Using history in mathematics teaching – some open education resources for the future

Noel-Ann Bradshaw[1], Mark McCartney[2] and Tony Mann[1]

[1]School of Computing and Mathematical Sciences, University of Greenwich and [2]School of Computing and Mathematics, University of Ulster

Abstract

A recent collaborative project produced both audio- and text-based resources on the history of mathematics to support the teaching of mathematics. The benefits of using the history of mathematics as a motivational pedagogic tool in the undergraduate curriculum are discussed, with specific reference to the use of MP3 formats as a technique to support different learning styles and future modes of learning. The deliverables produced by the project are discussed.

1. Background: history and teaching mathematics

Mathematics is usually, and of course correctly, presented "ready-made" to students, with techniques and applications presented systematically and in logical order. However, like any other academic subject, mathematics has a history which is rich in astonishing breakthroughs, false starts, mis-attributions confusions and dead ends. This history gives a narrative and human context which adds colour and context to the discipline.

While some students may be attracted to mathematics by the apparently impersonal nature of the subject, the experience of the authors is that many students are engaged by historical information. Learning about the history of the subject shows that it is living and changing rather than being a fixed set of results known forever. Understanding that great mathematicians of the past struggled and sometimes made mistakes helps learners appreciate that errors are part of doing mathematics. For some, the human stories make mathematics exciting. For the potential research mathematician, becoming a member of this community includes buying into the stories mathematicians tell about the history of their subject. The fact that some of these stories are mythical makes them no less important in a shared mathematical culture [1].

The authors have noted in their own teaching that motivating students to the study of a new topic in mathematics can often be helped by setting the subject in a historical context. Indeed, it has been shown by Hagerty *et al* [2] that "the inclusion of historical modules caused positive changes in mathematical communication, student achievement and attitudes". This echoes the views of many other academics and educators at all levels from primary to higher education and teacher training [3],[4],[5],[6],[7]. In England the National Curriculum for Key Stage 3 Mathematics includes "the rich historical and cultural roots of mathematics" [8].

The authors' experience of teaching first year undergraduates is that presenting brief accounts of the history of the subject can be a valuable tool for another reason. Students start mathematics degrees at Universities like Greenwich with very diverse mathematical backgrounds: their previous study may or may not have included A (or AS) level Further Mathematics, they may instead have taken the International Baccalaureate, they may have studied Scottish qualifications or they may be international students with a completely different background. For mature students, it may be some years since they last studied mathematics. Similarly, for engineering students at the University of

Ulster, there can be a wide range of mathematical knowledge and ability amongst freshers, with some students being surprised and concerned at having to study more mathematics. Inevitably much first year material will be new to some students but very familiar to others, and presenting the historical background to the subject can provide new *and* stimulating material for those who are not being stretched by the technical content, without intimidating those for whom the main material is either new or barely remembered from distant schooldays.

Setting historical context can motivate and enthuse learners, but it also enriches the curriculum, shows connections between different branches of the subject, and helps to produce students with a greater sense of the breadth and, what might be termed, the creative life of mathematics as a discipline. We believe that many mathematics lecturers in higher education would like to include historical elements in their courses but lack either the subject knowledge, the time to prepare such material, or in some cases the confidence to engage with history.

There are, of course, many valuable general resources on the history of mathematics, such as book-length histories of mathematics ([9], [10], [11]), source books ([12], [13]) and reference works [14]. On the web the MacTutor history of mathematics website created by E. F. Robertson and J.J. O'Connor is widely used by academics and students [15]. Resources like these provide a great deal of information but we feel that, for lecturers seeking to provide snippets of historical material while presenting mathematics to their students, there is a need for "bite-sized" resources specifically relating to the undergraduate curriculum. The MacTutor article on Newton, for example, runs to six close-packed pages, which would not easily fit into a lecture on calculus. As van Brummelen notes, "school teachers and educational associations demand easily digested 'sound bites' that may be inserted with little fuss into an existing curriculum" [16], and the requirements of those teaching mathematics to undergraduates are similar.

Increasingly, there are opportunities to exploit students' familiarity with multimedia technology for learning purposes. Today's students are used to accessing material in audio format – for many, their headphones are always in place (even when they are following lectures!). Podcasts such as Math/Maths [17] and Travels in a Mathematical World [18] have gathered a following amongst undergraduate mathematicians. One member of the project team (Bradshaw) provided a series of history of mathematics podcasts for the latter, which were favourably received by students. Students already use their mobile devices to support their learning in sophisticated and imaginative ways, and the importance of resources for mobile devices can only increase as these become ever more powerful and flexible.

2. Methodology

These considerations motivated the authors to initiate a project, which was generously supported by the HEA MSOR Network with a mini-project grant, to produce resources to help lecturers incorporate history of mathematics into their teaching of mathematics. The overall aim of the project was the creation of a set of short "stand-alone" two-page documents together with MP3 audio files on a range of topics from the history of mathematics to supplement, and help motivate the teaching of, mathematical topics covered in the undergraduate curriculum.

By undergraduate curriculum we mean both mathematics degree programmes, and relevant service teaching. We have provided resources to facilitate the inclusion of historical material in modules covering mathematical methods and ideas in both mathematics degree programmes and service teaching. The materials are not designed for use by academics teaching modules on mathematics degrees which are specifically on the history of mathematics, where a more complex analysis of the issues would be required. (There are around 20 such modules embedded within mathematics degree programmes in British universities.)

We use the term "reusable learning objects" (RLOs) to describe these documents and MP3 files as each is relatively brief, self contained and independent of each other, providing an efficient way for teachers to embed the materials into their lecture courses and supporting teaching websites.

The material is designed to support the teaching of mathematics: it is not intended for historians. While the material aims to be historically accurate, we aim to present simple accounts that are entertaining, readable, and put the mathematics our students are learning into context. They are not the place for detailed consideration of sources or careful analysis of opposing views on controversial topics. We hope that interested students may be led to more detailed historical sources and that some (like the authors) will develop a fascination for the history of mathematics which they will pursue in more rigorous historical ways. But the purpose of these RLOs is to help students learn mathematics, and the history has to be presented with that end in view.

As part of the project the project team organised a one-day event run jointly under the auspices of the MSOR Network and the British Society for the History of Mathematics (BSHM). Entitled *The History of Mathematics in the Undergraduate Curriculum*, this event took place at the University of Greenwich on the 30th March 2010. Amongst those attending were recognised leaders in the field of history of mathematics and academics who teach the subject, or use history in teaching mathematics, at various universities in the UK and Ireland. This conference offered an overview of how the history of mathematics can be used, and identified some of the opportunities and issues. The feedback from like-minded practitioners was encouraging and had a strong influence on the development of the resources.

3. The RLOs: Benefits and Issues

Twenty RLOs have been produced, all of which are available on the project web pages [19]. These are "branded" with a logo designed by a mathematics undergraduate at Greenwich, Adam Sebestyen (Figure 1). They aim to provide support for teaching many different parts of the undergraduate curriculum. Topics such as "Mathematical Notation", and "π" present general background. "Archimedes" and "Islamic Mathematics" present mathematicians of different cultures. Core material is addressed by "Complex Numbers" and "Newton" and "Maclaurin and Taylor" support the teaching of calculus. "Euler", "Fourier" and "Laplace" present overviews of mathematicians whose names will occur during a mathematics degree in many different contexts. "Non-Euclidean Geometry", "Florence Nightingale", "Gödel's Incompleteness Theorems", "Galois", "Einstein and Relativity" and "The Schrödinger Equation" cover more specialist topics. The misconception that mathematics was all done in the past is addressed by "The Logistic Map", showing the recent development of Chaos Theory.

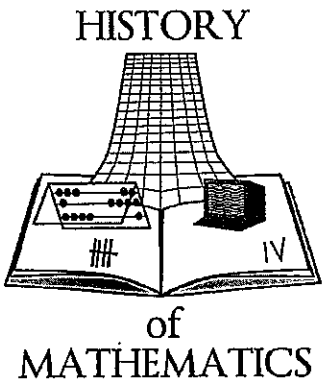

Figure 1: Project Logo

Our recent experience is that some mathematics undergraduates are hostile to the computing elements of the curriculum. We are often asked by students, whose experience of computing in school has not enthused them, why they need to acquire these skills. Our RLOs "From Beans to Bytes" and "Maskelyne and Comrie" attempt to address these negative attitudes by showing the long history of mathematicians turning to automatic calculation.

The text files are made available to be used in different ways. They provide a ready-made resource for a tutor to provide as background material for interested students. They provide a brief summary of historical context for a tutor who might adapt the contents in their own lecture material. Alternatively they could be cut and pasted directly into lecture notes.

The material is also available in audio format as MP3 files. These can be downloaded by students who might wish to listen to the background material on their journey to or from University, or while they are working at their computer. Each audio file lasts between 6 and 8 minutes, which the authors consider to be a length which

students will find they can use in various ways – as a quick break from doing mathematics, or to fill a bus journey or a walk across campus. The MP3 and text files also address students' different learning styles.

The authors considered providing Powerpoint slides as an additional resource for lecturers. Our conclusion was that this was impractical for a number of reasons. Powerpoint is by no means universally used as a tool for delivering mathematical content, so such a resource would be of limited value. Furthermore, different lecturers use Powerpoint in very different ways so it is unlikely that resources could be "dropped into" lecture material without considerable adaptation, which would defeat the purpose of the RLOs.

A number of issues arise regarding the RLOs. That there is no standard curriculum for undergraduate mathematics programmes obviously presents difficulty for a project like this. However there is a large amount of core material that will be covered on any mathematics degree – calculus and complex numbers, for example. Other RLOs may not be relevant for some undergraduates – for example, statistics and computing are core in some programmes but some undergraduates, particularly those on combined degrees, may not study these topics. While the RLOs on calculus and complex numbers will be relevant for engineering students learning mathematics, they will be less meaningful for business students, but the situation will be reversed for the statistics and computing material. Clearly we cannot expect that every RLO produced will be useful for every student, but the diversity of topics covered should mean that some are relevant.

The subjects are predominantly white, Western and male. While we have attempted to address this deficiency, only the RLOs on π, Fibonacci, mathematical notation and Islamic Mathematics feature mathematics from outside the Western tradition, and Florence Nightingale is the only female mathematician featured. Sadly, we feel that this lack of diversity reflects the mathematics curriculum, which these resources are intended to support. Future additions might include figures like Emmy Noether and Ramanujan, who are important figures in mathematical culture but whose mathematics may not be encountered by most undergraduates.

While we have tried to cater for mobile devices by creating audio files as well as texts, we have not attempted to produce full multimedia presentations incorporating large amounts of visual images or video. These would be very time-consuming to produce, whether the visual elements were created for the purpose or were sourced from existing material, and would present copyright issues. (The images used to illustrate the text files are all out of copyright and sourced from Wikimedia Commons [20] to avoid these complications.)

The question of dissemination is vital, since the intention of the project is that these resources should be widely available. They are being promoted through conferences such as that at Greenwich mentioned above and the annual 2010 MSOR Network conference. The authors are also promoting them through professional bodies [21] and by posting to the MATHSHEADS email list.

4. Evidence of Success

Feedback from undergraduates who have used early versions of these materials has been very positive. Comments include:

- *"I want to do maths because I am inspired by knowing about the people behind it"*- First year undergraduate 2010. Module: Mathematical Technology and Thinking, B.Sc. Mathematics, Greenwich.
- *"A quick and easy educational tool."* First year undergraduate 2011. Module: Mathematical Technology and Thinking, B.Sc. Mathematics, Greenwich.
- Listed as one of the three best things about the module: *"The extra information about people like Newton."* - First year undergraduate 2010. Module: Introductory Mathematics, B.Sc. Civil Engineering, Ulster.

We note that in all these cases the comments were made as part of the overall module evaluation process: the students were not specifically prompted for their views on the history of mathematics materials which had been

embedded in the lecture notes. The fact that students drew positive attention to the materials 'unprovoked', as it were, is itself encouraging. Further feedback will be obtained in future from students and tutors, and a hyperlink has been added to the project web pages inviting feedback from users.

5. Reflections

This project has attempted to address the needs of future learners of mathematics. While we will not know for some time the extent of the uptake for our resources, the work has identified a number of issues about teaching mathematics in the future. One is the need to present learning materials in a variety of formats to take advantage of new technologies, and the changes in learning patterns resulting from the use of mobile devices. These will present challenges for those teaching mathematics in higher education. A second is the way in which these technologies facilitate the use of material like this to motivate the study of mathematics, and the possibility that different students will benefit from different motivating material. In the brave new world of UK higher education post-2012, the need to maintain the enthusiasm of learners will be paramount. Resources which complement traditional curriculum material offer an opportunity to do this. For some learners these resources might relate to the history of the subject; for others, resources connecting the course material to applications, showing how it can be used in the workplace, or connecting it to other branches of mathematics, might be more motivational. This project may suggest a way forward which has wider application than the historical context of mathematics.

References

1. Alexander, Amir (2010) *Duel at Dawn: Heroes, Martyrs, and the Rise of Modern Mathematics.* Cambridge, Mass.: Harvard.

2. Hagerty, G.W., S. Smith, and D. Goodwin (2007), "The Unique Effects of Including History in College Algebra." *Convergence: Where Mathematics, History and Teaching Interact.* USA: MAA.

3. Katz, Victor J. (ed) (2000) *Using History to Teach Mathematics – An International Perspective.* USA: MAA.

4. Clark, Kathleen M. (2010) "Connecting local history, ancient history, and mathematics: the Eustis Elementary School pilot project." *BSHM Bulletin: Journal of the British Society for the History of Mathematics* 25(3), 132-143.

5. Musto, Garrod (2010) "Mathematical Timelines." *BSHM Bulletin: Journal of the British Society for the History of Mathematics*, 25(3), 162-167.

6. Furinghetti, Fulvia (2007) "Teacher education through the history of mathematics." *Educational Studies in Mathematics*, 66(2) 131-143.

7. Kourkoulos, Michael, and Constantinos Tzanakis (2010). "History, and students' understanding of variance in statistics." *BSHM Bulletin: Journal of the British Society for the History of Mathematics*, 25(3), 168-178.

8. Qualifications and Curriculum Authority. "Mathematics: Programme of Study for Key Stage 3 and Attainment Targets." *Qualifications and Curriculum Development Agency.* 2007. Accessed via http://curriculum.qcda.gov.uk/uploads/QCA-07-3338-p_Maths_3_tcm8-403.pdf (28th January 2011).

9. Katz, Victor J. (2008) *The History of Mathematics: an Introduction.* Oxford: Pearson, 3rd edition.

10. Hodgkin, Luke (2005) *A History of Mathematics: From Mesopotamia to Modernity.* Oxford: Oxford University Press.

11. Suzuki, Jeff (2009) *Mathematics in Historical Context.* USA: MAA.

12. Fauvel, John and Jeremy J. Gray (1987) *The History of Mathematics: A Reader. London*: Palgrave Macmillan.

13. Stedall, Jacqueline (2008) *Mathematics Emerging: A Sourcebook 1540 – 1900.* Oxford: Oxford University Press.

14. Grattan-Guinness, Ivor (1997) *The Fontana History of the Mathematical Sciences.* London: Fontana.

15. O'Connor, John J. and Edmund F. Robertson, The MacTutor History of Mathematics Archive. Accessed via http://www-history.mcs.st-and.ac.uk/ (28th January 2011).

16. Van Brummelen, Glen (2010) "Filling in the short blanks: musings on bringing the historiography of mathematics to the classroom", *BSHM Bulletin: Journal of the British Society for the History of Mathematics*, 25(1), 2 – 9.

17. Hansen, Samuel and Peter Rowlett, *Pulse-Project Math/Maths Podcast: 5136 miles of mathematics.* Accessed via http://www.pulse-project.org/pulsemathsmaths/ (28th January 2011).

18. Rowlett, Peter (ed), *Travels in a Mathematical World* (podcast). Accessed via http://www.travelsinamathematicalworld.co.uk/ (28th January 2011).

19. Bradshaw, Noel-Ann, Mark McCartney and Tony Mann, *'Bite-sized' resources for using the history of mathematics in mathematics teaching.* Accessed via http://www.infj.ulst.ac.uk/~mmccart/hom.htm (28th January 2011).

20. Wikimedia Commons Accessed via http://commons.wikimedia.org/wiki/Main_Page. (28th January 2011).

21. Bradshaw, Noel-Ann (2010), "Communicating History of Mathematics", *Mathematics Today* 46(5), 249.

Acknowledgements

The authors would like to thank the staff of the MSOR Network for their support, encouragement and guidance for this project. We are grateful to many students at Greenwich and Ulster whose comments and enthusiasm have inspired and encouraged this work. Finally, thanks to Adam Sebestyen for designing the project logo.