Collaborative collection effort strategies based on the "Internet + recycling" business model

ABSTRACT

"Internet + recycling", a new and emerging collecting mode, is booming in conjunction with widespread Internet use in China. For the recycling of waste electrical and electronic equipment (WEEE), this paper studies collaborative collection effort strategies in a collection system consisting of a third-party and an e-tailer based on the "Internet + recycling" business model. Considering the collaboration occurring during collecting and selling and mutual influences of partners on the recycling of old products, the paper applies collection effort cost sharing mechanisms to promote recycling. Four models, namely, the centralized model (C-Model), unit transfer price model (P-Model), unilateral cost sharing model (U-Model) and bilateral cost sharing model (B-Model), are established, and optimal decisions and members' profits in various collaborative models are derived and compared. The results show that there exists an interval of profit sharing proportions in which each of the two cost sharing models is a Pareto improvement of the P-Model, and the total collection volume and profit of the collecting system increase in the B-Model relative to those in the U-Model under the same proportion of profit sharing. However, the B-Model is not necessarily a Pareto improvement of the U-Model. The results also show that profit improvements of both parties can be achieved without the third-party sharing the e-tailer's collection effort cost in the B-Model when the collaborative marginal profit is large enough. The paper further explores the impact of the collaborative marginal profit and third-party's market influence on the total collection volume and the efficiency of the collecting system. This study provides insight into the promotion of WEEE recycling and into the selection of collaborative strategies for Internet recycling enterprises. The work will prove beneficial to the development of the WEEE

"Internet + recycling" industry.

Keywords: WEEE; Internet + recycling; Collaboration; Collection effort; Cost sharing; Bilateral participation

1. Introduction

Waste electrical and electronic equipment (WEEE) has increased sharply with the rapid updating of products and with the shortening of product life cycles. It is estimated that the number of smartphone and panel computer users reached 2.16 billion and 1.2 billion in 2016, accounting for 20% and 15% of the world's population, respectively (Greenpeace, 2016). Globally, approximately 30–50 million tons of WEEE are disposed of each year, with an estimated annual growth rate of 3–5% (Afroz, 2013). WEEE may contain valuable substances and even precious metals such as Au and Ag (Cucchiella, 2015). At the same time, WEEE can contain complex mixtures of potential environmental contaminants (Robinson, 2009). Under the dual effects of the resource crisis and environmental pollution, increasing attention has been dedicated to the recycling and reuse of WEEE.

As one of the world's largest developing countries, China accounts for approximately 20% of the global volume of WEEE (Awasthi and Li, 2017) and has become the largest producer and consumer of electrical and electronic equipment (Zeng et al., 2017). In the past, most residents in China preferred to sell their WEEE to informal peddlers or to store them at home. A recent questionnaire survey conducted in Hong Kong and Shenzhen also shows that more than 75% of the respondents prefer to store their obsolete mobile phones at home rather than recycle them (Deng et al., 2017). An online survey of lithium-ion battery (LIB) recycling also shows that 59.6% of respondents in China store their spent LIBs at home, whereas only 29.5% recycle spent LIBs with whole electronics units (Gu et al., 2017). Even so, only a small quantity of collected e-waste reaches authorized recyclers, and such waste flowing into the informal processing sector is sorted and dismantled using primitive methods in open air

(Awasthi and Li, 2017). Recovery price, convenience and personal information security are the main factors that influence customers' willingness to engage in e-waste recycling (Deng et al., 2017).

As is widely known, "Internet plus" has become China's national development strategy and has been highly encouraged through a series of policies and measures, such as "Guidance on actively promoting the 'Internet +' action" (SC, 2015) and "'Internet +' three-year action plan for green ecology" (NDRC, 2016). "Internet + recycling" refers to an O2O business model for online trading and offline recycling based on the use of Internet technology. The "Internet + recycling" industry is booming with strong support from government policies, widespread Internet use and the rapid evolution of smartphones in China. In recent years, many "Internet + recycling" enterprises have come into being; well-known examples include Huishouge, based in Wuhan (www.huishouge.cn); Aihuishou, based in Shanghai (www.aihuishou.com); Kuaishou, based in Beijing (www.kuaishou365.com); and Taolv365, based in Shenzhen (www.taolv365.com). "Internet + recycling" online platforms can be built by manufacturers, retailers, certified waste recyclers or third-party collectors, and platforms built by third-party collectors are the most common in practice. Recyclable goods include various types of items, such as intelligent digital products, notebook computers, household electronics, and clothes. This paper focuses on the "Internet + recycling" of WEEE provided by third parties such as Aihuishou.

Compared to the traditional recycling mode, the "Internet + recycling" mode is more convenient, and recycling prices are more transparent. In addition, the collector's professional data deletion service reduces consumers' worries concerning the leakage of private data stored in their digital products. More importantly, the new mode is more environmentally friendly and sustainable. It helps the Chinese government regulate recycling channels and guarantees that recycled products are delivered to qualified processing enterprises. Due to the use of advanced information technologies and automatic data processes, recovery efficiency can be greatly enhanced. Consequently, the "Internet + recycling" mode has been aggressively promoted by the Chinese government and in venture capital investments. Although the mode is still being popularized, its potential has already begun to show. For example, after the 2012 creation of Taolv365 (www. taolv365.com), an Internet trading platform for old products, the quantity of reclaimed mobile phones increased rapidly over the following three years (see Fig. 1).

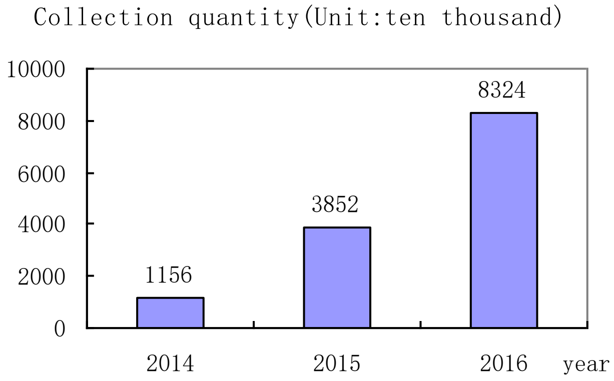


Fig. 1. The quantity of mobile phones reclaimed through Taolv365 (data source: Xue, Y., 2017)

Generally, customers often need to buy new electric and electronic equipment (EEE) when they return their old EEE, and vice versa. Accordingly, a win-win result can be achieved when third-party collectors cooperate with e-tailers, as such cooperation can not only increase the recovery of old products and the sales of new ones but also provide customers with one-stop recycling and upgrading services. Therefore, such cooperation is often adopted in practice. For example, Aihuishou (www.aihuishou.com), the largest O2O electronic product collection company in China, strategically cooperates with Jd (www.jd.com), a famous e-commerce company. Fig. 2 illustrates the typical logical trajectory of this form of cooperation. First, customers place orders for returned items through the e-tailer’s or third party’s platform, and all orders are aggregated to the third party. Next, consumers send recyclable goods to the third-party collector via third-party logistics, through outlets of the third-party collector or through door-to-door collection. Then, the third-party collector confirms the recycling price and completes the payment based on a quality inspection of the returned products, and customers receive money in cash or in coupon form, where the coupon can be used to buy new products from the e-tailer. In the end, the collected WEEE is sold to various parties, including certified disassembly plants, the second-hand market or remanufacturers (see Fig. 2).

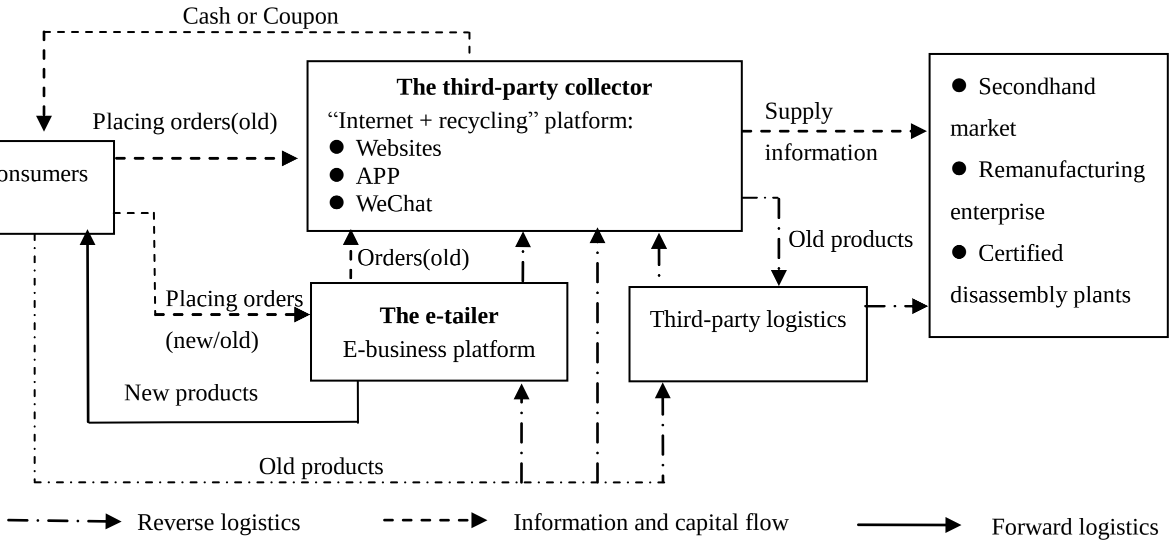


Fig. 2. The logical flow of the cooperative "Internet + recycling" between a third-party and an e-tailer

The performance of the reverse channel strongly relies on collectors' collection efforts, including their investments in advertising and promotional services, which motivate consumers to return their old products (Savaskan et al., 2004). Consumers can take express interest in returning their used products after receiving information through advertisements (Jena and Sarmah, 2015). Recycling price incentives, trading in the "old-for-new" model and coupons are all feasible means of promotion (Tong et al., 2018). Under the "Internet + recycling" mode, collection efforts can have several purposes, such as improving service quality and enhancing user experiences. For example, by 2018, Aihuishou had opened more than 300 outlets to provide face-to-face communication and transactions across 35 cities (Sun et al., 2018), while an outlet based in a downtown area itself serves as a good brand advertisement in addition to enhancing user experiences.

Motivated by the above, this paper studies collaborative collection effort strategies employed in a collecting system involving an e-tailer and a third-party under the “Internet + recycling” business model. To the best of our knowledge, such a comprehensive examination of this issue has not been undertaken in the literature. To this end, the paper develops models of the collecting system, considering collaboration occurring during collecting and selling and collection effort cost sharing mechanisms facilitating the return of used products. The optimal collection efforts are examined and compared within the framework of game theory, and members’ profits and system performance are analysed under different collaborative strategies.

The paper is organized as follows. In section 2, a relevant literature review is provided. Section 3 describes the problem and modelling assumptions. In section 4, four collaborative collection models based on the “Internet+ recycling” mode are examined, and the optimal decisions for each party are derived. Section 5 compares recycling quantities, collection effort levels and profits in the four models and presents the analytical and numerical results. A sensitivity analysis is conducted in section 6. Section 7 finally concludes this work and discusses further research.

2. Literature review

The related literature can be classified into three research streams: collection channels, collection efforts and cooperative strategies of supply chains.

Collection channel management is very central to reverse supply chains. Savaskan et al. (2004) proposed three models based on different reverse channels involving manufacturers, retailers and third parties in closed-loop supply chains (CLSCs) and found that retailer collection is the most effective means of product collection activity for the manufacturer. Savaskan and Wassenhove (2006) further extended the above models to multiple settings for the case of competing retailers and studied strategic product pricing decisions and the manufacturer’s reverse channel choices. Atasu et al. (2013) investigated the impact of collection cost structures on optimal reverse channel decisions based on the work of Savaskan et al. (2004). Mohan et al. (2018) analysed the effects of recycling and product quality levels on pricing decisions in a CLSC and showed that the unit price of the returned product paid to the retailer serves as an important determinant when selecting best channel structures between retailer- and manufacturer-led collection. Some works have focused on dual recycling or hybrid collection channels. Huang et al. (2013) investigated the channel configuration strategy of a CLSC with a dual recycling channel in which the retailer and third-party competitively collect used products and derived a parameter domain of competing intensity at which the dual recycling channel strategy outperforms the use of a single recycling channel. Hong et al. (2013) investigated three reverse hybrid collection channel structures in a manufacturer-oriented CLSC and showed that the retailer’s and manufacturer’s hybrid collection channel is the most effective. Liu et al. (2017) extended the work of Hong et al. (2013) and Huang et al. (2013) by comparing three types of hybrid competitive dual-recycling channel structures in a CLSC and found that the OEM and retailer dual collecting channel are the best tools regardless of the degree of competition intensity involved.

While the above literature provides models for studying the channel decisions made in a reverse supply chain, it mainly discusses this issue within the framework of CLSCs and with reference to traditional recycling channels. In a recent work, Feng et al. (2017) explored the recycling channel decisions of a recyclables dealer using traditional recycling and online recycling channels, and they investigated the strategic planning regarding the optimal design and coordination decisions of the dealer. Gu et al. (2019) assessed the overall environmental performance of “Internet + recycling” through a case study and concluded that the disposal of

WEEE incurs the highest environmental savings. Tong et al. (2018) identified three types of business models for recyclables using Internet technologies in China and evaluated the performance of these models. Wang et al. (2018) investigated “Internet + recycling” practices in China and made some suggestions regarding the sustainable development of “Internet + recycling”. Sun et al. (2018) analysed the structures, digital empowerment activities and types of WEEE collection business ecosystems through a study of two typical Internet-based collection enterprises. It can be observed that the literature focusing on “Internet + recycling” has grown dramatically over the past year. However, far too little attention has been paid to quantitative research regarding how to increase the quantity of WEEE acquired. Moreover, the recycling channel structure examined in this paper is different from that examined in the above literature, which includes a direct third-party online channel and an indirect e-tailer channel. The e-tailer’s platform acts as an important entry for recycling traffic, the e-tailer works together with the third party to provide consumers with one-stop services for recycling WEEE and for purchasing new ones, and the relationship between the third party and e-tailer is collaborative rather than competitive (see Fig. 2).

Many studies have considered collection efforts employed in reverse channels. Savaskan et al. (2004) first modelled the return rate of used products as a function of collection efforts and set the structure of the collection effort cost. Later, similar structures of collection effort cost have been widely used in the analysis of recycling problems related to the recycling channel, pricing, remanufacturing decisions and coordination mechanisms. For example, Gao et al. (2016) explored the influence of different channel power structures on optimal CLSC pricing decisions, collection efforts, sales efforts and performance. However, none of these studies considered cost sharing employed for collection efforts.

Cooperative strategies used in SCs have been comprehensively researched in the literature. Huang and Li (2001) investigated the efficiency of transactions for the system of manufacturer – retailer co-op advertising in the context of game theory. Ahmadi-Javid and Hoseinpour (2012) analyzed the co-op advertising model under nonnegative constraints of the sales function based on the work of Huang and Li (2001). Hong et al. (2015) incorporated advertising effects into CLSC models. In these works involving cooperative advertising, unilateral cost sharing is frequently used. Zhang et al. (2013) extended the popular unilateral participation strategy to bilateral participation in cooperative advertising and showed that properly designed bilateral participation offers several advantages relative to unilateral participation. Li et al. (2017) examined cooperative advertising strategies used in an O2O supply chain and found that bilateral cooperative advertising can offer significant benefits to the seller and to the entire channel relative to unilateral cooperative advertising. However, the above literature examines issues regarding cooperative advertising in terms of promoting the sale of new products. In a recent work, Jena et al. (2017) considered advertising as a means to entice consumers to return their used items in a CLSC, and they investigated the impacts of sharing or not sharing advertisement costs on total profits gained and on the quantity of used items acquired. Giovanni (2018) investigated whether retailers engage manufacturers to invest more heavily in green activity programmes by offering a joint incentive and showed that a joint maximization incentive always increases the manufacturers' investments made in green efforts. Ghosh et al. (2018) studied competition and collaboration between an OEM and remanufacturer. Ma et al. (2016) investigated various cooperative strategies in a three-echelon CLSC; they mainly focused on cooperative interactions occurring among members rather than cooperative collection efforts. Hence, collaborative collection effort strategies with cost sharing in an "Internet + recycling" environment have not been addressed in the reverse supply chain literature. This paper considers the effects of collaboration between the third party and e-tailer on collecting and selling and investigates how collaborative collection strategies without cost sharing, with unilateral cost sharing or with bilateral cost sharing affect the decisions of members and the performance of a collecting system.

3. Problem description

This paper considers a third-party, T, who collects used items from the market by using the "Internet + recycling" business model. To increase the volume of the recovery and to provide a better re-buy service, T cooperates with e-tailer R to collect recyclables. The logical flow of the cooperation mode is shown in Fig. 2.

Both T and R make efforts to motivate consumers to return their old products and provide consumers with related services for the purchase of new ones. A and a denote the collection effort investments of T and R, respectively. The direct collection volume through T is denoted as q_t, and the indirect volume through R is denoted as q_r. Since R does not provide the complete recycling process alone but rather cooperates with T to complete it, each member's collection efforts not only affect the collection volume of its own channel but also affect that of the other side. On one hand, the level of T's collection efforts determines its service quality and brand reputation and thus affects the recycling willingness of consumers directly or indirectly. On the other hand, because there are more opportunities for R to reach consumers, R's advertising and promoting of recycling activities not only enhance her own recovery of old products and her sales of new ones but are also conducive to increasing the popularity of T, thus indirectly enhancing the click rate of T's recycling platform. Hence, direct and indirect collection volumes travelling through the two recycling channels are respectively formulated as

$$q_t = s_t + \sqrt{A} + k_1\sqrt{a}\,, \tag{1}$$

$$q_r = s_r + \sqrt{a} + k_2\sqrt{A}\,. \tag{2}$$

The square root formulation of response functions denotes diminishing returns to collection effort expenses (Zhang et al., 2013), and $\sqrt{A}$ and $\sqrt{a}$ can be regarded as the two parties' levels of collection efforts. The additive function is also used in Jena et al. (2017). s_r

and s_t are positive constants representing the returned quantities when each member's collection efforts are valued at zero; to facilitate calculation, the values of s_r and s_t are set to zero, which does not affect the conclusions of this study. k_1 and k_2 represent the influencing coefficients of each member's collection efforts on the other side. Assume that each member's collection efforts boost the other party's collection volumes, so $k_1, k_2 \in (0,1)$.

Eqs. (1) - (2) indicate that the collection volume is a joint effort employed by T and R, and the values of A and a are related to the collaborative collection effort strategies adopted. Meanwhile, increasing the collection volume will increase the sales of new products and overall profits. To this end, four collaborative collection models are developed. The first model is a centralized model (C-Model) in which both T and R agree to make efforts to maximize the whole profits of the collecting system in an integrated manner. The second model is a unit transfer price model (P-Model) in which T pays a unit transfer price b_r to R for items returned through the R channel. The third model is a unilateral cost sharing model (U-Model) in which T not only invests in her own channel but also bears part of R's collection effort expenses. The fourth model is a bilateral cost sharing model (B-Model) in which each member shares partial costs of the other member, or rather, T shares a fraction, t_1 ($t_1 \in [0,1]$), of R's collection effort costs a, and R shares a fraction, t_2 ($t_2 \in [0,1]$), of T's costs A. Consistent with Zhang et al. (2013), t_1 and t_2 are referred to as T's participation rate and R's participation rate, respectively. Accordingly, the collaborative strategies based on the three decentralized decision models are referred to as the P-strategy, U-strategy and B-strategy, respectively.

Let b be the marginal profit generated from recycling per unit of used product. The appropriate allocation of recycling profit, i.e., b, between T and R is investigated in this paper. R not only shares income from the recovery of old products but also earns "old-for-new" profits. Let u be the collaborative marginal profit derived from the additional sale of new products caused by the recovery of per unit of old ones, and assume that $b \geq u \geq 0$. Generally, the higher the value of a product, the higher the collaborative marginal profit u. In addition, the stronger the level of coordination between T and R, the greater the probability of converting from recovery to purchasing and thus the greater the value of u.

The symbols used for the development of collaborative collection models are presented in Table 1.

Table 1. Descriptions of the symbols.

Symbol	Description
A	Collection effort investments of the third-party, decision variable
a	Collection effort investments of the e-tailer, decision variable
k_1	Influence coefficient of the e-tailer 's collection efforts to the third-party
k_2	Influence coefficient of the third-party's collection efforts to the e-tailer
q_t	Direct collection volume through the third-party's channel
q_r	Indirect collection volume through the e-tailer's channel
b	Marginal profit by recycling one unit of used products
u	Collaborative marginal profit for the sale of new products through the recovery of per unit of old ones
b_r	Unit transfer price paid to the e-tailer by the third-party, decision variable
β	Proportion of profit sharing for the e-tailer
t_1	Proportion of the e-tailer's collection effort investments shared by the third-party, decision variable
t_2	Proportion of the third-party's collection effort investments shared by the e-tailer, decision variable
π_i^j	Profit of channel member i in model j. Subscript $i \in \{t, r, s\}$ refers to the third-party, the e-tailer and the whole collecting system separately. Superscript $j \in \{C, P, U, B\}$ refers to the C-Model, P-Model, U-Model and B-Model separately.
η^j	Efficiency of the collecting system, $\eta = \pi_s^{j*} \big/ \pi_s^{C*}$.

4. Collaborative collection effort models

In this section, four collection effort models, namely, the centralized model (C-Model), unit transfer price model (P-Model), unilateral cost sharing model (U-Model) and bilateral cost sharing model (B-Model), are established, the optimal decisions are derived, and the influences of the key parameters on the optimal decisions are discussed. In the decentralized models, T is regarded as the Stackelberg game leader and R as the follower.

4.1 C-Model

In this case, T and R belong to the same business conglomerate and act as a single entity, and thus only one decision maker determines A and a to maximize the total profits of the collection system. The total profit of the system is denoted as

$$\pi_s^C = (b+u)\left[(1+k_2)\sqrt{A} + (1+k_1)\sqrt{a}\right] - A - a \tag{3}$$

Thus, from the first-order condition, i.e. $\frac{\partial \pi_s^C}{\partial A} = 0$, and $\frac{\partial \pi_s^C}{\partial a} = 0$, an optimal solution (A^{C*}, a^{C*}) is defined as follows:

$$\begin{cases} A^{C*} = \left(\dfrac{(b+u)(1+k_2)}{2}\right)^2 \\ a^{C*} = \left(\dfrac{(b+u)(1+k_1)}{2}\right)^2 \end{cases} \tag{4}$$

The optimal collection volumes are obtained based on collection effort levels, which are given by

$$q_t^{C*} = \frac{(b+u)(1+k_1+k_2+k_1^2)}{2}, \tag{5}$$

$$q_r^{C*} = \frac{(b+u)(1+k_1+k_2+k_2^2)}{2} \tag{6}$$

The total profit of the collecting system is

$$\pi_s^{C*} = \frac{(b+u)^2[(1+k_2)^2 + (1+k_1)^2]}{4} \tag{7}$$

These acquired closed-form solutions in the C-Model offer benchmarking for designing cooperative collection effort models.

4.2 P-Model

In this model, both T and R make efforts to motivate consumers to return WEEE, but they must address their collection effort expenses individually. T provides a unit transfer price b_r to R to induce her to collect used products. In addition, R earns additional profits from increased sales of new products due to the recovery of old products. The profit expressions of T and R can be written as

$$\pi_t^P = b(\sqrt{A} + k_1\sqrt{a}) + (b - b_r)\left(\sqrt{a} + k_2\sqrt{A}\right) - A \tag{8}$$

$$\pi_r^P = (b_r + u)\left(\sqrt{a} + k_2\sqrt{A}\right) + u\left(\sqrt{A} + k_1\sqrt{a}\right) - a \tag{9}$$

As the Stackelberg leader, T first proposes collection effort A and unit transfer price b_r, and then R determines the collection effort a.

Via standard backward induction, the optimal solution of the collection efforts from the first-order condition is given by

$$a^P = \left(\frac{b_r + u(1+k_1)}{2}\right)^2 \tag{10}$$

Proposition 1. Let $u^P = b(1 - \frac{k_2 + k_2^2}{1+k_1})$. In the P-Model, the optimal collection efforts of T and R, the optimal unit transfer price paid to R are given by

$$b_r^{P*} = \begin{cases} \dfrac{(b-u)(1+k_1) - k_2 b(1+k_2)}{2-k_2^2}, & u < u^P \\ 0 & , u \geq u^P \end{cases} \tag{11}$$

$$\sqrt{A^{P*}} = \begin{cases} \dfrac{2b(1+k_2) - k_2(1+k_1)(b-u)}{2(2-k_2^2)}, & u < u^P \\ \dfrac{b(1+k_2)}{2} & , u \geq u^P \end{cases}, \tag{12}$$

$$\sqrt{a^{P*}} = \begin{cases} \dfrac{(b + u - uk_2^2)(1+k_1) - k_2 b(1+k_2)}{2(2-k_2^2)}, & u < u^P \\ \dfrac{u(1+k_1)}{2} & , u \geq u^P \end{cases} \tag{13}$$

Proof. See Appendix A.

Proposition 1 implies that there is always an optimal combination of (A^{P*}, b_r^{P*}) for maximizing the profit of T in the P-Model. The condition $u < u^P$ guarantees that the optimal unit transfer price is greater than zero. When $u \geq u^P$, even when R cannot obtain a transfer payment for her collection efforts, she still gains quite good returns due to a high added collaborative profit.

The proportion of profit sharing for R’s collecting can be calculated as

$$\beta^{P*} = \frac{b_r^{P*}}{b} = \frac{(b-u)(1+k_1) - k_2 b(1+k_2)}{(2-k_2^2)b}, \ \beta^{P*} \in [0,1) \quad . \tag{14}$$

The optimal collection volumes of direct and indirect channels can be computed from Eqs. (1) - (2), and the optimal profits of T and R in the P-Model are obtained from Eqs. (8) - (9).

It is easy to observe that in the P-Model, the optimal unit transfer price b_r^{P*} is monotonically decreasing in u and k_2, whereas the optimal collection efforts and optimal profits of both T and R increase with increasing u. For the collection of products with high collaborative marginal profits, T can pay R a low transfer payment because R can obtain compensation from increasing sales of new products. In addition, in early stages when T enters the recovery market, which involve a lower value of k_2, T should pay R a higher transfer price to attract R to participate in collecting. Similarly, both T and R invest more in the collection of highly profitable items such as smartphones. All of these principles are consistent with observable reality.

4.3 U-Model

Collaborative collecting involves the joint efforts of T and R to increase collection volumes, the sales of new products and overall profits. To achieve better performance, a unilateral cost sharing model (U-Model) is proposed, in which dominant party T not only invests in his own channel collection efforts but also bears a fraction t_1 ($t_1 \in [0,1]$) of R’s collection effort expenses. Meanwhile, T shares a proportion $1-\beta$ of the R collection channel's profits, and the value of β ($\beta \in [0,1]$) is determined by both T and R.

The profit functions of T and R are formulated as

$$\pi_t^U = b(\sqrt{A} + k_1\sqrt{a}) + (1-\beta)b\left(\sqrt{a} + k_2\sqrt{A}\right) - A - t_1 a \tag{15}$$

$$\pi_r^U = \beta b\left(\sqrt{a} + k_2\sqrt{A}\right) + u\left(\sqrt{A} + k_1\sqrt{a} + \sqrt{a} + k_2\sqrt{A}\right) - (1-t_1)a \tag{16}$$

T first discloses his collection effort level and participation rate, and then R determines her collection effort level.

Taking the derivative of π_r^U with respect to a yields

$$\frac{\partial \pi_r}{\partial a} = \frac{\beta b + u(1+k_1)}{2\sqrt{a}} - (1-t_1)\text{, and } \frac{\partial^2 \pi_r}{\partial a^2} = -\frac{1}{4}[\beta b + u(1+k_1)]a^{-3/2} < 0$$

This implies that π_r^U is a concave function, and from the first-order condition, the optimal collection efforts of R is as follows:

$$a^U = \left[\frac{\beta b + u(1+k_1)}{2(1-t_1)}\right]^2 \tag{17}$$

By substituting a^U into Eq. (15) and solving T's problem, the optimal result is presented by Proposition 2.

Proposition 2. Let $\beta^U = \min\{\frac{(2b-u)(1+k_1)}{3b}, 1\}$; in the U-Model, the optimal participation rate of T is

$$t_1^{U*} = \begin{cases} \dfrac{(2+2k_1-3\beta)b - u(1+k_1)}{(2+2k_1-\beta)b + u(1+k_1)}, & \beta < \beta^U \\ 0, & \beta \geq \beta^U \end{cases}, \tag{18}$$

and the optimal collection efforts of T and R are given by

$$\sqrt{A^{U*}} = \frac{b[1+k_2(1-\beta)]}{2}, \beta \in [0,1] \tag{19}$$

$$\sqrt{a^{U*}}=\begin{cases}\dfrac{(2+2k_1-\beta)b+(1+k_1)u}{4}, & \beta<\beta^U\\ \dfrac{\beta b+u(1+k_1)}{2}, & \beta\ge\beta^U\end{cases} \tag{20}$$

Proof. See Appendix B.

Proposition 2 indicates that when the proportion of profit sharing for R is not too great (i.e., $\beta<\beta^U$), T has an incentive to share R's collection effort expenses to promote collecting for both direct and indirect channels. Otherwise, when the proportion is dominant enough ($\beta\ge\beta^U$), T will not participate in R's expenses ($t_1=0$), and so the U-Model is transformed into the P-Model; then, the value of β can be determined from Eq. (14). To distinguish it from the P-Model, the U-Model described below refers to a situation in which t_1 is greater than 0.

β^U denotes a critical value. The smaller the collaborative profit u is, the larger β^U is and the more likely T is willing to share R's collection effort expenses. In contrast, R's influence coefficient k_1 has a positive effect on the critical value β^U.

The formulation of an optimal collection volume can be computed from Eqs. (1) - (2), and the optimal profits of T and R can be determined from Eqs. (15) - (16).

From Proposition 2, Corollaries 1- 3 can be easily obtained.

Corollary 1 In the U-Model, the optimal participation rate t_1 is monotonically decreasing in u and is independent of k_2.

Corollary 1 implies that T should share more collection effort expenses of R for the sake of maximizing his profit when the collaborative marginal profit is small. For example, in the early stages of their cooperation, the conversion rate derived from the recovery of old products to the sale of new ones may be low due to poor coordination, which results in a small value of u. Under such conditions, T should undertake more collection effort investments of R. However, an increase in k_2, which can be regarded as the strengthening influence of T on the recycling market, does not affect T’s participation rate.

Corollary 2 In the U-Model, the optimal collection effort of R increases in u and is independent of k_2, whereas the optimal collection effort of T is monotonically increasing in k_2 and is independent of u.

Corollary 2 indicates that the collaborative marginal profit u has a positive impact on R's collection effort but has no effect on T’s collection effort. In contrast, an increase in k_2 does not cause R to increase her collection effort level, but it will increase T’s collection effort level.

Corollary 3 In the U-Model, the profits of both T and R are monotonically increasing functions of u and k_2.

Although only R’s collection effort increases with an increase of u, the profits of both T and R still grow as the direct and indirect collection volumes increase with respect to u, which implies that a higher collaborative marginal profit is beneficial not only to R but also to T. The same is true for the influence coefficient k_2.

4.4 B-Model

Studies have shown that bilateral participation can improve the channel efficiency of cooperative advertising strategies (Zhang et al., 2013). During the cooperative collection between T and R, as shown in Fig. 2, is R willing to share a portion of T’s collection costs to increase collection volumes and to thus promote the sale of new products? This is what the paper investigates regarding the B-Model. In this case, both members not only invest in their own channel collecting efforts but also bear a fraction t_1 / t_2 ($t_1, t_2 \in (0,1)$) of the other side’s collection expenses. They share the collecting profit, and β ($\beta \in (0,1)$) is the proportion of profit sharing for R.

The profit functions of T and R are formulated as follows:

$$\pi_t^B = b(\sqrt{A}+k_1\sqrt{a})+(1-\beta)b\left(\sqrt{a}+k_2\sqrt{A}\right)-(1-t_2)A-t_1a \tag{21}$$

$$\pi_r^B = \beta b\left(\sqrt{a}+k_2\sqrt{A}\right)+u[(1+k_2)\sqrt{A}+(1+k_1)\sqrt{a}]-(1-t_1)a-t_2A \tag{22}$$

There are four decision variables in the B-Model, including the collection effort investments of T and R, A and a, and the bilateral participation rates t_1 and t_2. According to Zhang et al. (2013), there are some rules regarding the allocation of decision-making power that game players should follow to avoid trivial or unreasonable game results. In applying these rules to the B-Model, suppose that the leader of the game makes a decision about participation rates, while the follower makes decisions about collection efforts.

Again, by using backward induction, the optimal result is presented by Proposition 3.

Proposition 3. Let $\beta^B = \min\{\frac{(2b-u)(1+k_2)}{3k_2b},1\}$. For any given β in the B-Model, the optimal participation rate of T is

$$t_1^{B*} = \begin{cases} \frac{(2b-u)(1+k_1)-3\beta b}{(2b+u)(1+k_1)-\beta b}, & \beta < \beta^U \\ 0, & \beta \geq \beta^U \end{cases} \tag{23}$$

The optimal participation rate of R is given by

$$t_2^{B*} = \begin{cases} \frac{2k_2\beta b+2u(1+k_2)}{(2b+u)(1+k_2)-k_2\beta b}, & \beta < \beta^B \\ 1, & \beta \geq \beta^B \end{cases} \tag{24}$$

The optimal collection efforts can be computed as follows:

$$\sqrt{a^{B*}} = \begin{cases} \frac{(2b+u)(1+k_1)-\beta b}{4}, & \beta < \beta^U \\ \frac{\beta b+u(1+k_1)}{2}, & \beta \geq \beta^U \end{cases} \tag{25}$$

$$\sqrt{A^{B*}} = \begin{cases} \frac{(2b+u)(1+k_2)-k_2\beta b}{4}, & \beta < \beta^B \\ \frac{k_2\beta b+u(1+k_2)}{2}, & \beta \geq \beta^B \end{cases} \tag{26}$$

Proof. See Appendix C.

Since $\beta^B \geq \beta^U$ and $t_2^{B*} > 0$ always hold, according to Proposition 3, the B-strategy would become another U-strategy when $\beta \geq \beta^U$. In other words, T may not need to share part of the collection effort cost of R ($t_1^{B*} = 0$), whereas R must share part of the cost of T($t_2^{B*} > 0$). This means that it is always beneficial to T when R bears a fraction of T's investment in collection efforts, while whether T has an incentive to share R's collection effort expense is related to the value of β, i.e., T has an incentive only when $\beta < \beta^U$.

Hence, under the B-strategy, the optimal collection volumes can be computed from Eqs. (1) - (2), and the optimal profits of T and R can be obtained from Eqs. (21) - (22).

From Proposition 3, Corollary 4 is easily obtained.

Corollary 4 In the B-Model, R's optimal participation rate t_2, T's collection effort A and the profits of both T and R are monotonically increasing with respect to k_2 and u.

Corollary 4 shows that higher collaborative profit and stronger influence of T can increase R's participation rate and T's collection effort investments. Consequently, the profits of both T and R can be improved.

5. Comparative analysis

According to the above results, some conclusions can be drawn through the comparison of different collaborative collection effort models. The following numerical analysis illustrates the results; the initial parameter setting is $b = 10$, $k_1 = 0.5, k_2 = 0.1, u = 3$.

5.1 Comparison of the U-Model and P-Model

Proposition 4. When $\beta = \beta^{P*}$ in the U-Model, relative to the P-Model, ordinal relationships of optimal collection efforts are related as $A^{P*} = A^{U*}$ and $a^{P*} < a^{U*}$. Consequently, collection volumes are related as follows: $q_r^{P*} < q_r^{U*}$ and $q_t^{P*} < q_t^{U*}$. The member's profits are related as follows: $\pi_t^{U*} > \pi_t^{P*}$, $\pi_r^{U*} > \pi_r^{P*}$ and $\pi_s^{U*} > \pi_s^{P*}$.

Proof. See Appendix D.

Proposition 4 implies that under the same profit share as the optimal one in the P-Model, R’s collection effort investments will be enhanced in the U-Model, whereas T’s collection effort investments remain the same. As T shares part of the collection effort investment of R, the total collection effort investment increases; thus, the collection volumes of the direct and indirect channels increase, and the profits of both T and R in the U-Model are greater than those in the P-Model. Therefore, the U-strategy is a Pareto improvement of the P-strategy when the profit share remains the same as that of P-strategy.

Corollary 5 Let $\beta^{UL}=\frac{\left(1+k_1+2k_2+2k_2^2\right)b-2k_2u(1+k_2)}{(1+4k_2^2)b}$. In the U-Model, the optimal profits of both T and the collecting system are monotonically decreasing in β, and the following hold:

(i) if $\beta^{UL}\geq\beta^{U}$, the optimal profit of R is an increasing function of β when $\beta\leq\beta^{U}$;

(ii) if $\beta^{UL}<\beta^{U}$, the optimal profit of R is an increasing function of β when $\beta\leq\beta^{UL}$ and a decreasing function of β when $\beta^{UL}<\beta\leq\beta^{U}$.

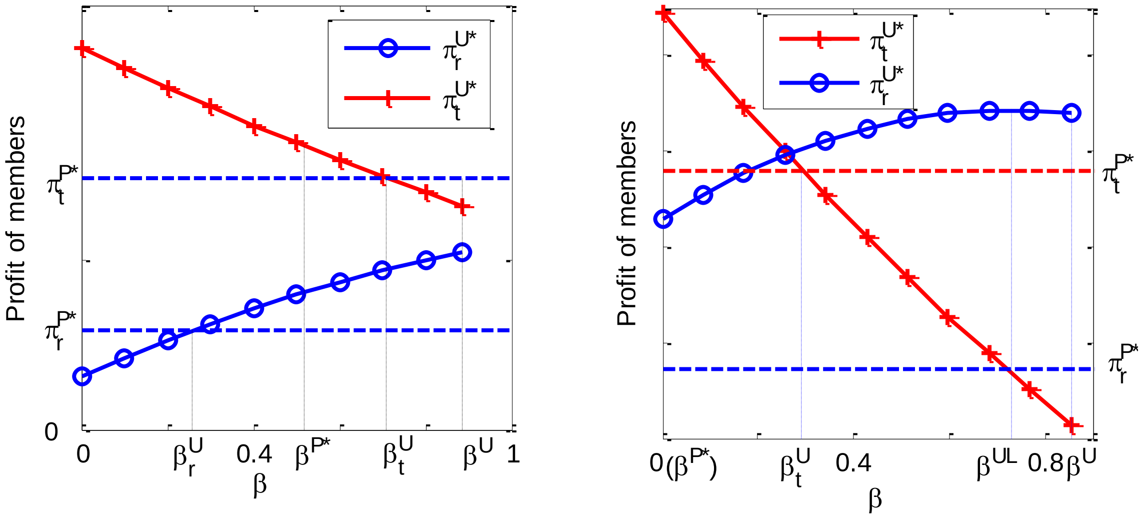
Proof. See Appendix E.

Corollary 5 shows that increasing the proportion of profit sharing for R is always disadvantageous to both T and the collecting system under U-strategy and is not always advantageous to R.

Corollary 6 There is always an interval (β_r^U,β_t^U) in the U-Model in which β_r^U and β_t^U satisfy $0\leq\beta_r^U\leq\beta^{P*}$ and $\beta^{P*}<\beta_t^U\leq\beta^U$, respectively. When the value of β falls within the range of (β_r^U,β_t^U), the optimal profits of both T and R will increase in the U-Model relative to those in the P-Model.

Proof. See Appendix F.

Corollary 6 extends the range of β in which the U-strategy is a Pareto improvement of the P-strategy. This also shows that when the value of β is within a certain range under the U-strategy, a win-win result can be achieved relative to that achieved with the P-strategy. In Fig. 3a, β_r^U and β_t^U are the proportions of profit sharing that give $\pi_r^{U*}\big|_{\beta=\beta^{P*}} = \pi_r^{P*}$ and $\pi_t^{U*}\big|_{\beta=\beta^{P*}} = \pi_t^{P*}$, respectively. When $\beta^{UL} \geq \beta^U$, the optimal profit of R is a monotonically increasing function of β under the U-strategy, thresholds β_r^U and β_t^U satisfy $0 \leq \beta_r^U < \beta^{P*}$, and $\beta^{P*} < \beta_t^U \leq \beta^U$, respectively, and thus the optimal profits of both T and R increase in the U-Model relative with those of the P-Model when $\beta \in (\beta_r^U, \beta_t^U)$ (see Fig. 3a). When $\beta^{UL} < \beta^U$, the optimal profit of R first increases and then decreases with increasing β. In this case, since $\beta^{P*} = 0$ and $\pi_r^{U*}\big|_{\beta=\beta^{P*}} > \pi_r^{P*}$, $\beta_r^U = \beta^{P*} = 0$ holds, and thus a win-win result can also be achieved when using the U-strategy rather than the P-strategy when $\beta \in (0, \beta_t^U)$ (see Fig. 3b; the parameter values are as follows: $b = 20$, $k_1 = 0.9$, $k_2 = 0.9, u = 13$).



(a) (b)

Fig. 3. Comparison between the U-Model and P-Model: (a) if $\beta^{UL} \geq \beta^U$ and (b) if $\beta^{UL} < \beta^U$

5.2 Comparison of the B-Model and P-Model

Proposition 5. When $\beta = \beta^{P*}$, relative to the P-Model, the ordinal relationships of the optimal collection efforts are $A^{P*} < A^{B*}$ and $a^{P*} < a^{B*}$. Consequently, the collection volumes are as follows: $q_r^{P*} < q_r^{B*}$ and $q_t^{P*} < q_t^{B*}$. The members' profits are related as follows: $\pi_t^{B*} > \pi_t^{P*}$, $\pi_r^{B*} \geq \pi_r^{P*}$ when $M \geq 0$, $\pi_r^{B*} < \pi_r^{P*}$ when $M < 0$, and $\pi_s^{B*} > \pi_s^{P*}$ where $M = (2b-4u)\beta^{P*}b[1+k_1-k_2(1+k_2)]+(2b-u)u[(1+k_1)^2-(1+k_2)^2]-3(1-k_2^2)(\beta^{P*}b)^2$.

Proof. See Appendix G.

Proposition 5 shows that with the same proportion of profit sharing as the optimal one in the P-Model, when both T and R share part of the collection investments of the other side, the collection efforts of both sides and the collection volumes of both channels will increase, and for T and the collecting system, the B-Model is more profitable than the P-Model. However, for R, only when $M \geq 0$ is the optimal profit of R for the B-Model higher than that of the P-Model. Through data simulations, it is also found that $M \geq 0$ almost always holds when $k_1 \geq k_2$, although it cannot be analytically proven due to the complexity of M.

Corollary 7 In the B-Model, the profit of R is an increasing function of β, and in contrast, the profits of both T and the collecting system are decreasing functions of β.

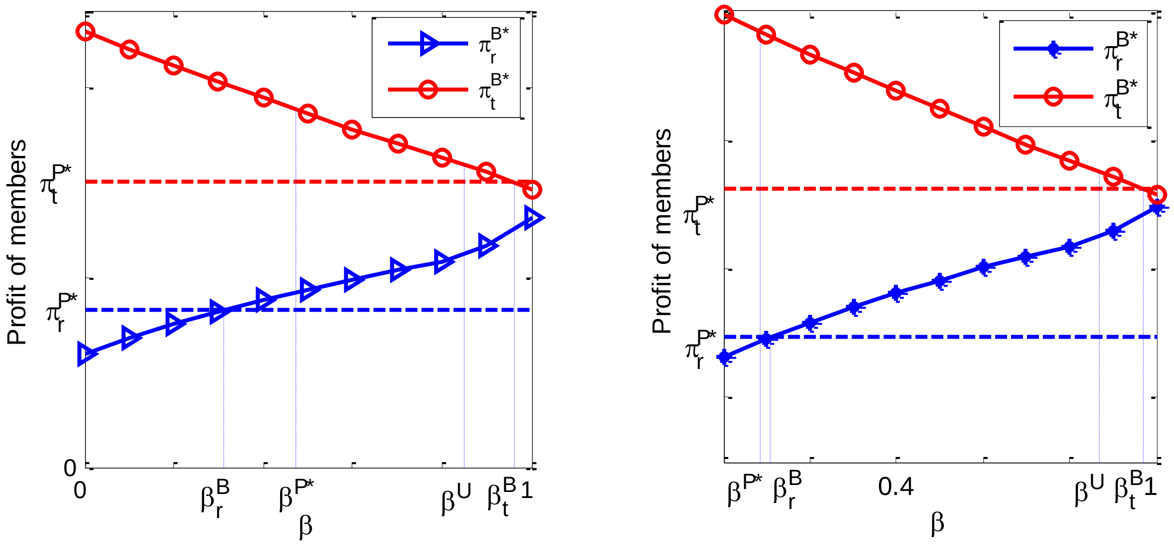
Proof. See Appendix H.

Corollary 7 indicates that increasing the proportion of profit sharing for R can increase R's profit, but it is at the expense of the profits of T and the collection system. There must be appropriate values of β for a trade-off between T and R.

Corollary 8 There is always an interval (β_r^B, β_t^B) in the B-Model in which $0 \leq \beta_r^B \leq \beta^{P*}$ and $\beta^{P*} < \beta_t^B \leq 1$ when $M \geq 0$ and in which $\beta^{P*} < \beta_r^B, \beta_t^B \leq 1$ when $M < 0$. When the value of β falls within the range of (β_r^B, β_t^B), the optimal profits of both T and R will increase relative to those of the P-Model.

Proof. As it is similar to the proof of Corollary 6, the proof is omitted here.

Corollary 8 gives the range of the profit share β in which the B-strategy is a Pareto improvement of P-strategy. In Fig. 4, β_r^B, β_t^B are the proportions of profit sharing that create $\pi_r^{B*}\big|_{\beta=\beta^{P*}}=\pi_r^{P*}$ and $\pi_t^{B*}\big|_{\beta=\beta^{P*}}=\pi_t^{P*}$, respectively. $M>0$ denotes that $\pi_r^{B*}\big|_{\beta=\beta^{P*}}>\pi_r^{P*}$, and so $0\le\beta_r^B<\beta^{P*}$ and $\beta^{P*}<\beta_t^B\le 1$ (see Fig. 4a). Keeping the values of other parameters unchanged and increasing the value of k_2 to 0.6 such that $M<0$, $\pi_r^{B*}\big|_{\beta=\beta^{P*}}<\pi_r^{P*}$, and thus $\beta^{P*}<\beta_r^B,\beta_t^B\le 1$ (see Fig. 4b).



(a)

(b)

Fig. 4. Comparison of the optimal profits between the B-Model and P-Model. (a) $M\ge 0$ and (b) $M<0$

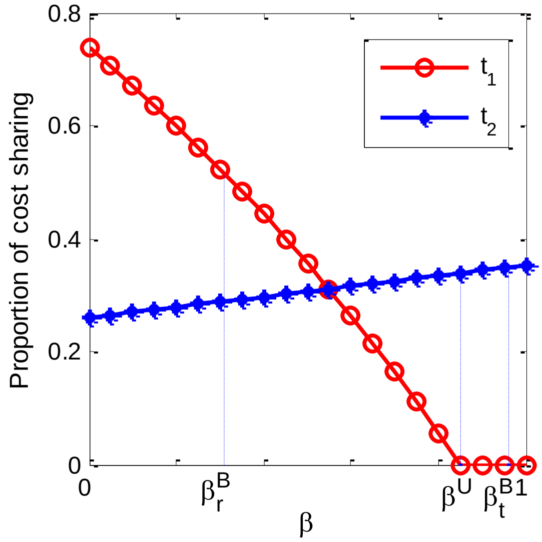


Fig. 5. The optimal proportion of cost sharing under the B-Model as β varies

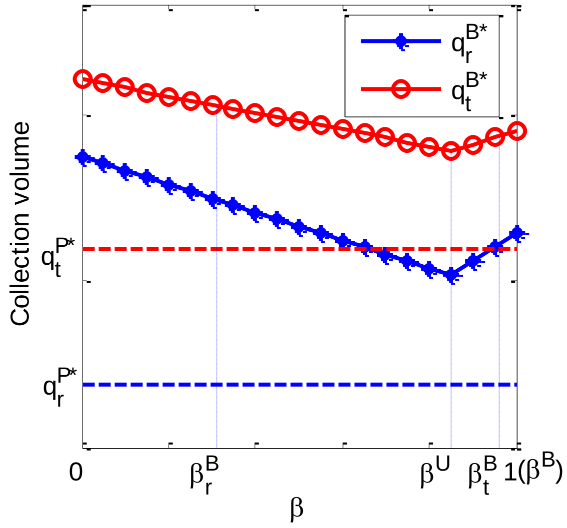
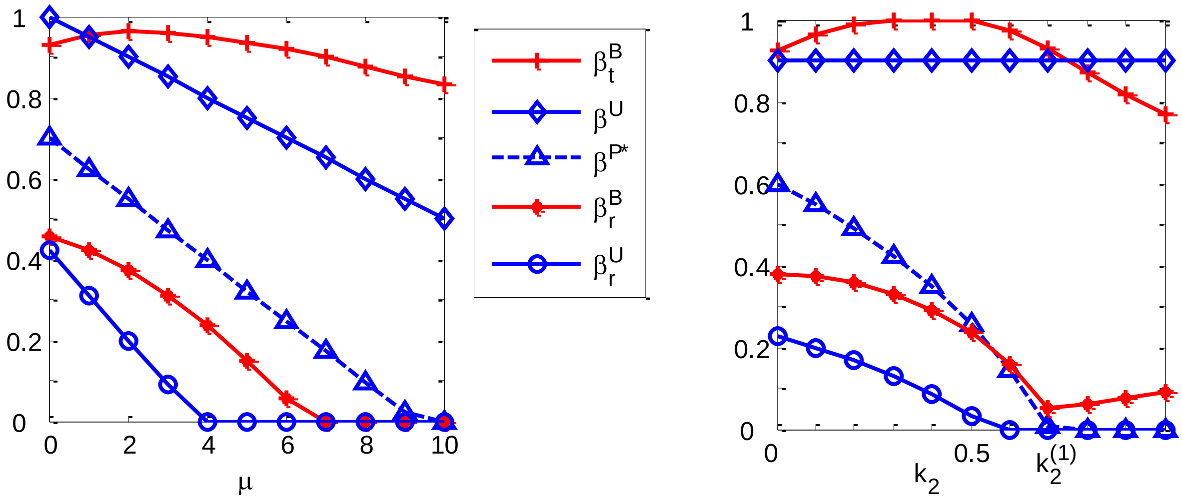


Fig. 6. Comparison of collection volumes between the B-Model and P-Model

Note that $\beta_t^B > \beta^U$ may be true (see Fig. 4b). When $\beta_t^B > \beta^U$ and $\beta \in (\beta^U, \beta_t^B)$, $t_1^* \equiv 0$ and $t_2^* > 0$ hold (see Fig. 5). Under such conditions, as the collection efforts of R increase with increasing β, both the direct and indirect collection volumes increase instead of decreasing (see Fig. 6). This result suggests that when $\beta_t^B > \beta^U$, T can afford R a higher proportion of profit sharing without sharing part of R's collection effort costs, and hence an improvement in profit for both parties and a significant increase in the total collection volume can be achieved.



(a) (b)

Fig. 7. The optimal value β^{P*} and the threshold values (β_r^U, β^U) and (β_r^B, β_t^B) (a) as u varies and (b) as k_2 varies

Through data simulations, it is found that the threshold value of β_t^B first increases and then decreases with increasing u, whereas the threshold value of β^U decreases more rapidly, and thus the higher the value of u, the more likely R is to share part of T’s collection effort cost unilaterally (see Fig. 7a). Similarly, the value of β_t^B first increases and then decreases with increasing in k_2, whereas the value of β^U is independent on k_2; thus, $\beta_t^B > \beta^U$ holds only when k_2 is not large enough (see Fig. 7b). This result explains why some e-tailers direct large amounts of capital to their recycling partners to facilitate the recovery of WEEE of high value, such as smartphones (which means that the value of u

may be higher), especially in early stages, when T is just entering the recycling market (which means that the value of k_2 may be lower).

5.3 Comparison of the B-Model and U-Model

Proposition 6. For any value of $\beta(\beta < \beta^U)$, the ordinal relationships of optimal collection efforts between the U-Model and B-Model are related as follows: $A^{U*} < A^{B*}$ and $a^{U*} = a^{B*}$. Consequently, the collection volumes are related as follows: $q_r^{U*} < q_r^{B*}$ and $q_t^{U*} < q_t^{B*}$. The ordinal relationships of the profits are as follows: $\pi_t^{B*} > \pi_t^{U*}$, $\pi_r^{B*} < \pi_r^{U*}$ and $\pi_s^{B*} > \pi_s^{U*}$.

Proof. See Appendix I.

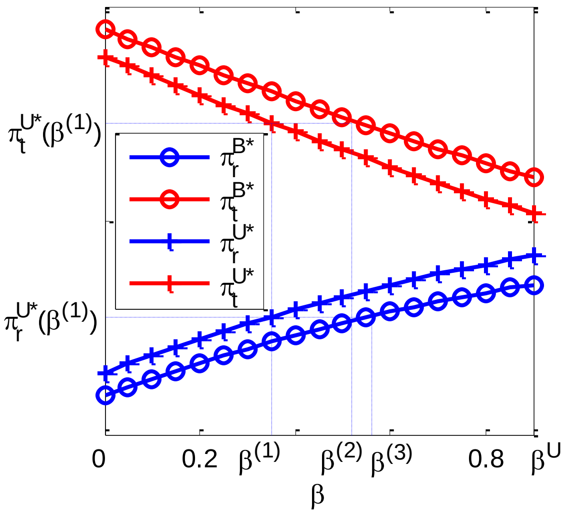


Fig. 8. Comparison of the member's optimal profits between the B-Model and U-Model

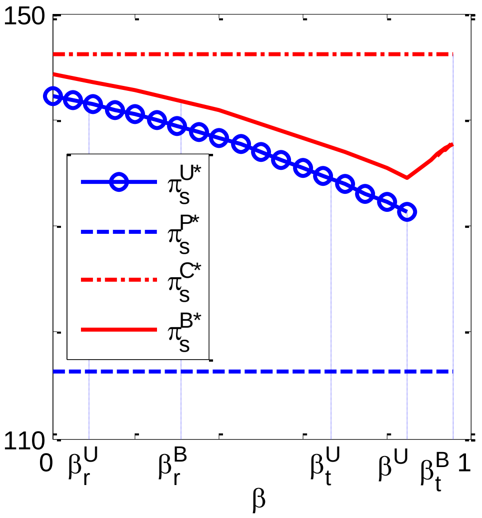


Fig. 9. Comparison of the profits of the collecting system under centralized and decentralized decisions

Proposition 6 indicates that under the same proportion of profit sharing, the B-Model is more profitable for T and the collection system but less profitable for R relative to the U-Model. In Fig. 8, since the profit of T decreases whereas the profit of R increases with respect to β, for a proportion of profit sharing $\beta^{(1)} \in [0,1]$ in the U-Model, $\pi_t^{B*}\big|_{\beta=\beta^i} > \pi_t^{U*}\big|_{\beta=\beta^{(1)}}$ implies $\beta^i < \beta^{(2)}$, and $\pi_r^{B*}\big|_{\beta=\beta^i} > \pi_r^{U*}\big|_{\beta=\beta^{(1)}}$ implies $\beta^i > \beta^{(3)}$. However, $\beta^{(3)} > \beta^{(2)}$ holds; thus, there is not necessarily a corresponding value β^i in the B-Model that supports $\pi_t^{B*}\big|_{\beta=\beta^i} > \pi_t^{U*}\big|_{\beta=\beta^{(1)}}$ and $\pi_r^{B*}\big|_{\beta=\beta^i} > \pi_r^{U*}\big|_{\beta=\beta^{(1)}}$ simultaneously. Clearly, whether the B-Model is a Pareto improvement of the U-Model depends on the crucial parameters of the collection system and the profit sharing proportion β.

According to Proposition 6, it is easy to see that $\beta_r^U \le \beta_r^B$ and $\beta_t^U \le \beta_t^B$.

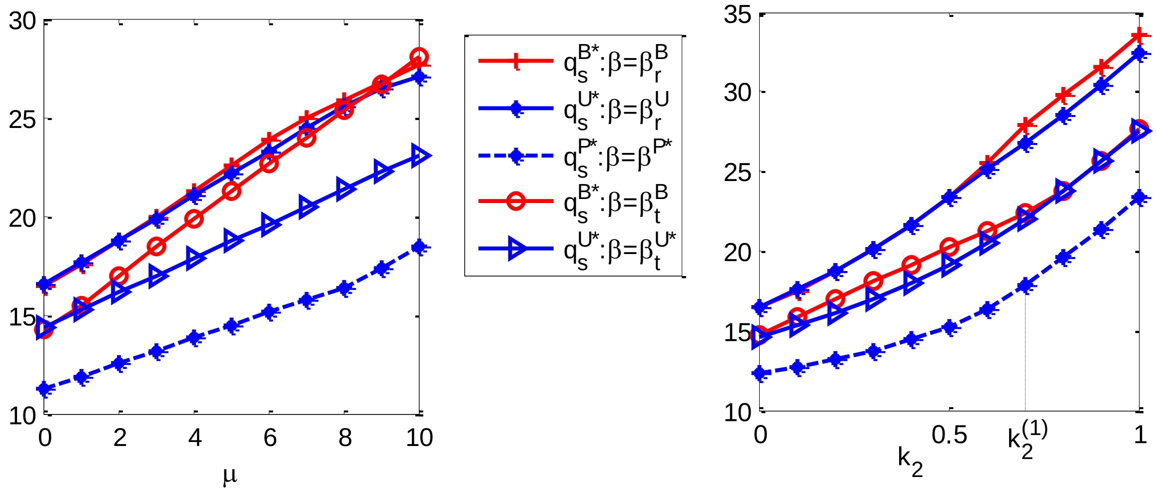
Through the above comparisons, it is obvious that the ordinal relationship of total profits for all of the collaborative collection effort models is $\pi_s^{C*} > \pi_s^{B*} > \pi_s^{U*} > \pi_s^{P*}$ when β falls within the range of $[0, \beta^U]$ (see Fig. 9).

6. Sensitivity analysis

The impacts of the influence coefficient k_2 and collaborative marginal profit u on the total collection volume and efficiency of the collecting system are further discussed. Since the direct and indirect collection volumes are monotonically decreasing functions of β, the values $q_s^{U*}\big|_{\beta=\beta_r^U}$ and $q_s^{U*}\big|_{\beta=\beta_t^U}$ respectively represent the highest and lowest collection volumes of the profit improvement interval (β_r^U, β_t^U) for the U-Model, and a similar conclusion is drawn for the B-Model. Thus, for the following analysis, in the cost sharing models, the proportions of profit sharing are set to the lower and upper bounds of the profit improvement interval, respectively.

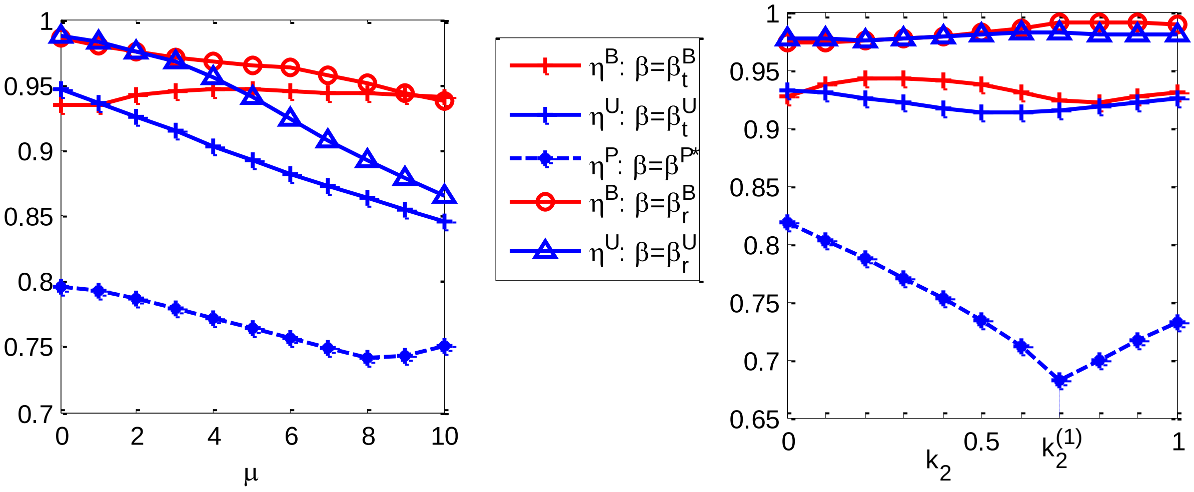
In Fig. 10, it is observed that total collection volumes in the three decentralized models increase with increasing u and k_2, and $q_s^{B*} > q_s^{P*}$ and $q_s^{U*} > q_s^{P*}$ always hold for any values of u and k_2. Fig. 10(a) shows that when u is low and when β is at the lower bound, the total collection volumes differ little between the B-Model and U-Model, whereas the total collection volume is significantly greater in the B-Model than that in the U-Model when the value of u is large and when the proportion of profit sharing β is at the upper bound. In contrast, the total collection volumes differ little between the B-Model and

U-Model when the value of β is at the lower bound and when the value of k_2 is low or when the value of k_2 is high while the value of β is at the upper bound (see Fig. 10b). Fig. 10 also shows that the difference between the U-Model and B-Model is more heavily affected by u than k_2 when the value of β is at the upper bound.



(a) (b)

Fig. 10. Comparison of the total collection volumes (a) as u varies and (b) as k_2 varies



(a) (b)

Fig. 11. Comparison of the collecting system efficiency (a) as u varies and (b) as k_2 varies

Fig. 11(a) shows that the collecting system efficiency of each cost sharing model is far higher than that of the P-Model, and the efficiency of the collecting system mainly follows a downward trend with increasing u in each of the three decentralized models. Fig. 11(a) also indicates that the efficiency of the B-Model is not higher than that of the U-Model when the value of u is very small. However, since the efficiency of the U-Model decreases more rapidly, the B-Model is more efficient than the U-Model when the value of u is large enough regardless of the profit sharing proportion involved. In Fig. 11(b), $k_2^{(1)}$ represents the threshold value that gives $\beta^{P*}=0$. Fig. 11(b) illustrates that the system's efficiency decreases with increasing k_2 in the P-Model when $k_2<k_2^{(1)}$, but when $k_2\geq k_2^{(1)}$, it increases as the collection volume increases more quickly with increasing in k_2. In addition, Fig. 11 shows that the system's efficiency in both the U-Model and B-Model is less affected by k_2.

The conclusions of the sensitivity analysis offer further guidance regarding how to make optimal decisions according to actual situations based on the market influences of T, levels of coordination, and types and values of collected products involved.

7. Conclusion

In this paper, collaborative collection effort strategies involving a third-party collector and an e-tailer based on the “Internet + recycling” business model are explored. The paper develops four cases of collaborative collection models, derives the optimal decisions, conducts a comparative analysis of these models and analyses the impact of crucial parameters on the collection volume and efficiency of the collecting system.

The main findings of this paper are as follows. (i) There exists an interval of profit sharing proportion in which each of the two cost sharing strategies is a Pareto improvement of the unit transfer price strategy. (ii) An increase in the collaborative marginal profit can increase the e-tailer's participation rate and her collection effort level under cost sharing strategies and thus improve the e-tailer's and third party's profits. (iii) An increase in the market influence of the third-party has no effect on the collection effort level of the e-tailer, but it can increase the participation rate of the e-tailer and thus improve the profits of both parties. (iv) Under the B-strategy, when the collaborative marginal profit is large enough, the third party can give the e-tailer a higher proportion of profit sharing but does not need to share part of the e-tailer’s collection effort cost, and thus a Pareto improvement of the P-strategy can also be achieved. (v) Although the total collection volume and profit of the collecting system increase under the B-strategy relative to those of the U-strategy under the same proportion of profit sharing, the B-strategy is not necessarily a Pareto improvement of the U-strategy.

The above conclusions provide some useful suggestions for "Internet + recycling" enterprises. First, it is more profitable for a third-party collector and an e-tailer to share a portion of the other's collection investments under the cooperative "Internet + recycling" mode. For instance, Jd.com, a famous e-tailer in China, cooperates with Aihuishou.com, a professional O2O electronic product collection company, in WEEE recycling. Jd.com has made several rounds of investment to Aihuishou.com to facilitate the recovery of WEEE of high value, such as smartphones, which can be explained by the B-strategy. Second, the third party should consider the types and values of WEEE involved when making the optimal choice. For example, for high-value WEEE collection, higher collection volumes and levels of system efficiency can be achieved under the B-strategy with a high profit sharing proportion than that involved when using the U-strategy, but for low-value WEEE collection, the third party may adopt the U-strategy with a low profit sharing proportion rather than the B-strategy with a high profit sharing proportion to obtain greater collection volume. Third, the third-party and e-tailer must strengthen coordination and resource integration to increase the probability of converting from recovery to purchasing with help of “Internet+”, which can improve not only the profit of the e-tailer but also the profit of the collector.

In future research, some assumptions may be relaxed to develop more comprehensive collaborative collection systems, such as a case in which a system includes e-tailers and third-party collectors in addition to consumers, where both the recycling price paid to customers and the discount for buying new products affecting the system should be considered. It would be interesting to study how partners make optimal decisions and how the consumer surplus changes during one-stop recycling and upgrading services under different collaborative strategies based on the “Internet + recycling” business model.

Acknowledgements

We thank the editors and anonymous referees for their helpful comments on earlier versions of the paper. This work was partially supported by the National Social Science Foundation of China [grant number 14BGL196], the Social Science Foundation of Hunan Province, China [grant number 17YBA406, 18YBA446], and the Hunan Social Science Achievement Assessment Committee [grant number XSP17YBZC201].

References

Afroz, R., Masud, M. M., Akhtar, R., Duasa, J. B., 2013. Survey and analysis of public knowledge, awareness and willingness to pay in Kuala Lumpur, Malaysia - a case study on household WEEE management. Journal of Cleaner Production. 52(4), 185-93.

Ahmadi-Javid, A., Hoseinpour, P., 2012. On a cooperative advertising model for a supply chain with one manufacturer and one retailer. European Journal of Operational Research. 219(2), 458–466.

Atasu, A., Toktay, L. B., 2013. How Collection Cost Structure Drives a Manufacturer’s Reverse Channel Choice. Production Operations Management. 22(5), 1089–1102.

Awasthi, A. K., Li, J., 2017. Management of electrical and electronic waste: A comparative evaluation of China and India. Renewable and Sustainable Energy Reviews. 76, 434–447.

Cucchiella, F., D’Adamo, I., Lenny Koh, S. C., Rosa, P., 2015. Recycling of WEEEs: An economic assessment of present and future e-waste streams. Renewable and Sustainable Energy Reviews. 51, 263-272.

Deng, W. J., Giesy, J. P., So, C. S., Zheng, H. L., 2017. End-of-life (EoL) mobile phone management in Hong Kong households. Journal of Environmental Management. 200, 22–28.

Feng, L., Govindan, K., Li, C., 2017. Strategic planning: Design and coordination for dual-recycling channel reverse supply chain considering consumer behavior. European Journal of Operational Research. 260(2), 601–612.

Gao, J., Han, H., Hou, L., Wang, H., 2016. Pricing and effort decisions in a closed-loop supply chain under different channel power structures. Journal of Cleaner Production. 112, 2043–2057.

Greenpeace, 2016. Summary of Report on Energy Resource Utilization Rate of Smartphones (In Chinese). http://www.greenpeace.org.cn/summary-report-smart-phone-recycle/ (accessed 15 Dec 2017).

Ghosh, D., Gouda, S., Shankar, R., Swami, S., Cheruvil, V., 2018. Strategic decision making under subscription-based contracts for remanufacturing. International Journal of Production Economics. 200, 134–150.

Giovanni, P. De., 2018. A joint maximization incentive in closed-loop supply chains with competing retailers : The case of spent-battery recycling. European Journal of Operational Research. 268(1), 128–147.

Gu, F., Guo, J., Yao, X., Summers, P. A., Widijatmoko, S. D., Hall, P., 2017. An investigation of the current status of recycling spent lithium-ion batteries from consumer electronics in China. Journal of Cleaner Production. 161, 765–780.

Gu, F., Zhang, W., Guo, J., Hall, P., 2019. Exploring “Internet + Recycling”: Mass balance and life cycle assessment of a waste management system associated with a mobile application. Science of the Total Environment. 649, 172–185.

Hong, X., Wang, Z., Wang, D. et al., 2013. Decision models of closed-loop supply chain with remanufacturing under hybrid dual-channel collection. International Journal of Advanced Manufacturing Technology. 68(5-8):1851-1865.

Hong, X., Xu, L., Du, P., Wang, W., 2015. Joint advertising, pricing and collection decisions in a closed-loop supply chain. International Journal of Production Economics. 167, 12–22.

Huang, M., Song, M., Lee, L. H., Ching, W. K., 2013. Analysis for strategy of closed-loop supply chain with dual recycling channel. International Journal of Production Economics. 144(2), 510–520.

Huang, Z., Li, S. X., 2001. Co-op advertising models in manufacturer-retailer supply chains: A game theory approach. European Journal of Operational Research. 135(3), 527–544.

Jena, S. K., Sarmah, S. P., 2015. Measurement of consumers' return intention index towards returning the used products. Journal of Cleaner Production. 108, 1–12.

Jena, S. K., Sarmah, S. P., Sarin, S. C., 2017. Joint-advertising for collection of returned products in a closed-loop supply chain under uncertain environment. Computers and Industrial Engineering. 113, 305–322.

Li, X., Li, Y., Cao, W., 2017. Cooperative advertising models in O2O supply chains. International Journal of Production Economics. 1-9.

Liu, L., Wang, Z., Xu, L., Hong, X., Govindan, K., 2017. Collection effort and reverse channel choices in a closed-loop supply chain. Journal of Cleaner Production. 144, 492–500.

Ma, Z.J., Zhang, N., Dai, Y., Hu, S., 2016. Managing channel profits of different cooperative models in closed-loop supply chains. Omega. 59, 251–262.

Mohan, N., Modak, N., Panda, S., 2018. Analyzing structure of two-echelon closed-loop supply chain for pricing, quality and recycling management. Journal of Cleaner Production. 171, 512–528.

NDRC (The National Development and Reform Commission of the People's Republic of China), 2016. "Internet +" three-year action plan for green ecology. http://www.gov.cn/xinwen/2016-01/21/content_5035064.htm (accessed 15 Dec 2017).

Robinson, B. H., 2009. E-waste: An assessment of global production and environmental impacts. Science of the Total Environment. 408(2), 183–191.

Savaskan, R. C., Bhattacharya, S., Van Wassenhove, L. N., 2004. Closed-Loop Supply Chain Models with Product Remanufacturing. Management Science. 50(2), 239–252.

Savaskan, R. C., Van Wassenhove, L. N., 2006. Reverse Channel Design: The Case of Competing Retailers. Management Science. 52(1): 1-14.

SC (The State Council of the People's Republic of China), 2015. Guidance on actively promoting the "Internet +" action. http://www.gov.cn/zhengce/content/2015-07/04/content_10002.htm (accessed 15 Dec 2017).

Sun, Q., Wang, C., Lu, F., 2018. Digital empowerment in a WEEE collection business ecosystem : A comparative study of two typical cases in China. Journal of Cleaner Production. 184, 414–422.

Tong, X., Tao, D., Lifset, R., 2018. Varieties of business models for post-consumer recycling in China. Journal of Cleaner Production. 170, 665–673.

Wang, H., Han, H., Liu, T., Tian, X., Xu, M., Wu, Y., Gu, Y., Liu, Y., Zuo, T., 2018. "Internet +" recyclable resources : A new recycling mode in China. Resources, Conservation & Recycling. 134, 44–47.

Xue, Y., 2017. Four new features and two new transformations of internet + resource recycling cases in 2017 (In Chinese). www.sohu.com/a/207126573_ 100014482 (accessed 15 Dec 2017).

Zeng, X., Duan, H., Wang, F., Li, J., 2017. Examining environmental management of e-waste: China's experience and lessons. Renewable and Sustainable Energy Reviews. 72, 1076–1082.

Zhang, J., Xie, J., Chen, B., 2013. Cooperative Advertising with Bilateral Participation. Decision Sciences. 44(1), 193–203.

Appendix A. The proof of Proposition 1

By substituting the value of a^P in Eq. (8), the problem of the third-party is written as

$$\underset{b_r, A}{Max}\,\pi_t^P = [b(1+k_2) - k_2 b_r]\sqrt{A} + \frac{[b(1+k_1) - b_r][b_r + u(1+k_1)]}{2} - A \tag{A1}$$

The first- and second-order derivatives of Eq. (A1) are given by

$$\frac{\partial \pi_t^p}{\partial A} = \frac{b(1+k_2) - k_2 b_r}{2\sqrt{A}} - 1\,,\quad \frac{\partial \pi_t^p}{\partial b_r} = -b_r - k_2\sqrt{A} + \frac{(1+k_1)(b-u)}{2}\,,$$

$$\frac{\partial^2 \pi_t^p}{\partial A^2} = -\frac{\left[b + k_2(b - b_r)\right]A^{-3/2}}{4} < 0\,, \frac{\partial^2 \pi_t^p}{\partial {b_r}^2} = -1 < 0\,,\quad \frac{\partial^2 \pi_t^p}{\partial A \partial b_r} = \frac{-k_2}{2\sqrt{A}}\,,\quad \frac{\partial^2 \pi_t^p}{\partial b_r \partial A} = -\frac{k_2}{2\sqrt{A}}\,.$$

The Hessian matrix of π_t^P is $\Omega = \begin{pmatrix} \frac{\partial^2 \pi_t^p}{\partial A^2} & \frac{\partial^2 \pi_t^p}{\partial A \partial b_r} \\ \frac{\partial^2 \pi_t^p}{\partial b_r \partial A} & \frac{\partial^2 \pi_t^p}{\partial {b_r}^2} \end{pmatrix}$. Let $Z^P = \frac{b(1+k_2) - k_2 b_r}{k_2^2}$; when $\sqrt{A} < Z^P$, Ω is negative definite, which shows that the objective function is concave with respect to (A, b_r). From the first-order conditions, the optimal solutions of the third-party are

$$\sqrt{A^{P*}} = \frac{2b(1+k_2) - k_2(1+k_1)(b-u)}{2(2-k_2^2)}\,, b_r^{P*} = \frac{(b-u)(1+k_1) - k_2 b(1+k_2)}{2-k_2^2}\,.$$

Substituting b_r^{P*} into a^P, the optimal collection effort of R is given by

$$\sqrt{a^{P*}} = \frac{(b + u - uk_2^2)(1+k_1) - k_2 b(1+k_2)}{2(2-k_2^2)}\,.$$

Substituting A^{P*} and b_r^{P*} into Z^P, one has $Z^{P*} = \frac{2b(1+k_2)-k_2(b-u)(1+k_1)}{k_2^2(2-k_2^2)}$; thus, $\sqrt{A} < Z^P$ always holds in the neighborhood of (A^{P*}, b_r^{P*}). Therefore, there is always an optimal combination of (A^{P*}, b_r^{P*}) to maximize the profit of the third party in the P-Model.

Since b_r^{P*} must be greater than or equal to zero, the condition $u < u^P = b(1-\frac{k_2+k_2^2}{1+k_1})$ guarantees it.

Appendix B. The proof of Proposition 2

Substituting a^U into Eq. (15), the first-order derivatives of π_t^U are given by

$\frac{\partial \pi_t^U}{\partial A} = \frac{b[1+k_2(1-\beta)]}{2\sqrt{A}} - 1$, and $\frac{\partial \pi_t^U}{\partial t_1} = \frac{NV}{4(1-t_1)^2} - \frac{t_1 V^2}{2(1-t_1)^3}$ where $N = b(2+2k_1-3\beta) - u(1+k_1)$, $V = \beta b + u(1+k_1)$. The two decision variables A and t_1 are not related and thus can be solved independently.

Because $\frac{\partial^2 \pi_t^U}{\partial A^2} = -\frac{b[1+k_2(1-\beta)]}{4} A^{-3/2} < 0$, with the first-order condition $\frac{\partial \pi_t^U}{\partial A} = 0$, one can easily show that $\sqrt{A^{U*}} = \frac{b[1+k_2(1-\beta)]}{2}$. Let $\beta^U = \min\{1, \frac{(2b-u)(1+k_1)}{3b}\}$. When $\beta \geq \beta^U$, $\frac{\partial \pi_t^U}{\partial t_1} < 0$ always holds, so $t_1^{U*} = 0$, and when $\beta < \beta^U$, $\frac{\partial^2 \pi_t^U}{\partial t_1^2} = \frac{1}{1-t_1} \cdot \left\{ \frac{NV}{2(1-t_1)^2} - \frac{V^2}{2} \cdot \frac{1+2t_1}{(1-t)^3} \right\}$; since $\frac{NV}{4(1-t_1)^2} = \frac{t_1 V^2}{2(1-t_1)^3}$ holds at the zero point of t_1, $\frac{\partial^2 \pi_t^U}{\partial t_1^2} < 0$ holds in the neighborhood of zero point, and thus the participation rate of the third party is given by $t_1^{U*} = \frac{(2+2k_1-3\beta)b - u(1+k_1)}{(2+2k_1-\beta)b + u(1+k_1)} \in (0,1)$.

Substituting t_1^{U*} into a^U, one has $\sqrt{a_t^{U*}} = \frac{(2+2k_1-\beta)b + (1+k_1)u}{4}$.

Appendix C. The proof of Proposition 3

Taking the first and second derivatives of π_r^B with respect to a yields

$\frac{\partial \pi_r^B}{\partial a} = \frac{\beta b + u(1+k_1)}{2\sqrt{a}} - (1-t_1)$, and $\frac{\partial^2 \pi_r^B}{\partial a^2} = -\frac{\beta b + u(1+k_1)}{4} a^{-\frac{3}{2}} < 0$.

Similarly, the derivatives of π_r^B with respect to A are given by

$$\frac{\partial \pi_r^B}{\partial A} = \frac{k_2\beta b + u(1+k_2)}{2\sqrt{A}} - t_2 \text{, and } \frac{\partial^2 \pi_r^B}{\partial A^2} = -\frac{k_2\beta b + u(1+k_2)}{4} A^{-\frac{3}{2}} < 0 .$$

From the first-order conditions, the optimal collection efforts of T and R satisfy

$$\sqrt{a^B} = \frac{\beta b + u(1+k_1)}{2(1-t_1)} \text{ and } \sqrt{A^B} = \frac{k_2\beta b + u(1+k_2)}{2t_2}\text{, respectively.}$$

Substituting a^B and A^B into the profit function of T, π_t^B is achieved, and taking the derivative of π_t^B with respect to t_1, one has $\frac{\partial \pi_t^B}{\partial t_1} = \frac{[(2b-u)(1+k_1) - 3\beta b]X}{4(1-t_1)^2} - \frac{t_1 X^2}{2(1-t_1)^3}$, where $X = \beta b + u(1+k_1)$, $Y = k_2\beta b + u(1+k_2)$, and $\frac{\partial^2 \pi_t^B}{\partial t_1^2} = \frac{2}{1-t_1} \times \frac{\partial \pi_t^B}{\partial t_1} - \frac{X^2}{2(1-t_1)^3} - \frac{t_1 X^2}{2(1-t_1)^4}$.

When $\beta \geq \beta^U$, $\frac{\partial \pi_t^B}{\partial t_1} < 0$, so $t_1^{B*} = 0$, and when $\beta < \beta^U$, the function π_t^B has zero points, and in the neighborhood of the zero points, $\frac{\partial^2 \pi_t^B}{\partial t_1^2} < 0$ holds, so $t_1^{B*} = \frac{(2b-u)(1+k_1) - 3\beta b}{(2b+u)(1+k_1) - \beta b}$.

Similarly, one has $\frac{\partial \pi_t^B}{\partial t_2} = \frac{Y}{2t_2^2}\left\{\frac{3k_2\beta b - 2b - 2bk_2 + u(1+k_2)}{2} + \frac{(1-t_2)Y}{t_2}\right\}$, and $\frac{\partial^2 \pi_t^B}{\partial t_2^2} = -\frac{2}{t_2} \times \frac{\partial \pi_t^B}{\partial t_2} - \frac{Y^2}{2t_2^4}$. From the formulation of the first order, it follows that the function π_t^B always has zero points of t_2 for any $\beta \leq 1$, and in the neighborhood of the zero points, $\frac{\partial^2 \pi_t^B}{\partial t_2^2} < 0$ holds, so $t_2^{B*} = \frac{2k_2\beta b + 2u(1+k_2)}{(2b+u)(1+k_2) - k_2\beta b}$. When $\beta < \beta^B$, $t_2^{B*} \in (0,1)$ holds; otherwise, $t_2^{B*} = 1$.

Substituting t_1^{B*} and t_2^{B*} into the functions for a^B and A^B, the optimal solutions of a^{B*} and A^{B*} are achieved.

Appendix D. The proof of Proposition 4

Since $b_r^* = \beta^{P*} b$, from the first-order conditions, one has $\sqrt{A^{P*}} = \frac{b(1+k_2) - k_2\beta^{P*} b}{2}$ and

$\sqrt{a^{P*}}=\frac{\beta^{P*}b+u(1+k_1)}{2}$. Let $\beta=\beta^{P*}$ in the U-Model. Since $\beta^{P*}<\beta^{U}$, it follows that $A^{P*}=A^{U*}$ and $\sqrt{a^{U*}}-\sqrt{a^{P*}}=\frac{(2b-u)(1+k_1)-3\beta^{P*}b}{4}>0$. Consequently, the collection volumes of the direct and indirect channels are as follows: $q_r^{P*}<q_r^{U*}$ and $q_t^{P*}<q_t^{U*}$.

According to the decision-making process, T will set $t_1^*=0$ if $\pi_t^{U*}\le\pi_t^{P*}$, and then the U-Model is transformed into the P-Model; if $t_1^*>0$, $\pi_t^{U*}>\pi_t^{P*}$ must hold. Since $\beta^{P*}<\beta^{U}$, $t_1^*>0$ holds, $\pi_t^{U*}\big|_{\beta=\beta^{P*}}>\pi_t^{P*}$ holds.

The optimal profit of R in the P-Model is given by

$$\pi_r^{P*}=\frac{[\beta^{P*}b+u(1+k_1)]}{2}\sqrt{a^{P*}}+[\beta^{P*}k_2b+u(1+k_2)]\sqrt{A^{P*}} \tag{D1}$$

Since $\beta^{P*}<\beta^{U}$, the optimal profit of R in the U-Model is given by

$$\pi_r^{U*}\big|_{\beta=\beta^{P*}}=\frac{\beta^{P*}b+u(1+k_1)}{2}\sqrt{a^{U*}}+[\beta^{P*}k_2b+(1+k_2)u]\sqrt{A^{U*}} \tag{D2}$$

Since $A^{P*}=A^{U*}$ and $a^{P*}<a^{U*}$, one has $\pi_r^{U*}\big|_{\beta=\beta^{P*}}>\pi_r^{P*}$.

Appendix E. The proof of Corollary 5

$$\frac{\partial\pi_t^{U*}}{\partial\beta}=\frac{b^2}{8}[(4k_2^2+1)\beta-4k_2(1+k_2)-2(1+k_1)]-\frac{bu(1+k_1)}{2},$$

$\frac{\partial\pi_r^{U*}}{\partial\beta}=\frac{\left(1+k_1+2k_2+2k_2^2-\beta-4k_2^2\beta\right)b^2-2k_2bu(1+k_2)}{4}$ and $\frac{\partial^2\pi_r^{U*}}{\partial\beta^2}=\frac{-\left(1+4k_2^2\right)b^2}{4}<0$. Since $\beta<\beta^{U}$ and $\frac{\partial\pi_t^{U*}}{\partial\beta}\Big|_{\beta=\beta^{U}}<0$, $\frac{\partial\pi_t^{U*}}{\partial\beta}<0$ holds. Let $\beta^{UL}=\frac{\left(1+k_1+2k_2+2k_2^2\right)b-2k_2u(1+k_2)}{(1+4k_2^2)b}$. If $\beta^{U}\le\beta^{UL}$, $\frac{\partial\pi_r^{U*}}{\partial\beta}>0$ holds when $\beta\le\beta^{U}$, and if $\beta^{U}>\beta^{UL}$, then $\frac{\partial\pi_r^{U*}}{\partial\beta}>0$ when $0\le\beta\le\beta^{UL}$, $\frac{\partial\pi_r^{U*}}{\partial\beta}<0$ when $\beta^{UL}<\beta\le\beta^{U}$ and $\frac{\partial\pi_r^{U*}}{\partial\beta}=0$ when $\beta=\beta^{UL}$.

Appendix F. The proof of Corollary 6

According to Proposition 4 and Corollary 5, one has $\pi_t^{U*}\big|_{\beta=\beta^{P*}}>\pi_t^{P*}$ and $\frac{\partial\pi_t^{U*}}{\partial\beta}<0$, and thus there is a threshold β_t^U ($\beta^{P*} < \beta_t^U \le \beta^U$) that makes $\pi_t^{U*}\big|_{\beta=\beta_t^U} = \pi_t^{P*}$ when $\beta_t^U < \beta^U$, or $\pi_t^{U*}\big|_{\beta=\beta_t^U} > \pi_t^{P*}$ when $\beta_t^U = \beta^U$. Hence, $\pi_t^{U*}(\beta) \ge \pi_t^{P*}$ when $\beta \le \beta_t^U$ ($\beta_t^U \in (\beta^{P*}, \beta^U]$).

In a similar manner, it can be proved that when $\beta \ge \beta_r^U$ ($\beta_r^U \in [0, \beta^{P*}]$), $\pi_r^U(\beta) \ge \pi_r^{P*}$ holds. Hence, when $\beta \in (\beta_r^U, \beta_t^U)$, the U-Model is a Pareto improvement of the P-Model.

Appendix G. The proof of Proposition 5.

Let $\beta = \beta^{P*}$ in the B-Model. Since $\beta^{P*} < \beta^U \le \beta^B$, $\sqrt{a^{B*}} = \frac{(2b+u)(1+k_1) - \beta^{P*}b}{4}$ and $\sqrt{A^{B*}} = \frac{(2b+u)(1+k_2) - k_2\beta^{P*}b}{4}$, one has $A^{P*} < A^{B*}$ and $a^{P*} < a^{B*}$. Consequently, the collection volumes of the direct and indirect channels are related as follows: $q_r^{P*} < q_r^{B*}$ and $q_t^{P*} < q_t^{B*}$.

It is obvious that $\pi_t^{B*}\big|_{\beta=\beta^{P*}} > \pi_t^{P*}$; otherwise, T would set $t_1^* = 0$ and $t_2^* = 0$, which is just the case of the P-Model.

Since $\pi_r^{B*} = \frac{\beta b + u(1+k_1)}{2}\sqrt{a^{B*}} + \frac{\beta k_2 b + u(1+k_2)}{2}\sqrt{A^{B*}}$, combined with Eq. (D1) of π_r^{P*}, it is easy to prove that $\pi_r^{B*} - \pi_r^{P} = \frac{M}{8}$, where

$$M = (2b-4u)\beta^{P*}b[1+k_1-k_2(1+k_2)] + (2b-u)u[\left(1+k_1\right)^2 - (1+k_2)^2] - 3(1-k_2^2)(\beta^{P*}b)^2$$

Thus $\pi_r^{B*}\big|_{\beta=\beta^{P*}} \ge \pi_r^{P*}$ when $M \ge 0$, and $\pi_r^{B*}\big|_{\beta=\beta^{P*}} < \pi_r^{P*}$ when $M < 0$.

When $\beta = \beta^{P*}$, the profits of the collection system in the B-Model and the P-Model are given by

$$\pi_s^{B*} = \frac{(2b+3u)(1+k_1) + \beta^{P*}b}{4}\sqrt{a^{B*}} + \frac{(2b+3u)(1+k_2) + \beta^{P*}k_2 b}{4}\sqrt{A^{B*}} \quad \text{and}$$

$$\pi_s^{P*} = \frac{(2b+u)(1+k_1) - \beta^{P*}b}{2}\sqrt{a^{P*}} + \frac{(b+2u)(1+k_2) + \beta^{P*}k_2 b}{2}\sqrt{A^{P*}}.$$

Since $\sqrt{A^{P*}} = \frac{b(1+k_2) - k_2\beta^{P*}b}{2}$ and $\sqrt{a^{P*}} = \frac{\beta^{P*}b + u(1+k_1)}{2}$, it is easy to see that

$$\pi_s^{B*} - \pi_s^{P*} = \frac{(2b-u)(1+k_1) - 3\beta^{P*}b}{4}\sqrt{a^{B*}} + \frac{(2b+3u)(1+k_2) + \beta^{P*}k_2 b}{4}\left(\sqrt{A^{B*}} - \sqrt{A^{P*}}\right) - \frac{u(1+k_2) + k_2\beta^{P*}b}{4}\sqrt{A^{P*}}.$$

Because $\beta^{P*} < \beta^U$ and $\sqrt{A^{B*}} - \sqrt{A^{P*}} = \frac{u(1+k_2) + k_2\beta b}{4}$, it is easy to prove that $\pi_s^{B*} - \pi_s^{P*} > 0$.

Appendix H. The proof of Corollary 7

Taking derivatives with respect to β yields

$$\frac{\partial \pi_t^{B*}}{\partial \beta} = \frac{-b}{8}\{(2b+u)(1+k_1+k_2+k_2^2) - \beta b(1+k_2^2)\} < 0, \text{ and}$$

$$\frac{\partial \pi_r^{B}}{\partial \beta} = \frac{b}{2}\sqrt{a^{B*}} + \frac{\beta b + u(1+k_1)}{2} \times \frac{\partial \sqrt{a^{B*}}}{\partial \beta} + \frac{k_2 b}{2}\sqrt{A^{B*}} + \frac{\beta k_2 b + u(1+k_2)}{2} \times \frac{\partial \sqrt{A^{B*}}}{\partial \beta}.$$

Since $\frac{\partial \sqrt{a^{B*}}}{\partial \beta} = \frac{-b}{4}$ and $\frac{\partial \sqrt{A^{B*}}}{\partial \beta} = \frac{-k_2 b}{4}$, $\frac{\partial \pi_r^{B}}{\partial \beta} = \frac{b^2}{4}\{1+k_1-\beta+k_2+k_2^2(1-\beta)\} > 0$, and

$$\frac{\partial \pi_t^{B*}}{\partial \beta} + \frac{\partial \pi_r^{B*}}{\partial \beta} < 0.$$

Appendix I. The proof of Proposition 6

From Propositions 4-5, one easily has $A^{U*} < A^{B*}$ and $a^{U*} = a^{B*}$ under the same profit sharing β ($\beta < \beta^U$). Consequently, the collection volumes are related as follows: $q_r^{U*} < q_r^{B*}$ and $q_t^{U*} < q_t^{B*}$.

When $\beta < \beta^U$, it is easy to obtain $\pi_t^{U*} = A^{U*} + a^{U*}$, where $\sqrt{A^{U*}} = \frac{b(1+k_2) - k_2 \beta b}{2}$ and $\sqrt{a^{U*}} = \frac{(1+k_1)(2b+u) - \beta b}{4}$, and $\pi_t^{B*} = A^{B*} + a^{B*}$, where $\sqrt{a^{B*}} = \sqrt{a^{U*}}$ and $\sqrt{A^{B*}} = \frac{(2b+u)(1+k_2) - k_2 \beta b}{4}$. Since $A^{U*} < A^{B*}$, $\pi_t^{B*} - \pi_t^{U*} > 0$.

For a given proportion of profit sharing, it is easy to obtain

$$\pi_r^{B*} - \pi_r^{U*} = \frac{\beta k_2 b + u(1+k_2)}{2}(\sqrt{A^{B*}} - 2\sqrt{A^{U*}}) \quad \text{and} \quad \sqrt{A^{B*}} - 2\sqrt{A^{U*}} = -\frac{(2b-u)(1+k_2) - 3k_2 \beta b}{4}.$$

When $\beta < \beta^B$, $\sqrt{A^{B*}} - 2\sqrt{A^{U*}} < 0$, and given $\beta^U \le \beta^B$, when $\beta < \beta^U$, $\pi_r^{B*} < \pi_r^{U*}$ holds.

Similarly, $\pi_s^{B*} - \pi_s^{U*} = A^{B*} - A^{U*} + \frac{u(1+k_2) + \beta k_2 b}{2}(\sqrt{A^{B*}} - 2\sqrt{A^{U*}})$.

Since $A^{B*} - A^{U*} = \frac{(4b+u)(1+k_2) - 3k_2 \beta b}{4} \times \frac{u(1+k_2) + k_2 \beta b}{4}$,

$$\pi_s^{B*} - \pi_s^{U*} = \frac{u(1+k_2) + k_2 \beta b}{4} \times \frac{3u(1+k_2) + 3k_2 \beta b}{4} > 0.$$