Journal Pre-proof
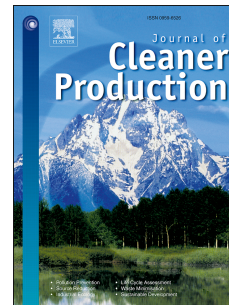
Collaborative collection effort strategies based on "Internet + recycling" business model

Huiyun Jian, Minli Xu, Li Zhou

Collaborative collection effort strategies based on "Internet + recycling" business model

Huiyun Jian[1], Minli Xu[1], Li Zhou[2]

1 Business School, Central South University, Changsha 410083 Hunan, PR China.

E-mail: jianhuiyun@163.com, xu_minli@163.com.

2 Faculty of Business, University of Greenwich, London, SE10 9LS, UK

E-mail: li.zhou@gre.ac.uk

Corresponding author：Minli Xu，*xu_minli@163.com*

1 **Collaborative collection effort strategies based on the "Internet + recycling"**

2 **business model**

3

4 **ABSTRACT**

5   "Internet + recycling", a new and emerging collecting mode, is booming in conjunction

6 with widespread Internet use in China. For the recycling of waste electrical and electronic

7 equipment (WEEE), this paper studies collaborative collection effort strategies in a collection

8 system consisting of a third-party and an e-tailer based on the "Internet + recycling" business

9 model. Considering the collaboration occurring during collecting and selling and mutual

10 influences of partners on the recycling of old products, the paper applies collection effort cost

11 sharing mechanisms to promote recycling. Four models, namely, the centralized model

12 (C-Model), unit transfer price model (P-Model), unilateral cost sharing model (U-Model) and

13 bilateral cost sharing model (B-Model), are established, and optimal decisions and members'

14 profits in various collaborative models are derived and compared. The results show that there

15 exists an interval of profit sharing proportions in which each of the two cost sharing models is

16 a Pareto improvement of the P-Model, and the total collection volume and profit of the

17 collecting system increase in the B-Model relative to those in the U-Model under the same

18 proportion of profit sharing. However, the B-Model is not necessarily a Pareto improvement

19 of the U-Model. The results also show that profit improvements of both parties can be

20 achieved without the third-party sharing the e-tailer's collection effort cost in the B-Model

21 when the collaborative marginal profit is large enough. The paper further explores the impact

22 of the collaborative marginal profit and third-party's market influence on the total collection

23 volume and the efficiency of the collecting system. This study provides insight into the

24 promotion of WEEE recycling and into the selection of collaborative strategies for Internet

25 recycling enterprises. The work will prove beneficial to the development of the WEEE

26 "Internet + recycling" industry.

27 *Keywords:* WEEE; Internet + recycling; Collaboration; Collection effort; Cost sharing;

28 Bilateral participation

1. Introduction

30 Waste electrical and electronic equipment (WEEE) has increased sharply with the rapid

31 updating of products and with the shortening of product life cycles. It is estimated that the

32 number of smartphone and panel computer users reached 2.16 billion and 1.2 billion in 2016,

33 accounting for 20% and 15% of the world's population, respectively (Greenpeace, 2016).

34 Globally, approximately 30–50 million tons of WEEE are disposed of each year, with an

35 estimated annual growth rate of 3–5% (Afroz, 2013). WEEE may contain valuable substances

36 and even precious metals such as Au and Ag (Cucchiella, 2015). At the same time, WEEE can

37 contain complex mixtures of potential environmental contaminants (Robinson, 2009). Under

38 the dual effects of the resource crisis and environmental pollution, increasing attention has

39 been dedicated to the recycling and reuse of WEEE.

40 As one of the world's largest developing countries, China accounts for approximately 20%

41 of the global volume of WEEE (Awasthi and Li, 2017) and has become the largest producer

42 and consumer of electrical and electronic equipment (Zeng et al., 2017). In the past, most

43 residents in China preferred to sell their WEEE to informal peddlers or to store them at home.

44 A recent questionnaire survey conducted in Hong Kong and Shenzhen also shows that more

45 than 75% of the respondents prefer to store their obsolete mobile phones at home rather than

46 recycle them (Deng et al., 2017). An online survey of lithium-ion battery (LIB) recycling also

47 shows that 59.6% of respondents in China store their spent LIBs at home, whereas only 29.5%

48 recycle spent LIBs with whole electronics units (Gu et al., 2017). Even so, only a small

49 quantity of collected e-waste reaches authorized recyclers, and such waste flowing into the

50 informal processing sector is sorted and dismantled using primitive methods in open air

51   (Awasthi and Li, 2017). Recovery price, convenience and personal information security are

52   the main factors that influence customers' willingness to engage in e-waste recycling (Deng et

53   al., 2017).

54   As is widely known, "Internet plus" has become China's national development strategy

55   and has been highly encouraged through a series of policies and measures, such as "Guidance

56   on actively promoting the 'Internet +' action" (SC, 2015) and "'Internet +' three-year action

57   plan for green ecology" (NDRC, 2016). "Internet + recycling" refers to an O2O business

58   model for online trading and offline recycling based on the use of Internet technology. The

59   "Internet + recycling" industry is booming with strong support from government policies,

60   widespread Internet use and the rapid evolution of smartphones in China. In recent years,

61   many "Internet + recycling" enterprises have come into being; well-known examples include

62   Huishouge, based in Wuhan (www.huishouge.cn); Aihuishou, based in Shanghai

63   (www.aihuishou.com); Kuaishou, based in Beijing (www.kuaishou365.com); and Taolv365,

64   based in Shenzhen (www.taolv365.com). "Internet + recycling" online platforms can be built

65   by manufacturers, retailers, certified waste recyclers or third-party collectors, and platforms

66   built by third-party collectors are the most common in practice. Recyclable goods include

67   various types of items, such as intelligent digital products, notebook computers, household

68   electronics, and clothes. This paper focuses on the "Internet + recycling" of WEEE provided

69   by third parties such as Aihuishou.

70   Compared to the traditional recycling mode, the "Internet + recycling" mode is more

71   convenient, and recycling prices are more transparent. In addition, the collector's professional

72   data deletion service reduces consumers' worries concerning the leakage of private data

73   stored in their digital products. More importantly, the new mode is more environmentally

74   friendly and sustainable. It helps the Chinese government regulate recycling channels and

75   guarantees that recycled products are delivered to qualified processing enterprises. Due to the

76   use of advanced information technologies and automatic data processes, recovery efficiency

77   can be greatly enhanced. Consequently, the "Internet + recycling" mode has been

78   aggressively promoted by the Chinese government and in venture capital investments.

79   Although the mode is still being popularized, its potential has already begun to show. For

80   example, after the 2012 creation of Taolv365 (www. taolv365.com), an Internet trading

81   platform for old products, the quantity of reclaimed mobile phones increased rapidly over the

82   following three years (see Fig. 1).

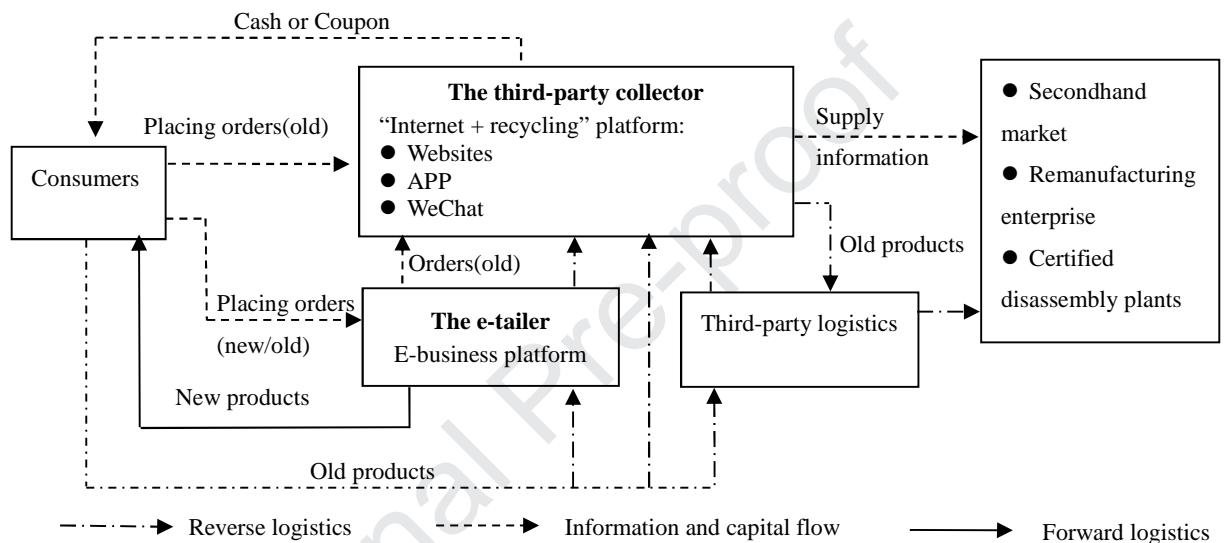Collection quantity(Unit:ten thousand)
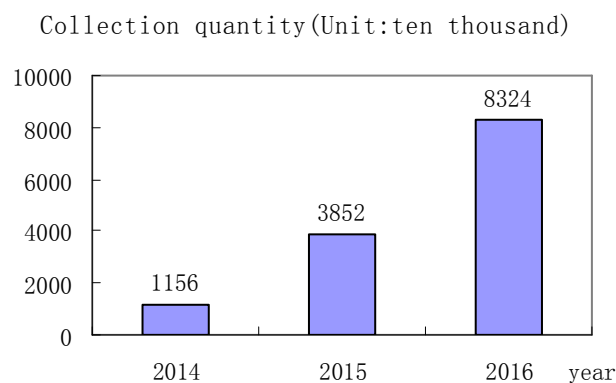
83

84   **Fig. 1.** The quantity of mobile phones reclaimed through Taolv365 (data source: Xue, Y., 2017)

85   Generally, customers often need to buy new electric and electronic equipment (EEE)

86   when they return their old EEE, and vice versa. Accordingly, a win-win result can be

87   achieved when third-party collectors cooperate with e-tailers, as such cooperation can not

88   only increase the recovery of old products and the sales of new ones but also provide

89   customers with one-stop recycling and upgrading services. Therefore, such cooperation is

90   often adopted in practice. For example, Aihuishou (www.aihuishou.com), the largest O2O

91   electronic product collection company in China, strategically cooperates with Jd

92   (www.jd.com), a famous e-commerce company. Fig. 2 illustrates the typical logical trajectory

93   of this form of cooperation. First, customers place orders for returned items through the

94   e-tailer's or third party's platform, and all orders are aggregated to the third party. Next,

95   consumers send recyclable goods to the third-party collector via third-party logistics, through

4

96   outlets of the third-party collector or through door-to-door collection. Then, the third-party

97   collector confirms the recycling price and completes the payment based on a quality

98   inspection of the returned products, and customers receive money in cash or in coupon form,

99   where the coupon can be used to buy new products from the e-tailer. In the end, the collected

100 WEEE is sold to various parties, including certified disassembly plants, the second-hand

101 market or remanufacturers (see Fig. 2).

Fig. 2. The logical flow of the cooperative "Internet + recycling" between a third-party and an e-tailer

102

103   The performance of the reverse channel strongly relies on collectors' collection efforts,

104 including their investments in advertising and promotional services, which motivate

105 consumers to return their old products (Savaskan et al., 2004). Consumers can take express

106 interest in returning their used products after receiving information through advertisements

107 (Jena and Sarmah, 2015). Recycling price incentives, trading in the "old-for-new" model and

108 coupons are all feasible means of promotion (Tong et al., 2018). Under the "Internet +

109 recycling" mode, collection efforts can have several purposes, such as improving service

110 quality and enhancing user experiences. For example, by 2018, Aihuishou had opened more

111 than 300 outlets to provide face-to-face communication and transactions across 35 cities (Sun

112 et al., 2018), while an outlet based in a downtown area itself serves as a good brand

5

113   advertisement in addition to enhancing user experiences.

114   Motivated by the above, this paper studies collaborative collection effort strategies

115   employed in a collecting system involving an e-tailer and a third-party under the "Internet +

116   recycling" business model. To the best of our knowledge, such a comprehensive examination

117   of this issue has not been undertaken in the literature. To this end, the paper develops models

118   of the collecting system, considering collaboration occurring during collecting and selling and

119   collection effort cost sharing mechanisms facilitating the return of used products. The optimal

120   collection efforts are examined and compared within the framework of game theory, and

121   members' profits and system performance are analysed under different collaborative

122   strategies.

123   The paper is organized as follows. In section 2, a relevant literature review is provided.

124   Section 3 describes the problem and modelling assumptions. In section 4, four collaborative

125   collection models based on the "Internet+ recycling" mode are examined, and the optimal

126   decisions for each party are derived. Section 5 compares recycling quantities, collection effort

127   levels and profits in the four models and presents the analytical and numerical results. A

128   sensitivity analysis is conducted in section 6. Section 7 finally concludes this work and

129   discusses further research.

130   **2. Literature review**

131   The related literature can be classified into three research streams: collection channels,

132   collection efforts and cooperative strategies of supply chains.

133   Collection channel management is very central to reverse supply chains. Savaskan et al.

134   (2004) proposed three models based on different reverse channels involving manufacturers,

135   retailers and third parties in closed-loop supply chains (CLSCs) and found that retailer

136   collection is the most effective means of product collection activity for the manufacturer.

137   Savaskan and Wassenhove (2006) further extended the above models to multiple settings for

6

138 the case of competing retailers and studied strategic product pricing decisions and the

139 manufacturer's reverse channel choices. Atasu et al. (2013) investigated the impact of

140 collection cost structures on optimal reverse channel decisions based on the work of Savaskan

141 et al. (2004). Mohan et al. (2018) analysed the effects of recycling and product quality levels

142 on pricing decisions in a CLSC and showed that the unit price of the returned product paid to

143 the retailer serves as an important determinant when selecting best channel structures between

144 retailer- and manufacturer-led collection. Some works have focused on dual recycling or

145 hybrid collection channels. Huang et al. (2013) investigated the channel configuration

146 strategy of a CLSC with a dual recycling channel in which the retailer and third-party

147 competitively collect used products and derived a parameter domain of competing intensity at

148 which the dual recycling channel strategy outperforms the use of a single recycling channel.

149 Hong et al. (2013) investigated three reverse hybrid collection channel structures in a

150 manufacturer-oriented CLSC and showed that the retailer's and manufacturer's hybrid

151 collection channel is the most effective. Liu et al. (2017) extended the work of Hong et al.

152 (2013) and Huang et al. (2013) by comparing three types of hybrid competitive

153 dual-recycling channel structures in a CLSC and found that the OEM and retailer dual

154 collecting channel are the best tools regardless of the degree of competition intensity

155 involved.

156   While the above literature provides models for studying the channel decisions made in a

157 reverse supply chain, it mainly discusses this issue within the framework of CLSCs and with

158 reference to traditional recycling channels. In a recent work, Feng et al. (2017) explored the

159 recycling channel decisions of a recyclables dealer using traditional recycling and online

160 recycling channels, and they investigated the strategic planning regarding the optimal design

161 and coordination decisions of the dealer. Gu et al. (2019) assessed the overall environmental

162 performance of "Internet + recycling" through a case study and concluded that the disposal of

163 WEEE incurs the highest environmental savings. Tong et al. (2018) identified three types of

164 business models for recyclables using Internet technologies in China and evaluated the

165 performance of these models. Wang et al. (2018) investigated "Internet + recycling" practices

166 in China and made some suggestions regarding the sustainable development of "Internet +

167 recycling". Sun et al. (2018) analysed the structures, digital empowerment activities and types

168 of WEEE collection business ecosystems through a study of two typical Internet-based

169 collection enterprises. It can be observed that the literature focusing on "Internet + recycling"

170 has grown dramatically over the past year. However, far too little attention has been paid to

171 quantitative research regarding how to increase the quantity of WEEE acquired. Moreover,

172 the recycling channel structure examined in this paper is different from that examined in the

173 above literature, which includes a direct third-party online channel and an indirect e-tailer

174 channel. The e-tailer's platform acts as an important entry for recycling traffic, the e-tailer

175 works together with the third party to provide consumers with one-stop services for recycling

176 WEEE and for purchasing new ones, and the relationship between the third party and e-tailer

177 is collaborative rather than competitive (see Fig. 2).

178   Many studies have considered collection efforts employed in reverse channels. Savaskan

179 et al. (2004) first modelled the return rate of used products as a function of collection efforts

180 and set the structure of the collection effort cost. Later, similar structures of collection effort

181 cost have been widely used in the analysis of recycling problems related to the recycling

182 channel, pricing, remanufacturing decisions and coordination mechanisms. For example, Gao

183 et al. (2016) explored the influence of different channel power structures on optimal CLSC

184 pricing decisions, collection efforts, sales efforts and performance. However, none of these

185 studies considered cost sharing employed for collection efforts.

186   Cooperative strategies used in SCs have been comprehensively researched in the literature.

187 Huang and Li (2001) investigated the efficiency of transactions for the system of

188   manufacturer – retailer co-op advertising in the context of game theory. Ahmadi-Javid and

189   Hoseinpour (2012) analyzed the co-op advertising model under nonnegative constraints of the

190   sales function based on the work of Huang and Li (2001). Hong et al. (2015) incorporated

191   advertising effects into CLSC models. In these works involving cooperative advertising,

192   unilateral cost sharing is frequently used. Zhang et al. (2013) extended the popular unilateral

193   participation strategy to bilateral participation in cooperative advertising and showed that

194   properly designed bilateral participation offers several advantages relative to unilateral

195   participation. Li et al. (2017) examined cooperative advertising strategies used in an O2O

196   supply chain and found that bilateral cooperative advertising can offer significant benefits to

197   the seller and to the entire channel relative to unilateral cooperative advertising. However, the

198   above literature examines issues regarding cooperative advertising in terms of promoting the

199   sale of new products. In a recent work, Jena et al. (2017) considered advertising as a means to

200   entice consumers to return their used items in a CLSC, and they investigated the impacts of

201   sharing or not sharing advertisement costs on total profits gained and on the quantity of used

202   items acquired. Giovanni (2018) investigated whether retailers engage manufacturers to

203   invest more heavily in green activity programmes by offering a joint incentive and showed

204   that a joint maximization incentive always increases the manufacturers' investments made in

205   green efforts. Ghosh et al. (2018) studied competition and collaboration between an OEM and

206   remanufacturer. Ma et al. (2016) investigated various cooperative strategies in a three-echelon

207   CLSC; they mainly focused on cooperative interactions occurring among members rather

208   than cooperative collection efforts. Hence, collaborative collection effort strategies with cost

209   sharing in an "Internet + recycling" environment have not been addressed in the reverse

210   supply chain literature. This paper considers the effects of collaboration between the third

211   party and e-tailer on collecting and selling and investigates how collaborative collection

212   strategies without cost sharing, with unilateral cost sharing or with bilateral cost sharing

213 affect the decisions of members and the performance of a collecting system.

214 **3. Problem description**

215 This paper considers a third-party, T, who collects used items from the market by using

216 the "Internet + recycling" business model. To increase the volume of the recovery and to

217 provide a better re-buy service, T cooperates with e-tailer R to collect recyclables. The logical

218 flow of the cooperation mode is shown in Fig. 2.

219 Both T and R make efforts to motivate consumers to return their old products and

220 provide consumers with related services for the purchase of new ones. A and a denote the

221 collection effort investments of T and R, respectively. The direct collection volume through T

222 is denoted as q_t, and the indirect volume through R is denoted as q_r. Since R does not

223 provide the complete recycling process alone but rather cooperates with T to complete it, each

224 member's collection efforts not only affect the collection volume of its own channel but also

225 affect that of the other side. On one hand, the level of T's collection efforts determines its

226 service quality and brand reputation and thus affects the recycling willingness of consumers

227 directly or indirectly. On the other hand, because there are more opportunities for R to reach

228 consumers, R's advertising and promoting of recycling activities not only enhance her own

229 recovery of old products and her sales of new ones but are also conducive to increasing the

230 popularity of T, thus indirectly enhancing the click rate of T's recycling platform. Hence,

231 direct and indirect collection volumes travelling through the two recycling channels are

232 respectively formulated as

233 $q_t = s_t + \sqrt{A} + k_1\sqrt{a}$, (1)

234 $q_r = s_r + \sqrt{a} + k_2\sqrt{A}$. (2)

235 The square root formulation of response functions denotes diminishing returns to

236 collection effort expenses (Zhang et al., 2013), and $\sqrt{A}$ and $\sqrt{a}$ can be regarded as the two

237   parties' levels of collection efforts. The additive function is also used in Jena et al. (2017). s_r

238   and s_t are positive constants representing the returned quantities when each member's

239   collection efforts are valued at zero; to facilitate calculation, the values of s_r and s_t are set

240   to zero, which does not affect the conclusions of this study. k_1 and k_2 represent the

241   influencing coefficients of each member's collection efforts on the other side. Assume that

242   each member's collection efforts boost the other party's collection volumes, so $k_1, k_2 \in (0,1)$.

243   Eqs. (1) - (2) indicate that the collection volume is a joint effort employed by T and R,

244   and the values of A and a are related to the collaborative collection effort strategies

245   adopted. Meanwhile, increasing the collection volume will increase the sales of new products

246   and overall profits. To this end, four collaborative collection models are developed. The first

247   model is a centralized model (C-Model) in which both T and R agree to make efforts to

248   maximize the whole profits of the collecting system in an integrated manner. The second

249   model is a unit transfer price model (P-Model) in which T pays a unit transfer price b_r to R

250   for items returned through the R channel. The third model is a unilateral cost sharing model

251   (U-Model) in which T not only invests in her own channel but also bears part of R's

252   collection effort expenses. The fourth model is a bilateral cost sharing model (B-Model) in

253   which each member shares partial costs of the other member, or rather, T shares a fraction, t_1

254   ($t_1 \in [0,1]$), of R's collection effort costs a, and R shares a fraction, t_2 ($t_2 \in [0,1]$), of T's costs

255   A. Consistent with Zhang et al. (2013), t_1 and t_2 are referred to as T's participation rate

256   and R's participation rate, respectively. Accordingly, the collaborative strategies based on the

257   three decentralized decision models are referred to as the P-strategy, U-strategy and

258   B-strategy, respectively.

259   Let b be the marginal profit generated from recycling per unit of used product. The

260   appropriate allocation of recycling profit, i.e., b, between T and R is investigated in this paper.

261 R not only shares income from the recovery of old products but also earns "old-for-new"

262 profits. Let u be the collaborative marginal profit derived from the additional sale of new

263 products caused by the recovery of per unit of old ones, and assume that $b \geq u \geq 0$. Generally,

264 the higher the value of a product, the higher the collaborative marginal profit u. In addition,

265 the stronger the level of coordination between T and R, the greater the probability of

266 converting from recovery to purchasing and thus the greater the value of u.

267   The symbols used for the development of collaborative collection models are presented

268 in Table 1.

269   Table 1. Descriptions of the symbols.

Symbol	Description
A	Collection effort investments of the third-party, decision variable
a	Collection effort investments of the e-tailer, decision variable
k_1	Influence coefficient of the e-tailer 's collection efforts to the third-party
k_2	Influence coefficient of the third-party's collection efforts to the e-tailer
q_t	Direct collection volume through the third-party's channel
q_r	Indirect collection volume through the e-tailer's channel
b	Marginal profit by recycling one unit of used products
u	Collaborative marginal profit for the sale of new products through the recovery of per unit of old ones
b_r	Unit transfer price paid to the e-tailer by the third-party, decision variable
β	Proportion of profit sharing for the e-tailer
t_1	Proportion of the e-tailer's collection effort investments shared by the third-party, decision variable
t_2	Proportion of the third-party's collection effort investments shared by the e-tailer, decision variable
π_i^j	Profit of channel member i in model j. Subscript $i \in \{t, r, s\}$ refers to the third-party, the e-tailer and the whole collecting system separately. Superscript $j \in \{C, P, U, B\}$ refers to the C-Model, P-Model, U-Model and B-Model separately.
η^j	Efficiency of the collecting system, $\eta = \pi_s^{j*} \big/ \pi_s^{C*}$.

270

12

271 **4. Collaborative collection effort models**

272   In this section, four collection effort models, namely, the centralized model (C-Model),

273 unit transfer price model (P-Model), unilateral cost sharing model (U-Model) and bilateral

274 cost sharing model (B-Model), are established, the optimal decisions are derived, and the

275 influences of the key parameters on the optimal decisions are discussed. In the decentralized

276 models, T is regarded as the Stackelberg game leader and R as the follower.

277 **4.1 C-Model**

278   In this case, T and R belong to the same business conglomerate and act as a single entity,

279 and thus only one decision maker determines A and a to maximize the total profits of the

280 collection system. The total profit of the system is denoted as

281
$$\pi_s^C = (b+u)\left[(1+k_2)\sqrt{A} + (1+k_1)\sqrt{a}\right] - A - a \tag{3}$$

282   Thus, from the first-order condition, i.e. $\dfrac{\partial \pi_s^C}{\partial A} = 0$, and $\dfrac{\partial \pi_s^C}{\partial a} = 0$, an optimal solution

283 (A^{C*}, a^{C*}) is defined as follows:

284
$$\begin{cases} A^{C*} = \left(\dfrac{(b+u)(1+k_2)}{2}\right)^2 \\ a^{C*} = \left(\dfrac{(b+u)(1+k_1)}{2}\right)^2 \end{cases} \tag{4}$$

285   The optimal collection volumes are obtained based on collection effort levels, which are

286 given by

287
$$q_t^{C*} = \frac{(b+u)(1+k_1+k_2+k_1^2)}{2}, \tag{5}$$

288
$$q_r^{C*} = \frac{(b+u)(1+k_1+k_2+k_2^2)}{2} \tag{6}$$

289   The total profit of the collecting system is

290
$$\pi_s^{C*} = \frac{(b+u)^2[(1+k_2)^2 + (1+k_1)^2]}{4} \tag{7}$$

13

291   These acquired closed-form solutions in the C-Model offer benchmarking for designing

292   cooperative collection effort models.

293   **4.2 P-Model**

294   In this model, both T and R make efforts to motivate consumers to return WEEE, but they

295   must address their collection effort expenses individually. T provides a unit transfer price b_r

296   to R to induce her to collect used products. In addition, R earns additional profits from

297   increased sales of new products due to the recovery of old products. The profit expressions of

298   T and R can be written as

299    $$\pi_t^P = b(\sqrt{A} + k_1\sqrt{a}) + (b - b_r)\left(\sqrt{a} + k_2\sqrt{A}\right) - A \tag{8}$$

300    $$\pi_r^P = (b_r + u)\left(\sqrt{a} + k_2\sqrt{A}\right) + u\left(\sqrt{A} + k_1\sqrt{a}\right) - a \tag{9}$$

301   As the Stackelberg leader, T first proposes collection effort A and unit transfer price b_r,

302   and then R determines the collection effort a.

303   Via standard backward induction, the optimal solution of the collection efforts from the

304   first-order condition is given by

305    $$\widetilde{a^P} = \left(\frac{b_r + u(1+k_1)}{2}\right)^2 \tag{10}$$

306   **Proposition 1.** Let $u^P = b(1 - \frac{k_2 + k_2^2}{1 + k_1})$. In the P-Model, the optimal collection efforts of T

307   and R, the optimal unit transfer price paid to R are given by

308    $$b_r^{P*} = \begin{cases} \dfrac{(b-u)(1+k_1) - k_2 b(1+k_2)}{2 - k_2^2}, & u < u^P \\ 0 & , u \geq u^P \end{cases} \tag{11}$$

309    $$\sqrt{A^{P*}} = \begin{cases} \dfrac{2b(1+k_2) - k_2(1+k_1)(b-u)}{2(2 - k_2^2)}, & u < u^P \\ \dfrac{b(1+k_2)}{2} & , u \geq u^P \end{cases} \tag{12}$$

14

310 $$\sqrt{a^{P*}} = \begin{cases} \dfrac{(b+u-uk_2^2)(1+k_1)-k_2b(1+k_2)}{2(2-k_2^2)}, & u < u^P \\ \dfrac{u(1+k_1)}{2} & , & u \geq u^P \end{cases}$$ (13)

311   **Proof.** See Appendix A.

312   Proposition 1 implies that there is always an optimal combination of (A^{P*}, b_r^{P*}) for

313   maximizing the profit of T in the P-Model. The condition $u < u^P$ guarantees that the optimal

314   unit transfer price is greater than zero. When $u \geq u^P$, even when R cannot obtain a transfer

315   payment for her collection efforts, she still gains quite good returns due to a high added

316   collaborative profit.

317   The proportion of profit sharing for R's collecting can be calculated as

318    $$\beta^{P*} = \frac{b_r^{P*}}{b} = \frac{(b-u)(1+k_1)-k_2b(1+k_2)}{(2-k_2^2)b}, \ \beta^{P*} \in [0,1) \ .$$ (14)

319   The optimal collection volumes of direct and indirect channels can be computed from Eqs.

320   (1) - (2), and the optimal profits of T and R in the P-Model are obtained from Eqs. (8) - (9).

321   It is easy to observe that in the P-Model, the optimal unit transfer price b_r^{P*} is

322   monotonically decreasing in u and k_2, whereas the optimal collection efforts and optimal

323   profits of both T and R increase with increasing u. For the collection of products with high

324   collaborative marginal profits, T can pay R a low transfer payment because R can obtain

325   compensation from increasing sales of new products. In addition, in early stages when T

326   enters the recovery market, which involve a lower value of k_2, T should pay R a higher

327   transfer price to attract R to participate in collecting. Similarly, both T and R invest more in

328   the collection of highly profitable items such as smartphones. All of these principles are

329   consistent with observable reality.

330   **4.3 U-Model**

331   Collaborative collecting involves the joint efforts of T and R to increase collection

15

332   volumes, the sales of new products and overall profits. To achieve better performance, a

333   unilateral cost sharing model (U-Model) is proposed, in which dominant party T not only

334   invests in his own channel collection efforts but also bears a fraction t_1 ($t_1 \in [0,1]$) of R's

335   collection effort expenses. Meanwhile, T shares a proportion $1-\beta$ of the R collection

336   channel's profits, and the value of β ($\beta \in [0,1]$) is determined by both T and R.

337   The profit functions of T and R are formulated as

338
$$\pi_t^U = b(\sqrt{A} + k_1\sqrt{a}) + (1-\beta)b(\sqrt{a} + k_2\sqrt{A}) - A - t_1 a \tag{15}$$

339
$$\pi_r^U = \beta b(\sqrt{a} + k_2\sqrt{A}) + u(\sqrt{A} + k_1\sqrt{a} + \sqrt{a} + k_2\sqrt{A}) - (1-t_1)a \tag{16}$$

340   T first discloses his collection effort level and participation rate, and then R determines

341   her collection effort level.

342   Taking the derivative of π_r^U with respect to a yields

343
$$\frac{\partial \pi_r}{\partial a} = \frac{\beta b + u(1+k_1)}{2\sqrt{a}} - (1-t_1) \text{, and } \frac{\partial^2 \pi_r}{\partial a^2} = -\frac{1}{4}[\beta b + u(1+k_1)]a^{-3/2} < 0$$

344   This implies that π_r^U is a concave function, and from the first-order condition, the

345   optimal collection efforts of R is as follows:

346
$$\widetilde{a^U} = \left[\frac{\beta b + u(1+k_1)}{2(1-t_1)}\right]^2 \tag{17}$$

347   By substituting $\widetilde{a^U}$ into Eq. (15) and solving T's problem, the optimal result is presented

348   by Proposition 2.

349   **Proposition 2.** Let $\beta^U = \min\{\frac{(2b-u)(1+k_1)}{3b}, 1\}$; in the U-Model, the optimal participation

350   rate of T is

351
$$t_1^{U*} = \begin{cases} \dfrac{(2+2k_1-3\beta)b - u(1+k_1)}{(2+2k_1-\beta)b + u(1+k_1)}, & \beta < \beta^U \\ 0 & , & \beta \geq \beta^U \end{cases} \tag{18}$$

352 and the optimal collection efforts of T and R are given by

353
$$\sqrt{A^{U*}} = \frac{b[1 + k_2(1-\beta)]}{2}, \beta \in [0,1] \tag{19}$$

354
$$\sqrt{a^{U*}} = \begin{cases} \dfrac{(2 + 2k_1 - \beta)b + (1+k_1)u}{4}, \beta < \beta^U \\ \dfrac{\beta b + u(1+k_1)}{2}, \qquad \beta \geq \beta^U \end{cases} \tag{20}$$

355 **Proof.** See Appendix B.

356 Proposition 2 indicates that when the proportion of profit sharing for R is not too great (i.e.,

357 $\beta < \beta^U$), T has an incentive to share R's collection effort expenses to promote collecting for

358 both direct and indirect channels. Otherwise, when the proportion is dominant enough

359 ($\beta \geq \beta^U$), T will not participate in R's expenses ($t_1 = 0$), and so the U-Model is transformed

360 into the P-Model; then, the value of β can be determined from Eq. (14). To distinguish it

361 from the P-Model, the U-Model described below refers to a situation in which t_1 is greater

362 than 0.

363 β^U denotes a critical value. The smaller the collaborative profit u is, the larger β^U is

364 and the more likely T is willing to share R's collection effort expenses. In contrast, R's

365 influence coefficient k_1 has a positive effect on the critical value β^U.

366 The formulation of an optimal collection volume can be computed from Eqs. (1) - (2), and

367 the optimal profits of T and R can be determined from Eqs. (15) - (16).

368 From Proposition 2, Corollaries 1- 3 can be easily obtained.

369 **Corollary 1** In the U-Model, the optimal participation rate t_1 is monotonically

370 decreasing in u and is independent of k_2.

371 Corollary 1 implies that T should share more collection effort expenses of R for the sake

372 of maximizing his profit when the collaborative marginal profit is small. For example, in the

373 early stages of their cooperation, the conversion rate derived from the recovery of old

17

374   products to the sale of new ones may be low due to poor coordination, which results in a

375   small value of u. Under such conditions, T should undertake more collection effort

376   investments of R. However, an increase in k_2, which can be regarded as the strengthening

377   influence of T on the recycling market, does not affect T's participation rate.

378   **Corollary 2** In the U-Model, the optimal collection effort of R increases in u and is

379   independent of k_2, whereas the optimal collection effort of T is monotonically increasing in

380   k_2 and is independent of u.

381   Corollary 2 indicates that the collaborative marginal profit u has a positive impact on

382   R's collection effort but has no effect on T's collection effort. In contrast, an increase in k_2

383   does not cause R to increase her collection effort level, but it will increase T's collection

384   effort level.

385   **Corollary 3** In the U-Model, the profits of both T and R are monotonically increasing

386   functions of u and k_2.

387   Although only R's collection effort increases with an increase of u, the profits of both T

388   and R still grow as the direct and indirect collection volumes increase with respect to u,

389   which implies that a higher collaborative marginal profit is beneficial not only to R but also to

390   T. The same is true for the influence coefficient k_2.

391   **4.4 B-Model**

392   Studies have shown that bilateral participation can improve the channel efficiency of

393   cooperative advertising strategies (Zhang et al., 2013). During the cooperative collection

394   between T and R, as shown in Fig. 2, is R willing to share a portion of T's collection costs to

395   increase collection volumes and to thus promote the sale of new products? This is what the

396   paper investigates regarding the B-Model. In this case, both members not only invest in their

397   own channel collecting efforts but also bear a fraction t_1 / t_2 ($t_1, t_2 \in (0,1)$) of the other side's

398   collection expenses. They share the collecting profit, and β ($\beta \in (0,1)$) is the proportion of

399   profit sharing for R.

400   The profit functions of T and R are formulated as follows:

401    $$\pi_t^B = b(\sqrt{A} + k_1\sqrt{a}) + (1-\beta)b(\sqrt{a} + k_2\sqrt{A}) - (1-t_2)A - t_1 a \tag{21}$$

402    $$\pi_r^B = \beta b(\sqrt{a} + k_2\sqrt{A}) + u[(1+k_2)\sqrt{A} + (1+k_1)\sqrt{a}] - (1-t_1)a - t_2 A \tag{22}$$

403   There are four decision variables in the B-Model, including the collection effort

404   investments of T and R, A and a, and the bilateral participation rates t_1 and t_2.

405   According to Zhang et al. (2013), there are some rules regarding the allocation of

406   decision-making power that game players should follow to avoid trivial or unreasonable game

407   results. In applying these rules to the B-Model, suppose that the leader of the game makes a

408   decision about participation rates, while the follower makes decisions about collection efforts.

409   Again, by using backward induction, the optimal result is presented by Proposition 3.

410   **Proposition 3.** Let $\beta^B = \min\{\dfrac{(2b-u)(1+k_2)}{3k_2 b}, 1\}$. For any given β in the B-Model, the

411   optimal participation rate of T is

412    $$t_1^{B*} = \begin{cases} \dfrac{(2b-u)(1+k_1) - 3\beta b}{(2b+u)(1+k_1) - \beta b}, & \beta < \beta^U \\ 0 & , \beta \geq \beta^U \end{cases} \tag{23}$$

413   The optimal participation rate of R is given by

414    $$t_2^{B*} = \begin{cases} \dfrac{2k_2\beta b + 2u(1+k_2)}{(2b+u)(1+k_2) - k_2\beta b}, & \beta < \beta^B \\ 1 & , \beta \geq \beta^B \end{cases} \tag{24}$$

415   The optimal collection efforts can be computed as follows:

416    $$\sqrt{a^{B*}} = \begin{cases} \dfrac{(2b+u)(1+k_1) - \beta b}{4}, & \beta < \beta^U \\ \dfrac{\beta b + u(1+k_1)}{2}, & \beta \geq \beta^U \end{cases} \tag{25}$$

19

417 $$\sqrt{A^{B*}} = \begin{cases} \dfrac{(2b+u)(1+k_2)-k_2\beta b}{4}, & \beta < \beta^B \\ \dfrac{k_2\beta b + u(1+k_2)}{2}, & \beta \geq \beta^B \end{cases} \qquad (26)$$

418   **Proof.** See Appendix C.

419   Since $\beta^B \geq \beta^U$ and $t_2^{B*} > 0$ always hold, according to Proposition 3, the B-strategy

420 would become another U-strategy when $\beta \geq \beta^U$. In other words, T may not need to share part

421 of the collection effort cost of R ($t_1^{B*} = 0$), whereas R must share part of the cost of T($t_2^{B*} > 0$).

422 This means that it is always beneficial to T when R bears a fraction of T's investment in

423 collection efforts, while whether T has an incentive to share R's collection effort expense is

424 related to the value of β, i.e., T has an incentive only when $\beta < \beta^U$.

425   Hence, under the B-strategy, the optimal collection volumes can be computed from Eqs.

426 (1) - (2), and the optimal profits of T and R can be obtained from Eqs. (21) - (22).

427   From Proposition 3, Corollary 4 is easily obtained.

428   **Corollary 4** In the B-Model, R's optimal participation rate t_2, T's collection effort A

429 and the profits of both T and R are monotonically increasing with respect to k_2 and u.

430   Corollary 4 shows that higher collaborative profit and stronger influence of T can increase

431 R's participation rate and T's collection effort investments. Consequently, the profits of both

432 T and R can be improved.

433 **5. Comparative analysis**

434   According to the above results, some conclusions can be drawn through the comparison

435 of different collaborative collection effort models. The following numerical analysis

436 illustrates the results; the initial parameter setting is $b = 10$, $k_1 = 0.5$, $k_2 = 0.1$, $u = 3$.

437 **5.1 Comparison of the U-Model and P-Model**

438   **Proposition 4.** When $\beta = \beta^{P*}$ in the U-Model, relative to the P-Model, ordinal

439 relationships of optimal collection efforts are related as $A^{P*} = A^{U*}$ and $a^{P*} < a^{U*}$.

20

440   Consequently, collection volumes are related as follows: $q_r^{P*} < q_r^{U*}$ and $q_t^{P*} < q_t^{U*}$. The

441   member's profits are related as follows: $\pi_t^{U*} > \pi_t^{P*}$, $\pi_r^{U*} > \pi_r^{P*}$ and $\pi_s^{U*} > \pi_s^{P*}$.

442   **Proof.** See Appendix D.

443   Proposition 4 implies that under the same profit share as the optimal one in the P-Model,

444   R's collection effort investments will be enhanced in the U-Model, whereas T's collection

445   effort investments remain the same. As T shares part of the collection effort investment of R,

446   the total collection effort investment increases; thus, the collection volumes of the direct and

447   indirect channels increase, and the profits of both T and R in the U-Model are greater than

448   those in the P-Model. Therefore, the U-strategy is a Pareto improvement of the P-strategy

449   when the profit share remains the same as that of P-strategy.

450   **Corollary 5** Let $\beta^{UL} = \dfrac{\left(1+k_1+2k_2+2k_2^2\right)b - 2k_2 u(1+k_2)}{(1+4k_2^2)b}$. In the U-Model, the optimal

451   profits of both T and the collecting system are monotonically decreasing in β, and the

452   following hold:

453   (i) if $\beta^{UL} \geq \beta^U$, the optimal profit of R is an increasing function of β when $\beta \leq \beta^U$;

454   (ii) if $\beta^{UL} < \beta^U$, the optimal profit of R is an increasing function of β when $\beta \leq \beta^{UL}$ and a

455   decreasing function of β when $\beta^{UL} < \beta \leq \beta^U$.

456   **Proof.** See Appendix E.

457   Corollary 5 shows that increasing the proportion of profit sharing for R is always

458   disadvantageous to both T and the collecting system under U-strategy and is not always

459   advantageous to R.

460   **Corollary 6** There is always an interval (β_r^U, β_t^U) in the U-Model in which β_r^U and

461   β_t^U satisfy $0 \leq \beta_r^U \leq \beta^{P*}$ and $\beta^{P*} < \beta_t^U \leq \beta^U$, respectively. When the value of β falls

462   within the range of (β_r^U, β_t^U), the optimal profits of both T and R will increase in the

U-Model relative to those in the P-Model.

Proof. See Appendix F.

Corollary 6 extends the range of β in which the U-strategy is a Pareto improvement of the P-strategy. This also shows that when the value of β is within a certain range under the U-strategy, a win-win result can be achieved relative to that achieved with the P-strategy. In Fig. 3a, β_r^U and β_t^U are the proportions of profit sharing that give $\pi_r^{U*}\big|_{\beta=\beta^{P*}} = \pi_r^{P*}$ and $\pi_t^{U*}\big|_{\beta=\beta^{P*}} = \pi_t^{P*}$, respectively. When $\beta^{UL} \geq \beta^U$, the optimal profit of R is a monotonically increasing function of β under the U-strategy, thresholds β_r^U and β_t^U satisfy $0 \leq \beta_r^U < \beta^{P*}$, and $\beta^{P*} < \beta_t^U \leq \beta^U$, respectively, and thus the optimal profits of both T and R increase in the U-Model relative with those of the P-Model when $\beta \in (\beta_r^U, \beta_t^U)$ (see Fig. 3a). When $\beta^{UL} < \beta^U$, the optimal profit of R first increases and then decreases with increasing β. In this case, since $\beta^{P*} = 0$ and $\pi_r^{U*}\big|_{\beta=\beta^{P*}} > \pi_r^{P*}$, $\beta_r^U = \beta^{P*} = 0$ holds, and thus a win-win result can also be achieved when using the U-strategy rather than the P-strategy when $\beta \in (0, \beta_t^U)$ (see Fig. 3b; the parameter values are as follows: $b = 20$, $k_1 = 0.9$, $k_2 = 0.9$, $u = 13$).
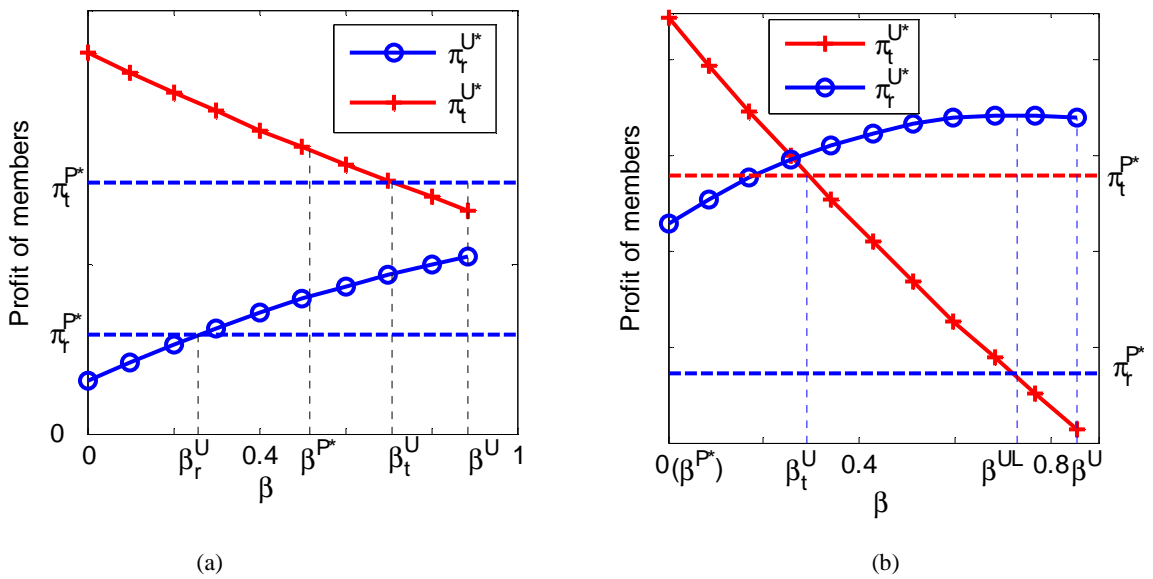

(a)   (b)

Fig. 3. Comparison between the U-Model and P-Model: (a) if $\beta^{UL} \geq \beta^U$ and (b) if $\beta^{UL} < \beta^U$

486

5.2 Comparison of the B-Model and P-Model

Proposition 5. When $\beta = \beta^{P*}$, relative to the P-Model, the ordinal relationships of the

optimal collection efforts are $A^{P*} < A^{B*}$ and $a^{P*} < a^{B*}$. Consequently, the collection volumes

are as follows: $q_r^{P*} < q_r^{B*}$ and $q_t^{P*} < q_t^{B*}$. The members' profits are related as follows:

$\pi_t^{B*} > \pi_t^{P*}$, $\pi_r^{B*} \geq \pi_r^{P*}$ when $M \geq 0$, $\pi_r^{B*} < \pi_r^{P*}$ when $M < 0$, and $\pi_s^{B*} > \pi_s^{P*}$ where

$M = (2b - 4u)\beta^{P*}b[1 + k_1 - k_2(1 + k_2)] + (2b - u)u[(1 + k_1)^2 - (1 + k_2)^2] - 3(1 - k_2^2)(\beta^{P*}b)^2$.

Proof. See Appendix G.

Proposition 5 shows that with the same proportion of profit sharing as the optimal one in

the P-Model, when both T and R share part of the collection investments of the other side, the

collection efforts of both sides and the collection volumes of both channels will increase, and

for T and the collecting system, the B-Model is more profitable than the P-Model. However,

for R, only when $M \geq 0$ is the optimal profit of R for the B-Model higher than that of the

P-Model. Through data simulations, it is also found that $M \geq 0$ almost always holds when

$k_1 \geq k_2$, although it cannot be analytically proven due to the complexity of M .

Corollary 7 In the B-Model, the profit of R is an increasing function of β , and in

contrast, the profits of both T and the collecting system are decreasing functions of β .

Proof. See Appendix H.

Corollary 7 indicates that increasing the proportion of profit sharing for R can increase

R's profit, but it is at the expense of the profits of T and the collection system. There must be

appropriate values of β for a trade-off between T and R.

Corollary 8 There is always an interval (β_r^B, β_t^B) in the B-Model in which

$0 \leq \beta_r^B \leq \beta^{P*}$ and $\beta^{P*} < \beta_t^B \leq 1$ when $M \geq 0$ and in which $\beta^{P*} < \beta_r^B, \beta_t^B \leq 1$ when $M < 0$.

When the value of β falls within the range of (β_r^B, β_t^B), the optimal profits of both T and R

510   will increase relative to those of the P-Model.

511   **Proof.** As it is similar to the proof of Corollary 6, the proof is omitted here.

512   Corollary 8 gives the range of the profit share β in which the B-strategy is a Pareto

513   improvement of P-strategy. In Fig. 4, β_r^B, β_t^B are the proportions of profit sharing that

514   create $\pi_r^{B*}\big|_{\beta=\beta^{P*}} = \pi_r^{P*}$ and $\pi_t^{B*}\big|_{\beta=\beta^{P*}} = \pi_t^{P*}$, respectively. $M>0$ denotes that $\pi_r^{B*}\big|_{\beta=\beta^{P*}} > \pi_r^{P*}$,

515   and so $0 \le \beta_r^B < \beta^{P*}$ and $\beta^{P*} < \beta_t^B \le 1$ (see Fig. 4a). Keeping the values of other parameters

516   unchanged and increasing the value of k_2 to 0.6 such that $M<0$, $\pi_r^{B*}\big|_{\beta=\beta^{P*}} < \pi_r^{P*}$, and thus

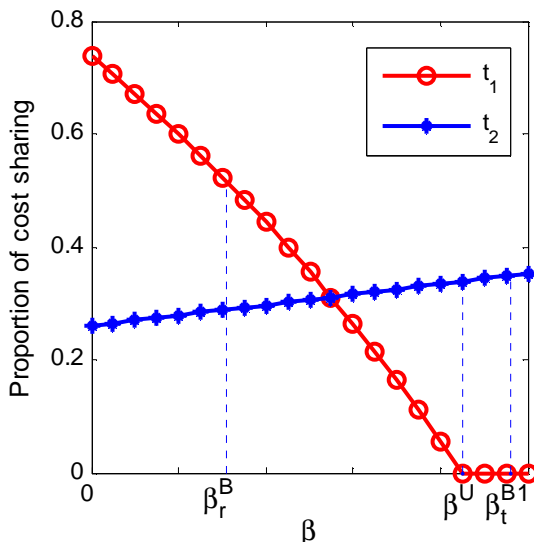517 $\beta^{P*} < \beta_r^B, \beta_t^B \le 1$ (see Fig. 4b).

518

(a)   (b)
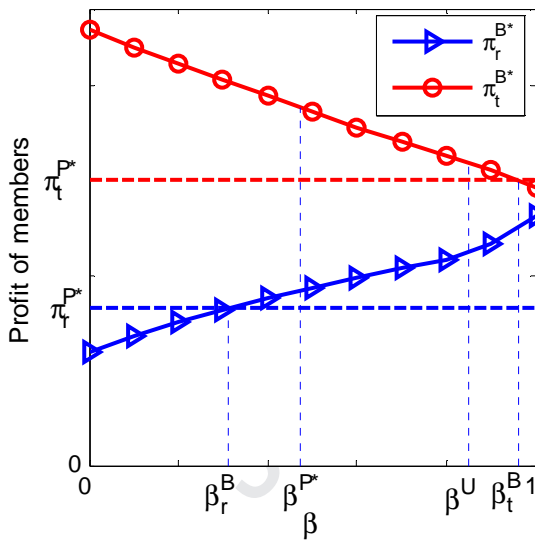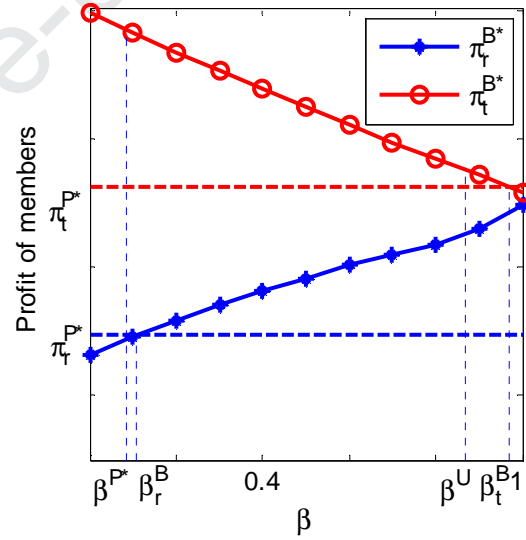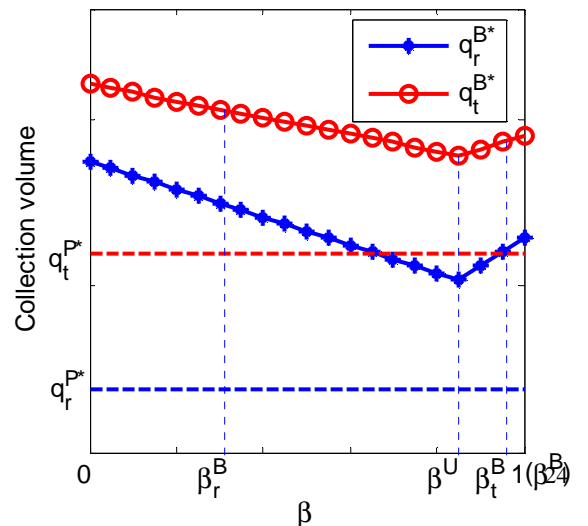
Fig. 4. Comparison of the optimal profits between the B-Model and P-Model. (a) $M \ge 0$ and (b) $M < 0$

Fig. 5. The optimal proportion of cost sharing under the B-Model as β varies

Fig. 6. Comparison of collection volumes between the B-Model and P-Model

534

535

536

Note that $\beta_t^B > \beta^U$ may be true (see Fig. 4b). When $\beta_t^B > \beta^U$ and $\beta \in (\beta^U, \beta_t^B)$, $t_1^* \equiv 0$

and $t_2^* > 0$ hold (see Fig. 5). Under such conditions, as the collection efforts of R increase

with increasing β, both the direct and indirect collection volumes increase instead of

decreasing (see Fig. 6). This result suggests that when $\beta_t^B > \beta^U$, T can afford R a higher

proportion of profit sharing without sharing part of R's collection effort costs, and hence an

improvement in profit for both parties and a significant increase in the total collection volume

can be achieved.

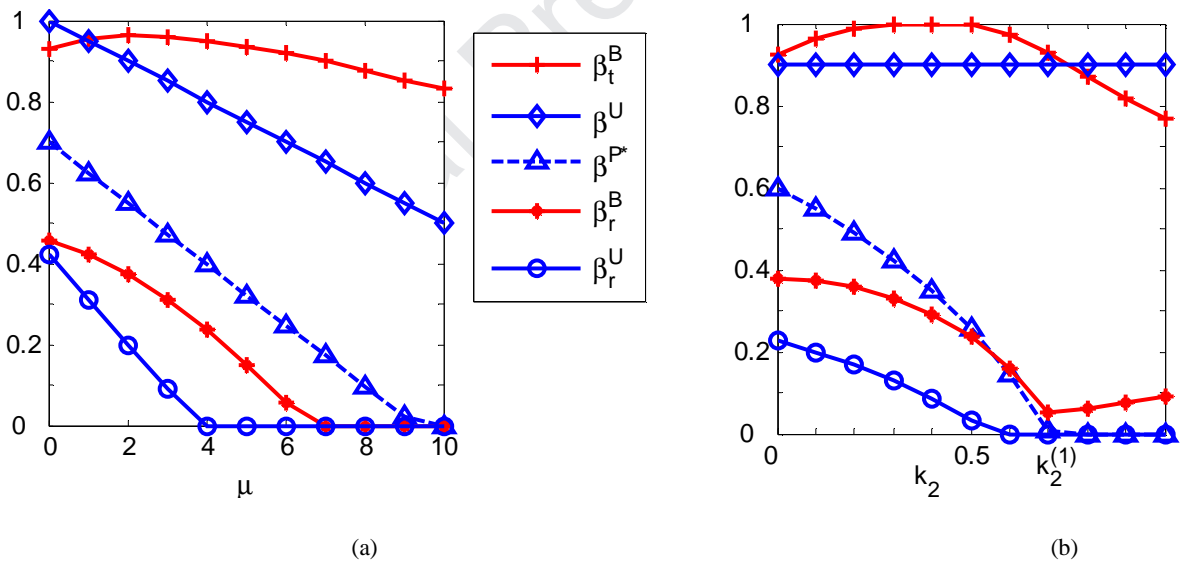(a)   (b)

Fig. 7. The optimal value β^{P*} and the threshold values (β_r^U, β^U) and (β_r^B, β_t^B) (a) as u varies and (b) as k_2 varies

Through data simulations, it is found that the threshold value of β_t^B first increases and

then decreases with increasing u, whereas the threshold value of β^U decreases more

rapidly, and thus the higher the value of u, the more likely R is to share part of T's

collection effort cost unilaterally (see Fig. 7a). Similarly, the value of β_t^B first increases and

then decreases with increasing in k_2, whereas the value of β^U is independent on k_2; thus,

25

558    $\beta_t^B > \beta^U$ holds only when k_2 is not large enough (see Fig. 7b). This result explains why

559   some e-tailers direct large amounts of capital to their recycling partners to facilitate the

560   recovery of WEEE of high value, such as smartphones (which means that the value of u

561   may be higher), especially in early stages, when T is just entering the recycling market (which

562   means that the value of k_2 may be lower).

563   **5.3 Comparison of the B-Model and U-Model**

564   **Proposition 6.** For any value of $\beta(\beta < \beta^U)$, the ordinal relationships of optimal

565   collection efforts between the U-Model and B-Model are related as follows: $A^{U*} < A^{B*}$ and

566    $a^{U*} = a^{B*}$. Consequently, the collection volumes are related as follows: $q_r^{U*} < q_r^{B*}$ and

567    $q_t^{U*} < q_t^{B*}$. The ordinal relationships of the profits are as follows: $\pi_t^{B*} > \pi_t^{U*}$, $\pi_r^{B*} < \pi_r^{U*}$ and

568    $\pi_s^{B*} > \pi_s^{U*}$.

569   **Proof.** See Appendix I.
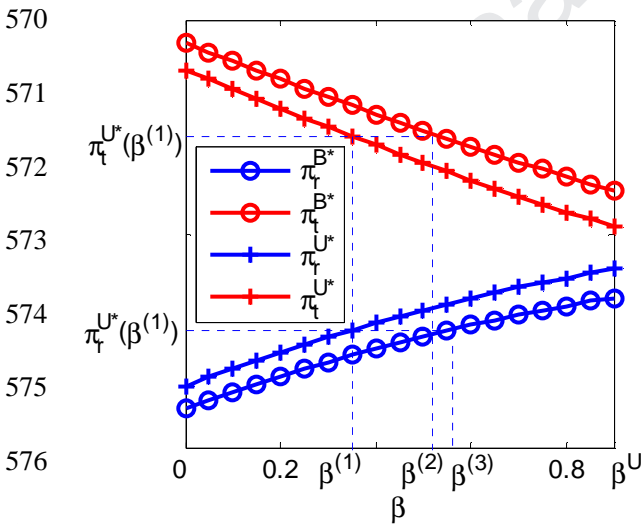
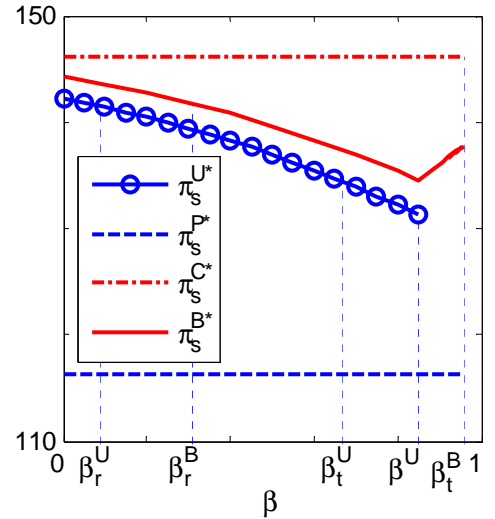**Fig. 8**. Comparison of the member's optimal profits between the B-Model and U-Model

Fig. 9. Comparison of the profits of the collecting system under centralized and decentralized decisions

579   Proposition 6 indicates that under the same proportion of profit sharing, the B-Model is

580   more profitable for T and the collection system but less profitable for R relative to the

581   U-Model. In Fig. 8, since the profit of T decreases whereas the profit of R increases with

582 respect to β, for a proportion of profit sharing $\beta^{(1)} \in [0,1]$ in the U-Model,

583 $\pi_t^{B*}\big|_{\beta=\beta^i} > \pi_t^{U*}\big|_{\beta=\beta^{(1)}}$ implies $\beta^i < \beta^{(2)}$, and $\pi_r^{B*}\big|_{\beta=\beta^i} > \pi_r^{U*}\big|_{\beta=\beta^{(1)}}$ implies $\beta^i > \beta^{(3)}$. However,

584 $\beta^{(3)} > \beta^{(2)}$ holds; thus, there is not necessarily a corresponding value β^i in the B-Model that

585 supports $\pi_t^{B*}\big|_{\beta=\beta^i} > \pi_t^{U*}\big|_{\beta=\beta^{(1)}}$ and $\pi_r^{B*}\big|_{\beta=\beta^i} > \pi_r^{U*}\big|_{\beta=\beta^{(1)}}$ simultaneously. Clearly, whether the

586 B-Model is a Pareto improvement of the U-Model depends on the crucial parameters of the

587 collection system and the profit sharing proportion β.

588   According to Proposition 6, it is easy to see that $\beta_r^U \leq \beta_r^B$ and $\beta_t^U \leq \beta_t^B$.

589   Through the above comparisons, it is obvious that the ordinal relationship of total profits

590 for all of the collaborative collection effort models is $\pi_s^{C*} > \pi_s^{B*} > \pi_s^{U*} > \pi_s^{P*}$ when β falls

591 within the range of $[0, \beta^U]$ (see Fig. 9).

6. Sensitivity analysis

593   The impacts of the influence coefficient k_2 and collaborative marginal profit u on the

594 total collection volume and efficiency of the collecting system are further discussed. Since

595 the direct and indirect collection volumes are monotonically decreasing functions of β, the

596 values $q_s^{U*}\big|_{\beta=\beta_r^U}$ and $q_s^{U*}\big|_{\beta=\beta_t^U}$ respectively represent the highest and lowest collection

597 volumes of the profit improvement interval (β_r^U, β_t^U) for the U-Model, and a similar

598 conclusion is drawn for the B-Model. Thus, for the following analysis, in the cost sharing

599 models, the proportions of profit sharing are set to the lower and upper bounds of the profit

600 improvement interval, respectively.

601   In Fig. 10, it is observed that total collection volumes in the three decentralized models

602 increase with increasing u and k_2, and $q_s^{B*} > q_s^{P*}$ and $q_s^{U*} > q_s^{P*}$ always hold for any

603 values of u and k_2. Fig. 10(a) shows that when u is low and when β is at the lower

604   bound, the total collection volumes differ little between the B-Model and U-Model, whereas

605   the total collection volume is significantly greater in the B-Model than that in the U-Model

606   when the value of u is large and when the proportion of profit sharing β is at the upper

607   bound. In contrast, the total collection volumes differ little between the B-Model and

608   U-Model when the value of β is at the lower bound and when the value of k_2 is low or

609   when the value of k_2 is high while the value of β is at the upper bound (see Fig. 10b). Fig.

610   10 also shows that the difference between the U-Model and B-Model is more heavily affected

611   by u than k_2 when the value of β is at the upper bound.

(a)   (b)
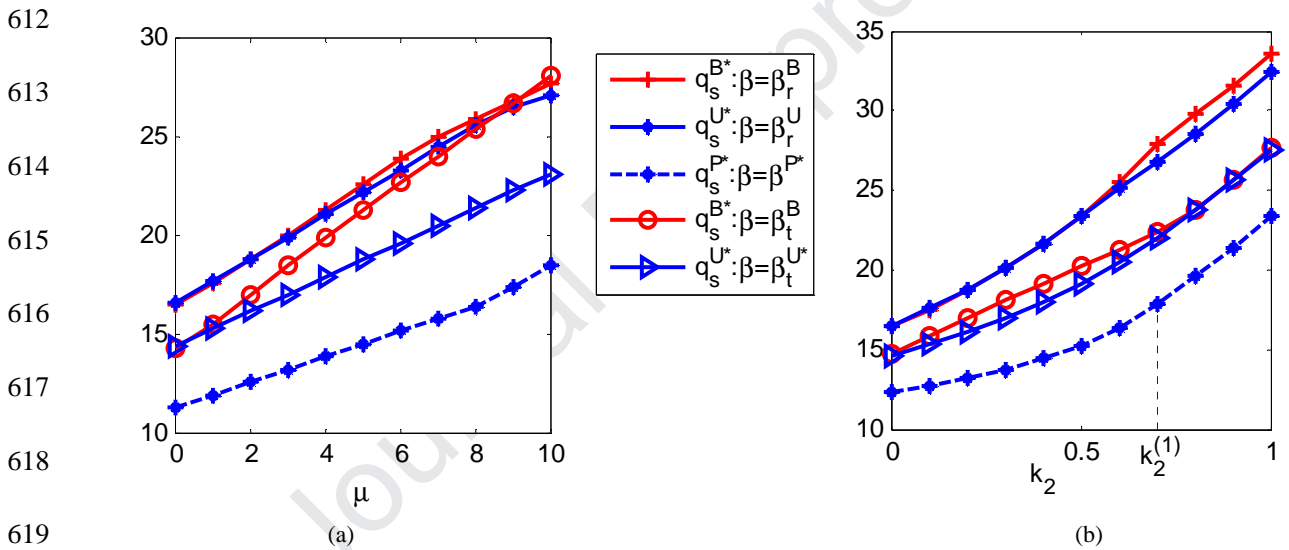
Fig. 10. Comparison of the total collection volumes (a) as u varies   and (b) as k_2 varies

(a)   (b)
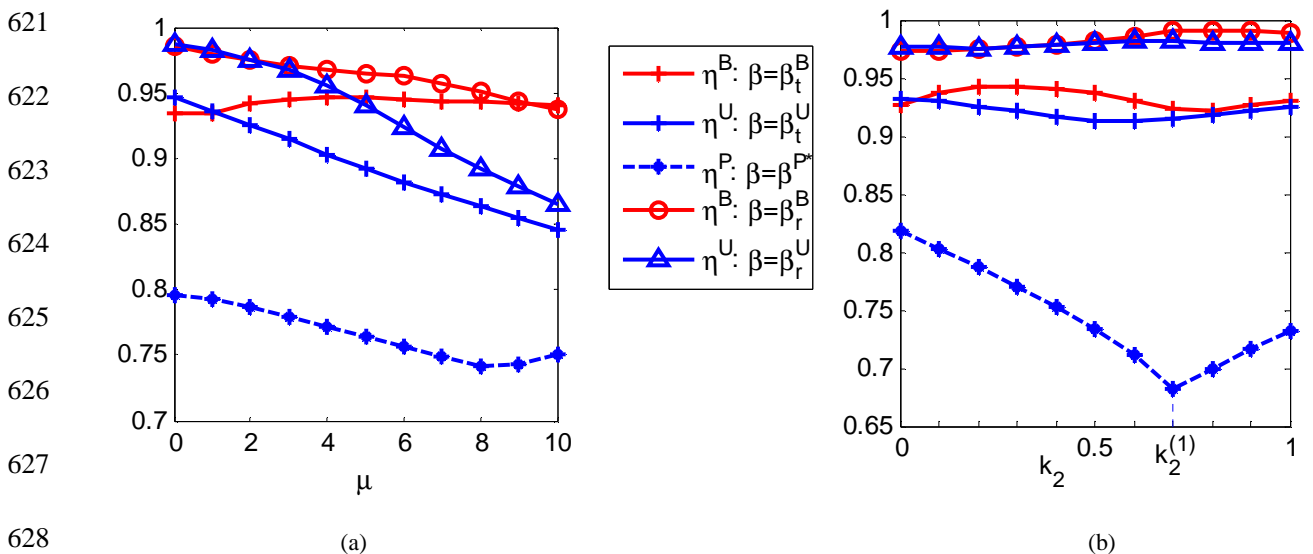
Fig. 11. Comparison of the collecting system efficiency (a) as u varies   and (b) as k_2 varies

28

629

630   Fig. 11(a) shows that the collecting system efficiency of each cost sharing model is far

631   higher than that of the P-Model, and the efficiency of the collecting system mainly follows a

632   downward trend with increasing u in each of the three decentralized models. Fig. 11(a) also

633   indicates that the efficiency of the B-Model is not higher than that of the U-Model when the

634   value of u is very small. However, since the efficiency of the U-Model decreases more

635   rapidly, the B-Model is more efficient than the U-Model when the value of u is large enough

636   regardless of the profit sharing proportion involved. In Fig. 11(b), $k_2^{(1)}$ represents the

637   threshold value that gives $\beta^{P*} = 0$. Fig. 11(b) illustrates that the system's efficiency decreases

638   with increasing k_2 in the P-Model when $k_2 < k_2^{(1)}$, but when $k_2 \geq k_2^{(1)}$, it increases as the

639   collection volume increases more quickly with increasing in k_2. In addition, Fig. 11 shows

640   that the system's efficiency in both the U-Model and B-Model is less affected by k_2.

641   The conclusions of the sensitivity analysis offer further guidance regarding how to make

642   optimal decisions according to actual situations based on the market influences of T, levels of

643   coordination, and types and values of collected products involved.

644   **7．Conclusion**

645   In this paper, collaborative collection effort strategies involving a third-party collector

646   and an e-tailer based on the "Internet + recycling" business model are explored. The paper

647   develops four cases of collaborative collection models, derives the optimal decisions,

648   conducts a comparative analysis of these models and analyses the impact of crucial

649   parameters on the collection volume and efficiency of the collecting system.

650   The main findings of this paper are as follows. (i) There exists an interval of profit

651   sharing proportion in which each of the two cost sharing strategies is a Pareto improvement

652   of the unit transfer price strategy. (ii) An increase in the collaborative marginal profit can

653 increase the e-tailer's participation rate and her collection effort level under cost sharing

654 strategies and thus improve the e-tailer's and third party's profits. (iii) An increase in the

655 market influence of the third-party has no effect on the collection effort level of the e-tailer,

656 but it can increase the participation rate of the e-tailer and thus improve the profits of both

657 parties. (iv) Under the B-strategy, when the collaborative marginal profit is large enough, the

658 third party can give the e-tailer a higher proportion of profit sharing but does not need to

659 share part of the e-tailer's collection effort cost, and thus a Pareto improvement of the

660 P-strategy can also be achieved. (v) Although the total collection volume and profit of the

661 collecting system increase under the B-strategy relative to those of the U-strategy under the

662 same proportion of profit sharing, the B-strategy is not necessarily a Pareto improvement of

663 the U-strategy.

664   The above conclusions provide some useful suggestions for "Internet + recycling"

665 enterprises. First, it is more profitable for a third-party collector and an e-tailer to share a

666 portion of the other's collection investments under the cooperative "Internet + recycling"

667 mode. For instance, Jd.com, a famous e-tailer in China, cooperates with Aihuishou.com, a

668 professional O2O electronic product collection company, in WEEE recycling. Jd.com has

669 made several rounds of investment to Aihuishou.com to facilitate the recovery of WEEE of

670 high value, such as smartphones, which can be explained by the B-strategy. Second, the third

671 party should consider the types and values of WEEE involved when making the optimal

672 choice. For example, for high-value WEEE collection, higher collection volumes and levels

673 of system efficiency can be achieved under the B-strategy with a high profit sharing

674 proportion than that involved when using the U-strategy, but for low-value WEEE collection,

675 the third party may adopt the U-strategy with a low profit sharing proportion rather than the

676 B-strategy with a high profit sharing proportion to obtain greater collection volume. Third,

677 the third-party and e-tailer must strengthen coordination and resource integration to increase

678 the probability of converting from recovery to purchasing with help of "Internet+", which can

679 improve not only the profit of the e-tailer but also the profit of the collector.

680   In future research, some assumptions may be relaxed to develop more comprehensive

681 collaborative collection systems, such as a case in which a system includes e-tailers and

682 third-party collectors in addition to consumers, where both the recycling price paid to

683 customers and the discount for buying new products affecting the system should be

684 considered. It would be interesting to study how partners make optimal decisions and how the

685 consumer surplus changes during one-stop recycling and upgrading services under different

686 collaborative strategies based on the "Internet + recycling" business model.

691 **References**

692 Afroz, R., Masud, M. M., Akhtar, R., Duasa, J. B., 2013. Survey and analysis of public

693   knowledge, awareness and willingness to pay in Kuala Lumpur, Malaysia - a case study

694   on household WEEE management. Journal of Cleaner Production. 52(4), 185-93.

695 Ahmadi-Javid, A., Hoseinpour, P., 2012. On a cooperative advertising model for a supply

696   chain with one manufacturer and one retailer. European Journal of Operational Research.

697   219(2), 458–466.

698 Atasu, A., Toktay, L. B., 2013. How Collection Cost Structure Drives a Manufacturer's

699   Reverse Channel Choice. Production Operations Management. 22(5), 1089–1102.

700 Awasthi, A. K., Li, J., 2017. Management of electrical and electronic waste: A comparative

701   evaluation of China and India. Renewable and Sustainable Energy Reviews. 76, 434–447.

702   Cucchiella, F., D'Adamo, I., Lenny Koh, S. C., Rosa, P., 2015. Recycling of WEEEs: An

703   economic assessment of present and future e-waste streams. Renewable and Sustainable

704   Energy Reviews. 51, 263-272.

705   Deng, W. J., Giesy, J. P., So, C. S., Zheng, H. L., 2017. End-of-life (EoL) mobile phone

706   management in Hong Kong households. Journal of Environmental Management. 200, 22–

707   28.

708   Feng, L., Govindan, K., Li, C., 2017. Strategic planning: Design and coordination for

709   dual-recycling channel reverse supply chain considering consumer behavior. European

710   Journal of Operational Research. 260(2), 601–612.

711   Gao, J., Han, H., Hou, L., Wang, H., 2016. Pricing and effort decisions in a closed-loop

712   supply chain under different channel power structures. Journal of Cleaner Production. 112,

713   2043–2057.

714   Greenpeace, 2016. Summary of Report on Energy Resource Utilization Rate of Smartphones

715   (In   Chinese).   http://www.greenpeace.org.cn/summary-report-smart-phone-recycle/

716   (accessed 15 Dec 2017).

717   Ghosh, D., Gouda, S., Shankar, R., Swami, S., Cheruvil, V., 2018. Strategic decision making

718   under subscription-based contracts for remanufacturing. International Journal of

719   Production Economics. 200, 134–150.

720   Giovanni, P. De., 2018. A joint maximization incentive in closed-loop supply chains with

721   competing retailers : The case of spent-battery recycling. European Journal of Operational

722   Research. 268(1), 128–147.

723   Gu, F., Guo, J., Yao, X., Summers, P. A., Widijatmoko, S. D., Hall, P., 2017. An

724   investigation of the current status of recycling spent lithium-ion batteries from consumer

725   electronics in China. Journal of Cleaner Production. 161, 765–780.

726 Gu, F., Zhang, W., Guo, J., Hall, P., 2019. Exploring "Internet + Recycling" : Mass balance

727   and life cycle assessment of a waste management system associated with a mobile

728   application. Science of the Total Environment. 649, 172–185.

729 Hong, X., Wang, Z., Wang, D. et al., 2013. Decision models of closed-loop supply chain with

730   remanufacturing under hybrid dual-channel collection. International Journal of Advanced

731   Manufacturing Technology. 68(5-8):1851-1865.

732 Hong, X., Xu, L., Du, P., Wang, W., 2015. Joint advertising, pricing and collection decisions

733   in a closed-loop supply chain. International Journal of Production Economics. 167, 12–

734   22.

735 Huang, M., Song, M., Lee, L. H., Ching, W. K., 2013. Analysis for strategy of closed-loop

736   supply chain with dual recycling channel. International Journal of Production Economics.

737   144(2), 510–520.

738 Huang, Z., Li, S. X., 2001. Co-op advertising models in manufacturer-retailer supply chains:

739   A game theory approach. European Journal of Operational Research. 135(3), 527–544.

740 Jena, S. K., Sarmah, S. P., 2015. Measurement of consumers' return intention index towards

741   returning the used products. Journal of Cleaner Production. 108, 1–12.

742 Jena, S. K., Sarmah, S. P., Sarin, S. C., 2017. Joint-advertising for collection of returned

743   products in a closed-loop supply chain under uncertain environment. Computers and

744   Industrial Engineering. 113, 305–322.

745 Li, X., Li, Y., Cao, W., 2017. Cooperative advertising models in O2O supply chains.

746   International Journal of Production Economics. 1-9.

747 Liu, L., Wang, Z., Xu, L., Hong, X., Govindan, K., 2017. Collection effort and reverse

748   channel choices in a closed-loop supply chain. Journal of Cleaner Production. 144, 492–

749   500.

750   Ma, Z.J., Zhang, N., Dai, Y., Hu, S., 2016. Managing channel profits of different cooperative

751   models in closed-loop supply chains. Omega. 59, 251–262.

752   Mohan, N., Modak, N., Panda, S., 2018. Analyzing structure of two-echelon closed-loop

753   supply chain for pricing, quality and recycling management. Journal of Cleaner

754   Production. 171, 512–528.

755   NDRC (The National Development and Reform Commission of the People's Republic of

756   China), 2016. "Internet +" three-year action plan for green ecology.

757   http://www.gov.cn/xinwen/2016-01/21/content_5035064.htm (accessed 15 Dec 2017).

758   Robinson, B. H., 2009. E-waste: An assessment of global production and environmental

759   impacts. Science of the Total Environment. 408(2), 183–191.

760   Savaskan, R. C., Bhattacharya, S., Van Wassenhove, L. N., 2004. Closed-Loop Supply Chain

761   Models with Product Remanufacturing. Management Science. 50(2), 239–252.

762   Savaskan, R. C., Van Wassenhove, L. N., 2006. Reverse Channel Design: The Case of

763   Competing Retailers. Management Science. 52(1): 1-14.

764   SC (The State Council of the People's Republic of China), 2015. Guidance on actively

765   promoting the "Internet +" action. http://www.gov.cn/zhengce/content/2015-07/04/

766   content_10002.htm (accessed 15 Dec 2017).

767   Sun, Q., Wang, C., Lu, F., 2018. Digital empowerment in a WEEE collection business

768   ecosystem : A comparative study of two typical cases in China. Journal of Cleaner

769   Production. 184, 414–422.

770   Tong, X., Tao, D., Lifset, R., 2018. Varieties of business models for post-consumer recycling

771   in China. Journal of Cleaner Production. 170, 665–673.

772   Wang, H., Han, H., Liu, T., Tian, X., Xu, M., Wu, Y., Gu, Y., Liu, Y., Zuo, T., 2018. "Internet

773   +" recyclable resources : A new recycling mode in China. Resources, Conservation &

774   Recycling. 134, 44–47.

775 Xue, Y., 2017. Four new features and two new transformations of internet + resource

776   recycling cases in 2017 (In Chinese). www.sohu.com/a/207126573_ 100014482 (accessed

777   15 Dec 2017).

778 Zeng, X., Duan, H., Wang, F., Li, J., 2017. Examining environmental management of

779   e-waste: China's experience and lessons. Renewable and Sustainable Energy Reviews. 72,

780   1076–1082.

781 Zhang, J., Xie, J., Chen, B., 2013. Cooperative Advertising with Bilateral Participation.

782   Decision Sciences. 44(1), 193–203.

783 **Appendix A. The proof of Proposition 1**

784   By substituting the value of $\widetilde{a^P}$ in Eq. (8), the problem of the third-party is written as

785 $$\underset{b_r, A}{Max}\, \pi_t^P = [b(1+k_2)-k_2 b_r]\sqrt{A} + \frac{[b(1+k_1)-b_r][b_r+u(1+k_1)]}{2} - A \qquad (A1)$$

786   The first- and second-order derivatives of Eq. (A1) are given by

787 $$\frac{\partial \pi_t^P}{\partial A} = \frac{b(1+k_2)-k_2 b_r}{2\sqrt{A}} - 1\,, \quad \frac{\partial \pi_t^P}{\partial b_r} = -b_r - k_2\sqrt{A} + \frac{(1+k_1)(b-u)}{2}\,,$$

788 $$\frac{\partial^2 \pi_t^P}{\partial A^2} = -\frac{\left[b+k_2(b-b_r)\right]A^{-3/2}}{4} < 0\,, \quad \frac{\partial^2 \pi_t^P}{\partial b_r^2} = -1 < 0\,, \quad \frac{\partial^2 \pi_t^P}{\partial A \partial b_r} = \frac{-k_2}{2\sqrt{A}}\,, \quad \frac{\partial^2 \pi_t^P}{\partial b_r \partial A} = -\frac{k_2}{2\sqrt{A}}\,.$$

789   The Hessian matrix of π_t^P is $\Omega = \begin{pmatrix} \dfrac{\partial^2 \pi_t^P}{\partial A^2} & \dfrac{\partial^2 \pi_t^P}{\partial A \partial b_r} \\ \dfrac{\partial^2 \pi_t^P}{\partial b_r \partial A} & \dfrac{\partial^2 \pi_t^P}{\partial b_r^2} \end{pmatrix}$. Let $Z^P = \dfrac{b(1+k_2)-k_2 b_r}{k_2^2}$; when $\sqrt{A} < Z^P$,

790 Ω is negative definite, which shows that the objective function is concave with respect to (A, b_r). From

791 the first-order conditions, the optimal solutions of the third-party are

792 $$\sqrt{A^{P*}} = \frac{2b(1+k_2)-k_2(1+k_1)(b-u)}{2(2-k_2^2)}\,, \quad b_r^{P*} = \frac{(b-u)(1+k_1)-k_2 b(1+k_2)}{2-k_2^2}\,.$$

793   Substituting b_r^{P*} into $\widetilde{a^P}$, the optimal collection effort of R is given by

794
$$\sqrt{a^{P*}} = \frac{(b+u-uk_2^2)(1+k_1) - k_2b(1+k_2)}{2(2-k_2^2)} .$$

795   Substituting A^{P*} and b_r^{P*} into Z^P, one has $Z^{P*} = \frac{2b(1+k_2) - k_2(b-u)(1+k_1)}{k_2^2(2-k_2^2)}$; thus, $\sqrt{A} < Z^P$ always

796   holds in the neighborhood of (A^{P*}, b_r^{P*}). Therefore, there is always an optimal combination of (A^{P*}, b_r^{P*})

797   to maximize the profit of the third party in the P-Model.

798   Since b_r^{P*} must be greater than or equal to zero, the condition $u < u^P = b(1 - \frac{k_2 + k_2^2}{1+k_1})$ guarantees it.

799 **Appendix B. The proof of Proposition 2**

800   Substituting $\widetilde{a^U}$ into Eq. (15), the first-order derivatives of π_t^U are given by

801      $\frac{\partial \pi_t^U}{\partial A} = \frac{b[1+k_2(1-\beta)]}{2\sqrt{A}} - 1$, and $\frac{\partial \pi_t^U}{\partial t_1} = \frac{NV}{4(1-t_1)^2} - \frac{t_1 V^2}{2(1-t_1)^3}$ where $N = b(2+2k_1-3\beta) - u(1+k_1)$,

802 $V = \beta b + u(1+k_1)$. The two decision variables A and t_1 are not related and thus can be solved

803 independently.

804   Because $\frac{\partial^2 \pi_t^U}{\partial A^2} = -\frac{b[1+k_2(1-\beta)]}{4} A^{-3/2} < 0$, with the first-order condition $\frac{\partial \pi_t^U}{\partial A} = 0$, one can easily

805   show that $\sqrt{A^{U*}} = \frac{b[1+k_2(1-\beta)]}{2}$. Let $\beta^U = \min\{1, \frac{(2b-u)(1+k_1)}{3b}\}$. When $\beta \geq \beta^U$, $\frac{\partial \pi_t^U}{\partial t_1} < 0$ always

806   holds, so $t_1^{U*} = 0$, and when $\beta < \beta^U$, $\frac{\partial^2 \pi_t^U}{\partial t_1^2} = \frac{1}{1-t_1} \cdot \left\{ \frac{NV}{2(1-t_1)^2} - \frac{V^2}{2} \cdot \frac{1+2t_1}{(1-t)^3} \right\}$; since

807      $\frac{NV}{4(1-t_1)^2} = \frac{t_1 V^2}{2(1-t_1)^3}$ holds at the zero point of t_1, $\frac{\partial^2 \pi_t^U}{\partial t_1^2} < 0$ holds in the neighborhood of zero point,

808 and thus the participation rate of the third party is given by $t_1^{U*} = \frac{(2+2k_1-3\beta)b - u(1+k_1)}{(2+2k_1-\beta)b + u(1+k_1)} \in (0,1)$.

809   Substituting t_1^{U*} into $\widetilde{a^U}$, one has $\sqrt{a_t^{U*}} = \frac{(2+2k_1-\beta)b + (1+k_1)u}{4}$.

810 **Appendix C. The proof of Proposition 3**

811   Taking the first and second derivatives of π_r^B with respect to a yields

36

812 $$\frac{\partial \pi_r^B}{\partial a} = \frac{\beta b + u(1+k_1)}{2\sqrt{a}} - (1-t_1) , \text{ and } \frac{\partial^2 \pi_r^B}{\partial a^2} = -\frac{\beta b + u(1+k_1)}{4} a^{-\frac{3}{2}} < 0 .$$

813 Similarly, the derivatives of π_r^B with respect to A are given by

814 $$\frac{\partial \pi_r^B}{\partial A} = \frac{k_2 \beta b + u(1+k_2)}{2\sqrt{A}} - t_2 , \text{ and } \frac{\partial^2 \pi_r^B}{\partial A^2} = -\frac{k_2 \beta b + u(1+k_2)}{4} A^{-\frac{3}{2}} < 0 .$$

815 From the first-order conditions, the optimal collection efforts of T and R satisfy

816 $$\sqrt{\widetilde{a^B}} = \frac{\beta b + u(1+k_1)}{2(1-t_1)} \text{ and } \sqrt{\widetilde{A^B}} = \frac{k_2 \beta b + u(1+k_2)}{2t_2} , \text{ respectively.}$$

817 Substituting $\widetilde{a^B}$ and $\widetilde{A^B}$ into the profit function of T, $\widetilde{\pi_t^B}$ is achieved, and taking the derivative of

818 $\widetilde{\pi_t^B}$ with respect to t_1, one has $\frac{\partial \pi_t^B}{\partial t_1} = \frac{[(2b-u)(1+k_1)-3\beta b]X}{4(1-t_1)^2} - \frac{t_1 X^2}{2(1-t_1)^3}$, where

819 $X = \beta b + u(1+k_1)$, $Y = k_2 \beta b + u(1+k_2)$, and $\frac{\partial^2 \pi_t^B}{\partial t_1^2} = \frac{2}{1-t_1} \times \frac{\partial \pi_t^B}{\partial t_1} - \frac{X^2}{2(1-t_1)^3} - \frac{t_1 X^2}{2(1-t_1)^4}$.

820 When $\beta \geq \beta^U$, $\frac{\partial \pi_t^B}{\partial t_1} < 0$, so $t_1^{B*} = 0$, and when $\beta < \beta^U$, the function π_t^B has zero points, and in

821 the neighborhood of the zero points, $\frac{\partial^2 \pi_t^B}{\partial t_1^2} < 0$ holds, so $t_1^{B*} = \frac{(2b-u)(1+k_1)-3\beta b}{(2b+u)(1+k_1)-\beta b}$.

822 Similarly, one has $\frac{\partial \pi_t^B}{\partial t_2} = \frac{Y}{2t_2^2} \left\{ \frac{3k_2 \beta b - 2b - 2bk_2 + u(1+k_2)}{2} + \frac{(1-t_2)Y}{t_2} \right\}$, and

823 $\frac{\partial^2 \pi_t^B}{\partial t_2^2} = -\frac{2}{t_2} \times \frac{\partial \pi_t^B}{\partial t_2} - \frac{Y^2}{2t_2^4}$. From the formulation of the first order, it follows that the function π_t^B always

824 has zero points of t_2 for any $\beta \leq 1$, and in the neighborhood of the zero points, $\frac{\partial^2 \pi_t^B}{\partial t_2^2} < 0$ holds, so

825 $t_2^{B*} = \frac{2k_2 \beta b + 2u(1+k_2)}{(2b+u)(1+k_2)-k_2 \beta b}$. When $\beta < \beta^B$, $t_2^{B*} \in (0,1)$ holds; otherwise, $t_2^{B*} = 1$.

826 Substituting t_1^{B*} and t_2^{B*} into the functions for $\widetilde{a^B}$ and $\widetilde{A^B}$, the optimal solutions of a^{B*} and

827 A^{B*} are achieved.

828 **Appendix D. The proof of Proposition 4**

829   Since $b_r^* = \beta^{P*}b$, from the first-order conditions, one has $\sqrt{A^{P*}} = \dfrac{b(1+k_2) - k_2\beta^{P*}b}{2}$ and

830 $\sqrt{a^{P*}} = \dfrac{\beta^{P*}b + u(1+k_1)}{2}$. Let $\beta = \beta^{P*}$ in the U-Model. Since $\beta^{P*} < \beta^U$, it follows that $A^{P*} = A^{U*}$

831 and $\sqrt{a^{U*}} - \sqrt{a^{P*}} = \dfrac{(2b-u)(1+k_1) - 3\beta^{P*}b}{4} > 0$. Consequently, the collection volumes of the direct and

832 indirect channels are as follows: $q_r^{P*} < q_r^{U*}$ and $q_t^{P*} < q_t^{U*}$.

833   According to the decision-making process, T will set $t_1^* = 0$ if $\pi_t^{U*} \le \pi_t^{P*}$, and then the U-Model is

834 transformed into the P-Model; if $t_1^* > 0$, $\pi_t^{U*} > \pi_t^{P*}$ must hold. Since $\beta^{P*} < \beta^U$, $t_1^* > 0$ holds,

835 $\pi_t^{U*}\big|_{\beta=\beta^{P*}} > \pi_t^{P*}$ holds.

836   The optimal profit of R in the P-Model is given by

837 $$\pi_r^{P*} = \frac{[\beta^{P*}b + u(1+k_1)]}{2}\sqrt{a^{P*}} + [\beta^{P*}k_2 b + u(1+k_2)]\sqrt{A^{P*}} \tag{D1}$$

838   Since $\beta^{P*} < \beta^U$, the optimal profit of R in the U-Model is given by

839 $$\pi_r^{U*}\big|_{\beta=\beta^{P*}} = \frac{\beta^{P*}b + u(1+k_1)}{2}\sqrt{a^{U*}} + [\beta^{P*}k_2 b + (1+k_2)u]\sqrt{A^{U*}} \tag{D2}$$

840   Since $A^{P*} = A^{U*}$ and $a^{P*} < a^{U*}$, one has $\pi_r^{U*}\big|_{\beta=\beta^{P*}} > \pi_r^{P*}$.

841 **Appendix E. The proof of Corollary 5**

842 $$\frac{\partial \pi_t^{U*}}{\partial \beta} = \frac{b^2}{8}[(4k_2^2+1)\beta - 4k_2(1+k_2) - 2(1+k_1)] - \frac{bu(1+k_1)}{2},$$

843 $\dfrac{\partial \pi_r^{U*}}{\partial \beta} = \dfrac{(1+k_1+2k_2+2k_2^2 - \beta - 4k_2^2\beta)b^2 - 2k_2 bu(1+k_2)}{4}$ and $\dfrac{\partial^2 \pi_r^{U*}}{\partial \beta^2} = \dfrac{-(1+4k_2^2)b^2}{4} < 0$. Since

844 $\beta < \beta^U$ and $\dfrac{\partial \pi_t^{U*}}{\partial \beta}\Big|_{\beta=\beta^U} < 0$, $\dfrac{\partial \pi_t^{U*}}{\partial \beta} < 0$ holds. Let $\beta^{UL} = \dfrac{(1+k_1+2k_2+2k_2^2)b - 2k_2 u(1+k_2)}{(1+4k_2^2)b}$. If

845 $\beta^U \le \beta^{UL}$, $\dfrac{\partial \pi_r^{U*}}{\partial \beta} > 0$ holds when $\beta \le \beta^U$, and if $\beta^U > \beta^{UL}$, then $\dfrac{\partial \pi_r^{U*}}{\partial \beta} > 0$ when $0 \le \beta \le \beta^{UL}$,

846 $\dfrac{\partial \pi_r^{U*}}{\partial \beta} < 0$ when $\beta^{UL} < \beta \le \beta^U$ and $\dfrac{\partial \pi_r^{U*}}{\partial \beta} = 0$ when $\beta = \beta^{UL}$.

847 **Appendix F. The proof of Corollary 6**

848   According to Proposition 4 and Corollary 5, one has $\left.\pi_t^{U*}\right|_{\beta=\beta^{P*}} > \pi_t^{P*}$ and $\dfrac{\partial \pi_t^{U*}}{\partial \beta} < 0$, and thus there is a

849   threshold β_t^U ($\beta^{P*} < \beta_t^U \leq \beta^U$) that makes $\left.\pi_t^{U*}\right|_{\beta=\beta_t^U} = \pi_t^{P*}$ when $\beta_t^U < \beta^U$, or $\left.\pi_t^{U*}\right|_{\beta=\beta_t^U} > \pi_t^{P*}$ when

850     $\beta_t^U = \beta^U$. Hence, $\pi_t^{U*}(\beta) \geq \pi_t^{P*}$ when $\beta \leq \beta_t^U$ ($\beta_t^U \in (\beta^{P*}, \beta^U]$).

851   In a similar manner, it can be proved that when $\beta \geq \beta_r^U$ ($\beta_r^U \in [0, \beta^{P*}]$), $\pi_r^U(\beta) \geq \pi_r^{P*}$ holds.

852   Hence, when $\beta \in (\beta_r^U, \beta_t^U)$, the U-Model is a Pareto improvement of the P-Model.

853   **Appendix G. The proof of Proposition 5.**

854   Let $\beta = \beta^{P*}$ in the B-Model. Since $\beta^{P*} < \beta^U \leq \beta^B$, $\sqrt{a^{B*}} = \dfrac{(2b+u)(1+k_1) - \beta^{P*}b}{4}$ and

855     $\sqrt{A^{B*}} = \dfrac{(2b+u)(1+k_2) - k_2\beta^{P*}b}{4}$, one has $A^{P*} < A^{B*}$ and $a^{P*} < a^{B*}$. Consequently, the collection

856   volumes of the direct and indirect channels are related as follows: $q_r^{P*} < q_r^{B*}$ and $q_t^{P*} < q_t^{B*}$.

857   It is obvious that $\left.\pi_t^{B*}\right|_{\beta=\beta^{P*}} > \pi_t^{P*}$; otherwise, T would set $t_1^* = 0$ and $t_2^* = 0$, which is just the case of

858   the P-Model.

859   Since $\pi_r^{B*} = \dfrac{\beta b + u(1+k_1)}{2}\sqrt{a^{B*}} + \dfrac{\beta k_2 b + u(1+k_2)}{2}\sqrt{A^{B*}}$, combined with Eq. (D1) of π_r^{P*}, it is

860   easy to prove that $\pi_r^{B*} - \pi_r^P = \dfrac{M}{8}$, where

861     $M = (2b-4u)\beta^{P*}b[1 + k_1 - k_2(1+k_2)] + (2b-u)u[(1+k_1)^2 - (1+k_2)^2] - 3(1-k_2^2)(\beta^{P*}b)^2$

862   Thus $\left.\pi_r^{B*}\right|_{\beta=\beta^{P*}} \geq \pi_r^{P*}$ when $M \geq 0$, and $\left.\pi_r^{B*}\right|_{\beta=\beta^{P*}} < \pi_r^{P*}$ when $M < 0$.

863   When $\beta = \beta^{P*}$, the profits of the collection system in the B-Model and the P-Model are given by

864     $\pi_s^{B*} = \dfrac{(2b+3u)(1+k_1) + \beta^{P*}b}{4}\sqrt{a^{B*}} + \dfrac{(2b+3u)(1+k_2) + \beta^{P*}k_2 b}{4}\sqrt{A^{B*}}$ and

865     $\pi_s^{P*} = \dfrac{(2b+u)(1+k_1) - \beta^{P*}b}{2}\sqrt{a^{P*}} + \dfrac{(b+2u)(1+k_2) + \beta^{P*}k_2 b}{2}\sqrt{A^{P*}}$.

866   Since $\sqrt{A^{P*}} = \dfrac{b(1+k_2) - k_2\beta^{P*}b}{2}$ and $\sqrt{a^{P*}} = \dfrac{\beta^{P*}b + u(1+k_1)}{2}$, it is easy to see that

867     $\pi_s^{B*} - \pi_s^{P*} = \dfrac{(2b-u)(1+k_1) - 3\beta^{P*}b}{4}\sqrt{a^{B*}} + \dfrac{(2b+3u)(1+k_2) + \beta^{P*}k_2 b}{4}\left(\sqrt{A^{B*}} - \sqrt{A^{P*}}\right)$

    $- \dfrac{u(1+k_2) + k_2\beta^{P*}b}{4}\sqrt{A^{P*}}$.   Because

868    $\beta^{P*} < \beta^{U}$ and $\sqrt{A^{B*}} - \sqrt{A^{P*}} = \dfrac{u(1+k_2)+k_2\beta b}{4}$, it is easy to prove that $\pi_s^{B*} - \pi_s^{P*} > 0$.

869   **Appendix H. The proof of Corollary 7**

870   Taking derivatives with respect to β yields

871    $\dfrac{\partial \pi_t^{B*}}{\partial \beta} = \dfrac{-b}{8}\{(2b+u)(1+k_1+k_2+k_2^2) - \beta b(1+k_2^2)\} < 0$, and

872    $\dfrac{\partial \pi_r^{B}}{\partial \beta} = \dfrac{b}{2}\sqrt{a^{B*}} + \dfrac{\beta b + u(1+k_1)}{2} \times \dfrac{\partial \sqrt{a^{B*}}}{\partial \beta} + \dfrac{k_2 b}{2}\sqrt{A^{B*}} + \dfrac{\beta k_2 b + u(1+k_2)}{2} \times \dfrac{\partial \sqrt{A^{B*}}}{\partial \beta}$.

873   Since $\dfrac{\partial \sqrt{a^{B*}}}{\partial \beta} = \dfrac{-b}{4}$ and $\dfrac{\partial \sqrt{A^{B*}}}{\partial \beta} = \dfrac{-k_2 b}{4}$, $\dfrac{\partial \pi_r^{B}}{\partial \beta} = \dfrac{b^2}{4}\{1 + k_1 - \beta + k_2 + k_2^2(1-\beta)\} > 0$,   and

874    $\dfrac{\partial \pi_t^{B*}}{\partial \beta} + \dfrac{\partial \pi_r^{B*}}{\partial \beta} < 0$.

875   **Appendix I. The proof of Proposition 6**

876   From Propositions 4-5, one easily has $A^{U*} < A^{B*}$ and $a^{U*} = a^{B*}$ under the same profit sharing β

877   ($\beta < \beta^U$). Consequently, the collection volumes are related as follows: $q_r^{U*} < q_r^{B*}$ and $q_t^{U*} < q_t^{B*}$.

878   When $\beta < \beta^U$, it is easy to obtain $\pi_t^{U*} = A^{U*} + a^{U*}$, where $\sqrt{A^{U*}} = \dfrac{b(1+k_2) - k_2\beta b}{2}$ and

879    $\sqrt{a^{U*}} = \dfrac{(1+k_1)(2b+u) - \beta b}{4}$,   and $\pi_t^{B*} = A^{B*} + a^{B*}$,   where $\sqrt{a^{B*}} = \sqrt{a^{U*}}$ and

880    $\sqrt{A^{B*}} = \dfrac{(2b+u)(1+k_2) - k_2\beta b}{4}$. Since $A^{U*} < A^{B*}$, $\pi_t^{B*} - \pi_t^{U*} > 0$.

881   For a given proportion of profit sharing, it is easy to obtain

882    $\pi_r^{B*} - \pi_r^{U*} = \dfrac{\beta k_2 b + u(1+k_2)}{2}(\sqrt{A^{B*}} - 2\sqrt{A^{U*}})$ and $\sqrt{A^{B*}} - 2\sqrt{A^{U*}} = -\dfrac{(2b-u)(1+k_2) - 3k_2\beta b}{4}$.

883   When $\beta < \beta^B$, $\sqrt{A^{B*}} - 2\sqrt{A^{U*}} < 0$, and given $\beta^U \leq \beta^B$, when $\beta < \beta^U$, $\pi_r^{B*} < \pi_r^{U*}$ holds.

884   Similarly, $\pi_s^{B*} - \pi_s^{U*} = A^{B*} - A^{U*} + \dfrac{u(1+k_2) + \beta k_2 b}{2}(\sqrt{A^{B*}} - 2\sqrt{A^{U*}})$.

885   Since $A^{B*} - A^{U*} = \dfrac{(4b+u)(1+k_2) - 3k_2\beta b}{4} \times \dfrac{u(1+k_2) + k_2\beta b}{4}$,

886    $\pi_s^{B*} - \pi_s^{U*} = \dfrac{u(1+k_2) + k_2\beta b}{4} \times \dfrac{3u(1+k_2) + 3k_2\beta b}{4} > 0$.

887

Highlights

- Collaborative collection effort strategies with/without cost sharing are studied.
- Optimal decisions and member's profits in various strategies are compared.
- Cost sharing strategies can achieve win-win results in a certain profit sharing range.
- Collaborative marginal profit has a greater impact on cost sharing strategies.