Structure-specified H_∞ loop shaping control for balancing of bicycle robots: A particle swarm optimization approach

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Abstract

In this paper, the particle swarm optimization (PSO) algorithm was used to design the structure-specified H_∞ loop shaping controllers for balancing of bicycle robots. The structure-specified H_∞ loop shaping controller design normally leads to a complex optimization problem. PSO is an efficient meta-heuristic search which is used to solve multi-objectives and non-convex optimizations. A model-based systematic procedure for designing the particle swarm optimization-based structure-specified H_∞ loop shaping controllers was proposed in this research. The structure of the obtained controllers are therefore simpler. The simulation and experimental results showed that the robustness and efficiency of the proposed controllers was gained when compared with the proportional plus derivative (PD) as well as conventional H_∞ loop shaping controller. The simulation results also showed a better efficiency of the developed control algorithm compared to the Genetic Algorithm based one.

Keywords: Bicycle robot; Structure-specified controller; H_∞ loop shaping control; Particle swarm optimization; Gyroscopic stabilizer.

1. Introduction

The electrical bicycle is a good mean of transportation because of its advantages in term of environmental friendliness, light weight, and capability of traveling in narrow roads. However, the bicycle is unstable in nature. Without a proper control, it easily falls down. Hence, the development of a self-balancing bicycle is an interesting topic for many researchers. An exciting example of bicycle robots is Murata Boy robot which was developed in Japan in 2005 [1].

There are many methods used to control balancing of the bicycle such as the flywheel balancing by Beznos et al. in 1998 [2], Gallaspy in 1999 [3], and Suprapto in 2006 [4], the mass balancing by Lee and Ham in 2002 [5], and the steering balancing by Tanaka and Murakami in 2004 [6]. Among these methods, the flywheel balancing method which uses a spinning wheel as a gyroscopic stabilizer is a good choice because the response time is short and the system can be stable even at the stationary position. The balancing principle using flywheel can also be applied to many other systems which require the dynamics balancing during movement, for example, the balancing of a biped robot [7].

Various balancing control algorithms have been proposed, such as the nonlinear control by Beznos et al. in 1998 [2] and Lee and Ham in 2002 [5], the compensator design using root locus approach by Gallaspy in 1999 [3], and the PD control by Suprapto in 2006 [4]. However, these control algorithms are not robust, the bicycles cannot carry loads with variable weights and cannot work in disturbance environments. Therefore, the robust control algorithm is necessary for the real applications of bicycle robots.

The H_∞ loop shaping control is a well-known and effective method. It is a robust control technique that is suitable for the systems with unstructured uncertainties. This approach was firstly developed by McFarlane and Glover in 1992 [8], and has been used successfully in many practical applications [9-12]. However, in the conventional H$_\infty$ loop shaping control design, the obtained controllers are normally high order ones, and it is difficult to implement in the reality.

The particle swarm optimization (PSO) is one of the most recent developed evolutionary techniques initially proposed by Kennedy and Eberhart in 1995 [14]. PSO is based on a model of a social interaction among independent particles. It uses social knowledge to find the global maximum or minimum of a generic function. It is fast and easy to implement because of its oriented searching and simple calculation [15,16]. In this paper, PSO is used to search for parameters of a structure-specified H$_\infty$ loop shaping controller. The remaining of this paper is organized as follows. In Section 2, a prototype of bicycle robot which is used as a platform to test control algorithm is described. Section 3 explains a systematic procedure for designing the proposed controller. Simulation and experimental results are presented in Section 4 and Section 5. Section 6 finally concludes the paper.

2. Configuration and dynamics model of the bicycle robot

2.1 Configuration of the bicycle robot

A bicycle robot was developed at Mechatronics Laboratory, Asian Institute of Technology (AIT), Thailand, as a platform to test the performance of the developed control algorithm of the study. A detail description of the robot is available in [22].

2.2 Dynamics model of the bicycle robot

A complete dynamics model of a bicycle as derived by Sharp in 1971 [18] is complicated since the system has many degrees of freedom, and not suitable for control purpose. Dynamics model of a bicycle is basically based on equilibrium of gravity forces and centrifugal forces. The dynamics model of the bicycle robot in state-space is shown by the following equation. More detail in how to derive the dynamics model of the robot is available in [22].

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \quad (1)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{g(m_b h_b + m_f h_f)}{m_b h_b^2 + m_f h_f^2 + I_b + I_r} & 0 & \frac{I_p \omega}{m_b h_b^2 + m_f h_f^2 + I_b + I_r} & 0 \\ 0 & -\frac{I_p \omega}{I_r} & -\frac{B_m}{I_r} & \frac{5K_m}{I_r} \\ 0 & 0 & -\frac{K_e}{L} & -\frac{R}{L} \end{bmatrix} \quad (2)$$

$B = [0 \quad 0 \quad 0 \quad 1/L]'$, $C = [1 \quad 0 \quad 0 \quad 0]$, and $D = [0]$. (3)

3. PSO-based structure-specified H$_\infty$ loop shaping control

3.1 H$_\infty$ loop shaping control

H$_\infty$ loop shaping control method is an effective approach for designing a robust controller. Let define the nominal model of a system as P, and the shaped plant with a pre-compensator, W_1, and a post-compensator, W_2, as P_s, thus,

$$P_s = W_2 P W_1 = \tilde{M}^{-1}\tilde{N} = \begin{bmatrix} A_s & B_s \\ C_s & D_s \end{bmatrix} \quad (4)$$

where A_s, B_s, C_s, and D_s are matrices of the shaped plant in state-space representation, $\tilde{M}$ and $\tilde{N}$ are the normalized left coprime factors of P_s. By assuming that the shaped plant is perturbed by unstructured uncertainties ΔM and ΔN, the perturbed plant, P_Δ, thus becomes

$$P_\Delta = (\tilde{M} + \Delta M)^{-1}(\tilde{N} + \Delta N) \quad (5)$$

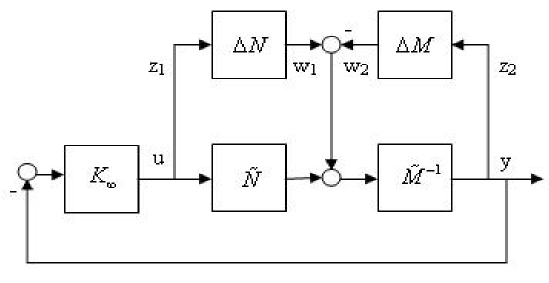

Figure 1. Robust stabilization with respect to the coprime factor uncertainties

It is proved from the small gain theorem that the shaped plant, P_s, is stable with all unknown but bound uncertainties $\|[\Delta M \quad \Delta N]\|_\infty < \varepsilon$ if and only if there exists an admissible controller, K_∞, such that

$$\|T_{zw}\|_\infty = \left\| \begin{bmatrix} I \\ K_\infty \end{bmatrix} \left(I + P_s K_\infty\right)^{-1} \tilde{M}^{-1} \right\|_\infty \le \gamma = 1/\varepsilon \quad (6)$$

Minimization of γ (maximization of ε) results in maximization of robustness of the system. A procedure called H_∞ loop shaping controller design was proposed by McFarlane and Glover [8] and further developed by Tang et al. [20]

3.2 Particle swarm optimization algorithm

PSO is one of the most recent evolutionary techniques. The method was developed by simulation of simplified social model, where each population is called a swarm. In PSO, multiple solutions are together and collaborate simultaneously. Each candidate, called a particle, flies through problem space to look for the optimal position, similar to food searching of bird swarm. A particle adapts its position based on its own knowledge, and knowledge of neighboring particles. The algorithm is initialized with a population of random particles. It searches for the optimal solution by updating particles in generations.

Let the search space be N-dimensional, then the particle i is represented by an N-dimensional position vector, $x_i = (x_{i1}, x_{i2}, ..., x_{iN})$. The velocity is represented also by an N-dimensional velocity vector, $v_i = (v_{i1}, v_{i2}, ..., v_{iN})$. The fitness of particles is evaluated by the objective function of the optimization problem. The best previously visited position of particle i is noted as its individual best position, $P_i = (p_{i1}, p_{i2}, ..., p_{iN})$. The position of the best individual of the whole swarm is noted as the global best position, $G = (g_1, g_2, ..., g_N)$. At each step of searching process, the velocity of particle and its new position are updated according to the following two equations [21].

$$\begin{aligned} v_i(k+1) &= w.v_i(k) + c_1.r_1.(P_i(k) - x_i(k)) \\ &+ c_2.r_2.(G(k) - x_i(k)) \end{aligned} \quad (7)$$

$$x_i(k+1) = x_i(k) + v_i(k) \quad (8)$$

where w, called inertia weight, controls the impact of previous velocity of the particle. r_1, r_2 are random variables in the range of [0,1]. c_1, c_2 are positive constant parameters called acceleration coefficients. The value of each component in v is limited to the range $[-v_{\max}, v_{\max}]$ to control excessive roaming of particles outside the search space.

3.3 Structure-specified H_∞ loop shaping controller design

3.3.1 Weighting functions selection

Since the algorithm is based on the H_∞ loop shaping method, the plant is firstly shaped by using the pre-compensator and post-compensator. In this paper, the lead/lag type compensators are used for weighting functions.

$$W_1 = K_1 \frac{s + \alpha_1}{s + \beta_1} \quad (9)$$

$$W_2 = K_2 \frac{s + \alpha_2}{s + \beta_2} \quad (10)$$

The shaped plant, thus, becomes

$$P_s = W_2 P W_1 \quad (11)$$

3.3.2 Structure-specified controller definition

The structure-specified controller, *K(s)*, is defined as follows.

$$K(s) = \frac{N_k(s)}{D_k(s)} = \frac{a_m s^m + a_{m-1} s^{m-1} + ... + a_0}{s^n + b_{n-1} s^{n-1} + ... + b_0} \quad (12)$$

The structure-specified controller can be in any forms such as PID, first order, second order controllers, etc., by selecting the suitable values of m and n.

3.3.3 Objective function definition

The structure-specified H_∞ loop shaping controller design problem can be defined as the problem of finding the parameters of all admissible controllers represented by equation (12) such that the H_∞ norm presented by equation (6), $\|T_{zw}\|_\infty$, is minimized.

Since $K_\infty = W_1^{-1} K(s) W_2^{-1}$, Then we have:

$$\|T_{zw}\|_\infty = \left\| \begin{bmatrix} I \\ K_\infty \end{bmatrix} (I + P_s K_\infty)^{-1} \tilde{M}^{-1} \right\|_\infty$$

$$= \left\| \begin{bmatrix} I \\ K_\infty \end{bmatrix} (I + P_s K_\infty)^{-1} \begin{bmatrix} I & P_s \end{bmatrix} \right\|_\infty$$

$$J_{\cos t} = \|T_{zw}\|_{\infty} = \left\| \begin{bmatrix} I \\ W_1^{-1}K(s)W_2^{-1} \end{bmatrix} (I + P_s W_1^{-1} K(s) W_2^{-1})^{-1} \begin{bmatrix} I & P_s \end{bmatrix} \right\|_{\infty} \tag{13}$$

The equation (13) is defined as the objective function of the optimization problem and it can be easily evaluated using the robust control toolbox in MATLAB.

3.3.4 Particle swarm optimization-based design

Once an objective function and a structure of the controller are defined, the procedure, using PSO to solve this optimization problem, is described as followings:

•**Step1**: Set particle *i* to $x_i = (x_{i1}, x_{i2}, ..., x_{iN}) = (a_0, a_1, ..., b_0, b_1, ..)$, the number of parameters of the controller in equation (12) is the dimension of particle, $N = m + n + 1$. Define maximum number of iterations as GenMax.

•**Step 2**: Initialize a random swarm of H particles as $[x_1 \quad x_2 \quad ... \quad x_H]$, when the swarm size is set to H.

•**Step 3**: For each generation, evaluate objective function for each particle using the objective function shown by equation (13), and determine individual best, $P_i(k)$, and global best, $G(k)$.

•**Step 4**: Update the velocity of particle and its new position using equations (7) and (8).

•**Step 5**: When the maximum number of iterations is arrived, stop the algorithm. Otherwise go to Step 3.

4. Simulation results

The nominal transfer function of bicycle robot is described in [22] as follows.

$$P = \frac{\theta(s)}{U(s)} = \frac{4887}{s^4 + 683.3s^3 + 1208s^2 + 109700s - 6949} \tag{14}$$

where U is the input voltage to the DC motor that controls the flywheel control axis, θ is the output lean angle of Bicyrobo. The weighting function, W_1, is selected by some trials for shaping the plant. W_2 is selected as identity matrix with an assumption that sensor noise is negligible. W_1 and W_2 are shown by the following equations.

$$W_1 = 40.6\frac{s + 0.09}{s + 0.085} \text{ and } W_2 = 1 \tag{15}$$

The full order controller is obtained as follows

$$K(s) = \frac{1275s^5 + 8.695e5s^4 + 5.151e5s^3 + 1.359e8s^2 + 2.435e7s + 1.091e6}{s^6 + 715.7s^5 + 2.355e4s^4 + 2.789e5s^3 + 3.802e6s^2 + 6.591e5s + 2.872e4}$$
(16)

The full order controller represented by equation (16) is sixth order, which is difficult to implement in reality.

4.1 First order controller design

The first order controller is selected as a structure-specified controller of the following form:

$$K_1(s) = \frac{a_0}{s + b_0} \tag{17}$$

The proposal algorithm is run with ten trials, and in all cases the same value of cost function $J_{\cos t} = \gamma_{opt} =$ 1.8365 ($\varepsilon_{opt} =$ 0.5445) is obtained. The obtained controller is shown by equation (18).

$$K_1(s) = \frac{135.2}{s + 4.63} \tag{18}$$

4.2 Second order controller design

A second order controller is selected as

$$K_2(s) = \frac{a_1 s + a_0}{s^2 + b_1 s + b_0} \tag{19}$$

The proposal algorithm is run with ten trials, and the controller as shown by equation (20) with $J_{\cos t} = \gamma_{opt} = 1.798$ ($\varepsilon_{opt} = 0.55617$) is obtained.

$$K_2(s) = \frac{129.7s + 499.6}{s^2 + 6.835s + 16.183} \tag{20}$$

4.3 Comparison

The step responses of the closed loop system using PD, first order, and full order H_∞ loop shaping controllers are compared in this Section. By tuning parameters K_P and K_D of PD controller, a satisfied step response with about the same response time as the proposed structure-specified controllers is obtained. This PD controller is expressed as (21):

$$K_{PD}(s) = 30 + 2.5s \tag{21}$$

The comparison is shown in Fig. 2 and Fig. 3. These simulations show that the step responses of the system using conventional H_∞ loop shaping controller and the proposed structure-specified H_∞ loop shaping controllers are similar. They are both better than the system using the PD controller.

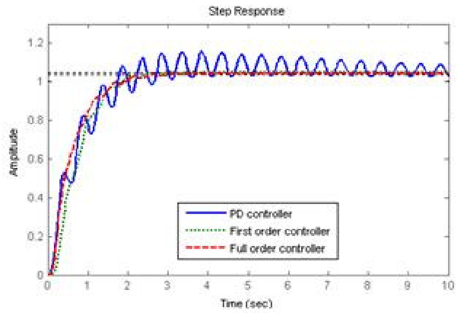

Figure 2. Step responses using PD, first order, and full order H_∞ loop shaping controllers

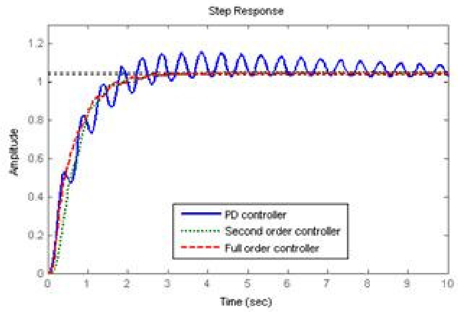

Figure 3. Step responses using PD, second order, and full order H_∞ loop shaping controllers

5. Experimental Results

Various experiments were conducted to evaluate the balancing performance and robustness of the proposed controllers. The first set of the experiments was tested on the system using the PD and the proposed first order controllers at a zero forward speed of bicycle robot without applied masses. The results showed that the proposed controller had a better balancing performance than the conventional PD controller.

Figure 4. Stationary experiment

Figure 5. Moving forward experiment

In order to show that the proposed controllers is robust to the parameter variations, the iron masses of 4kg and 8kg were applied on the system at a zero forward speed of the bicycle robot. The experiments on the bicycle robot using the proposed first order controller were tested. In both cases, the system was stable against these parameter variations.

6. Conclusion

The PSO-based structure-specified H_∞ loop shaping controller design method to control balancing of bicycle robots was successfully developed and presented in this paper. The first order and second order controllers were designed with the obtained stability margins ε_{opt} are 0.5445 and 0.55617 respectively. The simulation results showed that the performance of the closed loop system using the proposed controllers and the full order controller are similar, and the performance of these controller are better than the closed loop system using the PD controller. The simulation results also showed that the closed loop system is robustly stable to parameter variations using the proposed controllers while it is unstable if using the PD controller. The experimental results without the masses applied on the bicycle robot proved that the proposed first order controller achieves a better balancing performance in which the lean angles less are than 0.5 degrees, while the maximum lean angle of the PD controller is about 1 degree. The experimental results with the masses of 4kg and 8kg applied on the system using the proposed first order controller showed that the system attained a good balancing performance and the robustness in which the obtained lean angles is less than 1 degree with the above loading changes.

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