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Multivariate range-based EGARCH models

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The dynamic conditional correlation (DCC) and co-range models are two main frameworks used to incorporate range-based univariate volatility. Using the two approaches, we construct novel multivariate range-based EGARCH (REGARCH) models: a DCC-REGARCH and co-range REGARCH (CRREGARCH) model, and a co-range CARR (CRCARR) model. We compare these models with five existing models over twelve forecast horizons, ranging from one to twelve weeks, covering currencies and ETFs. Among the eight models, the DCC-REGARCH and CRREGARCH models show the best performance in out-of-sample forecasting of the variance-covariance matrix across a range of market conditions and forecast horizons. These models also generate the lowest variance and turnover for global minimum-variance (GMV) portfolios in the majority of cases.

1. Introduction

Extensive literature has developed various models aimed at improving volatility estimates to facilitate more informed financial decisions, particularly in asset allocation, risk management, and futures hedging (Kim et al., 2022; Koutmos et al., 2021; Wan, 2019). Among these, high-low range based models are notably informative and efficient (Bollerslev et al., 2024; Chou, 2005; Parkinson, 1980, among others). Portfolios constructed using these models tend to generate higher alpha than those using return-based ones (Lehnert, 2023). In particular, the range-based EGARCH (REGARCH) model (Brandt & Jones, 2006) is found to have a superior estimation and forecast accuracy due to its ability to capture key stylised features of volatility, including an approximation to normality (Alizadeh et al., 2002), fewer estimation constraints due to the guaranteed positive conditional volatility, and the ability to capture the leverage effect. Despite its strengths, the REGARCH model has not been extensively applied in a multivariate context. To address this gap, we develop multivariate range-based EGARCH (REGARCH) models by integrating the dynamic conditional correlation (DCC) model (Engle, 2002) and the co-range framework (Brandt & Jones, 2006).

The use of the DCC model is popular in multivariate GARCH modelling for three main reasons: first, it guarantees a positive definite variance-covariance matrix; second, the time-varying conditional variance and covariance can be captured; and third, it is a parsimonious model, requiring fewer parameters to be estimated relative to other approaches. Leveraging these benefits, several multivariate range-based models have been built in the literature using the DCC framework. For example, Chou et al. (2009) combine the DCC with the conditional autoregressive range (CARR) model (Chou, 2005) to create the DCC-CARR model, Fiszeder et al. (2019) integrate the DCC model with the range-based GARCH (RGARCH) model (Molnár, 2016) to develop the DCC-RGARCH model, and Fiszeder & Fałdziński (2019) introduce the co-range DCC (or DCC-CR)[1] model.

The co-range model, introduced by Brandt and Diebold (2006), offers a unique multivariate framework for measuring covariance. This model is built by taking the linear combination of the ranges of individual assets. It capitalises on the strength of the range estimator, which is not only efficient in estimation but also affected less by microstructure noise. A significant advantage of this model is that it does not require parameter estimation at the multivariate level. Building on this and the idea of EWMA, Harris and Yilmaz (2010) develop multivariate range-based EWMA models. They first construct a univariate model by combining the Parkinson range with the EWMA model, creating what they term the hybrid EMWA (HEWMA) model. They then incorporate the HEWMA model into both the co-range and return-based multivariate

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[1] Fiszeder and Fałdziński (2019) refer to the developed model as the co-range DCC model. We relabel it as 'DCC-CR' to ensure consistency with the naming conventions of the other DCC models used in this paper.

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EWMA frameworks, resulting in two multivariate HEWMA models: the co-range-hybrid-EWMA (CRHEWMA) model, and the multivariate HEMWA (MHEWMA) model. They were among the first to employ the co-range framework for a range-based multivariate model. However, their evaluation of the models' forecasting performance is limited to currencies.

Overall, the contribution of this paper is threefold. First, we develop three new multivariate models, including two multivariate REGARCH models - the DCC-REGARCH and the co-range REGARCH (CRREGARCH) - and one multivariate CARR model, the co-range CARR (CRCARR) model. The first two models newly extend the REGARCH model to the multivariate domain, essentially incorporating the REGARCH model into the popular DCC and co-range models described above and generating the DCC-REGARCH and CRREGARCH respectively. Furthermore, we introduce the co-range-CARR (CRCARR) model by combining the co-range framework and CARR model. This not only broadens the selection of multivariate models but also facilitates a comparative evaluation of the DCC and co-range models.

The literature has demonstrated that at least three types of range-based multivariate models (i.e., DCC-RGARCH, DCC-CR and MHEWMA) are superior to return-based alternatives such as DCC-GARCH and EWMA models (see Chou et al., 2009; Fiszeder et al., 2019; Fiszeder & Fałdziński, 2019; Harris & Yilmaz, 2010). However, a comprehensive comparison of all range-based models is lacking. Therefore, our second contribution is to find the best performing range-based covariance estimator in out-of-sample forecasts by comparing eight models: specifically, five existing range-based multivariate models (i.e., DCC-CARR, DCC-RGARCH, DCC-CR, MHEWMA, and CRHEWMA) and the three newly developed models (i.e., DCC-REGARCH, CRREGARCH, and CRCARR).

In a comprehensive forecasting exercise, we use currencies[2] (i.e., GBP/USD and EUR/USD) and ETFs (i.e., S&P 500 ETF Trust and United States Oil Fund), to evaluate if models perform consistently across different asset types. Moreover, unlike previous studies that typically focus on one-day or one-week ahead forecasts (e.g., Fiszeder et al., 2019; Fiszeder & Fałdziński, 2019; Harris & Yilmaz, 2010), we extend the scope to twelve forecast horizons, ranging from one week to twelve weeks. This extension is underpinned by two main considerations. First, in forecasting S&P 500 volatility, Chou (2005) uses forecast horizons of up to 13 weeks noting that at longer horizons both the absolute and relative performance of rival models can differ to those at shorter horizons. Second, we suggest that this approach is more representative of the range of hedging or investment positions an economic agent may encounter in real-world applications. Prior studies (e.g., Dichtl et al., 2016; Dunis et al., 2003) highlight the benefits of longer-term rebalancing in achieving optimal risk-adjusted portfolio returns. By evaluating the performance of the eight models across different forecast horizons, our study offers insight into the optimal model selection for portfolio management.

In addition to comparing the forecasting accuracy of the eight models by using purely statistical evaluation methods, we also compare their economic value in the context of asset allocation. The statistical criteria include the root mean squared error (RMSE), the Diebold-Mariano test, forecast regression test, Euclidean, Frobenius, and Quasi likelihood (QLIKE) loss functions (Laurent et al., 2012; Laurent et al., 2013), and Model Confidence Set (MCS) approach (Hansen et al., 2011). We also construct out-of-sample global minimum-variance portfolios based on the eight models and compare the portfolios' variance and turnover. Strikingly, the multivariate REGARCH models (i.e., CRREGARCH and DCC-REGARCH) generate the lowest variance and turnover in the majority of cases.

[2] Harris and Yilmaz (2010) and Fiszeder and Fałdziński (2019) use only currencies in their forecasting exercises, with the cross rate calculated under the condition of no-arbitrage.

Finally, we conduct a series of additional tests to assess the performance of the models we have developed under different conditions. First, we examine the impact of the weight of individual assets on the performance of the CRREGARCH model. Consistent with Bannouh et al. (2009), we find that the impact of the weight is limited. Next, by changing the asset combination to gold and oil ETFs, we find that the robust performance of our models is not dependent on specific assets. Further, after adjusting the forecast period and the in-sample size, the models' performance remains relatively unchanged. These tests confirm the consistently strong performance of the multivariate REGARCH models.

The paper is organised as follows. Section 2 covers the methodology behind constructing a proxy of the true covariance and the three newly developed models. Section 3 presents the evaluation methods and Section 4 presents the data used and the descriptive statistics of the covariance of different assets. The forecast performance and economic value of the competing models are analysed in Section 5 and Section 6, respectively. The robustness tests are discussed in Section 7. Finally, Section 8 concludes.

2. Methodology

This section presents the proxy of the true covariance, the realised covariance (Barndorff-Nielsen & Shephard, 2004), introduces three new estimators and subsequently reviews five existing covariance estimators.

2.1. Realised variance and covariance

We construct the realised variance-covariance matrix as the proxy of the true variance-covariance matrix. The realised variance (RV_t) introduced by Andersen et al. (2001) is specified as,

$$RV_t = \sum_{m=1}^{M} \left(lnP_{t,m} - lnP_{t,m-1}\right)^2 \tag{1}$$

where in our case, $P_{t,m}$ is the last observed price of the mth intraday trading interval[3] on trading day t. The daily realised variance can be further aggregated to obtain weekly realised variance. Likewise, the realised covariance (RCV_t) between assets i and j can be obtained by aggregating the cross product of returns in a specific trading interval,

$$RCV_t = \sum_{m=1}^{M} r_{i,t,m} r_{j,t,m} \tag{2}$$

where $r_{i,t,m}$ and $r_{j,t,m}$ denote the log return of the assets i and j in the mth trading interval on trading day t.

2.2. New covariance estimators

In this paper, we introduce three innovative covariance estimators: the DCC-REGARCH, the co-range REGARCH (CRREGARCH), and the co-range CARR (CRCARR) models. The development of these estimators also facilitates a comparison among range-based multivariate models. In particular, given the REGARCH model captures useful volatility characteristics such as log normality and the leverage effect, its extension into multivariate versions potentially enhances forecasting accuracy, particularly if these adaptations preserve the intrinsic benefits of the REGARCH model while integrating the depth of multivariate structures. In the following subsections, we embed the REGARCH model within the DCC and co-range frameworks. In a similar vein, we integrate the CARR model with the co-range framework, to allow comparison with the DCC-CARR model.

[3] In this paper, we use 5-min data to estimate realised variance and covariance. See Section 4 for details.

2.2.1. The DCC-REGARCH model

We introduce the DCC-REGARCH model by incorporating the REGARCH model developed by Brandt and Jones (2006) into the DCC framework (Engle, 2002). The REGARCH model is based on the finding that the log range is approximately Gaussian with a mean value of $0.43 + ln\sigma_{RE,t}$ (conditional log volatility) and a standard deviation of 0.29 (Alizadeh et al., 2002). Unlike the alternative range-based models, the REGARCH model captures the log normality and the leverage effect of asset volatility. This suggests that models based on the log range estimator might yield more accurate estimates. In addition, no parameter constraints are required to ensure positive volatility, and such advantages suggest that the REGARCH model is also well-suited for a multivariate context. The REGARCH (1,1) is given by,

$$ln\sigma_{RE,t} - ln\sigma_{RE,t-1} = k(\theta - ln\sigma_{RE,t-1}) + \phi X_{t-1} + \delta \frac{r_{t-1}}{\sigma_{RE,t-1}} \tag{3}$$

where $X_t = \frac{D_t - 0.43 - ln\sigma_{RE,t}}{0.29}$, denotes the standardised demeaned log range; k measures the speed of the log conditional volatility reverting to long-run mean θ, ϕ measures the sensitivity of log conditional volatility to lagged log range, and δ measures the sensitivity of log conditional volatility to the lagged return.

Next, we specify the new DCC-REGARCH as,

$$H_t^{DCC} = D_t^{REGARCH} R_t D_t^{REGARCH}, \tag{4}$$

$$R_t = diag\{Q_t\}^{-1/2} Q_t diag\{Q_t\}^{-1/2}, \tag{5}$$

$$Q_t = S \circ (\iota\iota' - A - B) + A \circ Z_{t-1}^{REGARCH} \left(Z_{t-1}^{REGARCH}\right)' + B \circ Q_{t-1} \tag{6}$$

where D_t is the diagonal matrix of adjusted standard deviations measured by the REGARCH model (3), that is, $D_t^{REGARCH} = diag\left(\sigma^*_{RE,1,t}, \sigma^*_{RE,2,t}, \cdots, \sigma^*_{RE,N,t}\right)$. $\sigma^*_{RE,i,t}$ is the adjusted REGARCH volatility to measure the return-based conditional covariance matrix (Q_t) and correlation matrix (R_t), that is, $\sigma^*_{RE,i,t} = \sigma_{RE,i,t} \times adj_{RE,i,t}$, where $adj_{RE,i,t}$ is a scale factor equal to the unconditional standard deviation over the mean value of the conditional volatility (i.e., $adj_{RE,i,t} = \overline{\sigma}_{i,t}/\overline{\sigma}_{RE,i,t}$). R_t is a time-varying correlation matrix, which can be obtained from the conditional covariance matrix (Q_t) of the standardised residuals. The ijth element of R_t is $\rho_{ij} = \frac{q_{ij,t}}{\sqrt{q_{ii,t}q_{jj,t}}}$. S is the unconditional covariance matrix containing the cross product of the standardised residuals, $\circ$ denotes the Hadamard product of two identically sized matrices and A and B are the parameter matrices. Lastly, ι is a vector of ones and $Z_t^{REGARCH}$ is a vector consisting of demeaned asset returns standardised by the conditional volatility. The standardised residuals of asset i at time t is given by, $z_{i,t}^{REGARCH} = \frac{r_{i,t}}{\sigma^*_{RE,i,t}}$. To obtain stationary series for the conditional covariance and variance, $\iota\iota' - A - B$ is required to be positive semi-definite, and A and B need to be positive definite, ensureing that Q_t remains positive semi-definite.

To estimate the parameters in the first and second steps, a quasi-maximum likelihood estimation (QMLE) is employed. The log likelihood function is divided into components of volatility and correlation in the following manner,

$$L(\theta_1, \theta_2) = L_{vol}(\theta_1) + L_{corr}(\theta_1, \theta_2) \tag{7}$$

$$L_{vol}(\theta_1) = -\frac{1}{2}\sum_t \left(klog(2\pi) + log|D_t|^2 + r_t' D_t^{-2} r_t\right) \tag{8}$$

$$L_{corr}(\theta_1, \theta_2) = -\frac{1}{2}\sum_t \left(log|R_t| + Z_t' R_t^{-1} Z_t - Z_t' Z_t\right) \tag{9}$$

For the standard DCC model (i.e., the return-based DCC model), the log likelihood function to estimate volatility is given by,

$$L_{vol}^{return}(\theta_1) = -\frac{1}{2}\sum_t \sum_{i=1}^{k} \left(log(2\pi) + log(\sigma_{i,t}) + \frac{r_{i,t}^2}{\sigma_{i,t}}\right) \tag{10}$$

However, to incorporate range-based volatility estimates into the DCC framework, the adjusted volatility $\sigma^*_{RE,i,t}$ replaces $\sigma_{i,t}$ in Eq. (10). The log likelihood function of the volatility component for DCC-REGARCH is therefore given by,

$$L_{vol}^{range}(\theta_1) = -\frac{1}{2}\sum_t \sum_{i=1}^{k} \left(log(2\pi) + log\left(\sigma^*_{i,t}\right) + \frac{r_{i,t}^2}{\sigma^*_{RE,i,t}}\right) \tag{11}$$

Finally, to forecast the conditional covariance of assets, the conditional covariance matrix of standardised residuals needs to be forecasted first. Suppose the sample size is T, the conditional covariance matrix at time $T+1$ is given by,

$$\begin{bmatrix} q_{11,T+1} & q_{12,T+1} \\ q_{21,T+1} & q_{22,T+1} \end{bmatrix} = (1-a-b)\begin{bmatrix} 1 & \overline{q}_{12} \\ \overline{q}_{12} & 1 \end{bmatrix} + a\begin{bmatrix} z_{1,T}^2 & z_{1,T}z_{2,T} \\ z_{1,T}z_{2,T} & z_{2,T}^2 \end{bmatrix} + b\begin{bmatrix} q_{11,T} & q_{12,T} \\ q_{21,T} & q_{22,T} \end{bmatrix} \tag{12}$$

where a, b are the parameters and $\overline{q}_{12}$ is the unconditional covariance of the standardised residuals of the two assets. Then the correlation at $T+1$ can be obtained by $\rho_{T+1} = \frac{q_{12,T+1}}{\sqrt{q_{11,T+1}q_{22,T+1}}}$ and the n-step ahead forecast of the conditional covariance matrix of the standardised residuals is specified as,

$$\begin{bmatrix} q_{11,T+n} & q_{12,T+n} \\ q_{21,T+n} & q_{22,T+n} \end{bmatrix} = (1-a-b)\begin{bmatrix} 1 & \overline{q}_{12} \\ \overline{q}_{12} & 1 \end{bmatrix} + (a+b)\begin{bmatrix} q_{11,T+n-1} & q_{12,T+n-1} \\ q_{21,T+n-1} & q_{22,T+n-1} \end{bmatrix} \tag{13}$$

2.2.2. The co-range REGARCH and co-range CARR model

In this subsection, we introduce the co-range REGARCH (CRREGARCH) and co-range CARR (CRCARR) model, which integrate the REGARCH and CARR model within the co-rang framework developed by Brandt and Diebold (2006). This framework constructs a linear combination of the variances of two individual assets and the variance of a "pseudo portfolio[4]" under the no-arbitrage condition. Within this framework, any pair of assets can be treated as a pseudo portfolio. The covariance between the two assets is derived from the variance of the pseudo portfolio and the variances of the individual assets. Specifically, for two currency assets, the variance of the pseudo portfolio is equivalent to the variance of the cross rate between the two currencies. For two different asset types, the variance of the pseudo portfolio is determined by the weighted average of their natural logarithm prices.

[4] Brandt and Diebold (2006) consider three portfolios, which consist of currencies, zero-coupon bonds and stocks. In the absence of arbitrage, the variance of the first two portfolios is equivalent to the variance of the cross rate of the two currencies and the variance of the forward contract of the two zero-coupon bonds. However, for the portfolio consisting of stocks, the close price of the portfolio needs to be computed first, which is equivalent to the weighted average price of the two assets.

Therefore, we specify the CRREGARCH model for measuring the covariance of two non-currency assets ($\sigma_{ij,t}^{CRREGARCH}$) and two currencies as follows,

$$\sigma_{ij,t}^{CRREGARCH} = \frac{1}{2\lambda_i\lambda_j}\left(\sigma_{RE,pf,t}^2 - \lambda_i^2\sigma_{RE,it}^2 - \lambda_j^2\sigma_{RE,jt}^2\right) \tag{14}$$

$$\sigma_{ij,t}^{CRREGARCH}\left(\frac{C_1}{\$}, \frac{C_2}{\$}\right) = \frac{1}{2}\left(\sigma_{RE,t}^2\left(\frac{C_1}{\$}\right) + \sigma_{RE,t}^2\left(\frac{C_2}{\$}\right) - \sigma_{RE,t}^2\left(\frac{C_1}{C_2}\right)\right) \tag{15}$$

where λ_i, λ_j denote the weights of assets i and j, respectively. Following Bannouh et al. (2009), we set equal weights for each asset (i. e., for the SPY and USO). $\sigma_{RE,pf,t}^2$, $\sigma_{RE,it}^2$, $\sigma_{RE,jt}^2$ represent the variance of pseudo portfolio and individual assets i and j obtained by the REGARCH model in week t, respectively. $\sigma_{RE,t}^2\left(\frac{C_1}{\$}\right)$ and $\sigma_{RE,t}^2\left(\frac{C_2}{\$}\right)$ are the variance of the two currencies C_1 and C_2 in dollar denomination. $\sigma_{RE,t}^2\left(\frac{C_1}{C_2}\right)$ is the variance of the cross rate between the two currencies (pseudo portfolio) under the no-arbitrage condition.

Similarly, we construct the CRCARR model by using the following formulations for two non-currency assets and two currencies,

$$\sigma_{ij,t}^{CRCARR} = \frac{1}{2\lambda_i\lambda_j}\left(\sigma_{CA,pf,t}^2 - \lambda_i^2\sigma_{CA,it}^2 - \lambda_j^2\sigma_{CA,jt}^2\right) \tag{16}$$

$$\sigma_{ij,t}^{CRCARR}\left(\frac{C_1}{\$}, \frac{C_2}{\$}\right) = \frac{1}{2}\left(\sigma_{CA,t}^2\left(\frac{C_1}{\$}\right) + \sigma_{CA,t}^2\left(\frac{C_2}{\$}\right) - \sigma_{CA,t}^2\left(\frac{C_1}{C_2}\right)\right) \tag{17}$$

The variance of an individual asset is determined by the CARR(1,1) model, which is given by,

$$\delta_t = \sigma_{CA,t}\varepsilon_t, \varepsilon_t|I_{t-1} \sim exp(1;\xi_t) \tag{18}$$

$$\sigma_{CA,t} = \omega + \alpha\delta_{t-1} + \beta\sigma_{CA,t-1} \tag{19}$$

where δ_t denotes the high-low price range (i.e., $\delta_t = lnp_{i,t}^H - lnp_{i,t}^L$), $\sigma_{CA,t}$ denotes the conditional mean of the range, and the error term ε_t is expoentially distributed with mean equal to 1 and time-varying variance ξ_t. For $\sigma_{CA,t}$ to be stationary, the sum of α and β needs to be less than unity and the parameters are estimated by quasi-maximum likelihood estimation (QMLE).

2.3. Existing models

In this subsection, we present an overview of existing covariance estimators used for comparison with our introduced models, including the DCC-CARR, DCC-RGARCH, DCC-CR, and the Hybrid EWMA model.

2.3.1. The DCC-CARR model

Chou et al. (2009) introduce a hybrid covariance estimator, the DCC-CARR model, by taking the product of the return-based correlation and scaled range-based standard deviation of two assets obtained by the CARR model. The DCC-CARR model is given by

$$H_t^{DCC} = D_t^{CARR}R_tD_t^{CARR}, \tag{20}$$

$$R_t = diag\{Q_t\}^{-1/2}Q_t diag\{Q_t\}^{-1/2}, \tag{21}$$

$$Q_t = S\circ(\iota\iota' - A - B) + A\circ Z_{t-1}^{CARR}\left(Z_{t-1}^{CARR}\right)' + B\circ Q_{t-1} \tag{22}$$

where D_t^{CARR} is the diagonal matrix of adjusted standard deviations measured by the CARR model Eq. (18) and (19), that is, $D_t^{CARR} = diag\left(\sigma_{CA,1,t}^*, \sigma_{CA,2,t}^*, \cdots, \sigma_{CA,N,t}^*\right)$. $\sigma_{CA,i,t}^*$ is the adjusted range, that is, $\sigma_{CA,i,t}^* = adj_{CA,i,t} \times \sigma_{CA,i,t}$, where $adj_{CA,i,t}$ is a scale factor defined as the ratio of the unconditional standard deviation to the mean value of the conditional mean range (i.e., $adj_{CA,i,t} = \overline{\sigma}_{i,t}/\overline{\sigma}_{CA,i,t}$). Z_t^{CARR} is the vector of standardised but correlated residuals. The standardised residuals of asset i at time t is given by, $z_{i,t}^{CARR} = \frac{r_{i,t}}{\sigma_{CA,i,t}^*}$. The QMLE functions Eq. (7), Eq. (8), Eq. (9) and Eq. (11) are employed to estimate the parameters of the DCC-CARR model.

2.3.2. The DCC-RGARCH model

Fiszeder et al. (2019) develop the DCC-RGARCH model by incorporating the RGARCH model (Molnár, 2016) within the DCC framework. The RGARCH model itself, introduced by Molnár (2016), is an extension of the GARCH model whereby the traditional variance estimator (i.e., the squared return) in the GARCH model is replaced by the Parkinson range and is given by,

$$r_t \sim N(0, \sigma_{RG,t}) \tag{23}$$

$$\sigma_{RG,t}^2 = \omega + \alpha PR_{t-1} + \beta\sigma_{RG,t-1}^2 \tag{24}$$

where r_t denotes the close-price return and of course, for $\sigma_{RG,t}$ to be stationary, the sum of α and β needs to be smaller than 1. Similarly to the DCC-CARR model, the volatilities of individual assets need to be obtained first before building the diagonal volatility matrix, D_t, and standardised residual matrix, Z_t. However, differently from the DCC-CARR model, and since the RGARCH model employs return-based volatility, the volatilities can be directly used to measure the standardised residual without adjustment, that is, $z_{RG,i,t} = r_{i,t}/\sigma_{RG,i,t}$, or to construct the diagonal volatility matrix, D_t. QMLE is employed to estimate the parameters of the DCC-RGARCH model.

2.3.3. The DCC-CR model

Fiszeder and Fałdziński (2019) develop the DCC-CR model, computing the range-based variance-covariance matrix by using range-based volatility and correlation directly without any adjustments. There is also a two-stage process for building this DCC model. In the first stage, the CARR model is employed to measure the volatility of individual assets. In the second stage, the time-varying range-based correlation matrix, computed by the co-range and the Parkinson range, is used to estimate the conditional covariance matrix. The DCC-CR model can be written as follows,

$$H_t^{DCC} = D_t^{CARR}R_tD_t^{CARR} \tag{25}$$

$$R_t = R\circ(\iota\iota' - \Theta_1 - \Theta_2) + A\circ\Phi_{t-1} + B\circ R_{t-1} \tag{26}$$

where D_t^{CARR} is k-by-k diagonal matrix of time-varying standard deviations generated from the univariate volatility model, a CARR (1,1), R is an unconditional correlation matrix, Θ_1 and Θ_2 are the parameter matrices, and Φ_t is the k-by-k range-based correlation matrix. We use the CARR and CRCARR model to construct Φ_t instead of the range and co-range model applied by Fiszeder and Fałdziński (2019) given the CARR model is used to construct D_t^{CARR}. The ijth element of Φ_t is given as, $\phi_{ij,t} = \frac{\sigma_{ij,t}^{CRCARR}}{\sigma_{CA,i,t}\sigma_{CA,j,t}}$.

2.3.4. The hybrid EWMA (HEWMA) model

Harris and Yilmaz (2010) develop the multivariate hybrid EWMA (MHEWMA) model by taking the cross product of the return-based correlation $(\rho_{EW,ij,t})$ obtained from the standard EWMA for assets i,j and the standard deviation of the assets measured by the univariate hybrid EWMA (HEWMA) model. The MHEWMA model is specified as,

$$\sigma_{ij,t}^{MHEWMA} = \rho_{EW,ij,t}\sigma_{hE,i,t}\sigma_{hE,j,t} \tag{27}$$

where $\sigma_{hE,i,t}$ and $\sigma_{hE,j,t}$ are the standard deviations of asset i and j, respectively, measured by the HEWMA model. The HEWMA model itself is specified as,

$$\sigma^2_{hE,t} = 0.94\sigma^2_{hE,t-1} + 0.06\sigma^2_{hP,t-1} \tag{28}$$

Following Harris and Yilmaz (2010), we set the smoothing factor as 0.94. $\sigma^2_{hP,t}$ is the hybrid Parkinson range obtained by using both high-low price range and open-to-close squared return. The hybrid Parkinson range estimator is given by,

$$\sigma^2_{hP,t} = \frac{1}{4ln2}\left(lnP^H_t - lnP^L_t\right)^2 + \left(lnP^O_t - lnP^C_{t-1}\right)^2 \tag{29}$$

where lnP^O_t denotes the log open price at week t, lnP^C_{t-1} denotes the log close price at week t-1. Harris and Yilmaz (2010) also develop the CRHEWMA model by applying the HEWMA model to the co-range framework. However, they only apply this model to measure the covariance of currencies. In this paper, we also use the model to calculate the covariance of non-currency assets. The models for two non-currency assets and two solely currencies are specified as follows,

$$\sigma^{CRHEWMA}_{ij,t} = \frac{1}{2\lambda_i\lambda_j}\left(\sigma^2_{hE,pf,t} - w_i^2\sigma^2_{hE,i,t} - w_j^2\sigma^2_{hE,j,t}\right) \tag{30}$$

$$\sigma^{CRHEWMA}_{ij,t}\left(\frac{C_1}{\$}, \frac{C_2}{\$}\right) = \frac{1}{2}\left(\sigma^2_{hE,t}\left(\frac{C_1}{\$}\right) + \sigma^2_{hE,t}\left(\frac{C_2}{\$}\right) - \sigma^2_{hE,t}\left(\frac{C_1}{C_2}\right)\right) \tag{31}$$

3. Evaluation approach

To assess the performance of the different models, we employ a comprehensive array of evaluation metrics. For assessing the accuracy of covariance forecast we employ the root mean squared error (RMSE), the Diebold-Mariano test, and the regression-based test. Furthermore, multivariate loss functions - Euclidean (Laurent et al., 2012), Frobenius and quasi likelihood (QLIKE) distance, along wiht the non-parametric Model Confidence Set (MCS) test (Hansen et al., 2011), are applied to evaluate the precision of the variance-covariance matrix forecasts.

RMSE is a standard metric that quantifies the discrepancy between the predicted covariance from the eight competing models and the true covariance. It is particularly useful in assessing the ability of these models to forecast the covariance. RMSE is given as,

$$RMSE = \left[\frac{1}{T}\sum\nolimits_{t=1}^{T}\left(\sigma_{ij,t} - \widehat{\sigma}_{ij,t}\right)^2\right]^{1/2} \tag{32}$$

where $\sigma_{ij,t}$ is the covariance measured by the realised covariance in Section 2.1, and $\widehat{\sigma}_{ij,t}$ denotes the forecasted covariance obtained from the eight models.

The Diebold-Mariano test is used for the pairwise comparison of models. We compute the t-test of the coefficient d_{kl}, which is the difference of the loss function between models k and l,

$$\left(\sigma_{ij,t} - \widehat{\sigma}^l_{ij,t}\right)^2 - \left(\sigma_{ij,t} - \widehat{\sigma}^k_{ij,t}\right)^2 = d_{kl,t} + \varepsilon_t \tag{33}$$

A significantly positive mean value of $d_{kl,t}$ indicates that model k is preferred to model l, whereas a significantly negative value indicates the opposite.

Following Brandt and Jones (2006), we use the regression-based tests to evaluate each model's capability to predict the "true" covariance. The test is specified as:

$$\sigma_{ij,t} = \alpha_k + \beta_k \times \widehat{\sigma}^k_{ij,t} + \varepsilon_{k,t} \tag{34}$$

For an unbiased forecast result, $\alpha_i = 0$ and $\beta_i = 1$. The regression R^2 is used to determine how effectively model k forecast the realised covariance ($\sigma_{ij,t}$).

To provide a more comprehensive assessment of the variance-covariance matrix forecast, we employ three multivariate loss functions that provide consistent model rankings (Laurent et al., 2012). The Euclidean loss function computes the linear distance between forecasted and true values within a multi-dimensional framework, providing a direct measure of forecast accuracy. The Frobenius Distance measures the magnitude of the difference between predicted and true variance-covariance metrics. Meanwhile, the QLIKE distance is derived from the log-likelihood of multivariate models, offering insights into a model's fit to the data. We also use the MCS test to determine whether a model significantly outperforms its competitors at a given confidence interval. This method offers a clear perspective on the top-performing model. The loss functions (i.e., Euclidean, Frobenius and QLIKE) are given as follows,

$$L^{Euclidean}_t = vech(\Sigma_t - H_t)' vech(\Sigma_t - H_t) \tag{35}$$

$$L^{Frobenius}_t = Tr\left[(\Sigma_t - H_t)'(\Sigma_t - H_t)\right] \tag{36}$$

$$L^{QLIKE}_t = log|H_t| + Tr\left(H_t^{-1}\Sigma_t\right) \tag{37}$$

where $vech(\Sigma_t - H_t)$ is the vector that stacks the difference of all the lower triangular and diagonal elements of the true and forecasted variance-covariance matrix (i.e., Σ_t and H_t, respectively) at time t and Tr denotes the trace of a matrix. We then use the non-parametric test statistics (i.e., the MCS) to identify models that are superior to the others at a particular significance level (i.e., 5 %). The initial model set M contains all eight competing models, which are compared with one of models from the set by taking the difference of the loss functions. If the difference is significantly different from zero, the inferior model is eliminated from the model set. The comparison procedure repeats until a subset of superior models reached. The null hypothesis is therefore set as,

$$H_0 : E\left(\Delta L_{ij,t}\right) = 0, \quad \text{for all } i,j \in M. \tag{38}$$

where $E\left(\Delta L_{ij,t}\right) = \frac{1}{n}\sum_{t=1}^{n}\Delta L_{ij,t}$ is the average loss difference between models i and j over the forecasting period. The null hypothesis is tested according to the semi-quadratic static, $t_{SQ} = \sum_{i<j}\frac{\left(E\left(\Delta L_{ij,t}\right)\right)^2}{\widehat{var}\left(\Delta L_{ij,t}\right)}$ where $\widehat{var}(\Delta L_{ij,t})$ is the asymptotic variance of $\Delta L_{ij,t}$. We choose 10,000 replication block bootstrap procedure and a block length of 2 observations following Laurent et al. (2012).

4. Data

We employ two data sets to compare the performance of the proposed models and their competitors: the most actively traded currencies in the Forex market (i.e., GBP/USD and EUR/USD) and widely recognised exchange-traded funds, S&P 500 ETF Trust (SPY) and United States Oil Fund (USO). We use these two types of assets to assess if the co-range-based models demonstrate different forecast accuracies with and without assets' weights being considered. To obtain synchronous trading prices[5] for the non-arbitrage portfolios when applying the co-range model, we extract 1-min open and close prices of currencies over the period 21 October 2002 to 30 June 2024, and ETFs from 10 April 2006 - the inception of USO trading - to 30 June 2024, from pitrading.com. These periods include significant market events that impact volatility, such as the global financial crisis (GFC), the European sovereign debt crisis, Brexit and the COVID-19 pandemic, as well as intervals of low volatility. This timeframe facilitates thorough evaluation of volatility models performance under different market conditions (Alves et al., 2024; Fiszeder et al., 2019; Fiszeder & Fałdziński, 2019; Symitsi et al., 2018).

We exclude all trading prices on days when at least one market is closed. The log price of the currency-based portfolio is equivalent to the log cross rate between the two currencies, which is obtained by taking

[5] As Bannouh et al. (2009) suggest, the high and low portfolio prices could not be obtained directly by taking the product of the corresponding individual asset prices because the time that the high and low price generated is different.

the difference between the USD-denominated prices of GBP and EUR. We construct a pseudo non-arbitrage portfolio by assigning a 50 % weight of the S&P 500 ETF, following Bannouh et al. (2009). If an asset lacks a trading price in a specific interval, we substitute the most recent availabe price from the previous trading interval. The log close price of the pseudo portfolio at each minute is calculated as the weighted average of the log prices of S&P 500 and crude oil ETFs. We then calculate the 5-min intraday returns of these assets.

We employ the realised variance/covariance matrix at a weekly frequency based on the work of Ferland and Lalancette (2006) and Chou et al. (2009). However, the weekly realised variance/covariance they estimate using a 30-min or daily sampling frequency is potentially less efficient than that obtained using 5-min returns (Andersen et al., 2001; Bannouh et al., 2009; Barndorff-Nielsen & Shephard, 2004). Therefore, we use 5-min returns to measure the daily realised variance and covariance, which is then used to construct realised variance/covariance at a weekly frequency. The number of weekly observations for the currencies and ETFs are 1131 and 950, respectively. Fig. 1 shows the weekly close price, absolute return, and range of the two individual currencies, two ETFs, and their corresponding pseudo portfolios. It is clearly shown that the prices of all the assets experienced significant volatility during the financial crisis over the period of 2007 to 2008 and during the pandemic years of 2020 and 2021. In addition, the Brexit referendum in 2016 had a significant impact on the GBP/USD.

Table 1 presents the summary statistics for returns, log absolute returns and log range of all the assets. In comparison to log returns and log absolute returns, the skewness of the log range is closer to zero, and the kurtosis is closer to three, which indicates that the log range is approximately normally distributed. In particular, the Jarque-Bera test statistic for the log range is much lower than that for log absolute returns.

Applying Eq. (1) and Eq. (2), we compute the realised variance/covariance and realised correlation of the assets. Figs. 2 and 3 present the realised covariance and realised correlation over the sample period, showing that the covariances between GBP/USD and EUR/USD, as well as between the S&P 500 and crude oil are strengthened during the GFC and the pandemic. In addition, the covariance of the currencies peaked during the Brexit in 2016. It is also worth noting that the realised

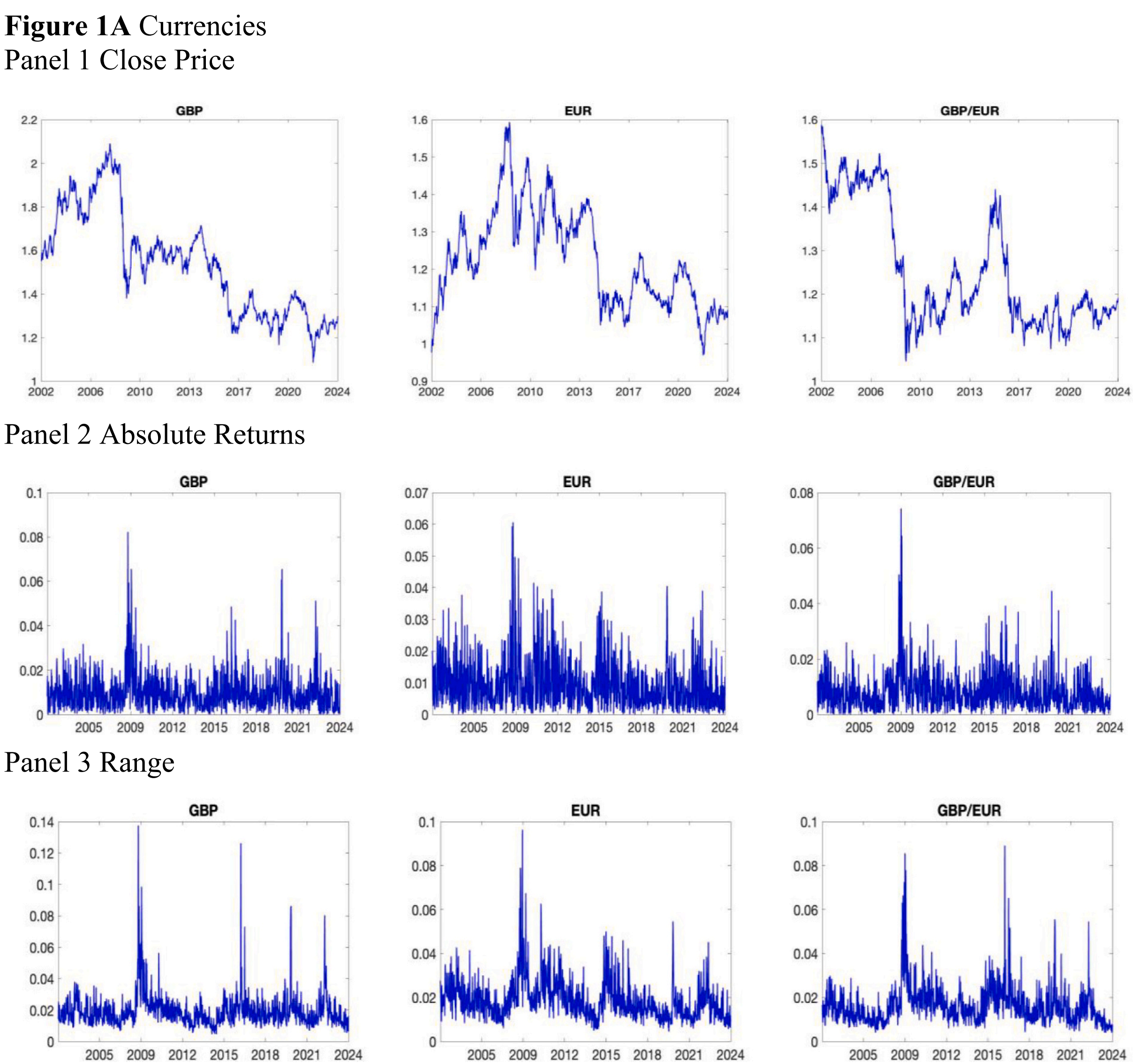

Fig. 1. Weekly log close prices, absolute returns and range.
Notes: The figures represent the weekly log close prices, absolute returns and range of currencies (i.e., GBP/USD and EUR/USD) and ETFs (i.e., S&P 500 ETF Trust (SPY) and United States Oil Fund (USO)), along with their pseudo portfolios over the periods 21 October 2002 to 30 June 2024, and 10 April 2006 to 30 June 2024, respectively. Intraday high and low prices and daily close prices are used to calculate the corresponding weekly price series. The weights of the S&P 500 and oil ETFs are both set at 50 %.

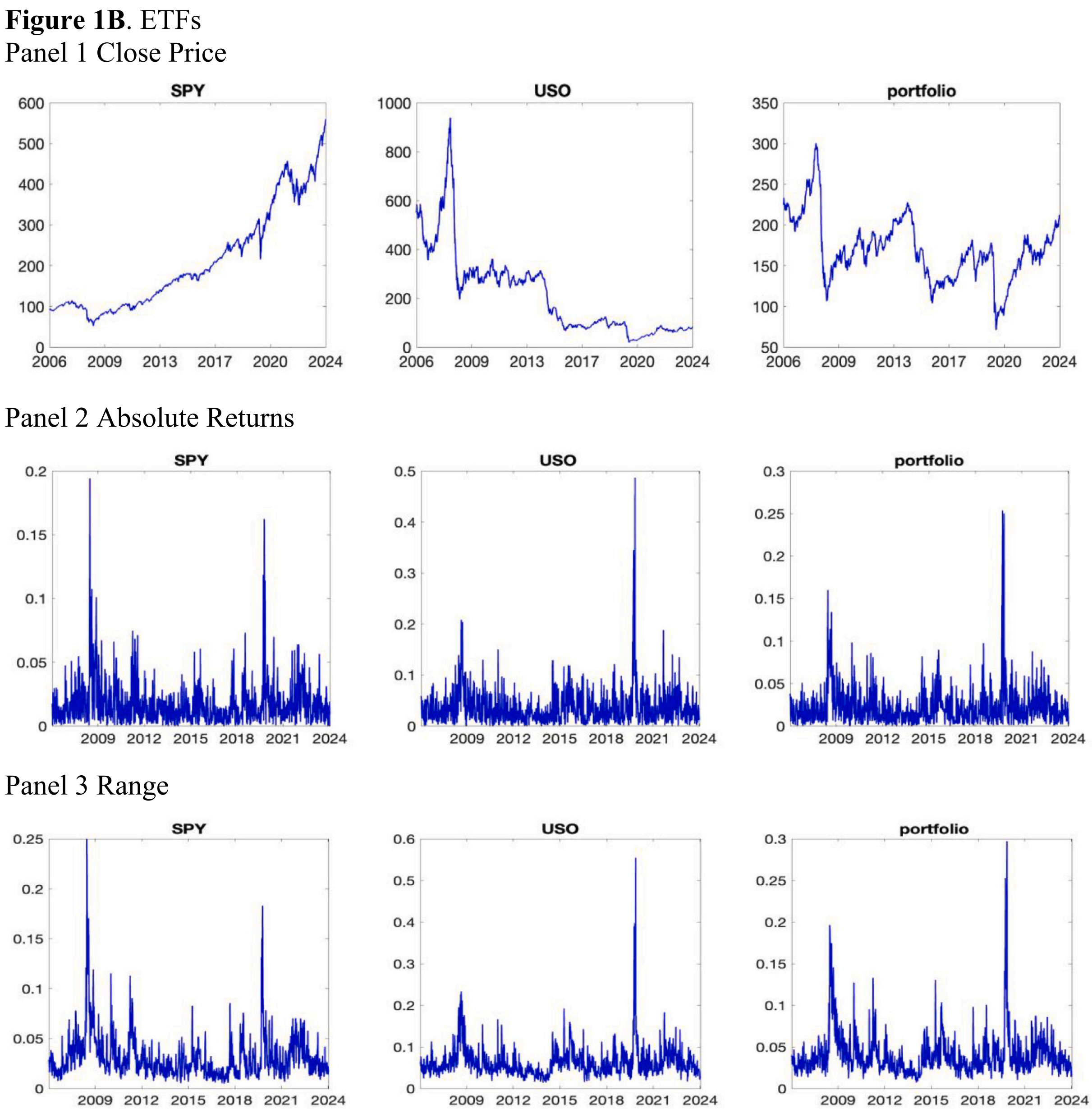

Fig. 1. (*continued*).

covariance between the S&P 500 and crude oil increased significantly towards the end of 2011, as the prices of the two assets rebounded from falling summer prices, partly due to the downgrade of the US credit rating (Afonso et al., 2014). Moreover, in Fig. 3 it is shown that the realised correlation between the S&P 500 and oil is more volatile than that of the currencies. This is in line with the higher standard deviation of the realised correlation between the former two assets, as shown in Table 2.

5. Forecast comparison

Following Chou et al. (2009), we use a rolling window of 500 observations to estimate the models, with the remaining 631 and 450 observations used to measure the forecast accuracy of the out-of-sample covariance for currencies and ETFs, respectively. We extend the forecasting horizon from one week that is commonly used in the literature to twelve weeks. Realised covariance is employed as the proxy of the true covariance. The performance of the eight models (i.e., MHEWMA, DCC-CARR, DCC-REGARCH, DCC-RGARCH, DCC-CR, CRHEWMA, CRCARR, and CRREGARCH) is evaluated by using RMSE, the Diebold-Mariano test, the regression-based R^2 test, three loss functions, and the MCS approach.

The RMSE values of the eight models reported in Table 3 show that the multivariate REGARCH models surpass the competitors. In particular, the CRREGARCH model consistently delivers the lowest RMSE over all twelve forecasting intervals when forecasting the covariance of the currencies, while the DCC-REGARCH model closely follows as the second best in 9 out of 12 cases. When forecasting the covariance of S&P 500 and crude oil ETFs, the DCC-REGARCH model outperforms the competitors in 6 cases. The CRREGARCH model is superior to the alternatives in 5 out of 12 cases, while the DCC-RGARCH model leads in one case.

Table 4 and Table 5 display the *t*-statistics of the Diebold-Mariano test for currencies and ETFs respectively. A *t*-statistic greater than 1.96 indicates that the model in the column is significantly preferred over the model in the row, while a *t*-statistic less than −1.96 indicate the opposite. Table 4 demonstrates that no competing models significantly outperform the multivariate REGARCH models in forecasting

Table 1
Summary statistics of the weekly returns, log absolute returns and log range.

Panel A. Summary statistics of currencies.

	Mean	Median	Maximum	Minimum	St. Dev.	Skewness	Kurtosis	Jarque-Bera
				Returns				
GBP/USD	0.0002	−0.0001	0.0822	−0.0656	0.0132	0.5080	6.3813	584
EUR/USD	−0.0001	−0.0002	0.0606	−0.0497	0.0126	0.2900	4.5148	123
GBP/EUR	0.0003	0.0005	0.0505	−0.0742	0.0110	−0.0573	6.8736	703
				Log absolute returns				
GBP/USD	−5.0351	−4.8031	−2.4980	−10.4952	1.1178	−1.4264	6.0857	828
EUR/USD	−5.0820	−4.8765	−2.8033	−10.9688	1.1024	−1.1542	4.9815	434
GBP/EUR	−5.2784	−5.0516	−2.6006	−12.3760	1.1855	−1.4514	6.5819	997
				Log range				
GBP/USD	−4.0530	−4.0816	−1.9836	−5.3971	0.4328	0.5601	4.6163	181
EUR/USD	−4.0783	−4.0936	−2.3408	−5.4104	0.4462	0.1538	3.1372	5
GBP/EUR	−4.2425	−4.2443	−2.4183	−5.5364	0.4757	0.2410	3.4309	19

Panel B. Summary statistics of ETFs

	Mean	Median	Maximum	Minimum	St. Dev.	Skewness	Kurtosis	Jarque-Bera
				Returns				
SPY	0.0019	0.0036	0.1142	−0.1942	0.0250	−0.9326	10.9353	2615
USO	−0.0020	0.0025	0.2776	−0.4873	0.0516	−1.3321	14.5168	5503
Portfolio	−0.0001	0.0026	0.1343	−0.2537	0.0323	−1.4986	12.5777	3966
				Log absolute returns				
SPY	−4.5687	−4.3646	−1.6387	−9.7216	1.1780	−0.9661	4.3954	223
USO	−3.8041	−3.6211	−0.7189	−9.2103	1.1508	−1.1216	5.2096	390
Portfolio	−4.3063	−4.1111	−1.3717	−10.0303	1.1500	−0.9092	4.3177	198
				Log range				
SPY	−3.6680	−3.6870	−1.3872	−5.3341	0.5946	0.2872	3.2708	16
USO	−2.8895	−2.9128	−0.5894	−4.2986	0.4912	0.3610	3.8814	51
Portfolio	−3.3629	−3.3973	−1.2134	−4.9416	0.5192	0.4116	3.6883	45

Notes: The table demonstrates the summary statistics for the weekly return, log absolute return and log range for currencies (GBP/USD, EUR/USD), ETFs (S&P 500 ETF Trust (SPY) and the United States Oil Fund (USO)), and the corresponding pseudo portfolios. The currency data covers the period from 21 October 2002 to 30 June 2024, while the ETF data spans 10 April 2006 to 30 June 2024. The weights of the S&P 500 and oil ETFs are both set at 50 %.

currencies. Notably, the CRREGARCH model significantly outperforms its competitors across all twelve forecast horizons, with exceptions: the DCC-CR, MHEWMA, and CRHEWMA in the 1-week ahead forecast, and the CRCARR model in the 1- to 2-week forecast horizons. The DCC-REGARCH model is significantly superior to all the competitors from the 4-week forecast horizon onwards, except when compared to the CRREGARCH model.

For forecasting the S&P 500 and oil ETFs, as shown in Table 5, no competing model significantly outperforms the CRREGARCH and DCC-REGARCH models. The CRREGARCH model significantly outperforms the DCC-CR and CRHEWMA across all twelve forecast horizons. In addition, it demonstrates significant superiority over the MHEWMA model starting from the 2-week forecast horizon, the DCC-CARR from the 3-week horizon, the DCC-RGARCH from the 4-week horizon, and the CRCARR from the 8-week horizon onward. Meanwhile, the DCC-REGARCH model significantly surpasses the DCC-CARR, DCC-CR, and CRHEWMA models across all twelve horizons. Furthermore, it significantly outperforms the MHEWMA and DCC-CR models from the 2-week to the 12-week forecast horizons and surpasses the DCC-RGARCH and CRCARR models from the 4-week and 8-week forecast horizons onward, respectively.

Table 6 reports the R^2 values of the forecast regressions, clearly demonstrating that the CRREGARCH achieves the highest R^2 across all twelve forecast horizons when forecasting the covariance of currencies. It is closely followed by the DCC-REGARCH model, which ranked second in 11 out of the 12 cases. In the context of forecasting the S&P 500 and oil ETFs, the DCC-REGARCH records the highest R^2 in 9 out of the 12 cases, while the MHEWMA model, leading in 3 out of the 12 cases.

We then compare the eight models in forecasting the variance-covariance matrix by employing the Euclidean, Frobenius, and QLIKE loss functions. Following Bollerslev et al. (2018), we employ the "in-sanity filter" to ensure all the matrices are positive definite. Specifically, we replace the out-of-sample negative-definite covariance matrix forecasts with the average value of the in-sample matrix estimated by the respective models. Table 7 demonstrates the forecast losses for the eight models when forecasting the covariance matrix of currencies over the twelve forecast horizons. The results show that the covariance matrices forecasted using the CRREGARCH model have the lowest forecast losses in all cases, with the exception of ranking second lowest under the QLIKE function at the 1-week forecast horizon and third lowest at the 3- and 7- week forecast horizons. The DCC-REGARCH generally performs the second best when using the Euclidean or Frobenius loss functions. When the QLIKE loss function is employed, the DCC-REGARCH model has the lowest or second-lowest forecast losses for forecast horizons beyond 6 weeks, whereas the CRCARR model achieves the third-lowest losses across almost all twelve forecast horizons.

The results of the MCS test for the three currency loss functions are shown in Table 8. The table reports the p-value for the null hypothesis, which tests whether the difference between the loss functions of the two models from the model set is significantly different from zero. It also includes the ranking of each model with a higher ranking indicates better forecast accuracy. Models that rank first are marked bold and italics. In line with the results shown in Table 7, the CRREGARCH model is the top-ranking model over all the twelve forecast horizons. The DCC-REGARCH model ranks second in the majority of cases when the Euclidean or Frobenius loss function is employed and is within top three in 6 out of 12 cases when the QLIKE loss function is employed.

The out-of-sample forecast losses and MCS results for the S&P 500 and oil ETFs are shown in Tables 9 and 10, respectively. The CRREGARCH model continues to outperform the competing models, with the DCC-REGARCH model closely following. Notably, the CRREGARCH model ranks first from the 4-week forecast horizons onwards under the Euclidean and Forbenius loss functions, and predominantly ranks second across the twelve forecast horizons when employing the QLIKE loss

Figure 2A. Currencies

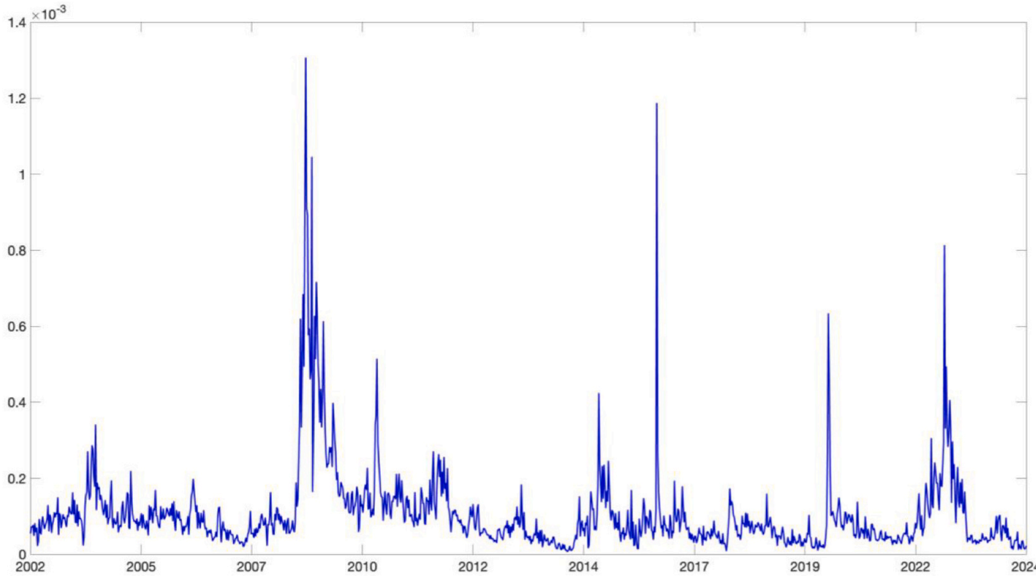

Figure 2B. ETFs

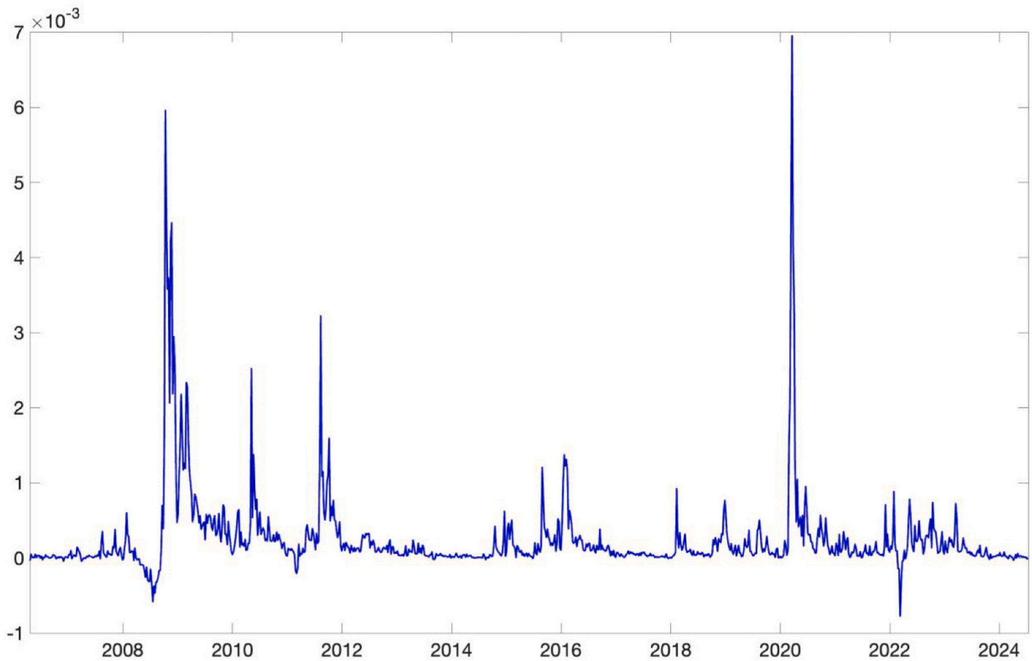

Fig. 2. Realised covariance of the assets.
*Notes: The figure shows the realised covariance of currencies (*i.e.*, GBP/USD and EUR/USD) and ETFs (*i.e.*, S&P 500 ETF Trust (SPY) and United States Oil Fund (USO)) over the sample period from 21 October 2002 to 30 June 2024 and from 10 April 2006 to 30 June 2024, respectively.*

function. The performance of the DCC-REGARCH model exhibits a complementary pattern: it generally ranks second under the Euclidean and Forbenius loss functions but ascends to first place under the QLIKE loss function from the 4-week to the 12-week forecast horizons.

In summary, in the empirical work so far, the multivariate REGARCH models demonstrate the best and most consistent performance across different datasets and forecast horizons. In particular, the CRREGARCH model achieves the lowest forecast error in the majority of cases, whereas the new DCC-REGARCH model consistently ranks first or second, especially when forecasting the covariance of the non-currency assets. The strong performance of the multivariate REGARCH models underscores the inherent strengths of the REGARCH model. This model effectively captures the stylised features of the asset volatility that the alternative models fail to account for. Moreover, the streamlined parameter estimation in the co-range model minimises estimation errors, further enhancing forecasting precision.

6. Minimum variance portfolios performance

Next, we construct Global Minimum Variance (GMV) portfolios based on the eight competing models and compare the variances and turnover of the out-of-sample portfolios. We choose the GMV optimal

Figure 3A. Currencies

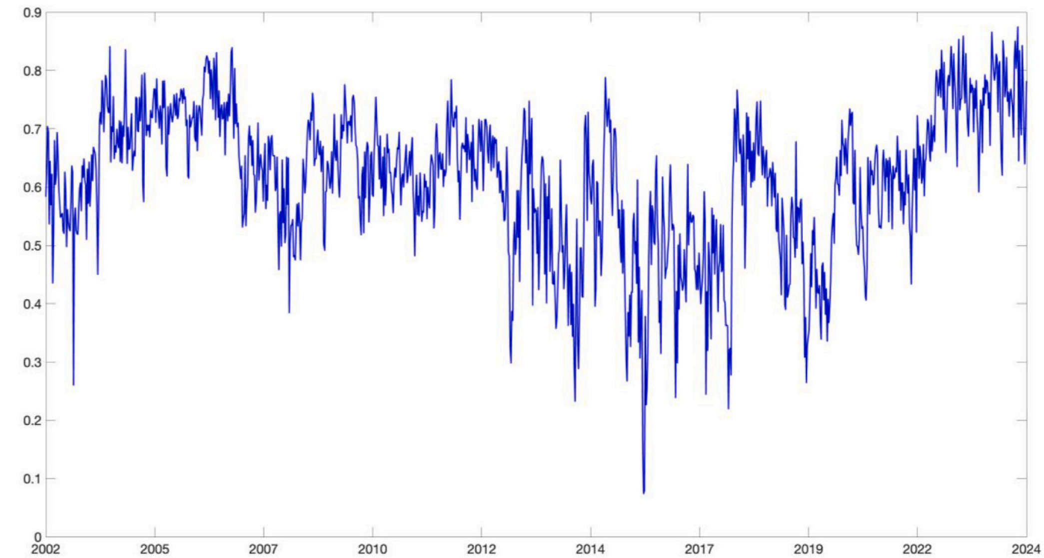

Figure 3B. ETFs

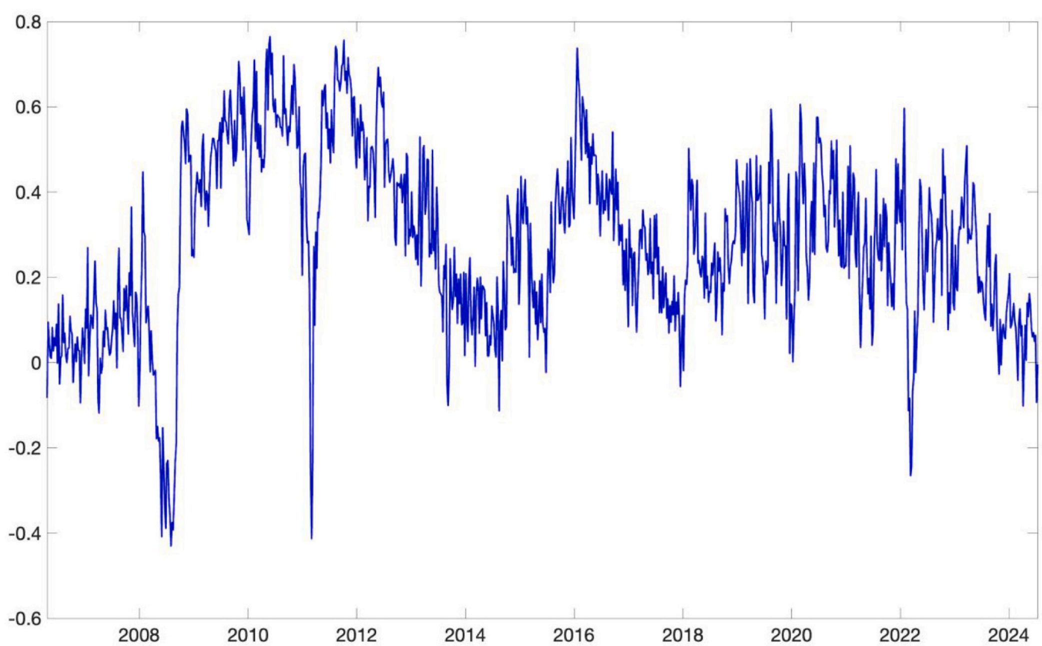

Fig. 3. Realised correlation based on realised variance.
Notes: The figure shows the realised correlation between GBP/USD and EUR/USD, S&P500 ETF Trust (SPY) and United States Oil Fund (USO), respectively, based on the realised covariance. The sample period for the currencies is from 21 October 2002 to 30 June 2024, while the sample period for ETFs is from 10 April to 30 June 2024. The weight of the S&P 500 is set at 50 %.

method, given that we mainly compare the variance and covariance forecast accuracy of various models (Symitsi et al., 2018). In addition, the introduction of expected return would affect the accuracy of the performance evaluation due to its estimation error (DeMiguel et al., 2009). We adjust the weight to minimise the portfolio variance over twelve different frequencies, ranging from 1 week to 3 months. In each period, investors aim to minimise the portfolio variance, that is,

$$min\ w_t' H_t w_t \tag{39}$$

$$s.t. w_t' \iota = 1$$

where w_t is an $N \times 1$ vector of GMV portfolio weights given N assets included, H_t is the $N \times N$ forecasted conditional variance-covariance matrix at time t obtained from the eight models, ι is an $N \times 1$ vector of ones. The optimal weight w_t is therefore given by,

$$w_t = \frac{H_t^{-1}\iota}{\iota' H_t^{-1}\iota} \tag{40}$$

We first use the rolling window forecasting method with a sample size of 500 observations to forecast the variance-covariance matrix H_t, before finding the optimal weights for each model using Eq. (40). The

Table 2
Summary statistics of realised volatility, realised covariance and realised correlation.

Panel A. Currencies			
	$\sqrt{RV}$		ρ_{RCV}
	GBP/USD	EUR/USD	
Mean	0.0124	0.0121	0.6125
St. Dev.	0.0054	0.0046	0.1230
Skewness	3.3190	1.4911	−0.7053
Kurtosis	21.5514	7.1373	3.6043

Panel B. ETFs			
	$\sqrt{RV}$		ρ_{RCV}
	SPY	USO	
Mean	0.0170	0.0332	0.2819
St. Dev.	0.0122	0.0175	0.2104
Skewness	3.5182	4.5342	−0.2149
Kurtosis	21.2914	41.6360	3.0766

Notes: The table demonstrates summary statistics for the square root of realised variance and correlation. The sample periods of the currencies and ETFs (S&P 500 ETF Trust (SPY) and United States Oil Fund (USO)) are from 21 October 2002 to 30 June 2024, and from 10 April 2006 to 30 June 2024, respectively. The weights of S&P 500 and oil ETFs are both set at 50 %.

estimated weight is then used to find the portfolio return at time $t+1$ given the assets return at the same time $t+1$, that is, $r^{(m)}_{pf,t+1} = \left(w^{(m)}_t\right)' r_{t+1}$, where $r^{(m)}_{pf,t+1}$ is the portfolio return for model m, r_{t+1} is a $N \times 1$ vector of assets returns.

To compare the out-of-sample portfolio performance of eight models over twelve rebalancing periods, we employ the portfolio variance and average portfolio turnover. The models which generate a more accurate forecast of the variance-covariance matrix tend to have lower portfolio variance and turnover. Out-of-sample portfolio variance is given by,

$$\widehat{\sigma}^2_m = \frac{1}{T}\sum_{t=1}^{T}\left(r^{(m)}_{pf,t} - \overline{r}^{(m)}_{pf}\right)^2 \tag{41}$$

where T is the size of the out-of-sample returns, $r^{(m)}_{pf,t}$ is the portfolio return at time t under model m, $\overline{r}^{(m)}_{pf}$ is the portfolio average return over the out-of-sample period. Average portfolio turnover is given by,

$$Turnover = \frac{1}{T-1}\sum_{t=1}^{T-1}\sum_{j=1}^{N}\left(\left|w_{m,j,t+1} - w_{m,j,t^+}\right|\right) \tag{42}$$

where $w_{m,j,t+1}$ is the desired portfolio weight of asset j measured by Eq. (40) at time $t+1$ under model m; w_{m,j,t^+} is the portfolio weight measured by the close price of the portfolio assets at time t before rebalancing at $t+1$. Finally, following DeMiguel et al. (2009), we compare the

Table 3
RMSE for Out-of-sample Covariance Forecast.

Panel A. Currencies (GBP and EUR)								
Forecast Horizon	Forecasting Models							
	Existing Models					New Models		
	DCC CARR	DCC RGARCH	DCC CR	MHEWMA	CR HEWMA	DCC REGARCH	CR REGARCH	CR CARR
1	0.7216	0.7363	0.7107	0.7821	0.7447	0.7010	***0.6715***	0.6939
2	0.7834	0.8063	0.7735	0.8172	0.7795	0.7525	***0.7255***	0.7661
3	0.8152	0.8404	0.8017	0.8316	0.8013	0.7797	***0.7553***	0.8021
4	0.8311	0.8569	0.8194	0.8445	0.8142	0.7912	***0.7627***	0.8172
5	0.8374	0.8622	0.8249	0.8463	0.8228	0.7975	***0.7678***	0.8196
6	0.8449	0.8680	0.8359	0.8538	0.8301	0.8021	***0.7673***	0.8213
7	0.8534	0.8797	0.8442	0.8678	0.8399	0.8107	***0.7789***	0.8288
8	0.8593	0.8846	0.8492	0.8738	0.8489	0.8180	***0.7903***	0.8361
9	0.8659	0.8914	0.8550	0.8886	0.8580	0.8252	***0.8012***	0.8429
10	0.8701	0.8949	0.8599	0.8911	0.8644	0.8285	***0.8024***	0.8419
11	0.8729	0.8995	0.8634	0.8979	0.8710	0.8331	***0.8077***	0.8419
12	0.8750	0.9049	0.8653	0.9056	0.8790	0.8362	***0.8136***	0.8441

Panel B. ETFs (SPY and USO)								
Forecast Horizon	Forecasting Model							
	Existing Models					New Models		
	DCC CARR	DCC RGARCH	DCC CR	MHEWMA	CR HEWMA	DCC REGARCH	CR REGARCH	CR CARR
1	5.7992	***4.5784***	6.6262	5.9820	7.3513	5.0089	5.2254	5.5260
2	6.0556	5.4036	6.6145	6.3958	7.7432	***5.1339***	5.2300	5.8487
3	6.1721	5.9103	6.5323	6.6889	7.9640	5.3542	***5.2779***	6.0330
4	6.4474	6.6770	6.5807	6.8889	8.1462	5.6806	***5.6419***	6.4648
5	6.4614	7.0492	6.4678	6.9879	8.2253	***5.7637***	5.7932	6.5452
6	6.4211	7.3208	6.3699	7.0399	8.2625	***5.7868***	5.8006	6.5356
7	6.3486	7.5370	6.2594	7.1075	8.3237	***5.7483***	5.8271	6.4746
8	6.2812	7.6788	6.1674	7.1546	8.3499	***5.7232***	5.7941	6.3928
9	6.2746	7.9210	6.1466	7.1878	8.3976	***5.7454***	5.7902	6.3820
10	6.2316	8.0750	6.0971	7.1319	8.3654	5.7317	***5.7173***	6.2364
11	6.1911	8.2521	6.0750	7.1341	8.3828	5.6928	***5.6242***	6.0849
12	6.1812	8.5172	6.0693	7.1983	8.4574	5.6705	***5.5748***	6.0300

Notes: The table shows the RMSE of the out-of-sample covarince forecasts for the currencies and ETFs obtained from the eight multivariate models. A rolling window forecast method is used, with a sample size of 500 observations. The number of forecast value is 631 for currencies and 450 for ETFs. The weight of the S&P 500 ETF is set at 50 %. All values are multiplied by 10,000.The model with the lowest RMSE is shown in bold and italics.

Table 4
Diebold-Mariano test for currencies (GBP and EUR).

Panel A. Forecast horizons 1 to 3 weeks

Forecasting Horizon		DCC CARR	DCC RGARCH	DCC CR	MHEWMA	CR HEWMA	DCC REGARCH	CR REGARCH	CR CARR
1	DCC-CARR	0	−1.99	1.49	1.27	1.4	1.91	2.81	2.39
	DCC-RGARCH		0	2.25	1.75	1.81	2.59	3.33	3.18
	DCC-CR			0	0.7	0.86	0.6	1.85	1.61
	MHEWMA				0	0.69	−0.46	0.73	−0.07
	CRHEWMA					0	−0.66	0.58	−0.21
	DCC-REGARCH						0	2.68	0.38
	CRREGARCH							0	−1.09
	CRCARR								0
2	DCC-CARR	0	−2.75	1.46	1.62	1.46	2.54	3.14	1.4
	DCC-RGARCH		0	2.85	2.53	2.28	3.34	3.8	2.8
	DCC-CR			0	0.91	0.85	1.27	2.21	0.64
	MHEWMA				0	−0.13	−0.05	2.11	−0.58
	CRHEWMA					0	0.01	2.35	−0.55
	DCC-REGARCH						0	2.43	−0.64
	CRREGARCH							0	−1.88
	CRCARR								0
3	DCC-CARR	0	−3.07	2.3	1.64	1.35	3.32	3.28	1.03
	DCC-RGARCH		0	3.49	2.65	2.29	3.8	3.9	2.75
	DCC-CR			0	0.62	0.49	1.56	2.15	−0.03
	MHEWMA				0	−0.23	1.21	2.75	−0.56
	CRHEWMA					0	1.11	3.56	−0.49
	DCC-REGARCH						0	2.04	−1.19
	CRREGARCH							0	−2.54
	CRCARR								0

Panel B. Forecast horizons 4 to 6 weeks

Forecasting Horizon		DCC CARR	DCC RGARCH	DCC CR	MHEWMA	CR HEWMA	DCC REGARCH	CR REGARCH	CR CARR
4	DCC-CARR	0	−3.01	2.53	1.13	0.81	3.93	4.2	1.15
	DCC-RGARCH		0	3.52	2.23	1.86	3.99	4.42	2.88
	DCC-CR			0	0.33	0.14	2.33	3.43	0.22
	MHEWMA				0	−0.52	2.16	5.55	−0.18
	CRHEWMA					0	1.98	6.46	−0.01
	DCC-REGARCH						0	2.93	−1.45
	CRREGARCH							0	−3.2
	CRCARR								0
5	DCC-CARR	0	−3.01	2.53	1.13	0.81	3.93	4.2	1.15
	DCC-RGARCH		0	3.52	2.23	1.86	3.99	4.42	2.88
	DCC-CR			0	0.33	0.14	2.33	3.43	0.22
	MHEWMA				0	−0.52	2.16	5.55	−0.18
	CRHEWMA					0	1.98	6.46	−0.01
	DCC-REGARCH						0	2.93	−1.45
	CRREGARCH							0	−3.2
	CRCARR								0
6	DCC-CARR	0	−2.87	1.82	0.83	0.18	4.47	5.32	2.64
	DCC-RGARCH		0	3.36	1.87	1.24	4	4.92	4.43
	DCC-CR			0	0.17	−0.36	3.04	5	2.44
	MHEWMA				0	−1.11	3.15	6.73	0.85
	CRHEWMA					0	2.72	6.52	1.3
	DCC-REGARCH						0	4.44	−1.31
	CRREGARCH							0	−3.57
	CRCARR								0

Panel C. Forecast horizons 7 to 9 weeks

Forecasting Horizon		DCC CARR	DCC RGARCH	DCC CR	MHEWMA	CR HEWMA	DCC REGARCH	CR REGARCH	CR CARR
7	DCC-CARR	0	−2.87	1.82	0.83	0.18	4.47	5.32	2.64
	DCC-RGARCH		0	3.36	1.87	1.24	4	4.92	4.43
	DCC-CR			0	0.17	−0.36	3.04	5	2.44
	MHEWMA				0	−1.11	3.15	6.73	0.85
	CRHEWMA					0	2.72	6.52	1.3
	DCC-REGARCH						0	4.44	−1.31
	CRREGARCH							0	−3.57
	CRCARR								0
8	DCC-CARR	0	−3.09	1.81	0.88	0.14	4.75	4.97	2.61
	DCC-RGARCH		0	3.65	2.09	1.34	4.27	4.99	4.76

(*continued on next page*)

Table 4 (*continued*)

Panel C. Forecast horizons 7 to 9 weeks

Forecasting Horizon		DCC CARR	DCC RGARCH	DCC CR	MHEWMA	CR HEWMA	DCC REGARCH	CR REGARCH	CR CARR
	DCC-CR			0	0.24	−0.41	3.35	4.95	2.43
	MHEWMA				0	−1.26	2.93	6.44	0.91
	CRHEWMA					0	2.65	6.98	1.44
	DCC-REGARCH						0	3.54	−1.33
	CRREGARCH							0	−3.67
	CRCARR								0
9	DCC-CARR	0	−3.29	2.72	0.27	−0.41	5.7	4.36	1.93
	DCC-RGARCH		0	4.14	1.81	0.94	4.76	4.84	3.92
	DCC-CR			0	−0.71	−1.2	4.18	4.32	1.29
	MHEWMA				0	−1.23	3.83	5.48	1.56
	CRHEWMA					0	3.23	7.24	2.38
	DCC-REGARCH						0	2.24	−1.33
	CRREGARCH							0	−4.12
	CRCARR								0

Panel D. Forecast horizons 10 to 12 weeks

Forecasting Horizon		DCC CARR	DCC RGARCH	DCC CR	MHEWMA	CR HEWMA	DCC REGARCH	CR REGARCH	CR CARR
10	DCC-CARR	0	−3.18	2.43	−0.32	−0.72	6.37	4.91	2.7
	DCC-RGARCH		0	3.89	1.31	0.59	4.96	5.2	4.55
	DCC-CR			0	−1.3	−1.5	5	5.31	2.37
	MHEWMA				0	−1.12	4.14	7.05	3.05
	CRHEWMA					0	3.36	7.49	3.31
	DCC-REGARCH						0	2.62	−1.15
	CRREGARCH							0	−4.44
	CRCARR								0
11	DCC-CARR	0	−3.33	2.32	−1.96	−0.95	6.09	4.26	3.01
	DCC-RGARCH		0	3.92	0.07	0.41	5	4.86	4.78
	DCC-CR			0	−3.03	−1.71	5.53	4.57	2.89
	MHEWMA				0	0.63	5.63	6.38	4.46
	CRHEWMA					0	3.55	7.43	3.83
	DCC-REGARCH						0	2.34	−0.88
	CRREGARCH							0	−4.04
	CRCARR								0
12	DCC-CARR	0	−3.48	2.68	−0.86	−1.17	5.61	3.74	2.98
	DCC-RGARCH		0	4.25	1.11	0.26	5	4.71	5.1
	DCC-CR			0	−1.8	−1.82	4.98	3.65	2.54
	MHEWMA				0	−1.32	4.57	6.41	4.19
	CRHEWMA					0	3.59	7.48	3.94
	DCC-REGARCH						0	2	−0.84
	CRREGARCH							0	−3.4
	CRCARR								0

Notes: The table shows the Diebold-Mariano statistics of the out-of-sample covariance forecast for currencies. A rolling window forecasting method is used, with a sample size of 500 observations and 631 forecast values. The performance of each pair of models is compared. A t-statistic > 1.96 indicates that the model in the column is significantly superior to the model in the row, whereas a t-statistic < -1.96 indicates the opposite.

performance of the aforementioned portfolios and the $1/N$ (i.e., equally weighted) portfolio. The portfolio variance obtained from Eq. (41) is tested using the null hypothesis that the variance of portfolio m is equivalent to the variance of the benchmark ($1/N$) portfolio i.e., $H_0: \widehat{\sigma}_m^2 - \widehat{\sigma}_{1/N}^2 = 0$. The robust non-parametric bootstrap method (Ledoit & Wolf, 2011) is employed to test the null hypothesis. The block size and number of bootstrap iterations are set as 10 and 10,000, respectively.

Table 11 panel A presents the out-of-sample portfolio variance for currencies. The portfolios derived from the multivariate REGARCH models dominate the competitors across the twelve rebalancing periods. Specifically, the CRREGARCH model generate the lowest annualised variance in 8 out 12 cases and ranks second lowest in 4 cases. Similarly, the DCC-REGARCH model performs competitively, generating the lowest and second lowest variance in 8 cases throughout the rebalancing periods. When comparing the portfolio variance of ETFs shown in Table 11 panel B, both multivariate REGARCH models have lower annualised variance than the benchmark portfolio at a 5 % significance level. In addition, the portfolios generated by the CRREGARCH model are in the top three for lowest turnover in most cases.

The results for portfolio turnover for currencies are given in Table 12 panel A. It shows that the turnover of the benchmark strategy is the lowest across the twelve rebalancing periods. Portfolios built using the CRREGARCH consistently rank among the top three for the lowest turnover in all cases. For ETFs, as shown in Table 12 panel B, the CRREGARCH model achieves the lowest turnover in 4 out of 12 cases and ranks as the second or third lowest in 5 additional cases. The DCC-RGARCH model also shows competitive performance, particularly from the 7-week rebalancing period onwards, recording the second or third lowest turnover.

7. Robustness tests

In this section, we investigate whether the performance of the developed multivariate REGARCH models remains consistent when we alter certain factors. These factors include changing the weight for S&P 500 and crude oil ETFs, modifying the combination of individual assets,

Table 5
Diebold-Mariano test for ETFs.

Panel A. Forecast horizons 1 to 3 weeks

Forecasting Horizon		DCC CARR	DCC RGARCH	DCC CR	MHEWMA	CR HEWMA	DCC REGARCH	CR REGARCH	CR CARR
1	DCC-CARR	0	5	−1.83	−0.3	−2.69	2.66	1.02	0.41
	DCC-RGARCH		0	−3.89	−2.29	−4.29	−1.05	−1.29	−1.53
	DCC-CR			0	1.07	−1.54	2.38	2.46	2.02
	MHEWMA				0	−4.19	1.46	0.94	0.53
	CRHEWMA					0	3.31	2.8	2.36
	DCC-REGARCH						0	−0.3	−0.57
	CRREGARCH							0	−0.72
	CRCARR								0
2	DCC-CARR	0	2.98	−1.85	−0.8	−3.5	3.17	1.84	0.31
	DCC-RGARCH		0	−3.99	−2.16	−4.49	0.59	0.5	−0.88
	DCC-CR			0	0.55	−3.19	2.68	2.99	1.37
	MHEWMA				0	−4.16	2.44	2.76	0.75
	CRHEWMA					0	3.98	4.56	2.79
	DCC-REGARCH						0	−0.15	−0.78
	CRREGARCH							0	−1.26
	CRCARR								0
3	DCC-CARR	0	1.16	−1.53	−1.29	−3.74	3.75	2.64	0.27
	DCC-RGARCH		0	−2.35	−1.68	−4.06	1.4	1.63	−0.33
	DCC-CR			0	−0.51	−4.16	2.76	3.32	1.09
	MHEWMA				0	−4.13	2.69	4.48	1.26
	CRHEWMA					0	4.15	5.14	3.55
	DCC-REGARCH						0	0.2	−1
	CRREGARCH							0	−1.65
	CRCARR								0

Panel B. Forecast horizons 4 to 6 weeks

Forecasting Horizon		DCC CARR	DCC RGARCH	DCC CR	MHEWMA	CR HEWMA	DCC REGARCH	CR REGARCH	CR CARR
4	DCC-CARR	0	−0.81	−0.82	−1.63	−3.88	4.02	3.51	−0.04
	DCC-RGARCH		0	0.43	−0.67	−3.82	2.17	2.86	0.76
	DCC-CR			0	−1.37	−4.45	2.74	3.02	0.27
	MHEWMA				0	−4.1	3.32	4.33	1.04
	CRHEWMA					0	4.26	4.83	3.67
	DCC-REGARCH						0	0.16	−1.25
	CRREGARCH							0	−1.8
	CRCARR								0
5	DCC-CARR	0	−1.87	−0.05	−2.04	−3.84	4.59	3.8	−0.2
	DCC-RGARCH		0	2.23	0.21	−3.21	2.81	3.34	2.04
	DCC-CR			0	−2.53	−4.48	2.77	2.82	−0.2
	MHEWMA				0	−4.04	3.57	3.92	1.34
	CRHEWMA					0	4.27	4.55	3.77
	DCC-REGARCH						0	−0.16	−1.49
	CRREGARCH							0	−1.86
	CRCARR								0
6	DCC-CARR	0	−2.67	0.47	−2.39	−3.86	5.05	4.25	−0.33
	DCC-RGARCH		0	3.44	1.13	−2.83	3.38	3.68	3.21
	DCC-CR			0	−3.06	−4.46	2.7	2.85	−0.51
	MHEWMA				0	−4.08	3.69	3.97	2.02
	CRHEWMA					0	4.29	4.6	4.16
	DCC-REGARCH						0	−0.1	−1.77
	CRREGARCH							0	−2.04
	CRCARR								0

Panel C. Forecast horizons 7 to 9 weeks

Forecasting Horizon		DCC CARR	DCC RGARCH	DCC CR	MHEWMA	CR HEWMA	DCC REGARCH	CR REGARCH	CR CARR
7	DCC-CARR	0	−3.15	0.92	−2.76	−3.88	5.58	3.88	−0.39
	DCC-RGARCH		0	3.82	1.72	−2.33	3.74	4.04	4.26
	DCC-CR			0	−3.65	−4.42	2.88	2.88	−0.71
	MHEWMA				0	−4.02	3.86	3.78	2.64
	CRHEWMA					0	4.28	4.46	4.21
	DCC-REGARCH						0	−0.51	−1.83
	CRREGARCH							0	−1.86
	CRCARR								0
8	DCC-CARR	0	−3.68	1.27	−3.05	−3.98	6.33	3.56	−0.43
	DCC-RGARCH		0	4.36	2.24	−2.03	4.26	4.5	4.78

(*continued on next page*)

Table 5 (*continued*)

Panel C. Forecast horizons 7 to 9 weeks

Forecasting Horizon		DCC CARR	DCC RGARCH	DCC CR	MHEWMA	CR HEWMA	DCC REGARCH	CR REGARCH	CR CARR
	DCC-CR			0	−3.96	−4.49	3.04	2.73	−0.91
	MHEWMA				0	−3.97	4.09	3.82	3.2
	CRHEWMA					0	4.4	4.49	4.26
	DCC-REGARCH						0	−0.47	−2.19
	CRREGARCH							0	−2.03
	CRCARR								0
9	DCC-CARR	0	−3.97	1.49	−3.03	−3.9	6.13	4.4	−0.43
	DCC-RGARCH		0	4.58	2.98	−1.48	4.42	4.73	5.06
	DCC-CR			0	−3.85	−4.37	2.87	3.29	−0.99
	MHEWMA				0	−3.89	4	3.99	3.63
	CRHEWMA					0	4.29	4.44	4.25
	DCC-REGARCH						0	−0.37	−2.17
	CRREGARCH							0	−2.14
	CRCARR								0

Panel D. Forecast horizons 10 to 12 weeks

Forecasting Horizon		DCC CARR	DCC RGARCH	DCC CR	MHEWMA	CR HEWMA	DCC REGARCH	CR REGARCH	CR CARR
10	DCC-CARR	0	−4.74	1.67	−3.29	−4.05	6.59	5.51	−0.02
	DCC-RGARCH		0	5.34	3.69	−0.9	5.24	5.64	5.45
	DCC-CR			0	−4.07	−4.49	3.03	4.23	−0.69
	MHEWMA				0	−3.91	4.5	4.64	3.84
	CRHEWMA					0	4.5	4.69	4.26
	DCC-REGARCH						0	0.15	−2.4
	CRREGARCH							0	−2.6
	CRCARR								0
11	DCC-CARR	0	−5.35	1.55	−3.44	−4.13	6.83	6.2	0.67
	DCC-RGARCH		0	5.89	4.43	−0.42	5.84	6.25	5.86
	DCC-CR			0	−4.11	−4.5	3.46	5.42	−0.06
	MHEWMA				0	−3.92	4.7	4.9	4.11
	CRHEWMA					0	4.59	4.81	4.36
	DCC-REGARCH						0	0.81	−2.61
	CRREGARCH							0	−3.11
	CRCARR								0
12	DCC-CARR	0	−5.41	1.55	−3.31	−4.05	7.06	6.44	0.98
	DCC-RGARCH		0	5.88	4.98	0.19	5.83	6.19	6.06
	DCC-CR			0	−3.9	−4.39	3.74	5.6	0.25
	MHEWMA				0	−3.91	4.41	4.61	4
	CRHEWMA					0	4.5	4.69	4.33
	DCC-REGARCH						0	1.19	−2.36
	CRREGARCH							0	−3.16
	CRCARR								0

Notes: The table shows the Diebold-Mariano statistics of the out-of-sample covariance forecast for ETFs (S&P 500 ETF Trust (SPY) and United States Oil Fund (USO)). A rolling window forecasting method is used, with a sample size of 500 observations and 450 forecast results. The performance of each pair of models is compared. A t-statistic > 1.96 indicates that the model in the column is significantly superior to the model in the row, whereas a t-statistic < −1.96 indicates the opposite. The weights for the S&P 500 and oil ETFs are both set at 50 %.

adjusting the out-of-sample forecast period, and varying the in-sample size.

7.1. Different weight set for the co-range-based models

Within the co-range framework, determining the initial weight for individual assets is necessary for calculating the price of a pseudo portfolio. Following the methodology of Bannouh et al. (2009), we initially set the weight for the S&P 500 at 50 %. To investigate the potential impact of varying weight assignments on forecasting performance, we conducted additional tests by assigning weights of 30 % and 70 % to the S&P 500, keeping other conditions constant.

The RMSE values for the eight competing models are presented in Table A1.1. With the S&P 500 weight set at 30 %, the DCC-REGARCH model outperforms the competitors in 9 out of 12 cases, followed by the CRREGARCH model, which is the best in 2 out of 12 cases and the second-best in 8 out of 12 cases. When the S&P 500 weight is set at 70 %, the CRREGARCH model dominates the competing models in most cases, followed by the DCC-REGARCH model.

Tables A1.2 and A1.3 show the Diebold-Mariano test results for the S&P500 weight set at 30 % and 70 %, respectively. Similar to the results shown in Table 5, none of the competing models significantly outperform the two multivariate REGARCH models. The DCC-REGARCH model is significantly superior to all the competitors for forecast horizons from 4 weeks onward, under both the 30 % and 70 % S&P 500 weightings. The CREGARCH model is significantly superior to all the competitors, except the DCC-REGARCH model, for forecast horizons starting at 6 weeks under both weightings. The R^2 value of the forecast regressions are shown in Table A1.4. The DCC-REGARCH demonstrates the highest predictive power, leading in 9 out of 12 cases across both 30 % and 70 % weightings of the S& P500.

The outcomes of the loss functions for the eight competing models, with the S&P 500 weight set at 30 % and 70 %, are shown in Table A1.5 and A1.7 of the online appendix. The values observed in these tables closely mirror those presented in Table 9, consistently showing the superior performance of the multivariate REGARCH models, especially for forecast horizon beyond 3 weeks. Notably, the DCC-REGARCH and CRREGARCH models exhibit closely matched results. The Model

Table 6
Forecast Regression R-squared Value.

Panel A. Forecast Regression R-squared Value for Currencies

Forecast Horizon	Forecasting Model							
	Existing Models					New Models		
	DCC CARR	DCC RGARCH	DCC CR	MHEWMA	CR HEWMA	DCC REGARCH	CR REGARCH	CR CARR
1	29.50	24.56	31.24	31.92	33.22	31.95	***35.40***	32.48
2	18.87	13.28	20.63	20.48	20.56	22.08	***24.98***	20.11
3	13.60	8.56	15.81	14.19	14.10	17.04	***19.45***	14.00
4	10.92	6.51	12.73	10.83	10.60	14.84	***18.00***	11.20
5	9.66	5.87	11.44	9.27	8.77	13.53	***16.89***	10.17
6	8.33	5.22	9.53	8.38	7.61	12.57	***16.86***	9.39
7	6.97	4.05	8.07	7.55	6.73	10.97	***14.70***	7.93
8	6.00	3.59	7.12	6.30	5.62	9.64	***12.66***	6.61
9	5.03	3.01	6.13	5.30	4.66	8.38	***10.74***	5.42
10	4.39	2.75	5.32	4.24	3.79	7.74	***10.44***	5.24
11	3.93	2.40	4.71	3.79	3.22	6.93	***9.48***	4.96
12	3.58	2.03	4.32	3.25	2.68	6.36	***8.46***	4.47

Panel B. Forecast Regression R-squared Value for ETFs (SPY and USO)

Forecast Horizon	Forecasting Model							
	Existing Models					New Models		
	DCC CARR	DCC RGARCH	DCC CR	MHEWMA	CR HEWMA	DCC REGARCH	CR REGARCH	CR CARR
1	45.73	48.12	30.46	16.07	12.64	***54.40***	42.14	40.48
2	24.36	25.36	13.31	7.56	5.52	***33.63***	24.78	22.13
3	11.62	11.75	6.00	3.16	2.36	***16.70***	11.61	8.42
4	5.10	4.61	2.81	1.92	1.44	***7.34***	5.35	3.01
5	2.79	2.49	1.53	1.07	0.89	***4.12***	3.03	1.69
6	1.36	1.02	0.77	0.72	0.64	***1.95***	1.28	0.46
7	1.02	0.72	0.65	0.53	0.50	***1.44***	0.74	0.30
8	0.66	0.56	0.46	0.30	0.36	***0.90***	0.37	0.12
9	0.27	0.26	0.19	0.29	0.31	***0.32***	0.04	0.01
10	0.11	0.18	0.07	***0.35***	0.32	0.09	0.01	0.02
11	0.04	0.16	0.01	***0.47***	0.38	0.03	0.00	0.06
12	0.01	0.17	0.00	***0.49***	0.39	0.00	0.01	0.31

Notes: The table shows the R^2 value of the forecast regression results for currencies and ETFs (S&P 500 ETF Trust (SPY) and United States Oil Fund (USO)). The dependent variable is realised covariance - the proxy of the true covariance, while the independent variable is the forecasted covariance obtained from the eight competing models. A rolling window forecasting method is used, with a sample size of 500 observations. The number of forecast results is 631 for currencies and 450 for ETFs. All results are multiplied by 100. The highest result is shown in bold and italics.

Confidence Set (MCS) results, as shown in Tables A1.6 and A1.8, further corroborate the dominance of the CRREGARCH and DCC-REGARCH models. These models consistently rank first or second over forecast horizons from three weeks to twelve weeks. Specifically, the CRREGARCH model outperforms its peers in most scenarios when evaluated using the Euclidean and Frobenius loss functions, whereas the DCC-REGARCH model takes the lead in the majority of cases when assessed using the QLIKE loss function.

Our findings underscore the robustness of the co-range models' forecasting performance, irrespective of the weight assignments to the assets. While the co-range framework requires the assignment of specific weights to assets, our empirical results indicate that forecasting accuracy remains consistent across different weight configurations.

7.2. Different asset combination

To assess whether varying asset combinations yield similar outcomes, we consider the combination of two alternative ETFs, SPDR Gold Shares (GLD) and United States Oil Fund (USO). The data spans from 10 April 2006 to 30 June 2024. In parallel with the previous covariance forecasting approach, we use a rolling window of 500 observations to forecast the variance-covariance matrix for gold and oil ETFs. Both assets are equally weighted at 50 %. The RMSE results for the eight competing models are presented in Table A2.1. Consistent with the forecasting results for the S&P 500 and oil ETFs shown in Table 3, the DCC-REGARCH model outperforms its competitors in most cases (7 out of 12), followed closely by the CRREGARCH model, which also demonstrates top or near-top performance in most instances.

The Diebold-Mariano test results, detailed in Table A2.2, indicate that none of the competing models significantly outperform the DCC-REGARCH model. There is only one instance where the CRREGARCH model is significantly outperformed by a competitor. Notably, the DCC-REGARCH model significantly dominates all competitors from the 6-week forecast horizon onwards. Table A2.4 presents the R^2 values of the forecast regressions. It shows that the CRREGARCH model has the better predictive power from the 6-week forecast horizon onward, outperforming other models in 7 out of 12 cases.

The results of loss functions and MCS test are shown in Table A2.4 and A2.5, revealing that the CRREGARCH is the top-performing model, followed by the DCC-REGARCH model. These findings suggest that the multivariate REGARCH models' efficacy remains consistent despite changes in portfolio composition.

7.3. Different out-of-sample period

In this subsection, we investigate whether the performance of the models varies over different forecast periods. Maintaining the in-sample size at 500, we split the initial forecast period in half. For currencies, the

Table 7
Out-of-sample forecast losses for currencies.

Panel A. Forecast horizons 1 to 4 weeks

Horizon	1 week			2 weeks			3 weeks			4 weeks		
	Euclidean	Frobenius	QLIKE	Euclidean	Frobenius	QLIKE	Euclidean	Frobenius	QLIKE	Euclidean	Frobenius	QLIKE
DCC-CARR	0.000629	0.000681	−16.482	0.000711	0.000772	−16.445	0.000752	0.000818	−16.423	0.000752	0.000818	−16.423
DCC-RGARCH	0.000678	0.000732	−16.455	0.000778	0.000843	−16.438	0.000828	0.000898	−16.415	0.000828	0.000898	−16.415
DCC-CR	0.000628	0.000678	***−16.509***	0.000709	0.000769	−16.496	0.000750	0.000814	−16.450	0.000750	0.000814	−16.450
MHEWMA	0.000780	0.000838	−15.801	0.000802	0.000863	−15.903	0.000812	0.000876	−15.926	0.000812	0.000876	−15.926
CRHEWMA	0.000783	0.000840	−15.875	0.000804	0.000866	−15.746	0.000815	0.000879	−15.940	0.000815	0.000879	−15.940
DCC-REGARCH	0.000609	0.000659	−16.476	0.000667	0.000724	−16.457	0.000702	0.000763	−16.414	0.000702	0.000763	−16.414
CRREGARCH	***0.000605***	***0.000651***	−16.508	***0.000663***	***0.000716***	−16.485	***0.000699***	***0.000756***	***−16.474***	***0.000699***	***0.000756***	***−16.474***
CRCARR	0.000625	0.000674	−16.507	0.000708	0.000767	***−16.503***	0.000750	0.000814	−16.461	0.000750	0.000814	−16.461

Panel B. Forecast horizons range from 5 to 8 weeks

Horizon	5 weeks			6 weeks			7 weeks			8 weeks		
	Euclidean	Frobenius	QLIKE	Euclidean	Frobenius	QLIKE	Euclidean	Frobenius	QLIKE	Euclidean	Frobenius	QLIKE
DCC-CARR	0.000778	0.000847	−16.397	0.000787	0.000858	−16.422	0.000799	0.000872	−16.383	0.000803	0.000877	−16.368
DCC-RGARCH	0.000862	0.000936	−16.358	0.000873	0.000947	−16.390	0.000890	0.000967	−16.356	0.000892	0.000970	−16.343
DCC-CR	0.000776	0.000843	−16.422	0.000785	0.000853	−16.420	0.000797	0.000869	−16.404	0.000801	0.000873	−16.362
MHEWMA	0.000821	0.000886	−15.885	0.000824	0.000890	−15.845	0.000849	0.000915	−15.756	0.000809	0.000882	−16.283
CRHEWMA	0.000823	0.000889	−15.801	0.000827	0.000893	−15.929	0.000851	0.000918	−15.717	0.000811	0.000887	−16.272
DCC-REGARCH	0.000722	0.000785	−16.394	0.000733	0.000797	−16.415	0.000744	0.000810	***−16.407***	0.000751	0.000817	−16.393
CRREGARCH	***0.000718***	***0.000776***	***−16.451***	***0.000728***	***0.000787***	***−16.426***	***0.000739***	***0.000800***	−16.391	***0.000746***	***0.000809***	***−16.403***
CRCARR	0.000776	0.000842	−16.434	0.000784	0.000852	−16.407	0.000795	0.000864	−16.367	0.000799	0.000869	−16.379

Panel C. Forecast horizons range from 9 to 12 weeks

Horizon	9 weeks			10 weeks			11 weeks			12 weeks		
	Euclidean	Frobenius	QLIKE	Euclidean	Frobenius	QLIKE	Euclidean	Frobenius	QLIKE	Euclidean	Frobenius	QLIKE
DCC-CARR	0.000802	0.000877	−16.330	0.000800	0.000876	−16.321	0.000792	0.000868	−16.317	0.000781	0.000857	−16.288
DCC-RGARCH	0.000892	0.000971	−16.305	0.000887	0.000967	−16.326	0.000879	0.000960	−16.297	0.000869	0.000951	−16.269
DCC-CR	0.000800	0.000873	−16.350	0.000799	0.000873	−16.341	0.000790	0.000865	−16.311	0.000779	0.000854	−16.308
MHEWMA	0.000846	0.000913	−15.775	0.000852	0.000921	−15.699	0.000852	0.000921	−15.731	0.000832	0.000911	−15.974
CRHEWMA	0.000847	0.000916	−15.954	0.000852	0.000922	−15.837	0.000853	0.000922	−15.842	0.000835	0.000916	−15.967
DCC-REGARCH	0.000752	0.000820	−16.351	0.000751	0.000820	−16.368	0.000745	0.000815	−16.337	0.000743	0.000813	−16.332
CRREGARCH	***0.000748***	***0.000812***	***−16.389***	***0.000747***	***0.000812***	***−16.380***	***0.000741***	***0.000806***	***−16.347***	***0.000740***	***0.000806***	***−16.341***
CRCARR	0.000798	0.000869	−16.365	0.000796	0.000866	−16.355	0.000786	0.000857	−16.323	0.000776	0.000847	−16.317

Notes: The table shows the out-of-sample forecast losses of the variance-covariance matrix for currencies over the forecast horizons ranging from 1 week to 12 weeks. A rolling window forecasting method with ta window size of 500 is used. The values of Euclidean and Frobenius loss functions are multiplied by 10,000. The model with the lowest value is shown in bold and italics.

forecast period is divided into 25 May 2012 to 01 June 2018 and 08 June 2018 to 30 June 2024. Each sub-period covers 315 weeks. Similarly, the forecast period for S&P 500 and the oil ETFs is segmented into 13 November 2015 to 28 February 2015 and 06 March 2015 to 30 June 2024, with each sub-period spanning 225 weeks.

Table A3.1 presents the RMSE results for the two currencies (GBP/USD and EUR/USD) over the two out-of-sample periods. The CRREGARCH dominates its competitors in both periods. The MHEWMA ranks second in the first period (25 May 2012 to 01 June 2018), while the DCC-REGARCH model has the second-best performance in the second period (08 June 2018 to 30 June 2024). The RMSE results for the S&P 500 and oil ETFs are shown in Table A4.1. The CRREGARCH model leads in the first period (13 November 2015 to 28 February 2020), whereas the DCC-REGARCH model exhibits the best forecast accuracy in the second period (06 March 2020 to 30 June 2024), with the CRREGARCH consistently ranking second.

The Diebold-Mariano test results for currencies, presented in Table A3.2 and Table A3.3, confirm the superior performance of the CRREGARCH model, which significantly outperforms other models, including the DCC-REGARCH model in most cases. The only exceptions occur over the 10-, 11- and 12-week forecast horizons during the first period, where the CRHEWMA model excels. The DCC-REGARCH model performs better in the second period from (08 June 2018 to 30 June 2024) than in the first period (25 May 2012 to 01 June 2018), where it is significantly outperformed by the MHEWMA and CRHEWMA models.

The Diebold-Mariano test results for the S&P 500 and oil ETFs, shown in Table A4.2 and Table A4.3, demonstrate that no competing models significantly outperform the two multivariate REGARCH models across the two out-of-sample periods, except in two cases where the DCC-REGARCH model is outperformed by the DCC-RGARCH model.

Regarding the R^2 values of the forecast regressions, as shown in Tables A3.4 and A4.4, the DCC-REGARCH model showcases the highest predictive power for ETFs across both periods and for currencies in the first period. In contrast, the CRREGARCH model demonstrates improved performance in the second period for currencies.

Tables A3.5 through A3.8 and A4.5 through A4.8 present the results for the loss functions and MCS tests for currencies and ETFs, respectively. These results consistently show that the CRREGARCH model

Table 8
Model Confidence Set results for currencies.

Panel A. Forecast horizons range from 1 to 3 weeks

Horizon	1 week						2 weeks						3 weeks					
	Euclidean		Frobenius		QLIKE		Euclidean		Frobenius		QLIKE		Euclidean		Frobenius		QLIKE	
	p-val	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank
DCC-CARR	0.06	5	0.06	5	0.01	4	0.06	5	0.06	5	0.01	4	0.02	5	0.02	5	0.00	6
DCC-RGARCH	0.04	6	0.03	6	0.00	6	0.04	6	0.03	6	0.00	6	0.01	7	0.01	8	0.01	4
DCC-CR	0.08	4	0.09	4	1.00	***1***	0.08	4	0.09	4	1.00	***1***	0.02	4	0.03	4	0.01	3
MHEWMA	0.01	7	0.00	7	0.00	8	0.01	7	0.00	7	0.00	8	0.01	6	0.01	6	0.00	8
CRHEWMA	0.00	8	0.00	8	0.00	7	0.00	8	0.00	8	0.00	7	0.00	8	0.01	7	0.00	7
DCC-REGARCH	0.08	2	0.09	2	0.00	5	0.08	2	0.09	2	0.00	5	0.08	2	0.08	2	0.01	5
CRREGARCH	1.00	***1***	1.00	***1***	0.97	2	1.00	***1***	1.00	***1***	0.97	2	1.00	***1***	1.00	***1***	1.00	***1***
CRCARR	0.08	3	0.09	3	0.97	3	0.08	3	0.09	3	0.97	3	0.02	3	0.03	3	0.01	2

Panel B. Forecast horizons range from 4 to 6 weeks

Horizon	4 weeks						5 weeks						6 weeks					
	Euclidean		Frobenius		QLIKE		Euclidean		Frobenius		QLIKE		Euclidean		Frobenius		QLIKE	
	p-val	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank
DCC-CARR	0.01	5	0.01	5	0.00	5	0.01	5	0.01	5	0.97	2	0.01	5	0.01	5	0.97	2
DCC-RGARCH	0.01	8	0.01	8	0.00	6	0.01	8	0.01	8	0.06	6	0.01	8	0.01	8	0.06	6
DCC-CR	0.01	4	0.01	4	0.00	3	0.01	4	0.01	4	0.97	3	0.01	4	0.01	4	0.97	3
MHEWMA	0.01	6	0.01	6	0.00	8	0.01	6	0.01	6	0.00	7	0.01	6	0.01	6	0.00	7
CRHEWMA	0.01	7	0.01	7	0.00	7	0.01	7	0.01	7	0.00	8	0.01	7	0.01	7	0.00	8
DCC-REGARCH	0.02	2	0.02	2	0.00	4	0.01	2	0.02	2	0.97	4	0.01	2	0.02	2	0.97	4
CRREGARCH	1.00	***1***	1.00	***1***	1.00	***1***	1.00	***1***	1.00	***1***	1.00	***1***	1.00	***1***	1.00	***1***	1.00	***1***
CRCARR	0.01	3	0.01	3	0.00	2	0.01	3	0.01	3	0.13	5	0.01	3	0.01	3	0.13	5

Panel C. Forecast horizons range from 7 to 9 weeks

Horizon	7 weeks						8 weeks						9 weeks					
	Euclidean		Frobenius		QLIKE		Euclidean		Frobenius		QLIKE		Euclidean		Frobenius		QLIKE	
	p-val	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank
DCC-CARR	0.00	5	0.01	5	0.01	4	0.01	5	0.01	5	0.01	4	0.01	5	0.01	5	0.00	6
DCC-RGARCH	0.00	8	0.01	8	0.00	6	0.01	8	0.01	8	0.00	6	0.01	8	0.01	8	0.00	5
DCC-CR	0.00	4	0.01	4	0.88	2	0.01	4	0.01	4	0.00	5	0.01	4	0.01	4	0.00	4
MHEWMA	0.00	6	0.01	6	0.00	8	0.01	6	0.01	6	0.00	8	0.01	6	0.01	6	0.00	8
CRHEWMA	0.00	7	0.01	7	0.00	7	0.01	7	0.01	7	0.00	7	0.01	7	0.01	7	0.00	7
DCC-REGARCH	0.01	2	0.01	2	1.00	***1***	0.03	2	0.02	2	0.72	2	0.07	2	0.06	2	0.00	3
CRREGARCH	1.00	***1***	1.00	***1***	0.88	3	1.00	***1***	1.00	***1***	1.00	***1***	1.00	***1***	1.00	***1***	1.00	***1***
CRCARR	0.00	3	0.01	3	0.01	5	0.01	3	0.01	3	0.01	3	0.01	3	0.01	3	0.00	2

Panel D. Forecast horizons range from 10 to 12 weeks

Horizon	10 weeks						11 weeks						12 weeks					
	Euclidean		Frobenius		QLIKE		Euclidean		Frobenius		QLIKE		Euclidean		Frobenius		QLIKE	
	p-val	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank
DCC-CARR	0.01	5	0.01	4	0.00	6	0.01	5	0.01	4	0.09	4	0.01	5	0.01	4	0.09	4
DCC-RGARCH	0.01	8	0.01	8	0.01	5	0.01	8	0.01	8	0.03	6	0.01	8	0.01	8	0.03	6
DCC-CR	0.01	4	0.01	5	0.01	4	0.01	4	0.01	5	0.03	5	0.01	4	0.01	5	0.03	5
MHEWMA	0.01	7	0.01	7	0.00	8	0.01	7	0.01	7	0.00	8	0.01	7	0.01	7	0.00	8
CRHEWMA	0.01	6	0.01	6	0.00	7	0.01	6	0.01	6	0.00	7	0.01	6	0.01	6	0.00	7
DCC-REGARCH	0.03	2	0.03	2	0.63	2	0.05	2	0.05	2	0.83	2	0.05	2	0.05	2	0.83	2
CRREGARCH	1.00	***1***	1.00	***1***	1.00	***1***	1.00	***1***	1.00	***1***	1.00	***1***	1.00	***1***	1.00	***1***	1.00	***1***
CRCARR	0.01	3	0.01	3	0.01	3	0.01	3	0.01	3	0.09	3	0.01	3	0.01	3	0.09	3

Notes: The table shows the results of Model Confidence Set (MCS) for currencies at a 5 % significance level. A rolling window forecasting method with a sample size of 500 is used. The number of forecast values is 631. The forecast horizon ranges from one week to twelve weeks. The model with the lowest value is shown in bold and italics.

maintains robust performance, outperforming competitors in the majority of cases. Similar to the results shown in Table 9 and 10, the CRREGARCH model ranks first under the Euclidean and Frobenius loss funcation, whereas the DCC-REGARCH model remains the best performing model when the QLIKE function is employed.

Overall, these findings highlight that the multivariate REGARCH models display consistent performance across varying forecast periods, whether the market exhibits volatile or relatively stable behaviour.

7.4. Different in-sample size

In line with Chou et al. (2009) and Harris and Yilmaz (2010), our comparison of the variances-covariance estimators is based on 500 observations. To assess weather a change of in-sample size affects the forecasting performance of the eight models, we increase the rolling sample size to 600 for both currencies and S&P 500 and oil ETFs. Consistent with the findings detailed in Table 3, the RMSE results shown

Table 9
Out-of-sample forecast losses for ETFs (SPY and USO).

Panel A. Forecast horizons range from 1 to 4 weeks

Horizon	1 week			2 weeks			3 weeks			4 weeks		
	Euclidean	Frobenius	QLIKE	Euclidean	Frobenius	QLIKE	Euclidean	Frobenius	QLIKE	Euclidean	Frobenius	QLIKE
DCC-CARR	0.2986	0.3026	−12.833	0.3064	0.3108	−12.690	0.3043	0.3090	−12.565	0.3375	0.3422	−12.454
DCC-RGARCH	0.1939	0.1966	***−13.035***	0.2153	0.2193	***−12.815***	0.2322	0.2374	−12.627	0.2897	0.2958	−12.462
DCC-CR	0.3005	0.3064	−12.843	0.3073	0.3127	−12.711	0.3046	0.3096	−12.589	0.3374	0.3421	−12.465
MHEWMA	***0.1905***	***0.1933***	−12.731	***0.2054***	***0.2090***	−12.583	***0.2179***	***0.2222***	−12.473	0.2373	0.2422	−12.374
CRHEWMA	0.2287	0.2444	−11.236	0.2345	0.2504	−11.008	0.2702	0.2864	−10.642	0.2758	0.2922	−10.327
DCC-REGARCH	0.2622	0.2655	−12.913	0.2493	0.2524	−12.711	0.2488	0.2520	−12.631	0.2365	0.2399	***−12.511***
CRREGARCH	0.2180	0.2212	−13.005	0.2144	0.2167	−12.810	0.2424	0.2451	***−12.669***	***0.2287***	***0.2318***	−12.031
CRCARR	0.2290	0.2329	−12.958	0.2511	0.2558	−12.784	0.2425	0.2473	−12.639	0.2698	0.2749	−12.246

Panel B. Forecast horizons range from 5 to 8 weeks

Horizon	5 weeks			6 weeks			7 weeks			8 weeks		
	Euclidean	Frobenius	QLIKE	Euclidean	Frobenius	QLIKE	Euclidean	Frobenius	QLIKE	Euclidean	Frobenius	QLIKE
DCC-CARR	0.3312	0.3359	−12.389	0.3512	0.3557	−12.310	0.3420	0.3463	−12.270	0.3365	0.3406	−12.223
DCC-RGARCH	0.2986	0.3051	−12.351	0.3623	0.3693	−12.226	0.3805	0.3877	−12.125	0.3987	0.4061	−12.031
DCC-CR	0.3311	0.3355	−12.404	0.3510	0.3553	−12.320	0.3418	0.3459	−12.277	0.3363	0.3402	−12.232
MHEWMA	0.2451	0.2502	−12.255	0.2605	0.2658	−12.232	0.2745	0.2799	−12.170	0.2801	0.2856	−12.098
CR-HEWMA	0.2800	0.2965	−10.504	0.2823	0.2990	−10.642	0.2846	0.3013	−10.635	0.2871	0.3040	−10.423
DCC-REGARCH	0.2243	0.2277	***−12.464***	0.2338	0.2373	***−12.439***	0.2291	0.2324	***−12.407***	0.2236	0.2269	***−12.368***
CRREGARCH	***0.2205***	***0.2237***	−12.430	***0.2136***	***0.2169***	−12.421	***0.2022***	***0.2055***	−12.182	***0.1990***	***0.2023***	−12.286
CRCARR	0.3012	0.3064	−12.181	0.2963	0.3003	−12.357	0.2864	0.2904	−12.332	0.2787	0.2826	−12.138

Panel C. Forecast horizons range from 9 to 12 weeks

Horizon	9 weeks			10 weeks			11 weeks			12 weeks		
	Euclidean	Frobenius	QLIKE	Euclidean	Frobenius	QLIKE	Euclidean	Frobenius	QLIKE	Euclidean	Frobenius	QLIKE
DCC-CARR	0.3310	0.3351	−12.176	0.3226	0.3266	−12.159	0.3150	0.3189	−12.129	0.3063	0.3101	−12.129
DCC-RGARCH	0.4151	0.4228	−11.951	0.4288	0.4368	−11.898	0.4423	0.4504	−11.841	0.4542	0.4625	−11.798
DCC-CR	0.3308	0.3346	−12.164	0.3224	0.3262	−12.147	0.3149	0.3186	−12.113	0.3061	0.3098	−12.114
MHEWMA	0.2848	0.2904	−12.107	0.2871	0.2928	−12.089	0.2893	0.2949	−12.077	0.2904	0.2959	−12.071
CRHEWMA	0.2888	0.3058	−10.457	0.2910	0.3081	−9.907	0.2931	0.3104	−10.209	0.2941	0.3116	−10.689
DCC-REGARCH	0.2209	0.2243	***−12.306***	0.2158	0.2191	−12.286	0.2117	0.2149	−12.259	0.2071	0.2103	***−12.264***
CRREGARCH	***0.1996***	***0.2030***	−12.300	***0.1965***	***0.1998***	***−12.297***	***0.1937***	***0.1968***	***−12.277***	***0.1909***	***0.1940***	−12.208
CRCARR	0.2762	0.2801	−12.222	0.2700	0.2737	−12.205	0.2653	0.2689	−12.174	0.2596	0.2631	−12.178

Notes: The table shows the out-of-sample forecast losses of the variance-covariance matrix for S&P 500 ETF Trust (SPY) and United States Oil Fund (USO) over the forecast horizons ranging from 1 week to 12 weeks. A rolling window forecasting method with a sample size of 500 is used. The values of Euclidean and Frobenius loss function are multiplied by 10,000. The model with the lowest value is shown in bold and italics. The weights for the S&P 500 and oil ETFs are both set at 50 %.

in Table A5.1 demonstrates that the multivariate REGARCH models outperform their competitors. In currencies forecasting, the CRREGARCH model is the top performer, followed by the DCC-REGARCH model, which ranks the second in all cases. For the S&P 500 and oil ETFs, the DCC-REGARCH model performs the best in 7 out of 12 cases, followed by the CRREGARCH model leading in 4 cases and the CRCARR model in 1 case.

The robust performance of the multivariate REGARCH models is further supported by the Diebold-Mariano test results shown in Table A5.2 and Table A5.3. No model significantly outperforms the two developed models. Notably, both models significantly dominate all competitors for forecast horizons of 3 weeks and onwards for currencies and ETFs. In addition, the CREGARCH model generates the highest R^2 values when forecasting currencies, as evidenced in Table 5.4, while the DCC-REGARCH model exhibits the highest predictive power when forecasting ETFs, echoing findings in Table 6.

Table A5.5, A5.6, A5.7 and A5.8 show that the CRREGARCH model consistently outperforms its counterparts in forecasting the variance-covariance matrix for both currencies and ETFs across three loss functions, with the DCC-REGARCH model ranking a close second. The results signify that the variation in in-sample size does not impede the robust performance of the multivariate REGARCH models.

8. Conclusion

Various range-based covariance estimators have been developed, as the range is more informative than the squared/absolute return when measuring the volatility of financial assets. The DCC, co-range, and hybrid EMWA models are the three main multivariate volatility frameworks used with univariate range-based volatility models, such as the Parkinson range, CARR and RGARCH models, to measure and forecast the variance-covariance matrix.

In this paper, we develop new DCC-REGARCH and CRREGARCH (i.e., co-range REGARCH) models by incorporating the REGARCH model into DCC and co-range frameworks. To thoroughly compare the performance of the new multivariate range-based models, we also develop a

Table 10
Model Confidence Set results for ETFs (SPY and USO).

Panel A. Forecast horizons range from 1 to 3 weeks

Horizon	1 week						2 weeks						3 weeks					
	Euclidean		Frobenius		QLIKE		Euclidean		Frobenius		QLIKE		Euclidean		Frobenius		QLIKE	
	p-val	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank
DCC-CARR	0.17	7	0.14	7	0.00	7	0.24	7	0.19	7	0.00	6	0.42	7	0.37	7	0.04	6
DCC-RGARCH	0.95	2	0.96	2	1.00	***1***	0.98	2	0.98	3	1.00	***1***	0.91	2	0.92	2	0.94	4
DCC-CR	0.12	8	0.11	8	0.00	5	0.19	8	0.16	8	0.02	5	0.35	8	0.32	8	0.49	5
MHEWMA	1.00	***1***	1.00	***1***	0.00	6	1.00	***1***	1.00	***1***	0.00	7	1.00	***1***	1.00	***1***	0.04	7
CRHEWMA	0.30	4	0.23	5	0.00	8	0.45	4	0.29	4	0.00	8	0.71	6	0.58	6	0.02	8
DCC-REGARCH	0.29	6	0.23	6	0.01	4	0.43	5	0.29	5	0.17	4	0.82	5	0.83	5	0.94	3
CRREGARCH	0.48	3	0.46	3	0.52	2	0.98	3	0.98	2	0.96	2	0.91	3	0.92	3	1.00	***1***
CRCARR	0.29	5	0.27	4	0.08	3	0.36	6	0.27	6	0.85	3	0.82	4	0.83	4	0.94	2

Panel B. Forecast horizons range from 4 to 6 weeks

Horizon	4 weeks						5 weeks						6 weeks					
	Euclidean		Frobenius		QLIKE		Euclidean		Frobenius		QLIKE		Euclidean		Frobenius		QLIKE	
	p-val	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank
DCC-CARR	0.17	8	0.15	8	0.52	4	0.22	8	0.18	8	0.31	4	0.09	7	0.08	7	0.01	5
DCC-RGARCH	0.30	6	0.24	6	0.70	3	0.27	5	0.39	4	0.29	5	0.08	8	0.07	8	0.01	7
DCC-CR	0.20	7	0.17	7	0.70	2	0.22	7	0.18	7	0.48	3	0.09	6	0.08	6	0.02	4
MHEWMA	0.96	3	0.96	3	0.52	5	0.76	3	0.73	3	0.15	7	0.17	3	0.16	3	0.01	6
CRHEWMA	0.39	5	0.28	5	0.07	8	0.27	4	0.18	5	0.04	8	0.09	5	0.08	5	0.01	8
DCC-REGARCH	0.96	2	0.96	2	1.00	***1***	0.93	2	0.92	2	1.00	***1***	0.19	2	0.19	2	1.00	***1***
CRREGARCH	1.00	***1***	1.00	***1***	0.52	7	1.00	***1***	1.00	***1***	0.72	2	1.00	***1***	1.00	***1***	0.82	2
CRCARR	0.58	4	0.56	4	0.52	6	0.25	6	0.18	6	0.29	6	0.12	4	0.12	4	0.25	3

Panel C. Forecast horizons range from 7 to 9 weeks

Horizon	7 weeks						8 weeks						9 weeks					
	Euclidean		Frobenius		QLIKE		Euclidean		Frobenius		QLIKE		Euclidean		Frobenius		QLIKE	
	p-val	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank
DCC-CARR	0.05	7	0.04	7	0.02	4	0.04	7	0.03	7	0.02	4	0.03	7	0.04	7	0.02	4
DCC-RGARCH	0.04	8	0.04	8	0.01	7	0.04	8	0.03	8	0.01	7	0.03	8	0.03	8	0.01	7
DCC-CR	0.05	6	0.04	6	0.02	3	0.04	6	0.03	6	0.02	3	0.03	6	0.04	6	0.01	5
MHEWMA	0.05	3	0.04	3	0.02	6	0.04	4	0.03	4	0.02	6	0.03	5	0.04	4	0.01	6
CRHEWMA	0.05	5	0.04	5	0.01	8	0.04	5	0.03	5	0.01	8	0.03	4	0.04	5	0.01	8
DCC-REGARCH	0.05	2	0.04	2	1.00	***1***	0.04	2	0.03	2	1.00	***1***	0.03	2	0.04	2	1.00	***1***
CRREGARCH	1.00	***1***	1.00	***1***	0.02	5	1.00	***1***	1.00	***1***	0.54	2	1.00	***1***	1.00	***1***	0.92	2
CRCARR	0.05	4	0.04	4	0.21	2	0.04	3	0.03	3	0.02	5	0.03	3	0.04	3	0.18	3

Panel D. Forecast horizons range from 10 to 12 weeks

Horizon	10 weeks						11 weeks						12 weeks					
	Euclidean		Frobenius		QLIKE		Euclidean		Frobenius		QLIKE		Euclidean		Frobenius		QLIKE	
	p-val	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank	*p-val*	Rank
DCC-CARR	0.03	7	0.03	7	0.02	4	0.03	7	0.03	6	0.04	4	0.03	7	0.02	6	0.05	4
DCC-RGARCH	0.03	8	0.03	8	0.01	7	0.03	8	0.03	8	0.01	7	0.03	8	0.02	8	0.01	7
DCC-CR	0.03	6	0.03	5	0.01	5	0.03	5	0.03	5	0.02	6	0.03	4	0.02	4	0.02	5
MHEWMA	0.03	5	0.03	4	0.01	6	0.03	4	0.03	4	0.03	5	0.03	6	0.02	5	0.02	6
CRHEWMA	0.03	4	0.03	6	0.01	8	0.03	6	0.03	7	0.01	8	0.03	5	0.02	7	0.01	8
DCC-REGARCH	0.03	2	0.03	2	0.84	2	0.03	2	0.03	2	0.72	2	0.03	2	0.02	2	1.00	***1***
CRREGARCH	1.00	***1***	1.00	***1***	1.00	***1***	1.00	***1***	1.00	***1***	1.00	***1***	1.00	***1***	1.00	***1***	0.59	2
CRCARR	0.03	3	0.03	3	0.11	3	0.03	3	0.03	3	0.10	3	0.03	3	0.02	3	0.30	3

Notes: The table shows the results of Model Confidence Set (MCS) for S&P 500 ETF Trust (SPY) and United States Oil Fund (USO) at a 5 % significance level. A rolling window forecasting method with a sample size of 500 is used. The number of forecast values is 450. The forecast horizon ranges from one week to twelve weeks. The model with the lowest value is shown in bold and italics. The weights for the S&P 500 and oil ETFs are both set at 50 %.

new CRCARR model. In total, we compare the forecast accuracy of eight range-based multivariate volatility models, including the three newly developed models and five existing models (i.e., DCC-CARR, DCC-RGARCH, DCC-CR, MHEWMA and CRHEWMA).

We employ two data sets, currencies (i.e., GBP/USD and EUR/USD) and ETFs (i.e., S&P 500 and crude oil), to have a comprehensive comparison of the eight models. In addition, we assess the performance of these models across twelve forecast horizons, ranging from one week to twelve weeks. Applying evaluation methods such as RMSE, the Diebold-Mariano test, the forecast regression test, the Euclidean, Frobenius and QILKE loss functions, and the Model Confidence Set, we find that the new CRREGARCH model is the best-performing model in most cases, followed by the DCC-REGARCH model. Moreover, we compare the variance and turnover of the global minimum variance portfolios constructed using the forecasted variance-covariance matrix from the eight models. The portfolios generated by the CRREGARCH and DCC-

Table 11
Out-of-sample Portfolio Variance.

Panel A. Out-of-sample Portfolio Variance for Currencies												
	Rebalancing Period											
	1 week	2 weeks	3 weeks	4 weeks	5 weeks	6 weeks	7 weeks	8 weeks	9 weeks	10 weeks	11 weeks	12 weeks
DCC-CARR	0.07240**	0.07307**	0.07294**	0.07192**	0.07190*	0.07159**	0.07096**	0.07216*	0.07256*	0.07307	0.07286	0.07286
DCC-RGARCH	0.07208**	0.07272**	0.07288**	0.07180**	0.07208*	0.07163**	0.07088**	0.07178*	0.07236*	0.07288	0.07275	0.07292
DCC-CR	0.07238**	0.07275**	0.07243**	0.07140**	0.07184**	0.07157**	0.07095**	0.07191*	0.07242*	0.07311	0.07289*	0.07283*
MHEWMA	***0.07153*****	0.07346	0.07127	0.07481	0.07397	0.07432	0.07548	0.07636	0.07515	0.07584	0.07537	0.07557
CRHEWMA	0.08098	0.09321	0.07561	0.08089	0.07486	0.07776	0.07461	0.08189	0.07560	0.08093	0.07967	0.07739
DCC-REGARCH	0.07186**	0.07241**	0.07228**	0.07170**	0.07157**	0.07100**	***0.07076*****	0.07198*	0.07225*	0.07247*	0.07211*	0.07196*
CRREGARCH	0.07167**	***0.07207*****	***0.07196*****	***0.07141*****	***0.07113*****	***0.07080*****	0.07083**	***0.07173*****	***0.07210*****	***0.07194*****	***0.07175*****	***0.07181*****
CRCARR	0.07192**	0.07260**	0.07276**	0.07212**	0.07205**	0.07171**	0.07126**	0.07208**	0.07259*	0.07278*	0.07269*	0.07269*
1/N	0.07853	0.07846	0.07852	0.07842	0.07847	0.07853	0.07812	0.07824	0.07836	0.07836	0.07848	0.07860

Panel B. Out-of-sample Portfolio Variance for ETFs												
	Rebalancing Period											
	1 week	2 weeks	3 weeks	4 weeks	5 weeks	6 weeks	7 weeks	8 weeks	9 weeks	10 weeks	11 weeks	12 weeks
DCC-CARR	0.4938**	***0.4105*****	0.4504**	0.4077**	0.4220**	0.4226**	0.4058**	0.4193**	0.4205**	0.4203**	0.4183**	0.4115**
DCC-RGARCH	0.5125**	0.4664**	0.5361*	0.4073**	0.4638**	0.5461**	0.4689**	0.4119	0.4439**	0.4610**	0.4470**	0.4449**
DCC-CR	0.4982**	0.4315**	0.4449**	***0.3846*****	***0.4036*****	***0.4083*****	***0.4014*****	0.4154**	0.4090**	0.4104**	0.4097**	0.4055**
MHEWMA	***0.4615*****	0.5172	0.5522	0.5382*	0.5811	0.4911**	0.4188**	0.4296**	0.4135**	0.4068**	0.4009**	***0.3948*****
CRHEWMA	0.4672	0.5445	0.4677**	0.5950	0.4407**	0.4421**	0.4150**	***0.4066*****	***0.4057*****	***0.3975*****	***0.3972*****	0.3992**
DCC-REGARCH	0.4823**	0.4702**	0.4754**	0.4099**	0.4094**	0.4095**	0.4054**	0.4184**	0.4232**	0.4211**	0.4185**	0.4080**
CRREGARCH	0.4721**	0.4347**	***0.4309*****	0.4277**	0.4202**	0.4338**	0.4141**	0.4200**	0.4109**	0.4097**	0.4130**	0.4092**
CRCARR	0.4953**	0.4159**	0.5600*	0.5216	0.4380**	0.4754**	0.4408**	0.5262**	0.4295**	0.4225**	0.4281**	0.4181**
1/N	0.8652	0.8670	0.8689	0.8700	0.48617	0.8630	0.8621	0.8635	0.8520	0.48496	0.8472	0.8479

Notes: This table presents the annualised variance of the minimum variance portfolios for currencies and ETFs constructed using the eight models. The 1/N strategy is employed as the benchmark to compare the performance of the minimum variance portfolios. The value in bold and italics indicates that the model achieves the lowest variance portfolios. * and ** indicate that the null hypothesis that the minimum variance portfolios and averaged weighted (1/N) portfolio have the same variance is rejected at the 10 % and 5 % levels, respectively. All values are multiplied by 100.

Table 12
Turnover.

Panel A. Turnover of the Currencies

	Rebalancing Period											
	1 week	2 weeks	3 weeks	4 weeks	5 weeks	6 weeks	7 weeks	8 weeks	9 weeks	10 weeks	11 weeks	12 weeks
DCC-CARR	0.1530	0.1464	0.1416	0.1373	0.1341	0.1315	0.1286	0.1258	0.1234	0.1215	0.1196	0.1174
DCC-RGARCH	0.1531	0.1325	0.1246	0.1196	0.1163	0.1142	0.1119	0.1097	0.1079	0.1065	0.1052	0.1040
DCC-CR	0.1501	0.1454	0.1420	0.1393	0.1368	0.1348	0.1322	0.1300	0.1278	0.1260	0.1242	0.1220
MHEWMA	0.1347	0.1345	0.1351	0.1351	0.1359	0.1346	0.1356	0.1353	0.1367	0.1360	0.1358	0.1363
CRHEWMA	0.1598	0.1597	0.1593	0.1585	0.1577	0.1580	0.1585	0.1580	0.1576	0.1569	0.1574	0.1572
DCC-REGARCH	0.1437	0.1396	0.1360	0.1328	0.1299	0.1280	0.1257	0.1236	0.1216	0.1199	0.1182	0.1167
CRREGARCH	0.1362	0.1323	0.1284	0.1252	0.1220	0.1195	0.1164	0.1138	0.1114	0.1093	0.1073	0.1057
CRCARR	0.1519	0.1453	0.1398	0.1346	0.1301	0.1267	0.1232	0.1196	0.1162	0.1136	0.1110	0.1085
1/N	***0.0849***	***0.0849***	***0.0849***	***0.0848***	***0.0848***	***0.0847***	***0.0847***	***0.0846***	***0.0846***	***0.0845***	***0.0844***	***0.0844***

Panel B. Turnover of ETFs

	Rebalancing Period											
	1 week	2 weeks	3 weeks	4 weeks	5 weeks	6 weeks	7 weeks	8 weeks	9 weeks	10 weeks	11 weeks	12 weeks
DCC-CARR	0.0980	0.0850	0.0749	0.0672	0.0609	0.0564	0.0531	0.0504	0.0482	0.0466	0.0452	0.0438
DCC-RGARCH	0.0909	0.0871	0.0842	0.0822	0.0803	0.0791	0.0782	0.0775	0.0771	0.0770	0.0768	0.0766
DCC-CR	0.0942	0.0787	0.0680	0.0602	0.0538	0.0489	0.0453	0.0425	***0.0402***	***0.0385***	***0.0369***	***0.0354***
MHEWMA	***0.0519***	***0.0517***	***0.0516***	***0.0514***	0.0515	0.0517	0.0518	0.0518	0.0516	0.0516	0.0514	0.0514
CRHEWMA	0.1627	0.1633	0.1640	0.1642	0.1648	0.1652	0.1654	0.1657	0.1658	0.1660	0.1665	0.1669
DCC-REGARCH	0.1074	0.0918	0.0787	0.0692	0.0620	0.0569	0.0531	0.0497	0.0469	0.0448	0.0429	0.0412
CRREGARCH	0.1068	0.0871	0.0710	0.0592	***0.0511***	***0.0459***	***0.0434***	***0.0421***	0.0417	0.0426	0.0435	0.0449
CRCARR	0.1409	0.1267	0.1168	0.1176	0.1033	0.0961	0.0936	0.0915	0.0894	0.0877	0.0857	0.0835
1/N	0.1458	0.1460	0.1462	0.1464	0.1465	0.1467	0.1469	0.1470	0.1472	0.1473	0.1474	0.1476

Notes: This table presents the turnover of minimum variance portfolios for currencies and ETFs (S&P 500 ETF Trust (SPY) and United States Oil Fund (USO)), constructed based on the eight models. The 1/N strategy is employed as the benchmark to compare the performance of the minimum variance portfolios. The value in bold and italics indicates that the model achieves the lowest turnover.

REGARCH models typically outperform their competitors (i.e., have the lowest variance or turnover) across most rebalancing periods.

Strikingly, our results demonstrate that integrating the REGARCH model within the co-range framework improves the forecast accuracy of the variance-covariance matrix, providing the most consistent performance across various forecast horizons and asset types. The superior performance of the CRREGARCH model underscores the strengths of both its univariate volatility estimator - the REGARCH model - and its multivariate counterpart - the co-range model. The REGARCH model effectively captures key volatility characteristics, enhancing univariate volatility estimation and forecasting. In addition, reducing the need for additional parameter estimations, the combination of the REGARCH and co-range model presents a reduced margin for estimation errors in contrast to the competing models. The strong performance of the multivarite REGARCH models suggests promising applications in portfolio management, asset allocation, risk management and contagion studies.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.irfa.2025.103983.

Data availability

Data will be made available on request.

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