Evaluating the Impact of Carbon Tax Policy on Manufacturing and Remanufacturing Decisions in a Closed-loop Supply Chain

ABSTRACT

This paper develops four game-theoretic models to evaluate the impact of carbon tax policy on manufacturing and remanufacturing decisions in a closed-loop supply chain (CLSC) consisting of a manufacturer and a retailer. To maximize profits, the decisions are made based on two scenarios, namely, no investment in carbon reduction technology and with investment, in the centralized and decentralized CLSC, respectively. The used products are collected for remanufacturing under three strategies, that is, no collection, partial collection, and full collection. The manufacturing and remanufacturing decisions are compared and analyzed. The results show: (1) Carbon tax can effectively promote manufacturers to invest in carbon reduction technology or remanufacture to reduce carbon emissions. However, it may demotivate manufacturers to remanufacture if a reasonable carbon tax is not designed. (2) Although a centralized model can achieve a higher total profit than a decentralized model, the carbon emission of the centralized CLSC will be higher than that of the decentralized CLSC when carbon tax is low. (3) Under the decentralized CLSC, the 'no collection' or 'full collection' decision depends only on the wholesale price between manufacturer and retailer rather than on the unit carbon saving of remanufactured products. Hence, policymakers should tailor carbon tax policy designs to different industries to promote remanufacturing such as tax reliefs, tax returns, and emissions reduction agreements. Furthermore, the government can advocate low-carbon consumptions and cultivate low-carbon preference among consumers by imposing taxes or subsidies for remanufactured products.

Keywords: manufacturing and remanufacturing decision; closed-loop supply chain; carbon tax; carbon reduction investment

1. Introduction

Global climate change has led to a higher probability of natural disasters and human diseases. One of the effective solutions to this problem is to curb carbon emissions. Many countries have enforced emission regulations and policies, such as carbon taxes and carbon cap and trade. Among these mechanisms, carbon tax is generally considered as one of the most effective market-based mechanisms and is widely accepted around the world. Currently, more than 20 countries, including Canada, Australia, the United Kingdom (UK), and the United States, have implemented carbon tax policies.

Under the carbon tax policy, more and more manufacturers are taking actions to reduce carbon emissions as much as they can. Generally, two methods are employed to realize this target. One is to introduce remanufacturing to form a closed-loop supply chain (CLSC); the other is to invest directly in low-carbon technology to reduce carbon emissions.

Without a doubt, remanufacturing operations have proven to be eco-efficient, low-carbon-producing ways to save energy and reduce carbon emissions effectively (Chang et al., 2017; Maiti and Giri, 2017; Xu and Wang, 2018). In the UK, remanufacturing is estimated to contribute to the reduction of 10 million tons of carbon dioxide every year (Yenipazarli, 2016). Some giant companies, like Hewlett-Packard Corporation (HP), IBM, Kodak, and Xerox, already engage in recycling and remanufacturing (Chen et al., 2018). In China, the government has introduced and implemented many policies and regulations to guide firms in remanufacturing. In 2019, the State Council officially promulgated the "measures for administering the retrieval of discarded motor vehicles" to promote the development of China's remanufacturing industry.

Low-carbon technology can also facilitate emission reduction regardless of whether manufacturers introduce remanufacturing operations. For example, Fuji Film curbs carbon emissions by improving the usage efficiency and product life of its toners. AU Optronics developed thin film transistor (TFT) display technology to curb the carbon emissions of its products by 30% (Li et al., 2018). Some governments also actively promote this strategy. In China, the National Development and Reform Commission released a "National key energy-saving and low-carbon technology promotion catalogue."

However, practices have shown that the effect of the carbon tax policy is not obvious because of interlinked factors in CLSC, such as the coordination of different members in a CLSC, optimization of decisions individually and collectively, returned product collection strategies, and remanufacturing magnitude. In terms of collecting used products, three operational strategies exist: no collection, partial collection, and full collection. Which strategy can best maximize profit is an important question. Considering whether or not to invest in carbon emission reduction technology makes the whole decision-making process more complex.

Therefore, this study takes the carbon tax policy into consideration while making decisions on the amount of manufacturing and remanufacturing under two scenarios, namely, no investment in carbon reduction technology and with investment. We will address the following questions:

(1) Under what conditions can manufacturers actively engage in remanufacturing to reduce carbon emissions with different collection strategies?

(2) What is the impact of carbon reduction investment in the manufacturing and remanufacturing decisions in centralized and decentralized models under the carbon tax policy?

(3) If the carbon reduction investment is more costly, can it still be profitable? To what extent can the manufacturer and retailer obtain more revenues while effectively reducing carbon emissions?

To answer these questions, we develop four game-theoretic models to evaluate the impact of carbon tax policy on manufacturing and remanufacturing decisions in a closed-loop supply chain (CLSC) consisting of a manufacturer and a retailer. To maximize profits, the decisions are made based on two scenarios, namely, no investment in carbon reduction technology and with investment, in the centralized and decentralized CLSC respectively. In the centralized CLSC, the manufacturer and retailer are vertically integrated as a whole system to make joint decisions that maximize the total SC profit. In the decentralized CLSC, the manufacturer and retailer make their own decisions to maximize their respective profits, with the manufacturer acting as a Stackelberg leader. The used products are collected for remanufacturing under three strategies, that is, no collection, partial collection, and full collection. Our model and results provide the optimal and pragmatic manufacturing and remanufacturing strategies for practitioners in the context of CLSC. The contributions of our work are summarized as follows: (1) Considering economic performance, we characterize under which conditions manufacturers are actively engaged in remanufacturing in the second period and investigate how carbon tax, carbon reduction technology investment, and consumer preference on remanufactured products affect the equilibriums of CLSC. (2) Unlike the work of Chang et al. (2017) and Chang et al. (2015), which found that carbon emission saving per remanufactured product affects the remanufacturing decision in the second period for one firm, our results show that the decisions of manufacturers are affected both by the carbon emission saving per remanufactured product and the wholesale price in the second period with decentralized CLSC. (3) Our results also suggest that both governmental carbon tax levy and CLSC members' individual decisions on carbon reduction technology investment play important roles in coordinating the supply chain and motivating remanufacturing operations. This finding deviates from that of Yang et al. (2016), Xu and Wang (2018), and Bulmuş et al. (2013) who emphasized only one influential aspect. (4) By analyzing consumer preference on remanufactured products, this research proves the necessity of cultivating customers' sustainable purchasing and consumption behavior.

The rest of the paper is organized as follows. Section 2 reviews the relevant literature. Section 3 introduces the problem and basic assumptions. Section 4 develops and analyzes four profit-maximization models. Sections 5 and 6 provide numerical examples and managerial applications, respectively. Finally, Section 7 summarizes the key findings and presents future research directions.

2. Literature review

In this section, we review the studies highly related to our work. These studies can be divided into three streams: supply chain management under carbon tax policy, operation and management of CLSC, and manufacturing/remanufacturing decisions of CLSC under carbon policies. A summary of relevant literature is presented in Table 1 to allow for a comparison of previous studies and position this research.

2.1 Supply chain management under the carbon tax policy

In recent years, the operational decisions of firms under the carbon tax policy and with carbon reduction technology investment have become one of the most important research fields. Benjaafar et al. (2013) studied how carbon footprint consideration can be incorporated into operational decision making to evaluate the impact of different regulatory policies (e.g., carbon tax, carbon cap and trade) and assessed the benefits of investments in more carbon-efficient technologies. Krass et al. (2013) considered fixed cost subsidies and consumer rebates to investigate several important aspects of using environmental taxes to motivate the choice of innovative and carbon-emission-reducing technologies. Du et al. (2016) presented a production optimization model to analyze the impacts of carbon footprint and low-carbon preference on the optimal decisions of the manufacturer and proposed some management insights on the formulation of a carbon emission mechanism. Zhang and Xu (2013) proposed a profit-maximization model to analyze optimal production and carbon trading decisions. They also compared the carbon cap and trade mechanism and carbon tax policy to show the effectiveness of the policy. Considering a complex supply chain composed of a single manufacturer and multiple retailers, Huang et al. (2016) established a game-theoretical model to investigate the impacts of product design, supplier selection, and pricing strategies on the profits and carbon emissions of the supply chain. Yang and Chen (2018) considered the carbon tax policy and consumers' environmental awareness to investigate the impact of revenue sharing and cost sharing offered by a retailer on the carbon emission reduction efforts of a manufacturer.

Meanwhile, some researchers have introduced carbon reduction technology investment into low-carbon supply chain management. Zhao et al. (2010) developed a carbon emission allowance allocation system with carbon reduction technology investment in the electric power market. Toptal et al. (2014) investigated the joint decisions of procurement and carbon emission reduction investment under three carbon emission policies. They also discussed the effect of different carbon policies on the decision regarding carbon reduction technology investment. Jiang and Chen (2016) investigated the production, pricing, carbon trading, carbon reduction technology investment strategies, and coordination of a low-carbon supply chain made up of a low-carbon manufacturer and a retailer. The above studies considered carbon emission factors and mainly focused on the forward supply chain. However, with the sustainable development of the economy and the environment, more and more enterprises are beginning to recycle and reuse waste materials, a decision that has an important impact on the operation strategy of enterprises.

2.2 Operations and management of the closed-loop supply chain

The CLSC has gradually become one of the most important research fields in recent years. Jena and Sarmah (2013) investigated the impact of the availability of returned products on acquisition price and channel profit by considering three collection methods (i.e., direct, indirect, and coordinated). Gan et al. (2015) built the Stackelberg game model to study the optimal decisions of new and remanufactured short life-cycle products. Considering that the demand function is a function of both selling price and advertising level, Taleizadeh et al. (2020) formulated a Stackelberg game model to investigate pricing and reverse channel selection decisions in a CLSC. Giovanni et al. (2015) considered a dynamic CLSC composed of a manufacturer and a retailer, both of whom invested in a product recovery program to increase the rate of returned products. The so-called incentive strategies were then used to realize the coordination of the supply chain. Maiti and Giri (2017) proposed four models to study the effects of key parameters on the decisions and feasibility of the cooperative game through a bargaining model. Miao et al. (2017) constructed three kinds of decision models for the CLSC, which consists of one manufacturer and one retailer, and compared the impact of trade-ins on these different supply chain models. Some literature argued that remanufacturing all returned products does not necessarily result in economic benefits (Hosoda et al., 2015; Zhou et al., 2017).

2.3 Manufacturing and remanufacturing decisions of CLSC under carbon policies

Research that addresses operational issues in remanufacturing under an emission regulation is scarce. However, emission regulations will lead to carbon cost and further complicate production decisions between new and remanufactured products (Yenipazarli, 2016). Some studies focused on the CLSC by considering carbon-emission-related factors. Yang et al. (2016) studied an acquisition and remanufacturing problem under the carbon tax scheme subject to budget and risk constraints. In addition, Liu et al. (2015) presented three optimization models to explore optimal remanufacturing decision making under three common carbon emission policies, namely, cap and trade, mandatory carbon emissions capacity, and carbon tax policy. Shu et al. (2018) investigated the optimal decisions in corporate social responsibility (CSR) with a policy of carbon emission constraints and discussed the impacts of such constraints and CSR strength on recycling and remanufacturing decisions. Considering different capital conditions, Wang and Chen (2017) proposed three mathematical profit-maximizing models to determine manufacturing/remanufacturing decisions under a carbon trading policy in a single period. Li et al. (2018) studied price and carbon emission reduction decisions in a two-echelon supply chain with a fairness-neutral manufacturer and a fairness-concerned retailer and examined the impacts of fairness concerns on the CLSC. Mohajeri and Fallah (2016) considered carbon emission constraints and adopted the fuzzy approach to develop an optimization model for a CLSC to cope with uncertain situations. Some researchers also studied the effects of different carbon policies, such as carbon tax and carbon cap and trade, on the manufacturing and recycling decisions of the CLSC (Bazan et al., 2017; Mohammed et al., 2017). However, the above studies focused on the decision making for CLSC with low-carbon policies during a single period and did not pay enough attention to two-period manufacturing/remanufacturing decisions.

Many studies have extended one-period decisions to two-period ones, which are closer to reality. Chang et al. (2015) investigated optimal manufacturing and remanufacturing decisions under the carbon cap and trade mechanism by considering a monopolist manufacturer who made new products in the first period and made new and remanufactured products simultaneously in the second period. Chang et al. (2017) studied a two-period problem to determine the optimal production quantities of new and remanufactured products. However, these prior works only considered one manufacturer who produced new and remanufactured products. Yenipazarli (2016) developed a Stackelberg game model between a profit-maximizing firm and a welfare-maximizing regulator to investigate the impact of carbon taxes and other factors on the optimal manufacturing/remanufacturing decisions in two periods. Bulmuş et al. (2013) established a two-period model in which new products are produced in both periods and remanufactured products are produced in the second period to investigate the effect of remanufacturing on capacity and production decisions.

Although the above studies examined manufacturing/remanufacturing decisions in two periods, they only considered a single oligopoly enterprise and ignored the cooperation between supply chain members. Few papers studied remanufacturing/manufacturing decisions that consider low-carbon factors between two firms in a supply chain in two periods. Xu and Wang (2018) presented centralized and decentralized models to investigate the decision strategy and profit distribution of a CLSC with retail price and emission reduction dependent on demand in two periods. However, they considered the retailer remanufacturing the used products in the second period but not the influence of carbon tax on the decisions of the supply chain.

Table 1. Summary of relevant literature and the position of our paper

Article	Low carbon factor		Supply chain type		Periods		Decision maker	
	Carbon policy	Carbon reduction investment	Forward supply chain	CLSC	Single period	Multi-period	One enterprise	Supply chain
Benjaafar et al. (2013)	✓		✓		✓			✓
Du et al. (2016)	✓		✓		✓			✓
Krass et al. (2013)	✓		✓		✓			✓
Zhang and Xu (2013)	✓		✓		✓			✓
Huang et al. (2016)	✓		✓		✓			✓
Yang and Chen (2018)	✓		✓		✓			✓
Jiang and Chen (2016)	✓	✓	✓		✓			✓
Zhao et al. (2010)	✓	✓	✓		✓			✓
Toptal et al. (2014)	✓	✓	✓		✓			✓
Gan et al. (2015)				✓	✓			✓
Taleizadeh et al. (2020)				✓	✓			✓
Giovanni et al. (2015)				✓	✓			✓
Maiti and Giri (2017)				✓	✓			✓
Miao et al. (2017)				✓	✓			✓
Zhou et al. (2017)				✓	✓			✓
Yang et al. (2016)	✓			✓	✓			✓
Li et al. (2018)	✓			✓	✓			✓
Shu et al. (2018)	✓			✓	✓			✓
Wang and Chen (2017)	✓			✓	✓			✓
Mohajeri and Fallah (2016)	✓			✓	✓			✓
Chang et al. (2015)				✓		✓	✓	
Chang et al. (2017)				✓		✓	✓	
Bulmuş et al. (2013)				✓		✓	✓	
Xu and Wang. (2018)		✓		✓		✓		✓
Our paper	✓	✓		✓		✓		✓

3. Problem description and notations

Inspired by real case studies in the manufacturing sector, such as Oracle, HP, Xerox, Wal-Mart, IKEA, and IBM, we analyze a two-period CLSC. In the first period, the manufacturer produces new products using virgin materials and the retailer sells the new products. In the second period, the manufacturer offers both new and remanufactured products. The remanufactured products in the second period originate from the products that were sold in the first period and then returned. The collection ratio is ψ ($0 < \psi \leq 1$), which refers to the proportion of new products made in period 1 that is available for remanufacturing in period 2. We assume that all collected products are successfully remanufactured. Here, given $0 < \psi \leq 1$, the ψ itself implies the loss of possibly salvageable products in the course of collecting, sorting, and remanufacturing. These uncollected products may end up in waste or disposal treatment. Hence, the products available for remanufacturing in the second period is less than or equal to ψQ_{1n}. The same assumption is adopted by many researchers, such as Chang et al. (2017), Wang et al. (2017), and Bulmuş et al. (2013).

New and remanufactured products have different characteristics and valuations and can thus substitute each other as competitors (Chang et al., 2017). Without a loss of generality, the prices for new and remanufactured products in the first and second periods can be given respectively as follows: $P_{1n}(Q_{1n}) = \alpha - Q_{1n}$, $P_{2n}(Q_{2n}, Q_{2r}) = \alpha - Q_{2n} - \beta Q_{2r}$, $P_{2r}(Q_{2r}, Q_{2n}) = \beta(\alpha - Q_{2r} - Q_{2n})$. Here, α is the size of the potential market, and β is the customers' preference for the remanufactured product ($0 < \beta \leq 1$). Similar assumptions have been widely adopted in previous studies (e.g., Chang et al., 2017; Xu and Wang, 2018). Given the carbon emission for producing one unit of new product e_0 (Yang and Chen, 2018), the carbon emission saving per remanufactured product is Δe (Chang et al., 2017). Therefore, the carbon emission for producing one unit of the remanufactured product is $e_0 - \Delta e$. To reduce the carbon emission, the manufacturer may pursue low-carbon production by investing in carbon reduction technology. We assume the cost function is convex and increases at the carbon emission reduction level e_i, that is, $\mathrm{C}(e_i) = \frac{1}{2}\xi(e_i)^2$, where i ($i = 1,2$) is the production period, and ξ is the carbon emission reduction cost factor. This assumption is reasonable since there is always a limit for abatement and a similar treatment can be found in previous studies (Xu et al., 2017; Yang et al., 2017). C_n and C_r are the production costs of new and remanufactured products, respectively, and $C_n > C_r + P_m$ and $C_n > C_r + P_{tc}$ indicate the cost-saving advantage of remanufacturing (Wang and Chen, 2017). The variables and notations used throughout this paper are shown in Table 2.

Table 2. Variables and notations

Parameters	Description
α	Size of the potential market.
β	Customers' preference for the remanufactured product ($0 < \beta \leq 1$).
e_0	Carbon emission for producing one unit of the new product e_0.
ψ	Collection ratio of the used products ($0 < \psi \leq 1$).
C_n, C_r	Unit cost to produce a new product and a remanufactured product.
ξ	Cost coefficient of carbon emission reduction.
τ	Carbon tax.
Δe	Carbon emission saving per unit remanufactured product.
E	Total carbon emission.
P_{tc} , P_m	Acquisition price per used product from customers by the centralized strategy and the manufacturer collection strategy, respectively.
P_{1n}, P_{2n}	Retail price of a new product in the first and second periods, respectively.
P_{2r}	Retail price of a remanufactured product in the second periods, respectively.
$\Pi_{SC}^i, \Pi_M^i, \Pi_R^i$	Profit of supply chain, manufacturer, and retailer.
Decision variables	
Q_{1n}^i, Q_{2n}^i	Production quantity of a new product in the first and second periods, respectively.
Q_{2r}^i	Quantity of remanufactured products.
e_1, e_2	Carbon reduction level of unit product in the first and second periods, respectively.
W_{1n}^i	Wholesale price of new products in the first period.
W_{2n}^i, W_{2r}^i	Wholesale price of new and remanufactured products in the second period.

Superscript $i \in \{NC, YC, NM, YM\}$ refers to the centralized CLSC without and with carbon reduction technology investment and the decentralized CLSC without and with carbon reduction technology investment.

4. Models and Analysis

4.1 Centralized decision-making model

Centralized decision-making model in CLSC is not new in practice. In this model, the manufacturer and merchandise retailer manage the returns as one entity. For example, the retail returns management of Oracle—a company that designs, manufactures, and sells both software and hardware products—is a centralized system designed to monitor and control the return of retail merchandise. Therefore, the manufacturer of Oracle and its retailers are treated as one entity (see Return management at Oracle®, 2020). According to their return quality, the returned products are repaired, refurbished, and remanufactured to the standard that can be used to satisfy customer demand again (Oracle Service Parts Planning Implementation and User's Guide, 2020). In this study, we take these three operations as interchangeable, similar to the practice in Oracle. Other examples of adopting a centralized return collection model can be found in Wal-Mart, IKEA, and IBM, all of which have implemented various preferential coordination policies to promote their upstream and downstream enterprises to work on low-carbon production together (Cohen and Vandenbergh, 2012; Zu et al., 2018).

In a centralized CLSC, whether or not a company invests in carbon reduction will influence their manufacturing and remanufacturing decisions. In the 2000s, Kodak implemented remanufacturing and did not invest in carbon reduction technology. By contrast, Fuji Film invested in carbon reduction by improving the usage efficiency and service life of its toners (Chen et al., 2018). Nevertheless, how investing in carbon reduction technology affects manufacturing/remanufacturing decisions under carbon tax remains unclear.

4.1.1 Model NC: Centralized model without carbon reduction technology investment

As illustrated in Fig. 1, the production of new products and the collection and remanufacture of used products are all conducted by an integrated supply chain in the centralized decision model (Miao et al., 2017). As no investment is made in carbon reduction technology, the optimal problem under Model NC in two periods can be expressed as follows:

$$Max\Pi_{\mathrm{SC}}^{\mathrm{NC}}(Q_{1n}^{NC},Q_{2n}^{NC},Q_{2r}^{NC}) = (P_{1n}^{NC} - C_n)Q_{1n}^{NC} + (P_{2n}^{NC} - C_n)Q_{2n}^{NC} + (P_{2r}^{NC} - C_r - \mathrm{P_{tc}})Q_{2r}^{NC} - \tau E^{NC} \quad (1)$$

$$\mathrm{s.t}\begin{cases} \mathrm{E}^{NC} = [\mathrm{e_0}(Q_{1n}^{NC} + Q_{2n}^{NC}) + (\mathrm{e_0} - \Delta e)Q_{2r}^{NC}] \\ Q_{2r}^{NC} \le \psi Q_{1n}^{NC} \\ Q_{1n}^{NC} \ge 0, Q_{2n}^{NC} \ge 0, Q_{2r}^{NC} \ge 0 \end{cases}$$

In the objective function, $(P_{1n}^{NC} - C_n)Q_{1n}^{NC}$, $(P_{2n}^{NC} - C_n)Q_{2n}^{NC}$, and $(P_{2r}^{NC} - C_r - P_{tc})Q_{2r}^{NC}$ represent the profits from selling products in both periods, and τE^{NC} is the total carbon tax in the two periods. Here, the equation $E^{NC} = \left[\mathrm{e_0}\left(Q_{1n}^{NC} + Q_{2n}^{NC}\right) + (\mathrm{e_0} - \Delta e)Q_{2r}^{NC}\right]$ describes the total carbon emissions in the two periods. The constraint $Q_{2r}^{NC} \le \psi Q_{1n}^{NC}$ implies that the production quantity of the remanufactured product in period 2 is no more than the collection of quantities from period 1.

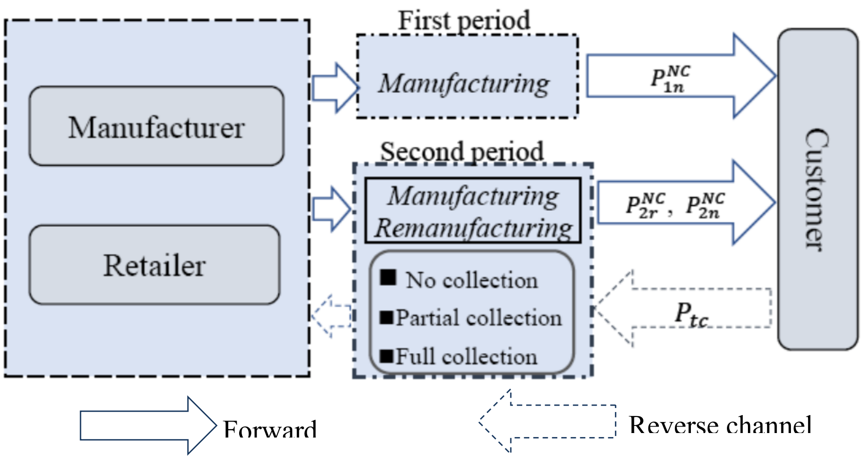

Fig. 1. Centralized CLSC without carbon reduction investment

On the basis of the Karush–Kuhn–Tucker (KKT) conditions, we can obtain the equilibrium decisions as presented in the following propositions.

Proposition 1. In Model NC, the optimal production quantity and retail price of the new products in period 1 are as follows.

$$\begin{cases} Q_{1n}^{NC^*} = \dfrac{1}{2}(a - C_n - e_0\tau) \\ P_{1n}^{NC^*} = \dfrac{1}{2}(a + C_n + e_0\tau) \end{cases}$$

Clearly, the production quantity decreases and the retail price increases with the increase of carbon tax.

In the second period, the supply chain needs to decide on the number of remanufactured products Q_{2r}^{NC} in addition to the number of new products Q_{2n}^{NC}. Substituting $Q_{1n}^{NC^*}$ and $P_{1n}^{NC^*}$ into the profit function of the supply chain, we obtain three collection scenarios based on the carbon saving per remanufactured product (Δe) by KKT conditions, namely, no collection (if $\Delta e < \Delta e_L^{NC^*}$), partial collection (if $\Delta e_L^{NC^*} \leq \Delta e \leq \Delta e_U^{NC^*}$), and full collection (if $\Delta e > \Delta e_U^{NC^*}$).

Proposition 2. The optimal production decisions and corresponding conditions in period 2 under Model NC are shown in Table 3, where the critical values of carbon saving per remanufactured product are $\Delta e_L^{NC^*} = \frac{C_r + P_{tc} + e_0\tau - \beta(C_n + e_0\tau)}{\tau}$ and $\Delta e_U^{NC^*} = \frac{C_r + P_{tc} + e_0\tau - \beta(C_n + e_0\tau) + (1-\beta)\beta(a - C_n - e_0\tau)\psi}{\tau}$.

The first column in Table 2 shows the equilibrium strategies with no collection, the second column displays the equilibrium decision under the partial collection strategy, and the third column is the equilibrium decision with a full collection strategy in the second period. (Proofs of Propositions 1 and 2 are presented in Appendix A.1 and A.2 respectively.)

Proposition 2 demonstrates that if the carbon emission saving per remanufactured product is too small ($0 < \Delta e < \Delta e_L^{NC*}$), then no used products will be collected in the second period. If it is above the critical value Δe_U^{NC*}, then all the available used products sold in the first period will be collected and remanufactured to gain profit in the second period. This outcome implies that when the carbon emission saving per remanufactured product is large enough, then the supply chain will prefer to collect more used products to gain more profit and, at the same time, reduce carbon emissions.

Table 3. Equilibrium optimal solutions in period 2 and total profits under Model NC

Optimal decision	No collection $\Delta e < \Delta e_L^{NC^*}$	Partial collection $\Delta e_L^{NC^*} \le \Delta e \le \Delta e_U^{NC^*}$	Full collection $\Delta e > \Delta e_U^{NC^*}$
$Q_{2n}^{NC^*}$	$\frac{1}{2}(a - C_n - e_0\tau)$	$\frac{a - C_n + C_r + P_{tc} - a\beta - \Delta e\tau}{2(1-\beta)}$	$\frac{1}{2}(a - C_n - e_0\tau)(1-\beta\psi)$
$P_{2n}^{NC^*}$	$\frac{1}{2}(a + C_n + e_0\tau)$	$\frac{1}{2}(a + C_n + e_0\tau)$	$\frac{1}{2}(a + C_n + e_0\tau)$
$Q_{2r}^{NC^*}$	0	$\frac{C_n\beta - e_0\tau + e_0\beta\tau + \Delta e\tau - C_r - P_{tc}}{2(1-\beta)\beta}$	$\frac{1}{2}(a - C_n - e_0\tau)\psi$
$P_{2r}^{NC^*}$	0	$\frac{1}{2}(C_r + P_{tc} + a\beta + e_0\tau - \Delta e\tau)$	$\frac{1}{2}\beta(a + C_n + e_0\tau - (1-\beta)(a - C_n - e_0\tau)\psi)$
E^{NC^*}	$e_0(a - C_n - e_0\tau)$	$\frac{1}{2(1-\beta)\beta}(\Delta e(P_{tc} - C_n\beta - \Delta e\tau) - C_r(e_0(1-\beta) - \Delta e) - {e_0}^2(1-\beta^2)\tau + e_0(1-\beta)(2a\beta - C_n\beta + 2\Delta e\tau - P_{tc}))$	$\frac{1}{2}(a - C_n - e_0\tau)(e_0(2 + (1-\beta)\psi) - \Delta e\psi)$
$\boldsymbol{\Pi}_{SC}^{NC^*}$	$\frac{1}{2}(a - C_n - e_0\tau)^2$	$\frac{1}{4(1-\beta)\beta}(2(a - C_n)^2\beta + (1 + 4(1-\beta)\beta)((C_r + P_{tc} - a\beta)(C_r + P_{tc} + (a - 2C_n)\beta) - 2(e_0(-1+\beta)(C_r + P_{tc} - a\beta) + (C_r + P_{tc} - C_n\beta)\Delta e)\tau + ({e_0}^2(1-\beta) - 2e_0(1-\beta)\Delta e + \Delta e^2)\tau^2))$	$\frac{1}{4}(a - C_n - e_0\tau)(2a - 2(C_n + e_0\tau) - 2(C_r + P_{tc} - C_n\beta + (e_0(1-\beta) - \Delta e)\tau)\psi - (1-\beta)\beta(a - C_n - e_0\tau)\psi^2)$

Corollary 1. In Model NC, the minimum critical value (i.e., Δe_L^{NC*}) that triggers remanufacturing operations decreases with the increase of consumer preference for the remanufactured product β.

Corollary 1 indicates that the remanufactured preference of consumers plays a key role in the course of the supply chain's remanufacturing decision making.

Corollary 2. In Model NC, the monotonicity results of $Q_{2n}^{NC^*}$, $Q_{2r}^{NC^*}$, $P_{2n}^{NC^*}$, $P_{2r}^{NC^*}$, and E^{NC^*} with respect to carbon tax τ and carbon saving per unit remanufactured product Δe are shown in Table 4.

Table 4. Monotonicity of the optimal decisions under Model NC

Parameters	$Q_{2n}^{NC^*}$			$Q_{2r}^{NC^*}$			$P_{2n}^{NC^*}$		
	No	Partial	Full	No	Partial	Full	No	Partial	Full
Δe ↗	→	↘	↘	→	↗	→	→	→	→
τ ↗	↘	↘	↘	→	↗ if $\Delta e > e_0(1-\beta)$ ↘ if $0 < \Delta e < e_0(1-\beta)$	↘	↗	↗	↗

Parameters	$P_{2r}^{NC^*}$			E^{NC^*}		
	No	Partial	Full	No	Partial	Full
Δe ↗	→	↘	→	→	↘	↘
τ ↗	→	↗	↗	↘	↘	↘

Note: ↗: increasing; ↘: decreasing; →: irrelevant.

From Corollary 2, some insights can be concluded as follows.

(i) In the non-collection case, the CLSC degenerates to a linear supply chain. Therefore, the equilibrium decisions are irrespective of Δe. As a result, remanufacturing can create real value only when partial collection or full collection is the optimal strategy.

(ii) In the case of partial collection, the remanufacturing decision is rather complex. The influence of carbon tax τ interrelates with carbon emission saving per remanufactured product (Δe). When Δe is relatively large ($\Delta e > e_0(1-\beta)$), a higher carbon tax can stimulate more remanufacturing. Otherwise, with a small Δe ($0 < \Delta e < e_0(1-\beta)$), supply chain members would rather pay carbon tax than become involved in remanufacturing because the saved carbon emission is not big enough to offset the carbon tax cost. Therefore, carbon tax can be a double-edged sword, promoting or demotivating remanufacturing. For those products whose carbon saving is high in the remanufacturing phase, carbon tax can result in better performance on environmental protection. Conversely, for those products whose carbon saving is small in the remanufacturing phase, carbon tax will discourage remanufacturing, which may actually worsen environmental performance. Hence, the regulator may intervene in the carbon tax to influence the optimal decision of the manufacturer to produce more remanufactured products.

(iii) If the full collection strategy dominates the others, then the greater the carbon saving per remanufactured product, the smaller the total carbon emission will be for the supply chain. In the meantime, with an increase in carbon tax, the total carbon emission in the supply chain in two periods decreases. However, assessing the economic benefit of carbon tax depends on the amount of emission saving per remanufacturing process. This finding suggests that if the manufacturer decides to collect used products in full, levying the carbon tax can indeed stimulate the remanufacturing process, hence benefitting the environment.

(iv) For the case of full collection, under the condition $\mathrm{e}_0(1-\beta) > \Delta e_U^{NC^*}$, when the carbon saving per remanufactured product satisfies $\Delta e > e_0(1-\beta)$, the total carbon emission in two periods is smaller than that for the no collection case (i.e., $\boldsymbol{E}^{\boldsymbol{NC}^*}|f < \boldsymbol{E}^{\boldsymbol{NC}^*}|n$). For the case of partial collection, under the condition $\mathrm{e}_0(1-\beta) < \Delta e_U^{NC*}$, when the carbon saving per remanufactured product Δe satisfies $\mathrm{e}_0(1-\beta) < \Delta e < \Delta e_U^{NC^*}$, the total carbon emission in two periods under the partial collection case is smaller than that for the no collection case (i.e., $\boldsymbol{E}^{\boldsymbol{NC}^*}|p < \boldsymbol{E}^{\boldsymbol{NC}^*}|n$). This result indicates that remanufacturing can encourage enterprises to effectively reduce carbon emissions. (Here, the superscripts “*n*”, “*p*”, and “*f*” represent “no collection,” “partial collection,” and “full collection,” respectively.)

4.1.2 Model YC: Centralized model with carbon reduction investment

Similar to 4.1.1, as illustrated in Fig. 2, in Model YC, the manufacturer and retailer make decisions in two periods as an integrated system, and investment in carbon reduction technology is made. The cost of carbon reduction investment in the first and second periods is $\mathrm{C}(e_1, e_2) = \frac{1}{2}\xi {e_1}^2 + \frac{1}{2}\xi {e_2}^2$, and the total carbon emission $E = (e_0 - e_1)Q_{1n} + (e_0 - e_2)Q_{2n} + (e_0 - e_1 - \Delta e)Q_{2r}$. Thus, the profit-maximization problem under Model YC can be written as follows:

$$Max\Pi_{\mathrm{SC}}^{\mathrm{YC}}(Q_{1n}^{YC}, Q_{2n}^{YC}, Q_{2r}^{YC}) = (P_{1n}^{YC} - \mathrm{C}_n)Q_{1n}^{YC} + (P_{2n}^{YC} - \mathrm{C}_n)Q_{2n}^{YC} + (P_{2r}^{YC} - C_r - \mathrm{P_{tc}})Q_{2r}^{YC} - \mathrm{C}(e_1, e_2) - \tau E^{YC} \quad (2)$$

$$\text{s.t} \begin{cases} E = (e_0 - e_1^{YC})Q_{1n}^{YC} + (e_0 - e_2^{YC})Q_{2n}^{YC} + (e_0 - e_1^{YC} - \Delta e)Q_{2r}^{YC} \\ C(e_1, e_2) = \frac{1}{2}\xi(e_1^{YC})^2 + \frac{1}{2}\xi(e_2^{YC})^2 \\ \psi Q_{1n}^{YC} \geq Q_{2r}^{YC} \\ Q_{1n}^{YC}, Q_{2n}^{YC}, Q_{2r}^{YC} \geq 0 \end{cases}$$

Applying the KKT conditions and backward induction methods can obtain the optimal equilibrium solution as the following proposition.

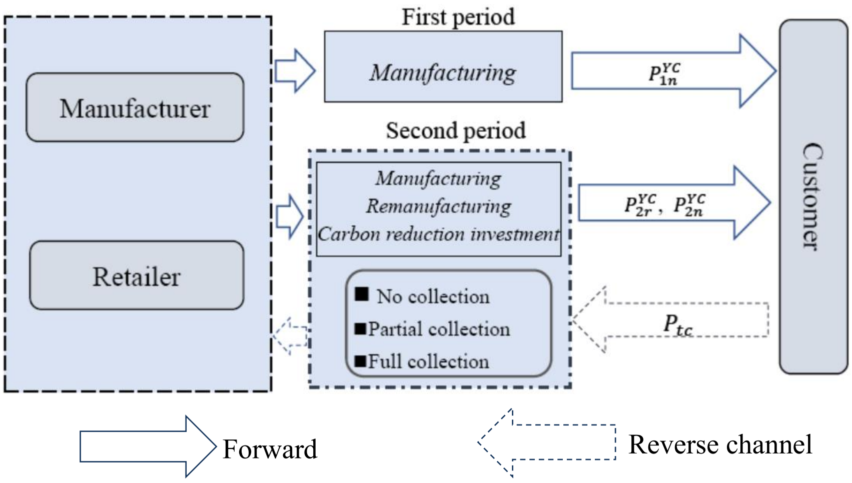

Fig. 2. Centralized CLSC with carbon reduction investment

Proposition 3. In a centralized supply chain with carbon emission reduction investment (Model YC), if the condition $\xi > \frac{1}{2}\tau^2$ is satisfied, then the optimal demand quantity, carbon reduction level, and retail price of the new products in period 1 will be as follows:

$$\begin{cases} Q_{1n}^{YC^*} = \dfrac{\xi(a - C_n - e_0\tau)}{2\xi - \tau^2} \\ e_1^{YC^*} = \dfrac{\tau(a - C_n - e_0\tau)}{2\xi - \tau^2} \\ P_{1n}^{YC^*} = \dfrac{C_n\xi + e_0\xi\tau + a(\xi - \tau^2)}{2\xi - \tau^2} \end{cases}$$

Proposition 4. In the second period, if the condition $\tau^2 - 2(1-\beta)\xi < 0$ is satisfied, then the optimal decisions will be as shown in Table 5, where the critical values of Δe defines three production decision regions. In Table 5, $\Delta e_L^{YC^*} = \frac{A}{\tau(2\xi-\tau^2)}$, $\Delta e_U^{YC^*} = \frac{1}{\tau(2\xi-\tau^2)^2}\left((2\xi - \tau^2)A + 2\beta\xi(a - C_n - e_0\tau)(2(1-\beta)\xi - \tau^2)\psi\right)$, and $A = 2(C_r + P_{tc} - C_n\beta)\xi + 2e_0(1-\beta)\xi\tau - (a - C_n + C_r + P_{tc} - a\beta)\tau^2$. (Proofs of Propositions 3 and 4 are presented in Appendix B.1 and B.2, respectively.)

Obviously, the manufacturing and remanufacturing decisions under Model YC are significantly affected by the carbon emission saving per unit remanufactured product (Δe).

Table 5. Equilibrium optimal solutions in period 2 under Model YC

Optimal decision	No collection $\Delta e < \Delta e_L^{YC^*}$	Partial collection $\Delta e_L^{YC^*} \leq \Delta e \leq \Delta e_U^{YC^*}$	Full collection $\Delta e > \Delta e_U^{YC^*}$
$Q_{2n}^{YC^*}$	$\frac{\xi(a-C_n-e_0\tau)}{2\xi-\tau^2}$	$\frac{1}{(2\xi-\tau^2)(2(\beta-1)\xi+\tau^2)}(\xi(2(C_n-C_r-P_{tc}+a(\beta-1))\xi$ $+2\Delta e\xi\tau+(\mathrm{C_r}-2\mathrm{C}_n+P_{tc}-a(\beta-2))\tau^2-(\mathrm{e_0}+\Delta e)\tau^3))$	$\frac{\xi(a-C_n-e_0\tau)(2\xi-\tau^2-2\beta\xi\psi)}{(2\xi-\tau^2)^2}$
$e_2^{YC^*}$	$\frac{\tau(a-C_n-e_0\tau)}{2\xi-\tau^2}$	$\frac{1}{(2\xi-\tau^2)(2(\beta-1)\xi+\tau^2)}\tau(2(C_n-C_r-P_{tc}+a(\beta-1))\xi$ $+2\Delta e\xi\tau+(C_r-2C_n+P_{tc}-a(\beta-2))\tau^2$ $-(e_0+\Delta e)\tau^3)$	$\frac{\tau(a-C_n-e_0\tau)(2\xi(1-\beta\psi)-\tau^2)}{(2\xi-\tau^2)^2}$
$P_{2n}^{YC^*}$	$\frac{a\xi+C_n\xi+e_0\xi\tau-a\tau^2}{2\xi-\tau^2}$	$\frac{1}{\left(2(2\xi-\tau^2)(2(\beta-1)\xi+\tau^2)\right)}(4(a+C_n)(\beta-1)\xi^2+4e_0(\beta$ $-1)\xi^2\tau+2(3a+C_n+C_r-(2a+C_n)\beta)\xi\tau^2$ $-2(e_0(\beta-2)+\Delta e)\xi\tau^3+(C_n-C_r+a(\beta$ $-3))\tau^4+\Delta e\tau^5)$	$\frac{a\left(2\xi^2+\tau^4+\xi\tau^2(\beta\psi-3)\right)+\xi(C_n+e_0\tau)\left(2\xi-\tau^2(1+\beta\psi)\right)}{(2\xi-\tau^2)^2}$
$Q_{2r}^{YC^*}$	0	$\frac{1}{2\beta(\tau^2-2\xi+2\beta\xi)}(2(C_r+P_{tc}-C_n\beta)\xi$ $+2(e_0(1-\beta)-\Delta e)\xi\tau$ $-(a-C_n+C_r+P_{tc}-a\beta)\tau^2+\Delta e\tau^3)$	$\frac{\xi\psi(a-C_n-e_0\tau)}{2\xi-\tau^2}$
$P_{2r}^{YC^*}$	0	$\frac{1}{2}(a-C_n+C_r+a\beta-\Delta e\tau-\frac{2\xi(a-C_n-e_0\tau)}{2\xi-\tau^2})$	$\frac{\beta(2\xi-\tau^2)\left((a+C_n)\xi+e_0\xi\tau-a\tau^2\right)-\beta\xi(a-C_n-e_0\tau)(2(1-\beta)\xi-\tau^2)\psi}{(2\xi-\tau^2)^2}$

Corollary 3. In Model YC, under the partial collection strategy, some conclusions can be summarized as follows.

(i) In the case of $\sqrt{2(1-\beta)\xi} \le \frac{e_0\xi+\sqrt{2aC_n\xi+e_0{}^2\xi^2}}{a}$, if $0<\tau<\sqrt{2(1-\beta)\xi}$, then the minimum critical value $\Delta e_L^{YC^*}$ that triggers remanufacturing operations will decrease with the increase in consumer preference for the remanufactured product β.

(ii) In the case of $2(1-\beta)\xi > \frac{e_0\xi+\sqrt{2aC_n\xi+e_0{}^2\xi^2}}{a}$, if $0<\tau<\frac{e_0\xi+\sqrt{2aC_n\xi+e_0{}^2\xi^2}}{a}$, then $\Delta e_L^{YC^*}$ will decrease with the increase of consumer preference for the remanufactured product β; otherwise, if $\frac{e_0\xi+\sqrt{2aC_n\xi+e_0{}^2\xi^2}}{a}<\tau<\sqrt{2(1-\beta)\xi}$, then it will increase in β. Thus, when the carbon tax is low, raising the remanufactured preference of consumers can encourage enterprises to produce more remanufactured products, but when the carbon tax is too high, even raising consumer preference for remanufactured products cannot encourage enterprises to remanufacture more products. Therefore, for the regulator, it is particularly important to formulate a reasonable carbon tax policy according to the nature of the enterprises.

(iii) The impact of carbon saving per remanufactured product Δe on manufacturing and remanufacturing decisions can be concluded as follows: $\frac{\partial Q_{2n}^{YC^*}}{\partial \Delta e}<0$, $\frac{\partial Q_{2r}^{YC^*}}{\partial \Delta e}>0$, $\frac{\partial P_{2n}^{YC^*}}{\partial \Delta e}>0$, $\frac{\partial P_{2r}^{YC^*}}{\partial \Delta e}<0$, $\frac{\partial e_2^{YC^*}}{\partial \Delta e}<0$. This outcome indicates that under a certain carbon tax policy, with the increase of carbon saving per unit remanufactured product, the quantity of new products decreases, the quantity of remanufacturing products increases and the carbon reduction level also decreases. Accordingly, the retail price of new products increases and the retail price of remanufactured products decreases. This means that the more the carbon saving per unit remanufactured product is, the more remanufactured products will be produced; more profit may also be gained by investing less in carbon reduction through remanufacturing.

Corollary 4. In Model YC, if full collection becomes the optimal strategy, then the optimal carbon reduction level of the unit new product in the second period (i.e., $e_2^{YC^*}$) will decrease with the increase in consumer preference for remanufactured products. The carbon reduction level in the second period will also be smaller than that in the first period (i.e., $e_2^{YC^*}<e_1^{YC^*}$). This result means that through remanufacturing, a higher profit can be made by investing less in carbon reduction technology.

Owing to the complexity of the problems, the impact of carbon tax τ on manufacturing and remanufacturing decisions cannot be obtained directly. Therefore, this study resorts to explicitly known solutions through numerical analysis, and the results are shown in the next section.

4.2 Decentralized Decision-making Model

In the decentralized decision-making model, the manufacturer and retailer make decisions respectively. The manufacturer decides on the wholesale price of new and remanufactured products as well as carbon reduction level while the retailer decides on the order quantity of new and remanufactured products. We consider that the manufacturer collects the used products for remanufacturing directly from the customers. In real cases, some manufacturers collect their used products directly from consumers. For instance, Xerox Corporation provides prepaid mailboxes so that customers can easily return their used copy or print cartridges to Xerox without incurring any costs. The green remanufacturing program saves the company 40%–65% in manufacturing costs through the reuse of parts and materials (Savaskan et al., 2004). HP encourages consumers to return their used computers and peripherals directly to the company (Miao et al., 2017).

In a decentralized CLSC, whether or not investments in carbon reduction are made will also influence the manufacturing and remanufacturing decisions significantly. In fact, Xerox Corporation collects customers' used copy or print cartridges and customers incur no costs. The company remanufactures these collected products without carbon reduction investment (Xerox Corporation, 2001). By contrast, AU Optronics invested in carbon reduction technology and developed TFT display technology to curb the carbon emissions of its products by 30% (Li et al., 2018). Responsibility for the operations may be taken by different supply chain members in the decentralized CLSC. Therefore, determining how to coordinate different members and optimize their manufacturing/remanufacturing decisions under carbon tax is a great challenge.

4.2.1 Model NM: Decentralized model without carbon reduction investment

As illustrated in Fig. 3, in Model NM, the manufacturer is responsible for the production of new products and the collection and remanufacture of used products while the retailer is responsible for selling new and remanufactured products to customers. No investment is made in carbon reduction technology. The game between the manufacturer and retailer is a Stackelberg game, in which the manufacturer is the leader and the retailer is the follower. Hence, the profit-maximization problem of the retailer and manufacturer can be expressed as follows:

$$Max\Pi_{\mathrm{R}}^{\mathrm{NM}}(Q_{1n}^{NM},Q_{2n}^{NM},Q_{2r}^{NM}) = (P_{1n}^{NM} - W_{1n}^{NM})Q_{1n}^{NM} + (P_{2n}^{NM} - W_{2n}^{NM})Q_{2n}^{NM} + (P_{2r}^{NM} - W_{2r}^{NM})Q_{2r}^{NM} \tag{3}$$

$$\mathrm{s.t}\ \begin{array}{c} \psi Q_{1n}^{NM} \geq Q_{2r}^{NM} \\ Q_{1n}^{NM}, Q_{2n}^{NM}, Q_{2r}^{NM} \geq 0 \end{array}$$

$$Max\Pi_M^{NM}(W_{1n}^{NM}, W_{2n}^{NM}, W_{2r}^{NM}) = (W_{1n}^{NM} - C_n)Q_{1n}^{NM} + (W_{2n}^{NM} - C_n)Q_{2n}^{NM} + (W_{2r}^{NM} - C_r - P_m)Q_{2r}^{NM} - \tau E^{NM} \quad (4)$$

$E^{NM} = [e_0(Q_{1n}^{NM} + Q_{2n}^{NM}) + (e_0 - \Delta e)Q_{2r}^{NM}]$ represents the total carbon emission in two periods. In the first period, the manufacturer first determines the wholesale price and then the retailer determines the order quantity. In the second period, the manufacturer first determines the wholesale price of new and remanufactured products and then the retailer determines the order quantity of new and remanufactured products. Similar to Model NC, the equilibrium optimal solution in each period under Model NM can be obtained based on the backward induction method. We then obtain the following propositions.

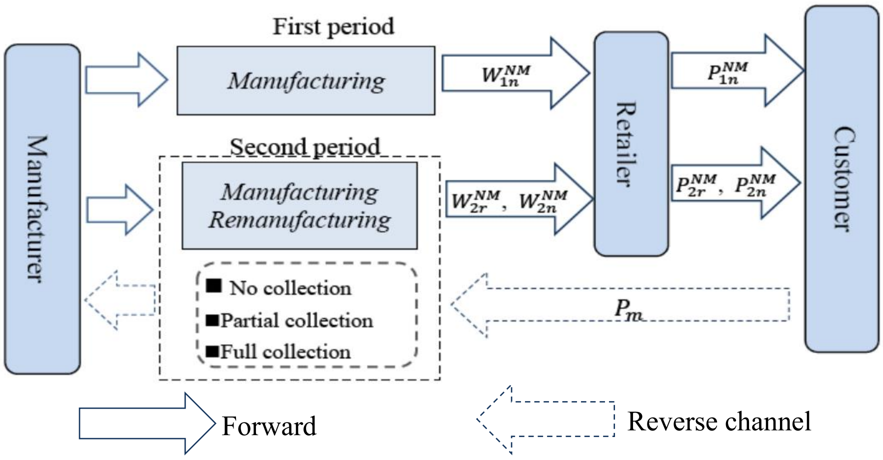

Fig. 3. Decentralized CLSC without carbon reduction investment

Proposition 5. In the manufacturer collecting model (Model NM), the optimal production quantity, wholesale price, and retail price for the new products in period 1 are as follows:

$$\begin{cases} Q_{1n}^{NM^*} = \dfrac{1}{4}(a - C_n - e_0\tau) \\ W_{1n}^{NM^*} = \dfrac{1}{2}(a + C_n + e_0\tau) \\ P_{1n}^{NM^*} = \dfrac{1}{4}(3a + C_n + e_0\tau) \end{cases}$$

In the second period, the manufacturer has the opportunity to collect some used products. Substituting $Q_{1n}^{NM^*}$, $W_{1n}^{NM^*}$, and $P_{1n}^{NM^*}$ into the supply chain members' profit of the second period, we can then obtain the next proposition.

Proposition 6. In Model NM, the carbon emission saving per unit remanufactured product and the wholesale price have different effects on the optimal manufacturing/remanufacturing decisions in period 2, as shown in Table 6. Here, $\Delta e_U^{NM^*} = \frac{C_r + P_m + e_0\tau - \beta(C_n + e_0\tau) + \beta(1-\beta)(a - C_n - e_0\tau)\psi}{\tau}$ and $\Delta e_L^{NM^*} = \frac{C_r + P_m - C_n\beta + e_0(1-\beta)\tau}{\tau}$. The critical values of the wholesale price for no collection and full collection are as follows: $W_L^{NM^*} = \frac{1}{2}\beta(a + C_n + e_0\tau)$ and $W_U^{NM^*} = \frac{1}{4}(2\beta(a + C_n + e_0\tau) - \beta(2-\beta)(a - C_n - e_0\tau)\psi)$. The wholesale price W^{NM} in the case of full collection is any value between the thresholds $(0, W_U^{NM^*})$, and the specific wholesale prices are determined by the bargaining power of the manufacturer and retailer, who can employ optimal production strategies to maximize their profit accordingly. (Proofs of Propositions 5 and 6 are presented in Appendix C.1 and C.2, respectively.)

From Proposition 6, we can also obtain some key insights as follows.

(i) In the no collection and full collection cases, the remanufacturing and manufacturing decisions are not affected by carbon saving per remanufactured product Δe but by the wholesale price. If $W_{2r}^{NM^*} > W_L^{NM^*}$, then no used products will be collected in the second period; if $0 < W_{2r}^{NM^*} < W_U^{NM^*}$, then all the available used products sold in the first period will be collected in the second period.

It is demonstrated that a wholesale price threshold exists. If the wholesale price is above the threshold $\mathrm{W}_{\mathrm{L}}^{\mathrm{NM}^*}$, then the manufacturer will only produce new products, and no used products will be collected in the second period. Similarly, if the wholesale price is lower than the threshold value $W_U^{NM^*}$, then a full collection strategy will be conducted; the wholesale price W^{NM} can be any value in the interval $\left(0, W_U^{NM^*}\right)$. The profit for the manufacturer and the retailer can be determined by the equations in Table 6.

The reason for this phenomenon is that a vertical competition exists between the retailer and the manufacturer under the manufacturer collection strategy. The manufacturer determines the wholesale price of new and remanufactured products, and according to the wholesale price, the retailer determines the order quantity of new and remanufactured products in the second period. Thus, when full collection is the optimal strategy, the wholesale price must be below the threshold. Similarly, if the wholesale price is over the threshold, then the retailer will not order the remanufactured product and so there is no remanufacturing.

(ii) When the carbon emission saving per unit remanufactured product Δe satisfies $\Delta e_L^{NM^*} \leq \Delta e \leq \Delta e_U^{NM^*}$, the manufacturer may partially collect the returned product and remanufacture more products in period 2 as Δe increases.

(iii) The minimum critical value of Δe for the existence of remanufacturing decreases along with customer preference for the remanufactured product (β), suggesting that the remanufacturing practice of the manufacturer is highly influenced by customer preference for the remanufactured product. Therefore, raising the environmental protection awareness of consumers can effectively promote the manufacturing of recycled products.

Table 6. Equilibrium optimal solutions in period 2 and total profit of supply chain members under Model NM

Optimal decision	No collection $W_{2r}^{NM^*} > W_L^{NM^*}$	Partial collection $\Delta e_L^{NM^*} \le \Delta e \le \Delta e_U^{NM^*}$	Full collection $W_{2r}^{NM^*} < W_U^{NM^*}$
$Q_{2n}^{NM^*}$	$\frac{1}{4}(a - C_n - e_0\tau)$	$\frac{a - C_n + C_r + P_m - a\beta - \Delta e\tau}{4 - 4\beta}$	$\frac{1}{8}(a - C_n - e_0\tau)(2 - \beta\psi)$
$W_{2n}^{NM^*}$	$\frac{1}{2}(a + C_n + e_0\tau)$	$\frac{1}{2}(a + C_n + e_0\tau)$	$\frac{1}{4}(a(2 - \beta\psi) + (C_n + e_0\tau)(2 + \beta\psi))$
$P_{2n}^{NM^*}$	$\frac{1}{4}(3a + C_n + e_0\tau)$	$\frac{1}{4}(3a + C_n + e_0\tau)$	$\frac{1}{8}(a(6 - \beta\psi) + (C_n + e_0\tau)(2 + \beta\psi))$
$Q_{2r}^{NM^*}$	0	$\frac{C_n\beta - (e_0(1 - \beta) - \Delta e)\tau - C_r - P_m}{4(1 - \beta)\beta}$	$\frac{1}{4}(a - C_n - e_0\tau)\psi$
$P_{2r}^{NM^*}$	0	$\frac{1}{4}(C_r + P_m + 3a\beta + e_0\tau - \Delta e\tau)$	$\frac{1}{8}\beta((C_n + e_0\tau)(2 + (2 - \beta)\psi) + a(6 - (2 - \beta)\psi))$
$W_{2r}^{NM^*}$	N/A	$\frac{1}{2}(C_r + P_m + a\beta + e_0\tau - \Delta e\tau)$	W^{NM}
E^{NM^*}	$\frac{1}{2}e_0(a - C_n - e_0\tau)$	$\frac{1}{4(\beta - 1)\beta}(C_r(e_0(1 - \beta) - \Delta e) + {e_0}^2(1 - \beta^2)\tau + e_0(1 - \beta)(P_m - 2a\beta + C_n\beta - 2\Delta e\tau) + \Delta e(C_n\beta + \Delta e\tau - P_m))$	$\frac{1}{8}(a - C_n - e_0\tau)(+e_0(4 + (2 - \beta)\psi) - 2\Delta e\psi)$
$\Pi_{\mathrm{M}}^{\mathrm{NM}^*}$	$\frac{1}{4}(a - C_n - e_0\tau)^2$	$\frac{1}{8(1 - \beta)\beta}({C_r}^2 + 2C_rP_m + {P_m}^2 + 2(a - C_n)^2\beta - 2C_nC_r\beta - 2C_nP_m\beta - 2a^2\beta^2 + 4aC_n\beta^2 - {C_n}^2\beta^2 - 2(e_0(\beta - 1)(C_r + P_m + (C_n - 2a)\beta) + (C_r + P_m - C_n\beta)\Delta e)\tau + ({e_0}^2(1 - \beta^2) - 2e_0(1 - \beta)\Delta e + \Delta e^2)\tau^2)$	$\frac{1}{32}(a - C_n - e_0\tau)(a(8 + \beta\psi(\beta\psi - 4)) + \psi(8\Delta e\tau - C_n\beta(\beta\psi - 4) - e_0\tau(8 + \beta(\beta\psi - 4))) - 8(C_n + e_0\tau + (C_r + P_m - W^{NM})\psi))))$
$\boldsymbol{\Pi}_{\mathrm{R}}^{NM^*}$	$\frac{1}{8}(a - C_n - e_0\tau)^2$	$\frac{1}{16(1 - \beta)\beta}({C_r}^2 + 2C_rP_m + {P_m}^2 + 2(a - C_n)^2\beta - 2C_nC_r\beta - 2C_nP_m\beta - 2a^2\beta^2 + 4aC_n\beta^2 - {C_n}^2\beta^2 + 2(e_0(1 - \beta)(C_r + Pm - (2a - C_n)\beta) + (C_r + P_m - C_n\beta)\Delta e)\tau + ({e_0}^2(1 - \beta^2) - 2e_0(1 - \beta)\Delta e + \Delta e^2)\tau^2)$	$\frac{1}{64}(a - C_n - e_0\tau)(-8(C_n + e_0\tau + 2\psi W^{NM}) + \beta(C_n + e_0\tau)\psi(4 + (4 - \beta)\psi) + a(8 + \beta\psi(12 - (4 - \beta)\psi)))$

Corollary 5. In Model NM, the monotonicity of the optimal manufacturing/remanufacturing decisions, such as $Q_{2n}^{NM^*}, Q_{2r}^{NM^*}, P_{2r}^{NM^*}$, and E^{NM^*}, with respect to carbon tax τ and carbon saving Δe are shown in Table 7.

Table 7. Monotonicity of the optimal decisions under Model NM

Parameters	$Q_{2n}^{NM^*}$			$Q_{2r}^{NM^*}$			$P_{2n}^{NM^*}$		
	No	Partial	Full	No	Partial	Full	No	Partial	Full
$\Delta e \nearrow$	→	↘	→	→	↗	→	→	→	→
$\tau \nearrow$	↘	↘	↘	→	↗ if $\Delta e > e_0(1-\beta)$ ↘ if $0 < \Delta e < e_0(1-\beta)$	↘	↗	↗	↗

Parameters	$P_{2r}^{NM^*}$			E^{NM^*}			$W_{2r}^{NM^*}$		
	No	Partial	Full	No	Partial	Full	No ($W_L^{NM^*}$)	Partial ($W_{2r}^{NM^*}$)	Full $W_U^{NM^*}$
$\Delta e \nearrow$	→	↘	→	→	↘	↘	→	↘	→
$\tau \nearrow$	→	↘	↗	↘	↘	↘	↗	↗	↗

Note: ↗: increasing; ↘: decreasing; →: irrelevant.

Most insights for Corollary 5 are similar to those for Corollary 2 under Model NC. We will now analyze the influence of carbon tax on the critical value of the wholesale price for full collection and no collection.

(i) Critical values of the wholesale price of remanufactured products for the full collection strategy $W_U^{NM^*}$ and the no collection strategy $W_L^{NM^*}$ in period 2 increase with respect to the carbon tax τ. This outcome indicates that carbon tax can urge a manufacturer to produce more remanufactured products to gain more profits under the background of the carbon tax policy.

(ii) Under the partial collection strategy, if the carbon saving per remanufactured product is above a certain threshold $\Delta e_L^{NM^*}$, then carbon tax can promote the manufacturer's practice of collecting more used products. Conversely, the manufacturer can pay the carbon tax rather than collect the used product. To regulators, it is important to design a reasonable carbon tax according to the nature of the enterprises to encourage them to remanufacture effectively and thus reduce carbon emissions.

4.2.2 Model YM: Decentralized model with carbon reduction investment

As illustrated in Fig. 4, in this case, the carbon reduction technology is invested. The problem is further modeled as a Stackelberg game with the manufacturer as the leader and the retailer as follower. In the first period, the manufacturer first determines the wholesale price and the carbon reduction level and then the retailer determines the order quantity. In the second period, the manufacturer first determines the wholesale price of the new and remanufactured products and the carbon reduction level of the new product and then the retailer determines the order quantity of the new and remanufactured products. The profit-maximization functions of the retailer and remanufacturer can be defined as follows:

$$Max\Pi_{\mathrm{R}}^{\mathrm{YM}}(Q_{1n}^{YM},Q_{2n}^{YM},Q_{2r}^{YM}) = (P_{1n}^{YM}-W_{1n}^{YM})Q_{1n}^{YM} + (P_{2n}^{YM}-W_{2n}^{YM})Q_{2n}^{YM} + (P_{2r}^{YM}-W_{2r}^{YM})Q_{2r}^{YM} \quad (5)$$

$$\text{s.t}\ \begin{array}{l} \psi Q_{1n}^{YM} \geq Q_{2r}^{YM} \\ Q_{1n}^{YM},Q_{2n}^{YM},Q_{2r}^{YM} \geq 0 \end{array}$$

$$Max\Pi_{M}^{YM}(W_{1n}^{YM},W_{2n}^{YM},W_{2r}^{YM}) = (W_{1n}^{YM}-\mathrm{C}_n)Q_{1n}^{YM} + (W_{2n}^{YM}-\mathrm{C}_n)Q_{2n}^{YM} + (W_{2r}^{YM}-\mathrm{C}_r-P_m)Q_{2r}^{YM} - \mathrm{C}(e_1^{YM},e_2^{YM}) - \tau E^{YM}$$

(6)

Here $\mathrm{C}(e_1^{YM},e_2^{YM}) = \frac{1}{2}\xi(e_1^{YM})^2 + \frac{1}{2}\xi(e_2^{YM})^2$, $E^{YM} = (\mathrm{e}_0-e_1^{YM})Q_{1n}^{YM} + (\mathrm{e}_0-e_2^{YM})Q_{2n}^{YM} + (\mathrm{e}_0-e_1^{YM}-\Delta e)Q_{2r}^{YM}$.

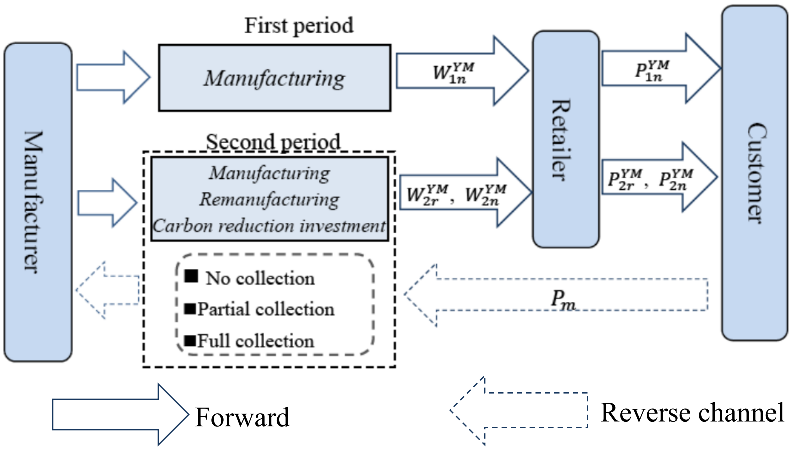

Fig. 4. Decentralized CLSC with carbon reduction investment

Similar to Model NM, this game model can be solved by using backward induction. From Eqs. (5) and (6), we obtain propositions as follows.

Proposition 7. In Model YM, the optimal demand quantity, carbon reduction level, wholesale price, and retail price of the new products in period 1 are as follows:

$$\begin{cases} Q_{1n}^{YM^*} = \dfrac{\xi(a-\mathrm{C}_n-\mathrm{e}_0\tau)}{4\xi-\tau^2} \\ e_2^{YM^*} = \dfrac{\tau(a-\mathrm{C}_n-\mathrm{e}_0\tau)}{4\xi-\tau^2} \\ W_{1n}^{YM^*} = \dfrac{2a\xi+2\mathrm{C}_n\xi+2\mathrm{e}_0\xi\tau-a\tau^2}{4\xi-\tau^2} \\ P_{1n}^{YM^*} = \dfrac{3a\xi+\mathrm{C}_n\xi+\mathrm{e}_0\xi\tau-a\tau^2}{4\xi-\tau^2} \end{cases}$$

Proposition 8. In Model YM, under the condition of $4(1-\beta)\xi-\tau^2>0$, the unique optimal solution is given in Table 7, where $\Delta e_L^{YM^*} = \frac{B}{(4\xi-\tau^2)\tau}$, $\Delta e_U^{YM^*} = \frac{1}{\tau(4\xi-\tau^2)^2}((4\xi-\tau^2)B + 4\beta\xi(a-\mathrm{C}_n-\mathrm{e}0\tau)(4(1-\beta)\xi-\tau^2)\psi)$, $B = 4(C_r+P_m-C_n\beta)\xi + 4e_0(1-\beta)\xi\tau - (a-C_n+C_r+P_m-a\beta)\tau^2$, and $\mathrm{Z} =$

$(4\xi - \tau^2)(4(\beta - 1)\xi + \tau^2)$. At the same time, the critical values of the wholesale price for the no collection and full collection strategies are $W_L^{YM^*} = \frac{\mathrm{H}}{4\xi - \tau^2}$ and $W_U^{YM^*} = \frac{(4\xi-\tau^2)\mathrm{H}-2\beta\xi(a-\mathrm{C}_n-\mathrm{e}_0\tau)(2(2-\beta)\xi-\tau^2)\psi}{(4\xi-\tau^2)^2}$, respectively, where $\mathrm{H} = \beta(2a\xi + 2\mathrm{C}_n\xi + 2\mathrm{e}_0\xi\tau - a\tau^2)$. The wholesale price W^{YM} in the case of full collection is any value in the interval $(0, W_U^{YM^*})$. (Proofs of Propositions 7 and 8 are presented in Appendix D.1 and D.2, respectively.)

Corollary 6. In Model YM, if partial collection is the best strategy, then some conclusions can be drawn as follows.

(i) When partial collection is the best strategy, we can conclude that $\frac{\partial Q_{2n}^{YM^*}}{\partial \Delta e} < 0$, $\frac{\partial Q_{2r}^{YM^*}}{\partial \Delta e} > 0$, $\frac{\partial P_{2n}^{YM^*}}{\partial \Delta e} > 0$, $\frac{\partial P_{2r}^{YM^*}}{\partial \Delta e} < 0$, $\frac{\partial W_{2n}^{YM^*}}{\partial \Delta e} > 0$, and $\frac{\partial W_{2r}^{YM^*}}{\partial \Delta e} < 0$.

(ii) If $0 < \tau < \frac{2(e_0\xi + \sqrt{aC_n\xi + {e_0}^2\xi^2})}{a}$, then with the increase in consumer preference for remanufactured product β, the minimum critical value of the carbon saving per unit remanufactured product ($\Delta e_L^{YM^*}$) will decrease; conversely, if $\tau > \frac{2(e_0\xi + \sqrt{aC_n\xi + {e_0}^2\xi^2})}{a}$, it will increase in β.

Therefore, when the carbon tax is low, consumer preference for remanufactured products can promote the manufacturer to produce more remanufactured products. When the carbon tax is high, consumer preference for remanufactured products is not conducive to the collection and remanufacture of products.

Table 8. Equilibrium optimal solutions in period 2 under Model YM

Optimal decision	No collection $W_{2r}^{YM^*} > W_L^{YM^*}$	Partial collection $\Delta e_L^{YM^*} \le \Delta e \le \Delta e_U^{YM^*}$	Full collection $W_{2r}^{YM^*} < W_U^{YM^*}$
$Q_{2n}^{YM^*}$	$\frac{\xi(a-C_n-e_0\tau)}{4\xi-\tau^2}$	$\frac{1}{Z}(\xi(4(C_n-C_r-P_m+a(\beta-1))\xi +4\Delta e\xi\tau+(C_r-2C_n+P_m-a(2-\beta))\tau^2-(e_0+\Delta e)\tau^3))$	$\frac{\xi(a-C_n-e_0\tau)(2\xi(2-\beta\psi)-\tau^2)}{(4\xi-\tau^2)^2}$
$e_2^{YM^*}$	$\frac{\tau(a-C_n-e_0\tau)}{4\xi-\tau^2}$	$\frac{1}{Z}(\tau(4(C_n-C_r-P_m+a(\beta-1))\xi+4\Delta e\xi\tau +(-2C_n+C_r+P_m+a(2-\beta))\tau^2-(e_0+\Delta e)\tau^3))$	$\frac{\tau(a-C_n-e_0\tau)(2\xi(2-\beta\psi)-\tau^2)}{(4\xi-\tau^2)^2}$
$W_{2n}^{YM^*}$	$\frac{2a\xi+2C_n\xi+2e_0\xi\tau-a\tau^2}{4\xi-\tau^2}$	$\frac{1}{2Z}\big(16(a+C_n)(\beta-1)\xi^2+16e_0(-1+\beta)\xi^2 + 4(3a+C_n+C_r+P_m-(2a+C_n)\beta)\xi\tau^2 + 4(e_0(2-\beta)+\Delta e)\xi\tau^3 + \big(C_n-C_r-P_m+a(-3+\beta)\big)\tau^4+\Delta e\tau^5\big)$	$\frac{1}{(4\xi-\tau^2)^2}(2\xi(C_n+e_0\tau)(2\xi(2+\beta\psi)-\tau^2(1+\beta\psi)) - a(2\xi-\tau^2)(\tau^2+2\xi(\beta\psi-2)))$
$P_{2n}^{YM^*}$	$\frac{3a\xi+C_n\xi+e0\xi\tau-a\tau^2}{4\xi-\tau^2}$	$\frac{1}{4Z}(16(3a+C_n)(\beta-1)\xi^2+16e_0(\beta-1)\xi^2\tau +4(7a+C_n+C_r+P_m-(4a+C_n)\beta)\xi\tau^2-4(e_0(\beta-2)+\Delta e)\xi\tau^3 +(C_n-C_r-P_m+a(\beta-5))\tau^4+\Delta e\tau^5)$	$\frac{1}{(4\xi-\tau^2)^2}(a(\tau^4-\xi\tau^2(7-\beta\psi)+2\xi^2(6-\beta\psi)) - \xi(C_n+e_0\tau)(\tau^2(1+\beta\psi)-2\xi(2 +\beta\psi)))$
$Q_{2r}^{YM^*}$	0	$\frac{1}{4\beta(4(\beta-1)\xi+\tau^2)}(4(C_r+P_m-C_n\beta)\xi-4(e_0(\beta-1)+\Delta e)\xi\tau -(a-C_n+C_r+P_m-a\beta)\tau^2+\Delta e\tau^3)$	$\frac{\psi\xi(a-C_n-e_0\tau)}{4\xi-\tau^2}$
$W_{2r}^{YM^*}$	N/A	$\frac{1}{2}(a-C_n+C_r+P_m+a\beta-\Delta e\tau-\frac{4\xi(a-C_n-e_0\tau)}{4\xi-\tau^2})$	W^{YM}
$P_{2r}^{YM^*}$	0	$\frac{1}{4}(a-C_n+C_r+P_m+3a\beta-\Delta e\tau-\frac{4\xi(a-C_n-e_0\tau)}{4\xi-\tau^2})$	$\frac{1}{(4\xi-\tau^2)^2}(\beta(4\xi-\tau^2)(3a\xi+C_n\xi+e_0\xi\tau-a\tau^2) +\beta\xi(a-C_n-e_0\tau)(2(\beta-2)\xi +\tau^2)\psi)$

4.3 Comparison of the models

In this section, we further compare the results regarding the triggering condition for remanufacturing before and after carbon reduction investment in different collection strategies.

Corollary 7. Under the decentralized CLSC, the following conclusions are drawn when the optimal decisions with and without carbon reduction investment are compared.

(i) A comparison of the maximum critical value of the wholesale price for full collection under models NM and YM reveals that when $0 < \tau < 2\sqrt{2}\sqrt{\frac{\xi(1+\psi-\beta\psi)}{2+(2-\beta)\psi}}$, the maximum critical value of the wholesale price for full collection with a carbon reduction investment is higher than that without carbon reduction investment. Thus, $W_U^{NM^*} > W_U^{YM^*}$. Conversely, when the carbon tax $\tau > 2\sqrt{2}\sqrt{\frac{\xi(1+\psi-\beta\psi)}{2+(2-\beta)\psi}}$, $W_U^{NM^*} < W_U^{YM^*}$.

This result demonstrates that when the carbon tax is high, it is easier for the manufacturer and retailer to reach an agreement on full collection by investing in carbon reduction technology to improve their profit and reduce carbon emissions. When the carbon tax is low, they are better off without investment in carbon reduction technology.

(ii) The minimum critical values of carbon saving Δe for remanufacturing with carbon reduction investment are lower than those without carbon reduction investment under two CLSC structures. This finding indicates that the manufacturer will be more easily motivated to engage in remanufacturing by investing in carbon reduction.

Corollary 8. The minimum critical value of carbon saving per unit remanufactured product Δe in the second period is smaller for Model YC than for Model NC.

Corollaries 7 and 8 demonstrate that carbon reduction technology investment is more effective in motivating the supply chain to engage in remanufacturing and promote the development of the remanufacturing industry.

5. Numerical examples

In this section, numerical examples are used to investigate comparatively the effects of carbon tax (τ) and consumer preference for remanufactured products (β) on the optimal decisions and profits of supply chain members in the proposed models. We start from adopting the parameter setting suggested by Chang et al. (2017) and combining it with the data by investigating the manufacturers in China and the actual situation in practice, $\alpha = 1000$, $\psi = 0.6$. According to the problem description and analysis above, the unit production cost of the new product is higher than that of the remanufactured product; referring to the studies of Wang and Chen, (2017), $C_n = 30$ and $C_r = 10$. Without loss of generality, manufacturers must make a profit by collecting and remanufacturing, so the collection price $\mathrm{P}_m = \mathrm{P}_{tc} = 10$. In addition, we know that the carbon emission per unit of the new product is higher than that of the remanufactured product, so $e_0 = 150,\ \Delta e = 130$. ξ is the carbon reduction cost factor, and according to Yang et al. (2017), we assume $\xi = 4$. Finally, note that the other parameters involved in the models will be determined around their value ranges, such as carbon tax and consumer preference for remanufactured product. To simplify the case and conform to the feasible region obtained from the earlier analysis, the condition $\tau^2 - 2(1-\beta)\xi < 0$ must be satisfied.

5.1 Effect of carbon tax on manufacturing/remanufacturing decisions

In this subsection, we analyze the effects of carbon tax on the equilibrium decisions of supply chain members. Here we consider that $\beta(\beta = 0.5)$ is fixed. The results are shown in Figs. 5–6 and Table 9.

Figure 5 shows the influence of carbon tax on the minimum critical values of carbon saving per remanufactured product, which triggers remanufacturing operations with different models. The following insights are revealed: (i) with the increase of carbon tax, the minimum critical value of carbon saving per unit remanufactured product for manufacturing decreases; and (ii) as Corollary 8 analyzed, the minimum critical values of carbon saving Δe for remanufacturing with carbon reduction investment are lower than those without a carbon reduction investment condition under centralized and decentralized CLSC. The result indicates that the manufacturer will be motivated more easily to engage in remanufacturing by investing in carbon reduction technology.

Figure 6 illustrates the impact of carbon tax on the critical value of wholesale price in the no collection (the letter “N” stands for no collection) and full collection (the letter “F” stands for no collection) scenarios under models NM and YM, respectively. It indicates that carbon tax is more effective in motivating the manufacturer to engage in remanufacturing.

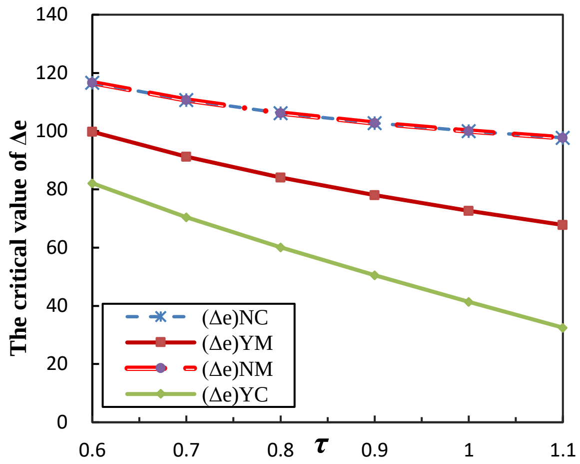

Fig. 5. Effect of τ on the minimum critical value of Δe for remanufacturing

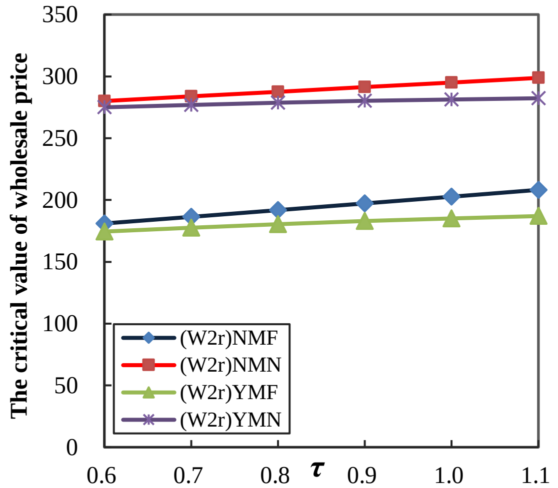

Fig. 6. Effect of τ on the critical value of wholesale price for full collection and no collection strategy

Table 9. Comparison of the optimal decisions with different models

	$\tau = 0.6$	$\tau = 0.7$	$\tau = 0.8$	$\tau = 0.9$	$\tau = 1.0$	$\tau = 1.1$
$Q_{2r}^{NC^*}$	16.00	27.00	38.00	49.00	60.00	71.00
$Q_{2r}^{YC^*}$	60.31	89.26	122.57	161.23	206.67	260.96
$Q_{2r}^{NM^*}$	8.00	13.50	19.00	24.50	30.00	35.50
$Q_{2r}^{YM^*}$	18.55	28.05	38.30	49.40	61.43	74.53
$Q_{2n}^{NC^*}$	432.00	419.00	406.00	393.00	380.00	367.00
$Q_{2n}^{YC^*}$	429.16	413.18	395.34	374.84	350.48	320.50
$Q_{2n}^{NM^*}$	216.00	209.50	203.00	196.50	190.00	183.50
$Q_{2n}^{YM^*}$	215.57	208.61	201.40	193.87	185.90	177.40
$e_2^{YC^*}$	64.37	72.31	79.07	84.34	87.62	88.14
$e_2^{YM^*}$	32.34	36.51	40.28	43.62	46.48	48.79
E^{NC^*}	131120	128265	125410	122555	119700	116845
E^{YC^*}	71054	68651	55781	42112	37182	21303
E^{NM^*}	65560	64133	62705	61277	59850	58421
E^{YM^*}	51271	47895	44571	41271	37962	34606
$P_{2r}^{NC^*}$	276.00	277.00	278.00	279.00	280.00	281.00
$P_{2r}^{YC^*}$	255.27	248.78	241.04	231.97	221.43	209.27
$P_{2r}^{NM^*}$	388.00	388.50	389.00	389.50	390.00	390.50
$P_{2r}^{YM^*}$	382.94	381.67	380.15	378.37	376.33	374.04
$W_{2n}^{NM^*}$	560.00	567.50	575.00	582.50	590.00	597.50
$W_{2n}^{YM^*}$	550.30	554.72	558.89	562.87	566.76	570.67
$W_{2r}^{NM^*}$	276.00	277.00	278.00	279.00	280.00	281.00
$W_{2r}^{YM^*}$	265.87	263.34	260.29	256.74	252.67	248.07

Table 9 shows the effects of carbon tax on the optimal decisions under the partial collection strategy of new and remanufactured products in the second period under different models. From the table, we can conclude the following insights: (1) Whether investing in carbon reduction technology or not, with the increase of carbon tax, the ordering quantity for remanufactured products increases and the retail price decreases accordingly; the ordering quantity for new products decreases, and the retail price increases. (2) When investment is made in carbon reduction technology, the quantity of new and remanufactured products in the second period are increased, the retail prices are decreased, and in turn the carbon emission is decreased. With carbon emission reduction technology investment, when the carbon tax rate is low, the carbon emission in the case of centralized CLSC is higher than that in the case of decentralized CLSC. On the contrary, when the carbon tax is high, the carbon emission under centralized CLSC is lower than that under decentralized CLSC. (3) In the decentralized CLSC, with the increase of carbon tax, the wholesale price of new products will increase regardless of the carbon reduction technology investment. However, through investing in carbon reduction technology, the wholesale price of remanufactured products will decrease accordingly. This finding suggests that investment in carbon reduction technologies can encourage manufacturers to produce more remanufactured products to reduce carbon emissions of the CLSC.

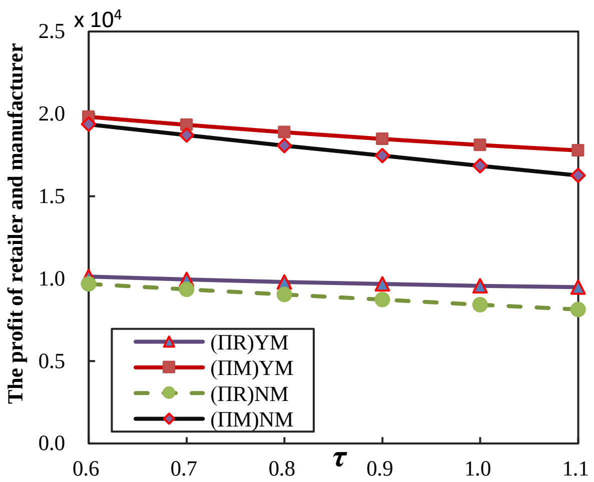

Fig. 7. Effect of carbon tax on the profits of the manufacturer and retailer

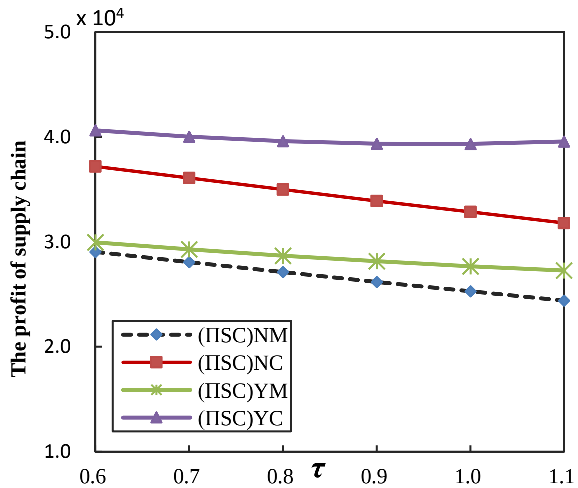

Fig. 8. Effect of carbon tax on the profits of the supply chain

Meanwhile, we compare the Π_{SC}, Π_{M}, and Π_{R} under four different models through numerical examples. From Figs. 7–8, we can conclude that with the increase of τ, the profits of the supply chain and the supply chain members decrease, and the profit of the manufacturer is far more than that of the retailer. We also observe that the supply chain can obtain more profit with the carbon reduction investment compared with the no carbon reduction investment.

From Fig. 8 and Table 9, we can also find that the centralized model can achieve a higher total profit than the decentralized model. However, the carbon emission on the CLSC is higher than that in the decentralized model when carbon tax is low.

5.2 Impact of β on manufacturing and remanufacturing decisions

Figures 9–11 show the impact of customer preference on remanufactured products on the optimal decisions of different models where $\tau = 1.25$

Figure 9 illustrates the impact of consumers' remanufactured preference coefficient β on the critical value of carbon saving per unit remanufactured product (Δe), which triggers remanufacturing in different models. Here, the carbon tax satisfies $0 < \tau < \frac{2(e_0\xi+\sqrt{aC_n\xi+e_0^2\xi^2})}{a}$, and it is obvious that with the increase of β, the minimum critical value of Δe decreases. (The results of the numerical examples are consistent with Corollaries 3 and 6.) When the manufacturer does not invest in carbon reduction technology, there is the same critical value of carbon savings that triggers remanufacturing under the centralized and decentralized CLSC. However, with the carbon emission reduction investment, the critical value of carbon emission saving per unit remanufactured product is less than that in the case of no carbon reduction investment. This outcome shows that consumer preference for remanufactured products and carbon reduction investment can promote the manufacturer to produce more remanufactured products.

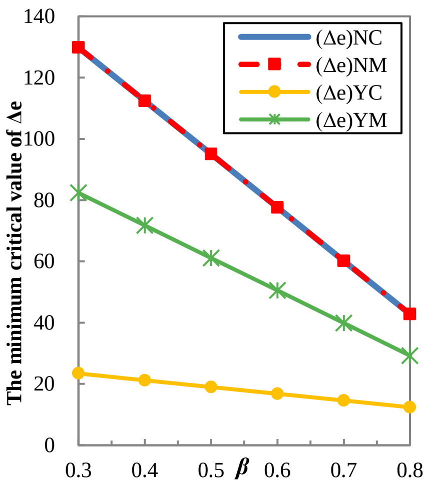

Fig. 9. Effect of β on the minimum critical value of Δe for remanufacturing

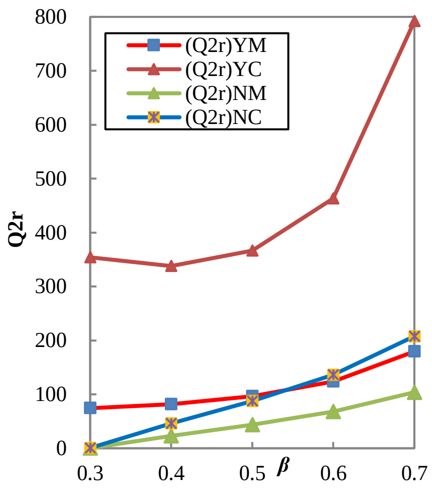

Fig. 10. Effect of β on Q_{2r}

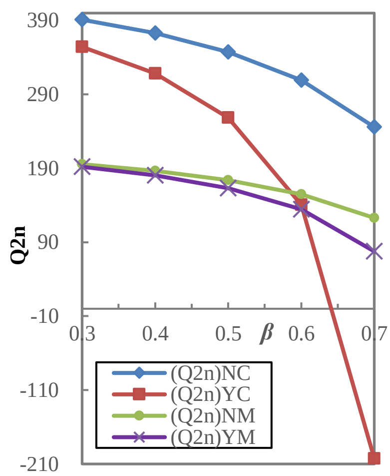

Fig.11. Effect of β on Q_{2n}

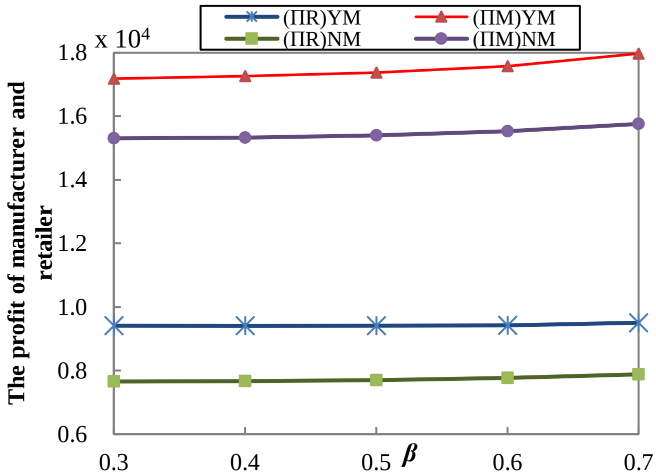

Fig. 12. Effect of β on the profits of the manufacturer and retailer

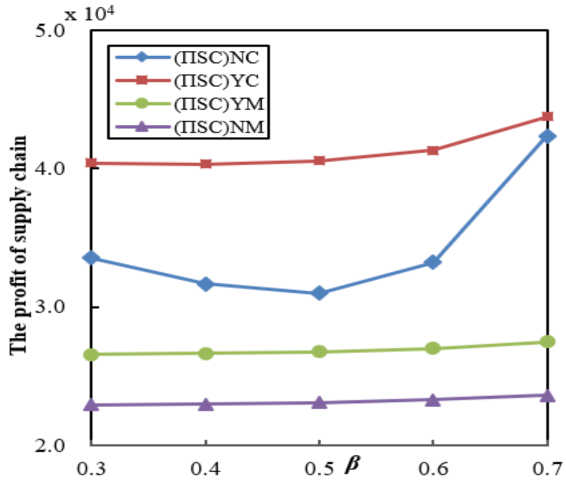

Fig.13. Effect of β on the profit of the supply chain

Figures 10–11 illustrate the comparison results of Q_{2n} and Q_{2r} under partial collection strategy under the four different decision models in the second period. With the increase of β, the quantity of new products decreases while that of remanufactured products increases. In particular, when the manufacturer invests in carbon reduction technologies, the number of new products decreases while the number of remanufactured products increases dramatically.

Figure 12 illustrates the comparison results of the supply chain members' profits in the case of a partial collection strategy in decentralized models before and after carbon reduction investment. Clearly, with the increase of β, the profit of the supply chain members increases. By investing in carbon reduction technology, the manufacturer and retailer can also obtain more profits than in the no carbon reduction investment.

Figure 13 illustrates the influence of β on the profit of the supply chain in the different models. It is obvious that in the centralized model, when there is no carbon reduction technology investment, the supply chain profit initially decreases and then increases with the increase of β. The reason is that when β is relatively small ($\beta \leq 0.5$), the supply chain will suffer a loss in profit because of the smaller quantity of remanufactured products and the sharp drop of new products. When β is large enough, more remanufactured products are produced because of the carbon and cost saving advantages of the remanufactured products; accordingly, the profits of the supply chain can be improved. However, under the carbon reduction investment condition, the profit of the supply chain increases with the increase of β. We also find that with a carbon reduction investment, the supply chain can gain more profits than without a carbon reduction investment. Without a loss of generality, the profit of a centralized supply chain is larger than that of a decentralized decision.

From the earlier analysis, it can be predicted that the consumer preference for remanufactured products will effectively urge manufacturers to produce more remanufactured products to reduce carbon emission.

6. Managerial implications

Based on the above theoretical and numerical analyses, we present some managerial implications for the government and CLSC.

6.1 Government perspective

Carbon tax policies should be designed appropriately, as the carbon tax is a double-edged sword. When such policies are not well-designed, they may demotivate manufacturers from remanufacturing, thereby increasing carbon emissions. Hence, policymakers should design different carbon tax policies according to industry to promote remanufacturing. Our results suggest that the carbon tax be high for products with high carbon savings during the remanufacturing phase (e.g., automobiles, refrigerators, air conditioners, and other high-energy-consuming products). By contrast, if carbon savings are low during the remanufacturing stage, such as in laptops and mobile phones, then the carbon tax should be relatively low.

In view of this phenomenon, the government can guide the implementation of carbon tax policies from the following aspects.

(1) A series of measures, such as tax reliefs, tax returns, and emissions reduction agreements, should be implemented to reduce carbon emissions without impacting economic effects. Several countries have taken such measures. For example, in Finland, the electricity industry as well as the raw materials used in the industry and fuel for international transportation are exempt from taxes. In Denmark, a 50% carbon tax rebate is given to companies that pay carbon taxes, and preferential tax rates are offered to high-energy-consuming enterprises that sign the voluntary emissions reduction agreement (Liu, 2016). In the United Kingdom, a "climate change levy" can be paid at "carbon price support rates" to encourage industries to use low-carbon technologies for electricity production (https://www.gov.uk/green-taxes-and-reliefs/climate-change-levy, accessed on April 26, 2021). The European Commission adopted the "European Green Deal" in 2019 with a twofold aim, that is, "economic growth" and "net-zero greenhouse gas emissions by 2050." As part of the deal, the EU will revise the Energy Taxation Directive (ETD) to accurately reflect the climate impact of various energy

sources and encourage consumers and businesses to change their behavior. The revised ETD is predicted to be in favor of remanufacturing operations, as manufacturing is a heavy-natural-resource-consuming sector; however, through remanufacturing, manufacturers can reduce raw material consumption by 70%, CO_2 emissions by 80%, and energy consumption by 60% (Wang and Hazen, 2016).

(2) One of the challenges in remanufacturing is the shortage of cores (i.e., used-product returns), which can hinder profitability. The government can implement taxes, regulations, and laws to promote remanufacturing. For instance, according to the “Motor Industry of Japan 2019” report, Japan implemented the end-of-life (EOL) vehicle recycling law in 2005, thereby standardizing the recycling rate for automobile shredder residue to 30%, which was amended to 50% in 2010 and to 70% in 2015. This law requires car owners to pay an environment tax upon car purchase for EOL recycling, which will be refunded upon the vehicle’s return to a designated remanufacturer at its end of use.

(3) The government can advocate low-carbon consumption and cultivate low-carbon preference among consumers through taxes or subsidies for remanufactured products. If the number of consumers with low-carbon preference increases, then demand for low-carbon products, including remanufactured products, may also increase. This market will in turn motivate manufacturers to introduce remanufacturing or invest in carbon reduction technologies. The research results show that according to the Office of Best Practice Regulation of the Australian Government (p. 14), the introduction of taxes or subsidies for remanufactured products will alter prices and consumer demand. In the United Kingdom, numerous tax incentives, grants, and subsidies are available to manufacturers to accelerate the development and implementation of environmental technologies. Specifically, loss-making high-tech (re-)manufacturers can surrender their losses and receive a cash payment of up to 33% (Findlay, 2021). Such incentives can ultimately reduce the cost of remanufactured products, thereby increasing market demand.

A summary of the policy implications is presented in Table 10.

Table 10. Summary of policy implications

	Measures	*Practical implications*
Government	Tax relief, tax returns, participation in voluntary emission reduction agreement, carbon emission reduction fund.	Encourage enterprises to implement carbon reduction technology investment, remanufacturing.
	Environmental tax, regulation and law enforce together.	Encourage return of used products, hence, increase remanufacturing profitability.
	Tax incentives, grants and subsidies to remanufacturers and remanufactured products.	Advocate low-carbon consumptions and cultivate low-carbon preferences among consumers, increasing market demand for remanufactured products.

6.2 Closed loop supply chain perspective

A carbon tax policy will inevitably change the operation mode of a closed-loop supply chain; thus, enterprises must adopt different carbon reduction and collection strategies. Based on the research results, we present the following suggestions.

(1) Generally, investing in carbon reduction technology is a favorable strategy. This type of investment would be beneficial to high-energy-consuming manufacturers and high-carbon-saving manufacturing. Such manufacturers and remanufacturers will be able to observe immediate investment returns. When carbon savings from remanufacturing are small, such investments can result in long-term economic and environmental benefits.

(2) The CLSC structure plays a key role in profitability. Intuitively, a centralized CLSC structure is more profitable than a decentralized structure. However, from an environmental perspective, a decentralized structure can outperform a centralized structure when the carbon tax is low. Hence, manufacturers can engage in a tradeoff between profitability and environmental impact.

(3) The collection and remanufacturing of used products at the end of the first period can generate more profits than if such operations are performed during the second period. However, if the unit carbon savings from remanufacturing are small, then partial collection would be ideal.

(4) Under a decentralized CLSC structure, a manufacturer, as a leader, will have the power to control collection and wholesale prices, which can help improve his/her profits. Therefore, the profit gained by the manufacturer will be higher than that gained by the retailer. Supply chain members can maximize their benefits through carbon reduction cost sharing and wholesale price contracts.

A summary of the practical guidance is shown in Table 11.

Table 11. Summary of practical guidance

<table>
<tr><th colspan="3" rowspan="2">Scenario</th><th colspan="2">Decision Strategies</th><th colspan="2">Effect</th></tr>
<tr><th>Carbon reduction investment</th><th>Collection strategy</th><th>Profit</th><th>Environment effect</th></tr>
<tr><td rowspan="4">High carbon tax</td><td rowspan="2">High carbon saving</td><td>Centralized CLSC</td><td rowspan="2">Yes</td><td>Full or partial</td><td>More profitable than decentralized CLSC.</td><td rowspan="4">(1) With carbon investment: the carbon emission under centralized CLSC is lower than decentralized CLSC.
(2) Without carbon investment: the carbon emission under centralized CLSC is higher than decentralized CLSC.</td></tr>
<tr><td>Decentralized CLSC</td><td>Full collection</td><td>The profit gained by the manufacturer is more than that of retailer.</td></tr>
<tr><td rowspan="2">Low carbon saving</td><td>Centralized CLSC</td><td rowspan="2">Yes</td><td>Partial or no collection</td><td>More profitable than decentralized CLSC.</td></tr>
<tr><td>Decentralized CLSC</td><td>Partial collection</td><td>The profit gained by the manufacturer is more than that of retailer.</td></tr>
<tr><td rowspan="4">Low carbon tax</td><td rowspan="2">High carbon saving</td><td>Centralized CLSC</td><td>No</td><td>Full or partial collection</td><td>More profitable than decentralized CLSC.</td><td rowspan="4">The carbon emission under centralized CLSC is higher than decentralized CLSC regardless of whether to invest in carbon emission reduction technology.</td></tr>
<tr><td>Decentralized CLSC</td><td>No</td><td>Partial collection</td><td>The profit gained by the manufacturer is more than that of retailer.</td></tr>
<tr><td rowspan="2">Low carbon saving</td><td>Centralized CLSC</td><td>Yes</td><td>Partial or no collection</td><td>More profitable than decentralized CLSC.</td></tr>
<tr><td>Decentralized CLSC</td><td>Yes</td><td>No collection</td><td>The profit gained by the manufacturer is more than that of retailer.</td></tr>
</table>

7. Conclusion

In both centralized and decentralized CLSC, whether or not investment is made in carbon reduction technology will greatly affect the manufacturing/remanufacturing decisions. Given three different collection strategies (no collection, partial collection, and full collection) and the responsibilities of supply chain members, determining how to coordinate different members and optimize their manufacturing/remanufacturing decisions under carbon tax is a great challenge.

Considering whether to invest in carbon reduction technology in centralized or decentralized CLSC, we develop four game models to study the remanufacturing/manufacturing strategies under carbon tax policy. We obtain the manufacturing and remanufacturing decisions in each model. Furthermore, we compare the optimal manufacturing/remanufacturing decisions with and without carbon reduction investment. Finally, we analyze the impacts of carbon tax and consumer's remanufactured preference on manufacturing/remanufacturing decisions of the supply chain. The key findings of our research are summarized as follows.

Carbon tax can effectively promote manufacturers to invest in carbon reduction technology or remanufacture to reduce carbon emissions. However, it may demotivate the CLSC to remanufacture if a reasonable carbon tax is not designed. In the centralized CLSC, full collection strategy is adopted when the unit carbon saving or carbon tax is higher. In the decentralized CLSC, no collection or full collection decision only depends on the wholesale price between the manufacturer and retailer rather than on the carbon saving of remanufactured products.

There are several topics for further research. In this study, we only analyze manufacturing/remanufacturing decisions in a CLSC under the carbon tax policy. However, the decision may be quite different under the carbon cap and trade policy. Thus, some studies about this topic can be studied in the future. The coordination of CLSC under the carbon reduction policy will likewise be examined extensively, for example, revenue sharing contract, price subsidy strategy, and so on.

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Appendix:

Appendix A: Model NC

A.1 Proof of Proposition 1

Proof: From the former analysis, the profit of the supply chain in the two periods is expressed as follows:

$$Max\Pi_{\mathrm{SC}}^{\mathrm{NC}}(Q_{1n}^{NC},Q_{2n}^{NC},Q_{2r}^{NC}) = (P_{1n}^{NC} - \mathrm{C}_n)Q_{1n}^{NC} + (P_{2n}^{NC} - \mathrm{C}_n)Q_{2n}^{NC} + (P_{2r}^{NC} - \mathrm{C}_r - \mathrm{P}_{\mathrm{tc}})Q_{2r}^{NC} - \tau E^{NC} \qquad \text{(A.1)}$$

$$\text{s.t} \begin{cases} E^{NC} = [\mathrm{e}_0(Q_{1n}^{NC} + Q_{2n}^{NC}) + (\mathrm{e}_0 - \Delta e)Q_{2r}^{NC}] \\ Q_{2r}^{NC} \leq \psi Q_{1n}^{NC} \\ Q_{1n}^{NC} \geq 0, Q_{2n}^{NC} \geq 0, Q_{2r}^{NC} \geq 0 \end{cases}$$

In the first period, by the second-order derivatives of Eq. (A.1) with respect to Q_{1n}^{NC} for the optimal solution, $\frac{\partial^2 \Pi_{\mathrm{SC}}^{\mathrm{NC}}}{\partial {Q_{1n}^{NC}}^2} = -2 < 0$. Thus, the objective function is concave in Q_{1n}^{NC}. In addition, by solving the first-order conditions of $\Pi_{\mathrm{SC}}^{\mathrm{NC}}$ with respect to Q_{1n}^{NC}, we can obtain the optimal decisions $Q_{1n}^{NC^*} = \frac{1}{2}(a - C_n - e_0\tau)$, $P_{1n}^{NC^*} = \frac{1}{2}(a + C_n + e_0\tau)$. Proposition 1 is thus proven.

A.2 Proof of Proposition 2

In the second period, substituting $Q_{1n}^{\mathrm{NC}^*}$ and $P_{1n}^{\mathrm{NC}^*}$ into Eq. (A.1) and taking the second-order derivatives of $\Pi_{\mathrm{SC}}^{\mathrm{NC}}$ with respect to Q_{2n}^{NC} and Q_{2r}^{NC}, the Hessian matrix is $\mathrm{H}(Q_{2n}^{NC}, Q_{2r}^{NC}) = \begin{pmatrix} -2 & -2\beta \\ -2\beta & -2\beta \end{pmatrix} = 4\beta(1-\beta)$; note that $0 < \beta < 1$. Thus, the Hessian matrix is negative definite and the profit function is concave in $(Q_{2n}^{NC}, Q_{2r}^{NC})$. Then by the KKT conditions, the Lagrangian function in the second period can be written as follows:

$$\mathcal{L}(Q_{2n}^{NC}, Q_{2r}^{NC}, \lambda_1, \lambda_2) = Q_{2r}^{NC}(P_{2r}^{NC} - C_r - P_{tc}) + Q_{2n}^{NC}(P_{2n}^{NC} - C_n) - (Q_{2r}^{NC}(e_0 - \Delta e)$$
$$+ \mathrm{e}_0 * Q_{2n}^{NC})\tau + \lambda_2 Q_{2r}^{NC} + \lambda_1(\psi Q_{1n}^{NC} - Q_{2r}^{NC}) \qquad \text{(A.2)}$$

By setting

$$\frac{\partial \mathcal{L}}{\partial Q_{2n}^{NC}} = a - C_n - 2Q_{2n}^{NC} - 2Q_{2r}^{NC}\beta - e_0\tau = 0 \qquad \text{(A.3)}$$

$$\frac{\partial \mathcal{L}}{\partial Q_{2r}^{NC}} = -C_r - P_{tc} - Q_{2n}^{NC}\beta + (a - Q_{2n}^{NC} - Q_{2r}^{NC})\beta - Q_{2r}^{NC}\beta - \lambda_1 + \lambda_2 - (e_0 - \Delta e)\tau = 0 \qquad \text{(A.4)}$$

$$\lambda_1(-Q_{2r}^{NC} + \psi Q_{1n}^{NC}) = 0 \qquad \text{(A.5)}$$

$$\lambda_2 Q_{2r}^{NC} = 0 \qquad \text{(A.6)}$$

$$\lambda_1, \lambda_2 \geq 0 \qquad \text{(A.7)}$$

Case 1: $\lambda_1 = \lambda_2 = 0$, from Eqs. (A.5) and (A.6), $Q_{2r}^{NC} \geq 0$ and $Q_{2r}^{NC} \leq \psi Q_{1n}^{NC}$. Combining Eqs. (A.3) and (A.4), we can obtain

$$Q_{2n}^{NC^*}=\frac{a-C_n+C_r+P_{tc}-a\beta-\Delta e\tau}{2(1-\beta)},\quad Q_{2r}^{NC^*}=\frac{C_n\beta-e_0\tau+e_0\beta\tau+\Delta e\tau-C_r-P_{tc}}{2(1-\beta)\beta}$$

$$\psi Q_{1n}^{NC}-Q_{2r}^{NC}\geq 0\Rightarrow \Delta e\leq\frac{C_r+P_{tc}+e_0\tau-\beta(C_n+e_0\tau)+(1-\beta)\beta(a-C_n-e_0\tau)\psi}{\tau}=\Delta e_U^{NC^*}$$

$$Q_{2r}^{NC^*}\geq 0\Rightarrow \Delta e\geq\frac{C_r+P_{tc}+e_0\tau-\beta(C_n+e_0\tau)}{\tau}=\Delta e_L^{NC^*}$$

Here $\Delta e_U^{NC^*}-\Delta e_L^{NC^*}>0$, so the carbon saving per unit remanufactured product Δe satisfies the condition $\Delta e_L^{NC^*}\leq\Delta e\leq\Delta e_U^{NC^*}$. In this case, the optimal price of new and remanufactured products and the profits of the supply chain in two periods can be obtained and shown in the second column of Table 3.

Case 2: $\lambda_1>0,\lambda_2=0$, then $Q_{2r}^{NC}=\psi Q_{1n}^{NC}$, $Q_{2r}^{NC}\geq 0$. Combining Eqs. (A.3) and (A.4), we can obtain $Q_{2n}^{NC^*}=\frac{1}{2}(a-\mathrm{C_n}-\mathrm{e_0}\tau)(1-\beta\psi)$.

$$\lambda_1=\frac{C_r+P_{tc}+e_0\tau-\beta(C_n+e_0\tau)+(1-\beta)\beta(a-C_n-e_0\tau)\psi}{\tau}$$

$$\lambda_1>0\Rightarrow\Delta e>\frac{C_r+Ptc+e_0\tau-\beta(C_n+e_0\tau)+(1-\beta)\beta(a-C_n-e_0\tau)\psi}{\tau}=\Delta e_U^{NC^*}$$

$$Q_{2r}^{NC^*}=\frac{1}{2}(a-C_n-e_0\tau)\psi>0$$

Similarly, the retail price of the new and remanufactured products and the profit of the supply chain can be obtained and shown in the third column of Table 3.

Case 3: $\lambda_1=0,\lambda_2>0$, then $Q_{2r}^{NC}\leq\psi Q_{1n}^{NC}$, $Q_{2r}^{NC}=0$. Combining Eqs. (A.3) and (A.4), we can obtain

$$Q_{2n}^{NC^*}=\frac{1}{2}(a-C_n-e_0\tau)$$

$$\lambda_2=C_r+P_{tc}-C_n\beta+e_0\tau-e_0\beta\tau-\Delta e\tau$$

$$\lambda_2>0\Rightarrow\Delta e<\frac{C_r+P_{tc}+e_0\tau-\beta(C_n+e_0\tau)}{\tau}=\Delta e_L^{NC^*}$$

Similarly, the other optimal decisions can be obtained and shown in the first column of Table 3. Thus, Proposition 2 is proven.

Appendix B: Model YC

B.1 Proof of Proposition 3

Proof: Recall the profit-maximization function of the supply chain:

$$Max\Pi_{\mathrm{SC}}^{\mathrm{YC}}\left(Q_{1n}^{YC},Q_{2n}^{YC},Q_{2r}^{YC},e_1^{YC},e_2^{YC}\right)=\left(P_{1n}^{YC}-\mathrm{C}_n\right)Q_{1n}^{YC}+\left(P_{2n}^{YC}-\mathrm{C}_n\right)Q_{2n}^{YC}+\left(P_{2r}^{YC}-C_r-P_{tc}\right)Q_{2r}^{YC}$$
$$-\mathrm{C}(e_1^{YC},e_2^{YC})-\tau E^{YC}\qquad\text{(B.1)}$$

$$\text{s.t}\begin{cases}E^{YC}=(\mathrm{e_0}-e_1^{YC})Q_{1n}^{YC}+(\mathrm{e_0}-e_2^{YC})Q_{2n}^{YC}+(\mathrm{e_0}-e_1^{YC}-\Delta e)Q_{2r}^{YC}\\ C(e_1^{YC},e_2^{YC})=\frac{1}{2}\xi(e_1^{YC})^2+\frac{1}{2}\xi(e_2^{YC})^2\\ \psi Q_{1n}^{YC}\geq Q_{2r}^{YC}\\ Q_{1n}^{YC}\geq 0,Q_{2n}^{YC}\geq 0,Q_{2r}^{YC}\geq 0\end{cases}$$

In the first period, the first and second derivatives to the decision variables Q_{1n}^{YC} and e_1^{YC} are as follows: $\frac{\partial \Pi_{SC}^{YC}}{\partial e_1^{YC}} = -\xi e_1^{YC} + Q_{1n}^{YC}\tau$ *and* $\frac{\partial \Pi_{SC}^{YC}}{\partial Q_{1n}^{YC}} = a - C_n - 2Q_{1n}^{YC} - (e_0 - e_1^{YC})\tau$.

The second derivatives to the decision variables Q_{1n}^{YC} and e_1^{YC} are as follows: $\frac{\partial^2 \Pi_{\mathrm{SC}}^{\mathrm{YC}}}{\partial Q_{1n}^{YC^2}} = -2$, $\frac{\partial^2 \Pi_{\mathrm{SC}}^{\mathrm{YC}}}{\partial e_1^{YC^2}} = -\xi$, $\frac{\partial^2 \Pi_{\mathrm{SC}}^{\mathrm{YC}}}{\partial Q_{1n}^{YC}\partial e_1^{YC}} = \frac{\partial^2 \Pi_{\mathrm{SC}}^{\mathrm{YC}}}{\partial e_1^{YC}\partial Q_{1n}^{YC}} = \tau$ and then the Hessian matrix is $\mathrm{H}\left(Q_{1n}^{YC}, e_1^{YC}\right) = \begin{pmatrix} -2 & \tau \\ \tau & -\xi \end{pmatrix} = 2\xi - \tau^2$. Clearly, the Hessian matrix is a negative definite and strictly joint concave in (Q_{1n}^{YC}, e_1^{YC}) if relevant coefficients satisfy $2\xi - \tau^2 > 0$. By solving the first-order conditions of $\Pi_{\mathrm{SC}}^{\mathrm{YC}}$, we can obtain $e_1^{YC^*} = \frac{\xi(a-C_n-e_0\tau)}{2\xi-\tau^2}$, $Q_{1n}^{YC^*} = \frac{\xi(a-C_n-e_0\tau)}{2\xi-\tau^2}$, and accordingly, the retail price $P_{1n}^{YC^*} = \frac{\xi(a-C_n-e_0\tau)}{2\xi-\tau^2}$. Proposition 3 is thus proven.

B.2 Proof of Proposition 4

In the second period, substituting $Q_{1n}^{YC^*}$, $e_1^{YC^*}$, and $P_{1n}^{\mathrm{YC}^*}$ into Eq. (1) and taking the second-order derivatives of Π_{SC}^{YC} with respect to e_2^{YC}, Q_{2n}^{YC}, and Q_{2r}^{YC}, the Hessian matrix is $\mathrm{H}(Q_{2n}^{YC}, Q_{2r}^{YC}, e_2^{YC}) = \begin{pmatrix} -2 & -2\beta & \tau \\ -2\beta & -2\beta & 0 \\ \tau & 0 & -\xi \end{pmatrix} = 2\beta(2\beta\xi + \tau^2 - 2\xi)$. Note that $0 < \beta < 1$; thus, if the condition $-2\xi + 2\beta\xi + \tau^2 < 0$, then Hessian matrix H is negative definite and the profit function is concave in $(Q_{2n}^{YC}, Q_{2r}^{YC})$. Thus, based on the KKT conditions, the Lagrangian function can be written as follows:

$$\mathcal{L}(Q_{2n}, Q_{2r}, \lambda_1, \lambda_2) = (P_{2n}^{YC} - \mathrm{C}_n)Q_{2n}^{YC} + (P_{2r}^{YC} - \mathrm{C}_r - \mathrm{P_{tc}})Q_{2r}^{YC} - \frac{1}{2}\xi e_1^{YC^2} - \tau[(\mathrm{e}_0 - e_2^{YC})Q_{2n}^{YC} + (\mathrm{e}_0 - e_1^{YC} - \Delta e)Q_{2r}^{YC}]$$

$$+\lambda_1(-Q_{2r}^{YC} + Q_{1n}^{YC}) + \lambda_2 Q_{2r}^{YC} \quad \text{(B.2)}$$

By setting

$$\frac{\partial \mathcal{L}}{\partial Q_{2n}^{YC}} = a - \mathrm{C}_n - 2Q_{2n}^{YC} - 2Q_{2r}^{YC}\beta + \lambda_2 - (e_0 - e_2^{YC}) \quad \text{(B.3)}$$

$$\frac{\partial \mathcal{L}}{\partial Q_{2r}^{YC}} = -a + C_n - C_r - P_{tc} + a\beta - 2Q_{2n}^{YC}\beta - 2Q_{2r}^{YC}\beta - \lambda_1 + \lambda_2 + \Delta e\tau + \frac{2\xi(-a+C_n+e_0\tau)}{\tau^2-2\xi} = 0 \quad \text{(B.4)}$$

$$\frac{\partial \mathcal{L}}{\partial e_2^{YC}} = -\xi e_2^{YC} + Q_{2n}^{YC}\tau = 0 \quad \text{(B.5)}$$

$$\lambda_1\left(Q_{1n}^{YC} - Q_{2r}^{YC}\right) = 0 \quad \text{(B.6)}$$

$$\lambda_2 Q_{2r}^{YC} = 0 \quad \text{(B.7)}$$

$$\lambda_1, \lambda_2 \geq 0 \quad \text{(B.8)}$$

Case 1: $\lambda_1 = \lambda_2 = 0$, from Eqs. (B.6) and (B.7), $Q_{2r} \geq 0$ and $Q_{2r}^{YC} \leq \psi Q_{1n}^{YC}$. Combining Eqs. (B.3), (B.4), and (B.5), we can obtain

$$Q_{2n}^{YC^*} = (\xi(2(C_n - C_r - P_{tc} + a(-1+\beta))\xi + 2\Delta e\xi\tau + (-2C_n + C_r + P_{tc} - a(-2+\beta))\tau^2 - (e_0 + \Delta e)\tau^3))$$

$$Q_{2r}^{YC^*} = \frac{1}{2\beta(\tau^2 - 2\xi + 2\beta\xi)}(2(C_r + P_{tc} - C_n\beta)\xi - 2(e_0(-1+\beta) + \Delta e)\xi\tau - (a - C_n + C_r + P_{tc} - a\beta)\tau^2 + \Delta e\tau^3)$$

$$e_2^{YC^*} = \frac{1}{(2\xi - \tau^2)(2(-1+\beta)\xi + \tau^2)}\tau(2(C_n - C_r - P_{tc} + a(-1+\beta))\xi + 2\Delta e\xi\tau + (-2C_n + C_r + P_{tc} - a(-2+\beta))\tau^2 - (e_0 + \Delta e)\tau^3)$$

$$\psi Q_{1n}^{YC^*} - Q_{2r}^{YC^*} \geq 0 \Rightarrow \Delta e \leq \frac{1}{\tau(2\xi-\tau^2)^2}\left((2\xi - \tau^2)A + 2\beta\xi(a - \mathrm{C}_n - \mathrm{e}_0\tau)(2(1-\beta)\xi - \tau^2)\psi\right) = \Delta e_U^{YC^*}.$$

Let $\mathrm{A} = 2(C_r + P_{tc} - C_n\beta)\xi + 2e_0(1-\beta)\xi\tau - (a - C_n + C_r + P_{tc} - a\beta)\tau^2$.

$$Q_{2r}^{YC^*} \geq 0 \Rightarrow \Delta e \geq \frac{2(a - C_n + C_r + P_{tc} - a\beta)\xi + (2C_n - C_r - P_{tc} + a(-2+\beta))\tau^2 + e_0\tau^3}{2\xi\tau - \tau^3} = \Delta e_L^{YC^*}$$

Here $\Delta e_U^{YC^*} - \Delta e_L^{YC^*} = \frac{2\beta\xi(a-\mathrm{C}_n-\mathrm{e}_0\tau)(\tau^2-2(1-\beta)\xi)\psi}{\tau(2\xi-\tau^2)^2}$, recall $(a - C_n - e_0\tau) > 0, (\tau^2 - 2(1-\beta)\xi)\psi > 0$. Thus, $\Delta e_U^{YC^*} - \Delta e_L^{YC^*} > 0$, and the other variables can be obtained, $\Delta e_L^{YC^*} \leq \Delta e \leq \Delta e_U^{YC^*}$. The conclusion of line 2 in Table 5 is likewise proven.

Case 2: $\lambda_1 > 0$, $\lambda_2 = 0$, then $Q_{2r}^{YC} = \psi Q_{1n}^{YC}$, $Q_{2r}^{YC} \geq 0$. Combining Eqs. (B.3), (B.4), and (B.5), we can obtain $Q_{2n}^{YC^*} = \frac{\xi(a-\mathrm{C}_n-\mathrm{e}_0\tau)(2\xi-\tau^2-2\beta\xi\psi)}{(2\xi-\tau^2)^2}$.

$$\lambda_1 = \frac{1}{(2\xi - \tau^2)^2}((2\xi - \tau^2)(-2(C_r + P_{tc} - C_n\beta)\xi + 2(e_0(-1+\beta) + \Delta e)\xi\tau + (a - C_n + C_r + P_{tc} - a\beta)\tau^2 - \Delta e\tau^3) - 2\beta\xi(-a + C_n + e_0\tau)(2(-1+\beta)\xi + \tau^2)\psi)$$

$$e_2^{YC^*} = \frac{\tau(a - C_n - e_0\tau)(2\xi(1 - \beta\psi) - \tau^2)}{(2\xi - \tau^2)^2}$$

$$\lambda_1 > 0 \Rightarrow \Delta e > \Delta e_U^{YC^*}$$

Recall $Q_{1n}^{YC^*} = \frac{\xi(a-\mathrm{C}_n-\mathrm{e}_0\tau)}{2\xi-\tau^2}$, so $Q_{2r}^{NC^*} = \frac{\psi\xi(a-\mathrm{C}_n-\mathrm{e}_0\tau)}{2\xi-\tau^2} > 0$

Similarly, the conclusion of column 3 in Table 5 is proven.

Case 3: $\lambda_1 = 0, \lambda_2 > 0$, then $Q_{2r}^{YC} \leq \psi Q_{1n}^{YC}$, $Q_{2r}^{YC} = 0$. Combining Eqs. (B.3), (B.4), and (B.5), we can obtain

$$Q_{2n}^{NC^*} = \frac{\xi(a - C_n - e_0\tau)}{2\xi - \tau^2}$$

$$e_2^{YC^*} = \frac{\tau(a - C_n - e_0\tau)}{2\xi - \tau^2}$$

$$\lambda_2 = \frac{2C_r\xi + 2P_{tc}\xi - 2C_n\beta\xi + 2e_0\xi\tau - 2e_0\beta\xi\tau - 2\Delta e\xi\tau - a\tau^2 + C_n\tau^2 - C_r\tau^2 - P_{tc}\tau^2 + a\beta\tau^2 + \Delta e\tau^3}{2\xi - \tau^2}$$

Recall $2\xi - \tau^2 > 0$, so $\lambda_2 > 0 \Rightarrow \Delta e < \Delta e_L^{YC^*}$.

Similarly, the other optimal decisions can be obtained and shown in the first column of Table 5. Thus, Proposition 4 is proven.

Appendix C: Model NM

C.1 Proof of Proposition 5

In Model NM, the profit of the manufacturer and retailer in two periods are expressed as follows:

$$Max\Pi_{\mathrm{R}}^{\mathrm{NM}}(Q_{1n}^{NM},Q_{2n}^{NM},Q_{2r}^{NM}) = (P_{1n}^{NM}-W_{1n}^{NM})Q_{1n}^{NM}+(P_{2n}^{NM}-W_{2n}^{NM})Q_{2n}^{NM}+(P_{2r}^{NM}-W_{2r}^{NM})Q_{2r}^{NM} \tag{C.1}$$

$$\text{s.t}\ \begin{array}{c}\psi Q_{1n}^{NM}\geq Q_{2r}^{NM}\\ Q_{1n}^{NM},Q_{2n}^{NM},Q_{2r}^{NM}\geq 0\end{array}$$

$$Max\Pi_{M}^{NM}(W_{2n}^{NM},W_{2r}^{NM}) = (W_{1n}^{NM}-\mathrm{C}_n)Q_{1n}^{NM}+(W_{2n}^{NM}-\mathrm{C}_n)Q_{2n}^{NM}+(W_{2r}^{NM}-\mathrm{C}_r-P_m)Q_{2r}^{NM}-\tau E^{NM} \tag{C.2}$$

Taking the second-order derivatives of Eq. (C.1) with respect to Q_{1n}^{NM}, we can get $\frac{\partial^2\Pi_R^{NM}}{\partial Q_{1n}^{NM^2}}=-2<0$. Thus, the objective function of the retailer in the first period is strictly concave in Q_{1n}^{NM}. In addition, by solving the first-order conditions of $\Pi_{\mathrm{R}}^{\mathrm{NM}}$ with respect to Q_{1n}^{NM}, we can obtain the optimal decisions, and the quantity of the new product can be obtained as $Q_{1n}^{NM}=\frac{a-W_{1n}^{NM}}{2}$. Substituting Q_{1n}^{NM} into Eq. (C.2) and taking the second-order derivatives of Eq. (C.2) with respect to W_{1n}^{NM} lead to $\frac{\partial^2\Pi_M^{NM}}{\partial W_{1n}^{NM^2}}=-1<0$. Thus, the objective function is concave in W_{1n}^{NM}, and by solving the first-order conditions of $\Pi_{\mathrm{M}}^{\mathrm{NM}}$ with respect to W_{1n}^{NM}, we can obtain the optimal decision $W_{1n}^{NM^*}=\frac{1}{2}(a+C_n+e_0\tau)$ and then $Q_{1n}^{NM^*}=\frac{1}{4}(a-C_n-e_0\tau)$ and $P_{1n}^{NM^*}=\frac{1}{4}(3a+C_n+e_0\tau)$. Proposition 5 is then proven.

C.2 Proof of Proposition 6

In the second period, substituting $Q_{1n}^{NM^*}$ and $P_{1n}^{NM^*}$ into Eq. (B.1) and taking the second-order derivatives of $\Pi_{\mathrm{R}}^{\mathrm{NM}}$ with respect to Q_{2n}^{NM} and Q_{2r}^{NM}, the Hessian matrix is $\mathrm{H}(Q_{2n}^{NM},Q_{2r}^{NM})=\begin{pmatrix}-2 & -2\beta\\ -2\beta & -2\beta\end{pmatrix}=4\beta(1-\beta)$. Note that $0<\beta<1$, so the Hessian is negative definite, and the profit function of the retailer is concave in $(Q_{2n}^{NM},Q_{2r}^{NM})$. Then the Lagrangian function can be written as follows:

$$\mathcal{L}(Q_{2n}^{NM},Q_{2r}^{NM},\lambda_1,\lambda_2)=(P_{2n}-W_{2n}^{NM})Q_{2n}^{NM}+(P_{2r}-W_{2r}^{NM})Q_{2r}^{NM}+\lambda_2 Q_{2r}^{NM}+\lambda_1(\psi Q_{1n}^{NM}-Q_{2r}^{NM}) \tag{C.3}$$

By setting

$$\frac{\partial\mathcal{L}}{\partial Q_{2r}^{NM}}=-W_{2r}^{NM}-Q_{2n}^{NM}\beta+(a-Q_{2n}^{NM}-Q_{2r}^{NM})\beta-Q_{2r}^{NM}\beta-\lambda_1+\lambda_2=0 \tag{C.4}$$

$$\frac{\partial\mathcal{L}}{\partial Q_{2n}^{NM}}=a-2Q_{2n}^{NM}-W_{2n}^{NM}-2\beta Q_{2r}^{NM}=0 \tag{C.5}$$

$$\lambda_1(\psi Q_{1n}^{NM}-Q_{2r}^{NM})=0 \tag{C.6}$$

$$\lambda_2\, Q_{2r}^{NM}=0 \tag{C.7}$$

$$\lambda_1,\lambda_2\geq 0 \tag{C.8}$$

Case 1: $\lambda_1=\lambda_2=0$, from Eqs. (C.6) and (C.7), $Q_{2r}^{NM}\geq 0$ and $Q_{2r}^{NM}\leq\psi Q_{1n}^{NM}$. Combining Eqs.

(B.4) and (B.5), we can obtain

$$Q_{2n}^{NM} = \frac{a-W_{2n}^{NM}+W_{2r}^{NM}-a\beta}{2(1-\beta)} \tag{C.9}$$

$$Q_{2r}^{NM} = \frac{\beta W_{2n}^{NM}-W_{2r}^{NM}}{2(1-\beta)} \tag{C.10}$$

The values of Q_{2n}^{NM} and Q_{2r}^{NM} are then substituted into the objective function of the manufacturer in the second period and the first and second partial derivatives of W_{2n}^{NM}, W_{2r}^{NM},

$\frac{\partial \Pi_M^{NM}}{\partial W_{2n}^{NM}} = \frac{C_r\beta+P_m\beta-(a-W_{2n}^{NM})\beta-(C_n-W_{2n}^{NM})\beta-2\beta W_{2r}^{NM}+a\beta^2-\beta\Delta e\tau}{2(\beta-1)\beta}$,

$\frac{\partial \Pi_M^{NM}}{\partial W_{2r}^{NM}} = \frac{2W_{2r}^{NM}-C_r-P_m+C_n\beta-2W_{2n}^{NM}\beta+e_0(-1+\beta)\tau+\Delta e\tau}{2\beta(\beta-1)}$ are taken. The Hessian matrix is $\mathrm{H}(W_{2n}^{NM}, W_{2r}^{NM}) = \begin{bmatrix} \frac{1}{-1+\beta} & \frac{1}{1-\beta} \\ \frac{1}{1-\beta} & \frac{1}{(-1+\beta)\beta} \end{bmatrix} = \frac{1}{\beta-\beta^2}$. Note that $0<\beta<1$, so the Hessian matrix is negative definite and the profit function is concave in $(W_{2n}^{NM}, W_{2r}^{NM})$. Solving the first-order conditions of $\Pi_{\mathrm{M}}^{\mathrm{NM}}$ with respect to W_{2n}^{NM} and W_{2r}^{NM}, we can obtain $W_{2n}^{NM^*} = \frac{1}{2}(a+C_n+e_0\tau)$, $W_{2r}^{NM^*} = \frac{1}{2}(C_r+P_m+a\beta+e_0\tau-\Delta e\tau)$. Therefore, substitute $W_{2n}^{NM^*}$ and $W_{2r}^{NM^*}$ into Eqs. (C.9) and (C.10), respectively, and then the optimal quantities of new and remanufactured products are as follows:

$Q_{2n}^{NM^*} = \frac{a-C_n+C_r+P_m-a\beta-\Delta e\tau}{4(1-\beta)}$, $\quad Q_{2r}^{NM^*} = \frac{C_n\beta-(e_0(1-\beta)-\Delta e)\tau-C_r-P_m}{4\beta(1-\beta)}$

$\because\ \psi Q_{1n}^{NM^*} - Q_{2r}^{NM^*} \geq 0$

$\therefore\ \Delta e \leq \frac{C_r+P_m-C_n\beta+e_0\tau-e_0\beta\tau+a\beta\psi-C_n\beta\psi-a\beta^2\psi+C_n\beta^2\psi-e_0\beta\tau\psi+e_0\beta^2\tau\psi}{\tau} = \Delta e_U^{NM^*}$

$\because\ Q_{2r} \geq 0$

$\therefore\ \Delta e \geq \frac{C_r+P_m-C_n\beta+e_0(1-\beta)\tau}{\tau} = \Delta e_L^{NM^*}$

$\Delta e_U^{NM^*} - \Delta e_L^{NM^*} = \frac{(1-\beta)\beta(a-\mathrm{C}_n-\mathrm{e}_0\tau)\psi}{\tau} > 0$

Thus, $\Delta e_U^{NM^*} > \Delta e_L^{NM^*}$, so the carbon saving per unit Δe satisfies the conditon $\Delta e_L^{NM*} \leq \Delta e \leq \Delta e_U^{NM*}$; in this case, the optimal decisions, such as the price of new and remanufactured products and the profits of the manufacturer and retailer in two periods can be obtained and shown in the second column of Table 6.

Case 2: $\lambda_1 > 0, \lambda_2 = 0$, then $Q_{2r}^{NM} = \psi Q_{1n}^{NM}$, $Q_{2r}^{NM} \geq 0$. Combining Eqs. (C.4) and (C.5), we can obtain

$$Q_{2n}^{NM^*} = \frac{1}{4}(2a-2W_{2n}^{NM}-a\beta\psi+\beta\psi\mathrm{C}_n+\mathrm{e}_0\beta\tau\psi) \tag{C.11}$$

$$\lambda_1 = \frac{1}{2}(-2W_{2n}^{NM} + 2W_{2n}^{NM}\beta - a\beta\psi + C_n\beta\psi + a\beta^2\psi - C_n\beta^2\psi + e_0\beta\tau\psi - e_0\beta^2\tau\psi) \quad (C.12)$$

Recall $Q_{1n}^{NM^*} = \frac{1}{4}(a - C_n - e_0\tau)$, so $Q_{2r}^{NM^*} = \frac{1}{4}\psi(a - C_n - e_0\tau)$. By substituting $Q_{2n}^{NM^*}$ and $Q_{2r}^{NM^*}$ into the profit of the manufacturer in the second period and taking the first and second partial derivatives of W_{2n}^{NM}, we can obtain

$$\frac{\partial \Pi_M^{NM}}{\partial W_{2n}^{NM}} = \frac{1}{4}(2(a - W_{2n}^{NM}) + 2(C_n - W_{2n}^{NM} + e_0\tau) - \beta(a - C_n - e_0\tau)\psi)$$

$$\frac{\partial^2 \Pi_M^{NM}}{\partial {W_{2n}^{NM}}^2} = -1$$

$$\frac{\partial \Pi_M^{NM}}{\partial W_{2r}^{NM}} = \frac{1}{4}(a - C_n - e_0\tau)\psi$$

Here we know that the manufacturer's profit function Π_M^{NM} is a concave function of W_{2n}^{NM} but not a joint concave function of W_{2n}^{NM} and W_{2r}^{NM}. According to Hua et al. (2010) and Guo et al. (2018), it was optimized by using two phases when given the wholesale price of manufactured products. First, we find the optimal wholesale price of the new product in the second period and then find the optimal wholesale price of the remanufactured product to maximize the manufacturer's profit function. Hence, we can obtain $W_{2n}^{NM^*} = \frac{1}{4}(a(2 - \beta\psi) + (C_n + e_0\tau)(2 + \beta\psi))$.

By substituting $W_{2n}^{NM^*}$ into Eqs. (C.11) and (C.12), we can obtain

$$Q_{2n}^{NM^*} = \frac{1}{8}(a - C_n - e_0\tau)(2 - \beta\psi)$$

$$\lambda_1 = \frac{1}{4}(-4W_{2r}^{NM} + 2\beta(a + C_n + e_0\tau) - (2 - \beta)\beta(a - C_n - e_0\tau)\psi)$$

$\because \lambda_1 > 0$, $\therefore W_{2r}^{NM^*} < \frac{1}{4}(2\beta(a + C_n + e_0\tau) - (2 - \beta)\beta(a - C_n - e_0\tau)\psi) = W_U^{NM^*}$. Under this condition, the full collection strategy is operated. The specific wholesale price is determined by the negotiating power of the manufacturer and retailer. The other optimal decisions can also be obtained easily and shown in the third column of Table 6.

Case 3: $\lambda_1 = 0, \lambda_2 > 0$, then $Q_{2r}^{NM} \le \psi Q_{1n}^{NM}$, $Q_{2r}^{NM} = 0$. Combining Eqs. (C.4) and (C.5), we can obtain

$$Q_{2n}^{NM^*} = \frac{a - W_{2n}^{NM}}{2} \quad (C.13)$$

$$\lambda_2 = W_{2r}^{NM^*} - \beta W_{2n}^{NM^*} \quad (C.14)$$

Substitute $Q_{2n}^{NM^*}$ and $Q_{2r}^{NM^*}$ into the profit function of the manufacturer in the second period and take the first and second partial derivatives of W_{2n}^{NM} and W_{2r}^{NM} as follows:

$$\frac{\partial \Pi_M^{NM}}{\partial W_{2n}^{NM}} = \frac{1}{2}(a + C_n - 2W_{2n}^{NM} + e_0\tau)$$
$$\frac{\partial^2 \Pi_M^{NM}}{\partial {W_{2n}^{NM}}^2} = -1$$

Similar to case 2, the manufacturer's profit function $\Pi_{\mathrm{M}}^{\mathrm{NM}}$ is concave in W_{2n}^{NM}. Hence, we can obtain $W_{2n}^{NM^*} = \frac{1}{2}(a + \mathrm{C}_n + \mathrm{e}_0\tau)$ and then $Q_{2n}^{NM^*} = \frac{1}{4}(a - \mathrm{C}_n - \mathrm{e}_0\tau)$, $\lambda_2 = W_{2r}^{NM} - \frac{1}{2}\beta(a + \mathrm{C}_n + \mathrm{e}_0\tau)$

$\because$ $\lambda_2 > 0$, $\therefore W_{2r}^{NM^*} > \frac{1}{2}\beta(a + \mathrm{C}_n + \mathrm{e}_0\tau) = W_L^{NM*}$. Thus, under the condition $W_{2r}^{NM^*} > \frac{1}{2}\beta(a + \mathrm{C}_n + \mathrm{e}_0\tau)$, the no collection strategy is operated. The optimal decisions are shown in the first column of Table 6. Proposition 6 is thus proven.

Appendix D: Model YM

D.1 Proof of Proposition 7

In Model YM, the profits of the manufacturer and retailer in two periods are expressed as follows:

$$Max\Pi_{\mathrm{R}}^{\mathrm{YM}}\left(Q_{1n}^{YM}, Q_{2n}^{YM}, Q_{2r}^{YM}\right) = \left(P_{1n}^{YM} - W_{1n}^{YM}\right)Q_{1n}^{YM} + \left(P_{2n}^{YM} - W_{2n}^{YM}\right)Q_{2n}^{YM} + \left(P_{2r}^{YM} - W_{2r}^{YM}\right)Q_{2r}^{YM} \qquad \text{(D.1)}$$
$$\text{s.t} \begin{array}{l} \psi Q_{1n}^{YM} \geq Q_{2r}^{YM} \\ Q_{1n}^{YM}, Q_{2n}^{YM}, Q_{2r}^{YM} \geq 0 \end{array}$$
$$Max\Pi_M^{YM}(W_{1n}^{YM}, W_{2n}^{YM}, W_{2r}^{YM}, e_1^{YM}, e_2^{YM}) = (W_{1n}^{YM} - \mathrm{C}_n)Q_{1n}^{YM} + (W_{2n}^{YM} - \mathrm{C}_n)Q_{2n}^{YM} + (W_{2r}^{YM} - \mathrm{C}_r - P_m)Q_{2r}^{YM}$$
$$-\mathrm{C}(e_1^{YM}, e_2^{YM}) - \tau E^{YM} \qquad \text{(D.2)}$$

Note that $\mathrm{C}(e_1^{YM}, e_2^{YM}) = \frac{1}{2}\xi(e_1^{YM})^2 + \frac{1}{2}\xi(e_2^{YM})^2$, $E^{YM} = (\mathrm{e}_0 - e_1^{YM})Q_{1n}^{YM} + (\mathrm{e}_0 - e_2^{YC})Q_{2n}^{YM} + (\mathrm{e}_0 - e_1^{YM} - \Delta e)Q_{2r}^{YM}$. Taking the second-order derivative of Eq. (D.1) with respect to Q_{1n}^{YM}, we can get $\frac{\partial^2 \Pi_{\mathrm{R}}^{\mathrm{YM}}}{\partial {Q_{1n}^{YM}}^2} = -2 < 0$, so the profit of the retailer in the first period is strictly concave in Q_{1n}^{YM}. Make the first partial derivative equal to 0 and the quantity of new products can be obtained as $Q_{1n}^{YM} = \frac{a - W_{1n}^{YM}}{2}$. Substituting Q_{1n}^{YM} into the manufacturer's profit function (i.e., Eq. (D.2)), the second-order derivatives of the equation with respect to W_{1n}^{YM} and e_1^{YC} can be obtained, and the Hessian matrix is $\mathrm{H}(W_{1n}^{YM}, e_1^{YM}) = \begin{pmatrix} -1 & -\frac{\tau}{2} \\ -\frac{\tau}{2} & -\xi \end{pmatrix} = \xi - \frac{\tau^2}{4}$. Therefore, under the condition $4\xi - \tau^2 > 0$, the optimal wholesale price and carbon reduction level in the first period can be obtained as follows: $W_{1n}^{YM^*} = \frac{2a\xi + 2\mathrm{C}_n\xi + 2\mathrm{e}_0\xi\tau - a\tau^2}{4\xi - \tau^2}$, $e_1^{YM^*} = \frac{\xi(a - \mathrm{C}_n - \mathrm{e}_0\tau)}{4\xi - \tau^2}$. Then $Q_{1n}^{YM^*} = \frac{\xi(a - C_n - e_0\tau)}{4\xi - \tau^2}$ and $P_{1n}^{YM^*} = \frac{\xi(a - \mathrm{C}_n - \mathrm{e}_0\tau)}{4\xi - \tau^2}$. Thus, Proposition 7 is proven.

D.2 Proof of Proposition 8

In the second period, substitute $Q_{1n}^{YM^*}$, $P_{1n}^{YM^*}$, and $W_{1n}^{YM^*}$ into Eq. (D.1), take the second-order derivatives of Π_R^{YM} with respect to Q_{2n}^{YM} and Q_{2r}^{YM}, and the Hessian matrix is $\mathrm{H}\left(Q_{2n}^{YM}, Q_{2r}^{YM}\right) =$

$\begin{pmatrix} -2 & -2\beta \\ -2\beta & -2\beta \end{pmatrix} = 4\beta(1-\beta)$. Note that $0 < \beta < 1$, so the Hessian matrix H is negative definite and the retailer's profit function is concave in $(Q_{2n}^{YM}, Q_{2r}^{YM})$. Then the Lagrangian function can be written as follows:

$$\mathcal{L}(Q_{2n}^{YM}, Q_{2r}^{YM}, \lambda_1, \lambda_2) = (P_{2n}^{YM} - W_{2n}^{YM})Q_{2n}^{YM} + (P_{2r}^{YM} - W_{2r}^{YM})Q_{2r}^{YM} + \lambda_2 Q_{2r}^{YM} + \lambda_1(\psi Q_{1n}^{YM} - Q_{2r}^{YM}) \quad \text{(D.3)}$$

By setting

$$\frac{\partial \mathcal{L}}{\partial Q_{2r}^{YM}} = -W_{2r}^{YM} - Q_{2n}^{YM}\beta + \left(a - Q_{2n}^{YM} - Q_{2r}^{YM}\right)\beta - \beta Q_{2r}^{YM} - \lambda_1 + \lambda_2 = 0 \quad \text{(D.4)}$$

$$\frac{\partial \mathcal{L}}{\partial Q_{2n}^{YM}} = a - 2Q_{2n}^{YM} - W_{2n}^{YM} - 2\beta Q_{2r}^{YM} = 0 \quad \text{(D.5)}$$

$$\lambda_1\left(\psi Q_{1n}^{YM} - Q_{2r}^{YM}\right) = 0 \quad \text{(D.6)}$$

$$\lambda_2 Q_{2r}^{YM} = 0 \quad \text{(D.7)}$$

$$\lambda_1, \lambda_2 \geq 0 \quad \text{(D.8)}$$

Case 1: $\lambda_1 = \lambda_2 = 0$, from Eqs. (D.6) and (D.7), $Q_{2r}^{YM} \geq 0$ and $Q_{2r}^{YM} \leq \psi Q_{1n}^{YM}$. Combining Eqs. (D.4) and (D.5), we can obtain

$$Q_{2n}^{YM} = \frac{a - W_{2n}^{YM} + W_{2r}^{YM} - a\beta}{2(1-\beta)} \quad \text{(D.9)}$$

$$Q_{2r}^{YM} = \frac{\beta W_{2n}^{YM} - W_{2r}^{YM}}{2(1-\beta)} \quad \text{(D.10)}$$

By substituting Q_{2n}^{YM} and Q_{2r}^{YM} into the profit of the manufacturer in the second period and taking the first and second partial derivatives of W_{2n}^{YM}, e_2^{YM}, and W_{2r}^{YM}, the Hessian matrix is $\mathrm{H}(W_{2n}^{YM}, W_{2r}^{YM}, e_2^{YM}) = \begin{bmatrix} -\frac{1}{1-\beta} & \frac{1}{1-\beta} & -\frac{\tau}{2(1-\beta)} \\ \frac{1}{1-\beta} & -\frac{1}{(1-\beta)\beta} & \frac{\tau}{2(1-\beta)} \\ -\frac{\tau}{2(1-\beta)} & \frac{\tau}{2(1-\beta)} & -\xi \end{bmatrix} = \frac{4(-1+\beta)\xi + \tau^2}{4(1-\beta)^2\beta}$. Note that $0 < \beta < 1$. Thus, the Hessian matrix is negative definite and the profit function is concave in $(W_{2n}^{YM}, W_{2r}^{YM}, e_2^{YM})$ under condition $4(-1+\beta)\xi + \tau^2 < 0$. Solving the first-order condition of $\Pi_{\mathrm{M}}^{\mathrm{YM}}$ with respect to W_{2n}^{YM}, W_{2r}^{YM}, and e_2^{YM}, we can obtain

$$\begin{aligned} W_{2n}^{YM^*} = &\frac{1}{2(4\xi - \tau^2)(4(-1+\beta)\xi + \tau^2)}\big(16(a + C_n)(-1+\beta)\xi^2 + 16e_0(-1+\beta)\xi^2\tau \\ &+ 4(3a + C_n + C_r + P_m - (2a + C_n)\beta)\xi\tau^2 - 4(e_0(-2+\beta) + \Delta e)\xi\tau^3 \\ &+ \left(C_n - C_r - P_m + a(-3+\beta)\right)\tau^4 + \Delta e\tau^5\big) \end{aligned}$$

$$W_{2r}^{YM^*} = \frac{1}{2}\left(a - C_n + C_r + P_m + a\beta - \Delta e\tau + \frac{4\xi(-a + C_n + e_0\tau)}{4\xi - \tau^2}\right),$$

$$e_2^{YM^*} = \frac{1}{(4\xi - \tau^2)(4(-1+\beta)\xi + \tau^2)}(\tau(4(C_n - C_r - P_m - a(1-\beta))\xi + 4\Delta e\xi\tau + +(-2C_n + C_r + P_m - a(-2+\beta))\tau^2 - (e_0 + \Delta e)\tau^3))$$

By substituting $W_{2n}^{YM^*}$ and $W_{2r}^{YM^*}$ into Eqs. (D.9) and (D.10), the optimal quantity of the new and remanufactured products are as follows:

$$Q_{2n}^{YM^*} = \frac{1}{(4\xi - \tau^2)(4(-1+\beta)\xi + \tau^2)}(\xi(4(C_n - C_r - P_m + a(-1+\beta))\xi + 4\Delta e\xi\tau + (-2C_n + C_r + P_m - a(-2+\beta))\tau^2 - (e_0 + \Delta e)\tau^3))$$

$$Q_{2r}^{YM^*} = \frac{4(C_r + P_m - C_n\beta)\xi - 4(e_0(-1+\beta) + \Delta e)\xi\tau - (a - C_n + C_r + P_m - a\beta)\tau^2 + \Delta e\tau^3}{4\beta(4(\beta-1)\xi + \tau^2)}$$

$\because\ \psi Q_{1n}^{YM^*} - Q_{2r}^{YM^*} \geq 0$

$\therefore\ \Delta e \leq \frac{1}{\tau(4\xi - \tau^2)^2}((4\xi - \tau^2)B + 4\beta\xi(a - C_n - e_0\tau)(4(1-\beta)\xi - \tau^2)\psi) = \Delta e_U^{YM^*}$

$\because\ Q_{2r}^{YM^*} \geq 0$

$\therefore\ \Delta e \geq \frac{B}{(4\xi - \tau^2)\tau} = \Delta e_L^{YM^*}$, let $B = 4(C_r + P_m - C_n\beta)\xi + 4e_0(1-\beta)\xi\tau - (a - C_n + C_r + P_m - a\beta)\tau^2$

$$\Delta e_U^{YM^*} - \Delta e_L^{YM^*} = \frac{4\beta\xi(a - C_n - e_0\tau)(4(1-\beta)\xi - \tau^2)\psi}{\tau(4\xi - \tau^2)^2}$$

Note that $a - C_n - e_0\tau > 0$ and $4(1-\beta)\xi - \tau^2 > 0$, thus $\Delta e_U^{YM^*} > \Delta e_L^{YM^*}$. Thus, the carbon saving per remanufactured product(Δe) satisfies the condition $\Delta e_L^{YM^*} \leq \Delta e \leq \Delta e_U^{YM^*}$. The optimal price of new and remanufactured products and the profits of the supply chain in two periods can be obtained and shown in the second column of Table 8 .

Case 2: $\lambda_1 > 0, \lambda_2 = 0$, then $Q_{2r}^{YM} = \psi Q_{1n}^{YM}$, $Q_{2r}^{YM} \geq 0$. Combining Eqs. (D.4) and (D.5), we can obtain

$$Q_{2n}^{YM^*} = \frac{4a\xi - 4W_{2n}^{YM}\xi - a\tau^2 + W_{2n}^{YM}\tau^2 - 2a\beta\xi\psi + 2C_n\beta\xi\psi + 2e_0\beta\xi\tau\psi}{2(4\xi - \tau^2)} \quad \text{(D.11)}$$

$$\lambda_1 = -\frac{4W_{2r}^{YM}\xi - 4W_{2n}^{YM}\beta\xi - W_{2r}^{YM}\tau^2 + W_{2n}^{YM}\beta\tau^2 + 2a\beta\xi\psi - 2C_n\beta\xi\psi - 2a\beta^2\xi\psi + 2C_n\beta^2\xi\psi - 2e_0\beta\xi\tau\psi + 2e_0\beta^2\xi\tau\psi}{4\xi - \tau^2} \quad \text{(D.12)}$$

Note that $Q_{1n}^{YM^*} = \frac{\xi(a - C_n - e_0\tau)\psi}{4\xi - \tau^2}$, so $Q_{2r}^{NM^*} = \frac{\xi(a - C_n - e_0\tau)\psi}{4\xi - \tau^2}$. By substituting $Q_{2n}^{YM^*}$ and $Q_{2r}^{YM^*}$ into the profit of the manufacturer in the second period and taking the first and second partial derivatives of W_{2n}^{YM}, W_{2r}^{YM}, we know that the manufacturer's profit function $\Pi_{\mathrm{M}}^{\mathrm{YM}}$ is a concave function of W_{2n}^{YM} and e_2^{YM} but not a joint concave function of $(W_{2n}^{YM},\ W_{2r}^{YM},\ e_2^{YM})$. Similar to C.2, it was optimized by using two phases. When given the wholesale price of new products W_{2n}^{YM} and carbon reduction level e_2^{YM}, we first find the optimal wholesale price of new product W_{2n}^{YM} and the carbon reduction level e_2^{YM} in the second period and then find the optimal wholesale price of remanufactured products W_{2r}^{YM} to maximize the manufacturer's profit function. The Hessian matrix $\mathrm{H}(W_{2n}^{YM}, e_2^{YM}) = \begin{bmatrix} -1 & -\frac{\tau}{2} \\ -\frac{\tau}{2} & -\xi \end{bmatrix} = \xi - \frac{\tau^2}{4}$, so under the condition $4\xi - \tau^2 > 0$, the manufacturer's profit function is concave in (W_{2n}^{YM}, e_2^{YM}). Solving the first partial derivatives of W_{2n}^{YM}, e_2^{YM} leads to

$$\frac{\partial \Pi_M^{YM}}{\partial e_2^{YM}} = -e_2^{YM}\xi - \frac{1}{2}\tau(-a + W_{2n}^{YM} + \frac{2\beta\xi(a - C_n - e_0\tau)\psi}{4\xi - \tau^2}) \tag{D.13}$$

$$\frac{\partial \Pi_M^{YM}}{\partial W_{2n}^{YM}} = \frac{1}{2}(a + C_n - 2W_{2n}^{YM} + (e_0 - e_2^{YM})\tau + \frac{2\beta\xi(-a + C_n + e_0\tau)\psi}{4\xi - \tau^2}) \tag{D.14}$$

Combining Eqs. (D.13) and (D.14), we can obtain

$$W_{2n}^{YM^*} = \frac{1}{(4\xi - \tau^2)^2}(2\xi(\mathrm{C}_n + \mathrm{e}_0\tau)(2\xi(2 + \beta\psi) - \tau^2(1 + \beta\psi)) - a(2\xi - \tau^2)(\tau^2 + 2\xi(-2 + \beta\psi)))$$

$$e_2^{YM^*} = \frac{\tau(a - \mathrm{C}_n - \mathrm{e}_0\tau)(2\xi(2 - \beta\psi) - \tau^2)}{(4\xi - \tau^2)^2}$$

By substituting $W_{2n}^{NM^*}$and $e_2^{YM^*}$ into Eqs. (D.11) and (D.12), we can obtain

$$Q_{2n}^{YM^*} = \frac{\xi(a - \mathrm{C}_n - \mathrm{e}_0\tau)(2\xi(2 - \beta\psi) - \tau^2)}{(4\xi - \tau^2)^2}$$

$$\lambda_1 = \frac{-W_{2r}^{YM}(-4\xi + \tau^2)^2 + \beta(4\xi - \tau^2)(2(a + \mathrm{C}_n)\xi + 2e_0\xi\tau - a\tau^2) - 2\beta\xi(-a + \mathrm{C}_n + e_0\tau)(2(-2 + \beta)\xi + \tau^2)\psi}{(\tau^2 - 4\xi)^2}$$

$\because\ \lambda_1 > 0$, $\therefore W_{2r}^{YM^*} < \frac{(4\xi - \tau^2)F - 2\beta\xi(a - \mathrm{C}_n - e_0\tau)(2(2 - \beta)\xi - \tau^2)\psi}{(4\xi - \tau^2)^2} = W_U^{YM^*}$, under this condition, the full collection strategy is operated. The specific wholesale price is determined by the negotiating power of the manufacturer and retailer. Furthermore, the other optimal decisions can be obtained easily and shown in the third column of Table 8.

Case 3: $\lambda_1 = 0, \lambda_2 > 0$, then $Q_{2r}^{YM} \le \psi Q_{1n}^{YM}$, $Q_{2r}^{YM^*} = 0$. Combining Eqs. (D.4) and (D.5), we can obtain

$$Q_{2n}^{NM^*} = \frac{a - W_{2n}^{YM}}{2} \tag{D.15}$$

$$\lambda_2 = W_{2r}^{YM} - \beta W_{2n}^{YM} \tag{D.16}$$

Substitute $Q_{2n}^{YM^*}$ and $Q_{2r}^{YM^*}$ into the profit function of the manufacturer in the second period and take the first and second partial derivatives of W_{2n}^{YM}, W_{2r}^{YM}. Similar to case 2, we solve the problem in two periods, and the Hessian matrix $\mathrm{H}(W_{2n}^{YM}, e_2^{YM}) = \begin{bmatrix} -1 & -\frac{\tau}{2} \\ -\frac{\tau}{2} & -\xi \end{bmatrix} = \xi - \frac{\tau^2}{4}$. Thus, under the condition $4\xi - \tau^2 > 0$, we can obtain

$W_{2n}^{YM^*} = \frac{2a\xi + 2C_n\xi + 2e_0\xi\tau - a\tau^2}{4\xi - \tau^2}$, $e_2^{YM^*} = \frac{\tau(a - C_n - e_0\tau)}{4\xi - \tau^2}$, so $Q_{2n}^{YM^*} = \frac{\xi(a - C_n - e_0\tau)}{4\xi - \tau^2}$,

$$\lambda_2 = W_{2r}^{YM} + \frac{\beta(-2a\xi - 2C_n\xi - 2e_0\xi\tau + a\tau^2)}{4\xi - \tau^2}$$

$\because\ \lambda_2 > 0$, $\therefore\ W_{2r}^{NM^*} > \frac{\beta(2a\xi+2C_n\xi+2e0\xi\tau-a\tau^2)}{4\xi-\tau^2} = W_L^{YM^*}$. Specifically, if the condition $W_{2r}^{YM^*} > \frac{\beta(2a\xi+2C_n\xi+2e_0\xi\tau-a\tau^2)}{4\xi-\tau^2}$ is satisfied, no collection strategy is operated. The optimal decisions are shown in the first column of Table 8. Thus, Proposition 8 is proven.