RELIABILITY ANALYSIS AND SERVICE LIFE PREDICTION OF PIPELINES

MOJTABA MAHMOODIAN

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DECLARATION

I certify that this work has not been accepted in substance for any degree, and is not concurrently being submitted for any degree other than that of Doctor of Philosophy being studied at the University of Greenwich. I also declare that this work is the result of my own investigations except where otherwise identified by references and that I have not plagiarised the work of others.

Student: *Mojtaba Mahmoodian* **Supervisor:** *Professor Amir Alani*

To my Mother and my Father

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ABSTRACT

Pipelines are extensively used engineering structures for conveying of fluid from one place to another. Most of the time, pipelines are placed underground, surcharged by soil weight and traffic loads. Corrosion of pipe material is the most common form of pipeline deterioration and should be considered in both the strength and serviceability analysis of pipes.

The study in this research focuses on two different types of buried pipes including concrete pipes in sewage systems (concrete sewers) and cast iron water pipes used in water distribution systems. This research firstly investigates how to involve the effect of corrosion as a time dependent process of deterioration in the structural and failure analysis of these two types of pipes. Then two probabilistic time dependent reliability analysis methods including first passage probability theory and the gamma distributed degradation model are developed and applied for service life prediction of the pipes. The obtained results are verified by using Monte Carlo simulation technique. Sensitivity analysis is also performed to identify the most important parameters that affect pipe failure.

For each type of the pipelines both individual failure mode and multi failure mode assessment are considered. The factors that affect and control the process of deterioration and their effects on the remaining service life are studied in a quantitative manner.

The reliability analysis methods which have been developed in this research, contribute as rational tools for decision makers with regard to strengthening and rehabilitation of existing pipelines. The results can be used to obtain a cost-effective strategy for the management of the pipeline system.

The output of this research is a methodology that will help infrastructure managers and design professionals to predict service life of pipeline systems and to optimize materials selection and design parameters for designing pipelines with longer service life.

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List of Symbols

a: depth of the equivalent rectangular stress block, (mm)

A: the acid-consuming capability of the wall material

A_s: Area of tension reinforcement in length b, (mm^2/m)

a_0: Concrete cover (mm)

b: unit length of pipe, 1000mm

B_1: crack control coefficient for effect of spacing and number of layers of reinforcement

c: the average rate of corrosion (mm/year)

C_1: Crack control coefficient for type of reinforcement

d: distance from compression face to centroid of tension reinforcement, (mm)

d_b: diameter of rebar in inner cage, (mm)

[DS]: Dissolved sulphide concentration (mg/l)

f_c': design compressive strength of concrete, (MPa)

f_y: design yield strength of reinforcement, (MPa)

F: crack width control factor

F_c : factor for effect of curvature on diagonal tension (shear) strength in curved components

F_d: factor for crack depth effect resulting in increase in diagonal tension (shear) strength with decreasing d

F_N: coefficient for effect of thrust on shear strength

h: overall thickness of member (wall thickness), (mm)

i: coefficient for effect of axial force at service load stress

k: Acid reaction factor

J: is pH-dependent factor for proportion of H_2S

w: the width of the stream surface

P': perimeter of the exposed wall

M_s: Service load bending moment acting on length b, (Nmm/m)

M_u: factored moment acting on length b, (Nmm/m)

N_s: Axial thrust acting on length b, service load condition (+ when compressive, - when tensile), (N/m)

N_u: Factored axial thrust acting on length b, (+ when compressive, - when tensile), (N/m)

s: is the slope of the pipeline

t: elapsed time

u: is the velocity of the stream (m/sec)

V_b: basic shear strength of length b at critical section

Φ: The average flux of H_2S to the wall

$\emptyset_f$: strength reduction factor for flexure

ϕ_v: strength reduction factor for shear

Δ: reduction in wall thickness due to corrosion, (mm)

Δ_{max}: Maximum permissible reduction in wall thickness (structural resistance or limit), (mm)

σ_F : hoop stress due to internal fluid pressure

σ_L : frost pressure

σ_P : axial stress due to internal fluid pressure

σ_S : soil pressure

σ_{T_e} : Thermal stress

σ_V , Traffic stress

1 INTRODUCTION

Buried pipes are subject to chemical and mechanical loading in their environment of service and these stresses cause failure that is costly to repair. Methods to predict pipe performance are poorly developed and require improvement, through the introduction of time dependent reliability analytical tools. This is the subject of this research.

This chapter presents the background and significance of the subject of this research. The need for improved reliability analysis and service life prediction for concrete sewers and cast iron water pipes is established. A review of pipe failures within water and wastewater systems is given, and the costs of failure are outlined.

The outcome of this research is a model for improved reliability analysis that is tested on real-life data. This model can help asset managers to develop a risk-informed and cost-effective strategy for the management and maintenance of corrosion-affected pipelines. Improved reliability analytical tools can assist design engineers to develop pipeline systems with longer service lives.

1.1 Background and research significance

Pipelines are widely used engineering structures for collecting wastewater or for the distribution of water in urban and/or rural areas. Most of the time, pipelines are placed underground, surcharged by soil weight and traffic loads. Evidently, underground pipelines are required to resist the influence of the external loads (soil and traffic) and internal fluid pressure (ASCE (60) 2007, ACPA 2007, Moser and Folkman (2008)).

In many cases underground pipelines are required to withstand particular environmental hazards. Corrosion of pipe material is the most common form of pipeline deterioration and should be considered in both strength and serviceability analysis of buried pipes (Ahammed

& Melchers (1997), Sharma et al. (2008)). The current study focuses on two categories of buried pipes: concrete sewers and cast iron water mains.

In the UK there is approximately 310,000 km of sewer pipes with an estimated total asset value of £110 billion (OFWAT 2000). The investment for repair and maintenance of this infrastructure is approximately £40 billion for the period of 1990 to 2015 (The Urban Waste Water Treatment Directive, 91/271/EEC, 2012). It has been known that sewer collapses are predominantly caused by the deterioration of the pipes. For cementitious sewers, sulphide corrosion is the primary cause of these collapses (Pomeroy (1976), ASCE (69) 2007).

In Los Angeles USA, approximately 10% of the sewer pipes are subject to significant sulphide corrosion, and the costs for the rehabilitation of these pipelines are roughly estimated at £325 million (Zhang et al. (2008)). As an example of an European country, in Belgium, the cost of sulphide corrosion of sewers is estimated at £4 million per year, representing about 10% of total cost for wastewater collection and treatment systems (Vincke (2002)). These statistics indicate that sewer systems are faced with high emergency repair and renewal costs, and frequent charges arising from increasing rates of deterioration. On the other hand, budget limitations are significantly restricting sewer systems and reducing their capabilities in terms of addressing these needs. Therefore to eliminate the high costs associated with sewer failures, sewer system managers need to generate proactive asset management strategies and prioritise inspection, repair, and renewal needs of sewer pipes by utilising reliability analysis. The failure assessment and reliability analysis of sewers can help asset managers to provide an improved level of service and publicity, gain approval and funding for capital improvement projects, and manage operations and maintenance practices more efficiently (Grigg (2003) and Salman and Salem (2012)).

In water distribution systems, although cast iron pipes are being phased out of the water pipeline network in the UK, a significant portion of current networks are comprised of cast iron pipes with some of them up to 150 years old. There are approximately 335,000 km of water mains in the UK and more than 60% is estimated to be cast iron pipes (Water UK 2007). In the UK, the failure rate of cast iron pipes can be as high as 3000 failures per year (i.e., 10 bursts/1000 km/year) (UKWIR 2002). Of many mechanisms for pipe failures, corrosion of cast iron has been found to be the most predominant, which is linked to almost all pipe failures (Misiunas (2005)).

Data from other countries in the world also shows that, on average, cast iron has been the dominating material for water distribution pipes before the 1960s. Therefore the average age of cast iron pipes in existing networks has been estimated to be 50 years (Rajani and Kleiner, (2004), Misiunas (2005)). Due to their long term use, the aging and deterioration of pipes are inevitable and indeed many failures have been reported worldwide (Atkinson et al. (2002), Misiunas (2005), Rajani *and* Tesfamariam (2007) and EPA/600 2012). Depending on the country, compared with other types of pipe material, cast iron pipes have the highest frequency of breaks as shown in Table 1.1. It has been established (e.g., Yamini and Lence (2010) and EPA/600 2012) that the corrosion of cast iron is the most common form of deterioration of the pipes and it is a matter of concern for both the safety and serviceability of pipes. It is also well known that the consequence of the failure of water pipes can be socially, economically and environmentally devastating, causing, e.g. enormous disruption of daily life, massive costs for repair, widespread flooding and then pollution, and so on. This warrants a thorough assessment of the likelihood of pipe failures and their remaining safe life which is the topic of the present research.

Table 1.1 Frequency of pipe breakage for different materials (Breaks/100km/year), Misiunas (2005)

Source	Cast Iron	Ductile Iron	PVC
NRC (1995)	36	9.5	0.7
Weimer (2001)	27	3	4
Pelletier et al. (2003)	55	20	2

Large investments are required for building new wastewater collection systems and/or water supply infrastructure. It is unlikely to replace the existing pipe networks completely over a short period of time. Therefore the solution is to maintain and rehabilitate the existing pipes. Accurate prediction of the service life of pipes is essential to optimize strategies for maintenance and rehabilitation in the management of pipe assets. Service life (of building component or material) is the period of time after installation during which all the properties exceed the minimum acceptable values when routinely maintained (ASTM E632-82(1996)). The basis for making quantitative predictions of the service life of structures is to understand the mechanisms and kinetics of many degradation processes of the material whether it is steel, concrete or other materials. Material corrosion in concrete sewers and/or cast iron water pipes is a matter of concern for both strength and serviceability functions. Loss of wall thickness through general corrosion affects the strength of the pipe. To that effect, incorporating the effect of corrosion into the structural analysis of a pipeline is of paramount importance. There are several parameters which may affect corrosion rate and hence the reliability of pipelines. To consider uncertainties and data scarcity associated with these parameters, various researches on probabilistic assessment of buried pipes have been undertaken (De Belie (2004), Sadiq et al. (2004), Davis et al. (2005), Kleiner et al. (2006), Davis et al. (2008), Salman and Salem (2012)).

Since the deterioration of buried pipelines is uncertain over time, it should ideally be represented as a stochastic process. A stochastic process can be defined as a random function of time in which for any given point in time the value of the stochastic process is a random variable depending on some basic random variables. Therefore a robust method for reliability analysis and service life prediction of corrosion affected pipes should be a time dependent probabilistic (i.e., stochastic) method which considers randomness of variables to involve uncertainties in a period of time.

In most of the literature, failure and reliability assessment of pipes has been carried out by considering one failure mode (Davis et al. (2005), Desilva et al. (2006), Moglia et al. (2008), Yamini (2009) and Zhou (2011)). However in reality, even in simple cases composed of just one element, various failure modes such as flexural failure, shear failure, buckling, deflection, etc, may exist. To have a more accurate reliability analysis and failure assessment, multi failure mode of concrete sewers and cast iron pipes are also considered in the current study.

For a comprehensive reliability analysis, evaluation of the contributions of various uncertain parameters associated with pipeline reliability can be carried out by using sensitivity analysis techniques. Sensitivity analysis is conducted as a main part of reliability analysis from which the effect of different variables on service life of pipes can be investigated. Sensitivity analysis is the study of how the variation in the output of a model (numerical or otherwise) can be apportioned, qualitatively or quantitatively, to different sources of variation (Saltelli et al. (2004)). Among the reasons for using sensitivity analysis are:

- To identify the factors that have the most influence on reliability of the pipe
- To identify factors that may need more research to improve confidence in the analysis.
- To identify factors that are insignificant to the reliability analysis and can be eliminated from further analysis.
- To identify which, if any, factors or groups of factors interact with each other.

1.2 Structure of thesis

This thesis is organised in 7 chapters as follows:

Chapter 1 - Introduction: This chapter describes the background and significance of the research and the structure of the thesis.

Chapter 2 – Scope of the research: aims and objectives of the research together with the methodologies which are used to address the research objectives are explained in Chapter 2.

Chapter 3 - Literature review: This chapter describes the relevant existing published research works in the areas of design of buried pipelines, corrosion mechanism, reliability analysis, service life prediction and sensitivity analysis methods for infrastructure management in general and for buried pipes in particular.

Chapter 4 – Developing methods for time dependent reliability analysis of pipes: in this chapter time dependent reliability analysis methods are introduced and developed for pipeline reliability analysis.

Chapter 5 – Application of the developed methods for concrete sewers: The two developed approaches in chapter 4 (i.e., first passage probability theory and gamma distributed degradation model) are applied for reliability analysis of a concrete sewer case study in the UK. The results are verified by using Monte Carlo simulation method.

Chapter 6 – Application of the developed methods for cast iron water pipes: The results of application of first passage probability theory and gamma distributed degradation model for reliability analysis of cast iron pipes in the UK are discussed in this chapter. A comparison between the methods is made and the results are verified by Monte Carlo simulation method.

Chapter 7 – Discussion and analysis of the results: The obtained results from application of the proposed methods in chapters 5 and 6 are discussed and analysed in this chapter. A comparison among the different purposed methods for reliability analysis of pipes in this research is presented and weakness and strengths of each method are emphasised. The chapter outlines how the results address the set objectives of the research and fills the gaps which had been found in literature review

Chapter 8 – Conclusion and recommendations: Conclusions and guidelines for reliability analysis and service life prediction of buried pipes as have been concluded from this study are presented in this chapter. Recommendations are also outlined to address the further research which is needed to develop the area of reliability analysis and service life prediction of corrosion affected buried pipes.

The significance of this research was described in this chapter. A review of failure of concrete sewers and cast iron water pipes reveals that there is a vital need for improved reliability analysis and service life prediction of buried infrastructure, to allow infrastructure managers to improve the management of these assets.

In the next chapter, the aims and objectives of the research are defined. The methodologies used to address the objectives are discussed

2 SCOPE OF THE RESEARCH

In the previous chapter the importance of reliability analysis of concrete sewers and cast iron pipes were described. To address the gap in knowledge outlined in chapter 1, the aims and objectives of the research are defined in this chapter. The methods proposed to meet the objectives of this research are presented.

2.1 Research Aim and Objectives

The significance and necessity of reliability analysis of concrete sewers and cast iron water pipes was discussed in the previous chapter. Apart from some sporadic research on the subject, there is a lack of a reliable methodology and a comprehensive research in the area of reliability analysis of corrosion affected concrete sewers and cast iron pipes. Therefore the aims of the current research were set as follows:

- To develop reliability methods for assessment of buried pipes (i.e., concrete sewers and cast iron water pipes)
- To apply the developed methods to predict service life of concrete sewers and cast iron water pipes in the UK

In order to achieve the aims of the research, the following research objectives were set for this study:

1) To understand and investigate the design procedure of buried pipes and their behaviour under various loading conditions.
2) To adopt models of structural deterioration (i.e., corrosion) for concrete sewer pipes and cast iron water mains.
3) To examine and understand reliability theory and methods in application to pipes.
4) To develop rational methods for reliability analysis and service life prediction of corrosion affected buried pipes.
5) To test the developed methods/models to concrete sewer pipes and cast iron water pipes

2.2 Research Methodology

The methodologies which are used to address each of the objectives of the research are explained below:

In addressing the appointed objective number 1, a comprehensive literature survey is carried out on the subject to acquire solid knowledge of the design procedure of buried pipes and their behaviour under various loading conditions. Structural reliability analysis and failure assessment of buried pipes can not be achieved without an extensive knowledge about loading and stresses conditions, design principles and failure modes of buried pipes. Current state-of-the-art of research on the design principles for buried pipes and their behaviour under various loads is reviewed. Recently published design manuals and codes of practice are used for this purpose (e.g., ASCE 15-98 (2000), ACPA (2007) and Moser and Folkman (2007)).

ASCE 15-98 (2000) presents the standard practice for direct design of buried precast concrete pipes. It is an appropriate design manual for the design of concrete sewers. Limit state functions (failure functions) which need to be considered in a comprehensive reliability analysis of concrete sewers can be extracted from this design manual. Other references such as ACPA (2007) and Moser and Folkman (2007), give more technical details which support the standardised procedures and formulations in the design manual.

Similarly, for cast iron pipes design handbooks, reports and frequently cited technical papers are used for understanding the design principles, stresses and failure modes which need to be considered for assessment and analysis of cast iron pipes (e.g., Ahammed and Melchers (1997), Rajani et al. (2000) and Moser and Folkman (2007)).

To adopt models of structural deterioration for concrete sewers and cast iron water mains (objective number 2), it is necessary to investigate how buried pipes including concrete sewers and cast iron water pipes deteriorate and how to incorporate the effect of corrosion as a time dependent process of deterioration in the structural analysis of the pipeline. Therefore, a comprehensive literature review is carried out to understand the chemical and mechanical corrosion mechanisms in concrete sewers and in cast iron water pipes.

Corrosion of buried pipelines is uncertain over time; therefore it should ideally be represented as a stochastic process. In this study, corrosion models taken from key reference works are used in the limit state functions (failure functions) developed in this work.

The general form of a sulphide corrosion model for concrete sewers has not changed since the mid 70's after Pomeroy (1976)'s work. The final form of the formulation for sulphide corrosion rate (ASCE No.60, 2007) is selected in this research and the corrosion depth is adopted from that formulae. By considering corrosion as a stochastic process, the variables in the formulations would be random variables and the corrosion model will have a form of stochastic model.

Likewise, corrosion models for cast iron water pipes are studied inclusively and the most acceptable form of the models is selected for further analysis. Recently published literature such as Rajani and Kleiner (2001), Melchers (2005 a, b) and Melchers (2008) are used to elaborate how a proper model for cast iron corrosion can be adopted.

A comprehensive study is carried out to understand the principles of structural reliability analysis (objective number 3). The focus of the literature review is on reliability analysis of corrosion affected structures in general and buried pipes in particular.

An in depth mathematical study and practice on probability theory and numerical methods should be carried out as a preface for reliability theory. References such as Papoulis and Pillai (2002) and Rubinstein and Kroese (2008) are used for this purpose. Reference books (such as Melchers (1999) and Ditlevsen and Madsen (1996)) and frequently cited papers are also used to understand the principles of reliability theory and the application history of the reliability analysis of pipelines.

A good understanding of probability theory especially in the area of statistical characteristics of random variables, probability density functions and stochastic processes is achieved by an extensive literature review. This approach facilitates the chance of quantifying the random variables associated with corrosion models leading to improved reliability analysis.

Since the Monte Carlo simulation method is used to verify the results obtained from the analytical reliability analysis methods, a full understanding of this method is required. References such as Melchers (1999) and Rubinstein and Kroese (2008) are used for learning and practicing Monte Carlo simulation technique and frequently cited literature (such as Sadiq et al. (2004) and Zhou (2011)) are used to investigate the adoptability of using this simulation method for pipeline reliability analysis.

To include corrosion mechanism as a time dependent process in reliability analysis of buried pipes, the focus should be on time dependent techniques to calculate the reliability and remaining service life of the pipes. After a comprehensive literature review the most adoptable time dependent methods are developed for reliability analysis of concrete sewers and cast iron pipes (objective number 4) by using advanced analytical mathematics. Probability theory is employed to develop analytical models for deterioration and reliability analysis of pipeline systems.

In addressing the appointed objective number 5, case studies on concrete sewers and cast iron water pipes in the UK are selected for application of the developed reliability analysis and service life prediction methods. A set of CCTV data on concrete sewers from city of Harrogate in the UK is used for the case study of reliability analysis of concrete sewers. Likewise, for cast iron pipes, a set of corrosion measurement data in the UK is taken from Marshall (2001)'s report.

The results from each analytical method and for each case (concrete sewer and/or cast iron pipe) are discussed and the methods are compared. Verification of the results is also carried out by using Monte Carlo simulation method.

MATLAB software is used as a strong programming tool for coding and calculations both for analytical methods and the numerical method (i.e., Monte Carlo).

Sensitivity analysis also is performed to identify the most important parameters that affect pipeline reliability and failure. Sensitivity indexes presented by frequently cited literature are used for this purpose.

To summarise, the application of these methods, the following subjects are investigated:

- The factors that affect and control the process of corrosion in concrete sewers and in cast iron water pipes
- Modelling corrosion process stochastically, to involve uncertainties of random variables which affect the corrosion rate
- Comparison between the developed time dependent reliability analysis methods
- Sensitivity analysis to assess the effectiveness of different parameters on reliability of concrete sewers and cast iron water pipes

Figure 2.1 briefly illustrates the process, steps and methodologies that are used in this research to reach the set aims and objectives.

The developed methodologies in this research can be used as rational tools for decision makers with regard to strengthening and rehabilitation of existing pipelines. Accurate prediction of service life of pipeline system can help structural engineers and asset managers to obtain a cost-effective strategy in the management of the system.

The output of this research will be a methodology that will permit infrastructure managers and construction professionals to:

- Predict service life of buried pipeline systems by a rational and reliable time dependent analysis
- Prioritise of design parameters and random variables by sensitivity analysis techniques.

The aims and objectives for this research were defined in this chapter and the methods outlined. However, to investigate the state-of-the-art of the reliability analysis of buried pipes, it is necessary to understand the engineering design of buried pipes, the corrosion mechanisms responsible for failure and principles of reliability analysis. This is dealt with in the next chapter, where a comprehensive literature review is presented.

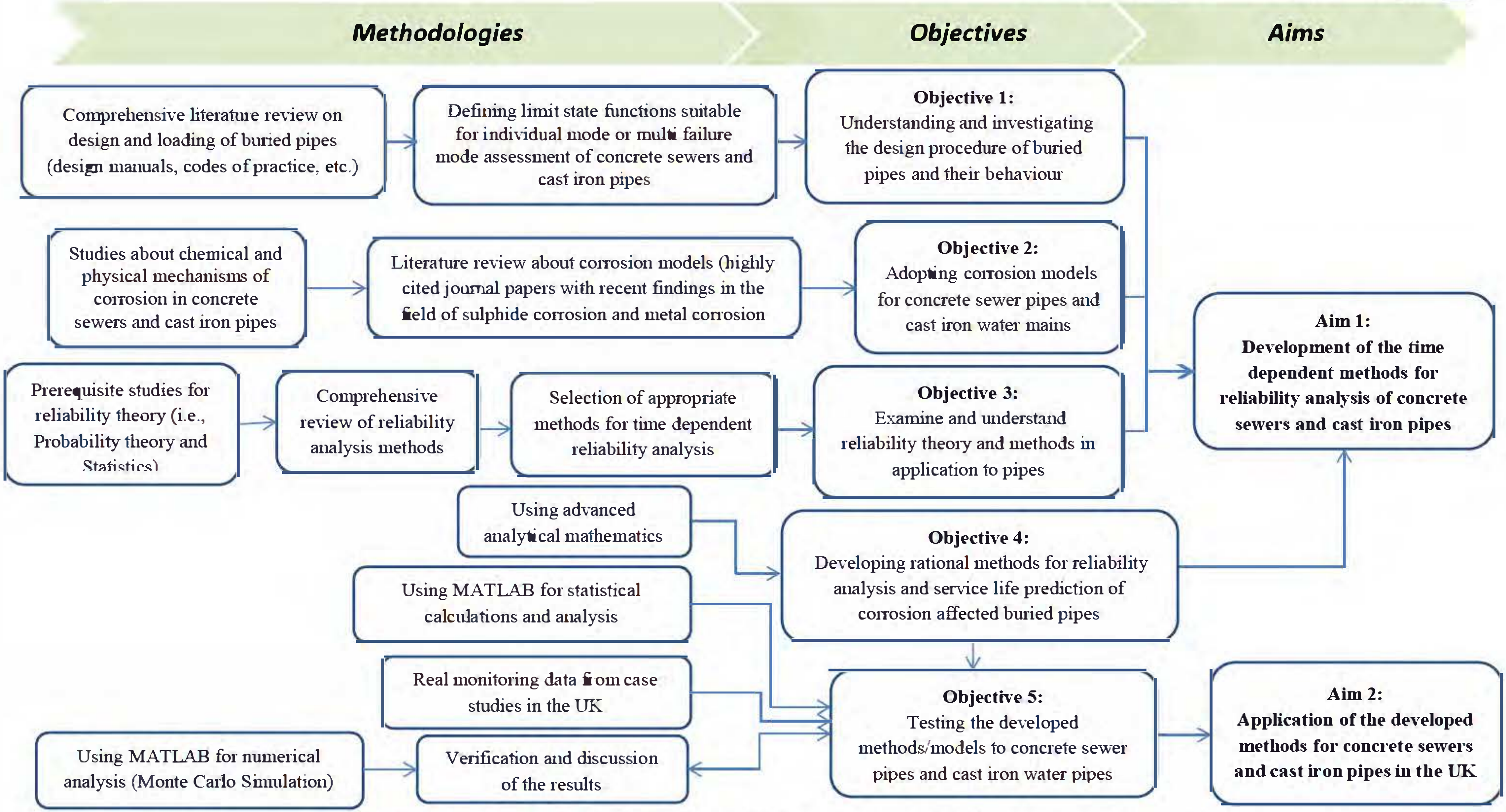


Figure 2.1 The process and methodologies that are used in current research to reach the aims and objectives

3 LITERATURE REVIEW

This chapter investigates the deficiencies in the field of reliability analysis and service life prediction of concrete sewers and cast iron pipes. A comprehensive literature survey is undertaken to gain solid knowledge of the design process, corrosion mechanism involved and methods of reliability analysis. The relevant literature on service life prediction and methods for sensitivity analysis for buried pipes is reviewed in this chapter, to enable the basis for a novel approach for reliability prediction to be established.

3.1 Design of buried pipes

3.1.1 Design principles

The design of buried pipes constitutes a wide ranging and complex field of engineering, which has been the subject of extensive study and research in the world over a period of many years.

There are two main stages for designing of water and wastewater pipes: a) Hydraulic design, and b) Structural design. In the hydraulic design stage, the focus is on determination of the demand of the system for collecting and conveying of water or wastewater. Based on this the diameter of the pipe is estimated. In the second stage, focus is on determination of structural capacity or strength, including details like wall thickness and/or reinforcement. This section discusses the structural design of buried rigid pipes. It introduces and compares different existing design methods. The structural properties of the pipe are analysed to ensure the pipe can safely sustain external and internal loads during its service life time, without loss of its function and without detriment to the environment.

A set of performance criteria must be met when the pipe is subjected to loads. As for other structures, there are two categories of performance criteria for underground rigid pipes: ultimate limit state and serviceability limit state.

The ultimate limit state is represented by the strength of the pipe and is reached when the pipe collapses or fails in general. Flexural and shear failures are two main ultimate limit states that are considered in design and assessment (ASCE 15-98, 2000). Serviceability limit states may be measured by cracking or other functional requirements (for example leakage, deformation beyond allowable limits (for flexible pipes) and excessive movement at the joints).

The principle for the design of a pipe is to ensure that both serviceability and ultimate limit states are not reached. This includes consideration of one or more of the following conditions: strain, stress, bending moment and normal force or load bearing capacity, in the ring or longitudinal direction as appropriate; and water tightness.

The design of a buried pipe involves the selection of an appropriate pipe strength and a bedding combination which is able to sustain the most adverse permanent and transient loads to which the pipeline will be subjected over its design life.

3.1.2 Loads on buried pipes

All pipes shall be designed to withstand the various external and internal loadings to which they are expected to be subjected, during construction and operation. The external loadings include loads due to the backfill, most severe surface surcharge or traffic loading (live load) likely to occur, and self-weight of the pipe and water weight. The internal pressure in the pipeline, if different from atmospheric, shall also be treated as a loading.

Earth load

Beginning in 1910, Anson Marston developed a method for calculating earth loads above a buried pipe based on the understanding of soil mechanics at that time. Marston's formula is considered for calculation of earth load on buried pipes in all codes of practice and manuals (such as BS EN 1295-1 and ASCE No.60). The general form of Marston's equation is:

$$W = C\gamma B^2 \tag{3.1}$$

Where W is the vertical load per unit length acting on the pipe because of gravity soil loads, γ is the unit weight of soil; B is the trench width or pipe width, depending on installation condition; and C is a dimensionless load coefficient depending on soil and installation type (available in design manuals).

The pressure distribution around the pipe from the applied loads (W) and bedding reaction shall be determined from a soil-structure analysis or a rational approximation. Acceptable pressure distribution diagrams from soil-structure analysis are the Heger Pressure Distribution (Figure 3.1a) for use with the Standard Installations; the Olander/Modified Olander Radial Pressure Distribution (Figure 3.1b) or the Paris/Manual Uniform Pressure Distribution (Figure 3.1c).

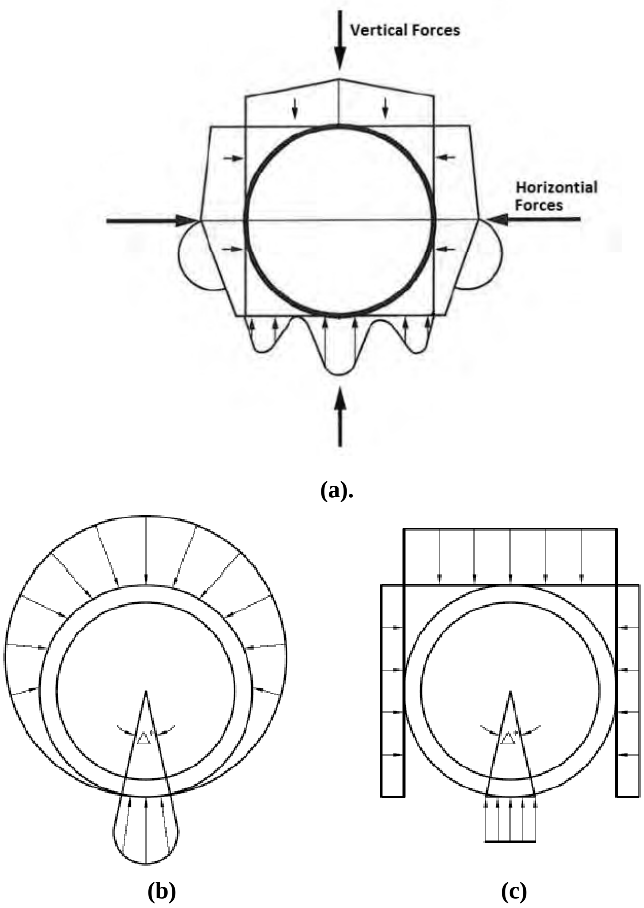


Figure 3.1 (a). Heger earth pressure distribution, (b). Olander/Modified Olander Radial Pressure Distribution, (c). Paris/Manual Uniform Pressure Distribution (ACPA, 1993)

Pipe and flow dead loads

The dead load of the pipe weight shall be considered in the design based on material density. The dead load of fluid in the pipe also shall be based on the unit weight of the stream (ASCE 15-98, 2000).

Live load

In designing buried pipes, it is necessary to consider the impact of live loads (surcharge) as well as the dead loads. Live loads become a greater consideration when a pipe is installed with shallow cover under un-surfaced road way, railroads and/or airport runways and taxiways. Surcharge loads are calculated using Boussinesq's theory (Moser and Folkman 2008), for various vehicle wheel loading patterns, representing the most severe loadings which might apply in various locations.

Both concentrated and distributed superimposed live loads should be considered in the structural design of sewers. The following equation for determining loads due to superimposed concentrated load, such as a truck wheel load has been presented by ASCE No.60 (2007):

$$W_{sc} = \frac{C_s P F}{L} \tag{3.2}$$

Where

W_{sc} = the live load on the sewer in kg/m of length

P= the concentrated load (kg)

F= the impact factor

C_s= the load coefficient, a function of $\frac{B_c}{2H}$ and $\frac{L}{2H}$ where H is the height of fill from the top of pipe to ground surface in (m) and B_c is the width of the sewer in (m)

L= the effective length of sewer in (m)

For the case of superimposed load distributed over an area of considerable extent, the formula for load on pipe is (ASCE No.60, 2007):

$$W_{sd} = C_s p F B_c \qquad (3.3)$$

Where

W_{sd}=the load on the pipe, (N/m)

p= the intensity of distributed load, (N/m^2)

F= impact factor

B_c= the width of the sewer pipe, (m)

C_s= the load coefficient, which is a function of $\frac{D}{2H}$ and $\frac{M}{2H}$, and D and M are width and length, respectively, of the area over which the distributed load acts .

H= height from the top of the sewer to ground surface, (m)

3.1.3 Stresses in buried pipes

Rajani et al. (2000) developed a formulation for total external stresses including all circumferential and axial stresses. σ_θ is hoop or circumferential stress, which is equal to $\sigma_F + \sigma_S + \sigma_L + \sigma_V$, where σ_F is hoop stress due to internal fluid pressure, σ_S is soil pressure, σ_L is frost pressure and σ_V is traffic stress.

Similarly axial stress, σ_x , would be equal to $\sigma_{Te} + \sigma_{\acute{F}} + (\sigma_S + \sigma_L + \sigma_V)\, \nu_p$ where σ_{T_e} is stress related to temperature difference, $\sigma_{\acute{F}}$ is axial stress due to internal fluid pressure, ν_p is pipe material Poisson's ratio and other parameters have already mentioned. Equations and references used for the above mentioned stresses have been presented in Table 3.1.

Table 3.1 Stresses on buried pipes

Stress Type	Model*	Reference
σ_F , hoop stress due to internal fluid pressure	$\frac{pD}{2d}$	Rajani et al. (2000)
σ_S , soil pressure	$\frac{3K_m\,\gamma\,B_d^2\,C_d\,E_P\,d\,D}{E_P\,d^3+3K_d\,p\,D^3}$	Ahammed & Melchers (1994)
σ_L , frost pressure	$f_{frost}\,.\,\sigma_S$	Rajani et al. (2000)
σ_V , Traffic stress	$\frac{3\,K_m\,I_c\,C_t\,F\,E_P\,d\,D}{A(E_P\,d^3+3K_d\,p\,D^3)}$	Ahammed & Melchers (1994)
σ_{T_e} , Thermal stress	$-\,E_P\,\alpha_P\,\Delta T_e$	Rajani et al. (2000)
σ_P , axial stress due to internal fluid pressure	$\frac{p}{2}\left(\frac{D}{d}-1\right)\nu_p$	Rajani et al. (2000)

*: Notations introduced in list of symbols

3.2 Corrosion of pipes

3.2.1 The corrosion mechanism of concrete sewers

Sewer pipes deteriorate at different rate depending on specific local conditions and are not determined by age alone. There have been numerous cases of severe damage to concrete pipes, where it has been necessary to replace the pipes before the desired service life has been reached. There are many cases in which sewer pipes designed to last 50 to 100 years have failed due to H_2S corrosion in 10 to 20 years. In extreme cases, concrete pipes have collapsed in as few as 3 years (Pomeroy 1976). The most corrosive agent that leads to the rapid deterioration of concrete pipelines in sewers is H_2S. Approximately 40% of the damage in concrete sewers can be attributed to biogenous sulphuric acid attack. Sulphide corrosion, which is often called microbiologically induced corrosion, has two distinct phases as follows:

- The conversion of sulphate in wastewater to sulphide, some of which is released as gaseous hydrogen sulphide

- The conversion of hydrogen sulphide to sulphuric acid, which subsequently attacks susceptible pipeline materials.

The surface pH of new concrete pipe is generally between 11 and 13. Cement contains calcium hydroxide, which neutralizes the acids and inhibits formation of oxidizing bacteria when the concrete is new. However, as the pipe ages, the neutralizing capacity is consumed, the surface pH drops, and the sulphuric acid-producing bacteria become dominant. In active corrosion areas, the surface pH can drop to 1 or even lower and can cause a very strong acid attack. The corrosion rate of the sewer pipe wall is determined by the rate of sulphuric acid generation and the properties of the cementitious materials. As sulphides are formed and sulphuric acid is produced, hydration products in the hardened concrete paste (calcium silicon, calcium carbonate and calcium hydroxide) are converted to calcium sulphate, more commonly known by its mineral name, gypsum (ASCE 69 1989). The chemical reactions involved in sulphide build-up can be explained as follows.

Sulphate, generally, abundant in wastewater, is usually the common sulphur source, though other forms of sulphur, such as organic sulphur from animal wastes, can also be reduced to sulphide. The reduction of sulphate in the presence of waste organic matter in a wastewater collection system can be described as follows:

$$SO_4^{-2} + \text{Organic matter} + H_2O \xrightarrow[\text{Bacteria}]{} 2HCO_3^- + H_2S \tag{3.4}$$

The H_2S gas in the atmosphere can be oxidized on the moist pipe surfaces above the water line by bacteria (Thiobacillus), producing sulphuric acid according to the following reaction (Meyer 1980):

$$H_2S + O_2 \xrightarrow[\text{Bacteria}]{} H_2SO_4 \tag{3.5}$$

As sulphides are formed and sulphuric acid is produced, hydration products in the hardened concrete paste (calcium silicate, calcium carbonate and calcium hydroxide) are converted to calcium sulphate. The chemical reactions involved in corrosion of concrete are

$$H_2SO_4 + CaO.SiO_2.2H_2O \rightarrow CaSO_4 + Si(OH)_4 + H_2O \tag{3.6}$$

$$H_2SO_4 + CaCO_3 \rightarrow CaSO_4 + H_2CO_3 \tag{3.7}$$

$$H_2SO_4 + Ca(OH)_2 \rightarrow CaSO_4 + 2H_2O \tag{3.8}$$

Gypsum does not provide much structural support, especially when wet. It is usually present as a pasty white mass on concrete surfaces above the water line. As the gypsum material is eroded, the concrete loses its binder and begins to spall, exposing new surfaces. This process will continue until the pipeline fails or corrective actions are taken. Sufficient moisture must be present for the sulphuric acid-producing bacteria to survive, however; if it is too dry, the bacteria will become desiccated, and corrosion will be less likely to occur. Figure 3.2 shows the process of sulphide build-up in a sewer system.

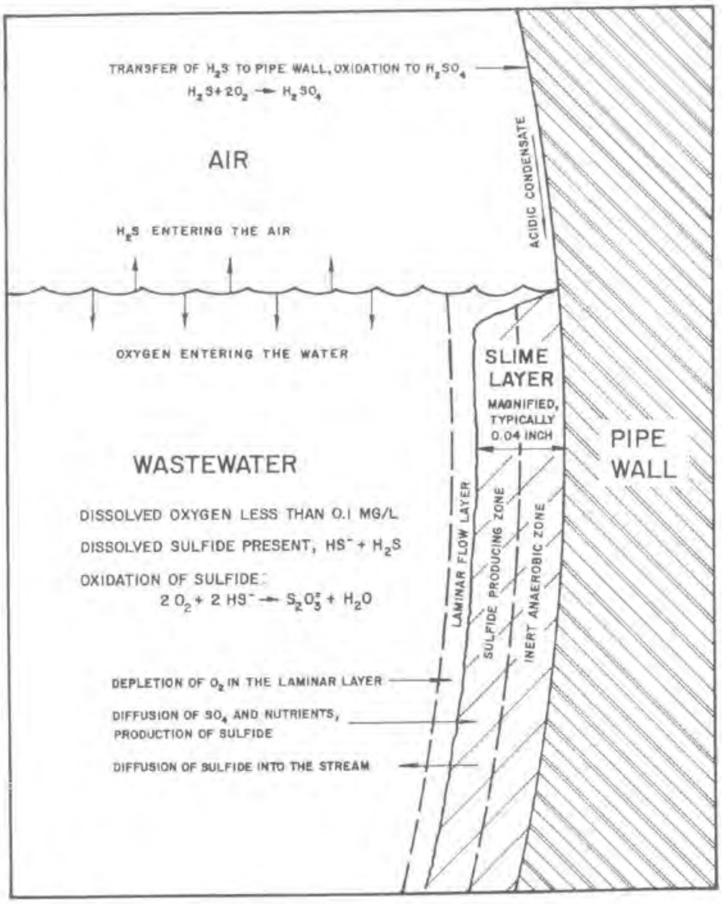


Figure 3.2 Process occurring in sewer under sulphide build up conditions, ASCE No.69, (1989)

Concrete corrosion rate

The rate of corrosion of a concrete sewer can be calculated from the rate of production of sulphuric acid on the pipe wall, which is in turn dependent upon the rate that H_2S is released from the surface of the sewage stream. The average flux of H_2S to the exposed pipe wall is equal to the flux from the stream into the air multiplied by the ratio of the surface area of the stream to the area of the exposed pipe wall, which is the same as the ratio of the width of the stream surface (b) to the perimeter of the exposed wall ($\acute{P}$). The average flux of H_2S to the wall is therefore calculated as follows (Pomeroy 1976):

$$\Phi = 0.7(su)^{3/8} j[DS](b/\acute{P}) \tag{3.9}$$

Where s is pipe slope, u is velocity of stream (m/s), j is pH-dependent factor for proportion of

H_2S, [DS] is dissolved sulphide concentration (mg/lit). A concrete pipe is made of cement-bonded material, or acid-susceptible substance, so the acid will penetrate the wall at a rate inversely proportional to the acid-consuming capability (A) of the wall material. The acid may partly or entirely react. The proportion of acid that reacts is variable (k), ranging from 100% when the acid formation is slow, to perhaps 30% to 40% when it is formed rapidly. Thus, the average rate of corrosion (mm/year) can be calculated as follows

$$c = 11.5k\Phi(1/A) \tag{3.10}$$

Where k is the factor representing the proportion of acid reacting, to be given a value selected by the judgement of the engineer and A is the acid-consuming capability, alkalinity, of the pipe material, expressed as the proportion of equivalent calcium carbonate. A value for granitic aggregate concrete ranges from 0.17 to 0.24 and for calcareous aggregate concrete, A ranges from 0.9 to 1.1 (ASCE No.60, 2007). Substituting Equation (3.9) into Equation (3.10):

$$\mathrm{c} = 8.05\mathrm{k} \times (\mathrm{su})^{3/8}\mathrm{j}.[\mathrm{DS}] \times \frac{\mathrm{b}}{\mathrm{P'A}} \tag{3.11}$$

Therefore the reduction in wall thickness in elapsed time t, is:

$$\mathrm{d(t)} = \mathrm{c.t} = 8.05\mathrm{k}.(\mathrm{su})^{3/8}\mathrm{j}.[\mathrm{DS}] \times \frac{\mathrm{b}}{\mathrm{P'A}}.\mathrm{t} \tag{3.12}$$

3.2.2 Corrosion mechanism of cast iron water mains

The predominant deterioration mechanism of iron-based pipes is electro-chemical corrosion with the damage occurring in the form of corrosion pits. The damage to iron is often identified by the presence of graphitisation, a result of iron being leached away by corrosion. Either form of metal loss represents a corrosion pit that grows with time and reduces the thickness and mechanical resistance of the pipe wall. This process eventually leads to the breakage of the pipe.

Corrosion pits have a variety of shapes with characteristic depths, diameters (or widths), and lengths. They can develop randomly along any segment of pipe and tend to grow with time at a rate that depends on environmental conditions in the immediate vicinity of the pipeline

(Rajani and Makar (2000)).

The corrosion rate of in-service cast iron pipes is believed to be higher in the beginning and then decreases over time as corrosion appears to be a self-inhibiting process (Shreir et al. (1994)). Furthermore, due to the variation of service environment it is rare that the corrosion occurs uniformly along the pipe but more likely locally in the form of a corrosion pit.

A number of models for corrosion of cast iron pipes have been proposed to estimate the depth of corrosion pit (e.g., Sheikh et al. (1990), Ahammed and Melchers (1997), Kucera and Mattsson (1987), Rajani et al. (2000) and Sadiq et al. (2004)). For example, Sheikh et al. (1990) suggested a linear model for corrosion growth in predicting the strength of cast iron pipes. A decade later, Rajani et al. (2000) proposed a two-phase corrosion model where the first phase is a rapid exponential pit growth and the second is a slow linear growth. There are debates in the research community as to whether the corrosion rate can be assumed linear or otherwise. A widely accepted model of corrosion as measured by the depth of corrosion pit is of a power law which was first postulated for atmospheric corrosion by Kucera and Mattsson (1987) and can be expressed as follows:

$$a = kt^n \tag{3.13}$$

Where t is exposure time and k and n are empirical constants largely determined from experiments and/or field data.

For underground corrosion, the constants are typically functions of localised conditions including soil type, the availability of oxygen and moisture and properties of pipeline material. In many cases it may be possible to use past experience to derive estimates for the two constants in Equation 3.13, but with somewhat more effort than would be necessary to estimate a constant corrosion rate as used conventionally (Ahammed and Melchers (1997)).

Rajani et al. (2000) proposed a two-phase corrosion model to accommodate this self-inhibiting process:

$$a = \alpha t + \beta\left(1 - e^{-\lambda t}\right) \tag{3.14}$$

Where α, β and λ are constant parameters.

In the first phase of the above equation there is a rapid exponential pit growth and in the second phase there is a slow linear growth. This model was developed based on a data set that lacked sufficient points in the early exposure times. Therefore prediction of pit depth in the first 15-20 years of pipe life should be considered highly uncertain when Equation 3.14 is used.

An example of field data which shows the rate of internal and external corrosion for cast iron pipes has been illustrated in Figure 3.3 (Marshall (2001)). As it can be concluded from this data, external corrosion has higher rate than internal corrosion especially during early stages. In Figure 3.4, a sample of a cast iron pipe taken from London water mains in Victorian time (i.e., 1800-1900) also shows the severity of external corrosion compared with internal corrosion.

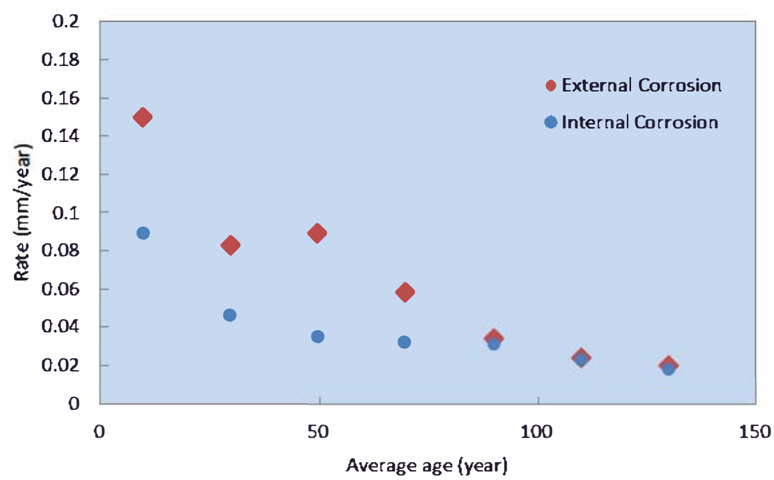


Figure 3.3 Rate of internal and external corrosion for cast iron pipes, Marshall (2001)

(a) (b)

Figure 3.4 A section of one of London's Victorian water mains, (a) External corrosion, (b) Internal corrosion

3.3 Service life prediction and structural reliability analysis

3.3.1 Background

Reliability analysis and the prediction of the service life of structures is one of the major challenges for infrastructure managers and structural engineers. Historically, reliability theory has most often been introduced in the military, aerospace and electronics fields (Cheung and Kyle (1996)). Over the past number of years, the significance of reliability theory has been increasingly realised in the area of civil engineering. The structural reliability began as a subject for academic research about 50 years ago (Freudenthal (1956)). The topic has grown rapidly during the last three decades and has evolved from being a topic for academic research to a set of well-developed or developing methodologies with a wide range of practical applications.

Structural reliability can be defined as the probability that the structure under consideration has a sufficient performance throughout its service life. Reliability methods are used to estimate the service life of structures.

In addition to the prediction of initial service life, reliability methods are effective tools to evaluate the efficiency of repair and replacement. The impact of any repair and maintenance option upon the future performance of the structure can be evaluated by decision makers using reliability analysis methods.

Furthermore, reliability analysis of a structure or a system can be used at the conceptual design stage to evaluate various design choices and to determine the impact that their implementation could have upon their service lives.

The uncertain nature of the loadings and the performance aspects of structures could have led the planners to probabilistic approaches for service life assessment. In probabilistic methods for dealing with uncertainties, the safety and service/performance requirements are measured by their reliabilities. The reliability of a structure or a component is defined as its probability of survival (Melchers (1999)):

$$P_s = 1 - P_f \tag{3.15}$$

Where

P_s: Probability of survival

P_f: Probability of failure

Failure can be defined in relation to different possible failure modes, commonly referred as limit states. Reliability is considered to be the probability that these limits will not be exceeded and is equal to the probability of survival. Each of the limit state function variables is attributed to a probability density function that presents its statistical properties.

To summarise, structural reliability analysis can be generally used for the following purposes:

- Service life prediction of existing structures, for funding allocation to most critical parts of the structure or infrastructure
- Evaluation of the effect of repair, maintenance and rehabilitation actions on the service life of the structure (ability to examine the consequences of potential action or inaction relative to operational and maintenance procedures).
- To be used at the conceptual design stage to evaluate various design choices and to determine the impact that their implementation would have upon the service lives

To predict the service life of existing structures, information is required on the present condition of the structure, rates of degradation, past and future loading, and definition of the failure of the structure. Based on remaining life predictions, cost-benefit analysis can also be made on whether or not a structure should be repaired, rehabilitated, or replaced.

3.3.2 Theory of reliability analysis

In the past, the design of structural systems considered all loads and strengths as deterministic values. The strength of an element was determined in such a way that it withstood the load within a certain margin. The ratio between the strength and the load was denoted a safety factor.

This safety factor was considered as a measure of the reliability of the structure. However, uncertainties in the loads, strengths and in the modelling of the systems require that methods based on probabilistic techniques in a number of situations have to be used. A structure is usually required to have a satisfactory performance in the expected service life, i.e. it is required that it does not collapse or becomes unsafe and that it fulfils certain functional requirements.

In order to estimate the reliability by using probabilistic concepts it is necessary to introduce stochastic variables and/or stochastic processes/fields and to introduce failure and non-failure behaviour of the structure under consideration.

Generally the main steps in a reliability analysis for service life prediction are:

1. Identify the significant failure modes of the structure.
2. Decompose the failure modes in series systems or parallel systems of single components (only needed if the failure modes consist of more than one component).
3. Formulate failure functions (limit state functions) corresponding to each component in the failure modes.
4. Identify the stochastic variables and the deterministic parameters in the failure functions. Further specify the distribution types and statistical parameters for the stochastic variables and the dependencies between them.

5. Estimate the reliability of each failure mode (illustration of how reliability or inversely the probability of failure changes with time).
6. Evaluate the reliability result by performing sensitivity analyses.

Typical failure modes to be considered in a structural reliability analysis are yielding, corrosion, buckling (local and global), fatigue and excessive deformations.

The failure modes (limit states) are generally divided in:

Ultimate limit states correspond to the maximum load carrying capacity which can be related to e.g. formation of a mechanism in the structure, excessive plasticity, rupture due to corrosion and instability (buckling).

Conditional limit states correspond to the load-carrying capacity if a local part of the structure has failed. A local failure can be caused by an accidental action or by fire. The conditional limit states can be related to e.g. formation of a mechanism in the structure, exceedance of the material strength or instability (buckling).

Serviceability limit states are related to normal use of the structure, e.g. excessive deflections, local damage and excessive vibrations.

The fundamental quantities that characterise the behaviour of a structure are called the basic variables and can be denoted X = (X1, ..., Xn) where n is the number of basic stochastic variables. Typical examples of basic variables are loads, strengths, dimensions and material properties. The basic variables can be dependent or independent. A stochastic process can be defined as a random function of time in which for any given point in time the value of the stochastic process is a random variable.

The uncertainty modelled by stochastic variables can be divided in the following groups:

Physical uncertainty: or inherent uncertainty is related to the natural randomness of a quantity, for example the uncertainty in the yield stress due to production variability.

Measurement uncertainty: is the uncertainty caused by imperfect measurements of for example a geometrical quantity.

Statistical uncertainty: is due to limited sample sizes of observed quantities.

Model uncertainty: is the uncertainty related to imperfect knowledge or idealizations of the mathematical models used or uncertainty related to the choice of probability distribution types for the stochastic variables.

All the above types of uncertainty can usually be treated by the reliability methods. Another type of uncertainty which is not covered by these methods is gross errors or human errors. These types of errors can be defined as deviation of an event or process from acceptable engineering practice.

Generally, methods to measure the reliability of a structure can be divided in four groups, see Madsen and Krenk (1986):

• Level I methods: The uncertain parameters are modelled by one characteristic value, as for example in codes of practice based on the partial safety factor concept.

• Level II methods: The uncertain parameters are modelled by the mean values and the standard deviations, and by the correlation coefficients between the stochastic variables. The stochastic variables are implicitly assumed to be normally distributed. The reliability index method is an example of a level II method.

• Level III methods: The uncertain quantities are modelled by their joint distribution functions. The probability of failure is estimated as a measure of the reliability.

• Level IV methods: In these methods the consequences (cost) of failure are also taken into account and the risk (consequence multiplied by the probability of failure) is used as a measure of the reliability. In this way different designs can be compared on an economic basis taking into account uncertainty, costs and benefits.

Level I methods can e.g. be calibrated using level II methods, level II methods can be calibrated using level III methods, etc.

Several techniques can be used to estimate the reliability for level II and III methods, including the following methods:

• **Simulation techniques**: Samples of the stochastic variables are generated and the relative number of samples corresponding to failure is used to estimate the probability of failure. The simulation techniques are different in the way the samples are generated. Monte Carlo method is the major simulation method for structural reliability analysis.

• **FORM techniques**: In First Order Reliability Methods the limit state function (failure function) is linearized and the reliability is estimated using level II or III methods.

• **SORM techniques**: In Second Order Reliability Methods a quadratic approximation to the failure function is determined and the probability of failure for the quadratic failure surface is estimated.

• **Time dependent reliability techniques**: when a structure is subjected to a time dependent degradation process, probabilistic time dependent methods can be used. First passage probability theory has been introduced for time dependent reliability analysis (Melchers (1999)). Gamma process concept also has the potential of usage as a model for reliability analysis of structures subject to monotonic degradation processes (van Noortwijk and Pandey (2003)). These methods are discussed and developed in Chapter 4 for reliability analysis of concrete sewers and cast iron pipes.

In level IV methods the consequences of failure can be taken into account. In cost-benefit analyses the expected total cost-benefit for a structure in its expected lifetime is maximized.

For a detailed introduction to structural reliability theory references are made to the following textbooks: Melchers (1999), Thoft-Christensen & Baker (1982) and Ditlevsen & Madsen (1996).

3.3.3 Generalisation of a basic reliability problem

In a basic reliability problem only one load effect, S, can be resisted by one resistance, R. The load and the resistance are expressed by a known probability density function, f_S and f_R

respectively.

Considering the definition of safety, the structure will be marked as failed if its resistance, R, is less than the stress resultant, S, action on it. Therefore the probability of failure can be stated as follows:

$$P_f = P[R - S \leq 0] = P[G(R,S) \leq 0] \tag{3.16}$$

Where $G(R,S)$ is termed the limit state function, and the probability of failure is identical with the probability of limit state violation. In figure 3.5, the above equation is represented by the hatched failure domain D, so that the failure probability becomes (Melchers (1999)):

$$P_f = P(R - S \leq 0) = \int_D \int f_{RS}(r,s)drds \tag{3.17}$$

where $f_{R,S}(r,s)$ is the joint (bivariate) density function.

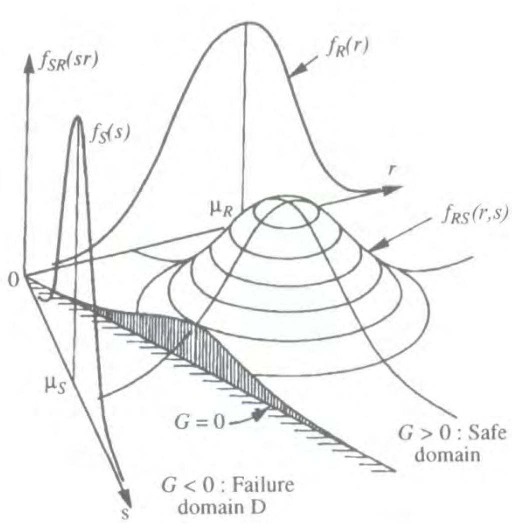


Figure 3.5 Two random variable joint density function $f_{RS}(r, s)$, marginal density functions f_R and f_S and failure domain D, (Melchers (1999))

With the limit state function expressed as G(X), the generalisation of the equation (3.17) becomes:

$$P_f = P[G(X) \leq 0] = \int \dots \int_{G(X)\leq 0} f_X(X) d_X \tag{3.18}$$

Here $f_X(X)$ is the joint probability density function for n-dimensional vector X of basic variables. Figure 3.6 shows generalisation of the reliability problem.

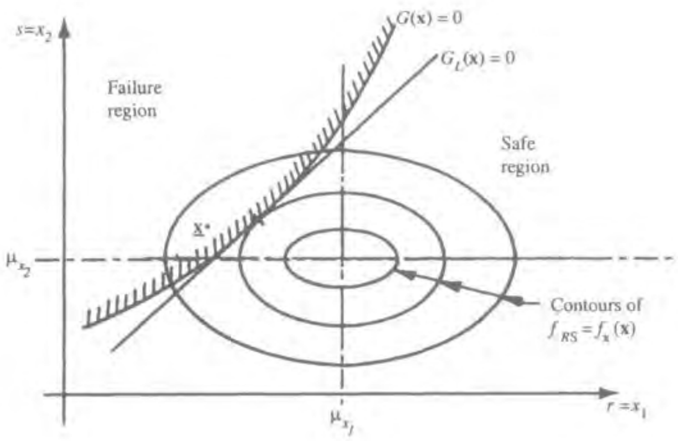


Figure 3.6 Limit state surface G(x)=0 and its linearised version G_L(x)=0 in the space of the basic variables, (Melchers (1999))

When both the load effect (S) and the pipe resistance (R) are independent and of normal distribution, the integral in Equation 3.17 can be determined from (Melchers, 1999):

$$P_f(t) = \Phi\left[\frac{-(\mu_R - \mu_S)}{(\sigma_S^2 + \sigma_R^2)^{1/2}}\right] = \Phi(-\beta) \tag{3.18}$$

where Φ is the standard normal distribution function, μ is the mean and σ is the standard deviation of random variables. β is known as safety index or reliability index.

3.3.4 Reliability of structural systems

In some cases of reliability analysis even in a simple structure composed of just one element, various limit states such as bending action, shear, buckling, axial stress, deflection, etc, may apply. Such a composition is referred to as the 'structural system'.

Individual structural components and subsystems have typical service lifespans that do not necessarily coincide with one another. In the reliability evaluation of structural systems it is described how the individual limit states interact on each other and how the overall systems reliability can be estimated when the individual failure modes are combined in a series or parallel system.

In a series system (also called a weakest link system), attainment of any one element limit state constitutes failure of the structure. All components of a parallel system (also called a redundant system) must fail for a system failure to occur. Combining parallel and series subsystems can make more complex systems (Figure 3.7).

If in a system reliability problem each failure mode is represented by a limit state equation Gi(x)=0 in basic variable space, the direct extension of the fundamental reliability problem (Equation 3.18) is

$$P_f = \int_{D \in X} \dots \int f_X(X) d_X \qquad (3.19)$$

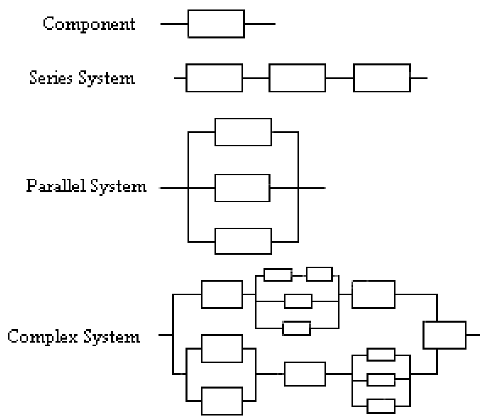


Figure 3.7 System definitions

Where X represents the vector of all basic random variables (load, strength of members, member properties, sizes, etc.) and D (and D1) is the domain in X defining failure of the system. This is defined in terms of the various failure modes as $G_i(X) \leq 0$.

In Figure 3.8 expression of Equation 3.20 is defined in a two dimensional X space.

As it was defined previously, a parallel system can fail only when all its contributory components have reached their limit states. This means that, in contrast with the situation for series systems, the behavioural characteristics of the system components are significantly important in defining system failure.

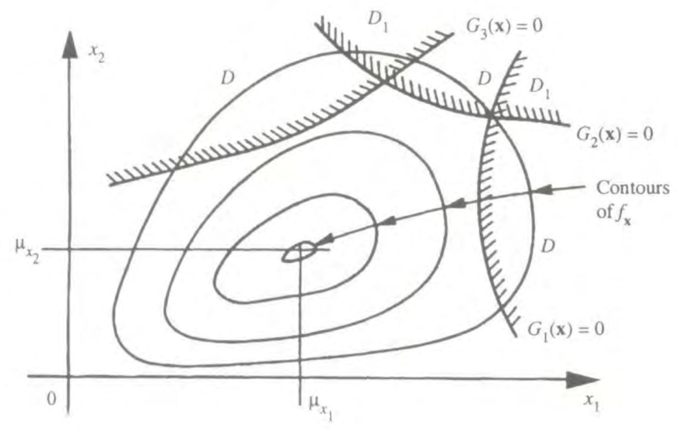


Figure 3.8 Basic structural system reliability problem in two dimensions showing failure domain D (and D1), (Melchers (1999))

3.3.5 Sensitivity analysis

Sensitivity analysis is widely accepted as a necessary part of reliability analysis of structures and infrastructure. The effect of variables on the reliability of a pipeline can be analysed by doing a comprehensive sensitivity analysis. In view of the large number of variables that affect the corrosion process and the limit state function, it is of interest to identify those variables that affect the failure most so that more research can focus on those variables.

Sensitivity analysis should be carried out to provide quantitative information necessary for classifying random variables according to their importance. These measures are essential for reliability-based service life prediction of deteriorating materials and structures.

Sensitivity analysis provides the degree of variation of limit state functions or measures at a specific point characterized by a realisation of all random variables. Similarly to the conventional sensitivity measure in the reliability approaches, the sensitivity measure, S, can be defined as follows (Kong and Frangopol (2005)):

$$S_{G(X)}(X_i) = \frac{\partial G(X)}{\partial X_i} = \lim_{\varepsilon \to 0} \frac{G(X+\varepsilon) - G(X)}{\varepsilon} \tag{3.20}$$

where G is a performance function of X; X and ε are vectors; and ε is a small perturbation. An element X_i of X can be any type of variable or parameter. For instance, it can be a mean or a standard deviation of a variable, or a deterministic parameter. For a complex system, the sensitivity measure can be computed by using the numerical differentiation method rather than by an analytical approach (Kong and Frangopol (2005)).

Different sensitivity indexes have been introduced. In the current research, relative contribution and sensitivity ratio will be discussed and will be used.

a) Relative Contribution

A sensitivity index that can be used in a comprehensive reliability analysis is the relative contribution of each variable in limit state function. The relative contribution (α_x^2) of each random variable (x) to the variance of the limit state function is introduced as follows (Ahammed and Melchers (1994)):

$$\alpha_x^2 = \frac{\left(\frac{\partial G}{\partial x}\sigma_x\right)^2}{\sigma_G^2} \tag{3.21}$$

where σ_x is standard deviation of the random variable x and σ_G^2 is the variance of the limit state function. Variables with higher values of α_x^2 contribute more in limit state function than other variables; therefore more focus and study needs to be carried out to determine the accurate values for such variables.

b) Sensitivity Ratio (SR)

A method of sensitivity analysis applied in many different models in science, engineering, and economics is the Sensitivity Ratio (SR), also known as the elasticity equation. The ratio is equal to the percentage change in output (e.g., probability of failure) divided by the percentage change in input for a specific input variable, as shown in the following equation (EPA 540, 2001):

$$\mathrm{SR} = \frac{\left(\frac{Y_2 - Y_1}{Y_1}\right)\times 100\%}{\left(\frac{X_2 - X_1}{X_1}\right)\times 100\%} \tag{3.22}$$

where, Y_1= the baseline value of the output variable using baseline values of input variables

Y_2= the value of the output variable after changing the value of one input variable

X_1= the baseline point estimate for an input variable

X_2= the value of the input variable after changing X_1

Risk estimates are considered most sensitive to input variables that yield the highest absolute value for SR. The basis for this equation can be understood by examining the fundamental concepts associated with partial derivatives. In fact, SR is equivalent to normalized partial derivative. Variables with higher values of sensitivity ratios are more effective on the limit state function or the probability of failure (EPA 540, 2001).

3.3.6 Background and methods for reliability analysis of pipes

Since large investment is required for building new urban water supply infrastructure it is unlikely to replace the existing pipe networks completely over a short period of time. Therefore the resort has to be maintenance and rehabilitation of existing pipes. To have an optimum strategy for maintenance and rehabilitation plans in the management of pipe asset, accurate prediction of the service life of pipes is essential. But this cannot be achieved without an accurate method for reliability analysis in which the likelihood of pipe failures is determined.

Reliability analysis can cover a wide domain of failure assessment of structures and infrastructure including both service life prediction and failure rate prediction. It should be noted that there is a clear distinction between the two terms:

Failure rate prediction of pipes: When the result of reliability analysis and/or failure assessment is presented as a number of failures within a period of time (e.g., breaks/year), it should be ideally considered as failure rate prediction.

Service life prediction of pipes: When in a study, service life of pipe(s), in terms of time, is investigated; the study should be named as service life prediction.

As a comparison, there is considerably less literature in the field of service life prediction compared to the failure rate prediction of pipes. It needs to be clarified that the focus of this study is on service life prediction of buried pipes. To that effect, the outcome of a comprehensive literature survey on reliability analysis and service life prediction of concrete sewers and iron based water pipes (including cast iron water pipes) is presented in this section.

Kleiner and Rajani (2001) define two classes of methods for service life prediction: deterministic and probabilistic methods. Deterministic methods do not consider variation in any variables that affect pipe behaviour and failure, whilst probabilistic methods consider some or all variables as random variables. The input parameters and output results of pipe deterioration models are heavily dependent on the type of methodology chosen. To find out the gaps and limitations of each model, the models are briefly explained here.

Deterministic models often use laboratory tests and sample specimens to find the necessary information, therefore the relationships between components are certain. Variations and uncertainties in variables are not considered in deterministic methods while probabilistic methods consider some or all variables as random variables.

Kaempfer and Berndt (1999) undertook a laboratory experiment to predict service life of concrete sewers subjected to sulphide corrosion. They used deterministic parameters from an accelerated laboratory test to predict the service life of concrete sewers. Since their study was in laboratory conditions, they suggested that for more accurate result the data from real sewage conditions is necessary.

Rajani et al. (2000) proposed a method to estimate the remaining service life of cast iron water pipes by considering that the corrosion pits reduce the structural capacity of the pipes. The residual capacity of the pipes was calculated by a reiterative model, based on corrosion pit measurement and the anticipated corrosion rate. The method was deterministic in the view that it does not consider the uncertainties involved in all factors contributing to the corrosion and subsequent failures.

A comprehensive deterministic life time assessment has been carried out by Kienow and Kienow (2001). They performed sulphide corrosion modelling as part of screening analyses to support the prioritization of sewer evaluation efforts for a sewer inspection and evaluation in the City of Fresno, California. The project consisted of the inspection and evaluation of approximately 90 kilometre of concrete sewers in sizes ranging from 30cm to 70cm in diameter.

Deb et al. (2002) presented a deterministic model based on analysing the growth of corrosion pits on cast iron (CI) pipes, loss of wall thickness and the strength reduction of the pipe over time. Kim et al. (2007) developed prediction models for CI pipes using the assessment of residual tensile strength based on pit characteristics and fracture toughness. Results illustrated that the proposed models using tensile strength and fracture toughness of CI pipes successfully estimated the residual life of water pipes. Analysis of the results showed that the determination of fracture toughness may be more reliable than considering only the pit depth.

Probabilistic models are often used when historical failure or inspection data is limited or unavailable. These models specifically analyse the effective parameters on pipe performance rather than evaluating the previous pipe failure history. Uncertainties are involved by considering random variables. Usually, these methods are applied to pipes where the process of deterioration and factors for failure are well understood.

Various frameworks have been proposed to model the behaviour of underground pipelines for different types of material, using the reliability-based concept. Ahammed and Melchers (1994, 1995 and 1997) reported a comprehensive and continuous study on the reliability analysis of underground steel pipelines. To consider uncertainty associated with the rate of corrosion and the uncertain location of its occurrence, they used a probabilistic approach (first-order second-moment reliability method, FOSM) for the analysis of pipeline reliability.

In 1994, they defined the failure mode as exceeding the sum of total stresses from the maximum allowable stress (yield strength of the pipe). They considered three types of existing stresses caused by internal pressure and external pressure:

- The circumferential stress due to internal fluid pressure
- The bending stress in the circumferential direction produced in the pipe wall by external soil loading
- The circumferential bending stresses produced in the pipe wall due to external traffic loads

Taking into account an empirical time-dependent corrosion model, resulted in a nonlinear limit state function that required an iterative solution technique for the calculation of reliability index and for sensitivity analysis.

In 1995 they modelled the growth of corrosion pits to assess the service life of liquid carrying metallic pipelines. In the pitting (localized corrosion) model, two corrosion related parameters (including pipeline dimension and liquid flow) were treated as probabilistic variables.

The limit state function was defined as the difference between the allowable fluid loss and the estimated fluid loss through the pit holes.

To calculate the probability of failure, they used the level II FOSM reliability method. This method requires an iterative solution, when the random variables are not normally distributed and the limit state function is nonlinear.

They developed their model in further studies (Ahammed and Melchers (1997)) by considering the effect of following longitudinal stresses:

- Longitudinal tensile stresses as a result of Poisson's ratio effect from the outward radial action of the internal fluid pressure
- Longitudinal thermal stresses
- Longitudinal stresses due to bending as a result of unevenness or settlement of the pipeline bedding

Camarinopoulos et al. (1999) used a combination of approximate quadrature analytical and Monte Carlo method to evaluate the multiple integrals in their reliability analysis for cast-iron buried water pipes. They also used the model to assess the sensitivity of structural reliability to the variation of some important parameters such as wall thickness, unsupported length and external corrosion coefficient.

Yves and Patrick (2000) also presented a method to calculate the reliability of the buried water pipes using maintenance records and the Weibull distribution for underlying variables. The method appears to rely entirely on the historical data, which in most cases is unknown.

Benmansour and Mrabet (2002) studied the reliability of buried concrete sewers by using the FOSM method. They studied two typical cases to assess the effect of loads on the circumferential and longitudinal behaviour of pipes. Therefore two limit state functions that they considered in their study were based on longitudinal cracking and circular cracking due to bending moments. They did not consider corrosion as a deterioration process, therefore their methodology was a simple time independent method.

Sadiq et al. (2004) used Monte Carlo simulations to perform the reliability analysis of cast iron water mains, considering axial and hoop stresses as acting loads in a limit state function. The reduction in the factor of safety (FOS) of water mains over time was computed, with a failure defined as a situation in which FOS becomes smaller than 1. The Monte Carlo simulations yielded an empirical probability density function of time to failure, to which a lognormal distribution was fitted leading to the derivation of a failure hazard function.

Davis et al (2005) used Weibull probability distribution to account for variation in the degradation rate of asbestos cement sewers. A tensile failure model was developed that simulates degradation until failure under in-service loading conditions. Simulated failure times were then fitted to a Weibull probability distribution, which allows the expected time to first failure to be calculated at different locations along the pipeline. They found a reasonable agreement between predicted failure times and recorded failures for a period of 8 years.

Amirat et al (2006) used reliability analysis to assess the effect of both the residual stresses generated during manufacturing process and in-service corrosion of underground steel pipes. During the service life of a pipe, residual stress relaxation occurs due to the loss of pipe thickness as material layers are consumed by corrosion. First they focused on the influence of residual stresses in uncorroded pipelines in order to identify the sensitivity of system parameters. In the second step, a probabilistic-mechanical model was used to couple the residual stress model with the corrosion model, in order to assess the aging effects through the pipe service life. For long term corrosion, the reliability analysis incorporated the residual stress relaxation resulting from wall thickness losses. The probability of failure of the pipeline was then evaluated for different corrosion rates varying from the atmospheric baseline to very active corrosion processes.

DeSilva et al. (2006) presented a condition assessment and probabilistic analysis to estimate failure rates in metallic pipelines. A Level II first-order-second moment (FOSM) analysis was combined with condition assessment data to determine the probability of failure. Davis and Marlow (2008) developed a physical probabilistic failure model for service life prediction of CI pipelines subject to corrosion under internal pressure and external loading. A limitation within their study was that the model only considered internal pressure and in-plane bending; therefore, the resulting failure mode was only shown with a longitudinal fracture, which is just one type of several failure modes. Other failure modes such as circumferential fractures were not considered.

Moglia et al. (2008) looked at the exploration of a CI pipe failure model utilising fracture mechanics of the pipe failure process. The first model generated was simple, which allows explorations of additional model assumptions. Throughout numerous assumptions, the model improved drastically. An elementary method, FOSM, was initially used but proved to yield inaccurate results. A new approach to the model evaluated the nominal tensile strength of pipes, which could determine the maximum corrosion defect. To account for the uncertainty or randomness within the data, a Weibull distribution was utilized adding stochasticity to the corrosion rate. The proposed model calculated failure rates based on historical data using a random Poisson statistical process. The maximum likelihood estimator used within the Poisson distribution was used to calculate the failure rate of the historical data sets. A case study was employed utilising small diameter reticulation mains. By modeling various assumptions into the simulated model, the predicted and observed failure rates yielded similar results. Only failure modes by corrosion or combined corrosion and fractures were included within the observed data model.

Yamini (2009) also used different reliability analysis methods (Monte Carlo simulation, First Order Reliability Method (FORM) and Second Order Reliability Method (SORM)) for failure analysis of cast iron water mains. In his study two failure modes were considered individually. A failure mode was defined as the point at which the corrosion depth is more than the maximum acceptable decrease in pipe wall thickness and another failure mode was defined as the time at which total stresses exceed the pipe strength capacity.

Lee et al (2010) also used a first order reliability method (FORM) to evaluate the time-dependent reliability index for a fully deteriorated piping component rehabilitated with Fibre

Reinforced Plastic (FRP), considering the demand of internal fluid pressure, external soil pressure and traffic loading.

Zhou (2011) developed a methodology to carry out the time-dependent reliability evaluation of a pressurized steel gas pipeline containing an active corrosion defect by taking into account the time-dependency of the internal pressure.

3.3.7 Gaps in the current state of the art of reliability analysis of concrete sewers and cast iron pipes

In the previous section, the former works on reliability analysis and service life prediction of underground pipes was reviewed. The comprehensive literature review revealed the areas lacking in knowledge in the subject. In this section the gaps that have been found specifically in the area of reliability analysis and service life prediction of concrete sewers and cast iron water pipes are declared.

a) Reliability analysis of concrete sewers

Despite some sporadic works on prediction of failure rates in sewer systems (Ana et al. (2008) and Savic et al. (2009)), there has not been sufficient research on service life prediction of concrete sewers. The lack of literature is more remarkable in the field of probabilistic service life prediction of concrete sewers.

The literature survey in this research also showed that although the mechanism of sulphide corrosion in concrete sewers has been reasonably understood, effectiveness and contribution of corrosion parameters on the service life of pipelines have not been studied by researchers. This lack of knowledge necessitates an extensive sensitivity analysis on reliability analysis of concrete sewers.

b) Reliability analysis of cast iron water pipes

Compared to concrete sewers, the study on probabilistic service life prediction of cast iron water pipes as a metal base infrastructure has been considered by several researchers in recent years (Rajani et al. (2000), Sadiq et al. (2004) and Davis and Marlow (2008)).

In most cases a Monte Carlo simulation method has been used as a strong numerical method for reliability analysis of buried pipes. The mathematical complexity of analytical methods for time dependent reliability analysis has made these methods less effective for failure assessment. This complexity is expanded when multi failure mode assessment (i.e., more than one limit state function) needs to be considered for a more realistic and comprehensive study.

Parameters which are involved in failure and service life of cast iron water pipes have been clarified in previous researches (e.g., Rajani et al. (2000) and Sadiq et al. (2004)). However, the significance and contribution level of these parameters on failure function(s) and service life of cast iron pipelines have not been studied previously. This is what is dealt with in sensitivity analysis of reliability of cast iron water pipes in chapter 6.

A review of most recent research literature (Sadiq et al. (2004), Moglia et al. (2008), Yamini (2009) and Clair and Sinha (2012)) also suggests that in most reliability analyses for buried pipes, multi failure modes are rarely considered; while the real condition in practice, necessitates consideration of multi failure modes analysis.

3.4 Summary

At the first step of reliability analysis and service life prediction of buried pipes, it is necessary to gain knowledge about the design principles and loads affecting buried pipes. The performance criteria for in service loads were introduced in this chapter as ultimate limit state and serviceability limit state. Flexural and shear failures are two main ultimate limit states that are considered in design and assessment. Serviceability limit states may be measured by cracking condition. These criteria should be met for a buried pipe to be safe and in service. This chapter reviewed the loads and stresses acting on buried pipes. The formulations for stresses will be used in reliability analysis, where the limit state function (failure mode) is checked.

Material corrosion is the most common form of buried pipe deterioration and is a matter of concern for both the strength and durability functions. Loss of wall thickness through general corrosion affects the strength of a pipe. Evidently, how to incorporate the effect of corrosion in the structural analysis of a pipeline is of practical importance. This chapter outlined the corrosion formulation for concrete sewers and for cast iron water mains.

Parameters affecting sulphide corrosion in a concrete sewer were introduced based on literature review and reliable references. Metal corrosion in cast iron water mains was also discussed and common formulation for corrosion in those pipes were outlined.

The principles of structural reliability analysis were discussed and the previous studies for reliability analysis and service life prediction of buried pipes were presented based on a comprehensive literature review.

Failure rate prediction has been considered in most of the previous researches in the field of reliability analysis and the failure assessment of sewers and water pipes. In contrast, service life prediction of these pipes has been considered less. This lack of literature is more remarkable for concrete sewers compared with cast iron water pipes. To that effect, one of the main contributions of the current research is to fill the gap in the area of service life prediction of both concrete sewers and cast iron pipes.

There are several parameters which may affect corrosion rate and hence the service lives of pipes. In conventional methods for their service life prediction, these parameters are considered to be deterministic. However, in reality there are uncertainties associated to these parameters. An approach to uncertainty representation is to represent each uncertain variable as a random variable with mean and standard deviation. Therefore for reliability analysis of corrosion affected pipes, using probabilistic methods is necessary. While the process of corrosion is time dependent, the reliability analysis methods which are used for corrosion affected pipes should also be time dependent. Therefore, among the reliability analysis techniques which were noted in section 3.3.2, using time dependent reliability techniques can be the best approach for dealing with reliability analysis and service life prediction of corrosion affected buried pipes. The methods are developed in chapter 4 to be applied for the reliability analysis of concrete sewers and cast iron water mains.

Another gap in the current state of the art of reliability analysis of pipes which was found in the literature review, is the lack of research on the multi failure mode analysis. Therefore the emphasis in the current study is to consider multi failure modes in reliability analysis and service life prediction of concrete sewers and cast iron water pipes.

The evaluation of the contributions of various uncertain parameters associated with pipeline life assessment can be carried out by using sensitivity analysis techniques. The effectiveness and contribution of corrosion parameters on the service life of concrete sewers and cast iron water pipes which have not been deeply studied by other researchers, are investigated in the current research.

In this chapter, the design of buried pipes was reviewed. The service loads, corrosion mechanism and principles of service life prediction were presented. A critical analysis of methods for reliability analysis and service life prediction of concrete sewers and cast iron water pipes was presented. In chapter 4, the novel methodologies proposed for development of two time-dependent reliability methods of analysis are presented.

4 DEVELOPING METHODS FOR TIME DEPENDENT RELIABILITY ANALYSIS OF PIPES

The deficiencies in the area of the reliability analysis and service life prediction of concrete sewers and cast iron pipes were identified and discussed in chapter 3. In this chapter, two new methods for the time-dependent reliability analysis of corrosion affected pipes are presented. These methods will allow for:

- Application to all type of corrosion-affected structures and/or pipelines,
- The capability to consider multi failure mechanisms/modes
- Consideration of the scarcity of monitoring data from real world examples

The outcome of this approach is potentially a robust tool for use by infrastructure managers to predict and mitigate pipe failure.

4.1 Background

As it was concluded in the previous chapter, probably the most viable approach to predict the structure's reliability or its service life under future performance conditions is through probability-based techniques involving time dependent reliability analyses.

By using these techniques a quantitative measure of structural reliability is provided to integrate information on design requirements, material and structural degradation, damage accumulation, environmental factors, and non-destructive evaluation technology. The technique can also investigate the role of in-service inspection and maintenance strategies in enhancing reliability and extending service life. Several non-destructive test methods that detect the presence of a defect in a structure tend to be qualitative in nature in that they indicate the presence of a defect but may not provide quantitative data about the defect's size, precise location, and other characteristics that would be needed to determine its impact on structural performance. None of these methods can detect a given defect with certainty. The imperfect nature of these methods can be described in statistical terms. This randomness affects the calculated reliability of a component.

Structural loads, engineering material properties, and strength-degradation mechanisms are random. The resistance, R(t), of a structure and the applied loads, S(t), both are stochastic functions of time. At any time, t, the safety limit state, $\mathrm{G(R,S,t)}$, is (Melchers (1999)):

$$G(R, S, t) = R(t) - S(t) \tag{4.1}$$

Making the customary assumption that R and S are statistically independent random variables, the probability of failure resulting from Equation 4.1, $P_f(t)$, is (Melchers (1999)):

$$P_f(t) = P[G(t) \leq 0] = \int_0^{\infty} F_R(x) f_s(x) dx \tag{4.2}$$

in which $F_R(x)$ and $f_S(x)$ are the probability distribution function of R and density function of S respectively. Equation 4.2 provides quantitative measure of structural reliability and performance, provided that P_f can be estimated and validated.

The probability that failure occurs for any one load application is the probability of limit state violation. Roughly, it may be represented by the amount of overlap of the probability density functions f_R and f_s in Figure 4.1. Since this overlap may vary with time, P_f also may be a function of time.

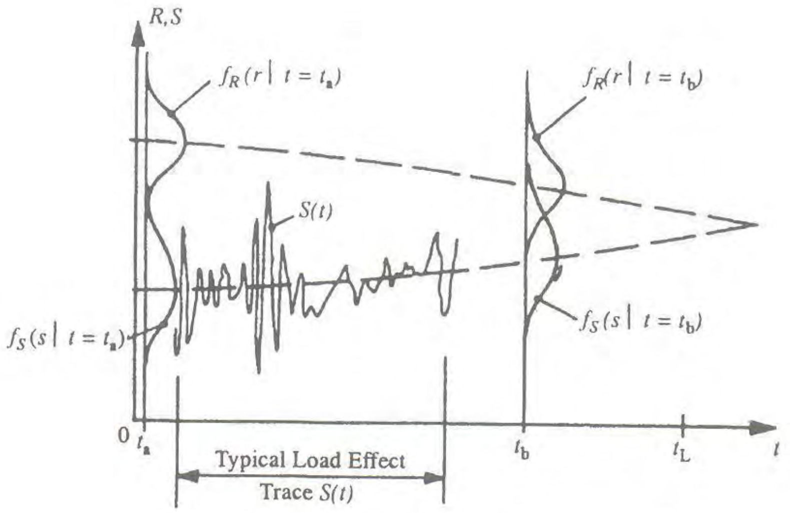


Figure 4.1 Schematic time dependent reliability problem, (Melchers (1999))

4.2 Selection of the appropriate method

As it was mentioned in section 3.3.6 of Chapter 3, between the two categories of reliability analysis methods, probabilistic methods should be considered for reliability analysis and service life prediction of buried pipes. For corrosion affected pipes, the method should also be time dependent.

It can be concluded from the a comprehensive literature review in chapter 3 that probabilistic time dependent methods need to be developed for use in reliability analysis of corrosion affected buried pipes. The methods include: first passage probability method and gamma process concept method.

In the following sections these methods are developed to be applied for corrosion affected pipes. Monte Carlo simulation method is also discussed to be used as a strong simulation technique for reliability analysis. This technique is used in the next chapters to verify the results obtained from the application of the two probabilistic time dependent methods (i.e., first passage probability and gamma process concept).

4.3 First passage probability method

The service life of a pipe or structure in general is a time period at the end of which the pipe stops performing the functions it is designed and built for. As it is mentioned earlier, to determine the service life for pipes, a limit state function ($G(t) = R(t) - S(t)$) is introduced. Where $S(t)$ is the action (load) or its effect at time t and R(t) is the acceptable limit (resistance) for the action or its effect. With the limit state function of Equation (4.1), the probability of pipe (structural) failure, p_f, can be determined by:

$$P_f(t) = P[G(t) \leq 0] = P[S(t) \geq R(t)] \tag{4.3}$$

At a time that $P_f(t)$ is greater than a maximum acceptable risk in terms of the probability of pipe failure, p_a, it is the time the pipe becomes unsafe or unserviceable and requires replacement or repairs. This can be determined from the following:

$$\mathrm{P_f(T_L) \geq P_a} \tag{4.4}$$

where T_L is the service life for the pipe for the given assessment criterion and acceptable risk. In principle, the acceptable risk, p_a, can be determined from a risk-cost optimization of the pipeline system during its whole service life. This is beyond the scope of this research and will not be discussed herein but can be referred to Mann and Frey (2011) and Dawotola et al. (2012).

Equation 4.3 represents a typical upcrossing problem in mathematics and can be dealt with using time-dependent reliability methods. Time-dependent reliability problems are those in which either all or some of basic variables are modeled as stochastic processes. In this method, the structural failure depends on the time that is expected to elapse before the first occurrence of the action process S(t) upcrossing an acceptable limit (the threshold) L(t) sometime during the service life of the structure [0, T_L]. Equivalently, the probability of the first occurrence of such an excursion is the probability of failure $P_f(t)$ during that time period. This is known as "first passage probability" and can be determined by (Melchers (1999)):

$$P_f(t) = 1 - [1 - P_f(0)]e^{-\int_0^t \nu d_t} \tag{4.5}$$

Where $P_f(0)$ is the probability of structural failure at time t = 0 and υ is the mean rate for the action process S(t) to upcross the threshold R(t).

The upcrossing rate in Equation 4.5 can be determined from the Rice formula (Melchers (1999)):

$$\nu = \nu_R^+ = \int_R^\infty (\dot{S} - \dot{R}) f_{S\dot{S}}(R, \dot{S}) d_{\dot{S}} \tag{4.6}$$

where ν_R^+ is the upcrossing rate of the action process *S*(t) relative to the threshold R, $\dot{R}$ is the slope of *R* with respect to time, $\dot{S}(t)$ is the time derivative process of *S*(t) and $f_{S\dot{S}}$ is the joint probability density function for S and $\dot{S}$. An analytical solution to Equation 4.6 has been derived for a deterministic threshold *R* in Li and Melchers (1993) as follows:

$$\upsilon_R^+ = \frac{\sigma_{\dot{S}|S}}{\sigma_S} \emptyset\left(\frac{R-\mu_S}{\sigma_S}\right)\left\{\emptyset\left(-\frac{\dot{R}- \mu_{\dot{S}|S}}{\sigma_{\dot{S}|S}}\right) - \frac{\dot{R}-\mu_{\dot{S}|S}}{\sigma_{\dot{S}|S}} \Phi\left(-\frac{\dot{R}-\mu_{\dot{S}|S}}{\sigma_{\dot{S}|S}}\right)\right\} \tag{4.7}$$

where $\emptyset$ and Φ are standard normal density and distribution functions respectively, μ and σ denote the mean and standard deviation of S and $\dot{S}$, represented by subscripts and " | " denotes the condition. For a given Gaussian stochastic process with mean function $\mu_S(t)$, and auto-covariance function $C_{SS}(t_i, t_j)$, all terms in Equation 4.7 can be determined, based on the theory of stochastic processes as detailed in Papoulis and Pillai (2002) as follows.

$$\mu_{\dot{S}|S} = E[\dot{S}|S = R] = \mu_{\dot{S}} + \rho\frac{\sigma_{\dot{S}}}{\sigma_S}(R - \mu_S) \tag{4.8a}$$

$$\sigma_{\dot{S}|S} = [\sigma_{\dot{S}}^2(1-\rho^2)]^{1/2} \tag{4.8b}$$

Where

$$\mu_{\dot{S}} = \frac{d\mu_S(t)}{dt} \tag{4.8c}$$

$$\sigma_{\dot{S}} = \left[\frac{\partial^2 C_{SS}(t_i,t_j)}{\partial t_i \partial t_j}\Big|_{\cdot i=j}\right]^{1/2} \tag{4.8d}$$

$$\rho = \frac{C_{S\dot{S}}(t_i,t_j)}{[C_{SS}(t_i,t_i).C_{\dot{S}\dot{S}}(t_j,t_j)]^{1/2}} \tag{4.8e}$$

And the cross-covariance function is:

$$C_{S\dot{S}}(t_i,t_j) = \frac{\partial C_{SS}(t_i,t_j)}{\partial t_j} \tag{4.8f}$$

Because it is unlikely that the corrosion depth in a given pipe exceeds the wall thickness at the beginning of structural service, the probability of failure due to corrosion at $t = 0$ is zero, i.e., $P_f(0) = 0$. The solution to Equation 4.5 can be expressed, after substituting Equation 4.7 into Equation 4.5, and considering that R is constant ($\dot{R} = 0$) therefore:

$$P_f(t) = \int_0^t \frac{\sigma_{\dot{S}|S}(t)}{\sigma_S(t)} \emptyset\left(\frac{R-\mu_S(t)}{\sigma_S(t)}\right)\left\{\emptyset\left(-\frac{\mu_{\dot{S}|S}(t)}{\sigma_{\dot{S}|S}(t)}\right) + \frac{\mu_{\dot{S}|S}(t)}{\sigma_{\dot{S}|S}(t)}\Phi\left(\frac{\mu_{\dot{S}|S}(t)}{\sigma_{\dot{S}|S}(t)}\right)\right\} d_\tau \tag{4.9}$$

For Equation (4.9) to be of practical use, i.e., determining the probability of serviceability failure due to corrosion over time, there is a requirement to develop a stochastic model for the corrosion depth. This is dealt with in section 5.3.1 in Chapter 5.

4.4 Gamma process concept

To deal with data scarcity and uncertainties, using stochastic models for time dependent reliability analysis of corrosion affected buried pipes can be considered. In order to model monotonic progression of a deterioration process, the stochastic gamma process concept can be used for modeling the reduction of pipe wall thickness due to corrosion. The gamma process is a stochastic process with independent, non-negative increments having a gamma distribution with an identical scale parameter and a time-dependent shape parameter.

A stochastic process model, such as gamma process, incorporates the temporal uncertainty associated with the evolution of deterioration (e.g. Bogdanoff and Kozin (1985), Nicolai et al. (2004), van Noortwijk and Frangopol (2004)).

The gamma process is suitable to model gradual damage monotonically accumulating over time, such as wear, fatigue, corrosion, crack growth, erosion, consumption, creep, swell, a degrading health index, etc. For the mathematical aspects of gamma processes, see Dufresne et al. (1991), Ferguson and Klass (1972), Singpurwalla (1997), and van der Weide (1997).

To the best of authors' knowledge, Abdel-Hameed (1975) was the first to propose the gamma process as a model for deterioration occurring randomly in time. In his paper he called this stochastic process the "gamma wear process". An advantage of modeling deterioration processes through gamma processes is that the required mathematical calculations are relatively straightforward.

4.4.1 Problem formulation

The mathematical definition of the gamma process is given in Equation (4.10). Given that a random quantity d has a gamma distribution with shape parameter $\alpha > 0$ and scale parameter $\lambda > 0$ if its probability density function is given by:

$$Ga(d|\alpha,\lambda) = \frac{\lambda^{\alpha}}{\Gamma(\alpha)} d^{\alpha-1} e^{-\lambda d} \tag{4.10}$$

Let $\alpha(t)$ be a non-decreasing, right continuous, real-valued function for $t \geq 0$, with $\alpha(0) \equiv 0$. $\Gamma(\alpha)$denotes gamma function of α with mathematical definition of $\Gamma(\alpha) = (\alpha-1)!$. The gamma process is a continuous-time stochastic process $\{d(t), t \geq 0\}$ with the following properties:

1. $d(0) = 0$ with probability one;
2. $d(\tau) - d(t) \sim Ga(\alpha(\tau) - \alpha(t), \lambda)$ for all $\tau > t \geq 0$;
3. $d(t)$ has independent increments.

Let $d(t)$ denote the deterioration at time t, $t \geq 0$, and let the probability density function of $d(t)$, in accordance with the definition of the gamma process, be given by

$$f_{d(t)}(d) = Ga(d|\alpha(t), \lambda) \tag{4.11}$$

with mean and variance as follows:

$$E\big(d(t)\big) = \frac{\alpha(t)}{\lambda} \tag{4.12}$$

$$Var\big(d(t)\big) = \frac{\alpha(t)}{\lambda^2} \tag{4.13}$$

A pipe is said to fail when its corrosion depth, denoted by $d(t)$, is more than a specific threshold (a_0). Assuming that the threshold a_0 is deterministic and the time at which failure occurs is denoted by the lifetime T. Due to the gamma distributed deterioration, Equation 4.11, the life time distribution can then be written as:

$$\mathrm{F(t)} = \Pr(\mathrm{T} \leq \mathrm{t}) = \Pr(d(t) \geq a_0) = \int_0^{a_0} f_{d(t)}(d) d_d = \frac{\Gamma(\alpha(t), a_o\lambda)}{\Gamma(\alpha(t))} \tag{4.14}$$

Where $\Gamma(\nu, x) = \int_{t=x}^{\infty} t^{\nu-1} e^{-t} dt$ is the incomplete gamma function for $x \geq 0$ and $\nu > 0$.

To model corrosion in a pipe, in terms of a gamma process, the question that remains to be answered is how its expected deterioration increases over time. The expected corrosion depth at time t may be modelled empirically by a power law formulation (Ahammed and Melchers (1997)):

$$\alpha(t) = ct^b \tag{4.15}$$

for some physical constants $c > 0$ and $b > 0$.

Because there is often engineering knowledge available about the shape of the expected deterioration in terms of the exponential parameter b in Equation 4.15, this parameter may be assumed constant. The typical values for b from some examples of expected deterioration according to a power law are presented in the Table 4.1.

Deterioration type	**Exponential parameter, *b***	**Reference**

degradation of concrete due to reinforcement corrosion	1	Ellingwood & Mori (1993)
sulfate attack	2	Ellingwood & Mori (1993)
diffusion-controlled aging	0.5	Ellingwood & Mori (1993)
creep	1/8	Cinlar et al. (1977)
expected scour-hole depth	0.4	Hoffmans & Pilarczyk (1995) and van Noortwijk & Klatter (1999)

Table 4.1 Typical values for exponential parameter, b, in different deterioration types

The reliability analysis approach which is developed in this section by using gamma process concept is entitled 'Gamma Distributed Degradation, GDD' model.

In the event of expected deterioration in terms of a power law (i.e., Equation 4.15), the parameters c and λ can be estimated by using statistical estimation methods. The estimation procedure is discussed for the two scenarios including a case with available corrosion depth data and a case of unavailability of corrosion depth data.

4.4.2 Developing gamma distributed degradation model with available corrosion depth data

In this section using gamma distributed degradation (GDD) model for reliability analysis of corrosion affected pipes in case of availability of corrosion depth data is discussed. The data of corrosion depth can be achieved by periodical inspections.

To model the corrosion as a gamma process with shape function $\alpha(t) = ct^b$ and scale parameter λ, the parameters c and λ should be estimated. For this purpose, statistical methods are suggested. The two most common methods that can be used for parameter estimation are the maximum likelihood and method of moments. Both methods for deriving the estimators of c and λ were initially presented by Cinlar et al. (1977) and were developed by van Noortwijk and Pandey (2003).

a) Maximum Likelihood Estimation

In statistics, maximum-likelihood estimation (MLE) is a method of estimating the parameters of a statistical model. When applied to a data set and given a statistical model, MLE provides estimates for the model's parameters.

In general, for a fixed set of data and underlying statistical model, the method of maximum likelihood selects values of the model parameters that produce a distribution that gives the observed data the greatest probability (i.e., parameters that maximize the likelihood function). Given that n observations are denoted by $x_1, x_2, \dots, x_n$, the principle of maximum likelihood assumes that the sample data set is representative of the population. This has a probability density function of $f_x(x_1, x_2, \dots, x_n; \theta)$, and chooses that value for θ (unknown parameter) that most likely caused the observed data to occur, i.e., once observations $x_1, x_2, \dots, x_n$ are given, $f_x(x_1, x_2, \dots, x_n; \theta)$ is a function of θ alone, and the value of θ that maximizes the above probability density function is the most likely value for θ.

In the current study a typical data set consists of inspection times $t_i, i = 1, \dots, n$ where $0 = t_0 < t_1 < t_2 < \cdots < t_n$, and corresponding observations of the cumulative amounts of deterioration $d_{i,}\ i = 1, \dots, n$ are assumed to be given as inputs of the model. Figure 4.2 schematically shows a time dependent degradation model in the case of two inspections with a deterministic path.

The maximum-likelihood estimators of c and λ can be determined by maximising the logarithm of the likelihood function of the increments. The likelihood function of the observed deterioration increments $\delta_i = d_i - d_{i-1},\ i = 1, \dots, n$ is a product of independent gamma densities (van Noortwijk and Pandey (2003)):

$$l(\delta_1, \dots, \delta_n | c, \lambda) = \prod_{i=1}^{n} f_{d(t_i)-d(t_{i-1})}(\delta_i) = \prod_{i=1}^{n} \frac{\lambda^{c[t_i^b - t_{i-1}^b]}}{\Gamma(c[t_i^b - t_{i-1}^b])} \delta_i^{c[t_i^b - t_{i-1}^b]} e^{-\lambda\delta_i} \quad (4.16)$$

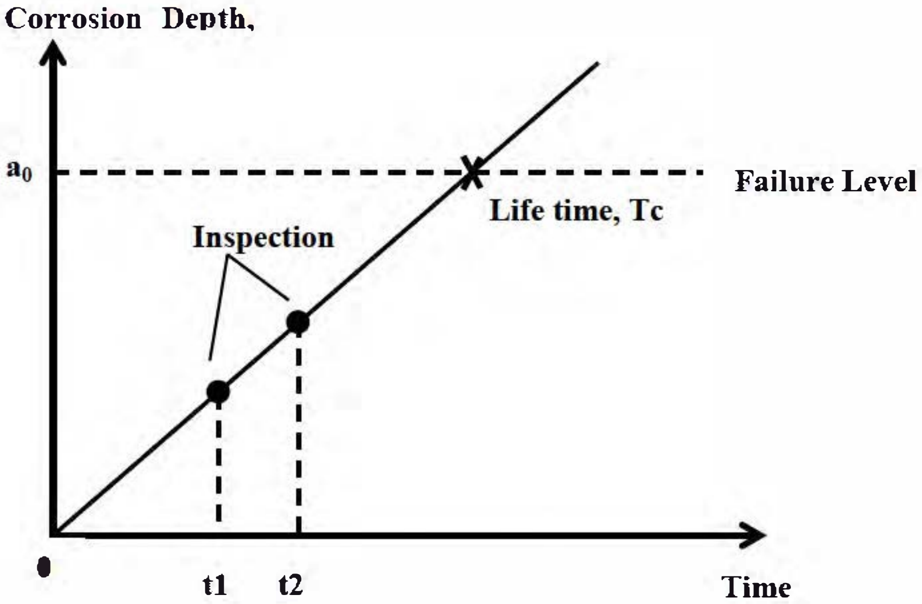


Figure 4.2 Time dependent degradation model in case of two inspections

To maximize the logarithm of the likelihood function, its derivatives are set to zero. It follows that the maximum likelihood estimator of λ is:

$$\hat{\lambda} = \frac{\hat{c}t_n^b}{d_n} \quad (4.17)$$

where $\hat{c}$ must be computed iteratively from the following equation:

$$\sum_{i=1}^{n}[t_i^b - t_{i-1}^b]\{\psi(\hat{c}[t_i^b - t_{i-1}^b]) - log\delta_i\} = t_n^b log\left(\frac{\hat{c}t_n^b}{d_n}\right) \quad (4.18)$$

where the function $\psi(x)$ is the derivative of the logarithm of the gamma function:

$$\psi(x) = \frac{\acute{\Gamma}(x)}{\Gamma(x)} = \frac{\partial log\Gamma(x)}{\partial x} \quad (4.19)$$

b) Method of Moments

In statistics, the method of moments is a method of estimation of population parameters such as mean and variance by equating sample moments with unobservable population moments and then solving those equations for the quantities to be estimated. Assuming transformed times between inspections as $w_i = t_i^b - t_{i-1}^b, i = 1, \dots, n$, the method-of-moments estimates c and λ can be found from (van Noortwijk and Pandey (2003)):

$$\frac{\hat{c}}{\hat{\lambda}} = \frac{\sum_{i=1}^{n} \delta_i}{\sum_{i=1}^{n} w_i} = \frac{d_n}{t_n^b} = \bar{\delta} \tag{4.20}$$

$$\frac{d_n}{\hat{\lambda}}\left(1 - \frac{\sum_{i=1}^{n} w_i^2}{\left[\sum_{i=1}^{n} w_i\right]^2}\right) = \sum_{i=1}^{n}\left(\delta_i - \bar{\delta} w_i\right)^2 \tag{4.21}$$

The first equation from both methods (i.e., Equations 4.17 and 4.20) are the same and the second equation in the method of moments is simpler since it does not necessarily require iterations to find the unknown parameter ($\hat{c}$).

The flowchart in Figure 4.3 illustrates the gamma distributed degradation model in case of availability of corrosion measurements. To use this procedure, at least two measures of corrosion depth should be available for calculation of δ_i in Equations 4.18 and 4.21.

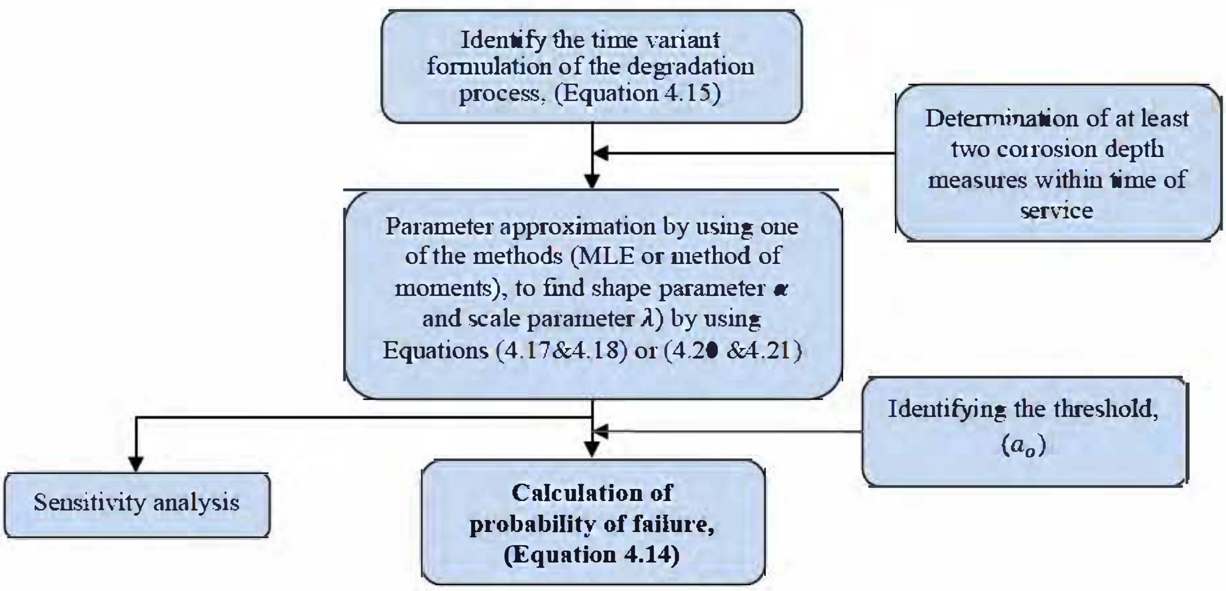


Figure 4.3 Gamma distributed degradation (GDD) Model in case of availability of corrosion depth data

4.4.3 Developing gamma distributed degradation model in case of unavailability of corrosion depth data

In practice, most of the time for reliability analysis of corrosion affected pipes, data such as corrosion depth are not available. Therefore, a method should be developed for such cases of using the gamma distributed degradation model. As it was mentioned in section 4.4.1, in order to calculate the probability of failure over elapsed time (Equation 4.14), the parameters corresponding to shape and scale parameters (α and λ) should be estimated. The steps for this purpose are:

a) Determining the approximate moments (mean and variance)

b) Estimating values for α and λ by using Equations 4.12 and 4.13

Assuming $X_1, X_2, \dots, X_n$ as basic random variables, moment approximation (i.e., step (a)) can be carried out by expanding the function $Y = Y(X_1, X_2, \dots, X_n)$ in a Taylor series about the point defined by the vector of the means$(\mu_{X_1}, \mu_{X_2}, \dots, \mu_{X_n})$. By truncating the series, the mean and variance are (Papoulis and Pillai (2002)):

$$\mathrm{E(Y)} \approx \mathrm{Y}\left(\mu_{\mathrm{X_1}}, \mu_{\mathrm{X_2}}, \dots, \mu_{\mathrm{X_n}}\right) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial^2 \mathrm{Y}}{\partial \mathrm{X_i}\,\partial \mathrm{X_j}}\mathrm{cov}(\mathrm{X_i}, \mathrm{X_j}) \tag{4.22}$$

$$\mathrm{var(Y)} \approx \sum_{i}^{n}\sum_{j}^{n} c_i c_j \mathrm{cov}(\mathrm{X_i}, \mathrm{X_j}) \tag{4.23}$$

The flowchart in Figure 4.4 illustrates the gamma distributed degradation model in case that corrosion measurements are not available. The procedure will be used for reliability analysis of concrete sewers and cast iron pipes in Chapters 5 and 6.

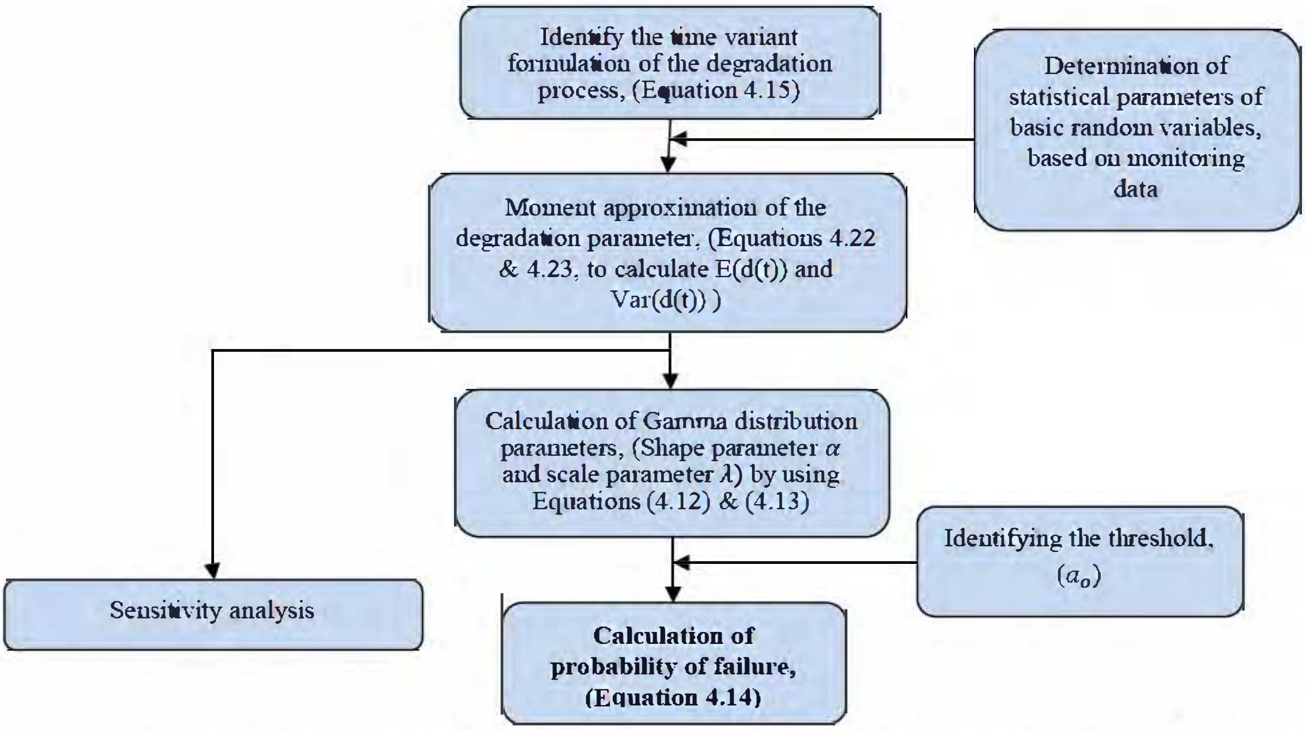


Figure 4.4 Gamma distributed degradation (GDD) Model in case of unavailability of corrosion depth data

4.5 Monte Carlo simulation method

Monte Carlo simulation has been successfully used for reliability analysis of different structures and infrastructure (e.g., Camarinopoulos et al. (1999), Melchers (1999), Sadiq et al. (2004) and Yamini (2009)). Hence, the method is used as a verification method to check the results which are obtained from application of the two time dependent analytical method (i.e, first passage probability method and gamma distributed degradation model).

Monte Carlo simulation techniques involve sampling at random to artificially simulate a large number of experiments and to observe the results. To use this method in structural reliability analysis, a value for each random variable is selected randomly ($\hat{x}_i$) and the limit state function ($G(\hat{x})$) is checked. If the limit state function is violated (i.e. $G(\hat{x}) \leq 0$), the structure or the system has failed. The experiment is repeated many times, each time with randomly chosen variables. If N trials are conducted, the probability of failure then can be estimated by dividing the number of failures to the total number of iterations:

$$P_f \approx \frac{n(G(\hat{x}) \leq 0)}{N} \tag{4.24}$$

The accuracy of Monte Carlo simulation result depends on the sample size generated and, in the case when the probability of failure is estimated, on value of the probability (the smaller the probability of failure, the larger the sample size needed to ensure the same accuracy). The accuracy of the failure probability estimates can be checked by calculating their coefficient of variation (e.g., Melchers (1999)).

In order to improve the accuracy of estimating the probability of ultimate strength failure, while keeping the computation time within reasonable limits, variance reduction techniques (e.g., importance sampling, Latin hypercube, and directional simulation) can be employed. However, in cases that the main emphasis is on serviceability failure, which can be estimated by a crude Monte Carlo simulation with very good accuracy within a relatively short computation time, such techniques are not necessary to be used (Val and Chernin (2009)).

Importance sampling is a variance reduction technique that can be used in the Monte Carlo method (Melchers (1999)). The idea behind importance sampling is that certain values of the input random variables in a simulation have more impact on the parameter being estimated than others. If these "important" values are emphasized by sampling more frequently, then the estimator variance can be reduced. Hence, the basic methodology in importance sampling is to choose a distribution which "encourages" the important values. The use of "biased" distributions will result in a biased estimator if it is applied directly in the simulation. However, the simulation outputs are weighted to correct use of the biased distribution, and this ensures that the new importance sampling estimator is unbiased.

The fundamental issue in implementing importance sampling simulation is the choice of the biased distribution which encourages the important regions of the input variables. Choosing or designing a good biased distribution is the "art" of importance sampling. The rewards for a good distribution can be significant run-time savings; the penalty for a bad distribution can be longer run times than for a general Monte Carlo simulation without importance sampling.

The details of the Monte Carlo method including sampling techniques can be found in Ditlevsen and Madesn (1996), Melchers (1999) and Rubinstein and Kroese (2008).

4.6 Summary

In this chapter, two probabilistic methods for time-dependent reliability analysis were presented. These two methods are the: (a) first passage probability method and (b) the gamma distributed degradation (GDD) model.

The gamma distributed degradation model was considered for two different scenarios: (a) where corrosion-depth data is available and, (b) where it is not.

The *availability* of deterioration data has already been considered in a previous study of sea defense structures (van Noortwijk et al. 2007). However, the current research will develop this approach for corrosion-affected buried pipes. In addition the same approach will be modified for corrosion scenarios where data is unavailable. This work has already been published in peer-reviewed journal papers, illustrating the novelty and robustness of this approach.

The application of first passage probability theory together with a Monte Carlo simulation, incorporating the new code developed in this work, the improved prediction of service performance has been achieved (Mahmoodian and Li 2011a and 2011b). Further work on the use of gamma process concept (Mahmoodian and Alani 2013a & 2013b)) has shown that this approach is very straight forward as the gamma distribution fits the monotonic progression of the corrosion processes.

In the next two chapters, the first passage probability method and the gamma distributed degradation model devised in this study will be applied to real case data from concrete sewers and cast iron water mains in the UK. This will check the capability of the methods for the reliability analysis of corrosion affected buried pipes in practice.

5 APPLICATION OF THE DEVELOPED METHODS TO CONCRETE SEWERS

5.1 Preface

In this chapter the two model approaches outlined in chapter 4 (i.e., first passage probability method and gamma distributed degradation model) are used for the reliability analysis and service life prediction of a concrete sewer pipeline in the UK. The results are verified by a Monte Carlo simulation and the weakness and strength of each method is discussed.

For a comprehensive time-dependent reliability analysis of corrosion-affected concrete sewers, the following steps need to be followed:

- Modelling the corrosion mechanism
- Defining the failure mode(s) (limit state function(s))
- Calculation of the probability of failure
- A sensitivity analysis

Two conditions are considered for the reliability analysis and service life prediction of the sewer pipeline in the UK. Firstly, the analysis is carried out based on an individual limit state function (i.e., considering one failure mode), and secondly, to have a more representative assessment of different failures to test reliability. The novelty of this approach is the consideration of all possible corrosion induced failure modes that exist in practice. The effectiveness and contribution of the different variables (on the failure probability) provides new insight to the sensitivity analysis of concrete sewers.

5.2 Case study

The two developed methodologies in chapter 4 for time dependent reliability analysis of pipes are applied on CCTV data from surveys of concrete sewers in Harrogate in the United Kingdom. Harrogate is a spa town in North Yorkshire, England. The town is a tourist destination with a population of approximately 75000.

The study considers concrete sewers with a 500mm diameter and 25mm of internal concrete cover.

To incorporate the effect of increments in populations of residents and essentially the increase in the flow rate during the system's life time, the modeling assumes flow rates corresponding to relative depths (i.e., depth/diameter) of 0.2, 0.4 and 0.6, each occurring over a period of 25 years.

Reliability analysis and service life prediction of this concrete sewer pipeline is of interest for asset managers in North Yorkshire, England, to develop a risk-informed and cost-effective strategy in the management and maintenance of concrete sewers. The analysis can also help infrastructure managers to develop rehabilitation or replacement strategies for existing pipe networks with a view to better management of the pipe asset.

In the following sections two conditions are considered for the assessment, firstly the pipeline reliability is analysed by considering individual failure mode and secondly the reliability analysis is carried out by considering multi failure mode.

5.3 Reliability analysis considering individual failure mode

5.3.1 Corrosion model and definition of failure mode (limit state function)

a) Corrosion Model

The corrosion mechanism in concrete sewers was discussed in section 3.2.1 and models for the rate of corrosion and wall thickness reduction were presented in the form of Equations 3.11 and 3.12 respectively.

Recalling the Equation 3.12 from chapter 3, the reduction in wall thickness in elapsed time t, is:

$$\mathrm{d(t) = c.t = 8.05k.(su)^{3/8}j.[DS] \times \frac{b}{P'A}.t} \tag{3.12}$$

Where k is the factor representing the proportion of acid reaction, s is pipe slope, u is the velocity of stream (m/s), j is pH-dependent factor for proportion of H_2S, [DS] is dissolved sulphide concentration (mg/lit), A is acid-consuming capability or Alkalinity, b is the width of the stream surface, $\acute{P}$ is the perimeter of the exposed wall and t is time.

To consider uncertainties about wall thickness reduction due to corrosion, a stochastic model is presented. Considering Equation 3.12, basic random variables affecting thickness reduction includes: k, u, j, [DS], b/P' and A.

The wall thickness reduction due to corrosion is a function of basic random variables as well as time. It can be expressed as:

$$d(t) = f(k, u, j, [DS], b/P', A, t) \quad (5.1)$$

Where $k, u, j, [DS], b/P'$ and A are the basic random variables, the probabilistic information of which are presumed available.

Values for the basic random variables in the current case study are presented in Table 5.1.

Table 5.1 Statistical data for the basic random variables

Basic Variables	Units	Mean	Standard deviation
K	-	0.8	0.1
j	-	0.2	0.04
[DS]	mg/L	1	0.5
u	m/sec	0.6	0.1
b/P’	-	h/D= 0.2 b/P’= 0.36	0.11
		h/D= 0.4 b/P’= 0.55	0.18
		h/D= 0.6 b/P’= 0.71	0.23
A	-	0.2	0.06

b) Definition of failure mode (limit state function)

According to the ASCE manual No.69 (1989), one of the performance criteria related to the stability of concrete sewers is to control the wall thickness reduction under an acceptable limit (normally concrete cover). In the theory of structural reliability this criterion can be expressed in the form of a limit state function as follows:

$$G(d_{max}, d, t) = d_{max}(t) - d(t) \quad (5.2)$$

where:

d: Reduction in wall thickness due to corrosion (or corrosion depth), (mm)

d_{max}: Maximum permissible reduction in wall thickness (structural resistance or limit), (mm)

t: elapsed time

d_{max} may change with time although in most practical cases it has a constant value prescribed in design codes of practice and manuals.

5.3.2 Calculation of the probability of failure

With the limit state function introduced in the form of Equation 5.2, the probability of failure of the concrete pipe due to the reduction of its wall thickness can be determined by:

$$P(t) = P[G(d_{max}, d, t) \leq 0] = P[d(t) \geq d_{max}(t)] \quad (5.3)$$

The two developed methods for time dependent reliability analysis in chapter 4 (i.e., first passage probability method and gamma distributed degradation model are applied for calculation of the probability of failure of the concrete sewer case study in Harrogate in the UK.

a) Using first passage probability method

Equation 5.3 is a typical upcrossing problem that can be solved by using first passage probability theory. In a time dependent reliability problem all or some of basic random variables are modelled as stochastic processes. For the above problem, the sewer failure depends on the time that is expected to elapse before the first occurrence of the stochastic process, d(t), upcrosses a critical limit (the threshold, d_{max}) sometime during the service life of the sewer.

As it was described in section 4.3, the probability of failure of a pipe can be determined by using first passage probability theory from Equation 4.9. Considering Equation 5.3 as the failure definition, reduction of wall thickness (d) is the action (load) and d_{max} is its effect or the acceptable limit (resistance). Therefore Equation 4.9 can be reproduced with d replacing S and d_{max} replacing R.

$$P_f(t) = \int_0^t \frac{\sigma_{\dot{d}|d}(t)}{\sigma_d(t)} \emptyset\left(\frac{d_{max}-\mu_d(t)}{\sigma_d(t)}\right)\left\{\emptyset\left(-\frac{\mu_{\dot{d}|d}(t)}{\sigma_{\dot{d}|d}(t)}\right) + \frac{\mu_{\dot{d}|d}(t)}{\sigma_{\dot{d}|d}(t)}\Phi\left(\frac{\mu_{\dot{d}|d}(t)}{\sigma_{\dot{d}|d}(t)}\right)\right\} d_\tau \quad (5.4)$$

For a given Gaussian stochastic process with mean function $\mu_d(t)$, and auto-covariance function $C_{dd}(t_i, t_j)$, all terms in Equation 5.4 can be determined as outlined in section 4.3 by using the following formulations:

$$\mu_{\dot{d}|d} = E[\dot{d} \mid d = d_{\max}] = \mu_{\dot{d}} + \rho_d \frac{\sigma_{\dot{d}}}{\sigma_d}(d_{\max} - \mu_d) \quad (5.5a)$$

$$\sigma_{\dot{d}|d} = [\sigma_{\dot{d}}^2(1-\rho_d^2)]^{1/2} \quad (5.5b)$$

where

$$\mu_{\dot{d}} = \frac{d\mu_d(t)}{dt} \quad (5.5c)$$

$$\sigma_{\dot{d}} = \left[\frac{\partial^2 C_{dd}(t_i,t_j)}{\partial t_i \partial t_j}\Big|_{i=j}\right]^{1/2} \quad (5.5d)$$

$$\rho_d = \frac{C_{d\dot{d}}(t_i,t_j)}{[C_{dd}(t_i,t_i)\cdot C_{\dot{d}\dot{d}}(t_j,t_j)]^{1/2}} \quad (5.5e)$$

and the cross-covariance function is

$$C_{d\dot{d}}(t_i,t_j) = \frac{\partial C_{dd}(t_i,t_j)}{\partial t_j} \quad (5.5f)$$

$$C_{dd}(t_i,t_j) = \lambda_d^2 \rho_d \mu_d(t_i)\mu_d(t_j) \quad (5.5g)$$

Where λ_d is the coefficient of variation of the wall thickness reduction which is determined based on Monte Carlo simulations and $\rho_d$is (auto-) correlation coefficient for the wall thickness reduction between two points in time t_i and t_j. Therefore all variables in Equation 5.4 can be determined.

To estimate the probability of failure due to corrosion, a critical limit for the wall thickness reduction should be established. ASCE manual No.69 (1989) considers exposure of reinforcement as a criterion for failure. Therefore the maximum acceptable limit for wall thickness reduction ($i.e.,\ d_{max}$) can be considered equal to the thickness of concrete cover

($d_{max} = a_0$, concrete cover).

The probability of failure due to wall thickness reduction is computed using Equation 5.4 and the results are shown in Figure 5.1. As can be concluded from this figure, the effect of auto-correlation coefficient (ρ_d) in the probability of failure is negligible, specially for the area of interest (i.e., lower probability of failure).

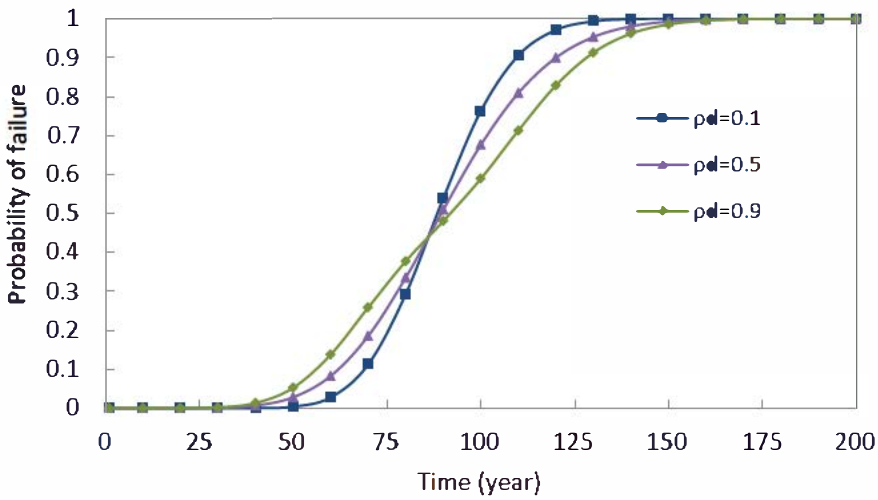


Figure 5.1 Probability of failure for different auto-correlation coefficient, ρ_d, from first passage probability method

b) Using Gamma distributed degradation (GDD) Model

The average rate of wall thickness reduction within time in a concrete sewer is calculated through Equation 3.12 as explained in section 3.2.1. Defining failure as the time when all concrete cover is corroded, the developed algorithm (GDD model) in section 4.4.3 is used for formulation of the probability of failure based on the gamma process concept.

Given that a random deterioration (i.e., corrosion depth, d) has a gamma distribution with shape parameter $\alpha > 0$ and scale parameter $\lambda > 0$, formulation for calculation of the probability of failure for a corrosion affected pipe is developed as it was mentioned in section 4.4.

The failure was defined as the time that all concrete cover on the reinforcement is corroded

(ASCE manual No.69, (1989)), therefore the concrete sewer is said to fail when its corrosion depth, denoted by $d(t)$, is more than a special value (for instance concrete cover, a_0). Assuming that the time at which failure occurs is denoted by the lifetime T, due to the gamma distributed deterioration, Equation 4.14 can be used for calculation of the probability of failure.

$$\mathrm{F(t)} = \Pr(\mathrm{T} \le \mathrm{t}) = \Pr(d(t) \ge a_0) = \int_0^{a_0} f_{d(t)}(d) d_d = \frac{\Gamma(\alpha(t), a_o\lambda)}{\Gamma(\alpha(t))} \tag{4.14}$$

Where $\alpha(t) = ct^b$ is the shape parameter with physical constants $c > 0$ and $b > 0$ and λ is the scale parameter. The parameters $\alpha(t)$ and λ can be estimated by using the estimation method explained in section 4.4.3. For the exponential parameter b, a value of one is assumed (b=1) based on some examples of expected deterioration that have been presented in the Table 4.1.

Figure 5.2 shows the results obtained for the probability of pipe failure by using the GDD method.

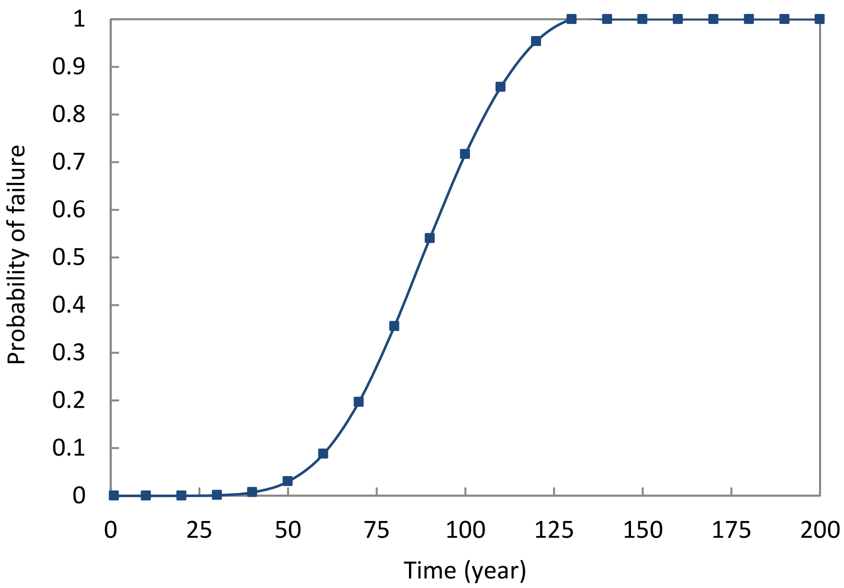


Figure 5.2 Probability of failure from gamma distributed degradation (GDD) model

5.3.3 Verification of the results of the probability of failure

To verify the results obtained from the two developed methods in the previous section, Monte Carlo simulation method (see section 4.5 for the detail) is used. The result of the probability of failure is presented in Figure 5.3.

To be able to compare the results obtained from the two different methods, graphs in Figures 5.1 and 5.2 are also illustrated in Figure 5.3. The graph for first passage probability method is taken from the result for $\rho_d = 0.5$.

The comparison shows that the probabilities of failure predicted by the two methods are in good agreements and they can be verified by the results of Monte Carlo simulation method, particularly for small probabilities which are of most practical interest.

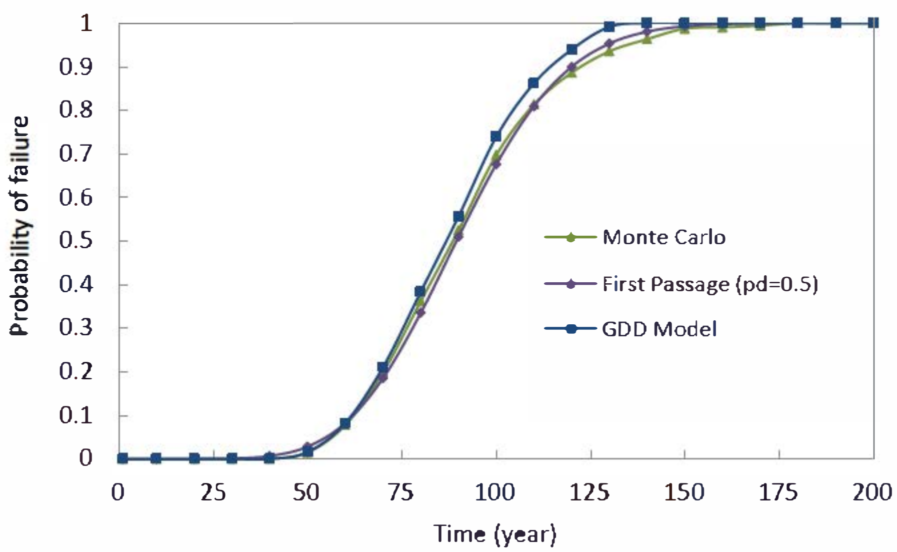


Figure 5.3 Verification of the results from the two methods by Monte Carlo simulation method

5.3.4 Sensitivity analysis

For a comprehensive reliability analysis it may be appropriate to assess the effect of different random variables on the service life of the sewer. It is of interest to identify those variables that affect the wall thickness reduction most, so that further studies can focus on those variables. For this purpose, relative contributions of variables to the variance of the limit state function were calculated using Equation 3.22.

Figure 5.4 illustrates sensitivities (α^2) or relative contribution of variables to the variance of the limit state function calculated based on the defmition in Equation 3.22, section 3.3.5. It can be concluded that while the contribution of some variables are relatively small, some other variables have considerable contribution in the limit state function. High values of contribution of variables mean that the sensitivity to those variables is more dependent on the actual value of their coefficient of variation. In such cases, more concern should be taken in order to determine relevant parameter values.

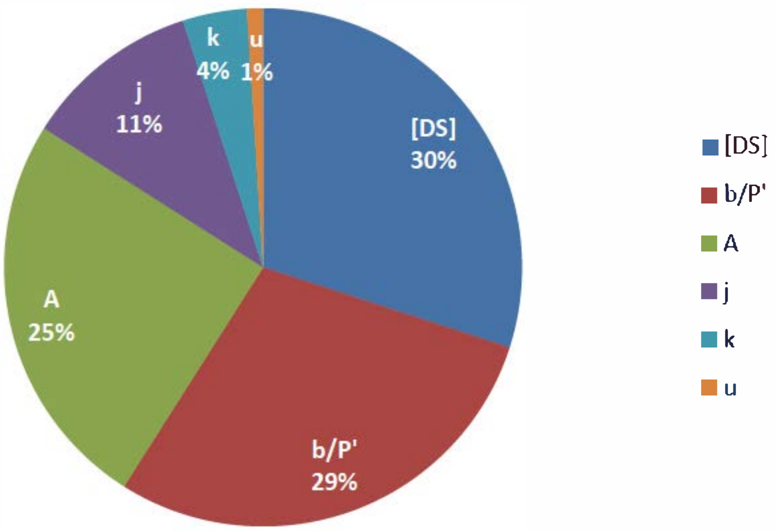


Figure 5.4 Relative contributions of random variables in failure function

The GDD model was run to elaborate on the effect of variables with high values of relative contribution (i.e., [DS], A and $b/\acute{P}$) on the probability of failure. As can be concluded from Figure 5.5, a change in sulphide concentration has a considerable effect on service life of the concrete sewer.

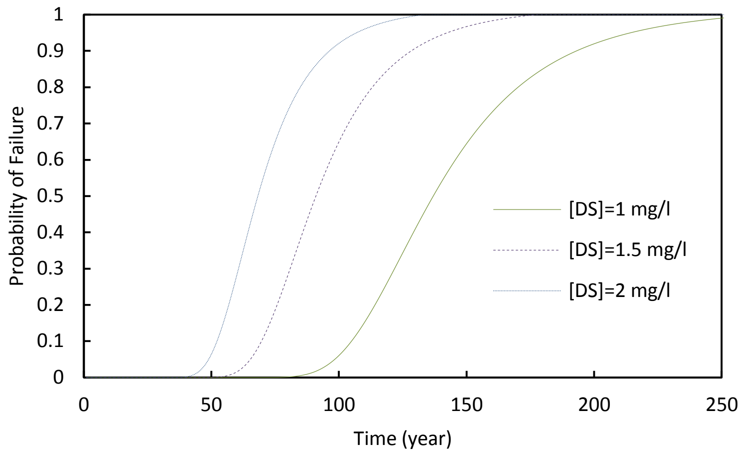


Figure 5.5 Effect of sulphide concentration on service life of the sewer

Figure 5.6 also shows the effect of sewage flow quantity on the service life of the sewer. The ratio of the width of the stream surface to the perimeter of the exposed wall ($b/\acute{P}$) has a significant effect on corrosion rate of concrete and consequently the service life of the sewer decreases dramatically by increasing the ratio of $b/\acute{P}$.

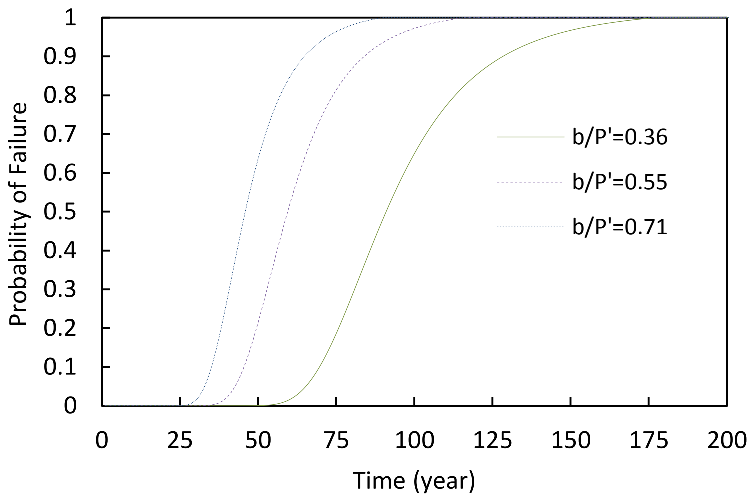


Figure 5.6 Effect of the ratio of the width of the stream surface to the perimeter of the exposed wall ($b/\acute{P}$) on service life of the sewer

To investigate more about the effect of concrete properties on the service life of concrete sewers, more studies on the effect of change in Alkalinity (acid consuming capability) of concrete on the probability of failure were carried out. The result of changing alkalinity from 0.14 to 0.22 is presented in Figure 5.7.

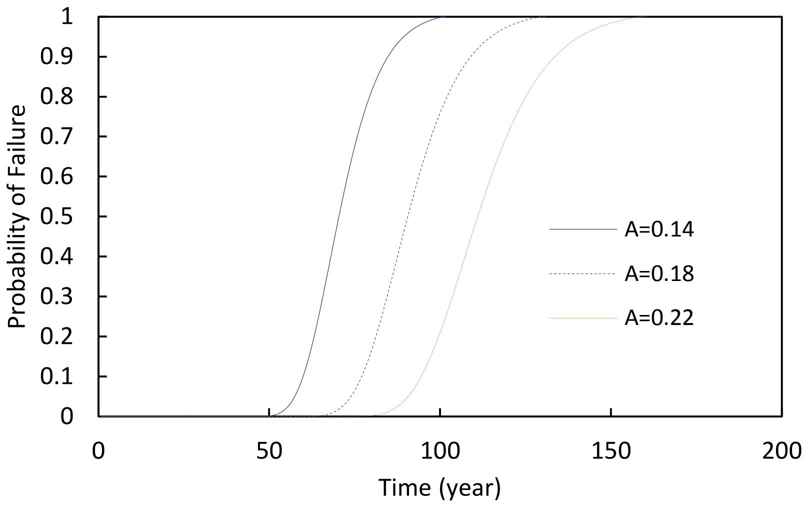


Figure 5.7 Probability of failure for different values of alkalinity, A

Further sensitivity studies were carried out to investigate the effect on the reliability index of the level of variability (i.e. coefficient of variation) of each of the major random variables. The reliability index (β) was chosen for this work in preference to the probability of failure, mainly to facilitate the interpretation of the results. Although these two quantities are directly related (Equation 3.19), the interpretation of results would be more appropriate when dealing with reliability index rather than probability of failure.

The random variables chosen for this study were three significant variables in terms of the α^2 contribution in Figure 5.4 (i.e., [DS], A, b/P').

The coefficient of variation for each of these random variables was varied from 0 to 0.5 in steps of 0.1. The coefficient of variation of all other variables was kept constant at the values given in Table 5.1. Figures 5.8 to 5.10 illustrate the results for four different pipeline lifespans (t). A period of service life from t=20yr to t=35yr which results in practical reliability indexes from 1.5 to 4.5 is selected for this study.

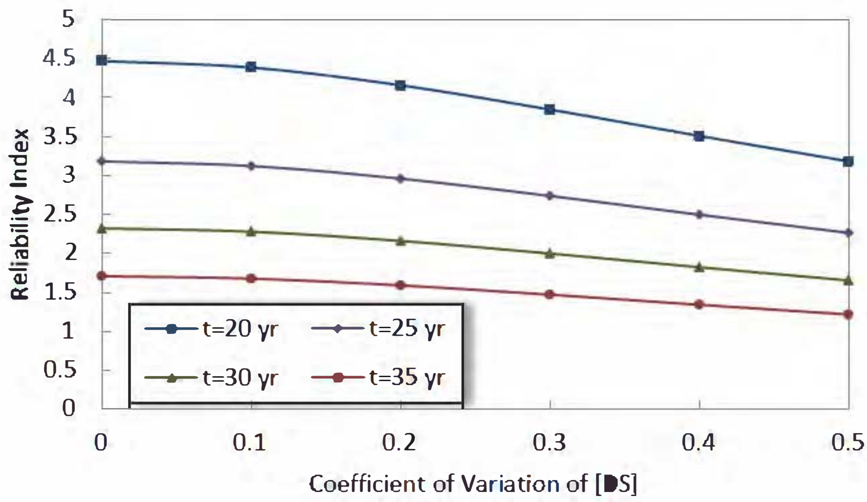


Figure 5.8 Reliability index vs. coefficient of variation of [DS] for various values of the pipeline elapsed time

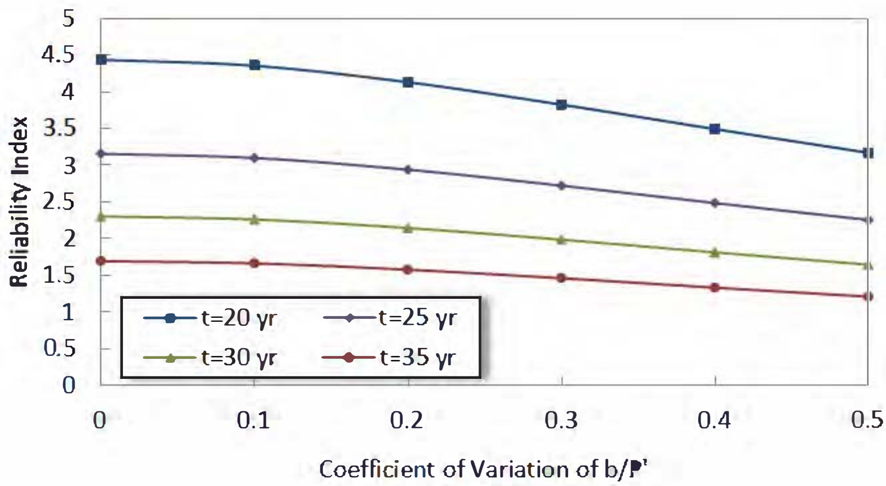


Figure 5.9 Reliability index vs. coefficient of variation of b/P' for various values of the pipeline elapsed time

The results show that the reliability index decreases as time and the coefficient of variation of random variables increases. It is also observed that the variability of these three random variables for low values of t has a more significant effect on the reliability index.

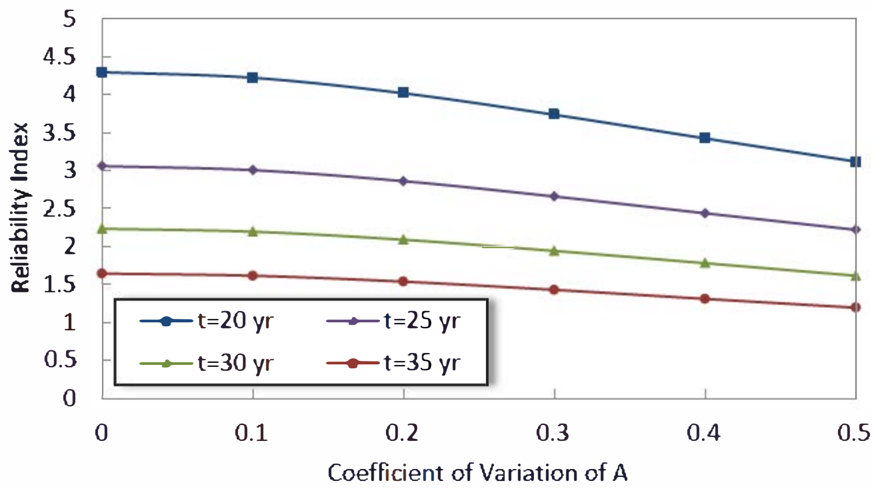


Figure 5.10 Reliability index vs. coefficient of variation of alkalinity (A) for various values of the pipeline elapsed time

5.4 Reliability analysis considering multi failure mode

In this section the case of the concrete sewer pipeline in Harrogate UK, is analysed by considering more than one possible failure mode. This study which is called multi failure mode reliability analysis will eventually end up with a more reliable service life estimation.

5.4.1 Corrosion model and definition of failure modes (limit state functions)

a) Corrosion model

The same corrosion model as presented in Equation (3.22) with $\mathrm{k}, \mathrm{u}, \mathrm{j}, [\mathrm{DS}], \mathrm{b}/\mathrm{P}'$ and A as the basic random variables is considered for the multi failure mode reliability analysis of the concrete sewer.

b) Definition of failure modes (limit state functions)

As it was mentioned earlier (section 4.1), failure can be defined in relation to different possible mechanisms and in the theory of structural reliability, it can be described by a limit state function as it was presented in Equation (4.1):

$$G(R, S, t) = R(t) - S(t) \tag{4.1}$$

Where $G(R, S, t)$ is limit state function, $S(t)$ is the action (load effect) at time t and $R(t)$ is the critical limit (resistance) for the action or its effect. The probability that failure occurs for any load application is the probability of the limit state violation.

For concrete sewers, failure does not necessarily imply structural collapse (ultimate strength failure) but in most cases is indicated by loss of structural serviceability, as characterised by concrete cracking and/or concrete cover loss. In a comprehensive reliability analysis it is of interest to take into account both serviceability failure and ultimate strength failure.

In some cases of reliability analysis of a structure, various limit states such as bending, shear, cracking and deflection may apply in a composition referred to as a 'system'. The detail of system reliability analysis was explained in section 3.3.4. As it was mentioned in that section, in a series system, attainment of any limit state constitutes failure of the structure. All components of a parallel system must fail for system failure to occur.

A concrete sewer can fail in multiple modes due to different limit state violations. Therefore, the probability of the sewer failure should be determined using the methods of systems reliability analysis.

Considering design codes of practice and manuals for reinforced concrete pipe design (ASCE 15-98, (2000) and ASCE 60, (2007)), the four following failure modes (limit state functions) should all be considered for buried concrete sewers:

Flexural limit state: $G_1(M_u, M_s, t) = M_u(t) - M_s(t)$ (5.6a)

Shear limit state: $G_2(V_b, V_s, t) = V_b(t) - V_s(t)$ (5.6b)

Excessive crack limit state: $G_3(F, F_{cr}, t) = F(t) - F_{cr}(t)$ (5.6c)

Cover loss limit state: $G_4(\Delta, a_o, t) = \Delta(t) - a_o(t)$ (5.6d)

where $M_u, V_b, F\ and\ \Delta$ are flexural strength, shear strength, crack control factor and concrete thickness reduction respectively. Formulisation of these four resistance modes for a reinforced concrete pipe are presented in Table 5.2.

$M_s, V_s, F_{cr}\ and\ a_o$ are flexural stress, shear stress, crack control limit and concrete cover which are considered as thresholds for limit state functions.

The formulae presented by design codes for the resistance modes have stationary formats; while in the case of corrosion, the wall thickness of the pipe is a time dependent parameter (i.e., decreases within the time). Hence, the time dependent format of each formula is given in the last column of Table 5.2.

Table 5.2 Formulisation of different resistance modes of the concrete sewer

Failure Mechanism		**Stationary formulisation**		**Time dependent formulisation**
		Equation	**Source**	
Strength	Flexural Failure	$M_u = A_s f_y\left(d - \frac{a}{2}\right) + N_u\left(\frac{h-a}{2}\right)$	ASCE 15-98 (2000)	$M_u(t) = A_s f_y\left(d - \frac{a}{2}\right) + N_u\left(\frac{h(t)-a}{2}\right)$
Strength	Shear Failure	$V_b = 0.083b\emptyset_v dF_{vp}\sqrt{f_c'}\left(1.1 + 63 \times \frac{A_s}{bd}\right)\left[\frac{F_d F_N}{F_c}\right]$ $F_N = 1 + \frac{N_u}{3.5bh}$	ASCE 15-98 (2000)	$V_b(t) = 0.083b\emptyset_v dF_{vp}\sqrt{f_c'}\left(1.1 + 63 \times \frac{A_s}{bd}\right)\left[\frac{F_d F_N(t)}{F_c}\right]$ $F_N(t) = 1 + \frac{N_u}{3.5bh(t)}$

Serviceability	Excessive Crack width	$F = \frac{B_1}{5250\emptyset_f d A_s}\left[\frac{M_s + N_s\left(d - \frac{h}{2}\right)}{ij} - \left(0.083 C_1 b h^2 \sqrt{f_c'}\right)\right]$ $j \cong 0.74 + 2.54e/d$ $i = \frac{1}{1 - \frac{jd}{e}}$ $e = \frac{M_s}{N_s} + d - \frac{h}{2}$	ASCE 15-98 (2000)	$F(t) = \frac{B_1}{5250\emptyset_f d A_s}\left[\frac{M_s + N_s\left(d - \frac{h(t)}{2}\right)}{i(t)j(t)} - \left(0.083 C_1 b h(t)^2 \sqrt{f_c'}\right)\right]$ $j(t) \cong 0.74 + 2.54\frac{e(t)}{d}$ $i(t) = \frac{1}{1 - \frac{j(t)d}{e(t)}}$ $e(t) = \frac{M_s}{N_s} + d - \frac{h(t)}{2}$
	Loss of Concrete Cover	$\Delta = h - d - \frac{d_b}{2}$	ASCE 60 (2007)	$\Delta(t) = h(t) - d - \frac{d_b}{2}$

Symbols

a: depth of the equivalent rectangular stress block, (mm)

A: the acid-consuming capability of the wall material

A_s: Area of tension reinforcement in length b, (mm^2/m)

b: unit length of pipe, 1000mm

B_1: crack control coefficient for effect of spacing and number of layers of reinforcement

c: the average rate of corrosion (mm/year)

C_1: Crack control coefficient for type of reinforcement

d: distance from compression face to centroid of tension reinforcement, (mm)

d_b: diameter of rebar in inner cage, mm

[DS]: Dissolved sulphide concentration (mg/l)

f_c': design compressive strength of concrete, (MPa)

f_y: design yield strength of reinforcement, (MPa)

F: crack width control factor

F_c : factor for effect of curvature on diagonal tension (shear) strength in curved components

F_d: factor for crack depth effect resulting in increase in diagonal tension (shear) strength with decreasing d

F_N: coefficient for effect of thrust on shear strength

h: overall thickness of member (wall thickness), (mm)

i: coefficient for effect of axial force at service load stress

k: Acid reaction factor

J: is pH-dependent factor for proportion of H_2S

w: the width of the stream surface

P': perimeter of the exposed wall

M_s: Service load bending moment acting on length b, (Nmm/m)

M_u: factored moment acting on length b, (Nmm/m)

N_s: Axial thrust acting on length b, service load condition (+ when compressive, - when tensile), (N/m)

N_u: Factored axial thrust acting on length b, (+ when compressive, - when tensile), (N/m)

s: is the slope of the pipeline

t: elapsed time

u: is the velocity of the stream (m/sec)

V_b: basic shear strength of length b at critical section

Φ: The average flux of H_2S to the wall

$\emptyset_f$: strength reduction factor for flexure

ϕ_v: strength reduction factor for shear

Δ: reduction in wall thickness due to corrosion, (mm)

Δ_{max}: Maximum permissible reduction in wall thickness (structural resistance or limit), (mm)

The four limit states (i.e., Equations 5.6a to 5.6d) can be classified in the two main categories of failure modes, namely serviceability limit states and ultimate strength limit states. If a pipe loses its flexural strength and/or its shear strength it has completely failed. Therefore flexural limit state and shear limit state are considered as ultimate strength limit state functions. On the other hand, if a pipe cracks or loses its cover, it is not necessarily failed structurally, but it is failed from a serviceability point of view. Therefore crack limit state and cover loss limit state can be considered as serviceability limit states.

As mentioned earlier (section 4.1), each failure mode happens when the limit state function is violated (i.e., $G_i \leq 0$). To consider all the four modes as a system, it is necessary to clarify the combination of the limit state functions. Figure 5.11 presents the combination which is suggested for a system reliability analysis of the concrete sewer; it is a combination of series and parallel systems. The two serviceability limit states (crack and cover loss) are considered parallel, because violation of them individually does not fail the whole system. On the other hand violation of flexural limit state and/or shear limit state will cause the failure of the whole system and therefore these two limit states are set in a series combination.

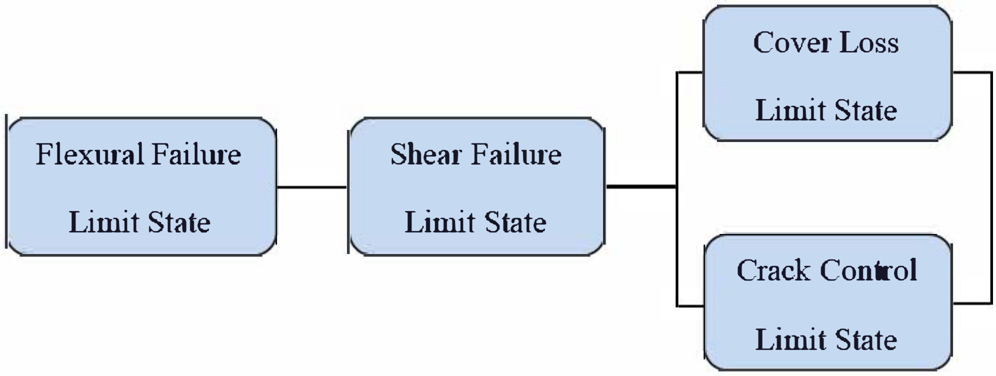


Figure 5.11 System combination of the four limit state functions for multi failure mode reliability analysis of the concrete sewer

5.4.2 Calculation of the probability of failure

According to the theory of systems reliability, the probability of failure for a series system ($P_{fs}(t)$) can be estimated by (Thoft-Christensen and Baker (1982)):

$$max[P_{f_i}(t)] \leq P_{fs}(t) \leq 1 - \prod_{i=1}^{m}[1 - P_{f_i}(t)] \tag{5.7}$$

where $P_{f_i}(t)$ is the probability of failure due to the i^{th} failure mode of pipe and m is the number of failure modes considered in the system.

Considering the system configuration presented in Figure 5.11 and the upper bound of Equation 5.7, the probability of failure of the whole concrete sewer system can be calculated by the following equation:

$$P_{fs}(t) = 1 - (1 - P_{f_1})(1 - P_{f_2}).(1 - P_{f_3}.P_{f_4}) \tag{5.8}$$

Where:

P_{f_1} is the probability of flexural failure

P_{f_2} is the probability of shear failure

P_{f_3} is the probability of crack failure

P_{f_4} is the probability of cover loss failure

For reliability analysis of the sewer, initially, the probability of failure in time t for each failure mode ($P_{f_i}(t)$, i=1,..,4) is estimated by using one of the two proposed methods (first passage probability and/or GDD model). Then Equation 5.8 is used for calculation of the probability of the sewer system failure ($P_f(t)$), considering all four possible failure modes.

a) Using first passage probability method

The results of using first passage probability method (section 4.3) for calculation of the probability of system failure is shown in Figure 5.12.

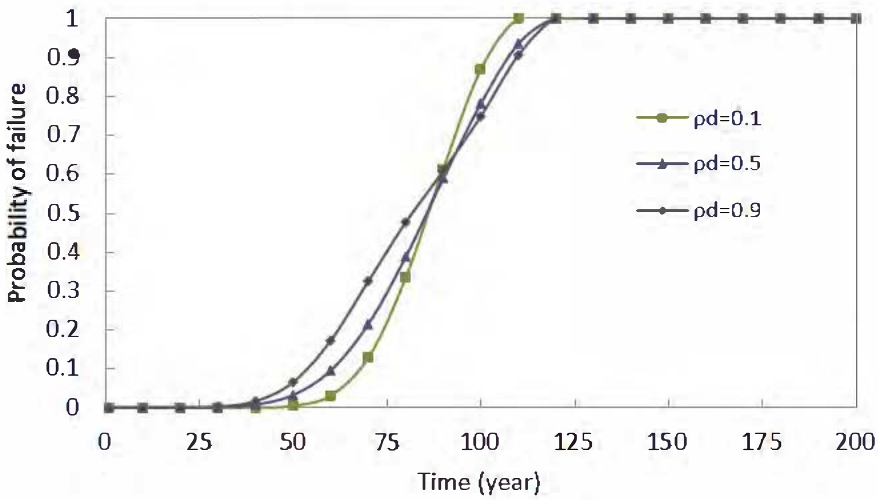


Figure 5.12 Probability of system failure from first passage probability method

b) Using Gamma Distributed Degradation (GDD) Model

The developed algorithm for GDD model in section 4.4.3 is used for calculation of the probability of system failure. The results of using gamma distributed degradation (GDD) model for calculation of the probability of the sewer system failure are shown in Figure 5.13.

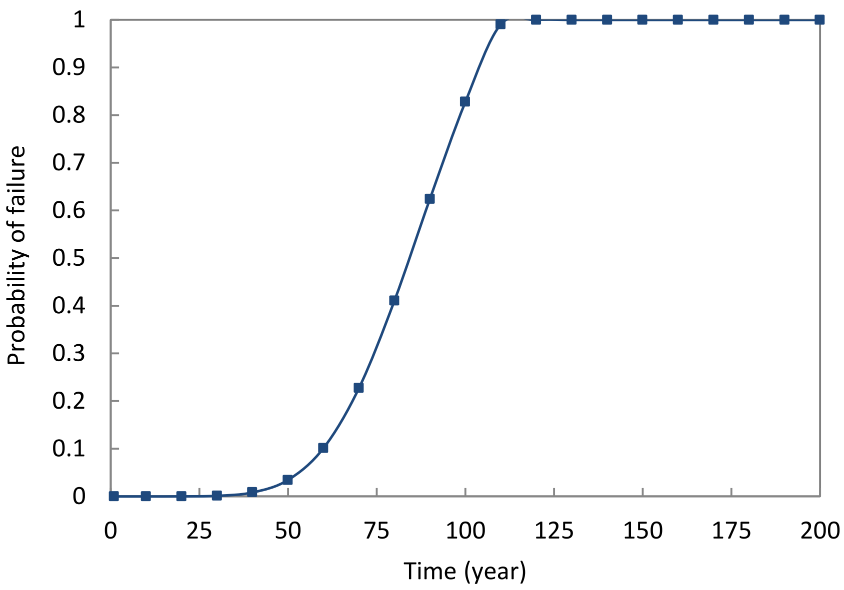


Figure 5.13 Probability of system failure from GDD model

5.4.3 Verification of the results of the probability of failure

The Monte Carlo simulation method (see section 4.5 for the detail) is performed for verification of the results from the two methods in the previous section. Figure 5.14 shows the comparison of the results of the probability of system failure from the three methods (first passage probability, GDD model and Monte Carlo simulation). The graph for first passage probability method is taken from the result for auto-correlations equal to 0.5.

The comparison shows that the probabilities of system failure predicted by the two methods are in good agreements and it can be verified by the results of the Monte Carlo simulation method, particularly for small probabilities which are of most practical interest.

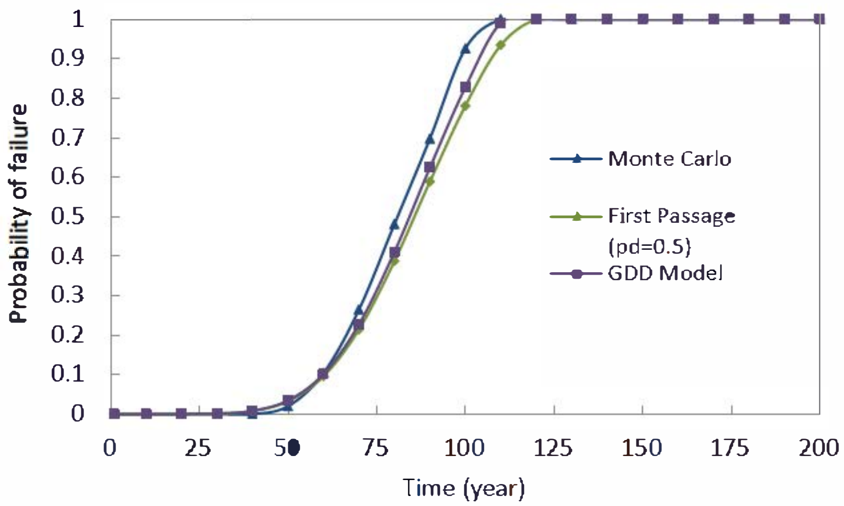


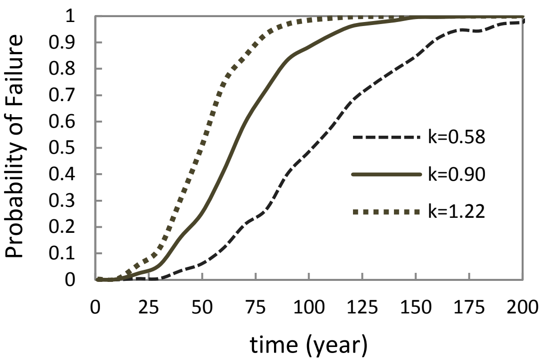
Figure 5.14 Verification of the results from the two methods by Monte Carlo simulation method

5.4.4 Sensitivity analysis

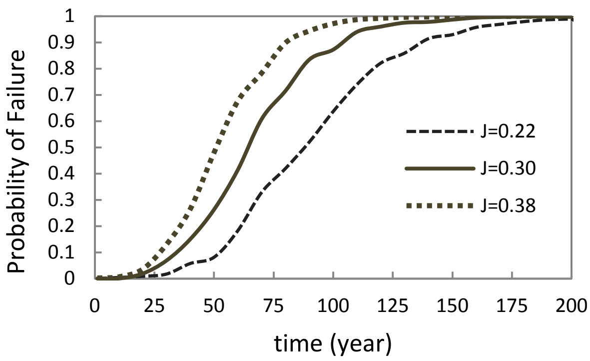
As it was mentioned earlier, for a comprehensive pipeline assessment, the effect of variables on the failure of the concrete sewer can be analysed by performing sensitivity analysis. In view of the large number of variables that affect the corrosion process, and hence the limit state functions, it is of interest to identify those variables that affect the failure most so that more research can focus on those variables.

Unlike individual failure mode assessment (section 5.3.5), the concept of relative contribution can not be used for multi failure mode analysis. It is simply because Equation 3.22 has been presented for calculation of relative contribution in case of individual limit state function. Hence, a new parametric method is developed and applied for sensitivity analysis of the concrete sewer in case of multi failure mode assessment.

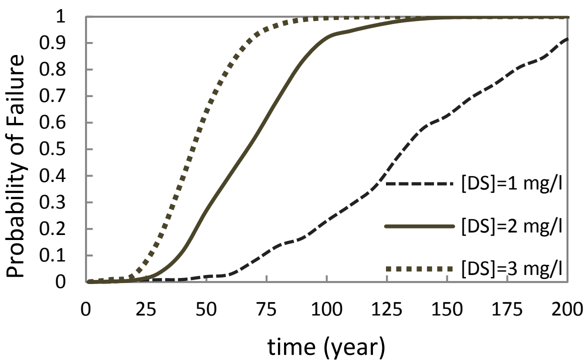
To assess how the change in the values of the six random variables (k, u, j, [DS], $b/\acute{P}$ and A) can affect the service life of the concrete sewer system, the values for each variable are changed from $\mu_i - 2\sigma_i$ to $\mu_i + 2\sigma_i$ (where μ_i is the mean of the random variable and σ_i is its standard deviation). Assuming a Gaussian distribution for the random variables, this range corresponds to 95.4 percent of the possible values of the variable. The results of the analysis by using Monte Carlo simulation method are illustrated in Figure 5.15 (a-f).



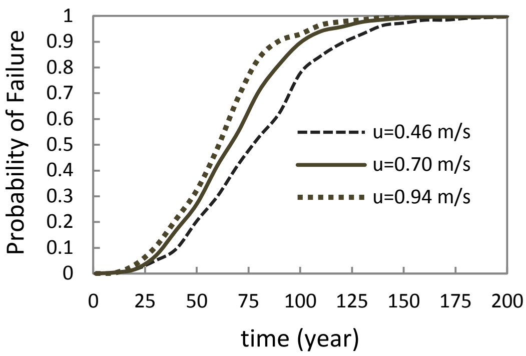
(a)



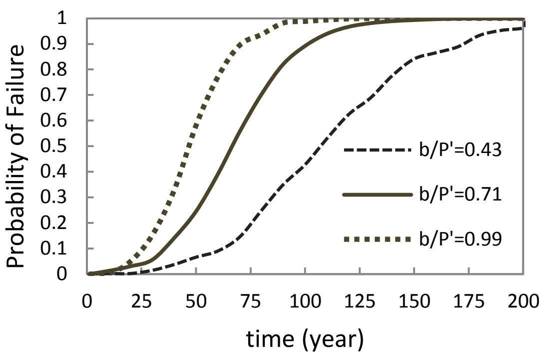
(b)



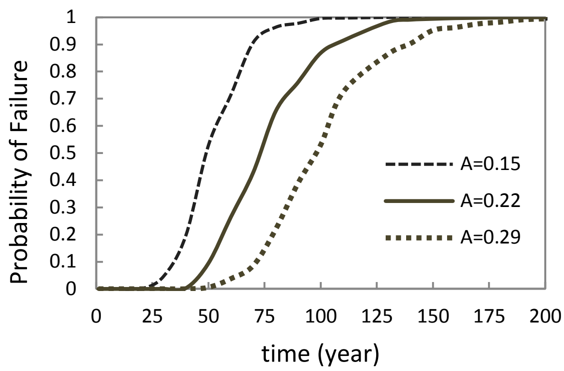
(c)



(d)



(e)



(f)

Figure 5.15 Probability of system failure of the concrete sewer for different values of basic random variables

It can be concluded that among all variables, the effect of [DS], A and $b/\acute{P}$ on the probability of failure of the sewer is highly remarkable. The disparity shown within the graphs for each of these three variables means that the sensitivity of the failure of the pipeline is more dependent on their actual value. In such cases, more concern should be taken in order to determine relevant parameter values.

For a better comparison of the effectiveness of random variables on the service life of the pipeline, the results in Figures 5.15(a-f) are summarised in the form of Figure 5.16. In this figure, a range value is defined for each random variable as the difference between the maximum and minimum values of the probability of failure in each elapsed time. Therefore a higher range value means wider resultant values for the probability of failures. This figure illustrates which variables contribute most to the probability of the failure of the system. To clarify, this means that the variables have more effect on the service life of the pipeline.

The significance of the three major variables (i.e, [DS], $b/\acute{P}$ and A) on the failure of the concrete sewer had also been concluded in individual failure mode analysis in section 5.3.5.

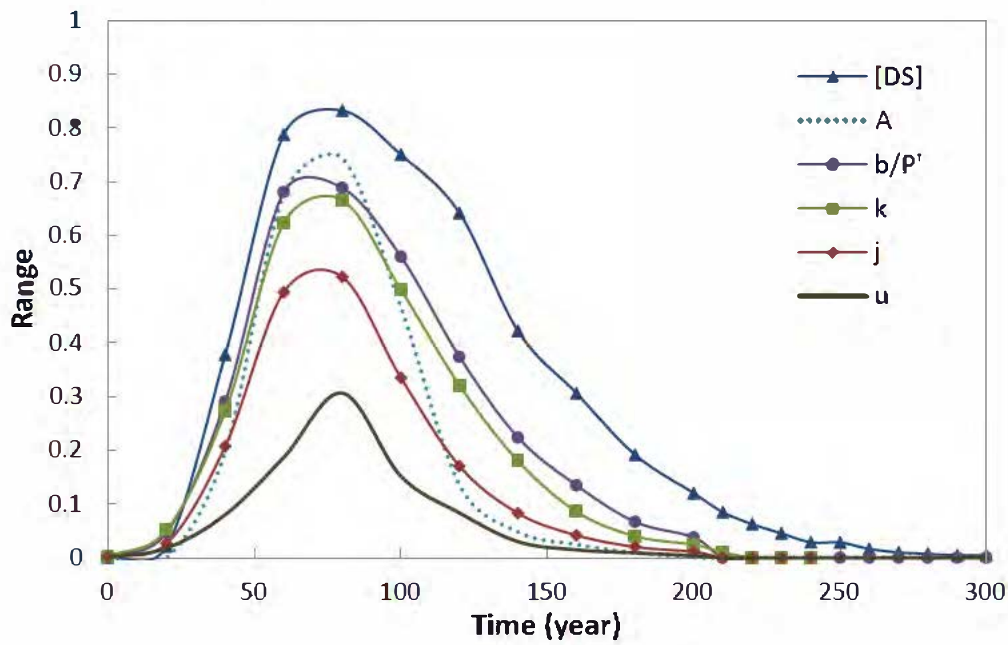


Figure 5.16 Comparison of range values of basic random variables

Of these three major random variables, the analysis shows the particular significance of dissolved sulfide concentration in the probability of failure of the pipeline, indicating that dissolved sulfide concentration is the most significant variable on the reliability of concrete sewers.

5.5 Summary

In this chapter a comprehensive study for the reliability analysis of a concrete sewer pipeline in the UK subjected to a time dependent deterioration (i.e. corrosion) was performed. Variables which affect the corrosion were investigated and a corrosion model for this type of buried pipe was presented. To deal with uncertainties and scarcity of monitoring data, the use of time dependent reliability analysis methods for failure analysis and service life prediction of the concrete sewer was considered.

The two developed methods for time dependent reliability analysis (i.e., first passage probability and GDD model) in Chapter 4, were applied for the concrete sewer pipeline in the UK. The assessment was carried out by considering two scenarios: individual failure mode and multi failure mode.

In an individual failure analysis scenario, loss of concrete cover was assumed as the criterion for failure of the sewer. Four failure modes including flexural, shear, cracking and cover loss were assumed as possible failure modes for multi failure mode analysis. These modes of failure, which are set into the two categories of serviceability and ultimate strength limit states, were then put into a system configuration consisting of a combination of series and parallel systems.

The results from the two developed methods in chapter 4 (i.e., first passage probability method and GDD model) showed a significant agreement with the results from Monte Carlo simulation method. Although the result from first passage probability method depends to the amount of assumed auto-correlation coefficient (ρ), this study showed that the effect of auto-correlation coefficient on the probability of failure is negligible.

Another concern is that to begin with first passage probability procedure, it is necessary to estimate the coefficient of variation of the deteriorating parameter (i.e. λ_d in Equation 5.5g). As it was mentioned in section 5.3.2 this estimation can be carried out by using a Monte

Carlo simulation. It means that using first passage probability method (as an analytical method) still needs application of a numerical simulation (e.g. Monte Carlo) to estimate required parameters (i.e. λ_d). This estimation would be time consuming and will involve errors in calculations.

The obtained results in this chapter showed that a gamma distributed degradation (GDD) model can appropriately model the monotonic behaviour of the aging and deterioration process of pipes. The service life of a concrete sewer can be predicted with high accuracy for different levels of probability of failure.

In addition to the assessment of existing pipes, the reliability analysis methods used in this case study can also show how changes in design parameters of sewers (such as concrete cover) can affect the service life of the pipeline.

Sensitivity analysis has also been undertaken to identify factors that affect the probability of pipe failure due to corrosion. The analysis showed less significant contribution of some variables in failure functions. Therefore, it would not be necessary to consider those parameters as random variables and they can be treated as deterministic constant values for further studies.

The results showed that among six random variables, [DS], $b/\acute{P}$ and A have the most effect on the probability of sewer failure. This effect is more considerable for lower values of time, which means special attention should be taken for accurate determination of these variables for new pipelines.

The new methods adopted in chapter 4 were applied to the reliability analysis of a UK concrete sewer case study. The results were verified by using a Monte Carlo simulation, and more realistic results from multi-failure assessment were achieved. The degree of sensitivity to different variables was also investigated. An extensive discussion and critical analysis of these results is presented in chapter 7.

6 APPLICATION OF THE DEVELOPED METHODS TO CAST IRON WATER PIPES

6.1 Preface

The applicability of the two time-dependent reliability analysis methods proposed in chapter 4 is tested by two UK case studies involving cast iron water pipes. Two different scenarios including individual and multi failure modes are considered during the analysis. The steps used for the reliability analysis of concrete sewers in chapter 5 are followed here, and involve:

- Modelling the corrosion mechanism of cast iron pipes
- Defining the failure mode(s) (limit state function(s))
- Calculation of the probability of failure
- A sensitivity analysis

In each case, both internal and external corrosion are considered and the results are compared and discussed. Although the research on reliability of cast iron pipes has been done by other researchers, the novelty of this study is consideration of both strength failure and fracture failure, in hoop and axial planes, leading to a more realistic and reliable outcome, as demonstrated from the Monte Carlo simulation.

6.2 Reliability analysis considering individual failure mode

6.2.1 Case Study

Underground cast iron pipes in a typical conurbation of the UK are considered here for illustration of the proposed methodologies. The pipes selected have a diameter ranging from 254 to 406 mm and the wall thickness ranging from 16 to 20.3 mm correspondently as shown in Table 6.1. The pipes are made of cast iron with the fracture toughness in a range of K_C = 7.66 to 9.25 MPa/m$^{0.5}$ (Marshall (2001)).

Table 6.1 Geometry and stresses in selected pipes

Diameter (mm)	Thickness (mm)	Fracture toughness Kc ($MPa/m^{0.5}$)	Hoop Stress (MPa)
254	16.0	7.66	44.9
305	17.5	7.86	49.6
406	20.3	9.25	62

Since the purpose is to demonstrate the application of the two time dependent reliability analysis methods for predicting the service life of cast iron pipes subjected to pitting corrosion, the loads that are applied to the pipes over the service life are taken from the design codes, BS EN 1295 (1997). It has been assumed that the overburden forces can be calculated considering main road traffic and a burial depth of 0.9 m, using the formula given in the national design standard BS EN 1295 (1997). A pressure surge of 1.6 MPa has also been assumed. The stresses produced by these loads are summarized in Table 6.1 for various diameters and thicknesses.

6.2.2 Corrosion model and definition of failure mode (limit state function)

a) Corrosion model

It has been well known that the predominant deterioration mechanism for cast iron pipes is electro-chemical corrosion in the form of corrosion pits. Each spot of metal loss represents a corrosion pit that grows with time and reduces the thickness and mechanical resistance of the pipe wall. This process eventually leads to the collapse of the pipe.

As it was mentioned in chapter 3, a number of models for corrosion of cast iron pipes have been proposed to estimate the depth of corrosion. There are debates in the research community as to whether the corrosion rate can be assumed linear or otherwise (e.g., Kucera and Mattsson (1987), Sheikh et al. (1990), Ahammed and Melchers (1997), Rajani et al.

(2000), Sadiq et al. (2004)). The widely used model for corrosion pit (as mentioned in chapter 3) is expressed in the form of the following:

$$a = kt^n \tag{6.1}$$

where t is the exposure time and K and n are empirical coefficients which in practice are obtained by fitting the model to experimental data.

To avoid controversy of employing a published model, which may have not gained public acceptance, and more importantly, since the purpose of this research is to propose reliability analysis methods that only use corrosion as an example, the modelling of corrosion pit is based on the experimental data from a UK Water Industry Research report (Marshall (2001)). This corrosion rate data for using in this case study has been illustrated in Figure 6.1. As the regression of available data fits a power law very well with high R-squire value (Figure 6.1, $R^2 = 0.959$ for internal corrosion and $R^2 = 0.857$ for external corrosion), the corrosion can be modelled, for both external and internal corrosion, as follows:

$$\mu_a = 2.54t^{0.32} \text{ for external corrosion} \tag{6.2}$$

$$\mu_a = 0.92t^{0.4} \text{ for internal corrosion} \tag{6.3}$$

where μ_a denotes the mean value for the depth of corrosion pit.

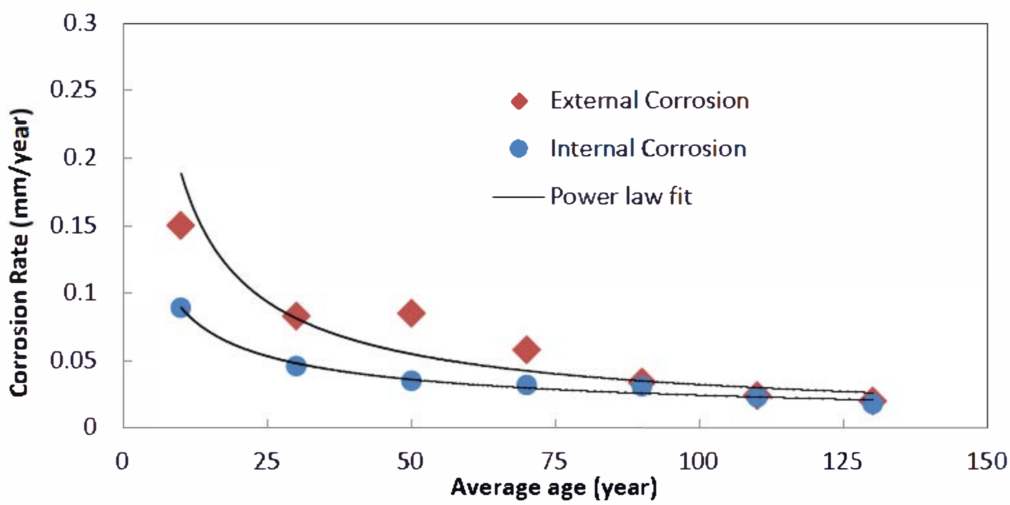


Figure 6.1 Corrosion rates for cast iron pipes (reproduced from Marshall (2001))

b) Definition of the failure mode (limit state function)

Cast iron is a brittle material and its failure can be characterised by fracture. The main effect of corrosion pits in cast iron pipes is the localised cracking (or fissuring) in the pipe wall which leads to stress concentration at or around the tip of the corrosion pit and associated reduction in the wall thickness of the pipe and hence the pipe capacity. With the growth of corrosion the pit propagates to a critical size such that the pipe cannot withstand the stresses incurred in the pipe and hence the wall is fractured and the pipe fails.

According to the theory of fracture mechanics, when a structural element (e.g., the pipe wall) under a nominal stress contains a sharp crack (e.g., the corrosion pit), the stress field ahead of the sharp crack (pit) can be described and measured by a single factor, known as stress intensity factor K. It is a parameter that amplifies the effect of stress field at the tips of crack leading to fracture. In essence, K serves as a scale factor to define the magnitude of the crack-tip stress field and is related to the geometrical parameters and stress types of the element.

The factor K is a function of far-field stress level σ, the size of the pit, a, the shape and orientation of the pit, and dimensions of the element where the pit occurs. This relationship can be expressed as a general form of (Hertzberg 1996):

$$K(t) = \sigma\sqrt{2\pi r} f(\theta, t) \tag{6.4}$$

where r and θ are the polar coordinates of the pit (crack) tip and f(θ, t) is the correction factor allowing for various geometries of the pit and structural element which also changes with time due to pit growth. Exact expressions for the stress intensity factor K(t) vary and depend on the mode of fracture and geometry of the pit and element. In general, there are three deformation modes of fracture (Hertzberg 1996), which are opening mode (Mode I), in-plane shear mode (Mode II) and out-of-plane shear or tear mode (Mode III). Since Mode I is found to be the dominant cracking condition in pipes under normal service conditions (Marshal 2001), in this research, only Mode I is considered in the failure analysis of the pipe (i.e., a crack plane is perpendicular to the direction of the stress incurred).

If K_I is stress intensity factor for crack in mode I and K_c is defined as the critical stress intensity factor, known as fracture toughness, beyond which the pipe cannot withstand the pit crack, the probability of pipe collapse (failure) can be defined as follows:

$$P_f(t) = P[K_I(t) \geq K_c] \tag{6.5}$$

In practical application of Equation 6.5 to service life prediction of cast iron pipes, the main effort lies in developing a model of stress intensity factor $K(t)$, i.e., the action process, since the fracture toughness K_c is a material property (ASTM-E1820-01 2001). In this research, two typical scenarios of corrosion pits are considered: (i) external corrosion; and (ii) internal corrosion. Since the focus is only on the hoop stress that leads to Mode I fracture, the following two cases are considered in determining the stress intensity factors of cast iron pipes.

Case 1 External surface pit under hoop stress. The typical example of this case is a pipe under internal pressure (e.g., water) and subjected to external corrosion. Figure 6.2 shows an idealised section of a cast iron pipe with external corrosion pit which is assumed to be semi-elliptical. In Figure 6.2, 2c is the length of the semi-elliptical pit, a is the radial depth of the pit through the pipe wall, R is the inner radius, d is the thickness of the pipe wall and the angle θ is used to describe the position around the semi-elliptical pit which varies between $0 \leq \theta \leq \pi$. In this case, the stress intensity factor for Mode I fracture of a semi-elliptical surface pit K_{I1} can be determined as follows (Raju and Newman 1982):

$$K_{I1} = \sigma_{hoop}\sqrt{\pi\frac{a}{Q}}F_e(\frac{a}{d},\frac{a}{c},\frac{d}{R},\theta) \tag{6.6}$$

where σ_{hoop} is the hoop stress caused by internal pressure in the pipe, F_e is the boundary correction factor for a semi-elliptical surface pit located outside the pipe. Since the growth of pit (i.e., a), increases with time and hence K_{I1} is time-variant.

In Equation (6.6), Q is the semi-elliptical pit correction factor and can be determined from (Raju and Newman 1982):

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} \tag{6.7}$$

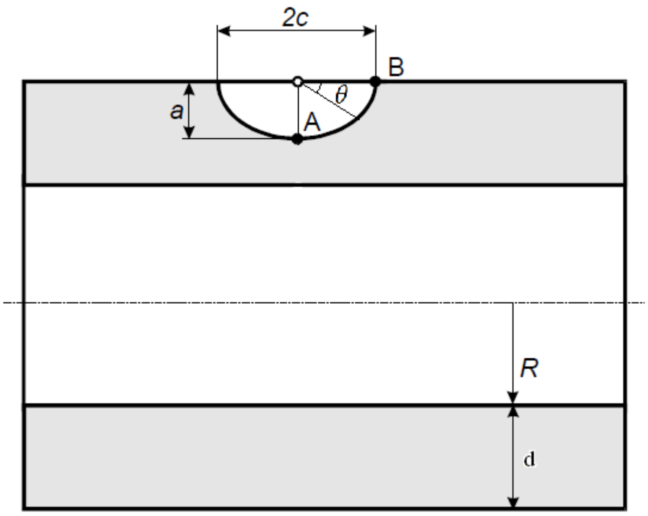


Figure 6.2 External semi-elliptical surface pit on a pipe

Also in Equation 6.6, F_e can be determined from

$$F_e = \frac{d}{R}\left(\frac{R^2}{R_o^2 - R^2}\right)\left[2G_0 + 2\frac{a}{R_o}G_1 + 3\left(\frac{a}{R_o}\right)^2 G_2 + 4\left(\frac{a}{R_o}\right)^3 G_3\right] \tag{6.8}$$

where R_o is outer radius of the pipe and G_j (j = 0, 1, 2, 3) are the influence coefficients for j$^{\text{th}}$ stress distribution on crack (pit) surface. The influence coefficients G_j depend on the geometry of the pits and the pipe as represented by *a, c, d* and *R*. Exact values of G_j for a semi-elliptical external surface pit with given geometric parameters can be obtained from finite element analysis as discussed in details in Raju and Newman (1982). For the calculation of probability of failure, using Equation 6.5, an analytical expression can be developed for G_j based on mathematical regression as outlined in the following.

Raju and Newman (1982) employed finite element method to determine the stress intensity factors for pipes with various geometry and corrosion pits. In their study the exact values of the influence coefficients, i.e., G_j (j = 0, 1, 2, 3) are tabulated. Based on these tables analytical expressions for G_j (j = 0, 1, 2, 3) can be obtained using mathematical regressions for given values of geometry of a pipe and corrosion pit. Using regression methods can change un-continuous data from the tables to continuous formulae which can be used in calculation of F_e and consequently in the reliability analysis method.

To obtain the maximum value of K_{I1}, it is measured at the deepest point of the pit, i.e., θ=90°. It is also assumed that c, the half-length, and a, the radial depth of the semi-elliptical surface pit, are equal (Hertzberg 1996). In addition for most pipes used by industry the ratio of wall thickness to radius, i.e., d/R, is in a range of 0.1– 0.2.

Using least squares regression method for the data from tables of Raju and Newman (1982) and for $\frac{d}{R} = 0.1$ and $\frac{2\theta}{\pi}$=1.0, the expressions for influence coefficients G_j for semi-elliptical surface crack on the outside surface of a pipe can be derived as follows:

$$G_0 = \left(5.7981\left(\frac{a}{c}\right)^2 - 9.1619\frac{a}{c} + 3.5187\right)\frac{a}{d} - 1.2154\left(\frac{a}{c}\right)^2 + 1.8072\frac{a}{c} + 0.4044 \tag{6.9a}$$

$$G_1 = \left(2.0031\left(\frac{a}{c}\right)^2 - 3.1684\frac{a}{c} + 1.2436\right)\frac{a}{d} - 0.3644\left(\frac{a}{c}\right)^2 + 0.6521\frac{a}{c} + 0.4135 \tag{6.9b}$$

$$G_2 = \left(1.0237\left(\frac{a}{c}\right)^2 - 1.6222\frac{a}{c} + 0.6468\right)\frac{a}{d} - 0.1531\left(\frac{a}{c}\right)^2 + 0.3594\frac{a}{c} + 0.3726 \tag{6.9c}$$

$$G_3 = \left(0.6387\left(\frac{a}{c}\right)^2 - 1.0082\frac{a}{c} + 0.4028\right)\frac{a}{d} - 0.0802\left(\frac{a}{c}\right)^2 + 0.2521\frac{a}{c} + 0.3328 \tag{6.9d}$$

Case 2 Internal surface pit under hoop stress. The internal surface corrosion pit can also be assumed to be semi-elliptical with length $2c$ and radial depth a as shown in Figure 6.3. In this case, the stress intensity factor for an internal semi-elliptical surface pit K_{I2} can also be determined from Equation 6.6 with F_i replacing F_e as follows (Raju and Newman 1982):

$$K_{I2} = \sigma_{hoop}\sqrt{\pi\frac{a}{Q}}F_i\left(\frac{a}{d},\frac{a}{c},\frac{d}{R},\theta\right) \tag{6.10}$$

where F_i is the boundary correction factor for a semi-elliptical surface pit located inside the pipe and can be determined from:

$$F_i = \frac{d}{R}\left(\frac{R_o^2}{R_o^2 - R^2}\right)\left[2G_0 - 2\frac{a}{R}G_1 + 3\left(\frac{a}{R}\right)^2 G_2 - 4\left(\frac{a}{R}\right)^3 G_3\right] \tag{6.11}$$

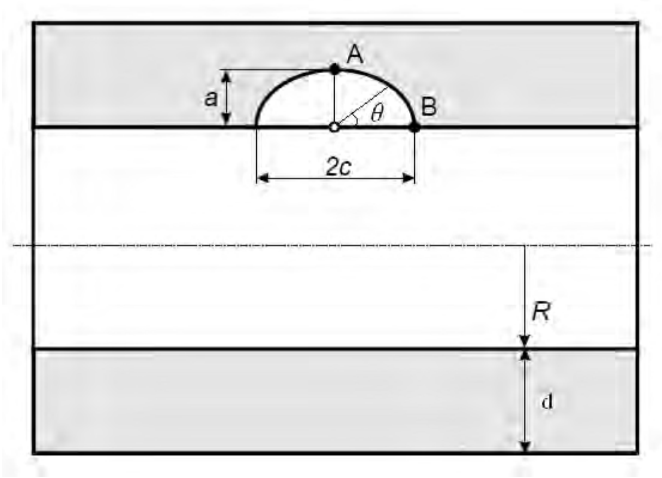


Figure 6.3 Internal semi-elliptical surface pit in a pipe

It is assumed that c, the half-length, and a, the radial depth, of the semi-elliptical surface pit, are equal (Hertzberg 1996). Likewise the external corrosion pit, the expressions for the influence coefficients G_j (j = 0, 1, 2, 3) for semi-elliptical surface crack on the inside surface of a pipe can be derived as follows:

$$G_0 = \left(3.6877\left(\frac{a}{c}\right)^2 - 5.84671\frac{a}{c} + 2.2834\right)\frac{a}{d} - 0.6844\left(\frac{a}{c}\right)^2 + 1.0071\frac{a}{c} + 0.6665 \tag{6.12a}$$

$$G_1 = \left(1.28448\left(\frac{a}{c}\right)^2 - 2.0459\frac{a}{c} + 0.8361\right)\frac{a}{d} - 0.1921\left(\frac{a}{c}\right)^2 + 0.3943\frac{a}{c} + 0.495 \tag{6.12b}$$

$$G_2 = \left(0.6521\left(\frac{a}{c}\right)^2 - 1.0462\frac{a}{c} + 0.4442\right)\frac{a}{d} - 0.0658\left(\frac{a}{c}\right)^2 + 0.2305\frac{a}{c} + 0.411 \tag{6.12c}$$

$$G_3 = \left(0.295\left(\frac{a}{c}\right)^2 - 0.502\frac{a}{c} + 0.2403\right)\frac{a}{d} + 0.051\left(\frac{a}{c}\right)^2 + 0.0669\frac{a}{c} + 0.3851 \tag{6.12d}$$

It should be noted that, in this study conservative assumptions and simplified form of Equations 6.6 to 6.12 have been adopted to determine the maximum values of K_{I1} and K_{I2}, which is based on the worst case scenario and measured at the deepest point of the pit, i.e., θ =90°.

6.2.3 Calculation of the probability of failure

a) Using first passage probability method

The first passage probability method is proposed in this section to quantitatively assess the probability of the pipe failure over a period of time. The concept of stress intensity in fracture mechanics is employed to establish the failure criterion, i.e., the limit state function used to determine the probability of pipe failure.

As it was already mentioned in section 4.2.1, probability of failure of a pipe can be determined by using first passage probability theory from Equation 4.9. Considering Equation 6.5 as the failure definition, stress intensity factor (K_I) is the action (load) and *Kc* is its effect or the acceptable limit (resistance). Therefore Equation (4.9) can be reproduced with K_I replacing *S* and *Kc* replacing *R*.

$$P_f(t) = \int_0^t \frac{\sigma_{\dot{K}_I|K_I}(t)}{\sigma_{K_I}(t)} \emptyset\left(\frac{K_c-\mu_{K_I}(t)}{\sigma_{K_I}(t)}\right)\left\{\emptyset\left(-\frac{\mu_{\dot{K}_I|K_I}(t)}{\sigma_{\dot{K}_I|K_I}(t)}\right)+\frac{\mu_{\dot{K}_I|K_I}(t)}{\sigma_{\dot{K}_I|K_I}(t)}\Phi\left(\frac{\mu_{\dot{K}_I|K_I}(t)}{\sigma_{\dot{K}_I|K_I}(t)}\right)\right\} d_\tau \tag{6.13}$$

For a given Gaussian stochastic process with mean function $\mu_{K_I}(t)$, and auto-covariance function $C_{K_I K_I}(t_i, t_j)$, all terms in Equation 6.13 can be determined as outlined in section 4.3 as follows:

$$\mu_{\dot{K}_I|K_I} = E[\dot{K}_I \mid K_I = Kc] = \mu_{\dot{K}_I} + \rho_{K_I}\frac{\sigma_{\dot{K}_I}}{\sigma_{K_I}}(Kc - \mu_{K_I}) \tag{6.14a}$$

$$\sigma_{\dot{K}_I|K_I} = [\sigma_{\dot{K}_I}^2(1-\rho_{K_I}^2)]^{1/2} \tag{6.14b}$$

where

$$\mu_{\dot{K}_I} = \frac{d\mu_{K_I}(t)}{dt} \tag{6.14c}$$

$$\sigma_{\dot{K}_I} = \left[\frac{\partial^2 C_{K_I K_I}(t_i, t_j)}{\partial t_i \partial t_j} \Big|_{i=j} \right]^{1/2} \tag{6.14d}$$

$$\rho_{K_I} = \frac{C_{K_I \dot{K}_I}(t_i, t_j)}{[C_{K_I K_I}(t_i, t_i) \cdot C_{\dot{K}_I \dot{K}_I}(t_j, t_j)]^{1/2}} \tag{6.14e}$$

and the cross-covariance function is

$$C_{K_I \dot{K}_I}(t_i, t_j) = \frac{\partial C_{K_I K_I}(t_i, t_j)}{\partial t_j} \tag{5.14f}$$

$$C_{K_I K_I}(t_i, t_j) = \lambda_{K_I}^2 \rho_{K_I} \mu_{K_I}(t_i) \mu_{K_I}(t_j) \tag{6.14g}$$

Where λ_{K_I} is the coefficient of variation of the stress intensity factor which is determined based on Monte Carlo simulations and $\rho_{K_I}$is (auto-) correlation coefficient for the stress intensity factor between two points in time t_i and t_j. Therefore all variables in Equation 6.13 can be determined.

Values of the influence coefficients, i.e., Gj (j = 0, 1, 2, 3) in Equations 6.8 and 6.11 are calculated by using equations 6.9(a-d) and 6.12(a-d) for external and internal corrosion respectively. The stresses produced by the existing loads are used from the values given in Table 6.1 for various diameters and thicknesses.

With this preparation, the stress intensity factors for two different cases can be calculated by Equations 6.6 and 6.10, respectively. The time-dependent probability of pipe failure can finally be calculated using Equation 6.13.

Typical results for each case are shown in Figures 6.4 and 6.5. These Figures indicate that the effect of the auto-correlation of the stress intensity factors between two points in time (i.e., ρ_{K_I}) on pipe collapse can be significant which justifies that using first passage probability method without having sufficient data (i.e., auto-correlation coefficient) for this example is not a desired method.

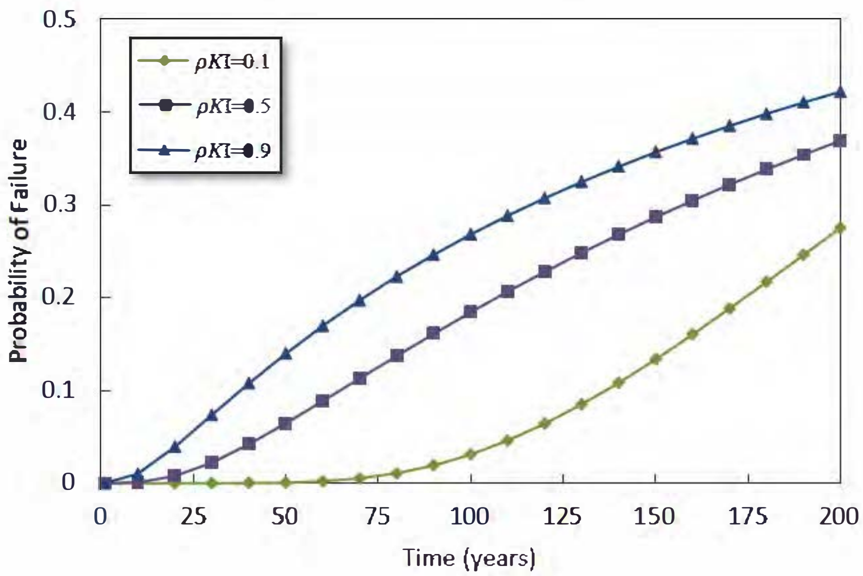


Figure 6.4 Probability of pipe collapse for different ρ_{K_I} , for case 1 (external corrosion), using first passage probability method (D = 305mm, λ_{K_I} = 0.41)

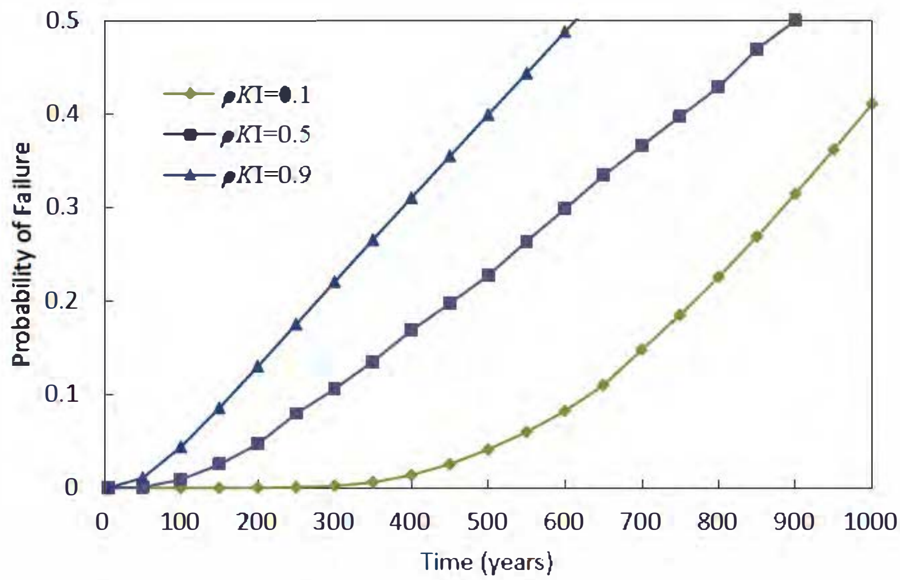


Figure 6.5 Probability of pipe collapse for different ρ_{K_I} , for case 2 (internal corrosion), using first passage probability method (D = 254mm, λ_{K_I} = 0.41)

b) Using Gamma distributed degradation (GDD) Model

With the failure definition in Equation 6.5, the developed algorithm (GDD model) in section 4.4.2 is used for formulation of the probability of failure based on gamma process concept. Given that a random process (i.e., stress intensity factor, K_I) has a gamma distribution with shape parameter $\alpha > 0$ and scale parameter $\lambda > 0$, formulation for calculation of the probability of failure for a corrosion affected pipe is developed as it was mentioned in section 4.4.

Assuming that the time at which failure occurs is denoted by the lifetime T, due to the gamma distributed deterioration, Equation 4.14 can be reproduced for calculation of the probability of failure.

$$\mathrm{F(t)} = \Pr(\mathrm{T} \leq \mathrm{t}) = \Pr(K_I(t) \geq K_c) = \int_0^{K_c} f_{K_I(t)}(K_I) d_{K_I} = \frac{\Gamma(\alpha(t), K_c\lambda)}{\Gamma(\alpha(t))} \tag{6.15}$$

Where $\alpha(t) = ct^b$ is the shape parameter with physical constants $c > 0$ and $b > 0$ and λ is the scale parameter. The parameters $\alpha(t)$ and λ can be estimated by using the estimation methods explained in section 4.4.2. For the exponential parameter b, a value of one is assumed (b=1) based on some examples of expected deterioration that have been presented in Table 4.1.

Figures 6.6 and 6.7 show the results obtained for the probability of pipe failure by using the GDD model for different cases.

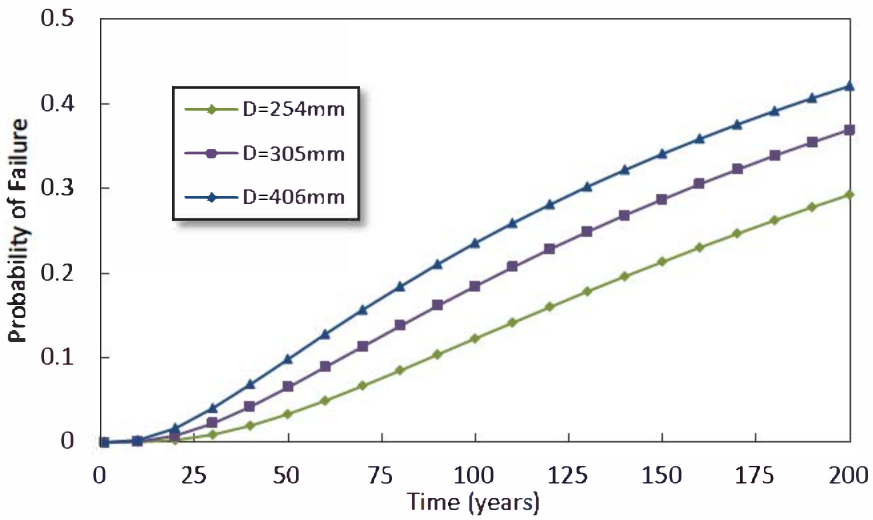


Figure 6.6 Probability of pipe collapse for Case 1 (external corrosion) with different diameters, using GDD Model

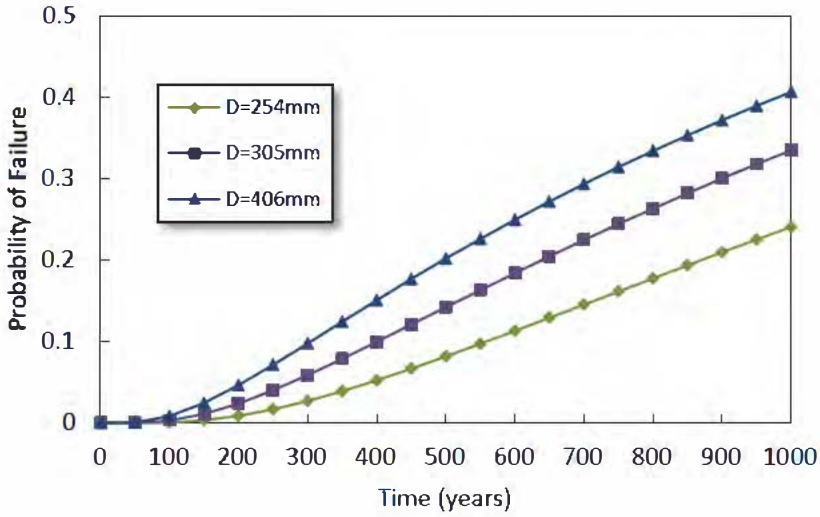


Figure 6.7 Probability of pipe collapse for Case 2 (internal corrosion) with different diameters, using GDD Model

6.2.4 Verification of the results of the probability of failure

To verify the results obtained from GDD model in the previous section, Monte Carlo simulation method is used. A MATLAB code was written to perform Monte Carlo simulation (see section 4.5 for the detail). The results are illustrated in Figures 6.8 and 6.9, involving all the results from the GDD model for each case. The results from using first passage probability are not considered for comparison because of high dependency to the value of auto-correlation.

The comparison shows that the probabilities of failure predicted by GDD model are in good agreements with the results of Monte Carlo simulation method, particularly for small probabilities which are of most practical interest.

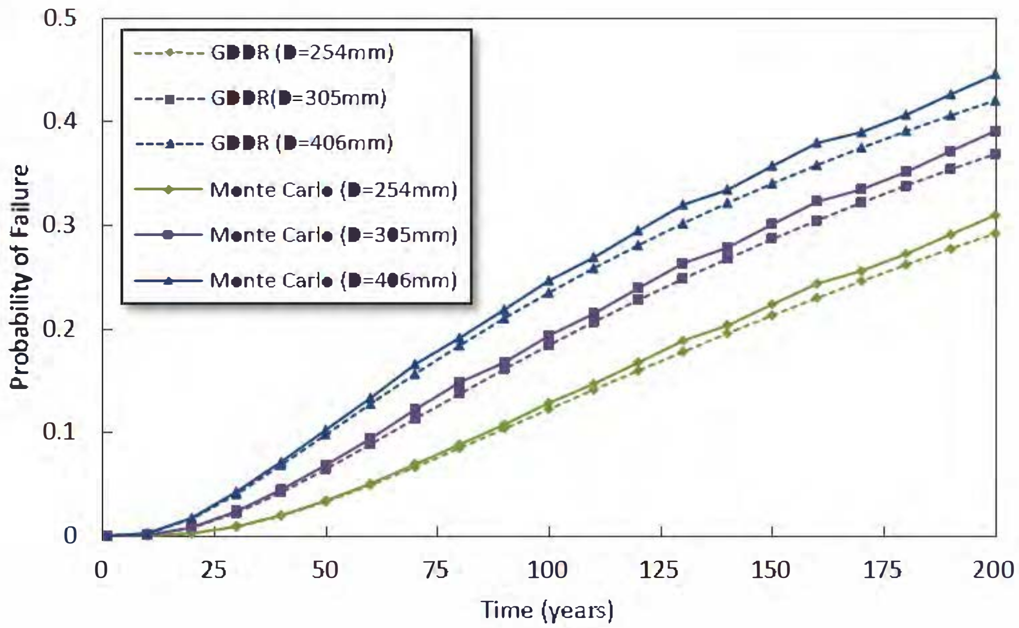


Figure 6.8 Verification of the results from GDD model by Monte Carlo simulation method for Case 1 (external corrosion) with different diameters

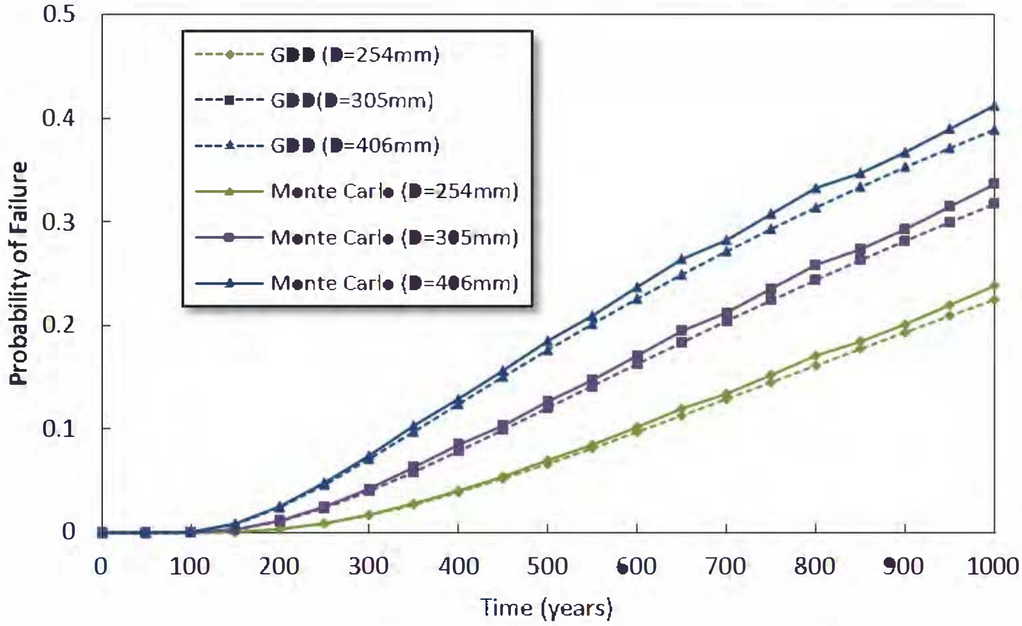


Figure 6.9 Verification of the results from GDD model by Monte Carlo simulation method for Case 2 (internal corrosion) with different diameters

6.2.5 Sensitivity analysis

To assess the effect of the pipe material property (i.e., toughness) on the probability of failure, the GDD model is run for different toughness of the pipe material (K_c). The results of this parametric sensitivity analysis are illustrated in Figure 6.10. These results are self-evident; the tougher the pipe is (i.e., the greater the fracture toughness), the smaller the probability of its failure.

In view of variables that affect the corrosion process, it is of interest to identify the degree of contribution of variables so that more research can focus on the most effective variable. The contribution of these variables in the failure function is calculated by using relative contribution concept (Chapter 3, Equation 3.22).

α_x^2 represents the relative contribution of random variables (k and n) in the violation of the limit state function. Figure 6.11 shows the degree of contribution of each variable during the time of service.

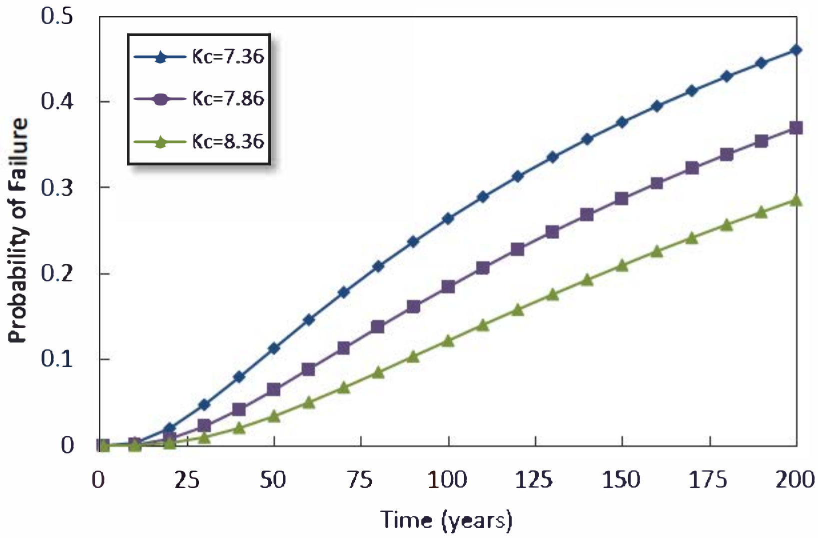
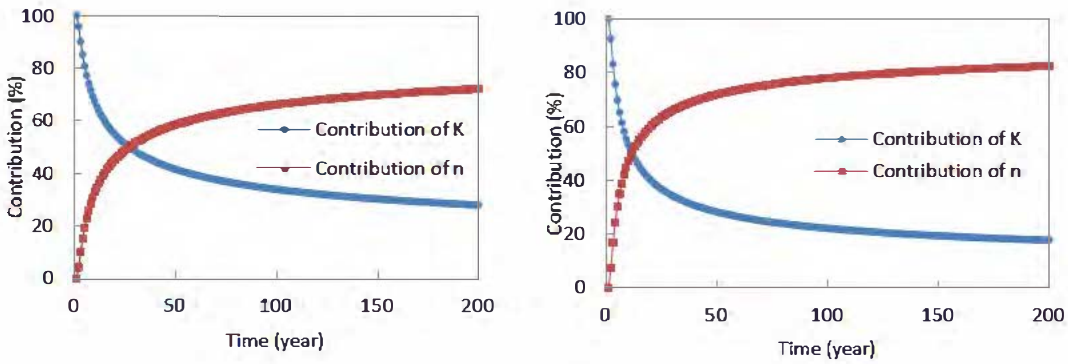


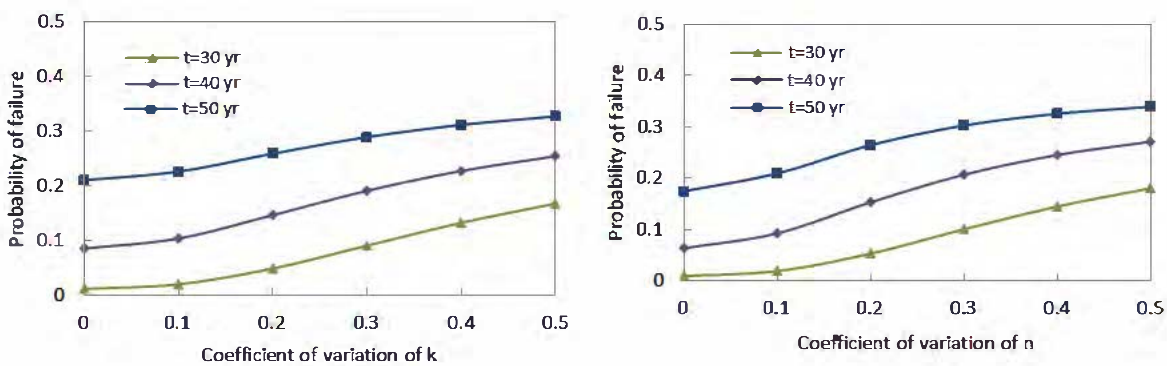
Figure 6.10 Probability of pipe failure for different K_c for case 1 (external corrosion), using GDD Model (D = 305mm)

Further sensitivity studies were carried out to investigate the effect on probability of failure of the level of variability (i.e. coefficient of variation) of each of corrosion model parameters (i.e. k and n). The coefficient of variation for each of these parameters was varied from 0 to 0.5 in steps of 0.1. Figures 6.12 and 6.13 illustrate the results for three different pipeline elapsed lives (t). Generally the probability of failure is more sensitive to the variation of the coefficient of variation of exponential constant (n).

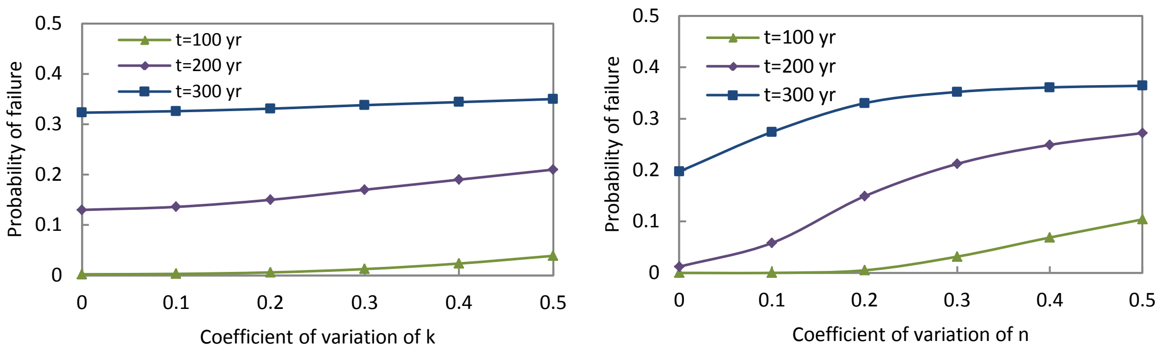


(a) (b)
Figure 6.11 Relative contribution to the variance of failure function for corrosion multiplying constant (K), and corrosion exponential constant (n), a) Case 1, External Corrosion, b) Case 2, Internal Corrosion

It is also observed that the variability of the parameters (k and n) for low values of t, has more significant effect on the probability of failure.



(a) (b)
Figure 6.12 Probability of failure due to external corrosion (case 1) Vs coefficient of variation for various values of pipeline elapsed life ((a) corrosion multiplying constant, K, and (b) corrosion exponential constant, n)



(a) (b)

Figure 6.13 Probability of failure due to internal corrosion (case 2) Vs coefficient of variation for various values of pipeline elapsed life ((a) corrosion multiplying constant, K, and (b) corrosion exponential constant, n)

6.3 Reliability analysis considering multi failure mode

Cast iron pipes can fail in many modes which in general can be summarized in two categories: loss of strength due to the reduction of wall thickness of the pipes, and loss of toughness due to the stress concentration at the tips of cracks or defects. Even in one category there can be many mechanisms that cause failure. For the example of strength failure it can be caused by hoop stress or axial stress in the pipes. A review of most recent research literature (Sadiq et al. (2004), Moglia et al. (2008), Yamini (2009) and Clair and Sinha (2012)) suggests that current research on pipe failures focuses more on loss of strength than loss of toughness. As it was mentioned in section 3.3.7(b) literature review also revealed that in most reliability analyses for buried pipes, multi failure modes are rarely considered even in practice this is the reality. Therefore the aim of this section is to consider multi failure modes in reliability analysis and service life prediction for cast iron pipes. Both loss of strength and toughness of the pipe are considered. A system reliability method is employed in calculating the probability of pipe failure over time, based on which the service life of the pipe can be estimated. Sensitivity analysis is also carried out to identify those factors that affect the pipe behavior most.

6.3.1 Case study

A cast iron pipe of 254 mm diameter and effective length of 6.5 m is considered as an example to illustrate the proposed method. Other data for the cast iron pipe required for calculation is presented in Table 6.2. Totally 15 random variables are involved in the problem in which their statistical data have been presented in the Table.

6.3.2 Corrosion model and definition of failure modes (limit state functions)

a) Corrosion model

Same as individual failure analysis, the widely used corrosion model (i.e., Equation (6.1)) is selected for the multi failure mode reliability analysis of the cast iron pipe. Therefore the statistical values (mean and standard deviation) for k and n in Equation (6.1) are again taken from the mathematical regression (Figure 6.1) to the data from Marshall (2001). Based on this data mean and standard deviation for corrosion coefficients (k and n) have been estimated (Table 6.2).

b) Definition of the failure modes (limit state functions)

Buried pipes are not only subjected to mechanical actions (loads) but also environmental actions that cause the corrosion of pipes. Corrosion related defects would subsequently cause fracture of cast iron pipes. In the presence of corrosion pit, failure of a pipe can be attributed to two mechanisms: (1) the stresses in the pipe exceed the corresponding strength or (2) the stress intensity exceeds fracture toughness of the pipe. Based on these two failure modes, two limit state functions can be established as follows.

Table 6.2 Values of basic random variables used in the case study

Symbol	Variable		Units	Min.	Mean	St. Dev.	Max	References
P	Internal Pressure		MPa	0.35	0.45	0.1	0.7	EPB 276 (2004)
D	Inner diameter		mm	240	254	14.28	260	BS78-2 (1965)
d	Wall thickness		mm	-	16	0.7	-	BS78-2 (1965)
K_m	Bending moment coefficient		-	-	0.235	0.04	-	Sadiq et al. (2004)
C_d	Calculation coefficient		-	-	1.32	0.20	-	Sadiq et al. (2004)
B_d	Width of ditch		mm	-	625	125	-	AWWA C600, (2005)
E_P	Modulus of elasticity of pipe		MPa	-	105000	15000	-	BS78-2 (1965)
K_d	Defection coefficient		-	-	0.108	0.0216	-	Sadiq et al. (2004)
I_c	Impact factor		-	-	1.5	0.375	-	Sadiq et al. (2004)
C_t	Surface load Coefficient		-	-	0.12	0.024	-	Sadiq et al. (2004)
F	Wheel load		N	30000	412000	20000	100000	Sadiq et al. (2004)
A	Pipe effective length		mm	-	6500	200	-	AWWA C600, (2005)
γ	Soil Unit weight		N/mm^3	-	18.85×10^{-6}	18.85×10^{-7}	-	Sadiq et al. (2004)
k	Multiplying constant	Internal	-	-	0.92	0.18	-	Marshall (2001)
		External	-	-	2.54	0.5	-	Marshall (2001)
n	Exponential constant	Internal	-	-	0.4	0.08	-	Marshall (2001)
		External	-	-	0.32	0.06	-	Marshall (2001)

Strength limit state. Rajani et al. (2000) developed a formula for total stresses in a buried pipe including both hoop and axial stresses (see Figure 6.14):

$$\sigma_h = \sigma_F + \sigma_S + \sigma_L + \sigma_V \tag{6.16}$$

where σ_h is the total hoop or circumferential stress in the pipe, σ_F, σ_S, σ_L and σ_V are the hoop stresses due to internal fluid pressure, soil pressure, frost pressure and traffic stresses respectively.

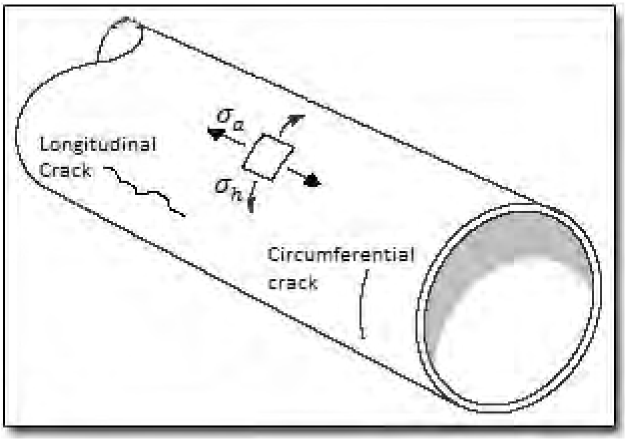


Figure 6.14 Stresses and cracks on a pipe wall (σ_h: hoop stress and σ_a: axial stress)

Similarly the total axial or longitudinal stress in the pipe can be expressed as:

$$\sigma_a = \sigma_{Te} + \sigma_P + (\sigma_S + \sigma_L + \sigma_V)\, \nu_p \tag{6.17}$$

where σ_a is the total axial stress in the pipe, σ_{T_e} is the stress related to temperature difference, σ_P is the axial stress due to internal fluid pressure, ν_p is Poisson's ratio of pipe material. Details of the equations and references used for determining the above stresses are presented in Table 6.3.

In practice, a pipe is usually under both axial and hoop stresses (σ_a and σ_h). If the yield strength of the pipe material is σ_y, the two limit state functions for strength can be established as follows

Hoop stress limit state: $G_1(\sigma_y, \sigma_h, t) = \sigma_y - \sigma_h(t)$ (6.18)

Axial stress limit state: $G_2(\sigma_y, \sigma_a, t) = \sigma_y - \sigma_a(t)$ (6.19)

Toughness limit state. For localised stress concentration caused by defects, e.g., corrosion pits, the term stress intensity factor, K_I, is used (as it was mentioned in section 6.2.2(b)) to more accurately predict the stress state ("stress intensity") near the tip of a crack (caused by applied or residual stresses).

Table 6.3 Models for stresses on buried pipes considered in this study

Stress Type	Model*	Reference
σ_F , hoop stress due to internal fluid pressure	$\frac{pD}{2d}$	Rajani et al. (2000)
σ_S , soil pressure	$\frac{3K_m \gamma B_d^2 C_d E_P d D}{E_P d^3 + 3K_d p D^3}$	Ahammed & Melchers (1994)
σ_V , Traffic stress	$\frac{3 K_m I_c C_t F E_P d D}{A(E_P d^3 + 3K_d p D^3)}$	Ahammed & Melchers (1994)
σ_{T_e} , Thermal stress	$-E_P \alpha_P \Delta T_e$	Rajani et al. (2000)
σ_P , axial stress due to internal fluid pressure	$\frac{p}{2}\left(\frac{D}{d}-1\right)\nu_p$	Rajani et al. (2000)

*Notations are in Table 6.2

Instead of using Equations 6.6 and 6.10 which have been developed just for hoop stresses by Raju and Newman (1982), the formulations presented by Laham (1999) are used for calculation of stress intensity factors for crack pits in a pipe under different stresses.

According to Laham (1999), the stress intensity factor for a crack pit in a pipe under hoop stress is as follows:

$$K_{I-h} = \sqrt{\pi a} \sum_{i=0}^{3} \sigma_i f_i \left(\frac{a}{d}, \frac{2c}{a}, \frac{R}{d}\right) \tag{6.20}$$

and the stress intensity factor for a crack pit in a pipe under axial stress:

$$K_{I-a} = \sqrt{\pi a} \left(\sum_{i=0}^{3} \sigma_i f_i \left(\frac{a}{d}, \frac{2c}{a}, \frac{R}{d}\right) + \sigma_{bg} f_{bg} \left(\frac{a}{d}, \frac{2c}{a}, \frac{R}{d}\right)\right) \tag{6.21}$$

where

K_{I-h} = Stress intensity factor for longitudinal crack in mode I, caused by hoop stress

K_{I-a} = Stress intensity factor for circumferential crack in mode I, caused by axial stress

a = Depth of the crack, i.e., corrosion pit

σ_i = Stress normal to the crack plane

f_i and f_{bg} = Geometry functions, depend on a, c (half-length of crack) and R (inner radius of pipe)

σ_{bg}= the global bending stress, i.e. the maximum outer fibre bending stress

For internal and/or external crack pits, the difference in formulations of stress intensity factor (Equations 6.20 and 6.21) lies in geometry functions (i.e., f_i and f_{bg}), which have been presented in different tables by Laham (1999). Due to the propagation of corrosion, a changes with time so the stress intensity factors are time variant.

If K_C is the critical stress intensity factor, known as fracture toughness, beyond which the pipe cannot sustain propagation of the crack pit, the two limit state functions for fracture toughness can be established as follows:

Axial fracture limit state: $G_3(K_C, K_{I-h}, t) = K_C - K_{I-h}(t)$ (6.22)

Hoop fracture limit state: $G_4(K_C, K_{I-a}, t) = K_C - K_{I-a}(t)$ (6.23)

6.3.3 Calculation of the probability of failure

In the case of a pipe with multiple modes of failure, the probability of pipe failure can be determined using the methods of system reliability. Since the occurrence of either failure mode will constitute the failure of the pipe, a series system is appropriate for the assessment of pipe failures. The description of series, parallel and complex systems have already been explained in section 3.3.4.

Equation 5.7 is used for calculation of the probability of series system failure. To estimate $P_{f_i}(t)$, the probability of failure due to the i^{th} failure mode, GDD model is considered in this study. The method of first passage probability is not considered for the analysis because the results of using this method for individual failure analysis in section 6.2.3 (a), showed that the dependency to auto-correlation coefficient affects the probability of failure significantly.

Therefore this method will not be used for multi failure analysis and only GDD model will be used for calculation of the probability of cast iron pipe system failure.

Using Gamma distributed degradation (GDD) Model

Considering the methodology presented in section 4.4.2, Equation 4.14 is reproduced for the all 4 limit state functions (i.e., $G_1(\sigma_y, \sigma_h, t)$, $G_2(\sigma_y, \sigma_a, t)$, $G_3(K_C, K_{I-h}, t)$ and $G_4(K_C, K_{I-a}, t)$). The results are used as $P_{f_i}(t)$ for i=1,…4 in Equation (5.7) and consequently the probability of the pipe system failure ($P_{fs}(t)$) is calculated.

The results of the probability of the pipe system failure are presented in Figures 6.15 and 6.16 for internal and external corrosion respectively.

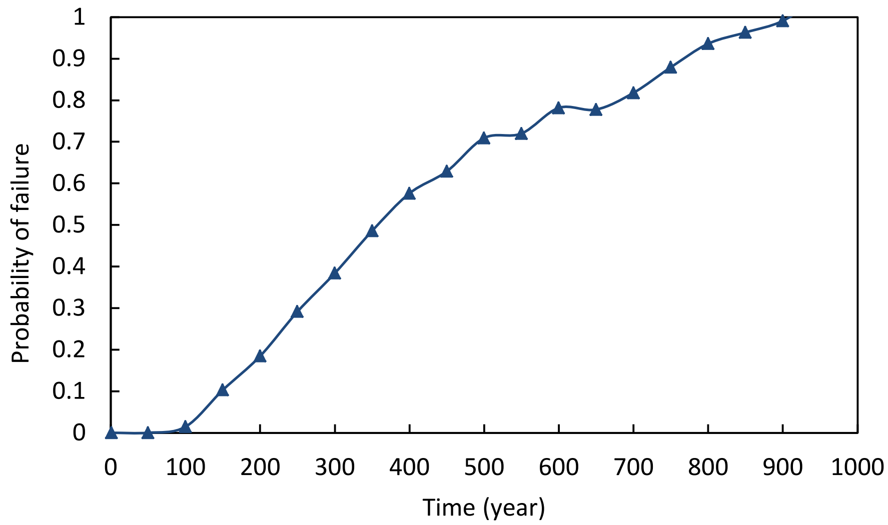


Figure 6.15 Probability of the pipe system failure (internal corrosion)

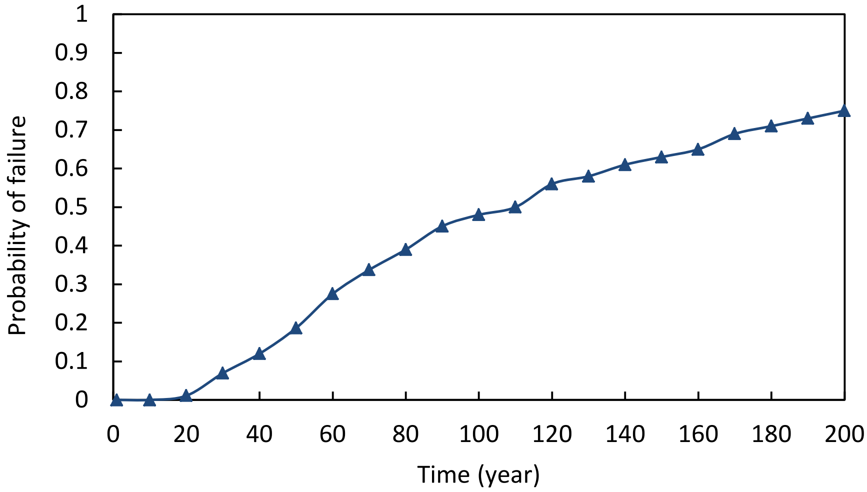


Figure 6.16 Probability of the pipe system failure (external corrosion)

6.3.4 Verification of the results of the probability of pipe system failure

The Monte Carlo simulation method (see section 4.5 for the detail) is performed for verification of the results from GDD model in the previous section. Figures 6.17 and 6.18 show the comparison of the results of the probability of system failure from GDD model and Monte Carlo simulation for internal and external corrosion respectively.

The comparison shows that the probabilities of system failure predicted by GDD model can be verified by the results of Monte Carlo simulation method, particularly for small probabilities which are of most practical interest.

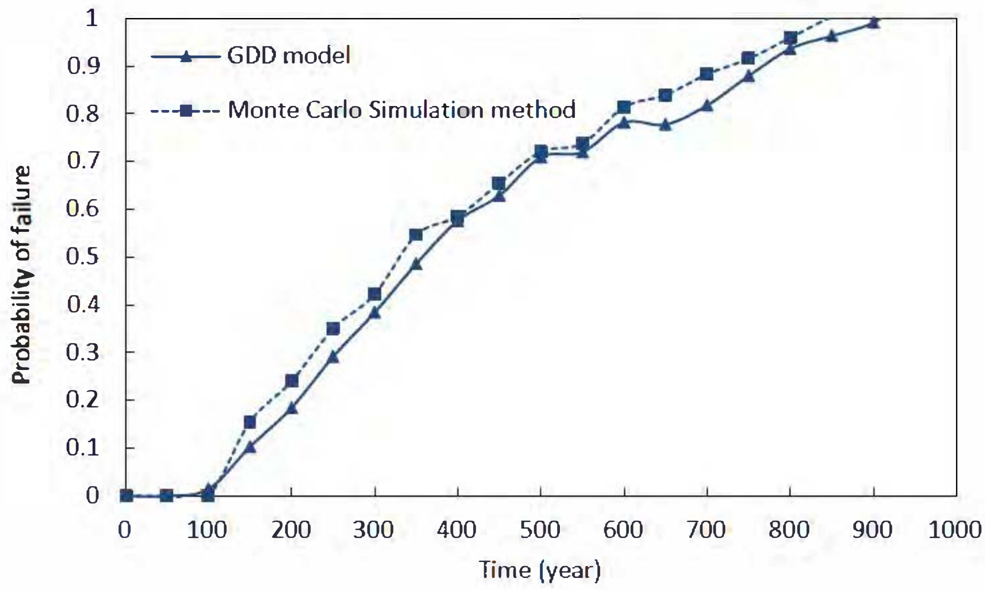


Figure 6.17 Verification of the results from GDD model by Monte Carlo simulation method (internal corrosion)

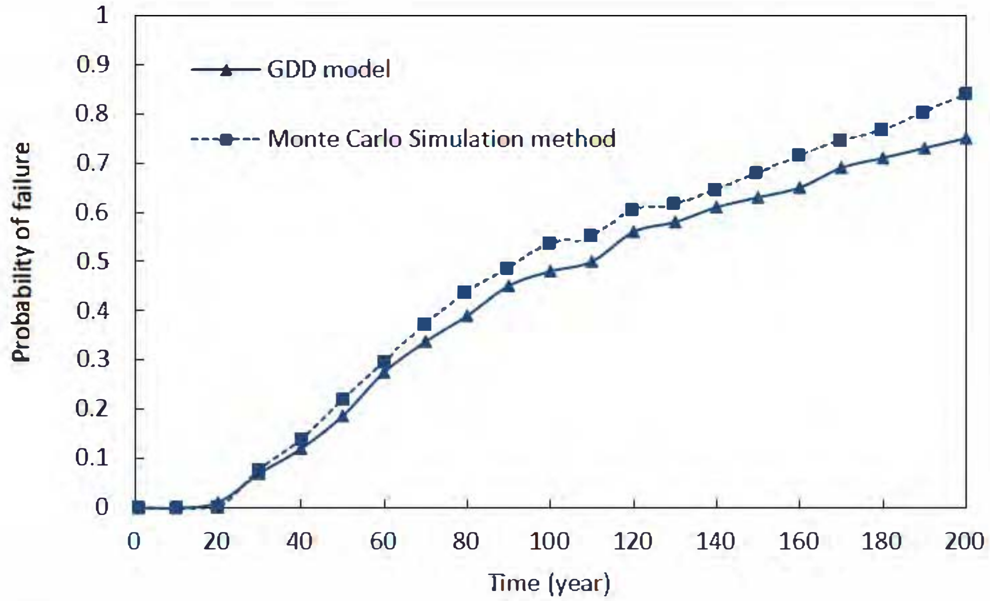


Figure 6.18 Verification of the results from GDD model by Monte Carlo simulation method (external corrosion)

6.3.5 Sensitivity analysis

It is known that the failure of a pipe can be affected by different factors, such as, pipe geometry, corrosion coefficients, soil properties and traffic loads. In view of large number of factors that affect the corrosion process and failure modes, it is of practical significance to identify those factors that affect the failure most so that more research can focus on those factors. The effect of each variable on the pipe failure can be estimated by reliability based sensitivity analysis as it was outlined in section 3.3.5.

To evaluate the sensitivity of the probability of failure to different random variables, sensitivity indexes are computed for the all 15 random variables. Figures 6.19 and 6.20 show relative contribution (α^2) and sensitivity ratios (SR) for 25-year time steps, respectively.

It is obvious from the results that the sensitivity indexes of internal pressure(P), modulus of elasticity(E_P), deflection coefficient(K_d), impact factor(I_c), surface load coefficient(C_t), wheel load(F) and pipe effective length (A) are very low for all values of time.

Among all variables, the relative contributions and sensitivity ratios of the corrosion parameters (k and n) are highly remarkable. This indicates that corrosion is very important factor for the design of underground pipelines with long lives.

Figure 6.19 also shows that the relative contribution of some other variables (e.g. wall thickness (d), bending moment coefficient (K_m), calculation coefficient (C_d) and width of ditch (B_d)) is large at early ages, but it gradually decreases within time. This suggests the relative unimportance of these variables particularly for old pipes.

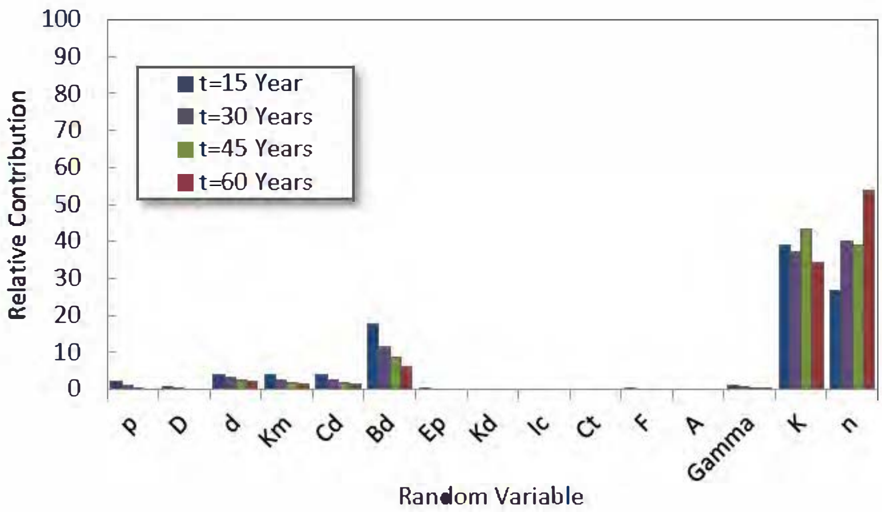


Figure 6.19 Relative contribution of random variables in pipe failure at different times for external corrosion

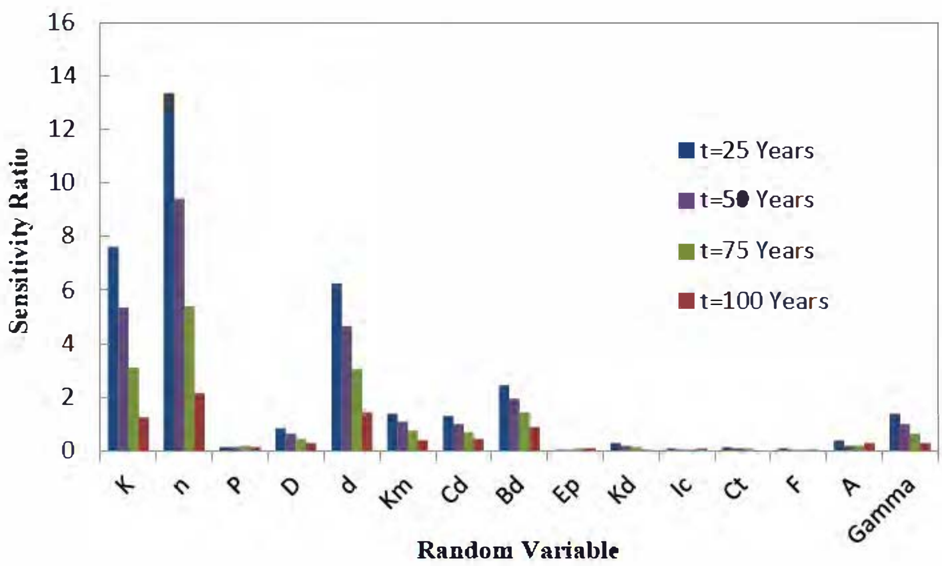


Figure 6.20 Sensitivity ratio of random variables subjected to external corrosion for different elapsed times

Further sensitivity studies were carried out to investigate the effect on probability of failure of the level of variability (i.e. coefficient of variation) of each of corrosion model coefficient (i.e. k and n) as major random variables. The coefficient of variation for each of these parameters was varied from 0 to 0.5 in steps of 0.1. The coefficient of variation of all other variables was kept constant at the values given in Table 6.3. Figures 6.21 and 6.22 illustrate the results for external corrosion for four different points of time (t). Generally the probability of failure increases while the coefficient of variation increases.

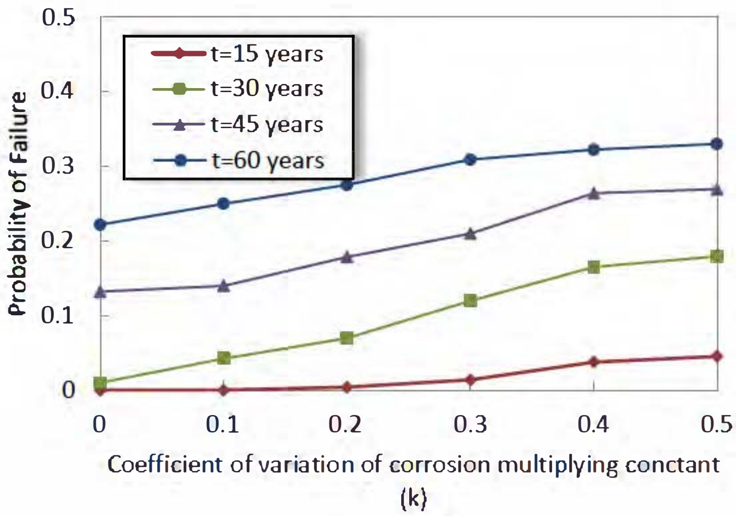


Figure 6.21 Probability of the pipe system failure due to external corrosion for different coefficients of variation of k at different times

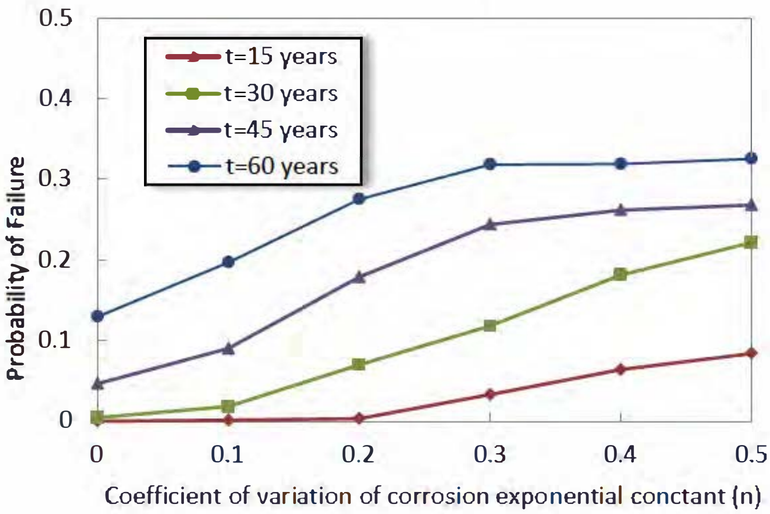


Figure 6.22 Probability of the pipe system failure due to external corrosion for different coefficients of variation of n at different times

6.4 Summary

The two developed time dependent reliability analysis methods in chapter 4 (first passage probability method and gamma distributed degradation model), were applied on two different case studies of reliability analysis of cast iron pipes in the UK. The two following scenarios were considered:

Individual failure mode:

The concept of stress intensity in fracture mechanics has been employed to establish the limit state function for determining the probability of cast iron pipe failure. A widely used model for corrosion pit was adopted based on data mining and mathematical regressions. Between the two methods for calculation of the probability of the pipe failure, first passage probability method did not attain reliable results. The results from this method were dependent to the exact value of auto-correlation coefficient which is not normally available. The results from the other method (i.e., GDD model) showed a good agreement with the results from Monte Carlo simulation method. From the results, it has been found that the probability of the pipe collapse increases with the increase of the diameter of the pipes for both external and internal corrosion and that for a given diameter, the probability of pipe failure for pipes with external corrosion is much higher than that for pipes with internal corrosion. It has also been found that the tougher the pipe, the smaller the probability of its failure. It can be concluded that GDD model is a very useful tool to predict the probability of cast iron pipe failure and its remaining service life.

Sensitivity analysis of the two random variables (i.e., corrosion parameters, k and n) showed that at early ages, the multiplying constant (k) has higher contribution in failure function and as time passes, the contribution of exponential constant (n) become higher.

Generally the probability of failure is more sensitive to the variation of the coefficient of variation of exponential constant (n). It is also observed that the variability of the parameters (k and n) for low values of t, has more significant effect on the probability of failure. In other word, the sensitivity of corrosion parameters is more dependent on the actual value of coefficient of variation in early ages.

<u>Multi failure mode:</u>

A system reliability method for reliability analysis and service life prediction for a corrosion affected cast iron pipe in the UK was proposed. A merit of this method is that it considers multi failure modes as a system, including both loss of strength and toughness of the pipe in assessing pipe failures. For calculation of the probability of the pipeline failure, GDD model was used and the results were verified by Monte Carlo simulation method. A sensitivity analysis was undertaken to identify the most significant factors among 15 random variables that affect the pipe behaviour and failure. Among all variables, sensitivity indexes of the corrosion parameters (k and n) was highly remarkable. This indicates that corrosion is very important factor for the design of underground pipelines with longer lives. High values of contribution for these two variables means that the sensitivity of these variables is more dependent on the actual value of their coefficient of variation. In such cases, more concern should be taken in order to determine relevant parameter values.

In this chapter, the applicability of the new approach to reliability analysis of cast iron water pipes in the UK was evaluated. The uniqueness of the approach, to the inclusion of key random variables impacting upon on the reliability of cast iron water obtained pipes, was demonstrated.

A comparison of the result of the first passage probability method and GDD model was also made. The results showed a preference for GDD over the other method, as it is less time consuming and more tolerant of data availability. In the next chapter, these results are discussed in detail.

7 DISCUSSION AND ANALYSIS OF THE RESULTS

The main contribution of this research is the development and successful testing of the two time dependent analytical reliability analysis methods. The suitability of these models was investigated for two different UK case studies of concrete sewers and cast iron water pipes. Chapters 5 and 6, discuss the application of these models to pipelines in the UK. A modified Monte Carlo simulation was used to verify the results obtained.

In this chapter a critical analysis of the findings of this research is given. The two models are compared, and the advantages and disadvantages of both are discussed.

The results of reliability and sensitivity analysis for the UK case studies are given in sections 7.2 and 7.3. The extensive analyses undertaken provide new insight in to pipeline assessment, and this has significant beneficial implications for pipeline managers and maintenance engineers.

7.1 Comparison of the two time developed methods

The results of applying first passage probability method to concrete sewers showed a reasonably good agreement with the results from the Monte Carlo simulation method.

A new approach based on gamma process concept for reliability analysis of corrosion affected pipes also introduced in this research. The proposed method showed that a gamma process can properly model the monotonic behavior of the ageing and deterioration process of pipes. The method was called gamma distributed degradation (GDD) model and it was applied for reliability analysis of concrete sewers and cast iron water pipes.

Both developed methods (i.e., first passage probability and GDD) were applied for two scenarios for reliability analysis of concrete sewers and cast iron water pipes. The two scenarios were: individual failure assessment and multi failure mode assessment.

In this section, the two methods are compared and the strengths and weak points of each method, based on the results of their application, are discussed.

7.1.1 First passage probability theory

To predict the service life of buried pipes, the first passage probability theory was developed. Failure due to corrosion was formulated and a stochastic model was proposed to consider uncertainties of basic random variables. The first passage probability method was employed to quantify the probability of failure, so that the time for the pipe to be unusable due to excessive corrosion can be determined.

Although the result of using first passage probability method was verified by Monte Carlo simulation method for the case of concrete sewer pipes, there are some weak points in the procedure which are discussed here.

a) Estimation of the coefficient of variation

To begin with first passage probability procedure, it is necessary to estimate the coefficient of variation of the deteriorating parameter (i.e. λ_{d} in Equation 5.5g and/or λ_{K_I} in Equation 6.14g). As it was mentioned in section 5.1.1 this estimation can be carried out by using a Monte Carlo simulation. It means that using first passage probability method (as an analytical method) still needs application of a numerical simulation (e.g. Monte Carlo) to estimate required parameters (i.e. λ_{d} and λ_{K_I}). This estimation is time consuming and will involve errors in calculations. In this case, the GDD model is less time consuming and a more reliable method for time dependent reliability analysis.

b) The effect of auto-correlation coefficient

Comparing results illustrated in Figures 5.1 and 5.12 with the results from Figures 6.4 and 6.5 shows that while in one case (concrete sewer case study) the effect of auto-correlation on probability of failure is negligible, in another case (cast iron pipe case study), significant effect for different auto-correlation coefficient can be concluded.

The effect of auto-correlation coefficient is not always negligible and the range of the results for different auto-correlation coefficients in some cases is remarkable. For instance, in the case study for cast iron pipe (section 6.2.3a, Figure 6.4), for t=100 years, the probability of failure can change dramatically from 0.030 to 0.267 for $\rho_a = 0.1$ to 0.9. This is of practical significance because auto-correlation coefficient is not readily available and therefore any wrong assumption can lead to unreliable results.

This finding shows that using first passage probability theory for time dependent reliability analysis in some cases is very dependent to assumed amount of auto-correlation coefficient.

7.1.2 Gamma distributed degradation (GDD) model

Corrosion in the two types of the buried pipes (i.e., concrete sewers and cast iron water pipes) was modeled using the gamma process concept. The proposed method (named as gamma distributed degradation model, GDD) showed that a gamma process can model the monotonic behavior of the ageing and deterioration process of pipes. Service life of a corrosion affected pipe can be predicted with high accuracy for different level of probability of failure by using gamma distributed degradation model.

Compared with first passage probability method, gamma distributed degradation model is a straight forward method with independency to estimate coefficient of variation and auto-correlation coefficient.

For degradation processes such as corrosion where gradual damage monotonically accumulates over time, the gamma process is suitable for modeling. However, the availability of monitoring data should be considered. Initially, the gamma distributed degradation model was developed assuming that corrosion measurement data are available (section 4.4.2). in that case, estimation of gamma process parameters (i.e. shape and scale parameters (α and λ)) is only possible if at least two corrosion measurements (depth of corrosion) are available.

To present more practical procedure, the proposed gamma distributed degradation model was developed in section 4.4.3 to be independent from data of corrosion measurements, because in most cases, such data is not available in the water or wastewater industry. It can be concluded that the GDD method can be used as a rational tool for a comprehensive assessment of corrosion affected concrete sewers.

7.2 Reliability analysis and service life prediction of concrete sewers

The two developed methods for time dependent reliability analysis (i.e., first passage probability and GDD model) in Chapter 4, were applied for the concrete sewer pipeline in the UK. The assessment was carried out by considering two scenarios: individual failure mode and multi failure mode.

7.2.1 Results of individual failure mode analysis

In individual failure analysis scenario, concrete cover loss was assumed as the definition for failure. Analysis and assessment for repair and rehabilitation planning of the pipe can be carried out by using the results obtained from application of the developed methods for reliability analysis of concrete sewers. From the results obtained in section 5.3, the time for the pipe to be unserviceable, i.e.,T_L, can be determined for a given acceptable probability of failure, P_a. For example, using the graph for GDD model in Figure 5.3, it can be obtained that T_L = 63 years for P_a= 0.1. If there is no intervention during the service period of (0, 63) years for the pipe, such as maintenance and repairs, T_L represents the time for the failure of the pipe, based on the reliability analysis. The information of T_L (i.e., time for interventions) is of practical importance to structural engineers and infrastructure managers with regard to planning for repairs and/or rehabilitation of the sewer. An optimum funding allocation for the pipeline system can be concluded by conducting a cost analysis for the repair and replacement of those corroded sewers with a higher risk of failure.

7.2.2 Results of multi failure mode analysis

Four failure modes including flexural, shear, cracking and cover loss were assumed as possible failure modes for multi failure mode analysis in section 5.4. As a comprehensive failure assessment, the time for the sewer to fail, i.e., T_L, due to concrete corrosion can be determined for a given acceptable probability of failure P_a. For example, using the graph for GDD model in Figure 5.14, it can be ascertained that $T_L = 58$ years for $P_a = 0.1$. If there is no intervention during the service period of (0, 58) years for the concerning pipe, such as maintenance and repairs, T_L represents the time for interventions or the end of service for the pipe; based on the performance criteria of the four assumed failure modes. The ascertainment of T_L (i.e., time for interventions) is of significant practical importance to structural engineers and asset managers of the concrete sewer in decision-making with regard to its repairs and/or rehabilitation, which are usually dependent on the budget situation of the day. Therefore, when to intervene is the first question of decision-makers.

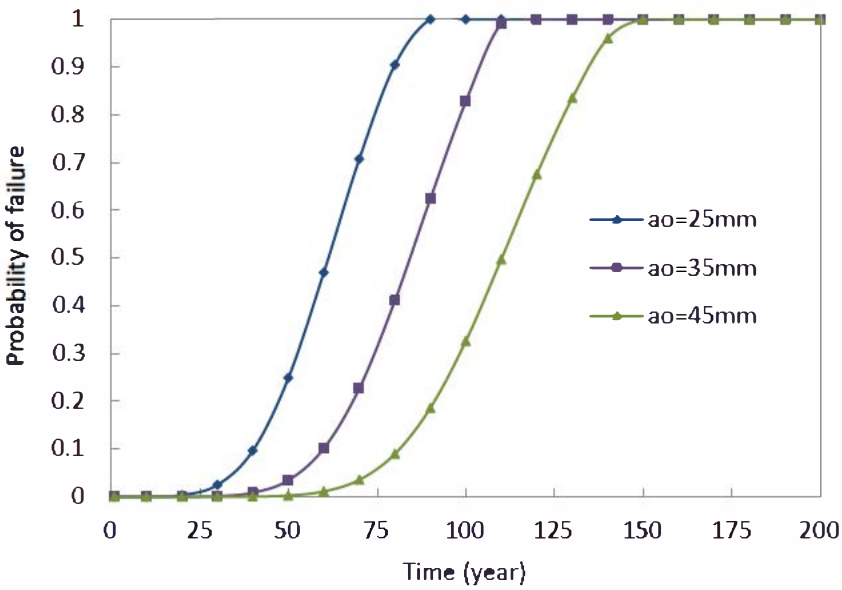


Figure 7.1 Probability of system failure for different concrete cover, a_o, GDD model

To evaluate how the thickness of concrete cover may affect the service life of the pipe, a parametric study was carried out. Figure 7.1 shows the probability of system failure over time for different thresholds (thicknesses of concrete cover, a_o) by using the GDD model. Although it is obvious that a concrete sewer with thicker concrete cover will last longer, the difference between the graphs in Figure 7.1 is a good quantitative indication to design engineers about the effectiveness of the thickness of concrete cover on service life of concrete sewers.

For instance, with an acceptable probability of failure of 10 percent (P_a=0.1), the service life for a pipe with 25mm concrete cover is 42 years. Increasing the concrete cover to 45mm will improve the service life to 82 years. This increment in service life can give a rational guidance for designing new pipes with longer service lives, considering more capital investment for production and using pipes with thicker concrete cover. Therefore a sewer designer can economically estimate the design and choose the optimum concrete cover with respect to higher service life in the conceptual design stage.

7.2.3 Sensitivity analysis and effectiveness of variables

A comprehensive sensitivity analysis was carried out in chapter 5 on reliability of concrete sewers. It can be identified from the results in Figure 5.4 that the relative contribution of some of the variables is considerably lower than other variables. These variables include stream velocity(u), the acid reaction factor (k) and the pH-dependent factor (j). This indicates that the inclusion of these as random variables has little influence on the probability of failure of the pipeline system. Therefore, in any future analysis, it would not be inaccurate to treat these variables as deterministic variables with constant quantities.

Among the variables, the relative contributions of the dissolved sulfide concentration ([DS]), the ratio of surface width of the stream to the perimeter of the exposed wall ($b/\acute{P}$) and alkalinity (A) are highly remarkable. Figures 5.5 to 5.7 presented a parametric sensitivity analysis of these three major variables. The graphs in these figures can be used for a quantitative sensitivity analysis. For instance, Figure 5.5 shows that considering 10 percent of acceptable probability of failure ($P_a = 0.1$), the service life increases from 50 to 105 years while sulphide concentration decreases from 2 mg/l to 1 mg/l. This is understandable since sulphide concentration is the main source of concrete corrosion in sewers.

Considerable effect of $b/\acute{P}$ on service life of the concrete sewer is concluded from Figure 5.6. This can be due to the presence of more sulphide for a higher amount of relative depth. To clarify, the more sewerage in the pipe, the more available sulphide and consequently more corrosion will take place.

Sensitivity of the pipe failure to other major variable, i.e., alkalinity, (A), was shown in Figure 5.7. Considering acceptable probability of failure ($\mathrm{P_a}$) equal to 0.1, Figure 5.7 shows, an increase of alkalinity (A) from 0.14 to 0.22 can lead to the increase of service life from approximately 60 to 95 years.

Alkalinity of concrete (A), which is expressed as the proportion of equivalent Calcium Carbonate (ASCE 60 2007), is the neutralizing capacity of concrete material. It needs to be noted that the results in Figure 5.7 can give a rational indication about the effect of concrete quality on the durability of concrete sewers. These results can be used for designing new concrete sewers with regard to concrete technology and the mix design of a durable concrete. For instance, the use of calcareous aggregate (limestone and dolomite) increases the alkalinity of concrete (Stutterheim and Van Aardt, (1953)) and thus prolongs the service life of concrete sewers. Results in Figure 5.7 can quantitatively show that how for each type of aggregate with specified alkalinity, the service life varies.

Further results of sensitivity analysis on major random variables illustrated in Figure 5.8 to 5.10 showed that the reliability index decreases as time and the coefficient of variation of random variables increases. It is also observed that the variability of these three random variables for low values of t has a more significant effect on the reliability index. To clarify, the sensitivity of these variables is more dependent on the actual value of coefficient of variation for lower values of time.

The effectiveness of random variables on multi failure mode of the concrete sewer was examined in section 5.4.5 and illustrated in Figure 5.15 and 5.16. Similar to individual failure mode analysis, the significance of the three major variables (i.e, [DS], $b/\acute{P}$ and A) on the system failure of the concrete sewer can be concluded for multi failure mode analysis.

Former studies also confirm this conclusion. For instance, research by Nielsen et al. (2009), Yongsiri et al. (2005) and De Belie et al. (2003) confirms the significance and effectiveness of the type of aggregates (i.e., alkalinity, A) and $b/\acute{P}$ ratio on the deterioration rate and service life of concrete sewers.

Among all variables, change in the velocity of the stream (u) has the least effect on the service life of the pipeline. For instance, for a given acceptable probability of failure of $\mathrm{P_a} = 0.1$, service life of the pipeline only changes from 28 to 37 years. In contrast, for the variable [DS] (Dissolved sulphide concentration), the service life of the pipeline varies from 27 to 78 years, which is a much wider domain compared with the velocity of the stream (u).

The significant effect of dissolved sulphide concentration, [DS], on service life of concrete sewers is consistent with practical experience and laboratory observations (De Belie et al. (2004), Ma Guadalupe et al. (2007), Antony et al. (2010) and Wells et al. (2009)).

Consequently, in order to achieve a more accurate reliability analysis and service life prediction of sewers, infrastructure managers must note the importance of monitoring data surrounding this particular parameter

7.3 Reliability analysis and service life prediction of cast iron water pipes

In chapter 6, the two developed time dependent reliability analysis methods in this study, were also applied on two different case studies of reliability analysis of cast iron water pipes in the UK, considering an individual failure and multi failure mode.

7.3.1 Results of individual failure mode analysis

From Figure 6.6 it can be concluded that the probability of pipe failure increases as the diameter of the pipe increases. This makes sense since the internal pressure is larger in pipes with larger diameter and hence larger stresses in the pipe. Figure 6.7 shows the same trend for internal corrosion as well, i.e., that probability of pipe failure increases with the increase of diameter of the pipes for both external and internal corrosion.

For the purpose of repair and rehabilitation planning, the time for the pipe to be unserviceable (i.e., T_L in Equation 4.4) needs to be determined for a given assessment criterion and acceptable probability of failure, P_a. For example it can be obtained for GDD model and external corrosion from Figure 6.6 that T_L = 67 years for P_a= 0.1 for a cast iron pipe with diameter D= 305mm. If there is no intervention during the service period of (0, 67) years for the pipe, such as maintenance and repairs, T_L represents the service life of the pipe, based on the criteria considered in the reliability analysis.

For the purpose of illustration, when P_a is taken to be 0.1, the service lives of selected pipes from GDD model are shown in Table 7.1. From the results, it can be seen that small pipes outlast large pipes considerably. The results in Table 7.1 appear to be in agreement with practical observations that cast iron pipes are as old as 150 years and there are fewer failures for small pipes than those of large ones (O’Day et al. (1986)).

Table 7.1 Service life (in years) for selected pipes

Diameter (mm)	For case 1	For case 2	For overall
254	85	590	83
305	67	402	65
406	51	340	49

Figure 7.2 compares the probability of failure for pipes with external corrosion to those with internal ones in case of individual failure mode assessment. The Figure shows the results obtained from using GDD model for cast iron pipe with diameter D=305mm. It is very clear that the likelihood of collapse for pipes with external corrosion is much higher than that for pipes with internal corrosion, in particular at initial stage. There are possibly two reasons for this. One is that the external corrosion grows faster than internal one as shown in Equations 6.2 and 6.3. The other more important reason is that mechanically the corrosion defect on external surface would make it easier for the wall of the pressurised pipe to crack inwardly than that with internal corrosion defect to crack outwardly.

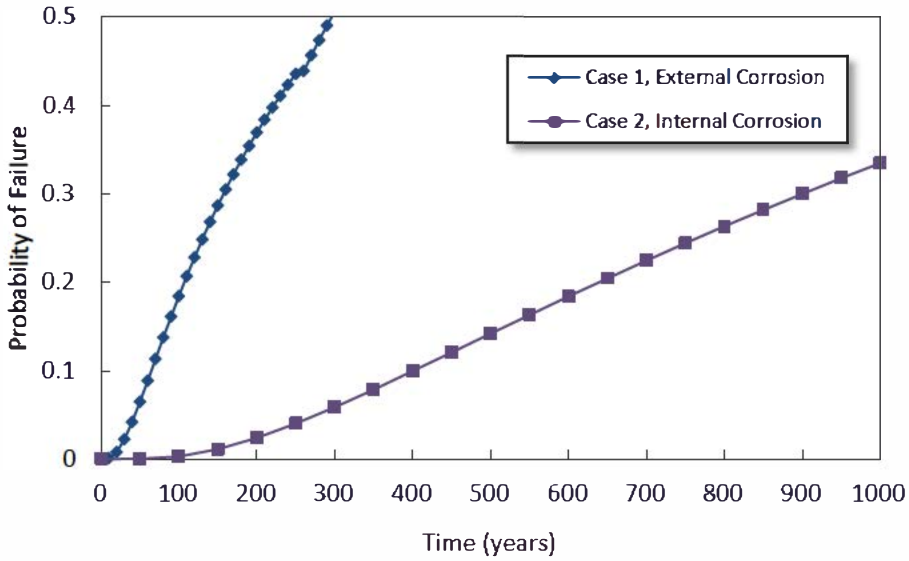


Figure 7.2 Probability of pipe failure for different cases, using GDD Model (D = 305mm), individual failure mode analysis

The result of a parametric sensitivity analysis on the effect of the pipe material property (i.e., toughness) on the probability of failure was illustrated in Figure 6.10. The figure shows that the tougher the pipe, the smaller the probability of its failure.

7.3.2 Results of multi failure mode analysis

More description and analysis of the results of the applied method (GDD model) are presented in this section. The service life of the pipe (i.e., T_L in Equation 4.4) can be determined for a given assessment criterion and acceptable probability of failure P_a. For example, using the criteria of system failure, it can be obtained from Figure 6.19 that T_L = 33 years for P_a= 10 from GDD model. If there is no intervention during the service period of (0, 33) years for the pipe, such as maintenance and repairs, based on the criteria considered in system reliability analysis, the pipe will not be serviceable after year $t > 33 years$. The information of T_L (i.e., time for intervention or service life) is of practical importance to structural engineers and asset managers with regard to planning for repairs and/or rehabilitation of the pipe networks. It can also help to achieve an optimum funding allocation for the repair and replacement of corroded pipes with higher risk of failure.

To compare the effect of internal and external corrosion on the safety of pipes, the probability of pipe failure due to external and internal corrosion from GDD model are shown in Figure 7.3. Considering Equation 6.1 as corrosion model, the difference for external and internal corrosion is in their multiplying and exponential coefficients, i.e. k and n. The results in Figure 7.3 shows that, compared with internal corrosion, external corrosion severely increases the probability of pipe failures and reduces its service life. This result is consistent with practical experience and observations (Figure 3.4 , Chapter 3). As an example, while for an acceptable probability of failure of 10 percent ($\mathrm{P_a} = 0.1$), the service life of the pipe subjected to internal corrosion is 148 years, for external corrosion, it reduces to 33 years (more than three times less).

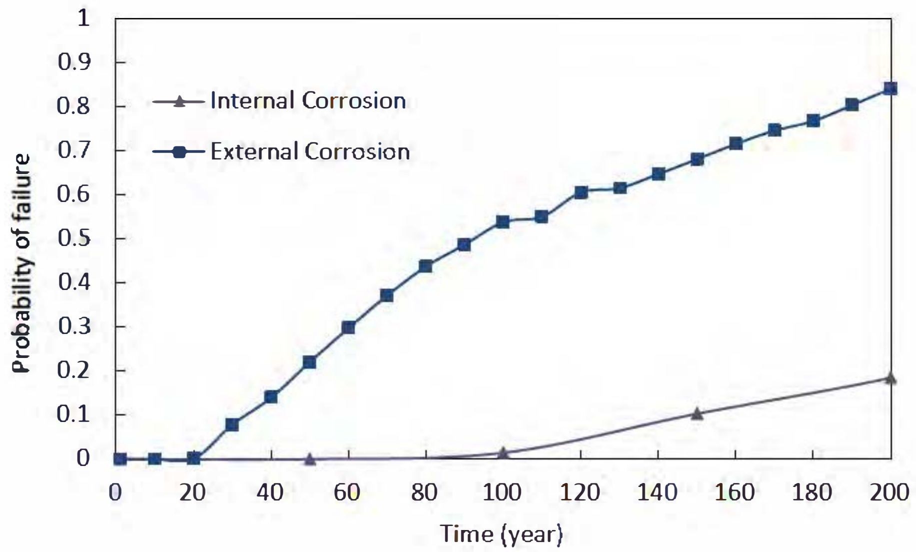


Figure 7.3 Probability of pipe system failure with different corrosion, multi failure mode analysis

To compare the result of considering individual failure mode with the result of multi failure modes analysis, the result of using GDD model for the system limit state and for each individual limit state are illustrated in Figure 7.4.

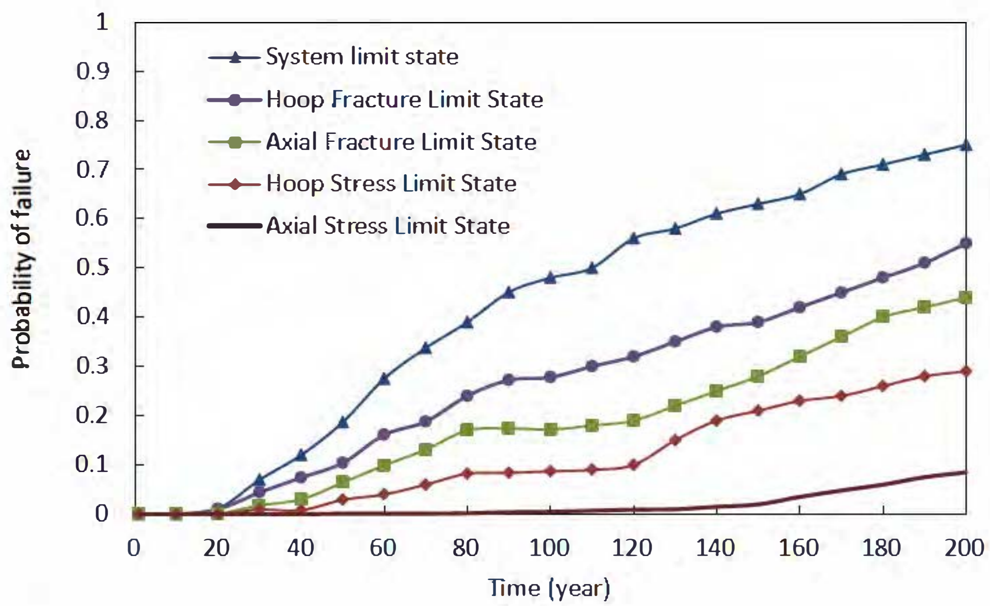


Figure 7.4 Probability of the pipe failure with different limit states (external corrosion)

It can be seen from this Figure that, the probability of the pipe failure is greater when multi failure modes are considered as a system than that when only each failure mode is considered individually. It is also seen that the likelihood of the pipe failure due to loss of toughness is much larger than that due to loss of strength, with the toughness caused by hoop fracture the largest. It is clear form Figure 7.4 that considering each failure mode individually in reliability analysis of pipes is not conservative which may pose undue risks to the public and subsequent disastrous consequences. The result of Figure 7.4 can also vindicate the significance of this study since it provides a more realistic and accurate method for the prediction of pipe failures.

7.3.3 Sensitivity analysis and effectiveness of variables

Sensitivity analysis on individual limit state analysis in section 6.2.6 showed that among the two random variable, the multiplying constant (k) has higher contribution in failure function at early ages and as time passes, the contribution of exponential constant (n) become higher (Figure 6.11). Further results from sensitivity analysis illustrated in Figures 6.12 and 6.13 showed that the variability of the parameters (k and n) for low values of t, has more significant effect on the probability of failure. It is to say, the sensitivity of corrosion parameters is more dependent on the actual value of coefficient of variation in early ages. In such cases, more concern should be taken in order to determine relevant parameter values.

A comprehensive sensitivity analysis on 15 random variables affecting multi failure mode of cast iron water pipes was carried out in section 6.3.6. The results presented in Figures 6.19 and 6.20 showed that the sensitivity indexes of some of the variables are very low for all values of time. These variables include internal pressure(P), modulus of elasticity($\mathrm{E_P}$), deflection coefficient($\mathrm{K_d}$), impact factor($\mathrm{I_c}$), surface load coefficient($\mathrm{C_t}$), wheel load(F) and pipe effective length (A). This indicates that the inclusion of these as variables has little influence on the probability of failure of the pipeline. Therefore, in any future analysis, it would not be inaccurate to treat these variables as deterministic variables with constant quantities.

Among all variables, the relative contribution of the corrosion parameters (k and n) is highly remarkable. Sensitivity ratios are also considerably high for these two variables. This indicates that corrosion is very important factor for the design of underground pipelines with long lives. High values of contribution for these two variables means that the sensitivity of these variables is more dependent on the actual value of their coefficient of variation. In such cases, more concern should be taken in order to determine relevant parameter values.

From Figures 6.19 and 6.20 it is also observed that the sensitivity indexes of some other variables (e.g. wall thickness (d), bending moment coefficient (K_m), calculation coefficient (C_d) and width of ditch (B_d)) is large at early ages. However, their contribution and effectiveness decrease gradually within time and after a long elapsed time, t, the contribution of these variables to the pipe failure is very low. This suggests the relative unimportance of these variables particularly for old pipes.

Further results of sensitivity analysis on the effect of the level of variability (i.e. coefficient of variation) of each of corrosion model coefficient (i.e. k and n) on the probability of failure were shown in Figure 6.21 and 6.22. Overall, the probability of failure increases while the coefficient of variation increases. Results in these figures again confirm that the uncertainty characteristics of k and n have considerable influence on the probability of pipe failure. Therefore, as it was suggested above, extreme care should be taken in determining the values of these variables for reliability studies of cast iron pipes subject to corrosion.

The results from the application of the two new models for reliability assessment, is described in chapters 5 and 6. The following key aspects were presented in this chapter:

- Strength and weakness of first passage probability method and GDD model

- Results of reliability analysis and service life prediction of concrete sewers for individual failure assessment and for multi failure mode assessment
- Contribution and effectiveness of different variables on reliability of concrete sewers
- Results of reliability analysis and service life prediction of cast iron water pipes for individual failure assessment and for multi failure mode assessment
- Contribution and effectiveness of different variables on reliability of cast iron water pipes

In the next chapter the general conclusions and recommendations drawn from this study are presented. Suggestions for further work are given.

8 CONCLUSION AND RECOMMENDATIONS

A comprehensive discussion and analysis of the results obtained from the application of two new methods for service life prediction were given in the previous chapter. The main conclusions and recommendations for improved reliability analysis and service life prediction of buried pipes are summarised in this chapter.

It is concluded that the new insight gained through this work, will provide for enhanced reliability analysis of corrosion affected pipes, and may be applicable for other structures in general. The key outcome of this research is a more accurate and realistic basis for reliability analysis and service life prediction of pipelines.

Recommendations are given that address the need for further improvement in the field of reliability analysis of corrosion affected buried pipes.

8.1 Conclusion

This research aimed to develop and apply reliability analysis methods for the assessment and service life prediction of corrosion affected buried pipes (concrete sewers and cast iron water pipes). In this section the outcomes of the research corresponding to each of the objectives are presented briefly.

a) Understanding and investigating the design procedure, adopting models for corrosion and examining and understanding of reliability theory for pipes

- Understanding of the design procedure of buried pipes and the behaviour of pipes under various load were obtained by a comprehensive literature review in Chapter 3. The primary principle for the design of a pipeline is to ensure that both serviceability and ultimate limit states are not reached. Flexural and shear failures are two main ultimate limit states that should be considered in design and assessment. Serviceability limit states may be measured by cracking or leakage for rigid pipes.

- In this research, structural deterioration (i.e., corrosion) in buried pipes as predominant cause of failure was studied. Variables which affect the corrosion of concrete sewers and cast iron water mains were investigated and corrosion models for these types of buried pipes were presented and adopted.
- Principles of structural reliability analysis, system reliability and sensitivity analysis were studied and a comprehensive literature review on reliability analysis of pipelines was carried out. To deal with uncertainties and scarcity of monitoring data, using time dependent methods for reliability analysis and service life prediction of corrosion affected buried pipes was intended.
- The literature review in this study also revealed that there is a lack of research on multi failure mode analysis of concrete sewers and cast iron pipes. Therefore the emphasis in the current research was on multi failure mode reliability analysis of the pipes.

b) Developing methods for reliability analysis of concrete sewers and cast iron pipes

- First passage probability theory and gamma distributed degradation (GDD) model were introduced and developed to be used for reliability analysis and service life prediction of concrete sewers and cast iron water pipes. To predict the probability of failure and service life, failure due to corrosion should be formulated in a form of limit state function and a stochastic model for corrosion should be presented to consider uncertainties of basic random variables. The first passage probability method and GDD model are employed to quantify the probability of failure, so that the time for the pipe to be unusable due to excessive corrosion can be determined.

- A weak point for first passage probability method compare with GDD model is dependency to estimation of coefficient of variation of the deterioration process. This estimation is time consuming and will potentially involve errors in calculations.

- Since, in practice, for reliability analysis of corrosion affected pipes, data such as corrosion depth is not available; therefore, the GDD model was extended and developed for such cases.

- For a comprehensive reliability analysis, the effect of variables on the reliability of a pipeline can be analysed by doing an in-depth sensitivity analysis. Sensitivity indexes were introduced and adopted in this study as a tool to identify those variables that affect the pipe failure most; so that more research can focus on those variables.

c) Application of methods for reliability analysis of concrete sewers and cast iron pipes

- First passage probability and GDD model were applied for two scenarios of individual failure assessment and multi failure mode assessment for concrete sewers and cast iron water pipes in the UK. The results of applying first passage probability method showed that the method in some cases is very dependent to assumed amount of auto-correlation coefficient.

- Compared with first passage probability method, gamma distributed degradation model is a straight forward method with independency to estimate coefficient of variation and auto-correlation coefficient. Service life of a corrosion affected pipe can be predicted with high accuracy for different level of probability of failure by using gamma distributed degradation model.

- The results of this study also indicate that considering each failure mode individually in reliability analysis of pipes is not conservative, which may pose undue risks to the public and subsequent disastrous consequences. In contrast, multi failure mode analysis can provide a more realistic and accurate method for the prediction of pipe failures. Accurate prediction of the service life of existing pipes has the potential to achieve risk-cost optimized strategy for the management of pipe asset.

- The results of a parametric sensitivity analysis can be used to assess how change in design parameters such as concrete cover of concrete sewers or toughness of cast iron pipes can affect the service life of the pipeline. Reliability indexes (relative contribution (α_x^2) and sensitivity ratios) were also estimated for both the concrete sewer case study and the cast iron water pipes case studies in the UK.

- For concrete sewers, the relative contributions of dissolved sulfide concentration, the ratio of surface width of the stream to the perimeter of the exposed wall and alkalinity ([DS], $b/\acute{P}$ and A), were highly remarkable. It was also observed that the variability of these three random variables for high values of time, has more significant effect on the reliability index. Thus, the sensitivity of these variables is more dependent on the actual value of the coefficient of variation for higher values of time.

- For a cast iron water pipe, the probability of the pipe failure increases with the increase of the diameter of the pipes for both external and internal corrosion and that for a given diameter, the probability of pipe failure for pipes with external corrosion is much higher than that for pipes with internal corrosion. It has also been found that the tougher the pipe, the smaller the probability of its failure. Among all variables, the relative contribution of the corrosion parameters (k and n) was highly remarkable. High values of contribution for these two variables means that the sensitivity of these variables is more dependent on the actual value of their coefficient of variation. Therefore, more concern should be taken in order to determine the values for the corrosion parameters (k and n).

The achievements of this research and its contribution to knowledge are summarised in Figure 8.1 (the original contribution of the research has been defined in bold text).

The results of this research have been disseminated in the form of 6 journal papers and 4 conference papers (as listed in Appendix 2), illustrating the importance of this work to practitioners and regulators of corrosion affected buried infrastructure, or to corrosion affected structures as a whole.

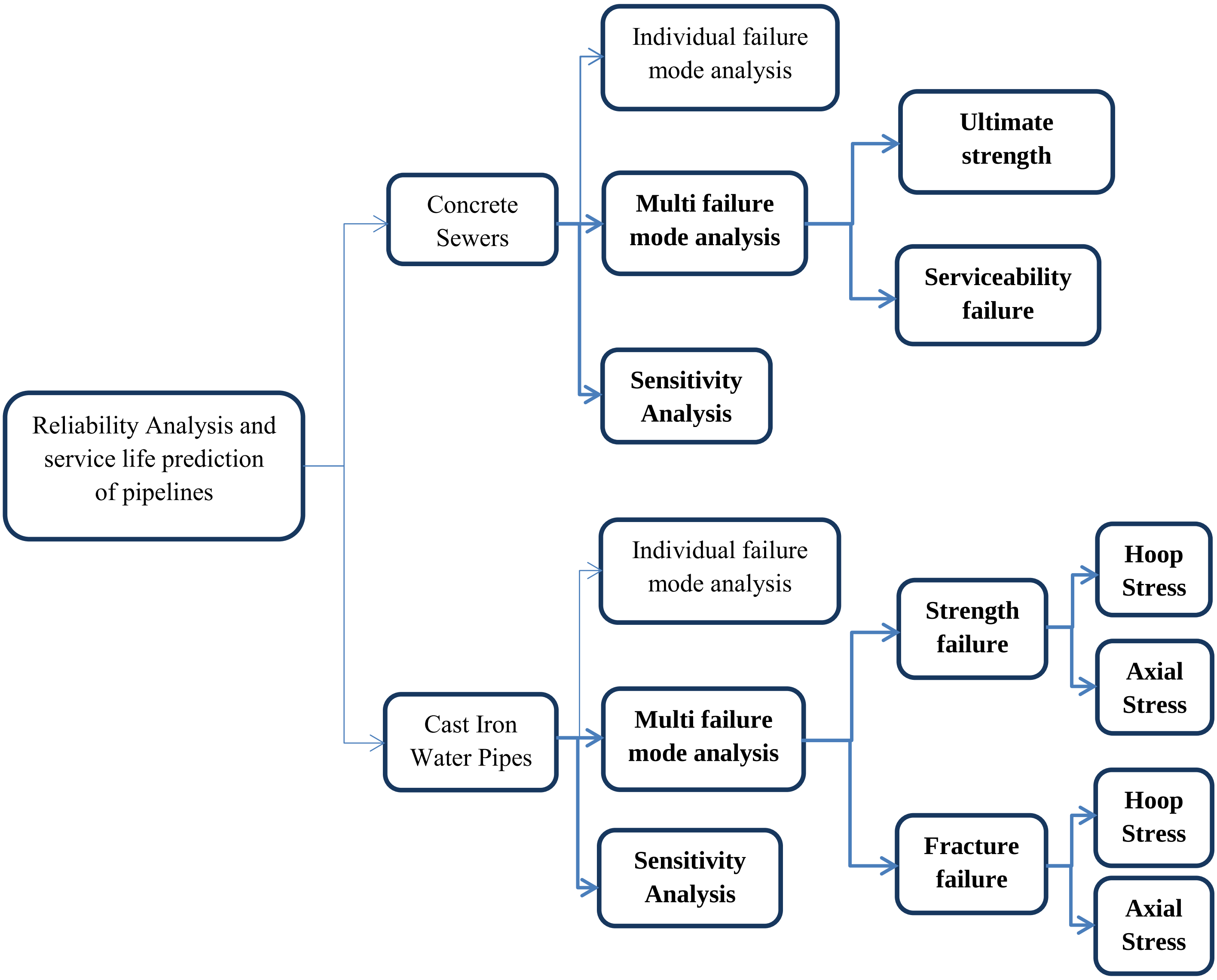


Figure 8.1Contribution of this research in reliability analysis and service life prediction of pipelines

(The original contribution of the research is defined in bold text)

8.2 Recommendations for further research

With the trend of deregulation of water and wastewater industry in developed countries, the onus would now fall on the industry to prove the safety and reliability of existing pipe networks. The proposed methods in this research would improve the ability of the water and wastewater industry in predicting and preventing catastrophic failures of the water and sewer pipes.

Between the two developed methods in this research, the first passage probability method was challenging. In addition to the mathematical complexity of the first passage probability method, lack of data about auto correlation coefficient makes this procedure an unreliable method for practical reliability analysis problems. Instead, GDD model showed a good capability for modeling the monotonic progression of the deterioration processes in buried pipes (i.e. corrosion).

For the first passage probability method to be of practical use, having information about the auto-correlation coefficient is necessary. This can not be achieved without a considerable amount of monitoring data about the corrosion depth increment within the time. Therefore it is suggested to propose further research for monitoring corrosion depth measurements for a period of time (for instance 10 years) from concrete sewers and/or cast iron water pipes. The result of this research could give a sufficient estimate of auto-correlation coefficient. The results can also be used for calibration and verification of the other time dependent reliability analysis methods (such as gamma distributed degradation model).

For further works in the field of failure assessment of buried pipes, it is suggested to model the pipeline and existing loads in a finite element software (such as Abaqus or ANSYS). Then the model can be coupled to the stochastic reliability analysis codes produced in this research. This can give a more accurate and extensive stress analysis in 3D planes compare to the equations used in Tables 5.2 and/or 6.3.

In this research, the progression of corrosion depth in concrete sewers was assumed linear with respect to time (Equation 3.12). The corrosion rate remains unlikely to be linear as environmental condition (Wastewater properties, pH and temperature) and concrete properties continue to evolve. The prediction of service life will be much more accurate if the non-linearity of corrosion process become clear. To obtain the realistic relationship between the corrosion depth and time in a concrete sewer, an extensive lab experiment is suggested. In the suggested experiment, concrete pipes are exposed to an accelerated corrosive environment. The corrosion depth in the concrete pipe sample is measured periodically and the monitoring data can be used for exploring the relationship between corrosion depth and time.

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APPENDIX 1- Codes and programming

A1.1Multi Failure Mode Reliability Analysis for Concrete Sewer

```
% system reliability analysis for 4 limit state functions (bending, shear,
% crack, cover loss)
%bP(ii) is b/P' and bPSD is its standard deviation,
        %for h/D=0.2==>>b/P'=0.36 and bPSD=0.072
        %for h/D=0.4==>>b/P'=0.55 and bPSD=0.11
        %for h/D=0.6==>>b/P'=0.71 and bPSD=0.14
        % revised 24 Feb 2012
clear

bP(1)=0.36;bP(2)=0.55;bP(3)=0.99;bPSD(1)=0.072;bPSD(2)=0.11;bPSD(3)=0.14;
%d is distance from compression face to centroid of tension
reinforcement(mm)
d=60;
% h overall thickness of member (wall thickness), (mm)
h=102;
db=12;   % diameter of rebar in inner cage, mm
%Ms= Service load bending moment acting on length b, (Nmm/m)
%Ns= Axial thrust acting on length b, service load condition (+ when
compressive, - when tensile), (N/m)
Ms=6992511/2;
Ns=16043;
Mu=0.95*(6992511/2);%  bending moment in job site
Msite=0.95*(6992511/2);% maximum bending moment in job site
Nu=0.9*16043;
Vsite=14219.5; % from file loads.xls, calculated from page 30 of ASCE 15-98
fy=270; %yeild strength of reinforcement (MPa)
tb=25;Sl=100; % tb: clear cover over reinforcement (mm), Sl: spacing of
circumferential reinforcement (mm)
B1=(25.4*tb*Sl/4)^(1/3);
Phif=0.95;% Phif is strength reduction factor for flexure, acording to
ASCE15-98 page 8 is 0.95
C1=1; % C1 is crack control coefficient for type of reinforcement, page 12,
ASCE15-98 (2000)
b=1000;
fc=27.6;
r=395; %radius to the centerline of pipe wall, mm
As=360; % As is the area of tension reinforcemnet in unit length, (mm2)
a=(fy*As+Nu)/(0.85*b*fc);
Phiv=0.9;Fvp=1;Fcr=1;
%%%%%%%%%%%%%%%%%
Ro=As/(b*d);
if (Ro>0.02)
    Ro=0.02;
end
%%%%%%%%%%%%%%%%%
Fd=0.8+(41/d);
if (Fd>1.3)
    Fd=1.3;
end
%%%%%%%%%%%%%%%%%%%%
Fc=1+(d/(2*r));
 for ii=3:3;
N=1000;
lgcl1=0;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
for t=1:200;
    lgcl1=0;     lgcl2=0;     lgcl3=0;  lgcl4=0;
    for i=1:N;
        for j=1:6;
            u(j)=rand;
            x(j)=norminv(u(j));
        end
        %calculation of basic random variables:

      k(i)=0.9+x(1)*0.16;
      A(i)=0.22+x(2)*0.07;
      j(i)=0.30+x(3)*0.04;
      DS(i)=2+x(4)*0.5;
      v(i)=0.7+x(5)*0.12;
      bp(i)=bP(ii)+x(6)*bPSD(ii)*0;

        %calculation of Phi (hydorgen sulfide flux to pipe surface)
        %the pipe slope= 0.0015
        Phi(i)=0.7*((0.0015*v(i))^(3/8))*j(i)*DS(i)*bp(i);
        % calculation of rate of corrosion c(i):
        c(t,i)=11.5*k(i)*Phi(i)*(1/A(i));
        Lossh(t,i)=h-c(t,i)*t;
         if (Lossh(t,i)>0)
    %%%%%%%%%%%%%%%%%%%%%%%%%%%BENDING%%%%%%%%%%%%%%%%%%%%%%%%%
        Mu(i)=As*fy*(d-(a/2))+Nu*((Lossh(t,i)-a)/2);
        if Mu(i)<0
            Mu(i)=0;
        end

        %%%%%%%%%%%%%%%%%%%%%%%%%%%SHEAR%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
       Fn(t,i)=1-(Nu/(3.5*b*Lossh(t,i)*t));

Vb(t,i)=0.083*b*Phiv*d*Fvp*(fc^0.5)*(1.1+(63*As)/(b*d))*((Fd*Fn(t,i))/Fc);
       if Vb(t,i)<0
           Vb(t,i)=0;
       end

       %%%%%%%%%%%%%%%%%%%%%%%%%%%%% Crack Control %%%%%%%%%%%%%%%%%%%%%%%%%%
        e(i)=(Ms/Ns)+d-(Lossh(t,i)/2);
% maximum for jjj(i) is 0.9
         jjj(i)=0.74+((2.54*e(i))/d);
         if (jjj(i)>0.9)
             jjj(i)=0.9;
         end
        iii(i)=1/(1-((jjj(i)*d)/e(i)));
        AA(i)=B1/(5250*Phif*d*As);
        BB(i)=(Ms+(Ns*(d-(Lossh(t,i))/2)))/(iii(i)*jjj(i));
        CC(i)=0.083*C1*b*((Lossh(t,i))^2)*(fc^0.5);
        F(i)=AA(i)*(BB(i)-CC(i));
        if F(i)<0
            F(i)=0;
        end
       %%%%%%%%%%%%%%%%%%%% cover theckness on reinforcement
%%%%%%%%%%%%%%%%%
      % cover(i)= Lossh(t,i)-d-(db/2);
       %if cover(i)<0
        %   cover(i)=0;
      % end
      cover(i)= c(t,i)*t;
```

```
%       *****************check for failure********************

       lgcl1=(Vb(t,i)<=Vsite)+lgcl1;
        lgcl2=(Mu(i)<=Msite)+lgcl2;
        lgcl3=(F(i)>=Fcr)+lgcl3;
        lgcl4=(cover(i)>=tb)+lgcl4;
      else
           lgcl1=1+lgcl1;    lgcl2=1+lgcl2; lgcl3=1+lgcl3;   lgcl4=1+lgcl4;
       end
  end
%calculation and plotting of probability of failure in time t
  Pf1(t)=lgcl1/N;    Pf2(t)=lgcl2/N;    Pf3(t)=lgcl3/N;     Pf4(t)=lgcl4/N;
  Pf(t)=1-(1-Pf1(t))*(1-Pf2(t))*(1-Pf3(t)*Pf4(t));  % considering crack and
cover are paralell together and totally series with bending and shear
    %Pf(t)=1-(1-Pf4(t))*(1-Pf2(t))*(1-Pf3(t))*(1-Pf1(t));
  end
hold on
end
T=1:200;
  plot(T,Pf(T))
  hold on
```

A1.2Multi Failure Mode Reliability Analysis for Cast Iron Water Pipe

```
% using Monte Carlo simulation to calculate probability of system failure
% limit state functions are strength<stress and KI>Kc means the stresses
should be more than strength and stress intensity factor should be more
than toughness to failure occur.
%when a cast iron pipe subjected to corrosion (external corrosion)
% written 29 June 2012
ffrost=0; % 0 or 1
%thickness is random therefore:

N=1000;

for t=1:60;
    lgcl1=0;
    lgcl2=0;
    lgcl3=0;
    lgcl4=0;
    for i=1:N;
% ****************(1), calculation of corrosion depth*********************
            u=rand;
            x1=norminv(u);

                Kconst(i)=2.54+x1*0.5;

            u=rand;
            x2=norminv(u);

        nconst(i)=0.32+x2*0.06;

        d(i)=Kconst(i)*t^nconst(i);  % condition 0<d(i)<wthickness, after
calculation of wthickness it is checked
        if d(i)<0
            d(i)=0;
        end
```

```
  % ***************(2-1), calculation of hoop stress**********************
            u=rand;
            x1=norminv(u);
%% to consider max and min of p=internal pressure,  max= 1.1, min =0.3
% therefore x1 should be greater than -2and less than 2
        while  (x1>2)|(x1<-2)
            u=rand;
            x1=norminv(u);
        end
                pinternal(i)=0.7+x1*0.2;
%%to consider max and min of D=internal diametere, max= 260, min=240
% therefore x2 should be greater than -0.98 and less than 0.42
            u=rand;
            x2=norminv(u);
        while  (x2>0.29)|(x2<-0.87)
            u=rand;
            x2=norminv(u);
        end
        D(i)=305+x2*17.14;
u=rand;
x3=norminv(u);
wthickness(i)=17.52+x3*0.7;
if d(i)>wthickness(i)
    d(i)=wthickness(i);
end
u=rand;
x4=norminv(u);
Km(i)=0.235+x4*0.04;    %Bending moment coefficient
u=rand;
x5=norminv(u);
Ep(i)=105000+x5*15000;      %modulus of elasticity of pipes
u=rand;
x6=norminv(u);
Kd(i)=0.108+x6*0.02;      %Deflection coefficient
u=rand;
x7=norminv(u);
Ic(i)=1.25+x7*0.20;           % Impact factor
u=rand;
x8=norminv(u);
Ct(i)=0.12+x8*0.025;            % surface load coefficient
%%  to consider max and min of F= wheel load of trafic,  max= 100000,
min=30000
% therefore x9 should be greater than -1 and less than 2.5
 u=rand;
 x9=norminv(u);
 while  (x9>2.5)|(x9<-1)
   u=rand;
   x9=norminv(u);
 end
  F(i)=50000+x9*20000;
u=rand;
x10=norminv(u);
A(i)=6500+x10*200;            % pipe effective length
u=rand;
x11=norminv(u);
Gama(i)=(18.2/1000000)+x11*(18.2/10000000);            % unit weight of
soil
u=rand;
x12=norminv(u);
Cd(i)=1.32+x12*0.25;            % calculation coefficient
```

```
u=rand;
x13=norminv(u);
Bd(i)=625+x13*125;            % width of ditch

        newthickness(i)=wthickness(i)-d(i);
        if d(i)==wthickness(i)
        SigmaF(i)=0;
        else
        SigmaF(i)=(pinternal(i)*D(i))/(2*newthickness(i));
        end

SigmaV(i)=(3*Km(i)*Ic(i)*Ct(i)*F(i)*Ep(i)*newthickness(i)*D(i))/(A(i)*(Ep(i
)*(newthickness(i)^3)+3*Kd(i)*pinternal(i)*D(i)^3));

SigmaS(i)=(3*Km(i)*Gama(i)*Bd(i)*Bd(i)*Cd(i)*Ep(i)*newthickness(i)*D(i))/(E
p(i)*newthickness(i)^3+3*Kd(i)*pinternal(i)*D(i)^3);
        Sigmahoop(i)=SigmaF(i)+SigmaV(i)+(1+ffrost)*SigmaS(i);

 % ***************(2-2), calculation of Axial Stress*********************
 Alfap=11/1000000;   %thermal coefficent of the pipe
 DeltaT=-10;     %min=-10, max=0,  Twater-Tground
 vp=0.21;  %poisson  ratio of pipe material
 SigmaT(i)=-Ep(i)*Alfap*DeltaT;
 if d(i)==wthickness(i)
    SigmaFprim(i)=0;
else
  SigmaFprim(i)=0.5*pinternal(i)*((D(i)/newthickness(i))-1)*vp;
 end
 SigmaAxial(i)=SigmaT(i)+SigmaFprim(i)+(SigmaV(i)+(1+ffrost)*SigmaS(i))*vp;

% ***************(2-3), calculation of maximum Stress*********************
           % Stress(i)=max(SigmaAxial(i),Sigmahoop(i));
          %  Stress(i)=(SigmaAxial(i)^2+Sigmahoop(i)^2)^0.5;

% ***************(3), calculation of  stength*********************

 % yeild strength of cast iron is 137 Mpa acording to ASTM A-48

Strength(i)=135;

% ************* calculation of stress intensity factor  KI-hoop***********
at(i)=d(i)/wthickness(i); % is the ratio of a/t , corrosion depth over wall
thickness
u=rand;
ca(i)=4*u+1;        % is the ratio of c/a , corrosion length over half
corrosion depth
Rt(i)=(D(i)/2)/wthickness(i);   % is the ratio of R/t , internal radious
over wall thickness
% fi from page AI.21 Laham 1998
fi(i)=0.076*at(i)^2+0.0125*at(i)+0.6554;
KIaxial(i)=(1/31.62)*1.772*Sigmahoop(i)*fi(i)*d(i)^0.5;    %  31.62 is
multiplied to change the dimension to MPa.m^0.5

% ************ calculation of stress intensity factor  KI-axial***********
at(i)=d(i)/wthickness(i); % is the ratio of a/t , corrosion depth over wall
thickness
u=rand;
ca(i)=4*u+1;        % is the ratio of c/a , corrosion length over half
corrosion depth
```

```
Rt(i)=(D(i)/2)/wthickness(i);   % is the ratio of R/t , internal radious
over wall thickness
% fi and fbg from page AI.33 Laham 1998
fi(i)=0.05*at(i)+.6564;
fbg(i)=-0.2188*at(i)^3+.268*at(i)^2-0.073*at(i)+0.6589;
KIhoop(i)=(1/31.62)*1.772*d(i)^0.5*((Sigmahoop(i)*fi(i))+(SigmaAxial(i)*fbg
(i)));    %  31.62 is multiplied to change the dimension to MPa.m^0.5

Kq=15;    % fracture toughness, Kq= 0.086*d^2-2.7d+29.3  MPa.m^0.5


    % *************(4), Checking limit state function**********************
lgcl1=(Sigmahoop(i)>Strength(i))+lgcl1;
lgcl2=(SigmaAxial(i)>Strength(i))+lgcl2;
lgcl3=(KIaxial(i)>Kq)+lgcl3;
lgcl4=(KIhoop(i)>Kq)+lgcl4;

    end
      Pf1(t)=lgcl1/N;  % for stress-strength limit state-hoop
      Pf2(t)=lgcl2/N;   %  for stress-strength limit state-Axial
     Pf3(t)=lgcl3/N;   % for stress intensity factor - toughness limit
state-hoop
          Pf4(t)=lgcl4/N;   % for stress intensity factor - toughness limit
state-Axial
Pf(t)=1-(1-Pf1(t))*(1-Pf2(t))*(1-Pf3(t))*(1-Pf4(t));
end

for t=1:60
  plot(t,Pf(t),'p')
  plot(t,Pf1(t),'r')
     plot(t,Pf2(t),'g')
       plot(t,Pf3(t),'y')
       plot(t,Pf4(t),'b')
%plot(t,Proabability(t))
  hold on
end
```

APPENDIX 2-List of Publications

a) Journal papers:

1- Mahmoodian, M. and Alani, A., (2013), Modelling deterioration in concrete pipes as a stochastic gamma process for time dependent reliability analysis, ASCE, Journal of pipeline systems engineering and practice. *http://ascelibrary.org/doi/abs/10.1061/%28ASCE%29PS.1949-1204.0000145*

2- Mahmoodian, M. and Alani, A., (2013), Multi failure mode assessment of buried concrete pipes subjected to time dependent deterioration using system reliability analysis, accepted for publication, *Journal of failure analysis and prevention*

3- Alani, A., Faramarzi, A., Mahmoodian, M., and Tee K. F., (2013), Prediction of sulphide build-up in filled sewer pipes, Under review *Journal of Environmental Technology*

4- Alani, A., and Mahmoodian, M., (2013), Sensitivity analysis for reliability assessment of concrete pipes subjected to sulphide corrosion, Under review, *Journal of Urban Water*

5- Mahmoodian, M. and Alani, A., (2013), A gamma distributed degradation rate (GDDR) model for time dependent structural reliability analysis of concrete pipes subject to sulphide corrosion, under review, *International Journal of Reliability and Safety*

6- Mahmoodian, M., Alani, A. and Tee K. F., (2012), Stochastic failure analysis of the gusset plates in the Mississippi River Bridge, *International Journal of Forensic Engineering*, Vol.1, No.2, pp.153 - 166

b) Conference papers:

1- Mahmoodian, M. and Alani, A., (2013), Time dependent reliability analysis of underground concrete pipes subjected to sulphide corrosion, 11th International Conference on Structural Safety & Reliability, June 2013, USA

2- Mahmoodian, M. and Li, C. Q. (2011), Structural System Reliability Analysis of Cast Iron Water Mains, 2nd Iranian Conference on Reliability Engineering, 24-26 October, Tehran, Iran.

3- Mahmoodian, M. and Li, C. Q. (2011), Service life prediction of underground concrete pipes subjected to corrosion, 4th International Conference on Concrete Repair, 26-28 September, Dresden, Germany.

4- Tee, K. F., Li, C. Q. and Mahmoodian, M. (2011), Prediction of Time-variant Probability of Failure for Concrete Sewer Pipes, International conference on durability of building material and components, 12-15 April, Porto, Portugal.

APPENDIX 3- Published Works