Integrated Strategic and Operational Planning of Dry Port Container Network in a Stochastic Environment

Kamran Sarmadi

k.sarmadi2@ncl.ac.uk

Business School, Newcastle University, Newcastle upon Tyne, UK

Mehdi Amiri-Aref

mehdi.amiri-aref@kedgebs.com

Centre of Excellence in Supply Chain and Transportation, KEDGE Business School, Bordeaux, France

Jing-Xin Dong*

jingxin.dong@ncl.ac.uk

Business School, Newcastle University, Newcastle upon Tyne, UK

Christian Hicks

chris.hicks@ncl.ac.uk

Business School, Newcastle University, Newcastle upon Tyne, UK

Abstract

This paper proposes a dry port network design model that integrates strategic and operational decision taking into account the stochastic nature of demand. The proposed model represents the forward and backward flows of both laden and empty containers as well as the inventory levels of empty containers throughout the network. This problem is formulated as a two-stage stochastic programming model where the dynamic decisions relating to containers' transportation and inventory are integrated with decisions that determine the number and location-allocation of dry ports in an uncertain environment. For solving this problem, a robust sample average approximation method enhanced with Benders decomposition algorithm which is accelerated by multi-cut framework, knapsack inequalities, and Pareto-optimal cut scheme is proposed. The model was tested using a hypothetical case relating to the design of a hinterland container shipping network in North Carolina, USA, that comprises a seaport and fifty manufacturing companies. The quality of solutions obtained was evaluated through extensive numerical experiments. The experimental results underline the sensitivity of network configuration and operational decisions to inventory holding costs and solution robustness. We also provide managerial insights that may lead to improvements in the network configuration, modality choice, service level, fill rate, and inventory turnover in container shipping industry.

Keywords: Dry port, Container shipping, Network design, Stochastic programming, Robust optimisation

1. Introduction

Globalisation has led to increases in trade volume, cargo size and the number of ships (Yang 2018). The shipping industry is subject to many short, medium and long-term risks and uncertainties arising from a range of factors including: technical faults, operational problems, finance, market conditions, competition and regulation (Thanopoulou and Strandenes 2017). The customers of shipping lines typically need door-to-door transportation, which requires intermodal transportation. For them to be successful, it is necessary for them to achieve a high level performance in the management of their assets within the context of inland networks (Olivo, Di Francesco, and Zuddas 2013). The inland parts of container shipping networks involve complex distribution in a stochastic environment. The adoption of appropriate hinterland operations leads to higher efficiency and responsiveness which improves competitiveness (Yu, Fransoo, and Lee 2018).

In this paper, we consider carrier haulage operations, where the containers are under the responsibility of shipping line throughout the whole process (Yu, Fransoo, and Lee 2018), which includes hinterland transportation and the storage of containers. Increasing container flows, driven by economic globalisation has given rise to the dry port concept as a joint seaport and hinterland approach (Crainic et al. 2015). The shipping line invests in establishing dry ports in the hinterland network in order to enhance the mentioned container operations. A dry port differs from traditional inland depots by providing more services including: the storage and consolidation of laden containers; depot-storage of empty containers; maintenance and the repair of containers; and customs clearance. The significant advantage of a dry port is the provision of high capacity for different transportation modes with direct connection to seaports, which enables customers to drop off/pick up their containers using dry ports rather than directly using seaports (Roso, Woxenius, and Lumsden 2009).

In order for a dry port to be competitive it needs to have enough volume and the operating costs need to be no more than a direct connection to the port (Lättilä, Henttu, and Hilmola 2013). The economic feasibility of an intermodal system may be evaluated in terms of the breakeven distance, which is the distance at which intermodal transport costs equal truck transport costs (Kim and Van Wee 2011). Therefore, the shipping line should identify the location of dry ports optimally in a container shipping network taking into account the trade-offs between the costs and savings. In this context, it is important to optimise networks by making appropriate strategic and tactical decisions including: the number and location of new dry ports, the allocation of customers to dry ports, the transportation modes for moving containers, as well as empty container inventory and leasing decisions.

Several studies have considered the dry port location problem. Roso, Woxenius, and Lumsden (2009) studied the concept of dry ports and categorised them according to their distance from seaports. Ambrosino and Sciomachen (2014) proposed a hub location model for identifying the best site for mid-range dry ports (normally located midway between the maritime terminals and the inland nodes) that took into account the flow of containers arriving by sea and moving to their final destination using road or rail. The model allowed demand to be split by modality or between hubs. However, direct flows between seaports and inland customer nodes were neglected. . Wang, Chen, and Huang (2018) developed a mathematical model that minimised the sum of transportation and fixed costs associated with opening and closing dry ports. They applied a sensitivity analysis of model parameters. However, this post-optimality approach was a reactive way to investigate uncertainties and it could not yield robust solutions proactively. These studies also neglected the effect of operational decisions and uncertainties associated with the location problem.

In addition to providing transhipment services, dry ports offer a range of other services including: consolidation; the storage of laden and empty containers; maintenance and repair of containers; and customs clearance (Roso, Woxenius, and Lumsden 2009, p.341). The provision of storage helps shipping line companies handle the Empty Container Repositioning (ECR) problem. In the literature the ECR problem has been considered independently from the dry port location problem. Since the ECR problem is mainly due to international trade imbalances, most studies have applied network flow optimisation models (see for example, Brouer, Pisinger, and Spoorendonk 2011; Song and Dong 2012) to address the problem. Jula, Chassiakos, and Ioannou (2006), Deidda et al. (2008), and Furió et al. (2013) analysed different policies for hinterland empty container repositioning within a deterministic environment. Erera, Morales, and Savelsbergh (2009) developed a robust optimisation framework for dynamic ECR that was modelled by time-space networks. Nonetheless, their approach was limited to the repositioning of empty containers without considering the strategic location decisions of ports or dry ports. In reality, in a containerised cargo transport chain the demand for empty containers is driven by laden container flows. In addition, both laden and empty containers flows should occur in the same network (Song and Dong 2015). This implies that the flow of laden and empty containers should be integrated into the network design problem to capture real-life practice. Xie et al. (2017) and Vojdani, Lootz, and Rösner (2013) studied the repositioning of empty containers in hinterland intermodal networks. Although the collaboration of different carriers was studied by Vojdani, Lootz, and Rösner (2013), their model was restricted to a deterministic environment. The flows of laden containers were assumed to be fixed parameters rather than decision variables. This ignored the necessity to simultaneously consider decisions relating to the flow of the laden and empty containers in the optimisation model. In summary, all the studies relating to ECR tried to address the problem within the container shipping context without considering the strategic decision of facility location. The models were mostly deterministic although uncertainty and fluctuations in container demand are key characteristics of the industry (Lee and Song 2017).

Strategic dry port location decisions should consider the impact of strategic and operational decisions since the deployment of dry ports, intermodal transportation planning and ECR are interdependent decisions (Lee and Song 2017). For instance, empty container repositioning decisions invole empty containers' waiting time, the amount of movement, as well as destinations (e.g. a manufacturer or a seaport). However, ECR decisions depend on the number and location of dry ports which determines the inbound and outbound transportation times and storage capacity utilisation. There is a temporal hierarchical structure and periodicity between strategic and tactical/operational decisions (Amiri-Aref, Klibi, and Babai 2018). It is therefore important to integrate strategic level decisions with the tactical and operational level decisions in containerised transport chain and distribution network models. This is addressed by this paper.

Table 1 summarises the literature related to container shipping, which is classified according to: the modelling approach; the structure and periodicity; mode of transport; decision level; and model decisions. Finally, this work is positioned relative to the gap in the academic literature. In contrast to earlier studies on container shipping network design, this work considers dry port location, which is a strategic decision, together with the operational decisions relating to the intermodal container flow management in a dynamic and stochastic environment.

The rest of this paper is organised as follows. Section 2 defines the problem and presents the proposed mathematical model in detail. Section 3 explains the solution procedure that includes the sample average approximation (SAA) approach, the robust counterpart of the SAA problem and the Benders decomposition algorithm that is used to address the computational challenges of two-stage stochastic programming model. Section 4 outlines the key performance indicators and provides practical insights. In section 5, comprehensive numerical experiments are presented that verify the practicability of the proposed model, as well as the efficiency of solution procedures. Finally, section 6 presents some conclusions and suggestions for possible future research.

Table 1 . A classification of relevant papers in the literature.

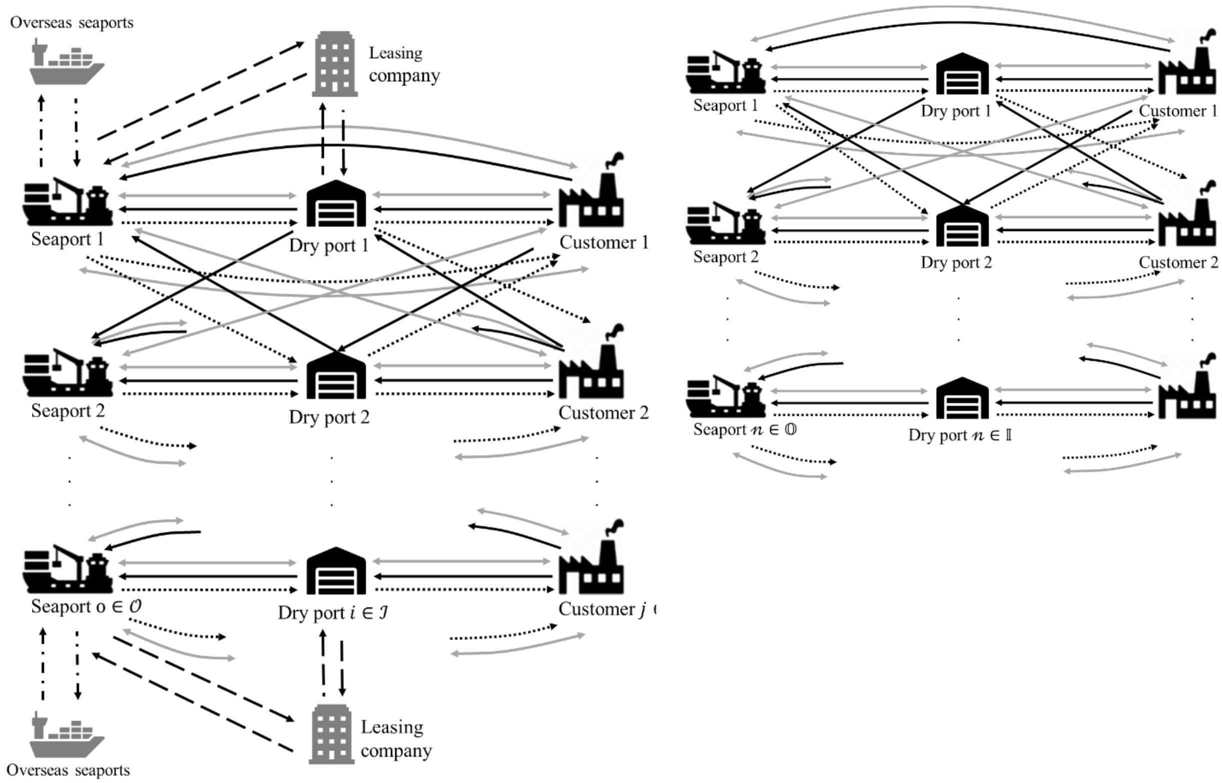
References	Modelling approach	Parameters features		Mode of transport	Decisions Level		Model Decisions			
		Structure	Periodicity		Strategic	Tactical/Operational	Location	Transportation	Inventory	ECR
Jula, Chassiakos, and Ioannou (2006)	LP	Deterministic	Multiple	Single modal	-	✓	-	✓	-	✓
Shintani et al. (2007)	LP	Deterministic	Single	Single modal	✓	✓	-	✓	-	✓
Deidda et al. (2008)	IP	Deterministic	Single	Single modal	-	✓	-	✓	-	✓
Erera, Morales, and Savelsbergh (2009)	IP	Stochastic	Multiple	Single modal	-	✓	-	✓	✓	✓
Zhang, Yun, and Moon (2009)	MILP	Deterministic	Single	Single modal	-	✓	-	✓	-	✓
Brouer, Pisinger, and Spoorendonk (2011)	LP	Deterministic	Single	Single modal	✓	-	-	✓	-	✓
Song and Dong (2012)	IP	Deterministic	Multiple	Single modal	-	✓	-	✓	✓	✓
Meng, Wang, and Liu (2012)	MIP	Deterministic	Single	Intermodal	-	✓	-	✓	-	✓
Furió et al. (2013)	IP	Deterministic	Multiple	Single modal	-	✓	-	✓	✓	✓
Vojdani, Lootz, and Rösner (2013)	IP	Deterministic	Single	Intermodal	✓	-	-	-	✓	✓
Bell et al. (2013)	LP	Deterministic	Single	Single modal	-	✓	-	✓	-	✓
Di Francesco, Lai, and Zuddas (2013)	SP	Stochastic	Single	Single modal	-	✓	-	✓	-	✓
Ambrosino and Sciomachen (2014)	MILP	Deterministic	Single	Intermodal	✓	-	✓	✓	-	-
Chang, Notteboom, and Lu (2015)	LP	Deterministic	Single	Intermodal	✓	-	✓	-	✓	-
Xie et al. (2017)	GM	Stochastic	Multiple	Intermodal	✓	-	-	-	✓	✓
Wang, Chen, and Huang (2018)	IP	Deterministic	Single	Intermodal	✓	-	✓	✓	-	-
Yu, Fransoo, and Lee (2018)	GM	Deterministic	Single	Intermodal	-	✓	-	-	✓	✓
Current Study	**SP**	**Stochastic**	**Multiple**	**Intermodal**	✓	✓	✓	✓	✓	✓

LP: Linear Programming; MILP: Mixed-integer Linear Programming; IP: Integer Programming; MIP: Mixed Integer Programming; SP: Stochastic programming; GM: Game model

2. Problem definition and formulation

2.1. Problem description

We propose a model for designing a container shipping network that includes the hierarchical decisions relating to dry port location and allocation and creation of arcs (links) between nodes in the network at the strategic level. At the operational level the model considers the intermodal transportation (flow) of laden and empty containers, as well as empty container repositioning and inventory planning. The model considers the uncertainties associated with containers' demand. It also establishes a hierarchy between different decision levels that uses a robust two-stage stochastic programming model in a multi-period setting. The purpose of the proposed model is to: optimise the location and allocation decisions relating to dry ports at the strategic level; and to minimise the costs associated with the container flow and empty container inventory at the operational level. This integrated approach will enhance the quality of the design and operation of hinterland container shipping networks. Figure 1 illustrates a typical hinterland container transport network. Figure 1a represents a traditional road and rail network that transports imported and exported raw materials, goods and empty containers between seaports and customers. A customer could be a producer of raw materials, a manufacturer, a distribution centre, freight forwarder, warehouse or retailer. An alternative is to add dry ports to the network as shown in Figure 1b. With this configuration there can still be direct flows to the seaport. The dry port can also be used to store empty containers. Hinterland empty container repositioning aims to satisfy the demand for empty containers whilst preventing unnecessary movements (Yu, Fransoo, and Lee 2018).



a. Traditional inland container network

b. Inland container network using dry ports

Intermodal flow of laden containers from customers
Intermodal flow of laden containers from seaports
Intermodal flow of empty containers

Figure 1. Hinterland container transport network

Our paper considers decisions associated with the transportation of empty containers throughout the network. The intermodal transportation of empty containers between the seaports, dry ports and customers are integrated with decisions related to the movement of laden containers throughout the network over the planning periods. The approach determines the optimal inventory levels of empty containers at the storage facilities located at the seaport, dry ports and customers for multiple periods. At the strategic level the model aims to determine the optimal location of dry ports from a given set of possible sites and decide the allocation of customers to dry ports. At the operational level it optimises: the intermodal transportation of laden and empty containers; and the inventory level of empty containers throughout the network. The model takes into account the uncertainties associated with the demand of containers.

2.2. Modelling approach

In this section, we propose a robust two-stage stochastic programming model to cope with the hierarchal decision-making structure. Let $\mathbb{N} = \mathbb{O} \cup \mathbb{I} \cup \mathbb{J}$ denote the complete set of nodes in the network, where $\mathbb{O}$ represents the set of seaports, $\mathbb{I}$ represents the set of candidate dry ports, and $\mathbb{J}$ represents the set of customers. The available capacity for storing empty containers at each node is given by Cap_n, where $n \in \mathbb{N}$. $\mathcal{A} = \{(p,q): p,q \in \mathbb{N}\}$ represents the set of all arcs (links) in the network, where p, q represent a pair of nodes.

The model includes the abovementioned strategic decisions in the first stage and considers uncertainties in the second stage. We denote the associated decision for the location of dry ports by the binary variable X_p with a fixed opening cost of f_p, where $p \in \mathbb{I}$. We define the binary variable Y_{pq}, where $(p,q) \in \mathcal{A}$, as the decision associated with the allocation of node p to node q in the considered network with a cost of d_{pq}. Similarly, Y_{pq} is the binary variable related to the allocation of node q to node p. It should be noted that p and q are used as indices to denote the nodes throughout the network ($i.e.\ n \in \mathbb{N} = \mathbb{O} \cup \mathbb{I} \cup \mathbb{J}$).

Uncertainty stems from the lack of, or poor coordination between demand and supply, which are subject to high variability and volatility over the planning horizon. We characterise the uncertainty in the model by a set of plausible scenarios, denoted by ω, where each scenario $\omega \in \Omega$ can occur with a probability of $\pi(\omega)$.

Let $\mathbf{X} = (X_p, Y_{pq})$ be the vector of all first-stage binary design variables relating to the network structure. The first-stage model optimises the strategic location-allocation decisions. The second-stage model optimises the expected operational planning costs. The first-stage objective function and constraints can be formulated as follows:

$$\min_{\mathbf{X}} \left\{ \sum_{\omega \in \Omega} \pi(\omega)\, \mathrm{q}(\mathbf{X},\omega) + \sum_{p \in \mathbb{I}} f_p X_p + \sum_{(p,q) \in \mathcal{A}} d_{pq} Y_{pq} \right\} \tag{1}$$

$$Y_{pq} \le X_p, \qquad \forall p \in \mathbb{I}, \forall q \in \mathbb{J} \tag{2}$$

$$\sum_{q \in \mathbb{O}} Y_{qp} \ge X_p, \qquad \forall p \in \mathbb{I} \tag{3}$$

$$\sum_{p \in \mathbb{O}} Y_{pq} + \sum_{p \in \mathbb{I}} Y_{pq} \ge 1, \qquad \forall q \in \mathbb{J} \tag{4}$$

$$X_p, Y_{pq} \in \{0,1\}, \qquad \forall p, q \in \mathbb{N} = \mathbb{O} \cup \mathbb{I} \cup \mathbb{J} \tag{5}$$

The objective function (1) includes three terms. The first term is the total expected operational costs of $q(\boldsymbol{X},\omega)$ which are optimized at the second stage. The second term is the fixed cost of opening a dry port at a candidate location $\forall p \in \mathbb{I}$. The third term indicates the relevant transportation cost relating to the allocation of node p to node q in the network, where $(p,q) \in \mathcal{A}$. Constraint (2) ensures that the customers are allocated to opened dry ports. Constraint (3) ensures that when a dry port location is chosen, it should be allocated to at least one seaport. Constraint (4) ensures that each customer is connected to at least one seaport or one dry port. This constraint also ensures that a given customer can be supplied by both seaports and dry ports. Constraint (5) represent first-stage binary variables.

It should be noted that the location and allocation strategic decisions that determine the configuration of the container shipping network, have a significant impact on the second-stage operational decisions. In this study, the operational decisions relating to the flow of empty and laden containers and the inventory levels of empty containers throughout the network are made/revisited on a periodic basis $t \in T$. The set of containers are denoted by $K = \{\ell, e\}$, where ℓ and e are associated with laden and empty containers of Twenty Equivalent Unit (TEU) size, respectively. A FEU is considered as two TEUs (Song & Dong, 2012). All links which connect node p to node q can utilise transportation mode $m \in M$, where M is the set of available transportation modes.

Let c^k_{pqm} be the unit cost for transporting container type $k \in K$ on arc $(p, q) \in \mathcal{A}$ when transportation mode m is used. Note that the loading, unloading and operational costs of containers at each node are also included in c^k_{pqm}. The unit cost of holding an empty container at each node is denoted by h_n, for all $n \in \mathbb{N}$. Decisions relating to the leasing of empty containers is also considered as a part of inventory planning. If the inventory level of empty containers at a dry-port is not enough to satisfy the demand in a specific period, the shipping line can lease empty containers from lessors or other companies with the cost of g^+_n, where $n \in \mathbb{N} \setminus \{\mathbb{O} \cup \mathbb{J}\}$, per unit of container. The shipping line returns the leased containers with a returning cost of g^-_n, where $n \in \mathbb{N} \setminus \{\mathbb{O} \cup \mathbb{J}\}$, per unit of container. In addition, if the shipping line has a deficiency of empty containers at a given seaport, it could import its own empty containers from overseas seaports with the unit cost of v^+_n, where $n \in \mathbb{N} \setminus \{\mathbb{I} \cup \mathbb{J}\}$. Accordingly, v^-_n shows the unit cost of returning (exporting) imported containers, where $n \in \mathbb{N} \setminus \{\mathbb{I} \cup \mathbb{J}\}$. A unit backorder cost per container b_q is applied for backordered incoming or outgoing demand at a given customer $q \in \mathbb{J}$. In addition, ν denotes the unit cost per container for rejected incoming or outgoing demand from customers. We denote the transportation lead-time by τ_{pqm}, where $(p, q) \in \mathcal{A}$ and $m \in M$. Moreover, θ denotes the container processing time associated with loading and unloading of finished goods/raw materials at customers sites.

We define $D^\ell_{qt}(\omega)$ and $S^\ell_{qt}(\omega)$ as the realisation of the demand for incoming and outgoing laden containers at customer q in period t under scenario ω. Hereafter, we refer to the demand for outgoing laden containers (i.e. $S^\ell_{qt}(\omega)$) as "supply" for convenience. Let $F^k_{pqtm}(\omega)$ be the decision variable denoting the flow of container type k on arc $(p, q) \in \mathcal{A}$ in period t using transportation mode m under scenario ω. Let $I^e_{nt}(\omega)$ denote the inventory level of empty containers at a given node n in period t under scenario ω. Furthermore, let $L^{e+}_{nt}(\omega)$ and $L^{e-}_{nt}(\omega)$ be the number of leased and returned empty containers, respectively, at $n \in \mathbb{N} \setminus \{\mathbb{O} \cup \mathbb{J}\}$ in period t under scenario ω. Due to the periodicity of decisions related to empty container leasing, it is important to consider the cost relating to the net number of leased containers at each period. Let us denote the net stock of leased empty containers by $L_{nt}^{e}(\omega) = L_{n,t-1}^{e}(\omega) + L_{nt}^{e+}(\omega) - L_{nt}^{e-}(\omega)$, with the corresponding cost of g_n, where $n \in \mathbb{N} \setminus \{\mathbb{O} \cup \mathbb{J}\}$. Note that g_n^{+} denotes the fixed leasing cost per container which is incurred only once when containers are leased, while g_n is the variable leasing cost per container per period for the net stock of leased empty containers remain in the network. Let $H_{nt}^{e+}(\omega)$ and $H_{nt}^{e-}(\omega)$ denote the number of empty containers of the shipping line which are imported and exported, respectively, at the seaport $n \in \mathbb{N} \setminus \{\mathbb{I} \cup \mathbb{J}\}$ in period t under scenario ω. The backordered incoming and outgoing demand at a customer $q \in \mathbb{J}$ in period t under scenario ω is denoted by $U_{qt}^{\ell}(\omega)$ and $B_{qt}^{\ell}(\omega)$, respectively. Finally, $\gamma_{qt}^{\ell}(\omega)$ and $\delta_{qt}^{\ell}(\omega)$ represent the rejected incoming and outgoing demand at a customer $q \in \mathbb{J}$ in period t under scenario ω, respectively. The sets, parameters, and decision variables used in the proposed model are summarized in Table 2.

Table 2. Notation of sets, parameters and decision variables

Sets	
$\mathbb{N}$	Set of nodes indexed by n.
$\mathbb{O}$	Set of seaports indexed by p, q.
$\mathbb{I}$	Set of candidate dry port locations indexed by p, q.
$\mathbb{J}$	Set of customers indexed by p, q.
$\mathcal{A}$	Set of arcs.
Ω	Set of scenarios indexed by ω.
T	Set of periods indexed by t.
K	Set of containers indexed by k, ℓ, e.
M	Set of available transportation modes indexed by m.
Parameters	
Cap_n	The storage capacity of node $n \in \mathbb{N}$
f_p	The fixed cost for opening a dry port at node $p \in \mathbb{I}$.
d_{pq}	The fixed cost for allocating node p to node q.
c_{pqm}^{k}	The unit cost of transporting container type k on arc $(p, q) \in \mathcal{A}$ using mode m .
h_n	The unit cost of holding an empty container at node $n \in \mathbb{N}$.
g_n^{+}/g_n^{-}	The unit cost of leasing/returning an empty container at node $n \in \mathbb{N} \setminus \{\mathbb{O} \cup \mathbb{J}\}$.
g_n	The unit cost of leased empty containers' net stock per container per period at node $n \in \mathbb{N} \setminus \{\mathbb{O} \cup \mathbb{J}\}$.
v_n^{+}/v_n^{-}	The unit importing/exporting cost of an empty container at node $n \in \mathbb{N} \setminus \{\mathbb{I} \cup \mathbb{J}\}$.
b_q	The unit backorder cost per container at customer $q \in \mathbb{J}$.
ν	The unit cost of rejected demand per container at customer $q \in \mathbb{J}$
τ_{pqm}	The transportation lead-time on arc $(p, q) \in \mathcal{A}$ using mode $m \in M$.

θ	The containers processing time associate with the loading and unloading of finished goods/raw materials at customers.
$\pi(\omega)$	The occurrence probability of scenario ω.
$D^{\ell}_{\mathscr{q}t}(\omega)$	The demand for incoming laden containers at customer $\mathscr{q}$ in period t under scenario ω.
$S^{\ell}_{\mathscr{q}t}(\omega)$	The demand for outgoing laden containers at customer $\mathscr{q}$ in period t under scenario ω.
First-stage Decision Variables	
$X_{\mathscr{p}}$	Binary variable associated with location of dry port $\mathscr{p}$.
$Y_{\mathscr{p}\mathscr{q}}$	Binary variable associated with the demand allocation of an arc from node $\mathscr{p}$ to $\mathscr{q}$.
Second Stage Decision Variables	
$F^{k}_{\mathscr{p}\mathscr{q}tm}(\omega)$	The flow of container type k on arc $(\mathscr{p},\mathscr{q}) \in \mathcal{A}$ in period t using mode m under scenario ω.
$I^{e}_{nt}(\omega)$	The inventory level of empty containers at node $n \in \mathbb{N}$ in period t under scenario ω.
$L^{e}_{nt}(\omega)$	The net stock of leased empty containers at node $n \in \mathbb{I}$ in period t under scenario ω.
$L^{e+}_{nt}(\omega)$	The number of leased empty containers at node $n \in \mathbb{I}$ in period t under scenario ω.
$L^{e-}_{nt}(\omega)$	The number of returned empty containers at node $n \in \mathbb{I}$ in period t under scenario ω.
$H^{e+}_{nt}(\omega)$	The number of imported empty containers at node $n \in \mathbb{O}$ in period t under scenario ω.
$H^{e-}_{nt}(\omega)$	The number of exported empty containers at node $n \in \mathbb{O}$ in period t under scenario ω.
$U^{\ell}_{\mathscr{q}t}(\omega)$	The backordered incoming demand at a customer $\mathscr{q}$ in period t under scenario ω.
$B^{\ell}_{\mathscr{q}t}(\omega)$	The backordered outgoing demand at a customer $\mathscr{q}$ in period t under scenario ω.
$\gamma^{\ell}_{\mathscr{q}t}(\omega)$	The rejected incoming demand at a customer $\mathscr{q}$ in period t under scenario ω.
$\delta^{\ell}_{\mathscr{q}t}(\omega)$	The rejected outgoing demand at a customer $\mathscr{q}$ in period t under scenario ω.

Below we present the second-stage model in which $\mathbf{C}(\omega) = \{F^{k}_{\mathscr{p}\mathscr{q}tm}(\omega),\ I^{e}_{nt}(\omega),\ L^{e}_{nt}(\omega), L^{e+}_{nt}(\omega), L^{e-}_{nt}(\omega), H^{e+}_{nt}(\omega), H^{e-}_{nt}(\omega), U^{\ell}_{\mathscr{q}t}(\omega), B^{\ell}_{\mathscr{q}t}(\omega), \gamma^{\ell}_{\mathscr{q}t}(\omega), \delta^{\ell}_{\mathscr{q}t}(\omega)\}$ denotes the vector of all continuous variables for each scenario ω. This model is applied after the strategic decisions from the first-stage model have been made and the uncertainty in demand is revealed. Therefore, the second-stage model is a collection of deterministic programming models over all possible scenarios. The objective function of the second-stage model for a single scenario $\omega \in \Omega$ is associated with all incurred operational costs, and can be formulated as (6.1) – (6.5):

$$\min_{\mathbf{C}} \mathrm{q}(\mathbf{X}, \omega) = \sum_{(\mathscr{p},\mathscr{q}) \in \mathcal{A}} \sum_{t \in T} \sum_{m \in M} \sum_{k \in K} c^{k}_{\mathscr{p}\mathscr{q}m} F^{k}_{\mathscr{p}\mathscr{q}tm}(\omega) \tag{6.1}$$

$$+ \sum_{t \in T} \sum_{n \in \mathbb{N}} h_n I^{e}_{nt}(\omega) \tag{6.2}$$

$$+\sum_{t\in T}\sum_{n\in\mathbb{N}\backslash\{\mathbb{O}\cup\mathbb{J}\}}\left(g_n^{+}L_{nt}^{e+}(\omega)+g_n^{-}L_{nt}^{e-}(\omega)+g_nL_{nt}^{e}(\omega)\right) \tag{6.3}$$

$$+\sum_{t\in T}\sum_{n\in\mathbb{N}\backslash\{\mathbb{I}\cup\mathbb{J}\}}\left(v_n^{+}H_{nt}^{e+}(\omega)+v_n^{-}H_{nt}^{e-}(\omega)\right) \tag{6.4}$$

$$+\sum_{t\in T}\sum_{\mathscr{q}\in\mathbb{J}} b_{\mathscr{q}}\left(U_{\mathscr{q}t}^{\ell}(\omega)+B_{\mathscr{q}t}^{\ell}(\omega)\right)+\nu\sum_{t\in T}\sum_{\mathscr{q}\in\mathbb{J}}\left(\gamma_{\mathscr{q}t}^{\ell}(\omega)+\delta_{\mathscr{q}t}^{\ell}(\omega)\right) \tag{6.5}$$

Expression (6.1) evaluates the total cost of intermodal transportation of laden and empty containers over all arcs involved in the considered network. This expression includes the transportation cost between dry ports and customers, between seaports and dry ports, and between seaports and customers, over all periods. Expression (6.2) represents the total holding cost of empty containers throughout all nodes in the network over all periods. Expression (6.3) evaluates the total cost associated with empty containers leasing operations, which include the cost of leases, returned containers, and net stock of leased empty containers over the planning periods within the network. Expression (6.4) refers to the total cost of importing/exporting empty containers at seaports over all periods. Finally, expression (6.5) indicates the total cost of backordered as well as rejected demand of incoming and outgoing containers at customers over the planning horizon. In the second stage of the model, the following constraints need to be followed.

- *Containers demand constraints*

Constraint (7) represents the flow of incoming laden containers from all possible dry ports and the set of seaports to a given customer during each period to fulfil its current uncertain demand. The possible backordered demand accumulated from previous periods as well as rejected demand are considered by this constraint. Note that $t-\tau_{\mathscr{p}\mathscr{q}m}$ represents the time when the containers are dispatched using transportation mode m to be delivered to a given customer in period t.

$$\sum_{\mathscr{p}\in\mathbb{I}}\sum_{m\in M} F_{\mathscr{p}\mathscr{q},t-\tau_{\mathscr{p}\mathscr{q}m},m}^{\ell}(\omega)+\sum_{\mathscr{p}\in\mathbb{O}}\sum_{m\in M} F_{\mathscr{p}\mathscr{q},t-\tau_{\mathscr{p}\mathscr{q}m},m}^{\ell}(\omega)+\gamma_{\mathscr{q}t}^{\ell}(\omega)=D_{\mathscr{q}t}^{\ell}(\omega)+U_{\mathscr{q},t-1}^{\ell}(\omega)-U_{\mathscr{q}t}^{\ell}(\omega)$$

$$\forall \mathscr{q}\in\mathbb{J}, t\in T, \omega\in\Omega \tag{7}$$

Constraint (8) refers to the flow of outgoing laden containers, which should be dispatched from a given customer to all possible dry ports and seaports. This constraint aims to meet the current uncertain demand of outgoing (supply) containers from customer $\mathscr{q}$ and their accumulated backordered units from previous periods. The possible rejected demand of outgoing containers is also included here. It should be noted that the customer's outgoing demand in period t would be satisfied, as soon as laden containers are dispatched from the customer in period t. That is why the lead time is excluded from this constraint.

$$\sum_{p \in \mathbb{I}} \sum_{m \in M} F^{\ell}_{qptm}(\omega) + \sum_{p \in \mathbb{O}} \sum_{m \in M} F^{\ell}_{qptm}(\omega) + \delta^{\ell}_{qt}(\omega) = S^{\ell}_{qt}(\omega) + B^{\ell}_{q,t-1}(\omega) - B^{\ell}_{qt}(\omega) \qquad \forall q \in \mathbb{J}, t \in T, \omega \in \Omega \quad (8)$$

- *Container flow conservation constraints*

The flow conservation of laden containers at any possible dry port for raw materials and finished goods are ensured by constraints (9) and (10), respectively. More specifically, constraint (9) specifies that the inward containers from all seaports to a given dry port should be equal to the outflow of those containers from the given dry port to all customers. It should be noted that this constraint corresponds to the forward flow of containers. Constraint (10) relates to the outgoing flow from customers to a given dry port, which is equal to the flow of those containers from the given dry port to all seaports. This constraint reflects the backward flow of containers in the network considered here. Constraints (9) and (10) consider the time lag between the decisions to deploy the containers, i.e. τ_{pqm}.

$$\sum_{q \in \mathbb{O}} \sum_{m \in M} F^{\ell}_{qp,t-\tau_{qpm},m}(\omega) = \sum_{q \in \mathbb{J}} \sum_{m \in M} F^{\ell}_{pqtm}(\omega), \qquad \forall p \in \mathbb{I}, t \in T, \omega \in \Omega \qquad (9)$$

$$\sum_{q \in \mathbb{O}} \sum_{m \in M} F^{\ell}_{pqtm}(\omega) = \sum_{q \in \mathbb{J}} \sum_{m \in M} F^{\ell}_{qp,t-\tau_{pqm},m}(\omega), \qquad \forall p \in \mathbb{I}, t \in T, \omega \in \Omega \qquad (10)$$

- *Container handling constraints*

Container handling constraints determine the inventory level of empty containers which are held in each period. This constraint is applied for each node of the network, including all seaports, dry ports, and customers, in (11), (12), and (13), respectively.

Constraint (11) quantifies the inventory level of available empty containers at a given customer at the end of each period. This is calculated by considering the accumulated inventory of empty containers from the previous periods as well as the inflow and outflow of laden and empty containers in the network. More specifically, the second (third) term of the right-hand side of this constraint includes: i) the inflow of laden and empty containers which are transported from all seaports (dry ports) to a given customer in periods $t - \tau_{pqm} - \theta$ and $t - \tau_{pqm}$, respectively; as well as ii) the outflow of laden and empty containers which are transported from a given customer to all seaports (dry ports) in periods $t + \theta$ and t, respectively. The arriving laden containers are emptied within a specific processing time θ at the given customer and then counted as empty containers for period t. Moreover, the inventory level of empty containers at a given customer is reduced when they are loaded with goods with a specific processing time of θ in order to be sent-out to the dry ports and seaports.

$$
\begin{aligned}
I^{e}_{q,t}(\omega) \\
&= I^{e}_{q,t-1}(\omega) + \sum_{p\in\mathbb{O}} \sum_{m\in M} \left(F^{\ell}_{pq,t-\tau_{pqm}-\theta,m}(\omega) + F^{e}_{pq,t-\tau_{pqm},m}(\omega) - F^{\ell}_{qp,t+\theta,m}(\omega) - F^{e}_{qptm}(\omega) \right) \\
&+ \sum_{p\in\mathbb{I}} \sum_{m\in M} \left(F^{\ell}_{pq,t-\tau_{pqm}-\theta,m}(\omega) + F^{e}_{pq,t-\tau_{pqm},m}(\omega) - F^{\ell}_{qp,t+\theta,m}(\omega) - F^{e}_{qptm}(\omega) \right), \forall q \in \mathbb{J}, t \\
&\in T, \omega \in \Omega \qquad (11)
\end{aligned}
$$

Constraint (12) represents the inventory level of available empty containers at each dry port in each period. This is equal to the accumulated inventories from previous periods as well as the inflow and outflow of empty containers in the network. The second (third) term of the right-hand side of this constraint considers the inflow of empty containers which are transported from the seaports (customers) to dry ports and the outflow of empty containers which are transported from the dry ports to the seaports. Note that, we also take the number of leased and returned empty containers into consideration at each dry port in each period.

$$
\begin{aligned}
I^{e}_{q,t}(\omega) \\
&= I^{e}_{q,t-1}(\omega) + \sum_{p\in\mathbb{O}} \sum_{m\in M} \left(F^{e}_{pq,t-\tau_{pqm},m}(\omega) - F^{e}_{qptm}(\omega) \right) \\
&+ \sum_{p\in\mathbb{J}} \sum_{m\in M} \left(F^{e}_{pq,t-\tau_{pqm},m}(\omega) - F^{e}_{qptm}(\omega) \right) + L^{e+}_{qt}(\omega) - L^{e-}_{qt}(\omega), \quad \forall q \in \mathbb{I}, t \in T, \omega \in \Omega \qquad (12)
\end{aligned}
$$

Constraint (13) specifies the inventory level of available empty containers at the seaport. It takes into account the accumulated inventories from the previous periods as well as the inflow and outflow of empty containers. The second (third) term of the right-hand side of this constraint is associated with the inflow of empty containers which are transported from the dry ports (customers) to the seaports and the outflow of empty containers which are transported from the seaports to the dry ports (customers). At each seaport, if the inventory level of empty containers is insufficient to meet the demand, empty containers can be imported from other seaports. The returned and exported empty containers are also considered in this constraint.

$$I^{e}_{qt}(\omega) = I^{e}_{q,t-1}(\omega) + \sum_{p\in\mathbb{I}}\sum_{m\in M}\left(F^{e}_{pq,t-\tau_{pqm},m}(\omega) - F^{e}_{qptm}(\omega)\right) + \sum_{p\in\mathbb{J}}\sum_{m\in M}\left(F^{e}_{pq,t-\tau_{pqm},m}(\omega) - F^{e}_{qptm}(\omega)\right) + H^{e+}_{qt}(\omega) - H^{e-}_{qt}(\omega), \quad \forall q \in \mathbb{O}, t\in T, \omega \in \Omega \qquad (13)$$

- *Container inter-balancing constraints*

Container inter-balancing constraints imply that the total outflow of containers at each node of the network, including all seaports, dry ports, and customers, should not exceed the available inventory level of empty containers, which are represented by (14), (15), and (16), respectively.

Constraint (14) ensures that the total outflow of containers from each customer cannot exceed its in-stock empty containers in period t. More specifically, the first term of the left-hand side refers to the total outflow of empty containers in period t and the number of empty containers which will be laden in $t+1$ from a given customer to all possible dry ports. Similarly, the total outflow of empty containers in period t and the number of empty containers which will be laden in $t+1$ from the customers to seaports is shown in the second term of the left-hand side of constraint (14). The inter-balancing constraints give a derived demand for empty containers. The flow of empty containers is driven by the flow of laden containers and determined internally by the shipping lines themselves rather than by external demand.

$$\sum_{q\in\mathbb{I}}\sum_{m\in M}\left(F^{e}_{pqtm}(\omega) + F^{\ell}_{pq,t+1,m}(\omega)\right) + \sum_{q\in\mathbb{O}}\sum_{m\in M}\left(F^{e}_{pqtm}(\omega) + F^{\ell}_{pq,t+1,m}(\omega)\right) \le I^{e}_{pt}(\omega), \quad \forall p \in \mathbb{J}, t\in T, \omega\in\Omega \qquad (14)$$

Constraint (15) ensures that the net outflow of empty containers from a given dry port to all seaports and customers in each period cannot exceed the available inventory level of empty containers at the given dry port. Correspondingly, constraint (16) represents that the net outflow of empty containers from a given seaport to all dry ports and customers should be equal or less than the available inventory level of empty containers at the given seaport.

$$\sum_{q\in\mathbb{O}}\sum_{m\in M} F^{e}_{pqtm}(\omega) + \sum_{q\in\mathbb{J}}\sum_{m\in M} F^{e}_{pqtm}(\omega) \le I^{e}_{pt}(\omega), \qquad \forall p \in \mathbb{I}, t\in T, \omega\in\Omega \qquad (15)$$

$$\sum_{q\in\mathbb{I}}\sum_{m\in M} F^{e}_{pqtm}(\omega) + \sum_{q\in\mathbb{J}}\sum_{m\in M} F^{e}_{pqtm}(\omega) \le I^{e}_{pt}(\omega), \qquad \forall p \in \mathbb{O}, t\in T, \omega\in\Omega \qquad (16)$$

- *Empty containers exchange constraints*

The empty containers exchange constraints guarantee equilibrium between the surplus and deficit of empty containers at the nodes of the network. More specifically, the net empty containers associated with the leased and returned decisions at each seaport and each dry port is given by (17). In addition, constraint (18) verifies that the total number of returned empty containers at each corresponding node (i.e. each seaport and each dry port) is equal or less than the total number of leased empty containers over the planning horizon. Constraint (19) ensures that at a given seaport, the total number of exported empty containers is equal or less than the total number of imported empty containers over the planning horizon.

$$L^{e}_{nt}(\omega) = L^{e}_{n,t-1}(\omega) + L^{e+}_{nt}(\omega) - L^{e-}_{nt}(\omega) \qquad \forall n \in \mathbb{N} \setminus \{\mathbb{O} \cup \mathbb{J}\}, t \in T, \omega \in \Omega \qquad (17)$$

$$\sum_{t \in T} L^{e-}_{nt}(\omega) \leq \sum_{t \in T} L^{e+}_{nt}(\omega) \qquad \forall n \in \mathbb{N} \setminus \{\mathbb{O} \cup \mathbb{J}\}, \omega \in \Omega \qquad (18)$$

$$\sum_{t \in T} H^{e-}_{nt}(\omega) \leq \sum_{t \in T} H^{e+}_{nt}(\omega) \qquad \forall n \in \mathbb{N} \setminus \{\mathbb{I} \cup \mathbb{J}\}, \omega \in \Omega \qquad (19)$$

- *Capacity constraints*

Constraints (20) and (21) represent the limited capacity for storing empty containers at each node involved in the network in each period.

$$I^{e}_{qt}(\omega) \leq Cap_{q} X_{q} \qquad \forall q \in \mathbb{I}, t \in T, \omega \in \Omega \qquad (20)$$

$$I^{e}_{nt}(\omega) \leq Cap_{n} \qquad \forall n \in \mathbb{N} \setminus \mathbb{I}, t \in T, \omega \in \Omega \qquad (21)$$

- *Standard constraints*

The flow of containers throughout the whole network is planned based on the allocation decisions which are made by the first-stage model. This is shown by constraint (22), where $\mathcal{M}$ is a sufficiently big number.

$$F^{k}_{pqtm}(\omega) \leq \mathcal{M}.Y_{pq} \qquad (p,q) \in \mathcal{A},\ t \in T, m \in M, k \in K, \omega \in \Omega \qquad (22)$$

Constraint (23) indicates the standard nonnegative continuous variables of the model.

$$F^{k}_{pqtm}(\omega), I^{e}_{pt}(\omega), L^{e}_{pt}(\omega), L^{e+}_{pt}(\omega), L^{e-}_{pt}(\omega), H^{e+}_{pt}(\omega), H^{e-}_{pt}(\omega), U^{\ell}_{pt}(\omega), B^{\ell}_{pt}(\omega), \gamma^{\ell}_{qt}(\omega), \delta^{\ell}_{qt}(\omega) \geq 0$$

$$\forall p, q \in \mathbb{N}, t \in T, m \in M, k \in K, \omega \in \Omega \qquad (23)$$

3. Solution Procedure

Solving the proposed two-stage stochastic programming model is difficult. The main reason for this difficulty is evaluating the expected value of the objective function for the ‘true’ model. For discrete distributions, the evaluation of expectation involves solving a large number of linear programming models for each scenario that corresponds to an uncertain parameter realisation. We cope with this difficulty using the Sample Average Approximation (SAA) approach (Santoso et al. 2005; Kleywegt, Shapiro, and Homem-De-Mello 2002). The solutions from the SAA approach converge to the optimal ‘true’ objective function value as the scenario sample size increases (Shapiro, Dentcheva, and Ruszczynski 2009). The set of sample scenarios are generated outside of the optimisation procedure using the Monte-Carlo sampling method (Shapiro 2003). The quality of the solutions obtained by SAA is evaluated through a validation analysis with respect to the sample size and the number of samples. This is conducted by calculating the optimality gap between the optimal ‘true’ solution for the problem and the SAA model solution from a given sample of scenarios. Moreover, the Benders decomposition (Benders 1962) is applied to enhance the computational performance of the developed SAA model. We also employ several acceleration methods within the Benders decomposition method to improve the computational time.

3.1. Robust SAA model

In the SAA approach, a set of N scenarios, denoted by $\Omega^N = \{\omega^1, \omega^2, \dots, \omega^N\}$, relating to the realisation of uncertain parameters, i.e. incoming and outgoing demand of customers, is randomly generated using the Monte-Carlo sampling method (Shapiro 2003). The expected value of the ‘true’ objective function is estimated by the average over objective values $\mathrm{q}(\boldsymbol{X}, \omega)$, where $\omega \in \Omega^N$ and N is the number of scenarios randomly generated in an equiprobable manner. Therefore, the expected value of the ‘true’ objective function can be represented by $\sum_{\omega\in\Omega^N} \pi(\omega)\, \mathrm{q}(\mathbf{X}, \omega)$, with $\pi(\omega) = {}^1/_N$. Given the original two-stage stochastic problem (1) – (23), the equivalent deterministic linear programming model is constructed as follows:

$$\begin{aligned}
\min \Bigg\{ & \sum_{p\in\mathbb{I}} f_{p} X_{p} + \sum_{(p,q)\in\mathcal{A}} d_{pq} Y_{pq} \\
& + {}^1/_N \sum_{\omega\in\Omega^N} \Bigg(\sum_{(p,q)\in\mathcal{A}} \sum_{t\in T} \sum_{m\in M} \sum_{k\in K} c^k_{pqm} F^k_{pqtm}(\omega) + \sum_{t\in T} \sum_{n\in\mathbb{N}} h_n I^e_{nt}(\omega) \\
& + \sum_{t\in T} \sum_{n\in\mathbb{N}\setminus\{\mathbb{O}\cup\mathbb{J}\}} \left(g^+_n L^{e+}_{nt}(\omega) + g^-_n L^{e-}_{nt}(\omega) + g_n L^e_{nt}(\omega) \right) + \sum_{t\in T} \sum_{n\in\mathbb{N}\setminus\{\mathbb{I}\cup\mathbb{J}\}} \left(v^+_n H^{e+}_{nt}(\omega) + v^-_n H^{e-}_{nt}(\omega) \right) \\
& + \sum_{t\in T} \sum_{q\in\mathbb{J}} b_{q} \left(U^{\ell}_{qt}(\omega) + B^{\ell}_{qt}(\omega) \right) \Bigg) \Bigg\}, \qquad (24)
\end{aligned}$$

subject to constraint sets (2) – (5), and constraint sets (7) – (23), where the first two terms in (24) represent the first-stage objective function and the third term represents the expected objective function of the second-stage problem.

The SAA method is applicable when a feasible solution exists and the problem has a finite objective value (Shapiro 2003). However, in the context considered here, the customers' uncertain incoming and outgoing demand may not have an identical distribution function or known distribution parameters. Hence, the SAA method is likely to obtain sub-optimal solutions in at least one scenario. To tackle this challenge, we present a robust counterpart problem for the mentioned SAA method to address solution robustness using an approach proposed by Mulvey, Vanderbei, and Zenios (1995).

In order to present the robust counterpart problem, we apply the modelling approach proposed by Yu and Li (2000) which is an extension to the work of Mulvey, Vanderbei, and Zenios (1995). This approach aims to achieve solution and model robustness. The former is aims to find solutions which are optimal for all possible scenario realisations of the uncertain parameters; the latter is aims to guarantee the feasibility of the obtained solutions over scenario realisations by considering a penalty function (see Mulvey et al. 1995). Below we discuss this robust modelling approach.

The solution robustness of the SAA model, denoted by $\Phi(\omega)$, is built using the absolute deviation of the second-stage objective values over the number of scenarios, as follows:

$$\Phi(\mathbf{X},\omega) = \left| \mathrm{q}(\mathbf{X},\omega) - \sum_{\omega' \in \Omega \setminus \{\omega\}} \pi(\omega')\mathrm{q}(\mathbf{X},\omega') \right|. \qquad \forall \omega \in \Omega \tag{25}$$

Expression (25), which computes the solution robustness, should be included in the objective function of the SAA model. However, this makes the SAA model non-linear due to the presence of absolute function. Hence, we apply a linearization method to ensure the solution space convexity.

Proposition 1. *As the minimisation objective function of the proposed robust SAA model contains the expression (25), we can substitute it by the following expressions:*

$$\Upsilon(\mathbf{X},\omega) = \mathrm{q}(\mathbf{X},\omega) - \sum_{\omega' \in \Omega - \{\omega\}} \pi(\omega')\mathrm{q}(\mathbf{X},\omega') + 2\rho(\omega), \qquad \forall \in \Omega^N \tag{26}$$

where,

$$\mathrm{q}(\mathbf{X},\omega) - \sum_{\omega' \in \Omega - \{\omega\}} \pi(\omega')\mathrm{q}(\mathbf{X},\omega') + \rho(\omega) \geq 0, \qquad \forall \omega \in \Omega^N \tag{27}$$

$$\rho(\omega) \geq 0. \qquad \forall \omega \in \Omega^N \tag{28}$$

Below, we verify two possible cases of the proposition.

Case 1 is where $\mathrm{q}(\mathbf{X},\omega)-\sum_{\omega'\in\Omega-\{\omega\}}\pi(\omega')\mathrm{q}(\mathbf{X},\omega')\geq 0$, then it is clear that $\rho(\omega)=0$, when minimizing expression (26). In this case, $\Upsilon(\mathbf{X},\omega)=\mathrm{q}(\mathbf{X},\omega)-\sum_{\omega'\in\Omega-\{\omega\}}\pi(\omega')\mathrm{q}(\mathbf{X},\omega')=\Phi(\mathbf{X},\omega)$.

Case 2 is where $\mathrm{q}(\mathbf{X},\omega)-\sum_{\omega'\in\Omega-\{\omega\}}\pi(\omega')\mathrm{q}(\mathbf{X},\omega')<0$. Considering the minimisation of $\Upsilon(\mathbf{X},\omega)$, we then have $\rho(\omega)=\sum_{\omega'\in\Omega-\{\omega\}}\pi(\omega')\mathrm{q}(\mathbf{X},\omega')-\mathrm{q}(\mathbf{X},\omega)$ which results in $\Upsilon(\mathbf{X},\omega)=\sum_{\omega'\in\Omega-\{\omega\}}\pi(\omega')\mathrm{q}(\mathbf{X},\omega')-\mathrm{q}(\mathbf{X},\omega)=\Phi(\mathbf{X},\omega)$. For more information regarding this linearisation method, refer to Yu and Li (2000).

The stochastic programming model is very likely to return infeasible solutions due to high variability of scenario realisations (Birge and Louveaux 2011). However, there is no constraint violation in our proposed model as it possesses complete recourse. More specifically, the incorporation of decisions regarding to backordered and rejected demand in constraints (7) and (8) prevents the model from being infeasible. The problem formulation considering solution robustness is addressed by the following equations:

$$\begin{aligned}\min\Bigg\{&\sum_{p\in\mathbb{I}} f_p X_p+\sum_{(p,q)\in\mathcal{A}} d_{pq}Y_{pq}\\ &+{}^{1}\!/_{N}\sum_{\omega\in\Omega^N}\Bigg(\sum_{(p,q)\in\mathcal{A}}\sum_{t\in T}\sum_{m\in M}\sum_{k\in K} c_{pqm}^{k}F_{pqtm}^{k}(\omega)+\sum_{t\in T}\sum_{n\in\mathbb{N}} h_n I_{nt}^{e}(\omega)\\ &+\sum_{t\in T}\sum_{n\in\mathbb{N}\backslash\{\mathbb{O}\cup\mathbb{J}\}}\left(g_n^{+}L_{nt}^{e+}(\omega)+g_n^{-}L_{nt}^{e-}(\omega)+g_n L_{nt}^{e}(\omega)\right)+\sum_{t\in T}\sum_{n\in\mathbb{N}\backslash\{\mathbb{I}\cup\mathbb{J}\}}\left(v_n^{+}H_{nt}^{e+}(\omega)+v_n^{-}H_{nt}^{e-}(\omega)\right)\\ &+\sum_{t\in T}\sum_{q\in\mathbb{J}} b_q\left(U_{qt}^{\ell}(\omega)+B_{qt}^{\ell}(\omega)\right)+\nu\sum_{t\in T}\sum_{q\in\mathbb{J}}\left(\gamma_{qt}^{\ell}(\omega)+\delta_{qt}^{\ell}(\omega)\right)+\lambda\Upsilon(\omega)\Bigg)\Bigg\},\end{aligned}\tag{29}$$

subject to constraints (2) – (5), (7) – (23), and (26) – (28), where λ is the cost of solution robustness. More precisely, λ refers to the weighting scale to measure the trade-off between the cost and dispersion minimization of second-stage objective values over all scenarios.

3.2. The SAA algorithm

In the previous subsection, we have obtained the robust counterpart problem formulation for the SAA model. Hereafter we refer to this model as the "robust SAA problem". In this section, a validation analysis is presented that estimates the optimality gap between the objective value associated with the robust SAA problem, and the expected value of the 'true' problem (1) – (23).

Let $\mathcal{Z}_N^*(\mathbf{X}_N^*, \mathbf{C}_N^*)$ be the optimal objective value of the robust SAA model with scenario sample size *N*, where $\mathbf{X}_N^*$ and $\mathbf{C}_N^*$ denote the optimal solution vector of the first-stage and the second-stage model, respectively. When the sample size *N* increases towards infinity, it leads $\mathcal{Z}_N^*(\mathbf{X}_N^*, \mathbf{C}_N^*)$ to converge to the optimal value for the 'true' solution. However, obtaining the optimal value of the true problem, $\mathcal{Z}^*(\mathbf{X}^*, \mathbf{C}^*)$, involves solving the problem for an enormously large number of scenarios. Therefore, we provide statistical confidence intervals that estimate lower and upper bounds on the quality of the approximate solutions based on the work of Shapiro, Dentcheva, and Ruszczynski (2009). We estimate the statistical lower bound and the statistical upper bound for the true optimal objective by *averaging* and *sampling* procedures, respectively. We finally calculate the optimality gap of the objective values. Below we represent these procedures.

3.2.1. Averaging procedure

In this procedure, a valid lower bound for the optimal value $\mathcal{Z}^*(\mathbf{X}^*, \mathbf{C}^*)$ for the 'true' problem is estimated. In order to do that, *R* independent samples with the size of *N* scenarios are generated. Let $\mathcal{Z}_N^r(\mathbf{X}_N^r, \mathbf{C}_N^r)$ be the optimal objective value of sample $r = 1, \dots, R$, of the robust SAA model with *N* scenarios, and $(\mathbf{X}_N^r, \mathbf{C}_N^r)$ be the corresponding solution vectors. We compute the average of the *R* replications of the robust SAA model as follows:

$$\bar{\mathcal{Z}}_N^R(\mathbf{X}_N^R, \mathbf{C}_N^R) = \frac{1}{R}\sum_{r=1}^{R} \mathcal{Z}_N^r(\mathbf{X}_N^r, \mathbf{C}_N^r) \tag{30}$$

which is an unbiased estimator of the lower bound for the expectation of optimal value $\mathcal{Z}^*(\mathbf{X}^*, \mathbf{C}^*)$. Since the generated samples are independent and have identical distribution, the standard deviation of this estimator can be computed as follows:

$$\hat{\sigma}_N^R = \sqrt{\frac{1}{(R-1)R}\sum_{r=1}^{R}\left(\mathcal{Z}_N^r(\mathbf{X}_N^r, \mathbf{C}_N^r) - \bar{\mathcal{Z}}_N^R(\mathbf{X}_N^R, \mathbf{C}_N^R)\right)^2} \tag{31}$$

We can apply an approximate 100(1-α) % confidence lower bound for the expectation of optimal $\mathcal{Z}^*(\mathbf{X}^*, \mathbf{C}^*)$ using the average and standard deviation of *R* SAA programs as follows:

$$\mathcal{L}_{N,1-\alpha}^R = \bar{\mathcal{Z}}_N^R(\mathbf{X}_N^R, \mathbf{C}_N^R) - \mathbf{t}_{\alpha,R-1}\hat{\sigma}_N^R \tag{32}$$

Here $\mathbf{t}_{\alpha,R-1}$ is the α-critical value of the t-distribution with $R-1$ degrees of freedom.

3.2.2. Sampling Procedure

The valid estimate of an upper bound on the expectation of optimal $\mathcal{Z}^*(\mathbf{X}^*, \mathbf{C}^*)$ can be computed by sampling. This can be obtained by solving the second-stage of the robust SAA model for a large enough sample of N' scenarios ($\omega \in \Omega^{N'} \subset \Omega$) generated independently, where $N' \gg N$. Let $\overline{\mathbf{X}}_N$ denote the first-stage solution of the initial robust SAA model with sample size N. In this case, we use the best computed $\overline{\mathbf{X}}_N$ among R replications as an input. We denote by $\hat{\mathcal{Z}}_{N'}(\overline{\mathbf{X}}_N, \hat{\mathbf{C}}_{N'})$ the optimal objective value of the robust SAA model with sample size N'. As a result, we have $\hat{\mathcal{Z}}_{N'}(\overline{\mathbf{X}}_N, \hat{\mathbf{C}}_{N'}) > \mathcal{Z}^*(\mathbf{X}^*, \mathbf{C}^*)$ since $\overline{\mathbf{X}}_N$ is a feasible solution of the true problem. Therefore, $\hat{\mathcal{Z}}_{N'}(\overline{\mathbf{X}}_N, \hat{\mathbf{C}}_{N'})$ is an estimation of an upper bound of $\mathcal{Z}^*(\mathbf{X}^*, \mathbf{C}^*)$. It should be clear that $\hat{\mathcal{Z}}_{N'}(\overline{\mathbf{X}}_N, \hat{\mathbf{C}}_{N'})$ involves solving N' second-stage models which produces an optimal objective value per scenario denoted by $\hat{\mathcal{Z}}_{\omega}(\overline{\mathbf{X}}_N, \hat{\mathbf{C}}_{\omega})$, where $\hat{\mathbf{C}}_{\omega}$ is the solution of the robust SAA second-stage model of scenario $\omega \in \Omega^{N'} \subset \Omega$. Obtaining $\hat{\mathcal{Z}}_{N'}(\overline{\mathbf{X}}_N, \hat{\mathbf{C}}_{N'})$ involves solving the second-stage model per scenario at a time, $\omega \in \Omega^{N'} \subset \Omega$. This is much easier to solve these problems for one scenario at a time. Additionally, since an independent and identical distribution is used to generate the sample N', we can estimate the variance of this upper bound as given in (33):

$$\sigma_{N'}^2(\overline{\mathbf{X}}) = \frac{1}{(N'-1)N'} \sum_{\omega=1}^{N'} (\hat{\mathcal{Z}}_{\omega}(\overline{\mathbf{X}}_N, \hat{\mathbf{C}}_{\omega}) - \hat{\mathcal{Z}}_{N'}(\overline{\mathbf{X}}_N, \hat{\mathbf{C}}_{N'}))^2 \tag{33}$$

Using the above mean and variance, an approximate 100 (1-α) % confidence upper bound for the expectation of optimal $\mathcal{Z}^*(\mathbf{X}^*, \mathbf{C}^*)$ is given as:

$$\mathcal{U}_{N',1-\alpha} = \hat{\mathcal{Z}}_{N'}(\overline{\mathbf{X}}_N, \hat{\mathbf{C}}_{N'}) + \mathbf{z}_{\alpha}\, \sigma_{N'}(\overline{\mathbf{X}}) \tag{34}$$

where $\mathbf{z}_{\alpha}$ is the standard normal critical value with a 100(1-α)% confidence level. Consequently, an approximate 100(1-α)% confidence interval for the expectation of optimal $\mathcal{Z}^*(\mathbf{X}^*, \mathbf{C}^*)$ is obtained by $(\mathcal{L}_{N,1-\alpha}^R, \mathcal{U}_{N',1-\alpha})$. Then, the statistical optimality gap and the statistical optimality gap percentage can be calculated by (35) and (36), respectively:

$$gap_{N,R,N'} = \mathcal{U}_{N',1-\alpha} - \mathcal{L}_{N,1-\alpha}^R \tag{35}$$

$$gap_{N,R,N'}\% = \frac{gap_{N,R,N'}}{\mathcal{U}_{N',1-\alpha}} \times 100\% \tag{36}$$

The SAA algorithm corresponding to the objective function (29) subject to constraints (2) – (5), (7) – (23), and (26) – (28) is summarized in Procedure 1.

```
Inputs:
```
$N, N', R \in \mathcal{N}, \alpha \in [0,1]$, where $\mathcal{N}$ represents the set of Natural numbers.

```
1. Pre-processing
```

For sample $r = 1 \dots R$

Run the Monte-Carlo sampling method to generate random numbers for demand of raw materials' containers with the given distribution function.

2. Averaging procedure (N, R, α)

a. For sample $r = 1 \dots R$

Solve the robust SAA model (29) s.t. (2) – (5), (7) – (23), and (26) – (28), and obtain the objective value $\mathcal{Z}_N^r(\mathbf{X}_N^r, \mathbf{C}_N^r)$ and solution $(\mathbf{X}_N^r, \mathbf{C}_N^r)$.

b. Calculate the average and variance of the objective functions of R robust SAA models using (30) and (31), respectively.

c. Compute an approximate 100(1-α) % confidence lower bound using (32).

d. Calculate $\mathcal{Z}_N^R = \min_r\{\mathcal{Z}_N^r(\mathbf{X}_N^r, \mathbf{C}_N^r)\}$ and $\overline{\mathbf{X}}_N = \text{argmin}\{\mathcal{Z}_N^R\}$, which the latter corresponds to the best first-stage design solution found among R samples of the robust SAA model with size N.

3. Sampling procedure ($\overline{\mathbf{X}}_N$, N', α)

a. For scenario $\omega = 1$ to N'

Solve the robust SAA model (29) s.t. (2) – (5), (7) – (23), and (26) – (28) for ω with the obtained first-stage design solution $\overline{\mathbf{X}}_N$ and get the optimal objective value $\hat{\mathcal{Z}}_\omega(\overline{\mathbf{X}}_N, \hat{\mathbf{C}}_\omega)$.

b. Compute the average of optimal objective values as $\hat{\mathcal{Z}}_{N'}(\overline{\mathbf{X}}_N, \hat{\mathbf{C}}_{N'})$ and its variance using (33).

c. Compute an approximate 100(1-α) % confidence upper bound using (34).

4. Optimality gap

Calculate the statistical optimality gap percentage using (36). If this gap is acceptable, stop. Otherwise, increase N and/or R and return to step 1.

Output: An approximate 100(1-α) % confidence interval for the expectation of optimal $\mathcal{Z}^*(\mathbf{X}^*, \mathbf{C}^*)$ and the associated robust solution $(\mathbf{X}^*, \mathbf{C}^*)$.

Procedure 1. The SAA algorithm corresponding to the robust model.

3.3. Benders Decomposition Algorithm

In this section, we improve the abovementioned SAA algorithm by employing the Benders decomposition (BD) method as well as several acceleration schemes. Step 2.a. of the outlined SAA algorithm in procedure 1 requests to solve the robust SAA model repetitively. Although this model contains much fewer scenarios than the true problem (1)–(23), this two-stage stochastic model is an NP-hard problem which has the same NP-hardness property as a classical capacitated facility location problem (see Balinski, 1970; Gen and Cheng, 2000). Therefore, we propose a solution approach inspired by a method introduced by Van Slyke and Wets (1969), which is the stochastic version of classical Benders decomposition applied to two-stage stochastic programming problems. The effectiveness of the method for solving various large optimisation problems including SAA two-stage stochastic programming has been demonstrated in the previous studies, e.g., Santoso et al (2005). In the following, we describe the Benders decomposition method applying to enhance the SAA algorithm corresponding to the robust model.

In order to implement the Benders decomposition method, we need to separate the robust SAA problem into a master problem (MP) that involves first-stage decision variables, and Benders sub problems (BSP) to optimize the second stage decision variables. The BSP of the proposed model can be formulated by fixing the first-stage variables to the given values at iteration it. The objective function of the BSP is:

$$\min\Big\{{}^{1}/_{N}\sum_{\omega\in\Omega^N}\Big(\sum_{(p,q)\in\mathcal{A}}\sum_{t\in T}\sum_{m\in M}\sum_{k\in K}c^k_{pqm}F^k_{pqtm}(\omega)+\sum_{t\in T}\sum_{n\in\mathbb{N}}h_n I^e_{nt}(\omega)+\sum_{t\in T}\sum_{n\in\mathbb{N}\backslash\{\mathbb{O}\cup\mathbb{J}\}}\big(g^+_n L^{e+}_{nt}(\omega)+g^-_n L^{e-}_{nt}(\omega)+g_n L^e_{nt}(\omega)\big)+\sum_{t\in T}\sum_{n\in\mathbb{N}\backslash\{\mathbb{I}\cup\mathbb{J}\}}\big(v^+_n H^{e+}_{nt}(\omega)+v^-_n H^{e-}_{nt}(\omega)\big)+\sum_{t\in T}\sum_{q\in\mathbb{J}}b_q\left(U^\ell_{qt}(\omega)+B^\ell_{qt}(\omega)\right)+\nu\sum_{t\in T}\sum_{q\in\mathbb{J}}\left(\gamma^\ell_{qt}(\omega)+\delta^\ell_{qt}(\omega)\right)+\lambda\Upsilon(\omega)\Big)\Big\} \quad (37)$$

The objective function (37) is constructed from objective function (29) by excluding the binary terms, i.e. $\sum_{p\in\mathbb{I}}f_p X_p+\sum_{(p,q)\in\mathcal{A}}d_{pq}Y_{pq}$. The constraints for the BSP are equations (7) – (23), and (26) – (28) in which the first-stage variables have been fixed to given values. The given values are updated at each iteration from solving the following MP. Before presenting the MP, it should be pointed out since our modelling approach possesses complete recourse, the BSP is feasible for the given values of first-stage variables, and an optimality cut can be deducted from an optimal solution to the dual of the sub-problem (DSP).

Let $\boldsymbol{\chi}$ be the vector of the DSP's variables corresponding to constraints (7) – (23), and (26) – (28). $\hat{\chi}$ denotes an element in $\boldsymbol{\chi}$, and also the extreme points of the dual polyhedron obtained from solving the dual sub-problem (DSP). The superscripts associated with $\hat{\chi}$ indicate the corresponding constraint. For example, $\hat{\chi}^{20}_{qt}(\omega)$ denotes the dual variable relating to constraint 20 for $\in\mathbb{I}, t\in T$. The MP, which produces a lower bound (LB) for the objective function of original robust SAA model at each iteration it, can be formulated based on the defined DSP variables as follows:

$$\text{MP: }\min\left\{\sum_{p\in\mathbb{I}}f_p X_p+\sum_{(p,q)\in\mathcal{A}}d_{pq}Y_{pq}+\gamma\right\} \quad (38)$$

s.t. (2) – (5),

$$\gamma \geq \sum_{\omega\in\Omega^N} {}^{1}/_{N}\Bigg(\sum_{t\in T}\Bigg[\sum_{k\in K}\sum_{\mathscr{p},\mathscr{q}\in\mathbb{N}}\sum_{m\in M} D^{\ell}_{\mathscr{q}t}(\omega)\,\hat{\chi}^{7}_{k\mathscr{p}\mathscr{q}tm}(\omega) + \sum_{k\in K}\sum_{\mathscr{p},\mathscr{q}\in\mathbb{N}}\sum_{m\in M} S^{\ell}_{\mathscr{q}t}(\omega)\,\hat{\chi}^{8}_{k\mathscr{p}\mathscr{q}tm}(\omega)$$
$$-\sum_{\mathscr{q}\in\mathbb{I}} Cap_{\mathscr{q}} X_{\mathscr{q},it}\,\hat{\chi}^{20}_{\mathscr{q}t}(\omega) - \sum_{\mathscr{q}\in\,\forall n\in\mathbb{N}\setminus\mathbb{I}} Cap_{\mathscr{q}}\,\hat{\chi}^{21}_{\mathscr{q}t}(\omega) \tag{39}$$
$$-\sum_{k\in K}\sum_{\mathscr{p},\mathscr{q}\in\mathbb{N}}\sum_{m\in M} \mathcal{M}.\,Y_{\mathscr{p}\mathscr{q},it}\,\hat{\chi}^{22}_{k\mathscr{p}\mathscr{q}tm}(\omega)\Bigg]\Bigg)$$

$$\gamma \geq 0 \tag{40}$$

Constraint (39) determines the optimality cut. At each iteration of BD, we first solve the MP to obtain the values of first-stage decisions. Then, these values are used to solve DSP to obtain an extreme point and a new optimality cut (39) is included in the MP. This procedure is conducted iteratively until the stopping criterion is met. The stopping criterion is established as a small percentage gap between the best upper and lower bounds. Although the described Benders decomposition method is a finite scheme, this algorithm may require a large number of iterations to converge for large optimization models (Santoso et al. 2005). In order to improve the slow convergence of outlined Benders decomposition method, we use a number of accelerating method in the follwing subsections.

3.3.1. Multi-cut framework

The number of iterations within Benders decomposition algorithm can be reduced significantly using the multi-cut framework since more dual information is provided for the MP as shown by (Birge and Louveaux 1988). In the abovementioned BD method, only one cut is added at each iteration, which approximates the sample average of the second-stage objective value function at the current solution. Instead, we add N cuts to the MP by decomposing the BSP into several independent BSPs in each iteration. These cuts approximate the independent second-stage objective functions corresponding to each of individual N scnearios. Thus, we determine the cuts for each scenario and define the new MP as follows:

$$\min\left\{\sum_{\mathscr{p}\in\mathbb{I}} f_{\mathscr{p}} X_{\mathscr{p}} + \sum_{(\mathscr{p},\mathscr{q})\in\mathcal{A}} d_{\mathscr{p}\mathscr{q}} Y_{\mathscr{p}\mathscr{q}} + \sum_{\omega\in\Omega^N}\gamma(\omega)\right\} \tag{41}$$

s.t. (2) – (5),

$$\gamma(\omega) \geq \sum_{t\in T}\Bigg[\sum_{k\in K}\sum_{p,q\in\mathbb{N}}\sum_{m\in M} D^{\ell}_{qt}(\omega)\,\hat{\chi}^{7}_{kpqtm}(\omega) + \sum_{k\in K}\sum_{p,q\in\mathbb{N}}\sum_{m\in M} S^{\ell}_{qt}(\omega)\,\hat{\chi}^{8}_{kpqtm}(\omega) - \sum_{q\in\mathbb{I}} Cap_{q} X_{q,it}\,\hat{\chi}^{20}_{qt}(\omega) - \sum_{q\in\,\forall n\in\mathbb{N}\setminus\mathbb{I}} Cap_{q}\,\hat{\chi}^{21}_{qt}(\omega) - \sum_{k\in K}\sum_{p,q\in\mathbb{N}}\sum_{m\in M} \mathcal{M}.Y_{pq,it}\,\hat{\chi}^{22}_{kpqtm}(\omega)\Bigg] \tag{42}$$

$$\gamma(\omega) \geq 0, \quad \omega \in \Omega^{N} \tag{43}$$

Applying this multi-cut framework can provide a better approximation of the sample average of the second-stage objective value functions due to the disaggregation of the optimulity cut. This accelerated method results in fewer number of BD iterations at the expense of a larger MP.

3.3.2. Knapsack Inequalities

Santoso et al (2005) showed that including knapsack inequalities together with optimality cut leads to an improved solution from the MP. They indicated that state-of-the-art solvers such as CPLEX can derive a variety of valid inequalities from the knapsack inequality, which accelerates the convergence of the Benders decomposition method. Let UB^{*} be the current best known upper bound. Since $UB^{*} > \sum_{p\in\mathbb{I}} f_{p}X_{p} + \sum_{(p,q)\in\mathcal{A}} d_{pq}Y_{pq} + \sum_{\omega\in\Omega^{N}}\gamma_{\omega}$, we can add the following valid knapsack inequality to the master problem in iteration $it+1$:

$$\sum_{p\in\mathbb{I}} f_{p}X_{p} + \sum_{(p,q)\in\mathcal{A}} d_{pq}Y_{pq} - \sum_{q\in\mathbb{I}} Cap_{q}X_{q,it}\,\hat{\chi}^{20}_{qt}(\omega) - \sum_{k\in K}\sum_{p,q\in\mathbb{N}}\sum_{m\in M}\mathcal{M}.Y_{pq,it}\,\hat{\chi}^{22}_{kpqtm}(\omega) \leq UB^{*} - \sum_{t\in T}\sum_{k\in K}\sum_{p,q\in\mathbb{N}}\sum_{m\in M} D^{\ell}_{qt}(\omega)\,\hat{\chi}^{7}_{kpqtm}(\omega) - \sum_{t\in T}\sum_{k\in K}\sum_{p,q\in\mathbb{N}}\sum_{m\in M} S^{\ell}_{qt}(\omega)\,\hat{\chi}^{8}_{kpqtm}(\omega) + \sum_{t\in T}\sum_{q\in\,\forall n\in\mathbb{N}\setminus\mathbb{I}} Cap_{q}\,\hat{\chi}^{21}_{qt}(\omega) \tag{44}$$

3.3.3. Pareto-optimal cuts generation scheme

In this section, we use an acceleration procedure proposed by Magnanti and Wong (1981) to streghnthen the optimality cuts of Benders decomposition method by generating Pareto-optimal cuts. An optimality cut is called pareto-optimal if there is no other cut to make it redundant. Likewise, the optimal dual solution corresponding to the Pareto-optiml cut is reffered to as Pareto-optimal. In BSPs with network structure as in our study, the DSP normally has multiple optimal solutions, among which the Pareto-optimal generates the strongest cut. To generate a Pareto-optimal cut, consider our robust two-stage stochastic model as the general problem $Min\ c_1^T \boldsymbol{C} + c_2^T \mathbf{X}$, s.t. $A\boldsymbol{C} + B\mathbf{X} \geq b, \boldsymbol{C} \geq 0, \mathbf{X} \in \{0,1\}$. Fixing integer variables $\mathbf{X} = \widetilde{\mathbf{X}}$, we can write the general form of SP as $Min\ c_1^T \boldsymbol{C}$, s.t. $A\boldsymbol{C} \geq b - B\,\widetilde{\mathbf{X}}$, $\boldsymbol{C} \geq 0$ and then its DSP is $Max\ \left(b - B\,\widetilde{\mathbf{X}}\right)^T \boldsymbol{\chi}$, s.t. $A^T\boldsymbol{\chi} \leq c_1^T, \boldsymbol{\chi} \geq 0$. Let $\mathbf{X}^c$ be a core point of the solution space of MP, and $\boldsymbol{\chi}^*$ be the optimal solution of the DSP. A Pareto-optimal cut can be obtained by solving the following problem, which is reffered to as Magnati-Wong problem:

$$Max\ (b - B\,\mathbf{X}^c\,)^T \boldsymbol{\chi}, \text{ s.t. } A^T\boldsymbol{\chi} \leq c_1^T, \left(b - B\,\widetilde{\mathbf{X}}\right)^T \boldsymbol{\chi} \leq \left(b - B\,\widetilde{\mathbf{X}}\right)^T \boldsymbol{\chi}^*, \boldsymbol{\chi} \geq 0 \tag{45}$$

At each iteration, the challeng is to identify and update a core point which should be lie inside the relative interior of the convex hull of the sub-region defined by the MP variables. To combat this problem, Papadakos (2008) proved that instead of a core point $\mathbf{X}^c$, one can use a convex combination of the current MP solution and the previously used core point to obtain a new core point at each iteration as $\mathbf{X}_{it}^c \leftarrow \varphi \mathbf{X}_{it-1}^c + (1-\varphi)\mathbf{X}_{it}^{MP}, 0 < \varphi < 1$. It should be noted that for the first iteration, $\mathbf{X}_0^c$ is set to the solution of MP.

In the following we summarize the proposed accelerated Benders decomposition algorithm. In the first step, we generate initial feasible solutions for DSP and MP. In step 2, the MP with multi-cut framework and knapsack inequalities is generated and then solved to obtain a lower bound and the first-stage solutions. In the next step, we solve the DSP, generate the Pareto-optimal cut from the corresponding Magnati-Wong problem's solutions, and update the upper bound. Finally, we add the Pareto-optimal cut to the MP and update the core point. The pseudo-code of the proposed accelerated Benders decomposition algorithm is illustrated in Procedure 2.

Inputs: $UB_0 = +\infty$, $LB_0 = -\infty$, $\widetilde{\mathbf{X}}_0$, φ, ϵ

1. Initialisation
   a. Solve the DSP corresponding to arbitrary feasible solutions of $\widetilde{\mathbf{X}}_0$ to obtain an initial dual feasible solution $\widetilde{\boldsymbol{\chi}}_0$
   b. Solve MP and set the core point equal to its solution

c. $it = 0$

While $(UB_{it} - LB_{it}) > \epsilon$

2. Master problem $(\widetilde{\boldsymbol{\chi}}_{it})$

Solve MP with multi-cut and knapsack inequalities, i.e. (41)– (44), using $\widetilde{\boldsymbol{\chi}}_{it}$

a. Update LB_{it}

b. Update $\widetilde{\mathbf{X}}_{it}$

3. Dual sub-problem $(\widetilde{\mathbf{X}}_{it}, \omega \in \Omega^N)$

For each scenario $\omega \in \Omega^N$ solve DSPs using $\widetilde{\mathbf{X}}_{it}$

If solved to optimality

a. Generate a Pareto-optimal cut

b. Update $\widetilde{\boldsymbol{\chi}}_{it}$

c. Update UB_{it}

End if

4. Update

Add generated cuts to the MP

a. $it = it + 1$

b. Update the core point $(\mathbf{X}^{c}_{it} \leftarrow \varphi \mathbf{X}^{c}_{it-1} + (1-\varphi)\mathbf{X}^{MP}_{it}, 0 < \varphi < 1)$

End while

Output: Return $\widetilde{\mathbf{X}}_{it}$ as the optimal solution, and UB_{it} as the optimal objective value.

Procedure 2. The accelerated Benders decomposition algorithm

4. Key performance indicators

In the robust SAA model presented above, we used a cost-based objective function for designing dry port container networks. In this section we define three further key performance indicators (KPIs): *service level*, *fill rate*, and *inventory turnover*, to provide further insights into the performance of network designs. The three KPIs are supplements to the cost-based objective function and can be calculated based on the optimal solution of the robust SAA model.

Let denote $\widehat{U}^{\ell}_{qt}(\omega)$ and $\widehat{B}^{\ell}_{qt}(\omega)$ as the solution value of variables $U^{\ell}_{qt}(\omega)$ and $B^{\ell}_{qt}(\omega)$, respectively. We also denote service level, which corresponds to the demand of raw materials and the supply of finished goods for each scenario by $\eta_r(\omega)$ and $\eta_s(\omega)$, respectively. Let $\xi_r(\omega)$ and $\xi_r(\omega)$ be the fill rate corresponding to the demand of raw materials and the supply of finished goods for each scenario, respectively. The service level is defined as the fraction of customer demand that is met through available stock. We calculate the service level for the demand of raw materials (in TEUs) for each scenario over all periods and all customers (manufacturers) as $\eta_r(\omega) = 1 - \frac{\sum_{\mathscr{q}\in\mathbb{J}}\sum_{t\in T}\widehat{U}^{\ell}_{\mathscr{q}t}(\omega)}{\sum_{\mathscr{q}\in\mathbb{J}}\sum_{t\in T}D^{\ell}_{\mathscr{q}t}(\omega)} \times 100\%$. We then estimate the expected value of $\eta_r(\omega)$ over all scenarios by:

$$\bar{\eta}_r = \frac{1}{|\Omega^N|}\sum_{\omega\in\Omega^N}\eta_r(\omega) \tag{46}$$

Likewise, we calculate the service level corresponding to the supply of finished goods for each scenario over all periods and all customers (manufacturers) by $\eta_s(\omega) = 1 - \frac{\sum_{\mathscr{q}\in\mathbb{J}}\sum_{t\in T}\hat{B}^{\ell}_{\mathscr{q}t}(\omega)}{\sum_{\mathscr{q}\in\mathbb{J}}\sum_{t\in T}D^{\ell}_{\mathscr{q}t}(\omega)} \times 100\%$, and its expected value over all scenarios by:

$$\bar{\eta}_s = \frac{1}{|\Omega^N|}\sum_{\omega\in\Omega^N}\eta_s(\omega) \tag{47}$$

The fill rate is defined as the percentage of demand orders which are satisfied on time and in full. We calculate the fill rate corresponding to the demand orders of raw materials for each scenario over all periods and all customers (manufacturers) by:

$$\xi_r(\omega) = \frac{\sum_{\mathscr{q}\in\mathbb{J}}\sum_{t\in T}\widetilde{U}^{\ell}_{\mathscr{q}t}(\omega)}{|\mathbb{J}||T|} \times 100\% \tag{48}$$

where $\widetilde{U}^{\ell}_{\mathscr{q}t}(\omega) = 1$ if $\widehat{U}^{\ell}_{\mathscr{q}t}(\mathscr{s}) = 0$ and zero, otherwise. We then compute the expected value of $\xi_r(\omega)$ over all scenarios by $\bar{\xi}_r = \frac{1}{|\Omega^N|}\sum_{\omega\in\Omega^N}\xi_r(\omega)$. Likewise, we calculate the fill rate corresponding to the supply orders of finished goods by $\xi_s(\omega) = \frac{\sum_{\mathscr{q}\in\mathbb{J}}\sum_{t\in T}\tilde{B}^{\ell}_{\mathscr{q}t}(\omega)}{|\mathbb{J}||T|} \times 100\%$, where $\tilde{B}^{\ell}_{\mathscr{q}t}(\omega) = 1$ if $\hat{B}^{\ell}_{\mathscr{q}t}(\omega) = 0$ and zero, otherwise, and its expected value by:

$$\bar{\xi}_s = \frac{1}{|\Omega^N|}\sum_{\omega\in\Omega^N}\xi_s(\omega) \tag{49}$$

The inventory turnover of empty containers reflects the number of times that empty containers were replenished at all dry ports over the planning horizon. We denote the total throughput of empty containers from all dry ports for each scenario $\omega \in \Omega^N$ by $\psi(\omega) = \sum_{\mathscr{p}\in\mathbb{I}}\sum_{t\in T}\sum_{m\in M}\big(\sum_{\mathscr{q}\in\mathbb{O}}\hat{F}^{e}_{\mathscr{p}\mathscr{q}tm}(\omega) + \sum_{\mathscr{q}\in\mathbb{J}}\hat{F}^{e}_{\mathscr{p}\mathscr{q}tm}(\omega)\big)$, where $\hat{F}^{e}_{\mathscr{p}\mathscr{q}tm}(\omega)$ represents the solution of the flow of empty containers from dry port $\mathscr{p} \in \mathbb{I}$ to the seaport $\mathscr{q} \in \mathbb{O}$ and manufacturer $\mathscr{q} \in \mathbb{J}$. We also denote the average inventory level of empty containers in each period by $\iota(\omega) = \frac{1}{|T|}\big(\sum_{\mathscr{p}\in\mathbb{I}}\sum_{t\in T}\hat{I}^{e}_{\mathscr{p}t}(\omega)\big)$, where $\hat{I}^{e}_{\mathscr{p}t}(\omega)$ represents the solution corresponding to the decision variable $I^{e}_{\mathscr{p}t}(\omega)$. Therefore, we can indicate the inventory turnover of empty containers for each scenario by $\varepsilon(\omega) = \frac{\psi(\omega)}{\iota(\omega)}$ and its expected value by:

$$\bar{\varepsilon} = \frac{1}{|\Omega^N|}\sum_{\omega\in\Omega^N}\varepsilon(\omega) \tag{50}$$

5. Computational Study

In this section, we describe a hypothetical case of designing a hinterland container shipping network and present the results obtained by the proposed solution procedure. We also discuss performance indicators and a sensitivity analysis to provide further managerial insights of interest to shipping line companies. Finally, the performance experiments of proposed accelerated Benders decomposition algorithm are provided.

5.1. Context for study

The hypothetical case study is based in North Carolina State which includes a seaport at Wilmington and fifty manufacturers. A set of eight predesignated locations are selected as the candidate points for establishing dry ports in the state. These points are chosen from sites in cities with more than 70,000 inhabitants. A one-year planning horizon ($T = 12$) is considered as the temporal scope of this study. Below we present the input data used in this study.

An average fixed cost of \$120 per TEU is considered for opening a dry port at each candidate location (Ambrosino and Sciomachen 2014). The available storage capacity at dry port candidate location $\mathcal{p} \in \mathbb{I}$ is randomly generated within [2.0, 5.0]×10^4 TEUs. A storage capacity of 2000 TEUs is taken for each manufacturer and 10,000 TEUs for the seaport. According to Ballou (2004), the unit cost for transporting container type k on arc $(\mathcal{p}, \mathcal{q}) \in \mathcal{A}$ follows a flat rate for each transportation mode, which is calculated as $c^k_{\mathcal{p}\mathcal{q}m} = \mathrm{t}_{\mathcal{p}\mathcal{q}m}\beta_m$, where $\mathrm{t}_{\mathcal{p}\mathcal{q}m}$ denotes the transportation time on arc $(\mathcal{p}, \mathcal{q}) \in \mathcal{A}$ using transportation mode m, and β_m denotes the transportation cost parameter of transportation mode m. The transportation time for mode m on arc $(\mathcal{p}, \mathcal{q}) \in \mathcal{A}$ is estimated by $\mathrm{t}_{\mathcal{p}\mathcal{q}m} = \frac{\Delta_{\mathcal{p}\mathcal{q}}}{V_m}$, where $\Delta_{\mathcal{p}\mathcal{q}}$ denotes the travel distance from node $\mathcal{p}$ to node $\mathcal{q}$, and V_m represents the average speed of transportation mode m. The set of transportation modes include road and rail, $M = \{1,2\}$. We set $\beta_m = \$3.88$ and $V_m = 60$ mph for the road mode ($m = 1$) and $\beta_m = \$0.05$ and $V_m = 24$ mph for the rail mode ($m = 2$) according to FAF3 dataset used in Ballou (2004). The unit cost of holding an empty container at the seaport, a dry port, and a manufacturer are set to \$2, \$4, and \$8, respectively.

The lognormal distribution is good for modelling economic variables such as demand (Kamath and Pakkala 2002). Therefore, incoming demand follows $D^{\ell}_{\mathcal{q}t}(\omega) \sim \text{Lognormal}\left(\mu_{D^{\ell}_{\mathcal{q}t}}, \sigma_{D^{\ell}_{\mathcal{q}t}}\right)$, where $\mu_{D^{\ell}_{\mathcal{q}t}}$ and $\sigma_{D^{\ell}_{\mathcal{q}t}}$ refer to the mean and standard deviation of lognormal probability distribution function, respectively. The supply of outgoing goods $S^{\ell}_{\mathcal{q}t}(\omega)$ could be calculated accordingly since a linear proportional relationship between incoming and outgoing goods is considered. With regards to the lognormal distribution, the random value of incoming goods demand for a given customer $\mathcal{q} \in \mathbb{J}$ over period $t \in T$ are given as $\mu_{D^{\ell}_{\mathcal{q}t}} \in [6000{,}7000]$, where the mean-variance ratio is set to $\mu_{D_{q,t}^{\ell}}/\sigma_{D_{q,t}^{\ell}} = 10$. Following the work of Ambrosino and Sciomachen (2014), we take into account different cost structures, with two levels of fixed cost for opening a dry port and two levels of empty container holding costs, as shown in Table 3.

Table 3. Cost structure.

Cost structures	fixed cost for opening a dry port ($)	cost of holding an empty container ($)*
(a)	$[1.8,4.5]\times10^{6}$	$[2, 4, 8]\times10^{-1}$
(b)	$[1.8,4.5]\times10^{6}$	$[2, 4, 8]\times10$
(c)	$[3.0;7.5]\times10^{6}$	$[2, 4, 8]\times10^{-1}$
(d)	$[3.0;7.5]\times10^{6}$	$[2, 4, 8]\times10$

* $[h_{o}, h_{i}, h_{j}]$, $\forall o \in \mathbb{O}, i \in \mathbb{I}, j \in \mathbb{J}$.

As mentioned before, the quality of solutions in the SAA method is related to the sample size. We calibrated, tested and validated the SAA model. For this validation, three different samples of size $N = 20$, $N = 40$, and $N = 50$ and the four cost structures as shown in Table 3 were considered. This design gave rise to 12 instances which were used to estimate the statistical optimality gap. Table 4 shows the results obtained from the SAA model, where $R = 4$ and $N' = 150$ were used to calculate the statistical optimality gap percentage with a 95% confidence interval using equation (36).

Table 4. Statistical optimality gap.

	Sample size											
	$N = 20$				$N = 40$				$N = 50$			
Cost structure	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)
Solution time (seconds)	242.4	156.9	261.4	217.7	93.2	614.5	603.6	2621.0	973.8	799.8	1056.4	3506.3
$gap_{N,R,N'}$%	1.08	1.07	1.08	1.08	1.03	1.02	1.03	1.02	**1.01**	**1.00**	**1.01**	**1.00**

When N scenarios were used in the proposed SAA model with a planning horizon of 12 periods; a combination of $12\times N$ sample scenarios were included for solving each instance. The results show that the quality of the SAA model solutions increased; the percentage optimality gap decreased as the sample size N increased. This validation analysis demonstrates that with the maximum considered sample size, the SAA method generated satisfactory optimality gaps of approximately 1%. Accordingly, we used the sample size $N = 50$ for our numerical experiments.

In order to attain comprehensive results, the problem was tested across different values of the solution robustness and rejected demand coefficients. Accordingly, a wide range of values of λ and ν were used for each problem instance which were $\lambda = \{10^{-3}, 10^{-2}, 10^{-1}, 1\}$ and$\nu = \{1000, 2000, 3000, 4000, 5000\}$. The combination of different values of these coefficients as well as the four cost structure levels shown in Table 3 yielded 80 problem instances. Furthermore, each problem instance included $|\mathbb{J}| = 50$ manufacturer locations over a planning period of $|T| = 12$ months, requiring the generation of $N.|\mathbb{J}|.\,|T| = 30{,}000$ sample scenarios to represent the entire market demand.

5.2. Numerical results and discussion

In section 5.2.1, the solutions obtained from the 80 explained instances are analysed regarding to the network configuration, container flow decisions, and key performance indicators. In section 5.2.2, we discuss the performance of the robustness approach. Finally, 5.2.3, is dedicated to the computational efficiency of applied solution procedure employing the proposed accelerated Benders decomposition algorithm.

5.2.1. Solution analysis

- *Network configuration*

We present dry ports location decisions for all the instances based on various cost structures and different values of λ shown in Figure 2. This illustrates the significant impact of operational costs on shipping network design. The horizontal axis represents the cost structures and the vertical axis shows the average number of dry ports to be opened over all instances. With low holding costs for keeping empty containers, i.e. cost structures (a) and (c), the shipping network tends to open more facilities (approximately 7 dry ports) in order to achieve more robust solutions. This outcome indicates that under these cost structures it is more cost-effective to decentralise the storage of empty containers by opening more dry ports across the region to cope with the existing uncertainty. On the other hand, centralisation of empty containers storage is recommended under cost structures (b) and (d) which aims to obtain more robust solutions with minimal cost.

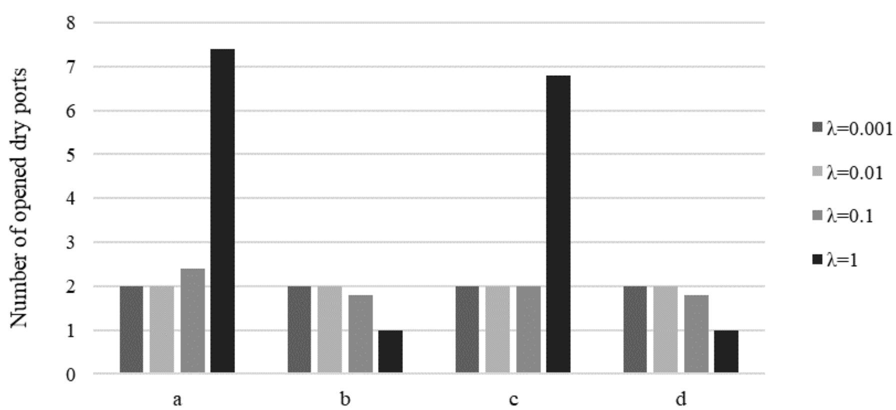


Figure 2. Location decisions on the number of opened dry ports.

To have a better view of the network configuration, we choose one of the instances described above and plot the corresponding solutions. Figure 3 shows the location of dry ports and their allocation to manufacturers. The location of the seaport, dry ports, and manufacturers are shown as orange, red, and blue icons, respectively. For this instance, our proposed model proposes two dry ports which are located fairly close to the seaport with a sufficient distance from/to all manufacturers. The distant manufacturers, located far away from the seaport, are mainly multiple-sourced by dry ports. The detailed network configuration results related to the location-allocation are provided in Tables A1-A4 of Appendix A.

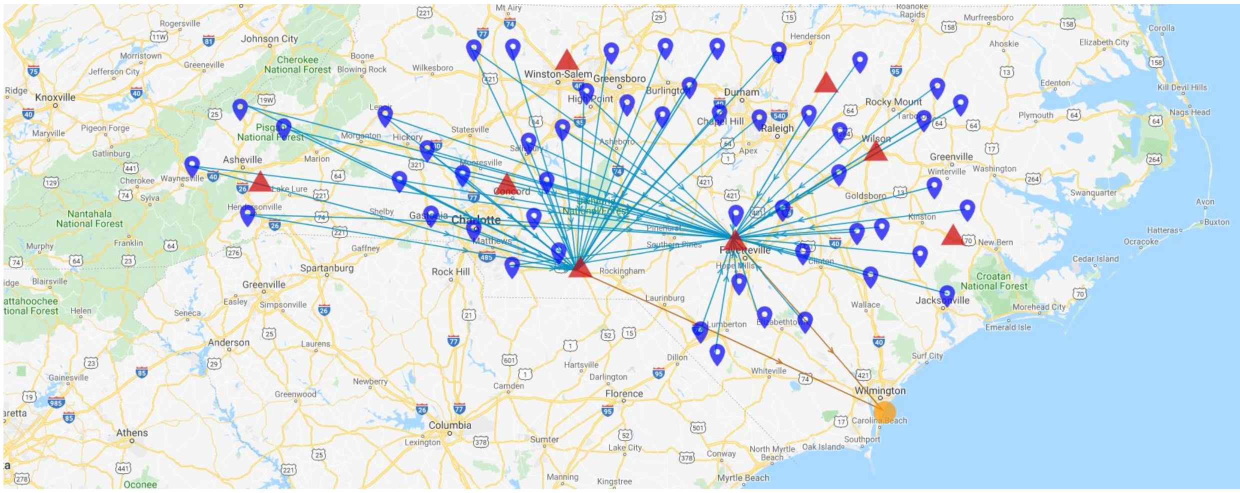
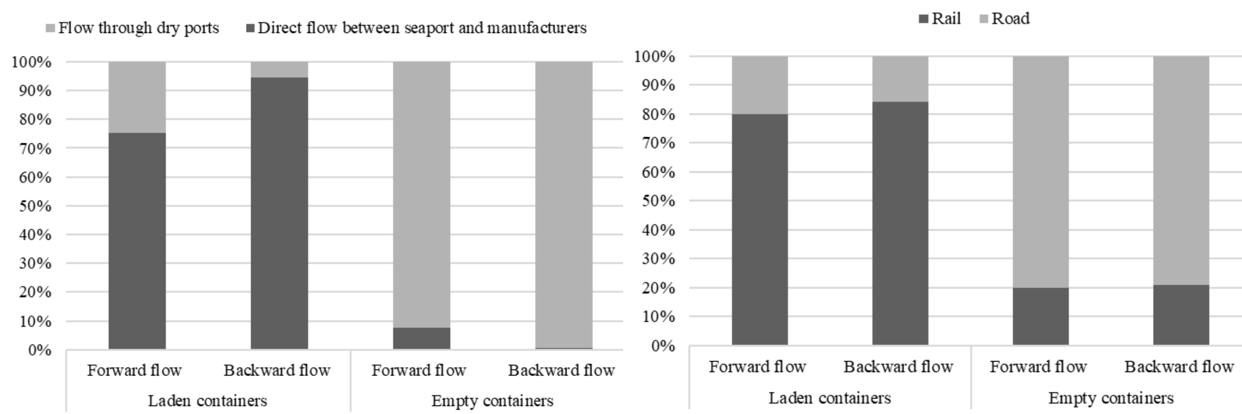


Figure 3. Geographical presentation of the network in North Carolina.

Containers flow decisions

Figure 4 summarises the flow of empty and laden containers in the network and the mode of transport applied. Figure 4(a) shows that the laden containers were mostly transported directly between the seaport and manufacturers, whilst the empty containers were mainly transported between these nodes through dry ports. Moreover, the results suggest that the backward flow of laden containers from manufacturers to the seaport contributed to a higher percentage of direct flow than the forward flow from the seaport to manufacturers. Figure 4(a) also shows that the usage of dry ports is more practical for empty container repositioning especially in the backward flow, i.e. from manufacturers to the seaport. This confirms the importance of deploying dry ports in the container shipping networks for the repositioning of empty containers. Figure 4(b) shows that a higher percentage of laden containers were transported by rail, whilst road was the main transportation mode for the flow of empty containers. Each transportation mode was used almost equally in the forward and backward laden containers flow. The results were similar for empty containers. The percentage flow using rail for laden containers was higher for empty containers, since the transportation costs were reduced by taking the advantage of the economies of scale for direct flows which involved longer distances than the indirect flows through dry ports.

Overall, rail was used more for the direct movement of laden containers, whilst road was employed more for the indirect flow of empty containers. The detailed results obtained for the flow of empty and laden containers using considered transportation modes for different cost structures are presented in Tables B1-B4 of Appendix B.



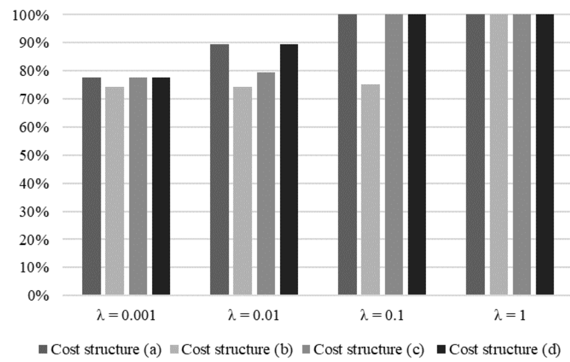
(a) Containers flow. (b) Transportation modality.

Figure 4. Transportation in the container shipping network.

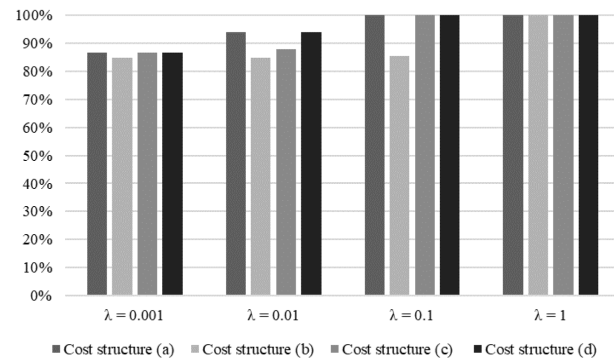
Key performance Indicators

We summarise the results obtained from computing the KPIs in Figure 5, in order to underline the impact of different cost structures and solution robustness coefficient. In the first glance, one can observe in Figures 5(a) and 5(b) that both $\bar{\eta}_r$ and $\bar{\eta}_s$, obtained from (46) and (47) respectively, were improved significantly as the solution robustness coefficient increased. Overall, the results show that increasing robustness led to higher service levels for the supply of finished goods compared to the one for the demand of raw materials. One also may argue that both $\bar{\xi}_r$ and $\bar{\xi}_s$, obtained from (48) and (49) respectively, had almost equal growth with respect to increase in the value of λ, as shown in Figures 5(c) and 5(d). This similarity in the value of fill rate is due to the linear proportional relationship between the demand and supply of containers at the manufacturers' locations. In general, this result confirms the importance of the incorporation of robustness considerations into container shipping network design in order to achieve the maximal desired service level and fill rate.

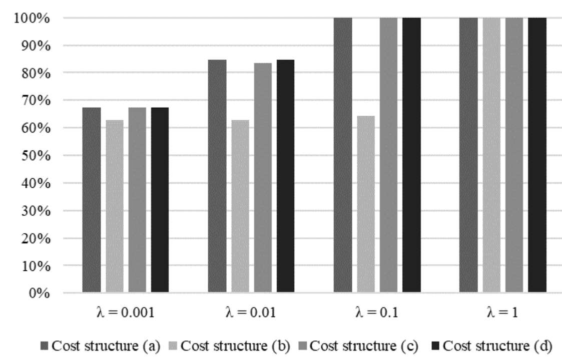
Moreover, Figures 5(a) and 5(b) show that when the fixed opening cost is relatively low, by increasing the holding cost of empty containers, both service levels $\bar{\eta}_r$ and $\bar{\eta}_s$ may decrease. This is because satisfying the demand and supply of laden containers is closely interrelated to the inventory level of empty containers across the shipping network. Figures 5(c) and 5(d) demonstrate a similar behaviour with regards to the cost structure.



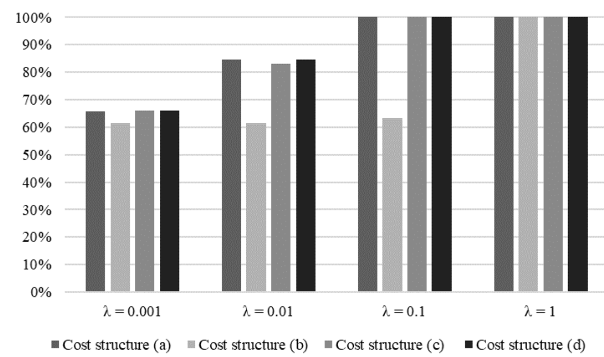
(a) Service level for the demand of raw materials.



(b) Service level for the supply of finished goods.



(c) Fill rate for the demand of raw materials.



(d) Fill rate for the supply of finished goods.

Figure 5. Service level and fill rate in considered shipping network.

We summarise the values of $\bar{\varepsilon}$ obtained from (50) with regards to different cost structures and solution robustness coefficients in Figure 6.

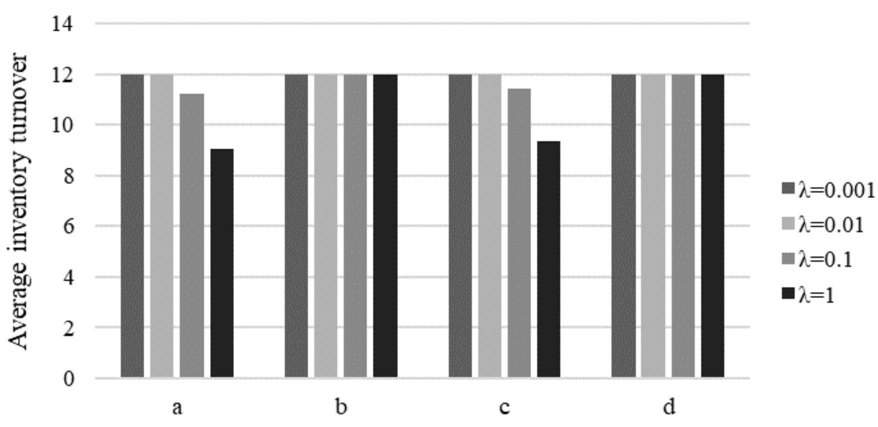


Figure 6. Average inventory turnover of empty containers.

Overall, it can be seen that the expected value of inventory turnover $\bar{\varepsilon}$ increased as the inventory holding cost of empty containers increased. This is due to the fact that when the inventory holding cost was high, a lower inventory level of empty containers was preferred at dry ports, increasing the number of replenishments. Moreover, Figure 6 shows that lower inventory turnover occured when seeking to achieve higher solution robustness, which implies that higher inventory levels were needed in order to avoid the effect of uncertainty which lowers the inventory turnover indicator. It is worthwhile to mention that the inventory turnover for instances with high holding costs suggests a monthly replenishment policy with the case study data, since a twelve-month planning horizon was considered. For detailed outputs regarding KPIs over all instances, please refer to Tables C1-C4 in Appendix C.

5.2.2. Performance of robustness

The proposed modelling approach provides decision-makers with reliable and robust solutions related to target inventory levels, empty container repositioning and intermodal transportation of laden containers. In relation to robust optimisation, we measured the performance and reliability of solutions by computing the standard deviation-to-mean ratio, which is referred to as the *coefficient of variation* (Birge 1982). Table 5 shows the minimum, mean, and maximum values over all instances. The results demonstrate that the minimum coefficient of variation (CV) for all of the operational decisions was less than 1%. However, the mean values of CV relating to the decision of empty containers inventory level at dry ports and direct flow of empty containers from seaports are 4.29% and 3.54% respectively. The former relatively high CV was due to the risk pooling effect relating to uncertain demand for empty containers at the dry ports. It implies that the uncertainty in laden container demand created high variability in decisions related to empty container repositioning, which emphasises the fact that the demand of empty and laden containers in the shipping network design were highly interrelated. The latter can be explained by the impact of network configuration on the variability of operational decisions. The repositioning of empty containers at the seaport which was coupled with importing and leasing operational decisions was another reason for this issue.

Table 5. Coefficient of variation values (%).

	$I^{e}_{qt}(\omega)$	$F^{\ell}_{pqtm}(\omega)$				$F^{e}_{pqtm}(\omega)$			
	$q \in \mathbb{I}$	$p \in \mathbb{O}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{O}$	$p \in \mathbb{I}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{I}$	$p \in \mathbb{O}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{O}$	$p \in \mathbb{I}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{I}$
Min	0.77	0.15	0.10	0.00	0.00	0.00	0.00	0.06	0.10
Mean	4.29	0.24	0.14	0.08	0.15	1.42	3.54	0.17	0.15
Max	5.58	0.52	0.38	0.16	0.98	2.80	19.61	0.38	0.19

In addition, we analysed the average value of CVs over ν associated with each cost structure and different values of λ. The corresponding value for the solution of variable $F^{\ell}_{pqtm}(\omega)$, $p \in \mathbb{I}, q \in \mathbb{J}$ is illustrated in Figure 7, where the horizontal axis represents the four cost structures considered and the vertical axis shows the average of CVs as a percentage. This shows that the value of CVs had a decreasing trend with regards to the incremental weight assigned to the solution robustness for all cost structures. This reduction of CVs was more significant when the cost of holding empty containers was relatively low, i.e. cost structures (a) and (c).

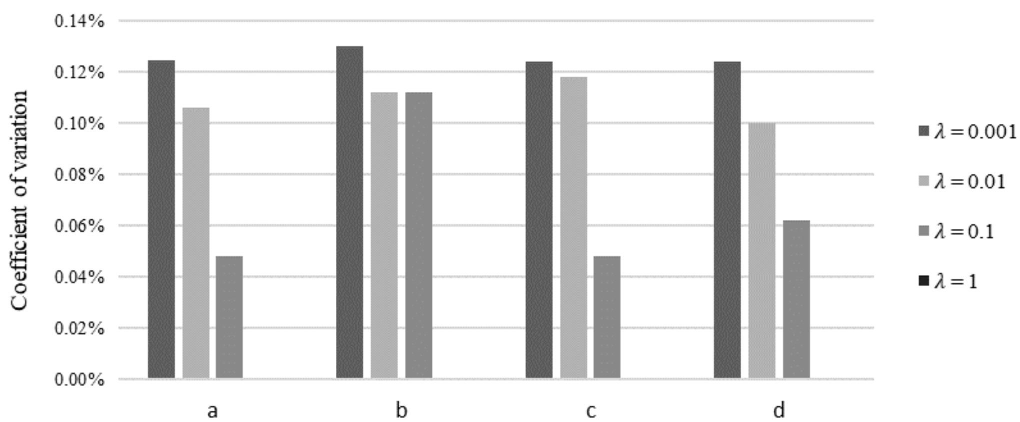


Figure 7. Coefficient of variations of flow for different cost structures.

We analysed the standard deviation-to-mean ratio for all decision variables for all instances (see Appendix D, Tables D1-D4). Overall, it can be concluded that the proposed robust modelling approach returned solutions with low variability.

5.2.3. Computational efficiency of the SAA with accelerated Benders decomposition algorithm

The computational efficienceis of the enhanced SAA procedure which is achieved by adding knapsack inequlities and utilizing Pareto-optimal custs are measured in terms of computational time. The accelerated BD algorithm is coded in CPLEX 12.8.0 and tested on a personal computer with a 3.2 gigahertz Intel Core 5 processor and 16 gigabytes of RAM.

We conduct two sorts of experiment in order to assess the efficiency of proposed accelerated BD method. The first set of experiments focuses on the impact of network size on the computational efficiency. Table 6 provides the characteristics of test sets for this experiment, and the effect of accleration methods on the computational times and the gap between the objective value obtained from Cplex and the one from the proposed accelerated BD. These experiments are conducted for $N = 20$ sampled scenarios. The results indicate that the model for smaller networks such as tests 1 and 2 can be solved directly by Cplex with smaller computational time compared to the BD method and its accelerated version. However, as the network size grows by increasing the number of nodes, both standard BD and the proposed accelerated BD strategy outperform Cplex in terms of computational time. The computational times given in Table 6 show that the proposed BD algorithm together with multi-cut schem, knapsack inequalities, and Pareto-cut strategy outperforms Cplex and standard BD for larger networks. Furthermore, it can be seen that the objective value gap is smaller when the accelerated BD method is applied to the instances.

It worths to mention that in our computational experiments, we initialise a core point approximation $\bar{\mathbf{X}}_{it}^{c}$ with a feasible solution to our MP and then update the approximation at each iteration by $\mathbf{X}_{it}^{c} \leftarrow \varphi \mathbf{X}_{it-1}^{c} + (1-\varphi)\mathbf{X}_{it}^{MP}$. According to emprical observations of Papadakos (2008) and Oliveira et al. (2014), φ is set to 0.5. Also, the stopping cirterion is set to when the objective value gap is below a threshold value 0.01.

Table 6. Computational results of the first experiment set.

Test set	$\lvert\mathbb{I}\rvert$	$\lvert\mathbb{J}\rvert$	**Number of variables**	**Number of constraints**	**Cplex**	**Bender decomposition**		**Accelerated Bender decomposition**	
					Time	Gap (%)	Time	Gap (%)	Time
1	2	10	82742	58550	4.09	0.02	22.14	0.001	22.06
2	3	20	202463	156020	15.20	0.11	26.28	0.023	25.09
3	5	30	429315	359160	77.40	0.06	36.19	0.001	32.92
4	6	40	653086	560680	198.13	0.05	47.09	0.022	44.73
5	8	50	1025608	909490	3043.26	0.74	2929.05	0.51	2436.92

The second experiment is designed to investigate the effect of the number of scenarios on the performance of the algorithm. Table 7 shows the characteristics of second group of test sets and compares the effect of accelerateion methods on the objective value gap, and computational times for different number of scenarios. We select a network instance with the size of $|\mathbb{I}| = 5$ and $|\mathbb{J}| = 20$ for this experiment and implement both standard and accelerated BD algorithms. Results suggest that both standard and the proposed acclerated Bender decomposition method outperform Cplex, specially for instances with higher number of scenarios. Yet, we observe that the computational time of the accelerated BD algortihm compare to the standard BD is slightly higher within instances with larger number of scenarios. The reason lies in the fact that despite employing Pareto-optimal cuts assists to reduce the number of iterationts, the computational time needed to solve the Magnati-Wong problem increases, when larger number of scenarios are involved. Nevertheless, the accelerated BD algorithm performs better than the standard version in terms of the solutions quality as the gaps are quite low. Overall, the results confirm the advantage of using several accelerating frameworks in the proposed solution strategy, since the accelerated BD algorithm is more time-efficient compare to Cplex and standard BD, and provides higher quality solutions with lower gaps as compare to standard BD method.

Table 7. Computational results of the second experiment set.

Test set	**Number of scenarios**	**Number of variables**	**Number of constraints**	**Cplex**	**Benders decomposition**		**Accelerated Benders decomposition**	
				Time	Gap (%)	Time	Gap (%)	Time
6	10	145985	121310	10.86	0.12	24.72	0.02	22.90
7	20	291865	242500	31.63	0.13	30.01	0.01	28.38
8	30	437745	363690	70.77	0.21	54.23	0.11	55.12
9	40	583625	484880	233.12	0.41	88.30	0.22	96.85
10	50	729505	606070	412.49	0.94	285.6	0.41	298.3

6. Conclusions

Previous research on inland container shipping network design has dealt with decision-making at different levels in an isolated manner. This paper proposes a novel integrated dry port network design model that integrates strategic decision making (dry port location, allocation and the provision of arcs between nodes in the network) with operational decision making (selection of transportation modes, empty container repositioning and inventory planning) that takes into account the stochastic nature of demand. The incoming and outgoing demand of laden containers were considered as uncertain parameters in this problem and were generated using Monte-Carlo sampling. This procedure resulted in a number of demand and supply realisations with high variability, which alongside with the hierarchical decision-making structure involved in this problem, led to us proposing a robust two-stage stochastic programming model that adopted the SAA method.

The experimental results revealed that the network configuration obtained from our proposed modelling approach varied according to the cost structure and the solution robustness coefficient. More specifically, in order to achieve more robust solutions, the number of opened dry ports declined when the holding cost of empty containers grew (and vice versa). The results, also, show that the direct transportation of laden containers between the seaport and manufacturers was mostly performed by rail, whereas road was mainly employed for the movement of empty containers between the seaport and manufacturers through dry ports, confirming the pivotal significance of dry ports relevant to ECR in the container shipping networks. Furthermore, service level, fill rate, and inventory turnover were evaluated. The results imply that both service level and fill rate improve when aiming for higher solution robustness by increasing the value of λ, whilst these KPIs may decrease with high holding costs. It was also observed that the inventory turnover of empty containers declined with high empty container holding costs, and also with high values of λ.

A validation procedure was adopted to investigate the performance of the enhanced SAA approach by approximating an optimality gap of 95%. The results from 80 instances based on a practical case study, disclosed some practical and managerial insights confirming the significance of hinterland container shipping network design and its operational decisions under uncertainty. The findings showed that the applied robust modelling approach produced solutions with low variability with the average CV of less than 1% under an uncertain environment. This was confirmed by the fact that this metric decreased for all operational decisions when the value of solution robustness coefficient rose.

Finally, two sets of experiment were tested and analysed to validate the effectiveness and efficiency of the proposed solution algorithm. The obtained objective value gaps, and computational times of Cplex, the standard Benders decomposition method, and the accelerated Benders decomposition algorithm were compared. The results confirmed that applying the acceleration methods to the Benders decomposition algorithm can improve the computational performance for various problems with different network sizes and different number of scenarios.

Overall, we believe the proposed model and solution procedure can be applied to address practical cases efficiently to achieve more reliable hinterland container shipping networks in terms of cost and operational performance. As future research directions, incorporating the inventory policies within the modelling process would be appealing. It also would be interesting to investigate the uncertainty in other parameters such as capacities, transportation and operational costs.

References

Ambrosino, Daniela, and Anna Sciomachen. 2014. "Location of Mid-range Dry Ports in Multimodal Logistic Networks." *Procedia - Social and Behavioral Sciences* 108:118-28. doi: https://doi.org/10.1016/j.sbspro.2013.12.825.

Amiri-Aref, M., W. Klibi, and M. Z. Babai. 2018. "The multi-sourcing location inventory problem with stochastic demand." *European Journal of Operational Research* 266 (1):72-87. doi: 10.1016/j.ejor.2017.09.003.

Ballou, R.H. 2004. *Business Logistics/Supply Chain Management: Planning, Organizing, and Controlling the Supply Chain* Pearson.

Bell, M. G. H., X. Liu, J. Rioult, and P. Angeloudis. 2013. "A cost-based maritime container assignment model." *Transportation Research Part B: Methodological* 58:58-70. doi: 10.1016/j.trb.2013.09.006.

Benders, J. F. 1962. "Partitioning procedures for solving mixed-variables programming problems." *Numerische Mathematik* 4 (1):238-52. doi: 10.1007/BF01386316.

Birge, J. R. 1982. "The value of the stochastic solution in stochastic linear programs with fixed recourse." *Mathematical Programming* 24 (1):314-25. doi: 10.1007/BF01585113.

Birge, J. R., and F. V. Louveaux. 1988. "A multicut algorithm for two-stage stochastic linear programs." *European Journal of Operational Research* 34 (3):384-92. doi: 10.1016/0377-2217(88)90159-2.

Birge, John R, and Francois Louveaux. 2011. *Introduction to stochastic programming*: Springer Science & Business Media.

Brouer, B. D., D. Pisinger, and S. Spoorendonk. 2011. "Liner shipping cargo allocation with repositioning of empty containers." *INFOR: Information Systems and Operational Research* 49 (2):109-24. doi: 10.3138/infor.49.2.109.

Chang, Z., T. Notteboom, and J. Lu. 2015. "A two-phase model for dry port location with an application to the port of Dalian in China." *Transportation Planning and Technology* 38 (4):442-64. doi: 10.1080/03081060.2015.1026103.

Crainic, T. G., P. Dell'Olmo, N. Ricciardi, and A. Sgalambro. 2015. "Modeling dry-port-based freight distribution planning." *Transportation Research Part C: Emerging Technologies* 55:518-34. doi: 10.1016/j.trc.2015.03.026.

Deidda, L., M. Di Francesco, A. Olivo, and P. Zuddas. 2008. "Implementing the street-turn strategy by an optimization model." *Maritime Policy and Management* 35 (5):503-16. doi: 10.1080/03088830802352145.

Di Francesco, M., M. Lai, and P. Zuddas. 2013. "Maritime repositioning of empty containers under uncertain port disruptions." *Computers and Industrial Engineering* 64 (3):827-37. doi: 10.1016/j.cie.2012.12.014.

Erera, A. L., J. C. Morales, and M. Savelsbergh. 2009. "Robust optimization for empty repositioning problems." *Operations Research* 57 (2):468-83. doi: 10.1287/opre.1080.0650.

Furió, S., C. Andrés, B. Adenso-Díaz, and S. Lozano. 2013. "Optimization of empty container movements using street-turn: Application to Valencia hinterland." *Computers and Industrial Engineering* 66 (4):909-17. doi: 10.1016/j.cie.2013.09.003.

Jula, H., A. Chassiakos, and P. Ioannou. 2006. "Port dynamic empty container reuse." *Transportation Research Part E: Logistics and Transportation Review* 42 (1):43-60. doi: 10.1016/j.tre.2004.08.007.

Kamath, K. Rajashree, and T. P. M. Pakkala. 2002. "A Bayesian approach to a dynamic inventory model under an unknown demand distribution." *Computers and Operations Research* 29 (4):403-22. doi: 10.1016/S0305-0548(00)00075-7.

Kim, Nam Seok, and Bert Van Wee. 2011. "The relative importance of factors that influence the break-even distance of intermodal freight transport systems." *Journal of Transport Geography* 19 (4):859-75. doi: https://doi.org/10.1016/j.jtrangeo.2010.11.001.
Kleywegt, A. J., A. Shapiro, and T. Homem-De-Mello. 2002. "The sample average approximation method for stochastic discrete optimization." *SIAM Journal on Optimization* 12 (2):479-502. doi: 10.1137/S1052623499363220.
Lättilä, L., V. Henttu, and O. P. Hilmola. 2013. "Hinterland operations of sea ports do matter: Dry port usage effects on transportation costs and CO2 emissions." *Transportation Research Part E: Logistics and Transportation Review* 55:23-42. doi: 10.1016/j.tre.2013.03.007.
Lee, C. Y., and D. P. Song. 2017. "Ocean container transport in global supply chains: Overview and research opportunities." *Transportation Research Part B: Methodological* 95:442-74. doi: 10.1016/j.trb.2016.05.001.
Magnanti, Thomas L, and Richard T Wong. 1981. "Accelerating Benders decomposition: Algorithmic enhancement and model selection criteria." *Operations research* 29 (3):464-84.
Meng, Q., S. Wang, and Z. Liu. 2012. "Network design for shipping service of large-scale intermodal liners." In *Transportation Research Record*, 42-50.
Mulvey, John M., Robert J. Vanderbei, and Stavros A. Zenios. 1995. "Robust Optimization of Large-Scale Systems." *Operations Research* 43 (2):264-81. doi: 10.1287/opre.43.2.264.
Oliveira, F., I. E. Grossmann, and S. Hamacher. 2014. "Accelerating Benders stochastic decomposition for the optimization under uncertainty of the petroleum product supply chain." *Computers and Operations Research* 49:47-58. doi: 10.1016/j.cor.2014.03.021.
Olivo, A., M. Di Francesco, and P. Zuddas. 2013. "An optimization model for the inland repositioning of empty containers." *Maritime Economics and Logistics* 15 (3):309-31. doi: 10.1057/mel.2013.12.
Papadakos, N. 2008. "Practical enhancements to the Magnanti-Wong method." *Operations Research Letters* 36 (4):444-9. doi: 10.1016/j.orl.2008.01.005.
Roso, V., J. Woxenius, and K. Lumsden. 2009. "The dry port concept: connecting container seaports with the hinterland." *Journal of Transport Geography* 17 (5):338-45. doi: 10.1016/j.jtrangeo.2008.10.008.
Santoso, T., S. Ahmed, M. Goetschalckx, and A. Shapiro. 2005. "A stochastic programming approach for supply chain network design under uncertainty." *European Journal of Operational Research* 167 (1):96-115. doi: 10.1016/j.ejor.2004.01.046.
Shapiro, Alexander. 2003. "Monte Carlo Sampling Methods." In *Handbooks in Operations Research and Management Science*, 353-425. Elsevier.
Shapiro, Alexander, Darinka Dentcheva, and Andrzej Ruszczynski. 2009. *Lectures on Stochastic Programming: Modeling and Theory*, *MOS-SIAM Series on Optimization*: Society for Industrial and Applied Mathematics.
Shintani, K., A. Imai, E. Nishimura, and S. Papadimitriou. 2007. "The container shipping network design problem with empty container repositioning." *Transportation Research Part E: Logistics and Transportation Review* 43 (1):39-59. doi: 10.1016/j.tre.2005.05.003.
Song, D. P., and J. X. Dong. 2012. "Cargo routing and empty container repositioning in multiple shipping service routes." *Transportation Research Part B: Methodological* 46 (10):1556-75. doi: 10.1016/j.trb.2012.08.003.
———. 2015. "Empty container repositioning." In *International Series in Operations Research and Management Science*, 163-208.
Thanopoulou, H., and S. P. Strandenes. 2017. "A theoretical framework for analysing long-term uncertainty in shipping." *Case Studies on Transport Policy* 5 (2):325-31. doi: 10.1016/j.cstp.2017.03.008.

Van Slyke, Richard M, and Roger Wets. 1969. "L-shaped linear programs with applications to optimal control and stochastic programming." *SIAM Journal on Applied Mathematics* 17 (4):638-63.

Vojdani, N., F. Lootz, and R. Rösner. 2013. "Optimizing empty container logistics based on a collaborative network approach." *Maritime Economics and Logistics* 15 (4):467-93. doi: 10.1057/mel.2013.16.

Wang, C., Q. Chen, and R. Huang. 2018. "Locating dry ports on a network: a case study on Tianjin Port." *Maritime Policy and Management* 45 (1):71-88. doi: 10.1080/03088839.2017.1330558.

Xie, Y., X. Liang, L. Ma, and H. Yan. 2017. "Empty container management and coordination in intermodal transport." *European Journal of Operational Research* 257 (1):223-32. doi: 10.1016/j.ejor.2016.07.053.

Yang, C. S. 2018. "An analysis of institutional pressures, green supply chain management, and green performance in the container shipping context." *Transportation Research Part D: Transport and Environment* 61:246-60. doi: 10.1016/j.trd.2017.07.005.

Yu, Chian-Son, and Han-Lin Li. 2000. "A robust optimization model for stochastic logistic problems." *International journal of production economics* 64 (1-3):385-97.

Yu, M., J. C. Fransoo, and C. Y. Lee. 2018. "Detention decisions for empty containers in the hinterland transportation system." *Transportation Research Part B: Methodological* 110:188-208. doi: 10.1016/j.trb.2018.02.007.

Zhang, R., W. Y. Yun, and I. Moon. 2009. "A reactive tabu search algorithm for the multi-depot container truck transportation problem." *Transportation Research Part E: Logistics and Transportation Review* 45 (6):904-14. doi: 10.1016/j.tre.2009.04.012.

Appendix A.

The results regarding to the number of opened dry ports and allocation decisions obtained from all described problem instances are presented in Tables A1-A4. The allocation decisions are shown according to the sourcing strategies of single-sourcing and multiple-sourcing, where the former shows the percentage of manufacturers who allocated to only one opened dry port, while the latter denotes the percentage of manufacturers allocated to more than one established dry ports. Furthermore, the average of mentioned results over different values of ν for a given λ are provided for further transparency.

Table A1. Network configuration decisions results of cost structure (a)

λ	ν	Nb Dry Ports	Allocation decision	
			Single-sourcing	Multiple-sourcing
0.001	1000	2	67%	33%
	2000	2	68%	32%
	3000	2	68%	32%
	4000	2	67%	33%
	5000	2	68%	32%
Average		2	68%	32%
0.01	1000	2	68%	32%
	2000	2	70%	30%
	3000	2	68%	32%
	4000	2	68%	32%
	5000	2	68%	32%
Average		2	68%	32%
0.1	1000	3	100%	0%
	2000	3	100%	0%
	3000	2	68%	32%
	4000	2	68%	32%
	5000	2	68%	32%
Average		2.4	81%	19%
1	1000	5	88%	12%
	2000	8	0%	100%
	3000	8	0%	100%
	4000	8	0%	100%
	5000	8	0%	100%
Average		7.4	18%	82%

Table A2. Network configuration decisions results of cost structure (b)				
λ	ν	Nb Dry Ports	Allocation/Sourcing rule	
			Single-sourcing	Multiple-sourcing
0.001	1000	2	68%	32%
	2000	2	68%	32%
	3000	2	68%	32%
	4000	2	68%	32%
	5000	2	68%	32%
Average		2	68%	32%
0.01	1000	2	67%	33%
	2000	2	68%	32%
	3000	2	68%	32%
	4000	2	68%	32%
	5000	2	68%	32%
Average		2	68%	32%
0.1	1000	1	100%	0%
	2000	2	68%	32%
	3000	2	68%	32%
	4000	2	68%	32%
	5000	2	68%	32%
Average		1.8	74%	26%
1	1000	1	100%	0%
	2000	1	100%	0%
	3000	1	100%	0%
	4000	1	100%	0%
	5000	1	100%	0%
Average		1	100%	0%

Table A3. Network configuration decisions results of cost structure (c)				
λ	ν	Nb Dry Ports	Allocation/Sourcing rule	
			Single-sourcing	Multiple-sourcing
0.001	1000	2	68%	32%
	2000	2	68%	32%
	3000	2	68%	32%
	4000	2	68%	34%
	5000	2	68%	32%
Average		2	68%	32%
0.01	1000	2	67%	33%
	2000	2	68%	32%
	3000	2	68%	32%
	4000	2	68%	32%
	5000	2	68%	32%
Average		2	68%	32%
0.1	1000	2	88%	12%
	2000	2	88%	12%
	3000	2	68%	32%
	4000	2	68%	32%
	5000	2	68%	32%
Average		2	76%	24%
1	1000	2	100%	0%
	2000	8	0%	100%
	3000	8	0%	100%
	4000	8	0%	100%
	5000	8	0%	100%
Average		6.8	20%	80%

Table A4. Network configuration decisions results of cost structure (d)				
λ	ν	Nb Dry Ports	Allocation/Sourcing rule	
			Single-sourcing	Multiple-sourcing
0.001	1000	2	68%	32%
	2000	2	70%	30%
	3000	2	68%	32%
	4000	2	68%	32%
	5000	2	68%	32%
Average		2	68%	32%
0.01	1000	2	68%	32%
	2000	2	68%	32%
	3000	2	68%	32%
	4000	2	68%	32%
	5000	2	68%	32%
Average		2	68%	32%
0.1	1000	1	100%	0%
	2000	2	68%	32%
	3000	2	68%	32%
	4000	2	68%	32%
	5000	2	68%	32%
Average		1.8	74%	26%
1	1000	1	100%	0%
	2000	1	100%	0%
	3000	1	100%	0%
	4000	1	100%	0%
	5000	1	100%	0%
Average		1	100%	0%

Appendix B.

The details of numerical results associated with the solution of the intermodal transportation of both laden and empty containers are provided in Tables B1-B4. More specifically, we present the percentage of laden containers flow (LCF) and empty containers flow (ECF) throughout the network using both available transportation modes of road and rail. Also, we categorized the flow solutions according to the dry-ports deployment, where "Direct" flows refer to the flows of containers which are transported between nodes without using dry ports, while "Indirect" flows denote the flows of containers which are moved via established dry ports. Moreover, the average of flow percentages over different values of ν for a given λ, as well as the total average of flow percentages over all different values of ν and λ, are provided for each cost structure.

Table B1. Flow decisions results for cost structure (a)

λ	ν	LCF (forward)				LCF (backward)			
		Direct		Indirect		Direct		Indirect	
		Road	Rail	Road	Rail	Road	Rail	Road	Rail
0.001	1000	6.30%	56.70%	18.50%	18.50%	9.67%	84.36%	4.59%	1.38%
	2000	6.53%	58.75%	17.36%	17.36%	9.68%	84.35%	4.59%	1.38%
	3000	6.21%	55.93%	18.93%	18.93%	9.68%	84.34%	4.61%	1.38%
	4000	6.18%	55.62%	19.10%	19.10%	9.68%	84.35%	4.59%	1.38%
	5000	6.34%	57.03%	18.32%	18.32%	9.62%	84.36%	4.65%	1.38%
Average		6.31%	56.81%	18.44%	18.44%	9.67%	84.35%	4.61%	1.38%
0.01	1000	6.35%	57.18%	18.23%	18.23%	9.62%	84.33%	4.67%	1.38%
	2000	6.30%	56.70%	18.50%	18.50%	9.66%	84.37%	4.59%	1.38%
	3000	6.26%	56.38%	18.68%	18.68%	9.59%	84.36%	4.67%	1.38%
	4000	6.24%	56.13%	18.82%	18.82%	9.60%	84.35%	4.67%	1.38%
	5000	6.21%	55.93%	18.93%	18.93%	9.58%	84.37%	4.67%	1.38%
Average		6.27%	56.46%	18.63%	18.63%	9.61%	84.36%	4.65%	1.38%
0.1	1000	10.00%	90.00%	0.00%	0.00%	9.59%	84.34%	6.07%	0.00%
	2000	10.00%	90.00%	0.00%	0.00%	9.68%	84.32%	6.00%	0.00%
	3000	6.35%	57.18%	18.23%	18.23%	9.62%	84.33%	4.67%	1.38%
	4000	6.35%	57.18%	18.23%	18.23%	9.62%	84.33%	4.67%	1.38%
	5000	6.35%	57.18%	18.23%	18.23%	9.62%	84.33%	4.67%	1.38%
Average		7.81%	70.31%	10.94%	10.94%	9.63%	84.33%	5.22%	0.83%
1	1000	10.00%	90.00%	0.00%	0.00%	10.16%	83.60%	6.24%	0.00%
	2000	10.00%	90.00%	0.00%	0.00%	9.72%	83.86%	6.41%	0.00%
	3000	10.00%	90.00%	0.00%	0.00%	9.49%	84.35%	6.16%	0.00%
	4000	10.00%	90.00%	0.00%	0.00%	9.49%	84.35%	6.16%	0.00%
	5000	10.00%	90.00%	0.00%	0.00%	9.49%	84.35%	6.16%	0.00%
Average		7.81%	70.31%	10.94%	10.94%	9.63%	84.33%	5.22%	0.83%
Total Average		**7.60%**	**68.39%**	**12.00%**	**12.00%**	**9.64%**	**84.29%**	**5.18%**	**0.90%**

Table B1. Flow decisions results for cost structure (a) (cont.).

λ	ν	ECF (forward)				ECF (backward)			
		Direct		Indirect		Direct		Indirect	
		Road	Rail	Road	Rail	Road	Rail	Road	Rail
0.001	1000	10.07%	0.00%	62.95%	26.98%	3.31%	0.00%	74.66%	22.03%
	2000	0.00%	0.00%	70.00%	30.00%	1.65%	0.00%	74.26%	24.08%
	3000	0.00%	0.00%	70.00%	30.00%	1.95%	0.00%	74.03%	24.01%
	4000	28.21%	0.00%	50.25%	21.54%	3.53%	0.00%	74.46%	22.02%
	5000	0.00%	0.00%	70.00%	30.00%	0.20%	0.00%	77.80%	22.00%
Average		7.66%	0.00%	64.64%	27.70%	2.13%	0.00%	75.04%	22.83%
0.01	1000	0.00%	0.00%	70.00%	30.00%	0.93%	0.00%	77.04%	22.02%
	2000	0.00%	0.00%	70.00%	30.00%	0.20%	0.00%	75.84%	23.96%
	3000	0.00%	0.00%	70.00%	30.00%	0.10%	0.00%	77.91%	21.99%
	4000	0.00%	0.00%	70.00%	30.00%	0.07%	0.00%	77.94%	21.99%
	5000	0.00%	0.00%	70.00%	30.00%	0.09%	0.00%	77.92%	21.99%
Average		0.00%	0.00%	70.00%	30.00%	0.28%	0.00%	77.33%	22.39%
0.1	1000	0.00%	0.00%	100.0%	0.00%	0.00%	0.00%	100.0%	0.00%
	2000	0.00%	0.00%	100.0%	0.00%	0.00%	0.00%	96.67%	3.33%
	3000	0.00%	0.00%	70.00%	30.00%	0.90%	0.00%	77.07%	22.02%
	4000	0.00%	0.00%	70.00%	30.00%	1.25%	0.00%	76.72%	22.03%
	5000	0.00%	0.00%	70.00%	30.00%	0.71%	0.00%	77.26%	22.03%
Average		0.00%	0.00%	82.00%	18.00%	0.57%	0.00%	85.54%	13.88%
1	1000	0.00%	0.00%	97.59%	2.41%	0.00%	0.00%	90.09%	9.91%
	2000	0.00%	0.00%	99.57%	0.43%	0.00%	0.00%	100.0%	0.00%
	3000	0.00%	0.00%	98.42%	1.58%	0.00%	0.00%	100.0%	0.00%
	4000	0.00%	0.00%	98.37%	1.63%	0.00%	0.00%	100.0%	0.00%
	5000	0.00%	0.00%	98.45%	1.55%	0.00%	0.00%	100.0%	0.00%
Average		0.00%	0.00%	82.00%	18.00%	0.57%	0.00%	85.54%	13.88%
Total Average		**1.91%**	**0.00%**	**78.78%**	**19.31%**	**0.74%**	**0.00%**	**83.98%**	**15.27%**

Table B2. Flow decisions results for cost structure (b)

λ	ν	LCF (forward)				LCF (backward)			
		Direct		Indirect		Direct		Indirect	
		Road	Rail	Road	Rail	Road	Rail	Road	Rail
0.001	1000	6.30%	56.70%	18.50%	18.50%	9.56%	84.41%	4.65%	1.38%
	2000	6.24%	56.13%	18.82%	18.82%	9.62%	84.39%	4.62%	1.38%
	3000	6.20%	55.76%	19.02%	19.02%	9.63%	84.38%	4.62%	1.38%
	4000	6.36%	57.26%	18.19%	18.19%	9.67%	84.37%	4.59%	1.38%
	5000	6.31%	56.81%	18.44%	18.44%	9.58%	84.38%	4.66%	1.38%
Average		6.28%	56.53%	18.59%	18.59%	9.61%	84.39%	4.63%	1.38%
0.01	1000	6.30%	56.70%	18.50%	18.50%	9.72%	84.36%	4.54%	1.38%
	2000	6.24%	56.13%	18.82%	18.82%	9.62%	84.39%	4.62%	1.38%
	3000	6.20%	55.76%	19.02%	19.02%	9.63%	84.38%	4.62%	1.38%
	4000	6.36%	57.26%	18.19%	18.19%	9.67%	84.37%	4.59%	1.38%
	5000	6.31%	56.81%	18.44%	18.44%	9.58%	84.38%	4.66%	1.38%
Average		6.28%	56.53%	18.59%	18.59%	9.64%	84.38%	4.61%	1.38%
0.1	1000	10.00%	90.00%	0.00%	0.00%	11.92%	83.64%	3.29%	1.15%
	2000	6.24%	56.13%	18.82%	18.82%	9.62%	84.39%	4.62%	1.38%
	3000	6.20%	55.76%	19.02%	19.02%	9.63%	84.38%	4.62%	1.38%
	4000	6.36%	57.26%	18.19%	18.19%	9.67%	84.37%	4.59%	1.38%
	5000	6.31%	56.81%	18.44%	18.44%	9.58%	84.38%	4.66%	1.38%
Average		7.02%	63.19%	14.89%	14.89%	10.08%	84.23%	4.36%	1.33%
1	1000	10.71%	89.29%	0.00%	0.00%	24.59%	75.41%	0.00%	0.00%
	2000	10.00%	90.00%	0.00%	0.00%	25.81%	74.19%	0.00%	0.00%
	3000	10.0%	90.0%	0.00%	0.00%	27.0%	73.0%	0.00%	0.00%
	4000	10.0%	90.0%	0.00%	0.00%	26.98%	73.02%	0.00%	0.00%
	5000	10.0%	90.0%	0.00%	0.00%	12.24%	83.32%	3.29%	1.15%
Average		10.14%	89.86%	0.00%	0.00%	23.32%	75.79%	0.66%	0.23%
Total Average		**7.43%**	**66.53%**	**13.02%**	**13.02%**	**13.17%**	**82.20%**	**3.56%**	**1.08%**

Table B2. Flow decisions results for cost structure (b) (cont.).									
λ	ν	ECF (forward)				ECF (backward)			
		Direct		Indirect		Direct		Indirect	
		Road	Rail	Road	Rail	Road	Rail	Road	Rail
0.001	1000	5.88%	0.00%	65.88%	28.24%	0.24%	0.00%	77.81%	21.95%
	2000	5.88%	0.00%	65.88%	28.24%	1.23%	0.00%	76.78%	21.98%
	3000	5.88%	0.00%	65.88%	28.24%	0.59%	0.00%	77.43%	21.98%
	4000	5.88%	0.00%	65.88%	28.24%	0.81%	0.00%	75.19%	24.01%
	5000	5.88%	0.00%	65.88%	28.24%	0.85%	0.00%	77.14%	22.01%
Average		5.88%	0.00%	65.88%	28.24%	0.74%	0.00%	76.87%	22.39%
0.01	1000	12.09%	0.00%	61.54%	26.37%	3.27%	0.00%	72.77%	23.96%
	2000	5.88%	0.00%	65.88%	28.24%	1.23%	0.00%	76.78%	21.98%
	3000	5.88%	0.00%	65.88%	28.24%	0.59%	0.00%	77.43%	21.98%
	4000	5.88%	0.00%	65.88%	28.24%	0.81%	0.00%	75.19%	24.01%
	5000	5.88%	0.00%	65.88%	28.24%	0.85%	0.00%	77.14%	22.01%
Average		7.12%	0.00%	65.01%	27.87%	1.35%	0.00%	75.86%	22.79%
0.1	1000	33.34%	0.00%	66.66%	0.00%	0.00%	0.00%	64.52%	35.48%
	2000	5.88%	0.00%	65.88%	28.24%	1.23%	0.00%	76.78%	21.98%
	3000	5.88%	0.00%	65.88%	28.24%	0.59%	0.00%	77.43%	21.98%
	4000	5.88%	0.00%	65.88%	28.24%	0.81%	0.00%	75.19%	24.01%
	5000	5.88%	0.00%	65.88%	28.24%	0.85%	0.00%	77.14%	22.01%
Average		11.37%	0.00%	66.04%	22.59%	0.70%	0.00%	74.21%	25.09%
1	1000	33.34%	0.00%	66.66%	0.00%	0.01%	0.00%	69.99%	30.00%
	2000	33.34%	0.00%	66.66%	0.00%	0.01%	0.00%	65.62%	34.37%
	3000	33.33%	0.0%	66.7%	0.0%	0.0%	0.0%	65.6%	34.4%
	4000	33.34%	0.00%	66.66%	0.00%	0.0%	0.0%	64.52%	35.48%
	5000	33.34%	0.00%	66.66%	0.00%	0.0%	0.0%	64.52%	35.48%
Average		33.34%	0.00%	66.67%	0.00%	0.00%	0.00%	66.05%	33.95%
Total Average		**14.43%**	**0.00%**	**65.90%**	**19.67%**	**0.70%**	**0.00%**	**73.25%**	**26.05%**

Table B3. Flow decisions results for cost structure (c)									
λ	ν	LCF (forward)				LCF (backward)			
		Direct		Indirect		Direct		Indirect	
		Road	Rail	Road	Rail	Road	Rail	Road	Rail
0.001	1000	6.30%	56.70%	18.50%	18.50%	9.59%	84.36%	4.67%	1.38%
	2000	6.53%	58.75%	17.36%	17.36%	9.61%	84.36%	4.66%	1.38%
	3000	6.21%	55.93%	18.93%	18.93%	9.61%	84.36%	4.65%	1.38%
	4000	6.18%	55.62%	19.10%	19.10%	9.60%	84.35%	4.68%	1.38%
	5000	6.34%	57.03%	18.32%	18.32%	9.63%	84.35%	4.65%	1.38%
Average		6.31%	56.81%	18.44%	18.44%	9.61%	84.36%	4.66%	1.38%
0.01	1000	6.30%	56.70%	18.50%	18.50%	9.70%	84.36%	4.56%	1.38%
	2000	6.30%	56.70%	18.50%	18.50%	9.59%	84.36%	4.68%	1.38%
	3000	6.26%	56.38%	18.68%	18.68%	9.59%	84.35%	4.68%	1.38%
	4000	6.36%	57.26%	18.19%	18.19%	9.59%	84.39%	4.64%	1.38%
	5000	6.31%	56.81%	18.44%	18.44%	9.63%	84.35%	4.64%	1.38%
Average		6.31%	56.77%	18.46%	18.46%	9.62%	84.36%	4.64%	1.38%
0.1	1000	10.00%	90.00%	0.00%	0.00%	9.59%	84.46%	4.70%	1.25%
	2000	10.00%	90.00%	0.00%	0.00%	9.61%	84.33%	4.68%	1.38%
	3000	6.35%	57.18%	18.23%	18.23%	9.62%	84.33%	4.67%	1.38%
	4000	6.35%	57.18%	18.23%	18.23%	9.62%	84.33%	4.67%	1.38%
	5000	6.35%	57.18%	18.23%	18.23%	9.62%	84.33%	4.67%	1.38%
Average		7.81%	70.31%	10.94%	10.94%	9.61%	84.36%	4.68%	1.35%
1	1000	10.00%	90.00%	0.00%	0.00%	10.52%	83.00%	5.32%	1.17%
	2000	10.00%	90.00%	0.00%	0.00%	9.72%	83.86%	6.41%	0.00%
	3000	10.00%	90.00%	0.00%	0.00%	9.49%	84.35%	6.16%	0.00%
	4000	10.00%	90.00%	0.00%	0.00%	9.49%	84.35%	6.16%	0.00%
	5000	10.00%	90.00%	0.00%	0.00%	9.49%	84.35%	6.16%	0.00%
Average		10.00%	90.00%	0.00%	0.00%	9.74%	83.98%	6.04%	0.23%
Total Average		**7.61%**	**68.47%**	**11.96%**	**11.96%**	**9.65%**	**84.26%**	**5.01%**	**1.09%**

Table B3. Flow decisions results for cost structure (c) (cont.).									
λ	ν	ECF (forward)				ECF (backward)			
		Direct		Indirect		Direct		Indirect	
		Road	Rail	Road	Rail	Road	Rail	Road	Rail
0.001	1000	0.00%	0.0%	70.00%	30.00%	1.97%	0.00%	76.03%	22.01%
	2000	0.00%	0.0%	70.00%	30.00%	0.21%	0.00%	77.79%	22.00%
	3000	0.00%	0.0%	70.00%	30.00%	1.97%	0.00%	76.02%	22.01%
	4000	0.00%	0.0%	70.00%	30.00%	0.01%	0.00%	77.99%	22.00%
	5000	0.00%	0.0%	70.00%	30.00%	1.82%	0.00%	76.14%	22.04%
Average		0.00%	0.00%	70.00%	30.00%	1.20%	0.00%	76.79%	22.01%
0.01	1000	0.00%	0.0%	70.00%	30.00%	0.18%	0.00%	75.87%	23.96%
	2000	0.00%	0.0%	70.00%	30.00%	0.07%	0.00%	77.94%	21.98%
	3000	0.00%	0.0%	70.00%	30.00%	0.12%	0.00%	77.89%	21.99%
	4000	0.00%	0.0%	70.00%	30.00%	0.20%	0.00%	77.81%	21.99%
	5000	0.00%	0.0%	70.00%	30.00%	0.23%	0.00%	77.77%	22.00%
Average		0.00%	0.00%	70.00%	30.00%	0.16%	0.00%	77.46%	22.38%
0.1	1000	0.00%	0.0%	70.00%	30.00%	0.00%	0.00%	76.67%	23.33%
	2000	0.00%	0.0%	70.00%	30.00%	0.00%	0.00%	76.93%	23.07%
	3000	0.00%	0.00%	70.00%	30.00%	0.03%	0.00%	77.97%	22.00%
	4000	0.00%	0.00%	70.00%	30.00%	0.85%	0.00%	77.12%	22.02%
	5000	0.00%	0.00%	70.00%	30.00%	0.98%	0.00%	76.99%	22.03%
Average		0.00%	0.00%	70.00%	30.00%	0.37%	0.00%	77.14%	22.49%
1	1000	0.00%	0.00%	100%	0.00%	0.00%	0.00%	81.63%	18.37%
	2000	0.00%	0.00%	99.57%	0.43%	0.00%	0.00%	100%	0.00%
	3000	0.00%	0.00%	98.42%	1.58%	0.00%	0.00%	100%	0.00%
	4000	0.00%	0.00%	98.37%	1.63%	0.00%	0.00%	100%	0.00%
	5000	0.00%	0.00%	98.45%	1.55%	0.00%	0.00%	100%	0.00%
Average		0.00%	0.00%	98.96%	1.04%	0.00%	0.00%	96.33%	3.67%
Total Average		**0.00%**	**0.00%**	**77.24%**	**22.76%**	**0.43%**	**0.00%**	**81.93%**	**17.64%**

1

Table B4. Flow decisions results for cost structure (d)									
λ	ν	LCF (forward)				LCF (backward)			
		Direct		Indirect		Direct		Indirect	
		Road	Rail	Road	Rail	Road	Rail	Road	Rail
0.001	1000	6.30%	56.70%	18.50%	18.50%	9.54%	84.41%	4.68%	1.38%
	2000	6.53%	58.75%	17.36%	17.36%	9.56%	84.40%	4.67%	1.38%
	3000	6.21%	55.93%	18.93%	18.93%	9.73%	84.36%	4.54%	1.38%
	4000	6.18%	55.62%	19.10%	19.10%	9.54%	84.40%	4.68%	1.38%
	5000	6.34%	57.03%	18.32%	18.32%	9.55%	84.40%	4.68%	1.38%
Average		6.31%	56.81%	18.44%	18.44%	9.58%	84.39%	4.65%	1.38%
0.01	1000	6.35%	57.18%	18.23%	18.23%	9.57%	84.34%	4.72%	1.38%
	2000	6.30%	56.70%	18.50%	18.50%	9.42%	84.41%	4.80%	1.38%
	3000	6.26%	56.38%	18.68%	18.68%	9.55%	84.39%	4.68%	1.38%
	4000	6.24%	56.13%	18.82%	18.82%	9.55%	84.39%	4.69%	1.38%
	5000	6.21%	55.93%	18.93%	18.93%	9.64%	84.36%	4.63%	1.38%
Average		6.27%	56.46%	18.63%	18.63%	9.55%	84.38%	4.70%	1.38%
0.1	1000	10.00%	90.00%	0.00%	0.00%	11.92%	83.64%	3.29%	1.15%
	2000	7.04%	63.33%	14.82%	14.82%	9.60%	84.33%	4.69%	1.38%
	3000	6.35%	57.18%	18.23%	18.23%	9.60%	84.33%	4.69%	1.38%
	4000	6.35%	57.18%	18.23%	18.23%	9.60%	84.33%	4.69%	1.38%
	5000	6.35%	57.18%	18.23%	18.23%	9.57%	84.34%	6.09%	0.00%
Average		7.22%	64.97%	13.90%	13.90%	10.06%	84.19%	4.69%	1.06%
1	1000	10.71%	89.29%	0.00%	0.00%	24.59%	75.41%	0.00%	0.00%
	2000	10.00%	90.00%	0.00%	0.00%	25.81%	74.19%	0.00%	0.00%
	3000	10.00%	90.00%	0.00%	0.00%	26.98%	73.02%	0.00%	0.00%
	4000	10.00%	90.00%	0.00%	0.00%	26.98%	73.02%	0.00%	0.00%
	5000	10.00%	90.00%	0.00%	0.00%	12.24%	83.32%	3.29%	1.15%
Average		10.14%	89.86%	0.00%	0.00%	23.32%	75.79%	0.66%	0.23%
Total Average		**7.49%**	**67.03%**	**12.74%**	**12.74%**	**13.13%**	**82.19%**	**3.68%**	**1.01%**

Table B4. Flow decisions results for cost structure (d) (cont.).

λ	ν	ECF (forward)				ECF (backward)			
		Direct		Indirect		Direct		Indirect	
		Road	Rail	Road	Rail	Road	Rail	Road	Rail
0.001	1000	5.88%	0.00%	65.88%	28.24%	2.18%	0.00%	75.88%	21.94%
	2000	5.88%	0.00%	65.88%	28.24%	0.14%	0.00%	77.88%	21.98%
	3000	5.88%	0.00%	65.88%	28.24%	0.32%	0.00%	73.70%	25.98%
	4000	5.88%	0.00%	65.88%	28.24%	0.43%	0.00%	77.59%	21.98%
	5000	5.88%	0.00%	65.88%	28.24%	0.09%	0.00%	77.92%	21.99%
Average		5.88%	0.00%	65.88%	28.24%	0.63%	0.00%	76.59%	22.77%
0.01	1000	5.88%	0.00%	65.88%	28.24%	2.06%	0.00%	75.97%	21.98%
	2000	5.88%	0.00%	65.88%	28.24%	1.59%	0.00%	76.46%	21.96%
	3000	5.88%	0.00%	65.88%	28.24%	0.16%	0.00%	77.87%	21.97%
	4000	5.88%	0.00%	65.88%	28.24%	0.15%	0.00%	77.86%	21.98%
	5000	5.88%	0.00%	65.88%	28.24%	1.97%	0.00%	74.04%	23.99%
Average		5.88%	0.00%	65.88%	28.24%	1.19%	0.00%	76.44%	22.38%
0.1	1000	33.34%	0.00%	66.66%	0.00%	0.00%	0.00%	63.89%	36.11%
	2000	5.88%	0.00%	65.88%	28.24%	0.00%	0.00%	76.67%	23.33%
	3000	5.88%	0.00%	65.88%	28.24%	0.87%	0.00%	77.14%	21.99%
	4000	5.88%	0.00%	65.88%	28.24%	1.16%	0.00%	76.85%	21.99%
	5000	5.88%	0.00%	94.12%	0.00%	0.12%	0.00%	99.88%	0.00%
Average		11.37%	0.00%	71.68%	16.94%	0.43%	0.00%	78.89%	20.68%
1	1000	33.34%	0.00%	66.66%	0.00%	0.01%	0.00%	69.99%	30.00%
	2000	33.34%	0.00%	66.66%	0.00%	0.01%	0.00%	65.62%	34.37%
	3000	33.34%	0.00%	66.66%	0.00%	0.00%	0.00%	65.62%	34.37%
	4000	33.34%	0.00%	66.66%	0.00%	0.00%	0.00%	64.52%	35.48%
	5000	33.34%	0.00%	66.66%	0.00%	0.00%	0.00%	64.52%	35.48%
Average		33.34%	0.00%	66.66%	0.00%	0.00%	0.00%	66.05%	33.94%
Total Average		**14.12%**	**0.00%**	**67.53%**	**18.36%**	**0.56%**	**0.00%**	**74.49%**	**24.94%**

Appendix C.

The obtained numerical results regarding to described KPIs of service level, fill rate, and inventory turnover are reported for different values of rejected demand and solution robustness coefficients, and different cost structures in Tables C1-C4. Also, the average of these KPIs over different values of ν is presented for each given λ.

Table C1. KPIs results for cost structure (a)

λ	ν	Service Level: Raw materials ($\bar{\eta}_r$)	Service Level: Finished goods ($\bar{\eta}_s$)	Fill Rate: Raw materials ($\bar{\xi}_r$)	Fill Rate: Finished Goods ($\bar{\xi}_s$)	Inventory Turnover ($\bar{\varepsilon}$)
0.001	1000	92%	92%	95%	97%	12
	2000	80%	77%	86%	91%	12
	3000	69%	69%	79%	88%	12
	4000	54%	54%	68%	82%	12
	5000	42%	38%	60%	76%	12
Average		67%	66%	78%	87%	12
0.01	1000	100%	100%	100%	100%	12
	2000	92%	92%	95%	97%	12
	3000	85%	85%	89%	94%	12
	4000	77%	77%	84%	91%	12
	5000	69%	69%	79%	88%	12
Average		85%	85%	89%	94%	12
0.1	1000	100%	100%	100%	100%	10.08
	2000	100%	100%	100%	100%	10.08
	3000	100%	100%	100%	100%	12
	4000	100%	100%	100%	100%	12
	5000	100%	100%	100%	100%	12
Average		100%	100%	100%	100%	11.232
1	1000	100%	100%	100%	100%	5.76
	2000	100%	100%	100%	100%	8.64
	3000	100%	100%	100%	100%	10.32
	4000	100%	100%	100%	100%	10.32
	5000	100%	100%	100%	100%	10.32
Average		100%	100%	100%	100%	9.072

Table C2. KPIs results for cost structure (b)						
λ	ν	Service Level		Fill Rate		Inventory Turnover ($\bar{\varepsilon}$)
		Raw materials ($\bar{\eta}_r$)	Finished goods ($\bar{\eta}_s$)	Raw materials ($\bar{\xi}_r$)	Finished Goods ($\bar{\xi}_s$)	
0.001	1000	92%	92%	95%	97%	12
	2000	77%	77%	84%	91%	12
	3000	62%	62%	73%	85%	12
	4000	49%	46%	65%	79%	12
	5000	34%	31%	54%	73%	12
Average		63%	62%	74%	85%	12
0.01	1000	92%	92%	95%	97%	12
	2000	77%	77%	84%	91%	12
	3000	62%	62%	73%	85%	12
	4000	49%	46%	65%	79%	12
	5000	34%	31%	54%	73%	12
Average		63%	62%	74%	85%	12
0.1	1000	100%	100%	100%	100%	12
	2000	77%	77%	84%	91%	12
	3000	62%	62%	73%	85%	12
	4000	49%	46%	65%	79%	12
	5000	34%	31%	54%	73%	12
Average		64%	63%	75%	86%	12
1	1000	100%	100%	100%	100%	12
	2000	100%	100%	100%	100%	12
	3000	100%	100%	100%	100%	12
	4000	100%	100%	100%	100%	12
	5000	100%	100%	100%	100%	12
Average		100%	100%	100%	100%	12

Table C3. KPIs results for cost structure (c)						
λ	ν	Service Level		Fill Rate		Inventory Turnover ($\bar{\varepsilon}$)
		Raw materials ($\bar{\eta}_r$)	Finished goods ($\bar{\eta}_s$)	Raw materials ($\bar{\xi}_r$)	Finished Goods ($\bar{\xi}_s$)	
0.001	1000	92%	92%	95%	97%	12
	2000	80%	77%	86%	91%	12
	3000	69%	69%	79%	88%	12
	4000	54%	54%	68%	82%	12
	5000	42%	38%	60%	76%	12
Average		67%	66%	78%	87%	12
0.01	1000	92%	92%	95%	97%	12
	2000	92%	92%	95%	97%	12
	3000	85%	85%	89%	94%	12
	4000	80%	77%	65%	79%	12
	5000	69%	69%	54%	73%	12
Average		70%	69%	80%	88%	12
0.1	1000	100%	100%	100%	100%	10.08
	2000	100%	100%	100%	100%	11.04
	3000	100%	100%	100%	100%	12
	4000	100%	100%	100%	100%	12
	5000	100%	100%	100%	100%	12
Average		100%	100%	100%	100%	11.42
1	1000	100%	100%	100%	100%	7.2
	2000	100%	100%	100%	100%	8.64
	3000	100%	100%	100%	100%	10.32
	4000	100%	100%	100%	100%	10.32
	5000	100%	100%	100%	100%	10.32
Average		100%	100%	100%	100%	9.36

Table C4. KPIs results for cost structure (d)

λ	ν	Service Level: Raw materials ($\bar{\eta}_r$)	Service Level: Finished goods ($\bar{\eta}_s$)	Fill Rate: Raw materials ($\bar{\xi}_r$)	Fill Rate: Finished Goods ($\bar{\xi}_s$)	Inventory Turnover ($\bar{\varepsilon}$)
0.001	1000	92%	92%	95%	97%	12
	2000	80%	77%	86%	91%	12
	3000	69%	69%	79%	88%	12
	4000	54%	54%	68%	82%	12
	5000	42%	38%	60%	76%	12
Average		67%	66%	78%	87%	12
0.01	1000	100%	100%	100%	100%	12
	2000	92%	92%	95%	97%	12
	3000	85%	85%	89%	94%	12
	4000	77%	77%	84%	91%	12
	5000	69%	69%	79%	88%	12
Average		85%	85%	89%	94%	12
0.1	1000	100%	100%	100%	100%	12
	2000	100%	100%	100%	100%	12
	3000	100%	100%	100%	100%	12
	4000	100%	100%	100%	100%	12
	5000	100%	100%	100%	100%	12
Average		100%	100%	100%	100%	12
1	1000	100%	100%	100%	100%	12
	2000	100%	100%	100%	100%	12
	3000	100%	100%	100%	100%	12
	4000	100%	100%	100%	100%	12
	5000	100%	100%	100%	100%	12
Average		100%	100%	100%	100%	12

Appendix D.

In this section, the numerical results related to the CV of the solution of second-stage decision variables, represented by $\hat{I}^{e}_{qt}(\omega), \hat{F}^{\ell}_{pqtm}(\omega), \hat{U}^{\ell}_{qt}(\omega)$, and $\hat{B}^{\ell}_{qt}(\omega)$ are provided in Tables D1-D4 for all 80 described instances.

Table D1. Coefficient of variation for cost structure (a)

λ	ν	$\hat{I}^{e}_{qt}(\omega)$	$\hat{F}^{\ell}_{pqtm}(\omega)$				$\hat{F}^{e}_{pqtm}(\omega)$				$\hat{U}^{\ell}_{qt}(\omega)$	$\hat{B}^{\ell}_{qt}(\omega)$
		$q \in \mathbb{I}$	$p \in \mathbb{O}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{O}$	$p \in \mathbb{I}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{I}$	$p \in \mathbb{O}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{O}$	$p \in \mathbb{I}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{I}$	$q \in \mathbb{J}$	$q \in \mathbb{J}$
0.001	1000	4.808%	0.182%	0.112%	0.094%	0.142%	1.596%	2.035%	0.155%	0.142%	0.540%	0.558%
	2000	4.808%	0.243%	0.112%	0.109%	0.142%	0.000%	2.693%	0.155%	0.141%	0.341%	0.316%
	3000	4.810%	0.270%	0.110%	0.130%	0.140%	0.000%	2.750%	0.150%	0.140%	0.260%	0.270%
	4000	4.810%	0.300%	0.110%	0.140%	0.140%	1.370%	2.050%	0.150%	0.140%	0.200%	0.200%
	5000	4.810%	0.330%	0.110%	0.150%	0.140%	0.000%	7.680%	0.150%	0.140%	0.170%	0.160%
0.01	1000	4.810%	0.150%	0.110%	0.080%	0.140%	0.000%	3.900%	0.150%	0.140%	0.000%	0.000%
	2000	4.810%	0.180%	0.110%	0.090%	0.140%	0.000%	4.630%	0.150%	0.140%	0.540%	0.560%
	3000	4.810%	0.210%	0.110%	0.110%	0.140%	0.000%	7.000%	0.150%	0.140%	0.390%	0.390%
	4000	4.810%	0.240%	0.110%	0.120%	0.140%	0.000%	10.360%	0.150%	0.140%	0.310%	0.320%
	5000	4.810%	0.270%	0.110%	0.130%	0.140%	0.000%	8.150%	0.150%	0.140%	0.260%	0.270%
0.1	1000	1.370%	0.150%	0.110%	0.000%	0.140%	0.000%	0.000%	0.070%	0.180%	0.000%	0.000%
	2000	1.330%	0.150%	0.110%	0.000%	0.140%	0.000%	0.000%	0.070%	0.180%	0.000%	0.000%
	3000	4.810%	0.150%	0.110%	0.080%	0.140%	0.000%	3.980%	0.150%	0.140%	0.000%	0.000%
	4000	4.810%	0.150%	0.110%	0.080%	0.140%	0.000%	3.370%	0.150%	0.140%	0.000%	0.000%
	5000	4.810%	0.150%	0.110%	0.080%	0.140%	0.000%	4.380%	0.150%	0.140%	0.000%	0.000%
1	1000	1.720%	0.150%	0.140%	0.000%	0.160%	0.000%	0.000%	0.120%	0.150%	0.000%	0.000%
	2000	0.770%	0.150%	0.110%	0.000%	0.140%	0.000%	0.000%	0.070%	0.160%	0.000%	0.000%
	3000	0.850%	0.150%	0.110%	0.000%	0.140%	0.000%	0.000%	0.060%	0.180%	0.000%	0.000%
	4000	0.830%	0.150%	0.110%	0.000%	0.140%	0.000%	0.000%	0.060%	0.180%	0.000%	0.000%
	5000	0.840%	0.150%	0.110%	0.000%	0.140%	0.000%	0.000%	0.060%	0.180%	0.000%	0.000%

Table D2. Coefficient of variation for cost structure (b)

λ	ν	$\hat{I}^{e}_{qt}(\omega)$	$\hat{F}^{\ell}_{pqtm}(\omega)$				$\hat{F}^{e}_{pqtm}(\omega)$				$\hat{U}^{\ell}_{qt}(\omega)$	$\hat{B}^{\ell}_{qt}(\omega)$
		$q \in \mathbb{I}$	$p \in \mathbb{O}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{O}$	$p \in \mathbb{I}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{I}$	$p \in \mathbb{O}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{O}$	$p \in \mathbb{I}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{I}$	$q \in \mathbb{J}$	$q \in \mathbb{J}$
0.001	1000	4.810%	0.180%	0.110%	0.090%	0.140%	2.770%	3.410%	0.150%	0.140%	0.540%	0.560%
	2000	4.810%	0.240%	0.110%	0.120%	0.140%	2.770%	3.350%	0.150%	0.140%	0.310%	0.320%
	3000	4.810%	0.290%	0.110%	0.140%	0.140%	2.770%	4.880%	0.150%	0.140%	0.230%	0.230%
	4000	4.810%	0.320%	0.110%	0.140%	0.140%	2.770%	4.050%	0.150%	0.140%	0.190%	0.180%
	5000	4.810%	0.340%	0.110%	0.160%	0.140%	2.770%	4.010%	0.150%	0.140%	0.150%	0.140%
0.01	1000	4.810%	0.180%	0.110%	0.000%	0.140%	2.520%	2.130%	0.150%	0.140%	0.540%	0.560%
	2000	4.810%	0.240%	0.110%	0.120%	0.140%	2.770%	3.350%	0.150%	0.140%	0.310%	0.320%
	3000	4.810%	0.290%	0.110%	0.140%	0.140%	2.770%	4.880%	0.150%	0.140%	0.230%	0.230%
	4000	4.810%	0.320%	0.110%	0.140%	0.140%	2.770%	4.050%	0.150%	0.140%	0.190%	0.180%
	5000	4.810%	0.340%	0.110%	0.160%	0.140%	2.770%	4.010%	0.150%	0.140%	0.150%	0.140%
0.1	1000	5.580%	0.150%	0.150%	0.000%	0.190%	2.770%	0.000%	0.380%	0.180%	0.000%	0.000%
	2000	4.810%	0.240%	0.110%	0.120%	0.140%	2.770%	3.350%	0.150%	0.140%	0.310%	0.320%
	3000	4.810%	0.290%	0.110%	0.140%	0.140%	2.770%	4.880%	0.150%	0.140%	0.230%	0.230%
	4000	4.810%	0.320%	0.110%	0.140%	0.140%	2.770%	4.050%	0.150%	0.140%	0.190%	0.180%
	5000	4.810%	0.340%	0.110%	0.160%	0.140%	2.770%	4.010%	0.150%	0.140%	0.150%	0.140%
1	1000	5.580%	0.520%	0.380%	0.000%	0.000%	2.770%	1.610%	0.380%	0.190%	0.000%	0.000%
	2000	5.580%	0.500%	0.380%	0.000%	0.000%	2.770%	1.960%	0.380%	0.180%	0.000%	0.000%
	3000	5.580%	0.500%	0.380%	0.000%	0.000%	2.770%	2.770%	0.380%	0.180%	0.000%	0.000%
	4000	5.580%	0.500%	0.380%	0.000%	0.980%	2.770%	0.000%	0.380%	0.180%	0.000%	0.000%
	5000	5.580%	0.150%	0.180%	0.000%	0.190%	2.770%	0.000%	0.380%	0.180%	0.000%	0.000%

Table D3. Coefficient of variation for cost structure (c)

λ	ν	$\hat{I}^{e}_{qt}(\omega)$	$\hat{F}^{\ell}_{pqtm}(\omega)$				$\hat{F}^{e}_{pqtm}(\omega)$				$\hat{U}^{\ell}_{qt}(\omega)$	$\hat{B}^{\ell}_{qt}(\omega)$
		$q \in \mathbb{I}$	$p \in \mathbb{O}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{O}$	$p \in \mathbb{I}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{I}$	$p \in \mathbb{O}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{O}$	$p \in \mathbb{I}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{I}$	$q \in \mathbb{J}$	$q \in \mathbb{J}$
0.001	1000	4.810%	0.180%	0.110%	0.090%	0.140%	0.000%	2.660%	0.150%	0.140%	0.540%	0.560%
	2000	4.810%	0.240%	0.110%	0.110%	0.140%	0.000%	6.820%	0.150%	0.140%	0.340%	0.320%
	3000	4.810%	0.270%	0.110%	0.130%	0.140%	0.000%	2.720%	0.150%	0.140%	0.260%	0.270%
	4000	4.810%	0.300%	0.110%	0.140%	0.140%	0.000%	8.010%	0.150%	0.140%	0.200%	0.200%
	5000	4.810%	0.330%	0.110%	0.150%	0.140%	0.000%	2.750%	0.150%	0.140%	0.170%	0.160%
0.01	1000	4.810%	0.180%	0.110%	0.090%	0.140%	0.000%	4.180%	0.150%	0.140%	0.540%	0.560%
	2000	4.810%	0.180%	0.110%	0.090%	0.140%	0.000%	6.540%	0.150%	0.140%	0.540%	0.550%
	3000	4.810%	0.210%	0.110%	0.110%	0.140%	0.000%	9.180%	0.150%	0.140%	0.390%	0.390%
	4000	4.810%	0.320%	0.110%	0.140%	0.140%	0.000%	6.890%	0.150%	0.140%	0.190%	0.180%
	5000	4.810%	0.340%	0.110%	0.160%	0.140%	0.000%	7.350%	0.150%	0.140%	0.150%	0.140%
0.1	1000	1.670%	0.150%	0.110%	0.000%	0.140%	0.000%	0.000%	0.070%	0.180%	0.000%	0.000%
	2000	1.700%	0.150%	0.110%	0.000%	0.140%	0.000%	0.000%	0.070%	0.190%	0.000%	0.000%
	3000	4.810%	0.150%	0.110%	0.080%	0.140%	0.000%	19.610%	0.150%	0.140%	0.000%	0.000%
	4000	4.810%	0.150%	0.110%	0.080%	0.140%	0.000%	4.070%	0.150%	0.140%	0.000%	0.000%
	5000	4.810%	0.150%	0.110%	0.080%	0.140%	0.000%	3.750%	0.150%	0.140%	0.000%	0.000%
1	1000	2.520%	0.150%	0.140%	0.000%	0.160%	0.000%	0.000%	0.150%	0.160%	0.000%	0.000%
	2000	0.770%	0.150%	0.110%	0.000%	0.140%	0.000%	0.000%	0.070%	0.160%	0.000%	0.000%
	3000	0.850%	0.150%	0.110%	0.000%	0.140%	0.000%	0.000%	0.060%	0.180%	0.000%	0.000%
	4000	0.830%	0.150%	0.110%	0.000%	0.140%	0.000%	0.000%	0.060%	0.180%	0.000%	0.000%
	5000	0.840%	0.150%	0.110%	0.000%	0.140%	0.000%	0.000%	0.060%	0.180%	0.000%	0.000%

Table D4. Coefficient of variation for cost structure (d)

λ	ν	$\hat{I}^{e}_{qt}(\omega)$	$\hat{F}^{\ell}_{pqtm}(\omega)$				$\hat{F}^{e}_{pqtm}(\omega)$				$\hat{U}^{\ell}_{qt}(\omega)$	$\hat{B}^{\ell}_{qt}(\omega)$
		$q \in \mathbb{I}$	$p \in \mathbb{O}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{O}$	$p \in \mathbb{I}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{I}$	$p \in \mathbb{O}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{O}$	$p \in \mathbb{I}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{I}$	$q \in \mathbb{J}$	$q \in \mathbb{J}$
0.001	1000	4.810%	0.180%	0.110%	0.090%	0.140%	2.770%	2.580%	0.150%	0.140%	0.540%	0.560%
	2000	4.810%	0.240%	0.110%	0.110%	0.140%	2.770%	5.010%	0.150%	0.140%	0.340%	0.320%
	3000	4.810%	0.270%	0.110%	0.130%	0.140%	2.770%	6.300%	0.150%	0.140%	0.260%	0.270%
	4000	4.810%	0.300%	0.110%	0.140%	0.140%	2.770%	5.630%	0.150%	0.140%	0.200%	0.200%
	5000	4.810%	0.330%	0.110%	0.150%	0.140%	2.770%	8.500%	0.150%	0.140%	0.170%	0.160%
0.01	1000	4.810%	0.150%	0.110%	0.080%	0.140%	2.770%	2.820%	0.150%	0.140%	0.000%	0.000%
	2000	4.810%	0.180%	0.110%	0.090%	0.140%	2.770%	3.050%	0.150%	0.140%	0.540%	0.560%
	3000	4.810%	0.210%	0.110%	0.110%	0.140%	2.770%	5.260%	0.150%	0.140%	0.390%	0.390%
	4000	4.810%	0.240%	0.110%	0.120%	0.140%	2.770%	7.000%	0.150%	0.140%	0.310%	0.320%
	5000	4.800%	0.300%	0.100%	0.100%	0.100%	2.800%	2.800%	0.200%	0.100%	0.300%	0.300%
0.1	1000	5.580%	0.150%	0.150%	0.000%	0.190%	2.770%	0.000%	0.380%	0.180%	0.000%	0.000%
	2000	4.810%	0.150%	0.110%	0.070%	0.140%	2.770%	0.000%	0.150%	0.180%	0.000%	0.000%
	3000	4.810%	0.150%	0.110%	0.080%	0.140%	2.770%	4.320%	0.150%	0.140%	0.000%	0.000%
	4000	4.810%	0.150%	0.110%	0.080%	0.140%	2.770%	3.730%	0.150%	0.140%	0.000%	0.000%
	5000	4.160%	0.150%	0.110%	0.080%	0.140%	2.770%	11.420%	0.150%	0.140%	0.000%	0.000%
1	1000	5.580%	0.520%	0.380%	0.000%	0.000%	2.770%	1.610%	0.380%	0.190%	0.000%	0.000%
	2000	5.580%	0.500%	0.380%	0.000%	0.000%	2.770%	1.960%	0.380%	0.180%	0.000%	0.000%
	3000	5.580%	0.500%	0.380%	0.000%	0.000%	2.770%	2.770%	0.380%	0.180%	0.000%	0.000%
	4000	5.580%	0.500%	0.380%	0.000%	0.980%	2.770%	0.000%	0.380%	0.180%	0.000%	0.000%
	5000	5.580%	0.150%	0.180%	0.000%	0.190%	2.770%	0.000%	0.380%	0.180%	0.000%	0.000%