Optimized design method for profile extrusion die based on NURBS modeling

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Abstract: For optimization design of polymer extrusion dies, dimensional accuracy is critical to product quality of the extrudate. The extrusion dies used to be a regular geometrical profile, which is mostly composed by a straight line. Traditional optimization methods for extrusion die design used to have poor controllability when dealing with a curved profile.

In this paper, the response surface optimization method is used to find out an optimal solution of the design of the extrusion die. Firstly, the Latin Hypercube Sampling method is used to generate the experiment samples for the design of experiments. Secondly, ANSYS Polyflow software is adopted to execute the computational fluid dynamics analysis. Thirdly, the Kriging method is used to generate the response surface. Finally, nonlinear programming by using Quadratic-Lagrangian algorithm is applied to find out the optimal solution. It is worth noting that Non-uniform Rational B-Splines (NURBS) modeling is used to optimize flow channel of an extrusion die in order to obtain a qualified extrudate. Thus, design variables for the optimization involve control points of the NURBS curve of the inlet cross-section. Meanwhile, two new objective functions, including minimization of point displacement and minimization of dimensional tolerance are proposed in the optimization process. Compared with existing objective functions of flow balancing and homogeneous die swell, the new objective functions of minimization of point displacement and minimization of dimensional tolerance have significant advantages of strong adaptability, more precise shape of the extrudate and fast convergence, which significantly improve efficiency of the optimization design and thus lower manufacturing costs of the extrusion die.

Keywords: Polymer Extrusion Die; Optimized Design Method; Dimensional Tolerance; NURBS; Response Surface Methodology.

1 Introduction

Polymer extrusion process has been widely used in industry, which is suitable for manufacturing products such as automobile weather-strip, tire components, and hollow profile. A well-designed extrusion die is critical for manufacturing high-quality extrudates, but very hard to achieve at the beginning. For design of extrusion dies, flow uniformity and extrudate swell at the die exit are two significant obstacles, especially for profiles with vast differences in wall thickness

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[1]. Therefore, prior goals for design of extrusion dies are to improve the flow uniformity at the die exit and eliminate the impact of extrusion swell by modifying the design of extrusion die.

In conventional designs, polymer extrusion dies design relies heavily on experiences of designers and the time and money consuming trial-and-error method [2]. Nowadays, the trial-and-error process can be replaced by computer optimization design based on computational fluid dynamics (CFD) tools to some extent, and reduce the number of mold rework to a minimum. With development of CFD codes, numerical simulation has been widely used for the design of profile extrusion die, such as ANSYS Polyflow, Dieflow, HyperXtrude, FLOW 2000 and so on [3]. Correspondingly, success of CFD software development in the extrusion simulation has significantly led towards potentially cost reduction and time savings.

In the past, a number of design methods for polymer profile extrusion dies have been reported [4], which are primarily classified in two kinds of approaches for the extrusion die design [5] [6].

The first approach aims to balance the flow at die exit by using optimization techniques. Rajkumar *et al.* [5] adopted a novel method to balance the flow distribution in the extrusion die, and the cross-section geometry was divided into L and T shaped profiles. Yilmaz *et al.* [7] proposed objective functions based on uniformity of velocity distribution at the die exit and adopted the Simulated Annealing-Kriging Meta-Algorithm for the optimization of extrusion die geometry. Sienz *et al.* [8] presented a procedure for design sensitivity analysis and used in the design of the extrusion process with satisfactory results. Mu et al. [9] studied the influence of geometric structure on flow balance, which benefited design of the complex hollow plastic profile extrusion.

The second approach is inverse design of extrusion dies which aims to get an expected profile of the extrudate. In other words, the inverse design starts from required extrudate's profile and eventually find out the best solution of the extrusion dies. For inverse problems, there are multiple solutions under different limiting factors. Pauli *et al* [10] proposed the homogeneous die swell as objective function. Siegbert *et al* [11] demonstrated an optimization design method that profile shape was used as an objective function. Gifford [12] demonstrated a method of compensation for die swell in the design of profile extrusion die.

Many methods have been used to improve efficiency of the optimization procedure [13]. The response surface method has been used by Pauline *et al.* [14], which benefits the optimization of the extrusion die design and process parameter determination. Zhang *et al.* [15] achieved a uniform velocity distribution by using the response surface method to aid the optimization design of a feeder extrusion.

Non-Uniform Rational B-Splines (NURBS) also has been widely used in CAD modeling optimization. The NURBS curve has properties with geometric invariance, convex hull property, symmetry, and locality so on. For any geometric optimization problem, the structure of extrusion dies can be local fine-tuning without affecting other parts.

In this work, some new objective functions have been used to find out design solution of an extrusion die. Firstly, governing equations used in the simulation are briefly introduced, and the finite element based on ANSYS Workbench is illustrated in details, including geometry modeling, meshing, the flow analysis in ANSYS Polyflow, and results are shown in CFD-Post. With several novel optimization approaches, the geometry optimization of the flow channel is used to achieve the design objectives. Finally, the advantage and disadvantage of the optimization approach are compared and discussed with other existing approaches.

2 Simulation Approach Based on CFD

2.1 Governing Equations

In this paper, the generalized Newtonian incompressible isothermal viscous flow is used for the simulation. Density of the fluid is assumed to be constant from the inlet to the outlet. The continuity equation and steady-state Navier-Stokes equations are shown in equation (1) and (2). The gravity and inertial forces are neglected compared to the viscous forces.

$$\nabla \cdot v = 0 \tag{1}$$

$$-\nabla p + \nabla \cdot \left(\mu(\nabla v + (\nabla v)^T)\right) = 0 \tag{2}$$

where ρ is fluid density, v is fluid velocity, p is pressure, μ is fluid dynamic viscosity.

High-density polyethylene (HDPE) is used in the CFD simulations, which has been used before [7]. The Bird-Carreau viscous model has been chosen for the shear rate dependence of viscosity in Polyflow (see equation 3).

$$\eta(\dot{\gamma}) = \eta_0[1 + (\lambda\dot{\gamma})^2]^{(1-n)/2} \tag{3}$$

where $\eta_0 = 8920\ Pa \cdot s$ is zero-shear-viscosity, $\lambda = 1.58\ \mathrm{s}$ is relaxation time, $\mathrm{n} = 0.496$ is the power-law index, $\dot{\gamma}$ is the shear rate [7] [16] [17]. The Picard iterations for viscosity have been activated as the power law index is lower below 0.75.

2.2 Finite Element Model

An extrusion die typically consists of an extruder mounting plate, die adapter plate, transition plate, pre-land plate, and die land plate. For the sake of simplicity, only the pre-land and the die land plate are considered, and the reservoirs before the pre-land plate are ignored in this simulation [18].

The extrudate studied is showed in **Fig. 1**, which has three sub-sections $(ES_i, i = 1, 2, 3)$ and 9 points indicated by black dots in the figure as the design objective points. The extrudate has two parts with different thickness, the thicker one (t_1) is 5 mm thick and the smaller one (t_2) is 3.5 mm thick. In the simulation, the pre-land section is modified to achieve a flow balance at the die exit or homogeneous extrudate swell, and thus both die and free surface domains are included in the computational domain. The definition of curves is as follows, L_1 (P_1P_2), L_2 (P_2P_3), L_3 (P_3P_4), L_4 (P_4P_5), L_5 (P_5P6), L_7 (P_7P_8), and L8 (P_8P_1).

The finite element model has Die Component and Extrudate Component, which are separated by the Die Lip, as shown in Fig. 2. The lengths of Die and Extrudate are 100 mm and 50 mm respectively. Length of the die land (L) and thickness (t) of the Die Inlet need to be changed to achieve a more balancing flow at the Die Lip.

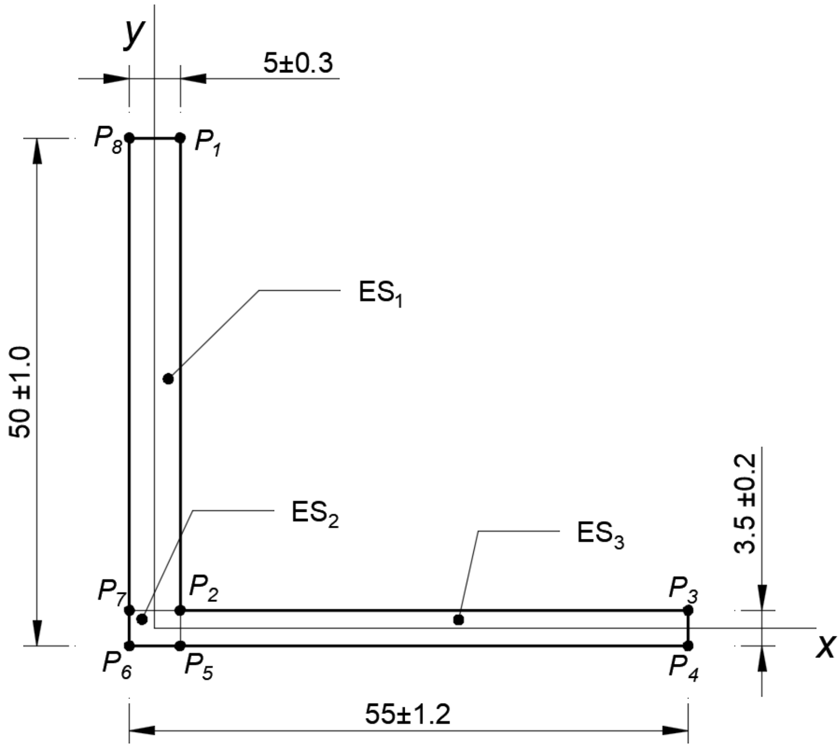


Fig. 1: Cross-section shape and dimensions of the extrudate (unit: mm)

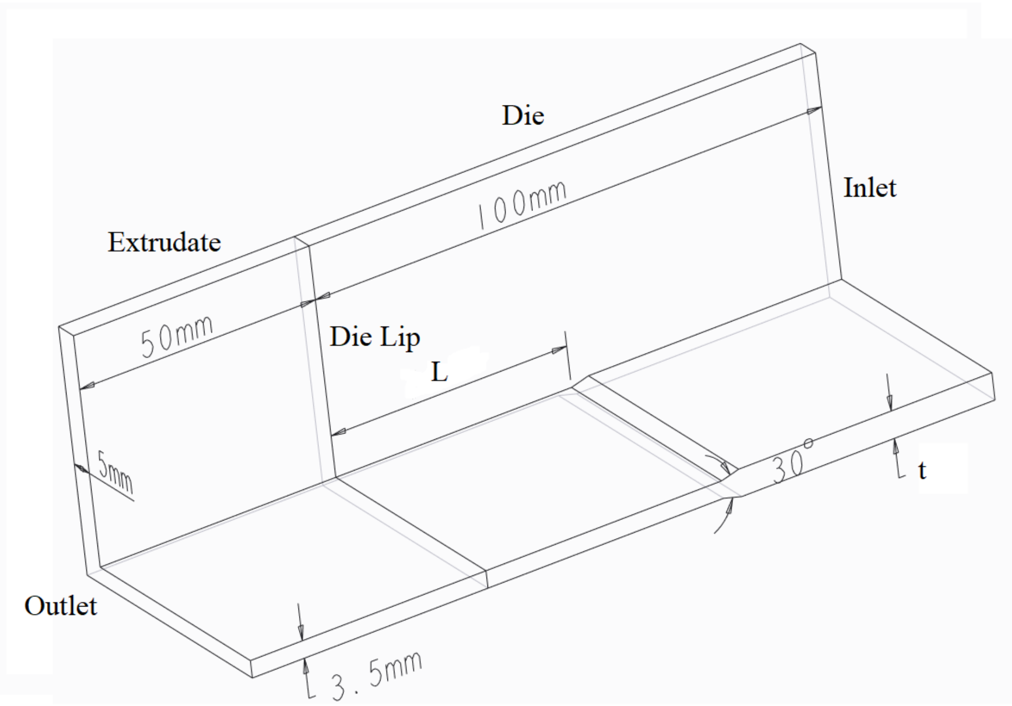


Fig. 2: Finite element model (Die and Extrudate)

The fluid flow analysis system is composed of various components that represent the workflow for performing the extrusion simulation. There are four main components used in the simulation as shown in **Fig. 3**, including Mesh Component (A), Polyflow Component (B), Results Component (C) and Response Surface Optimization Component (D).

The Polyflow Component (B) can obtain mesh data from the Mesh Component (A), and transfer results data to the Results Component (C) after the finite element analysis. Simultaneously, the input parameters used in the optimization can be obtained from the Parameter Set, and the output parameters can be exported into the Parameter Set.

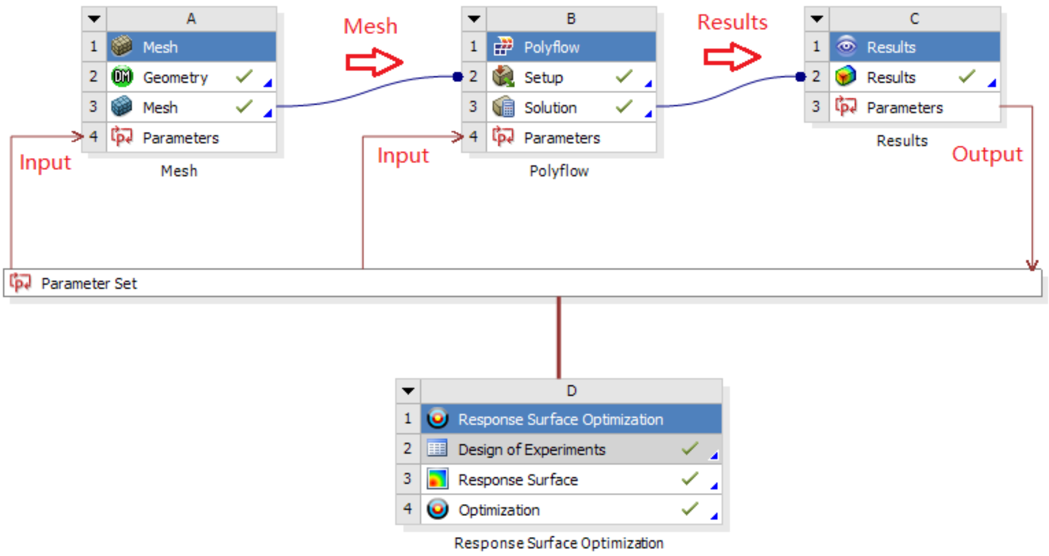


Fig. 3: Fluid Flow (Polyflow) Analysis System in ANSYS Workbench

The Mesh Component (A) has Geometry Cell and Mesh Cell (see **Fig. 3**). The CAD model is created in Geometry Cell using ANSYS Design-Modeler, which has the ability of parametric geometric modelling and NURBS modelling. The CAD parameters are firstly inputted from the Parameter Set for parametric geometric modeling. The geometric model subsequently transfers to the Mesh Cell for parametric meshing by using ANSYS Meshing. According to the results of mesh convergence study, corner region of the cross-section in the extrusion direction and the intersection region between different die plates have been refined to improve the mesh quality, as shown in **Fig. 2**. It is noticed that the mesh can be parametrically refined to accommodate the changes in geometry, and thus accuracy and quality of the mesh in the optimization process can be ensured. The mesh type generated by the sweep method is 8-node hexahedron and the total number of initial element and node is 86304 and 98982, respectively.

The Polyflow Component (B) executed the CFD simulation with the mesh from Mesh Component (A) and the parameters such as material parameters and boundary conditions from the Parameter Set, and then output the results to the Results Component (C). The boundary conditions defined in Polydata are shown as follows.

(1) Inflow boundary condition: The Inlet flow is fully developed flow, and the volumetric flow rate is specified as 120000 mm^3/s.

(2) No-slip boundary condition, where normal and tangential velocity of the fluid is specified as zero.

(3) Free surface boundary condition,

(4) Outflow boundary condition, where normal velocity and zero-tangential forces are imposed. The constant normal velocity has been applied at the outlet, which value is -20 m/min.

The Results Component (C) use the CFD-Post software for the post-processing and then output the results to the Parameter Set, which can be used to calculate objective functions of the optimal design and determine whether the optimization process converges.

3 Optimization Methods

The purpose of optimization and design exploration for polymer extrusion die design is evaluating multiple designs and optimizing geometric parameters of the die, finally find an optimal solution which can output an expected geometry of the extrudate. In other words, the objective function of the optimization is the geometric parameters of the extrudate, and design variables are the geometric parameters of the extrusion die. The bound for objective function is the geometric limited which composed a manufacturable extrusion die. A typical flowchart of steps involved in optimization design is shown in **Fig. 4**.

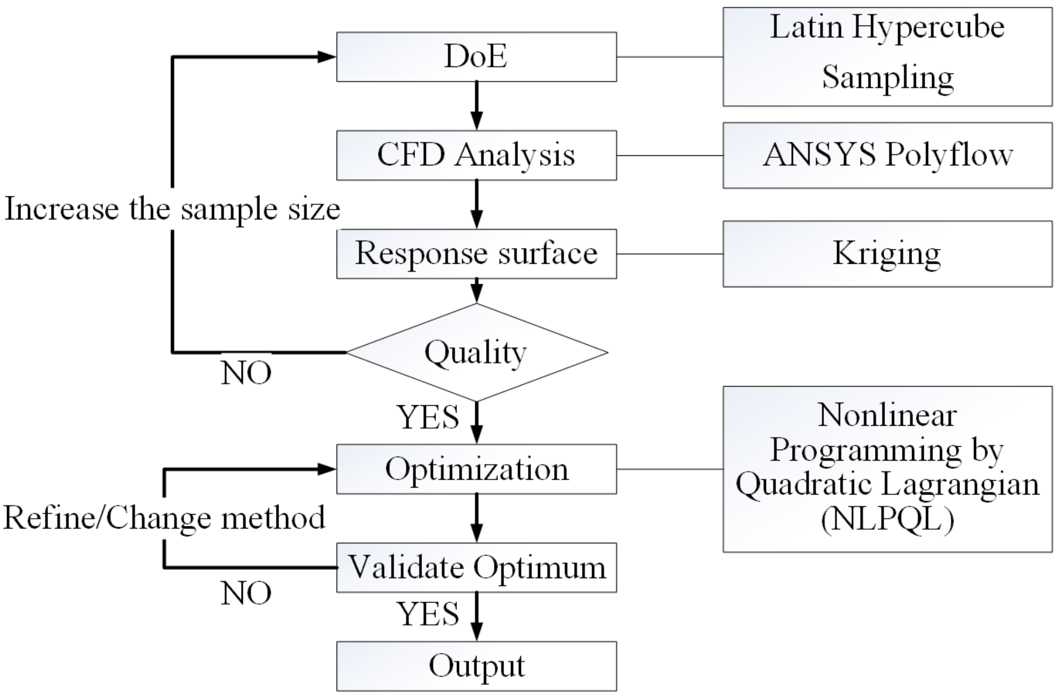


Fig. 4: the Process of Response Surface Optimization

The response surface methodology (RSM) is a method developed to combine mathematics and statistics, which is very suitable for optimal experimental design. Primarily, it can be used to find the relationships between several explanatory variables and one or more response variables. For an RSM problem, a general assumption is that the problem is the best restrictive optimization problem. In general, the Response Surface Optimization showed in **Fig. 4** can be roughly divided into three stages:

The first step is the design of experiments (DoE) which acquisition of experimental samples by Latin Hypercube Sampling (LHS). The LHS is employed to generate samples which used in the following response surface design and optimization with the corresponding objective function. According to Latin Hypercube Sampling (LHS) design of experiments, 100 experiments are designed and corresponding Polyflow models are built for the simulation of the polymer extrusion process.

The second step is response surface design. The Min-Max values of the objective functions can be achieved by using genetic aggregation method. The Kriging method is used to construct the response surface. By using the Sensitivity Analysis Method (SAM), the sensitive dimensions that have an essential influence on the extrusion process from the uncertain dimensions and analyzing.

The final step is response surface optimization. In this stage, the Nonlinear Programming by Quadratic Lagrangian (NLPQL) option which based on Gradient optimization algorithm can be used. Based on previous DoE and Response Surface Methodology (RSM), the relationship between objective function and design variable could be found, and the optimum solution can be chosen from the design spaces.

3.1 Objective Functions

The principle of extrusion die design mainly includes: balanced velocity distribution at the die exit, minimizing pressure drop and homogeneous die swell [19] [20]. It is noticed that geometric accuracy of the extrudate is a critical factor which heavily influences the quality of the products. In this paper, several fundamental objective functions have been compared for the optimization design of extrusion dies, including the flow balance coefficient, the die swell coefficient, the square Euclidean distance of corresponding corner point, and the dimensional tolerances index number.

Recently researches showed that flow distribution should be as balanced as possible [21], and thus the first objective function is the flow balance coefficient at the die exit, which bases on the deviation of the velocity ratio with respect to the average value.

$$J_{balance} = \frac{\int_{\partial\Omega}\left(\frac{v \cdot n}{\bar{v}} - 1\right)^2 ds}{S} \tag{4}$$

where v is fluid velocity, $\bar{v}$ is average velocity along the die exit, and n is the outward normal along the die exit, S is area of the die lip. Compared with the quantification of die balancing in ANSYS Polyflow software, the impact of the velocity term has been eliminated in the Equation 4. A uniform velocity distribution at the die exit will lead to that the value of the coefficient is equal to zero. However, larger the extrusion productivity is, more unbalanced the flow distribution will be. In order to increase the productivity, value of flow balance coefficient should be as small as possible. Traditionally, increase the flow supply in thinner regions, and decrease the flow supply in thicker regions, and thus make the different regions to have the same average velocity.

The second objective function is the homogeneous extrudate swell. The die swell ratio (B) is equal to the area of the extrudate (S_{ex}) divided by the area of the die lip (S_0). Ideally, the expected value of total die swell ratio is close to one. Otherwise, it should be controlled by adjusting the haul-off speed. In the meantime, the die swell ratio in different subareas ES_i also should be close to one. The equation is defined as follows:

$$J_{swell} = \sum_{i=1}^{3} \left(\frac{B_i}{B} - 1\right)^2 \tag{5}$$

where B and B_i is the die swell ratio of total and subareas, respectively. The expected value of B_i/B is one.

The third objective function is the minimization of point displacement, which finds out the sum of the squared Euclidean distance of the corresponding point (shown in **Fig. 1**) between the expected location and the actual location after die swell. Due to the dimensional accuracy should be within the design-allowable range, the objective function $J_{position}$ is proposed for quantifying the swell deformation, as shown in Equation 6.

$$J_{position} = \sum_{i=1}^{8} d^2(q_i - p_i) \tag{6}$$

where q_i is the expected location and q_i is the real location.

More point used in the optimization, more accurate results can be. However, only the corner points are used for the limitation on the scale of the optimization design.

The fourth objective function is the minimization of dimensional tolerance, which bases on the deviation of the actual value of dimension tolerance band concerning the expectation. The maximum and the minimum values of the abscissa and the ordinate of each curve of the profile can be obtained through numerical simulation. Thus, the actual dimension tolerance band is equal to the difference between the maximum and minimum dimensions. The expected value of dimension tolerance band is given by the product design, as shown in **Fig. 1**.

$$\mathrm{J}_{tolerance} = \sum_{i=1}^{4} \left(\frac{\Delta x_i^1}{\Delta x_i^0} - 1\right)^2 \tag{7}$$

where Δx_i^0 and Δx_i^1 are the expectation and actual value of dimension tolerance band, respectively.

3.2 Design Variables

In many cases, extrusion die design is still more an art than a science [22]. For optimization design of an extrusion die, design variables are usually the geometric parameters of the extrusion die and process parameters. Generally, the process parameters are adjustable, including the inlet volumetric flow rate of fluid, extrusion temperature and haul-off speed. Comparatively, geometric parameters of an extrusion die are fixed after manufacturing and more important than process parameters. Here, the design variables at the inlet region which used in the optimization are demonstrated in **Fig. 5**. The geometric parameters are the control point in the two symmetrical NURBS curves P_2P_3 and P_4P_5. The X coordinates of the curves remain unchanged, while the Y coordinate of control point of NURBS curves are used as the design variables. Constraints of the variables are shown in **Table 1**.

Table 1. Design Variables and Constraints of the NURBS curves

Design variables	**C_{21}. (mm)**	**C_{22}. (mm)**	**C_{23}. (mm)**
Unchanged	C_{21}.x=12	C_{22}.x=25	C_{23}.x=38
Constraints	1.8≤C_{21}.y≤3.0	1.8≤C_{22}.y≤3.0	1.8≤C_{23}.y≤3.0

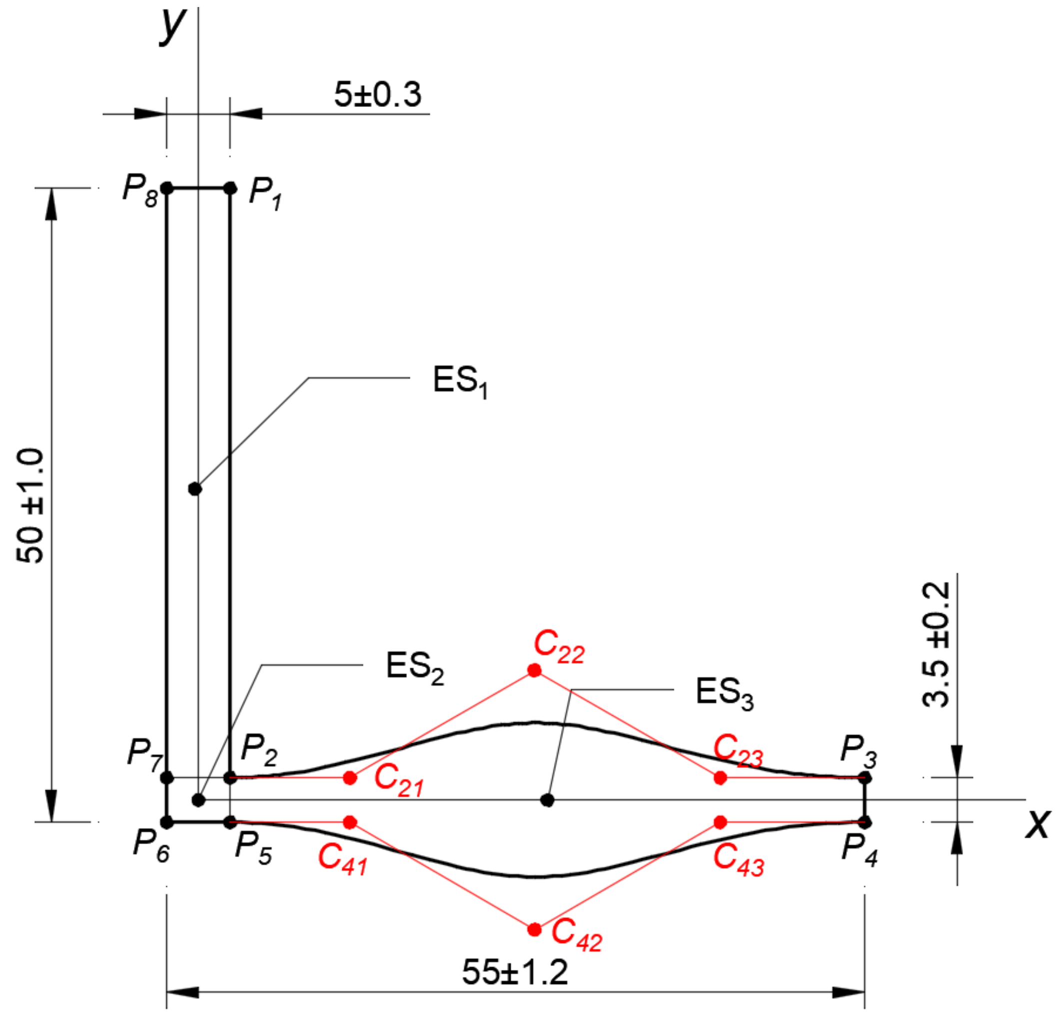


Fig. 5: Design variables at the die inlet region

The geometric parameters including t_l ($5mm \le t_1 \le 5.5mm$) (maximum and minimum) L_1, t_2 (maximum and minimum), L_2. Here, the same bound for the thickness t (3.5 mm $\le t \le 15$ mm) of the ES3 region and the length of die land L (15 mm $\le L \le 30$ mm) is used.

In order to reduce optimization scale, the optimization is split into two stages. Firstly, the optimization is based on the direct lines for the reduction of design spaces. Secondly, the optimization is based on the NURBS curves for more accurate optimization results.

In the meantime, the length of the die land should not be too long that will decrease the effect of flow compensation. The length of the die land is usually not less than ten times the minimum wall thickness of the profile. The appropriate geometric parameters of the die need to be appropriately selected to ensure the manufacturability of the extrusion die. In general, the maximum taper angle that can be machined by wire EDM is ±15° degree [23]. As a result, the length of Land 2 should be carefully chosen to fulfill the limitation of taper angle.

4 Optimum Die Design

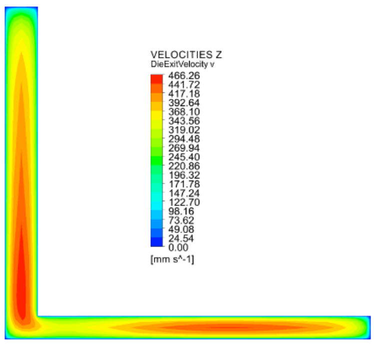
The optimum dies with values of design variables and objective functions are shown in **Table 2**. The optimization has been executed in a Lenovo D30 Workstation.

Table 2: Optimization results by NURBS model compared to Yilmaz's model

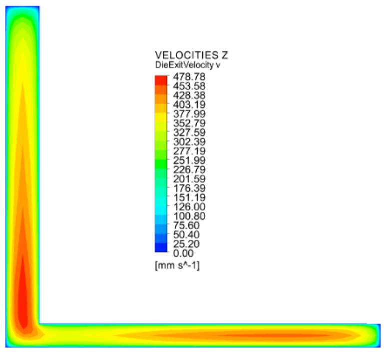
		Yilmaz's model[7]	**NURBS model**			
		Flow Balancing method	Flow Balancing method	Homogeneous Die Swell method	Position control method	Tolerance control method
Design variables	P3.Y (mm)	5.9	1.9483	2.7183	2.6483	5.0517
	C1.Y (mm)	5.9	2.415	3.4417	2.0883	4.795
	C2.Y (mm)	5.9	3.3717	2.0883	2.6017	2.2283
	C3.Y (mm)	5.9	2.9283	3.1617	2.9517	1.9717
	L (mm)	15	16	16	16	16
Objective Functions	$J_{balance}$	0.11056	0.10007	0.1007	0.1026	0.11509
	J_{swell}	0.01519	0.0007	0.00047	0.0071	0.019
	$J_{position}$ (mm^2)	430.29	106.6	157.84	36.907	558.63
	$J_{tolerance}$	2264.9	493.77	809.18	358.28	41.101

Firstly, the results show that the flow balance has been significantly improved by optimizing the geometry compared with the initial geometry. The cross-section of the extrudate has been modified as shown below. From the results in Table. 2, the flow balancing result based on NURBS modeling is better than the common modelling method (Yilmaz [7]). The main reason is that the use of NURBS modeling is conducive to the minimization of cross flow [24] [25]. Secondly, homogeneous die swell method has a little bigger flow balance coefficient value. However, the corner point position and dimension tolerance of this method are better than flow balance method. Meanwhile, this method firstly proposed by L. Pauli *et al.* [10] needs more time to solve free surface problem of die swell. Thirdly, position control method has the lowest corner point position value, which means that all the corner point is close to the expected position. Finally, the tolerance control method has the best dimension tolerance, which will have the best cross-section profile compared with the expectation.

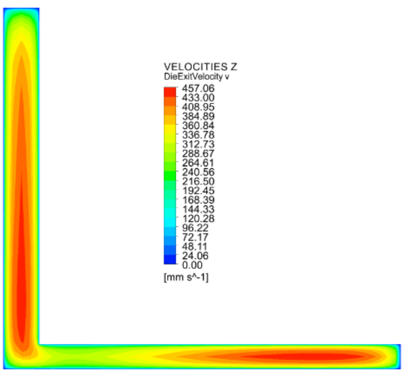
From the results in **Fig. 6**, the velocity distributions at the die exit have a similar trend. The distributions have been improved compared with the origin design. Also, the average velocity in vertical section is more significant than horizontal section except the result shown in **Fig, 6 (d)**. However, the profile of extrudate using the position and tolerance as design targets is more accurate than the other methods. As W. Gifford says "The design of profile dies involves more than just "balancing" the die but also compensating for the effects of die swell" [12]. As a result, the swell deformation cannot be compensated by using the flow balance method.



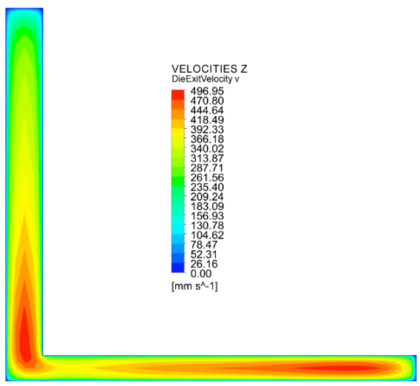
(a): Flow Balancing Method



(b): Homogeneous Die Swell Method

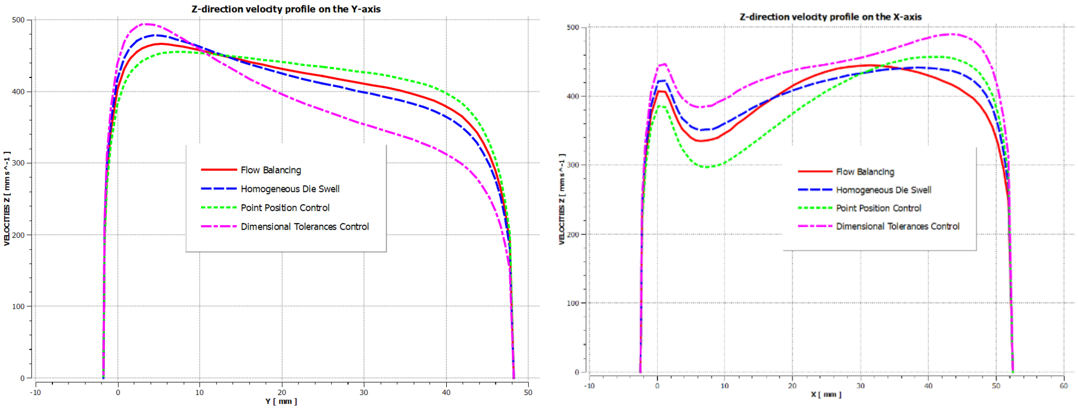


(c): Point Position Control Method



(d); Dimensional Tolerance Control Method

Fig. 6: Z-component of the flow velocity at the die exit with four simulation methods



(a): Vertical section (b) Horizontal section

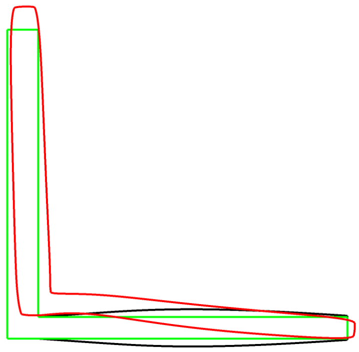
Fig. 7: Comparison of flow velocity distributions at the middle of two sections for four methods

From the results in **Fig. 7**, the average flow velocity is about 400 mm/s in the center line of the vertical section and horizontal section. The maximum velocity in the vertical section is close to the coordinate origin. Also, the maximum velocity appears at the far end of the horizontal arm. While by the flow balance, the maximum velocity appears at the middle of the horizontal arm. The flow balance method obtains a proper flow distribution in two arms, which is shown with the red line [26]. However, the balanced flow in vertical arm and horizontal arm are shown with the green line and blue line, respectively. The worst flow balance appears when using the tolerance method, which is shown with purple lines.

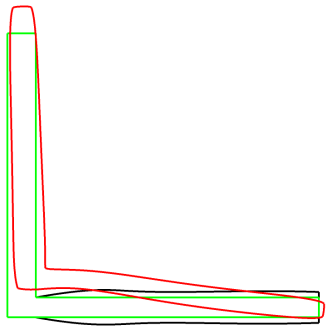
From the simulation results, by using the flow balancing method as shown in **Fig. 8 (a)**, the extrudate still have a certain deformation after die swell. Also, the homogeneous die swell method as shown in **Fig. 8 (b)** has similar results. However, with the point position control method showed in **Fig. 8 (c)**, the extrudate has a slight deformation out of the die lip. The best result is the one shown in **Fig. 8 (d)**, which uses the dimensional tolerance control method. The tolerance method takes all the points on the curve into account, while the point position control method only takes the corner points on the curve into account. Simultaneously, the dimension tolerances of the product which are important attributes of end product have been indicated as weight coefficients of optimization target in equation (7). The dimensional tolerances of the product in the paper are L1=50±1.0 mm, L2= 55±1.2 mm, t1=5±0.3 mm, t2= 3.5±0.2 mm, as shown in Fig.1. From the optimization results, actual value of dimension tolerance band are $\Delta x_1^1 = 1.5\,mm$, $\Delta x_2^1 = 1.6\,mm$, $\Delta x_3^1 = 2.8\,mm$, $\Delta x_4^1 = 2.5\,mm$.

Compared with existing objective function of flow balancing, the new objective functions for minimization of point displacement and minimization of dimensional tolerance have significant advantages of strong adaptability, the more precise shape of the extrudate and fast convergence, which can significantly improve efficiency of the optimization design and thus lower manufacturing costs of the extrusion die. As Szarvasy et al. say "The correct profile has been obtained when the skeleton line of the cross section matches the designed one" [27]. However, compared with the Point

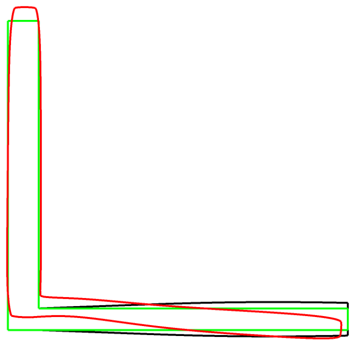
Position Control Method, the Dimensional Tolerance Control Method do not need to define several points for the optimization. In the meantime, the significance of dimension and the size of the tolerance band can be reflected in the optimization process. This method is equivalent to adding weights to the objective functions, which is advantageous for dimensions with high precision requirements.



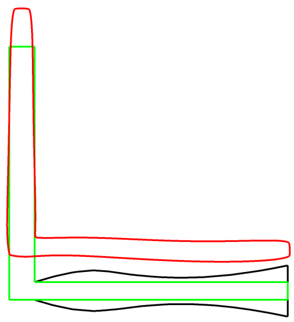
(a): Flow Balancing Method



(b): Homogeneous Die Swell Method



(c): Point Position Control Method



(d): Dimensional Tolerance Control Method

Fig. 8: profile of extrudate compared (expected and actually);
Black line: Flow Inlet; Green line: Die Exit; Red line: Extrudate

5 Conclusions

For this study, some important conclusions are addressed here:

1. The work has demonstrated that the optimized design method for extrusion die design based on NURBS modeling is feasible and effective compared to the existing methods.
2. The flow uniformity of profile dies has been improved by using NURBS modeling. Compared to flow balancing methodology, the optimization method based on Point Position Control and Dimensional Tolerance Control can achieve similar or even better results.
3. This method only needs an ANSYS Polyflow software or other CFD codes with the Response Surface Methodology (RSM).
4. The optimum design can be directly used for manufacturing the extrusion, while the solution

of inverse extrusion using ANSYS Polyflow need to be re-modeling by CAD software.

5. A drawback, however, is only the viscosity model which was considered in the simulation and the viscoelastic properties were not taken into account. In the meantime, not all of the control points have been used as the design variables, due to the limitations of the scale of optimization problems and the computing power.
6. Compared with the inverse method using ANSYS Polyflow, the optimization aimed to decrease the swell distortion is more time-consuming. However, the geometry is under-control by the engineer.

6 Acknowledgments

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