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APPLICATION OF SUBSET SIMULATION IN RELIABILITY ESTIMATION OF UNDERGROUND PIPELINES

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ABSTRACT

This paper presents a computational framework for implementing an advanced Monte Carlo simulation method, called Subset Simulation (SS) for time-dependent reliability prediction of underground flexible pipelines. The SS can provide better resolution for low failure probability level of rare failure events which are commonly encountered in pipeline engineering applications. Random samples of statistical variables are generated efficiently and used for computing probabilistic reliability model. It gains its efficiency by expressing a small probability event as a product of a sequence of intermediate events with larger conditional probabilities. The efficiency of SS has been demonstrated by numerical studies and attention in this work is devoted to scrutinise the robustness of the SS application in pipe reliability assessment and compared with direct Monte Carlo simulation (MCS) method. Reliability of a buried flexible steel pipe with time-dependent failure modes, namely, corrosion induced deflection, buckling, wall thrust and bending stress has been assessed in this study. The analysis indicates that corrosion induced excessive deflection is the most critical failure event whereas buckling is the least susceptible during the whole service life of the pipe. The study also shows that SS is robust method to estimate the reliability of buried pipelines and it is more efficient than MCS, especially in small failure probability prediction.

***Key Words*:** Subset Simulation; Probability of failure; Markov Chain Monte Carlo Simulation; Reliability; Failure modes; Underground Pipes

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1. INTRODUCTION

Structural reliability algorithms have been received greater attention over the world, though prediction techniques of small failure probabilities are very few till now. In recent years, attention has been focused on reliability problems with complex system characteristics in high dimensions (i.e., with a large number of uncertain or random variables) (Schueller and Pradlwarter, 2007). Prediction of small failure probabilities is one of the most important and challenging computational problems in reliability engineering (Zuev et al, 2012). The probabilistic assessment of engineering systems may involve a significant number of uncertainties in their behaviour. To implement probabilistic assessment for an engineering system, main difficulties arise from: (1) the relationship between the random variables, (2) too many random variables involved, (3) information about rare scenarios and (4) many interactive response variables in the description of performance criteria.

Like other engineering systems, reliability analysis of buried pipeline systems are characterised by a large number of degrees of freedom, time-varying and response dependent nonlinear behaviour. In the presence of uncertainty, the performance of an underground pipeline can be quantified in terms of 'performance margin' with respect to specified design objectives. In reliability engineering, 'performance margin' is denoted as reliability index, probability of failure, safety margin, etc. Failure events in pipe reliability analysis can be formulated as exceedance of a critical response variable over a specified threshold level. By predicting pipeline reliability, the safe service life can be estimated with a view to prevent unexpected failure of underground pipelines by prioritising maintenance based on failure severity and system reliability (Tee and Li, 2011; Khan et al, 2013).

There is no general algorithm available to estimate the reliability of a buried pipeline system. The pipeline reliability is usually given by an integral over a high dimensional uncertain parameter space. Methods of reliability analysis such as first order reliability method (FORM), second-order reliability method (SORM), point estimate method (PEM), Monte Carlo simulation (MCS), gamma process, probability density evolution method (PDEM), etc. are available in literature (Sivakumar Babu and Srivastava, 2010; Tee et al, 2014; Mahmoodian et al, 2012; Fang et al, 2013a, 2013b). In this context, a robust uncertainty propagation method whose applicability is insensitive to complexity nature of the problem is most desirable. Many methods are

inefficient when there are a large number of random variables and/or failure probabilities are small. Moreover, some methods need a large number of samples which is time-consuming.

Advanced Monte Carlo methods, often called 'variance reduction techniques' have been developed over the years. In this respect, a promising and robust approach is Subset Simulation (SS) which is originally developed to solve the multidimensional problems of engineering structural reliability analysis (Au and Beck 2001; Au et al, 2007). A structural system fails when the applied load or stress level exceeds the capacity or resistance. SS is well suited for quantitative analysis of functional failure systems, where the failures are specified in terms of one or more safety variables, e.g., temperatures, pressures, flow rates, etc. In the SS approach, the functional failure probability is expressed as a product of conditional probabilities of adaptive chosen intermediate events. The problem of evaluating small probabilities of functional failures is thus tackled by performing a sequence of simulations of more frequent events in their conditional probability spaces; then the necessary conditional samples are generated through successive Markov Chain Monte Carlo (MCMC) simulations in a way to gradually populate the intermediate conditional regions until the final functional failure region is reached (Zio and Pedroni, 2008).

Many researchers, such as Au and Beck (2001), Au et al (2007), Ching et al (2005), Song et al (2009) and Zhao et al (2011) have used SS in reliability analysis of engineering structures, such as bridges and buildings. However, according to authors' knowledge, no such work has been found in the literature on time-dependent reliability analysis of buried pipeline systems. This paper focuses on application of SS for computing time-dependent reliability of flexible buried metal pipelines. Failure probabilities for corrosion induced multi-failure events, namely deflection, buckling, wall thrust and bending have been predicted in this study. Firstly, the SS is applied for estimating the failure probabilities for each failure case individually and then due to multi-failure modes, an upper and lower bounds of failure probabilities are predicted as a series system. Besides that, coefficients of variation (COVs) and a sensitivity analysis of pipe failure due to corrosion induced deflection, as an example of failure event, have also been assessed to illustrate the robustness and effectiveness of SS method. The application of SS method is verified with respect to the standard MCS.

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2. FORMULATION FOR PIPE FAILURE

A system failure occurs when a system does not meet its requirement. The number of potential failure modes is very high for buried pipe structures. This is true in spite of the simplifications imposed by assumptions such as having a finite number of failure elements at given points of the structure and only considering the proportional loadings. It is, therefore, important to have a method by which the most critical failure modes can be identified. When the residual ultimate strength of a buried pipeline is exceeded, breakage becomes imminent and the overall reliability of the pipe is reduced. The critical failure modes are those contributing significantly to the reliability of the system at the chosen level. The failure criteria adopted here are due to loss of structural strength of pipelines by corrosion through reduction of the pipe wall thickness which then lead to pipe failure by excessive deflection, buckling, wall thrust and bending.

2.1 Corrosion of metal pipes

Buried pipes are made of plastic, concrete or metal, e.g. steel, galvanized steel, ductile iron, cast iron or copper. Plastic pipes tend to be resistant to corrosion. Damage in concrete pipes can be attributed to biogenous sulphuric acid attack (Tee et al, 2011; Alani et al, 2014). On the other hand, metal pipes are susceptible to corrosion. Metal pipe corrosion pit is a continuous and variable process. Under certain environmental conditions, metal pipes can become corroded based on the properties of the pipe, soil, liquid properties and stray electric currents. The corrosion pit depth can be modelled with respect to time as shown in Eq. (1) (Ahammed and Melchers, 1994; Sadiq et al, 2004).

The corrosion pit depth,

$$D_T = kT^n \tag{1}$$

where D_T is pit depth and T is exposure time. The parameters k and n are corrosion empirical constants and depend on pipe materials and surrounding environments.

For a plain pipe, due to reduction of wall thickness given by Eq. (1), the moment of inertia of pipe wall per unit length, I and the cross-sectional area of pipe wall per unit length, A_s can be defined as below (Watkins and Anderson, 2000; Tee and Khan, 2012).

Moment of inertia, $I = (t - D_T)^3 / 12 \tag{2}$

Cross-sectional area, $A_s = t - D_T$ (3)

where t is the thickness of the pipe wall. The pipe is assumed as a thin-walled pipe with $D/t > 10$ where D is mean diameter. The corrosion empirical constants (k and n) and pipe wall thickness (t) are considered as random variables.

2.2 Pipe failure criteria

In this paper, the chosen dominating failure criteria of flexible pipes are characterised by corrosion induced deflection, buckling, wall thrust and bending stress.

Deflection

The performance of flexible pipes in its ability to support load is typically assessed by measuring the deflection from its initial shape. Deflection is quantified in terms of the ratio of the horizontal (or vertical) increased diameter to the original pipe diameter. The critical or allowable deflection for flexible pipe, Δy_{cr} is normally determined as 5% - 7% of inside diameter of pipe (Gabriel, 2011). The actual deflection, Δ_y can be calculated as shown in Eq. (4) (BS EN 1295:1, 1997; Watkins and Anderson, 2000). $Z(X) = \Delta y_{cr} - \Delta_y = 0$ is the limit state function for this failure mode where $Z(X) < 0$ represents failure state and $Z(X) > 0$ indicates a safe state.

$$\Delta_y = \frac{K_b(D_L W_c + P_s)D}{\left(\dfrac{8EI}{D^3} + 0.061E'\right)}$$ (4)

where K_b is deflection coefficient, D_L is deflection lag factor, D is mean diameter $= D_i + 2c$ where D_i is inside diameter and c is distance from inside diameter to neutral axis, E is modulus of elasticity of pipe material and E' is modulus of soil reaction $= \dfrac{k' E_s(1-v_s)}{(1+v_s)(1-2v_s)}$ where E_s is modulus of soil and k' is a numerical value depends on poison's ratio, v_s (Babu and Rao, 2005).

The loads acting on the pipe are governed by the term $D_L W_c + P_s$ where W_c is soil load and P_s is live load. Soil load can be calculated by multiplying unit weight of soil (γ_s) by the height of soil on the top of pipe invert (H) (Sarplast, 2008).

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Buckling pressure

Buckling is a premature failure in which the pipe is not able to maintain its initial circular shape and the structure becomes unstable at a stress level that is well below the yield strength of the structural material (Sivakumar Babu and Srivastava, 2010). The actual buckling pressure should be less than the critical buckling pressure for the safety of structure. The actual buckling pressure, p and the critical buckling pressure, p_{cr} can be calculated as shown in Eqs. (5) and (6), respectively (AWWA, 1999). $Z(X) = p_{cr} - p = 0$ is the limit state function for this failure mode where $Z(X) < 0$ represents failure state and $Z(X) > 0$ indicates a safe state.

$$p = R_w \gamma_s + \gamma_w H_w + P_s \tag{5}$$

$$p_{cr} = \sqrt{\left(32 R_w B' E_s \frac{EI}{D^3} \right)} \tag{6}$$

where R_w is water buoyancy factor = $1 - 0.33$ (H_w/H), γ_w is unit weight of water, H_w is height of groundwater above the pipe and B' is empirical coefficient of elastic support $= 1/(1 + 4e^{-0.213H})$.

Wall stress/thrust

If the buried depth is not enough then the pipe wall can crush due to earth and surface loading. Buried depth should be sufficient to avoid the crushing of the side wall. Two wall thrust analyses are required: (1) accounts both the dead load and live load and employs the short term material properties throughout the procedure, (2) accounts only the dead load and employs the long-term material properties throughout the process. Then, the most limiting value is used for reliability analysis. The critical and actual wall thrust can be estimated as shown in Eqs. (7) and (8), respectively (Hancor, 2009). $Z(X) = T_{cr} - T_a = 0$ is the limit state function for this failure mode where $Z(X) < 0$ represents failure state and $Z(X) > 0$ indicates a safe state.

The critical wall thrust, $T_{cr} = F_y A_s \phi_p$ (7)

where F_y is the minimum tensile strength of pipe and ϕ_p is capacity modification factor for pipe.

The actual wall thrust, $T_a = (W_A + P_s C_L + P_w)(D_0 / 2)$ (8)

where D_o is outside diameter and C_L is live load distribution coefficient. The loads acting on the pipe considered in wall thrust analysis are soil arch load W_A, live load P_s and hydrostatic pressure P_w. Hydrostatic pressure P_w can be calculated by multiplying unit weight of water (γ_w) by the height of groundwater above the pipe (H_w) whereas soil arch load W_A can be calculated by multiplying geostatic load (P_{sp}) by the vertical arching factor (V_{AF}) where P_{sp} $= \gamma_s (H + 0.11 \times 10^{-7} (D_0))$, $V_{AF} = 0.76\text{-}0.71((S_h - 1.17)/(S_h + 2.92))$, S_h is hoop stiffness factor $= \varphi_s M_s R / EA_s$, φ_s is soil capacity modification factor, M_s is secant constrained soil modulus and R is effective radius of pipe.

Bending

A pipe subjected to increasing pure bending will fail as a result of increased ovalisation of the cross section and reduced slope in the stress-strain curve. Under the effect of earth and surface loads, the buried pipe may bend through pipe wall. The allowable bending stress σ_{cr} is the long term tensile strength of the pipe material whereas the allowable strain ε_{cr} for flexible pipes is 0.15% to 2% (Mohr, 2003). The bending stress and strain are important to ensure that these are within material capability. Actual bending stress and bending strain can be calculated using Eqs. (9) and (10), respectively (Gabriel, 2011). $Z(X) = \sigma_{cr} - \sigma_b = 0$ or $Z(X) = \varepsilon_{cr} - \varepsilon_b = 0$ is the limit state function for this failure mode where $Z(X) < 0$ represents failure state and $Z(X) > 0$ indicates a safe state.

Bending stress, $\sigma_b = 2D_f E \Delta_y y_0 / D^2$ (9)

Bending strain, $\varepsilon_b = 2D_f \Delta_y y_0 / D^2$ (10)

where D_f is shape factor and y_0 is distance from centroid of pipe wall to the furthest surface of the pipe. Δ_y is pipe deflection which can be calculated as shown in Eq. (4). In this study, K_b, γ_s, E, E_s, P_s and H are assumed as random variables.

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3. RELIABILITY PREDICTION

3.1 Basic equations for Subset Simulation

Subset Simulation is an adaptive stochastic simulation procedure for efficiently computing a small failure probability. For simplification, F is denoted as the failure event as well as its corresponding failure region in the uncertain parameter space. Given a failure event F, let $F_1 \supset F_2 \supset F_3 .... \supset F_m = F$. If the failure of a system is defined as an exceedance of one uncertain demand Y over a given capacity y, that is $F = (Y > y)$, then a sequence of decreasing failure events can simply be defined as $F_i = \{Y > y_i\}$ where $y_1 < y_2 < y_3 < \cdots\cdots < y_m = y$ and $i = 1, 2, 3,..., m$ where m is the number of conditional events. In this study, Y is the actual value of structural performance such as corrosion-induced deflection, buckling, wall thrust or bending stress whereas y represents the allowable or critical limit for the considered failure modes. A conceptual illustration of the SS method is presented in Figure 1 for a two-dimensional case (Song et al, 2009).
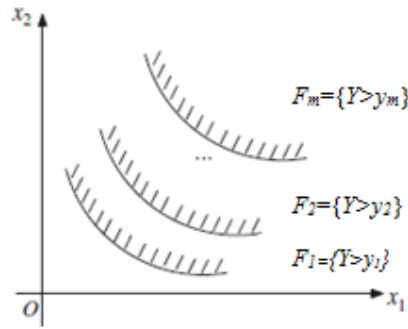

Figure 1: Illustration of failure events in SS method

The probability of failure (P_f) can be calculated based on the above sequence of failure domains (or subsets) which enables computation of P_f as a product of conditional probabilities $P(F_1)$ and $P(F_{i+1} | F_i)$ as follows (Schueller and Pradlwarter, 2007; Phoon, 2008).

$$P_f = P(F_m) = P(F_m | F_{m-1})P(F_{m-1} | F_{m-2})......P(F_2 | F_1)P(F_1) \qquad (11)$$
$$= P(F_1)\prod_{i=1}^{m-1} P(F_{i+1} | F_i)$$

When $P(F_1)$ is denoted by P_1 and $P(F_i \mid F_{i-1})$ for $i = 2,3,....m$ is denoted by P_i, Eq. (11) expresses the failure probability as a product of conditional probabilities P_1 and P_i $(i = 2,3,...,m)$. In the first step, it is natural to compute conditional failure probabilities based on an estimator similar to Eq. (12), which requires simulation of samples according to the conditional distribution θ that lies in F_i (Au and Beck, 2001). The probability P_1 can be determined by application of the direct MCS simulation as shown in Eq. (12).

$$P_1 = \frac{1}{N_1} \sum_{k=1}^{N_1} I_{F_1}(\theta_k^{(1)})$$

(12)

where $\theta_k^{(1)} (k = 1,2,3,......,N_1)$ are independent and identically distributed samples simulated according to probability density function (PDF) q. $I_{F_1}(\theta_k^{(1)})$ is an indicator function, when $\theta_k^{(1)} \in F_1$, $I_{F_1}(\theta_k^{(1)})=1$, otherwise 0.

The conditional distribution of θ lies in F_i, that is $q(\theta \mid F_i) = q(\theta)I_{F_i}(\theta)/P(F_i)$. Computing the conditional probabilities, Markov Chain Monte Carlo (MCMC) simulation provides a powerful method for generating conditional samples on the failure region (Au and Beck, 2001; Au and Beck, 2003). With the application of the MCMC simulation by the modified Metropolis-Hastings algorithm, samples can be generated as follows.

$$P_i = \frac{1}{N_i} \prod_{k=1}^{N_i} I_{F_i}(\theta_k^{(i)}) \qquad (i = 2,3,....,m)$$

(13)

where $\theta_k^{(i)} (k = 1,2,3,......,N_i; i = 2,3,....,m)$ are independent and identically distributed conditional samples. $I_{F_i}(\theta_k^{(i)})$ is an indicator function which is equal to 1 when $\theta_k^{(i)} \in F_i$, otherwise 0.

Based on Eqs. (12) and (13), Eq. (11) can be rewritten as follows

$$P_f = \frac{1}{N_1} \sum_{k=1}^{N_1} I_{F_1}(\theta_k^{(1)}) \prod_{i=1}^{m-1} \frac{1}{N_i} \prod_{k=1}^{N_i} I_{F_i}(\theta_k^{(i)})$$

(14)

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On the basis of reliability analysis using SS, the failure probability P_f can be transformed into a set of conditional failure probabilities P_i (i =1, 2, 3,…,m). Based on Eq. (14), the partial derivative of the failure probability with respect to distributional parameter α (the mean μ or the standard deviation σ) of normal random variables can be obtained, which is so-called reliability sensitivity as shown in Eq. (15) (Song et al, 2009).

$$\frac{\partial P_f}{\partial \alpha} = \sum_{i=1}^{m} \frac{P_f}{P_i} \frac{\partial P_i}{\partial \alpha} \tag{15}$$

Reliability sensitivity analysis can reflect the significance of the distributional parameter with respect to the failure probability. According to sample means, reliability sensitivity of Eq. (15) for each variable can be obtained using Eq. (16) and (17) as follows (Song et al, 2009).

$$\frac{\partial (P_1)}{\partial \alpha} = \frac{1}{N_1} \sum_{k=1}^{N_1} \left[\frac{I_{F_1}(\theta_k^{(1)}) \partial q(\theta_k^{(1)})}{q(\theta_k^{(1)}) \partial \alpha} \right] \tag{16}$$

$$\frac{\partial (P_i)}{\partial \alpha} = \frac{1}{N_i} \sum_{k=1}^{N_i} \left\{ I_{F_i}(\theta_k^{(i)}) \left[\frac{1}{q(\theta_k^{(i)})} \frac{\partial q(\theta_k^{(i)})}{\partial \alpha} - \sum_{j=1}^{i-1} \frac{1}{P_j} \frac{\partial P_j}{\partial \alpha} \right] \right\} \tag{17}$$

3.2 Methodology

Subset Simulation expresses the failure probability as a product of larger conditional failure probabilities for a sequence of intermediate failure events, thereby converting a rare event simulation problem into a sequence of more frequent ones (Au et al, 2007). During the simulation process, the conditional samples are generated from specially designed Markov chains (MC), so that they gradually populate each intermediate failure region until they reach the final target failure region (Au and Beck, 2001). In this study, the intermediate threshold values are chosen adaptively in such a way that the estimated conditional probabilities are equal to a fixed value which is $p_0 = 0.1$ (Au and Beck, 2001; Au and Beck, 2003; Zio and Pedroni, 2008).

Procedure of SS algorithm for adaptively generating samples corresponding to specified target probabilities can be summarised as follows.

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1. Generate N samples $\theta_{0,k}(k=1,2,.....,N)$ by direct MCS, i.e., from the original PDF $q(.)$. The subscript '0' denotes the samples corresponding to conditional level 0;

2. Set $i = 0$;

3. Compute the corresponding response variables $Y_{i,k}(k=1,2,.....,N)$;

4. The value of y_{i+1} is chosen as the $(1-p_0)N$ th value in the ascending order of $Y_{i,k}(k=1,2,.....,N)$ (from step 3), so that the sample estimate of $P(F_{i+1}) = P(Y > y_{i+1})$ is always equal to p_0. p_0 and N are chosen such a way that p_0N is always an integer;

5. If $y_{i+1} \geq y_m$, proceed to step 10 below;

6. On the other hand, if $y_{i+1} < y_m$, with the choice of y_{i+1} performed at step 4 above, identify the p_0N samples $\theta_i^u(u=1,2,.....,p_0N)$ among $\theta_{i,k}(k=1,2,.....,N)$ whose response Y lies in $F_{i+1} = \{Y > y_{i+1}\}$, these samples are at 'conditional level $i + 1$' and distributed as $q(.\,|\,F_{i+1})$;

7. Starting from each one of the samples $\theta_i^u(u=1,2,.....,p_0N)$ (from step 6), use MCMC simulation to generate $(1-p_0)N$ additional conditional samples distributed as $q(.\,|\,F_{i+1})$, so that there are a total of N conditional samples $\theta_{i+1,k}(k=1,2,.....,N) \in F_{i+1}$, at conditional level $i+1$;

8. Set $i \leftarrow i+1$;

9. Return to step 3 above;

10. Stop the algorithm.

Note that the total number of samples employed is $N_T = N + (m-1)(1-p_0)N$.

3.3 Advantages of Subset Simulation

Estimating small failure probabilities to precisely assess the risk involved in a system remains quite a challenging task in structural reliability engineering. FORM, SORM or PEM are suitable solutions to estimate reliability of large-scale systems. Due to their inherent assumptions, these methodologies are sometimes lead to incorrect results which are involved with multiple design points and/or non smooth failure domains. On the other hand, MCS is a traditional simulation algorithm to compute failure probabilities in structural systems, which in spite of being robust to solve the problem; it becomes computationally expensive where small failure probabilities to be calculated, since it requires a large number of evaluations of the system to achieve a suitable accuracy.

SS requires much less samples to achieve a given accuracy. It can be used to obtain conditional samples in each simulation level to compute efficiently the probabilities related to rare events in reliability problems with complex system characteristics and with a large number of uncertain or random variables in failure events. Choosing the intermediate failure events $F_i(i = 1,2,3,...,m)$ appropriately, the conditional probabilities involved in Eq. (11) can be made sufficiently by subset simulation process (Ching et al, 2005). For example, probability of failure $P_f = 10^{-4}$ is too small for efficient estimation by direct Monte Carlo simulation. However, the conditional probabilities, which are the order of 0.1, can be evaluated efficiently by simulation because the failure events are more frequent as supported by the results in Figure 8. The problem of simulating the rare events in the original probability space is thus replaced by a sequence of simulations of more frequent events in the conditional probability spaces.

4 NUMERICAL EXAMPLE

The time-dependent structural reliability for an underground flexible metal pipe has been predicted in this example, where pipe failure probability, sensitivity and COV analysis are conducted by applying SS and MCS. Calculations are presented for a buried steel pipe under a heavy roadway subject to corrosion and external loadings. A typical pipe section is shown in Figure 2. Numerical values are based on industrial practice and have been obtained from the literature (Ahammed and Melchers, 1997; Sadiq et al, 2004). The materials properties and parameters are listed in Table 1. There are 9 random variables where the means and COVs are listed in Table 2.

The pipe corrosion rate is modelled using Eq. (1). Assuming the change of pipe surface due to corrosion is uniform over the entire surface area. It is assumed that the pipe is thin-walled circular (plain) and placed above ground water level, i.e. $H_w = 0$. According to the references by Ahammed and Melchers (1997) and Sadiq et al (2004), most of the random variables in Table 2 are normally distributed as these variables are found symmetric around their mean. However, the deflection coefficient (K_b) accounts for the bedding support which varies with the bedding angle and this variable's logarithm is found normally distributed.
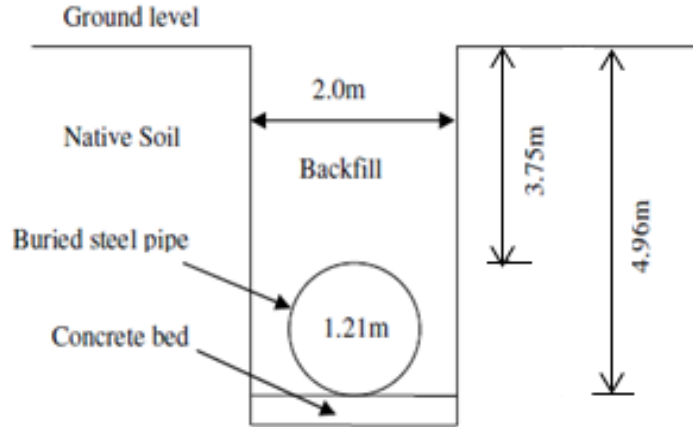
Figure 2: Geometrical details of the buried steel pipe section (not to scale)

Table 1: Material properties and parameters

Symbol description	Value
Buoyancy factor, R_w	1.00
Trench width, B_d	2.00 m
Outside pipe diameter, D_o	1.231 m
Inside pipe diameter, D_i	1.189 m
Deflection lag factor, D_L	1
Soil constrained modulus, M_s	2.02×10^3 kPa
Shape factor, D_f	4.0
Allowable deflection, Δy_{cr}	5% of D_i
Capacity modification factor for pipe, ϕ_p	1.00
Capacity modification factor for soil, ϕ_s	0.90
Poisson's ratio, ν_s	0.3
Live load distribution coefficient, C_L	1
k'	1.5
Allowable strain	0.2%
Minimum tensile strength of pipe, F_y	450 MPa

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Table 2: Statistical properties of random variables

Material properties	Mean (μ)	COV (%)	Standard Deviation (σ)
Elastic modulus of pipe, E	213.74×10^6 kPa (Normal)	1.0	2.1374×10^6 kPa
Backfill soil modulus, E_s	10^3 kPa (Normal)	5.0	50 kPa
Unit of weight of soil, γ_s	18.0 kN/m^3 (Normal)	2.5	0.45 kN/m^3
Wheel load (Live load), P_s	80.0 kPa (Normal)	3.0	2.4 kPa
Multiplying constant, k	2.0 (Normal)	10.0	0.1
Exponential constant, n	0.3 (Normal)	5.0	0.015
Thickness of pipe, t	0.021 m (Normal)	1.0	0.00021 m
Height of the backfill, H	3.75 m (Normal)	1.0	0.00375 m
Deflection coefficient, K_b	0.11 (Lognormal)	1.0	0.0011

5   RESULTS AND DISCUSSION

In the case of buried pipes, the assessment of P_f on yearly basis is useful because it enables calculation of reliability over time. The P_f for corrosion induced excessive deflection, buckling, wall thrust and bending stress with respect to time have been estimated using SS and MCS with material properties and random variables presented in Tables 1 and 2. In SS, the P_f is predicted as a sum of the sub failure events within each failure mode. The simple but pivotal idea behind SS is that a small failure probability can be expressed as a product of larger conditional failure probabilities for some intermediate failure events, suggesting the possibility of converting a problem involving rare events simulation into a sequence of problems involving more frequent events. SS is applied in this study with a conditional failure probability at each level equal to $p_0 = 0.1$. The total number of samples, N used in MCS is 10^6 for all the failure modes whereas SS needs 500 samples to achieve the similar accuracy of the results. The results presented in Figures 3 to 8 are in log scale of P_f to scrutinise the effectiveness of SS method in the region of small failure probability (< 0.1).
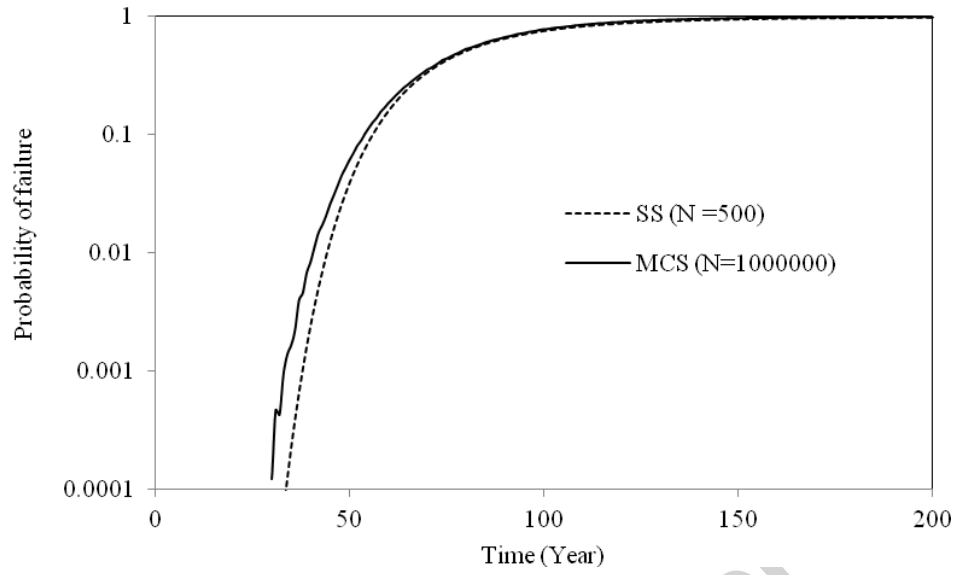
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Figure 3: Probability of failure due to corrosion induced deflection

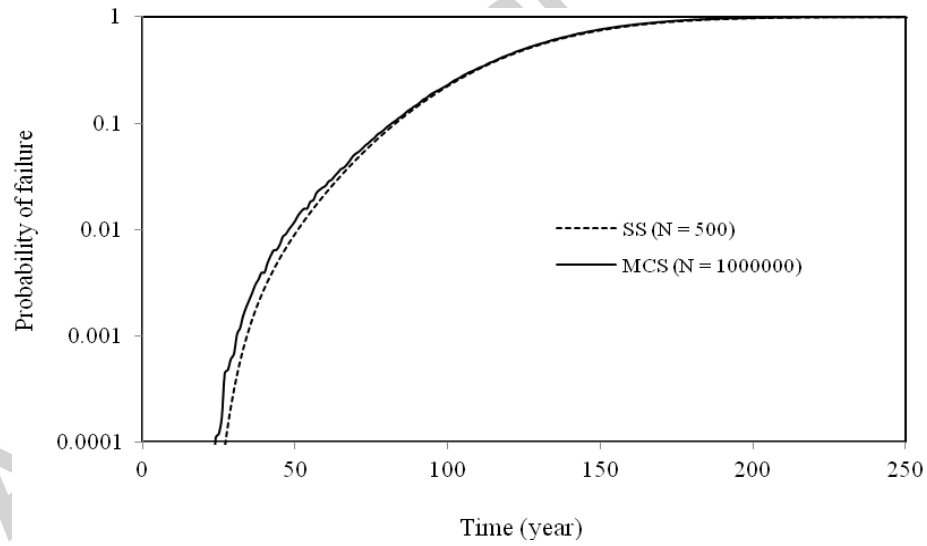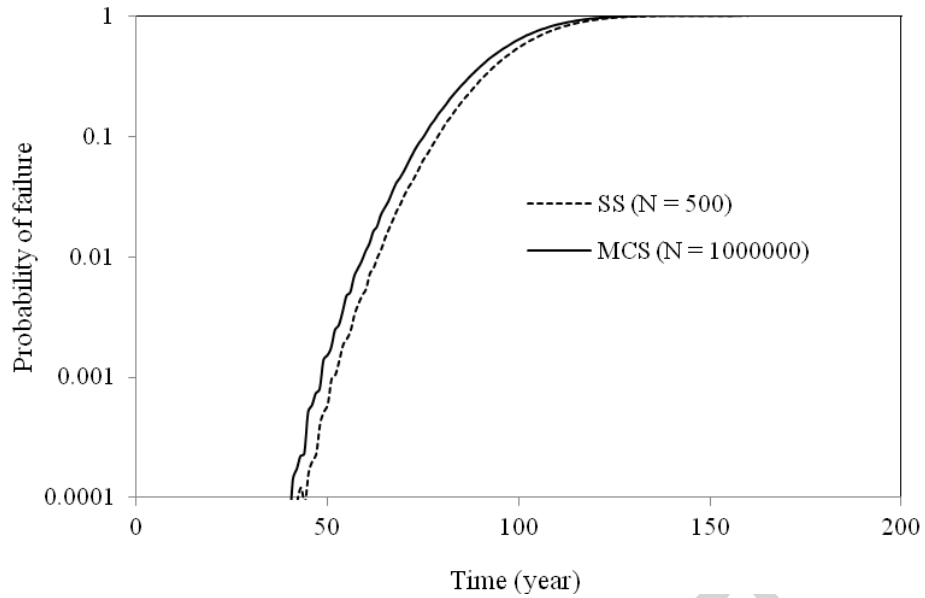Figure 4: Probability of failure due to corrosion induced buckling

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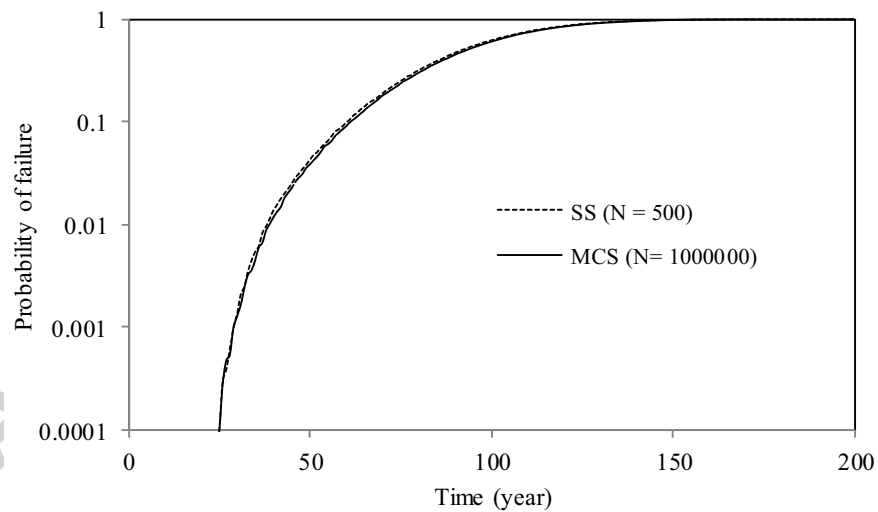Figure 5: Probability of failure due to corrosion induced wall thrust

Figure 6: Probability of failure due to corrosion induced bending stress

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As shown in Figures 3 to 6, the results reveal that corrosion-induced excessive deflection is the most critical failure event whereas buckling has the lowest P_f during the whole service life of the pipe. Considering the failure probability of 0.1 (10%) as a threshold level for the safe service life (Babu and Srivastava, 2010), the study illustrates that the safe service life in the worst case scenario is about 50 years.
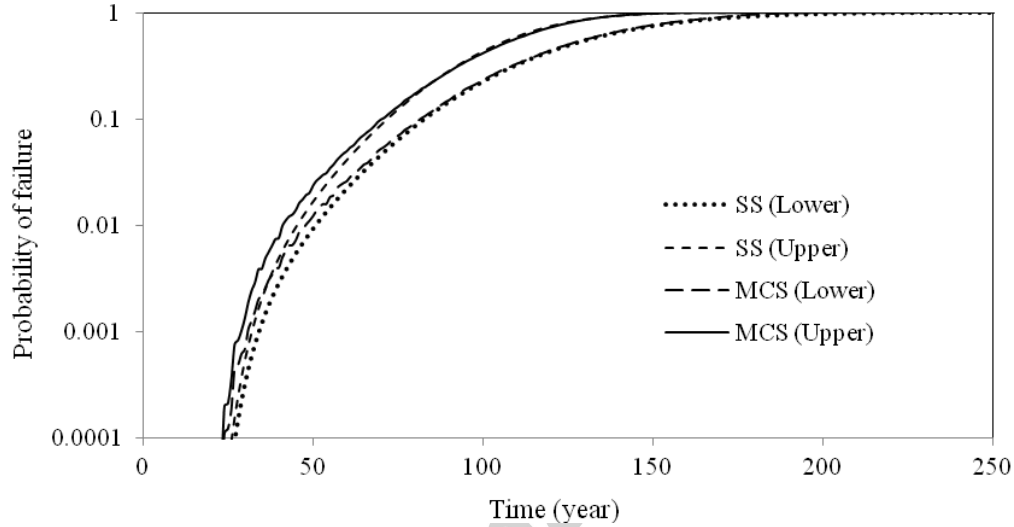

Figure 7: Probability of failure in series system due to corrosion induced multi-failure modes

Pipeline contains multiple failure events in which any of the modes can lead to a system failure. The failure modes are correlated due to common random variables between the failure events. Therefore, a series system is considered for pipe failures prediction. The correlation coefficients between different failure modes show that all the failure modes are strongly correlated positively, i.e., where the failure modes might happen concurrently within a buried pipeline system (Tee and Khan, 2013). Thus, applying the theory of systems reliability, the probability of failure for a series system, $P_{f,s}$ can be estimated by Eq. (18) (Fetz and Tonon, 2008).

$$Max[P_{f,j}] \le P_{f,s} \le 1 - \prod_{j=1}^{r}[1 - P_{f,j}] \tag{18}$$

where $P_{f,j}$ is the probability of failure due to j^{th} failure mode of pipe and r is the number of failure modes considered in the system. The expected value of P_f for series system is determined in-between upper and lower bounds using Eq. (18) and the results are shown in Figure 7. The number of conditional levels is chosen to cover the required response level whose failure

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probability is estimated. The results show that the P_f values using MCS and SS have a good agreement over the pipe service life.
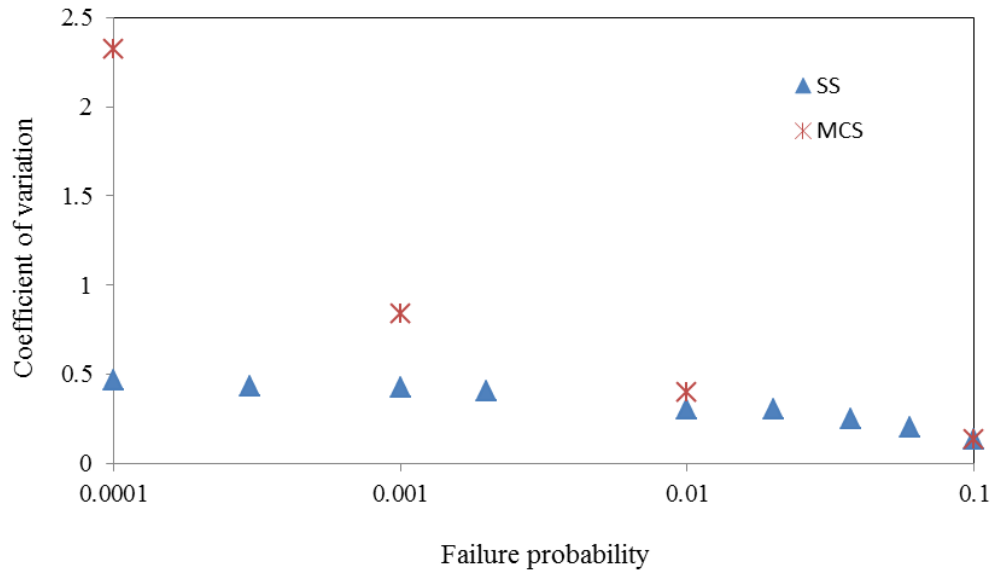

Figure 8: COV of pipe failure probability due to corrosion induced deflection for 50-year of service life

Nevertheless, one of the advantages of SS over MCS is that SS is able to estimate small failure probability more efficiently which is demonstrated in Figure 8. In this analysis, the sample average values and COVs of failure probabilities are calculated using 50 independent simulation runs. For comparison, the same numbers of samples are used for both MCS and SS methods. The total numbers of samples, N used for obtaining estimates of failure probability at 0.1, 0.01, 0.001 and 0.0001 are 500, 950, 1400 and 1850, respectively. The COVs of failure probability estimates produced by MCS can be calculated based on $\sqrt{\{1 - P(F)\} / P(F)N}$ (Au and Beck, 2007). COVs of failure probabilities due to corrosion induced deflection for 50-year of service life are plotted in Figure 8 for both SS and MCS. The results show that COVs achieved by SS and MCS are approximately the same in the large probability region. The values of COV for SS and MCS coincide at P_f = 0.1, since according to the SS procedure with $p_0 = 0.1$, this probability is computed based on an initial MCS. The study shows that the COVs are increased with decreasing failure probabilities because it is more difficult to estimate smaller failure probability, which is the main concern of SS. The value of COV estimated using SS are always less than that using MCS and the difference is larger when the failure probability is getting smaller as shown

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in Figure 8. Thus, it is inefficient to use ordinary MCS when the failure probabilities are small. SS is robust and more accurate and efficient compared to MCS in the prediction of small failure probabilities.

The improvement in accuracy also comes with considerable saving in computational time mainly due to smaller samples involved. The computational speed is measured in terms of Central Processing Unit (CPU) time on a 1.6-GHz Pentium IV personal computer. The study illustrates that SS (with 500 samples) needs 5–6 minutes to obtain the results whereas MCS (with 10^6 samples) spends 15–17 minutes to achieve the similar accuracy. Therefore, on the same computer, the saving in computational time of SS is about 67% as compared to MCS, which indicates the supremacy and accurateness of the proposed SS method. The computational time for MCS is generally higher than SS due to the high number of samples needed.
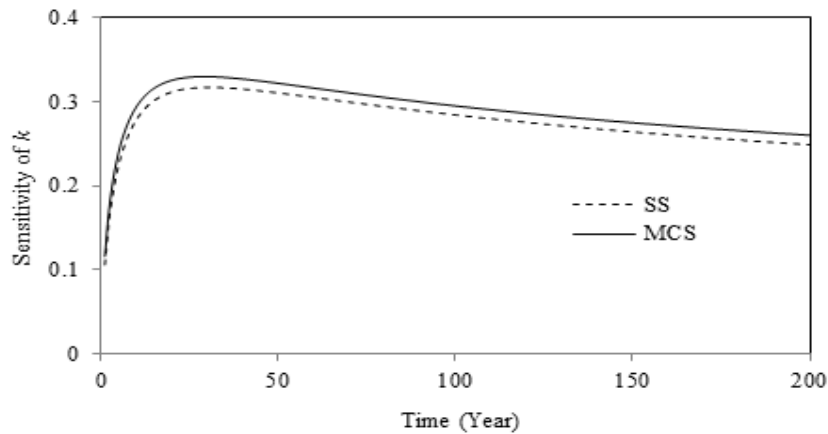

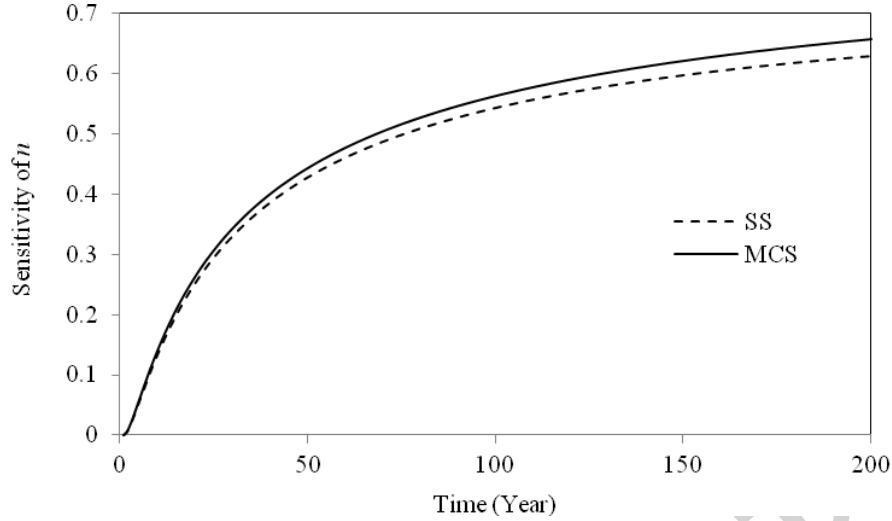Figure 9: Sensitivity of multiplying constant (k) for corrosion induced deflection during pipe service life

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Figure 10: Sensitivity of exponential constant (*n*) for corrosion induced deflection during pipe service life

Finally, two sensitivity analyses based on sample means are selected to evaluate the relative contribution of each random variable in pipe reliability estimation throughout the service life by applying Eqs. (15-17) and the results are shown in Figures 9 and 10. Note that due to page constraint, the COV and sensitivity analyses have been presented only for failure due to corrosion induced deflection. Corrosion constants (multiplying constant, *k* and exponential constant, *n*) in Eq. (1) are considered as the dominant influencing parameters in pipe reliability (Tee et al, 2013). The study shows that, at the early stage of pipe service life, multiplying constant (*k*) and exponential constant (*n*) have a negligible effect on pipe reliability but the effect increases significantly with the pipe age as shown in Figures 9 and 10. The similar trend has been found for other failure criteria, i.e., buckling, wall thrust and bending stress. This is attributed to the fact that corrosion does not cause any problem to new pipes but is mainly the root cause of failure and collapse for aging pipes.

6 CONCLUSIONS

A Subset Simulation approach is proposed for time-dependent reliability estimation of buried pipeline system subject to corrosion induced failures modes. The results show that this method is robust to the choice of the intermediate failure events. One of the major complications to estimating small failure probabilities is to simulate rare events. SS resolves this by breaking the

problem into the estimation of a sequence of larger conditional probabilities. It is found that the reliability analysis calculated by SS is in good agreement with that from MCS, while the efficiency of the SS method, which is indicated by the sample size and computational time, is higher than that of MCS. The study also shows that SS is robust and more accurate than MCS in small failure probability prediction based on COV analysis. The analysis shows that behaviour of buried pipes is considerably influenced by uncertainties due to external loads, corrosion parameters, pipe materials and surrounding soil properties etc. where excessive deflection is the most critical failure event whereas buckling is the least susceptible during the whole service life of the pipe. The estimation of failure probability can be utilised to form a maintenance strategy and to avoid unexpected failure of pipeline networks during service life.

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