UNIVERSITY of GREENWICH

ILD | Institute for Lifecourse Development

Whole School Approach to Teaching, Learning and Understanding TIMES TABLES

Year-Long CPD Innovation Project (2016-Current Day)

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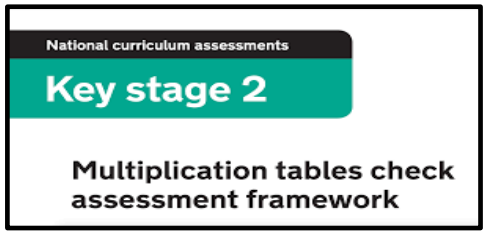
‘*Nine-year-olds should recite times tables by heart … with an emphasis on memorising*’ (2014)
Schools Minister Nick Gibb



My Initial Thoughts

1. Not more ‘*timed*’ maths tests…even worse… ‘*individually timed*’ questions!

2. Schools might focus on ‘rote’ learning without understanding

3. This will cause more anxiety an increase negative attitudes?

SCHOOLS NEED MORE SUPPORT

That being said …I do believe times tables (multiplication bonds) are important, and that knowing them (*fairly* quickly) eases cognitive load, allowing children to get on to more interesting maths.

But in 2016 I had to consider why, over the years, I had frequently heard teachers say in despair … '*if only they knew their times tables!'*

Did schools have a structured approach?
Had schools considered how they teach tables?

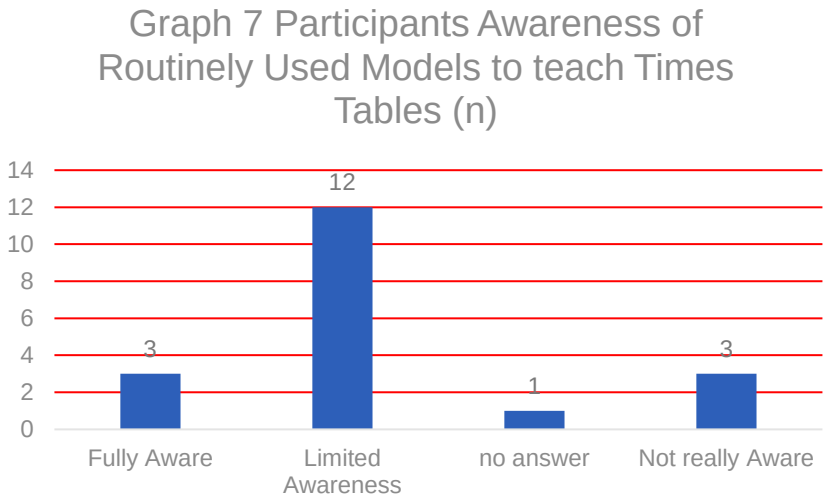
http://saimagesutaswa.blogspot.com/2015/10/free-images-elephant-in-room.html

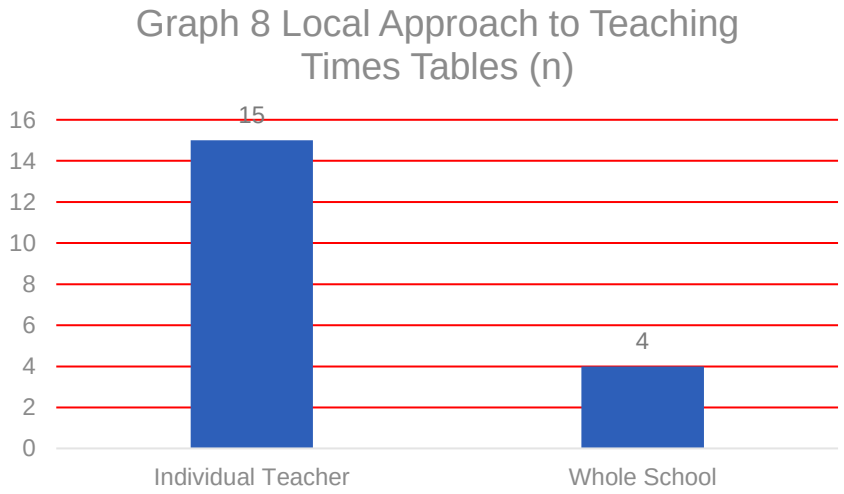
'*Frequently children arrive in upper KS2 with very poor knowledge of multiplication facts. Part of this may be due to … the reticence of some teachers to dedicate time for the teaching and learning of these facts within school time.*

Times tables are often viewed as the equivalent of spelling in Literacy – it is a homework task, to be supported by parents'

Richards, A. MA Primary Mathematics Journal 2015

So in 2016 I undertook some research with 19 schools





86% of Maths Leads were **not really aware** of how TTs were taught in their School

Only 21% of Maths Leads had any whole school approaches to teaching TTs in their School

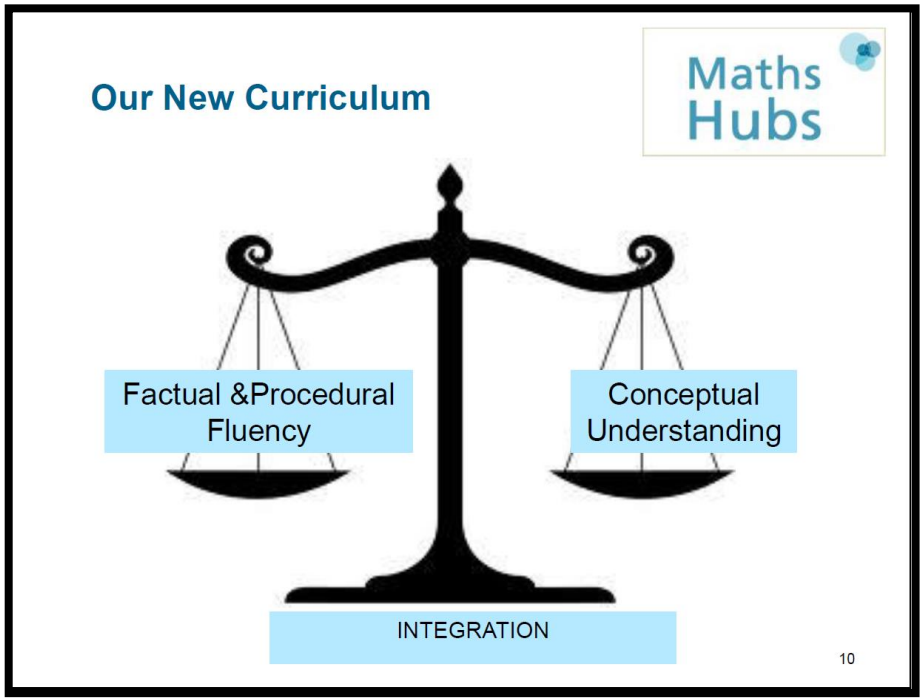
This was not a surprise to be as Maths Leads are overworked and have been stretched in many directions with too many priorities

Where I started …

The 3 aims of the NC for maths

1. **Fluency** **(what does this mean?)**
2. Reasoning
3. Problem Solving

Jane Jones HMI, Ofsted National Lead for Maths until 2018



Describes fluency as a *'blend of conceptual understanding and procedural flexibility'*

NCETM blog November 2014

Do these words represent FLUENCY to you?
‘rote learning’ ‘knowing off by heart’ ‘memorization’

Tweet David Martin (2019)

If we only want people who can recite multiplication facts, then use flash cards … but what if we want more? Transferability?

Watched a student tell me $12 \times 13 = 156$ then asked her 1.2×13

She responded ‘*we don’t do those*’

Does she need more flashcards?’

PISA research into memorisation in OECD countries (2019):

Organisation for Economic Co-operation and Development

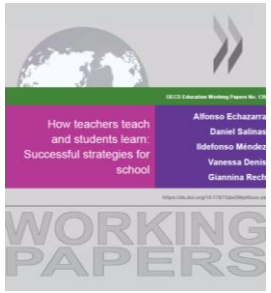
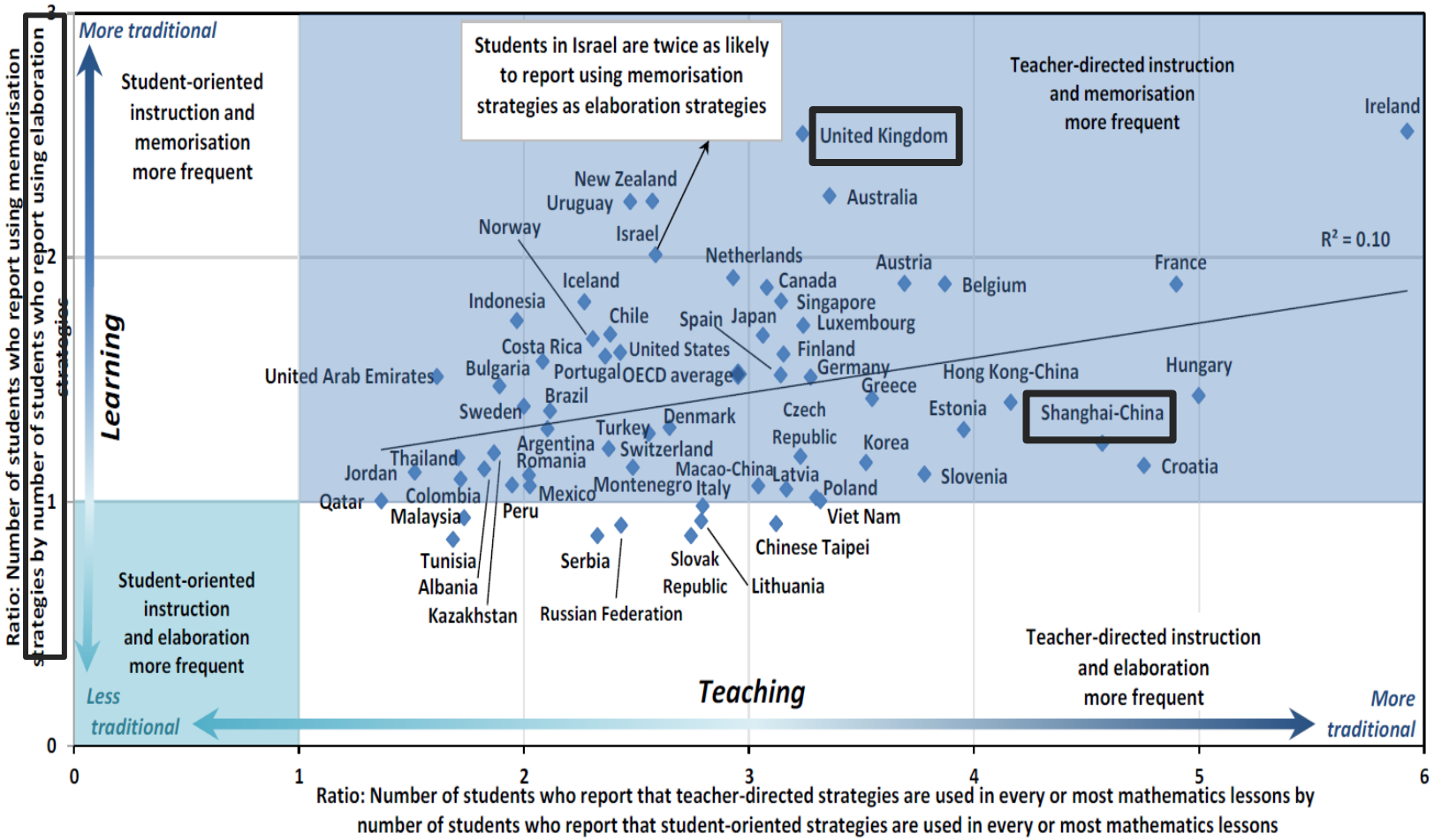


Figure 1.2. Traditional and modern mathematics teaching and learning



Perhaps we need a different word … automaticity?

Memorization or Automaticity?

'***Memorization*** *of basic facts usually refers to committing the result of operations to memory so that thinking is unnecessary'*

'Teaching facts for ***automaticity*** *in contrast relies on thinking. Answers to facts must be automatic, but thinking about the relationships among the facts is critical.*
A child can then think of 9x6 as (10x6)-6'

Twomey Fosnot, C. and Dolk, M. (2000:p85)

The Project I designed was based on this definition of automaticity.

To be '*fairly quick'* … but for the foundation of this knowledge to be built on reasoning about properties, relationships, connections and patterns

Approached in ways which enable '*the Explored*' to become '*the Known'*

Year-Long Action Research Project

Whole School Approaches to
Teaching, Learning and Understanding Times Tables
Focus on a Consistent Approach across School

Maths Leads attend 4 half days over the year
Gap Tasks: Practical things to do in school between training
Professional Log completed before each session
Blended Learning: A repository of materials and a support forum

Delivery

- Focus on Deep Understanding and Mastery
- Research Driven
- Focus on conceptual understanding, leading to automaticity
- Advocates progression and consistency across the school
- Supports schools in taking control of their own curriculum

Project comprises
4 PRE-REQUISITES followed by 8 WHOLE SCHOOL STEPS

2 COMPONANT APPROACH

Dedicated Times Tables Practice Time
High quality activities for automaticity, 10 mins 3 days per week

AND

Specific Whole Maths Lessons
Approx. three lessons per half term focussed on a new times table for exploration and mastery

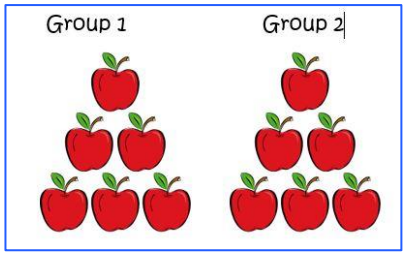
Researchers (e.g., Baroody, 1999; Steel & Funnell, 2001) believe that the development of multiplication recall is in part related to the frequency with which problems and opportunities for repeated practice are provided.
However, it is not simply repetition that leads to improved performance.

Brief overview of content …

4 PRE-REQUISITES

1. Unitizing

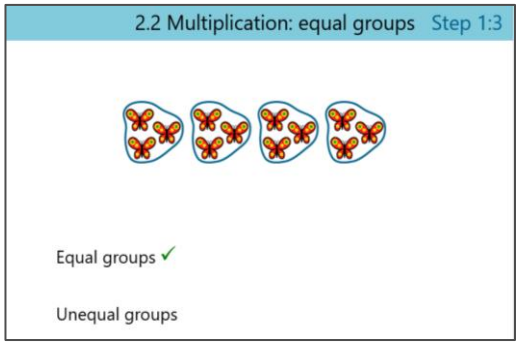
'*First being able to consider many as one, such as one group, one basket of thing*' MA and Kessel (2018) *Building the Foundations*

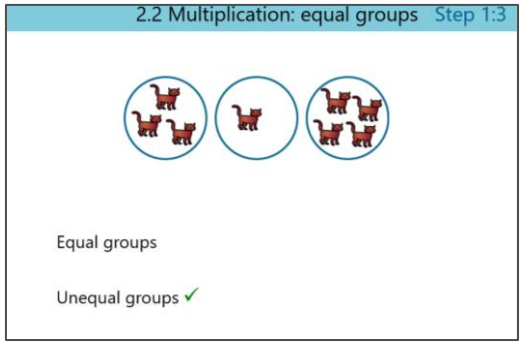


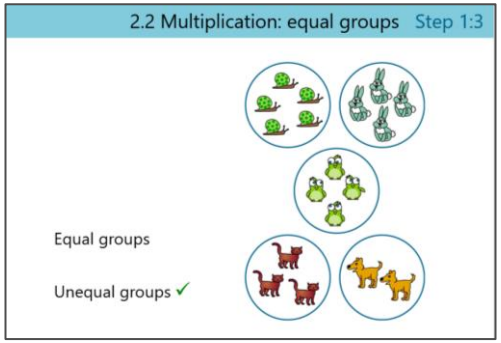
2. Bringing together more than one unit

Counting in groups and seeing each group as one unit

3. Understanding Equal and NOT Equal Groups







NCETM Spine Resources

4. Understanding the early relationship between + and ×

8 WHOLE SCHOOL STEPS

Step 1

Decide the order in which your school will teach times tables, which tables and why.

I believe the National Curriculum in this respect is not well considered

MULTIPLICATION & DIVISION FACTS					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
count in multiples of twos, fives and tens (copied from Number and Place Value)	*count in steps of 2, 3, and 5 from 0, and in tens from any number, forward or backward* (copied from Number and Place Value)	*count from 0 in multiples of 4, 8, 50 and 100* (copied from Number and Place Value)	*count in multiples of 6, 7, 9, 25 and 1 000* (copied from Number and Place Value)	*count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000* (copied from Number and Place Value)	
	recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers	recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables	recall multiplication and division facts for multiplication tables up to 12 × 12		

How I might plan it …discuss

Focus on **ONE times table each half term** – with opportunities built in to also practise those learnt previously

YEAR	First half term	Second half term	Third half term	Fourth half term	Fifth half term	Sixth half term
Year 1	Experience of counting in 1s, 2s, 5s, 10s					
Year 2	1×	(1×) 2×	5×	(5×)10×	0× and revision	revision
Year 3	(2×) 4 ×	(4×) 8 ×	3×	(3×) 6×	(6×) 12×	revision
Year 4	9×	7×	11×	Squares	revision	Test: June

Why a focus one TT per half term? Plasticity of the brain

Neuroscientists tell us it takes approximately **8 weeks of repetition to make a new neural pathway with a myelin sheath** - making this 'go to' automated thinking**!** Then continued practice makes the sheath thicker

Step 2

Decide on whole school **Presentation and Language**

Step 3

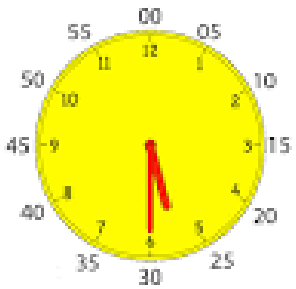
Begin by building each ‘new’ table systematically with the children, considering what they **already know and previously met.**

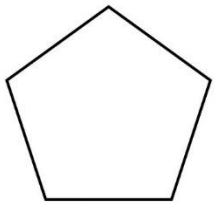
Step 4

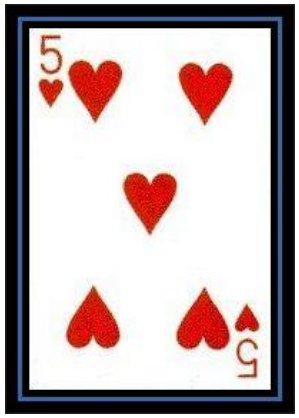
Introduce a new times table by first making clear conceptual **links to the real world** – half termly display of ‘what comes in …’.

WHAT COMES IN 5s?

Make a class display for half a term of children's ideas, photos and resources







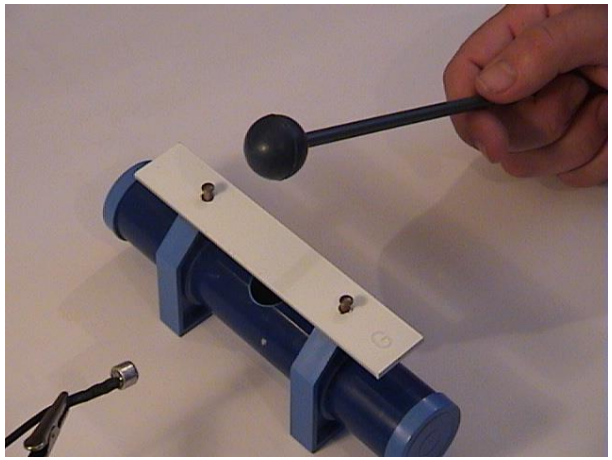
1. Conceptual understanding
2. Great source for word problems
3. Keep adding to it – focus on one table each month

Step 5

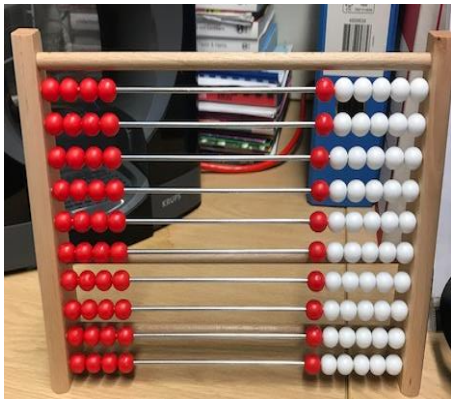
Highly important and happening alongside the other Steps

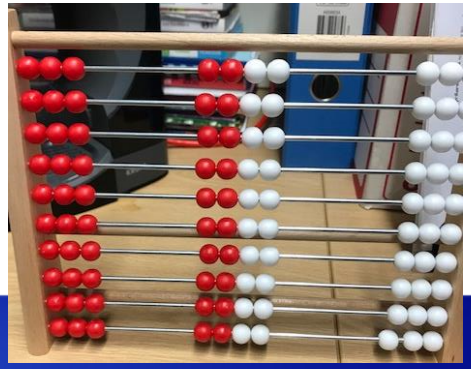
Regular retrieval practice to develop fluency (5-10 minutes)
Provide teachers across school with a ‘Bank of high quality activities for retrieval’

Include conceptual support (at least initially)
Include full verbal patterning (saying whole calculation) and also step counting
First in order then out of order
Build in tests but **NOT** as the main activity



What’s the same?
what’s different?





Step 6

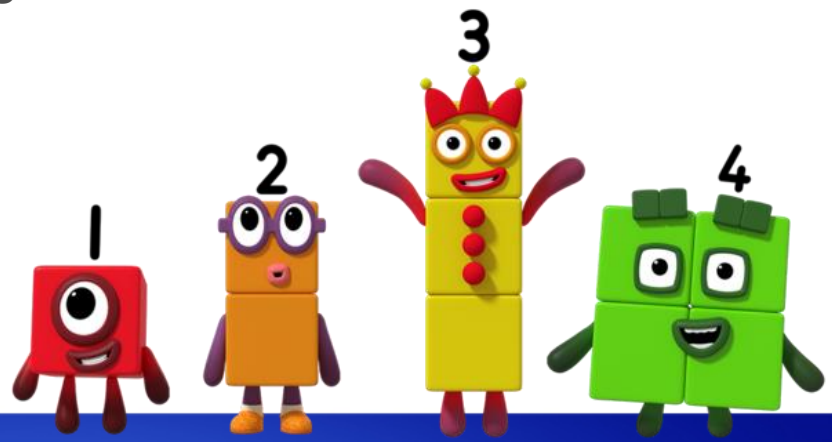
The Array as a Key Representation for tables, to expose properties and laws of multiplication and enable deeper understanding of relationships and connections

Choice of representation is **not about quantity of models, but about quality and progression.**

Research by Barmby et al (2009) exposes the power of the array
Further research undertaken by Huntley (2019:7) states that '*greater exposure to arrays will offer significant benefits'.*

A key finding from my research has been that Numberblocks appears to have made the array more accessible for younger children

Whole School Progression of the Array from EYFS to Year 6



Step 7

Explore the many patterns within each new times table; repeating digits, reversing digits, addition of digits, divisibility and how each table relates to several others.

An example

×1	0,1,2,3,4,5,6,7,8,9,0
×9	0,9,8,7,6,5,4,3,2,1,0
×2	0,2,4,6,8,0
×8	0,8,6,4,2,0
×3	0,3,6,9,2,5,8,1,4,7,0
×7	0,7,4,1,8,5,2,9,6,3,0
×4	0,4,8,2,6,0
×6	0,6,2,8,4,0

Step 8

This relates to all the other Steps and runs concurrently – considering mastery through the use of variation (rather than variety) and through intelligent practice as they plan their TT Curriculum.

A selection of key impact themes:

- Long term high quality CPD made Maths Leads more confident when working with staff, creating opportunities for deepening subject knowledge and improving pedagogy

- Staff across schools are more receptive when things are well structured, are in small steps and do-able

- Schools involved are now actually teaching times tables with a whole school approach which has improved consistency across the school (or are working towards this)

Post intervention:

95% stated that there had been changes in the teaching of times tables

100% felt that their subject knowledge had deepened

84% said that subject knowledge of their staff had grown

In addition Maths Leads from Cohort 2 undertook a times tables check test with their year 3 children pre & post intervention (circa 1500 children)

100% of children increased their scores over 5 months, and 75% saw their score increase between 20-30%.

A few quotes …

'*This Project has been a perfect balance of rationale, pedagogy, subject knowledge development, examples and ideas to take back to school, the importance of teaching times tables well, and deepening understanding – now it's our turn to implement it and make sure it has lasting impact'*

'In the 16 years I have been a teacher, this has been by far the most interesting, practical and useful course I have ever attended. It has clearly suggested and reasoned why this model is good practice and will have a definite impact on my own teaching and my whole school staff. Thank you!'

'It has been inspiring. It has enabled me to pass on a joy of teaching times tables to my children and colleagues and enabled a deeper understanding of mastering tables and mathematical concepts.'

'*I just wanted to let you know that Holly introduced the teaching of times tables to staff yesterday. The agreed practice is exceptionally clear and usable, her subject knowledge and enthusiasm shone through and I would like to thank you. Your times tables project has been highly impactful at ****'

Key Limitations

- **Time needed to see impact:**
  Careful planning necessary, and acceptance that whole school change and impact take time to introduce, deliver and embed.

  'School Staff are willing to take on small changes, one bit at a time'

- **Maths Lead and staff time limited - other priorities and staff changes**

  'Only limitation is time, haven't done as much as would have liked'

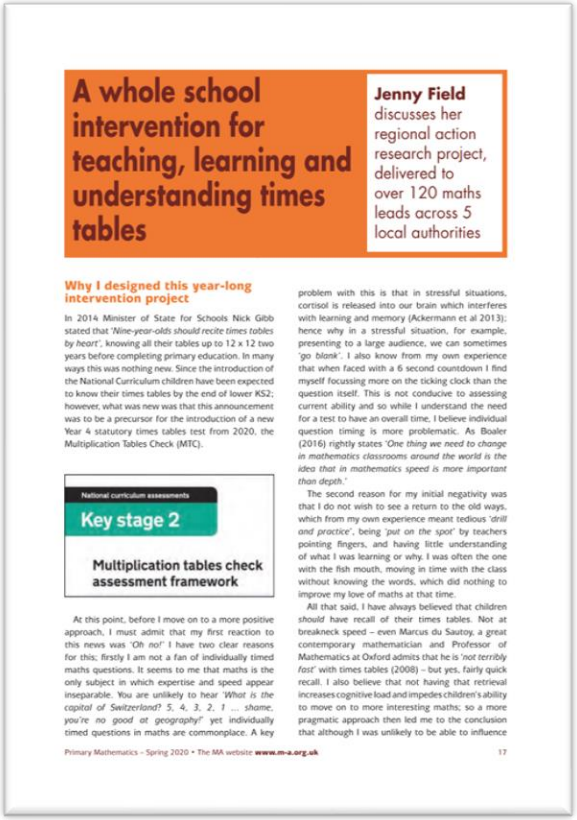
- **Staff subject knowledge – several teachers spoke of this issue**

  'Exposes limited knowledge in teachers / staff.
  Children picked up terms and their meanings quickly.'

The Journey So Far …

- Over 6 years direct participation has grown to over 300 schools nationally
- Attended by all the maths leaders in at least 3 large academies
- Train the Trainer model for United Learning (Cumbria to Kent 40,000 children)
- Influenced design of two ‘spin off’ pedagogical research projects
- Two publications – journal articles, one joint written with participants
- Presented at several Conferences
- Co-author of Position Statement on Multiplication Bonds for MA and ATM
- NCETM Endorsement https://www.ncetm.org.uk/features/whole-school-approach-to-learning-times-tables/
- National Impact from those attending conferences or reading articles
- Endorsed by NCETM
- Boroughs in London South East Plus Maths Hub and London Thames Maths Hub do better than average nationally in MTC

My publications – endorsed by NCETM and MA/ATM



A whole school intervention for teaching, learning and understanding times tables

Jenny Field discusses her regional action research project, delivered to over 120 maths leads across 5 local authorities

Why I designed this year-long intervention project

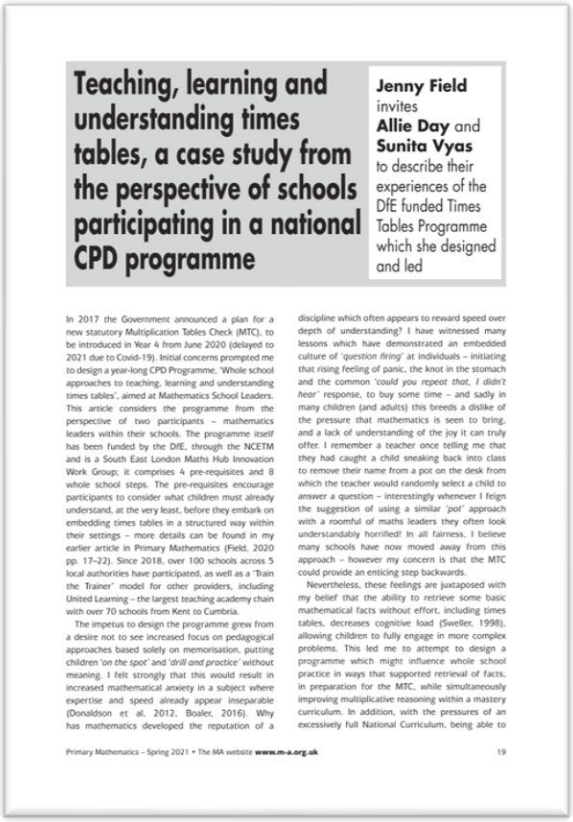
In 2014 Minister of State for Schools Nick Gibb stated that *'Nine-year-olds should recite times tables by heart'*, knowing all their tables up to 12 x 12 two years before completing primary education. In many ways this was nothing new. Since the introduction of the National Curriculum children have been expected to know their times tables by the end of lower KS2; however, what was new was that this announcement was to be a precursor for the introduction of a new Year 4 statutory times tables test from 2020, the Multiplication Tables Check (MTC).

National curriculum assessments
Key stage 2
Multiplication tables check assessment framework

At this point, before I move on to a more positive approach, I must admit that my first reaction to this news was 'Oh no!' I have two clear reasons for this; firstly I am not a fan of individually timed maths questions. It seems to me that maths is the only subject in which expertise and speed appear inseparable. You are unlikely to hear *'What is the capital of Switzerland? 5, 4, 3, 2, 1 ... shame, you're no good at geography!'* yet individually timed questions in maths are commonplace. A key problem with this is that in stressful situations, cortisol is released into our brain which interferes with learning and memory (Ackermann et al 2013); hence why in a stressful situation, for example, presenting to a large audience, we can sometimes *'go blank'*. I also know from my own experience that when faced with a 6 second countdown I find myself focussing more on the ticking clock than the question itself. This is not conducive to assessing current ability and so while I understand the need for a test to have an overall time, I believe individual question timing is more problematic. As Boaler (2016) rightly states *'One thing we need to change in mathematics classrooms around the world is the idea that in mathematics speed is more important than depth.'*

The second reason for my initial negativity was that I do not wish to see a return to the old ways, which from my own experience meant tedious *'drill and practice'*, being *'put on the spot'* by teachers pointing fingers, and having little understanding of what I was learning or why. I was often the one with the fish mouth, moving in time with the class without knowing the words, which did nothing to improve my love of maths at that time.

All that said, I have always believed that children *should* have recall of their times tables. Not at breakneck speed – even Marcus du Sautoy, a great contemporary mathematician and Professor of Mathematics at Oxford admits that he is *'not terribly fast'* with times tables (2008) – but yes, fairly quick recall. I also believe that not having that retrieval increases cognitive load and impedes children's ability to move on to more interesting maths; so a more pragmatic approach then led me to the conclusion that although I was unlikely to be able to influence



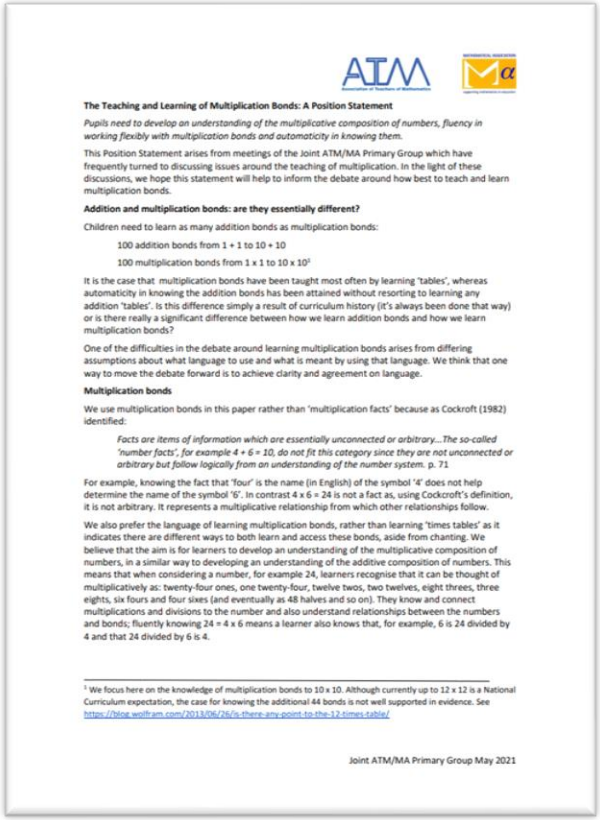
Teaching, learning and understanding times tables, a case study from the perspective of schools participating in a national CPD programme

Jenny Field invites **Allie Day** and **Sunita Vyas** to describe their experiences of the DfE funded Times Tables Programme which she designed and led

In 2017 the Government announced a plan for a new statutory Multiplication Tables Check (MTC), to be introduced in Year 4 from June 2020 (delayed to 2021 due to Covid-19). Initial concerns prompted me to design a year-long CPD Programme, 'Whole school approaches to teaching, learning and understanding times tables', aimed at Mathematics School Leaders. This article considers the programme from the perspective of two participants – mathematics leaders within their schools. The programme itself has been funded by the DfE, through the NCETM and is a South East London Maths Hub Innovation Work Group; it comprises 4 pre-requisites and 8 whole school steps. The pre-requisites encourage participants to consider what children must already understand, at the very least, before they embark on embedding times tables in a structured way within their settings – more details can be found in my earlier article in Primary Mathematics (Field, 2020 pp. 17–22). Since 2018, over 100 schools across 5 local authorities have participated, as well as a 'Train the Trainer' model for other providers, including United Learning – the largest teaching academy chain with over 70 schools from Kent to Cumbria.

The impetus to design the programme grew from a desire not to see increased focus on pedagogical approaches based solely on memorisation, putting children *'on the spot'* and *'drill and practice'* without meaning. I felt strongly that this would result in increased mathematical anxiety in a subject where expertise and speed already appear inseparable (Donaldson et al, 2012, Boaler, 2016). Why has mathematics developed the reputation of a discipline which often appears to reward speed over depth of understanding? I have witnessed many lessons which have demonstrated an embedded culture of *'question firing'* at individuals – initiating that rising feeling of panic, the knot in the stomach and the common *'could you repeat that, I didn't hear'* response, to buy some time – and sadly in many children (and adults) this breeds a dislike of the pressure that mathematics is seen to bring, and a lack of understanding of the joy it can truly offer. I remember a teacher once telling me that they had caught a child sneaking back into class to remove their name from a pot on the desk from which the teacher would randomly select a child to answer a question – interestingly whenever I feign the suggestion of using a similar 'pot' approach with a roomful of maths leaders they often look understandably horrified! In all fairness, I believe many schools have now moved away from this approach – however my concern is that the MTC could provide an enticing step backwards.

Nevertheless, these feelings are juxtaposed with my belief that the ability to retrieve some basic mathematical facts without effort, including times tables, decreases cognitive load (Sweller, 1998), allowing children to fully engage in more complex problems. This led me to attempt to design a programme which might influence whole school practice in ways that supported retrieval of facts, in preparation for the MTC, while simultaneously improving multiplicative reasoning within a mastery curriculum. In addition, with the pressures of an excessively full National Curriculum, being able to



The Teaching and Learning of Multiplication Bonds: A Position Statement

Pupils need to develop an understanding of the multiplicative composition of numbers, fluency in working flexibly with multiplication bonds and automaticity in knowing them.

This Position Statement arises from meetings of the Joint ATM/MA Primary Group which have frequently turned to discussing issues around the teaching of multiplication. In the light of these discussions, we hope this statement will help to inform the debate around how best to teach and learn multiplication bonds.

Addition and multiplication bonds: are they essentially different?

Children need to learn as many addition bonds as multiplication bonds:

100 addition bonds from 1 + 1 to 10 + 10

100 multiplication bonds from 1 x 1 to 10 x 10[1]

It is the case that multiplication bonds have been taught most often by learning 'tables', whereas automaticity in knowing the addition bonds has been attained without resorting to learning any addition 'tables'. Is this difference simply a result of curriculum history (it's always been done that way) or is there really a significant difference between how we learn addition bonds and how we learn multiplication bonds?

One of the difficulties in the debate around learning multiplication bonds arises from differing assumptions about what language to use and what is meant by using that language. We think that one way to move the debate forward is to achieve clarity and agreement on language.

Multiplication bonds

We use multiplication bonds in this paper rather than 'multiplication facts' because as Cockroft (1982) identified:

> *Facts are items of information which are essentially unconnected or arbitrary...The so-called 'number facts', for example 4 + 6 = 10, do not fit this category since they are not unconnected or arbitrary but follow logically from an understanding of the number system.* p. 71

For example, knowing the fact that 'four' is the name (in English) of the symbol '4' does not help determine the name of the symbol '6'. In contrast 4 x 6 = 24 is not a fact as, using Cockcroft's definition, it is not arbitrary. It represents a multiplicative relationship from which other relationships follow.

We also prefer the language of learning multiplication bonds, rather than learning 'times tables' as it indicates there are different ways to both learn and access these bonds, aside from chanting. We believe that the aim is for learners to develop an understanding of the multiplicative composition of numbers, in a similar way to developing an understanding of the additive composition of numbers. This means that when considering a number, for example 24, learners recognise that it can be thought of multiplicatively as: twenty-four ones, one twenty-four, twelve twos, two twelves, eight threes, three eights, six fours and four sixes (and eventually as 48 halves and so on). They know and connect multiplications and divisions to the number and also understand relationships between the numbers and bonds; fluently knowing 24 = 4 x 6 means a learner also knows that, for example, 6 is 24 divided by 4 and that 24 divided by 6 is 4.

[1] We focus here on the knowledge of multiplication bonds to 10 x 10. Although currently up to 12 x 12 is a National Curriculum expectation, the case for knowing the additional 44 bonds is not well supported in evidence. See https://blog.wolfram.com/2013/06/26/is-there-any-point-to-the-12-times-table/

Joint ATM/MA Primary Group May 2021

ATM/MA Position Statement on Multiplication Bonds

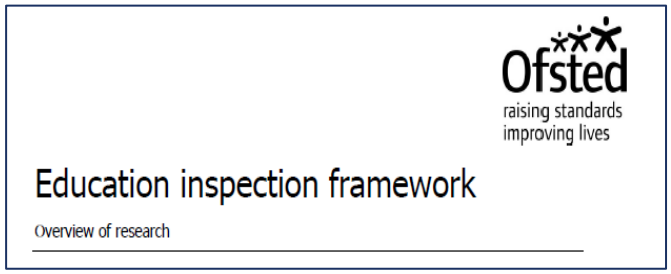
Authors: Mike Askew, Jenni Back, Jenny Field, Ruth Trundley, Christina Wood

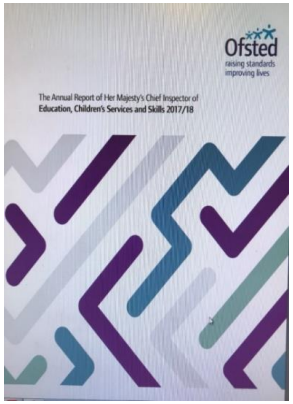
https://www.atm.org.uk/write/MediaUploads/News/The_teaching_and_learning_of_multiplication_bonds_ATM_MA_may__22_final.pdf

Articles in MA Primary Mathematics Journals

https://www.ncetm.org.uk/features/whole-school-approach-to-learning-times-tables/

Great time for change:
Ofsted now has more focus on research-based curriculum design and ownership





Curriculum Knowledge and Expertise Ofsted Annual Report

'A striking conclusion that we have drawn from the findings is that, despite the fact that the curriculum is what is taught, there ***is little debate or reflection about it****'*

'There is ***limited evidence of a thoughtful approach to curriculum,*** *which is often equated with the timetable and discussed in a generic fashion. Schools reported that few teachers are trained in curriculum development or theory.'*

This TT Project gives schools an opportunity to discuss and decide… shaping their own curriculum with their own clear rationales

Whatever we feel about the new MTC ***(and I am not a fan of the test itself)***

It IS an opportunity to ***reflect on our curriculum*** *and make positive changes in an area that has probably* ***not had enough focussed attention*** *in the past*

We must not return to drill and practice alone, without understanding
or
Take it home on a Friday for a test on Monday

THANK YOU FOR LISTENING ☺

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