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Machine Learning and Mechanics Based Soft Computing Applications

A Review of Mathematical Methods for Flexible Robot Dynamics Modeling and Simulation

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Abstract

In recent decades, lots of robots are designed and produced all over the world because of their important applications. Nowadays, using the robot is more and more popular in many different fields. In practice, the modeling and control of most of the robots are performed with an important assumption that all links of a robot are rigid bodies. This is to simplify the modeling, analysis, and control for a robot. The elastic deformation of a link always exists during a robot's operation. This elastic deformation of a flexible robot has significant effects on several characterizations and specifications of the robot such as the robot strength, the accuracy of the robot motion, the robot control, etc. In the literature, there have been many studies addressing the dynamics modeling and control of flexible robots. This paper presents an overview of the mathematical methods which have been used for the kinematic and dynamic modeling of the flexible manipulators.

Keywords

- **Flexible robot**
- **Dynamic modeling**
- **FEM**

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