\lambda_u^2)\\ +\beta c_u\left(\lambda_l-1\right)\left(\lambda_u-1\right))+4(\alpha-1)\theta(Hkq_{EV2})(c_{ntr}\left(\lambda_l+1\right)\\ +\left(\lambda_l-1\right)\delta_m-2)\end{pmatrix} \quad \text{(D.54)}$$

$$N_{162}=\begin{pmatrix}(\alpha-1)\theta(Hkq_{EV2})(\alpha\theta(Hkq_{EV2})((\beta-1)(\lambda_h-2)\lambda_h\\ -\beta\lambda_u^2+2\beta\lambda_u)+\alpha\theta(-(Hkq_{EV2}))-4)\end{pmatrix} \quad \text{(D.55)}$$

$$D_{160}=\begin{pmatrix}\theta(Hkq_{EV2})(\alpha(\beta-1)\lambda_h^2-2\alpha(\beta-1)\lambda_h+(\alpha-1)(\lambda_l-2)\lambda_l\\ -\alpha\beta\lambda_u^2+2\alpha\beta\lambda_u)+\theta(-(Hkq_{EV2}))-4\end{pmatrix} \quad \text{(D.56)}$$

Relationship between c_h and Π_3

$$\Pi_3\left(c_h\right)=\frac{N_{170}+N_{171}c_h+N_{172}c_h^2}{4kD_{170}} \quad \text{(D.57)}$$

$$\begin{cases}-4\le N_{170}\le 20\\ 8.57\times10^{-8}\le N_{171}\le 12\\ 6.89\times10^{-8}\le N_{172}\le 4\\ -5\le D_{170}\le -4\end{cases} \quad \text{(D.58)}$$

We find The first derivative $-4.23/\left(4k\right)\le\Pi_3'\left(c_l\right)\le-8.43\times10^{-8}/\left(4k\right)$ and the linearity is $-24.91\%\le\eta_{3c_h}\le-6.26\%$. Therefore, the relationship between c_l and Π_3 cannot be treated as negative linear.

$$N_{170}=\left(\begin{array}{l}\alpha\theta^2(Hkq_{EV2})^2(2(\alpha-1)c_l(\beta c_u(\lambda_l-1)(\lambda_u-1)-(c_{ntr}-1)\\ \cdot\left((\beta-1)\lambda_h\left(\lambda_h-\lambda_l-1\right)-\lambda_l+\beta\left(\lambda_l+1\right)\lambda_u-\beta\lambda_u^2\right))+2\beta\\ \cdot\left(c_{ntr}-1\right)c_u(\alpha(\beta-1)\lambda_h^2-\alpha(\beta-1)\lambda_h\left(\lambda_u+1\right)+(\alpha-1)\left(\lambda_l-1\right)\lambda_l\\ +\lambda_u\left(\alpha\beta-\alpha\lambda_l+\lambda_l-1\right))-\left(c_{ntr}-1\right)^2((\beta-1)(\alpha(\beta-1)+1)\lambda_h^2\\ -2(\beta-1)\lambda_h\left(-\alpha\lambda_l+\lambda_l+\alpha\beta\lambda_u\right)+(\alpha-1)\lambda_l^2-2(\alpha-1)\beta\lambda_l\lambda_u\\ +\beta(\alpha\beta-1)\lambda_u^2)+(\alpha-1)c_l^2\left((\beta-1)\left(\lambda_h-2\right)\lambda_h-\beta\lambda_u^2+2\beta\lambda_u-1\right)\\ +\beta c_u^2(-\alpha\beta+(\alpha-\alpha\beta)\lambda_h^2+2\alpha(\beta-1)\lambda_h-(\alpha-1)(\lambda_l-2)\lambda_l+1))\\ +4\theta(Hkq_{EV2})(-c_{ntr}(\alpha\beta c_u(\lambda_u+1)+\alpha(\beta-1)\lambda_h^2\delta_m-\alpha(\beta-1)\lambda_h\\ \cdot(\delta_m+1)+(\alpha-1)\lambda_l((\lambda_l-1)\delta_m-1)-\alpha\beta\delta_m\lambda_u^2+\alpha\beta(\delta_m+1)\lambda_u+1)\\ +c_{ntr}^2\left(\left(\alpha-\alpha\beta\right)\lambda_h-\left(\alpha-1\right)\lambda_l+\alpha\beta\lambda_u\right)-\left(\alpha-1\right)c_l^2+\left(\alpha-1\right)c_l\\ \cdot(c_{ntr}(\lambda_l+1)+(\lambda_l-1)\delta_m-2)-\alpha\beta c_u\delta_m\lambda_u+\alpha\beta c_u\delta_m+\alpha\beta c_u^2\\ +2\alpha\beta c_u+\alpha\beta\lambda_h\delta_m-\alpha\lambda_h\delta_m+\alpha\lambda_l\delta_m-\lambda_l\delta_m+\delta_m-\alpha\beta\delta_m\lambda_u+1)\\ -4(c_{ntr}-\delta_m)^2\end{array}\right) \quad (D.59)$$

$$N_{171}=\left(\begin{array}{c}4\alpha\left(\beta-1\right)\theta\left(Hkq_{EV2}\right)\left(c_{ntr}\left(\lambda_h+1\right)+\left(\lambda_h-1\right)\delta_m-2\right)-2\alpha\left(\beta-1\right)\theta^2\\ \cdot(Hkq_{EV2})^2((\alpha-1)c_l(\lambda_h-1)(\lambda_l-1)+(c_{ntr}-1)(\lambda_h(\alpha(-\beta)+\alpha\\ -\alpha\lambda_l+\lambda_l+\alpha\beta\lambda_u-1)+(\alpha-1)(\lambda_l-1)\lambda_l-\alpha\beta\lambda_u^2+\alpha\beta\lambda_u)\\ -\alpha\beta c_u(\lambda_h-1)(\lambda_u-1))\end{array}\right) \quad (D.60)$$

$$N_{172}=\left(\begin{array}{c}\alpha(\beta-1)\theta(-(Hkq_{EV2}))(\theta(Hkq_{EV2})(\alpha(\beta-1)+1)\\ +\theta(Hkq_{EV2})(-(\alpha-1)(\lambda_l-2)\lambda_l+\alpha\beta\lambda_u^2-2\alpha\beta\lambda_u)+4)\end{array}\right) \quad (D.61)$$

$$D_{170}=\left(\begin{array}{c}\theta(Hkq_{EV2})(\alpha(\beta-1)\lambda_h^2-2\alpha(\beta-1)\lambda_h+(\alpha-1)(\lambda_l-2)\lambda_l\\ -\alpha\beta\lambda_u^2+2\alpha\beta\lambda_u)+\theta(-(Hkq_{EV2}))-4\end{array}\right) \quad (D.62)$$

Relationship between c_u and Π_3

$$\Pi_3\left(c_u\right)=\frac{N_{180}+N_{181}c_u+N_{182}c_u^2}{4kD_{180}} \quad (D.63)$$

$$\begin{cases}-4\le N_{180}\le 20\\ 1.82\times10^{-8}\le N_{181}\le 12\\ 9.09\times10^{-8}\le N_{182}\le 4\\ -5\le D_{180}\le -4\end{cases} \quad (D.64)$$

We find The first derivative $-4.24/\left(4k\right)\le\Pi_3'\left(c_u\right)\le-9.08\times10^{-8}/\left(4k\right)$ and the linearity is $-33.33\%\le\eta_{3c_u}\le-6.29\%$. Therefore, the relationship between c_u and Π_3 cannot be treated as negative linear.

$$N_{180}=\begin{pmatrix} \alpha\theta^2(Hkq_{EV2})^2(-2(\beta-1)c_h((\alpha-1)c_l(\lambda_h-1)(\lambda_l-1)+(c_{ntr}-1) \\ \cdot(\lambda_h\left(\alpha(-\beta)+\alpha-\alpha\lambda_l+\lambda_l+\alpha\beta\lambda_u-1\right)+(\alpha-1)\left(\lambda_l-1\right)\lambda_l \\ -\alpha\beta\lambda_u^2+\alpha\beta\lambda_u))-2(\alpha-1)c_l\left(c_{ntr}-1\right)((\beta-1)\lambda_h\left(\lambda_h-\lambda_l-1\right)-\lambda_l \\ +\beta\left(\lambda_l+1\right)\lambda_u-\beta\lambda_u^2)-\left(c_{ntr}-1\right)^2((\beta-1)(\alpha(\beta-1)+1)\lambda_h^2 \\ -2\left(\beta-1\right)\lambda_h\left(-\alpha\lambda_l+\lambda_l+\alpha\beta\lambda_u\right)+\left(\alpha-1\right)\lambda_l^2-2\left(\alpha-1\right)\beta\lambda_l\lambda_u \\ +\beta(\alpha\beta-1)\lambda_u^2)-(\beta-1)c_h^2(\alpha(\beta-1)-(\alpha-1)\left(\lambda_l-2\right)\lambda_l+\alpha\beta\lambda_u^2 \\ -2\alpha\beta\lambda_u+1)+(\alpha-1)c_l^2((\beta-1)(\lambda_h-2)\lambda_h-\beta\lambda_u^2+2\beta\lambda_u-1)) \\ +4\theta(Hkq_{EV2})((\alpha-\alpha\beta)c_h^2+\alpha(\beta-1)c_h(c_{ntr}(\lambda_h+1)+(\lambda_h-1)\delta_m \\ -2)-\alpha\beta c_{ntr}\lambda_h^2\delta_m+\alpha\beta c_{ntr}\lambda_h\delta_m+\alpha c_{ntr}\lambda_h^2\delta_m-\alpha c_{ntr}\lambda_h\delta_m-\alpha\beta c_{ntr}^2\lambda_h \\ +\alpha\beta c_{ntr}\lambda_h+\alpha c_{ntr}^2\lambda_h-\alpha c_{ntr}\lambda_h-\left(\alpha-1\right)c_l^2-\alpha c_{ntr}\lambda_l^2\delta_m+\alpha c_{ntr}\lambda_l\delta_m \\ +\left(\alpha-1\right)c_l\left(c_{ntr}\left(\lambda_l+1\right)+\left(\lambda_l-1\right)\delta_m-2\right)+c_{ntr}\lambda_l^2\delta_m-c_{ntr}\lambda_l\delta_m \\ -\alpha c_{ntr}^2\lambda_l+\alpha c_{ntr}\lambda_l+c_{ntr}^2\lambda_l-c_{ntr}\lambda_l+\alpha\beta c_{ntr}\delta_m\lambda_u^2 \\ +\alpha\beta\lambda_u\left(\left(c_{ntr}-1\right)c_{ntr}-\left(c_{ntr}+1\right)\delta_m\right)-c_{ntr}+\alpha\beta\lambda_h\delta_m-\alpha\lambda_h\delta_m \\ +\alpha\lambda_l\delta_m-\lambda_l\delta_m+\delta_m+1) \end{pmatrix} \quad \text{(D.65)}$$

$$N_{181}=\begin{pmatrix} 2\alpha\beta\theta^2(Hkq_{EV2})^2((c_{ntr}-1)(\alpha(\beta-1)\lambda_h^2-\alpha(\beta-1)\lambda_h(\lambda_u+1) \\ +(\alpha-1)\left(\lambda_l-1\right)\lambda_l+\lambda_u\left(\alpha\beta-\alpha\lambda_l+\lambda_l-1\right))+\alpha(\beta-1)c_h\left(\lambda_h-1\right) \\ \cdot\left(\lambda_u-1\right)+(\alpha-1)c_l\left(\lambda_l-1\right)\left(\lambda_u-1\right))-4\alpha\beta\theta(Hkq_{EV2}) \\ \cdot\left(c_{ntr}\left(\lambda_u+1\right)+\delta_m\left(\lambda_u-1\right)-2\right) \end{pmatrix} \quad \text{(D.66)}$$

$$N_{182}=\begin{pmatrix} \alpha\beta\theta(Hkq_{EV2})(\theta(Hkq_{EV2})(-\alpha(\beta-1)(\lambda_h-2)\lambda_h-(\alpha-1)\lambda_l^2 \\ +2(\alpha-1)\lambda_l)+(Hkq_{EV2})(\theta-\alpha\beta\theta)+4) \end{pmatrix} \quad \text{(D.67)}$$

$$D_{180}=\begin{pmatrix} \theta(Hkq_{EV2})(\alpha(\beta-1)\lambda_h^2-2\alpha(\beta-1)\lambda_h+(\alpha-1)(\lambda_l-2)\lambda_l \\ -\alpha\beta\lambda_u^2+2\alpha\beta\lambda_u)+\theta(-(Hkq_{EV2}))-4 \end{pmatrix} \quad \text{(D.68)}$$