Stamataki, Ioanna; Zang, Jun; Bazeley, William; Morgan, Gerald

Study of Flow Over Weirs such as Pulteney Weir

Stamataki, Ioanna; Zang, Jun; Bazeley, William; Morgan, Gerald (2014): Study of Flow Over Weirs such as Pulteney Weir. In: Lehfeldt, Rainer; Kopmann, Rebekka (Hg.): ICHE 2014. Proceedings of the 11th International Conference on Hydroscience & Engineering, September 28 - October 2, 2014, Hamburg, Germany. Karlsruhe: Bundesanstalt für Wasserbau. S. 295-302.

ICHE 2014, Hamburg - Lehfeldt & Kopmann (eds) - © 2014 Bundesanstalt für Wasserbau ISBN 978-3-939230-32-8

Study of Flow Over Weirs such as Pulteney Weir

I. Stamataki, J. Zang & W.D. Bazeley
Department of Architecture and Civil Engineering, University of Bath, Bath, United Kingdom
G.C.J. Morgan
Edenvale Young Associates Ltd, Bristol, United Kingdom

ABSTRACT: The present research paper looks into the expression of a head-discharge relationship for Pulteney Weir, constructed in Bath in 1975 as a part of a flood protection scheme to reduce the city's risk of flooding, and initiates research for the computation of its flow. To achieve this, the study looks at the flow of three known shapes of weirs – a sharp-crested rectangular, a sharp-crested 45° oblique weir and a sharp-crested 60° oblique weir - and compares experimental and analytical expressions, to eventually test "Half" Pulteney and Pulteney Weir models. The series of tests were conducted in the University of Bath's Hydraulic Laboratory. For each experiment, upstream and downstream water levels were measured for different flow rates and boundary conditions. Head – discharge relationships were established for all weirs tested. The results showed that for the sharp-crested 45° and 60° oblique weirs, the discharge coefficient is simply a function of H/P. The derived equation for Pulteney Weir indicated that the impact of a serious flood situation in Bath could be predicted. Although the "Half" Pulteney experiments presented that the turbulent flow was not as symmetrical as it was expected to be. All equations found, proved a very a good agreement with the experimental data.

Keywords: Open channel flow, Labyrinth weir, Head-discharge relationship, Flow discharge, Weir, Sharp-crested rectangular weir, Oblique weir, Pulteney weir

1 INTRODUCTION

Global warming and the rise in temperature initiate extreme weather condition changes. One expected consequence of this is an increase in the instance of flooding with 50,000 hectares in Great Britain being at risk (HM Gonvernment, 2012).

The Avon is the river that part of it flows from Bath to Bristol. Pulteney Weir was constructed in Bath in 1975 as a part of a flood protection scheme to reduce the risk of flooding of the city. It is one of the many engineering obstructions built in the river's path to control the flow.

Figure 1. Pulteney Weir 1975.

Figure 2. Pulteney Weir 2013.

Figure 3. Flood Protection Scheme Bath.

One of the main issues in civil engineering is the measurement of discharge in open channels. Weirs are hydraulic structures used not only for flow measurement but also for flow regulation and flood protection, whilst at the same time having the ability to completely alter a river's flow regime.

The aim of this experimental project was to model Pulteney Weir, at University of Bath's Hydraulic Laboratory using a 7.5 m Armfield S6-MkII glass-sided tilting flume, and establish a complete head-discharge relationship for Pulteney Weir, which would allow the computation of its flow. Such computation could be a way to predict and save Bath from possible flooding. First, the accuracy of the method was validated. The head-discharge relationship of two different known shapes of weirs were then determined, and experimental results were compared with their corresponding analytical expressions. This paper outlined the process of finding a head discharge equation and initiated research into knowledge of new labyrinth weir shapes.

Sharp crested weirs are simple shaped weirs, widely used, helpful in the acquisition of discharge in open channels only by measuring the height of the water head upstream of the weir. Thus, the analysis of a sharp-crested weir was the starting point of this research (Ackers et al., 1978).

The most known discharge equation (1) used for sharp crested rectangular weirs that is derived from energy equilibrium is the following (Bos, 1989):

$$Q = C_d \frac{2}{3} \cdot \sqrt{2gbh^{3/2}} \tag{1}$$

Complex "labyrinth" weirs continue to create significant problems for river modelling and flood prediction (Simon & Korom, 1997), as equations for their behaviour are uncommon. It has been suggested that Pulteney Weir might not be considered as a labyrinth weir. None the less, since the labyrinth is the closest weir shape that we can compare it with, this study's data will be presented in a labyrinth weir's equation. Labyrinth weirs are used as structures that help increase the specific discharge. In 2000, as a progression of the conventional labyrinth weirs, a new solution was introduced (Figure 5).

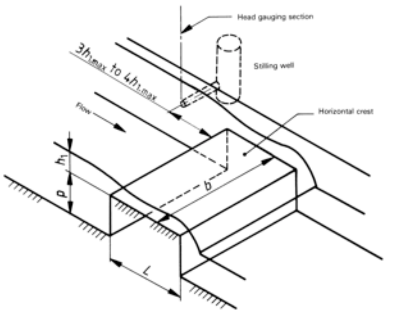


Figure 4. Sharp Crested Rectangular Weir (British Standards, 1990).

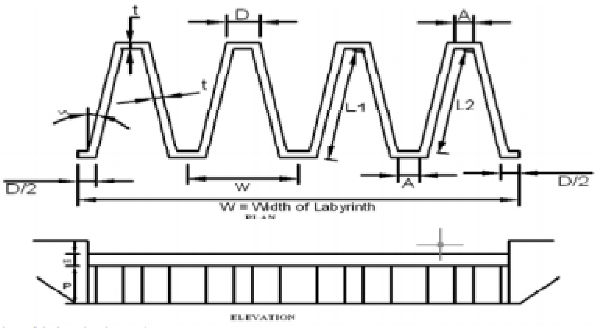


Figure 5. Layout of Labyrinth Weir (Khode & Tembhurkar, 2010).

Prominent research carried out in this field continues and the latest analysis is that of Tullis et al (Tullis et al., 2007) where they have generated the following labyrinth weir submerge equations:

$$\frac{H^*}{H_o} = 0.3320\left(\frac{H_d}{H_o}\right)^4 + 0.2008\left(\frac{H_d}{H_o}\right)^2 + 1 \qquad \text{where } 0 \le \frac{H_d}{H_o} \le 1.53 \tag{2}$$

$$\frac{H^*}{H_o} = 0.9379\left(\frac{H_d}{H_o}\right) + 0.2174 \qquad \text{where } 0 \le \frac{H_d}{H_o} \le 1.53 \tag{3}$$

$$H^* = H_d \qquad \text{where } 3.5 \le \left(\frac{H_d}{H_o}\right) \tag{4}$$

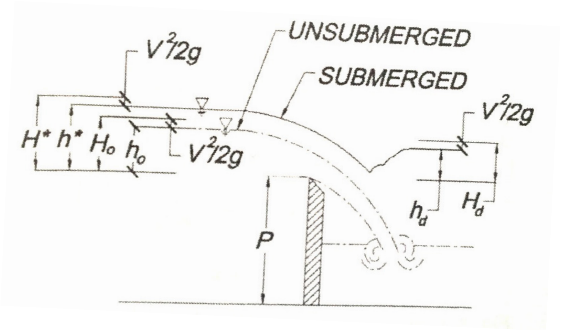


Figure 6. Submerged and free-flow parameters (Tullis et al., 2007).

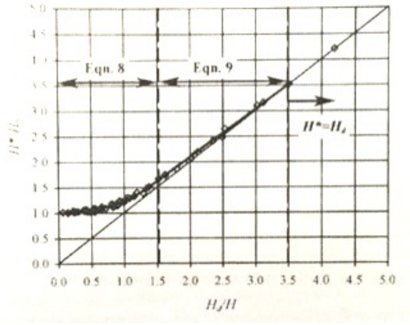


Figure 7. Limits of H^*/H_o submerge equations by Tullis et al. (Tullis et al., 2007).

2 EXPERIMENTAL SETUP

Four wave gauges were placed in pairs before and after the weirs, to measure water height. Point gauges were also used along the flume to confirm the measurements.

Five models were fabricated. The first one was a sharp crested rectangular weir, the second was a sharp crested 45° oblique rectangular weir and the third was a sharp crested 60° oblique rectangular weir. These three models were made from concrete in 18mm phenolic-faced plywood formwork. The fourth model assembled was a “Half” Pulteney (Figure 9) in a 1:50 scale, and the fifth was “Pulteney Weir” (Figure 10) in a 1:100 scale. The last two, and harder, models were made in Bath University’s timber laboratory using plywood and contact glue. They were then both filled and painted to give them the appropriate shape and waterproof effect.

Figure 8. University of Bath’s Hydraulic Flume.

Figure 9. “Half” Pulteney Model.

Figure 10. Full Pulteney Model.

3 EXPERIMENTAL RESULTS AND DISCUSSION

3.1 *Sharp-Crested Rectangular Weir*

The Rehbock equation (5) was used on a sharp crested rectangular weir in order to confirm the accuracy of the experiment.

$$C_d = 0.611 + 0.075\left(\frac{H}{P}\right) \tag{5}$$

There was a very good agreement between theoretical and experimental results. Additionally, looking at the C_d versus H/P curve we found a linear relationship from which, a general equation (6), was established with very close agreement to the theoretical equation, therefore validating our experiment.

$$C_d = 0.602 + 0.083\left(\frac{H}{P}\right) \tag{6}$$

3.2 *Sharp-Crested Oblique Rectangular Weirs*

Rehbock's equation was presented in its more general form and the values of a and b were determined for the different oblique weirs:

$$C_d = a + b\left(\frac{H}{P}\right) \tag{7}$$

The results obtained by these experiments proved that the discharge coefficient was, in fact, only a function of H/P when surface tension effects are minimised. Eight experiments were carried out for the sharp-crested 45° oblique rectangular weir and seven for the sharp-crested 60° oblique rectangular weir, alternating the flow rate between 2.5 and $20\,\mathrm{l/s}$. Cd was plotted against H/P in figures 11 and 12 using only the results between $3-20\,\mathrm{l/s}$. The $2.5\,\mathrm{l/s}$ is a very small flow rate and surface tension effects were too significant to be ignored, therefore it was not included in the graphical representation. The best linear fits were found, giving the following equations:

For a sharp-crested 45° oblique rectangular weir:

$$C_d = 0.00001 + 0.000005\frac{H}{P} \tag{8}$$

For a sharp-crested 60° oblique rectangular weir:

$$C_d = 0.00002 + 0.000002\frac{H}{P} \tag{9}$$

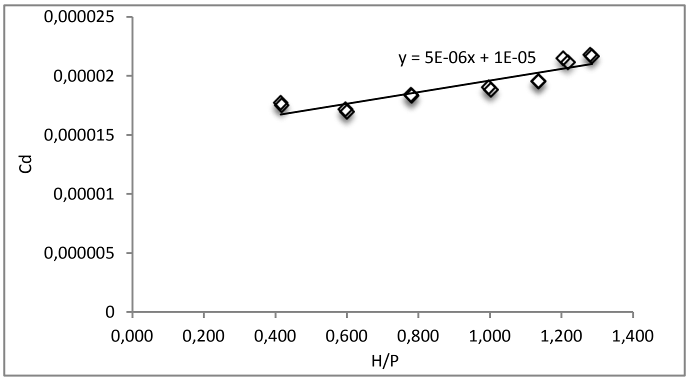


Figure 11. Coefficient discharge vs. H/P for sharp crested 45 oblique rectangular weir.

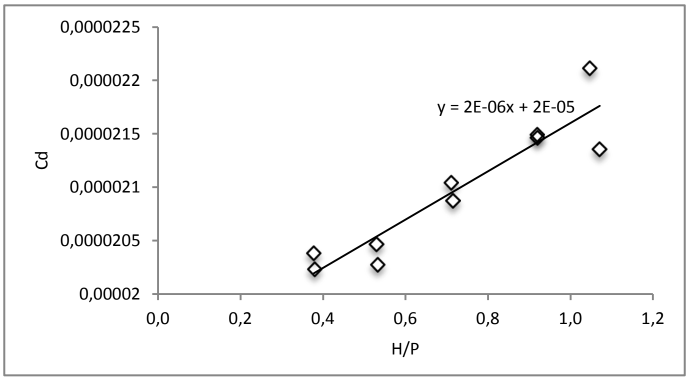


Figure 12. Coefficient discharge vs. H/P for sharp crested 60 oblique rectangular weir.

Furthermore, the results for the sharp-crested 45° oblique rectangular weir verified the decrease in water head H, using an oblique weir instead of a simple rectangular one, implemented in previous research (Borghei et al., 2003) opposite to the sharp-crested 60° oblique rectangular weir that did not prove it. The decrease in head, H, shown confirmed that a sharp-crested 45° oblique rectangular weir is a suitable discharge meter device and in some cases is more appropriate than the plain sharp-crested rectangular weir. The sharp-crested 60° oblique rectangular weir is a suitable discharge meter device, but it is definitely not more appropriate than the plain sharp-crested rectangular weir, as the water head, H, is increased.

3.3 *Pulteney Weir*

For Pulteney weir, 22 experiments were performed in 1:250 slope with flow rates between 1.5 and $31\,\mathrm{l/s}$ in the same boundary conditions, and a further 15 experiments in slow flow rates, between 1.5 and $5\,\mathrm{l/s}$, this time changing the boundary conditions. The different boundary conditions used were: a sluice gate at

19 mm from the bottom of the flume, a 30x30 mm weir, a 30x40 mm weir and a 30x50 mm weir. The approach was used to fit a general weir equation to the data assimilated from the experiments.

The general weir equation is:

$$Q = C_d \frac{2}{3} \cdot \sqrt{2gbh^{3/2}}$$

By adjusting both the discharge coefficient and the exponent in this formula, an equation was obtained. Looking at this formula in a more general form, we develop the following:

$$y = ax^b \tag{10}$$

Where:

$y = \frac{Q}{\frac{2}{3}\sqrt{2gb}}$, $a = C_d$, $x = H$ and $b =$ exponent

The H data plotted against y for all different flow rates is shown in Figure 13. As observed from this graph, there are two different equations and there is quite a strong correlation between the data points and the fitted lines.

The general form of the equation for flow rates between 1.5 and 23 l/s is therefore $y = 3.9119x^{2.8098}$ and for flow rates between 25 and 31 l/s is therefore $y = 0.1254x^{0.9437}$.

Thus, the equations for the model of Pulteney Weir are:

$$Q = 3.9119\frac{2}{3}\sqrt{2g}bH^{2.8098} \text{ for flow rates between 1.5 and 23 l/s and} \tag{11}$$

$$Q = 0.1254\frac{2}{3}\sqrt{2g}bH^{0.9437} \text{ for flow rates between 25 and 31 l/s.} \tag{12}$$

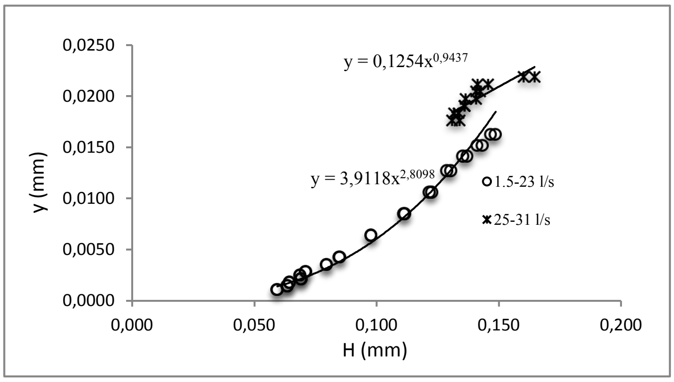


Figure 13. H versus y.

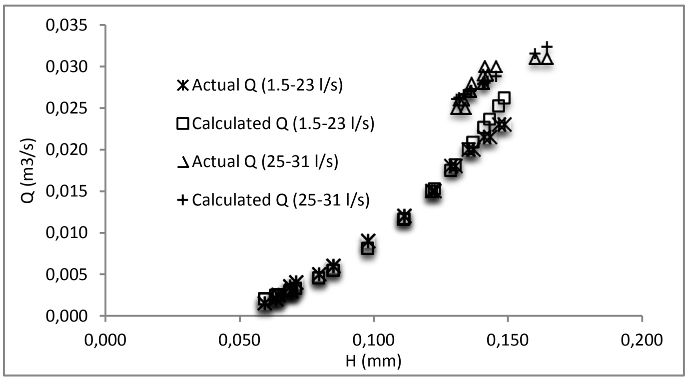
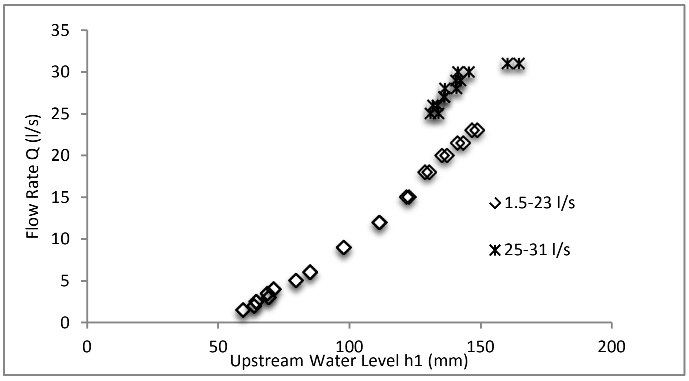


Figure 14. Actual and Calculated Q versus H for Pulteney Weir.

The discharge coefficient has a value of 3.91 but as suggested by Crookston (Crookston, 2010), labyrinth weirs with small angles and small H/P ratios are expected to have 2-3 times the discharge coefficient a rectangular weir would have, therefore is not considered very large. Using these equations to calculate the flow rates, it was observed (Figure 14) that there is a very good correspondence between analytical and experimental results.

It is also interesting to observe the upstream water level, h1, plotted against flow rate, Q (Figure 15). In this specific graphical representation, very extreme flood conditions are apparent as after 25 l/s there is a decrease in h1, which means that h2, becomes equal to h1.

From figure 16 what is interesting to observe, is that all different weirs used for boundary conditions, have the same effect on the upstream water level. Although the sluice gate follows a different linear equation and has a bigger effect on the upstream water level than the weirs have. This is a very interesting observation as in the prototype there is a sluice gate parallel to Pulteney Weir and this proves that the effect that sluice gate has is important.



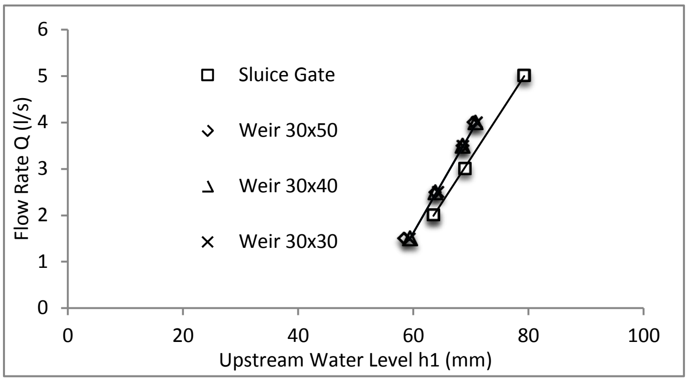


Figure 15. Flow rate, Q, versus upstream water level, h1, for Pulteney Weir.

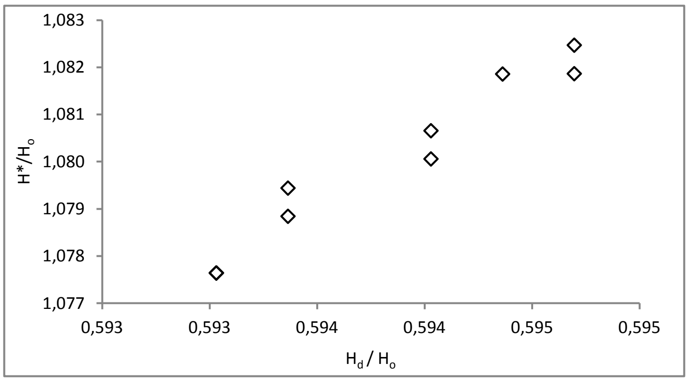
Figure 16. Flow rate, Q, versus upstream water level, h1, for different boundary conditions for Pulteney Weir.

Comparing now the experimental data with Tullis' equations (Tullis et al., 2007) we can only use the following equation, as our $\mathrm{H_d}/\mathrm{H_o}$ range is limited:

$$\frac{\mathrm{H}^*}{\mathrm{H_o}} = 0.3320\left(\frac{\mathrm{H_d}}{\mathrm{H_o}}\right)^4 + 0.2008\left(\frac{\mathrm{H_d}}{\mathrm{H_o}}\right)^2 + 1 \tag{13}$$

where $0 \le \frac{\mathrm{H_d}}{\mathrm{H_o}} \le 1.53$

The experimental results have only a 3% difference with the calculated results, which proves that this equation is a very good fit for our experimental results.



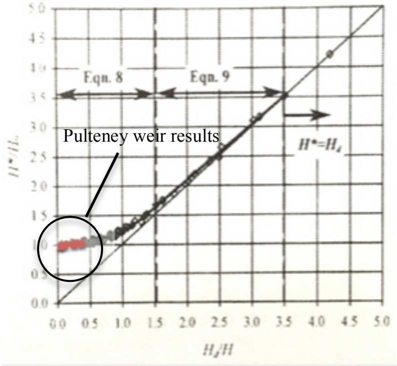


Figure 17. $\mathrm{H_d}/\mathrm{H_o}$ versus $\mathrm{H}^*/\mathrm{H_o}$.

Figure 18. Pulteney weir results plotted on Tullis' graph.

3.4 *"Half" Pulteney Weir*

11 experiments were performed for "Half" Pulteney weir in 1:250 slope with flow rates between 1.5 and 25 l/s, in same boundary conditions. By using the equation (11) expressed for Pulteney Weir, for flow rates between 1.5 and 23 l/s, it was observed that there was a relationship between the analytical and experimental results for low flow rates but as the flow rate increased, the relationship between the analytical and experimental results became weaker. The fact that the results did not match up properly meant that the turbulent flow was not as symmetrical as it was expected it to be.

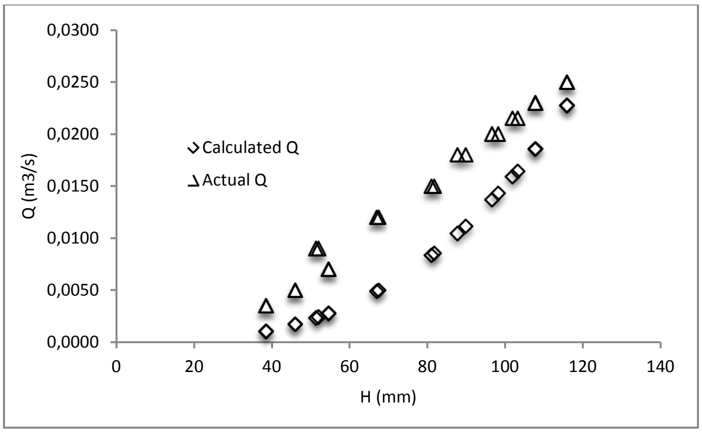


Figure 19. Actual and calculated Q versus H for "Half" Pulteney Weir.

3.5 *Prototype Scale*

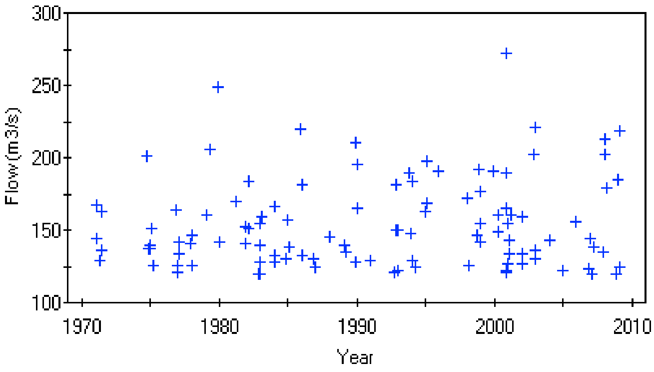
Using dimensional similarity, we acquire that the length scale for Pulteney Weir is λ=100 and the length scale for "Half" Pulteney is λ=50, as the models were built in 1:100 and 1:50 scale respectively. Therefore the equations for Pulteney Weir become:

$$\frac{\lambda^3}{\sqrt{\lambda}}Q = 3.9119\frac{2}{3}\sqrt{2\times 1\times g}(\lambda b)(\lambda H)^{2.8098}$$

$$\Rightarrow 10^5Q = 3.9119\frac{2}{3}\sqrt{2g}(100b)(100H)^{2.8098} \text{ for flow rates between 1.5 and 23 l/s and} \tag{14}$$

$$\frac{\lambda^3}{\sqrt{\lambda}}Q = 0.1254\frac{2}{3}\sqrt{2\times 1\times g}(\lambda b)(\lambda H)^{0.9437}$$

$$\Rightarrow 10^5Q = 0.1254\frac{2}{3}\sqrt{2g}(100b)(100H)^{0.9437} \text{ for flow rates between 25 and 31 l/s.} \tag{15}$$



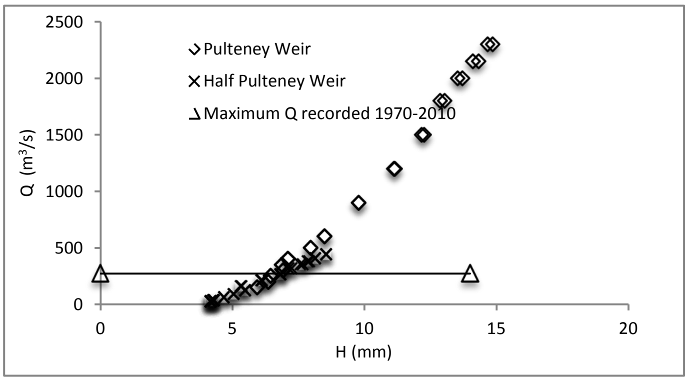


Figure 20. Flow data Bathford (Anon., 2013).

Figure 21. Prototype Q versus H.

Plotting now real scale Q versus H (Figures 19 and 20), for both "Half" and Full Pulteney, we can observe that the data obtained, especially from the Pulteney Weir model, are a lot larger than the maximum flow rate recorded between 1970 and 2010, $272.66\,m^3/s$ the 10th October 2000.

4 CONCLUSION

This paper examines flow over weirs such as Pulteney Weir and equations were successfully derived for the weirs tested. The experiments were first validated, by testing a sharp-crested rectangular weir. The tests showed a remarkable agreement between analytical and experimental results ensuring therefore the accuracy of the equipment and measuring techniques. Equations were found for both 45 and 60° oblique weirs, proving that the discharge coefficient is only a function of H/P. A head - discharge equation for Pulteney Weir was also found which satisfied the initial goal of this research, although the "Half" Pulteney model did not prove to demonstrate its predicted turbulent flow symmetry. However in low flow rates, surface tension was too great to be ignored, and large flow rates could not be very realistic as in such flow rates, the sluice gate parallel to Pulteney Weir would have a very big impact.

Further research is needed to examine the head discharge relationships for all labyrinth weirs of this type, using different alternatives of the weirs' curves and dimensions to establish eventually a general equation for this type of weirs. From the derived equation, the impact of a serious flood situation in Bath can be predicted and shows that Pulteney Weir will probably be able to prevent flooding even for a lot higher flow rates than the one recorded in October 2000. Furthermore, OpenFOAM is a CFD simulation tool that using the equation derived in this research, can model Pulteney Weir including its surroundings; the sluice gate, Pulteney Bridge and the rest of the engineering obstructions built in the river Avon's flow. Further research is therefore suggested for an OpenFOAM model to be constructed to examine the possibility of Bath's flooding in more detail.

Flooding is a very important consequence of climate change. It is therefore essential to understand how to prevent future flooding by using structures, like weirs. Pulteney Weir is not only a weir, but is a beautifully made structure fulfilling its goal whilst pleasing with its appearance. It has become one of Bath's wonders without anyone even knowing it has such an important purpose.

NOTATION

Cd	Discharge coefficient
wc/p	Vertical aspect ratio
H*	Total upstream head on a submerged weir relative to the crest elevation
Hd	Total downstream head of submerged weir relative to the crest elevation
Ho	Total head upstream of a weir operating in a free-flow condition and measured relative to the crest elevation
Ht	Total head
Le	Effective length of labyrinth weir
Ts	Surface Tension
A	Area
b	Breadth
CL	Crest coefficient per unit length of the labyrinth weir
F	Force
g	Gravitational acceleration constant
H	Total Head
K	Constant
n	Number of cycles
P	Weir's Height
P	Pressure
Q	Discharge over weir
u	Velocity
W	Width of the weir
z	Head
ρ	Density

REFERENCES

Ackers , P., White, W.R., Perkins, J.A. & Harrissson, A.J., 1978. *Weirs and Flumes for Flow Measurement*. Bath: Pitman Press.

Anon., 2013. *POT for Bathford*. [Online] Available at: http://www.environment-agency.gov.uk/hiflows/apr.aspx?53018_pot [Accessed 19 April 2013].

Borghei, S.M., Vatannia, Z., Ghodsian, M. & Jalili, M.R., 2003. Oblique rectangular sharp crested weir. *Water &Maritime Engineering*, 156(WM2), pp.185-91.

Bos, M., 1989. *Discharge Measurement Structures*. Wageningen: International Institute for Land Reclamation and Improvement, ILRI.

British Standards, 1990. BS 3680-4E *Measurement of liquid flow in open channels has some applicable strategies*.

Crookston, B., 2010. *Labyrinth weirs*. PhD Dissertation. Logan: Utah State University.

Crookston, B.M., 2010. *Labyrinth Weirs*. Dissertation. Utah State University.

HM Gonvernment, 2012. *UK Climate Change Risk Assessment: Government Report*.

Khode, B.V. & Tembhurkar, A.R., 2010. Evaluation and analysis of crest coefficient for labyrinth weirs. *World applied sciences journal*, 11(7), pp.835-39.

Simon, A. & Korom, S., 1997. *Hydraulics*. United States of America: Prentice.

Tullis, B., Young, J. & Chandler, M., 2007. Head-discharge relationships for submerged labyrinth weirs. *Journal of Hydraulic Engineering*, 133(3), pp.248-54.