

# Optimization of bus stop locations for on-demand public bus service

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**Abstract:** To serve the planning needs of on-demand public bus service, this paper proposed an optimization model for on-demand public bus stop location. The objective is to minimize the total travel time of all the passengers. According to the unique characteristic of on-demand public bus service, the model constraints include the length of segment, relation of the shed line and the bus stop location, minimum stop spacing, vehicle capacity and nonnegative constraints. Numerical results show that the bus stop location planning for on-demand public bus should be different with the conventional bus stop location planning. The passenger density, travel time perception weight ratio, access speed, and the time loss near and at the stop will affect the on-demand public bus stop spacing.

**Keywords:** bus stop location, on-demand public bus service, minimum travel time, passenger density

## 1. Introduction

With high concern on sustainable transportation development, public transport receives priority in both planning and management in most of the cities around the world. To attract more types and more amounts of public transport users, two innovative public transport services have been launched, that are, shared-vehicle system and on-demand vehicle service. The core concept of the two services is to utilize the public resources efficiently through sharing the fleet with multiple users. Shared-vehicle, like bicycle-sharing and car-sharing, is a self-service public transport system that gives users access to a bicycle or private car for a short and viable period of time. On-demand vehicle service includes on-demand car service (e.g. Uber, Didi) and on-demand public bus service (e.g. customized bus), which allows users could obtain the public transport services according to their personal requests. These emerging urban mobility services present challenges to traffic planners to design the systems that can be well accepted by users.

On-demand public bus service is a type of demand-responsive public transport system, which provides high level of services to specific passenger groups, especially those who have the similar long-distance travel pattern starting from same original area to same destination area within same time period. The planners design the bus stop location and arrange the fleet based on the user demand, which is gathered through online information platforms. The scheduled bus collects the subscribed users from the first certain stops, then drives on bus lane or expressway directly to the destination without stopping in between. The advantages of reliability, rapidness and convenience of on-demand public bus draw lots of attention from the conventional public bus passengers who do not satisfy with the bus delay and the crowdedness. Some on-demand public buses like customized buses are designed to attract more private car users who are suffering the traffic congestion in peak hour. Each user on this kind of bus is ensured to have a cosy seat, Free-WIFI, air-condition, phone charger, entertainment amenities (e.g. TV, radio, newspaper, magazine), drink water, tissue and so on.

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Because of the various merits, on-demand public bus service becomes more and more popular in China recently. For example, as of March 2016, more than 70 cities in China have operated customized buses programmes. Due to the on-demand public bus service's recent emergence, little researches have been carried out about its planning and operation strategy. Bus stop is the place where bus stops there for passengers' boarding and alighting. Bus stop location selection problem is one of the important elements in planning and management of a public transport system, which has been studied for several decades. The suitable bus stop spaces could improve the accessibility of the service, maximum the service coverage, minimum the dwelling time, and then attract more users. The current on-demand public bus stop location selection in China mostly is decided by the planners by utilising the existing conventional bus stop. With the increase of user demand, on-demand public bus stop could be planned more feasible. Therefore, it is envisioned to design a corresponding location determine method according to on-demand public bus service's characteristic.

Three unique operational features of on-demand public bus service need to be considered during the modelling. Take the service line from residential area to central business district (CBD) as example, firstly, the bus stops only located at the two ends of the service, where the first few stops are only for passenger boarding and the rest few stops are only for passenger alighting. Secondly, the demand is large and concentrated within certain period, which is not suitable to dispatch vehicles every certain minute. Thirdly, on-demand public bus service could skip the stops where there is not passenger subsidized there. This paper aims to develop an optimization model to determine on-demand public bus's stop location based on the existing research outcomes for conventional bus stop. The rest of the paper is organized as follows: Section 2 reviews the current literature on solving the bus stop location problem. Section 3 models the optimization framework for on-demand public bus stop location. Section 4 outlines the solution characteristic. Section 5 designs 6 scenarios to discuss the relationship between the bus stop space and various influencing factors, and conclusions are drawn in Section 6.

## **2. Related work**

Spacing and location of stop problem has been growing as lots of attention has been received by public transport system. Vuchic and Newell (1968) systematically summarized the findings from notable publications in this topic between 1913 and 1966. These papers mostly assumed that the population distribution is uniform along the bus service line, and the interstation spacing is constant. To extend this assumption to a more realistic situation, Vuchic and Newell (1968) then proposed a general model to fit any type of population distribution and interstation spacing. The objective function is to minimize the total passenger travel time with considering the impact from the operational factors (e.g. access time, acceleration and deceleration time). Furthermore, after relaxing the restriction on interstation spacing, they obtained some convincing conclusions such as the density of stations depends on the relation of the passengers along the line and those on the vehicle.

Vuchic and Newell's paper caused a stirring of interest in this topic, where various objective functions has been proposed by researchers, such as total cost minimization, user cost minimization, social welfare maximization, private profit maximization, area coverage maximization, ridership maximization, with consideration more practical influence factors, such as network design, bus frequency, route density and bus size, access cost, waiting cost and so on. Mohring (1972) proposed the first microeconomic model, in which the bus schedule and bus stop spacing problems were taken into account simultaneously. Wirasinghe and Ghoneim (1981) initially developed the optimal spacing model with non-uniform many to many travel demand. Ceder et al. (1983) concentrated their efforts on a model aiming to minimize the number of bus stops where the passenger could access the public

bus stop within certain distance. Kuah and Perl (1988) presented an analytic model for a feeder bus network and showed that the optimal route spacing and headway are cubic root functions of the spacing and operating headway. The models proposed in this decade usually assumed the demand follows a continuous or uniform distribution along a route or over an urban area. The continuous model could analyse and investigate the optimal bus spacing and the effect caused by related parameters.

The resurgence of this topic in this century tends to build the model to selection the optimal bus stop location from the limited numbers of potential bus stop locations, which is more realistic in application (Furth and Rahbee, 2000; Furth et al., 2007). More and more researchers tend to consider the discrete demand and system situation, and would like to make use of auxiliary tools (e.g. geographic information system, GIS) to solve the problem. For example, Murray (2003) proposed a hybrid set covering model which was applied in GIS to determine bus stops under the objection of maximize transit demand coverage over a study area in the city of Brisbane, Australia. In the same year, Sanka et al. (2003) considered various parameters and used the multi-criteria analysis in GIS to decide the optimal bus stop locations in an Indian city. Chien and Qin (2004) developed a more realistic mathematical model with demand not distributed uniformly along the route. Furth and Mekuria (2005) developed a software package based on GIS system to evaluate the effect of bus stop spacing. The proposed model was tested by analysing two lines in Boston and New York. Alterkawi (2006) also developed a computer program to determine optimal bus stop spacing and applied in Saudi Arabia. Furth (2007) proposed a parcel-level model base on street network and discrete demand area in GIS, which can relocate the stops and be used to evaluate the influence of removing stops for a Boston line. Ibeas et al. (2010) presented a bi-level optimization for locating bus stops which considered possible changes under different bus stop configurations. And the technique is applied in a Spanish city to analyse the variables influence.

More recently, Medina et al. (2013) considered boarding and alighting demand, and stop density as continuous functions along the route and proposed a model. It was solved by two-phase solution methodology and applied in a Greater Santiago corridor. Zheng et al. (2015) developed the model based on game theory to get the optimal stop spacing, where the Thiessen polygon method was applied to divide the service areas of the stations. Ceder et al. (2015) developed a mathematical modelling approach to bus stop placement which includes considerations of uneven topography which were applied as case study in the Auckland. Stewart and El-Geneidy (2016) proposed a new methodology for bus-stop consolidation to remove unnecessary stops, where the objective to estimate the bus stop spacing is whether the operating cost savings will come at the expense of passengers. Moccia and Laporte (2016) extended the basic optimization model for transit line account for optimal stop spacing. Results showed that the ratio of optimal stop spacings among different modes follows a square root formula.

### **3. Model development**

#### **3.1. Assumption**

Several assumptions are made before the modeling as follows.

A1: The line is servicing from residential area to CBD with large commuting demand in morning peak hours.

A2: The service area is a linear corridor as shown in Figure 1, where the demand follows uniform distribution along the corridor. The left rectangle is the boarding area, where the bus stops in the boarding area are only for passenger boarding. Likewise, the bus stops in the right rectangle are only

for passenger alighting. After collecting all the passenger in the boarding area, the vehicle drives directly to the alighting area without stopping in between.

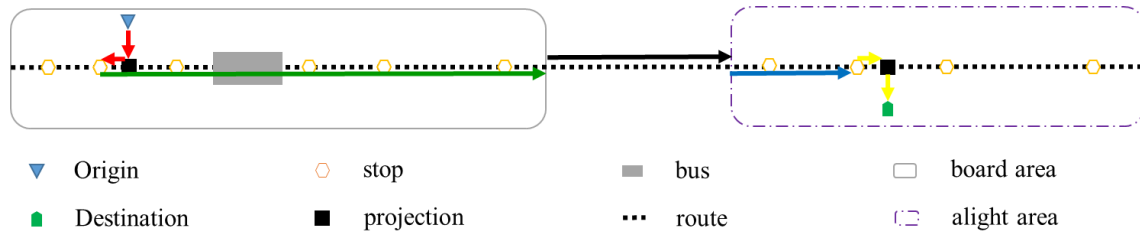


Figure 1 A diagrammatic sketch of the linear corridor

A3: The distance from the passenger's origin point to the boarding station is simplified as the sum of the horizontal and vertical distances between the start point and the boarding station, as shown in Figure 1. Similarly, the distance from the alighting station to the destination point is simplified as the sum of the horizontal and vertical distances between the alighting station and the destination point.

A4: Vehicle operates under the same condition at each station, e.g. uniform acceleration to operating speed, uniform deceleration until stopping.

A5: The generation of passenger demand is not depending on time accumulation.

A6: All the on-demand public buses are the same type.

A7: The queuing and delay time caused by signal control is not considered.

A8: If the demand between specific one origin and destination (OD) pair is over the capacity of one bus, planners shall dispatch the direct bus for this OD pair. This direct bus will not affect the optimization results of the proposed model.

A9: The access mode to the bus station only takes walking into account.

### 3.2. Objective function

The objective of our model is to minimize the total travel time of all the passengers. Let  $(x, y)$  represent the coordinate of origin point,  $p(x, y)$  represent the passenger density function at point  $(x, y)$ , the total travel time  $TT$  of all passengers from the origin to destination could be expressed as:

$$TT = \iint_{x,y} p(x, y) t(x, y) dx dy \quad (1)$$

where  $t(x, y)$  is the shortest travel time from origin to destination. According to assumption A3,  $t(x, y)$  could be transformed to one-dimensional expression as:

$$t(x, y) = t(x) + t(y) \quad (2)$$

where  $t(x)$  and  $t(y)$  are the shortest travel time in horizontal and vertical directions from origin to destination.  $t(y)$  is purely the walking time, while  $t(x)$  including walking time and bus travel time.

Substituting Eq.(2) to Eq.(1), we could obtain:

$$TT = \iint_{x,y} t(x) p(x, y) dx dy + \iint_{x,y} t(y) p(x, y) dx dy \quad (3)$$

Since  $t(y)$  is purely the walking time, which is only affected by the width of the corridor. Thus only the second part in the equation will influence the optimization results for bus stop spacing. In this case, we can simplify Eq.(3) with merely considering the effect from horizontal direction as follows:

$$TT = \int t(x) p(x) dx \quad (4)$$

The total travel time could also be calculated by summing up the travel time at each trip stage, including the walking time from origin point to the boarding station  $\bar{T}_w$ , the in-vehicle time in the boarding area  $\bar{T}_v$ , the main haul in-vehicle time  $\bar{T}_v$ , the in-vehicle time in alighting area  $T_v$ , and the walking time from the alighting station to the destination point  $T_w$ , that is:

$$TT = \omega_w \bar{T}_w + \omega_v \bar{T}_v + \omega_v \bar{T}_v + \omega_v T_v + \omega_w T_w \quad (5)$$

where  $\omega_w$  and  $\omega_v$  are the perception weights for walking time and in-vehicle time. Usually, passenger perceived that he spend more time than he actually is during walking than in-vehicle, and  $0 < \omega_v < \omega_w$ . The main haul in-vehicle time  $\bar{T}_v$  is not affected by the location of the residential area and CBD, which could be expressed as:

$$\bar{T}_v = PL_m / v_{veh} \quad (6)$$

where  $P$  is the total number of the passengers,  $L_m$  is the length of the main haul,  $v_{veh}$  is the average operation speed of the on-demand public bus. Since the travel pattern in the boarding and alighting areas are symmetrical, to calculate the minimum total travel time  $TT_{min}$ , we could basically minimize the travel time in boarding area. Thus the objective function could be simplified as:

$$TT_{min} = Min\{\omega_w \bar{T}_w + \omega_v \bar{T}_v\} \quad (7)$$

### 3.3. Model constraints

#### **Vehicle and segment**

Since each vehicle has the same capacity limitation, the required number of vehicle  $N_{veh}$  could be calculated by:

$$N_{veh} = \left\lceil \int_0^L \frac{p(x)}{C_{veh}} dx \right\rceil \quad (8)$$

where  $C_{veh}$  is the capacity of the vehicle,  $L$  is the length of the boarding area on the corridor. When  $p(x)$  is small, one vehicle is enough to service this corridor,  $N_{veh} = 1$ . When  $p(x)$  becomes larger, more vehicles are needed,  $N_{veh} > 1$ . The first vehicle starts its service at the first stop in the residential area until it becomes full occupied at certain stop, then this vehicle will not service for the rest passengers and will go directly to the destination without stopping in the middle. Since the vehicle could start the service at any station in the residential area, the second vehicle will start from the boundary of the previous servicing area and follow the same operational pattern. Likewise, we can divide the corridor into multi-segments. With the increase of  $p(x)$ , the number of segments increases, and the length of each segment shortens. As the bus stop spacing could not be reduced without limit, we assume the minimum bus stop spacing is  $S_*$ . When the length of the segment shorten to  $S_*$ , this segment is only serviced by one stop. It means that the demand at this stop is exactly equal to the capacity of one vehicle. Then this stop will dispatch one direct service to destination, where the passenger density  $p_*(x)$  in this segment at this moment satisfied the following equation:

$$\int_{S_*} p_*(x) dx = C_{veh} \quad (9)$$

According to assumption A2 and A8, besides the passengers that are serviced by the direct service, the distribution function for the rest passengers is  $p'(x)$  which can be represented as:

$$p'(x) = p(x) - \frac{C_{veh}}{S_*} k \quad (10)$$

where  $k$  is the number of the direct bus in one segment. Then, the number of non-direct vehicle is:

$$n_{veh} = \left\lceil \int_0^L \frac{p'(x)}{C_{veh}} dx \right\rceil \quad (11)$$

Base on the discussion above, we can conclude that each non-direct vehicle services one separate segment alone. The number of segments is equal to the number of the non-direct vehicles:

$$n = n_{veh} \quad (12)$$

The capability of the vehicle is limited, thus the length of the segment  $l_j$  should meet the following requirement:

$$\int_{l_j} p'(x) dx \leq C_{veh} \quad (13)$$

In the real operation, the length of the segment could be divided by: (a) uniformly-spaced calculation, that is  $l_j = L/n$ . In this situation, each vehicle will have serval empty seats to attract and arrange the casual user. (b) the closer to CBD, the longer distance of  $l_j$ , that is  $l_{j-1} < l_j$ . In this situation, long-distance user could save the travel time without standing the stopping time in the middle of the trip. (c) all the segment are fully occupied, except one segment which is servicing the users less than the vehicle's capacity, that is,  $l_{j=\eta} < C_{veh}/p'(x)$ ,  $l_{j \neq \eta} = C_{veh}/p'(x)$ . In this situation, this segment could be scheduled with specific vehicle with small capacity.

### ***Shed line***

Shed line is the boundary which separates the passenger bus stop selection from two adjacent boarding stops into two parts. In this study, the boundary of each segment is a special shed line, therefore we defined that there are two types of shed line: one is stop shed line within one segment, the other is the segment shed line.

For the stop shed line within one segment, the shed line separates the bus stop spacing into two parts:

$$S_i^j = a_{i+1}^j + b_i^j \quad (14)$$

where  $S_i^j$  is the bus stop spacing from stop  $i$  to stop  $i+1$  in segment  $j$ ,  $a_{i+1}^j$  is the distance from the stop shed line to stop  $i+1$  in segment  $j$ ,  $b_i^j$  is the distance from the stop  $i$  to stop shed line in segment  $j$ .

Passenger on the stop shed line perceives that the walking time from the origin point to stop  $i+1$  is equal to the sum of walking time from origin point to stop  $i$  and the in-vehicle time from stop  $i$  to stop  $i+1$ , that is:

$$T_{veh}^{S_i^j} \omega_v + \frac{b_i^j}{v_{walk}} \omega_w = \frac{a_{i+1}^j}{v_{walk}} \omega_w \quad (15)$$

where  $v_{walk}$  is the average walking speed.  $T_{veh}^{S_i^j}$  is the in-vehicle time from stop  $i$  to stop  $i+1$  in segment  $j$ , which includes the normal operation time and the loss time near and at the bus stop:

$$T_{veh}^{S_i^j} = \frac{S_i^j}{v_{veh}} + T_{lost,i}^j \quad (16)$$

where  $v_{veh}$  is the average normal operational speed of the vehicle.  $T_{lost,i}^j$  is the loss time near and at the bus stop  $i$  in segment  $j$ , which includes the deceleration time, acceleration time, stopping time for boarding, and buffer time, as follows:

$$T_{lost,i}^j = \frac{v_{veh}}{2\sigma_1} + \frac{v_{veh}}{2\sigma_2} + \frac{n_{p,i} t_p}{n_{door}} + t_f \quad (17)$$

where  $\sigma_1$  and  $\sigma_2$  are the deceleration and acceleration speed,  $n_{p,i}$  is the number of boarding passenger at stop  $i$ ,  $t_p$  is the average boarding time per passenger,  $n_{door}$  is the number of the doors of the vehicle. To ensure the bus can arrive at each stop at the fixed time, we consider a buffer time  $t_f$  at each stop. Since all the stops in residential are only for passenger boarding, the capacity of each vehicle is limited, and the passenger density is uniform, we could regard the time loss at each bus stop is the same, that is:

$$T_{lost,i}^j = T_{lost} \quad (18)$$

Assume that the relationship between the weight of perceived walking time and the weight of perceived in-vehicle time meets the following relation:

$$\omega = \frac{\omega_w}{\omega_v} \quad (19)$$

Combining Eqs.(14)-(16) and Eq.(19), we could obtain:

$$a_{i+1}^j = \frac{\omega v_{veh} + v_{walk}}{\omega v_{veh} - v_{walk}} b_i^j + \frac{v_{veh} v_{walk}}{\omega v_{veh} - v_{walk}} T_{lost,i}^j \quad (20)$$

Passenger on the segment shed line perceives that the walking time from the origin point to the first stop in segment  $j+1$  is equal to the sum of the walking time from the origin point to the last stop in segment  $j$  and the in-vehicle time from the last stop in segment  $j$  to the first stop in segment  $j+1$ :

$$\frac{b_{m(j)}^j}{v_{walk}} \omega_w + \frac{b_{m(j)}^j}{v_{veh}} \omega_v + \frac{l^{j+1}}{v_{veh}} \omega_v = \frac{a_1^{j+1}}{v_{walk}} \omega_w + \frac{l^{j+1} - a_1^{j+1}}{v_{veh}} \omega_v + m_{(j+1)} T_{lost} \quad (21)$$

where  $m_{(j+1)}$  is the number of bus stops in segment  $j+1$ ,  $b_{m(j)}^j$  is the distance from the last stop in segment  $j$  to the segment  $j$ 's shed line,  $l^{j+1}$  is the length of the segment  $j+1$ ,  $a_1^{j+1}$  is the distance from the segment  $j$ 's shed line to the first stop in segment  $j+1$ . Similarly, Eq.(21) could be simplified as:

$$a_1^{j+1} = \frac{\omega v_{veh} + v_{walk}}{\omega v_{veh} - v_{walk}} b_{m(j)}^j - \frac{v_{veh} v_{walk} T_{lost}}{\omega v_{veh} - v_{walk}} m_{(j+1)} \quad (22)$$

### **Minimum stop spacing**

Since  $b_i^j \geq 0$ , from Eq.(20), we can obtain that:

$$a_{i+1}^j \geq \frac{v_{veh} v_{walk}}{\omega v_{veh} - v_{walk}} T_{lost} \quad (23)$$

Combing with Eq. (14) leads to:

$$S_i^j \geq \frac{v_{veh} v_{walk}}{\omega v_{veh} - v_{walk}} T_{lost} \quad (24)$$

According to the assumption A4, the bus stop spacing should meet the requirement of:

$$S_i^j \geq \frac{(v_{veh})^2}{2\sigma_1} + \frac{(v_{veh})^2}{2\sigma_2} \quad (25)$$

Therefore, combining Eqs.(24) and (25), the minimum bus spacing is:

$$S_{min} = \text{Max} \left\{ \left[ \frac{v_{cb} v_{walk}}{\omega v_{cb} - v_{walk}} T_{lost} \right], \left[ \frac{(v_{cb})^2}{2\sigma_1} + \frac{(v_{cb})^2}{2\sigma_2} \right] \right\} \quad (26)$$

### **Total travel time**

As mentioned, the corridor is divided into  $n$  segments, where each segment includes several bus stops. According to the location diagram in Figure 2, the distance relationship among the segments could be expressed as:

$$A_i^j = \sum_{\xi=0}^i a_{\xi+1}^j + \sum_{\xi=0}^i b_{\xi}^j \quad (27)$$

$$B_i^j = \sum_{\xi=0}^i (a_{\xi}^j + b_{\xi}^j) \quad (28)$$

$$l_j = \sum_{i=1}^{m(j)} (a_i^j + b_i^j) \quad (29)$$

$$L = \sum_{j=1}^n l_j \quad (30)$$

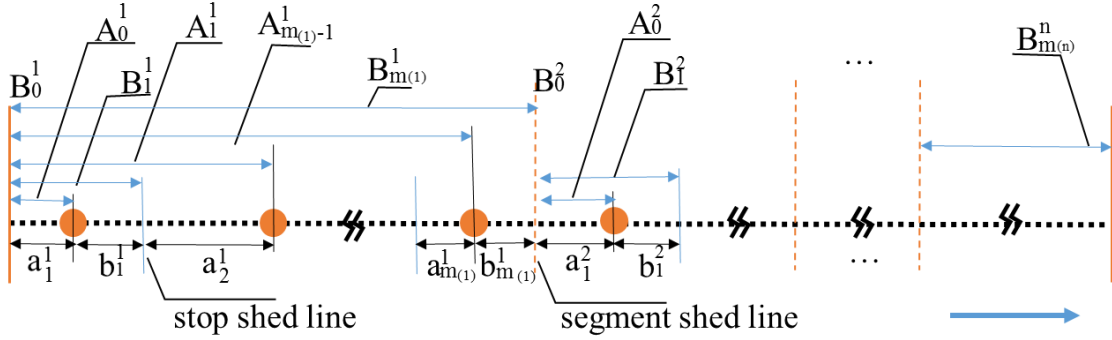


Figure 2 Diagram of the stop locations

The total travel time includes total walking time and total in-vehicle time. All the passengers walk from their origin points to the nearest bus stop. As shown in Figure 2, the total walking time of all the passengers is the sum of the travel time from all the shed lines to the next adjacent bus stops and the travel time from all the stops to the next adjacent shed lines, as follows:

$$\bar{T}_W = \frac{1}{v_{walk}} \sum_{j=1}^n \sum_{i=0}^{m(j)-1} \left[ \int_{B_i^j}^{A_i^j} (A_i^j - x) p'(x) dx + \int_{A_i^j}^{B_{i+1}^j} (x - A_i^j) p'(x) dx \right] \quad (31)$$

The total in-vehicle time consists of three parts: the first part is the total in-vehicle time from the origin point to the last stop in the segment, the second part is the total in-vehicle time from the last stop in the segment to the segment shed line, and the third part is the total in-vehicle time from the segment shed line to the boundary of boarding area, that is:

$$\bar{T}_V = \sum_{j=1}^n \left[ \sum_{i=1}^{m(j)-1} T_{veh}^{S_i^j} p'(x) B_i^j + \frac{(B_{m(j)}^j - A_{m(j)-1}^j)}{v_{veh}} p'(x) l_j + \left( L - \sum_{i=1}^j l_i \right) p'(x) l_j \right] \quad (32)$$

### 3.4. Optimization model

The optimization model could be written as:

$$TT_{\min} = \text{Min} \left\{ \begin{array}{l} \frac{\omega_w}{v_{walk}} \sum_{j=1}^n \sum_{i=0}^{m(j)-1} \left[ \int_{B_i^j}^{A_i^j} (A_i^j - x) p'(x) dx + \int_{A_i^j}^{B_{i+1}^j} (x - A_i^j) p'(x) dx \right] + \\ \omega_v \sum_{j=1}^n \left[ \sum_{i=1}^{m(j)-1} T_{veh}^{S_i^j} p'(x) B_i^j + \frac{(B_{m(j)}^j - A_{m(j)-1}^j)}{v_{veh}} p'(x) l_j + \left( L - \sum_{i=1}^j l_i \right) p'(x) l_j \right] \end{array} \right\} \quad (33)$$

Besides the constraints Eqs. (10) -(30), the following constraints are also needed to be satisfied:

$$S_i^j \geq S_{\min} \quad (34)$$

$$a_i^j \geq 0 \quad (35)$$

$$b_i^j \geq 0 \quad (36)$$



$$a_0^j = 0 \quad (37)$$

$$b_0^j = 0 \quad (38)$$

#### 4. Algorithm and solution characteristic

Dynamic programming is one of the typical methods to solve the proposed model. The detailed derivative process could be found in Vuchic and Newell (1968), where that paper is the special case of our model when  $n_i = 1$ ,  $\omega_w = \omega_v = 1$ ,  $b_m = 0$ . We apply the dynamic programming function in the Mathematica software to solve the proposed model.

The dynamic programming gives out the numerical results directly, it is not easy to analyse the characteristic of the solution. Therefore, the method of Lagrange multiplier is introduced to analyse the characteristic of the solution. Base on the model, the corresponding Lagrange function of the objective function could be written as:

$$F(a_1^j, b_i^j, \lambda_u, \kappa) = \Gamma - \sum_{u=1}^U \lambda_u g_u(x) - \kappa \left[ L - \sum_{j=1}^{n_j} \sum_{i=1}^{n_j} (a_i^j + b_i^j) \right] \quad (39)$$

The bus stop spacing is limited within  $[S_{\min}, L]$ . The reducing of bus stop spacing increases the number of bus stop number and decreases the total walking time. While due to the increase of the total loss time near and at the bus stops, the total in-vehicle time increases. Therefore, the objective function is convex and the model has at least one optimal solution, which satisfies the following equation:

$$\frac{\partial F(a_1^j, b_i^j, \lambda_u, \kappa)}{\partial b_i^j} = 0 \quad (40)$$

Then,

$$\frac{\partial F(a_1^j, b_i^j, \lambda)}{\partial b_\xi^j} - \frac{\partial F(a_1^j, b_i^j, \lambda)}{\partial b_{\xi+1}^j} = 0, \text{ for } \xi = 1, 2, \dots, n_j - 2 \quad (41)$$

Simplify Eq.(40), we have:

$$\frac{2p' \omega_w \left[ v_{veh} v_{walk} \omega_v T_{lost} + (v_{walk} \omega_v + v_{veh} \omega_w) b_\xi^j - (v_{walk} \omega_v + v_{veh} \omega_w) b_{\xi+1}^j \right]}{v_{walk} (v_{walk} \omega_v - v_{veh} \omega_w)} = 0 \quad (42)$$

According to Eqs.(14)-(15), Eqs.(18)-(20) and Eq.(28), we obtain that:

$$B_i^j = \frac{\omega v_{veh} - v_{walk}}{2\omega v_{veh}} A_i^j + \frac{\omega v_{veh} + v_{walk}}{2\omega v_{veh}} A_{i-1}^j - \frac{v_{walk} T_{lost}}{2\omega} \quad (43)$$

Let  $\frac{\omega v_{veh} - v_{walk}}{2\omega v_{veh}} = \alpha$ ,  $\frac{\omega v_{veh} + v_{walk}}{2\omega v_{veh}} = \beta$ ,  $\frac{v_{walk} T_{lost}}{2\omega} = \gamma$ , Eq.(20) can be rewritten as:

$$a_{i+1}^j = \frac{\beta}{\alpha} b_i^j + \frac{\gamma}{\alpha} \quad (44)$$

Combining Eq.(19), Eq.(42) can be simplified as:

$$b_{i+1}^j - b_i^j = \frac{\gamma}{\beta}, \text{ for } i = 1, 2, \dots, m_{(j)} - 2 \quad (45)$$

Combining Eq.(44) and Eq.(45) leads to:

$$\frac{a_{i+1}^j}{b_{i+1}^j} = \frac{\beta}{\alpha}, \text{ for } i = 1, 2, \dots, m_{(j)} - 2 \quad (46)$$

Considering Eq.(14), we can conclude that:

$$S_{i+1}^j - S_i^j = \frac{\gamma}{\alpha\beta}, \text{ for } i = 1, 2, \dots, m_{(j)} - 2 \quad (47)$$

Therefore, the differences of bus stop spacing in each segment only related with  $\alpha$ ,  $\beta$  and  $\gamma$ . As discussed above, when the passenger follows uniform distribution, the bus stop spacing in each segment is in an arithmetic progression, and the common difference is  $\frac{\gamma}{\alpha\beta}$ , which is not affected by the length of corridor and the density of passenger. The common difference could be simplified as:

$$\frac{\gamma}{\alpha\beta} = \frac{2\omega T_{lost}}{\omega^2 - \left(\frac{v_{walk}}{v_{veh}}\right)^2} v_{walk} \quad (48)$$

It shows that if the loss time at the bus stop is neglected  $T_{lost} = 0$ , the bus stop spacing at each segment is the same. For the conventional bus, passenger could board and alight at each stop. When the passenger in the vehicle could maintain balance, the optimal bus stop spacing is the same. This result reflects that the accumulation of the passenger on the vehicle is the main factor that makes the bus stop spacing in an arithmetic progression. Meanwhile, according to Eq.(48), with the increase of operational speed  $v_{veh}$ , the stand deviation will not increase significantly, which means that only increase the vehicle operational speed will not significantly reduce the bus stop density.

## 5. Numerical results

In this section, a set of numerical results are presented. Parameters in this model are set as in Table 1.

Table 1 parameter setting

| Parameter  | Value | Description   |
|------------|-------|---|
| $v_{veh}$  | 5.56  | Average speed of bus operating (m/s)                                  |
| $v_{walk}$ | 1.39  | Average speed of walking (m/s)  |
| $\omega$   | 2.5   | Ratio of perceived walking time weight to perceived in-vehicle weight |
| $C_{veh}$  | 40    | Capacity of the bus (per)   |
| $P$        | 0.033 | Passenger density (per/m)   |
| $L$        | 3000  | Length of the residential area?(m)                                    |
| $T_{lost}$ | 60    | Loss time at the bus stop (s)   |

As concluded before, the bus stop spacing is in an arithmetic progression, where the common difference is not affected by passenger density  $p$ . But the location and the density of the bus stop are influenced by passenger density  $p$ . Figure 3 depicts the number of bus stops and the bus stop location results from different passenger density. It could be found that when the passenger density is low ( $p < 13$ ), only one bus is need to service this corridor. With the increase of passenger density, more buses are required and thus more segments are divided. The length of each segment and the number of bus stops in each segment are correspondingly reduced. When  $p$  is small ( $p < 26$ ), bus stop location is related with the number of segment, and does not affected by the value of  $p$ , which is in line with the results from Vuchic and Newell (1968). It is because that the small value of  $p$  will lead to a larger service distance  $C_{veh}/p$ , the optimal distance of each segment will be never longer than  $C_{veh}/p$ . As shown in Figure 4, when  $p$  becomes larger ( $p > 26$ ), the constraint Eq.(13) starts to limit the objective function. With the increase of  $p$ , the length of the last segment becomes longer, while the length of other segments become shorter and are all equal to  $C_{veh}/p$ . When the length of all the segments are

equal, the total passenger demand meets  $nC_{veh} = pL$ . Another conclusion can be found is that when  $C_{veh}/p$  remains the same, the location of bus stops are fixed.

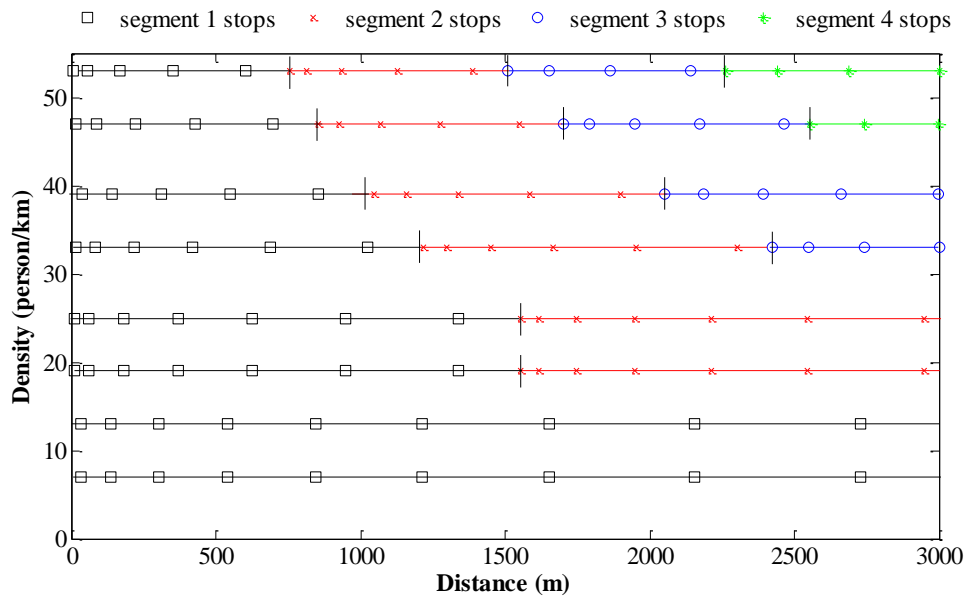


Figure 3 Bus stops locations resulting from passenger density

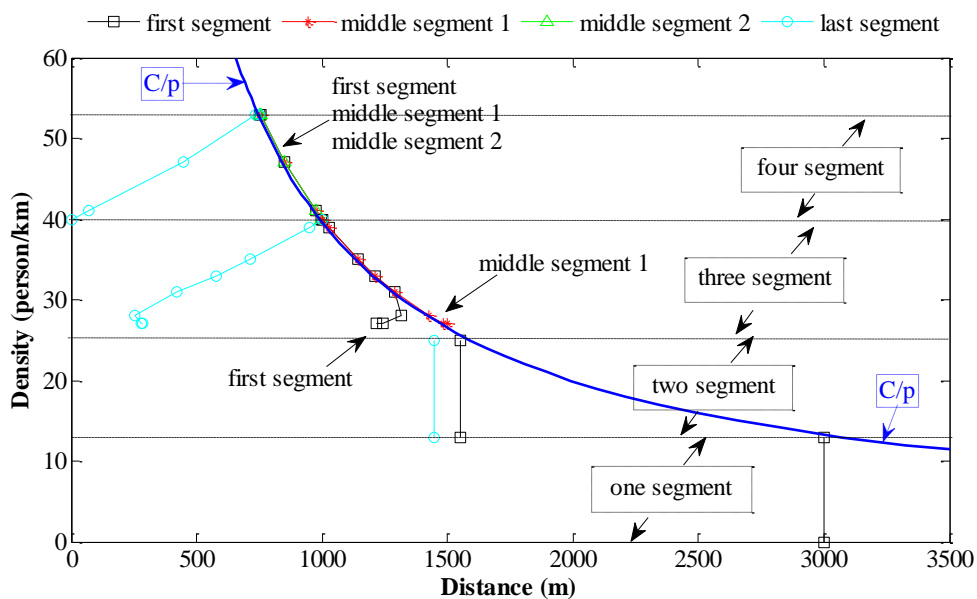


Figure 4 The optimal length of the segment resulting from passenger density

As defined before, the number of segment is only related with the total passenger demand in the study area, while the total passenger demand is the function of corridor length and the demand density. Take the total demand density is 33 as example, comparing routes A and B as shown in Figure 5, if the length of corridor increases from 3000 to 4000, the total number of segment increases from 3 to 4. But the number of the bus stops and the location in the first two segments only has slightly changes. Comparing routes A and C, when the total demand are same, although the length of the corridor increases, the total number of segment is not changed, but the length of the segment and the stop density increase.

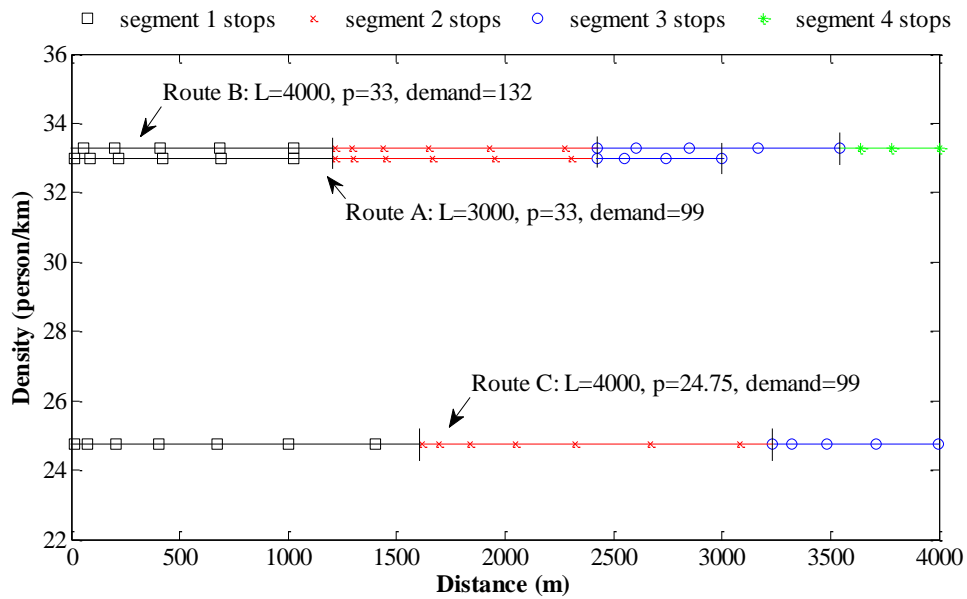


Figure 5 Bus stops locations resulting from passenger density and length of corridor

People usually have different time perception in different situation. Usually people perceived they spend more than they actually spend during walking and in-vehicle. The perception ratio  $\omega$  is related with the weather, geometry, level of service, and even the duration of the travel time. Figure 6 shows the optimal bus stop location resulting from different time perception weights. Although there is little difference in the length of each segment, the stop density varies significantly. when the weight is small ( $\omega < 2.5$ ), the influence of  $w$  on the bus stop location and density is obvious where the increase of the ratio will increase the bus stop density. when the  $w$  increases to a certain number ( $\omega > 2.5$  in this application), the influence is not distinct.

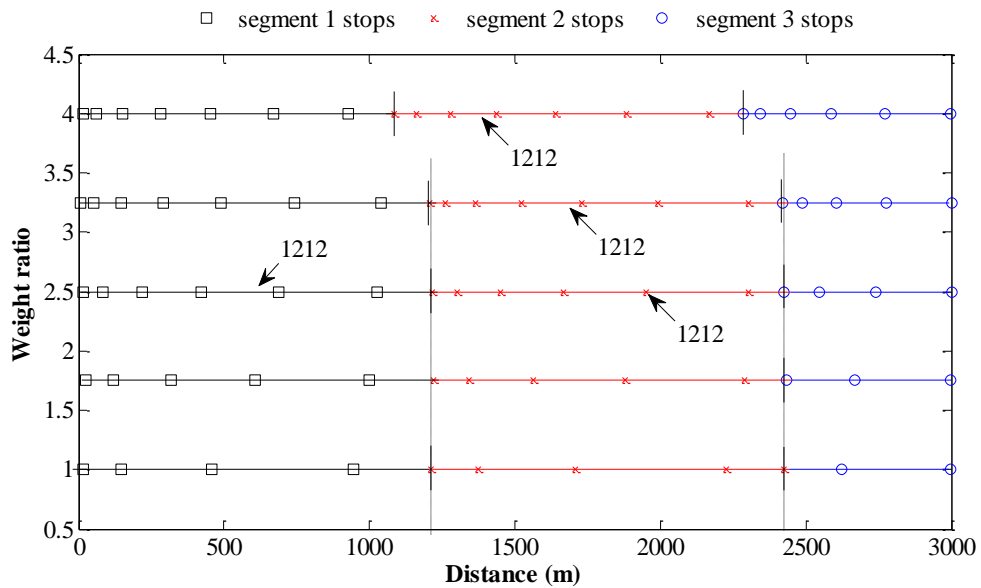


Figure 6 Bus stops locations resulting from time perception ratio

Previous researches in bus stop locations also have the analysis on the access speed, normal operating speed and the time loss at the bus stop. If we release the assumption A9 and more access modes with different speed are considered. Results from Figure 7(a) show that the increase of the access speed will reduce the bus stop density, which is in line with the previous researches in conventional bus. However, as shown in Figure 7(b), the changes in normal operating speed will not affect the bus stop spacing and density significantly. Only when the speed raises by 50%, the length of the first segment becomes slightly shorter and the length of the last segment becomes slightly longer, while the length of the middle segment maintains the same. And the locations of the stops in different segment do not change too much, which also seconds the solution characteristics conclusions obtained by Lagrange multiplier method.

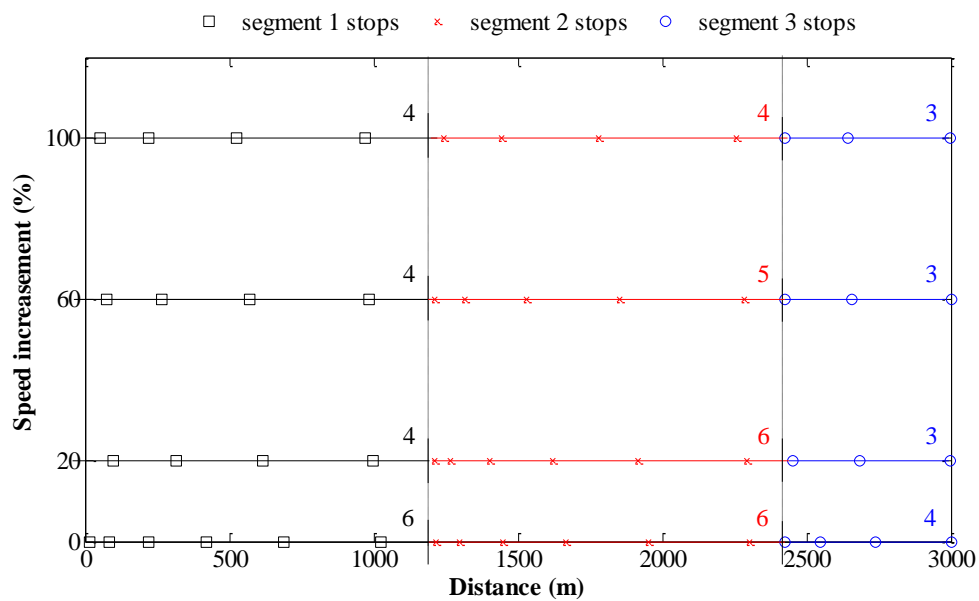


Figure 7(a) Bus stops locations resulting from walking speed

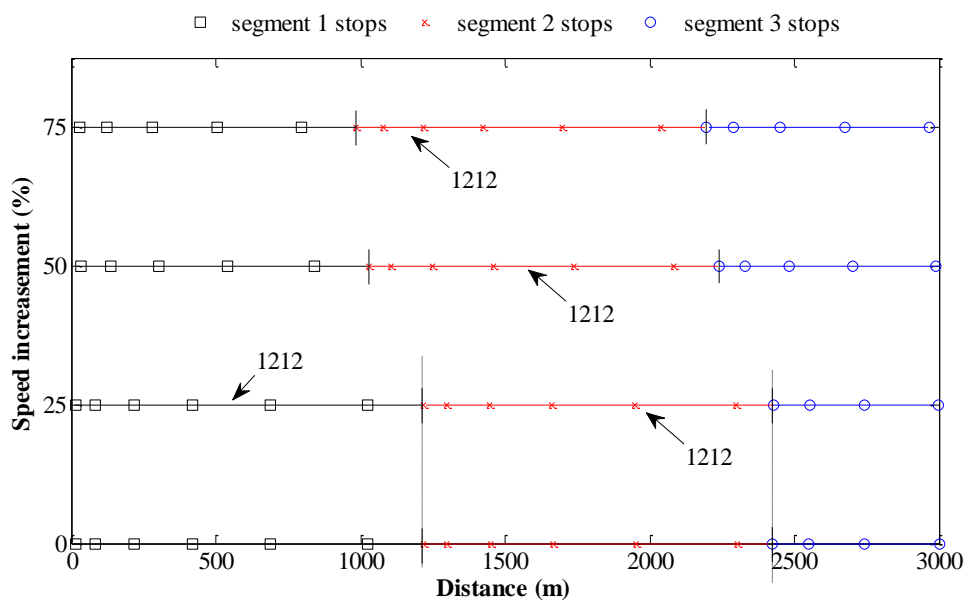


Figure 7(b) Bus stops locations resulting from vehicle speed

Figure 8 shows that the optimal bus stop location caused by different loss time at the bus stop. The increase of the loss time at the bus stop has no influence for the length of segments, but significantly reduce the number of bus stops in each segment. The density of the bus stop first drops drastically, then becomes stable. The changes of number of bus stops follow an approximate exponential distribution. When the loss time at the bus stop is large (>60), the location of bus stop is relatively fixed. Meantime, the Figure 8 also proved the conclusion by Lagrange multiplier method. The common difference of bus stop spacing reduces and tends to be 0 when the loss time tends to be 0.

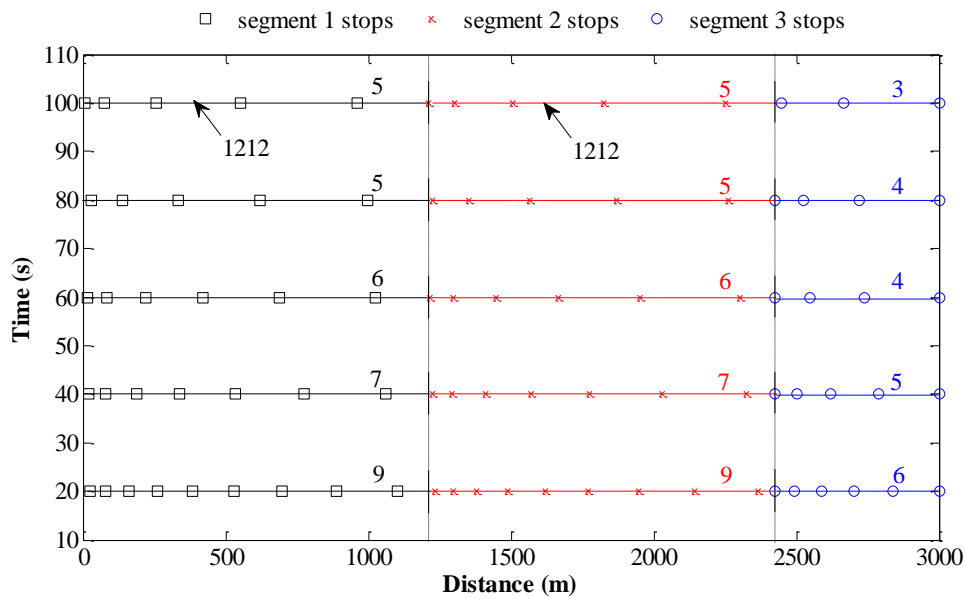


Figure 8 Bus stops locations resulting from time loss

From Figures 6-8, it could be noted that if the number of segments is fixed, the time perception weight ratio, access speed, normal operating speed and the time loss at the stop will not influence the length of the middle segment. Figure 9 shows that when the number of segments is 3, the effect of the above factors on the length of segment. The increase of time perception weight ratio or the normal operating speed will shorten the length of the first segment and elongate the length of the last segment, while the length of the middle segment maintains the same. Another interesting point is that although access speed and the time loss at the stop will affect the stop density and location in one segment, these two factors will not affect on the length of the segments.

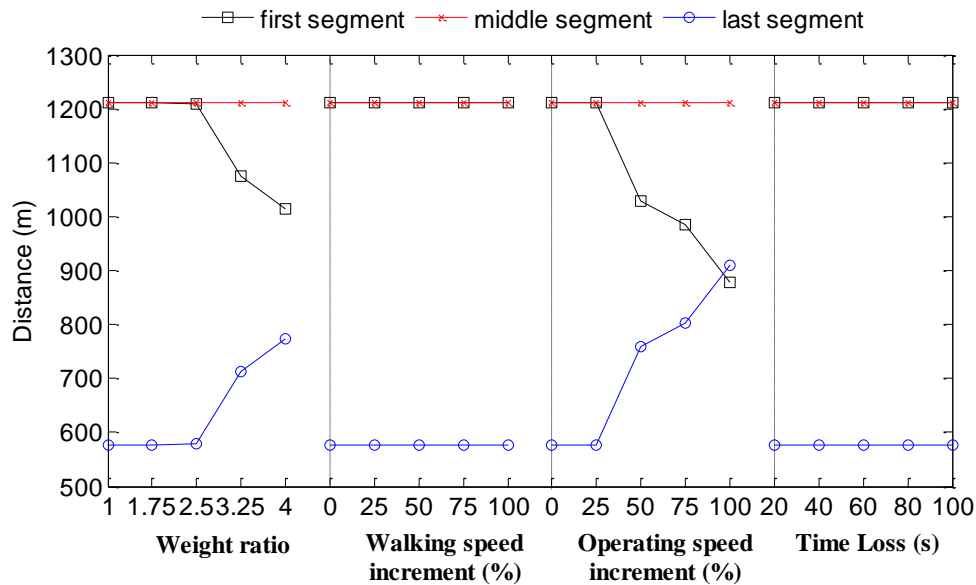


Figure 9 The length of segment resulting from four factors

## 6. Conclusions

An optimization model for bus stop spacing is proposed for on-demand public bus service. Several unique operational characteristics of on-demand public bus are considered. The model is solved by the dynamic programming function in the Mathematica software. To the best of our knowledge, this is the first attempt to systematically plan the bus stop location and spacing for on-demand public bus service.

The results of the example application show that the plan of on-demand public bus stop location should be different with conventional bus stop location plan. First, the on-demand public bus service could dispatch the direct service when the demand from certain stop to destination is greater than the capacity of the vehicle. The non-direct vehicle services for the rest demand and each non-direct vehicle services one separate segment alone. The number of segment is equal to the number of the non-direct vehicle. Second, the bus stop spacing in one segment is in an arithmetic progression and the common differences of the bus stop spacing in all segments are the same. The common difference is affected by the number of passengers on the vehicle. Third, the operation speed of the vehicle does not impact the bus stop spacing. Fourth, the ratio of the walking time perception to the in-vehicle time perception has the significant influence on the bus stop density. Five, the reduce of time loss will decrease the common difference of bus stop spacing and increase the bus stop density.

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