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# Stochastic Dynamic Traffic Assignment Model under Emergent Incidents

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## Abstract

Urban emergent incidents affect transportation operation and result in the rapid spread of traffic congestion in network, so it's necessary to analyze the dynamic changes of traffic flow distribution under emergent incidents. Therefore, model and algorithm for the dynamic traffic assignment problem under emergent incidents have been highly concerned by government and scholars. This paper proposes a stochastic dynamic traffic assignment (SDTA) model based user optimum considering the loss of node capacity and change of network structure under traffic and environment emergencies. The Nested Logit model is used to describe the departure time and path choice. Then, the variational inequality formulation is proposed and discrete dynamic network loading algorithm is designed and validated by a numerical example. The results show that the model and algorithm can be used to express the development trend of actual dynamic network under emergency.

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*Keywords:* Stochastic dynamic traffic assignment; User optimum; Variational inequality formulation; Emergent incidents

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## 1. Introduction

With the rapid urbanization process and urban size expansion, many emergencies occur frequently in the big city. Taking the 2012 Beijing rainstorm (Jul. 21st- Jul. 22nd) as an example, 95 roads were cut off because of the road flooded and large traffic congestion emerged in traffic network. There were 63 waterlogging roads in Beijing, and the rainwater depth of 30 waterlogging roads were above 30 cm. 5 metro lines and 12 stations had to be closed due to the leakage of rain [1]. Thus the urban traffic system becomes vulnerable under the influence of emergency, and

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its operation reliability and bearing capacity are greatly reduced. Traffic congestion spreads rapidly as a result. Therefore, it is necessary to provide an insight into the dynamic traffic assignment (DTA) problem under emergent incidents. And the real-time traffic flow distribution will provide a data foundation for the application of traffic intervention countermeasures.

In this study, the emergency is an sudden incident, which can induce the disruption of road notes and loss of road capacity, such as the traffic accident, deep waterlogging caused by heavy rain, sudden traffic control, and etc. Existing researches focus on the evacuation mechanism, evacuation path and evacuation organizations. Cova and Johnson thought the conflict of the intersection traffic flow was the main reason for the evacuation delay (Cova and Johnson, 2003). They proposed a lane-based model to optimize the evacuation path under the complex network. Hamza-Lup built up an emergency management system (EMS) to organize emergent traffic under disaster or terrorist attack based on real-time traffic information [10]. Gao and He investigated the optimization of evacuation routes and organizations considering the interaction characteristics, interaction delay and signal control [13]. Gao proposed a contraflow implementation strategy to improve traffic efficiency under emergency [15]. Zhang and Gao addressed a temporary vehicle movement bans measurement to alleviate congestion under emergency [16]. However, these studies only investigate the evacuation path to quickly evacuate traffic demand, and the DTA problem under emergency is seldom investigated.

There are a variety of DTA models, such as mathematical programming model, the optimal control model, and the variational inequality model [2-4]. These models have developed from static to dynamic and from certainty to randomness [6]. Stochastic Dynamic Traffic Assignment (SDTA) based the variational inequality formulation has received increasing attention recently [11]. Moreover, some researchers focus on the improvement of the algorithm efficiency, which can provide support for the big data and complex network [9,12]. Zhao and Huang introduced the concept of satisfaction, and built the user equilibrium traffic assignment model for the bounded rationality users [19]. The studies above promote the research of DTA. However, the emergency factor is seldom taken into account.

Chen and Xiao proposed a system optimization DTA model based on the shortest evacuation time [14]. However, the model was only applied to specified area based on system optimization. Although some researchers investigated the relationship between dynamic user optimal traffic assignments and emergency, the emergency influence was merely examined through the sensitivity analysis, and the DTA model and algorithm directed to emergency was not established [17]. Furthermore, some researchers studied the traffic assignment problem in a degradable transportation network, but they merely considered the degradation degree of the network capacity and ignored the change of the road capacity and network structure under incidents [18].

Therefore, this study connects the DTA problem and emergent incidents to establish SDTA model based variational inequality formulation. Considering the loss of the road node capacity under emergency, the traffic assignment procedure is implemented by a discrete dynamic network loading algorithm to show the actual dynamic traffic distribution.

This study is organized as follows. Section 2 describes the equilibrium condition and explains the constraint condition used in the models. Section 3 contains the dynamic traffic equilibrium assignment model and a solution algorithm is developed in Section 4. Numerical examples are then provided in Section 5. Finally, the important findings and recommendations for future study are summarized.

## 2. Equilibrium condition and constraints

### 2.1. Equilibrium condition

The stochastic dynamic user optimum equilibrium condition can be described as follows: in an equilibrium traffic network, no traveler can improve his perceived travel cost by unilaterally changing his departure time and path. Moreover, the probability of minimum perceived travel cost on one path is equivalent to the probability of path choice. Therefore this study uses the Nested Logit model to describe departure time and path choice.

$A$  is the set of links and  $N$  is the set of nodes.  $O$  is the set of origin nodes and  $D$  is the set of destination nodes.  $P^{od}$  is the set of paths for  $OD$  pair  $od$ ,  $p \in P^{od}$ .  $P_b^{od}$  is the set of disable paths,  $P_c^{od}$  is the set of paths except disable paths, and  $A_b$  is the set of links which are connected with the disabled node.

This study adopts discrete mathematics method to build the model, considering the period  $[K_0, K_1]$  is long enough allowing all of the departure vehicles from the origin node leave from the network in this period. The period  $[K_0, K_1]$  is divided into  $K$  intervals with length of  $\delta = (K_1 - K_0) / K$ . Then period can be expressed  $[k_0, k_1]$  and  $k \in K$ .  $[k_b, k_r]$  is the duration of network node capacity loss.

Assume that the vehicles cannot enter and leave a link at the same time interval. In the equilibrium state of a traffic network, no traveler can decrease perceived travel cost by unilaterally changing departure time or path.

Moreover, this study uses  $C^{od}(k)$  to describe the minimal expected travel cost at different time  $k$ .

$$C^{od}(k) = \begin{cases} -\frac{1}{\theta_{\tau_1}} \ln \sum_p \exp[-\theta_{\tau_1} C_p^{od}(k)], \forall k \subseteq [k_0, k_b], \\ -\frac{1}{\theta_{\tau_2}} \ln \sum_p \exp[-\theta_{\tau_2} C_p^{od}(k)], \forall k \subseteq [k_b, k_r], \\ -\frac{1}{\theta_{\tau_3}} \ln \sum_p \exp[-\theta_{\tau_3} C_p^{od}(k)], \forall k \subseteq [k_r, k_1] \end{cases} \quad (1)$$

where  $C^{od}(k)$  is the minimal expected travel cost for  $OD$  pair  $od$  at time  $k$ .  $C_p^{od}(k)$  is the travel cost of path  $p$  for  $OD$  pair  $od$  at time  $k$ .  $\theta_{\tau_i}$  is the path perceived error for the traveler which has different values under different network states.

The Nested Logit model is introduced to describe the preference of departure time and travel path choice.

$$f^{od}(k) = T^{od} \frac{\exp[-\theta_t C^{od}(k)]}{\sum_{k=1}^k \exp[-\theta_t C^{od}(k)]}, \forall k \quad (2)$$

$$f_p^{od}(k) = f^{od}(k) \times \frac{\exp[-\theta_{\tau_i} \times C_p^{od}(k)]}{\sum_{p \in P^{od}} \exp[-\theta_{\tau_i} \times C_p^{od}(k)]}, \forall k \quad (3)$$

where  $f^{od}(k)$  is the departure flow rate at time  $k$  for  $OD$  pair  $od$ ,  $f_p^{od}(k)$  is the flow rate of path  $p$  at time  $k$  for  $OD$  pair  $od$ , and  $T^{od}$  is the traffic demand for  $OD$  pair  $od$ .  $\theta_t$  is the perceived error of departure time for the traveler, and  $\theta_{\tau_i}$  is the path perceived error for the traveler which has different values under different network states.

## 2.2. Cost function

In actual traffic network, the perceived path travel cost  $\tau_p(k)$  is the only cost traveler uses, which is the estimated value to the real cost  $C_p(k)$ . And a random variable  $\varepsilon_p(k)$  is equal to the difference value between perceived cost and real cost. Then, the perceived path travel cost can be expressed as [11]

$$\tau_p(k) = C_p(k) + \varepsilon_p(k), \forall p \in P^{od}, \forall k \quad (4)$$

The path travel cost function is given by

$$C_p(k) = \sum_{a \in p} t_a(k), \forall k \quad (5)$$

Considering the exit flow rate from one link in  $A_b$ , which is the set of links connected with the disabled node is 0 at emergent incidents time  $[k_b, k_r]$ , and link cost will not be infinity but add the during time of emergent incidents. This situation is different from the usual constrain of interrupted flow. Then this study uses the improved cost function to calculate the link travel time, and the link travel cost  $t_a(k)$  is decided by the link travel time. So the link travel cost can be expressed as

$$t_a(k) = \begin{cases} \frac{L_a}{v_a^{\min} + (v_a^{\max} - v_a^{\min})[1 - (\frac{X_a(k)}{L_a w_{aj}})^\alpha]^\beta} + k_c - k_b, \forall a \in A_b, k \in [k_b, k_c] \\ \frac{L_a}{v_a^{\min} + (v_a^{\max} - v_a^{\min})[1 - (\frac{X_a(k)}{L_a w_{aj}})^\alpha]^\beta}, \text{others} \end{cases} \quad (6)$$

where  $C_p(k)$  is the travel cost on path  $p$  at time  $k$ ,  $\tau_p(k)$  is the perception travel cost on path  $p$  at time  $k$ ,  $t_a(k)$  is the travel cost on link  $a$  at time  $k$ ,  $L_a$  is the length of link  $a$ ,  $v_a^{\min}$  is the minimal velocity on link  $a$ , and  $v_a^{\max}$  is the free stream velocity on link  $a$ .  $X_a(k)$  is the number of vehicles on link  $a$  at time  $k$ , and  $w_{aj}$  is the jamming density on link  $a$  at time  $k$ .

### 2.3. Link state equations

The link state is described by the traffic loading in the SDTA model. Traffic loading is a spatial indicator, which refers to the total number of vehicles on the link. The discrete link state equation is as follows (Zhou 2003):

$$X_{ap}^{od}(k) = B_{ap}^{od}(k) - E_{ap}^{od}(k), \forall (o, d), \forall p \in P^{od}, \forall a \in A, \forall k \quad (7)$$

where  $X_{ap}^{od}(k)$  is the number of vehicles on link  $a$  following path  $p$  at time  $k$  for  $OD$  pair  $od$ ,  $B_{ap}^{od}(k)$  is the accumulative inflow amount into link  $a$  following path  $p$  at time  $k$  for  $OD$  pair  $od$ , and  $E_{ap}^{od}(k)$  is accumulative exit amount from link  $a$  following path  $p$  at time  $k$  for  $OD$  pair  $od$ .

### 2.4. Flow propagation function

The flow propagation function is given by

$$E_{ap}^{od}(k) = \sum_{j \in \{0 \leq j \delta + t_a(j) \leq k \delta\}} b_{ap}^{od}(j) \delta, \forall (o, d), \forall p \in P^{od}, \forall a \in A, \forall k \quad (8)$$

$$b_{ap}^{od}(k) = \begin{cases} f_p^{od}(k), & \text{the first link of path } p \\ e_{a'p}^{od}(k), & a' \text{ is the after link of } a \end{cases}, \forall (o, d), \forall p \in P^{od} \quad (9)$$

$$e_{ap}^{od}(k) = \frac{E_{ap}^{od}(k) - E_{ap}^{od}(k-1)}{\delta}, \forall (o, d), \forall p \in P^{od} \quad (10)$$

In the stage of node capacity loss, the parameters are defined as follows:

$$e_{ap}^{od}(k) = 0, \forall p \in P_b^{od}, \forall k \in [k_b, k_r], \forall a \in A_b$$

$$E_{ap}^{od}(k) = E_{ap}^{od}(k_b), \forall p \in P_b^{od}, \forall k \in [k_b, k_r], \forall a \in A_b$$

where  $b_{ap}^{od}(k)$  is the inflow rate into link  $a$  following path  $p$  at time  $k$  for  $OD$  pair  $od$ ,  $e_{ap}^{od}(k)$  is the exit flow rate from link  $a$  following path  $p$  at time  $k$  for  $OD$  pair  $od$ , and  $f_p^{od}(k)$  is the flow rate using path  $p$  at time  $k$  for  $OD$  pair  $od$ .

### 2.5. General constraints

Except for the above constraints, other general constraints including initial constraints, flow conservation constraints and nonnegative constraints, given as follows

$$B_{ap}^{od}(0) = 0, E_{ap}^{od}(0) = 0, V_{ap}^{od}(0) = 0, \forall (o, d), \forall p \in P^{od}, \forall a \in A \quad (11)$$

$$B_{ap}^{od}(k) = \sum_{j=0}^k b_{ap}^{od}(j) \delta \quad (12)$$

$$\sum_k f_p^{od}(k) = T^{od}, \forall k \quad (13)$$

$$\sum_{p \in P_{od}} f_p(k) = f^{od}(k), \forall k \quad (14)$$

$$B_{ap}^{od}(k) \geq 0, b_{ap}^{od}(k) \geq 0, E_{ap}^{od}(k) \geq 0, e_{ap}^{od}(k) \geq 0, X_{ap}^{od}(k) \geq 0 \\ \forall (o, d), \forall p \in P^{od}, \forall a \in A, \forall k \quad (15)$$

### 3. Model Formulation

Considering the user optimum equilibrium condition, cost function and constrains, the variational inequality formulation that equivalent to the stochastic dynamic user equilibrium condition can be expressed as follows:

$$\sum_{od} \sum_{p \in P^{od}} \sum_k \left\{ [C_p(k) + \frac{1}{\theta_r} \ln \frac{f_p^*(k)}{f^{od*}(k)}] [f_p(k) - f_p^*(k)] + \frac{1}{\theta_t} \ln \frac{f^{od*}(k)}{T^{od}} [f^{od}(k) - f^{od*}(k)] \right\} \geq 0 \quad (16)$$

Where the feasible set satisfies to constrains, variables marked with “\*” are the solution of the model.

Then this study proves the equivalence between the variational inequality formulation (16) and the equilibrium condition of (2) and (3). Analyzing the Karush-Kuhn-Tucker conditions of model (16) (Ren and Gao 2004), can be expressed as

$$C_p(k) + \frac{1}{\theta_r} \ln \frac{f_p(k)}{f^{od}(k)} - l_k^{od} - \lambda_p^{od} = 0 \quad (17)$$

$$\frac{1}{\theta_t} \ln \frac{f^{od}(k)}{T^{od}} + l_k^{od} - l^{od} = 0 \quad (18)$$

$$f_p(k) \lambda_p^{od} = 0 \quad (19)$$

$$\lambda_p^{od} \geq 0 \quad (20)$$

The formula 17 ensures that  $f_p(k) > 0$ , then we substitute (19) and (20) into (17) and obtain

$$f_p(k) = f^{od}(k) \exp\{\theta_r [I_k^{od} - C_p(k)]\} = f^{od}(k) \exp(\theta_r I_k^{od}) \cdot \exp[-\theta_r C_p(k)] \quad (21)$$

With summation of all the paths and all the OD pairs with the consideration of conservation conditions (14), we can conclude that

$$\sum_{p \in P^{od}} f_p(k) = \sum_{p \in P^{od}} f^{od}(k) \exp(\theta_r I_k^{od}) \cdot \exp[-\theta_r C_p(k)] = f^{od}(k) \quad (22)$$

And then

$$\exp(\theta_r I_k^{od}) \sum_{p \in P^{od}} \exp[-\theta_r C_p(k)] = 1 \quad (23)$$

After substituting (23) into (21), we can obtain the Logit model (3). Similarly, the Logit model (2) can be got from (18). Therefore, the variational inequality formulation is equivalent to the stochastic dynamic user equilibrium condition.

#### 4. Solution algorithm

This study designs a discrete dynamic network loading algorithm to solve the SDTA model, considering the network structure changes under node capacity loss. The detailed steps can be described as follows.

Step 1: Initialization

- Set an empty network, where

$$B_{ap}^{od}(0) = 0, E_{ap}^{od}(0) = 0, X_{ap}^{od}(0) = 0, \forall (o, d), \forall p \in P^{od}, \forall a \in A$$

- Set the duration of time interval  $\delta$  and convergence parameter  $\varepsilon$ .
- Set the iteration number  $n = 1$ , the start time  $k \neq 1$ , and the duration of the node capacity loss  $[k_b, k_r]$ .

Step 2: Seek for effective paths

- In the normal network, the effective path set  $P^{od}$ , i.e. link-disjoint paths, are obtained by traversing graph method. Calculate each path travel cost  $C_p^{od}(k)$  for OD pair  $od$ .
- In the state of network node capacity loss, the path set  $P^{od}$  is divided into  $P_b^{od}$  and  $P_c^{od}$ . Among the disable paths, the exit flow rate of disable links is 0. Calculate each path travel cost  $C_p^{od}(k)$ .
- After the network recovers to normal state, the exit flow rate of disable links recovers normal. Calculate each path travel cost  $C_p^{od}(k)$  for OD pair  $od$ .

Step 3: Path flow assignment

- Calculate the link travel cost  $t_a(k)$  on link  $a$  based on formula (6).
- Calculate  $C^{od}(k)$  for OD pair  $od$  based on formula (1).
- Calculate the departure flow rate  $f^{od}(k)$  for OD pair  $od$  based on formula (2).
- Calculate the path flow  $f_p^{od}(k)$  for OD pair  $od$  based on formula (3).

Step 4: Dynamic Stochastic Network Loading

- Calculate the accumulative exit amount  $E_{ap}^{od}(k)$  from link  $a$  following path  $p$  for OD pair  $od$  with formula (8). Calculate the exit flow rate  $e_{ap}^{od}(k)$  from link  $a$  following path  $p$  with formula (10).
- Calculate the accumulative inflow amount  $B_{ap}^{od}(k)$  into link  $a$  following path  $p$  for OD pair  $od$  with formula (12). Calculate the inflow rate  $b_{ap}^{od}(k)$  into link  $a$  following path  $p$  with formula (9).
- Calculate the number of vehicles on link  $a$  and path  $p$  for OD pair  $od$  with formula (7).

Step 5: Modify the path flow

$$f_p^{(n+1)} = f_p^{(n)} + \lambda^{(n)}(f_p^{(n)} - f_p^{(n)}), \lambda^{(n)} = \frac{n^d}{1^d + 2^d + 3^d + \dots + n^d}, d = 2$$

Step 6: Convergence judgment

$$\text{If } \frac{\sum_{od} \sum_{p \in P^{od}} \sum_{k=1}^K |f_p^{od(n)}(k) - f_p^{od(n-1)}(k)|}{\sum_{od} \sum_{p \in P^{od}} \sum_{k=1}^K |f_p^{od(n)}(k)|} \leq \mu, \text{ then stop; otherwise } n = n + 1, k = 1 \text{ and turn to step 2.}$$

### 5. Numerical example

Figure 1 shows the structure of test network. There are one origin and one destination, so the OD pair is (1, 9). The network has 9 nodes and 24 directed links. Table 1 shows the length of each link in the test network.

Parameter setting: The traffic demand is  $T^{19} = 1500$  vehicle/h. The test during are 7200s and time interval  $\delta$  is 10s. The minimal velocity is  $v_a^{min} = 5km/h$  and free stream velocity is  $v_a^{max} = 60km/h$ . Other parameters are as follows:  $\theta_r = 0.01$ ,  $\theta_i = 0.005$ ,  $\alpha = 1.4$ ,  $\beta = 3$ . Assume that node 5 is a disabled node during (240, 480) intervals, and the paths containing note 5 are break paths.

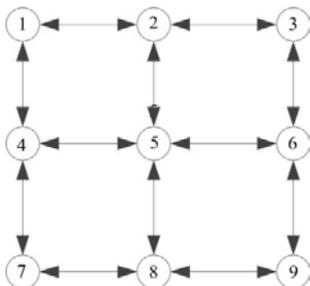


Fig. 1. The structure of test network.

Table 1. The length of links in the test network (m).

Node ID	1	2	3	4	5	6	7	8	9
1	Inf	4000	Inf	5000	Inf	Inf	Inf	Inf	Inf
2	4000	Inf	5000	Inf	3600	Inf	Inf	Inf	Inf
3	Inf	5000	Inf	Inf	Inf	5000	Inf	Inf	Inf
4	5000	Inf	Inf	Inf	3500	Inf	5000	Inf	Inf
5	Inf	3600	Inf	3500	Inf	4000	Inf	4000	Inf
6	Inf	Inf	5000	Inf	4000	Inf	Inf	Inf	4000
7	Inf	Inf	Inf	5000	Inf	Inf	Inf	4000	Inf
8	Inf	Inf	Inf	Inf	4000	Inf	4000	Inf	5000
9	Inf	Inf	Inf	Inf	Inf	4000	Inf	5000	Inf

After about 35 iterations, the program reaches the convergence condition in 3.5seconds. Figure 2 reveals the flow rates of all paths in the network. Similarly, the flow rates of break paths containing loss node 5 are shown in Figure 3 and the flow rates of paths except break paths are shown in Figure 4. Figure 5 shows the total flow rates for OD pair. In total tendency, paths which have high cost are under a lower influence in emergence incidents, such as these paths have approximately zero flow rates with low volatility. Moreover, the path flow rates show different trends at the three intervals, normal, emergency and recover.

During the normal intervals (0, 240), the tendency of every path decline to equilibrium and is almost same. The initial flow rates are high because of the empty network leading to low travel costs and large demand. As time goes on, the vehicles in network increase and travel costs rise. Then the flow rates decline gradually and reach equilibrium status.

During the emergency intervals (240, 480), the flow rates tendency of break paths containing loss node 5 declines immediately and then slowly rises. At the same time, other paths show an opposite tendency. Emergent incidents generate the loss node 5 and the travel costs of break paths become high. So the demand is transferred to other paths. With the passage of time the other paths travel costs increase to some degree. The vehicles in break paths decline gradually as the same of travel costs. Then flow rates of break paths rise gradually to equilibrium status. Especially, this study supposes the time duration of emergent incidents is predictable. Therefore at the time of 240, the flow rate of break paths is not zero because of some travelers understanding the incidents time and considering the traffic may recover normal when they reach the loss node.

During the recover intervals (480, 720), there is a buffer intervals existing obviously in Figure 2, Figure 3 and Figure 4. It is shown that the flow rates of break paths increase fast and then decline, while other paths show the opposite tendency. It is the results of the quickly decline travel costs when incidents time is over and the node 5 recovers. And then travel demand increases fast and lead to the vehicles rise fast. Therefore the costs of the whole network become high and then result in the decline tendency of flow rate, which is shown obviously in Figure 5. However, as the buffer intervals are over and the perception of traveler recovers normal without the effect of emergent incidents, each path recovers gradually to equilibrium status. Moreover, in Figure 2 the flow rate of paths in equilibrium status is the same to the equilibrium values of normal intervals.

Above all, the results of the numerical example conform to the development trend of actual network under emergency and prove the validity of the mode and solution algorithm.

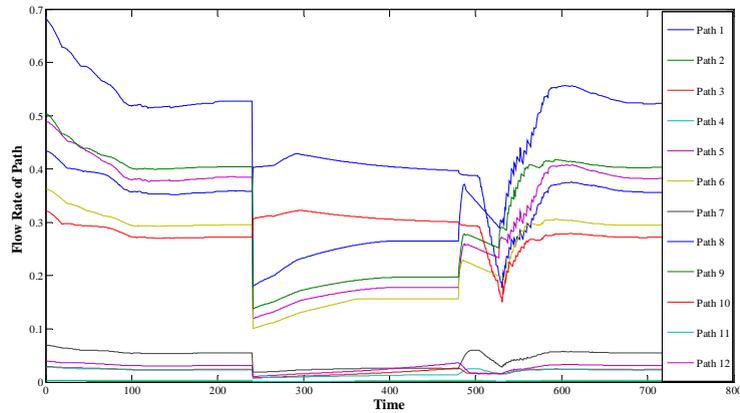


Fig. 2. Flow rates of all paths for the OD pair.

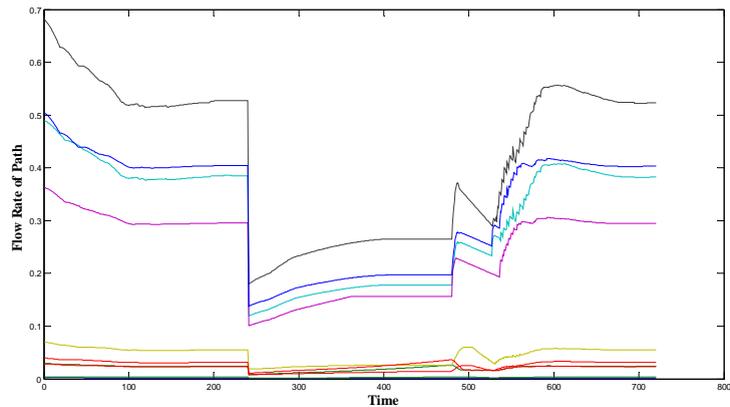


Fig. 3. Flow rates of break paths containing loss node for the OD pair.

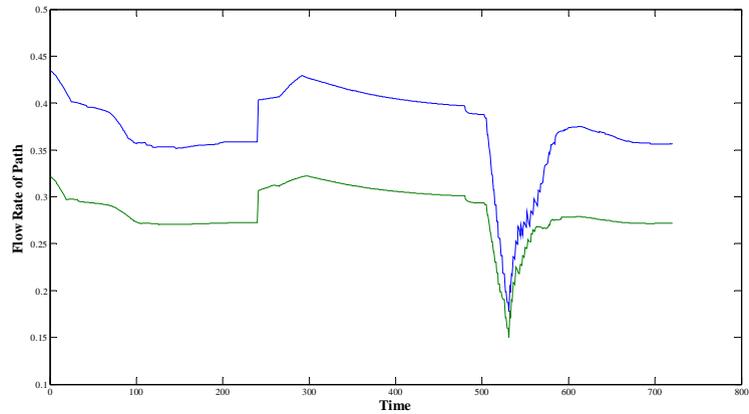


Fig. 4. Flow rates of paths except break paths for the OD pair.

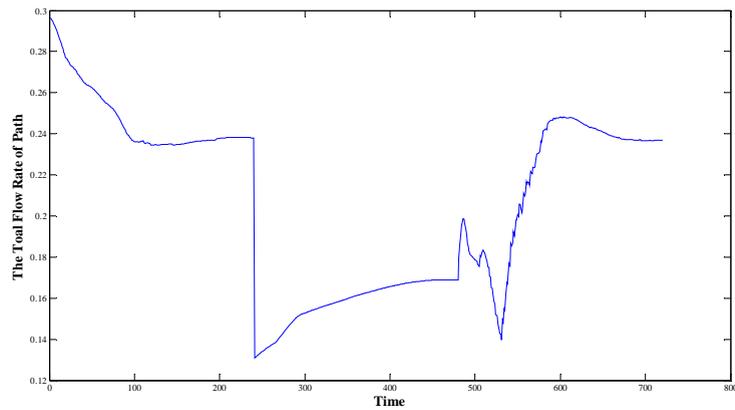


Fig. 5. The total flow rates of all paths for the OD pair.

## 6. Conclusions

This study proposes a SDTA model based on the variational inequality formulation under emergent incidents, with a model formulation and solution algorithm. The algorithm is designed in different stages, considering the network structure changes under emergent incidents. Results from numerical experiment show that the algorithm is effective for SDTA model under emergency, and valid to express the development trend of actual dynamic network.

The results of this study can provide an approach to solve the problem of network node capacity loss and a data foundation for policy makers to apply traffic intervention countermeasures. There are some recommendations for effectively alleviating the traffic problem under emergency in China. Firstly, policy makers should focus on the construction of Intelligent Transportation System (ITS) and pay more attention to the reliability of infrastructure. In addition, intervention countermeasures should be considered and analyzed with dynamic traffic network.

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