Analysis of Bullwhip Effect and Inventory Cost in the Online Closed-Loop Supply Chain

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Abstract

This paper focuses on the impact of product return on the bullwhip effect in the online closed-loop supply chain. We investigate the optimal return modes when the inspection system is undertaken by the logistics center or the remanufacturer. Then, we optimize the online retailers' return decisions of minimizing the inventory cost and bullwhip effect under different supply chain circumstances. Distinctive from previous conclusion that the inspection operation is usually undertaken by the remanufacturer to sort the returned products, the analysis results reveal that the optimal decision of mitigating the bullwhip effect in most cases is to set the inspection system on logistics center. Besides, consumers' return and exchange behaviors in e-commerce have different impacts on the supply chain efficiency. The product return can not necessarily mitigate the information distortion in online closed-loop supply chain. The research provides insights for managers to strategize about return policies and resource allocation in different e-commerce contexts.

Key Words: closed-loop supply chain; inspection**;** e-commerce; return; bullwhip effect

1. Introduction

The logistics and remanufacturing costs incurred by returns cause serious economic loss and resource waste for supply chains. Product returns in the retailing industry have led to huge economic losses, up to $260.5 billion for the U.S. and $28.3 billion for Canada, respectively (Guo et al. 2018). Moreover, the return rate in e-commerce is up to 30%, compared to 8.89% in offline retailing (https://www.invespcro.com/blog/ecommerce-productreturn-rate-statistics/). The product return usually signifies potential costs for the supply chain, including inactive transportation, wasted resources, misguided demand information and customer defection. In addition, different return policies and return modes in e-commerce will have different impacts on supply chain performance. Therefore, it is desirable to explore the impact of product returns in e-business to promote the efficiency of the online supply chain.

When products' quality varies greatly, it is necessary to develop an inspection system to sort the products in the forward supply chain (Yoo et al. 2009; Khan et al. 2011) and the reverse supply chain (Hammond and Beullens 2007; Kannan et al. 2010; Pishvaee et al. 2011; Amin and Zhang 2013). Products from consumers include both intact and defective items, so the returned products are usually inspected and sorted before they are pushed into remanufacturing. Some online retailers operate their private logistics centers. While most retailers do not own ones, they choose the third-party logistics services (Tarn et al. 2003; Joong-Kun et al. 2008; Ramanathan 2011; Liu et al. 2021). Particularly, online merchants can streamline the returns process and enhance the processing capacity during the peak season through outsourcing logistics and offering designated staffs to inspect and process the returned products. The self-operated e-commerce platforms sell and ship the products in their own warehouses. However, part of the pre-sale, post-sale and return services are still managed by the third-party retailers. For self-operated e-commerce platforms such as JD.com, there are inspectors designated to the logistic center to detect the quality of returned products. For third-party e-retailers in Amazon and Tmall, who do not own their logistic center, the quality inspection will be undertaken by the remanufacturer. Therefore, when the inspection operation is undertaken by different participants, i.e., the logistics center or remanufacturer, the supply chain network and model will change accordingly, thus resulting in different return modes. In this paper, we define two return modes in the online closed-loop supply chain: the return mode when the inspection system is undertaken by the remanufacturer or the logistics center. By comparing the supply chain performances of different return modes, we can optimize the decisions of online retailers in different supply chain contexts.

Then, we extend the supply chain model by incorporating return policies. Most online retailers offer return policies with a return and/or exchange service. Usually, adopting a tolerant return policy increases the return amount and reverse logistics cost, although it can promote online sales and improve the customer satisfaction and loyalty (Mukhopadhyay and Setoputro 2005; Ketzenberg and Zuidwijk 2009; Yoo et al. 2015). Online retailers need to balance the two aspects to maximize profits and improve the supply chain performance. Some retailers may support both the return and exchange services to attract new customers by enhancing the buying intention (Ramanathan 2010, 2011). While others only allow their consumers to exchange the items but not return them to avoid return fraud and logistics loss. The return and exchange of products will have different influences on the retailers' ordering decisions so that the impacts of different return policies on the supply chain performance are also different. Finally, the relationships between the manufacturer/remanufacturer's lead times and the return lead time will influence the structure of online supply chain. Therefore, supply chain performances vary in different supply chain contexts, which needs to be further explored. We build three CL (closed-loop) online supply chains to investigate the optimal return policies and the optimal return modes by minimizing the bullwhip effects and the expected costs under different e-commerce circumstances.

There are few researches on bullwhip effects in online CL supply chains. As far as we know, most studies on bullwhip effects are based on offline supply chains. Clearly, there are three main differences between offline and online supply chains with return. First, the quality conditions of the items are checked by the remanufacturer directly in offline return operation. However, the self-operated e-commerce platforms generally designate the logistics centers to inspect the quality conditions of the returned self-operated commodities. Thus, the supply chain models will be different when the inspection system is undertaken by different participants. The different inspectors and inspection systems are the main distinctions between online and offline closed-loop supply chains. Second, product exchange in online supply chain will amplify the practical demand of the e-retailer and deliver the signal of increasing demand to the upstream supplier. Third, in traditional offline retailing, the customers can directly return the purchased products to the real stores, while online consumers have to deliver the returns to the logistics center first. The time delay will enlarge the information distortion in the online CL supply chain. Therefore, the return lead time as a major operational element will influence the online supply chain performance.

Previous researches on bullwhip effects in CL supply chains are mainly based on the offline retailing issues. We innovatively investigate the bullwhip effects in the online CL supply chains and analyze the influence of product return in e-commerce on the supply chain performance under different supply chain contexts. We propose three research problems in this paper:

(1) What are the optimal return modes in different e-commerce settings when the inspection system is undertaken by a logistics center or a remanufacturer?

(2) How does the relationship between the remanufacturer/manufacturer's lead times and the return lead time affect the efficiency of online CL supply chain?

(3) What are the differences in the influence of consumers' return and exchange behaviors in e-commerce on the bullwhip effects of the online CL supply chain?

The key contributions of the research are threefold. First, we originally quantify and investigate the expected costs and bullwhip effects in the online CL supply chain. Most studies on bullwhip effects are based on offline supply chains. Second, we explore the optimal return modes in the different supply chain contexts with different inspection systems, which would help to promote the supply chain efficiency. Last, we present the different influences of consumers' return and exchange behaviors in e-business on bullwhip effects of the online CL supply chains.

The rest of the research is structured in the following order. Section 2 examines existing literature. Section 3 investigates optimal return modes in different e-commerce contexts with different inspection systems. Section 4 analyzes the inventory cost and bullwhip effect. Section 5 extends the CL supply chain model. Section 6 concludes the research and presents the limitations.

2. Literature review

Researchers identify the bullwhip effect as a main factor that results in decline of operational performance. The true demand dynamics will be contorted when propagating upstream throughout the supply chain, which leads to serious operational issues of the upstream participants (Lee et al. 1997b, 1997a, 2000). By far, bullwhip effects have been widely investigated in offline supply chains (Ketzenberg 2009; Ma et al. 2013; Hosoda et al. 2015; Dai et al. 2016; Wang and Disney 2016; Naim et al. 2017; Raghunathan et al. 2017; Nagaraja and McElroy 2018; Teunter et al. 2018; Dolgui et al. 2020; Feng et al. 2021). However, studies on bullwhip effects in more realistic and complex structures, i.e., CL and online supply chains, are still rare.

Bullwhip effects in offline CL supply chains have been extensively addressed (Pati et al. 2010; Ramírez 2012; Sheng et al. 2015; Zou et al. 2016; Li et al. 2017; Braz et al. 2018; Hosoda and Disney 2018; Qiu et al. 2018; Ponte et al. 2020; Tombido & Baihaqi 2020; Tombido et al. 2020; Zhang & Zhang 2020; Cannella et al. 2021; Saffari et al. 2021; Yang et al. 2021; Papanagnou 2022; Ponte et al. 2022; Tombido et al. 2022). Most studies on bullwhip effects in the CL supply chain focus on products' return and recycling for environmental and resource concerns. Tang and Naim (2004) drew the remanufacturing and manufacturing process into a system dynamics model to measure the supply chain characteristic. The results proved that the two main factors affecting bullwhip effects are the customer holding time of the returned products and the remanufacturer's lead time. A similar setting can be seen in Zhou and Disney (2006). They found that both order and inventory variances decrease with the return rate. Therefore, product returns can mitigate the information distortion in supply chains. Hosoda et al. (2015) investigated impacts of sharing the information between remanufacturer and retailer on bullwhip effects through mathematical modeling. They found that higher return rates can sometimes lower the supply chain performance. Besides, the results proved that the lead times and random yields are significant factors in information sharing decisions. Zhou et al. (2017) measured how the remanufacturing uncertainties and product returns in CL supply chains affect bullwhip effects.

In recent years, more and more researches put emphases on the product return and remanufacturing in offline closed-loop supply chains. Ponte et al. (2020) indicated that the influence of lead times and return rates on the supply chain efficiency is up to the degree of information transparency. Tombido & Baihaqi (2020) investigated the different influence of market segmentation with reproduced and new products. The analysis results proved that splitting the market will amplify the ordering information distortion. Tombido et al. (2020) showed that having a responsible collector providing old products than multiple collectors with uncertainties would be more profitable for the offline close-loop supply chain. Ponte et al. (2022) indicated that a larger batch size will lead to higher bullwhip effect when the retailers adopt the order-up-to inventory policy. The analysis also presented that lower manufacturing batch sizes are more encouraged than remanufacturing batch sizes especially when both are large.

By contrast, we observe that researchers investigated the bullwhip effect of the closed-loop supply chain in following issues: the information sharing and supply chain visibility (Ponte et al. 2020), the lead time variability (Dominguez et al. 2020), the order-quantity batching (Ponte et al. 2022), the segmentation of markets (Tombido & Baihaqi 2022), the number of collectors (Tombido et al. 2020), the collection and remanufacturing capacity limits (Tombido et al. 2022), etc. Different from the previous studies, in this research we focus on optimization of the inspection system decisions, and analyze the impact of the return and exchange policies on the bullwhip effect and the inventory cost in the online closed-loop supply chain. Besides, all above researches highlighted the issues of recycling the used products for environmental consideration. All in all, previous researches on CL supply chains proved the mitigating effects of the product return on expected inventory costs and bullwhip effects. However, we put emphasis on the newly purchased products with quality problems. We analyze the different impacts of return and exchange rates in e-commerce on supply chain performance. Our results show that product return in e-commerce could offset the fluctuation of market demand to some degree and mitigate the information distortion only in certain supply chain contexts.

Bullwhip effects in online supply chains have recently been considered by scholars. Online supply chains differ from offline ones in many aspects; i.e., the frequent price discount and promotion, transportation loss, high return rate, delivery delay etc. Gao et al. (2017) addressed the difference of offline and online supply chains to explore impacts of price discounts in e-business on the bullwhip effects of the online supply chains. Analysis data proved that the information distortion of the online supply chain is generally amplified due to price discount in e-market. Zhao et al. (2018) investigated the information sharing mechanisms by minimizing bullwhip effects in the online supply chains. Their analytical results present that the information symmetry between the retailers and the manufacturer is profitable for the bullwhip reduction in e-commerce. Gao et al. (2020) explored the product loss in e-commerce and analyzed its impacts on the information sharing decisions of the wholesaler and the online retailer. They concluded that the product loss information may play a major role in improving the supply chain efficiency and should not be neglect. Except for the price promotion, transportation loss and information sharing, there are more problems in online supply chains that deserve further exploration. This research also focuses on issues in e-commerce and analyses impacts of product return on information distortion and inventory costs. By stressing the difference between online and offline supply chain, we hope to fill the gap of researches on bullwhip effects in CL online supply chain and provide insights on the optimal decisions for online retailer in different e-commerce settings.

We initially investigate the bullwhip effects in the online CL supply chains considering the return issues in e-commerce under different supply chain contexts. This paper investigates the impacts of the different return policies and different return modes on bullwhip effects and the inventory costs through three CL online retail supply chains with product returns. Furthermore, we explore how the relationships between the manufacturer/remanufacturer's lead times and the return period will influence the operational performance. The optimal decisions of return policies and return modes are analyzed in different supply chain settings.

3 Optimal return modes in CL online supply chains

Products returned from consumers include both intact and defective ones. Items need to be inspected and sorted before they are sent into remanufacturing process. Therefore, when the inspection operation is undertaken by different participants, i.e., the logistics center or remanufacturer, the supply chain model will change accordingly, thus resulting in two different return modes. We establish two CL online supply chain networks to analyze the optimal return modes in different e-commerce contexts, shown in **Fig.1** and **Fig.2**.

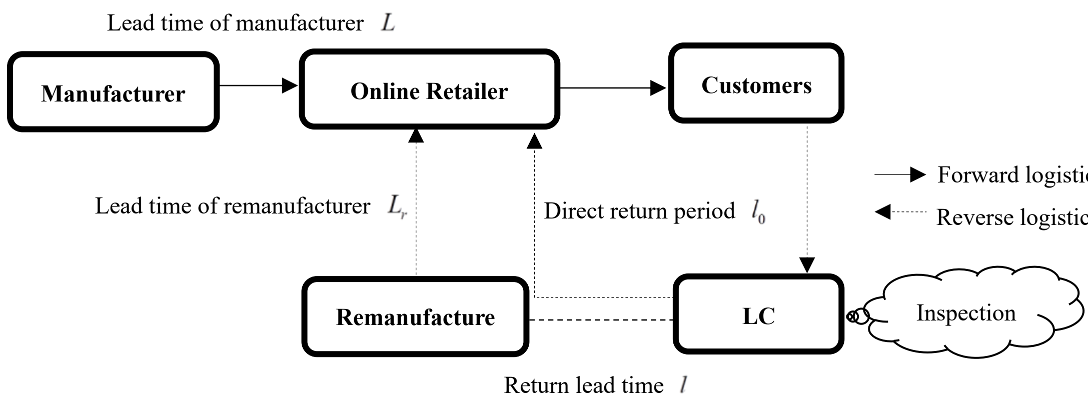

Fig. 1 The CL online supply chain network with an inspection system in the logistics center

Fig. 1 Alt Text: A CL online supply chain network showing the ordering process in the forward logistics and the returning process in the reverse logistics when setting an inspection system in the logistics center.

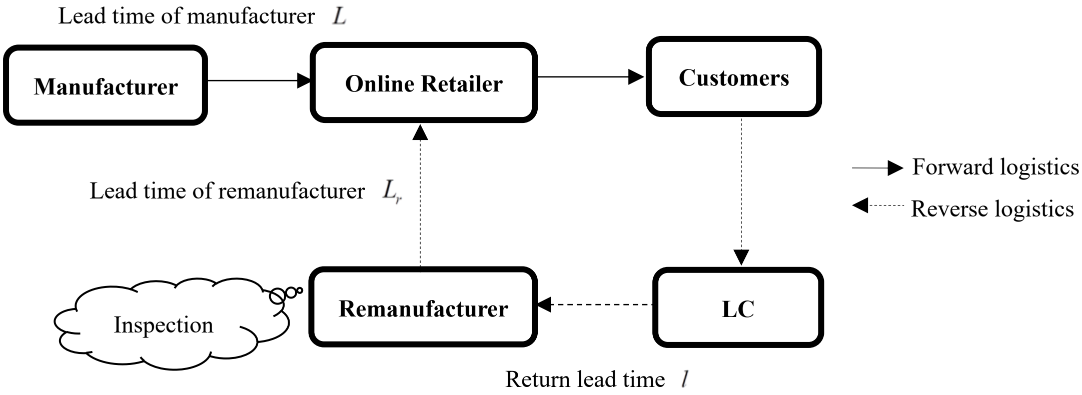

Fig. 2 The CL online supply chain network with an inspection system in the remanufacturing process

Fig. 2 Alt Text: A CL online supply chain network presenting the ordering process in the forward logistics and the returning channel in the reverse logistics when setting an inspection system at the remanufacturer.

Considering a CL online supply chain with a manufacturer, a remanufacturer, a logistics center and an online retailer, the external demand of a single product in e-market is as follows:

$$d_t = \mu + \varepsilon_t \tag{1}$$

where μ is the constant term of the e-market demand and $\varepsilon_t \sim N\left(0, \sigma^2\right)$ is the demand shock. Besides, inappreciable probability of negative demand is due to a great constant term of demand (Lee et al. 1997b). The notations and definitions of lead times are listed in Table 1.

Table 1. Notations and definitions of lead times

Notations	Phrases	Definitions
L	Lead time of the manufacturer	The delivery lead time from the manufacturer to the retailer
L_r	Lead time of the remanufacturer	The delivery lead time from the remanufacturer to the retailer
l	Return lead time	The delivery lead time from the consumers, via the logistics center to the remanufacturer
l_0	Direct return period	The delivery lead time from the consumers, via the logistics center to the retailer
$L_r + l$	Indirect return period	The delivery lead time from the consumers, via the logistics center and remanufacturer, finally to the retailer

3.1 Inspection system in logistics center

We first establish a CL online supply chain network to investigate the bullwhip effects and expected costs when setting the inspection system in logistics center, shown as **Fig.1.** The online retailer observes the market demand of previous period and delivers products to consumers. When consumers receive the products, some of them are unsatisfied with the received products due to quality problems. After negotiation with the online retailer, consumers are required to return the items for refund. The logistics center will collect all of the returned products from consumers. There is an inspection system in the logistics center to determine whether the returned products are defective products $r_{t,1}$ or intact ones $r_{t,2}$:

$$r_{t,1} = \theta_1 d_{t-l-1} + \zeta_{t,1} \tag{2}$$

$$r_{t,2} = \theta_2 d_{t-l_0-1} + \zeta_{t,2} \tag{3}$$

where $0 \le \theta_1 \le 1$ and $0 \le \theta_2 \le 1$ are the defective and intact rates, respectively, $\zeta_{t,1}$ and $\zeta_{t,2}$ are random disturbances, $\zeta_{t,1} \sim N\left(0, \sigma_{\xi_1}^2\right)$, $\zeta_{t,2} \sim N\left(0, \sigma_{\xi_2}^2\right)$. The direct return period l_0 is the delivery lead time via the logistics center to the online retailer. The intact products will be shipped directly to the online retailer after a direct return period l_0, while the defective products will be delivered to the remanufacturer to undergo the remanufacturing process after l periods. There is no inspection error in the inspection system (Chen et al. 1998; Yao and Zheng 1999; Kakade et al. 2004). Furthermore, the inspection results (i.e., the defective rate and intact rate) are the same regardless of whether the inspection operation is undertaken by the remanufacturer or the logistics center. Assume that shock terms $\zeta_{t,1}$ and $\zeta_{t,2}$ have no relations with each other and the online market demand d_t. Then, the covariances are zero as: $Cov\left(\zeta_{t,1}, \zeta_{t',2}\right) = 0, \left(\forall t, t'\right)$ and $Cov\left(d_t, \zeta_{t',i}\right) = 0, \left(\forall t, t'\right), i = 1, 2$. Similarly, the probability of negative returns is negligible.

When a remanufacturer receives defective returns $r_{t,1}$ at period t, the actual output in remanufacturing process will be:

$$M_t = \xi r_{t,1} + \varsigma_t \quad (4)$$

where ξ is the average yield rate of the remanufacturer and $\varsigma_t \sim N\left(0, \sigma_\varsigma^2\right)$ is the random term irrelevant with demand or the return shocks. Therefore, we obtain the covariance $Cov\left(\varsigma_t, r_{t',i}\right) = 0, \left(\forall t, \forall t'\right), i = 1,2$. The total quantity delivered by the remanufacturer to the online retailer M_t, the remanufacturing lead time L_r and the manufacturing lead time L are well known by the online retailer. This research adopts the assumption that the time delay of the remanufacturing process is neglected to simplify the supply chain model. The similar assumption can be found in Hosoda et al. (2015). The remanufacturer informs the retailer as soon as the remanufacturing process is finished. Thus, there is no asymmetric information between the remanufacturer and the e-retailer. The reproduced items are then sent into the retailer's stock to partially satisfy the market demand supposing that those reproduced products function as well as new products (Atasu et al. 2013; Jena and Sarmah 2014). Therefore, the returned products received by the online retailer in period t is:

$$r_t = M_{t-L_r} + r_{t,2} = \xi\left(\theta_1 d_{t-L_r-l-1} + \zeta_{t-L_r,1}\right) + \varsigma_{t-L_r} + \left(\theta_2 d_{t-l_0-1} + \zeta_{t,2}\right) \quad (5)$$

3.1.1 Ordering process

The online retailer adopts the MMSE (Minimum Mean Squared Error) technique to make demand prediction (Box et al. 1994). Accordingly, the prediction of the e-market demand $\hat{d}_{t+i}$ is $\hat{d}_{t+i} = E\left(d_{t+i} \middle| d_{t-1}\right)$ made at the end of period $t-1$ (Lee et al. 2000). The online retailer's estimated lead-time demand is $D_t^L = \sum_{i=0}^{L-1} d_{t+i} = L\mu + \sum_{i=0}^{L-1} \varepsilon_{t+i}$.

In a traditional offline supply chain without product returns, the retailer's ordering decision is $q_t = y_t - (y_{t-1} - d_{t-1})$ with the order-up-to replenishment policy (Lee et al. 1997b, 1997a, 2000), where $y_t = \hat{D}_t^L + z\hat{\sigma}_t^L$ is the order-up-to level, z is a safety factor, and $\hat{\sigma}_t^L$ is the predicted standard deviation of forecasting error. However, in a CL online supply chain, items returned from the remanufacturer can partly satisfy the actual demand of the online retailer assuming that the remanufactured products function as well as new products (Atasu et al. 2013; Jena and Sarmah 2014). Thus, the practical lead time demand should be the total demand short of the total return quantity as $\hat{D}_t^L - \hat{R}_t^L$. Therefore, the actual order-up-to level of the CL online supply chain is:

$$y_t = \hat{D}_t^L - \hat{R}_t^L + z\hat{\sigma}_t^L \tag{6}$$

where $\hat{D}_t^L = E\left(\sum_{i=0}^{L-1} d_{t+i}\right)$ is a prediction of lead time demand during the time interval $[t, t+L)$, and $\hat{R}_t^L = E\left(\sum_{i=0}^{L-1} r_{t+i}\right)$ is an estimate of the total return quantity of L periods from the remanufacturer during interval $[t, t+L)$. $\hat{\sigma}_t^L = \sqrt{Var\left(\left(D_t^L - \hat{D}_t^L\right) - \left(R_t^L - \hat{R}_t^L\right)\right)}$ is the prediction for the standard deviation of the forecasting error during L periods. In addition, $z = \Phi^{-1}[P/P+H]$ is a safety factor with the standard normal distribution Φ (Chen et al. 2000; Chen and Simchi-Levi 2000). P and H denote the penalty and holding costs of the online retailer, respectively. Accordingly, the ordering decision of the online retailer is:

$$q_t = y_t - \left(y_{t-1} - \left(d_{t-1} - r_{t-1}\right)\right) \ , \tag{7}$$

where r_{t-1} is the return volume received by the online retailer at t.

Substituting (6) into (7), the ordering level of the online retailer is rewritten as:

$$q_t = \hat{D}_t^L - \hat{D}_{t-1}^L - \left(\hat{R}_t^L - \hat{R}_{t-1}^L\right) + d_{t-1} - r_{t-1} + z\left(\hat{\sigma}_t^L - \hat{\sigma}_{t-1}^L\right) \tag{8}$$

Apparently, because the total return quantity of L periods R_t^L is different in different supply chain contexts, the expected costs of the online retailer are also different. When the manufacturing lead time is larger than that of the remanufacturer, the product returns contain unknown information that needs to be estimated and considered into the ordering decisions for the retailer. The estimate of the total return quantity of L periods of the online retailer is:

$$\hat{R}_t^L = E\left(\sum_{i=0}^{L-1} r_{t+i}\right) = E\left(\sum_{i=1}^{L_r} M_{t-i}\right) + E\left(\sum_{i=0}^{L-L_r-1} M_{t+i}\right) + E\left(\sum_{i=0}^{L-1} r_{t+i,2}\right) \tag{9}$$

When $L > L_r$, the total return quantity from the remanufacturer during periods $[t, t+L)$ includes the remanufactured quantity $\sum_{i=1}^{L_r} M_{t-i}$ and the future yield $\sum_{i=0}^{L-L_r-1} M_{t+i}$. Because $\sum_{i=1}^{L_r} M_{t-i}$ is the known information, we have $E\left(\sum_{i=1}^{L_r} M_{t-i}\right) = \sum_{i=1}^{L_r} M_{t-i}$. Besides, the online retailer has to forecast the future returns from the remanufacturer during $[t, t+L)$: $\sum_{i=0}^{L-L_r-1} M_{t+i} = \sum_{i=0}^{L-L_r-1} \left(\xi\left(\theta_1 d_{t+i-l-1} + \zeta_{t+i,1}\right) + \varsigma_{t+i}\right)$. The expected of the future return from the remanufacturer is $E\left(\sum_{i=0}^{L-L_r-1} M_{t+i}\right) = \theta_1 \xi E\left(\sum_{i=0}^{L-L_r-1} d_{t+i-l-1}\right)$, where

$$E\left(\sum_{i=0}^{L-L_r-1} d_{t+i-l-1}\right) = \begin{cases} \sum_{i=0}^{L-L_r-1} d_{t+i-l-1}, & l \ge L - L_r - 1 \\ \sum_{i=0}^{l} d_{t+i-l-1} + \left(L - L_r - l - 1\right)\mu, & L - L_r - 1 > l \end{cases} \tag{10}$$

Similarly, the total return quantity directly from the logistic center during periods $[t, t+L)$ also needs to be predicted as $E\left(\sum_{i=0}^{L-1} r_{t+i,2}\right) = \theta_2 E\left(\sum_{i=0}^{L-1} d_{t-l_0-1+i}\right) + E\left(\sum_{i=0}^{L-1} \zeta_{t+i,2}\right)$, where

$$\sum_{i=0}^{L-1} r_{t+i,2} - E\left(\sum_{i=0}^{L-1} r_{t+i,2}\right) = \begin{cases} \sum_{i=0}^{L-1} \zeta_{t+i,2}, & l_0 \ge L - 1 \\ \theta_2 \sum_{i=l_0+1}^{L-1} \varepsilon_{t-l_0-1+i} + \sum_{i=0}^{L-1} \zeta_{t+i,2}, & L - 1 > l_0 \end{cases} \tag{11}$$

When the lead time of the manufacturer is smaller than that of the remanufacturer, which means $L \le L_r$, the total return quantity of L periods of the online retailer is known information. The estimate of the total return quantity is $\hat{R}_t^L = E\left(\sum_{i=0}^{i=L-1} r_{t+i}\right) = \sum_{i=L_r-L+1}^{L_r} M_{t-i} + \sum_{i=0}^{L-1} r_{t+i,2}$.

Therefore, the differences of estimated total returns at period t and period $t-1$ are expressed as:

When $L>L_r$,

$$\hat{R}_t^L-\hat{R}_{t-1}^L=\begin{cases} M_{t-1}-M_{t-L_r-1}+\theta_1\xi\left(d_{t+L-L_r-l-2}-d_{t-l-2}\right)+\theta_2\left(d_{t+L-l_0-2}-d_{t-l_0-2}\right), l\ge L-L_r-1\wedge l_0\ge L-1 \\ M_{t-1}-M_{t-L_r-1}+\theta_1\xi\left(d_{t-1}-d_{t-l-2}\right)+\theta_2\left(d_{t-1}-d_{t-l_0-2}\right), \quad L-L_r-1>l\wedge L-1>l_0 \\ M_{t-1}-M_{t-L_r-1}+\theta_1\xi\left(d_{t-1}-d_{t-l-2}\right)+\theta_2\left(d_{t+L-l_0-2}-d_{t-l_0-2}\right), \quad L-L_r-1>l\wedge l_0\ge L-1 \\ M_{t-1}-M_{t-L_r-1}+\theta_1\xi\left(d_{t+L-L_r-l-2}-d_{t-l-2}\right)+\theta_2\left(d_{t-1}-d_{t-l_0-2}\right), \quad l\ge L-L_r-1\wedge L-1>l_0 \end{cases}. \quad (12)$$

When $L\le L_r$,

$$\hat{R}_t^L-\hat{R}_{t-1}^L=\begin{cases} M_{t+L-L_r-1}-M_{t-L_r-1}+\theta_2\left(d_{t+L-l_0-2}-d_{t-l_0-2}\right), l_0\ge L-1 \\ M_{t+L-L_r-1}-M_{t-L_r-1}+\theta_2\left(d_{t-1}-d_{t-l_0-2}\right), \quad L-1>l_0 \end{cases}. \quad (13)$$

Lemma 1: *Variances of the forecasting error of lead-time demand in two return modes and policies under different e-business contexts remain constant.*

Proof*:* see *Appendix A.*

Lemma 1 proves that $\hat{\sigma}_t^L=\hat{\sigma}_{t'}^L,(\forall t,t')$. Therefore, when setting the inspection system in logistic center, the ordering quantity of the online retailer in period t is derived as:

When $L>L_r$,

$$q_t=\begin{cases} d_{t-1}-\theta_1\xi d_{t+L-L_r-l-2}-\theta_2 d_{t+L-l_0-2}-\xi\zeta_{t-1,1}-\varsigma_{t-1}-\zeta_{t-1,2}, l\ge L-L_r-1\wedge l_0\ge L-1 \\ (1-\theta_1\xi-\theta_2)d_{t-1}-\xi\zeta_{t-1,1}-\varsigma_{t-1}-\zeta_{t-1,2}, \quad L-L_r-1>l\wedge L-1>l_0 \\ (1-\theta_1\xi)d_{t-1}-\theta_2 d_{t+L-l_0-2}-\xi\zeta_{t-1,1}-\varsigma_{t-1}-\zeta_{t-1,2}, \quad L-L_r-1>l\wedge l_0\ge L-1 \\ (1-\theta_2)d_{t-1}-\theta_1\xi d_{t+L-L_r-l-2}-\xi\zeta_{t-1,1}-\varsigma_{t-1}-\zeta_{t-1,2}, \quad l\ge L-L_r-1\wedge L-1>l_0 \end{cases} \quad (14)$$

When $L\le L_r$,

$$q_t=\begin{cases} d_{t-1}-\xi\theta_1 d_{t-L_r+L-l-2}-\theta_2 d_{t+L-l_0-2}-\xi\zeta_{t-L_r+L-1,1}-\varsigma_{t-L_r+L-1}-\zeta_{t-1,2}, l_0\ge L-1 \\ (1-\theta_2)d_{t-1}-\xi\theta_1 d_{t-L_r+L-l-2}-\xi\zeta_{t-L_r+L-1,1}-\varsigma_{t-L_r+L-1}-\zeta_{t-1,2}, \quad L-1>l_0 \end{cases} \quad (15)$$

3.2 Inspection system in the remanufacturer

Shown as **Fig.2,** a CL online supply chain network is built to analyze the return and ordering processes when setting the inspection system in the remanufacturer. The total lead time in the reverse logistics in **Fig.2** is defined as the indirect return period. Thereinto, L_r is the lead time of remanufacturer and l is the return lead time, which refers to the delivery lead time from the consumers via the logistic center to the remanufacturer. Since there is only one return channel, both intact and defective products will be delivered to the remanufacturer by the logistics center. Then, after l periods, the remanufacturer receives the total returned products from the consumers. There is an inspection system in the remanufacturing to determine whether the returned products are defective products $r_{t,1}$ or intact products $r_{t,2}$, before the defective ones are sent into the remanufacturing operation.

When the remanufacturer receives the returned products from the consumers, only the defective products are put into the remanufacturing process. Thus, the actual output of remanufacturing in period t is $M_t = \xi r_{t,1} + \varsigma_t + r_{t,2}$. Then, the total return received by the online retailer is:

$$r_t = M_{t-L_r} = \xi r_{t-L_r,1} + \varsigma_{t-L_r} + r_{t-L_r,2} \tag{16}$$

Similarly, when the manufacturing lead time is larger than the remanufacturing lead time, the product returns also contain unknown information that needs to be estimated and considered into ordering decisions for the retailer. The estimate of the total return quantity of L periods of the online retailer is:

$$\hat{R}_t^L = E\left(\sum_{i=0}^{L-1} r_{t+i}\right) = E\left(\sum_{i=1}^{L_r} M_{t-i}\right) + E\left(\sum_{i=0}^{L-L_r-1} M_{t+i}\right) \tag{17}$$

When $L > L_r$, the total return quantity from the remanufacturer during periods $[t, t+L)$ includes the remanufactured quantity $\sum_{i=1}^{L_r} M_{t-i}$ and the future yield $\sum_{i=0}^{L-L_r-1} M_{t+i}$. Because $\sum_{i=1}^{L_r} M_{t-i}$ is the known information, we have $E\left(\sum_{i=1}^{L_r} M_{t-i}\right) = \sum_{i=1}^{L_r} M_{t-i}$. Besides, the online retailer has to forecast the future returns from the remanufacturer during $[t, t+L)$: $\sum_{i=0}^{L-L_r-1} M_{t+i} = (\theta_1\xi + \theta_2) \sum_{i=0}^{L-L_r-1} d_{t+i-l-1} + \xi \sum_{i=0}^{L-L_r-1} \zeta_{t+i,1} + \sum_{i=0}^{L-L_r-1} \varsigma_{t+i} + \sum_{i=0}^{L-L_r-1} \zeta_{t+i,2}$. The expected of the future return from the remanufacturer is:

$$E\left(\sum_{i=0}^{L-L_r-1} M_{t+i}\right) = \begin{cases} (\theta_1\xi + \theta_2) \sum_{i=0}^{L-L_r-1} d_{t+i-l-1}, & l \geq L - L_r - 1 \\ (\theta_1\xi + \theta_2) \sum_{i=0}^{l} d_{t+i-l-1} + (\theta_1\xi + \theta_2)(L - L_r - l - 1)\mu, & L - L_r - 1 > l \end{cases} \tag{18}$$

When the manufacturing lead time is smaller than the remanufacturing lead time, which means $L \leq L_r$, the total return quantity of L periods of the online retailer is known information. The estimate of the total return quantity is $\hat{R}_t^L = E\left(\sum_{i=0}^{L-1} r_{t+i}\right) = \sum_{i=L_r-L+1}^{L_r} M_{t-i}$.

Similar to the supply chain model with the inspection system in logistic center, the forecasting errors of total returns in different e-commerce settings can be derived as:

$$R_t^L - \hat{R}_t^L = \begin{cases} \xi \sum_{i=0}^{L-L_r-1} \zeta_{t+i,1} + \sum_{i=0}^{L-L_r-1} \zeta_{t+i,2} + \sum_{i=0}^{L-L_r-1} \varsigma_{t+i}, & L > L_r \wedge l \geq L - L_r - 1 \\ (\theta_1\xi + \theta_2) \sum_{i=l+1}^{L-L_r-1} \varepsilon_{t+i-l-1} + \xi \sum_{i=0}^{L-L_r-1} \zeta_{t+i,1} + \sum_{i=0}^{L-L_r-1} \varsigma_{t+i} + \sum_{i=0}^{L-L_r-1} \zeta_{t+i,2}, & L > L_r \wedge L - L_r - 1 > l \\ 0, & L \leq L_r \end{cases} \tag{19}$$

And the difference of estimated total returns in period t and period $t-1$ is:

$$\hat{R}_t^L - \hat{R}_{t-1}^L = \begin{cases} M_{t-1} - M_{t-L_r-1} + (\theta_1\xi + \theta_2)(d_{t+L-L_r-l-2} - d_{t-l-2}), & L > L_r \wedge l \geq L - L_r - 1 \\ M_{t-1} - M_{t-L_r-1} + (\theta_1\xi + \theta_2)(d_{t-1} - d_{t-l-2}), & L > L_r \wedge L - L_r - 1 > l \\ M_{t+L-L_r-1} - M_{t-L_r-1}, & L \leq L_r \end{cases} . \tag{20}$$

According to **Lemma 1**, and substituting (20) into (8), the order level in period t is expressed as:

$$q_t = \begin{cases} d_{t-1} - (\theta_1\xi + \theta_2) d_{t+L-L_r-l-2} - \xi\zeta_{t-1,1} - \varsigma_{t-1} - \zeta_{t-1,2}, & L > L_r \wedge l \geq L - L_r - 1 \\ (1 - \theta_1\xi - \theta_2) d_{t-1} - \xi\zeta_{t-1,1} - \varsigma_{t-1} - \zeta_{t-1,2}, & L > L_r \wedge L - L_r - 1 > l \\ d_{t-1} - (\theta_1\xi + \theta_2) d_{t+L-L_r-l-2} - \xi\zeta_{t+L-L_r-1,1} - \varsigma_{t+L-L_r-1} - \zeta_{t+L-L_r-1,2}, & L \leq L_r \end{cases} \tag{21}$$

4 Analysis of bullwhip effect and expected cost

4.1 Bullwhip effect and expected cost

In this section, the online retailer's bullwhip effects and inventory costs under different supply chain contexts in two return modes will be computed.

The differences of bullwhip effects in two return modes are dependent on the online retailer's ordering variances $Var(q_t)$. The order variances of the retailer in two return modes under different supply chain contexts can be computed by using Equations (14), (15) and (21), in the assumption that the online retailer employs the order-up-to policy and the MMSE estimating method.

When setting the inspection system in the logistics center, the order variances of the online retailer are:

When $L > L_r$,

$$\sigma_{q,l}^2 = \begin{cases} \sigma^2 + (\theta_1\xi)^2 \sigma^2 + \theta_2{}^2\sigma^2 + \xi^2\sigma_{\zeta_1}^2 + \sigma_\varsigma^2 + \sigma_{\zeta_2}^2, l \geq L - L_r - 1 \wedge l_1 \geq L-1 \\ (1-\theta_1\xi-\theta_2)^2 \sigma^2 + \xi^2\sigma_{\zeta_1}^2 + \sigma_\varsigma^2 + \sigma_{\zeta_2}^2, \quad L - L_r - 1 > l \wedge L-1 > l_1 \\ (1-\theta_1\xi)^2 \sigma^2 + \theta_2{}^2\sigma^2 + \xi^2\sigma_{\zeta_1}^2 + \sigma_\varsigma^2 + \sigma_{\zeta_2}^2, \quad L - L_r - 1 > l \wedge l_1 \geq L-1 \\ (1-\theta_2)^2 \sigma^2 + (\theta_1\xi)^2 \sigma^2 + \xi^2\sigma_{\zeta_1}^2 + \sigma_\varsigma^2 + \sigma_{\zeta_2}^2, l \geq L - L_r - 1 \wedge L-1 > l_1 \end{cases} \tag{22}$$

When $L \leq L_r$,

$$\sigma_{q,l}^2 = \begin{cases} \sigma^2 + (\theta_1\xi)^2 \sigma^2 + \theta_2{}^2\sigma^2 + \xi^2\sigma_{\zeta_1}^2 + \sigma_\varsigma^2 + \sigma_{\zeta_2}^2, l_1 \geq L-1 \\ (1-\theta_2)^2 \sigma^2 + (\theta_1\xi)^2 \sigma^2 + \xi^2\sigma_{\zeta_1}^2 + \sigma_\varsigma^2 + \sigma_{\zeta_2}^2, L-1 > l_1 \end{cases} \tag{23}$$

When setting the inspection system in the remanufacturer, the order variances of the online retailer are:

$$\sigma_{q,r}^2 = \begin{cases} \left(1+(\theta_1\xi+\theta_2)^2\right)\sigma^2 + \xi^2\sigma_{\zeta_1}^2 + \sigma_\varsigma^2 + \sigma_{\zeta_2}^2, L > L_r \wedge l \geq L - L_r - 1 \\ (1-\theta_1\xi-\theta_2)^2 \sigma^2 + \xi^2\sigma_{\zeta_1}^2 + \sigma_\varsigma^2 + \sigma_{\zeta_2}^2, \quad L > L_r \wedge L - L_r - 1 > l \\ \left(1+(\theta_1\xi+\theta_2)^2\right)\sigma^2 + \xi^2\sigma_{\zeta_1}^2 + \sigma_\varsigma^2 + \sigma_{\zeta_2}^2, L \leq L_r \end{cases} \tag{24}$$

As the shipment inventory during the replenishment lead time is normally distributed with mean $\hat{D}_t^L - \hat{R}_t^L$ and standard deviation $\hat{\sigma}_t^L$, expected inventory cost for the retailer is given as:

$$\begin{aligned} C_t &= E\left[P\int_{y_t}^{\infty}\left(D_t^L - R_t^L - y_t\right)d\bar{F}_t\left(D_t^L - R_t^L\right) + H\int_{-\infty}^{y_t}\left(y_t - \left(D_t^L - R_t^L\right)\right)d\bar{F}_t\left(D_t^L - R_t^L\right)\right] \\ &= \hat{\sigma}_t^L\left[\left(H+P\right)L\left(z\right) + Hz\right] \end{aligned} \quad (25)$$

where $\bar{F}_t\left(D_t^L - R_t^L\right)$ is the true distribution of $D_t^L - R_t^L$ with mean $\hat{D}_t^L - \hat{R}_t^L$ and variance $\left(\hat{\sigma}_t^L\right)^2$. $L(x)$ is $L(x) = \int_x^{\infty}(y-x)d\Phi(y)$ convex and decreasing in x, and $H\left(z+L(z)\right) + PL(z) \le H\left(x+L(x)\right) + PL(x) \forall x \ge z$ (Lee et al. 1997b, 1997a, 2000). Variances of the forecasting error $\left(\hat{\sigma}_t^L\right)^2$ in different supply chain contexts are expressed in **Appendix A**.

4.2 Comparative analysis

Usually, the market demand in e-commerce is more drastic due to the frequent price discount and promotion (Gao et al. 2017). While, the return and remanufacturing quantities are relative stationary because the device condition, product quality and yield are more stable so that the random shocks are sometimes even neglected (Akcali and Cetinkaya 2011). Therefore, the market demand is much more fluctuant than return and yield from remanufacturer in these supply chain models. For bullwhip effects of the online retailer in two return modes, we have the following properties: if $l \ge L - L_r - 1$, $Var(q_t)_r > Var(q_t)_l$; if $L - L_r - 1 > l \wedge L - 1 \le l_0$, $Var(q_t)_r < Var(q_t)_l$; or else, $Var(q_t)_r = Var(q_t)_l$. For inventory costs of the online retailer in two return modes, we have the following properties: if $L - 1 > l_0$, $C_{t,r} > C_{t,l}$; if $L - 1 \le l_0$, $C_{t,r} < C_{t,l}$ ($\sigma^2 >> \sigma_{\zeta_2}^2 \approx 0$). Comparing the order variances and inventory costs in different e-business settings, we conclude the following propositions on the optimal decisions on return modes in CL supply chains:

Proposition 1: When $l \ge L - L_r - 1$, the optimal return mode to minimize the bullwhip effect is to set the inspection system in the logistics center rather than in the remanufacturing process. When $L-L_r-1>l \wedge L-1\le l_0$, the optimal return mode to minimize the bullwhip effect is to set the inspection system in the remanufacturing process. In other supply chain settings, two return modes are equally optimal.

Proposition 2: When $L-1>l_0$, the optimal return mode to minimize the expected cost is to set the inspection system in the logistics center rather than in the remanufacturing process. When $L-1\le l_0$, the optimal return mode to minimize the expected cost is to set the inspection system in the remanufacturing process.

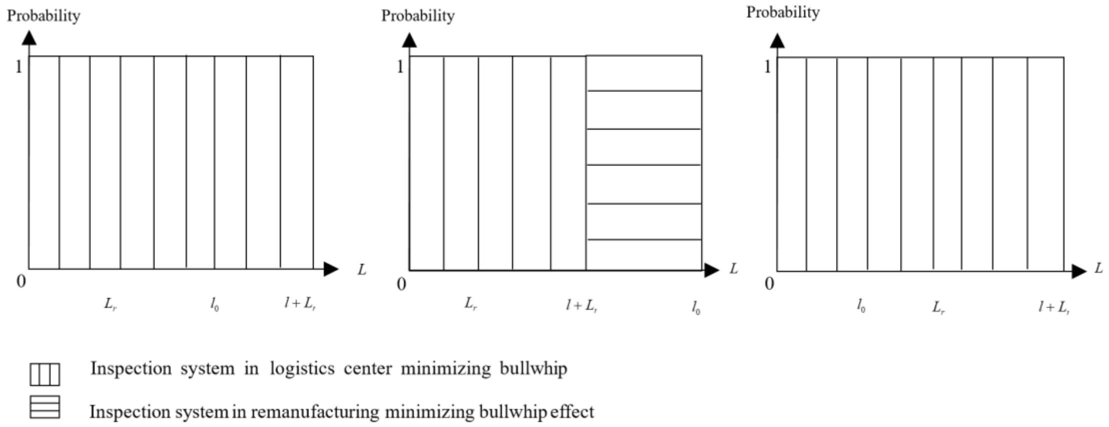

Fig. 3 Optimal decisions of minimizing the bullwhip effect under different supply chain

Fig. 3 Alt Text: The optimal decisions of choosing the return mode minimizing the bullwhip effect in three different supply chain contexts, depending on the relationship between the manufacturing lead time and the indirect return period.

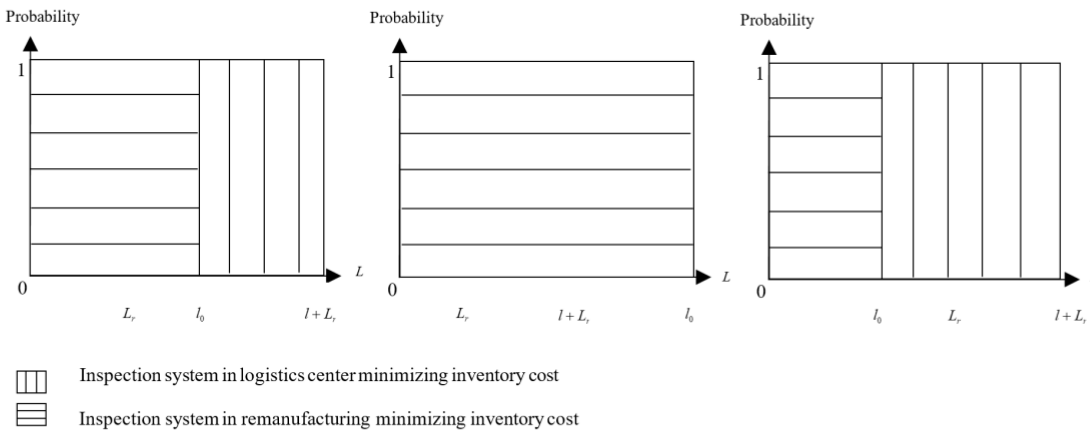

Fig. 4 Optimal decisions of minimizing the inventory cost under different supply chain circumstances

Fig. 4 Alt Text: The optimal decisions on the return mode minimizing the inventory cost in three different supply chain contexts, depending on the relationship between the replenishment lead time and the direct return period.

Proposition 1 and **Proposition 2** exhibit optimal return modes in different supply chain settings when the inspection system is undertaken by a logistics center or a remanufacturer. From **Proposition 1**, setting the inspection system in the logistics center will efficiently mitigate information distortion in the CL online supply chain, if the supply chain satisfies the condition that the manufacturer's lead time is smaller than the indirect return period. However, this operation will certainly amplify the demand information distortion and lead to disrupted production schedules in a rare supply chain setting that the manufacturer's lead time is longer than indirect return period and shorter than direct return period. **Proposition 2** indicates that if the inventory cost reduction is the primary consideration of the online retailer, the critical factor of decision is up to the relationship between the direct return period and replenishment lead time. The retailer should preferentially designate the inspectors to the logistics center when having a shorter return period, and vice versa.

Then, to more clearly display the optimal return decisions under different supply chain circumstances, two propositions are illustrated in **Fig. 3** and **Fig. 4. Fig. 3** depicts the optimal policies of receding information distortion in different e-commerce settings. Obviously, only when the lead time of forward logistics is greater than that of reverse logistics, setting the inspection system in remanufacturing will effectively alleviate the bullwhip effects and inventory backlog. In this case, the replenishment lead time is much larger than the remanufacturing period, which means that the total return period is enclosed in an ordering period. Thus, returned products will be delivered into the retailer’s inventory to partly meet the market demand of current period. Otherwise, it will be more beneficial for the online retailer to assign inspectors in the logistics center. In the circumstance, the remanufacturing process is more complicated and time-consuming. Therefore, to improve the supply chain efficiency, the inspection operation should be finished before the returned products undergo remanufacturing.

Fig. 4 describes the online retailer’s optimal decisions of controlling inventory costs in different e-commerce settings. Generally, the direct return period is shorter than the indirect return period in CL supply chains. In the premise, we will discuss the optimization condition of minimizing the expected costs. The inspection system should be set in the remanufacturing when the manufacturing lead time is smaller than the direct return period. The total return period is beyond an ordering period so that the returned products will be delivered into the retailer’s stock to partly supply the future demand instead of current demand. Otherwise, the supply chain will benefit more when the logistics center undertakes the inspection operation. However, on rare occasion that the direct return period is longer than the indirect one, the optimal return mode to minimize inventory cost will always be setting the inspection system in the remanufacturing.

Overall, **Fig. 3** and **Fig. 4** reveal the optimal return modes of CL online supply chains in different contexts. Relationships of lead times between the reverse and forward supply chains will significantly influence the information distortion and inventory cost. The analysis is profitable for supply chain managers to make optimal return and exchange strategies and inspection decisions. When the manufacturer's lead time is larger (smaller) than the indirect return period, the optimal return mode to minimize bullwhip effect is to set up the quality inspection system at the remanufacturer (logistics center). While, if the manufacturer's lead time is larger (smaller) than the direct return period, the optimal return mode to minimize the inventory cost is to set up the quality inspection system in the logistics center (remanufacturer). In brief, the optimal decision of minimizing the bullwhip effect (inventory cost) in most cases is setting the inspection system on logistics center (remanufacturing). Thus, sometimes the merchants will benefit more if the inspectors are designated to the logistics center to process the returned products.

5 Extended CL online supply chain model

The different return policies of online retailers will make significant impacts on the closed-loop supply chain model by allowing consumers to return and/or exchange products. An online CL supply chain with a remanufacturer, a manufacturer, a logistics center and an online retailer is constructed, as shown in **Fig. 5**, to analyze the different impact of return policies on the bullwhip effects and the inventory costs of online retailers. We extend the supply chain network by measuring impacts of different return policies on the operational performance in this section.

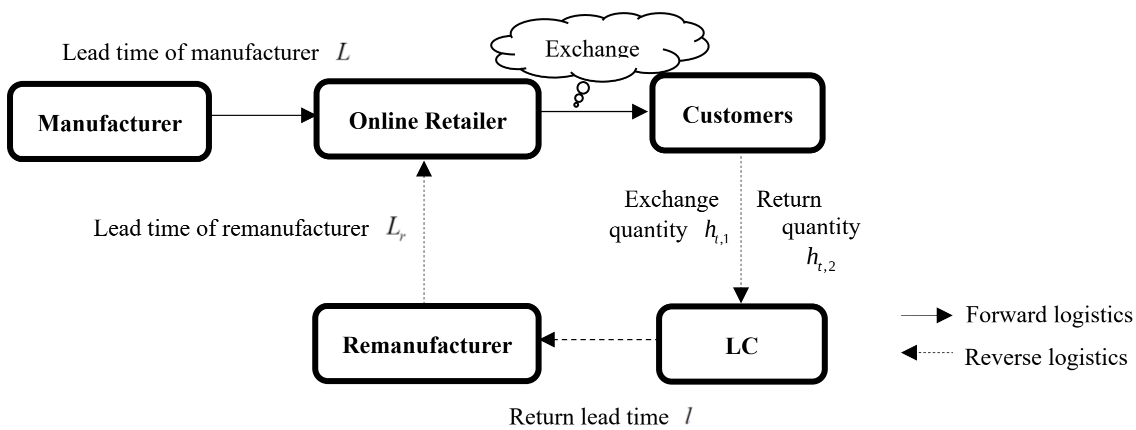

Fig. 5 The online closed-loop supply chain network with return and exchange

Fig. 5 Alt Text: A CL online supply chain network showing the ordering process in the forward logistics and the returning process in the reverse logistics, including the product return and exchange information.

5.1 Basic model

In e-commerce, dealing with consumers' return and exchange is one of the essential operations of online retailers. Consumers are enjoying e-shopping more than ever with the convenient return services provided by online sellers. They can select to return or exchange the unsatisfactory purchases with a fraction of the cost. Then, the logistics center will collect the returned items and deliver them to the remanufacturer. As soon as the remanufacturing department obtains the returns, the online retailer will confirm the return and replacement information. New products need to be reshipped to consumers who apply for the exchange service, which would correspondingly amplify the practical demand of the online retailer. Assume that those reshipped products are all intact. Therefore, the exchange amount will only depend on the actual market demand level and the exchange rate.

The online retailer delivers actual demand quantity to consumers at the end of period $t-l-1$. After a return lead time l, the online retailer observes the exchange quantity $h_{t,1}=\alpha_1 d_{t-l-1}+\upsilon_{t,1}$ at the beginning of period t, where $0<\alpha_1<1$ is the exchange rate of the online retailer. $\upsilon_{t,1}\sim N\left(0,\sigma_{\upsilon_1}^2\right)$ is the random shock of the exchange quantity. Besides, inappreciable probability of a negative exchange is due to a great constant term of demand.

Therefore, the online retailer's actual demand includes the exchange quantity required by consumers and the e-market demand:

$$D_t=d_t+h_{t,1}=d_t+\alpha_1 d_{t-l-1}+\upsilon_{t,1} \tag{26}$$

The remanufacturer receives total returned products from consumers at period t, including the exchange quantity $h_{t,1}$ and the return quantity $h_{t,2}=\alpha_2 d_{t-l-1}+\upsilon_{t,2}$, where $0<\alpha_2<1$ is the return rate of the online retailer. $\upsilon_{t,2}\sim N\left(0,\sigma_{\upsilon_2}^2\right)$ is the random term of return quantity. Assume that the shock terms $\upsilon_{t,i}$ are irrelevant with each other and the e-market demand d_t. Thus, the covariance between the random shock and market demand is $Cov\left(d_t,\upsilon_{t',i}\right)=0,(\forall t,t',i=1,2)$. Also, we obtain the covariance $Cov\left(\upsilon_{t,1},\upsilon_{t,2}\right)=0$. The remanufacturer receives the returned products from consumers at the beginning of period t. Thus, the actual output of the remanufacturer is $M_t=\xi\left(h_{t,1}+h_{t,2}\right)+\varsigma_t$. Thus, the return quantity received by the online retailer in period t is $r_t=M_{t-L_r}=\xi\left(h_{t-L_r,1}+h_{t-L_r,2}\right)+\varsigma_{t-L_r}$.

5.2 Ordering process

From Equation (26), we can obtain the predicted e-market demand in period $t+i$ as $\hat{d}_{t+i}=\mu$, and the total demand prediction $\hat{D}_{t+i}=\hat{d}_{t+i}+\alpha_1\hat{d}_{t+i-l-1}$. If the manufacturing lead time is larger than return lead time, market demand contains unknown information that needs to be estimated and considered into the ordering decisions for the retailer; or else, the demand information is already realized and known to the online retailer. Therefore, when $i>l$, the information of $d_{t+i-l-1}$ needs to be estimated; when $i\le l$, $d_{t+i-l-1}$ is already known to the online retailer. Thus, the estimation of market demand in period $t+i-l-1$ is $\hat{d}_{t+i-l-1}=\begin{cases}\mu, & i>l\\ d_{t+i-l-1}, & i\le l\end{cases}$.

Then, the online retailer's prediction of actual demand in period $t+i$ is:

$$\hat{D}_{t+i}=\begin{cases}\mu+\alpha_1\mu, & i>l\\ \mu+\alpha_1 d_{t+i-l-1}, & i\le l\end{cases} \tag{27}$$

And the predicted mean lead-time demand is:

$$\hat{D}_t^L = \sum_{i=0}^{L-1} \hat{D}_{t+i} = \begin{cases} L\mu + (L-l-1)\alpha_1\mu + \alpha_1 \sum_{i=0}^{l} d_{t+i-l-1}, & L > l \\ L\mu + \alpha_1 \sum_{i=0}^{L-1} d_{t+i-l-1}, & L \le l \end{cases} \tag{28}$$

Therefore, the difference of estimated total returns in period t and period $t-1$ is:

$$\hat{R}_t^L - \hat{R}_{t-1}^L = \begin{cases} M_{t-1} - M_{t-L_r-1} + \xi(\alpha_1+\alpha_2)\left(d_{t+L-L_r-l-2} - d_{t-l-2}\right), L > L_r \wedge l \ge L - L_r - 1 \\ M_{t-1} - M_{t-L_r-1} + \xi(\alpha_1+\alpha_2)\left(d_{t-1} - d_{t-l-2}\right), \quad L > L_r + l + 1 \\ M_{t-L_r+L-1} - M_{t-L_r-1}, \quad L \le L_r \end{cases} \tag{29}$$

Besides, the forecasting error of total returns in different e-commerce settings can be derived as:

$$R_t^L - \hat{R}_t^L = \begin{cases} \xi \sum_{i=0}^{L-L_r-1} \upsilon_{t+i,1} + \xi \sum_{i=0}^{L-L_r-1} \upsilon_{t+i,2} + \sum_{i=0}^{L-L_r-1} \varsigma_{t+i}, \quad L > L_r \wedge l \ge L - L_r - 1 \\ \xi(\alpha_1+\alpha_2) \sum_{i=0}^{L-L_r-l-2} \varepsilon_{t+i} + \xi \sum_{i=0}^{L-L_r-1} \upsilon_{t+i,1} + \sum_{i=0}^{L-L_r-1} \varsigma_{t+i} + \xi \sum_{i=0}^{L-L_r-1} \upsilon_{t+i,2}, L > L_r + l + 1 \\ 0, \quad L \le L_r \end{cases} \tag{30}$$

Lemma 2: *Variances of the forecasting error of the lead-time demand in CL online supply chains with product return and exchange under different e-business contexts are constant.*

Proof*:* see *Appendix B****.***

Lemma 2 proves that $\hat{\sigma}_t^L = \hat{\sigma}_{t'}^L, (\forall t, t')$. In addition, the ordering quantity of the online retailer is derived as:

When $L > l$,

$$q_t = \begin{cases} (1+\alpha_1) d_{t-1} - \xi(\alpha_1+\alpha_2) d_{t+L-L_r-l-2} + (1-\xi)\upsilon_{t-1,1} - \xi\upsilon_{t-1,2} - \varsigma_{t-1}, L > L_r \wedge l \ge L - L_r - 1 \\ \left(1+\alpha_1 - \xi(\alpha_1+\alpha_2)\right) d_{t-1} + (1-\xi)\upsilon_{t-1,1} - \xi\upsilon_{t-1,2} - \varsigma_{t-1}, \quad L > L_r + l + 1 \\ (1+\alpha_1) d_{t-1} - \xi(\alpha_1+\alpha_2) d_{t+L-l-L_r-2} + \upsilon_{t-1,1} - \xi\upsilon_{t+L-L_r-1,1} - \xi\upsilon_{t+L-L_r-1,2} - \varsigma_{t+L-L_r-1}, L \le L_r \end{cases} \tag{31}$$

When $L \le l$,

$$q_t = \begin{cases} d_{t-1} + \alpha_1 d_{t+L-l-2} - \xi(\alpha_1+\alpha_2) d_{t+L-L_r-l-2} + (1-\xi)\upsilon_{t-1,1} - \xi\upsilon_{t-1,2} - \varsigma_{t-1}, L > L_r \wedge l \ge L - L_r - 1 \\ d_{t-1} + \alpha_1 d_{t+L-l-2} - \xi(\alpha_1+\alpha_2) d_{t+L-l-L_r-2} + \upsilon_{t-1,1} - \xi\upsilon_{t+L-L_r-1,1} - \xi\upsilon_{t+L-L_r-1,2} - \varsigma_{t+L-L_r-1}, L \le L_r \end{cases}$$

(32)

5.3 Comparative analysis

Similarly, the online retailer's order variances and inventory costs under different supply chain contexts in different return policies will be computed. When the online retailer employs the order-up-to policy and the minimum mean square error estimating method, order variances in the CL online supply chain could be derived by using Equations (31) and (32):

When $L > l$,

$$\sigma_q^2 = \begin{cases} (1+\alpha_1)^2\sigma^2 + \xi^2(\alpha_1+\alpha_2)^2\sigma^2 + (1-\xi)^2\sigma_{\upsilon_1}^2 + \sigma_\varsigma^2 + \xi^2\sigma_{\upsilon_2}^2, L > L_r \wedge l \ge L - L_r - 1 \\ (1+\alpha_1 - \xi(\alpha_1+\alpha_2))^2\sigma^2 + (1-\xi)^2\sigma_{\upsilon_1}^2 + \sigma_\varsigma^2 + \xi^2\sigma_{\upsilon_2}^2, \quad L > L_r + l + 1 \\ (1+\alpha_1)^2\sigma^2 + \xi^2(\alpha_1+\alpha_2)^2\sigma^2 + (1+\xi^2)\sigma_{\upsilon_1}^2 + \sigma_\varsigma^2 + \xi^2\sigma_{\upsilon_2}^2, L \le L_r \end{cases} \tag{33}$$

When $L \le l$,

$$\sigma_q^2 = \begin{cases} (1+\alpha_1^2)\sigma^2 + \xi^2(\alpha_1+\alpha_2)^2\sigma^2 + (1-\xi)^2\sigma_{\upsilon_1}^2 + \sigma_\varsigma^2 + \xi^2\sigma_{\upsilon_2}^2, L > L_r \wedge l \ge L - L_r - 1 \\ (1+\alpha_1^2)\sigma^2 + \xi^2(\alpha_1+\alpha_2)^2\sigma^2 + (1+\xi^2)\sigma_{\upsilon_1}^2 + \sigma_\varsigma^2 + \xi^2\sigma_{\upsilon_2}^2, L \le L_r \end{cases} \tag{34}$$

As the shipment inventory in the replenishment lead time is normally distributed with mean $\hat{D}_t^L - \hat{R}_t^L$ and standard deviation $\hat{\sigma}_t^L$, expected inventory cost for the retailer is given as $C_t = \hat{\sigma}_t^L\left[(H+P)L(z) + Hz\right]$. The forecasting error $\hat{\sigma}_t^L$ in different supply chain contexts are expressed in *Appendix B*.

By derivation, we conclude the following propositions about impacts of different return policies on the CL online supply chain:

***Proposition 3*:** The impact of the product exchange on the bullwhip effect of the online retailer has following property: α_1 in the interval $(0,1)$, $\frac{\partial\sigma_q^2}{\partial\alpha_1} \ge 0$. Thus, the bullwhip effect in the online CL supply chain increases with the exchange rate.

Proposition 4: The impact of the product return on the bullwhip effect of the online retailer has following property:

(a) When $L-L_r-1>l$, α_2 in the interval $(0,1)$, $\frac{\partial\sigma_q^2}{\partial\alpha_2}\leq 0$. Thus, the bullwhip effect in the online CL supply chain decreases with the return rate.

(b) When $L-L_r-1\leq l$, α_2 in the interval $(0,1)$, $\frac{\partial\sigma_q^2}{\partial\alpha_2}>0$. Thus, the bullwhip effect in the online CL supply chain increases with the return rate.

Propositions 3 and **4** indicate that product return and exchange have different impacts on the supply chain efficiency. Consumers' exchange behavior can amplify the bullwhip effect in the online CL supply chain. Product exchange will lower the supply chain efficiency and performance. It indicates that the product exchange in e-commerce will damage the benefits of the online retailer and increase the inventory costs of the online CL supply chain. Therefore, the online retailer should try to reduce the defective rate and control the product quality. Besides, product return will lower the information distortion of the e-retailer only when the total return period is enclosed in an ordering period. However, if the return period is beyond an ordering period, consumers' return behavior will amplify the information distortion of the online retailer. Moreover, relationships between the manufacturer/remanufacturer's lead times and return lead time have remarkable effect on operational efficiency, i.e., expected cost and bullwhip effect. It follows that when the total lead time of forward logistics is greater than that of reverse logistics, a return policy that allows consumers' return would be more beneficial for the online retailer.

To more precisely verify the propositions, contour plots are drawn to investigate the influence of return policies on bullwhip effects. **Fig.6-8** illustrate the impacts of product return and exchange on the bullwhip effect of the online retailer in e-commerce under different supply chain contexts, for $\xi = 0.5, \sigma = 2, \sigma_{\upsilon_1} = 1, \sigma_{\varsigma} = 0.5, \sigma_{\upsilon_2} = 0.5$. The 'areas' (different combinations of the exchange rate α_1 and the return rate α_2) for bullwhip effects in the online closed-loop supply chain are shown.

Furthermore, **Fig.6-8** indicate that under the supply chain context that the manufacturer's lead time is greater than the indirect return lead time, when more consumers choose the exchange service and less select the return service, the online retailer will suffer a larger bullwhip effect. Otherwise, the effect of ordering information distortion will be relieved. Moreover, on the supply chain context that the replenishment lead time is shorter than the indirect return lead time, the scenarios are similar to some extent. Besides, from (33) and (34), it can be observed that the product exchange has a larger amplifying effect than the product return. Consumers' exchange behavior will significantly increase the practical demand of the online retailer, resulting in more drastic information distortion in supply chain. Thus, if more consumers choose the exchange service and less select the return service, the bullwhip effect will be larger. Therefore, a generous return policy will be more effective in restraining the bullwhip effect in the online closed-loop supply chain. Besides, the bullwhip effect could be completely eliminated in the online closed-loop supply chain when the total return period is enclosed in an ordering period. Otherwise, the bullwhip effect of the online retailer will always exist. The information fluctuation caused by the signal of increasing demand dominates and can't be offset.

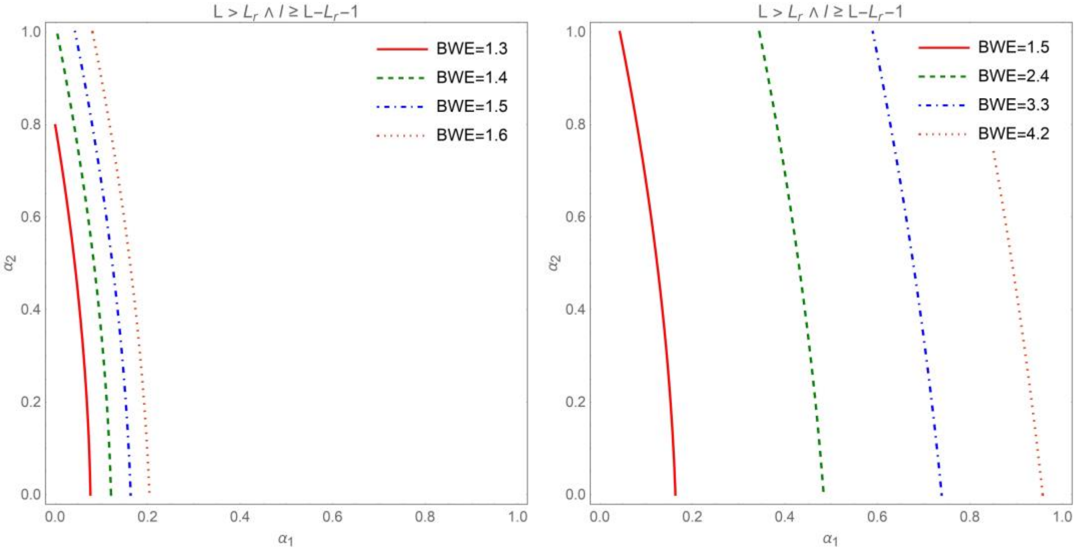

Fig. 6 Impacts of return and exchange rates on bullwhip effect when $L > L_r \wedge l \geq L - L_r - 1$

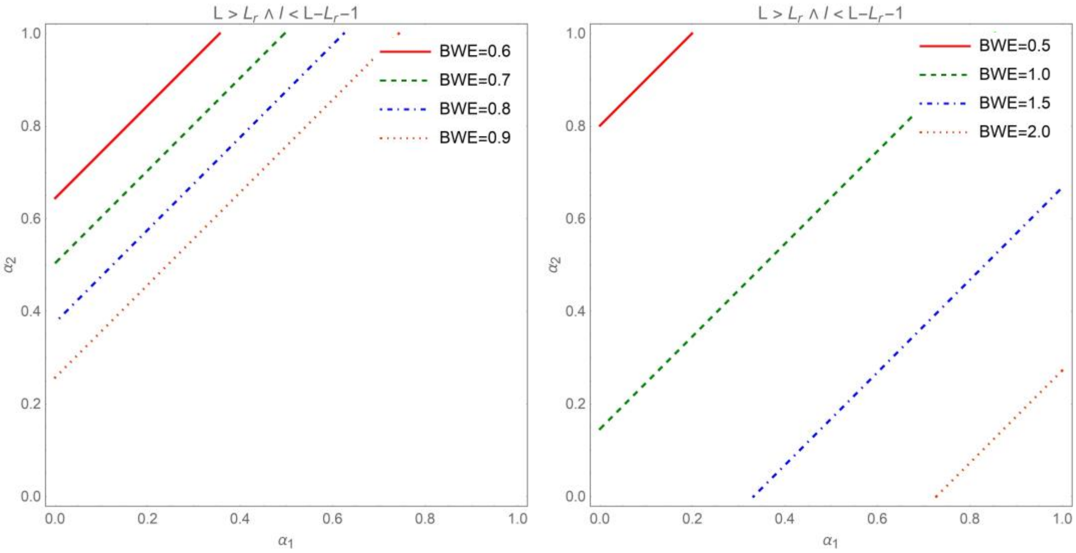

Fig. 7 Impacts of return and exchange rates on bullwhip effect when $L > L_r + l + 1$

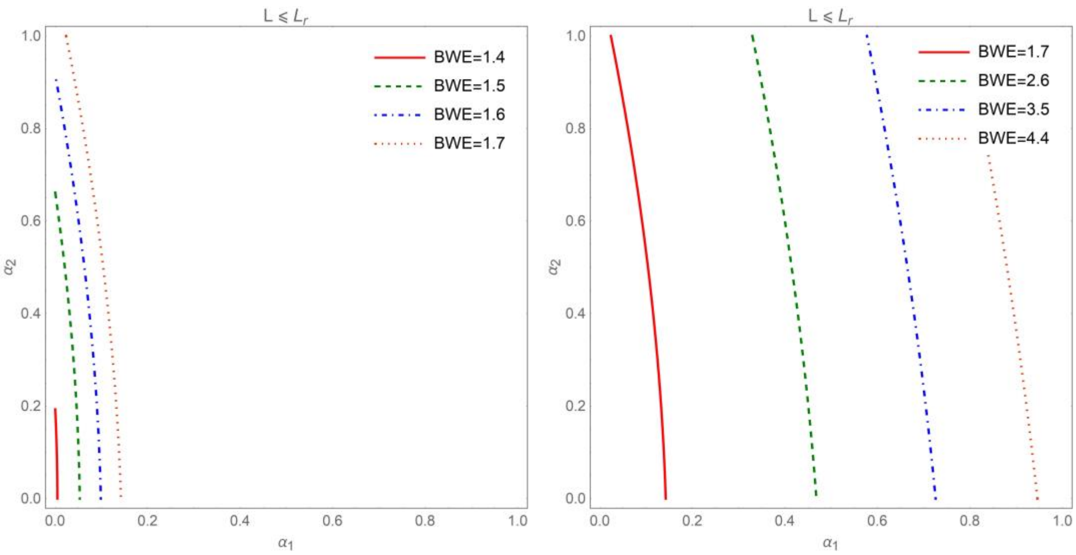

Fig. 8 Impacts of return and exchange rates on bullwhip effect when $L \leq L_r$

Fig.6-8 Alt Text: When more consumers choose the exchange service and less select the return service, the online retailer in the CL supply chain will suffer a worse bullwhip effect.

Proposition 5**:** The impact of the replenishment lead time on the expected inventory cost of the online retailer has following property: L in the interval $\left(0,+\infty\right)$, $\frac{\partial C_t}{\partial L} \geq 0$. Thus, the inventory cost in the online CL supply chain increases with the replenishment lead time.

Propositions 5 presents that in different supply chain scenarios, the expected inventory cost in the online CL supply chain is positively correlated with the manufacturing lead time. Apparently, a longer lead time will lead to higher inventory cost for e-retailers.

6 Conclusion

This research explores the impacts of two return modes and different return policies on the supply chain performance through three CL online supply chains. We investigate the optimal return modes by minimizing expected costs and bullwhip effects under different supply chain contexts. In addition, we explore the influence of the relationships between the return lead time and the ordering lead times on the operational efficiency. From the research results, we obtain meaningful conclusions and management implications. The research provides suggestions for online supply chain executives to optimize their resolutions about return modes and policies in e-business, which can help to reduce inventory costs and promote the supply chain efficiency in different e-commerce contexts.

6.1 Theoretical contribution

Previous researches on bullwhip effects in CL supply chains are mainly based on the offline retailing issues. We innovatively investigate the bullwhip effects in the online CL supply chains and analyze optimization of the inspection system decisions and the influence of product return in e-commerce on the supply chain performance under different supply chain contexts. The key contributions of the research are threefold:

First, the research explores the optimal return modes in the different supply chain contexts with different inspection systems, which would help to promote the supply chain efficiency. The analysis results show that if the inventory cost reduction is the primary consideration of the online retailer, the critical factor of decision is up to the relationship between the direct return period and replenishment lead time. The retailer should preferentially designate the inspectors to the logistics center when having a shorter return period, and vice versa. When the bullwhip reduction is the major concern, the retailer optimize the decision of inspection system based on the relationship between the indirect return period and replenishment lead time. Relationships of lead times between the reverse and forward supply chains will significantly influence the information distortion and inventory cost.

Second, the amplifying effect of the behaviors of consumers' product exchange on the bullwhip effect in online CL supply chains are proved. Thus, a higher exchange rate always leads to a larger bullwhip effect no matter how long the replenishment lead time is. New products need to be reshipped to consumers who apply for the exchange service, which would correspondingly amplify the practical demand of the online retailer. Therefore, product exchange tends to encourage the decision maker to order more to meet the inventory loss. The signal of increasing demand is descended to the upstream supplier through the ordering decisions of the online retailer, resulting in more drastic information distortion in supply chain.

Finally, the impact mechanisms between the product return and the bullwhip effect in online CL supply chains are investigated. Consumers' product return behavior can diminish the information distortion of the online supply chain only if the ordering lead time is larger than the return period, which means that the total return period is enclosed in an ordering period. Returned products will be delivered into the retailer's inventory to partly balance out the fluctuation of current demand and thus can alleviate the information distortion. In addition, online retailers can timely adjust the forecast of future actual demand, so as to improve the accuracy of ordering decision when the return period is included in an ordering period. The mitigating effect of product return on bullwhip effects have been proved by previous studies (Zhou and Disney 2006; Pati et al. 2010; Hosoda et al. 2015). However, product return will amplify the information distortion of the online retailer if the return period is beyond an ordering period. The returned products will be used to partly supply the future demand instead of current demand after delivered into the retailer's stock. Consequently, product return can not efficiently recede the mutation of demand information. Besides, the bullwhip effect of the online retailer could be completely eliminated only when the total return period is enclosed in an ordering period. Otherwise, bullwhip effect in the online closed-loop supply chain always exists.

6.2 Managerial implications

This research investigates the optimal return modes when the inspection system is undertaken by the logistics center or the remanufacturer. Then, we optimize the online retailers' return decisions of minimizing the inventory cost and bullwhip effect under different supply chain circumstances. The research provides several managerial implications:

First, the supply chain managers need to optimize the decisions of setting the inspection system according to different e-commerce contexts. In practice, the inspection process is usually undertaken by the remanufacturer (Kannan et al. 2010; Pishvaee et al. 2011; Amin and Zhang 2013). However, according to analytical results, sometimes merchants will benefit more if the inspectors are designated to the logistics center to process the returned products. For those retailers who operate their private logistics centers, it would be much easier to assign inspectors to the logistics centers. While, most retailers do not own the logistics centers, they usually outsource the logistics service to the 3PLs (Tarn et al. 2003; Joong-Kun et al. 2008). Just as shown in the commercial practice report from Reverse Logistics Magazine that it would be more profitable for retailers to outsource the return service and offer designated staffs to the 3PLs to receive, assess and process the returns to the warehouse especially during and after peak seasons.

Second, the online retailer should try to reduce the exchange rate of returned products and control the product quality. If necessary, the retailer should look for a more qualified supplier. Consumers' product exchange behavior will enlarge the information distortion and amplify the demand fluctuation in the online CL supply chain. The product exchange has a negative impact on mitigating bullwhip effects of the online retailer. It follows that the product exchange will harm the profits and increase the inventory costs of the online CL supply chain.

Finally, online retailer should implement the return policies in e-commerce more flexibly. A generous return policy is more effective in eliminating demand information disturbance when the total return period is enclosed in an ordering period. Intuitively, product return can weaken bullwhip effects in CL supply chains. However, according to the analysis results, only when the total lead time of the forward logistics is larger than that of the reverse logistics, the consumers' product return behavior can reduce the information distortion and the expected costs of the online retailer. Besides, allowing product return will not always benefit the performance of the closed-loop supply chain. Product return may not effectively counteract the information distortion of current demand due to an overlong return period. Returned products can partly satisfy the future demand instead of current demand after shipped into the e-retailer's warehouse. Furthermore, when more consumers choose the exchange service and less select the return service, the retailer in the online closed-loop supply chain will suffer a worse bullwhip effect. Otherwise, the effect of ordering information distortion can be relieved.

6.3 Limitations

The main limitations in the research are threefold. First, we divide the product return issue into two subparts without unifying them into a systematical model. Although these assumptions will simplify the supply chain models to provide focus on the key impact factors, some of them are still harsh. If we relax some assumptions, the modeling may need some corresponding modifications. Second, due to the complexity of CL supply chains, we quantify demand process as a simple normal distribution to simplify the calculation. In some studies on bullwhip modeling, it is also common to depict the demand function as an auto-regressive moving average model or a more complicated form. The managerial implications will be different if the demand function becomes more complicated. Finally, we assume that if a consumer requires the product exchange service, the reshipped products are all intact and will not be returned or exchanged again. However, some consumers may still request a second return attempt. If the model includes this section of the demand, the analysis results could subsequently produce potential differences. These limitations additionally provide some future research directions to improve and extend the theory of the CL online retail supply chains, which is worth further investigation in this area.

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Data Availability Statement

The authors confirm that the data supporting the findings of this study are available within the article.

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Appendix A

The variances of the estimating error of the lead-time demand of the e-retailer in two return modes under different supply chain contexts are derived as follows.

(1) When setting the inspection system in the logistic center, the forecasting errors of the online retailer can be computed as $\left(\hat{\sigma}_t^L\right)^2 = Var\left(\left(D_t^L - \hat{D}_t^L\right) - \left(R_t^L - \hat{R}_t^L\right)\right)$:

When $L > L_r \wedge l \geq L - L_r - 1 \wedge l_0 \geq L - 1$,

$$\left(\hat{\sigma}_t^L\right)^2 = Var\left(\sum_{i=0}^{L-1} \varepsilon_{t+i} - \xi \sum_{i=0}^{L-L_r-1} \zeta_{t+i,1} - \sum_{i=0}^{L-L_r-1} \varsigma_{t+i} - \sum_{i=0}^{L-1} \zeta_{t+i,2}\right) = L\sigma^2 + \left(L - L_r\right)\xi^2\sigma_{\zeta_1}^2 + \left(L - L_r\right)\sigma_{\varsigma}^2 + L\sigma_{\zeta_2}^2$$

When $L > L_r \wedge L - L_r - 1 > l \wedge L - 1 > l_0$,

$$\begin{aligned}\left(\hat{\sigma}_t^L\right)^2 &= Var\left(\sum_{i=0}^{L-1} \varepsilon_{t+i} - \left(\theta_1\xi \sum_{i=0}^{L-L_r-l-2} \varepsilon_{t+i} + \xi \sum_{i=0}^{L-L_r-1} \zeta_{t+i,1} + \sum_{i=0}^{L-L_r-1} \varsigma_{t+i} + \theta_2 \sum_{i=0}^{L-l_0-2} \varepsilon_{t+i} + \sum_{i=0}^{L-1} \zeta_{t+i,2}\right)\right) \\ &= Var\left(\left(1 - \theta_1\xi - \theta_2\right) \sum_{i=0}^{L-L_r-l-2} \varepsilon_{t+i} + \left(1 - \theta_2\right) \sum_{i=L-L_r-l-1}^{L-l_0-2} \varepsilon_{t+i} + \sum_{i=L-l_0-1}^{L-1} \varepsilon_{t+i} - \xi \sum_{i=0}^{L-L_r-1} \zeta_{t+i,1} - \sum_{i=0}^{L-L_r-1} \varsigma_{t+i} - \sum_{i=0}^{L-1} \zeta_{t+i,2}\right) \\ &= \left(L - L_r\right)\xi^2\sigma_{\zeta_1}^2 + L\sigma_{\zeta_2}^2 + \left(L - L_r\right)\sigma_{\varsigma}^2 + \left(1 - \theta_1\xi - \theta_2\right)^2\left(L - L_r - l - 1\right)\sigma^2 + \left(1 - \theta_2\right)^2\left(L_r + l - l_0\right)\sigma^2 + \left(l_0 + 1\right)\sigma^2\end{aligned}$$

When $L > L_r \wedge L - L_r - 1 > l \wedge l_0 \geq L - 1$,

$$\begin{aligned}\left(\hat{\sigma}_t^L\right)^2 &= Var\left(\sum_{i=0}^{L-1} \varepsilon_{t+i} - \left(\theta_1\xi \sum_{i=0}^{L-L_r-l-2} \varepsilon_{t+i} + \xi \sum_{i=0}^{L-L_r-1} \zeta_{t+i,1} + \sum_{i=0}^{L-L_r-1} \varsigma_{t+i} + \sum_{i=0}^{L-1} \zeta_{t+i,2}\right)\right) \\ &= Var\left(\left(1 - \theta_1\xi\right) \sum_{i=0}^{L-L_r-l-2} \varepsilon_{t+i} + \sum_{i=L-L_r-l-1}^{L-1} \varepsilon_{t+i} - \xi \sum_{i=0}^{L-L_r-1} \zeta_{t+i,1} - \sum_{i=0}^{L-L_r-1} \varsigma_{t+i} - \sum_{i=0}^{L-1} \zeta_{t+i,2}\right) \\ &= \left(L - L_r\right)\xi^2\sigma_{\zeta_1}^2 + L\sigma_{\zeta_2}^2 + \left(L - L_r\right)\sigma_{\varsigma}^2 + \left(1 - \theta_1\xi\right)^2\left(L - L_r - l - 1\right)\sigma^2 + \left(L_r + l + 1\right)\sigma^2\end{aligned}$$

When $L > L_r \wedge l \geq L - L_r - 1 \wedge L - 1 > l_0$,

$$\begin{aligned}\left(\hat{\sigma}_t^L\right)^2 &= Var\left(\sum_{i=0}^{L-1} \varepsilon_{t+i} - \xi \sum_{i=0}^{L-L_r-1} \zeta_{t+i,1} - \sum_{i=0}^{L-L_r-1} \varsigma_{t+i} - \theta_2 \sum_{i=0}^{L-l_0-2} \varepsilon_{t+i} - \sum_{i=0}^{L-1} \zeta_{t+i,2}\right) \\ &= Var\left(\left(1 - \theta_2\right) \sum_{i=0}^{L-l_0-2} \varepsilon_{t+i} + \sum_{i=L-l_0-1}^{L-1} \varepsilon_{t+i} - \xi \sum_{i=0}^{L-L_r-1} \zeta_{t+i,1} - \sum_{i=0}^{L-L_r-1} \varsigma_{t+i} - \sum_{i=0}^{L-1} \zeta_{t+i,2}\right) \\ &= \left(1 - \theta_2\right)^2\left(L - l_0 - 1\right)\sigma^2 + \left(l_0 + 1\right)\sigma^2 + \left(L - L_r\right)\xi^2\sigma_{\zeta_1}^2 + L\sigma_{\zeta_2}^2 + \left(L - L_r\right)\sigma_{\varsigma}^2\end{aligned}$$

When $L \le L_r \wedge l_0 \ge L-1$,

$$\left(\hat{\sigma}_t^L\right)^2 = Var\left(\sum_{i=0}^{L-1}\varepsilon_{t+i} - \sum_{i=0}^{L-1}\zeta_{t+i,2}\right) = L\sigma^2 + L\sigma_{\zeta_2}^2$$

When $L \le L_r \wedge L-1 > l_0$,

$$\begin{aligned}\left(\hat{\sigma}_t^L\right)^2 &= Var\left(\sum_{i=0}^{L-1}\varepsilon_{t+i} - \theta_2\sum_{i=0}^{L-l_0-2}\varepsilon_{t+i} - \sum_{i=0}^{L-1}\zeta_{t+i,2}\right)\\ &= Var\left(\left(1-\theta_2\right)\sum_{i=0}^{L-l_0-2}\varepsilon_{t+i} + \sum_{i=L-l_0-1}^{L-1}\varepsilon_{t+i} - \sum_{i=0}^{L-1}\zeta_{t+i,2}\right)\\ &= \left(1-\theta_2\right)^2\left(L-l_0-1\right)\sigma^2 + \left(l_0+1\right)\sigma^2 + L\sigma_{\zeta_2}^2\end{aligned}$$

(2) When setting the inspection system in the remanufacturing, the estimating errors of the online retailer can be computed as:

When $L > L_r \wedge l \ge L-L_r-1$,

$$\left(\hat{\sigma}_t^L\right)^2 = Var\left(\sum_{i=0}^{L-1}\varepsilon_{t+i} - \xi\sum_{i=0}^{L-L_r-1}\zeta_{t+i,1} - \sum_{i=0}^{L-L_r-1}\zeta_{t+i,2} - \sum_{i=0}^{L-L_r-1}\varsigma_{t+i}\right) = L\sigma^2 + \left(L-L_r\right)\xi^2\sigma_{\zeta_1}^2 + \left(L-L_r\right)\sigma_{\varsigma}^2 + \left(L-L_r\right)\sigma_{\zeta_2}^2$$

When $L > L_r \wedge L-L_r-1 > l$,

$$\begin{aligned}\left(\hat{\sigma}_t^L\right)^2 &= Var\left(\sum_{i=0}^{L-1}\varepsilon_{t+i} - \left(\theta_1\xi+\theta_2\right)\sum_{i=0}^{L-L_r-l-2}\varepsilon_{t+i} - \xi\sum_{i=0}^{L-L_r-1}\zeta_{t+i,1} - \sum_{i=0}^{L-L_r-1}\varsigma_{t+i} - \sum_{i=0}^{L-L_r-1}\zeta_{t+i,2}\right)\\ &= Var\left(\left(1-\theta_1\xi-\theta_2\right)\sum_{i=0}^{L-L_r-l-2}\varepsilon_{t+i} + \sum_{i=L-L_r-l-1}^{L-1}\varepsilon_{t+i} - \xi\sum_{i=0}^{L-L_r-1}\zeta_{t+i,1} - \sum_{i=0}^{L-L_r-1}\varsigma_{t+i} - \sum_{i=0}^{L-L_r-1}\zeta_{t+i,2}\right)\\ &= \left(L-L_r\right)\xi^2\sigma_{\zeta_1}^2 + \left(L-L_r\right)\sigma_{\zeta_2}^2 + \left(L-L_r\right)\sigma_{\varsigma}^2 + \left(1-\theta_1\xi-\theta_2\right)^2\left(L-L_r-l-1\right)\sigma^2 + \left(L_r+l+1\right)\sigma^2\end{aligned}$$

When $L \le L_r$,

$$\left(\hat{\sigma}_t^L\right)^2 = Var\left(\sum_{i=0}^{L-1}\varepsilon_{t+i}\right) = L\sigma^2$$

Therefore, the variances of the forecasting error of the lead-time demand of the retailer in two return modes under different supply chain contexts are constant. This completes the proof.

Appendix B

When considering product return and exchange in the CL online supply chains, the estimating errors for the online retailer can be computed as:

When $L > l$,

$$
R_t^L - \hat{R}_t^L = \begin{cases} \xi \sum_{i=0}^{L-L_r-1} \zeta_{t+i,1} + \xi \sum_{i=0}^{L-L_r-1} \upsilon_{t+i,2} + \sum_{i=0}^{L-L_r-1} \varsigma_{t+i}, & L > L_r \wedge l \geq L - L_r - 1 \\ \xi(\alpha_1 + \alpha_2) \sum_{i=0}^{L-L_r-l-2} \varepsilon_{t+i} + \xi \sum_{i=0}^{L-L_r-1} \upsilon_{t+i,1} + \sum_{i=0}^{L-L_r-1} \varsigma_{t+i} + \xi \sum_{i=0}^{L-L_r-1} \upsilon_{t+i,2}, & L > L_r \wedge L - L_r - 1 > l \\ 0, & L \leq L_r \end{cases}
$$

When $L > L_r \wedge l \geq L - L_r - 1$,

$$
\begin{aligned}
\left(\hat{\sigma}_t^L\right)^2 &= Var\left(\sum_{i=0}^{L-1} \varepsilon_{t+i} + \alpha_1 \sum_{i=l+1}^{L-1} \varepsilon_{t+i-l-1} - \xi \sum_{i=0}^{L-L_r-1} \upsilon_{t+i,1} - \xi \sum_{i=0}^{L-L_r-1} \upsilon_{t+i,2} - \sum_{i=0}^{L-L_r-1} \varsigma_{t+i} \right) \\
&= Var\left((1+\alpha_1) \sum_{i=0}^{L-l-2} \varepsilon_{t+i} + \sum_{i=L-l-1}^{L-1} \varepsilon_{t+i} - \xi \sum_{i=0}^{L-L_r-1} \upsilon_{t+i,1} - \xi \sum_{i=0}^{L-L_r-1} \upsilon_{t+i,2} - \sum_{i=0}^{L-L_r-1} \varsigma_{t+i} \right) \\
&= (1+\alpha_1)^2 (L-l-1)\sigma^2 + (l+1)\sigma^2 + \xi^2 (L-L_r)\sigma_{\upsilon_1}^2 + \xi^2 (L-L_r)\sigma_{\upsilon_2}^2 + (L-L_r)\sigma_{\varsigma}^2
\end{aligned}
$$

When $L > L_r \wedge L - L_r - 1 > l$,

$$
\begin{aligned}
\left(\hat{\sigma}_t^L\right)^2 &= Var\left(\sum_{i=0}^{L-1} \varepsilon_{t+i} + \alpha_1 \sum_{i=l+1}^{L-1} \varepsilon_{t+i-l-1} - \xi(\alpha_1+\alpha_2) \sum_{i=0}^{L-L_r-l-2} \varepsilon_{t+i} - \xi \sum_{i=0}^{L-L_r-1} \upsilon_{t+i,1} - \xi \sum_{i=0}^{L-L_r-1} \upsilon_{t+i,2} - \sum_{i=0}^{L-L_r-1} \varsigma_{t+i} \right) \\
&= Var\left(\left(1+\alpha_1 - \xi(\alpha_1+\alpha_2)\right) \sum_{i=0}^{L-L_r-l-2} \varepsilon_{t+i} + (1+\alpha_1) \sum_{i=L-L_r-l-1}^{L-l-2} \varepsilon_{t+i} + \sum_{i=L-l-1}^{L-1} \varepsilon_{t+i} - \xi \sum_{i=0}^{L-L_r-1} \upsilon_{t+i,1} - \xi \sum_{i=0}^{L-L_r-1} \upsilon_{t+i,2} - \sum_{i=0}^{L-L_r-1} \varsigma_{t+i} \right) \\
&= \left(1+\alpha_1 - \xi(\alpha_1+\alpha_2)\right)^2 (L-L_r-l-1)\sigma^2 + (1+\alpha_1)^2 L_r \sigma^2 + (l+1)\sigma^2 + \xi^2 (L-L_r)\sigma_{\upsilon_1}^2 + (L-L_r)\sigma_{\varsigma}^2 + \xi^2 (L-L_r)\sigma_{\upsilon_2}^2
\end{aligned}
$$

When $L \leq L_r$,

$$
\begin{aligned}
\left(\hat{\sigma}_t^L\right)^2 &= Var\left(\sum_{i=0}^{L-1} \varepsilon_{t+i} + \alpha_1 \sum_{i=l+1}^{L-1} \varepsilon_{t+i-l-1} \right) \\
&= Var\left((1+\alpha_1) \sum_{i=0}^{L-l-2} \varepsilon_{t+i} + \sum_{i=L-l-1}^{L-1} \varepsilon_{t+i} \right) \\
&= (1+\alpha_1)^2 (L-l-1)\sigma^2 + (l+1)\sigma^2
\end{aligned}
$$

When $L \leq l$,

$$
R_t^L - \hat{R}_t^L = \begin{cases} \xi \sum_{i=0}^{L-L_r-1} \upsilon_{t+i,1} + \sum_{i=0}^{L-L_r-1} \upsilon_{t+i,2} + \sum_{i=0}^{L-L_r-1} \varsigma_{t+i}, & L > L_r \wedge l \geq L - L_r - 1 \\ \xi(\alpha_1 + \alpha_2) \sum_{i=0}^{L-L_r-l-2} \varepsilon_{t+i} + \xi \sum_{i=0}^{L-L_r-1} \upsilon_{t+i,1} + \sum_{i=0}^{L-L_r-1} \varsigma_{t+i} + \sum_{i=0}^{L-L_r-1} \upsilon_{t+i,2}, & L > L_r \wedge L - L_r - 1 > l \\ 0, & L \leq L_r \end{cases}
$$

When $L > L_r \wedge l \geq L - L_r - 1$,

$$
\left(\hat{\sigma}_t^L\right)^2 = Var\left(\sum_{i=0}^{L-1} \varepsilon_{t+i} - \xi \sum_{i=0}^{L-L_r-1} \upsilon_{t+i,1} - \xi \sum_{i=0}^{L-L_r-1} \upsilon_{t+i,2} - \sum_{i=0}^{L-L_r-1} \varsigma_{t+i} \right) = L\sigma^2 + \xi^2 (L-L_r)\sigma_{\upsilon_1}^2 + (L-L_r)\sigma_{\varsigma}^2 + \xi^2 (L-L_r)\sigma_{\upsilon_2}^2
$$

When $L \leq L_r$,

$$
\left(\hat{\sigma}_t^L\right)^2 = Var\left(\sum_{i=0}^{L-1} \varepsilon_{t+i} \right) = L\sigma^2
$$

Therefore, when considering product return and exchange in the CL online supply chains, variances of the estimating error of lead-time demand of the online retailer are constant. This completes the proof.