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Research Article

Driver behaviour modelling of vehicles at signalized intersection with heterogeneous traffic

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ABSTRACT

Roads network is composed of mid-blocks and intersections. The part where two roads cross is called an intersection whereas the straight sections without intersection or any other interuptions is called mid-block. It can be observed that the vehicles on the mid-blocks tend to achieve their free-flow speeds while those at the intersections are forced to decelerate. Modelling of these sections needs to separate the intersections from mid-block. Further, drivers behave differently at these two locations. Present study attempts to separate the intersection zone of influence (IZOI) and mid-block using the manoeuvring characteristics of drivers in terms of acceleration/deceleration. These were captured through a global positioning system (GPS) device in the vehicle after sighting a red signal at the intersection. Further, this study also tried to observe whether different classes of drivers such as aggressive, normal or timid drivers, based on acceleration/deceleration behaviour exists. A junction with 1-km straight stretch in R. K. Puram New Delhi (India) was chosen for the study to find the IZOI. After identifying IZOI a video data was collected in Mumbai (India) for a stretch more than 200-m long near intersection where the red signal was visible; This enabled observing the driver behaviour more closely. Around 900 drivers of different modes were analysed to understand their behaviour. It was found that cars start reducing its speed at 160 m, motorized three-wheelers at 124 m and buses start reducing their speeds at 98 m distance from the intersection. The driver behaviours were distinct in each of the mode (Bus, Car and motorized-there-wheelers), but it emerges that the drivers cannot be classified into finite number of clusters based on the fitted normal distribution. Thus it can be seen that there are no clearly demcarcated driver behaviours irrespective of the vehicle type, such as aggressive, normal and timid categories as the intersection approaches. A normal distribution model can classify the drivers satisfactorily.

1. Introduction

Drivers behave diffirently at the intersections and mid-blocks. Usually drivers at the mid-block tend to acheive free flow speeds. Whereas at the intersections they are forced to reduce their speeds and pass the intersection carefully. However, all drivers do not behave uniformly. Some of them are cautious or normal and while others could be aggressive. Driver behaviour is crucial in simulation, emissions and fuel consumption models. A report by Toroyan (2015) [1] showed that in 2015, around 1.25 million fatal crashes were observed worldwide. Another study [2] reported that more than 95% of the crashes can be attributed to the driver behavioural factors. Though various governments have taken several steps to reduce the fatalities, but no traffic deaths reduction has been observed yet [2]. Design of intersections and deceleration lanes also involves an understanding of the driver behaviour (deceleration) of vehicles [3]. Driver behaviour studies are helpful in monitoring and training of drivers for safe and effective traffic management. It is not known when drivers change their behaviour after sighting a signal at the intersection. Hence the present study tries to find the answers related to the type of drivers and the distance after which drivers behave differently compared to the mid-blocks.

Based on the type of data required at traffic facilities, it can be collected using several instruments such as using video cameras, inductive loop detectors, pressure detectors, Passive infrared (PIR) sensors, microwave radar, active infrared detectors and many others. Section data can be collected using licence plate detection, loop detector, Bluetooth scanning. Further trajectories data can be collected using GPS, video cameras [4]. As the video camera and other conventional data collection methods

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cover a small length of road which is not sufficient to record the driver manoeuvring at intersections, GPS is an alternate tool to record the manoeuvring (acceleration, deceleration speeds) of drivers. The advantages of using GPS data is that long portions of roads can be covered and it is not dependent on any other software or machine to extract the data from GPS device. GPS data gives dynamic properties of travel behaviour, including temporal, spatial and attributes conditions for full travelling stretch [5]. Some countries such as Canada have insurance plans to encourage drivers to drive safely and efficiently. Drivers are monitored using GPS and based on how they drive and where they drive they are rewarded. This monitoring has 3-fold benefits, firstly it motivates drivers to earn more rewards; secondly, it reduces greenhouse gases because drivers are rewarded based on emission also. Thirdly drivers are cautious so the wear and tear in the vehicles is also less [6]. Several studies are available for analysing the deceleration behaviour of drivers' at mid-blocks and intersections in homogenous traffic conditions [7–9]. However, no study attempts to analyse the deceleration behaviour of vehicles at signalized intersection. Once the data from GPS has been collected, then this data can be used to plot simple trajectories, to calculate acceleration-deceleration, speeds and driver behaviour of vehicles. Machine learning and neural networks have been used to analyse traffic data for quite some time [10]. Moreira-Matias and Farah [11] discussed issues related to the existing techniques of driver identification and used machine learning algorithms to identify the gender of drivers with In-Vehicle Data Recorder (IVDR).

As the drivers play an important role for the traffic movement, safety and capacity of roads. It is important to study the behaviour of drivers of different modes and in each mode the classification of driving behaviour. Present study tries to model the deceleration and acceleration behaviour at the signalized junction when the signal is red. The data was collected at a signalized junction in Delhi for a stretch of 1 km. One hundred samples of each vehicle type such as passenger car, bus and motorized three-wheeler were obtained using Global Positioning System (GPS) while the signal was red in off-peak hours. Further, the trend of vehicles deceleration/acceleration was seen over the 1 km stretch and influencing zone of the intersection was found. This zone separates the intersection from mid-block. When the vehicles are in the zone of influence of intersection, they are forced to reduce their speeds till they stop, while some of them try to accelerate and cross the intersection fast. This data was analysed to check whether clusters emerge into various type of driver categories such as aggeressive, timid and normal. After trying several clustering methods no grouping of drivers was found, hence another video data set was collected in Mumbai (India) covering over 200 m distance. This data was extracted with the help of Manual Traffic Data Extractor (MTraDE) [12]. However, drivers could not be clustered with this data as well. Hence, drivers were modelled as distribution function. To get the driver behaviour of each type of vehicles first, the acceleration and deceleration behaviour of drivers was analysed using the cluster analysis. Classification of drivers' acceleration and deceleration behaviour is helpful to model the behaviour in simulation model. Unsupervised learning where objects are classified without external help [13] can be used to classify the drivers. The classes of drivers are not pre-determined hence unsupervised learning technique self-organizing maps was used to classify the drivers. A method which is not influenced by the outliers named as 'Robust Local Regression- rloess' [14] was used to filter and smoothen the data. This study will be useful as an input for the simulation models in terms of driving behaviour. The IZOI could help in deciding a suitable intersection dimensions while modelling.

1.1. Organization of paper

Rest of the paper is organized as follows. Next section 2 presents the literature review for idenitifying the available gaps and techniques, followed by section 3 explaining the data colletion. Results and methodology used to evaluate and separate the driver behaviours at mid-block and IZOI are discussed in section 4. Lastly, in section 5, conclusions and scope for further research are discussed.

2. Literature review

Intersections are the unavoidable part of transportation systems. Drivers behave diffirenlty at the intersections. Studies show that the drivers with a long red light become impatient and try to cross the intersection [15–17]. Driver behaviour can be studied in several perspectives such as keeping environmental issues (emissions) [18,19], or safety of vehicles [20,21] or accurate simulation model in consideration [22,23]. For instance, studies discuss the prediction of stop-go behaviour of drivers at the intersecion [24,25]. This behaviour is very important in many of countries, as vehicles try to cross the red light. It is one of the serious known problem related with crashes [25,26]. To prevent the red light running a system has also been developed with the help of machine learning [27] and Bayesian network techniques [28]. Further, researchers have also modelled the dynamic decision making of the drivers in the phase transition period [29]. In another study a head-on colflict solving algorithm was developed for an intersection with heterogenous traffic conditions [30]. The study was calibrated and validated with the field data. Most of the past studies thus far addressed driver behaviour with homogenous traffic condtions.

2.1. Driver behaviour modelling

As discussed above, driver behaviour is one of the key characteristics to be incorporated in the simulation models. Many researchers have tried incorporating the basic behaviour such as car-following to machine learning to adapt the driver manoeuvring sequence. Driver behaviour can be studied in several perspectives such as keeping environmental issues (emissions) in consideration [18,19], or safety of vehicles [20,21] or accurate simulation model in consideration [22,23]. Bennett and Dunn [7] studied the driver deceleration behaviour at a freeway in New Zealand. They kept a series of data loggers at 500-m intervals and recorded the speed data. It was found that the deceleration length of vehicles was same regardless of their initial speeds and hence speed and deceleration rate were proportional to each other. An initial speed dependent relation was developed to get the speed at desired time. Oliver and Pentland [31] developed a smart car for live data acquisition and playback with the help of machine learning (dynamic graphical models and Hidden Markov Models (HMM)) to observe and modelling of driver behaviour. 70 drivers rode the car for 1.25 h for 2 months in Greater Boston area to facilitate in collecting various manoeuvres data. Camera was used to collect the data of traffic, drivers view point, drivers head position. Other instruments were used to record the data of cars brake, steering wheel angle, gear, speed and acceleration. Similar study was done by Mitrovic [32], lateral and longitudinal acceleration and speed data was collected in normal driving situation. This data was trained using HMM to predict driver manoeuvres. A review study by Toledo [33] explains that capturing tactical manoeuvring decisions of drivers in different traffic conditions is called driver behaviour model. This study has classified driver behaviour in two parts such as acceleration and lane changing models (Fig. 1).

With the advancement of time and development of ITS, more sophisticated techniques have emerged to understand the driver behaviour from the vehicle manoeuvring sequence. For instance Levesque [34] modelled and evaluated driver behaviour with the help of a driver risk assessment tool. This tool was constructed using neural networks, elderly people were given special attention in the model. Data for this model was acquired with the help of simulator named as "Groupe de Recherche en Analyse du Mouvement et Ergonomie (GRAME, translated: Movement and Ergonomics Analysis Research Group)". Longitudinal speed, acceleration and lateral data position, speeds acceleration data was collected, speed limits, road curvature were also collected. After collecting the data they used a neural network to construct and

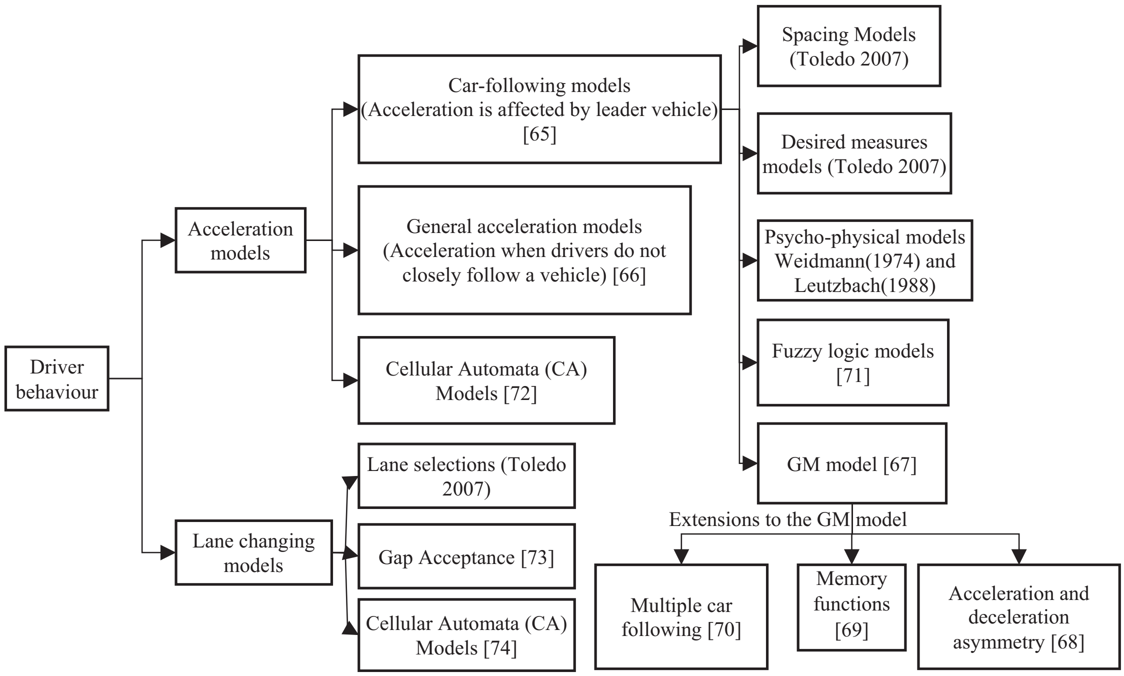

Fig. 1. Driver behaviour models (Toledo 2007), [65–74].

train the model. Few machine learning algorithms such as self-organizing map (SOM) and linear vector quantization (LVQ) were used to classify the drivers. Driver behaviour was classified as acceptable, unacceptable and marginal. Few other studies have adopted the driver behaviour with clearly demarcated classes such as aggressive, timid or normal driver behaviour [35]. Al-Doori [36] used computation intelligence particularly SOM to classify the drivers. This driver behaviour was used to find the relation with emission. Calm, normal and aggressive driver behaviour were given as the input to the fuzzy model. Jasinski and Baldo [9] collected everyday trips data of drivers using GPS to analyse their movement behaviour. Further, this study tries to classify aggressive drivers. Driver behaviour was based on Trajectory Aggressivity Indicator (TAI). The scale of TAI was between '0' to '100', 0 means no aggressiveness whereas 100 means aggressive behaviour. Derbel [6] used GPS to assessed driver behaviour along with Kalman filtering technique to eliminate the missing signal points of GPS at several locations such as tunnel, urban canyon and bridges. They further fused data obtained from GPS, Inertial Navigation System (INS) and driver data in terms of sex and age and this global fusion data produced driver vehicles environment. Driver behaviour was assessed in terms of risk which was subdivided into low risk (LR), medium risk (MR) and high risk (HR). A recent study by Guo [37] modelled and classified driver behaviour using autoencoder and SOM (AESOM). Author collected 4021 observation data using GPS. Driver behaviours were classified as slight moderate and heavy deceleration, acceleration and speeds.

2.2. Zone of influence of intersections

The part of the road where the drivers change their behaviour after seeing the intersection is called influence zone of intersections [38]. These are the locations where most of the acceleration/deceleration manoeuvre takes place. Further, another study [39] reveals that the acceleration/decelerations are the major manouvers causing the emissions of pollutants on the road. Minh [40] studied longitudinal threshold distance between two motorcycles, which is defined as the distance at which the follower motorized two-wheelers is affected by the leader motorized two-wheeler in terms of deceleration in order to stop at stop line and maintain safe gap. Present study attempts to apply the methodology to find the threshold gap of intersection and other different vehicular modes (bus, motorized three-wheelers, motorized two-wheelers and car). Another study by Chauhan [38] also tries to find the intersection influence zone to study the emission behaviour of vehicles, acceleration and deceleration behaviour of vehicles were considered to find the influence zones. This study also tries to find the distance from the intersection influencing the vehicles particularly after sighting the red signal.

2.3. Data filtering and smoothing

GPS contains error due to several factors such as environment, receiving a smaller number of satellite signals, GPS handling. Before further analysis these errors need to be filtered and smoothed. Few studies have used variety of methods for clustering and filtering the data, for instance authors [41] used several methods to smooth the GPS data and recommended that modified Kalman filter is better than other filtering and smoothing methods. Some researchers [42] used local regression methods (loess or lowess) to smooth the time series data. Adaptive Kalman filter with wavelet analysis was recommended in another sudy by Wu et al. [43]. Here the authors claimed an improvement of 43% in the accuracy using this method. Jiang [44] integrated adaptive Kalman filter with H-infinity filter to construct a robust filter to remove outliers. The scheme was validated with field data collected using integration of Inertial Navigation System (INS) and Global Positioning System (GPS). Kaloop and Kim [45] used wavelets transformation to filter and smooth the data. Seventh-degree spherical simplex-radial cubature Kalman filter was modelled by Feng [46] author demonstrated that this filter is better than normal Kalman filter. There are several methods to cluster the data, few of them such as 'moving average', 'lowess', 'loess' were tested. As other methods are susceptible outliers, 'rloess' was chosen as suitable method for the current study. This method is less affected to the outliers [14,47].

2.4. Data clustering

Once the data has been filtered, it can be clustered subsequently to find the type of drivers in different zones. For clustering any data, number of clusters should be known a priori. Several methods to get the appropriate number of clusters are available for instance elbow method, silhouette method, gap statistic. Milligan and Cooper [48] demonstrated around 30 methods to cluster the data, but studies suggest that none of these methods are the best in terms of deciding the number of clusters [49]. Elbow method is one of the methods to determine the number of clusters in the data. In 'elbow' method several numbers of clusters are tried (from 1 to any number n) and the explanation of variance is observed. While increasing the number of clusters when the substantial changes in the variance stop to happen that many clusters are optimal clusters [50]. In other words, the point at which the second derivative of the sum of distances is maximum that point is called optimal number of clusters. These methods were further extended and named as gap statistic by Tibshirani [51] and was tested over simulations and found better to estimate the number of clusters. Gap statistic method was further modified by Yan and Ye [52] using weighted gap statistic. Once the number of clusters are determined there are several methods to cluster the data. All clusters approached can be divided into two group names as 'hierarchical clustering' and partitioning clustering. These groups can be further subdivided into other classifications methods [53,54]. Several review papers and books are available on the methodology of clustering and selection of suitable clustering methods [54–61]. Present study started clustering first by visually inspecting the data, there were no visible clusters. Then simple k-means was tried, but to use k-means it was required to know the number of clusters, hence 'elbow' and 'gap statistic' methods were tried in order to be sure about the number of clusters.

2.5. Summary

Hence, it can be concluded that the driver behaviour at the intersections is an important characterstic to study. It has been tried to model them into several classifications, such as timid, normal and aggressive [35] or Calm, normal and aggressive [36]. However, the field verified thresholds of separating these is not available in the existing studies. Futher, these driving behaviour could be diffirent for several modes of transportation. Forintsace the truck driver may have diffirent behaviour than the car. Hence, a study is needed to separate their behaviour in a mode (diffirent driving behaviours of car) as well as in diffirent modes (car driving behaviour would not be same as truck). Futher, drivers are forced to reduce their speed when they are near to the signalized intersections. Hence proper dimensions are required to simulate an intersection model. Moreover, the intersections are the segments of roads where drivers spent their time in deceleration/acceleration and waiting for clearance. These are crash prone areas and huge amout of pollutants are emitted at these locations [39]. Hence, study is required for the analysis of influcen zone of intersections such that some measures can be taken to overcome the issues. Difirent methods to analyse the data were used in the study discussed above. Following section 3 gives the details regardin the data collection.

3. Data collection

Data was collected at signalized intersection in Delhi, India. Sample size was calculated using sample of proportion method (Eq. 1) [62]. At 95% confidence interval (CI) and 10% margin of error (ME) following sample size was calculated (Table 1).

$$n = \frac{z^2 p(1-p)}{E^2} \tag{1}$$

where 'n' is sample size, z is z-statistics, 'p' is sample proportion and 'E' is margin of error. With a conservative guess 'p' was taken as 0.5 and data of 100 sample vehicles for each type was collected using GPS.

Vehicles were fitted with GPS device for some period to collect the data of 1 km stretch on the approach road to the intersection. Different

Table 1
Calculation of sample size.

Modes	Proportion	Sample size (calculated)
Bus	0.02	100 (8)
Cars	0.57	100 (94)
Motorized Three Wheelers	0.12	100 (40)

vehicles of each mode were chosen for the sampling. This data was then extracted and smoothed with 'rloess' method as discussed above. Further the data was separated from intersection and mid blocks for further analysis. A sequence of the data analysis is given in the following Fig. 2.

Fig. 3 shows the distance from intersection at which vehicles start reducing their speeds after seeing the red signal, here traffic is moving left to right in the graph. Red mark (●) in Fig. 3 shows the starting location of intersection and star (✳) shows the starting point of deceleration of vehicles. Intermediate drops in the speeds of different modes are due to the presence of a bus stop in the selected study stretch. More variation in terms of speeds can be observed in cars, whereas most of the buses have less variation in speeds. Motorized three wheelers have intermediate speed variations.

4. Results and methodology

4.1. Methodology for the calculation of influence zone of intersections (IZOI)

To find the influence of intersection, the distance from the starting point of the deceleration to the stop line was observed, when the vehicles faced red signal at the intersection. The distance from the starting point of deceleration to stop line was named as influence zone of intersection (IZOI) (Fig. 4). Further, the section between two intersections where there is no affect of intersection is called mid-block. Present study has found that IZOI is different for different type of vehicles.

As the distance from the intersection was following normal distribution with 'p' values given in Table 2. The zone of influence was calculated using following equation (Eq. 2).

$$Influence\ Zone\ of\ Intersection\ (IZOI) = \overline{X} \pm \frac{t * S}{\sqrt{N}} \tag{2}$$

where $\overline{X}$ is the the mean of the data, t is t-statistic, S is sample standard deviation and N is the number of samples.

Following Table 2 shows the descriptive statistics of the stopping distance from the intersection. It can be seen that the distance from the intersection is following normal distribution (based on AD statistic and $p > 0.05$) and hence the range was calculated with Eq. 2. Futher, this can also be observed that different vehicles start reducing speeds at different locations.

Above Fig. 5a shows the 95% confidence interval (CI) range for the vehicles to reduce their speeds and stop at intersections. Car drivers see the intersection and start reducing their speeds from a distance while other vehicles start reducing their speeds at a closer distance. Buses starts reducing their speed when it is nearest to the intersection compared to other modes. This can be observed at any confidence interval (Fig. 5a). This behaviour of bus is unexpected and may lead to safety issues. The cars start reducing their speeds from afar, one reason of this could be the cautiousness of the car drivers. Fig. 5b shows the speed profiles of the vehicle on a stretch of 1 km road. It can be seen that cars drivers were cautious while approaching the intersection as they start reducing their speeds much before intersections (around 160 m away), whereas bus and motorized three-wheeler drivers were aggressive with decelerating distance of 98.83 and 124.65 m. However maximum speeds of taxi-cabs were slightly more than bus and motorized three-wheeler Fig. 5b. It can be said that car drivers were cautious while driving on the road, whereas bus and motorized

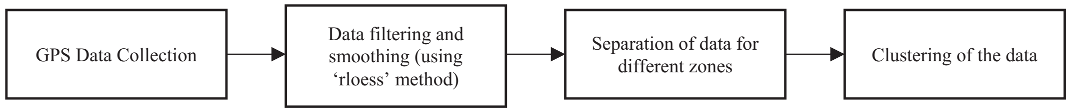

Fig. 2. Data collection and analysis methodology.

three-wheeler drivers were aggressive comparatively. However maximum speeds of cabs were slightly more than bus and motorized three-wheeler.

Similar behaviour of drivers with different thresholds of maximum speeds and acceleration was observed for vehicles. When the vehicles are away from intersection, they have maximum free speeds and when they see the intersection, they start reducing their speeds which can be seen in the centre of following Fig. 6 for different vehicles. Vehicles start applying deceleration and increase deceleration rate while going near to intersection. Further, once they achieve a certain speed, they reduce their deceleration rate and while stopping they nearly '0' deceleration. Cars drivers were observed to be more cautious in terms of acceleration/deceleration having a deceleration range between 0.2 to -1.6 m/s^2 but their acceleration/deceleration rate was higher than bus and motorized three-wheelers (around $-1/250$ in Fig. 6). It can be observed in Fig. 6 that cars behaviour is different than other vehicles. Interestingly some of the car drivers have higher speed than the rest in their vehicle category, while in other modes all the drivers are similar to each other. It was found that the rate of acceleration was modest in motorized three-wheelers and highest in bus.

Identical pattern of speeds-deceleration graph have been observed in existing studies [63,64]. Following Fig. 7 shows the comparision of current and existing study. It can be observed in the Fig. 7 that the trend in both the graphs is similar and it redues with the increment in the speeds.

4.2. Speed-acceleration behaviour at mid-blocks

The behaviour of vehicles at mid-blocks can be different from that of at signalized intersection. The speed and acceleration behaviour at mid blocks were observed in present study (Fig. 8). It can be seen that both the behaviour (acceleration and deceleration) are present in the mid-block opposite to the behaviour in IZOI in above sections. Motorized-three-wheeler drivers at mid-block tend to have constant acceleration/deceleration rate behaviour between 1.5 to -2 m/s^2 whereas bus drivers have wider and more scattered acceleration/deceleration (2 to -3 m/s^2) behaviour with steep acceleration/deceleration rate with respect to speed. Car drivers have cautious acceleration/deceleration behaviour compare to motorized three-wheelers and buses with a less range of 1 to -1 m/s^2.

It can be observed in Figs. 6 and 8 that once the vehicles are in the IZOI, the tendency of acceleration at intersections is significantly low in comparison to the mid-block. This behaviour separates intersection from mid-block. Further the behaviour of acceleration and deceleration is different in both the cases, the acceleration/deceleration behaviour of vehicles follows a wide' shape pattern at intersection, whereas at mid-block the shapes of modes is different, for instance the shape of acceleration/deceleration-speed curve for motorized three-wheeler is different than that of cars and buses, similarly for other modes. Once data was found not to cluster into groups it was modelled as distribution function. Table 4 summarizes the normality test of the acceleration behaviour of drivers. More than 100 drivers for each mode were considered. It can be observed that acceleration and deceleration range is different for different modes near the intersection. Highest acceleration and deceleration can be observed in motorized two-wheelers whereas lowest acceleration and deceleration is observed in cars.

4.3. Clustering and validation of clustering

Clustering was attempted on acceleration and speeds vehicles with respect to IZOI, there was no visual cluster in the data (Fig. 6). As discussed above, elbow, gap static and weighted gap statistic methods were used to determine the number of clusters in the data. Seeing the Table 3, it can be decided that there are no particular classes of clusters. However, a pattern of acceleration with IZOI and speed with acceleration can be observed in Fig. 6. It thus emerges when vehicles are at certain distance (different for different modes) from the intersection they increase their deceleration rate and when they are near to the intersection, they reduce their acceleration rate and finally stop at the intersection. Table 3 shows the calculated cluster using different methods, and it can be seen that by using different methodologies, different number of clusters emerge.

GPS data was considered for only three modes namely cars, buses and three-wheelers. After identifying IZOI, additional video data was collected at Powai, Mumbai (India), covering the stretch more than IZOI as discussed above and data (acceleration/deceleration characteristics) was extracted for 4 modes (car, bus, motorized two-wheeler and motorized three-wheeler). The scatter plot of the acceleration data when the signal has turned green has been given in Fig. 9, it is evident from the data that there are not visually separable thresholds to

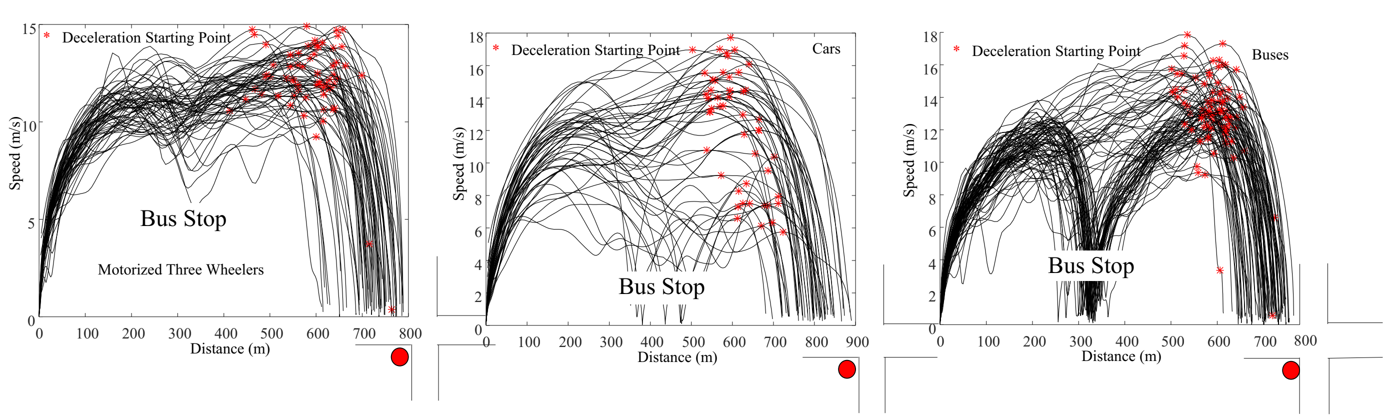

Fig. 3. Deceleration Distance from intersection (● indicates starting of intersection).

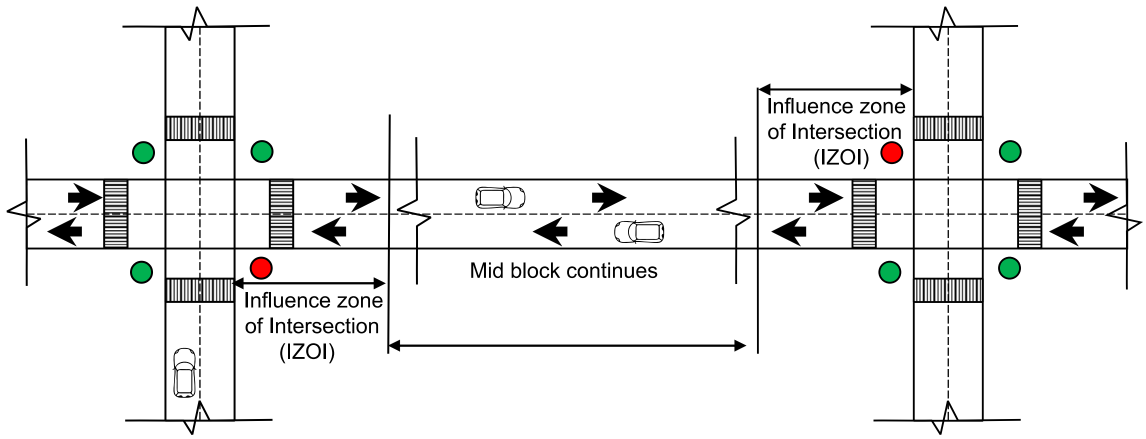

Fig. 4. Mid-block and Influence zone of intersection.

Table 2
Zone of influence of intersection.

Mode	Descriptive statistics						N. D. GOF		IZOI ($\overline{X} \pm 1.96^{*}S/\sqrt{N}$)	
	N	$\overline{X}$	S	Median	Minimum	Maximum	AD	P	Min	Max
Car	86	187.4	126.3	184.1	39.1	314.9	0.7	0.1	160.7	214.1
Bus	98	111.3	62.8	113.6	0.5	206.4	0.23	0.9	98.8	123.7
MThW*	79	141.1	74.7	132.6	1.3	212.6	0.5	0.1	124.7	157.6

MThW*- Motorized Three-Wheeler, *N.D.GOF-normal distribution goodness of fit, Where '$\overline{X}$' is sample mean, 'S' is sample standard deviation and N is number of samples. AD-Anderson Darling test statistic, p-p value. 1.96 is 't' value at 95% confidence interval*

separate the drivers with the acceleration. Hence several methods were tried to group drivers (k-means, Gap statistic, Weighted gap statistic), but suitable group numbers were not found hence probability models were used to model the driver behaviour. It would be required to see the video beyond 250 m to get the deceleration of vehicles that would stop at around 250 m, hence the data only shows acceleration.

5. Driver behaviour with normal distributions

After trying to group the acceleration/deceleration behaviour, when no grouping was found then drivers were modelled as a distribution function. It was found that acceleration/deceleration of drivers follow Normal distribution, hence there are two extreme of acceleration/deceleration behaviour followed by either aggressive drivers or timid drivers. Present study assumes that the acceleration/deceleration behaviour adopted by most of the drivers is normal driving behaviour and acceleration/deceleration followed by least number of drivers is either aggressive (if it is more than acceleration of normal drivers) or timid driving behaviour (when it is less than acceleration of normal driving driver). Following Fig. 10 can be used to inderstand this behaviour.

Following Fig. 11 shows the behaviour observed in the field. As discussed above while considering the acceleration, the mode (statistics)

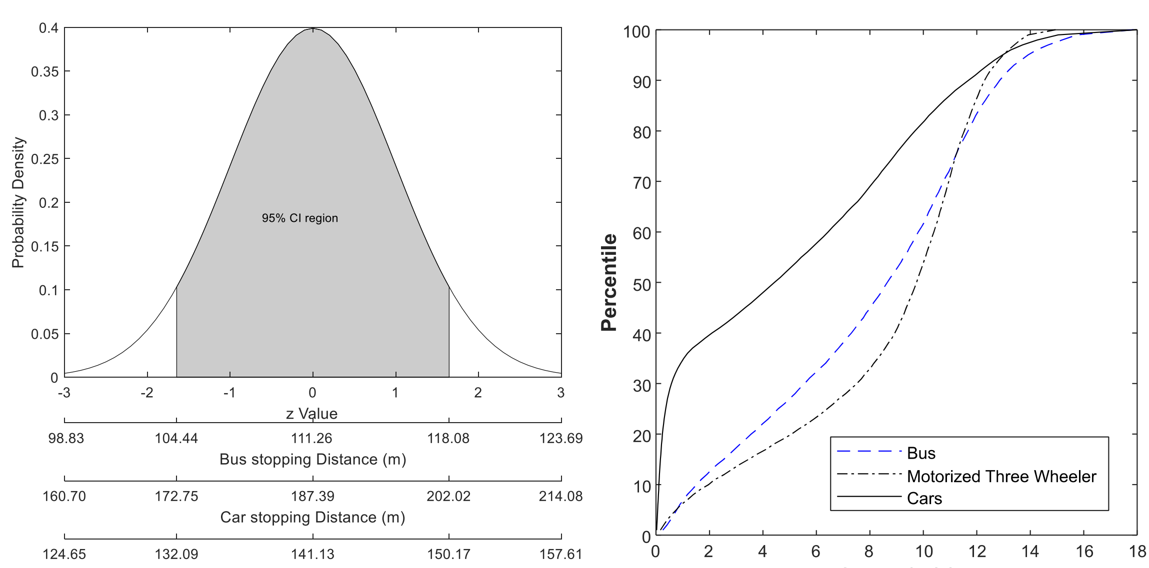

Fig. 5. (a) length of Zone of Influence of vehicles (b) Overall Speed Profiles (from starting to stopping at intersection).

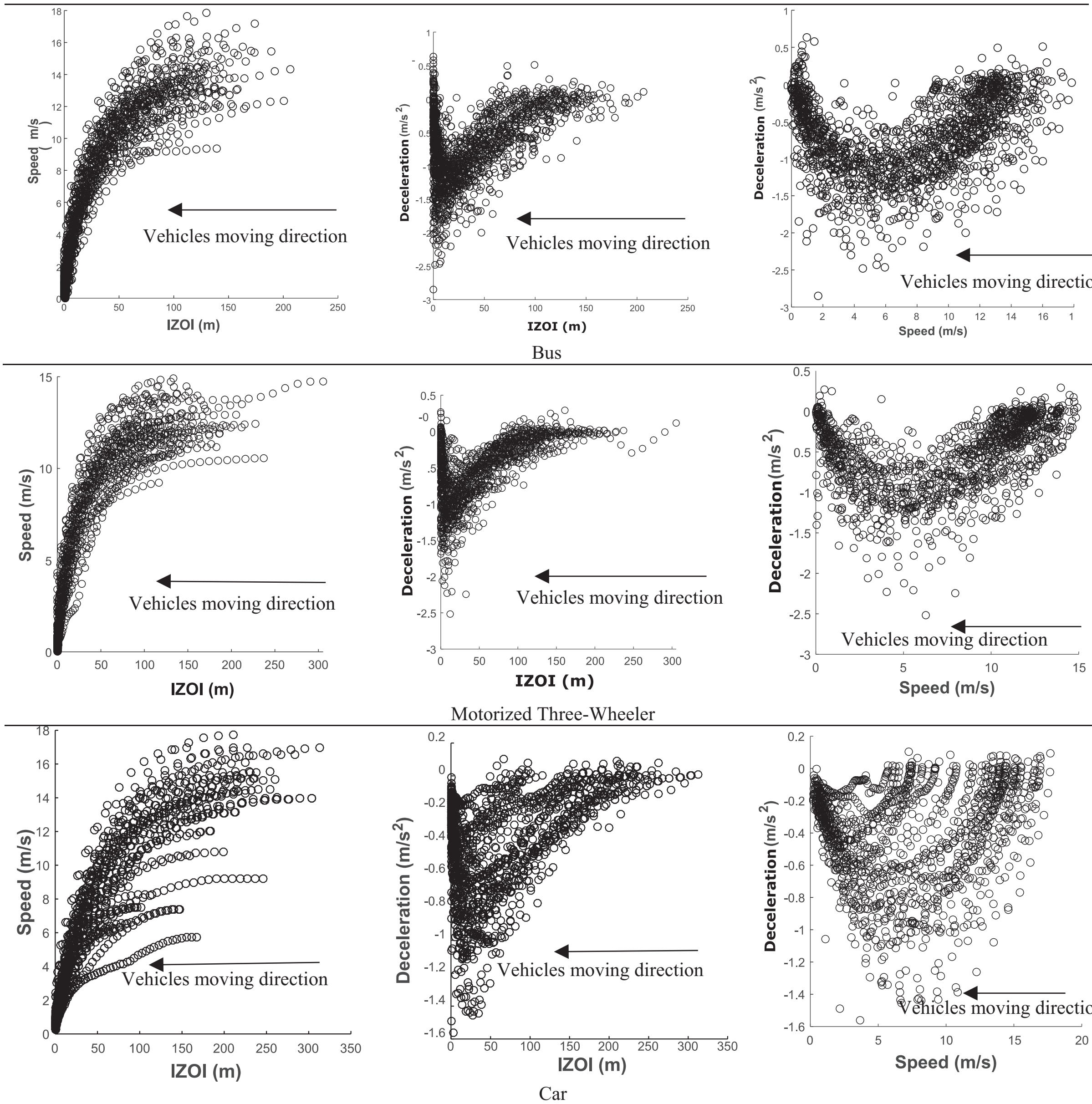

Fig. 6. Speed and deceleration behaviour of vehicles in zone of influence of intersection.

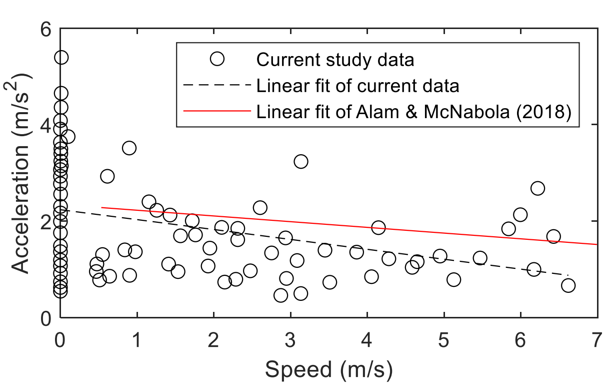

Fig. 7. Comparision of current and existing driving behveiour.

of acceleration is followed by normal drivers whereas aggressive drivers follow more acceleration than normal drivsers and timid divers adopt lower acceleration behaviour. Similarly, while considering the deceleration, drivers following less acceleration than normal drivers are called aggressive drivers and timid drivers follow more deceleration than normal drivers. Hence in the present study it appears that there are finitely large number of driver classes to be simulated on the road network.

6. Conclusion and discussion

Often when the researchers simulating an isolated signalized intersection, the length of the of intersection approaches to be considered is not known a pirori. Adopting longer length of the approaches would incur additional time and resources. Hence a suitable length of the approaches to be chosen as this would immensely help in driver behaviour modelling. This study attempts to decide the suitable approach length for the simulation of intersections which would influence the driver

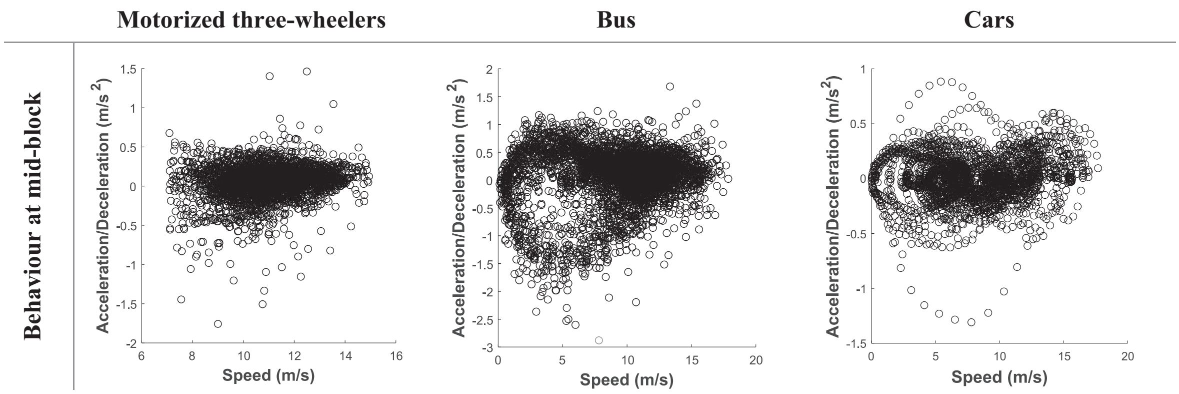

Fig. 8. Acceleration/Deceleration behaviour of vehicles at mid-block.

Table 3
Number of clusters with different methods for different modes.

Method	Number of clusters								
	Bus			Cars			Motorized three-wheeler		
	IZOI-Speed	IZOI-Acceleration	Speed-Acceleration	IZOI-Speed	IZOI-Acceleration	Speed-Acceleration	IZOI-Speed	IZOI-Acceleration	Speed-Acceleration
Elbow	5	5	5	5	5	5	6	6	5
Gap statistic	1	1	2	1	1	3	1	1	2
Weighted gap statistic	n	n	n	n	n	3	N	n	n

n- any number.

behaviour. Further, it is important to observe the driver behaviour as it could help in deciding the effectiveness of policies, safety and emissions. Further these are required as an input into the simulation models. As different drivers may not accept the neighbouring gaps to move hence, driver behaviour may affect the capacities of the facilities as well. Following conclusions can be drawn from the present study.

- Cars start decelerating at the intersection much before than other modes (at around 160.70 to 214.08 m), then motorized three-wheelers start reducing their speeds (at 124.65 to 157.61 m) and lastly bus start reducing their speeds at 98.83 to 123.69 m. This suggests that bus drivers are either aggressive/courageous, or they take advantage of the less speed and more load leading to a less stopping distance required (Fig. 6).
- The behaviour of drivers at mid-block and signalized intersection are inherently different. At mid-block, both acceleration and deceleration are observed, whereas at IZOI mostly deceleration is observed (Figs. 5, 6 and 8).
- Bus drivers also show a wide range of acceleration/deceleration behaviour from 2 to -3 m/s^2 in mid-blocks and 1 to -3 at IZOIs, which is more than other modes studied. This confirms the aggressive behaviour of bus drivers, Figs. 6 and 8). Motorized three-wheelers are calm at mid-block, whereas cars are calm at signalized intersections.
- It is not necessary to have different types or classes of drivers for a mode, aggressive drivers are rare and may not be found while doing the survey (Figs. 6 and 8). One possibility of this behaviour is congestion of vehicles near the intersection, thus forcing them all to reduce their speeds and behave similar to each other.

6.1. Applications

Few major applications of the study are:

- To do the intersection modelling studies, these IZOI limits can be used to collect data without losing necessary (or adding unnecessary) details of the traffic and driver behaviour of vehicles.

Table 4
Discriptive statistics of the driver behaviour data with its goodness of fit.

Mode	Descriptive statistics						N. D. GOF*	
	N	Mean (μ)	St. Dev (σ)	Median	Minimum	Maximum	AD	p
Acceleration Behaviour								
Car	142	2.04	0.69	1.99	0.12	4.45	0.29	0.63
Motorized two-wheeler	121	2.81	0.79	2.85	0.70	5.15	0.25	0.76
Motorized three-wheeler	168	2.65	0.86	2.64	0	5.46	0.28	0.65
Bus	127	2.40	0.90	2.33	0	4.69	0.50	0.21
Deceleration Behaviour								
Car	142	−1.29	0.51	−1.27	−3.07	−0.12	0.30	0.60
Motorized two-wheeler	121	−2.82	0.88	−2.81	−5.16	−0.37	0.37	0.43
Motorized three-wheeler	168	−2.62	0.85	−2.63	−4.58	0	0.19	0.9
Bus	126	−2.12	0.84	−2.11	−4.46	0	0.50	0.20

*N.D.GOF-normal distribution goodness of fit, AD-Anderson darling test statistic, *p-p* value

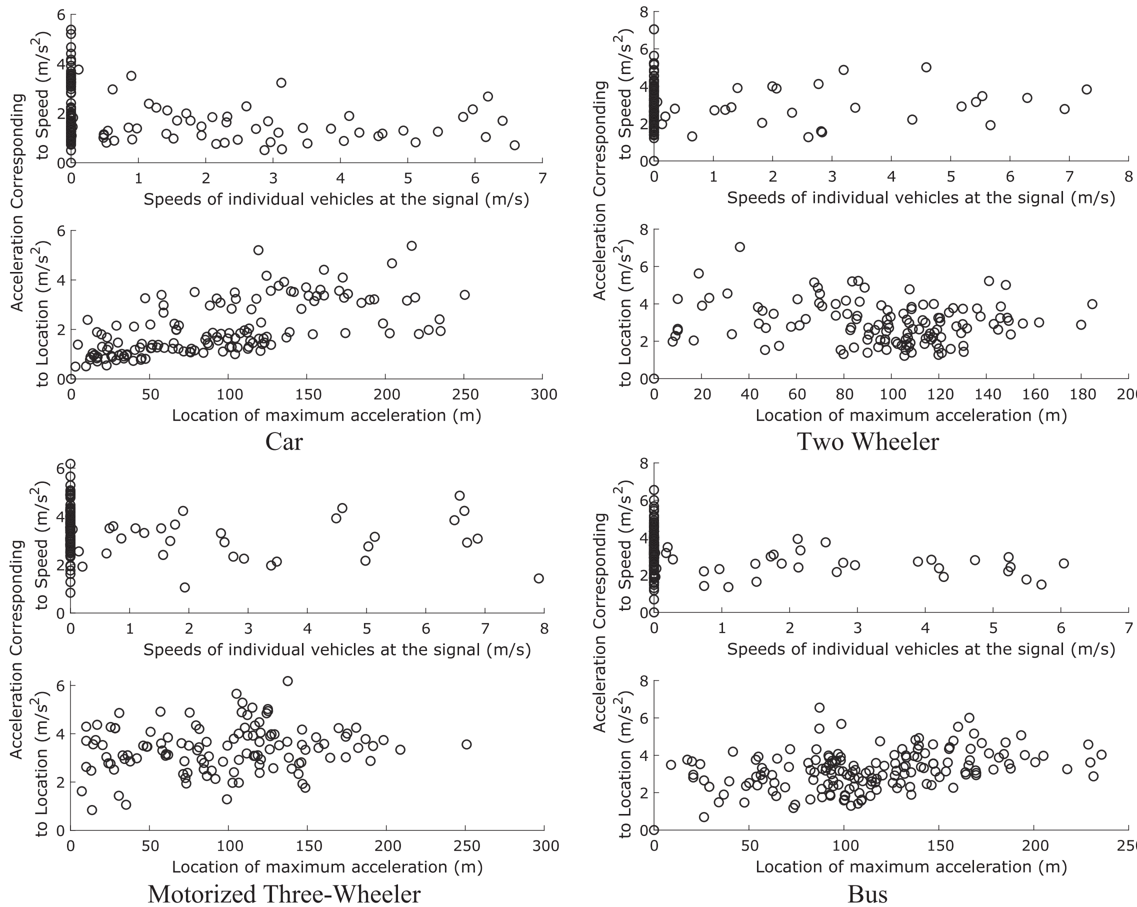

Fig. 9. Location of maximum acceleration corresponding speeds.

- Mode wise inputs of the acceleration and deceleration to the simulation models can be adopted from the study. Further, while modelling the imultion, it can be considered not to take 3 classifications of driers rather, a distribution can be adopted.
- As discussed in past studies, most of the emissions is generated in the manoeuvre of acceleration/deceleration of vehicles [39] which happens mostly in the IZOI. Hence emission control devices can be kept at these places to reduce emissions.

6.2. Further research

Studies classify drivers as aggressive, normal or cautious [35] or Calm, normal and aggressive driver (Al-Doori [36]. Which was not seen in the current study. As it is not always possible to model driver behaviour as distribution, classification of drivers may exist at some locations and can be analysed. Furthermore GPS data can be collected to get more realistic behaviour of the traffic and drivers. The study takes a case

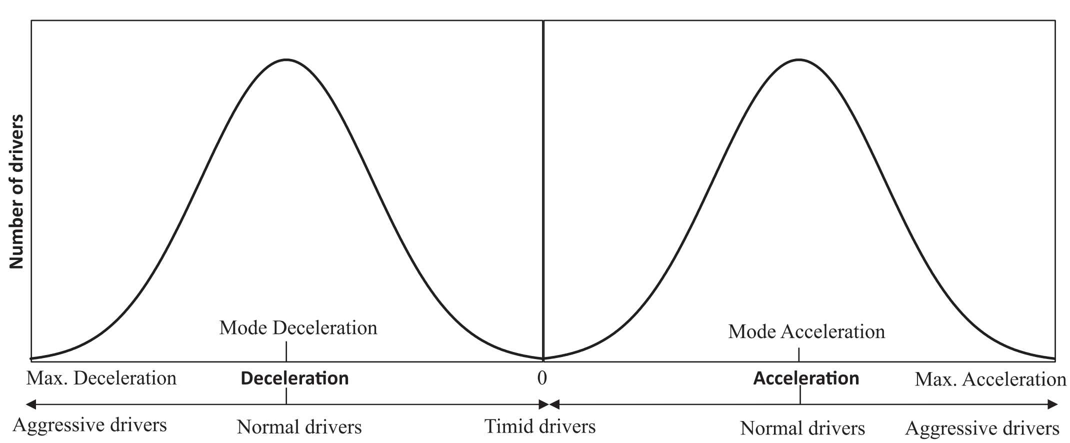

Fig. 10. Driver behaviour distribution.

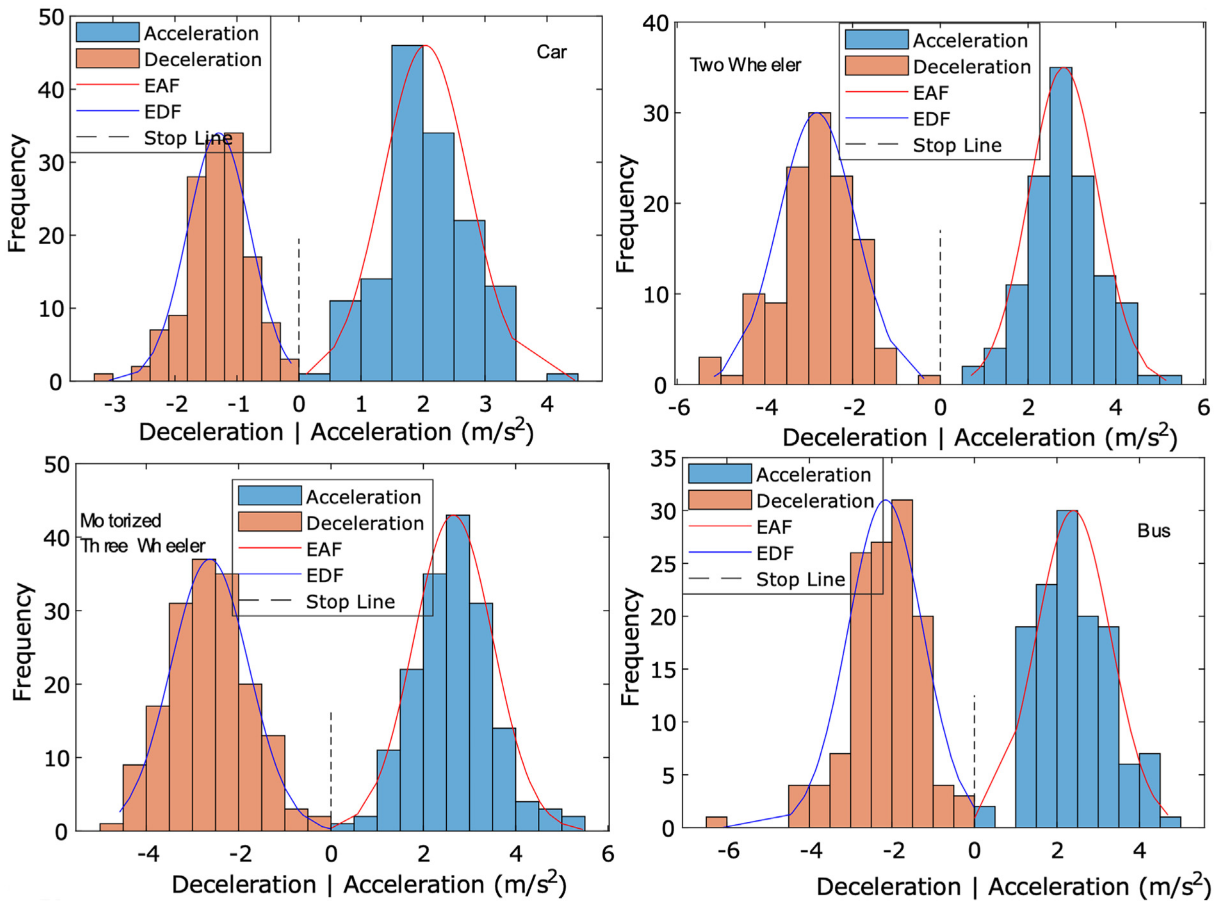

Fig. 11. Field (bars) and estimated (lines) histogram for acceleration and deceleration (EAF: Estimated Acceleration Fit, EDF: Estimated Decleration Fit).

where there is no flyover, a study with flyover can also be conducted in future. Moreover, Emission behaviour of the vehicles in IZOI can be analysed as most of pollutant emission emitting activity takes place in this area.

Declaration of Competing Interest

None.

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