Mapping and Optimizing an Electric Vehicle Triple Supply Chain: Electric Vehicles, Energy Supply and Batteries

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DECLARATION

I certify that the work contained in this thesis, or any part of it, has not been accepted in substance for any previous degree awarded to me, and is not concurrently being submitted for any degree other than that of Doctor of Philosophy being studied at the University of Greenwich. I also declare that this work is the result of my own investigations, except where otherwise identified by references and that the contents are not the outcome of any form of research misconduct.

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Abstract

Several governments worldwide have set up measures to promote the electric vehicle (EV) as one option to address environmental issues. However, EV sales figures suggest that the EVs have not been widely accepted by consumers due to cost (notably battery cost), the limited range of products, long recharge times and the limited availability of recharging infrastructure. This indicates the need for in-depth research on the relevant EV supply chains (SCs) to help overcome these barriers.

This thesis provides a systematic process for the analysis of EV triple SCs, namely the vehicles, the energy supply and the EV batteries. Three models have been developed for the triple SC—specifically, an EV subsidy model for the vehicle SC; a charging station (CS) selection scheme model for the energy SC; and a closed-loop supply chain (CLSC) model for the EV battery SC. Game theory is used in both EV subsidy model and the EV battery CLSC model, and the stochastic analysis with a Monte-Carlo simulation is used in the CS selection scheme model. Numerical analysis is an inherent element of each of the models to demonstrate their applications.

The EV subsidy model involves the government, the gasoline vehilce (GV)/EV manufacturer and retailer and the vehicle customer. The optimum profit for each party and the optimal government subsidy are discussed based on mathematical formulas. The results show that the optimal subsidy is the same, whether it is for the EV manufacturer or the EV customer. In the EV energy supply, the author proposes three EV charging station (CS) selection schemes— per-time selection, bulk selection and combined per-time and bulk selection. Using mathematical analysis and simulation, it can be found that the Per-time selection has the best performance. However, combined selection could be the optimal choice for policymakers as it performs similarly to per-time selection while reducing the system cost. Last, the EV battery CLSC model involves the EV battery manufacturer and remanufactuer and divides the entire life cycle into three periods, the author discusses two optimization decisions: to optimize the profit of each party and to optimize the optimal profit. Although the results and relationships are complex, they can be simplified as being linear or quadratic, which is helpful for the stakeholders in the SC.

In summary, this research proposes and discusses the EV triple SC that includes the vehicle, the energy supply and the EV battery. However, the interrelationship between internal sub-supply chains is less studied and will be the direction of future research.

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Love and Peace
Studying at Greenwich
Around squirrels and flowers
It is a place where I wish

Knowledgeable Petros
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Supportive and kindness
So valuable to me

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Warm heart of Faculty
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But always with me

The world maybe smaller
With you all, life becomes better

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Chapter 1

Introduction

This chapter establishes the context of the study of the electric vehicle (EV) supply chain (SC) and outlines the background, motivation, and significance of the research. In order to better understand the EV SC and expound on challenges related to it, an overview of the EV industry is first provided in Section 1.1. The research aims and objectives are then presented in Section 1.2. Fianlly, the whole structure of the thesis is summarised in Section 1.3, showing how the chapters link with the research aims and objectives.

1.1 An overview of electric vehicles

1.1.1 Definition of electric vehicles

EVs are defined as follows (Faiz et al., 1996):

> The Electric vehicle (EV) refers to an electric drive vehicle, which uses electric motors or traction motors for propulsion. An electric vehicle may be powered through a collector system by electricity from off-vehicle sources, or may be self-contained with a battery or generator to convert fuel to electricity.

Based on the electricity producing method, there are three types of EVs (Faiz et al., 1996):

(1) The vehicle that relies on continuous electricity supply from an off-board generation system, such as a tramcar with overhead wires.

(2) The vehicle that relies on stored electricity from an off-board generation system. This means the vehicle uses a battery or other electricity storage media located on itself to store energy that comes from off-board energy sources.

(3) The vehicle relies on on-board electricity generation to supply its demand. This refers to a series of vehicles that could be driven by both an internal combustion engine and an electro-motor powered by energy that may be lost in a normal car (e.g., energy of motion when braking).

This work mainly studies the SC for the second type of EV, the most extensive EV, which normally includes hybrid EVs (HEVs), plug-in hybrid EVs (PHEVs) and pure battery EVs (BEVs). HEVs, such as the Toyota Prius (Toyota, 2017), combines an internal combustion engine and an electric propulsion system. The electricity, as the auxiliary power, comes from the transformation of kinetic energy, such as the regenerative brakes. By using this system, the fuel economy can be improved; however the power still comes from the gasoline and the HEV design is complex (Denton, 2016). PHEVs are driven by both electricity and gasoline (internal combustion) (Dickerman and Harrison, 2010). The electricity comes from charging the battery; this method can be used for short journeys in order to reduce emissions instead of just improving the fuel economy like HEVs. For example, the Chevrolet Volt, one of the top selling PHEVs, has only 85 km pure electric miles (Chevrolet, 2017). Furthermore, with the internal combustion engine, customers can drive long distances. However, PHEVs are still too complex, and customers do not use the charging feature (Chan, 2002).

Pure BEVs are only powered by electric batteries. They have the most energy conversion efficiency and the least emissions compared to PHEVs and HEVs (Markowitz, 2013). BEVs also have quite a simple structure (Chan, 2002). However, currently, BEVs are not popular because of the long charging time and short driving mileage. Tesla has released BEVs with long driving mileage and has constructed 1130 super charging stations (CSs) all over the world; however, these stations are too few in number. In addition, BEVs remain very expensive. For example, the Tesla Model X costs £79000 in the UK and has a driving mileage of 552km (Tesla, 2017*a*,*b*).

1.1.2 History of EV development

The first EV was invented by an American named Thomas Davenport in 1834 (Nye, 1992), approximate 50 years before the invention of the first vehicle powered by an internal combustion engine in 1886 (Benz, C, 1886). In 1881, Gustave Trouv built the first rechargeable vehicle (Wakefield, 1993). However, with the rapid development of the internal combustion engine and the oil industry, EVs could not compete with oil-driven vehicles. Specifically, from the 1800s to the end of the last century, more than 95% of vehicles manufactured by American companies were fossil fuel vehicles (Kirsch, 2000).

Beyond the development of the economy, environmental issues have been the main reason for focusing on EVs once again. As the economy developed, people began to pay attention to sustainability, making EVs an option. In comparison to gasoline vehicles (GVs), the benefits of using EVs are two-fold. First, both the tank-to-wheel and well-to-wheel energy conversion efficiency of EVs are higher than that of GVs (Markowitz, 2013): By processing from chargers to batteries, inverters, motors and drivetrains, the total tank-to-wheel efficiency of EVs is approximately 73%. However, the tank-to-wheel efficiency of GVs is just 16%, which is one quarter that of EVs. If considering well-to-wheel efficiency, which refers to electricity generation or oil supply to the vehicle, as the entire energy conversion chain from source to vehicles, the efficiency of EVs is 42% and that of GVs is 18% (Asahinet, n.d.). Second, EVs produce lower carbon emissions than GVs. According to Aguirre et al. (2012), in terms of CO_2 emissions during a full lifecycle, which includes vehicle parts and battery or engine manufacture, transportation, use phase and disposal process, the emission of GVs are 63,000 kg and those of EVs are 32,000 kg of EV, although the CO_2 emissions from EVs' battery manufacturing process is high. Moreover, in regard to other pollution emissions (VOC, CO, NOx, PM10, PM2.5, and SOx), EVs produce less pollution than GVs. Therefore, in terms of both efficiency and pollution emission, EVs are better than conventional GVs and are becoming more of an option in the sustainable development of the automobile industry.

1.1.3 Global sales outlook and current situation

This section comprises an overview of EV sales and market share over recent years. Table 1.1 presents the stock data for EVs (BEVs and PHEVs) from 2005 to 2016, and Table 1.2 shows the market share by country (International Energy Agency, 2017). The total market share is calculated based on the total market size of all the countries in the table. Note that in each of the tables below, "others" comprises Austria, Belgium, Bulgaria, Croatia, Cyprus, the Czech Republic, Denmark, Estonia, Finland, Greece, Hungary, Iceland, Italy, Ireland, Latvia, Lichtenstein, Lithuania, Luxemburg, Malta, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Switzerland, and Turkey.

Table 1.1: The EV stock (BEV and PHEV) by country from 2005 to 2016 (thousands)

	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
Canada							0.52	2.54	5.66	10.73	17.69	29.27
China					0.48	1.91	6.98	16.88	32.22	105.39	312.77	648.77
France	0.01	0.01	0.01	0.01	0.12	0.3	3.03	9.29	18.91	31.54	54.49	84.00
Germany	0.02	0.02	0.02	0.09	0.10	0.25	1.89	5.26	12.19	24.93	48.12	72.73
India				0.37	0.53	0.88	1.33	2.76	2.95	3.35	4.35	4.80
Japan					1.08	3.52	16.14	40.58	69.46	101.74	126.40	151.25
Korea						0.06	0.34	0.85	1.45	2.76	5.95	11.21
Netherlands				0.01	0.15	0.27	1.14	6.26	28.67	43.76	87.53	112.01
Norway			0.01	0.26	0.40	3.35	5.38	9.89	20.37	44.21	84.18	133.26
Sweden							0.18	1.11	2.66	7.32	15.91	29.33
United Kingdom	0.22	0.55	1.00	1.22	1.40	1.68	2.89	5.59	9.34	24.08	48.51	86.42
United States	1.12	1.12	1.12	2.58	2.58	3.77	21.50	74.74	171.44	290.22	404.09	563.71
Others					0.64	0.83	3.25	6.90	12.76	25.35	52.63	87.48
Total	1.37	1.69	2.15	4.54	7.47	16.81	64.58	182.64	388.07	715.39	1262.61	2014.22

Table 1.2: The EV market share (BEV and PHEV) by country from 2005 to 2016

	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
Canada								0.15%	0.20%	0.29%	0.39%	0.59%
China						0.01%	0.04%	0.06%	0.09%	0.38%	0.99%	1.37%
France						0.01%	0.13%	0.34%	0.55%	0.72%	1.22%	1.46%
Germany						0.00%	0.05%	0.11%	0.23%	0.42%	0.72%	0.73%

India				0.02%	0.01%	0.01%	0.02%	0.05%	0.01%	0.02%	0.04%	0.02%
Japan					0.03%	0.06%	0.35%	0.53%	0.63%	0.68%	0.58%	0.59%
Korea							0.02%	0.04%	0.05%	0.09%	0.21%	0.34%
Netherlands					0.01%	0.02%	0.16%	1.02%	5.38%	3.89%	9.74%	6.39%
Norway			0.01%	0.22%	0.15%	0.31%	1.33%	3.27%	6.00%	13.71%	23.63%	28.76%
Sweden						0.00%	0.05%	0.31%	0.53%	1.44%	2.37%	3.41%
United Kingdom	0.01%	0.01%	0.02%	0.01%	0.01%	0.01%	0.06%	0.13%	0.17%	0.6%	1.11%	1.41%
United States	0.01%			0.01%		0.01%	0.17%	0.44%	0.75%	0.74%	0.67%	0.91%
Others	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.04%	0.06%	0.10%	0.21%	0.38%	0.52%
Total	0.00%	0.00%	0.00%	0.01%	0.01%	0.01%	0.10%	0.23%	0.38%	0.54%	0.85%	1.10%

As can be seen from Table 1.1, EV stock increased rapidly from 2005 to 2016. However, compared with GVs, the market share of EVs is still very low, as shown in Table 1.2. The market shares for most countries are less than 2%, except those of Norway, the Netherlands, and Sweden, although governments and vehicle manufacturers are expected to stimulate diffusion and market share in the very early years. For example, in 1991, a group called the United States Council for Automotive Research LLC (USCAR) was established by the Chrysler Group LLC, the Ford Motor Company, and General Motors to conduct research and develop an energy system for EVs and hybrid EVs and thus advance sustainable transportation. The low diffusion of EVs may due to EVs' high price, lower mileage and under-developed CS networks. As Reed (2010) said, Consumers around the world are interested in buying electric cars, but they are unwilling to pay a higher price for them and are not satisfied with the performance of currently available models. This motivates researches to develop strategies promoting the uptake ot EVs, such as by creating incentive policies and speeding up the construction of charging networks.

1.1.4 EV subsidy policy

In this study, the author selects four countries or economies as examples—the USA, the European Union (EU), China and Japan—to elaborate the incentive policies and promote EVs.

Subsidy policy in the USA

The US Department of Energy (DoE) has supported research and development on innovative vehicle and energy-ecient transportation technologies at national laboratories and associated industry enterprises (Abkemeier et al., 2012). In terms of developing a pure EV company, Tesla, co-financed by Martin Eberhard and Marc Tarpenning and developed as a result of Elon Musk's involvement, is the first company to sell fully electric sports cars worldwide. The Tesla Model S is ranked as the world's second best selling BEV, with a sales volume of 31,655 in 2014 (Cobb, 2015*a*). Its global sales passed 90,000 by October 2015 (Cobb, 2015*b*).

As a result of the 2008 financial crisis, the US Congress approved the Bush Administration's $700 billion bailout plan, the Emergency Economic Stabilization Act of 2008 (Clark, 2008; Lawder, 2008). This plan included a tax breaks for each PHEV to a maximum of $7500. By 2009, the Consumer Assistance to Recycle and Save Act was implemented. The US federal government allocated $3 billion as a subsidy for vehicle customers to encourage them to replace old cars with an environmentally friendly fuel-efficient car (Administration et al., 2009).

Table 1.3: The subsidy upper limit for PHEV and PEV

Gross Vehicle Weight (GVW/pounds)	Upper limit ($)
GVW≤10000	7500
10000<GVW ≤14000	10000
14000<GVW≤26000	12500
GVW>26000	15000

For the non-PHEVs, based on the Energy Policy Act of 2005 (Energy Policy Act, 2005), the Internal Revenue Service (IRS) allows a tax deductible amount. Specifically, if the maximum output of an electric power driven system is more than 5% of the total power, then the amount will range from $250 to $3400. Moreover, in order to avoid automotive companies' dependence on subsidies, government regulations set out that: the first 60,000 hybrid vehicles will be eligible for the whole subsidy. After selling those vehicles, the subsidy will be decreased to 50% in the subsequent two seasons and to 25% for the following two seasons. While after that, the subsidy will be withdrawn.

For PHEVs and pure EVs, if the battery power is no less than 4 kWh, the basic exemption limit is $2500 plus $417 per kWh for the extra power. The upper bound is shown in Table 1.3 (China Automotive Technology and Research Center et al., 2013). Government regulations for plug-in and pure EVs are similar to those for the non-plug-in vehicles, but the cumulative number of sales figure is 250,000 instead of 60,000.

Subsidy policy in the EU

The EU is an alliance of independent countries in Europe comprising 28 member countries. The EU cannot introduce an energy policy with a unified law. Therefore, the European Commission, which, among other things, is responsible for proposing legislation can only suggest goals and help direct the development of all countries and new energy companies. The EU is interested in both hybrid vehicle and EVs, particularly zero-emission vehicles. Transport solutions involving renewable and non-pollution energy sources to address climate change are attracting increasing attention in the EU (Abkemeier et al., 2012). The BMW i3, a plug-in model, is the first zero-emission BMW vehicle to be widely sold worldwide. It has a 25 kWh electric motor, and the mileage is approximately 150 km. It was the Green Car of the year in 2014 and to date, the sales volume is approximately 38,600 units (BMW Group, 2015*a*,*b*).

Most European Union countries provide subsidies for EV users or manufacturers. In Austria, EVs are exempt from fuel consumption tax. While hybrid vehicles and other low-emission vehicles are also exempt from fuel consumption tax and qualify for as much as €800 subsidy (Hockenos, 2011). In Denmark, EVs that weigh under 2,000 kg have been exempt from the new car registration tax since 1985. The government also provides free parking in the centre of Copenhagen for EVs, which unfortunately does not apply to HEVs (European Automobile Manufacturers Association, 2014; Dealbook, 2009). France provides a subsidy of no more than €5000 for cars with CO_2 emissions less than 60 g/

km, no matter whether they are EVs, HEVs, or PHEVs. If CO_2 emissions are less than 125 g/ km, then the maximum subsidy granted before August 2012 was €2000. However, the subsidy has been decreasing. As of 2013, the subsidy is €5000 if the CO_2 emissions are in the range of 20–50 g/km, while a €4500 subsidy is given if the emissions are 50–60 g (ABC Carbon, n.d.; European Automobile Manufacturers Association, 2014).

In Italy, EVs are exempt from the annual circulation tax or ownership tax for five years from the date of their first registration (European Automobile Manufacturers Association, 2014). The incentive in the Netherlands is exemption from the registration fee and road tax for EVs, low-emission hybrid cars. The government also offers €3,000 subsidy to the EV charging infrastructure construction. If a vehicle is operated in Amsterdam, Rotterdam, The Hague, Utrecht and the Arnhem-Nijmegen metropolitan area, the subsidy for charging infrastructure is increased to €5,000 per vehicle (AutomotiveWorld, 2014). In Norway, the government's goal is that there should be 50,000 zero-emission cars on the road by 2018. EVs are exempt from all non-recurring vehicle fees, including purchase taxes, which are extremely high for ordinary cars. They are also exempt from the 25% VAT on purchase (Valøen and AS, 2012).

The incentive policy in Germany is different than in most other countries. From early May 2010, the German government decided to provide a budget to invest in research and development. As the policymakers thought that only providing a subsidy to customers was not conducive to the development of the while automotive industry. The Nationale Plattform Elektromobilitt (NPE) covers EV products and solutions with its international standards dealing with EV technology (drive, batteries, materials, recycling, charging technology), while also emphasizing resource and battery recycling technology (Nationale Plattform Elektromobilitaet, n.d.). Furthermore, besides R&D support in EVs, projects connected to electricity supply and transportation communication network techniques are also encouraged. The German Federal Ministry of Economics and Technology and the Federal Ministry for the Environment have invested €47 million in these projects. One of the projects involves Vehicle-to-Grid (V2G) and Grid-to-vehicle (G2V) system that realize two-directional flows of electricity both from the grid to batteries and from batteries to the grid (China Automotive Technology and Research Center et al., 2013). In terms of tax relief, Germany focuses on carbon dioxide emissions and engine displacement. Taxes on EVs are nil in the first five years.

EV incentive policies are also strong in the UK. Nissan's Sunderland plant was granted £20.7 million of subsidies from the UK government and up to £220 million from the European Investment Bank. By November 2013, the British government had guaranteed £400 million to support the development of the plug-in EV industry from 2010 to 2015. However, as of the end of September 2013, only £2 million had been spent and an additional 82 million allocated for R&D, 16 million for infrastructure, such as public charging points, and £25 million on consumer purchase incentives (Foy, 2013; Kane, 2013).

Incentive plans applied to plug-in EVs in the UK started in January 2011, providing subsidies for new vehicles for a maximum of 25% of the PEV price, with an upper bound of £5000 (Green Car Congress, 2011). From April 2015, the maximum was raised to 35% of the sale price with the same upper bound price of £5000. The government decided that this policy would end in 2017 or when 50,000 incentive cars were sold (Kramer, 2015). However, forecasting found that the 50,000 limit would be reached by November 2015, meaning that the government had to make available a minimum total of a £200 million to continue this policy. The basic terms and conditions for vehicles to which this subsidy would apply are as follows: (1) The CO_2 emissions must be less than 75 g/km. (2) The minimum range is 110 km for a pure EV and 16 km for PHEVs powered by electric motors. (3) The minimum speed is 97 km/h (UK Government, 2015). By early 2012, the incentive policy was extended to vans. For vans, the subsidy is 20% of the sale price, with an upper bound of £8000. The London congestion charge is not applied to EVs or PHEVs, if the vehicle is registered with the Driver and Vehicle Licensing Agency (DVLA) and has a fuel type of 'electric' or if the vehicle is a 'plug-in hybrid' and is on the Government's list of PHEVs eligible for the Office of Low Emission Vehicles (OLEV) grant (Transport for London, n.d.).

Subsidy policy in China

China is one of the fastest developing economics in the world. Therefore, the quality and reliability of PHEVs has improved significantly, but pure battery EVs are still the main sellers. The current situation of the Chinese EV industry is that, the whole R&D capacity, especially battery and electric motor techniques, has increased; however,related electronic chip design is still not strong enough. In addition to the technology, EV business model innovation also plays a greater role in promoting the popularization and application of EVs. Moreover, the instalment of private and personal charging points in most cities is relatively slower than the instalment of public charging infrastructure. The government is still improving the policy environment, and support has intensified (China Automotive Technology and Research Center et al., 2013). In China, one of the largest EV manufacturing companies is BYD. As BYD was previously a lithium battery manufacturer, it is accomplished at EV production whether BEVs or PHEVs. The three famous models are BYD Qin, BYD Tang and BYD E5.

The first subsidy policy for EVs was on 23 January 2009 and was only applied to public institutions. The first subsidy documents were published in May 2010 on the Notice on Private Purchase of New Energy Vehicles to Subsidise Pilot (Ministry of Finance of the PRC, 2010) and Private Purchase of New Energy Vehicles Pilot Financial subsidies for Interim Measures (Ministry of Industry and Information Technology of the PRC, 2010). The subsidy policy was first piloted in six cities, which were Shanghai, Beijing, Changchun, Shenzhen, Hangzhou, and Hefei.

In regard to reducing the vehicle purchase tax, the nineteenth meeting of the Standing Committee of the 11th National People's Congress adopted the People's Republic of China Travel Tax Law on 25 February 2011, which emphasised that travel tax for the vehicles that use new energy sources can be reduced or exempted. In 2012, the Ministry of Finance, the State Administration of Taxation and the Ministry of Industry and Information Technology released the Notice on Energy Conservation, the Use of New Energy Transportation Travel Tax Policy (State Administration of Taxation of the PRC, 2012), which states the travel tax for transportation vehicles should be halved in order to save energy and that new energy transportation tools should be exempt from the travel tax. The Notice of Public Transit Enterprises Exempting from Vehicle Purchase Tax When Purchase Public Vehicles (State Administration of Taxation of the PRC, 2015) was also released, exempting gas-electric vehicles from the purchase tax for public transit enterprises (including various types of electric buses) from 2012 to the end of 2015.

Table 1.4: The EV subsidy standards in China

Type of vehicle	Mileage driven by electricity (R/km)			
	80≤R<150	150≤R<250	R≥250	R≥50
Pure electric vehicle (RMB)	35000 (£3500)	50000 (£5000)	60000 (£6000)	-
Plug-in hybrid electric vehicle(RMB)	-	-	-	35000 (£3500)

There are three main kinds of subsidies: direct purchase, vehicle leasing and battery leasing. The central government will give a subsidy to EV manufacturers so they can sell or rent EVs to customers or leasing companies. In the case of battery leasing, the government first gives the subsidy to the battery leasing company and they will rent batteries at a reduced price and also provide battery maintenance, and replacement services. Furthermore, the local financial department in different cities should arrange certain funds, focusing on sup-porting the construction of CSs and other infrastructure, the purchase of new energy vehicles and the repurchasing of batteries.

The subsidy standards before 2013 were as follows: 1. The subsidy for a hybrid vehicle (excluding plug-in hybrid vehicles) that the displacement of which is less than 1.6 L and that satisfy state-four emission standards is RMB 3000 (£300); 2. The subsidy for plug-in EVs is RMB 4000-50000 (£400–5000); 3. The subsidy for a pure EV is RMB 60000 (£6000). The 2014 subsidy was reduced by 5% based on the 2013 standard, while the subsidy in 2015 was reduced by 10% compared with 2013 (The Central Peoples Government of PRC, 2014) . The details of the subsidies are shown in Table 1.4 above.

The first list of EVs subject to a reduced vehicle purchase tax was released in August 2014 and until now, more than 100 EVs have been in the shortlist for reductions. Most of them are made in China; however, more and more imported cars will be listed to reduce the vehicle purchase tax.

Another policy concerns vehicle licensing. In China, in order to solve urban road congestion and control the growth of car ownership, a policy limiting vehicle licensing has been implemented in several cities (Shanghai, Beijing, Guangzhou, Tianjin, Guiyang, Hangzhou and Shenzhen). There are two main methods in this policy. Shanghai uses a car license auction, while Beijing uses a ballot method. Other cities use a mix of auctions and lotteries, such as the 50% auction and 50% lottery system in Shenzhen. However, customers have to pay to wait or for other costs. For instance, the success rate of the license-plate lottery in Beijing for August 2015 was only 0.52%, which equals 1/191, while the average auction price in Shanghai in July 2015 was 83,000 RMB, which is approximately equivalent to £8300. Fortunately, in Shanghai a new energy vehicle can be licenced for free. Beijing, the success rate is 38.1% (around 19/50) (Sohu, 2015). Other cities that implement the policy have been very loose in applying it to new EVs. In the State Council executive meeting held on 29 September 2015, Premier Keqiang Li said that the restriction should not be implemented because of the limit diffusion of new energy vehicles until the target been achieved.

Subsidy policy in Japan

Japan is a small island country that has few natural resources. Therefore, the government is paying close attention to the development of new energy vehicles. Japanese electric automotive industry has formed a part of industrialisation in the battery R&D and hybrid cars aspect. Hybrid EVs made by Toyota, Honda and Nissan are not only famous in Japan but also sell worldwide. In terms of battery manufacturing, there are about 20 global companies in the field including A123 (acquired by the Wanxiang Group in China in 2012), Johnson Controls in America, BYD in China and LG Chemical in South Korea. However, in Japan alone there are the Nippon Electric Company (NEC), Toshiba, Hitachi, GSYuasa, and Panasonic EV Energy associated with Toyota Motor and Matsushita Electric among others.

Nissan Leaf (LEAF) was the sales champion before May 2013 for the global BEV market, with 65,000 vehicle being sold (Nissan USA, 2013). This model also received the 2010 Green Car Vision Award, the 2011 European Car of the Year, the 2011 World Car of the Year, and the 2011/2012 Car of the Year Japan.

Japan's central government and local governments have provided support to purchase EVs, to provide charging infrastructure and to facilitate business model innovation by offering financial subsidies, tax breaks and municipal plans relating to pure EVs and PHEVs (China Automotive Technology and Research Center et al., 2013). Starting with

2009, the Japanese government has invested JPY 370 billion (£2025.3) to support the purchasing of new energy vehicles. Green taxation was implemented on April 1, 2004 for pure EVs, clean diesel vehicles, hybrid vehicles and approved low-emission vehicles. The specific policy is as follows. In cases of replacing old vehicles and buying a new car that meets the 2010 emission stadnards, the subsidy is JPY 250,000 for ordinary cars, JPY 125,000 for lightweight passenger cars (i.e. the cars length is less than 3.3 m, wide is less than 1.4 m, height is less than 2 m and displacement is less than 660 cc) and from JPY 400,000–1,800,000 for buses and trucks. In cases of not replacing used vehicles, if cars with emissions more than 15% of the 2010 pollution emission standards are bought, the subsidy is JPY 100,000 for ordinary cars and JPY 50,000 for lightweight cars. Furthermore, if the newly bought car's emission level is 75% less than the 2005 standard and 25% or 15% less than the 2010 standard, then the motor vehicle tax and vehicle purchase tonnage tax will be decreased to 25% or 50% (Japan Automobile Manufacturers Association, 2009; Kuruma, 2013).

The charger is also a subsidy item to promote the EV. Detailed information of subsidy amount is shown in Table 1.5 (China Automotive Technology and Research Center et al., 2013).

Table 1.5: Charger's subsidy limit in Japan

Kind of charger	Output power	The amount limit of subsidies (yen)
Fast charger	>50kW	1500000 (£10062)
	40∼50kW	1250000 (£8837)
	30∼40kW	1000000 (£6710)
	10∼30kW	750000 (£5032)
Common charger	High performance smart charger	400000 (£2684)
	General performance charger	200000 (£1341)

1.1.5 Charging station status quo

Charging infrastructure is indispensable for operating EVs, whether at public locations, at workplaces or at home. According to the International Energy Agency (2017), the availability of chargers has emerged as one of the key factors contributing to the market penetration of EVs. Ensuring the availability of chargers is also essential to enable the diversification of the transport fuel mix and to catalyse its transition towards clean energy.

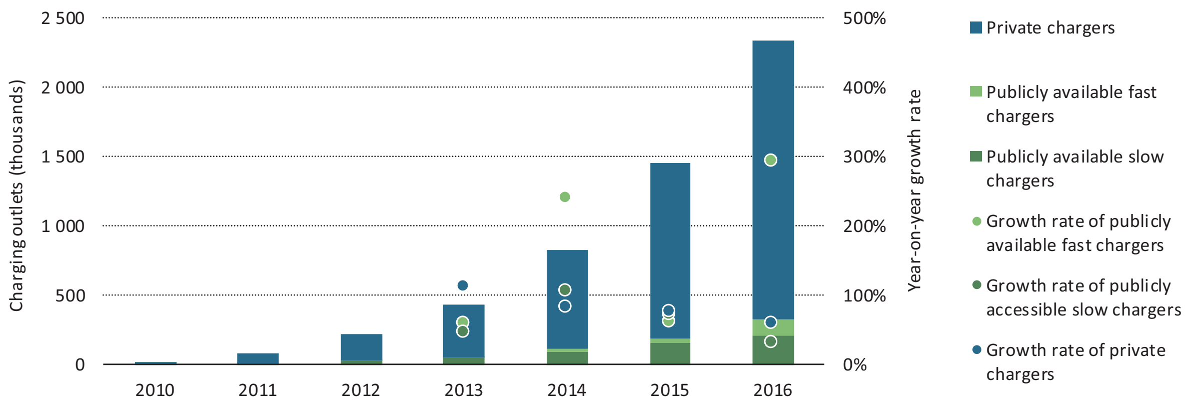

Fig. 1.1: Global EV charging station outlook 2010-2016

Figure 1.1 shows the EV CS outlets from 2010 to 2016 (International Energy Agency, 2017). It is obvious that the higher rate of growth for EV chargers compared to the EV growth rate is consistent with the need to deploy CS networks.

China has been the fastest growing country in terms of charging infrastructure construction. By the end of 2016, there were more than 150,000 CSs operating in China, which is the country (or economy) with the largest EV ownership, as can be seen in Table 1.6.

Table 1.6: Amount of public charging stations

Country	Japan	USA	EU	China
The number of public EV charging stations	22,000	40,000	106,000	150,000

Government subsidies have also been offered for charging facilities. For example:

- In the United Kingdom, the grant is set as a 75% contribution to the cost of one chargepoint and its installation fee with a cap of £500 (including VAT) per eligible EV (Office for Low Emission Vehicles, 2016).

- In the US, the US Department of Energy updated the electricity grid to adapt to the cgarhing demand with an investment of \$10 million.The government has also provided a subsidy that equals 30% of the total investment, no matter whether the investor is an individual or a company, with an upper limit of \$1000 for an individual or \$30,000 for companies. Clean Cities Coalition Awards are also worth \$8.5 million to support new energy vehicles and infrastructure (U.S. Department of Energy, 2001). The government also offered a 30% tax credit against the cost of EV charging equipment, which includes both conventional corded and wireless home CSs. Stations installed before the end of 2016 may qualify for up to a $1,000 tax credit for individuals. Businesses could recoup up to $30,000 in charger and installation costs (Yamauchi, 2017).

- Accroding to Metering (2017), the German government is going to implement a project to enhance the country's EV charging infrastructure, which is expected to cost €300 million ($315 million) and will be deployed over the next four years. This is open to German auto giants Volkswagen, Daimler and BMW, as well as all other national and foreign brands.

- In China, most cities have set subsidies for EV chargers. Some cities provide a 20–30% subsidy for the charger (e.g. Chengdu and Wenzhou), and some cities pay the subsidy based on the power of the charger. For instance, Shanghai plans to install 211,000 CSs by 2020. For direct current CSs, the subsidy is RMB 600/kWh, while the subsidy is RMB 300/kWh for alternating current CSs (Wang, 2017).

1.1.6 Current battery recycling situation

The battery is one of the key parts in EVs. Increasing EV sales means the rapid growth of vehicle power batteries. However, the disposal of waste batteries may increase resources and environmental issues. From the resource point of view, the manufacture of power batteries requires nickel, cobalt, lithium and other elements that are scarce and difficult to mine. From an environmental point of view, the components for waste power batteries are complex, and random disposal will cause environmental pollution and threaten human health. Therefore, reasonable recycling of waste power batteries can help alleviate the issue of limited resources and minimize environmental pollution and damage to human health.

Normally, an EV battery should be removed when its capacity falls to 70–80% (Arcus, 2016). In order to maximize a battery's usage efficiency, the used battery can be reused (cascade utilization) if it is still in good condition. Otherwise, it must be recycled directly through dismantling and smelting.

In many countries, as with normal batteries, used automotive batteries are not recommended to go into landfills or to be incinerated. In response to this regulation, many EV battery collection and recycle schemes have been set up. For instance, in North America, Tesla, working with Kinsbursky Brothers, recycles about 60% of its battery packs, In Europe, Tesla started working with Umicore on recycling (Kelty, 2011). Nissan and Volkswagen require their EV customers to return used batteries to licensed depots or via local authority battery collection schemes (Nissan, 2015; Volkswagen, 2016). Most used batteries are recycled into materials rather than being reused. It is gratifying that more and more EV manufacturers are starting to reuse EV batteries: BMW and Nissan are expected to reuse returned batteries as home energy storage systems (Ayre, 2016; Dalton, 2016). In Michigan, Chevrolet has set up an energy storage station using old EV batteries at the General Motors facility (Voelcker, 2016). However, our preliminary search of the literature suggests that there is little research examining this topic.

1.1.7 The big map

A big map of the EV triple SC is shown in Fig. 1.2 below. There are three sub-SCs: the EV production and sales SC, the energy SC and the battery closed-loop SC (CLSC). The customer is the core of this big map, while the government is a kind of policy maker for the whole triple SC.

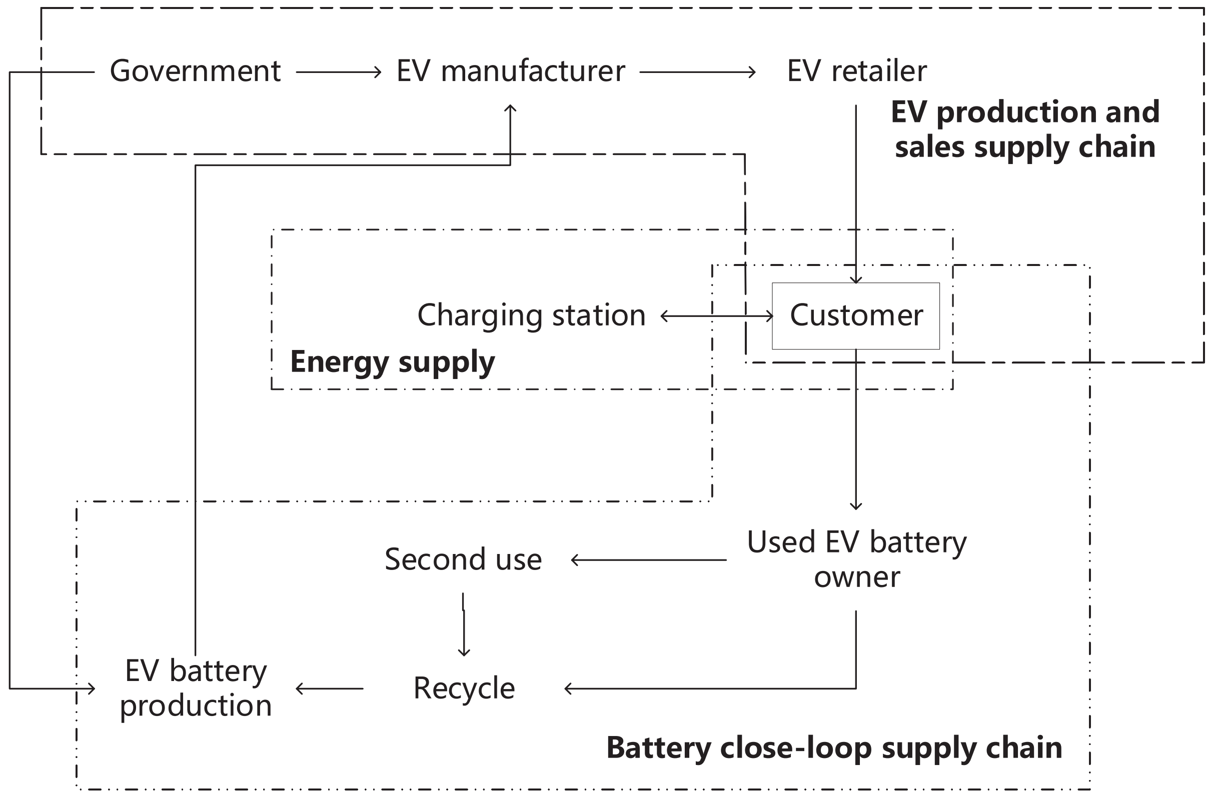

Fig. 1.2: The electric vehicle triple supply chain

1.1.8 Challenges

Based on the big map of the EV triple SC, the author suggest there are three challenges:

(1) Regarding the vehicle SC, in promoting EV production and sales, most incentive policies only have a general perspective. The impact of subsidies on both the entire EV SC and the players in the SC, particularly the optimal amount of subsidies and the impact on the SC, requires further study.

(2) Regarding the energy SC, the EV CS network is currently developing rapidly. How to improve the efficiency and profitability for both EV users and service providers on requires more research.

(3) Regarding the EV battery CLSC, although the EV and EV battery industries are in a period of rapid growth, the commercial operation of cascade utilization of used EV batteries is underdeveloped. Therefore, some research should be conducted in this respect. For example, a business model combining the second use and recycling the returned EV batteries can be designed to study optimal utilization.

1.2 Research aims and objectives

Based on the challenges above, the aim of this research is to map the relationship and interaction among the EV SC, the energy SC and the used EV battery CLSC. In order to study and optimise the dynamic performance of the triple SC, this research will:

(1) Investigate the impact of government incentives for end users on the EV SC (i.e., the optimal amount and allocation of subsidies).

(2) Design schemes to improve the use and efficiency of CS networks.

(3) Develop strategies to improve the system performance and profitability of EV battery reuse and the recycling SC.

This research will mainly study pure BEVs. Based on the discussion in Section 1.1.1 and according to Björnsson and Karlsson (2017), it is likely that BEVs will perform better compared to PHEVs over time. Therefore, for sustainability reason and because the price for battery energy capacity is anticipated to decrease, the automotive industry will likely focus on developing pure EVs in the future.

1.3 Thesis structure

A brief overview of the structure of this thesis and how each chapter connects to the research is given below.

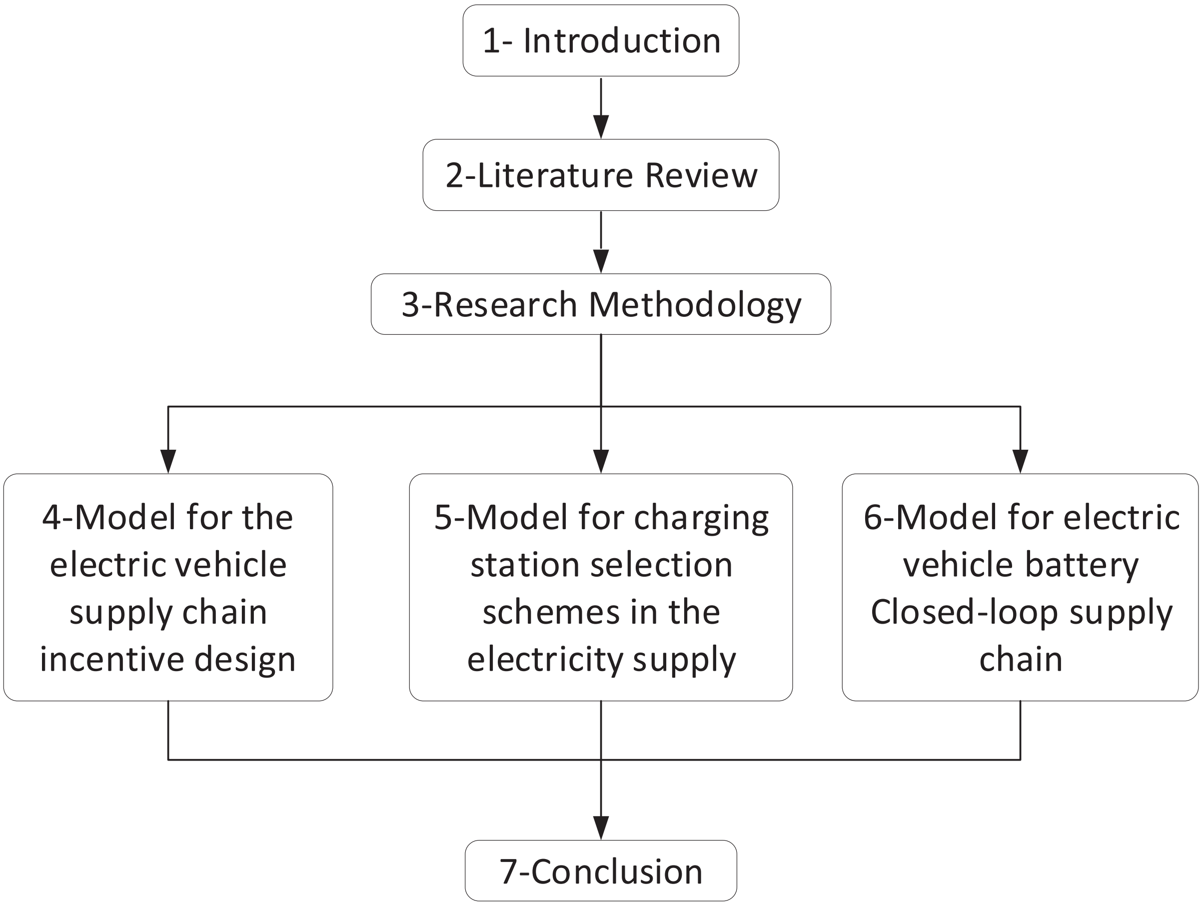

Fig. 1.3: Schematic of this thesis

Chapter 1: Introduces background information relating to the EV SC and the situation regarding EV subsidies, CSs and EV battery recycling. The industry challenges and research aims are then formulated.

Chapter 2: Contains the literature review that provides an overview of previous research in the three topics: EV subsidies, CS networks and the EV battery recycling SC. It then identifies the research gaps in the three main topics.

Chapter 3: Outlines the research methodology used to conduct this research-including the research philosophies and paradigms, the research design, the methods, and the tools used.

Chapter 4: Proposes a SC model involving government, the GV/EV stakeholders, and the end customer to study subsidy policies in the EV SC.

Chapter 5: Presents three CS selection schemes for the EV driver and discusses their performance.

Chapter 6: Details a multi-period CLSC model for EV battery manufacturing, return, reuse and and remanufacturing. Two conditions are discussed which are optimize the entire CLSC and optimize each part, the manufacturer and remanufacturer, in the SC. The optimal profit is also analysed for each period.

Chapter 7: Concludes the whole thesis. This chapter summarizes the contributions of the research to academia and the industry. Finally, the limitations and potential further research opportunities are discussed.

1.4 Summary

This chapter has provided a brief history of EVs, the EV sales status quo and a review of the subsidy policies, the CS networks and EV batteries, as the background information of this research, as well as the research aims and objectives. The structure of this thesis and a summary of the chapters' contents were also given. The next chapter will provide more context for the thesis via a literature review.

1.5 List of the publications over the PhD study period

This section lists publications over the author's PhD research period. They are classified as journal papers and conference papers, ordered by the publication date.

1.5.1 Papers published in academic journals

- Gu, X., Ieromonachou, P., Zhou, L., 2017, Optimizing Manufacturing and Remanufacturing Total Profits in an Electric Vehicle Battery Closed-loop Supply Chain, *Journal of Cleaner Production* (Minor revision).

- The right format: Xiaoyu Gu; Petros Ieromonachou; Li Zhou; MingLang Tseng, (2018) Optimising Quantity of Manufacturing and Remanufacturing in an Electric Vehicle Battery Closed-loop Supply Chain, *Industrial Management and Data Systems*, Vol.118(1), pp.283-302, doi:10.1108/IMDS-04-2017-0132..

- Li Zhou, Jiaping Xie, Xiaoyu gu, Yong Lin, Petros Ieromonachou (2016), Forecasting Return of Used Products for Remanufacturing Using Graphical Evaluation and Review Technique (GERT), *International Journal of Production Economics*, 181(B), 315-324, doi: 10.1016/j.ijpe.2016.04.016..

1.5.2 Papers published in academic conferences

- Gu, X., Ieromonachou, P., Zhou, L., An Imperfect Information Game in Subsidizing Electric Vehicle Supply Chain, Proceedings of *the 20th International Working Seminar on Production Economics*, Innsbruck, Austria, February 19-23, 2018.

- Gu, X., Ieromonachou, P., Zhou, L., 2017, A three-period model for returned electric vehicle batteries. *8th International Congress on Transportation research*, The Hellenic Institute of Transportation Engineers and the Hellenic Institute of Transport, Greece

- Gu, X., Ieromonachou, P. and Zhou, L., 2016, July. A game theory approach in subsidizing electric vehicle supply chain. *2016 International Conference in Logistics, Informatics and Service Sciences (LISS)*, IEEE, Beijing, pp. 1-7, doi: 10.1109/LISS.2016.7854375

Chapter 2

Literature Review

This chapter will provide an overview of previous research related to the themes of this thesis. Incentive policy and subsidy design for the EV SC; CS network design and the EV battery CLSC. Moreover, this chapter will identify the research gaps related to the research aims in the previous chapter. Some relevant definitions are given first. Next sections 2.2, 2.3, and 2.4 will review the three themes and defines the research gap.

The literature review was conducted using keyword searches of multiple databases, including Science Direct, Emerald, Scopus, and IEEE Xplore. Google Scholar was also used to locate some papers and reports. The author used the following keywords: 'subsidy', 'industry', 'supply chain' , 'electric vehicle', 'electric car'. 'charging station', 'charging station network', 'charging station selection', 'allocation', 'EV battery', 'recycling', 're-manufacturing', 'closed-loop supply chain' and 'reverse logistics'.

2.1 Definitions

This section will provide a quick overview of key definitions and terminologies used frequently throughout the thesis.

2.1.1 Industrial economics

The concept of industrial economics emerged in the early 1950s in the writing of Andrews (Katzenstein, 1985). It adds some complications to the perfect competitive model, such as transaction costs, limited information, and barriers to the entry of new firms—that may be associated with imperfect competition (Coase, 1937). Industrial economics analyses the behaviour of firms and the market based on government actions (Durlauf et al., 2008; Devine et al., 2018). In summary, industrial economics is the study of interactions among enterprises, the development of industry, the interactions between industries and the distribution of industry (Devine et al., 2018; Fishman, 1975). In this thesis, the models will study the relationship among departments in the EV triple SC based on industrial economics.

2.1.2 Operations research

According to Taha (2005), operations research (OR) is a discipline that helps in making better deterministic and probabilistic decisions using a series of analytical methods. Using mathematical modelling, statistical analysis and mathematical optimization, OR aims to find optimal or near-optimal solutions for complex problems. OR is often concerned with determining how to maximize profit and performance or how to minimize the cost and the loss associated with some real-world objective (Winston and Goldberg, 2004; Hillier, 2012). In this thesis, OR will be used to find the optimal solution in the most efficient way for the triple SC.

2.1.3 Supply chain management

Generally speaking, an SC is 'a system of organizations, people, activities, information, and resources involved in moving a product or service from supplier to customer which involves the transformation of natural resources, raw materials, and components into a finished product that is delivered to the end customer' (Nagurney, 2006).

In this thesis, an SC is narrowly defined as 'the network of organisations that are involved, through upstream and downstream linkages, in the different processes and activities that produce value in the form of products and services delivered to the ultimate consumer' (Christopher, 2016). The objective of SC research is to 'coordinate activities within the supply chain to maximize the supply chain's competitive advantage and benefits' (Heizer, 2016).

2.1.4 Closed-loop supply chain

A CLSC, also called reverse logistics, is the process of 'moving goods from their typical final destination for the purpose of capturing value, or proper disposal. And remanufacturing, refurbishing and recycling activities is also included in the definition of reverse logistics.' (Hawks, 2006). Through reverse logistics, the SC will be completed as a loop. More and more studies are dealing with remanufacturing, refurbishing and recycling to foster green and environmentally friendly aims and economic benefit in SC management. For example, vehicle tire manufacturers in Europe arrange for the recycling of one used tire for every new tire they sell. Kodak remanufactures its single-use cameras after the film has been developed. In the 1990s, the company recycled more than 310 million cameras in more than 20 countries (Guide and Wassenhove, 2002).

2.1.5 Subsidy

According to Myers (2001), a subsidy is a form of financial aid or support extended to an industrial economic sector (or institution, business or individual), generally with the aim of promoting economic and social policy. Normally, a subsidy is given to the manufacturer or the consumer (Myers, 2001). Although there are many reasons for subsidies, the subsidies researched here seek to improve the competitiveness of the product. For example, in order to promote the diffusion of EVs, some subsidy policies need to be employed.

2.1.6 Game theory

Game theory is 'the study of mathematical models of conflict and cooperation between intelligent rational decision-makers' (Myerson, 2013). The idea of game theory appeared very early on the first known discussion of game theory can be found in a letter written by Charles Waldegrave, a British diplomat, in 1713 (Bellhouse, 2007). In the letter, Charles provides a mixed strategy solution to a two-person version of a card game known as the Waldegrave problem. However, game theory did not really exist as a unique field until John von Neumann published a paper in the 1900s on the theory of games and economic behaviour (Neumann et al., 1953).

The Nash equilibrium is one of the key foundational concepts in game theory (Nash et al., 1950; Nash, 1951, 1953). If each player has chosen a strategy and no player can benefit by changing strategies while the other players stick to their decisions, then the current set of strategy choices and the corresponding payoffs constitute a Nash equilibrium. In this thesis, game theory and the Nash equilibrium will be used to study the "best response" to the departments in the triple SC.

2.1.7 Information pass-through effect

According to Graham and King (1996), the information pass-through effect means that before the actual transaction occurs in the market, one of the two parties has information and the other party lacks information. When the party with superior information sends some relevant information to the party lacking information, it is possible for the latter to analyse the situation and make a better decision. As stated in Andrew and Dollery (1990), in the SC, the information normally passes from the manufacturer (as the upstream player with more information) to the retailer (who has less information) to the entire SC.

2.1.8 Rebound effect

In conservation and energy economics, the rebound effect is the reduction, which may result from changes in behaviour or other actions, in expected gains from new technologies that increase the efficiency of resource use (Brookes, 1992; Grubb, 1990). For example, the rebound effect is 60%, if with a 5% improvement in vehicle efficiency, there is only a 2% increase in energy using utility.

2.1.9 Planned behaviour

In psychology, the theory of planned behaviour links one's beliefs and behaviour (Ajzen, 1991). It states that attitudes toward behaviour, subjective norms, and perceived behavioural control together shape an individual's behavioural intentions and behaviours. In this research, based on the theory of planned behaviour, when customers decide whether to buy a GV or an EV, they will examine their attitude towards environmental awareness and preferences, their acceptance of new technology, the use cost, convenience, etc (Liao et al., 2017).

2.2 Review of electric vehicle supply chain incentive policy research

This section first reviews some relevant research in about EV subsidies and then discusses some research gaps. Some detailed research questions are given at the end of this section.

2.2.1 Review

First, some subsidy models are reviewed. According to our research, most subsidy models use the System Dynamics (SD) or the Game Theory model. SD comprise 'a perspective and a set of conceptual tools that enable us to understand the structure and dynamics of complex systems' (Sterman, 2000). Based on the close mutual dependency in the internal mechanism of system behaviour, SD studies causal relationships by establishing and employing a mathematical model. Briefly, it is a process-oriented research method using causal loop diagrams and stock and flow diagrams as techniques to frame, understand, and discuss complex problems or issues. Currently, it is widely used by managers and stakeholders for policy analysis and design purposes (Radzicki and Taylor, 2008). In the field of subsidy incentive research, Jeon et al. (2015) designed an SD model to capture the complex interactions among investors, consumers and policymakers, as well as future uncertainties of key energy, economic and environmental factors to optimize financial subsidies and public R&D investments to achieve the Korean photovoltaic (PV) diffusion target. Wang et al. (2014) studied the impact of recycling and remanufacturing in China. They introduced four subsidy policies—an initial subsidy policy, a recycling subsidy policy, a R&D subsidy policy and a production subsidy policy. They then built SD models relating to manufacturing and remanufacturing subsystems, taking the subsidies into consideration. Chang et al. (2013) simulated the effect of government subsidy policies on the installation of solar water heaters in Taiwan by considering factors such as product type, safety, energy awareness, raw material prices, R&D and its associated investment costs, installation costs, duration of sunlight and system cost subsidies.

Some researchers have used the game theory to study and design their subsidy models. The benefit of applying game theory to incentive subsidy research is that it can be used to find the optimal equilibrium point as the most reasonable course of action amongst game players, no matter whether they are in a cooperative or a non-cooperative (or competitive) situation. For instance, Chen et al. (2016) studied subsidy contract design problems using game theory. In their game, the players are a telecom service operator (TSO) and a mobile phone manufacturer (MPM), and the non-cooperative game is applied both in a decentralized and a centralized mobile phone supply chain (MPSC). In this situation, the MPM makes a decision about the retail price of its phones and the TSO separately decides the cost of its service for customers. Other relevant studies are as follows: Kahil et al. (2016) developed a cooperative game framework to analyse water management policies, dealing with a case about a typical arid and semi-arid basin in south-eastern Spain. Janssens and Zaccour (2014) used the Stackelberg game method to investigate strategic price subsidies for new technologies within firms and government. They believed that 'the subsidy is not a parameter, but a variable to be minimized'. Dong et al. (2010) looked at the electroplating industry in China and analysed the implementation of cleaner production policies by local government for possibility polluting firms using the Nash equilibrium. Zhang et al. (2013) used evolutionary game theory to study the impacts of subsidy policies on vaccination decisions in contact networks.

Several other methods have also been used in incentive mechanism design problems. Kuiran et al. (2009) used gradient dynamics in their study. Jeanjean (2010) created a math model to define whether subsidies should be given to the customer side or the supply side. He found that in the case of broadband coverage speed consumer-side subsidies are more effective, while demand-side subsidies encourage investment in no-service areas. Tomohara and Ohno (2013) analysed the shirking model, the gift exchange model and the reciprocity model in relation to work incentives in real labour markets.

There is also a fair amount of research regarding the subsidy model in the EV and SC fields. Raz and Ovchinnikov (2015) used the newsvendor model with pricing to account for and analyse government performance concerning subsidies and rebates in coordinating the price and supply of EVs. They considered three mechanisms as incentive policies to achieve maximal welfare in a SC system that consists of a manufacturer/seller, consumers and the government. The three mechanisms are the joint mechanism that uses both subsidies and rebates and two simplified mechanisms that use only rebates or only subsidies. The evaluation criterion is a welfare function that consists of a firm's profit, consumer surplus, an external benefit and government costs. Using the example of the Chevy Volt, the results show that the welfare loss associated with the rebates-only mechanism is smaller than that associated with the joint mechanism or the subsidy-only mechanism. Zhang (2014) also used the newsvendor model to study the performance of EVs and internal combustion engine vehicles (ICEVs) and their influence on EV production decisions by observing customers trade-offs and governments subsidies. The main conclusions are: (1) The subsidies given by government 'should be worthwhile to reduce the environmental impact and to promote the energy efficiency from an environmental cost perspective'; and (2) In terms of expected utility aspects, a subsidy or high ICEV price may offset the impact of loss aversion, while a high EV price could heighten loss aversion.

Regarding EV SC management, Huang et al. (2013) and Luo et al. (2014) studied government subsidy and incentive schemes. Huang et al. (2013) analysed the subsidy incentive policy in a duopoly market setting with a fuel automobile SC and an electric-fuel hybrid automobile SC (just involving the manufacturer and the retailer). The Nash equilibrium of the whole sale price in these two supply chains is achieved when the government provides subsidies to promote the EV market. The conclusion is that subsidy incentives help increase EV sales and reduce the negative impact of GVs, that is, the environmental hazard. Based on this research, Luo et al. (2014) focused on the EV SC only. In the model, they investigated a SC where a manufacturer and a retailer serve heterogeneous consumers with EVs under a governments price-discount incentive scheme that involves a price discount rate and a subsidy ceiling. The optimal EV purchase discount rate and subsidy ceiling were determined using the Nash bargaining scheme cooperative game. The results show that the subsidy ceiling is more effective in influencing the optimal wholesale pricing decision of the manufacturer, resulting in a higher unit production cost.

The literature also contains several case studies/surveys regarding subsidy policy. Gnann et al. (2015) used alternative automobile diffusion and infrastructure models to analyse the market evolution of EVs in Germany until 2020. They concluded that future shares of PEVs (pure EVs) in Germany will range from 0.4% to almost 3% by 2020. In addition energy prices have a large impact on PEV market evolution, as a 25% increase in fuel prices would double the number of PEVs in stock by 2020. Sierzchula et al. (2014) used linear regression to analyse 30 national EV market shares in 2012. They concluded that financial incentives, charging infrastructure and the local presence of production facilities are significant and positively correlated to a country's EV market share. The results suggest that charging infrastructure is most strongly related to EV adoption. However, descriptive analysis suggests that high electric vehicle adoption rates cannot be assured. Shin et al. (2012) used an environmental impact case study in South Korea and concluded that offering purchase price subsidies is more helpful to environmental improvements and has more benefits than offering tax incentives.

2.2.2 Research gap and summary

The previous section reviewed some of the literature about subsidies, incentive models and about relational incentive models in the EV supply chain.

While EV manufactures and customers have been discussed, EV retailers have not received as much attention. In addition, while case study research has found that the energy price is an important parameter in relation to EVs, how fuel and electricity prices affect the EV industry has been quantitatively under-researched. Although various works have discussed different subsidy policies, optimal subsidy allocation in relation to the EV SC has been less studied, However optimal subsidy allocation has been studied in other industries (e.g. Internet infrastructure; see Chen et al. (2016)). But because of the different features and frameworks of different industries, the subsidy allocation in the EV SC specifically must be discussed

Therefore, based on research aim and objectives in Section 1.2, the author puts forward three research questions: (1) Can the retailer be involved in the EV SC incentive design; (2) How do the energy price and the electricity price impact the probability of purchasing

EVs and government subsidies; (3) How should the subsidies be allocated to the EV supply chain? These questions will be discussed in Chapter 4.

2.3 Review of electric vehicle charging planning and scheduling

2.3.1 Review

This section reviews studies relevant to the EV CS mapping, planning and scheduling aspects. Normally, the GV petrol stations have been developed so people can choose any station to refuel their cars without waiting a long time. However, as mentioned in the previous chapter, the EV CS network is less developed. Moreover, it normally takes a longer time (because of queueing time and charging time) to charge an EV.

Some studies have focused on the optimal location of EV CSs,such as those by Alegre et al. (2017); Wang et al. (2016); Brandstätter et al. (2017); Ge et al. (2011); You and Hsieh (2014); Alhazmi et al. (2017); Arslan and Karaşan (2016); Zhu et al. (2016). Ge et al. (2011) proposed a method of locating and sizing CSs for EVs based on the grid partition by minimizing users' loss on the way to the CS. They used a genetic algorithm to select the best location for each partition, considering traffic density and CSs' capacity constraints. In order to maximize the population coverage in the EV CS, You and Hsieh (2014) used a hybrid heuristic mixed-integer programming model. Alhazmi et al. (2017) simulated a new plug-in EV CS allocation model by considering trip distances and the EV's remaining electric ranges combined with driving habits, battery capacities, the state of charge and trip classes. Moreover, because of the peak and off-peak effect in using electricity, several researchers have discussed CS scheduling problems in terms of using a smart grid for supply-side management; see, for example, Hafez and Bhattacharya (2017); Awasthi et al. (2017); Alonso et al. (2014).

Research about EV CS optimization from the perspective of EV users (demand side) is lacking. Yet, there has been some relevant research: in order to improve the charging utility, Shinde and Swarup (2016) proposed a vehicle-to-grid (V2G) control strategy (which allows the EV to communicate with the power grid to sell demand response services by either returning electricity to the grid or by throttling their charging rate) to optimize the charging schedule for the demand side by valley filling and peak shaving. To enhance customer satisfaction, Yang et al. (2017) developed a heurstic algorithm and studied the EV swapping/CS network design problem using the leasing or electric car sharing service business model. Hu et al. (2011) proposed an approach to optimize the charging schedule of an EV fleet with the goal of minimizing the charging cost paid by costumers. Several studies have discussed the EV routing problem for charging. Lin et al. (2014) conducted a survey of the green vehicle routing problem. Artmeier et al. (2010) studied a model to help EV drivers find the shortest path while considering energy consumption. Goeke and Schneider (2015) researched charging time windows and a mixed fleet to optimize the EV routing problem.

2.3.2 Research gap and summary

This section has reviewed several models relating to CS location, scheduling, and planning. Current studies have also covered CS experiences on the demand side by using different methodologies. However, most use the simulation method to represent the model, which means theoretical research in this area is not as prevalent. Also, with the existed charging station, how to select an optimal one is less studied. Based on the research aim and objectives, the author proposes one research question: Is there any possibility of developing a theoretical model for charging station selection and how to evaluate the selection's performance? These questions will be discussed in Chapter 5.

2.4 Review of electric vehicle battery closed-loop supply chain

2.4.1 Review

This section reviews some of the research about EV battery recycling, and remanufacturing aspects. Most of the research on the reuse of EV batteries has focused on technical aspects; see, for example, Lih et al. (2012); Neubauer and Pesaran (2011); Patten et al. (2011); Yu et al. (2013). There are very few studies focusing on the economic benefits. Patten et al. (2011) discussed how EV batteries can be used in a wind storage system to improve utilisation and make more efficient use of EV batteries prior to recycling. Assução et al. (2016) built a model of how re-purposed EV batteries can be used to support solar energy. The model is analysed from both technical and economic perspectives. In their model, the authors allow users to generate energy and inject it into the grid for economic benefits. As shown in that paper, the used batteries can be reused for more than 10 years and will bring a good payback to customers. Ahmadi et al. (2014) and Aziz et al. (2015) studied the performance of reused EV batteries in rebalancing the electricity grid. They state that if re-purposed batteries are used to store off-peak electricity to serve peak demand, carbon dioxide (CO_2) emissions will be reduced by around 56%. However, some people have different views. Although used EV batteries could be a potential enhancement of cost-effective energy storage systems, Neubauer and Pesaran (2011) argue that even though the second use of the batteries is not expected to significantly affect PHEV/EV price, long-term battery degradation and second-use applications need to be investigated in more detail.

The CLSC is a well-studied yet challenging area, particularly when it becomes a multi-period model. A number of papers have studied a two-period model (Atasu et al., 2008; Ferguson and Toktay, 2006; Mitra and Webster, 2008; Webster and Mitra, 2007; Majumder and Groenevelt, 2001), most of which focus on the relationship and decision making between manufacturer and remanufacturer. Specifically, both Atasu et al. (2008) and Ferguson and Toktay (2006) designed a two-period competition model. In the first period, only the new product exists on the market, and in the second period the remanufactured product competes with the new product. Majumder and Groenevelt (2001) studied a two-period competition model involving an original equipment manufacturer (OEM) and a local remanufacturer in which the total cost for dealing with the returned items was fixed. In the first period as he defined, only the OEM manufacturers sells new products. In the second period, a fraction of these items are returned for remanufacturing. The model developed by Webster and Mitra (2007) details the impact of take-back laws in remanufacturing competitive strategy in two periods. The first period is the life of using the product. At the end of the first period, some or all of the usable returns may be purchased by the remanufacturer, and the manufacturer and remanufacturer will compete for sales in the second period. Next, the authors developed another model to analyse the regulation of remanufacturing activities in two periods. In the first period, a manufacturer introduces a new generation of a product. The length of this period corresponds to the useful life of the product. After that, some of the products are returned, and a remanufacturer enters the market in the second period (Mitra and Webster, 2008). Ferrer and Swaminathan (2010) analyse the (re)manufacturer monopoly environment from a two-period to a multi-period planning horizon and develop a strategy for optimizing the price for the firm in the model. In the first period is the new product is used, and in the subsequent periods there is an option of making new and remanufactured products.

2.4.2 Research gap and summary

The previous section reviewed some multi-period CLSC. However, based on the review, the existing multi-period CLSC models are not able to reflect the practices relating to used EV battery reuse and recycling and the characteristics of such CLSCs. That is, unlike normal goods, EV batteries cannot be reused for their original purpose, which complicates normal CLSC operations. Therefore, the following research questions are put forth: (1) How can we illustrate multi-period CLSCs for used EV batteries? and (2) What about the relationship between EV battery manufacturer and remanufacturer in this CLSC? These two questions will be discussed in Chapter 6.

2.5 Conclusion of literature review

This chapter provided several definitions related to the research and reviewed the literature related to EV supply chain subsidies, EV CS quality of service and the EV used battery CLSC. Based on the research gap defined above, three models will be designed and developed:

(1) In chapter 4, we will propose a model to illustrate a SC subsidy model involving government, vehicle manufacturers, retailers and end users.

(2) In chapter 5, we will discuss a general theoretical CS model in the context of customer selection schemes.

(3) In chapter 6, we will present a model for the purpose of studying the reuse, reycling and remanufacturing of used EV batteries in a CLSC.

Chapter 3

Research Methodology

The chapters above discussed the research topic and highlighted the research gaps that will be considered in the form of research questions. This chapter will explain how this research was conducted, including the research methods, and the tools used.

3.1 Research philosophies and paradigms

A research paradigm consists of ontology, epistemology and methodology (Guba, 1990). Ontology refers to the 'nature of reality and knowledge'. Specifically, ontology is the philosophical study of the nature of being, becoming, existence, or reality, as well as the basic categories of being and their relationships (Dictionary, 2002). Ontology examines whether reality is observed or viewed from an objective or a subjective perspective. Epistemology deals with 'How to know something', refering to 'the assumptions and declarations made about the ways in which knowledges of the reality is obtained' (Sauders et al., 2003). Research methodology is influenced by ontology and epistemology, 'how the knowledge of the world is gained', and this is 'the basis and rationale behind the selection of methods and collection of concepts, ideas and theories' (Bryman and Bell, 2015).

It is important to understand the implications of the epistemology and the methodology chosen for the research topic. Normally, qualitative research endeavours to maximize realism, while quantitative research tries to omtimize the generalization (Golicic et al., 2005). qualitative research normally analyses open-ended questions through observations and interviews by focusing on experiences, opinions, feelings and knowledge. The findings are presented as quotes, narratives and explanations. Quantitative research usually answers closed questions by collecting data from participants directly or by measuring things. This method focuses on measurement, comparison and generalisation using statistical or mathematical analysis. Results and findings are presented in figures and tables (Creswell and Creswell, 2017). In the present EV triple SC research, the author uses a quantitative method. With this method, it is more convenient to observe relationships between variables and the target by establishing causes and effects in controlled circumstances. The author uses deductive logic and the conceptual research approach to study the research questions. Conceptual research approach is an appropriate method that crosses both scale in lots of large and small theories and can be used to describe the SC through visual or graphical representations (Maxwell, 2008). As the EV industry is developing with inexplicit policies or schemes, inductive reasoning may not be suitable. Deductive logic, as a kind of top-down reasoning, can be used. It is the process of reasoning from one or more statements to reach a logically certain conclusion. Deductive reasoning links premises with conclusions. If all premises are true, the terms will be clear, and if the rules of deductive logic were followed, then the conclusion reached is necessarily true (Hendricks, 2006). Both mathematical models and simulation analysis are deductive reasoning approaches.

3.2 Research methods and tools

3.2.1 Modelling

Models are of central importance in many scientific contexts, such as the Bohr model of the atom, the double helix model of DNA, agent-based and system dynamics models in the social sciences, and general equilibrium models of markets. Philosophers are increasingly acknowledging the importance of models and are investigating the various roles models play in scientific practice (Zalta et al., 2003).

Many things can be thought of as models: physical objects, fictional objects, set-theoretic structures, descriptions, equations or combinations of some of these (Berto and Plebani, 2015). According to Swoyer (1991); Zalta et al. (2003), models are vehicles for learning about the world. Significant parts of scientific investigation are carried out on models rather than on reality itself because by studying a model we can discover features of and facts about the system the model stands for. In brief, models allow for surrogative reasoning. Three steps of learning models are denotation, demonstration, and interpretation (Hughes, 1997). First, establish a representative relationship between the model and the target (denotation). Second explore the parameters or features of the model to demonstrate some theoretical claims about the model (demonstration). Third, conclude with findings about the system and the model (interpretation).

3.2.2 Simulation

Simulation is the imitation of the operation about a real-world process or system over time. It represents the key characteristics, behaviours and functions of the system process (Ross, 1990; Jerry, 1984).

According to Trochim and Davis (1986), simulation is useful for (1) improving understanding of the principles and implementation of research; (2) investigating the effects of problems and (3) exploring the accuracy and utility of the simulated model. First, the simulation model should be designed using the relevant software. Second, the user should create the data and feed them to the model. Third, the results should be analysed.

Simulations have been widely used in research (Guetzkow, 1962). For some purposes, simulation are better than the analysis of real data (Trochim and Davis, 1986). With real data, the analyst 'never perfectly knows the real-world processes that caused the particular measured values to occur'. In a simulation, the analyst is able to 'control all of the factors making up the data and can manipulate these systematically to see directly how specific problems and assumptions affect the analysis' (Trochim and Davis, 2016).

3.2.3 Monte Carlo method

The Monte Carlo method (Monte Carlo experiments) represents a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results (Rubinstein and Kroese, 2016). When the deterministic principle is difficult or impossible to apply, using randomness to solve problems could be an option, and by using the Monte

Carlo method, the probability distribution of the result can be generated (Kroese et al., 2014). This method is useful in simulations and can be used to solve any problem having a probabilistic interpretation of the law of large numbers.

The steps involved can be summarized as follows: (1) Define the domain of possible inputs. (2) Based on a probability distribution, generate inputs randomly from the domain. (3) Perform or simulate the deterministic computation. (4) Aggregate the results and conduct an analysis. In this thesis, the Monte Carlo method will be used with a simulation to evaluate the performance of selecting the EV CS.

3.2.4 Game theory

The basic definition of game theory can be found in Section 2.1.6. In simple terms, a game includes six elements, which are described below (Harrington, 2009; Myerson, 2013):

(1) Player

The player is the participant in the game who makes decisions using certain rules to maximize his own profit. It can either be an agent or a group, or even a typical situation.

(2) Information

Information refers to a set of knowledge pertaining to the game's situation.

(3) Action and strategy space

Action constitutes a choice that the player must make in a particular situation, no matter how it is made. Strategy is the only action the player can choose in that particular situation. Action and strategy are different concepts but are related to each other.

(4) Order

In the process of decision making, the order of players making decisions is based on different priorities, and a different order often results in a different strategy and different outcomes.

(5) Payoff

Payoff, as the quantitative value of a game, is the definite or expected effect of players.

(6) Equilibrium and outcome

Equilibrium occurs when the strategy set is best for each player.

Based on the six elements above, game theory can be classified as including cooperative games and non-cooperative games. A cooperative game is a game in which players have a binding agreement with each other. Generally speaking, a cooperative game emphasizes collective rationality, efficiency, fairness, and equality. A non-cooperative game pays more attention to personal rationality and optimal decisions and the result may either be effective or ineffective. Most studies focus on non-cooperative games (Selten, 1965*a*,*b*; Harsanyi et al., 1988; Harsanyi, 2004). Game can also be classified as static or dynamic. In a static game, each player makes a single decision and has no knowledge of the decisions the other players make before making their own. Decisions are made simultaneously; that is, order is irrelevant. In a dynamic game, the players have some information about the strategies chosen by others and therefore can base how they play on previous moves (Osborne and Rubinstein, 1994). Moreover, based on the degree of knowing other players' information, a game can also be categorized as an incomplete information game and a complete information game. A complete information game is a game in which the players know everyone else's information, strategy space, and payoff functions. Otherwise, the game can be thought of as an incomplete information game and can be divided into four categories, as shown in Table 3.1.

Table 3.1: Classification for the Game

	Static order	Dynamic order
Complete information	Static Game with Complete Information; Nash equilibrium (Nash et al., 1950; Nash, 1951)	Dynamic Games with Complete Information; Subgame perfect Nash equilibrium(Selten, 1965*a*)
Incomplete information	Static Game with Incomplete Information; Bayesian Nash equilibrium (Harsanyi et al., 1988)	Dynamic Games with Incomplete Information; PerfectBayesianequilibrium(Selten, 1975; Kreps and Wilson, 1982)

3.2.5 Stackelberg Competition Game

In 1934, Heinrich Freiherr von Stackelberg created a useful game model for SC management called Stackelberg competition (Von Stackelberg, 2010). In this dynamic type of game, there is a leader and a follower who compete with each other. The leader knows that the follower observes his action. Indeed, if the follower commits to the Stackelberg leader's action and the leader knows this, the leader's best response is to play a Stackelberg follower action.

In the SC, the upstream department, for example, the manufacturer, is normally able to obtain more information about the supply chain than the downstream department (e.g. the retailer, the customer). In this case, the manufacturer will make decision first as a leader, and the retailer, as a follower, will take action based on the manufacturer's decision. Using Stackelberg competition, subgame perfect Nash equilibrium in the SC can be solved for players, which means the optimal action for each player will be figured out. There is plenty of research that uses the Stackelberg competition game to obtain the optimal decision for each player in the SC (Chen et al., 2017; Ma et al., 2013; Atasu et al., 2008).

3.2.6 Stochastic process analysis

A stochastic process is a group of random variables indexed by a set of numbers, usually viewed as points in a time series (Brzezniak and Zastawniak, 2000). It has been widely used as a mathematical model of systems such as in chemistry, physics, signal processing, information theory, computer science, and telecommunications.

For the EV CS selection, lots of parameters would affect the driving experience and utility. Most parameters are random but obey some distributions. A distribution refers to the probability that a parameter takes a value, where it can be either continuous or discrete.

3.2.7 Grid search

According to algorithm theory (Sedgewick, 1988), a grid search is an ergodic global searching method used to find the target value. In this thesis, a grid search is used to find the maximum and minimum value of linearity by searching the values of all possible parameters based on the resolution.

3.2.8 Linearity

According to Fraden (2004) and Cooper (1970), linearity is defined as a ratio of maximum deviation between the practical curve and fitted straight line with full scale output, that is $\eta = max(\Delta Y)/Y * \%100$. Figure 3.1 is a schematic diagram of linearity. By using linear regression, the optimal fitted line can be solved. For simplification, we draw a simple fitted line just connecting two points on the curve with a cross point between $x = 0$ and the original curve and a cross point between $x = 1$ and the original curve. It is easy to prove that the optimal linearity of the curve is always less than or equal to the linearity for the simple fitted line. In this research, the author asserts that the curve is approximately linear if the linearity of the simple fitted line is $\pm\%10$.

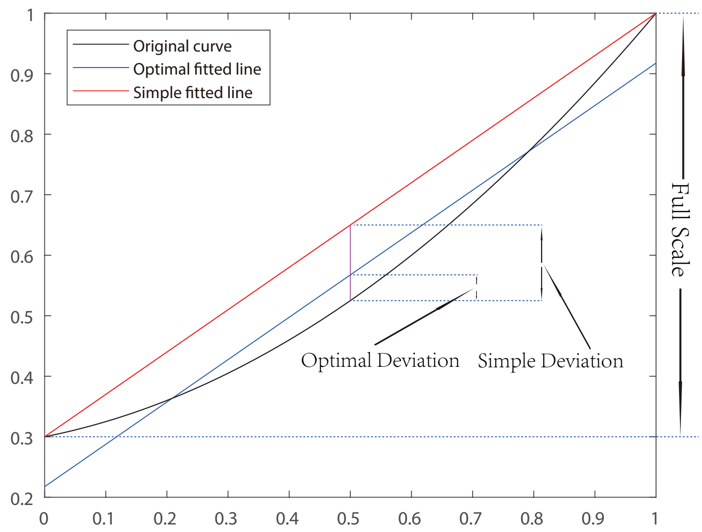

Fig. 3.1: Example for linearity

3.2.9 Software and tools

Different software will be used for simulation and mathematical analyses.

- MATLAB™: Used for modelling, simulation, and drawing graphs

- Wolfram Mathematica™: Used for calculating and simplifying mathematical expressions.

3.3 Summary

This chapter has explained how this research was carried out, including the research philosophies and paradigms, the research methods and the tools used. The next chapter will describe and discuss models in different CSs.

Chapter 4

Model for the electric vehicle supply chain incentive design

In this chapter, we build a model to describe a government subsidy model for the EV SC that includes government, EV/GV manufacturers, EV/GV retailers and potential customers. The Stackelberg model with incomplete information will be implemented for this.

Section 4.1 introduces the model. Section 4.1.1 lists notations used in the model. Section 4.1.2 provides equations of vehicle use cost, which includes time cost and money cost. In section 4.1.3, the utility functions for customers using GVs or EVs are proposed based on the six development periods in the EV industry. In section 4.2, the author chooses two periods as representative of the early and later EV development stages to analyse the model using the Stackelberg game with incomplete information. Sections 4.3 and 4.4 respectively analyse the results numerically and mathematically, and discuss the analysis from a management point of view. The chapter concludes with a summary.

4.1 Introduction

Figure 4.1 shows a four-echelon SC structure of this model. There are six players in this model: the government, the EV manufacturer, the GV manufacturer, the EV retailer, the GV retailer and the customer. Government is responsible for making policy and overseeing the entire EV system. The reason for government provides subsidies (either to the EV manufacturer or the EV customer) is to maximise the entire social profit (particularly of the entire vehicle SC). In each echelon, the objective is to maximize the profit, while customers buy a vehicle that makes them happiest.

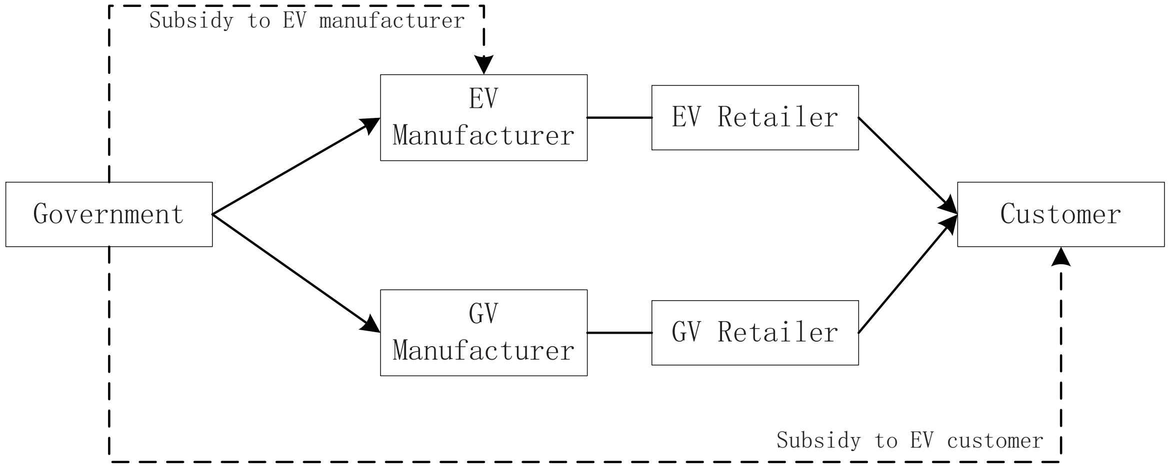

Fig. 4.1: Structure for the subsidy model

There are two paths in the structure, the EV manufacturer and retailer path and the GV manufacturer and retailer path. Government will pay close attention to both paths and the entire structure. Customer has two options, that is, purchase a GV or purchase an EV. As discussed in the previous chapters, government has an incentive to promote the EV supply chain. To simplify the model, the author assumes two directions for the subsidy: subsidize the EV manufacturer or subsidize the EV customer (as shown by the dotted lines in the figure).

4.1.1 Notations

Table 4.1 lists all parameters for this model.

Table 4.1: Notations in the subsidy model

Input parameters			
k	Customer's intention about buying a vehicle	M_L	Average mileage driven per year (km/year)
L	Vehicle lifecycle time (year)	p_{fuel}	Price of fuel (/L)
p_{elec}	Price of electricity (£/kWh)	E_{fuel}	Mileage per fuel unit(km/L)

E_{elec}	Mileage per electricity unit (km/kWh)	v_t	Time value (£/h)
h_{refuel}	Time cost in each fuel refilling (hour)	H_{rechr}	Time cost in each electricity rechagring (hour)
V_{gv}	GV driving experience utility (£)	V_{ev}	Battery volume (kWh)
π_{gv}	GV driving experience utility (£)	π_{ev}	EV driving experience utility (£)
θ_{gv}	Environmental awareness level for GV user	θ_{ev}	Environmental awareness level for EV user (£)
p_{Mgv}	GV manufacturer cost (£)	p_{Mev}	EV manufacturer cost (£)
c_{envir}	Average environmental management cost for each GV (£)	$[c_{e0}, c_{e1}]$	Lower and upper limits of the EV manufacturing cost
$[c_{g0}, c_{g1}]$	Lower and upper limits of the GV manufacturing cost	$[r_{e0}, r_{e1}]$	Lower and upper limits of the price sold from EV manufacturer to the retailer
$[r_{g0}, r_{g1}]$	Lower and upper limits of the price sold from GV manufacturer to the retailer		
Intermediate variables			
M	The entire mileage vehicle will be driven over lifecycle(km)	V_{fuel}	Fuel cost in the vehicle's lifecycle(£)
V_{elec}	Electricity cost in the vehicle's lifecycle(£)	V_{rf_time}	Refuelling time cost in the vehicle's lifecycle (£)

V_{rc_time}	Recharging time cost in the vehicle's lifecycle (£)	C_{gv}	The total operation cost of GV (£)
C_{ev}	The total operation cost of EV (£)	v_{gv}	GV user's profit (£)
v_{ev}	EV user's profit (£)	U_{Cgv}	Utility function of using the GV
U_{Cev}	Utility function of using the EV	$f_{re}(x)$	The probability distribution function (PDF) of EV price sold from EV manufacturer to the retailer
$f_{rg}(x)$	The PDF of GV price sold from GV manufacturer to the retailer	$f_{ce}(x)$	The PDF of EV manufacturing cost
$f_{cg}(x)$	The PDF of GV manufacturing cost	P_{gve}, P_{gvl}	The probability of buying the GV in the early/later development stage
P_{eve}, P_{evl}	The probability of buying the EV in the early/later development stage	C_G	The cost paid from the government
Decision variables			
s_c	Subsidy to customer given by the government (£)	s_m	Subsidy to manufacturer given by the government (£)
p_{Cgv}	GV price that customer paid to the retailer	p_{Cev}	EV price that customer paid to the retailer
p_{Rgv}	GV price that retailer bought from manufacturer	p_{Rev}	EV price that retailer bought from manufacturer
Output			

π_{Cgv}	Profit for GV customer (£)	π_{Cev}	Profit for EV customer (£)
π_{Rgv}	Profit for GV retailer (£)	π_{Rev}	Profit for EV retailer (£)
π_{Mgv}	Profit for GV manufacturer (£)	π_{Mev}	Profit for EV manufacturer (£)
π_{entire}	The social entire profit (£)		

Note:

1) k satisfies $0 \leq k \leq 1$, where $k = 0$ means the customer will not buy a car and $k = 1$ means the customer will buy a car.

2) Input variables π_{gv}, π_{ev}, θ_{gv}, θ_{ev} are intangible, but they are converted to tangible values to reflect people's driving experience and environment protection awareness—which is similar to the assumptions in Greene et al. (2004).

4.1.2 EV and GV costs in the lifecycle

According to Greene et al. (2004), the mileage that a vehicle will accumulate over time is defined as $M = M_L L$. In the whole lifetime, the money cost equals total mileage (M) multiplied by the unit price of fuel or electricity (p_{fuel} or p_{elec}) divided by the efficiency (mileage per unit, i.e., E_{fuel} or E_{elec}). Therefore, money cost of fuel for GV and money cost are respectively

$$V_{fuel} = \frac{p_{fuel} M}{E_{fuel}} \tag{4.1}$$

$$V_{elec} = \frac{p_{elec} M}{E_{elec}} \tag{4.2}$$

The time value is equal to time cost (v_t) multiplies the time duration of each re-fuel/re-charge (H_{refuel}/H_{rechr}) multiplied entire refuelling or recharging time in the whole life cycle, where the time equals to total mileage M divided by the mileage after the single refuel or recharge ($V_{gv}E_{fuel}$ or $V_{ev}E_{elec}$), which can be expressed as

$$V_{rf\_time} = v_t H_{refuel} \frac{M}{V_{gv} E_{fuel}} \tag{4.3}$$

$$V_{rc\_time} = v_t H_{rechr} \frac{M}{V_{ev} E_{elec}} \tag{4.4}$$

Therefore, the total cost of GV and EV are

$$C_{gv} = V_{fuel} + V_{rf\_time} \tag{4.5}$$

$$C_{ev} = V_{elec} + V_{rc\_time} \tag{4.6}$$

4.1.3 Customer choice

Different people have different intentions to buy an automobile. Based on Shafiei et al. (2012) and Helveston et al. (2015), the author argues that customers will consider vehicle price, the cost of use, subsidies and even environmental impact. The author defines the two parts of the utility function which are the using profit across the lifecycle and the purchase price. As different people have different sensitivity about the profit, we use $k, k \in [0, 1]$ to denote the intention to profit over the whole use period, where profit is the using profit (including the environment awareness) minus the use cost. The utility functions are described as follows:

$$U_{Cgv}(k) = (\pi_{gv} + \theta_{gv} - C_{gv})k - P_{Cgv} = v_{gv}k - p_{Cgv} \tag{4.7}$$

$$U_{Cev}(k) = (\pi_{ev} + \theta_{ev} - C_{ev})k - P_{Cev} + s_c = v_{ev}k - p_{Cev} + s_c \tag{4.8}$$

Moreover, we have some assumptions here:

- User's profit of GV and EV is unequal, which means $v_{gv} \neq v_{gv}$;
- The subsidy to the EV customer is lower than the EV price, i.e. $s_c < p_{Cev}$;
- The subsidy to the EV manufacturer is lower than the EV manufacturing cost, which is $s_m < p_{Mev}$;
- The customer is rational and will always choose a type of vehicle based on the utility he or she will obtain.

Based on the assumptions above, we have two stages, which are the EV early development stage and the EV later development stage. More details are given in Appendix A.1. In the appendix, the early stage is defined as Case V and the later stage is defined as Case II. The schematic diagram of purchasing probability for the early and later stages are shown below:

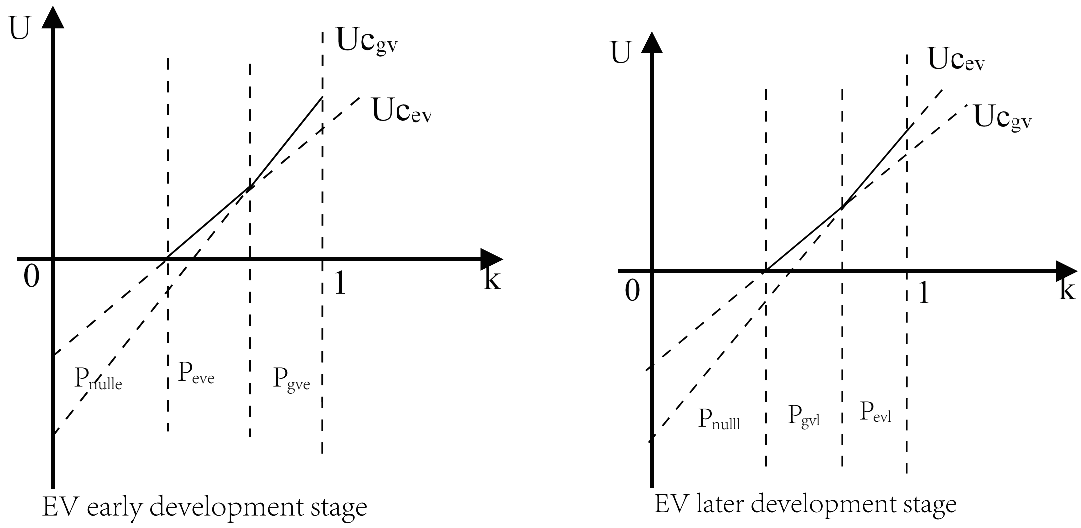

Fig. 4.2: Possibility of relationships between U_{Cgv} and C_{Cev}

4.2 Model under incomplete information condition

Unlike the normal assumption in which every player knows others' selling price and purchase price, based on the incomplete information condition assumption, retailers only know their own EV/GV customer price, which is closer to reality. Although the retailers do not know the manufacturing cost of the GV/EV, which indicates that they do not know the exact price the manufacturer charged for the vehicle, they know both the probability distribution of the GV/EV manufacturing selling price and the probability distribution of the manufacturing cost. If the EV price sold from EV manufacturer probability distribution to retailer is $f_{re}(x)$, the GV price probability distribution sold from GV manufacturer to retailer $f_{rg}(x)$, the EV manufacturing cost probability distribution is $f_{ce}(x)$ and the GV manufacturing cost is $f_{cg}(x)$. Therefore, we have

$$prop(p_{Rgv}|p_{Mgv}) = \frac{f_{rg}(p_{Mgv}|p_{Rgv}) \lim_{\triangle\varepsilon\to 0} \int_{p_{Rgv}}^{p_{Rgv}+\triangle\varepsilon} f_{cg}(x)dx}{\int_0^{+\infty} f_{rg}(p_{Rgv}|p_{Mgv}) f_{cg}(p_{Mgv}) dp_{Mgv}} \quad (4.9)$$

and

$$prop(p_{Rev}|p_{Mev}) = \frac{f_{re}(p_{Mev}|p_{Rev}) \lim_{\triangle\varepsilon \to 0} \int_{p_{Rev}}^{p_{Rev}+\triangle\varepsilon} f_{ce}(x)dx}{\int_0^{+\infty} f_{re}(p_{Rev}|p_{Mev}) f_{ce}(p_{Mev}) dp_{Mev}} \tag{4.10}$$

Therefore, the profit function of retailers for selling GV and EV are

$$\pi_{Rgv}(p_{Cgv}) = \int_0^{+\infty} P_{gv}(p_{Cgv} - p_{Rgv}) prop(p_{Rgv}|p_{Mgv}) dp_{Mgv} \tag{4.11}$$

$$\pi_{Rev}(p_{Cev}) = \int_0^{+\infty} P_{ev}(p_{Cev} - p_{Rev}) prop(p_{Rev}|p_{Mev}) dp_{Mev} \tag{4.12}$$

For simplification, we assume the price probability distribution function obeys uniform distribution. We have four distributions here: $f_{re}(x) \sim U[r_{e0}, r_{e1}]$, $f_{rg}(x) \sim U[r_{g0}, r_{g1}]$, $f_{ce}(x) \sim U[c_{e0}, c_{e1}]$, $f_{cg}(x) \sim U[c_{g0}, c_{g1}]$. As the manufacturer makes decision first, the retailer will know the exact price p_{Rev} and p_{Rgv}. So, we have $prop(p_{Mev}|p_{Rev}) = \frac{1}{c_{e1}-c_{e0}}$, $prop(p_{Mgv}|p_{Rgv}) = \frac{1}{c_{g1}-c_{g0}}$, $prop(p_{Rev}|p_{Mev}) = \frac{1}{r_{e1}-r_{e0}}$ and $prop(p_{Rgv}|p_{Mgv}) = \frac{1}{r_{g1}-r_{g0}}$.

Moreover, the profit functions for GV and EV manufacturer are

$$\pi_{Mgv}(p_{Rgv}) = P_{gv}(p_{Rgv} - p_{Mgv}) \tag{4.13}$$

$$\pi_{Mev}(p_{Rev}) = P_{ev}(p_{Rev} - p_{Mev} + s_m) \tag{4.14}$$

4.2.1 Early stage of EV development

The purchase probabilities and the profit for each player are shown below. All details of substitutions from B_0 to B_1 and from D_0 to D_{26} are described in Table A-1.

Customer

In this case, the probabilities of people buying the GV and EV are

$$P_{gve} = 1 - \frac{p_{Cgv} - p_{Cev} + s_c}{v_{gv} - v_{ev}} = 1 - B_0(p_{Cgv} - p_{Cev} + s_c) \tag{4.15}$$

$$P_{eve} = \frac{p_{Cgv} - p_{Cev} + s_c}{v_{gv} - v_{ev}} - \frac{p_{Cev} - s_c}{v_{ev}} = B_0 p_{Cgv} - (B_0 + B_1)p_{Cev} + (B_0 + B_1)s_c \tag{4.16}$$

Retailer

By applying Eq. 4.16, Eq. 4.12 and Eq. 4.10, the profit function for the EV retailer is

$$\begin{aligned}
\pi_{Rev}(p_{Cev}) &= \int_{c_{e0}}^{c_{e1}} P_{eve}(p_{Cev} - p_{Rev})\frac{1}{r_{e1} - r_{e0}}dp_{Mev} \\
&= \frac{P_{eve}}{r_{e1} - r_{e0}}(p_{Cev} - p_{Rev})(c_{e1} - c_{e0}) \\
&= \left(\begin{array}{l} -\frac{(B_0+B_1)(c_{e0}-c_{e1})}{r_{e0}-r_{e1}}p_{Cev}^2 \\ \frac{(c_{e0}-c_{e1})\left(B_0\left(s_c+p_{Cgv}+p_{Rev}\right)+B_1(s_c+p_{Rev})\right)}{r_{e0}-r_{e1}}p_{Cev} \\ -\frac{p_{Rev}(c_{e0}-c_{e1})\left(B_0\left(s_c+p_{Cgv}\right)+B_1 s_c\right)}{r_{e0}-r_{e1}} \end{array} \right)
\end{aligned} \tag{4.17}$$

and similarly, by applying Eq. 4.15, Eq. 4.11 and Eq. 4.9, the profit function for GV retailer is

$$\begin{aligned}
\pi_{Rgv}(p_{Cgv}) &= \int_{c_{g0}}^{c_{g1}} P_{gve}(p_{Cgv} - p_{Rgv})\frac{1}{r_{g1} - r_{g0}}dp_{Mgv} \\
&= \frac{P_{gve}}{r_{g1} - r_{g0}}(p_{Cgv} - p_{Rgv})(c_{g1} - c_{g0}) \\
&= \left(\begin{array}{l} \frac{B_0(c_{g1}-c_{g0})}{r_{g0}-r_{g1}}p_{Cgv}^2 \\ \frac{(c_{g0}-c_{g1})\left(B_0\left(-s_c+p_{Cev}+p_{Rgv}\right)+1\right)}{r_{g0}-r_{g1}}p_{Cgv} \\ -\frac{p_{Rgv}(c_{g0}-c_{g1})(B_0(p_{Cev}-s_c)+1)}{r_{g0}-r_{g1}} \end{array} \right)
\end{aligned} \tag{4.18}$$

It is easy to find $\frac{\partial^2 \pi_{Rgv}(p_{Cgv})}{\partial p_{Cgv}^2} < 0$ and $\frac{\partial^2 \pi_{Rev}(p_{Cev})}{\partial p_{Cev}^2} < 0$. So, by using first order condition $\frac{\partial \pi_{Rgv}(p_{Cgv})}{\partial p_{Cgv}} = 0$ and $\frac{\partial \pi_{Rev}(p_{Cev})}{\partial p_{Cev}} = 0$, we have

$$\begin{cases} p_{Cgv} = \frac{(B_0+B_1)\left(B_0\left(-s_c+p_{Rev}+2p_{Rgv}\right)+2\right)}{B_0(3B_0+4B_1)} \\ p_{Cev} = \frac{B_0\left(s_c+2p_{Rev}+p_{Rgv}\right)+2B_1(s_c+p_{Rev})+1}{3B_0+4B_1} \end{cases} \tag{4.19}$$

Through substituting equation above into P_{gve} and P_{eve}, we have

$$P_{gve} = -D_0 p_{Rgv} + D_1\left(p_{Rev} - s_c\right) + 2D_2 \tag{4.20}$$

$$P_{eve} = D_1 p_{Rgv} + D_3(s_c - p_{Rev}) + D_2 \tag{4.21}$$

Manufacturer

The profits for GV and EV manufacturer are

$$\pi_{Mgv}(p_{Rgv}) = P_{gve}(p_{Rgv} - p_{Mgv}) = (-D_0 p_{Rgv} + D_1 (p_{Rev} - s_c) + 2D_2)(p_{Rgv} - p_{Mgv}) \tag{4.22}$$

$$\pi_{Mev}(p_{Rev}) = P_{eve}(p_{Rev} - p_{Mev} + s_m) = (D_1 p_{Rgv} + D_3(s_c - p_{Rev}) + D_2)(p_{Rev} - p_{Mev} + s_m) \tag{4.23}$$

As $\frac{\partial^2 \pi_{Mgv}(p_{Rgv})}{\partial p_{Rgv}^2} < 0$ and $\frac{\partial^2 \pi_{Mev}(p_{Rev})}{\partial p_{Rev}^2} < 0$ as well, by using the first order condition, we have

$$\begin{cases} p_{Rgv} = \frac{D_1(D_3(-s_c - s_m + p_{Mev}) + D_2) + 2D_3\left(D_0 p_{Mgv} + 2D_2\right)}{4D_0 D_3 - D_1 D_4} = D_4 s_c + D_4 s_m - D_5 \\ p_{Rev} = \frac{D_0\left(2D_3(s_c - s_m + p_{Mev}) + D_4 p_{Mgv} + 2D_2\right) + D_4(2D_2 - D_1 s_c)}{4D_0 D_3 - D_1 D_4} = D_6 s_c + D_7 s_m + D_8 \end{cases} \tag{4.24}$$

For simplification, then rewrite the purchase probability P_{gve} and P_{eve} as

$$\begin{cases} P_{gve} = D_9 s_c + D_{10} s_m + D_{11} \\ P_{eve} = D_{12} s_c + D_{13} s_m + D_{14} \end{cases} \tag{4.25}$$

Government

The government is expected to pay close attention to the entire social profit:

$$\pi_{entire} = \pi_C + \pi_R + \pi_M - C_G \tag{4.26}$$

Where C_G is the subsidy payment and the cost of environmental protection:

$$C_G = P_{gve} c_{envir} + P_{eve}(s_c + s_m) \tag{4.27}$$

The utility for the customer is

$$\pi_c = \begin{pmatrix} \frac{1}{2}P_{gve}(v_{gv} - p_{Cgv} + \frac{v_{gv}(p_{Cgv}-p_{Cev}+s_c)}{v_{gv}-v_{ev}} - p_{Cgv}) \\ +\frac{1}{2}P_{eve}(\frac{v_{gv}(p_{Cgv}-p_{Cev}+s_c)}{v_{gv}-v_{ev}} - p_{Cgv}) \end{pmatrix} = \frac{\begin{pmatrix} (D_{12}s_c + D_{13}s_m + D_{14})(D_{15}s_c + D_{16}s_m + D_{17}) \\ +(C_9 s_c + D_{10}s_m + D_{11})(D_{18}s_c + D_{19}s_m + D_{20}) \end{pmatrix}}{-2B_0(3B_0 + 4B_1)(v_{ev} - v_{gv})} \tag{4.28}$$

The utility for the manufacturer, retailer and government together is

$$\begin{aligned} &\pi_M + \pi_R - P_{gve}c_{envir} - P_{eve}(s_c + s_m) \\ &= \begin{pmatrix} \frac{P_{eve}(p_{Cev}-p_{Rev})(c_{e1}-c_{e0})}{r_{e1}-r_{e0}} + \frac{P_{gve}(p_{Cgv}-p_{Rgv})(c_{g1}-c_{g0})}{r_{g1}-r_{g0}} \\ +P_{gve}(p_{Rgv} - p_{Mgv} - c_{envir}) + P_{eve}(p_{Rev} - p_{Mev} - s_c) \end{pmatrix} \\ &= \begin{pmatrix} (D_{12}s_c + D_{13}s_m + D_{14})(D_{21}s_c + D_{22}s_m + D_{23}) \\ +(D_9 s_c + D_{10}s_m + D_{11})(D_{24}s_c + D_{25}s_m + D_{26}) \end{pmatrix} \end{aligned} \tag{4.29}$$

Therefore, the entire utility is

$$\pi_{entire} = \begin{pmatrix} (D_{12}s_c + D_{13}s_m + D_{14})(D_{21}s_c + D_{22}s_m + D_{23}) \\ +(D_9 s_c + D_{10}s_m + D_{11})(D_{24}s_c + D_{25}s_m + D_{26}) \\ +\frac{\begin{pmatrix} (D_{12}s_c + D_{13}s_m + D_{14})(D_{15}s_c + D_{16}s_m + D_{17}) \\ +(C_9 s_c + D_{10}s_m + D_{11})(D_{18}s_c + D_{19}s_m + D_{20}) \end{pmatrix}}{-2B_0(3B_0+4B_1)(v_{ev}-v_{gv})} \end{pmatrix} \tag{4.30}$$

Discussion

i. Subsidy given to EV customer only

The subsidy given to the EV customer should be satisfied with

$$\begin{cases} \frac{\partial \pi_{entire}}{\partial s_c} = 0 \\ s_m = 0 \end{cases} \tag{4.31}$$

So,

$$\pi_{entire} = \begin{pmatrix} (D_{12}s_c + D_{14})(D_{21}s_c + D_{23}) + (D_9 s_c + D_{11})(D_{24}s_c + D_{26}) \\ + \frac{\left((D_{12}s_c + D_{14})(D_{15}s_c + D_{17}) + (C_9 s_c + D_{11})(D_{18}s_c + D_{20}) \right)}{-2B_0(3B_0+4B_1)(v_{ev}-v_{gv})} \end{pmatrix} \tag{4.32}$$

We have $-\frac{2D_{12}D_{15}+2D_9D_{18}}{2B_0(3B_0+4B_1)(v_{ev}-v_{gv})} + 2D_{12}D_{21} + 2D_9D_{24} < 0$. Therefore, by using the first order condition:

$$\frac{\partial \pi_{entire}}{\partial s_c} = \begin{pmatrix} \frac{D_{12}(2D_{15}s_c+D_{17})+D_{18}(2D_9s_c+D_{11})+D_{14}D_{15}+D_9D_{20}}{2B_0(3B_0+4B_1)(v_{gv}-v_{ev})} \\ +2D_{12}D_{21}s_c + 2D_9D_{24}s_c + D_{14}D_{21} + D_{12}D_{23} + D_{11}D_{24} + D_9D_{26} \end{pmatrix} = 0 \tag{4.33}$$

So, the optimal subsidy to the customer s_c is

$$s_c = \frac{\begin{pmatrix} D_{14}\left(D_{15} - 2B_0\left(3B_0 + 4B_1\right) D_{21}\left(v_{ev} - v_{gv}\right)\right) \\ +2B_0\left(3B_0 + 4B_1\right)\left(D_{12}D_{23} + D_{11}D_{24} + D_9D_{26}\right)\left(v_{gv} - v_{ev}\right) \\ +D_{12}D_{17} + D_{11}D_{18} + D_9D_{20} \end{pmatrix}}{-2\begin{pmatrix} D_{12}\left(D_{15} - 2B_0\left(3B_0 + 4B_1\right) D_{21}\left(v_{ev} - v_{gv}\right)\right) \\ +D_9\left(D_{18} - 2B_0\left(3B_0 + 4B_1\right) D_{24}\left(v_{ev} - v_{gv}\right)\right) \end{pmatrix}} \tag{4.34}$$

The expansion of s_c is shown in Appendix A.2.2.

ii. Subsidy given to EV manufacturer only

The subsidy given to the EV manufacturer should be satisfied with

$$\begin{cases} \frac{\partial \pi_{entire}}{\partial s_m} = 0 \\ s_c = 0 \end{cases} \tag{4.35}$$

So,

$$\pi_{entire} = \begin{pmatrix} (D_{13}s_m + D_{14})(D_{22}s_m + D_{23}) + (D_{10}s_m + D_{11})(D_{25}s_m + D_{26}) \\ + \frac{\left((D_{13}s_m + D_{14})(D_{16}s_m + D_{17}) + (D_{10}s_m + D_{11})(D_{19}s_m + D_{20}) \right)}{-2B_0(3B_0+4B_1)(v_{ev}-v_{gv})} \end{pmatrix} \tag{4.36}$$

We have $-\frac{2D_{13}D_{16}+2D_{10}D_{19}}{2B_0(3B_0+4B_1)(v_{ev}-v_{gv})} + 2D_{13}D_{22} + 2D_{10}D_{25} < 0$. Therefore, by using the first order condition:

$$\frac{\partial \pi_{entire}}{\partial s_m} = \left(\begin{array}{l} \frac{D_{13}(2D_{16}s_m+D_{17})+D_{19}(2D_{10}s_m+D_{11})+D_{14}D_{16}+D_{10}D_{20}}{2B_0(3B_0+4B_1)(v_{gv}-v_{ev})} \\ +2D_{13}D_{22}s_m + 2D_{10}D_{25}s_m + D_{14}D_{22} + D_{13}D_{23} + D_{11}D_{25} + D_{10}D_{26} \end{array} \right) = 0 \tag{4.37}$$

So, the optimal subsidy to the EV manufacturer is

$$s_m = \frac{\left(\begin{array}{l} D_{14}\left(D_{16} - 2B_0\left(3B_0 + 4B_1\right)D_{22}\left(v_{ev} - v_{gv}\right)\right) \\ +2B_0\left(3B_0 + 4B_1\right)\left(D_{13}D_{23} + D_{11}D_{25} + D_{10}D_{26}\right)\left(v_{gv} - v_{ev}\right) \\ +D_{13}D_{17} + D_{11}D_{19} + D_{10}D_{20} \end{array} \right)}{-2\left(\begin{array}{l} D_{13}\left(D_{16} - 2B_0\left(3B_0 + 4B_1\right)D_{22}\left(v_{ev} - v_{gv}\right)\right) \\ +D_{10}\left(D_{19} - 2B_0\left(3B_0 + 4B_1\right)D_{25}\left(v_{ev} - v_{gv}\right)\right) \end{array} \right)} \tag{4.38}$$

The expansion of s_m is shown in Appendix A.2.3.

By comparing Appendix A.2.2 and Appendix A.2.3, it can be concluded that $s_c = s_m$ in the early EV development stage.

Summary

By substituting table A-1 to P_{eve}, P_{gve}, p_{Cgv}, p_{Cev}, p_{Rgv}, p_{Rev}, we have

$$P_{gve} = \frac{\left(\begin{array}{l} \left(v_{ev} - 2v_{gv}\right)\left(v_{gv}^2\left(2\left(s_c + s_m\right) + 11v_{ev} - 2p_{Mev} + 8p_{Mgv}\right)\right. \\ \left. -v_{ev}v_{gv}\left(s_c + 3v_{ev} + s_m - p_{Mev} + 9p_{Mgv}\right) + 2v_{ev}^2 p_{Mgv} - 8v_{gv}^3\right) \end{array} \right)}{\left(v_{ev} - 4v_{gv}\right)\left(v_{ev} - v_{gv}\right)\left(-17v_{ev}v_{gv} + 4v_{ev}^2 + 16v_{gv}^2\right)} \tag{4.39}$$

$$P_{eve} = \frac{\left(\begin{array}{l} v_{gv}(v_{ev}^2 v_{gv}\left(13\left(s_c + s_m\right) + 12v_{ev} - 13p_{Mev} - 4p_{Mgv}\right) \\ -2v_{ev}v_{gv}^2\left(13\left(s_c + s_m\right) + 11v_{ev} - 13p_{Mev} - 2p_{Mgv}\right) \\ +4v_{gv}^3\left(4\left(s_c + s_m\right) + 3v_{ev} - 4p_{Mev}\right) + v_{ev}^3\left(-2\left(s_c + v_{ev} + s_m\right) + 2p_{Mev} + p_{Mgv}\right)) \end{array} \right)}{v_{ev}\left(v_{ev} - 4v_{gv}\right)\left(v_{ev} - v_{gv}\right)\left(-17v_{ev}v_{gv} + 4v_{ev}^2 + 16v_{gv}^2\right)} \tag{4.40}$$

$$p_{Cgv} = \frac{\left(\begin{array}{l} v_{gv}(2\left(v_{ev} - 3v_{gv}\right)\left(-v_{gv}\left(2\left(s_c + s_m\right) + 11v_{ev}\right)\right. \\ \left. +v_{ev}\left(s_c + 3v_{ev} + s_m\right) + 8v_{gv}^2\right) - 2p_{Mev}\left(v_{ev} - 2v_{gv}\right)\left(v_{ev} - 3v_{gv}\right) \\ -p_{Mgv}\left(v_{ev} - 2v_{gv}\right)\left(3v_{ev} - 8v_{gv}\right)) \end{array} \right)}{\left(v_{ev} - 4v_{gv}\right)\left(-17v_{ev}v_{gv} + 4v_{ev}^2 + 16v_{gv}^2\right)} \tag{4.41}$$

$$p_{Cev} = \frac{\left(\begin{array}{l} v_{ev}^2 v_{gv} \left(-30s_c - 28v_{ev} + 3s_m - 3p_{Mev} + 10p_{Mgv}\right) \\ +2v_{ev}v_{gv}^2 \left(35s_c + 30v_{ev} - 7s_m + 7p_{Mev} - 6p_{Mgv}\right) \\ -4v_{gv}^3 \left(12s_c + 9v_{ev} - 4s_m + 4p_{Mev}\right) - 2v_{ev}^3 \left(p_{Mgv} - 2\left(s_c + v_{ev}\right)\right) \end{array} \right)}{\left(v_{ev} - 4v_{gv}\right)\left(-17v_{ev}v_{gv} + 4v_{ev}^2 + 16v_{gv}^2\right)} \tag{4.42}$$

$$p_{Rgv} = \frac{\left(\begin{array}{l} v_{gv}(v_{gv} \left(-2\left(s_c + s_m\right) - 11v_{ev} + 2p_{Mev}\right) \\ +v_{ev}\left(s_c + 3v_{ev} + s_m - p_{Mev}\right) + 8v_{gv}^2) + 2p_{Mgv}\left(v_{ev} - 2v_{gv}\right)^2 \end{array} \right)}{-17v_{ev}v_{gv} + 4v_{ev}^2 + 16v_{gv}^2} \tag{4.43}$$

$$p_{Rev} = \frac{\left(\begin{array}{l} v_{ev}v_{gv}\left(-9s_c - 8v_{ev} + 8s_m - 8p_{Mev} + 2p_{Mgv}\right) \\ +2v_{gv}^2\left(4\left(s_c - s_m + p_{Mev}\right) + 3v_{ev}\right) + v_{ev}^2\left(2\left(s_c + v_{ev} - s_m\right) + 2p_{Mev} - p_{Mgv}\right) \end{array} \right)}{-17v_{ev}v_{gv} + 4v_{ev}^2 + 16v_{gv}^2} \tag{4.44}$$

$$\Pi_{Rgv} = -\frac{\left(\begin{array}{l} \left(c_{g0} - c_{g1}\right)\left(v_{ev} - 2v_{gv}\right)^2 (v_{gv}^2\left(2\left(s_c + s_m\right) + 11v_{ev} - 2p_{Mev} + 8p_{Mgv}\right) \\ -v_{ev}v_{gv}\left(s_c + 3v_{ev} + s_m - p_{Mev} + 9p_{Mgv}\right) + 2v_{ev}^2 p_{Mgv} - 8v_{gv}^3)^2 \end{array} \right)}{\left(v_{ev} - 4v_{gv}\right)^2\left(v_{ev} - v_{gv}\right)\left(-17v_{ev}v_{gv} + 4v_{ev}^2 + 16v_{gv}^2\right)^2\left(r_{g0} - r_{g1}\right)} \tag{4.45}$$

$$\Pi_{Rev} = -\frac{\left(\begin{array}{l} v_{gv}\left(c_{e0} - c_{e1}\right)(v_{ev}^2 v_{gv}\left(-13\left(s_c + s_m\right) - 12v_{ev} + 13p_{Mev} + 4p_{Mgv}\right) \\ +2v_{ev}v_{gv}^2\left(13\left(s_c + s_m\right) + 11v_{ev} - 13p_{Mev} - 2p_{Mgv}\right) \\ -4v_{gv}^3\left(4\left(s_c + s_m\right) + 3v_{ev} - 4p_{Mev}\right) + v_{ev}^3\left(2\left(s_c + v_{ev} + s_m\right) - 2p_{Mev} - p_{Mgv}\right))^2 \end{array} \right)}{v_{ev}\left(r_{e0} - r_{e1}\right)\left(v_{ev} - 4v_{gv}\right)^2\left(v_{ev} - v_{gv}\right)\left(-17v_{ev}v_{gv} + 4v_{ev}^2 + 16v_{gv}^2\right)^2} \tag{4.46}$$

$$\Pi_{Mgv} = -\frac{\left(\begin{array}{l} \left(v_{ev} - 2v_{gv}\right)(v_{gv}^2\left(2\left(s_c + s_m\right) + 11v_{ev} - 2p_{Mev} + 8p_{Mgv}\right) \\ -v_{ev}v_{gv}\left(s_c + 3v_{ev} + s_m - p_{Mev} + 9p_{Mgv}\right) + 2v_{ev}^2 p_{Mgv} - 8v_{gv}^3)^2 \end{array} \right)}{\left(v_{ev} - 4v_{gv}\right)\left(v_{ev} - v_{gv}\right)\left(-17v_{ev}v_{gv} + 4v_{ev}^2 + 16v_{gv}^2\right)^2} \tag{4.47}$$

$$\Pi_{Mev} = -\frac{\left(\begin{array}{l} v_{gv}\left(v_{ev} - 2v_{gv}\right)(v_{ev}v_{gv}\left(-9\left(s_c + s_m\right) - 8v_{ev} + 9p_{Mev} + 2p_{Mgv}\right) \\ +2v_{gv}^2\left(4\left(s_c + s_m\right) + 3v_{ev} - 4p_{Mev}\right) + v_{ev}^2\left(2\left(s_c + v_{ev} + s_m\right) - 2p_{Mev} - p_{Mgv}\right))^2 \end{array} \right)}{v_{ev}\left(v_{ev} - 4v_{gv}\right)\left(v_{ev} - v_{gv}\right)\left(-17v_{ev}v_{gv} + 4v_{ev}^2 + 16v_{gv}^2\right)^2} \tag{4.48}$$

4.2.2 Later stage of EV development

The purchase probabilities and the profit for each player are shown below. All details of substitutions from A_0 to A_1 and from C_0 to C_{27} are described in Table A-2.

Customer

In this case, the probabilities of people buying GV and EV are

$$P_{gvl} = \frac{p_{Cgv} - p_{Cev} + s_c}{v_{gv} - v_{ev}} - \frac{p_{Cgv}}{v_{gv}} = A_0(p_{Cgv} - p_{Cev} + s_c) - A_1 p_{Cgv} \tag{4.49}$$

$$P_{evl} = 1 - \frac{p_{Cgv} - p_{Cev} + s_c}{v_{gv} - v_{ev}} = 1 - A_0(p_{Cgv} - p_{Cev} + s_c) \tag{4.50}$$

Retailer

Through applying Eq. 4.50, Eq. 4.12 and Eq. 4.10, the profit function for the EV retailer is

$$\begin{aligned}
\pi_{Rev}(p_{Cev}) &= \int_{c_{e0}}^{c_{e1}} P_{evl}(p_{Cev} - p_{Rev}) \frac{1}{r_{e1} - r_{e0}} dp_{Mev} \\
&= \frac{P_{evl}}{r_{e1} - r_{e0}} (p_{Cev} - p_{Rev})(c_{e1} - c_{e0}) \\
&= \left(\begin{array}{l} \frac{A_0(c_{e0}-c_{e1})}{r_{e0}-r_{e1}} p_{Cev}^2 \\ -\frac{(c_{e0}-c_{e1})\left(A_0\left(s_c+p_{Cgv}+p_{Rev}\right)-1\right)}{r_{e0}-r_{e1}} p_{Cev} \\ +\frac{p_{Rev}(c_{e0}-c_{e1})\left(A_0\left(s_c+p_{Cgv}\right)-1\right)}{r_{e0}-r_{e1}} \end{array} \right)
\end{aligned} \tag{4.51}$$

and similarly, by applying Eq. 4.49, Eq. 4.11 and Eq. 4.9, the profit function for the GV retailer is

$$\begin{aligned}
\pi_{Rgv}(p_{Cgv}) &= \int_{c_{g0}}^{c_{g1}} P_{gvl}(p_{Cgv} - p_{Rgv}) \frac{1}{r_{g1} - r_{g0}} dp_{Mgv} \\
&= \frac{P_{gvl}}{r_{g1} - r_{g0}} (p_{Cgv} - p_{Rgv})(c_{g1} - c_{g0}) \\
&= \left(\begin{array}{l} \frac{(A_0-A_1)(c_{g0}-c_{g1})}{r_{g0}-r_{g1}} p_{Cgv}^2 \\ +\frac{(c_{g0}-c_{g1})\left(A_1 p_{Rgv}-A_0\left(-s_c+p_{Cev}+p_{Rgv}\right)\right)}{r_{g0}-r_{g1}} p_{Cgv} \\ +\frac{A_0 p_{Rgv}(c_{g0}-c_{g1})(p_{Cev}-s_c)}{r_{g0}-r_{g1}} \end{array} \right)
\end{aligned} \tag{4.52}$$

It is easy to find $\frac{\partial^2 \pi_{Rgv}(p_{Cgv})}{\partial p_{Cgv}^2} < 0$ and $\frac{\partial^2 \pi_{Rev}(p_{Cev})}{\partial p_{Cev}^2} < 0$. So, by using the first order condition

$\frac{\partial \pi_{Rgv}(p_{Cgv})}{\partial p_{Cgv}} = 0$ and $\frac{\partial \pi_{Rev}(p_{Cev})}{\partial p_{Cev}} = 0$, we have

$$\begin{cases} p_{Cgv} = \frac{A_0\left(-s_c+p_{Rev}+2p_{Rgv}\right)-2A_1p_{Rgv}-1}{3A_0-4A_1} \\ p_{Cev} = \frac{A_0(A_0-2A_1)s_c+(A_0-A_1)\left(A_0\left(2p_{Rev}+p_{Rgv}\right)-2\right)}{A_0(3A_0-4A_1)} \end{cases} \tag{4.53}$$

By substituting the equation above into P_{evl} and P_{gvl}, we have

$$\begin{aligned} P_{gvl} &= \frac{(A_0^2 - 3A_1A_0 + 2A_1^2)}{3A_0 - 4A_1} p_{Rgv} - \frac{A_0(A_0 - A_1)\left(p_{Rev} - s_c\right)}{3A_0 - 4A_1} + \frac{(A_0 - A_1)}{3A_0 - 4A_1} \\ &= C_0 p_{Rgv} - C_1\left(p_{Rev} - s_c\right) + C_2 \end{aligned} \tag{4.54}$$

$$\begin{aligned} P_{evl} &= \frac{A_0^2 - 2A_1A_0}{3A_0 - 4A_1} p_{Rev} + \frac{(A_0A_1 - A_0^2)}{3A_0 - 4A_1} p_{Rgv} - \frac{A_0^2 - 2A_1A_0}{3A_0 - 4A_1} s_c + \frac{2(A_0 - A_1)}{3A_0 - 4A_1} \\ &= C_3(p_{Rev} - s_c) + C_4 p_{Rgv} + 2C_2 \end{aligned} \tag{4.55}$$

Manufacturer

The formulas of profit for GV and EV manufacturer are

$$\pi_{Mgv}(p_{Rgv}) = P_{gvl}(p_{Rgv} - p_{Mgv}) = \left(C_0 p_{Rgv} - C_1\left(p_{Rev} - s_c\right) + C_2\right)\left(p_{Rgv} - p_{Mgv}\right) \tag{4.56}$$

$$\pi_{Mev}(p_{Rev}) = P_{evl}(p_{Rev} - p_{Mev} + s_m) = \left(C_3(p_{Rev} - s_c) + C_4 p_{Rgv} + 2C_2\right)\left(p_{Rev} - p_{Mev} + s_m\right) \tag{4.57}$$

As $\frac{\partial^2 \pi_{Mgv}(p_{Rgv})}{\partial p_{Rgv}^2} < 0$ and $\frac{\partial^2 \pi_{Mev}(p_{Rev})}{\partial p_{Rev}^2} < 0$ as well, by using the first order condition, we have

$$\begin{cases} p_{Rgv} = -\frac{C_1(C_3(s_c+s_m-p_{Mev})+2C_2)+2C_3\left(C_2-C_0p_{Mgv}\right)}{4C_0C_3+C_1C_4} = -C_5 s_c - C_5 s_m + C_6 \\ p_{Rev} = \frac{C_4(C_1s_c+C_2)-C_0\left(-2C_3(s_c-s_m+p_{Mev})+C_4p_{Mgv}+4C_2\right)}{4C_0C_3+C_1C_4} = C_7 s_c - C_8 s_m + C_9 \end{cases} \tag{4.58}$$

It can be rewritten the purchase probability as

$$\begin{aligned} P_{gvl} &= C_0 p_{Rgv} - C_1\left(p_{Rev} - s_c\right) + C_2 \\ &= (C_1 - C_7C_1 - C_0C_5)s_c + (C_1C_8 - C_0C_5)s_m + C_2 + C_0C_6 - C_1C_9 \\ &= C_{10}s_c + C_{11}s_m + C_{12} \end{aligned} \tag{4.59}$$

$$
\begin{aligned}
P_{evl} &= C_3(p_{Rev} - s_c) + C_4 p_{Rgv} + 2C_2 \\
&= (C_7C_3 - C_3 - C_4C_5)s_c - (C_4C_5 + C_3C_8)s_m + 2C_2 + C_4C_6 + C_3C_9 \\
&= C_{13}s_c - C_{14}s_m + C_{15}
\end{aligned} \tag{4.60}
$$

Government

The government is expected to pay close attention to the entire society profit:

$$
\pi_{entire} = \pi_{Cgv} + \pi_{Cev} + \pi_{Rgv} + \pi_{Rev} + \pi_{Mgv} + \pi_{Mev} - C_G \tag{4.61}
$$

Where C_G is the subsidy payment and the cost of environmental protection:

$$
C_G = P_{gvl}c_{envir} + P_{evl}(s_c + s_m) \tag{4.62}
$$

The utility for the customer is

$$
\begin{aligned}
\pi_{Cgv} + \pi_{Cev} &= \begin{pmatrix} \frac{1}{2}P_{gvl}(\frac{v_{gv}(p_{Cgv}-p_{Cev}+s_c)}{v_{gv}-v_{ev}} - p_{Cgv}) \\ +\frac{1}{2}P_{evl}(\frac{v_{gv}(p_{Cgv}-p_{Cev}+s_c)}{v_{gv}-v_{ev}} - p_{Cgv} + v_{ev} - p_{Cev} + s_c) \end{pmatrix} \\
&= \frac{\begin{pmatrix} (C_{10}s_c + C_{11}s_m + C_{12})(C_{16}s_c + C_{17}s_m + C_{18}) \\ + (C_{13}s_c - C_{14}s_m + C_{15})(C_{19}s_c + C_{20}s_m + C_{21}) \end{pmatrix}}{2A_0(3A_0 - 4A_1)(v_{ev} - v_{gv})}
\end{aligned} \tag{4.63}
$$

The utility for the manufacturer, retailer and government together is

$$
\begin{aligned}
&\pi_{Mgv} + \pi_{Mev} + \pi_{Rgv} + \pi_{Rev} - P_{gvl}c_{envir} - P_{evl}(s_c + s_m) \\
&= \begin{pmatrix} \frac{P_{evl}(p_{Cev}-p_{Rev})(c_{e1}-c_{e0})}{r_{e1}-r_{e0}} + \frac{P_{gvl}(p_{Cgv}-p_{Rgv})(c_{g1}-c_{g0})}{r_{g1}-r_{g0}} \\ +P_{gvl}(p_{Rgv} - p_{Mgv} - c_{envir}) + P_{evl}(p_{Rev} - p_{Mev} - s_c) \end{pmatrix} \\
&= \begin{pmatrix} (C_{10}s_c + C_{11}s_m + C_{12})(C_{22}s_c + C_{23}s_m + C_{24}) \\ +(C_{13}s_c - C_{14}s_m + C_{15})(C_{25}s_c + C_{26}s_m + C_{27}) \end{pmatrix}
\end{aligned} \tag{4.64}
$$

Therefore, the entire utility is

$$\pi_{entire} = \left(\begin{array}{l} \left(C_{10}s_c + C_{11}s_m + C_{12}\right)\left(C_{22}s_c + C_{23}s_m + C_{24}\right) \\ + \left(C_{13}s_c - C_{14}s_m + C_{15}\right)\left(C_{25}s_c + C_{26}s_m + C_{27}\right) \\ + \frac{\left(\begin{array}{l} \left(C_{10}s_c + C_{11}s_m + C_{12}\right)\left(C_{16}s_c + C_{17}s_m + C_{18}\right) \\ + \left(C_{13}s_c - C_{14}s_m + C_{15}\right)\left(C_{19}s_c + C_{20}s_m + C_{21}\right) \end{array} \right)}{2A_0(3A_0-4A_1)(v_{ev}-v_{gv})} \end{array} \right) \tag{4.65}$$

Discussion

i. Subsidy given to EV customer only

The subsidy given to the EV customer should be satisfied with

$$\begin{cases} \frac{\partial \pi_{entire}}{\partial s_c} = 0 \\ s_m = 0 \end{cases} \tag{4.66}$$

So,

$$\pi_{entire} = \left(\begin{array}{l} \left(C_{10}s_c + C_{12}\right)\left(C_{22}s_c + C_{24}\right) + \left(C_{13}s_c + C_{15}\right)\left(C_{25}s_c + C_{27}\right) \\ + \frac{\left(C_{10}s_c + C_{12}\right)\left(C_{16}s_c + C_{18}\right) + \left(C_{13}s_c + C_{15}\right)\left(C_{19}s_c + C_{21}\right)}{2A_0(3A_0-4A_1)(v_{ev}-v_{gv})} \end{array} \right) \tag{4.67}$$

We have $\frac{\partial^2 \pi_{entire}}{\partial s_c^2} = \frac{2C_{10}C_{16}+2C_{13}C_{19}}{2A_0(3A_0-4A_1)(v_{ev}-v_{gv})} + 2C_{10}C_{22} + 2C_{13}C_{25} < 0$. Therefore, by using the first order condition:

$$\frac{\partial \pi_{entire}}{\partial s_c} = \left(\begin{array}{l} \frac{C_{10}(2C_{16}s_c+C_{18})+C_{19}(2C_{13}s_c+C_{15})+C_{12}C_{16}+C_{13}C_{21}}{2A_0(3A_0-4A_1)(v_{ev}-v_{gv})} \\ +C_{22}\left(C_{10}s_c + C_{12}\right) + C_{10}\left(C_{22}s_c + C_{24}\right) \\ +C_{25}\left(C_{13}s_c + C_{15}\right) + C_{13}\left(C_{25}s_c + C_{27}\right) \end{array} \right) = 0 \tag{4.68}$$

So, in the later stage, the s_c is

$$s_c = \frac{\begin{pmatrix} C_{12}\left(2A_0\left(3A_0-4A_1\right)C_{22}\left(v_{ev}-v_{gv}\right)+C_{16}\right) \\ +2A_0\left(3A_0-4A_1\right)\left(C_{10}C_{24}+C_{15}C_{25}+C_{13}C_{27}\right)\left(v_{ev}-v_{gv}\right) \\ +C_{10}C_{18}+C_{15}C_{19}+C_{13}C_{21} \end{pmatrix}}{-2\begin{pmatrix} C_{10}\left(2A_0\left(3A_0-4A_1\right)C_{22}\left(v_{ev}-v_{gv}\right)+C_{16}\right) \\ +C_{13}\left(2A_0\left(3A_0-4A_1\right)C_{25}\left(v_{ev}-v_{gv}\right)+C_{19}\right) \end{pmatrix}} \tag{4.69}$$

ii. Subsidy given to EV manufacturer only

The subsidy given to the EV manufacturer should be satisfied with

$$\begin{cases} \frac{\partial \pi_{entire}}{\partial s_m} = 0 \\ s_c = 0 \end{cases} \tag{4.70}$$

So,

$$\pi_{entire} = \begin{pmatrix} \left(C_{11}s_m+C_{12}\right)\left(C_{23}s_m+C_{24}\right)+\left(-C_{14}s_m+C_{15}\right)\left(C_{26}s_m+C_{27}\right) \\ +\frac{\left(C_{11}s_m+C_{12}\right)\left(C_{17}s_m+C_{18}\right)+\left(-C_{14}s_m+C_{15}\right)\left(C_{20}s_m+C_{21}\right)}{2A_0(3A_0-4A_1)(v_{ev}-v_{gv})} \end{pmatrix} \tag{4.71}$$

We have $\frac{2C_{11}C_{17}-2C_{14}C_{20}}{2A_0(3A_0-4A_1)(v_{ev}-v_{gv})}+2C_{11}C_{23}-2C_{14}C_{26}<0$. Therefore, by using the first order condition:

$$\frac{\partial \pi_{entire}}{\partial s_m} = \begin{pmatrix} \frac{C_{11}(2C_{17}s_m+C_{18})-C_{14}(2C_{20}s_m+C_{21})+C_{12}C_{17}+C_{15}C_{20}}{2A_0(3A_0-4A_1)(v_{ev}-v_{gv})} \\ +C_{23}\left(C_{11}s_m+C_{12}\right)+C_{11}\left(C_{23}s_m+C_{24}\right) \\ +C_{26}\left(C_{15}-C_{14}s_m\right)-C_{14}\left(C_{26}s_m+C_{27}\right) \end{pmatrix} = 0 \tag{4.72}$$

So, in the later stage, the s_m is

$$s_m = \frac{\begin{pmatrix} C_{12}\left(-2A_0\left(3A_0-4A_1\right)C_{23}\left(v_{ev}-v_{gv}\right)-C_{17}\right) \\ +2A_0\left(3A_0-4A_1\right)\left(C_{11}C_{24}+C_{15}C_{26}-C_{14}C_{27}\right)\left(v_{gv}-v_{ev}\right) \\ -C_{11}C_{18}-C_{15}C_{20}+C_{14}C_{21} \end{pmatrix}}{2\begin{pmatrix} C_{11}\left(2A_0\left(3A_0-4A_1\right)C_{23}\left(v_{ev}-v_{gv}\right)+C_{17}\right) \\ +C_{14}\left(-2A_0\left(3A_0-4A_1\right)C_{26}\left(v_{ev}-v_{gv}\right)-C_{20}\right) \end{pmatrix}} \tag{4.73}$$

By comparing Appendix A.3.3 and Appendix A.3.2, we can conclude that $s_c = s_m$ in the later EV development stage.

Summary

By substituting table A-2 to P_{evl}, P_{gvl}, p_{Cgv}, p_{Cev}, p_{Rgv}, p_{Rev}, we have

$$P_{gvl} = \frac{\begin{pmatrix} v_{ev}\left(2v_{ev}-v_{gv}\right)\left(v_{gv}^2\left(s_c-8v_{ev}+s_m-p_{Mev}-2p_{Mgv}\right)\right. \\ \left.+v_{ev}v_{gv}\left(-2\left(s_c-3v_{ev}+s_m\right)+2p_{Mev}+9p_{Mgv}\right)-8v_{ev}^2p_{Mgv}+2v_{gv}^3\right) \end{pmatrix}}{v_{gv}\left(v_{ev}-v_{gv}\right)\left(4v_{ev}-v_{gv}\right)\left(-17v_{ev}v_{gv}+16v_{ev}^2+4v_{gv}^2\right)} \tag{4.74}$$

$$P_{evl} = \frac{\begin{pmatrix} -2v_{ev}^2v_{gv}\left(13\left(s_c+s_m\right)+15v_{ev}-13p_{Mev}+2p_{Mgv}\right) \\ +v_{ev}v_{gv}^2\left(13\left(s_c+s_m\right)+17v_{ev}-13p_{Mev}+p_{Mgv}\right) \\ +v_{gv}^3\left(-2\left(s_c+s_m\right)-3v_{ev}+2p_{Mev}\right) \\ +4v_{ev}^3\left(4\left(s_c+v_{ev}+s_m\right)-4p_{Mev}+p_{Mgv}\right) \end{pmatrix}}{\left(v_{ev}-v_{gv}\right)\left(4v_{ev}-v_{gv}\right)\left(-17v_{ev}v_{gv}+16v_{ev}^2+4v_{gv}^2\right)} \tag{4.75}$$

$$p_{Cgv} = \frac{\begin{pmatrix} 2v_{gv}\left(3v_{ev}-v_{gv}\right)\left(v_{gv}\left(s_c-8v_{ev}+s_m-p_{Mev}\right)\right. \\ \left.+2v_{ev}\left(-s_c+3v_{ev}-s_m+p_{Mev}\right)+2v_{gv}^2\right) \\ +v_{ev}p_{Mgv}\left(2v_{ev}-v_{gv}\right)\left(8v_{ev}-3v_{gv}\right) \end{pmatrix}}{\left(4v_{ev}-v_{gv}\right)\left(-17v_{ev}v_{gv}+16v_{ev}^2+4v_{gv}^2\right)} \tag{4.76}$$

$$p_{Cev} = \frac{\begin{pmatrix} -2v_{ev}^2v_{gv}\left(35s_c+41v_{ev}-7s_m+7p_{Mev}+5p_{Mgv}\right) \\ +v_{ev}v_{gv}^2\left(30s_c+40v_{ev}-3s_m+3p_{Mev}+2p_{Mgv}\right) \\ -2v_{gv}^3\left(2s_c+3v_{ev}\right)+4v_{ev}^3\left(12s_c+12v_{ev}-4s_m+4p_{Mev}+3p_{Mgv}\right) \end{pmatrix}}{\left(4v_{ev}-v_{gv}\right)\left(-17v_{ev}v_{gv}+16v_{ev}^2+4v_{gv}^2\right)} \tag{4.77}$$

$$p_{Rgv} = \frac{\left(\begin{array}{l} v_{gv}(v_{gv}\left(s_c - 8v_{ev} + s_m - p_{Mev}\right) \\ +2v_{ev}\left(-s_c + 3v_{ev} - s_m + p_{Mev}\right) \\ +2v_{gv}^2) + 2p_{Mgv}\left(v_{gv} - 2v_{ev}\right)^2 \end{array}\right)}{-17v_{ev}v_{gv} + 16v_{ev}^2 + 4v_{gv}^2} \tag{4.78}$$

$$p_{Rev} = \frac{\left(\begin{array}{l} -v_{ev}v_{gv}\left(9s_c + 11v_{ev} - 8s_m + 8p_{Mev} + p_{Mgv}\right) \\ +v_{gv}^2\left(2\left(s_c - s_m + p_{Mev}\right) + 3v_{ev}\right) \\ +2v_{ev}^2\left(4\left(s_c + v_{ev} - s_m\right) + 4p_{Mev} + p_{Mgv}\right) \end{array}\right)}{-17v_{ev}v_{gv} + 16v_{ev}^2 + 4v_{gv}^2} \tag{4.79}$$

$$\Pi_{Rgv} = \frac{\left(\begin{array}{l} v_{ev}\left(c_{g0} - c_{g1}\right)\left(v_{gv} - 2v_{ev}\right)^2(v_{gv}^2\left(s_c - 8v_{ev} + s_m - p_{Mev} - 2p_{Mgv}\right) \\ +v_{ev}v_{gv}\left(-2\left(s_c - 3v_{ev} + s_m\right) + 2p_{Mev} + 9p_{Mgv}\right) - 8v_{ev}^2p_{Mgv} + 2v_{gv}^3)^2 \end{array}\right)}{v_{gv}\left(v_{ev} - v_{gv}\right)\left(v_{gv} - 4v_{ev}\right)^2\left(-17v_{ev}v_{gv} + 16v_{ev}^2 + 4v_{gv}^2\right)^2\left(r_{g0} - r_{g1}\right)} \tag{4.80}$$

$$\Pi_{Rev} = \frac{\left(\begin{array}{l} \left(c_{e0} - c_{e1}\right)(-2v_{ev}^2v_{gv}\left(13\left(s_c + s_m\right) + 15v_{ev} - 13p_{Mev} + 2p_{Mgv}\right) \\ +v_{ev}v_{gv}^2\left(13\left(s_c + s_m\right) + 17v_{ev} - 13p_{Mev} + p_{Mgv}\right) \\ +v_{gv}^3\left(-2\left(s_c + s_m\right) - 3v_{ev} + 2p_{Mev}\right) \\ +4v_{ev}^3\left(4\left(s_c + v_{ev} + s_m\right) - 4p_{Mev} + p_{Mgv}\right))^2 \end{array}\right)}{\left(\left(r_{e0} - r_{e1}\right)\left(v_{ev} - v_{gv}\right)\left(v_{gv} - 4v_{ev}\right)^2\left(-17v_{ev}v_{gv} + 16v_{ev}^2 + 4v_{gv}^2\right)^2\right)} \tag{4.81}$$

$$\Pi_{Mgv} = \frac{\left(\begin{array}{l} v_{ev}\left(2v_{ev} - v_{gv}\right)(v_{gv}^2\left(s_c - 8v_{ev} + s_m - p_{Mev} - 2p_{Mgv}\right) \\ +v_{ev}v_{gv}\left(-2\left(s_c - 3v_{ev} + s_m\right) + 2p_{Mev} + 9p_{Mgv}\right) - 8v_{ev}^2p_{Mgv} + 2v_{gv}^3)^2 \end{array}\right)}{v_{gv}\left(v_{ev} - v_{gv}\right)\left(4v_{ev} - v_{gv}\right)\left(-17v_{ev}v_{gv} + 16v_{ev}^2 + 4v_{gv}^2\right)^2} \tag{4.82}$$

$$\Pi_{Mev} = \frac{\left(\begin{array}{l} \left(2v_{ev} - v_{gv}\right)(v_{ev}v_{gv}\left(-9\left(s_c + s_m\right) - 11v_{ev} + 9p_{Mev} - p_{Mgv}\right) \\ +v_{gv}^2\left(2\left(s_c + s_m\right) + 3v_{ev} - 2p_{Mev}\right) \\ +2v_{ev}^2\left(4\left(s_c + v_{ev} + s_m\right) - 4p_{Mev} + p_{Mgv}\right))^2 \end{array}\right)}{\left(-5v_{ev}v_{gv} + 4v_{ev}^2 + v_{gv}^2\right)\left(-17v_{ev}v_{gv} + 16v_{ev}^2 + 4v_{gv}^2\right)^2} \tag{4.83}$$

4.3 Numerical example

In this section, we conduct several groups of numerical examples to illustrate the impacts of relevant parameters and profit. The average mileage driven per year and the general vehicle's life cycle can be found from UK Government (2017), and the prices of energy and the energy efficiencies are shwon in UK Power (2017) and China Automotive Technology and Research Center et al. (2016). Moreover, the time value is simplified as the hourly wage in UK (UK Government, 2018).

Table 4.2: The value of notations in numerical example

Average mileage driven per year (km/year)	$M_L = 12900$	Vehicle's lifecycle(Year)	$L = 11$
Price of fuel (£/L)	$p_{fuel} = 1.16$	Price of electricity (£/kWh)	$p_{elec} = 0.2$
Mileage per fuel unit(km/L)	$E_{fuel} = 12.5$	Mileage per electricity unit (km/kWh)	$E_{elec} = 7$
Time value (£/h)	$v_t = 8$	Time cost in each fuel refilling (hour)	$h_{refuel} = 0.1$
Time cost in each electricity rechagring (hour)	$h_{rechr} = 0.5$	GV driving experience utility (£)	$V_{gv} = 50$
Battery volume (kWh)	$V_{ev} = 85$	Environmental awareness level for GV user	$\theta_{gv} = 3000$
Environmental awareness level for EV user (£)	$\theta_{ev} = 5000$	GV manufacturer cost (£)	$p_{Mgv} = 10000$
EV manufacturer cost (£)	$p_{Mev} = 12000$	Average environmental management cost for each GV (£)	$c_{envir} = 1000$
$[c_{g0}, c_{g1}]$	$[c_{g0}, c_{g1}] = [10000, 14000]$	$[r_{g0}, r_{g1}]$	$[r_{g0}, r_{g1}] = [11000, 15000]$
$[c_{e0}, c_{e1}]$	$[c_{e0}, c_{e1}] = [13000, 18000]$	$[r_{e0}, r_{e1}]$	$[r_{e0}, r_{e1}] = [16000, 24000]$

4.3.1 Early stage

The driving experience in the early stage is shown in Table 4.3.

Table 4.3: The vehicle driving experience in the early stage

GV driving experience utility (£)	$\pi_{gv} = 120000$
EV driving experience utility (£)	$\pi_{ev} = 70000$

The optimal values are shown below for giving subsidy to EV customer and manufacturer in Table 4.4 and 4.5 severally.

Table 4.4: The optimal values in early stage when subsidy is given to EV customer

$s_c = 4311.59$	$P_{gve} = 0.33$	$P_{eve} = 0.25$	$p_{Rgv} = 42617.1$
$p_{Rev} = 27440.1$	$p_{Cgv} = 55829$	$p_{Cev} = 33694$	$\Pi_{Rgv} = 4401.44$
$\Pi_{Rev} = 695.70$	$\Pi_{Mgv} = 10866.2$	$\Pi_{Mev} = 3814.56$	$\Pi_{total} = 32630.9$

Table 4.5: The optimal values in early stage when subsidy is given to EV manufacturer

$s_m = 4311.59$	$P_{gve} = 0.33$	$P_{eve} = 0.25$	$p_{Rgv} = 42617.1$
$p_{Rev} = 23128.5$	$p_{Cgv} = 55829$	$p_{Cev} = 29382.6$	$\Pi_{Rgv} = 4401.44$
$\Pi_{Rev} = 965.70$	$\Pi_{Mgv} = 10866.2$	$\Pi_{Mev} = 3814.56$	$\Pi_{total} = 32630.9$

4.3.2 Later stage

The driving experience for GV and EV in the later stage are shown in Table 4.6, and the optimal results are shown in Table 4.7 and 4.8.

Table 4.6: The vehicle driving experience in later stage

GV driving experience utility (£)	$\pi_{gv} = 125000$
EV driving experience utility (£)	$\pi_{ev} = 118000$

Table 4.7: The optimal values in later stage when subsidy is given to EV customer

$s_c = 1344.57$	$P_{gvl} = 0.40$	$P_{evl} = 0.47$	$p_{Rgv} = 13801.9$
$p_{Rev} = 16632.9$	$p_{Cgv} = 15092.9$	$p_{Cev} = 18206$	$\Pi_{Rgv} = 513.29$
$\Pi_{Rev} = 462.87$	$\Pi_{Mgv} = 1511.6$	$\Pi_{Mev} = 2180.99$	$\Pi_{total} = 47234$

Table 4.8: The optimal values in later stage when subsidy is given to EV manufacturer

$s_m = 1344.57$	$P_{gvl} = 0.40$	$P_{evl} = 0.47$	$p_{Rgv} = 13801.9$
$p_{Rev} = 15288.3$	$p_{Cgv} = 15092.9$	$p_{Cev} = 16861.5$	$\Pi_{Rgv} = 513.29$
$\Pi_{Rev} = 462.87$	$\Pi_{Mgv} = 1511.6$	$\Pi_{Mev} = 2180.99$	$\Pi_{total} = 47234$

4.4 Numerical experiment and analysis

In this section, the purchase price, and profits for the retailer and the manufacturer will be analysed. The first part compares the relationships between optimal vehicle purchase probability, sales prices and optimal profit for retailers and manufacturers in two situations, which are to give the subsidy to the EV customer and to give the subsidy to EV the manufacturer. In the next part, the relationships between energy price (fuel price and electricity price) and the optimal values are discussed.

4.4.1 Profit analysis

In the EV early development stage, and by substituting the equivalent value of s_c and s_m, it can be found that the probability of choosing vehicles, the GV sales price and the profit for each agent are equivalent, no matter whether the subsidy is given to the EV customer or the EV manufacturer. In other words, P_{gve}, P_{eve}, p_{Rgv}, p_{Cgv}, Π_{Rgv}, Π_{Rev}, Π_{Mgv}, Π_{Mgv} are equivalent in both situations. But the EV sales price, that is, the price the EV manufacturer charged to EV retailer(p_{Rev}) and the price the EV retailer charged the EV customer(p_{Cev}), are different if subsidies are given to different participants (the EV manufacturer or the EV customer) in the SC.

The same observations can be found in the later development stage (case II). That is, P_{gvl}, P_{evl}, p_{Rgv}, p_{Cgv}, Π_{Rgv}, Π_{Rev}, Π_{Mgv}, Π_{Mgv} are equivalent no matter to whom the subsidy is given. However, the EV sales price, p_{Rev} and p_{Cev}, are different.

This conclusion indicates that the recipient of the subsidy(the EV manufacturer or the EV customer) does not affect the allocation of the optimal yield. It only impacts the EV sales price. For this reason, the government can pay more attention to the other aspects of the SC rather than consider who should be subsidized, which may help them to save resources and focus on key points, such as the energy price.

4.4.2 Impact of the energy price in the early stage

In this section, both the relationship between the energy price (both the fuel price and the electricity price) and related optimal parameters for the early development stage are analysed using graphs.

Impact of fuel price

In the early stage, as shown in Fig 4.3, the subsidy decreases with the increasing fuel price, and the GV purchasing probability increases. However, the EV purchasing probability decreases slightly The GV probability is greater than the EV purchase probability. Fig 4.4 shows that most sales prices decrease except the EV sales price for the retailer when the subsidy is given to the EV manufacturer. In addition, the prices the manufacturer charges the retailer are always less than the prices the retailer charges the customers which is consistent with reality.

Looking at Fig 4.5, with regard to the profit when the fuel price increases, both the agents' profits and the overall profit decreases. In this case, the GV manufacturer will obtain the most profit, and the EV retailer will receive the least profit.

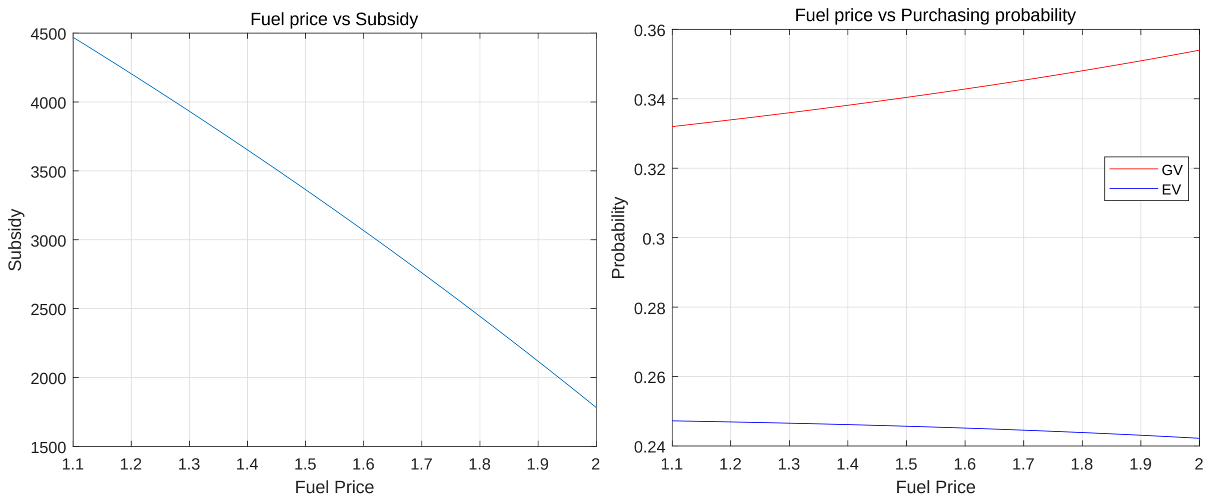

Fig. 4.3: Fuel price vs. subsidy and purchasing probability in the early stage

Therefore, in the early stage, a higher fuel price will result in a lower subsidy from the government. This explains why the EV purchase probability decreases slightly—to keep the balance between the GV and EV SC and to acquire the optimal profit for the entire industry. However, the effect is offset by the fuel price and the subsidy. In order to increase the profit for the chain and the agents, the government should control the increase in fuel prices and invest in more subsidies.

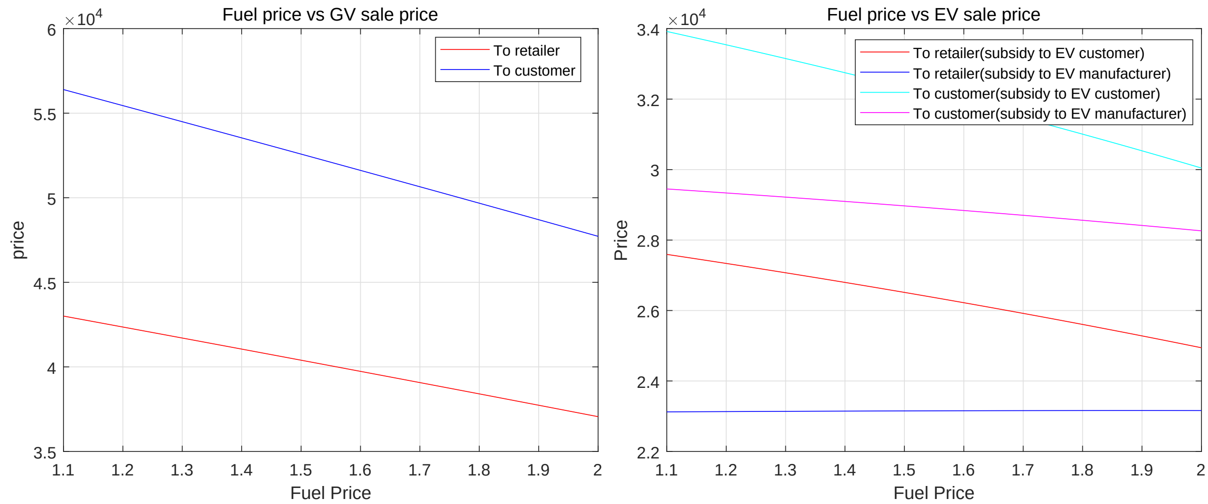

Fig. 4.4: Fuel price vs. sale price in the early stage

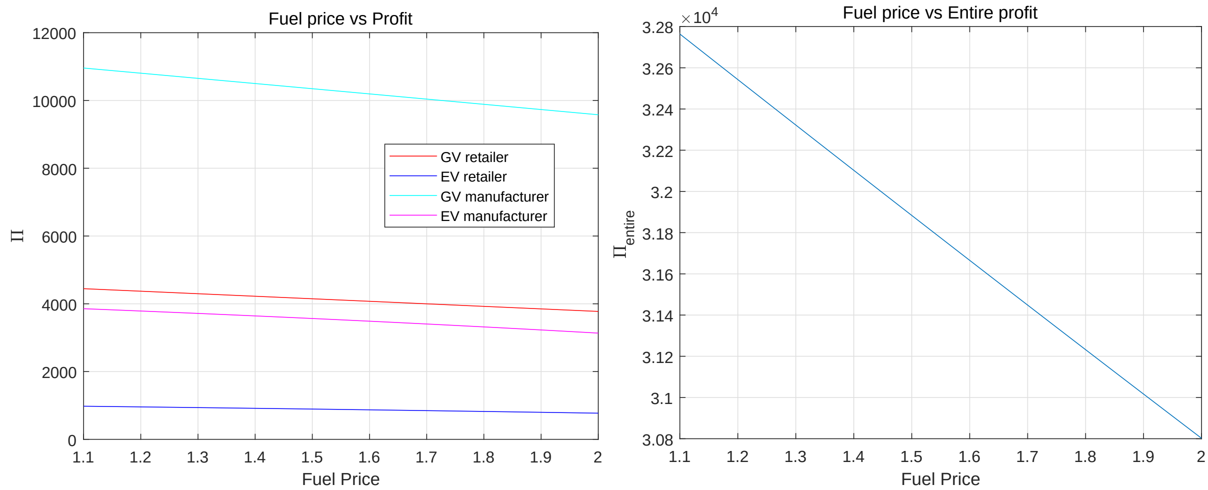

Fig. 4.5: Fuel price vs. profit in the early stage

Impact of electricity price

As can be seen in Fig 4.6, the higher the electricity price, the higher the subsidy amount and the lower the vehicle purchase probability for both GVs and EVs. The GV purchase probability is still greater than the EV purchase probability. Based on the sale price, the GV sales price increases slightly, but the EV sales price remains stable, which is shown in Figure 4.7.

Fig. 4.8 indicates that the profits for agents are stable as well, but the total profit is decreasing, which means the customer's experience decreases sharply with the increasing electricity price.

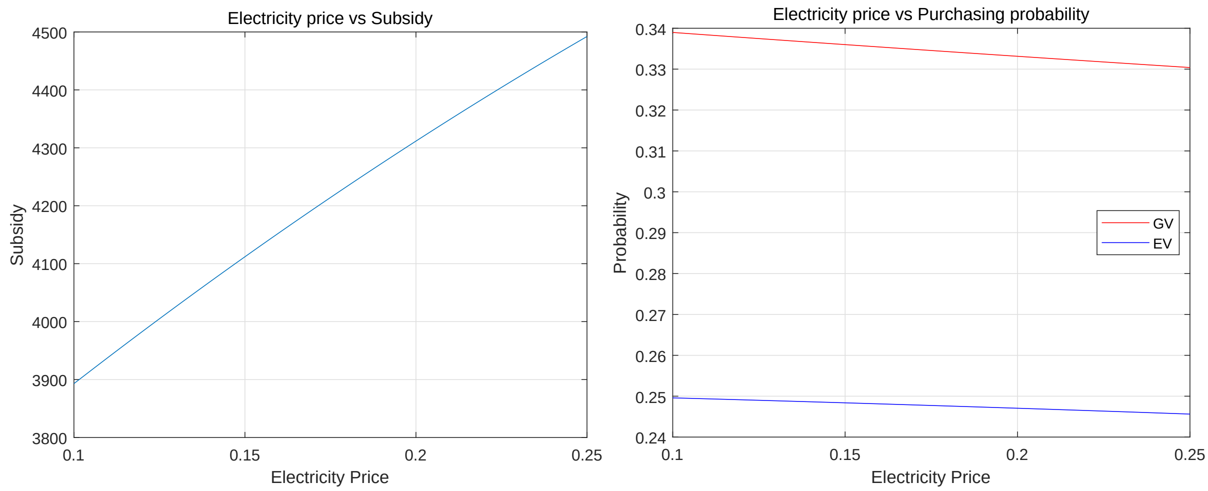

Fig. 4.6: Electricity price vs. subsidy and Purchasing probability in the early stage

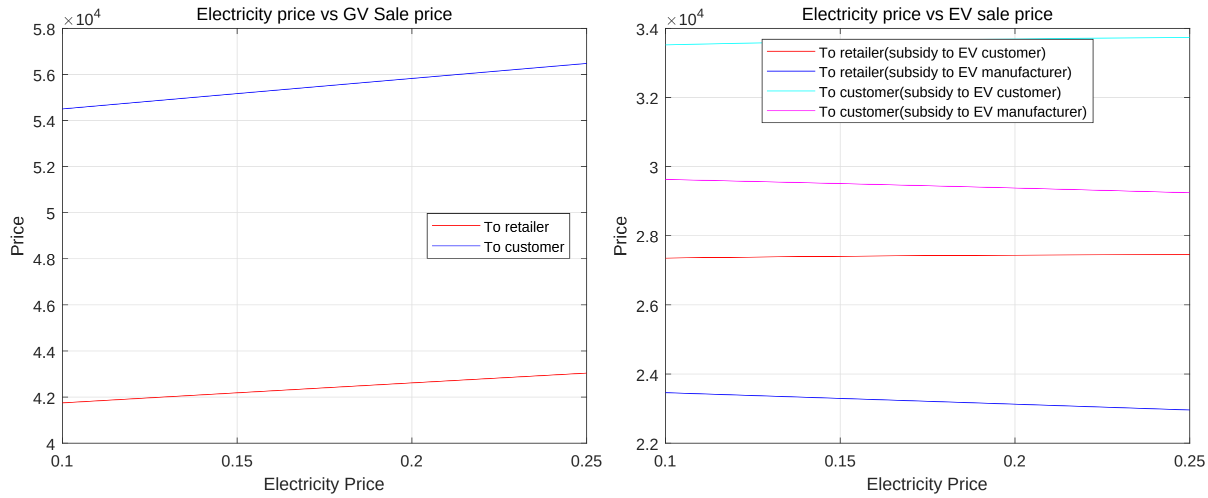

Fig. 4.7: Electricity price vs. sale price in the early stage

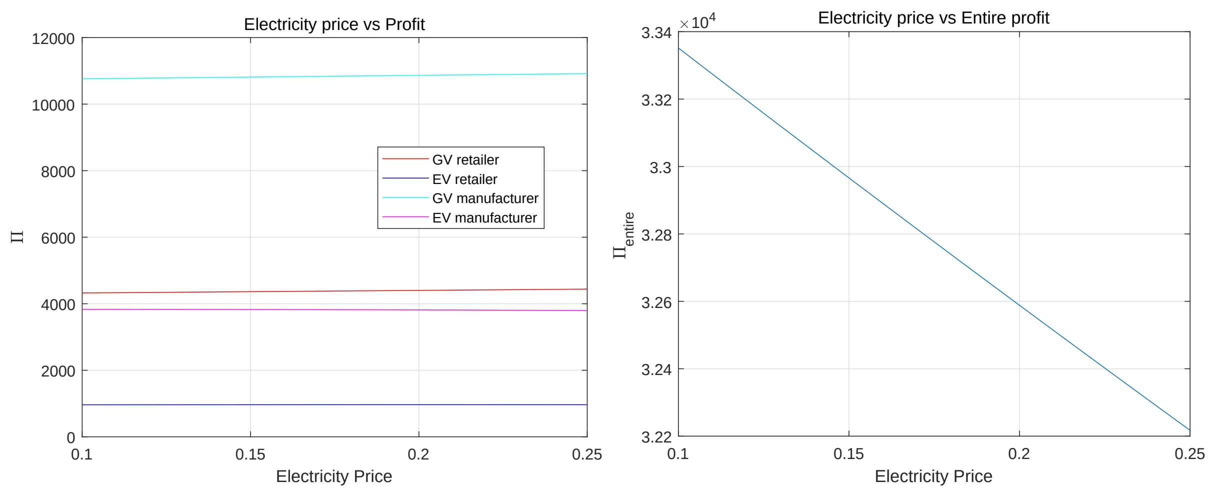

Fig. 4.8: Electricity price vs. profit in the early stage

In summary, in promoting EVs it must be remembered that higher electricity prices result in larger subsidies in the EV SC and higher vehicle sales prices. With the rising electricity unit price, some potential customers may not choose to buy a vehicle. Therefore, the government may be concerned about the increase in the electricity price. What is more, with a lower electricity price, the government does not need to budget more subsidies.

4.4.3 Impact of energy price in the later stage

This section analyses the relationship between fuel/electricity price and related optimal parameters for the EV later development stage using graphs.

Impact of fuel price

In the later stage, the subsidy increases as the price of fuel increases. And of course, the probability of buying an EV goes up as the GV purchasing probability decreases, as shown in Fig 4.9.

As can be seen in Fig. 4.10, both GV and EV sales prices increasing as the fuel prices increases, except the price that the EV manufacturer charges the retailer if the EV manufacturer is given a subsidy. As seen in Fig 4.11, when the fuel price increases, the profit for the EV SC increases with a stable GV chain, while the total profit increases with a slower rate of increase.

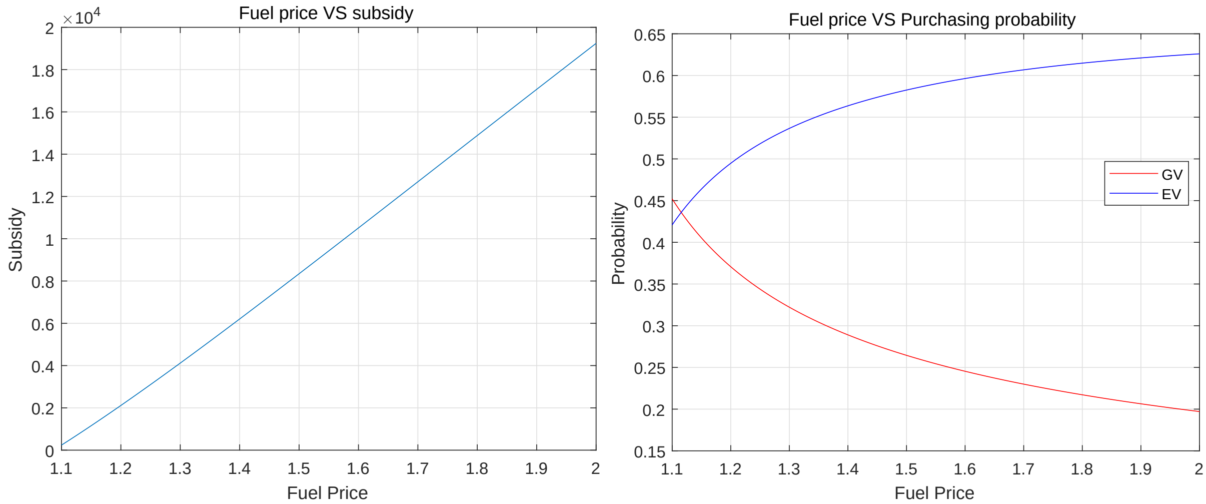

Fig. 4.9: Fuel price vs. subsidy and purchasing probability in the later stage

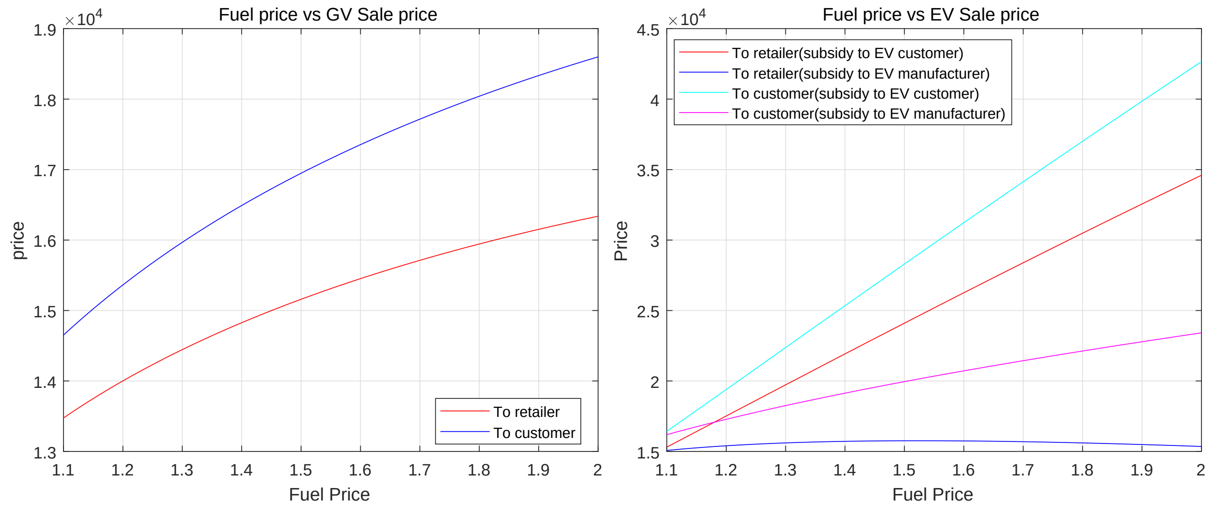

Fig. 4.10: Fuel price vs. sale price in the later stage

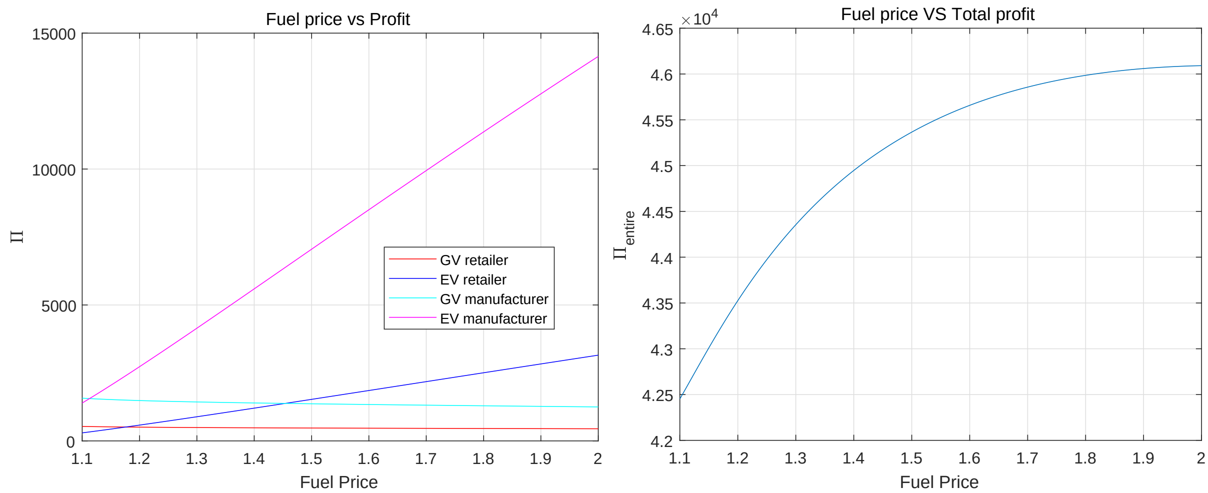

Fig. 4.11: Fuel price vs. profit in the later stage

By increasing the fuel price, EVs can be diffused quickly. And because of the increasing sales prices of vehicles, the government will have to pay more subsidies while the total profit increases.

Impact of electricity price

To summarize Figs. 4.12, 4.12 and 4.14, higher electricity prices lead to lower subsidies, lower EV purchasing probability and higher GV purchasing desire. A higher electricity price results in a lower sales price. What is more, as the electricity price increases, the total profit decreases and the profit of the EV chain also decreases. Meanwhile, the profit in the GV SC has a very slow rising trend.

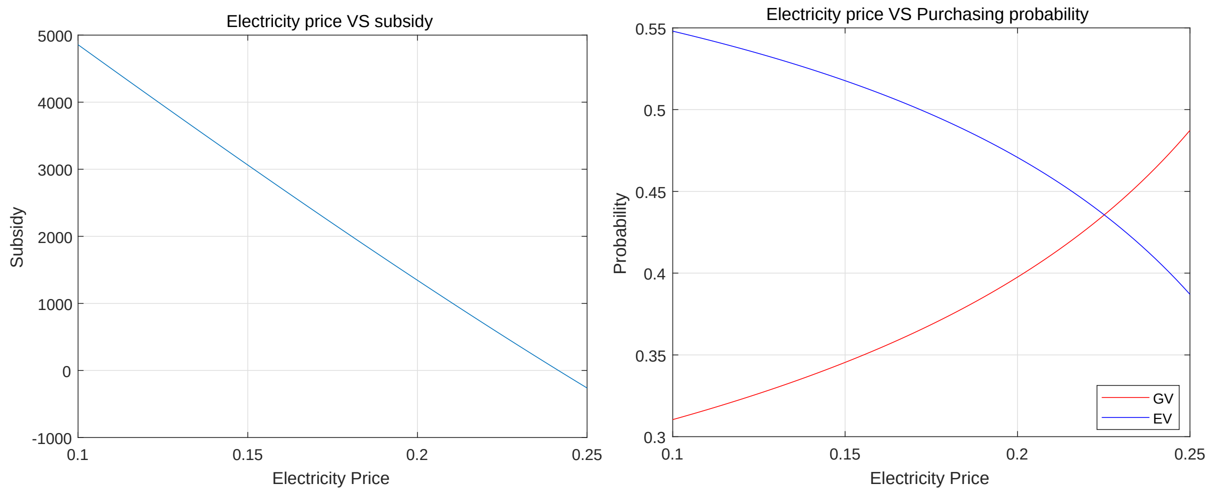

Fig. 4.12: Electricity price vs. subsidy and purchasing probability in the later stage

Therefore, the government should control the electricity price to ensure a stable profit and does not need to promote more incentives in the EV SC.

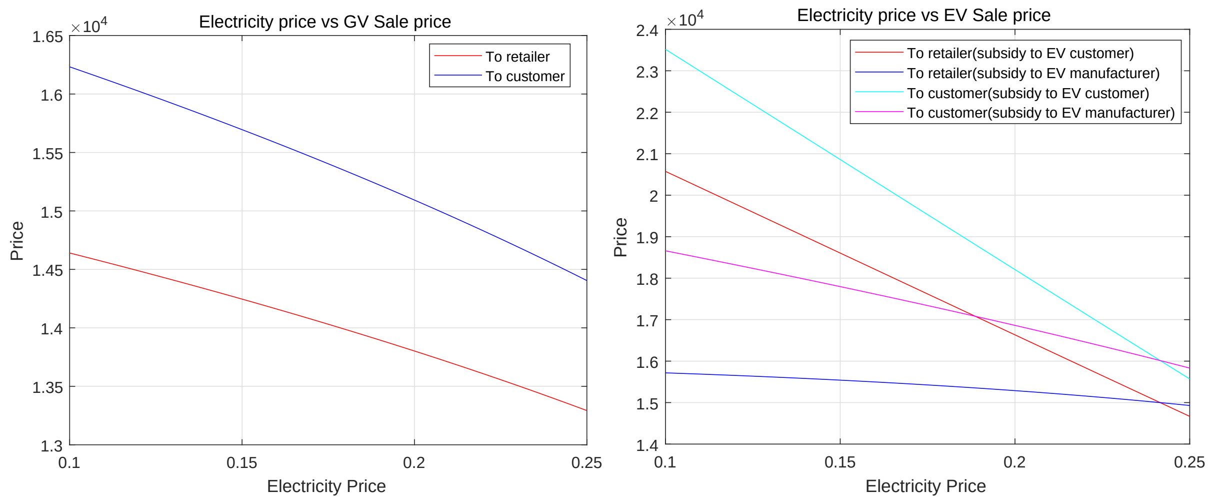

Fig. 4.13: Electricity price vs. sale price in the later stage

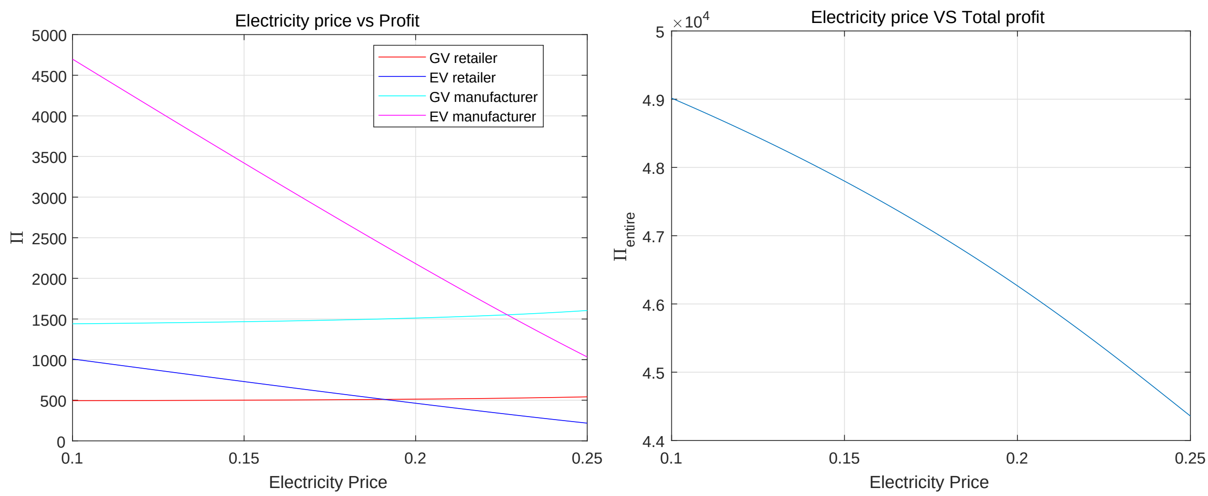

Fig. 4.14: Electricity price vs. profit in the later stage

4.5 Summary

In this chapter, we designed a multi-echelon SC that includes the government, EV/GV manufacture, EV/GV retailers and the end vehicle user. In order to promote the EV industry for sustainable development purposes, the author designs two mechanisms, which are given the subsidy to the EV manufacturer and giving the subsidy to the EV customer in different EV development stages. The results show that the effect of the subsidy given to the EV customer or the manufacturer is equivalent in the optimal equilibrium point. This includes the GV sales price, the profit for the GV/EV manufacturer, the profit for the GV/EV retailer and vehicle purchase probability. The EV sales price the manufacturer to the retailer and the retailer charges the customer are different if the subsidy is given to different agents. If the subsidy is given to the EV manufacturer, the EV price will be reduced-and if the subsidy is given to the EV customer the sales price will be increased.

Regarding the energy price, in the early EV development stage, the government should focus more on the fuel price compared to the electricity price, as the fuel price has more impact and will lead to a lower sales price and a lower subsidy paid by the government-as well as a lower total profit. Higher electricity prices result in higher GV sales price and higher EV sales price. Higher electricity prices also lead to higher government subsidies to promote EVs. In the later stage, government should focus more on the electricity price, as the higher electricity price will more strongly affect the EV diffusion, the profit for both agents in the SC and the entire SC.

The limitation of this research is that because more attention is being paid to the impacts of energy prices on subsidies in promoting the EV SC, vehicle driving experience and customer environmental awareness parameters are simply converted into tangible variables. Future research should attempt to find some functions to quantify experience and customer awarenesses and should discuss their influence on subsidies.

Chapter 5

Model for charging station selection schemes in the electricity supply chain

In this chapter, we develop a model for designing several CS selection schemes. This study is party about EV charging optimisation, trying to guide the EV user to find the optimal CS. The author presents three different schemes as options for EV users, which are the Per-time selection scheme (to select the optimal CS on each time slot of charging), the Bulk selection scheme (to select the optimal CS based on a whole charging time slots' evaluation) and the Combined Per-time/Bulk CS selection scheme by analogy from telecommunication technology. The remainder of the chapter is organised as follows. Section 5.1 describes the three schemes. Section 5.2 analyses the complexity of the three schemes. In Section 5.3, a mathematical program is developed to evaluate the performance. Section 5.4 analyses the results, and the Monte Carlo simulation is used to demonstrate the model. Section 5.5 consists of the discussion and the conclusion.

5.1 Model description

An EV driver is considering where to charge his EV. He/she starts at point $m(m \in 1, 2, ..., m, ...M)$. He also has $N = 1, 2, ..., n, ...N$ possible charging stations to charge the EV. In one time period, he may charge his car at time $t(t \in 1, 2, ...t, ...T)$ which means that the EV driver has T possible time slots to charge his own EV. We define the entire cost for him to charge the EV, which is shown below:

$$r_{m,n,t} = h_{m,n,t}s_t + v_{m,n,t}, m = 1, ..., M; n = 1, ..., N; t = 1, ...T; \tag{5.1}$$

Where s_t denotes the uniform the cost of charging the EV and $h_{n,t}$ is the expected demand of charging volume. And $v_{m,n,t}$ is an extra cost, i.e., driving cost from the starting point to the charging station and waiting cost when charging. The coefficients $h_{m,n,t}$ are assumed to be independent and identically distributed (i.i.d.) and $v_{m,n,t}$ is i.i.d. as well.

In science and engineering, Signal-to-Noise Ratio (abbreviated SNR) is defined as the ratio of signal power to the noise power, where the signal power represents the effective part and the noise power is the unrelated and interference part. Similar to SNR, we then define a parameter to evaluate the EV charging economic efficiency, which is called Revenue-to-Cost Ratio (RCR). RCR is defined as a ratio of effective cost (i.e., charging cost) to extra cost (i.e. driving cost and time cost, etc.):

$$\gamma_{m,n,t} = \frac{|h_{m,n,t}| \cdot |s_t|}{|v_{m,n,t}|} \tag{5.2}$$

The average RCR is defined as

$$\bar{\gamma} = \frac{E\{|h_{m,n,t}|\}E\{|s_t|\}}{E\{|v_{n,t}|\}}, m = 1, ..., M; n = 1, ..., N; t = 1, ...T; \tag{5.3}$$

And the RCR about charging from the mth starting point to the nth charging station at time t can also be expressed as

$$\gamma_{m,n,t} = \beta_{m,n,t}\bar{\gamma} \tag{5.4}$$

Here, we refer to $\beta_{m,n,t}$ as the normalized RCR. Similar to Sandell and Coon (2012), we define three charging station schemes: Per-time, Bulk and Combined per-time and bulk selection. As an example, as shown in Fig 5.1, assume that there are four charging stations (i.e. from CS1 to CS4), which is $N = 4$, for him to select and he can charge his vehicle at six possible time slots, i.e., $T = 6$.

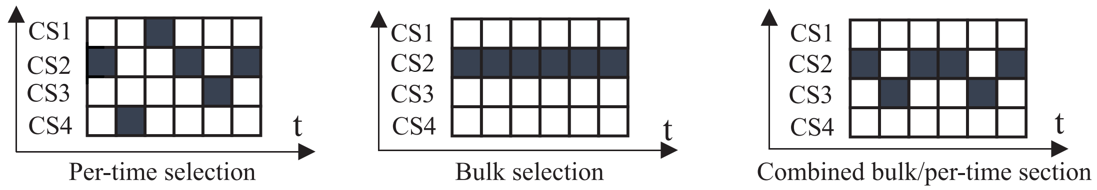

Fig. 5.1: Three charging station selection schemes

Three charging station schemes are shown below:

1) Per-time selection: one specific CS will be selected at each time slot by choosing a CS with the greatest RCR. In the first picture in Fig. 5.1, the driver will select CS2 at time slot 1 and CS4 in time slot 2, etc.;

2) Bulk selection: first, to mathematical sum up different time slot's RCR for each CS as an accumulated RCR, and the CS with greatest accumulated RCR will be chosen as the target charging station for drivers to charge the EV. So, In the second picture of Fig. 5.1, the driver will only choose CS2 no matter when he wants to charge his EV;

3) Combined bulk and per-time selection: a hybrid selection scheme is introduced which means first, more than one charging station will be selected as a subset by using the bulk selection scheme and per-time selection is conducted in the subset. With the example of the third figure in Fig. 5.1, first the author select CS2 and CS3, and then we do per-time selection in CS2 and CS3. We can conclude that the EV driver will charge his car at CS2 in time slot 1, 2, 3 and 5 while charging his car at CS3 in the remaining time slots.

Therefore, in one sentence, the basic charging station selection algorithm is to choose the CS with optimal RCR:

(1) Per-time selection

In this scheme, CS with greater RCR will be selected for each time slot:

$$S_t = \arg\max_n \gamma_{n,t} \tag{5.5}$$

The accumulated RCR is the sum of each optimal RCR at time:

$$\gamma_{pt} = \sum_t \max_n \gamma_{n,t} \tag{5.6}$$

(2) Bulk selection

Bulk selection is a cheaper option where the CS with the greatest accumulated RCR will be used for charging at each time slot:

$$S_t = \arg\max_n \sum_t \gamma_{n,t} \tag{5.7}$$

The accumulated RCR is

$$\gamma_{bulk} = \max_n \sum_t \gamma_{n,t} \tag{5.8}$$

(3) Combined bulk and per-time selection

First, by using the bulk selection method, a subset of L$(L < N)$ better RCR CSs are chosen among whole N CSs:

$$G_j = \arg\max_{k \in G_j} \sum_t \gamma_{n,t}, j = \{1, ..., L\} \tag{5.9}$$

Where $G_j \in 1, 2, ..., N$ and $|G_j| = L$. After that, per-time selection is performed among those chosen L CSs. The scheme is expressed as:

$$S_t = \arg\max_k \gamma_{k,t}, k \in G_j \tag{5.10}$$

The accumulated RCR is

$$\gamma_{cmd} = \sum_t \max_k \left(\max_{k \in G_j} \sum_t \gamma_{n,t} \right) \tag{5.11}$$

5.2 Complexity

This section discusses the complexity of each selection scheme algorithm. We still fix the starting point and assume the driver will charge his car at T time slots with N charging station options as well. In the combined selection, we are supposed to select L CSs, where $L = \lfloor N/k \rfloor, 1 < k < N$, e.g., if $N = 8$ and $k = 1.5$, we have $L = 5.33 = 5$. Based on Cormen (2009), we have two principles:

- The complexity for summing N numbers $\sum_{i=1}^{N} x_i$ is $N-1$.
- The complexity of sorting N numbers (whether sorting by ascending or descending) is N^2.

Based on the principles above, the complexity for each selection scheme is expressed below.

(1) Per-time selection

At each time slot, sorting for N CSs will be conducted to find the maxima. So, the complexity is

$$C_{pt} = TN^2 \tag{5.12}$$

(2) Bulk selection

First, we will calculate the accumulated RCR, which is $(T-1)$ additions, then we will pick up the CS with the greatest accumulated RCR through sorting. So, the complexity is

$$C_{bulk} = T - 1 + N^2 \tag{5.13}$$

(3) Combined per-time and bulk selection

Bulk scheme will be conducted for selecting L stations and T times searching among L stations will be done:

$$C_{cmd} = T - 1 + N^2 + TL^2 \tag{5.14}$$

We have three propositions in the complexity study:

1. $C_{pt} > C_{bulk}$.

   If $N > 1$ and $T > 1$, we have

$$C_{pt} - C_{bulk} = TN^2 - (T - 1 + N^2) = (N^2 - 1)(T - 1) > 0 \tag{5.15}$$

   Proposition proved.

2. $C_{cmd} > C_{bulk}$.

   The reason is $C_{cmd} - C_{bulk} = TL^2 > 0$.

3. $C_{pt} > C_{cmd}$ is satisfied when $k > \sqrt{\frac{TN^2}{TN^2-T+1-N^2}}$.

We have

$$C_{pt} - C_{cmd} = TN^2 - T + 1 - N^2 - TL^2 = TN^2 - T + 1 - N^2 - \frac{TN^2}{k^2} \tag{5.16}$$

if want $C_{pt} > C_{cmd}$, we get $k > \sqrt{\frac{TN^2}{TN^2-T+1-N^2}}$. Therefore, the complexity of C_{cmd} will be less than C_{pt} only when $k > \sqrt{\frac{TN^2}{TN^2-T+1-N^2}}$.

5.3 Performance

Here, we use the outage probability to describe the acceptance probability of RCR for charging. By setting γ_{out} as a limit, people can only charge their EV when instantaneous RCR is greater than γ_{out}.

Therefore, the probability of refusing to use this scheme to find the optimal charging station is

$$P_{out} = P(\gamma < \gamma_{out}) = \int_0^{\gamma_{out}/\bar{\gamma}} f(\beta_{m,n,t})dx \tag{5.17}$$

Based on Wang and Giannakis (2003); Tarokh et al. (1998); Proakis (1995) and for simplification, the probability density function (PDF) of $\beta_{m,n,t}$ is $f(\beta_{m,n,t}) = a\beta^b_{m,n,t} + o(\beta^{b+\varepsilon}_{m,n,t}) \approx a\beta^b_{m,n,t}$ for $\varepsilon > 0$, where $o(\beta^{b+\varepsilon}_{m,n,t})$ is an infinitesimal in $f(\beta_{m,n,t})$.

Before studying the performance of each selection scheme, we start with four lemmas below.

Lemma 1: If the PDF of independent variables X_j is $f_X(x) = ax^b + o(x^{b+\varepsilon})$, then the PDF of $\max X_1, X_2, ..., X_j$ is

$$f_Y(x) = \frac{J\alpha^J}{(b+1)^{J-1}} x^{J(b+1)-1} + o(x^{J(b+1)-1+\varepsilon}) \tag{5.18}$$

Proof: According to Papoulis and Pillai (2002), the cumulative density function (CDF) for $F_Y(x)$ maximum is

$$\begin{aligned} F_Y(x) = (F_X(x))^J = (\frac{a}{b+1}x^{b+1} + o(x^{b+1+\varepsilon}))^J = \frac{a^J}{(b+1)^J}x^{J(b+1)} + o(x^{J(b+1)+\varepsilon}) \\ \Rightarrow f_Y(x) = \frac{dF_Y(x)}{dx} = \frac{Ja^J}{(b-1)^{J-1}}x^{J(b+1)-1} + o(x^{J(b+1)-1+\varepsilon}) \end{aligned} \tag{5.19}$$

Lemma 2: If the PDF of the independent variables X_j is $f_X(x) = ax^b + o(x^{b+\varepsilon})$, the PDF of $Y = \sum_{j=1}^{J} X_j$ is

$$f_Y(x) = \frac{(ab!)^J}{(J(b+1)-1)!}x^{J(b+1)-1} + o(x^{J(b+1)-1+\varepsilon}) \tag{5.20}$$

Proof: By using the Laplace transform, we have

$$f_X(x) = ax^b + o(x^{b+\varepsilon}) \Leftrightarrow \mathcal{L}_X(s) = ab!s^{-(b+1)} + o(s^{-(b+1)}) \tag{5.21}$$

Based on Papoulis and Pillai (2002), we have

$$Y = \sum_{j=1}^{J} X_j \Leftrightarrow \mathcal{L}_Y(s) = (\mathcal{L}_X(s))^J = (ab!)^J s^{-J(b+1)} + o(s^{-J(b+1)}) \tag{5.22}$$

Therefore,by matching Eq. 5.21, we have

$$f_Y(x) = \frac{(ab!)^J}{(J(b+1)-1)!}x^{J(b+1)-1} + o(x^{J(b+1)-1+\varepsilon}) \tag{5.23}$$

Lemma 3: If the PDF of independent variables X_j has the formula $f_X(x) = ax^b + o(x^{b+\varepsilon})$, the P_{out} as shown in Eq. 5.17 can be expressed as

$$P_{out} = (A\bar{\gamma})^{-B} + o(\bar{\gamma}^{-B}) \tag{5.24}$$

Where $A = \frac{1}{\gamma_{out}}(\frac{a}{b+1})^{-\frac{1}{b+1}}$; $B = b+1$. Proof: The proof is shown in Section IV in Wang and Giannakis (2003).

Lemma 4: Let $\alpha(x) = (x!)^{1/x}$, it can be approximately equal to $\frac{x+1}{e}$.

Proof: With Gamma function $x! = \Gamma(x+1)$ and Stirling's formula $x! \approx \sqrt{2\pi x}(x/e)^x$

((Råde and Westergren, 1990), we have

$$\begin{aligned}\Gamma(x) &= (x-1)! = \sqrt{2\pi}e^{-x}x^{x-1/2}(1+O(x^{-1})); x \to \infty \\ \Rightarrow (x!)^{1/x} &= e^{-1-1/x}x^{1+1/2x} \approx \frac{x+1}{e}\end{aligned} \tag{5.25}$$

Now study the PDF and P_{out} for each selection scheme.

1. Per-time selection

   By using Lemma 1 and 2, the PDF of RCR for Per-time selection is

$$f_{pt}(x) = \frac{TN(\alpha^{TN})((N(b+1))!)^T}{(b+1)^{NT-1}(TN(b+1))!}x^{TN(b+1)-1} + o(x^{TN(b+1)-1+\varepsilon}) \tag{5.26}$$

   By using Lemma 3,

$$A_{pt} = \frac{1}{\gamma_{out}}\left(\frac{a^{TN}}{(b+1)^{TN}}\frac{((N(b+1))!)^T}{(TN(b+1))!}\right)^{-\frac{1}{TN(b+1)}} \tag{5.27}$$

$$B_{pt} = TN(b+1) \tag{5.28}$$

2. Bulk selection

   The PDF of RCR in this scheme is

$$f_{bulk}(x) = \frac{NT(ab!)^{NT}(b+1)}{(T(b+1)!)^N}x^{NT(b+1)-1} + o(x^{NT(b+1)-1+\varepsilon}) \tag{5.29}$$

   and

$$A_{bulk} = \frac{1}{\gamma_{out}}\left(\frac{(ab!)^T}{(T(b+1))!}\right)^{-\frac{1}{T(b+1)}} \tag{5.30}$$

$$B_{bulk} = TN(b+1) \tag{5.31}$$

3. Combined Per-time and Bulk selection

   In this scheme, we will first choose L charging stations from N of them ($L < N$). So, with L stations, the PDF is

$$f_\beta(x) = \frac{LTa^{LT}((L(b+1))!)^T}{(b+1)^{LT-1}(LT(b+1))!}x^{LT(b+1)-1} + o(x^{LT(b+1)-1+\varepsilon}) \tag{5.32}$$

However, all the possible subsets are non-independent, as different subset may share some charging stations, if only consider N/L disjoint charging stations subsets (Sandell and Coon, 2012). Therefore, the PDF for the combined selection is

$$f_c(x) = \frac{NTa^{NT}((L(b+1))!)^{NT/L}}{(b+1)^{NT-1}((LT(b+1))!)^{N/L}} x^{NT(b+1)-1} + o(x^{NT(b+1)-1+\varepsilon}) \tag{5.33}$$

In this scheme A and B could be

$$A_{cmd} = \frac{1}{\gamma_{out}} \left(\frac{a^{TL}((L(b+1))!)^{T}}{(b+1)^{TL}(TL(b+1))!} \right)^{-\frac{1}{TL(b+1)}} \tag{5.34}$$

$$B_{cmd} = TN(b+1) \tag{5.35}$$

5.4 Analysis and simulation

5.4.1 Performance analysis

To analyse the performance of each charging seletion scheme, as discussed before, the author uses the Probability Distribution Function (PDF) to describe the outage probaility. According to the Section 5.3, the PDF for three selection schemes are defined in equations Eq. 5.26, Eq. 5.29, Eq. 5.33 with parameters A_{pt}, B_{pt}, A_{bulk}, B_{bulk}, A_{cmd}, B_{cmd}. By comparing these equations, it can be found

$$B_{bulk} = B_{pt} = B_{cmd} \tag{5.36}$$

Now, turning to the ratio A_{cmd}/A_{pt}, A_{cmd}/A_{bulk} and A_{pt}/A_{bulk}. If $b = N - 1$, we have

1. $\frac{A_{cmd}}{A_{pt}}$

$$\begin{aligned} \frac{A_{cmd}}{A_{pt}} &= \frac{((TLN)!)^{\frac{1}{TLN}}}{((LN)!)^{\frac{1}{LN}}} \frac{((NN)!)^{\frac{1}{NN}}}{((TNN)!)^{\frac{1}{TNN}}} \\ &\approx \frac{(TLN+1)}{(LN+1)} \frac{(NN+1)}{(TNN+1)} \end{aligned} \tag{5.37}$$

2. $\frac{A_{cmd}}{A_{bulk}}$

$$\begin{aligned}\frac{A_{cmd}}{A_{bulk}} &= \frac{(N!)^{\frac{1}{N}}((TLN)!)^{\frac{1}{TLN}}}{((TN)!)^{\frac{1}{TN}}((LN)!)^{\frac{1}{LN}}} \\ &\approx \frac{(N+1)(TLN+1)}{(TN+1)(LN+1)}\end{aligned} \tag{5.38}$$

3. $\frac{A_{pt}}{A_{bulk}}$

$$\begin{aligned}\frac{A_{pt}}{A_{bulk}} &= \frac{(N!)^{\frac{1}{N}}((TNN)!)^{\frac{1}{TNN}}}{((TN)!)^{\frac{1}{TN}}((NN)!)^{\frac{1}{NN}}} \\ &\approx \frac{(N+1)(TNN+1)}{(TN+1)(NN+1)}\end{aligned} \tag{5.39}$$

If keep the ratio N/L fixed to k ($N/L = k$), we have

$$\lim_{T\to\infty} \frac{A_{cmd}}{A_{pt}} = \frac{L(NN+1)}{N(LN+1)} = 1 - \frac{k-1}{NN+k} > 0 \tag{5.40}$$

$$\lim_{T\to\infty} \frac{A_{cmd}}{A_{bulk}} = \frac{L(N+1)}{(LN+1)} = 1 + \frac{L-1}{LN+1} > 1 \tag{5.41}$$

$$\lim_{T\to\infty} \frac{A_{pt}}{A_{bulk}} = \frac{N(N+1)}{(NN+1)} = 1 + \frac{N-1}{NN+1} > 1 \tag{5.42}$$

$$\lim_{N\to\infty} \frac{A_{cmd}}{A_{pt}} = \lim_{N\to\infty} \frac{A_{cmd}}{A_{bulk}} = \lim_{N\to\infty} \frac{A_{pt}}{A_{bulk}} = 1 \tag{5.43}$$

Eq. 5.43 indicates that if charging stations are located everywhere, the performance of the three schemes is equivalent. However, it is impossible that EV driver can start at everywhere to charge the car or the charging stations are located everywhere. But the driver can charge his car at any time. We have

$$\lim_{T\to\infty} \frac{A_{pt}}{A_{bulk}} - \lim_{T\to\infty} \frac{A_{cmd}}{A_{bulk}} = \frac{N(N+1)(k-1)}{(NN+1)(NN+k)} > 0 \tag{5.44}$$

We have $A_{pt} > A_{cmd} > A_{bulk}$, which means that the performance of the Per-time charging station selection scheme is the best. Bulk selection is not that good and the Combined selection scheme's performance is between the two.

5.4.2 Simulation

In this section, the Monte Carlo method with a simulation will be used to visualise the results. The relationship between three parameters, which are the number of CSs, the number of time slots and the number of chosen stations in combined selection. The ratio of parameter A for different schemes are discussed. After that, the simulation experiment for the three schemes' performance is implemented.

i. Number of charging stations vs. ratio

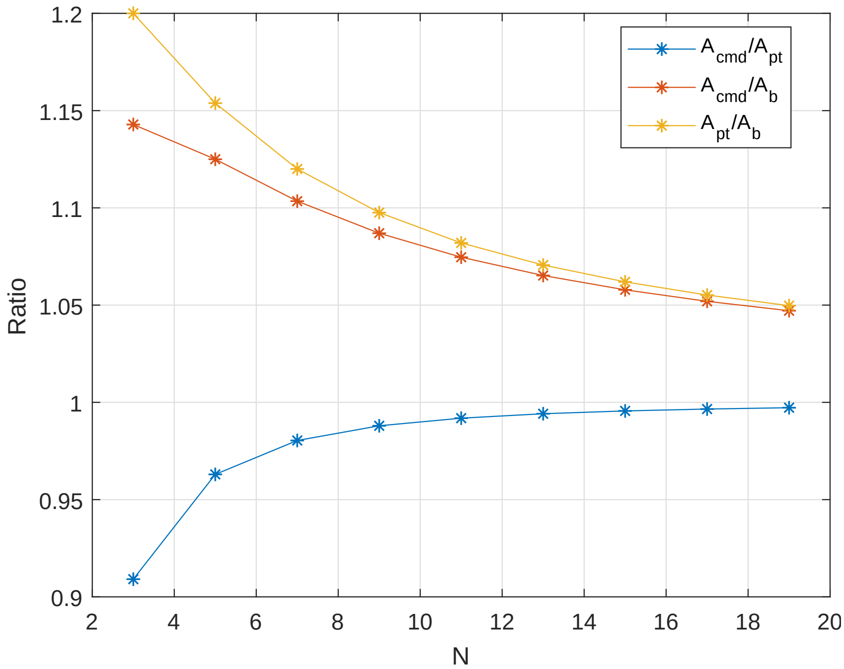

Fig. 5.2: N (Number of charging stations) vs. ratio

This section discusses the relationship between the number of CSs and the ratio of A. Fig. 5.2 shows the ratio of A for each scheme. We set $k = 2$, which means in the Combined selection, half of the CSs from the whole station set will be chosen. As can be seen, as the number of possible CSs N increases, the ratio of A for Per-time selection and combined selection approaches 1, which means the performance of combined selection and per-time selection tends to be the same. Moreover, in observing the ratio between combined selection and pet-time or bulk selection, combined selection is 5% better than the others when the number of chosen available stations reaches 18.

ii. Number of time slots vs. ratio

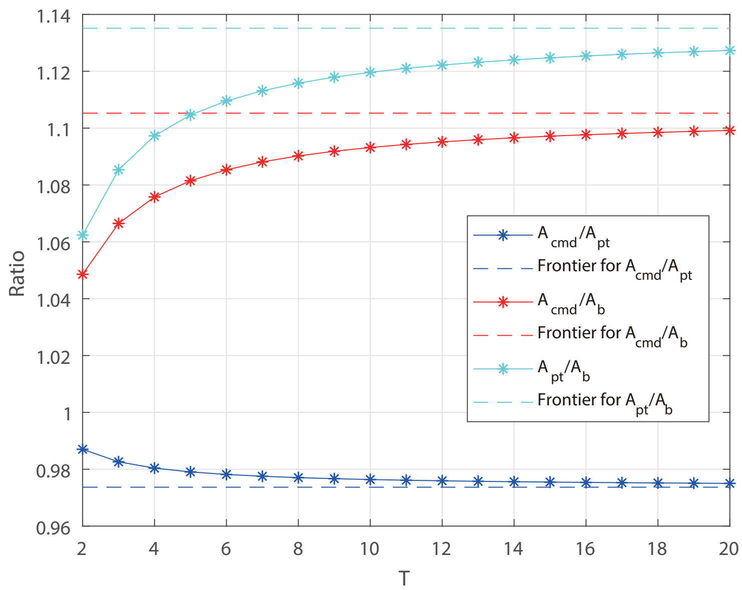

Fig. 5.3: T (number of time slots) vs. ratio

This section discusses the relationship between the number of time slots T and the ratio of A. In Fig. 5.3, with the initial setting $N = 6$ and $k = 2$, when increase the number of time slot T, with frontier by formula Eq. 5.40, Eq. 5.41 and Eq. 5.42, the ratio of A between Per-time selection and Combined selection is close to 0.97, the ratio A between Per-time and Bulk is close to 1.13 and the ratio A_{cmd}/A_b is close to 1.10. Therefore, in this case, the performance between Combined selection and Per-time selection is closer than the others.

iii. Number of chosen charging stations in combined selection vs. ratio

This section discusses the number of selected CSs as a sub-group in the combined selection scheme. With the setting of $N = 20$, Fig. 5.4 shows the ratio A_{cmd}/A_{pt} if we choose a different L with the increasing T. If we only choose 10 CSs as a sub-group in Combined selection scheme, the performance, the red curve shown in the figure, is still quite close to Per-time selection $A_{cmd}/A_{pt} > 0.997$. Moreover, if we only select two CSs as a sub-group among 20 (L=2), the performance ratio could still be around 0.98, which is fine.

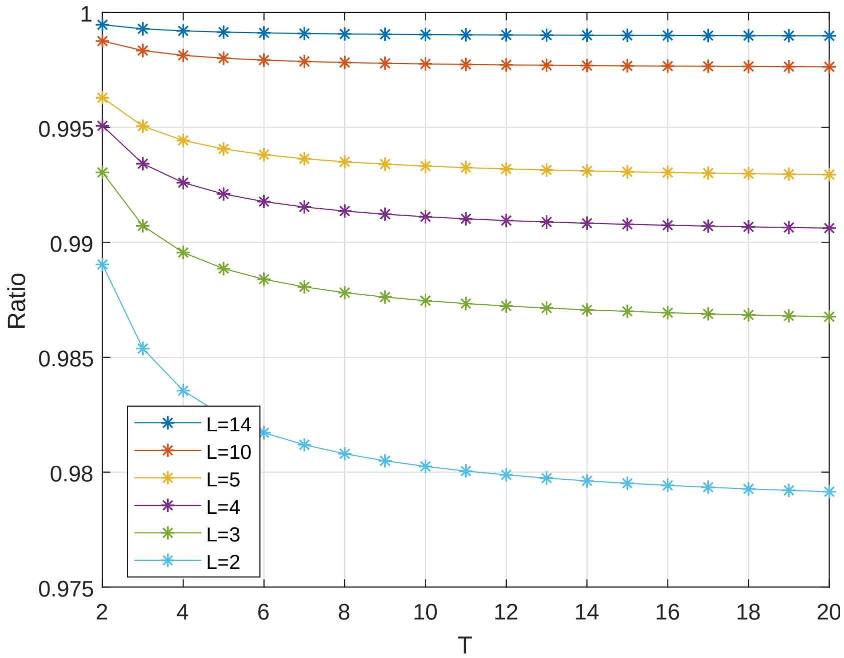

Fig. 5.4: L (number of chosen CSs in Combined scheme) vs. ratio

iv. Three schemes simulation

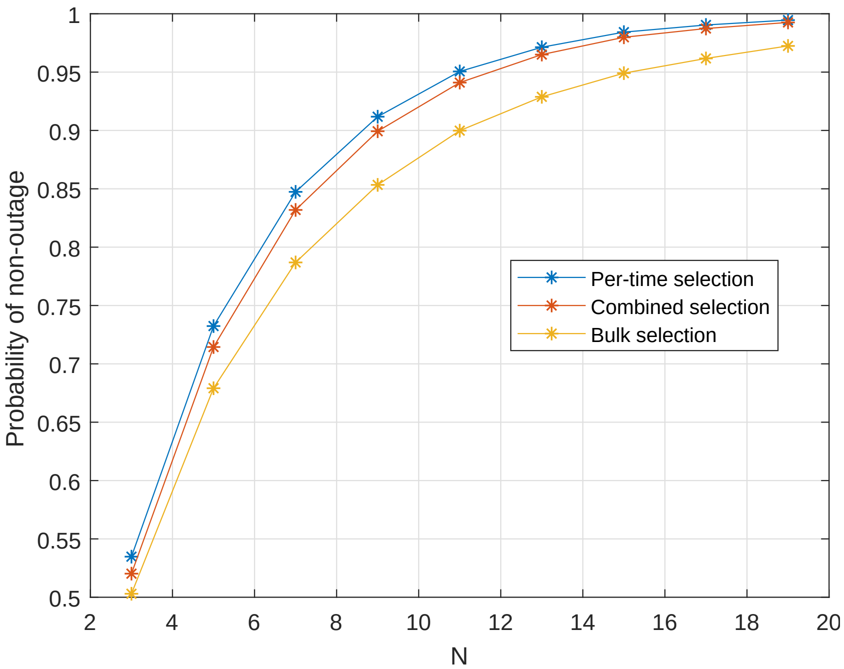

Fig. 5.5: Simulation for three schemes

In this section, we perform a realistic simulation. Fig. 5.5 simulates the non-outage probability, which means an EV driver will find the CS that meets his requirements. It can be concluded that Bulk selection may not perform as well as Per-time selection and that the performance between Combined and Per-time selection is very close. Therefore, Per-time or Combined selection can be used to find the optimal CS for the EV driver. In order to reduce system overhead and the use of resources, the combined selection would be a better choice than the Per-time selection.

5.5 Summary

Using the antenna selection telecommunication technique originally developed by Sandell and Coon (2012), we built three real-time CS selection schemes for EV users: Bulk selection, Per-time selection and Combined bulk and per-time selection. First, the selection method was proposed, followed by the selection complexity. Per-time selection is more complex than the Bulk selection, and the Combined selection is more complex than the Bulk selection. The complexity between Combined and Per-time selection depends on the number of time slots and the number of selectable CSs.

After that, the performance of the three selection schemes was discussed and compared through ratios.Based on mathematical justification and Monte Carlo simulation, it can be concluded that the Per-time selection has the best performance compared to the other two. The Combined selection falls in between Bulk selection and Per-time selection. Furthermore, when the number of charging slots and the number of CS increases, the performance between Per-time selection and combined selection gets closer, although the Combined selection only uses a subset group from among all the selectable CS. To conclude, the research in this chapter shows that combined selection could be an optimal choice to be implemented in the CS network.

However, this model is a theoretical model that does not distinguish the specific effective cost and the extra cost. Further research is needed to implement this model when combined with the geographic information system to help EV drivers find an optimal station and to help managers in better scheduling the whole network.

Chapter 6

Model for electric vehicle battery closed-loop supply chain

This chapter will discuss a game theory model for the EV battery recycling supply chain. Nash Equilibrium will be used for the optimal situation in maximizing the profit. Two situations are discussed, which are to optimize the profit for the entire supply chain and to optimize the profit for each participant(player) in the supply chain.

Section 6.1 describes the model. Section 6.2 studies the case of optimizing the individual's profit and Section 6.3 studies the whole supply chain's profit optimization. Section 6.4 summarises this chapter. Furthermore, all the proofs are provided in appendix sections B.1 and B.2.

6.1 Model description

We consider a three-period model to describe an EV battery manufacturing/ remanufacturing system as shown in Fig. 6.1. Initially, the demand for EV battery raw material is based on the quantity of EV batteries required. Furthermore, the demand for EV batteries depends on the EV market size.

- In period 1, all EV batteries are made from raw materials. The battery manufacturing quantity is based on EV demand.
- In period 2, batteries are made from raw materials, and high- and low-quality returns. First, a proportion of θ of used EV batteries is returned. We categorise

returns into two classes (Cai et al., 2014; Gaines and Singh, 1995): a proportion (α) of high-quality returns and a proportion ($1-\alpha$) of low-quality returns. Then, high-quality returns are sorted again: a proportion of β will be reused, meanwhile ($1-\beta$) will be recycled directly. Because of depletion in the battery recycling process, we set λ_l and λ_h as the remanufacturing rate for low- and high-quality returns. This indicates that λ_l (or λ_h) of low- (or high-) quality returns can be recycled into materials.

- In period 3, batteries are made from raw materials, high- and low-quality returns and reused batteries. Those reused batteries reach their end of life and will be recycled as well. The recycling rate for reused batteries is λ_u. The other returns will be recycled as indicated in period 2.

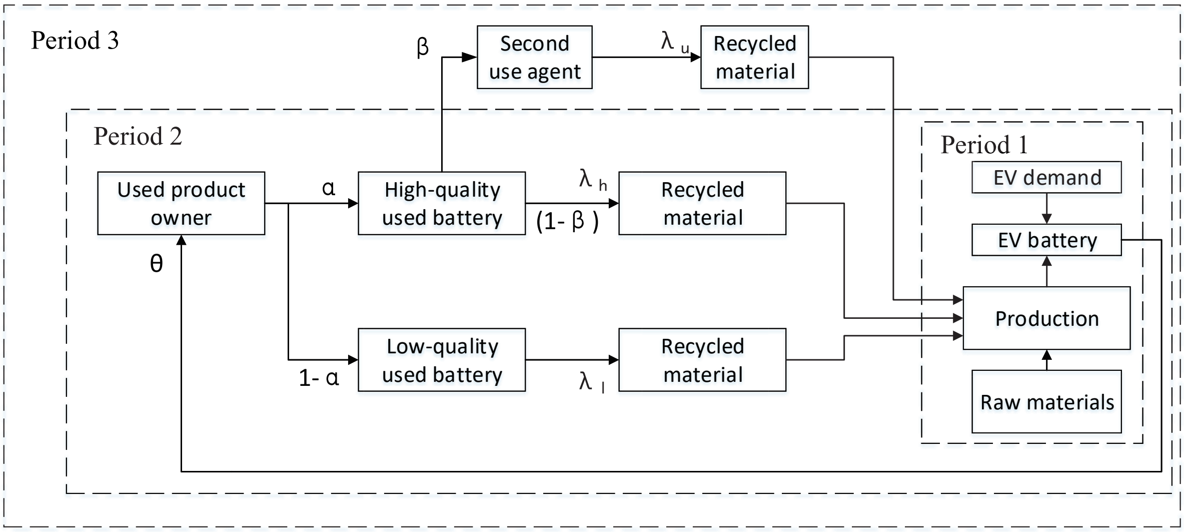

Fig. 6.1: A three-period model in manufacturing/remanufacturing system

The notations defined for this model are listed in Table 6.1.

Table 6.1: Notations

Input parameters			
θ	Battery return yield ($0 < \theta < 1$)	α	High quality return yield ($0 < \alpha < 1$)
β	Reusable return yield ($0 < \beta < 1$)	c_{ntr}	cost for new EV battery (including material cost and manufacturing cost)

c_h	Remanufacturing cost for high-quality returns	c_l	Remanufacturing cost for low-quality returns
c_u	Remanufacturing cost for reused battery	λ_h	High quality returns recycling rate $(0 < \lambda_h < 1)$
λ_l	Low quality returns recycling rate $(0 < \lambda_l < 1)$	λ_u	Reused EV battery recycling rate $(0 < \lambda_u < 1)$
Decision variables			
$p_{EVi}; i = 1, 2, 3$	EV price in period i	$q_{EVi}; i = 1, 2, 3$	EV demand in period i
$p_i; i = 1, 2, 3$	Battery price in period i	$q_{in}; i = 1, 2, 3$	Battery quantity made from raw material in period i
$q_{il}; i = 2, 3$	Quantity of batteries re-manufactured from low-quality returns in period i	$q_{ih}; i = 2, 3$	Quantity of batteries re-manufactured from high-quality returns in period i
q_{3u}	Quantity of batteries re-manufactured from reused batteries in period 3	$s_{il}; i = 2, 3$	Cost of purchasing low-quality returns in period i
$s_{ih}; i = 2, 3$	Cost of purchasing high-quality returns in period i	s_{3u}	Cost of purchasing reused batteries in period 3
$\Pi_{in}; i = 2, 3$	Profit from new battery manufacturer in period i	$\Pi_{il}; i = 2, 3$	Profit from low-quality returns re-manufacturer in period i
$\Pi_{ih}; i = 2, 3$	Profit from high-quality returns remanufacturer in period i	Π_{3u}	Profit for reused battery returns remanufacturer in period 3
Π_2	Total profit in period 2	Π_3	Total profit in period 3
Common variables			

M_{EV}	EV market size	k	For simplification, suppose $k = \delta_m/(HM_{EV})$
H	Coefficient between battery material quantity and EV sold quantity: $q_i = Hq_{EVi}, (H > 0)$	δ_m	Coefficient between EV sale price and material price: $p_i = \delta_m p_{EVi}, (0 < \delta_m < 1)$

6.2 Model under condition of individual profit optimisation

6.2.1 Nash equilibrium in period 1

Assume the EV market is M_{EV} with the EV sale price p_{EV1}. A austomer's willingness to pay for the EV is v. We uniformly distributed both p_{EV1} and v between 0 and 1 (i.e., $v \in [0, 1]$ and $p_{EV1} \in [0, 1]$). By adopting the same utility-based approach as Bulmus et al. (2014); Ferguson and Toktay (2006); Debo et al. (2005), the customers utility of buying EV is $U = v - p_{EV1}$. The quantity of EV that is sold in this period becomes

$$q_{EV1} = M_{EV}(1 - p_{EV1}) \tag{6.1}$$

We assume the demand for battery material is $q_1 = Hq_{EV1}(H > 0)$ and the EV battery price accounts for δ_m of the total car price ($p_1 = \delta_m p_{EV1}$). In period 1, all EV batteries are made from the raw materials, that is $q_{1n} = q_1$. Hence, through substituting q_1 and p_1 into Eq. 6.1, we have

$$q_{1n} = HM_{EV}(1 - p_1/\delta_m) \tag{6.2}$$

Let $k = \delta_m/(HM_{EV})$, and through formula transformation, the battery price in period 1 is

$$p_1 = \delta_m - kq_{1n} \tag{6.3}$$

The battery manufacturer's profit is the sale price minus the new EV battery cost (including both raw material cost and manufacturing cost), then multiplied by the quantity of sold. Through substituting Eq. 6.3, the profit can be expressed as

$$\Pi_1 = \Pi_{1n} = (p_1 - c_{ntr})q_{1n} = (\delta_m - kq_{1n} - c_{ntr})q_{1n} \tag{6.4}$$

6.2.2 Nash equilibrium in period 2

Similar to period 1, the entire demand for EV depends on market size and EV price in period 2:

$$q_{EV2} = M_{EV}(1 - p_{EV2}) \tag{6.5}$$

With $q_2 = Hq_{EV2}$ and $p_2 = \delta_m p_{EV2}$, the quantity of EV batteries required in this period is

$$q_2 = HM_{EV}(1 - p_{EV2}/\delta_m) \tag{6.6}$$

Let $k = \delta_m/(HM_{EV})$, then we can derive the EV battery cost function by inversing Eq. 6.6:

$$p_2 = \delta_m - kq_2 \tag{6.7}$$

In this period, θ of batteries will be returned. These returned batteries will be sorted into three classes: reusable returns, high-quality returns and low-quality returns. As shown in Fig. 6.1, the reusable returns will be reused to other places, for example, to generate a battery pack as a energy storage device. Both high- and low-quality returns will be recycled into battery materials directly. Therefore, in period 2, the battery materials come from three sources: raw natural materials and material recycled from both high-quality returns and low-quality returns. The amount of raw natural materials required amounts to the material demand of making a battery minus the quantity of materials recycled from the returned batteries:

$$q_{2n} = q_2 - q_{2l} - q_{2h} \tag{6.8}$$

We can derive the inverse of the demand function Eq. 6.8 by substituting Eq. 6.7 as follows:

$$p_2 = \delta_m - k(q_{2n} + q_{2l} + q_{2h}) \tag{6.9}$$

The total return at period 2 is the return rate θ multiplied by the quantity of battery material in the previous period, i.e. $H\theta q_{EV1}$. As mentioned, $(1-\alpha)$ of them are classified as low-quality returns. For the other returns, β of them are high-quality returns which will be recycled, while $(1-\beta)$ of them will be sorted as reusable returns. If set $A_{2h} = H\theta q_{EV1}\alpha(1-\beta)$ and $A_{2l} = H\theta q_{EV1}(1-\alpha)$, the demands for high-quality returned batteries and low-quality returned batteries are, respectively:

$$\begin{cases} q_{2h} = H\theta q_{EV1}\alpha(1-\beta)(1-s_{2h}) = A_{2h}(1-s_{2h}) \\ q_{2l} = H\theta q_{EV1}(1-\alpha)(1-s_{2l}) = A_{2l}(1-s_{2l}) \end{cases} \tag{6.10}$$

The quantity of materials made from different categories of returns is the quantity of returns multiplied by the returned batteries' recycling rate, λ_h and λ_l. The profit for the new battery manufacturer and the low-quality battery remanufacturer is defined as battery sale revenue minus recycling cost and returned battery purchase cost. Hence, by substituting Eq. 6.9, and supposing $A_2 = A_{2h} + A_{2l}$, the profit functions for the new battery manufacturer (Π_{2n}), low-quality and high-quality battery remanufacturer in period 2, i.e. Π_{2l} and Π_{2h}, are

$$\begin{cases} \Pi_{2n} = (p_2 - c_{ntr})q_{2n} \\ \qquad = -kq_{2n}^2 + (-c_{ntr} - kq_{2h} - kq_{2l} + \delta_m)\, q_{2n} \\ \Pi_{2l} = (\lambda_l p_2 - c_l - s_{2l})q_{2l} \\ \qquad = -k\lambda_l q_{2l}^2 + (-c_l - kq_{2h}\lambda_l - k\lambda_l q_{2n} + \lambda_l\delta_m - s_{2l})\, q_{2l} \\ \Pi_{2h} = (\lambda_h p_2 - c_h - s_{2h})q_{2h} \\ \qquad = -k\lambda_h q_{2h}^2 + (-c_h - k\lambda_h q_{2l} - k\lambda_h q_{2n} + \lambda_h\delta_m - s_{2h})\, q_{2h} \end{cases} \tag{6.11}$$

By using the second order condition, we have $\frac{\partial^2\Pi_{2n}}{\partial q_{2n}^2} = -k < 0$, $\frac{\partial^2\Pi_{2l}}{\partial s_{2l}^2} = -k\lambda_l < 0$ and $\frac{\partial^2\Pi_{2h}}{\partial s_{2h}^2} = -k\lambda_h < 0$. Therefore, the optimal profit can be achieved by using the first order condition, which is $\frac{\partial \Pi_{2n}}{\partial q_{2n}} = 0$, $\frac{\partial \Pi_{2l}}{\partial s_{2h}} = 0$ and $\frac{\partial \Pi_{2h}}{\partial s_{2l}} = 0$:

$$\begin{cases} \frac{\partial \Pi_{2n}}{\partial q_{2n}} = -2kq_{2n} - c_{ntr} - kq_{2h} - kq_{2l} + \delta_m = 0 \\ \frac{\partial \Pi_{2l}}{\partial s_{2l}} = -2k\lambda_l q_{2l} - c_l - kq_{2h}\lambda_l - k\lambda_l q_{2n} + \lambda_l \delta_m - s_{2l} = 0 \\ \frac{\partial \Pi_{2h}}{\partial s_{2h}} = -2k\lambda_h q_{2h} - c_h - k\lambda_h q_{2l} - k\lambda_h q_{2n} + \lambda_h \delta_m - s_{2h} = 0 \end{cases} \tag{6.12}$$

Combining Eq. 6.12 and Eq. 6.10, the optimal values are shown below:

$$s_{2l}^* = \frac{\lambda_l\left(-kA_{2h}\left(\lambda_h\left(c_{ntr}+\delta_m\right)+c_h+4\right)+4k^2A_{2h}^2\lambda_h+c_{ntr}+\delta_m\right)+c_l\left(3kA_{2h}\lambda_h-2\right)}{kA_{2h}\left(\lambda_h\left(4kA_{2h}\lambda_l-3\right)-3\lambda_l\right)+2} \tag{6.13}$$

$$s_{2h}^* = \frac{\lambda_h\left(-kA_{2h}\left(\lambda_l\left(c_{ntr}+\delta_m\right)+c_l+4\right)+4k^2A_{2h}^2\lambda_l+c_{ntr}+\delta_m\right)+c_h\left(3kA_{2h}\lambda_l-2\right)}{kA_{2h}\left(\lambda_h\left(4kA_{2h}\lambda_l-3\right)-3\lambda_l\right)+2} \tag{6.14}$$

$$q_{2n}^* = \frac{\left(\begin{array}{l} k^2A_{2h}^2\left(\left(c_h+1\right)\lambda_l+\lambda_h\left(\lambda_l\left(\delta_m-3c_{ntr}\right)+c_l+1\right)\right) \\ -kA_{2h}\left(-\left(\lambda_h+\lambda_l\right)\left(2c_{ntr}-\delta_m\right)+c_h+c_l+2\right)-c_{ntr}+\delta_m \end{array}\right)}{k\left(kA_{2h}\left(\lambda_h\left(4kA_{2h}\lambda_l-3\right)-3\lambda_l\right)+2\right)} \tag{6.15}$$

$$q_{2l}^* = A_{2l}(1 - s_{2l}^*) \tag{6.16}$$

$$q_{2h}^* = A_{2h}(1 - s_{2h}^*) \tag{6.17}$$

6.2.3 Nash equilibrium in period 3

Similarly, the EV quantity in period 3 is

$$q_{EV3} = M_{EV}(1 - p_{EV3}) \tag{6.18}$$

With $q_3 = Aq_{EV3}$ and $p_3 = \delta_m p_{EV3}$, the total demand for EV batteries in this period is

$$q_3 = HM_{EV}(1 - p_3/\delta_m) \tag{6.19}$$

Let $k = \delta_m/(HM_{EV})$, then we can achieve the price function by deriving from Eq. 6.19:

$$p_3 = \delta_m - kq_3 \tag{6.20}$$

In this period, the battery material consists of raw natural materials, high and low-quality returns and end-of-life reused battery returns. The demand for batteries made from raw natural materials is the total market demand minus all EV batteries made from returns:

$$q_{3n} = q_3 - q_{3l} - q_{3h} - q_{3u} \tag{6.21}$$

And the price

$$p_3 = \delta_m - k(q_{3n} + q_{3l} + q_{3h} + q_{3u}) \tag{6.22}$$

The return quantity in period 3 is new batteries manufactured in period 2 multiplied by the return rate, i.e. $H\theta q_{EV2}$. In this period, all returns in the three categories (low-quality, high-quality and reused returns) will be recycled with the quantity:

$$\begin{cases} q_{3l} = H\theta q_{EV2}(1-\alpha)(1-s_{3l}) \\ q_{3h} = H\theta q_{EV2}\alpha(1-\beta)(1-s_{3h}) \\ q_{3u} = H\theta q_{EV2}\alpha\beta(1-s_{3u}) \end{cases} \tag{6.23}$$

For simplification, let $A_{3l} = H\theta q_{EV2}(1-\alpha)$,$A_{3h} = H\theta q_{EV2}\alpha(1-\beta)$ and $A_{3u} = H\theta q_{EV2}\alpha\beta$, and $A_3 = A_{3l} + A_{3h} + A_{3u}$, then Eq. 6.23 can be rewritten as:

$$\begin{cases} q_{3l} = A_{3l}(1-s_{3l}) \\ q_{3h} = A_{3h}(1-s_{3h}) \\ q_{3u} = A_{3u}(1-s_{3u}) \end{cases} \tag{6.24}$$

The entire profit for the new product manufacturer is new EV battery demand multiplied by each new EV battery's profit that can be earned in manufacturing. The profits for batteries made from recycled or reused returns are the revenues minus all the costs. By supposing Eq. 6.24, the profits are:

$$\begin{cases} \Pi_{3n} = (p_3 - c_{ntr})q_{3n} \\ \quad = -kq_{3n}^2 + (-c_{ntr} - kq_{3h} - kq_{3l} - kq_{3u} + \delta_m)\, q_{3n} \\ \Pi_{3l} = \lambda_l q_{3l} p_3 - (c_l + s_{3l})q_{3l} \\ \quad = -k\lambda_l q_{3l}^2 + (-c_l - kq_{3h}\lambda_l - k\lambda_l q_{3n} - k\lambda_l q_{3u} + \lambda_l \delta_m - s_{3l})\, q_{3l} \\ \Pi_{3h} = \lambda_h q_{3h} p_3 - (c_h + s_{3h})q_{3h} \\ \quad = -k\lambda_h q_{3h}^2 + (-c_h - k\lambda_h q_{3l} - k\lambda_h q_{3n} - k\lambda_h q_{3u} + \lambda_h \delta_m - s_{3h})\, q_{3h} \\ \Pi_{3u} = \lambda_u q_{3u} p_3 - (c_u + s_{3u})q_{3u} \\ \quad = -kq_{3u}^2\lambda_u + (-c_u - kq_{3h}\lambda_u - kq_{3l}\lambda_u - kq_{3n}\lambda_u + \delta_m\lambda_u - s_{3u})\, q_{3u} \end{cases} \tag{6.25}$$

Similar to Period 2, we have $\frac{\partial^2\Pi_{3n}}{\partial q_{3n}^2} = -k < 0$, $\frac{\partial^2\Pi_{3l}}{\partial s_{3l}^2} = -k\lambda_l < 0$, $\frac{\partial^2\Pi_{3h}}{\partial s_{3h}^2} = -k\lambda_h < 0$ and $\frac{\partial^2\Pi_{3u}}{\partial s_{3u}^2} = -k\lambda_u < 0$. Therefore, the optimal profit for each (re)manufacturer is obtained when the conditions below are:

$$\begin{cases} \frac{\partial\Pi_{3n}}{\partial q_{3n}} = -2kq_{3n} - c_{ntr} - kq_{3h} - kq_{3l} - kq_{3u} + \delta_m = 0 \\ \frac{\partial\Pi_{3l}}{\partial s_{3l}} = -2k\lambda_l q_{3l} - c_l - kq_{3h}\lambda_l - k\lambda_l q_{3n} - k\lambda_l q_{3u} + \lambda_l\delta_m - s_{3l} = 0 \\ \frac{\partial\Pi_{3h}}{\partial s_{3h}} = -2k\lambda_h q_{3h} - c_h - k\lambda_h q_{3l} - k\lambda_h q_{3n} - k\lambda_h q_{3u} + \lambda_h\delta_m - s_{3h} = 0 \\ \frac{\partial\Pi_{3u}}{\partial s_{3u}} = -2k\lambda_u q_{3u} - c_u - kq_{3h}\lambda_u - kq_{3l}\lambda_u - kq_{3n}\lambda_u + \delta_m\lambda_u - s_{3u} = 0 \end{cases} \tag{6.26}$$

By combining Eq. 6.26 and Eq. 6.24, the optimal values are:

$$s_{3l}^* = \frac{\left(\begin{array}{l} \lambda_l(k(A_{3h}\left(\lambda_h\left(-k\left(4A_{3l} + A_{3u}c_u + A_{3u}\right) + c_{ntr} + \delta_m\right) + c_h\right) \\ +A_{3u}\lambda_u\left(A_{3h}k\left(\lambda_h\left(5A_{3l}k - c_{ntr} - \delta_m\right) - c_h\right) + k(-(A_{3h} + 4A_{3l})) + c_{ntr} + \delta_m\right) \\ +A_{3u}c_u) + k(A_{3h} + 3A_{3l} + A_{3u}) - c_{ntr} - \delta_m) + c_l(A_{3h}k\lambda_h\left(4A_{3u}k\lambda_u - 3\right) \\ -3A_{3u}k\lambda_u + 2) \end{array}\right)}{A_{3h}k\lambda_h\left(A_{3l}k\lambda_l\left(5A_{3u}k\lambda_u - 4\right) - 4A_{3u}k\lambda_u + 3\right) + A_{3l}k\lambda_l\left(3 - 4A_{3u}k\lambda_u\right) + 3A_{3u}k\lambda_u - 2} \tag{6.27}$$

$$s_{3h}^* = \frac{\left(\begin{array}{l} \lambda_h(k(A_{3l}(\lambda_l(-k(4A_{3h}+A_{3u}c_u+A_{3u})+c_{ntr}+\delta_m)+c_l) \\ +A_{3u}\lambda_u(A_{3l}k(\lambda_l(5A_{3h}k-c_{ntr}-\delta_m)-c_l)+k(-(4A_{3h}+A_{3l}))+c_{ntr}+\delta_m) \\ +A_{3u}c_u)+k(3A_{3h}+A_{3l}+A_{3u})-c_{ntr}-\delta_m)+c_h(A_{3l}k\lambda_l(4A_{3u}k\lambda_u-3) \\ -3A_{3u}k\lambda_u+2) \end{array}\right)}{A_{3h}k\lambda_h(A_{3l}k\lambda_l(5A_{3u}k\lambda_u-4)-4A_{3u}k\lambda_u+3)+A_{3l}k\lambda_l(3-4A_{3u}k\lambda_u)+3A_{3u}k\lambda_u-2} \tag{6.28}$$

$$s_{3u}^* = \frac{\left(\begin{array}{l} \lambda_u(k(A_{3h}(\lambda_h(k(-(A_{3l}+4A_{3u}))-A_{3l}kc_l+c_{ntr}+\delta_m)+c_h) \\ +A_{3l}\lambda_l(A_{3h}k(\lambda_h(5A_{3u}k-c_{ntr}-\delta_m)-c_h)+k(-(A_{3h}+4A_{3u}))+c_{ntr}+\delta_m) \\ +A_{3l}c_l)+k(A_{3h}+A_{3l}+3A_{3u})-c_{ntr}-\delta_m)+c_u(A_{3h}k\lambda_h(4A_{3l}k\lambda_l-3) \\ -3A_{3l}k\lambda_l+2) \end{array}\right)}{A_{3h}k\lambda_h(A_{3l}k\lambda_l(5A_{3u}k\lambda_u-4)-4A_{3u}k\lambda_u+3)+A_{3l}k\lambda_l(3-4A_{3u}k\lambda_u)+3A_{3u}k\lambda_u-2} \tag{6.29}$$

$$q_{3n}^* = \frac{-\left(\begin{array}{l} (A_{3l}k\lambda_l-1)((A_{3h}k\lambda_h-1)(A_{3u}k\lambda_u(\delta_m-2c_{ntr})+A_{3u}k(c_u+1)+c_{ntr}-\delta_m) \\ +A_{3h}k(A_{3u}k\lambda_u-1)(-c_{ntr}\lambda_h+c_h+1))+A_{3l}k(A_{3h}k\lambda_h-1)(A_{3u}k\lambda_u-1) \\ \cdot(-c_{ntr}\lambda_l+c_l+1) \end{array}\right)}{k(A_{3h}k\lambda_h(A_{3l}k\lambda_l(4-5A_{3u}k\lambda_u)+4A_{3u}k\lambda_u-3)+A_{3l}k\lambda_l(4A_{3u}k\lambda_u-3)-3A_{3u}k\lambda_u+2)} \tag{6.30}$$

$$q_{3l}^* = A_{3l}(1-s_{3l}^*) \tag{6.31}$$

$$q_{3h}^* = A_{3h}(1-s_{3h}^*) \tag{6.32}$$

$$q_{3u}^* = A_{3u}(1-s_{3u}^*) \tag{6.33}$$

6.2.4 Analysis

In this section, we analyse the relationships between the parameters (i.e., θ, α, β, λ_l, λ_h, λ_u, c_{ntr}, c_l, c_h, c_u) and the total profit for different (re)manufacturers in both period 2 and 3.

In order to obtain the relationships, the author uses the grid search, as shown in Section 3.2.7, to look through (i.e., traversal) all possible values within the set range as describled in Table 6.1. To reduce the length of chapter, all proofs of relationship between each parameter and optimal profit for each player are shown in the appendix Section B.1. From all proofs, the result can be concluded and encapsulated in Table 6.2. The author use "$\uparrow$/$\downarrow$" to represent the positive/negative correlation and use "N/A" to indicate that the relationship between those two parameters is inapplicable and "-" indicates that the relationship is indeterminate.

Table 6.2: Summary for theorems

	θ	α	β	λ_l	λ_h	λ_u	c_{ntr}	c_l	c_h	c_u
Π_{2n}	-	-	-	-	$\nearrow$	N/A	-	-	-	N/A
Π_{2l}	$\nearrow$	-	-	-	-	N/A	-	$\nearrow$	-	N/A
Π_{2h}	$\nearrow$	-	-	$\searrow$	-	N/A	-	$\nearrow$	$\nearrow$	N/A
Π_{3n}	-	-	-	-	-	-	-	-	-	-
Π_{3l}	$\nearrow$	-	-	$\nearrow$	-	-	-	$\nearrow$	$\nearrow$	$\nearrow$
Π_{3h}	$\nearrow$	$\nearrow$	-	-	$\nearrow$	-	-	$\nearrow$	$\nearrow$	$\nearrow$
Π_{3u}	$\nearrow$	$\nearrow$	$\nearrow$	-	-	$\nearrow$	-	$\nearrow$	$\nearrow$	$\nearrow$

Therefore, we reach six observations, as shown in the remarks below:

Remark 1 Return yield for the whole used batteries θ has positive relationships with the profits for all remanufacturers, but its relationships with manufacturer are uncertain.

Remark 2 Return yield for the high quality returns α has positive relationships with high quality return remanufacturer and reused battery remanufacturer in period 3.

Remark 3 Return yield for the re-usable returns β has a positive relationship with Π_{3u}.

Remark 4 The higher recycling rate λ_l, λ_h, λ_u for remanufacturers, the higher profits they will obtain in period 3.

Remark 5 The more expensive the remanufacturing cost (c_l, c_h, c_u), the more profit for the remanufacturer except the relationship between c_h and Π_{2l}.

Remark 6 The relationship between other parameters and the profit for EV battery manufacturer is normally indeterminate.

6.2.5 Management implication

In this section, we analyse the management insight based on the remarks from section 3. Specifically, we consider the following findings:

- For the new EV battery manufacturer, the performance is quite uncertain and it only depends on the specific case.
- For low quality returned batteries' remanufacturer, the profit can be improved if there are increases in the total return rate and his processing cost.
- For high quality returned batteries' remanufacturer, in order to increase his profit, his processing cost should be increased.
- Through increasing the return rate and the processing cost, the profit of reused batteries' remanufacturer can be raised.

Therefore, based on the findings above, in the economic aspect, improving the recycle and battery reuse rates as well as raising the price in remanufacturing will lead to more optimal profit for the remanufacturers. Their own profit level will improve if the waste in the remanufacturing process can be reduced.

6.2.6 Numerical experiment

Implementation of the model

According to International Energy Agency (2017), EV market size is predicted to be 18,000,000 in 2020, i.e. $M_{EV} = 18,000,000$. And EV battery price accounts for around 30% of the electric car price i.e. $\delta_m = 0.3$. According to Fred Lambert (2017) and Mark (2014), using the Tesla Model as an example, the whole cost for each EV battery is £11700. According to Binks (2016) and Date (2015), the average cost to process a used battery is £860. We assume the low-quality used battery recycling processing cost is £950, high-quality is £850 and reusable battery is £800. To normalise the cost into the same scale without losing generality, we set $c_{ntr} = 0.2$ as a benchmark, other costs against the benchmark $c_l = 0.2 * 950/11700 = 0.016$, $c_h = 0.2 * 850/11700 = 0.015$, $c_u = 0.2 * 800/11700 = 0.014$. We come up with a numerical example to demonstrate the model. All numerical parameters are shown in Table 6.3:

Table 6.3: Numerical example parameters

$M_{EV} = 18000000$	$H = 4$	$\delta_m = 0.4$	$\theta = 0.3$	$\alpha = 0.8$
$\beta = 0.7$	$c_{ntr} = 0.2$	$c_h = 0.015$	$c_l = 0.016$	$c_u = 0.014$
$\lambda_l = 0.8$	$\lambda_h = 0.9$	$\lambda_u = 0.85$		

In **period 1**, the optimal quantity for battery raw materials (for period 1, this is also the optimal total quantity) is $q_1^* = q_{1n}^* = 18,000,000$. The optimal sale price is $p_1^* = 17550$ and the optimal profit in this stage is $\Pi_1 = \Pi_{1n} = 1.8 \times 10^6$. Moreover, the optimal EV sale quantity is $q_{EV1}^* = 4.5 \times 10^6$. In **period 2** we substitute q_{EV1}^* as the initial input parameter for this period. By applying Eq. 6.44 to Eq. 6.48, the optimal values are as follows in Table 6.4.

Table 6.4: Optimal values in period 2

$q_{2n}^* = 1.70 \times 10^7$	$q_{2l}^* = 8.48 \times 10^5$	$q_{2h}^* = 9.78 \times 10^5$	$q_2^* = 1.88 \times 10^7$
$p_2^* = 17539$	$\Pi_{2n}^* = 1.62 \times 10^6$	$\Pi_{2l}^* = 4.47 \times 10^3$	$\Pi_{2h}^* = 5.62 \times 10^3$
$q_{EV2}^* = 4.5 \times 10^6$			

And in **period 3**, we substitute q_{EV2}^* as the initial EV quantity in this period. By applying Eq. 6.49 to Eq. 6.56, the optimal values in this period are shown in Table 6.5.

Table 6.5: Optimal values in period 3

$q_{3n}^* = 1.58 \times 10^7$	$q_{3l}^* = 8.92 \times 10^5$	$q_{3h}^* = 1.03 \times 10^6$	$q_{3u}^* = 2.47 \times 10^6$
$q_3^* = 2.02 \times 10^7$	$p_3^* = 17088$	$\Pi_{3n}^* = 1.39 \times 10^6$	$\Pi_{3l}^* = 3.54 \times 10^3$
$\Pi_{3h}^* = 5.32 \times 10^3$	$\Pi_{3u}^* = 2.88 \times 10^4$	$q_{EV3}^* = 5.05 \times 10^6$	

Numerical study

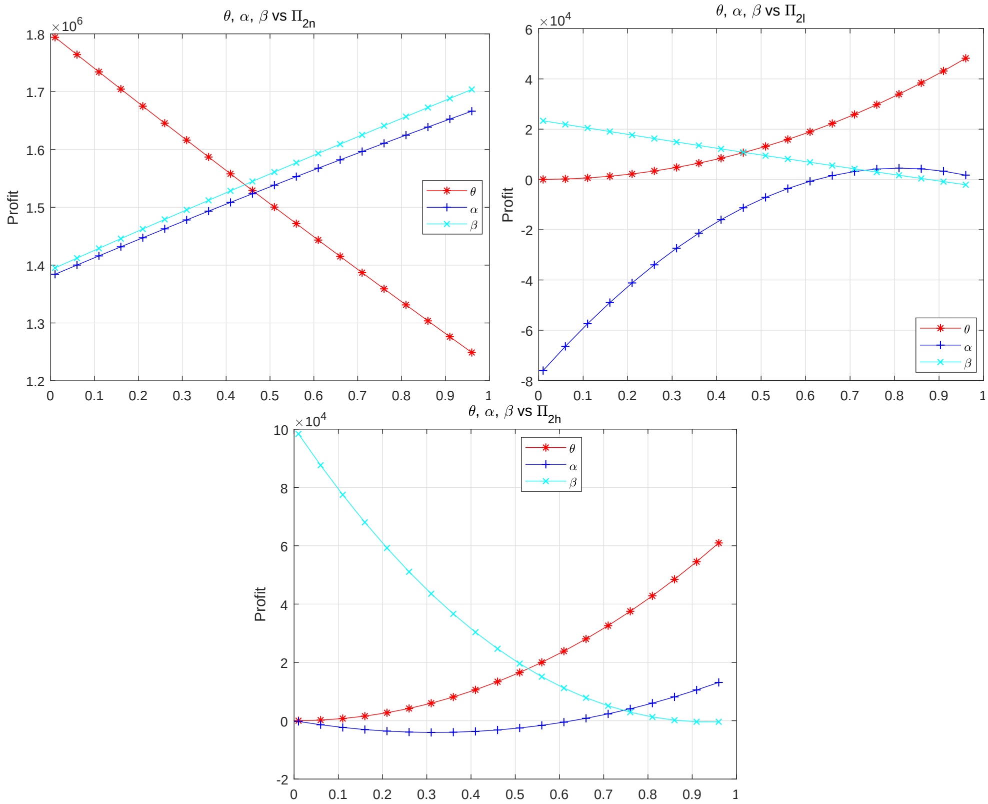

Fig. 6.2: θ, α, β vs. profit in period 2

This subsection shows the trend in the specific case with the initial value in table 6.3. Figure 6.2 and 6.3 show the relationships between θ, α, β and the profit for each player in the model. In the case of period 2, the return rate θ is positive with Π_{2l} and Π_{2h} while more returns will generate less profit for the manufacturer. Low quality sorting rate α has a positive trend with new battery producer. Furthermore a high quality classification rate (β) has positive relationship with Π_{2n} and a negative relationship with Π_{2l} and Π_{2h}.

In period 3, the tendency is similar to period 2. Moreover, for the profit of reused batteries' remanufacturer, Π_{3u}, all relationships are positive.

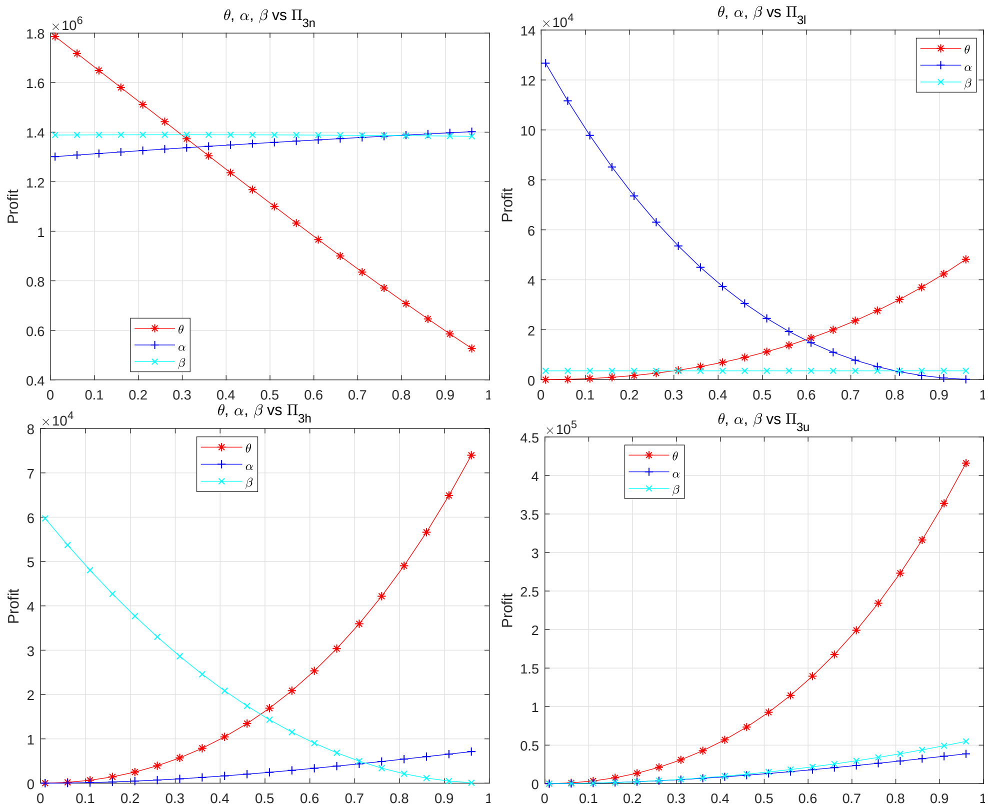

Fig. 6.3: θ, α, β vs. profit in period 3

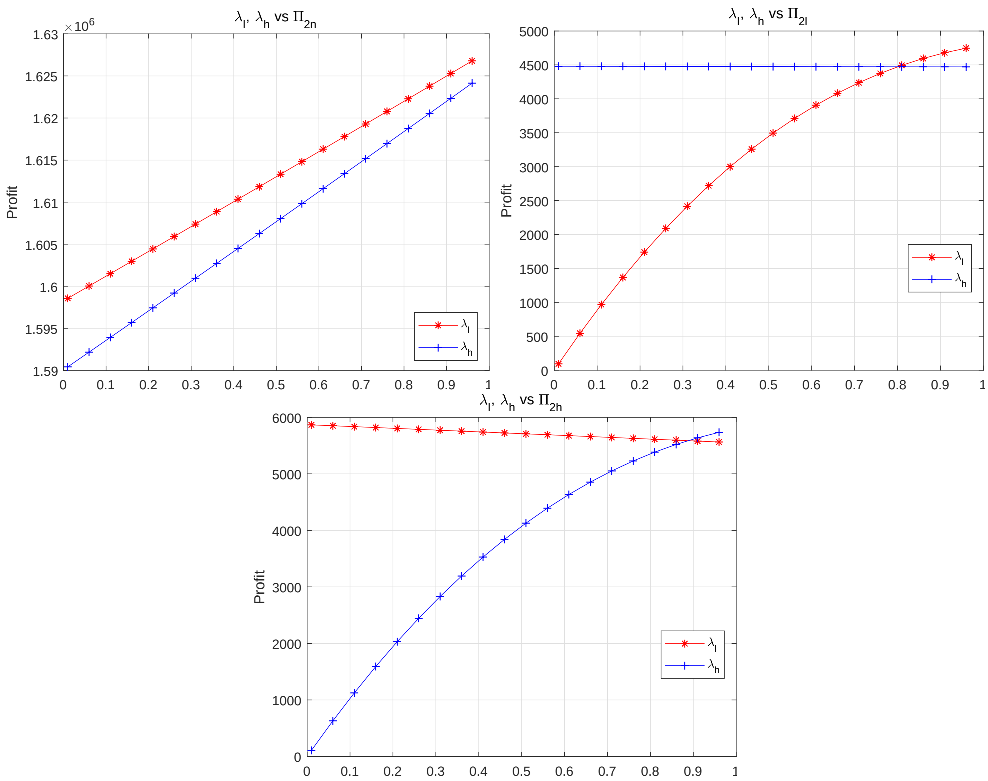

Fig. 6.4: λ_l, λ_h, λ_u vs. profit in period 2

Figure 6.4 and 6.5 study the remanufacturing rate. Generally speaking, the higher the remanufacturing rate, which also can be thought of less waste in the remanufacturing rate, the better the profit. However, as can be seen in the figures, the relationship between λ_h and Π_{2l}, Π_{3l}, Π_{3u}; between λ_u and Π_{3l}, Π_{3h}; between λ_l and Π_{3h}, Π_{3u} are not quite relevant.

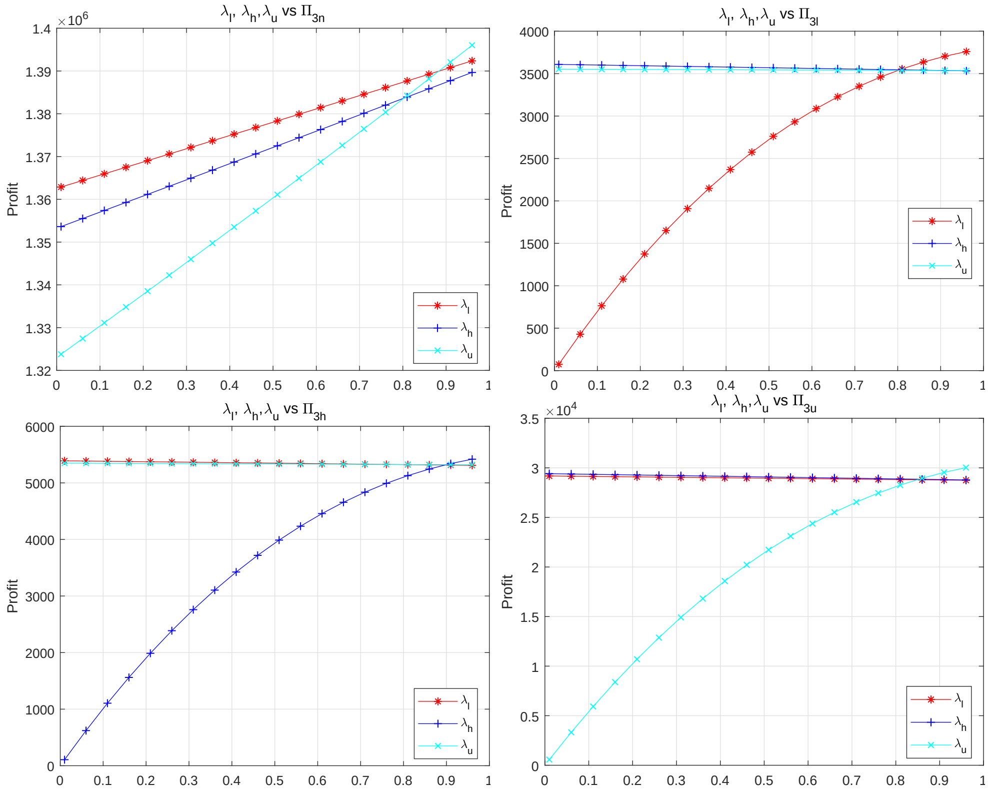

Fig. 6.5: λ_l, λ_h, λ_u vs. profit in period 3

The figures below represent the relations about the price of nature resources to manufacture the new EV battery. It has a tendency of dropping first and then rising with all profits.

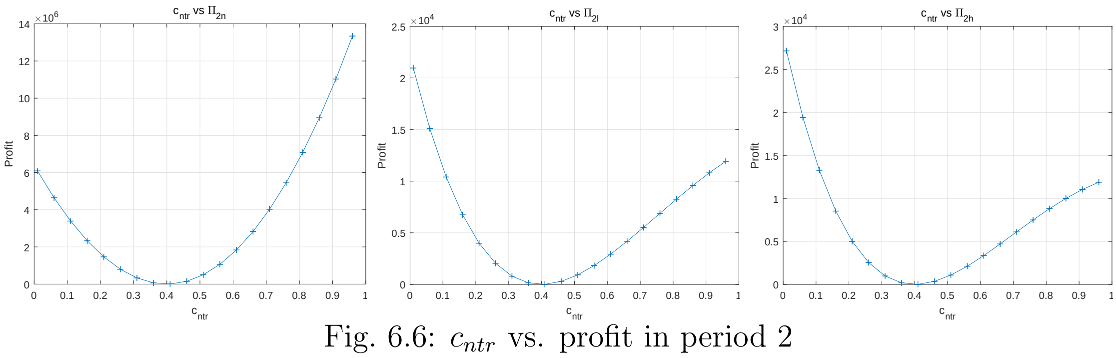

Fig. 6.6: c_{ntr} vs. profit in period 2

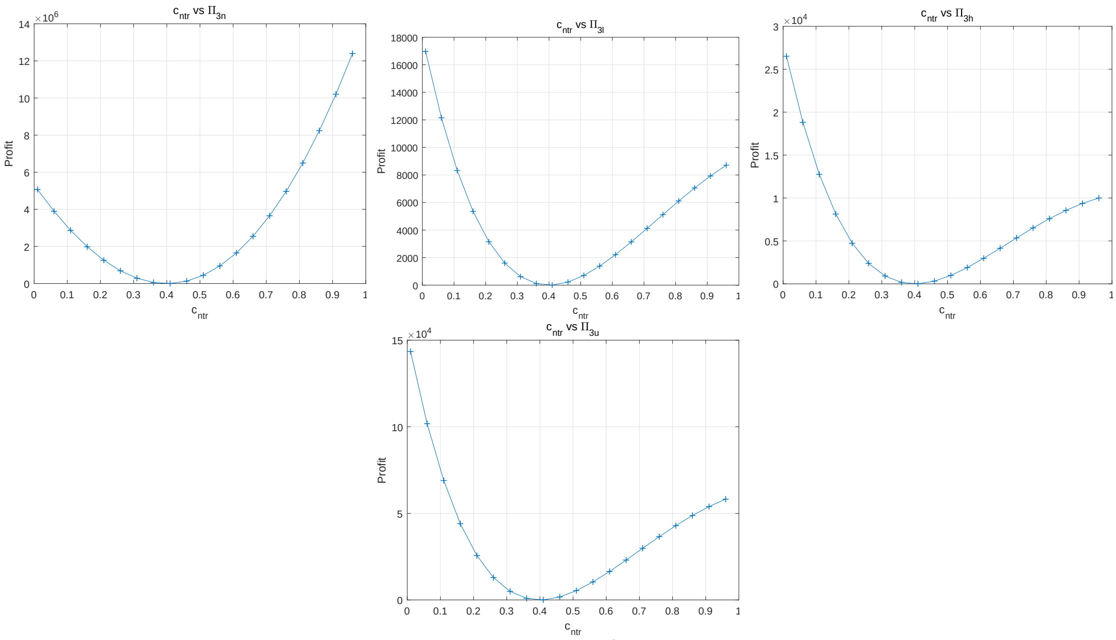

Fig. 6.7: c_{ntr} vs. profit in period 3

Lastly, the figure 6.8 and 6.9 discuss the remanufacturing cost. Higher remanufacturing cost will bring lower profit for the new battery manufacturer, while if any remanufacturers increase their processing cost, their profit will be increased as well. This observation is the same in both period 2 and 3.

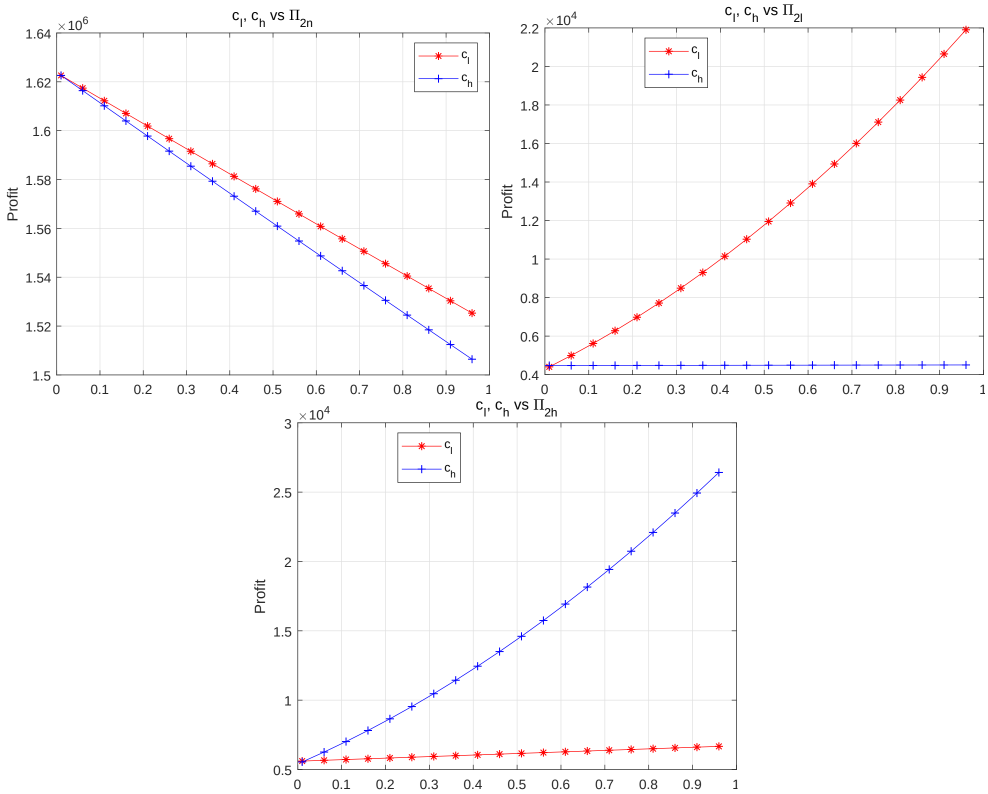

Fig. 6.8: c_l, c_h vs. profit in period 2

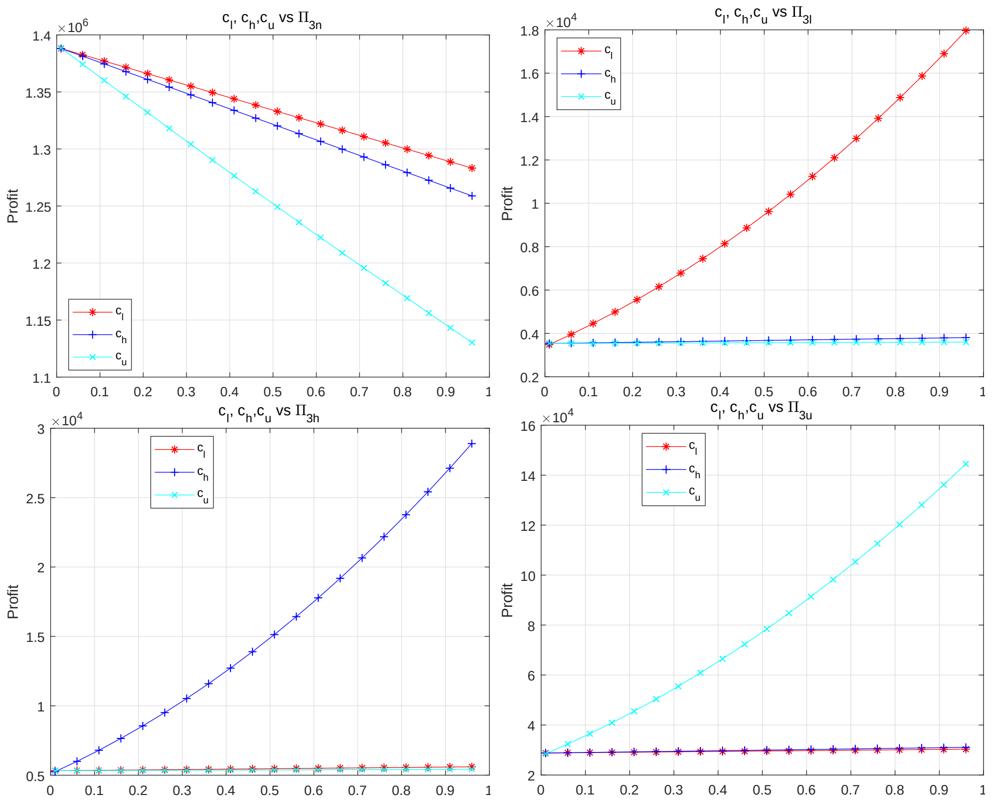

Fig. 6.9: c_l, c_h, c_u vs. profit in period 3

6.2.7 Conclusion

This section has studied the model optimizing each (re)manufacturer's profit. Analysis and management implication is given first, followed by the numerical experiment and discussion.

6.3 Model under condition of the whole chain profit optimization

6.3.1 Nash equilibrium in period 1

The Nash equilibrium in period 1 is as same as Section 6.2.1, where we have

$$q_{EV1} = M_{EV}(1 - p_{EV1}) \tag{6.34}$$

$$q_{1n} = HM_{EV}(1 - p_1/\delta_m) \tag{6.35}$$

$$p_1 = \delta_m - kq_{1n} \tag{6.36}$$

While the profit is expressed as

$$\Pi_1 = \Pi_{1n} = (p_1 - c_{ntr})q_{1n} = (\delta_m - kq_{1n} - c_{ntr})q_{1n} \tag{6.37}$$

6.3.2 Nash equilibrium in period 2

Similar with Section 6.2.2, the profit functions for the new battery manufacturer (Π_{2n}), low-quality and high-quality battery remanufacturer in period 2, i.e. Π_{2l} and Π_{2h}, are rewritten as

$$\begin{cases} \Pi_{2n} = (p_2 - c_{ntr})q_{2n} \\ \quad = q_{2n}\left(-k\left(A_2 + q_{2n} - A_{2h}s_{2h} - A_{2l}s_{2l}\right) - c_{ntr} + \delta_m\right) \\ \Pi_{2l} = (\lambda_l p_2 - c_l - s_{2l})q_{2l} \\ \quad = A_{2l}\left(1 - s_{2l}\right)\left(\lambda_l\left(\delta_m - k\left(A_2 + q_{2n} - A_{2h}s_{2h} - A_{2l}s_{2l}\right)\right) - c_l - s_{2l}\right) \\ \Pi_{2h} = (\lambda_h p_2 - c_h - s_{2h})q_{2h} \\ \quad = A_{2h}\left(1 - s_{2h}\right)\left(\lambda_h\left(\delta_m - k\left(A_2 + q_{2n} - A_{2h}s_{2h} - A_{2l}s_{2l}\right)\right) - c_h - s_{2h}\right) \end{cases} \tag{6.38}$$

Unlike Section 6.2, which studies the model under condition of optimizing the individual's profit, the entire profit in period 2 is

$$\begin{aligned} \Pi_2 &= \Pi_{2n} + \Pi_{2l} + \Pi_{2h} \\ &= \left(\begin{aligned} & q_{2n}\left(\delta_m - k\left(-A_{2h}s_{2h} - A_{2l}s_{2l} + A_2 + q_{2n}\right) - c_{ntr}\right) \\ & + A_{2h}\left(1 - s_{2h}\right)\left(\lambda_h\left(\delta_m - k\left(-A_{2h}s_{2h} - A_{2l}s_{2l} + A_2 + q_{2n}\right)\right) - c_h - s_{2h}\right) \\ & + A_{2l}\left(1 - s_{2l}\right)\left(\lambda_l\left(\delta_m - k\left(-A_{2h}s_{2h} - A_{2l}s_{2l} + A_2 + q_{2n}\right)\right) - c_l - s_{2l}\right) \end{aligned} \right) \end{aligned} \tag{6.39}$$

In terms of the convexity about Π_2, we find the Hessian matrix for Π_2:

$$H(\Pi_2) = \begin{bmatrix} 0 & kA_{2h} & kA_{2l} \\ kA_{2h} & -2A_{2h}(kA_{2h}\lambda_h - 1) & -kA_{2h}A_{2l}(\lambda_h + \lambda_l) \\ kA_{2l} & -kA_{2h}A_{2l}(\lambda_h + \lambda_l) & -2A_{2l}(kA_{2l}\lambda_l - 1) \end{bmatrix} \tag{6.40}$$

It can be found that $-2A_{2h}(kA_{2h}\lambda_h - 1) > 0$ and $-2A_{2l}(kA_{2l}\lambda_l - 1) > 0$. Therefore,

$H(\Pi_2)$ is positive-semidefinite, and the optimal total profit will be achieved by using the first-order condition, that is $\frac{\partial \Pi_2}{\partial q_{2n}} = \frac{\partial \Pi_2}{\partial s_{2h}} = \frac{\partial \Pi_2}{\partial s_{2l}} = 0$:

$$\frac{\partial \Pi_2}{\partial q_{2n}} = \begin{pmatrix} -k\left(-s^*_{2h}A_{2h} - s^*_{2l}A_{2l} + A_2 + q^*_{2n}\right) - k(1-s^*_{2h})A_{2h}\lambda_h \\ -k(1-s^*_{2l})A_{2l}\lambda_l - c_{ntr} - kq^*_{2n} + \delta_m \end{pmatrix} = 0 \tag{6.41}$$

$$\frac{\partial \Pi_2}{\partial s_{2h}} = \begin{pmatrix} -A_{2l}\left(\lambda_l\left(\delta_m - k\left(-s^*_{2h}A_{2h} - s^*_{2l}A_{2l} + A_2 + q^*_{2n}\right)\right) - c_l - s^*_{2l}\right) \\ +k(1-s^*_{2h})A_{2h}A_{2l}\lambda_h + kq^*_{2n}A_{2l} + (1-s^*_{2l})A_{2l}\left(kA_{2l}\lambda_l - 1\right) \end{pmatrix} = 0 \tag{6.42}$$

$$\frac{\partial \Pi_2}{\partial s_{2l}} = \begin{pmatrix} -A_{2h}\left(\lambda_h\left(\delta_m - k\left(-s^*_{2h}A_{2h} - s^*_{2l}A_{2l} + A_2 + q^*_{2n}\right)\right) - c_h - s^*_{2h}\right) \\ +k(1-s^*_{2l})A_{2h}A_{2l}\lambda_l + kq^*_{2n}A_{2h} + (1-s^*_{2h})A_{2h}\left(kA_{2h}\lambda_h - 1\right) \end{pmatrix} = 0 \tag{6.43}$$

The optimal values are shown below:

$$q^*_{2n} = \frac{\begin{pmatrix} k(A_{2l}\lambda_l^2\left(kA_{2h}\left(-c_h + c_{ntr} + 1\right) + 2\left(kA_{2l} + \delta_m\right)\right) \\ +A_{2l}\lambda_l(kA_{2h}\left(\lambda_h\left(c_h + c_l - 2c_{ntr} + 2\right) + c_h - c_l - 2\right) \\ -2\left(kA_{2l} + c_l - 2c_{ntr} + \delta_m + 1\right)) + A_{2h}\lambda_h(kA_{2l}(\lambda_h\left(-c_l + c_{ntr} + 1\right) \\ -c_h + c_l - 2) + 2kA_{2h}\left(\lambda_h - 1\right) - 2\left(c_h - 2c_{ntr} + \delta_m\right) + 2\lambda_h\delta_m) \\ -2\left(A_{2h}\left(c_h + \lambda_h - 1\right) + A_{2l}\left(c_l - 1\right)\right) - 2A_2(kA_{2h}\left(\lambda_h - 1\right)\lambda_h \\ +kA_{2l}\left(\lambda_l - 1\right)\lambda_l + 2)) - 4c_{ntr} + 4\delta_m \end{pmatrix}}{2k\left(kA_{2h}\left(\lambda_h - 1\right)^2 + kA_{2l}\left(\lambda_l - 1\right)^2 + 4\right)} \tag{6.44}$$

$$s^*_{2l} = \frac{\begin{pmatrix} kA_{2h}(-c_l\left(\lambda_h - 1\right)^2 + c_h\left(\lambda_h - 1\right)\left(\lambda_l - 1\right) + \lambda_l(c_{ntr}\left(-\lambda_h\right) \\ +c_{ntr} + \lambda_h + 1) + \lambda_h\left(c_{ntr}\left(\lambda_h - 1\right) + \lambda_h - 3\right)) + 2(kA_{2l}\left(\lambda_l - 1\right)\lambda_l \\ +c_{ntr}\left(\lambda_l + 1\right) - 2c_l + \left(\lambda_l - 1\right)\delta_m + 2) - 2A_2k\left(\lambda_l - 1\right) \end{pmatrix}}{2\left(kA_{2h}\left(\lambda_h - 1\right)^2 + kA_{2l}\left(\lambda_l - 1\right)^2 + 4\right)} \tag{6.45}$$

$$s^*_{2h} = \frac{\begin{pmatrix} kA_{2l}(-c_h\left(\lambda_l - 1\right)^2 + c_l\left(\lambda_h - 1\right)\left(\lambda_l - 1\right) - \lambda_l\left(\left(c_{ntr} - 1\right)\lambda_h + c_{ntr} + 3\right) \\ +\left(c_{ntr} + 1\right)\lambda_h + \left(c_{ntr} + 1\right)\lambda_l^2) + 2\lambda_h\left(kA_{2h}\left(\lambda_h - 1\right) + c_{ntr} + \delta_m\right) \\ -2A_2k\left(\lambda_h - 1\right) - 4c_h + 2c_{ntr} - 2\delta_m + 4 \end{pmatrix}}{2\left(kA_{2h}\left(\lambda_h - 1\right)^2 + kA_{2l}\left(\lambda_l - 1\right)^2 + 4\right)} \tag{6.46}$$

$$q^*_{2h} = A_{2h}(1 - s^*_{2h}) \tag{6.47}$$

$$q_{2l}^* = A_{2l}(1 - s_{2l}^*) \tag{6.48}$$

6.3.3 Nash equilibrium in period 3

Again, similar with Section 6.2.3, the formulas of profit for (ra)manufacturers are rewritten as:

$$\begin{cases} \Pi_{3n} = (p_3 - c_{ntr})q_{3n} \\ \quad = (\delta_m - k(q_{3n} + A_3 - A_{3l}s_{3l} - A_{3h}s_{3h} - A_{3u}s_{3u}) - c_{ntr})\, q_{3n} \\ \Pi_{3l} = \lambda_l q_{3l} p_3 - (c_l + s_{3l})q_{3l} \\ \quad = \begin{pmatrix} \lambda_l A_{3l}(1 - s_{3l})(\delta_m - k(q_{3n} + A_3 - A_{3l}s_{3l} - A_{3h}s_{3h} - A_{3u}s_{3u})) \\ -(c_l + s_{3l})A_{3l}(1 - s_{3l}) \end{pmatrix} \\ \Pi_{3h} = \lambda_h q_{3h} p_3 - (c_h + s_{3h})q_{3h} \\ \quad = \begin{pmatrix} \lambda_h A_{3h}(1 - s_{3h})(\delta_m - k(q_{3n} + A_3 - A_{3l}s_{3l} - A_{3h}s_{3h} - A_{3u}s_{3u})) \\ -(c_h + s_{3h})A_{3h}(1 - s_{3h}) \end{pmatrix} \\ \Pi_{3u} = \lambda_u q_{3u} p_3 - (c_u + s_{3u})q_{3u} \\ \quad = \begin{pmatrix} \lambda_u A_{3u}(1 - s_{3u})(\delta_m - k(q_{3n} + A_3 - A_{3l}s_{3l} - A_{3h}s_{3h} - A_{3u}s_{3u})) \\ -(c_u + s_{3u})A_{3u}(1 - s_{3u}) \end{pmatrix} \end{cases} \tag{6.49}$$

The entire profit is a sum profit for manufacturers/remanufacturers:

$$\begin{aligned} \Pi_3 &= \Pi_{3n} + \Pi_{3l} + \Pi_{3h} + \Pi_{3u} \\ &= \begin{pmatrix} q_{3n}(k(A_{3h}s_{3h} + A_{3l}s_{3l} + A_{3u}s_{3u} - A_3 - q_{3n}) - c_{ntr} + \delta_m) \\ +A_{3h}(s_{3h} - 1)(c_h + s_{3h}) + A_{3l}(s_{3l} - 1)(c_l + s_{3l}) + A_{3u}(s_{3u} - 1)(c_u + s_{3u}) \\ -A_{3h}\lambda_h(s_{3h} - 1)(k(A_{3h}s_{3h} + A_{3l}s_{3l} + A_{3u}s_{3u} - A_3 - q_{3n}) + \delta_m) \\ -A_{3l}\lambda_l(s_{3l} - 1)(k(A_{3h}s_{3h} + A_{3l}s_{3l} + A_{3u}s_{3u} - A_3 - q_{3n}) + \delta_m) \\ -A_{3u}(s_{3u} - 1)\lambda_u(k(A_{3h}s_{3h} + A_{3l}s_{3l} + A_{3u}s_{3u} - A_3 - q_{3n}) + \delta_m) \end{pmatrix} \end{aligned} \tag{6.50}$$

The Hessian matrix for Π_3 is

$$H(\Pi_3) =$$

$$\begin{bmatrix} 0 & kA_{3h}(1+\lambda_h) & kA_{3l}(1+\lambda_l) & kA_{3u}(1+\lambda_u) \\ kA_{3h}(1+\lambda_h) & -2A_{3h}(kA_{3h}\lambda_h - 1) & -kA_{3h}A_{3l}(\lambda_h+\lambda_l) & -kA_{3h}A_{3u}(\lambda_h+\lambda_u) \\ kA_{3l}(1+\lambda_l) & -kA_{3h}A_{3l}(\lambda_h+\lambda_l) & -2A_{3l}(kA_{3l}\lambda_l - 1) & -kA_{3l}A_{3u}(\lambda_l+\lambda_u) \\ kA_{3u}(1+\lambda_u) & -kA_{3h}A_{3u}(\lambda_h+\lambda_u) & -kA_{3l}A_{3u}(\lambda_l+\lambda_u) & -2A_{3u}(kA_{3u}\lambda_u - 1) \end{bmatrix} \tag{6.51}$$

We have $-2A_{3h}(kA_{3h}\lambda_h - 1) > 0$, $-2A_{3l}(kA_{3l}\lambda_l - 1) > 0$ and $-2A_{3u}(kA_{3u}\lambda_u - 1) > 0$, therefore $H(\Pi_3)$ is positive-semidefinite as with $H(\Pi_2)$. By using the first-order condition to acquire the optimal profit for each agent:

$$\begin{cases} \left(\begin{array}{l} k\left(s^*_{3h}A_{3h} + s^*_{3l}A_{3l} + s^*_{3u}A_{3u} - A_3 - q^*_{3n}\right) + k(s^*_{3h} - 1)A_{3h}\lambda_h \\ + ks^*_{3l} - 1)A_{3l}\lambda_l + k(s^*_{3u} - 1)A_{3u}\lambda_u - c_{ntr} - kq^*_{3n} + \delta_m \end{array} \right) = 0 \\ \left(\begin{array}{l} A_{3l}\left(c_l + s^*_{3l}\right) - A_{3l}\lambda_l\left(k\left(s^*_{3h}A_{3h} + s^*_{3l}A_{3l} + s^*_{3u}A_{3u} - A_3 - q^*_{3n}\right) + \delta_m\right) + kq^*_{3n}A_{3l} \\ - k(s^*_{3h} - 1)A_{3h}A_{3l}\lambda_h - k(s^*_{3l} - 1)A^2_{3l}\lambda_l - k(s^*_{3u} - 1)A_{3l}A_{3u}\lambda_u + (s^*_{3l} - 1)A_{3l} \end{array} \right) = 0 \\ \left(\begin{array}{l} A_{3h}\left(c_h + s^*_{3h}\right) - A_{3h}\lambda_h\left(k\left(s^*_{3h}A_{3h} + s^*_{3l}A_{3l} + s^*_{3u}A_{3u} - A_3 - q^*_{3n}\right) + \delta_m\right) + kq^*_{3n}A_{3h} \\ - k(s^*_{3l} - 1)A_{3h}A_{3l}\lambda_l - k(s^*_{3h} - 1)A^2_{3h}\lambda_h - k(s^*_{3u} - 1)A_{3h}A_{3u}\lambda_u + (s^*_{3h} - 1)A_{3h} \end{array} \right) = 0 \\ \left(\begin{array}{l} A_{3u}\left(c_u + s^*_{3u}\right) - A_{3u}\lambda_u\left(k\left(s^*_{3h}A_{3h} + s^*_{3l}A_{3l} + s^*_{3u}A_{3u} - A_3 - q^*_{3n}\right) + \delta_m\right) + kq^*_{3n}A_{3u} \\ - k(s^*_{3h} - 1)A_{3h}A_{3u}\lambda_h - k(s^*_{3l} - 1)A_{3l}A_{3u}\lambda_l - k(s^*_{3u} - 1)A^2_{3u}\lambda_u + (s^*_{3u} - 1)A_{3u} \end{array} \right) = 0 \end{cases} \tag{6.52}$$

Through solving the above equations, the optimal values in period 3 are

$$q_{3n}^{*} = \frac{\left(\begin{array}{l} k(A_{3h}(kA_{3l}(\lambda_h^2\left(-c_l + c_{ntr} + 1\right) + \lambda_h\left(\lambda_l\left(c_h + c_l - 2c_{ntr} + 2\right) - c_h + c_l - 2\right) \\ +\lambda_l\left(\lambda_l\left(-c_h + c_{ntr} + 1\right) + c_h - c_l - 2\right)) + kA_{3u}(\lambda_u^2\left(-c_h + c_{ntr} + 1\right) \\ +\lambda_u\left(\lambda_h\left(c_h - 2c_{ntr} + c_u + 2\right) + c_h - c_u - 2\right) + \lambda_h(\lambda_h\left(c_{ntr} - c_u + 1\right) \\ -c_h + c_u - 2)) + 2\lambda_h\left(-c_h + 2c_{ntr} + \left(\lambda_h - 1\right)\delta_m\right)) - 2(A_{3h}\left(c_h + \lambda_h - 1\right) \\ +A_{3l}\left(c_l - 1\right) + A_{3u}\left(c_u - 1\right)) + A_{3u}\lambda_u^2\left(kA_{3l}\left(-c_l + c_{ntr} + 1\right) + 2\left(kA_{3u} + \delta_m\right)\right) \\ +A_{3l}\lambda_l(kA_{3u}\left(\lambda_l\left(c_{ntr} - c_u + 1\right) - c_l + c_u - 2\right) + 2kA_{3l}\left(\lambda_l - 1\right) \\ -2\left(c_l - 2c_{ntr} + \delta_m + 1\right) + 2\lambda_l\delta_m) + A_{3u}\lambda_u(kA_{3l}(\lambda_l\left(c_l - 2c_{ntr} + c_u + 2\right) \\ +c_l - c_u - 2) - 2\left(kA_{3u} - 2c_{ntr} + c_u + \delta_m + 1\right)) + 2kA_{3h}^2\left(\lambda_h - 1\right)\lambda_h \\ -2A_3\left(kA_{3h}\left(\lambda_h - 1\right)\lambda_h + kA_{3l}\left(\lambda_l - 1\right)\lambda_l + kA_{3u}\left(\lambda_u - 1\right)\lambda_u + 2\right)) \\ -4c_{ntr} + 4\delta_m \end{array}\right)}{2k\left(kA_{3h}\left(\lambda_h - 1\right)^2 + kA_{3l}\left(\lambda_l - 1\right)^2 + kA_{3u}\left(\lambda_u - 1\right)^2 + 4\right)} \tag{6.53}$$

$$q_{3l}^{*} = A_{3l}(1 - s_{3l}^{*}) \tag{6.54}$$

$$q_{3h}^{*} = A_{3h}(1 - s_{3h}^{*}) \tag{6.55}$$

$$q_{3u}^{*} = A_{3u}(1 - s_{3u}^{*}) \tag{6.56}$$

$$s_{3l}^{*} = \frac{\left(\begin{array}{l} kA_{3h}(-c_l\left(\lambda_h - 1\right)^2 + c_h\left(\lambda_h - 1\right)\left(\lambda_l - 1\right) + \lambda_l\left(c_{ntr}\left(-\lambda_h\right) + c_{ntr} + \lambda_h + 1\right) \\ +\lambda_h\left(c_{ntr}\left(\lambda_h - 1\right) + \lambda_h - 3\right)) + kA_{3u}c_{ntr}\lambda_l + kA_{3u}\lambda_u^2\left(-c_l + c_{ntr} + 1\right) \\ +kA_{3u}\lambda_u\left(-c_{ntr}\left(\lambda_l + 1\right) + c_u\left(\lambda_l - 1\right) + 2c_l + \lambda_l - 3\right) - kA_{3u}c_u\lambda_l - kA_{3u}c_l \\ +kA_{3u}c_u + 2kA_{3l}\lambda_l^2 - 2kA_{3l}\lambda_l - 2A_3k\left(\lambda_l - 1\right) + kA_{3u}\lambda_l + 2c_{ntr}\lambda_l - 4c_l \\ +2c_{ntr} + 2\lambda_l\delta_m - 2\delta_m + 4 \end{array}\right)}{2\left(kA_{3h}\left(\lambda_h - 1\right)^2 + kA_{3l}\left(\lambda_l - 1\right)^2 + kA_{3u}\left(\lambda_u - 1\right)^2 + 4\right)} \tag{6.57}$$

$$s^*_{3h} = \frac{\left(\begin{array}{l} kA_{3l}(-c_h\left(\lambda_l-1\right)^2+c_l\left(\lambda_h-1\right)\left(\lambda_l-1\right)-\lambda_l\left(\left(c_{ntr}-1\right)\lambda_h+c_{ntr}+3\right) \\ +\left(c_{ntr}+1\right)\lambda_h+\left(c_{ntr}+1\right)\lambda_l^2)+kA_{3u}c_{ntr}\lambda_h+kA_{3u}\lambda_u^2\left(-c_h+c_{ntr}+1\right) \\ +kA_{3u}\lambda_u\left(-c_{ntr}\left(\lambda_h+1\right)+c_u\left(\lambda_h-1\right)+2c_h+\lambda_h-3\right)-kA_{3u}c_u\lambda_h \\ -kA_{3u}c_h+kA_{3u}c_u+2kA_{3h}\lambda_h^2-2kA_{3h}\lambda_h-2A_3k\left(\lambda_h-1\right)+kA_{3u}\lambda_h \\ +2c_{ntr}\lambda_h-4c_h+2c_{ntr}+2\lambda_h\delta_m-2\delta_m+4 \end{array}\right)}{2\left(kA_{3h}\left(\lambda_h-1\right)^2+kA_{3l}\left(\lambda_l-1\right)^2+kA_{3u}\left(\lambda_u-1\right)^2+4\right)} \tag{6.58}$$

$$s^*_{3u} = \frac{\left(\begin{array}{l} \lambda_u(k(A_{3h}\left(\lambda_h\left(c_h-c_{ntr}+1\right)-c_h+c_{ntr}+1\right)+A_{3l}(\lambda_l\left(c_l-c_{ntr}+1\right)-c_l \\ +c_{ntr}+1)-2A_{3u}-2A_3)+2\left(c_{ntr}+\delta_m\right))+k(A_{3h}(\lambda_h^2\left(c_{ntr}-c_u+1\right) \\ -\lambda_h\left(c_h+c_{ntr}-2c_u+3\right)+c_h-c_u)+A_{3l}(\lambda_l^2\left(c_{ntr}-c_u+1\right) \\ -\lambda_l\left(c_l+c_{ntr}-2c_u+3\right)+c_l-c_u)+2A_3)+2kA_{3u}\lambda_u^2 \\ +2\left(c_{ntr}-2c_u-\delta_m+2\right) \end{array}\right)}{2\left(kA_{3h}\left(\lambda_h-1\right)^2+kA_{3l}\left(\lambda_l-1\right)^2+kA_{3u}\left(\lambda_u-1\right)^2+4\right)} \tag{6.59}$$

6.3.4 Analysis

In this section, we analyse the relationships between the parameters (i.e., θ, α, β, λ_l, λ_h, λ_u, c_{ntr}, c_l, c_h, c_u) and the total profit in both period 2 and 3 (Π_2 and Π_3). To reduce the length of chapter, all proofs are shown in the appendix B.2. In the appendix, the relationship between each parameter and the total optimal profit in both period 2 and 3, especially the range of first-order derivative and range of linearity. Although the profit functions are too complex, they can be approximated. For example, the character of first-order derivative represents the monotonicity and the range of linearity indicates how possible relationships can be treated as a straight line.

To summarise, all the relationships between parameters and the total profit in period 2 and 3 are encapsulated in Table 6.6. The author uses "L" to express the linear relationship, "Q" to describe the quadratic relation and "$\nearrow$/$\searrow$" to represent the positive/negative correlation. Moreover, the author uses "N/A" to indicate that this relationship is inapplicable.

Table 6.6: Summary for theorems

	θ	α	β	λ_l	λ_h	λ_u	c_{ntr}	c_l	c_h	c_u
Π_2	$\searrow$&L	L	$\nearrow$&L	$\nearrow$	$\nearrow$	N/A	Q	$\searrow$&Q	$\searrow$&Q	N/A
Π_3	$\searrow$&L	L	L	$\nearrow$	$\nearrow$	$\nearrow$	Q	$\searrow$&Q	$\searrow$&Q	$\searrow$&Q

Therefore, we reach six observations, as shown in the remarks below:

Remark 1. The relationship between return yeild θ, α, β and total optimal profit in period 2 and 3 (Π_2/Π_3) can be treated as linear.

Remark 2. The relationships between all the (re)manufacturing cost c_{ntr}, c_l, c_h, c_u and optimal profit Π_2, Π_3 are quadratic.

Remark 3. The reusable yield β has a positive correlation with Π_2.

Remark 4. All return recycling rates (λ_l, λ_h, λ_u) positively correlated with both Π_2 and Π_3.

Remark 5. The relationships between return rate θ, remanufacturing cost c_l, c_h, c_u and the total profit Π_2, Π_3 are negative.

6.3.5 Management implication

In this section, we analyse the management insight based on the remarks from section 3. Specifically, we consider the following findings:

- θ has a negative linear relationship between optimal total profits in both period 2 and 3. This indicates that the higher the return rate, the lower the total optimal profit. This is due to the higher costs involved in the recycling process throughout the entire supply chain. Furthermore, this also explains why EV battery recycling has not been adopted widely.

- The relationship between β and optimal profit in periods 2 and 3 is different. It is positive with the optimal profit in period 2. But in period 3 the relationship depends on the value of the initial parameters. Therefore, managers should pay close attention to β when analysing the CLSC for each specific case.

- λ_l, λ_h and λ_u have a positive relationship with optimal profit. This is because high λ presents less waste in the recycling process, which also demonstrates high recycling efficiency. As an example, Gaines (2014) looks ahead at how to improve the recycling efficiency technically.

- According to King et al. (2006), remanufacturing could be the best solution to deal with the returns. In EV battery remanufacturing CLSC, the higher c_l, c_h and c_u will cause the lower optimal profit in both period 2 and 3, as higher recycling processing costs may reduce profits.

Therefore, based on the findings above, in the economic aspect, improving the recycle and battery reuse rates lead to less optimal profit throughout the entire supply chain, but in order to promote EV used battery recycle and reuse rates, the entire supply chain should increase the high-quality recycling rate, reduce the level of waste in the recycling process and the remanufacturing and recycling processing costs.

6.3.6 Numerical experiments

Implementation of the model

The author use the same initial numerical parameters as shown in Table 6.3.

In **period 1**, the optimal quantity for battery raw materials (for period 1, this is also the optimal total quantity) is $q_1^* = q_{1n}^* = 18,000,000$. The optimal sale price is $p_1^* = 17550$ and the optimal profit in this stage is $\Pi_1 = \Pi_{1n} = 1.8 \times 10^6$. Moreover, the optimal EV sale quantity is $q_{EV1}^* = 4.5 \times 10^6$.

While in **period 2** we substitute q_{EV1}^* as the initial input parameter for this period. By applying Eq. 6.44 to Eq. 6.48, the optimal values are as follows in Table 6.7.

Table 6.7: Optimal values in period 2

$q_{2n}^* = 1.70 \times 10^7$	$q_{2l}^* = 4.73 \times 10^5$	$q_{2h}^* = 5.48 \times 10^5$	$q_2^* = 1.81 \times 10^7$
$p_2^* = 17526$	$\Pi_2^* = 1.36 \times 10^6$	$q_{EV2}^* = 4.52 \times 10^6$	

And in **period 3**, we substitute q_{EV2}^* as initial EV quantity in this period. By applying Eq. 6.49 to Eq. 6.56, the optimal values in this period are as shown in Table 6.8.

Table 6.8: Optimal values in period 3

$q_{3n}^* = 1.58 \times 10^7$	$q_{3l}^* = 4.75 \times 10^5$	$q_{3h}^* = 5.50 \times 10^5$	$q_{3u}^* = 1.30 \times 10^6$
$q_3^* = 1.82 \times 10^7$	$p_3^* = 20781$	$\Pi_3^* = 17491$	$q_{EV3}^* = 4.54 \times 10^6$

Analysis

Based on the initial numeric input in 6.7, this subsection shows the relationship between parameters and total profits in charts and figures with the accurate expressions and the fitted straight line expressions if relationship is not quadratic.

Period 2

- Relationship between θ, α, β and total profit in period 2

  The original curves are accurate curves measured by equations from section 4 and the appendix while the fitted curves with '*' are drawn by linear regression. In addition, three functions below describe all of the curves.

$$\begin{aligned}\Pi_2^*(\theta) &= -137616.\theta + \frac{1.95883\times 10^{13}}{\theta+3846.15} - 5.09116\times 10^9 \\ &\approx -1.46\times 10^6\theta + 1.80\times 10^6\end{aligned} \tag{6.60}$$

$$\Pi_2^*(\alpha) = \frac{1.06\times 10^{13}}{3604.68-\alpha} - 72527.4\alpha - 2.95\times 10^9 \approx 7.46\times 10^5\alpha + 7.64\times 10^5 \tag{6.61}$$

$$\Pi_2^*(\beta) = \frac{2.1422\times 10^{14}}{16668.7-\beta} - 1.29\times 10^{10} \approx 7.71\times 10^5\beta + 8.22\times 10^5 \tag{6.62}$$

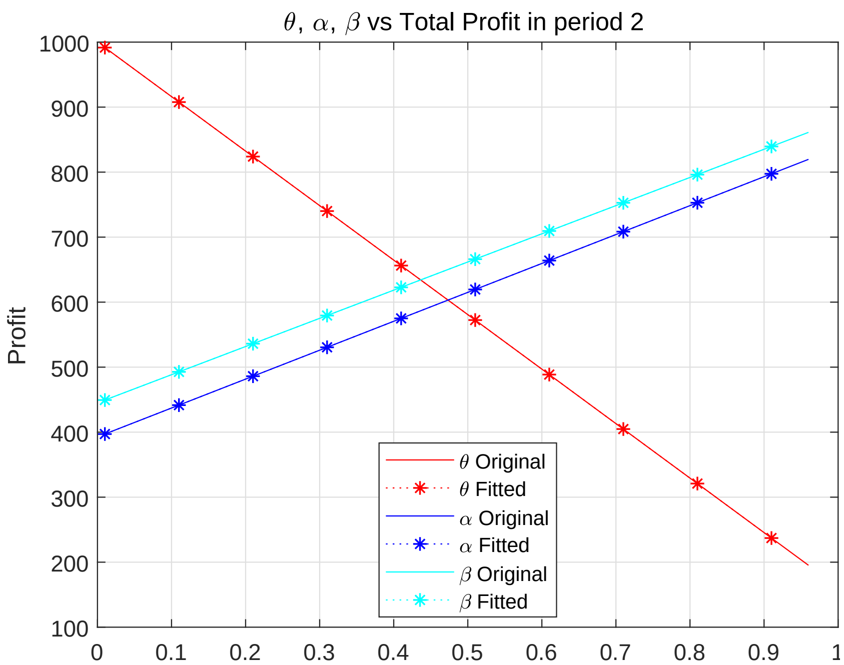

Fig. 6.10: θ, α, β vs. total profit in period 2

- Relationships between λ_l, λ_h and total profit in period 2

The figure and equations below show functions λ_l and λ_h with optimal profit in period 2.

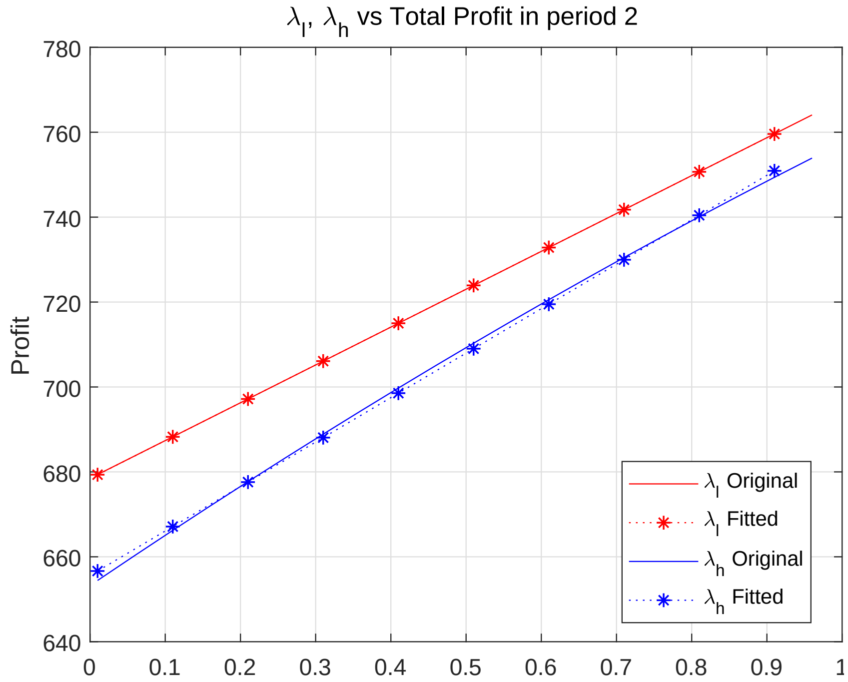

Fig. 6.11: λ_l, λ_h vs. total profit in period 2

$$\Pi_2^*(\lambda_l) = \frac{8.81 \times 10^7 \lambda_l + 1.06 \times 10^{10}}{\lambda_l\,(\lambda_l - 2) + 667.68} - 1.46 \times 10^7 \approx 1.57 \times 10^5 \lambda_l + 1.24 \times 10^6 \quad (6.63)$$

$$\Pi_2^*(\lambda_h) = \frac{8.79 \times 10^7 \lambda_h + 8.78 \times 10^9}{\lambda_h\,(\lambda_h - 2) + 556.59} - 1.46 \times 10^7 \approx 1.88 \times 10^5 \lambda_h + 1.20 \times 10^6 \quad (6.64)$$

- Relationship between c_{ntr}, c_n, c_l, c_h and total profit in period 2

  Through Eq. B-46, Eq. B-52 and Eq. B-58, we can describe the quadratic relationships using the figures and expressions below:

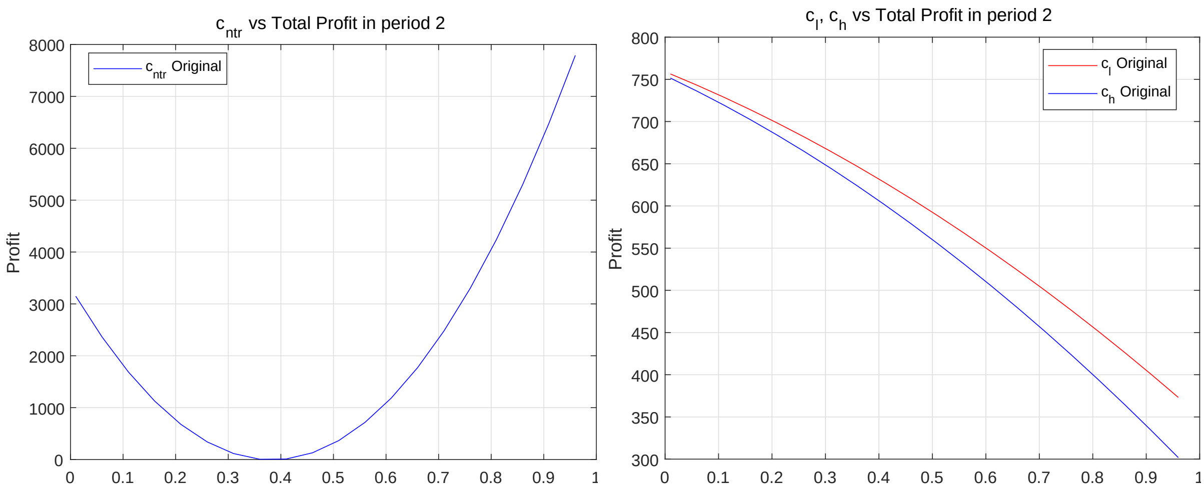

Fig. 6.12: c_{ntr}, c_h, c_l, c_n vs. total profit in period 2

$$\Pi_2^*(c_{ntr}) = 4.18 \times 10^7 c_{ntr}^2 - 3.19 \times 10^7 c_{ntr} + 0.61 \times 10^7 \tag{6.65}$$

$$\Pi_2^*(c_l) = -0.27 \times 10^6 c_l^2 - 0.46 \times 10^6 c_l + 1.37 \times 10^5 \tag{6.66}$$

$$\Pi_2^*(c_h) = -0.32 \times 10^6 c_h^2 - 0.54 \times 10^6 c_h + 1.37 \times 10^6 \tag{6.67}$$

Period 3 All relationships and description functions for period 3 are given below.

- Relationships between θ, α, β and total profit in period 3

$$\begin{aligned}\Pi_3^*(\theta) &= \frac{\begin{pmatrix} 3.13 \times 10^{-13}\theta^6 - 1.54 \times 10^7\theta^5 + 1.89 \times 10^{26}\theta^4 \\ +9.68 \times 10^{31}\theta^3 + 1.43 \times 10^{37}\theta^2 + 5.02 \times 10^{41}\theta - 2.72 \times 10^{41} \end{pmatrix}}{\theta^4 - 2.46 \times 10^{19}\theta^3 - 3.47 \times 10^{26}\theta^2 - 8.70 \times 10^{31}\theta - 1.51 \times 10^{35}} \\ &\approx -3.36 \times 10^6\theta + 1.81 \times 10^6\end{aligned} \tag{6.68}$$

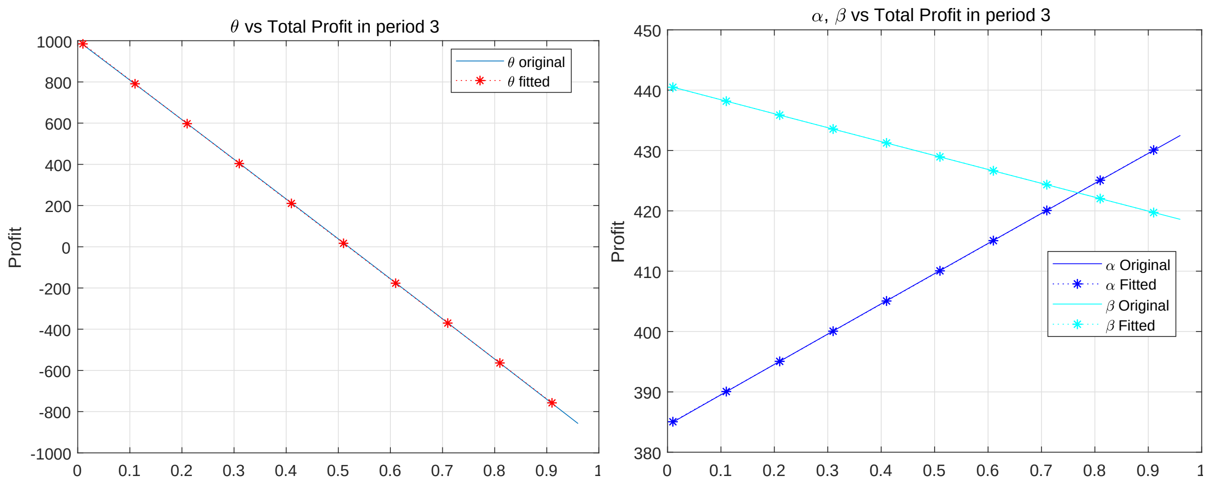

Fig. 6.13: α, β, θ vs. total profit in period 3

$$\begin{aligned}\Pi_3^*(\alpha) &= \frac{\begin{pmatrix} -1.66 \times 10^{-14}\alpha^6 - 1.47 \times 10^7\alpha^5 - 3.24 \times 10^{27}\alpha^4 \\ +2.02 \times 10^{33}\alpha^3 - 3.15 \times 10^{38}\alpha^2 - 5.53 \times 10^{41}\alpha - 8.59 \times 10^{42} \end{pmatrix}}{\alpha^4 + 4.42 \times 10^{20}\alpha^3 - 5.87 \times 10^{27}\alpha^2 + 1.79 \times 10^{33}\alpha - 1.12 \times 10^{37}} \\ &\approx 0.61 \times 10^5\alpha + 7.51 \times 10^5\end{aligned} \tag{6.69}$$

$$\Pi_3^*(\beta) = \frac{\begin{pmatrix} 2.40 \times 10^{-14}\beta^6 + 2.47 \times 10^7\beta^5 + 6.33 \times 10^{27}\beta^4 - 8.89 \times 10^{33}\beta^3 \\ +3.13 \times 10^{39}\beta^2 - 7.39 \times 10^{42}\beta + 2.37 \times 10^{44} \end{pmatrix}}{\left(\beta^4 + 5.14 \times 10^{20}\beta^3 - 3.12 \times 10^{28}\beta^2 + 2.16 \times 10^{34}\beta + 2.88 \times 10^{38} \right)}$$
$$\approx -0.21 \times 10^5\beta + 8.14 \times 10^5 \tag{6.70}$$

- Relationships between λ_l, λ_h, λ_u and total profit in period 3

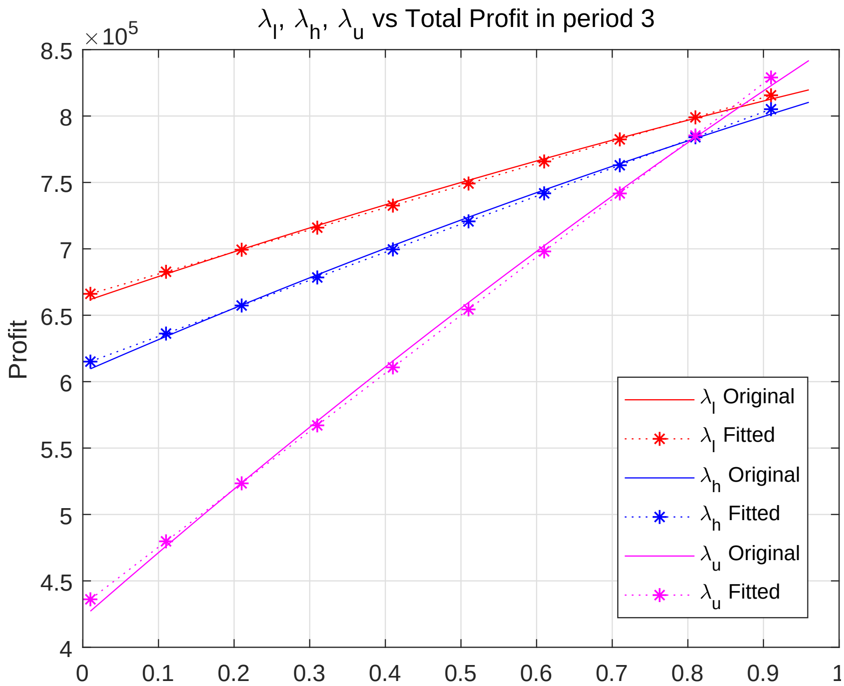

Fig. 6.14: λ_l, λ_h, λ_u vs. total profit in period 3

$$\Pi_3^*(\lambda_l) = \frac{\begin{pmatrix} 2.32 \times 10^8\lambda_l^5 - 1.73 \times 10^7\lambda_l^6 - 4.66 \times 10^{13}\lambda_l^4 + 5.28 \times 10^{14}\lambda_l^3 \\ -2.18 \times 10^{19}\lambda_l^2 + 1.70 \times 10^{20}\lambda_l + 6.50 \times 10^{20} \end{pmatrix}}{\begin{pmatrix} \lambda_l^6 - 8.04\lambda_l^5 + 3 \times 10^6\lambda_l^4 - 1.69 \times 10^7\lambda_l^3 + 1.44 \times 10^{12}\lambda_l^2 \\ -2.88 \times 10^{12}\lambda_l + 9.62 \times 10^{14} \end{pmatrix}}$$
$$\approx 1.66 \times 10^5\lambda_l + 6.64 \times 10^5 \tag{6.71}$$

$$\Pi_3^*(\lambda_h) = \frac{\begin{pmatrix} 2.30\times10^8\lambda_h^5 - 1.71\times10^7\lambda_h^6 - 3.87\times10^{13}\lambda_h^4 \\ +4.38\times10^{14}\lambda_h^3 - 1.51\times10^{19}\lambda_h^2 + 1.18\times10^{20}\lambda_h + 3.53\times10^{20} \end{pmatrix}}{\begin{pmatrix} \lambda_h^6 - 8.03\lambda_h^5 + 2.50\times10^6\lambda_h^4 - 1.41\times10^7\lambda_h^3 + 1\times10^{12}\lambda_h^2 \\ -2\times10^{12}\lambda_h + 5.57\times10^{14} \end{pmatrix}} \approx 2.11\times10^5\lambda_h + 6.13\times10^5 \tag{6.72}$$

$$\Pi_3^*(\lambda_u) = \frac{8.78\times10^7\lambda_u + 3.64\times10^9}{\lambda_u(\lambda_u - 2) + 239.11} - 1.48\times10^7 \approx 4.37\times10^5\lambda_u + 4.32\times10^5 \tag{6.73}$$

- Relationships between c_{ntr}, c_l, c_h, c_u and total profit in period 3

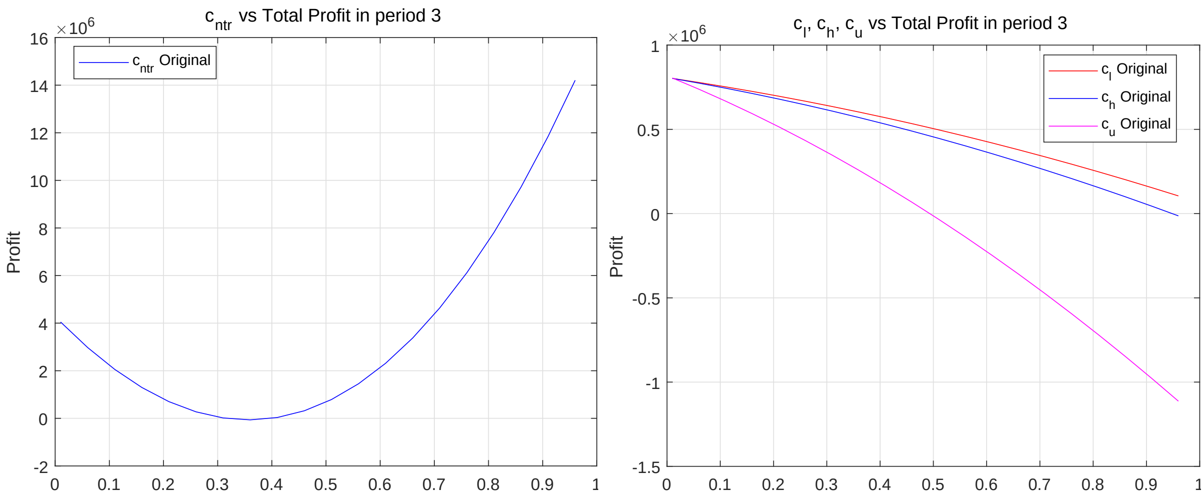

Fig. 6.15: c_{ntr}, c_h, c_l, c_u vs. total profit in period 3

$$\Pi_3^*(c_{ntr}) = 3.81\times10^7c_{ntr}^2 - 2.67\times10^7c_{ntr} + 0.45\times10^7 \tag{6.74}$$

$$\Pi_3^*(c_l) = -2.74\times10^5c_l^2 - 4.69\times10^5c_l + 8.07\times10^5 \tag{6.75}$$

$$\Pi_3^*(c_h) = -3.27\times10^5c_h^2 - 5.41\times10^5c_h + 8.08\times10^5 \tag{6.76}$$

$$\Pi_3^*(c_u) = -0.76\times10^6c_u^2 - 1.28\times10^6c_u + 0.82\times10^6 \tag{6.77}$$

This section revalidates the remarks in section 3. Figures 6.10 to 6.15 show the relationships between independent variables and dependent variables, Π_2 and Π_3. Profit functions are complex and non-linear. As can be seen from figures 6.10 to 6.13, in this given case, all sorting rates (θ, α, β) and all λ can still be regressed with a straight line. All the costs have a quadratic relation with the total profit.

6.3.7 Conclusion

This section has studied the model optimizing the total closed-loop supply chain's profit. Analysis and management implication is given first, followed by the numerical experiment and discussion.

6.4 Summary

Considering the variation in the quality of the returned EV batteries, in this chapter, the author develops a model to analyse the recycling and reusing process, which includes the EV battery's return, recycle, remanufacture and reuse. Based on two targets (1) to optimize the parties' profit in different period and (2) to optimize the total profit in different period, the optimal profit for each (re)manufacturers are given, meanwhile the numerical experiments are included as well to explain the results with real case figures. As can be seen that the relationship between return rate, recycling rate, reuse rate, cost and optimal profits in period 2 or 3 are complex, but some of their variation tendencies are predicted and some can be simplified into linear or quadratic relationships. Though this study, relationships are explicit and simplified which are helpful for managers, researchers and stakeholders.

Furthermore, the internal logic of result under those two different optimization conditions is requisite for the further investigation. As well as how to realise and balance the environmental and economic benefits.

Chapter 7

Conclusion

This chapter will relate the findings to the research motivation and objectives, as well as to the questions defined in Chapter 2. In addition, the contributions of this research to theory and the industry will be summarized. Fiinally, the limitations and potential topics for further research are discussed.

7.1 Contribution to the electric vehicle supply chain

In order to investigate the impact of incentivization of the EV SC from government to the end customer, a multi-echelon model about subsidizing the EV SC was designed, which includes the government, the EV/GV manufacturers, the EV/GV retailers and the customer. Based on the model, the author has answered the three questions below:

- **Question 1: Can the retailer be involved in the EV SC incentive design?**
  There have been several studies of in EV supply chain incentive design (Luo et al., 2014; Huang et al., 2013; Zhang, 2014; Raz and Ovchinnikov, 2015). However, the retailer is not involved in the model although it is an indispensable part. In order to figure out the optimal profit and the subsidy for each party in the SC, we refer to the Stackelberg game, where the retailer is defined as the follower of the manufacturer and the leader of the customer.

- **Question 2: How do the energy price and the electricity price impact the probability of purchasing EVs and government subsidies?**
  Although how the energy price impacts EVs has been mentioned in a case study by

Gnann et al. (2015), how to quantify the impact between the two is lack of study. In the early EV development stage, the EV use experience has been under-researched, and the fuel price has more impact on the EV SC. In the later EV development stage, the electricity price has a greater impact. In the early stage with the increasing fuel price, the optimal subsidy decreases rapidly, and more customers will still select the GV. In the later stage, the higher electricity price will bring about a reduction in the purchase probability of EVs.

- **Question 3: How should the subsidies be allocated to the EV supply chain?**

  Some research has discussed the influence on different kinds of incentive policies, for example, Raz and Ovchinnikov (2015); Luo et al. (2014). But in the EV SC, who is the best party to receive the subsidy is still a question. In this research, it was found that whether the subsidy is given to the EV manufacturer or to the EV customer, the optimal subsidy paid by the government is equivalent. In addition, the EV/GV purchase probability, the optimal profit for each party are equivalent no matter who receives the subsidy. The difference between these two situations is that the price the EV manufacturer charges the EV retailer is less than the price the EV retailer charges the customer.

In summary, the EV SC incentive model answered three questions new to the literature. Furthermore, the effect of the subsidy given to the EV customer or to the manufacturer is equivalent at the optimal equilibrium point. This is a new finding.

In terms of practical industrial relevance, this model will help government to better observe and understand the EV SC from manufacturing to the end user. The government does not need to care about who should be given the subsidy. The government should rather pay more attention to the fuel and electricity prices in both the early and later EV development stages, as the energy price has a relatively large impact on the EV SC. In most cases, the higher the energy price, the lower profit for the entire SC.

7.2 Contribution to the energy supply chain

The literature has discussed the influencing in for locating the EV CSs (Wang et al., 2016) and in optimizing the location for CSs (Ge et al., 2011; You and Hsieh, 2014; Alhazmi et al., 2017; Arslan and Karaşan, 2016) with simulation. In order to design a kind of theritical schemes to improve the use efficiency for charging station networks, the author puts forward this research question:

- **Is there any possibility of developing a theoretical model for charging station selection and how to evaluate the selection's performance?**
  Regarding the EV energy SC, the author developed three EV CS selection schemes—the Per-time selection scheme, the Bulk selection scheme, and the Combined per-time and bulk selection scheme. Per-time selection scheme searches the whole CSs in real time to find the best one. The Bulk selection scheme summarises the performance for each CS and selects the best one as a fixed option for the EV driver. The Combined selection scheme first uses Bulk selection to select some optimal stations as a set and then uses the Per-time selection scheme to find the best one in real time from within the selected station set. Based on both mathematical analysis and simulation, Per-time selection has the best performance; however, it uses too many system resources because of its real-time complexity. The Combined selection uses fewer system resources and has a better performance than Per-time selection. Therefore, the conclusion is that the Combined per-time and bulk selection might be the best option.

To conclude, this model could be an innovative model in EV SC selection. In practice, the schemes will help EV drivers find the optimal CS and will help managers more efficiently schedule the charging network at a lower system cost.

7.3 Contribution to the electric vehicle battery closed-loop supply chain

In order to develop strategies to improve the system performance and profitability for EV battery reuse and the recycling SC, the author designed a closed-loop supply chain CLSC for used EV batteries. Based on the literature review, two questions are proposed:

- **How can we illustrate multi-period CLSCs for used EV batteries?**
  Currently, most studies are investigating the technical aspects of reuse and recycling (Lih et al., 2012; Neubauer and Pesaran, 2011; Patten et al., 2011). In terms of a closed-loop multi-period SC model, most research discussed two-period CLSC, e.g., Atasu et al. (2008); Ferguson and Toktay (2006); Mitra and Webster (2008), etc. However, the EV battery life cycle normally has three stages: new product use, reuse and recycle. Therefore, it is necessary to promote a three-period EV battery CLSC to describe these processes. In the author's model, the first period consists of new battery manufacturing and use period for the EV battery; when battery's life falls to around 70%, the battery has to be removed from the vehicle but can be reused for other purposes (Arcus, 2016). In the second period, some batteries will be returned and sorted as high and low quality. High-quality returned batteries will be reused, while low-quality batteries will be remanufactured directly. In the third period, the reused batteries will be remanufactured.

- **What about the relationship between EV battery manufacturer and remanufacturer in this CLSC?**
  In the three-period CLSC, the parties in different periods is complex. Specifically, in the first period, there is only the battery manufacturer. In the second period, the high-quality remanufacturer and low-quality remanufacturer are involved, and in the third period, high-quality, low-quality and reused battery remanufacturers are involved. Based on two targets, which are to optimize the individual parties' profit and to optimize the total profit in the SC, the interrelationship between parties' profit and related parameters (i.e., return yield, recycling rate, remanufacturing

cost) are analysed.

With the target of optimizing the individual parties' profit in different period, the higher return yield will normally bring higher profit, especially in terms of the return rate for the used EV batteries. In addition, the higher remanufacturing cost will result in more profit for the renumafcuters. With the goal of maximizing the total profit in the SC in different periods, the higher used battery return yield, the lower total profit in both second and third periods. In order to increase the total profit, the recycling rate should be increased and the remanufacturing cost should be reduced.

In conclusion, this model has described the EV battery from the perspective of the CLSC. This model has extended the previous two-period model to a three-period model to better reflect reality.

In industrial practise, some mathematical interrelationships between profit and parameters are too complex. A simplified linear or quadratic relationship between the processing cost, the return rate and the profit would make it much easier for managers and other stakeholders in the SC to understand the system better, and thus, make quicker decisions.

7.4 Limitations and future research opportunities

This research is limited in the mathematical modelling and simulation. Although each model does provide managerial insights, the models are conceptual. It would be best if they were to be verified by case studies. Further research could include:

1. Exploring the relationship between some more specific parameters and the optimal subsidy, for example, quantifying the customer's environmental awareness with discussions.

2. Exploring the rebound effect on EVs and between GVs and EVs. This research is more about the promotion of the vehicle SC, and based on Font Vivanco et al. (2014)'s research, the environmental rebound effect on EVs must be a potential research direction.

3. Extending the theoretical charging station selection scheme model to a real application with GIS.

4. Exploring the profitability under optimizing different goals.

5. Exploring the interrelationship among different sub-supply chains in the EV industry.

Moreover, this research lacks the use of real-world data to conduct the whole EV triple supply chain in the framework of Fig. 1.2. In order to implement this, surveys and cases studies with automotive enterprises are recommended.

7.5 Summary

This chapter concludes the thesis, by highlighting the overall findings and the contributions to the EV SC research and industrial practice. The limitations of this research due to the methods adopted and the time constraints were discussed, along with future research opportunities.

Overall, this research explored the current situation of EV SC and the existing EV SC-related literature and then developed three models—an EV subsidy model for the vehicle SC; a charging station selection scheme model for the energy SC; and a model for the EV battery CLSC. The profit of parties in the EV triple SC were also investigated.

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Appendix A: Electric vehicle supply chain incentive design model

A.1 Relationships between U_{Cev} and U_{Cgv}

From Eq. 4.7 and Eq. 4.8, it can be drawn the utility-intention diagram for $U_{Cgv}(k)$ and $U_{Cev}(k)$ together. The intercepts of $U_{Cgv}(k)$ and $U_{Cgv}(k)$ are p_{Cgv} and $(p_{Cev} - s_c)$, respectively. The slopes of them are v_{gv} and v_{ev}. Therefore, there are six possibilities of relationships between $U_{Cgv}(k)$ and $C_{Cev}(k)$, which are shown in Fig. A-1.

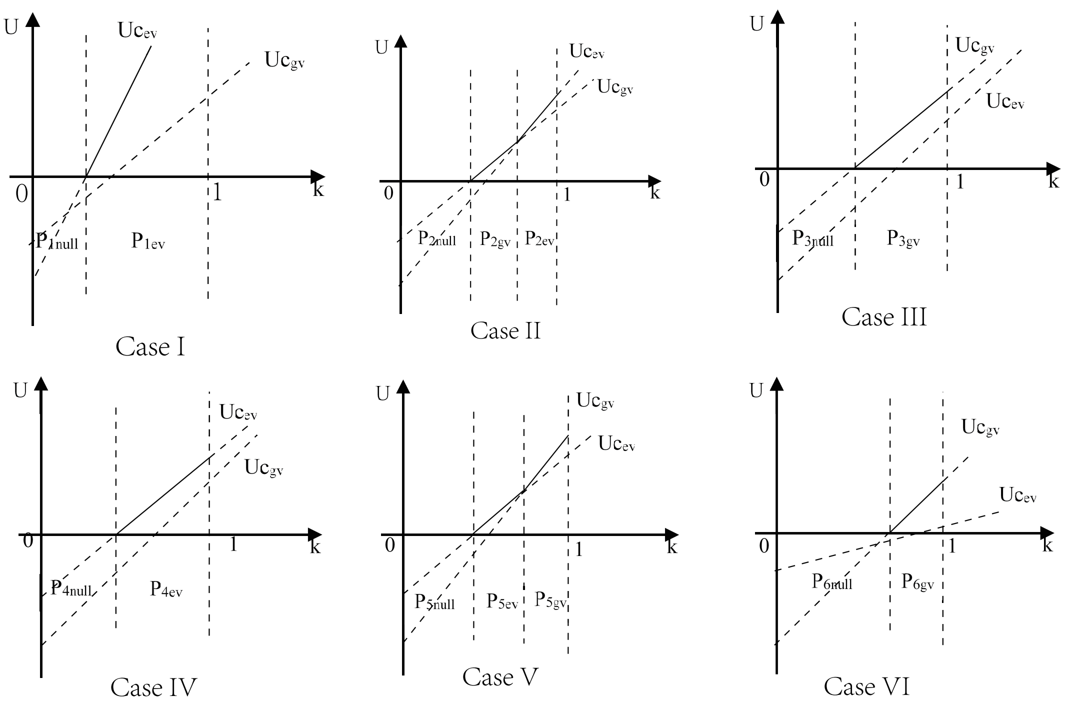

Fig. A-1: Possibility of relationships between U_{Cgv} and C_{Cev}

The probabilities of buy GV, EV or not buying vehicle from Case I to Case VI are defined from P_{1null}, P_{1gv}, P_{1ev} to P_{6null}, P_{6gv}, P_{6ev}. And all the probabilities are described below:

- Case I

  The condition of this case is

$$\begin{cases} p_{Cgv} < p_{Cev} - s_c \\ v_{gv}(p_{Cev} - s_c) \leq p_{Cgv} v_{ev} \end{cases} \tag{A-1}$$

  In this case, the probability that people will buy EV is

$$P_{1ev} = 1 - \frac{p_{Cev} - s_c}{v_{ev}} \tag{A-2}$$

  The probability people will buy GV is 0 and the probability they will not buy a vehicle is

$$P_{1null} = \frac{p_{Cev} - s_c}{v_{ev}} \tag{A-3}$$

- Case II

  The condition in this case is

$$\begin{cases} p_{Cgv} < p_{Cev} - s_c \\ \frac{p_{Cgv}}{v_{gv}} < \frac{p_{Cev} - s_c}{v_{ev}} < 1 \end{cases} \tag{A-4}$$

  The probabilities are

$$P_{2null} = \frac{p_{Cgv}}{v_{gv}} \tag{A-5}$$

$$P_{2gv} = \frac{p_{Cgv} - p_{Cev} + s_c}{v_{gv} - v_{ev}} - \frac{p_{Cgv}}{v_{gv}} \tag{A-6}$$

$$P_{2ev} = 1 - \frac{p_{Cgv} - p_{Cev} + s_c}{v_{gv} - v_{ev}} \tag{A-7}$$

- Case III

  The condition in this case is

$$\begin{cases} p_{Cgv} < p_{Cev} - s_c \\ v_{ev} - p_{Cev} + s_c \leq v_{gv} - p_{Cgv} \end{cases} \tag{A-8}$$

The probabilities are

$$P_{3null} = \frac{p_{Cgv}}{v_{gv}} \tag{A-9}$$

$$P_{3gv} = 1 - \frac{p_{Cgv}}{v_{gv}} \tag{A-10}$$

$$P_{3ev} = 0 \tag{A-11}$$

- Case IV

  The condition in this case is

$$\begin{cases} p_{Cgv} > p_{Cev} - s_c \\ v_{ev} - p_{Cev} + s_c \geq v_{gv} - p_{Cgv} \end{cases} \tag{A-12}$$

  The probabilities are

$$P_{4null} = \frac{p_{Cev} - s_c}{v_{ev}} \tag{A-13}$$

$$P_{4gv} = 0 \tag{A-14}$$

$$P_{4ev} = 1 - \frac{p_{Cev} - s_c}{v_{ev}} \tag{A-15}$$

- Case V

  The condition in this case is

$$\begin{cases} p_{Cgv} > p_{Cev} - s_c \\ \frac{p_{Cgv}}{v_{gv}} > \frac{p_{Cev} - s_c}{v_{ev}} > 1 \end{cases} \tag{A-16}$$

  The probabilities are

$$P_{5null} = \frac{p_{Cev} - s_c}{v_{ev}} \tag{A-17}$$

$$P_{5gv} = 1 - \frac{p_{Cgv} - p_{Cev} + s_c}{v_{gv} - v_{ev}} \tag{A-18}$$

$$P_{5ev} = \frac{p_{Cgv} - p_{Cev} + s_c}{v_{gv} - v_{ev}} - \frac{p_{Cev} - s_c}{v_{ev}} \tag{A-19}$$

- Case VI

The condition of this case is

$$\begin{cases} p_{Cgv} > p_{Cev} - s_c \\ v_{gv}(p_{Cev} - s_c) \geq p_{Cgv} v_{ev} \end{cases} \tag{A-20}$$

The probabilities are

$$P_{6null} = \frac{p_{Cgv}}{v_{gv}} \tag{A-21}$$

$$P_{6gv} = 1 - \frac{p_{Cgv}}{v_{gv}} \tag{A-22}$$

$$P_{6ev} = 0 \tag{A-23}$$

In case I and IV, the customer will only select EV as its utility is always better than GV. And in case III and VI, customer will only buy the GV. To ensure a competitive market, the author will only consider case II and case V. In case V, the experience of driving GV is better than EV but the initial payment (the original cost and subsidy) of EV is cheaper. While in case II, the situation is inverse. This is in line with the reality of industrial development. Therefore, the author defines **Case V** as the EV early development period and **Case II** as the late EV development period.

A.2 EV early development stage

A.2.1 Substitutions in the electric vehicle early development stage

Table A-1: Substitutions in the electric vehicle early development stage

$B_0 = \frac{1}{v_{gv} - v_{ev}}$	$B_1 = \frac{1}{v_{ev}}$	
$D_0 = \frac{B_0(B_0+2B_1)}{3B_0+4B_1}$	$D_1 = \frac{B_0(B_0+B_1)}{3B_0+4B_1}$	$D_2 = \frac{(B_0+B_1)}{3B_0+4B_1}$
$D_3 = \frac{(B_0+2B_1)(B_0+B_1)}{3B_0+4B_1}$	$D_4 = \frac{D_1 D_3}{D_1^2 - 4D_0 D_3}$	$D_5 = \frac{D_1(D_3 p_{Mev}+D_2)+2D_3\left(D_0 p_{Mgv}+2D_2\right)}{D_1^2-4D_0D_3}$
$D_6 = \frac{\left(D_1^2-2D_0D_3\right)}{D_1^2-4D_0D_3}$	$D_7 = \frac{2D_0D_3}{D_1^2-4D_0D_3}$	$D_8 = \frac{D_0\left(2D_3 p_{Mev}+D_1 p_{Mgv}+2D_2\right)+2D_1D_2}{4D_0D_3-D_1^2}$
$D_9 = (D_1(D_6 - 1) - D_0 D_4)$	$D_{10} = (D_1 D_7 - D_0 D_4)$	$D_{11} = (2D_2+D_0D_5+D_1D_8)$

$D_{12} = (D_1D_4 - D_3(D_6 - 1))$	$D_{13} = (D_1D_4 - D_3D_7)$	$D_{14} = (D_2 - D_1D_5 - D_3D_8)$
$D_{15} = \left(B_0((B_0 + B_1) \cdot (2D_4 + D_6 - 1)\, v_{ev} - (2B_1 (D_6 - 1) + B_0 \cdot (D_4 + 2D_6 - 2))v_{gv}) \right)$	$D_{16} = \left(B_0((B_0 + B_1) \cdot (2D_4 + D_7)\, v_{ev} - 2B_1D_7v_{gv} - B_0 (D_4 + 2D_7)\, v_{gv}) \right)$	$D_{17} = \left(B_0(B_0D_5 - 2(B_0 + B_1) \cdot D_8 - 1)v_{gv} - (B_0 + B_1) \cdot (B_0 (2D_5 - D_8) - 2)\, v_{ev} \right)$
$D_{18} = \left(B_0(2(B_0 + B_1) \cdot (2D_4 + D_6 - 1) \cdot (v_{ev} - v_{gv}) + B_0 (D_4 - D_6 + 1)\, v_{gv} + B_1 (2D_4 - D_6 + 1) \cdot v_{gv}) \right)$	$D_{19} = \left(B_0(2(B_0 + B_1) \cdot (2D_4 + D_7)\, v_{ev} - 3B_0 (D_4 + D_7)\, v_{gv} - B_1 (2D_4 + 3D_7)\, v_{gv}) \right)$	$D_{20} = \left(B_0^2(-3v_{gv}(-D_5 + D_8 + v_{ev}) + 2(D_8 - 2D_5) \cdot v_{ev} + 3v_{gv}^2) + B_0(v_{ev}(2B_1(-2D_5 + D_8 - 2v_{gv}) + 4) + v_{gv}(B_1(2D_5 - 3D_8 + 4v_{gv}) - 3)) + 2B_1 (2v_{ev} - v_{gv}) \right)$
$D_{21} = D_6 - 1 + \frac{(B_0(D_4-D_6+1)-2B_1(D_6-1))(c_{e0}-c_{e1})}{(3B_0+4B_1)(r_{e0}-r_{e1})}$	$D_{22} = D_7 + \frac{(B_0D_4-(B_0+2B_1)D_7)(c_{e0}-c_{e1})}{(3B_0+4B_1)(r_{e0}-r_{e1})}$	$D_{23} = D_8 - p_{Mev} - \frac{(2B_1D_8+B_0(D_5+D_8)-1)(c_{e0}-c_{e1})}{(3B_0+4B_1)(r_{e0}-r_{e1})}$
$D_{24} = \frac{(B_0+B_1)(2D_4+D_6-1)(c_{g0}-c_{g1})}{(3B_0+4B_1)(r_{g0}-r_{g1})} + \frac{D_4(c_{g0}-c_{g1})}{r_{g1}-r_{g0}} + D_4$	$D_{25} = D_4 - \frac{((B_0+2B_1)D_4-(B_0+B_1)D_7)(c_{g0}-c_{g1})}{(3B_0+4B_1)(r_{g0}-r_{g1})}$	$D_{26} = \frac{\left(B_0^2(3(r_{g1} - r_{g0}) \cdot (c_{envir} + D_5 + p_{Mgv}) + (D_5 + D_8)\, c_{g0} - (D_5 + D_8)\, c_{g1}) + B_0(4B_1 (r_{g1} - r_{g0}) \cdot (c_{envir} + D_5 + p_{Mgv}) + (B_1 (2D_5 + D_8) + 2) \cdot c_{g0} - (B_1 (2D_5 + D_8) + 2)c_{g1}) + 2B_1 (c_{g0} - c_{g1}) \right)}{B_0(3B_0+4B_1)(r_{g0}-r_{g1})}$

A.2.2 The numerator and denominator for s_c in Eq. 4.34

According to Eq. 4.34, the denominator of s_c in the early stage can be expanded as

$$\left(\begin{array}{l}(v_{ev}-2v_{gv})\left(2c_{e0}\left(r_{g0}-r_{g1}\right)\left(2v_{ev}^2-9v_{gv}v_{ev}+8v_{gv}^2\right)^2-2c_{e1}\left(r_{g0}-r_{g1}\right)\right.\\ \cdot\left(2v_{ev}^2-9v_{gv}v_{ev}+8v_{gv}^2\right)^2+\left(r_{e0}-r_{e1}\right)\left(8\left(r_{g1}-r_{g0}\right)v_{ev}^4\right.\\ +2\left(c_{g0}-c_{g1}+39r_{g0}-39r_{g1}\right)v_{gv}v_{ev}^3+\left(-8c_{g0}+8c_{g1}-271r_{g0}\right.\\ \left.\left.\left.+271r_{g1}\right)v_{gv}^2v_{ev}^2+4\left(2c_{g0}-2c_{g1}+97r_{g0}-97r_{g1}\right)v_{gv}^3v_{ev}+192\left(r_{g1}-r_{g0}\right)v_{gv}^4\right)\right)\end{array}\right) \tag{A-24}$$

And the numerator can be expanded as

$$\left(\begin{array}{l}2c_{g1}\left(r_{e0}-r_{e1}\right)v_{ev}(8v_{gv}^3+\left(2p_{Mev}-8p_{Mgv}-11v_{ev}\right)v_{gv}^2+v_{ev}\left(9p_{Mgv}+3v_{ev}-p_{Mev}\right)v_{gv}\\ -2p_{Mgv}v_{ev}^2)\left(v_{ev}-2v_{gv}\right)^2+2c_{g0}\left(r_{e0}-r_{e1}\right)v_{ev}(-8v_{gv}^3+\left(-2p_{Mev}+8p_{Mgv}+11v_{ev}\right)v_{gv}^2\\ +v_{ev}\left(p_{Mev}-3\left(3p_{Mgv}+v_{ev}\right)\right)v_{gv}+2p_{Mgv}v_{ev}^2)\left(v_{ev}-2v_{gv}\right)^2+\left(r_{g0}-r_{g1}\right)\\ \cdot(2c_{e0}\left(v_{ev}-2v_{gv}\right)\left(2v_{ev}^2-9v_{gv}v_{ev}+8v_{gv}^2\right)\left(\left(2p_{Mev}+p_{Mgv}-2v_{ev}\right)v_{ev}^2\right.\\ \left.+\left(-9p_{Mev}-2p_{Mgv}+8v_{ev}\right)v_{gv}v_{ev}+2\left(4p_{Mev}-3v_{ev}\right)v_{gv}^2\right)-2c_{e1}\left(v_{ev}-2v_{gv}\right)\\ \cdot\left(2v_{ev}^2-9v_{gv}v_{ev}+8v_{gv}^2\right)\left(\left(2p_{Mev}+p_{Mgv}-2v_{ev}\right)v_{ev}^2\right.\\ \left.+\left(8v_{ev}-9p_{Mev}-2p_{Mgv}\right)v_{gv}v_{ev}+2\left(4p_{Mev}-3v_{ev}\right)v_{gv}^2\right)+\left(r_{e0}-r_{e1}\right)(4c_{envir}v_{ev}^5\\ -(41c_{envir}+8\left(p_{Mev}+p_{Mgv}-v_{ev}\right))v_{gv}v_{ev}^4+2(75c_{envir}+35p_{Mev}+26p_{Mgv}\\ -31v_{ev})v_{gv}^2v_{ev}^3-2\left(116c_{envir}+109p_{Mev}+52p_{Mgv}-83v_{ev}\right)v_{gv}^3v_{ev}^2\\ +8\left(16c_{envir}+35p_{Mev}+8p_{Mgv}-22v_{ev}\right)v_{gv}^4v_{ev}+64\left(v_{ev}-2p_{Mev}\right)v_{gv}^5))\end{array}\right) \tag{A-25}$$

A.2.3 The numerator and denominator for s_m in Eq. 4.38

According to Eq. 4.38, the numerator of s_m in the early stage can be expanded as

$$\left(\begin{array}{l} 2c_{g1}\left(r_{e0}-r_{e1}\right)v_{ev}(8v_{gv}^{3}+\left(2p_{Mev}-8p_{Mgv}-11v_{ev}\right)v_{gv}^{2}+v_{ev}\left(9p_{Mgv}+3v_{ev}-p_{Mev}\right)v_{gv} \\ -2p_{Mgv}v_{ev}^{2})\left(v_{ev}-2v_{gv}\right)^{2}+2c_{g0}\left(r_{e0}-r_{e1}\right)v_{ev}(-8v_{gv}^{3}+\left(-2p_{Mev}+8p_{Mgv}+11v_{ev}\right)v_{gv}^{2} \\ +v_{ev}\left(p_{Mev}-3\left(3p_{Mgv}+v_{ev}\right)\right)v_{gv}+2p_{Mgv}v_{ev}^{2})\left(v_{ev}-2v_{gv}\right)^{2}+\left(r_{g0}-r_{g1}\right)(2c_{e0}\left(v_{ev}-2v_{gv}\right) \\ \cdot\left(2v_{ev}^{2}-9v_{gv}v_{ev}+8v_{gv}^{2}\right)(\left(2p_{Mev}+p_{Mgv}-2v_{ev}\right)v_{ev}^{2}+\left(-9p_{Mev}-2p_{Mgv}+8v_{ev}\right)v_{gv}v_{ev} \\ +2\left(4p_{Mev}-3v_{ev}\right)v_{gv}^{2})-2c_{e1}\left(v_{ev}-2v_{gv}\right)\left(2v_{ev}^{2}-9v_{gv}v_{ev}+8v_{gv}^{2}\right)((2p_{Mev}+p_{Mgv} \\ -2v_{ev})v_{ev}^{2}+\left(-9p_{Mev}-2p_{Mgv}+8v_{ev}\right)v_{gv}v_{ev}+2\left(4p_{Mev}-3v_{ev}\right)v_{gv}^{2}))+\left(r_{e0}-r_{e1}\right) \\ \cdot(4c_{envir}v_{ev}^{5}-(41c_{envir}+8\left(p_{Mev}+p_{Mgv}-v_{ev}\right))v_{gv}v_{ev}^{4}+2(75c_{envir}+35p_{Mev}+26p_{Mgv} \\ -31v_{ev})v_{gv}^{2}v_{ev}^{3}-2\left(116c_{envir}+109p_{Mev}+52p_{Mgv}-83v_{ev}\right)v_{gv}^{3}v_{ev}^{2}+8(16c_{envir} \\ +35p_{Mev}+8p_{Mgv}-22v_{ev})v_{gv}^{4}v_{ev}+64\left(v_{ev}-2p_{Mev}\right)v_{gv}^{5})) \end{array}\right) \tag{A-26}$$

And the denominator of s_m can be expanded as

$$\left(\begin{array}{l} \left(v_{ev}-2v_{gv}\right)(2c_{e0}\left(r_{g0}-r_{g1}\right)\left(2v_{ev}^{2}-9v_{gv}v_{ev}+8v_{gv}^{2}\right)^{2}-2c_{e1}\left(r_{g0}-r_{g1}\right) \\ \cdot\left(2v_{ev}^{2}-9v_{gv}v_{ev}+8v_{gv}^{2}\right)^{2}+\left(r_{e0}-r_{e1}\right)(8\left(r_{g1}-r_{g0}\right)v_{ev}^{4} \\ +2\left(c_{g0}-c_{g1}+39r_{g0}-39r_{g1}\right)v_{gv}v_{ev}^{3}+\left(-8c_{g0}+8c_{g1}-271r_{g0}+271r_{g1}\right)v_{gv}^{2}v_{ev}^{2} \\ +4\left(2c_{g0}-2c_{g1}+97r_{g0}-97r_{g1}\right)v_{gv}^{3}v_{ev}+192\left(r_{g1}-r_{g0}\right)v_{gv}^{4})) \end{array}\right) \tag{A-27}$$

A.3 EV later development stage

A.3.1 Substitutions in the electric vehicle later development stage

Table A-2: Substitutions in the electric vehicle later development stage

$A_0=\frac{1}{v_{gv}-v_{ev}}$	$A_1=\frac{1}{v_{gv}}$	
$C_0=\frac{\left(A_0^2-3A_1A_0+2A_1^2\right)}{3A_0-4A_1}$	$C_1=\frac{A_0(A_0-A_1)}{3A_0-4A_1}$	$C_2=\frac{(A_0-A_1)}{3A_0-4A_1}$
$C_3=\frac{A_0^2-2A_1A_0}{3A_0-4A_1}$	$C_4=\frac{\left(A_0A_1-A_0^2\right)}{3A_0-4A_1}$	$C_5=\frac{C_1C_3}{4C_0C_3+C_1C_4}$

$C_6 = \dfrac{\begin{pmatrix} C_1C_3p_{Mev} - 2C_1C_2 \\ -2C_3(C_2 - C_0p_{Mgv}) \end{pmatrix}}{4C_0C_3+C_1C_4}$	$C_7 = \frac{2C_0C_3+C_1C_4}{4C_0C_3+C_1C_4}$	$C_8 = \frac{2C_0C_3}{4C_0C_3+C_1C_4}$
$C_9 = \dfrac{\begin{pmatrix} 2C_0C_3p_{Mev} + C_4C_2 \\ -C_0C_4p_{Mgv} - 4C_0C_2 \end{pmatrix}}{4C_0C_3+C_1C_4}$	$C_{10} = (C_1 - C_7C_1 - C_0C_5)$	$C_{11} = (C_1C_8 - C_0C_5)$
$C_{12} = C_2 + C_0C_6 - C_1C_9$	$C_{13} = (C_7C_3 - C_3 - C_4C_5)$	$C_{14} = (C_4C_5 + C_3C_8)$
$C_{15} = 2C_2 + C_4C_6 + C_3C_9$	$C_{16} = \begin{pmatrix} A_0(-2A_1C_5v_{ev} \\ +A_0\left(2C_5 - C_7 + 1\right)v_{ev} \\ -\left(A_0 - A_1\right) \\ \cdot\left(C_5 - 2C_7 + 2\right)v_{gv}) \end{pmatrix}$	$C_{17} = \begin{pmatrix} A_0(-2A_1C_5v_{ev} \\ +A_0\left(2C_5 + C_8\right)v_{ev} \\ -\left(A_0 - A_1\right) \\ \cdot\left(C_5 + 2C_8\right)v_{gv}) \end{pmatrix}$
$C_{18} = \begin{pmatrix} A_0(2A_1C_6 \\ -A_0\left(2C_6 + C_9\right) + 1) \\ \cdot v_{ev} + (A_0 - A_1) \\ \cdot(A_0\left(C_6 + 2C_9\right) - 2) \\ \cdot v_{gv} \end{pmatrix}$	$C_{19} = \begin{pmatrix} A_0(A_1((-3C_5 + 2C_7 \\ -2)v_{ev} + 2(C_5 - 2C_7 \\ +2)v_{gv}) + A_0(3(C_5 \\ -C_7 + 1)v_{ev} - 2(C_5 \\ -2C_7 + 2)v_{gv})) \end{pmatrix}$	$C_{20} = \begin{pmatrix} A_0((3A_0\left(C_5 + C_8\right) \\ -A_1\left(3C_5 + 2C_8\right))v_{ev} \\ -2\left(A_0 - A_1\right)\left(C_5 + 2C_8\right) \\ \cdot v_{gv}) \end{pmatrix}$
$C_{21} = \begin{pmatrix} A_0^2(v_{gv}(2C_6 + 4C_9 \\ -3v_{ev}) - 3v_{ev}(C_6 + C_9 \\ -v_{ev})) + A_0(v_{ev}(A_1 \\ \cdot(3C_6 + 2C_9 - 4v_{ev}) \\ +3) - 2v_{gv}(A_1(C_6 \\ +2C_9 - 2v_{ev}) + 2)) \\ -2A_1\left(v_{ev} - 2v_{gv}\right) \end{pmatrix}$	$C_{22} = \frac{(A_0(C_5+C_7-1)-2A_1C_5)(c_{g0}-c_{g1})}{(3A_0-4A_1)(r_{g0}-r_{g1})} - C_5$	$C_{23} = \frac{(A_0(C_5-C_8)-2A_1C_5)(c_{g0}-c_{g1})}{(3A_0-4A_1)(r_{g0}-r_{g1})} - C_5$

$C_{24} = -c_{envir} + C_6 - p_{Mgv} + \frac{(2A_1C_6 + A_0(C_9 - C_6) - 1)(c_{g0} - c_{g1})}{(3A_0 - 4A_1)(r_{g0} - r_{g1})}$	$C_{25} = C_7 - 1 - \frac{\left(\begin{array}{l} A_0(C_5 + C_7 - 1) \\ -A_1(C_5 + 2C_7 - 2)\end{array}\right)(c_{e0} - c_{e1})}{(3A_0 - 4A_1)(r_{e0} - r_{e1})}$	$C_{26} = -C_8 + \frac{(A_0(C_8 - C_5) + A_1(C_5 - 2C_8))(c_{e0} - c_{e1})}{(3A_0 - 4A_1)(r_{e0} - r_{e1})}$
$C_{27} = \frac{\left(\begin{array}{l} A_0^2((C_6 - C_9)c_{e0} \\ +(C_9 - C_6)c_{e1} \\ +3(C_9 - p_{Mev})(r_{e0} - r_{e1})) \\ +A_0((A_1(C_6 - 2C_9) + 2) \\ \cdot(-c_{e0}) + (A_1(C_6 - 2C_9) \\ +2)c_{e1} - 4A_1(C_9 - p_{Mev}) \\ \cdot(r_{e0} - r_{e1})) \\ +2A_1(c_{e0} - c_{e1})\end{array}\right)}{A_0(3A_0 - 4A_1)(r_{e0} - r_{e1})}$		

A.3.2 The numerator and denominator for s_c in Eq. 4.69

According to Eq. 4.69, the numerator of s_c can be expanded as

$$\left(\begin{array}{l} (r_{g0} - r_{g1})((r_{e0} - r_{e1})(v_{ev}^2 v_{gv}^3(41c_{envir} + 1403v_{ev} - 70p_{Mev} + 8p_{Mgv}) - v_{ev}^3 v_{gv}^2 \\ \cdot(150c_{envir} + 1635v_{ev} - 218p_{Mev} + 52p_{Mgv}) + 8v_{ev}^4 v_{gv}(29c_{envir} + 126v_{ev} - 35p_{Mev} \\ +13p_{Mgv} - 4v_{ev}v_{gv}^4(c_{envir} + 167v_{ev} - 2p_{Mev}) - 64v_{ev}^5(2c_{envir} + 4v_{ev} - 2p_{Mev} + p_{Mgv}) \\ +164v_{ev}v_{gv}^5 - 16v_{gv}^6) + 2c_{e0}(2v_{ev} - v_{gv})\left(-9v_{ev}v_{gv} + 8v_{ev}^2 + 2v_{gv}^2\right)(v_{ev}v_{gv}(11v_{ev} - 9p_{Mev} \\ +p_{Mgv}) + v_{gv}^2(2p_{Mev} - 3v_{ev}) + 2v_{ev}^2(-4v_{ev} + 4p_{Mev} - p_{Mgv})) - 2c_{e1}(2v_{ev} - v_{gv}) \\ \cdot\left(-9v_{ev}v_{gv} + 8v_{ev}^2 + 2v_{gv}^2\right)(v_{ev}v_{gv}(11v_{ev} - 9p_{Mev} + p_{Mgv}) + v_{gv}^2(2p_{Mev} - 3v_{ev}) \\ +2v_{ev}^2(-4v_{ev} + 4p_{Mev} - p_{Mgv}))) - 2c_{g0}v_{ev}(r_{e0} - r_{e1})(v_{gv} - 2v_{ev})^2(v_{gv}^2(8v_{ev} + p_{Mev} + 2p_{Mgv}) \\ -v_{ev}v_{gv}(6v_{ev} + 2p_{Mev} + 9p_{Mgv}) + 8v_{ev}^2 p_{Mgv} - 2v_{gv}^3) + 2c_{g1}v_{ev}(r_{e0} - r_{e1})(v_{gv} - 2v_{ev})^2 \\ \cdot(v_{gv}^2(8v_{ev} + p_{Mev} + 2p_{Mgv}) - v_{ev}v_{gv}(6v_{ev} + 2p_{Mev} + 9p_{Mgv}) + 8v_{ev}^2 p_{Mgv} - 2v_{gv}^3) \end{array}\right) \tag{A-28}$$

and the denominator of s_c can be expanded as

$$\left(\begin{array}{l} (2v_{ev}-v_{gv})\left((r_{e0}-r_{e1})\left(4v_{ev}^{3}v_{gv}\left(2c_{g0}-2c_{g1}+97r_{g0}-97r_{g1}\right)\right.\right. \\ +v_{ev}^{2}v_{gv}^{2}\left(-8c_{g0}+8c_{g1}-271r_{g0}+271r_{g1}\right) \\ +2v_{ev}v_{gv}^{3}\left(c_{g0}-c_{g1}+39r_{g0}-39r_{g1}\right)+192v_{ev}^{4}\left(r_{g1}-r_{g0}\right) \\ \left.+8v_{gv}^{4}\left(r_{g1}-r_{g0}\right)\right)+2c_{e0}\left(-9v_{ev}v_{gv}+8v_{ev}^{2}+2v_{gv}^{2}\right)^{2}\left(r_{g0}-r_{g1}\right) \\ \left.-2c_{e1}\left(-9v_{ev}v_{gv}+8v_{ev}^{2}+2v_{gv}^{2}\right)^{2}\left(r_{g0}-r_{g1}\right)\right) \end{array}\right) \tag{A-29}$$

A.3.3 The numerator and denominator for s_m in Eq. 4.73

According to Eq. 4.73, the numerator of s_m can be expanded as

$$\left(\begin{array}{l} (r_{g0}-r_{g1})\left((r_{e0}-r_{e1})\left(v_{ev}^{2}v_{gv}^{3}\left(41c_{envir}+1403v_{ev}-70p_{Mev}+8p_{Mgv}\right)\right.\right. \\ -v_{ev}^{3}v_{gv}^{2}\left(150c_{envir}+1635v_{ev}-218p_{Mev}+52p_{Mgv}\right) \\ +8v_{ev}^{4}v_{gv}\left(29c_{envir}+126v_{ev}-35p_{Mev}+13p_{Mgv}\right) \\ -4v_{ev}v_{gv}^{4}\left(c_{envir}+167v_{ev}-2p_{Mev}\right)-64v_{ev}^{5}\left(2c_{envir}+4v_{ev}-2p_{Mev}+p_{Mgv}\right) \\ \left.+164v_{ev}v_{gv}^{5}-16v_{gv}^{6}\right)+2c_{e0}\left(2v_{ev}-v_{gv}\right)\left(-9v_{ev}v_{gv}+8v_{ev}^{2}+2v_{gv}^{2}\right) \\ \cdot\left(v_{ev}v_{gv}\left(11v_{ev}-9p_{Mev}+p_{Mgv}\right)+v_{gv}^{2}\left(2p_{Mev}-3v_{ev}\right)\right. \\ \left.+2v_{ev}^{2}\left(-4v_{ev}+4p_{Mev}-p_{Mgv}\right)\right)-2c_{e1}\left(2v_{ev}-v_{gv}\right) \\ \cdot\left(-9v_{ev}v_{gv}+8v_{ev}^{2}+2v_{gv}^{2}\right)\left(v_{ev}v_{gv}\left(11v_{ev}-9p_{Mev}+p_{Mgv}\right)\right. \\ \left.\left.+v_{gv}^{2}\left(2p_{Mev}-3v_{ev}\right)+2v_{ev}^{2}\left(-4v_{ev}+4p_{Mev}-p_{Mgv}\right)\right)\right) \\ -2c_{g0}v_{ev}\left(r_{e0}-r_{e1}\right)\left(v_{gv}-2v_{ev}\right)^{2}\left(v_{gv}^{2}\left(8v_{ev}+p_{Mev}+2p_{Mgv}\right)\right. \\ \left.-v_{ev}v_{gv}\left(6v_{ev}+2p_{Mev}+9p_{Mgv}\right)+8v_{ev}^{2}p_{Mgv}-2v_{gv}^{3}\right) \\ +2c_{g1}v_{ev}\left(r_{e0}-r_{e1}\right)\left(v_{gv}-2v_{ev}\right)^{2}\left(v_{gv}^{2}\left(8v_{ev}+p_{Mev}+2p_{Mgv}\right)\right. \\ \left.-v_{ev}v_{gv}\left(6v_{ev}+2p_{Mev}+9p_{Mgv}\right)+8v_{ev}^{2}p_{Mgv}-2v_{gv}^{3}\right) \end{array}\right) \tag{A-30}$$

And the denominator of s_m can be expanded as

$$\left(\begin{array}{l} (2v_{ev}-v_{gv})\left((r_{e0}-r_{e1})\left(4v_{ev}^3 v_{gv}\left(2c_{g0}-2c_{g1}+97r_{g0}-97r_{g1}\right)\right.\right.\\ +v_{ev}^2 v_{gv}^2\left(-8c_{g0}+8c_{g1}-271r_{g0}+271r_{g1}\right)\\ +2v_{ev}v_{gv}^3\left(c_{g0}-c_{g1}+39r_{g0}-39r_{g1}\right)+192v_{ev}^4\left(r_{g1}-r_{g0}\right)\\ \left.+8v_{gv}^4\left(r_{g1}-r_{g0}\right)\right)+2c_{e0}\left(-9v_{ev}v_{gv}+8v_{ev}^2+2v_{gv}^2\right)^2\left(r_{g0}-r_{g1}\right)\\ \left.-2c_{e1}\left(-9v_{ev}v_{gv}+8v_{ev}^2+2v_{gv}^2\right)^2\left(r_{g0}-r_{g1}\right)\right) \end{array}\right) \tag{A-31}$$

Appendix B: Electric vehicle battery closed-loop supply chain model

B.1 Proof for model under condition of individual profit optimization

This section will prove all the relationships between parameters and profit for each (re)manufacturers in period 2 and 3 severally. One lemma is first proposed as a preparation:

Lemma: Function with format

$$f(x) = \frac{N_0 + N_1 x + N_2 x^2}{D_0 + D_1 x} \tag{B-1}$$

If $(N_0 + N_1 x) \gg N_2 x^2$ holds, then $f(x)$ approaches

$$f(x) \approx \frac{N_0 + N_1 x}{D_0 + D_1 x} \tag{B-2}$$

(1) $\boldsymbol{\Pi_{2n}}$

$$\begin{aligned}
&\Pi_{2n}(\theta, \alpha, \beta, \lambda_l, \lambda_h, c_{ntr}, c_l, c_h) \\
&= \frac{\left(\begin{array}{l}
\alpha(\beta-1)\theta H k q_{EV1}(\lambda_l\left(\alpha(\beta-1)\theta H k q_{EV1}\left(\lambda_h\left(\delta_m - 3c_{ntr}\right) + c_h\right) - 2c_{ntr} + \delta_m\right) \\
+c_l\left(\alpha(\beta-1)\theta H k q_{EV1}\lambda_h + 1\right) + \lambda_h\left(-2c_{ntr} + \alpha(\beta-1)\theta H k q_{EV1} + \delta_m\right) \\
+c_h + \alpha(\beta-1)\theta H k q_{EV1}\lambda_l) - c_{ntr} + 2\alpha(\beta-1)\theta H k q_{EV1} + \delta_m
\end{array}\right)^2}{k\left(\alpha(\beta-1)\theta H k q_{EV1}\left(\lambda_h\left(4\alpha(\beta-1)\theta H k q_{EV1}\lambda_l + 3\right) + 3\lambda_l\right) + 2\right)^2}
\end{aligned} \tag{B-3}$$

- when θ is an unknown variable

  Let other variables belong to $(0,1)$, we have $-2 < \frac{\partial \Pi_{2n}}{\partial \theta} < 471.39$, and the linearity is $-\infty < \eta_{2\theta} < +\infty$.

- when α is an unknown variable

  Let other variables belong to $(0,1)$, we have $-2 < \frac{\partial \Pi_{2n}}{\partial \alpha} < 5.44 \times 10^6$, and the linearity is $-\infty < \eta_{2\alpha} < +\infty$.

- when β is an unknown variable

  Let other variables belong to $(0,1)$, we have $-4.66 \times 10^7 < \frac{\partial \Pi_{2n}}{\partial \beta} < 2$, and the linearity is $-\infty < \eta_{2\beta} < 1396.94$.

- when λ_l is an unknown variable

  Let other variables belong to $(0,1)$, we have $-0.32 < \frac{\partial \Pi_{2n}}{\partial \lambda_l} < 0.03$, and the linearity is $-0.14 < \eta_{2\lambda_l} < +\infty$.

- when λ_h is an unknown variable

  Let other variables belong to $(0,1)$, we have $0 < \frac{\partial \Pi_{2n}}{\partial \lambda_h} < 0.03$, and the linearity is $-\infty < \eta_{2\lambda_h} < +\infty$. Therefore, λ_h has positive relationship with Π_{2n}.

- when c_{ntr} is an unknown variable

  Let other variables belong to $(0,1)$, we have $-0.5 < \frac{\partial \Pi_{2n}}{\partial c_{ntr}} < 2.57$, and the linearity is $-\infty < \eta_{2c_{ntr}} < +\infty$.

- when c_l is an unknown variable

  Let other variables belong to $(0,1)$, we have $-0.12 < \frac{\partial \Pi_{2n}}{\partial c_l} < 1921.58$, and the linearity is $-\infty < \eta_{2c_l} < +\infty$.

- when c_h is an unknown variable

  Let other variables belong to $(0,1)$, we have $-0.12 < \frac{\partial \Pi_{2n}}{\partial c_h} < 580702$, and the linearity is $-\infty < \eta_{2c_h} < +\infty$.

(2) Π_{2l}

$$\Pi_{2l}(\theta,\alpha,\beta,\lambda_l,\lambda_h,c_{ntr},c_l,c_h) = \frac{\begin{pmatrix} \alpha^2(\beta-1)^2\theta^2H^2kq_{EV1}^2\lambda_l(\lambda_l(\alpha(\beta-1)\theta Hkq_{EV1}\left(\lambda_h\left(c_{ntr}+\delta_m\right)+c_h\right)+c_{ntr} \\ +\alpha(\beta-1)\theta Hkq_{EV1}+\delta_m)+\left(c_l+1\right)\left(-3\alpha(\beta-1)\theta Hkq_{EV1}\lambda_h-2\right)) \end{pmatrix}^2}{\left(\alpha(\beta-1)\theta Hkq_{EV1}\left(\lambda_h\left(4\alpha(\beta-1)\theta Hkq_{EV1}\lambda_l+3\right)+3\lambda_l\right)+2\right)^2} \tag{B-4}$$

- when θ is an unknown variable

  Let other variables belong to $(0,1)$, we have $0 < \frac{\partial\Pi_{2l}}{\partial\theta} < +\infty$, and the linearity is $0.25 < \eta_{2\theta} < +\infty$. Therefore, θ has positive relationship with Π_{2l}.

- when α is an unknown variable

  Let other variables belong to $(0,1)$, we have $-0.24 < \frac{\partial\Pi_{2l}}{\partial\alpha} < +\infty$, and the linearity is $0.25 \leq \eta_{2\alpha} < +\infty$.

- when β is an unknown variable

  Let other variables belong to $(0,1)$, we have $-\infty < \frac{\partial\Pi_{2l}}{\partial\beta} < 0.27$, and the linearity is $-\infty < \eta_{2\beta} < -0.25$.

- when λ_l is an unknown variable

  Let other variables belong to $(0,1)$, we have $-0.016 < \frac{\partial\Pi_{2l}}{\partial\lambda_l} < +\infty$, and the linearity is $0 < \eta_{2\lambda_l} < 12.28$.

- when λ_h is an unknown variable

  Let other variables belong to $(0,1)$, we have $-2.05\times10^{-8} < \frac{\partial\Pi_{2l}}{\partial\lambda_h} < 2.05\times10^{-8}$, and the linearity is $-\infty < \eta_{2\lambda_h} < 0$.

- when c_{ntr} is an unknown variable

  Let other variables belong to $(0,1)$, we have $-303.22 < \frac{\partial\Pi_{2l}}{\partial c_{ntr}} < 2.34$, and the linearity is $-0.25 < \eta_{2c_{ntr}} < 0$.

- when c_l is an unknown variable

  Let other variables belong to $(0,1)$, we have $0 < \frac{\partial\Pi_{2l}}{\partial c_l} < 694594$, and the linearity is $-9212.11 < \eta_{2c_l} < 0.25$. Therefore, c_l has positive relationship with Π_{2l}.

- when c_h is an unknown variable

  Let other variables belong to $(0,1)$, we have $-0.82 < \frac{\partial \Pi_{2l}}{\partial c_h} < 538082$, and the linearity is $0 < \eta_{2c_h} < +\infty$.

(3) Π_{2h}

$$\Pi_{2h}(\theta,\alpha,\beta,\lambda_l,\lambda_h,c_{ntr},c_l,c_h)$$
$$= \frac{\left(\begin{array}{l}\alpha^2(\beta-1)^2\theta^2H^2kq_{EV1}^2\lambda_h(\lambda_h(\alpha(\beta-1)\theta Hkq_{EV1}\left(\lambda_l\left(c_{ntr}+\delta_m\right)+c_l\right)+c_{ntr}\\ +\alpha(\beta-1)\theta Hkq_{EV1}+\delta_m)+c_h\left(-3\alpha(\beta-1)\theta Hkq_{EV1}\lambda_l-2\right)\\ -3\alpha(\beta-1)\theta Hkq_{EV1}\lambda_l-2)\end{array}\right)^2}{\left(\alpha(\beta-1)\theta Hkq_{EV1}\left(\lambda_h\left(4\alpha(\beta-1)\theta Hkq_{EV1}\lambda_l+3\right)+3\lambda_l\right)+2\right)^2} \tag{B-5}$$

- when θ is an unknown variable

  Let other variables belong to $(0,1)$, we have $0 < \frac{\partial \Pi_{2h}}{\partial \theta} < +\infty$, and the linearity is $0.25 < \eta_{2\theta} < +\infty$. Therefore, θ has positive relationship with Π_{2h}.

- when α is an unknown variable

  Let other variables belong to $(0,1)$, we have $-\infty < \frac{\partial \Pi_{2h}}{\partial \alpha} < +\infty$, and the linearity is $0.25 < \eta_{2\alpha} + \infty$.

- when β is an unknown variable

  Let other variables belong to $(0,1)$, we have $-\infty < \frac{\partial \Pi_{2h}}{\partial \beta} < 0.73$, and the linearity is $-\infty < \eta_{2\beta} < -0.25$.

- when λ_l is an unknown variable

  Let other variables belong to $(0,1)$, we have $-2.48 \times 10^{-4} < \frac{\partial \Pi_{2h}}{\partial \lambda_l} < 0$, and the linearity is $-\infty < \eta_{2\lambda_l} < 0$.

- when λ_h is an unknown variable

  Let other variables belong to $(0,1)$, we have $-0.33 < \frac{\partial \Pi_{2h}}{\partial \lambda_h} < +\infty$, and the linearity is $0 < \eta_{2\lambda_h} < 61.79$.

- when c_{ntr} is an unknown variable

Let other variables belong to $(0,1)$, we have $-151.26 < \frac{\partial \Pi_{2h}}{\partial c_{ntr}} < 151.26$, and the linearity is $-0.22 < \eta_{2c_{ntr}} < 0$.

- when c_l is an unknown variable

  Let other variables belong to $(0,1)$, we have $0 < \frac{\partial \Pi_{2h}}{\partial c_l} < 375512$, and the linearity is $0 < \eta_{2c_l} < +\infty$. Therefore, c_l has positive relationship with Π_{2h}.

- when c_h is an unknown variable

  Let other variables belong to $(0,1)$, we have $0 < \frac{\partial \Pi_{2h}}{\partial c_h} < 1309.29$, and the linearity is $0.08 < \eta_{2c_h} < 0.23$. Therefore, c_h has positive relationship with Π_{2h}.

(4) Π_{3n}

$$\Pi_{3n}(\theta,\alpha,\beta,\lambda_l,\lambda_h,\lambda_u,c_{ntr},c_l,c_h,c_u)$$

$$= \frac{\left(\begin{array}{l} \theta Hkq_{EV2}(\alpha(1-\beta)\lambda_h(\theta Hkq_{EV2}((1-\alpha)\lambda_l(\alpha\beta\theta Hkq_{EV2}(c_u+1)+3c_{ntr} \\ -\delta_m)+\alpha\beta\lambda_u((1-\alpha)\theta Hkq_{EV2}(\lambda_l(\delta_m-4c_{ntr})+c_l)+3c_{ntr}+(\alpha-1)\theta \\ \cdot(-H)kq_{EV2}-\delta_m)+(\alpha-1)c_l-\alpha\beta c_u)-2c_{ntr}+\theta Hkq_{EV2}(-\alpha\beta+\alpha-1) \\ +\delta_m)+\alpha(1-\beta)c_h((\alpha-1)\theta(-H)kq_{EV2}\lambda_l-1)(\alpha\beta\theta Hkq_{EV2}\lambda_u-1) \\ +(\alpha-1)\lambda_l(\alpha\theta Hkq_{EV2}(\beta c_u+1)+2c_{ntr}-\delta_m)+\alpha\beta\lambda_u((\alpha-1)\theta \\ \cdot Hkq_{EV2}(\lambda_l(-3c_{ntr}+\alpha(\beta-1)\theta Hkq_{EV2}+\delta_m)+c_l)-2c_{ntr}+\theta Hk \\ \cdot q_{EV2}(\alpha\beta-1)+\delta_m)-(\alpha-1)c_l+\alpha\beta c_u)+c_{ntr}+\theta Hkq_{EV2}-\delta_m \end{array}\right)^2}{k\left(\begin{array}{l} \theta Hkq_{EV2}(\alpha(\beta-1)\lambda_h(\theta Hkq_{EV2}((\alpha-1)\lambda_l(5\alpha\beta\theta Hkq_{EV2}\lambda_u-4) \\ +4\alpha\beta\lambda_u)-3)+(\alpha-1)\lambda_l(4\alpha\beta\theta Hkq_{EV2}\lambda_u-3)+3\alpha\beta\lambda_u)-2 \end{array}\right)^2} \tag{B-6}$$

- when θ is an unknown variable

  Let other variables belong to $(0,1)$, we have $-0.89 < \frac{\partial \Pi_{3n}}{\partial \theta} < +\infty$, and the linearity is $-\infty < \eta_{3\theta} < +\infty$.

- when α is an unknown variable

  Let other variables belong to $(0,1)$, we have $-\infty < \frac{\partial \Pi_{3n}}{\partial \alpha} < +\infty$, and the linearity is $-\infty < \eta_{3\alpha} < 369.60$.

- when β is an unknown variable

  Let other variables belong to $(0, 1)$, we have $-\infty < \frac{\partial \Pi_{3n}}{\partial \beta} < +\infty$, and the linearity is $-\infty < \eta_{3\beta} < +\infty$.

- when λ_l is an unknown variable

  Let other variables belong to $(0, 1)$, we have $-0.41 < \frac{\partial \Pi_{3n}}{\partial \lambda_l} + \infty$, and the linearity is $-\infty < \eta_{3\lambda_l} < 514857$.

- when λ_h is an unknown variable

  Let other variables belong to $(0, 1)$, we have $-3.51 < \frac{\partial \Pi_{3n}}{\partial \lambda_h} < +\infty$, and the linearity is $-\infty < \eta_{3\lambda_h} < +\infty$.

- when λ_u is an unknown variable

  Let other variables belong to $(0, 1)$, we have $-0.05 < \frac{\partial \Pi_{3n}}{\partial \lambda_u} < +\infty$, and the linearity is $-0.43 < \eta_{3\lambda_u} < +\infty$.

- when c_{ntr} is an unknown variable

  Let other variables belong to $(0, 1)$, we have $-\infty < \frac{\partial \Pi_{3n}}{\partial c_{ntr}} < 1.6$, and the linearity is $-\infty < \eta_{3c_{ntr}} < +\infty$.

- when c_l is an unknown variable

  Let other variables belong to $(0, 1)$, we have $-0.25 < \frac{\partial \Pi_{3n}}{\partial c_l} < +\infty$, and the linearity is $-\infty < \eta_{3c_l} < +\infty$.

- when c_h is an unknown variable

  Let other variables belong to $(0, 1)$, we have $-0.25 < \frac{\partial \Pi_{3n}}{\partial c_h} < +\infty$, and the linearity is $-\infty < \eta_{3c_h} < +\infty$.

- when c_u is an unknown variable

  Let other variables belong to $(0, 1)$, we have $-0.25 < \frac{\partial \Pi_{3n}}{\partial c_u} < +\infty$, and the linearity is $-\infty < \eta_{3c_u} < +\infty$.

(5) Π_{3l}

$$\Pi_{3l}(\theta,\alpha,\beta,\lambda_l,\lambda_h,\lambda_u,c_{ntr},c_l,c_h,c_u)$$
$$=\frac{\left(\begin{array}{l}(\alpha-1)^2\theta^2H^2kq_{EV2}^2\lambda_l(\lambda_l(-(\alpha(\beta-1)\theta Hkq_{EV2}c_h-\alpha\theta Hkq_{EV2}\left(\beta c_u+1\right)\\ +c_{ntr}+\delta_m))+\alpha\beta\theta Hkq_{EV2}\lambda_u(\lambda_l(\alpha(\beta-1)\theta Hkq_{EV2}c_h+c_{ntr}+\alpha(\beta-1)\\ \cdot\theta Hkq_{EV2}+\delta_m)-3)+\alpha(\beta-1)\theta Hkq_{EV2}\lambda_h(\lambda_l(\alpha\beta\theta Hkq_{EV2}\left(c_u+1\right)-c_{ntr}\\ -\delta_m)+\alpha\beta\theta Hkq_{EV2}\lambda_u\left(\lambda_l\left(c_{ntr}+\delta_m\right)-4\right)+3)+c_l(\alpha\theta Hkq_{EV2}(-(\beta-1)\lambda_h\\ \cdot\left(4\alpha\beta\theta Hkq_{EV2}\lambda_u-3\right)-3\beta\lambda_u)+2)+2)\end{array}\right)^2}{\left(\begin{array}{l}\theta Hkq_{EV2}(\alpha(\beta-1)\lambda_h(\theta Hkq_{EV2}\left((\alpha-1)\lambda_l\left(5\alpha\beta\theta Hkq_{EV2}\lambda_u-4\right)+4\alpha\beta\lambda_u\right)\\ -3)+(\alpha-1)\lambda_l\left(4\alpha\beta\theta Hkq_{EV2}\lambda_u-3\right)+3\alpha\beta\lambda_u)-2\end{array}\right)^2} \tag{B-7}$$

- when θ is an unknown variable

  Let other variables belong to $(0,1)$, we have $0<\frac{\partial\Pi_{3l}}{\partial\theta}<+\infty$, and the linearity is $0.25<\eta_{3\theta}<+\infty$. Therefore, θ has positive relationship with Π_{3l}.

- when α is an unknown variable

  Let other variables belong to $(0,1)$, we have $-\infty<\frac{\partial\Pi_{3l}}{\partial\alpha}<+\infty$, and the linearity is $-\infty<\eta_{3\alpha}<-0.25$.

- when β is an unknown variable

  Let other variables belong to $(0,1)$, we have $-\infty<\frac{\partial\Pi_{3l}}{\partial\beta}<+\infty$, and the linearity is $-\infty<\eta_{3\beta}<+\infty$.

- when λ_l is an unknown variable

  Let other variables belong to $(0,1)$, we have $0<\frac{\partial\Pi_{3l}}{\partial\lambda_l}<+\infty$, and the linearity is $0<\eta_{3\lambda_l}<5.43$. Therefore, λ_l has positive relationship with Π_{3l}.

- when λ_h is an unknown variable

  Let other variables belong to $(0,1)$, we have $-\infty<\frac{\partial\Pi_{3l}}{\partial\lambda_h}<+\infty$, and the linearity is $-0.46<\eta_{3\lambda_h}<0$.

- when λ_u is an unknown variable

Let other variables belong to $(0,1)$, we have $-\infty < \frac{\partial \Pi_{3l}}{\partial \lambda_u} < +\infty$, and the linearity is $-0.37 < \eta_{3\lambda_u} < +\infty$.

- when c_{ntr} is an unknown variable

  Let other variables belong to $(0,1)$, we have $-\infty < \frac{\partial \Pi_{3l}}{\partial c_{ntr}} < +\infty$, and the linearity is $-0.25 < \eta_{3c_{ntr}} < 0$.

- when c_l is an unknown variable

  Let other variables belong to $(0,1)$, we have $0 < \frac{\partial \Pi_{3l}}{\partial c_l} < 94045.6$, and the linearity is $-1.87 < \eta_{3c_l} < 0.25$. Therefore, c_l has positive relationship with Π_{3l}.

- when c_h is an unknown variable

  Let other variables belong to $(0,1)$, we have $0 < \frac{\partial \Pi_{3l}}{\partial c_h} < 66980$, and the linearity is $0 < \eta_{3c_h} < 0.78$. Therefore, c_h has positive relationship with Π_{3l}.

- when c_u is an unknown variable

  Let other variables belong to $(0,1)$, we have $0 < \frac{\partial \Pi_{3l}}{\partial c_u} < +\infty$, and the linearity is $0 < \eta_{3c_u} < 0.15$. Therefore, c_u has positive relationship with Π_{3l}.

(6) Π_{3h}

$$\Pi_{3h}(\theta,\alpha,\beta,\lambda_l,\lambda_h,\lambda_u,c_{ntr},c_l,c_h,c_u)$$
$$= \frac{\left(\begin{array}{l} \alpha^2(\beta-1)^2\theta^2H^2kq_{EV2}^2\lambda_h(\lambda_h((\alpha-1)\theta Hkq_{EV2}c_l-\theta Hkq_{EV2}(\alpha(\beta-1) \\ +\alpha\beta c_u+1)+c_{ntr}+\delta_m)+\theta Hkq_{EV2}(-(\alpha-1)\lambda_h\lambda_l(\alpha\beta\theta Hkq_{EV2}\left(c_u+1\right) \\ -c_{ntr}-\delta_m)-\alpha\beta\lambda_u(\lambda_h((\alpha-1)\theta Hkq_{EV2}\left(\lambda_l\left(c_{ntr}+\delta_m\right)+c_l\right)+c_{ntr} \\ +(\alpha-1)\theta Hkq_{EV2}+\delta_m)-4(\alpha-1)\theta Hkq_{EV2}\lambda_l-3))+c_h(\theta Hkq_{EV2}((\alpha-1) \\ \cdot\lambda_l\cdot\left(4\alpha\beta\theta Hkq_{EV2}\lambda_u-3\right)+3\alpha\beta\lambda_u)-2)-3(\alpha-1)\theta Hkq_{EV2}\lambda_l-2) \end{array}\right)^2}{\left(\begin{array}{l} \theta Hkq_{EV2}(\alpha(\beta-1)\lambda_h(\theta Hkq_{EV2}\left((\alpha-1)\lambda_l\left(5\alpha\beta\theta Hkq_{EV2}\lambda_u-4\right)+4\alpha\beta\lambda_u\right) \\ -3)+(\alpha-1)\lambda_l\left(4\alpha\beta\theta Hkq_{EV2}\lambda_u-3\right)+3\alpha\beta\lambda_u)-2 \end{array}\right)^2} \tag{B-8}$$

- when θ is an unknown variable

  Let other variables belong to $(0,1)$, we have $0 < \frac{\partial \Pi_{3h}}{\partial \theta} < +\infty$, and the linearity is

$0.25 < \eta_{3\theta} < +\infty$. Therefore, θ has positive relationship with Π_{3h}.

- when α is an unknown variable

  Let other variables belong to $(0, 1)$, we have $0 < \frac{\partial \Pi_{3h}}{\partial \alpha} < +\infty$, and the linearity is $0.25 < \eta_{3\alpha} < +\infty$. Therefore, α has positive relationship with Π_{3h}.

- when β is an unknown variable

  Let other variables belong to $(0, 1)$, we have $-\infty < \frac{\partial \Pi_{3h}}{\partial \beta} < +\infty$, and the linearity is $-\infty < \eta_{3\beta} < -0.25$.

- when λ_l is an unknown variable

  Let other variables belong to $(0, 1)$, we have $-\infty < \frac{\partial \Pi_{3h}}{\partial \lambda_l} < +\infty$, and the linearity is $-0.17 < \eta_{3\lambda_l} < +\infty$.

- when λ_h is an unknown variable

  Let other variables belong to $(0, 1)$, we have $0 < \frac{\partial \Pi_{3h}}{\partial \lambda_h} < +\infty$, and the linearity is $0 < \eta_{3\lambda_h} < 14765.7$. Therefore, λ_h has positive relationship with Π_{3h}.

- when λ_u is an unknown variable

  Let other variables belong to $(0, 1)$, we have $-\infty < \frac{\partial \Pi_{3h}}{\partial \lambda_u} < +\infty$, and the linearity is $-0.48 < \eta_{3\lambda_u} < +\infty$.

- when c_{ntr} is an unknown variable

  Let other variables belong to $(0, 1)$, we have $-\infty < \frac{\partial \Pi_{3h}}{\partial c_{ntr}} < +\infty$, and the linearity is $-0.24 < \eta_{3c_{ntr}} < 0$.

- when c_l is an unknown variable

  Let other variables belong to $(0, 1)$, we have $0 < \frac{\partial \Pi_{3h}}{\partial c_l} < 61847.2$, and the linearity is $0 < \eta_{3c_l} < 47.56$. Therefore, c_l has positive relationship with Π_{3h}.

- when c_h is an unknown variable

  Let other variables belong to $(0, 1)$, we have $0 < \frac{\partial \Pi_{3h}}{\partial c_h} < 74806.7$, and the linearity is $-1.32 < \eta_{3c_h} < 0.21$. Therefore, c_h has positive relationship with Π_{3h}.

- when c_u is an unknown variable

  Let other variables belong to $(0,1)$, we have $0 < \frac{\partial \Pi_{3h}}{\partial c_u} < +\infty$, and the linearity is $0 < \eta_{3c_u} < 0.37$. Therefore, c_u has positive relationship with Π_{3h}.

(7) Π_{3u}

$$\Pi_{3u}(\theta, \alpha, \beta, \lambda_l, \lambda_h, \lambda_u, c_{ntr}, c_l, c_h, c_u)$$
$$= \frac{\left(\begin{array}{l} \alpha^2\beta^2\theta^2H^2kq_{EV2}^2\lambda_u((c_u+1)\left(\theta Hkq_{EV2}(\alpha(\beta-1)\lambda_h(4(\alpha-1)\theta Hkq_{EV2}\lambda_l\right. \\ +3)+3(\alpha-1)\lambda_l)+2)-\lambda_u(\theta Hkq_{EV2}(\alpha((\beta-1)c_h+(\beta-1)\lambda_h \\ \cdot((\alpha-1)\theta Hkq_{EV2}c_l+c_{ntr}+(\alpha-1)\theta Hkq_{EV2}+\delta_m)+c_l)+(\alpha-1)\lambda_l \\ \cdot\left(\alpha(\beta-1)\theta Hkq_{EV2}\left(\lambda_h\left(c_{ntr}+\delta_m\right)+c_h\right)+c_{ntr}+\alpha(\beta-1)\theta Hkq_{EV2}+\delta_m\right) \\ -c_l)+c_{ntr}+\theta Hkq_{EV2}(\alpha\beta-1)+\delta_m)) \end{array}\right)^2}{\left(\begin{array}{l} \theta Hkq_{EV2}(\alpha(\beta-1)\lambda_h(\theta Hkq_{EV2}((\alpha-1)\lambda_l\left(5\alpha\beta\theta Hkq_{EV2}\lambda_u-4\right) \\ +4\alpha\beta\lambda_u)-3)+(\alpha-1)\lambda_l\left(4\alpha\beta\theta Hkq_{EV2}\lambda_u-3\right)+3\alpha\beta\lambda_u)-2 \end{array}\right)^2} \tag{B-9}$$

- when θ is an unknown variable

  Let other variables belong to $(0,1)$, we have $0 < \frac{\partial \Pi_{3u}}{\partial \theta} < +\infty$, and the linearity is $0.25 < \eta_{3\theta} < +\infty$. Therefore, θ has positive relationship with Π_{3u}.

- when α is an unknown variable

  Let other variables belong to $(0,1)$, we have $0 < \frac{\partial \Pi_{3u}}{\partial \alpha} < +\infty$, and the linearity is $0.08 < \eta_{3\alpha} < +\infty$. Therefore, α has positive relationship with Π_{3u}.

- when β is an unknown variable

  Let other variables belong to $(0,1)$, we have $0 < \frac{\partial \Pi_{3u}}{\partial \beta} < +\infty$, and the linearity is $0.25 < \eta_{3\beta} < +\infty$. Therefore, β has positive relationship with Π_{3u}.

- when λ_l is an unknown variable

  Let other variables belong to $(0,1)$, we have $-0.02 < \frac{\partial \Pi_{3u}}{\partial \lambda_l} < +\infty$, and the linearity is $-0.21 < \eta_{3\lambda_l} < +\infty$.

- when λ_h is an unknown variable

Let other variables belong to $(0, 1)$, we have $-\infty < \frac{\partial \Pi_{3u}}{\partial \lambda_h} < +\infty$, and the linearity is $-0.47 < \eta_{3\lambda_h} < 0$.

- when λ_u is an unknown variable

  Let other variables belong to $(0, 1)$, we have $0 < \frac{\partial \Pi_{3u}}{\partial \lambda_u} < +\infty$, and the linearity is $0 < \eta_{3\lambda_u} < 25.03$. Therefore, λ_u has positive relationship with Π_{3u}.

- when c_{ntr} is an unknown variable

  Let other variables belong to $(0, 1)$, we have $-\infty < \frac{\partial \Pi_{3u}}{\partial c_{ntr}} < +\infty$, and the linearity is $-0.23 < \eta_{3c_{ntr}} < 0$.

- when c_l is an unknown variable

  Let other variables belong to $(0, 1)$, we have $0 < \frac{\partial \Pi_{3u}}{\partial c_l} < +\infty$, and the linearity is $0 < \eta_{3c_l} < 59.91$. Therefore, c_l has positive relationship with Π_{3u}.

- when c_h is an unknown variable

  Let other variables belong to $(0, 1)$, we have $0 < \frac{\partial \Pi_{3u}}{\partial c_h} < +\infty$, and the linearity is $0 < \eta_{3c_h} < 4.45$. Therefore, c_h has positive relationship with Π_{3u}.

- when c_u is an unknown variable

  Let other variables belong to $(0, 1)$, we have $0 < \frac{\partial \Pi_{3u}}{\partial c_u} < +\infty$, and the linearity is $-7.09 < \eta_{3c_u} < 0.24$. Therefore, c_u has positive relationship with Π_{3u}.

B.2 Proof for model under condition of whole chain profit optimization

This Section will prove all relationships between parameters and total profit in period 2 and 3. The grid search and Linearity will be used as well (see Section 3.2.7 and 3.2.8). And one lemma is first presented as preparation:

Lemma: Given $q_{EV} < M_{EV}$ and $0 < \delta_m < 1$, therefore $0 < Hkq_{EV} < 1$ holds.

Proof: Substituting $k = \delta_m/(HM_{EV})$ into Hkq_{EV}, we get $Hkq_{EV} = \frac{q_{EV}}{M_{EV}}\delta_m$. As $q_{EV} < M_{EV}$ and $0 < \delta_m < 1$, we have $0 < \frac{q_{EV}}{M_{EV}}\delta_m < 1$. Hence, $0 < Hkq_{EV} < 1$.

B.2.1 Proof for parameters vs total optimal profits in Period 2

(1) $\boldsymbol{\theta}$ The total optimal profit in period 2 can be rewritten as

$$\Pi_2(\theta) = \frac{N_{10} + N_{11}\theta + N_{12}\theta^2}{4k(-4 + D_{11}\theta)} \tag{B-10}$$

$$\begin{cases} 0 \leq N_{10} \leq 0 \\ 1.23 \times 10^{-7} \leq N_{11} \leq 24 \\ 0 \leq N_{12} \leq 0 \\ -1 \leq D_{11} \leq -2.86 \times 10^{-7} \end{cases} \tag{B-11}$$

Expressions of N_{10}, N_{11}, N_{12}, and D_{11} are shown from Eq. B-13 to Eq. B-16 in Appendix. Based on the range above, we have

(i) Because of $N_{12} \approx 0$, $N_{10} \approx 0$ and Lemma 1, $\Pi_2(\theta) = \frac{N_{11}\alpha}{4k(-4+D_{11}\alpha)}$

(ii) By using grid search, the range of first order of Π_2 on θ is $-6/(4k) \leq \Pi_2'(\theta) \leq -8.53/(4k) \times 10^{-8}$, also $\Pi_2'(\theta) < 0$. Therefore, the relationship is decremented.

(iii) For simplification, we assume the fitted line crosses (a_1, b_1) and (a_2, b_2), where $a_1 = 0$, $b_1 = \Pi_2(a_1)$, $a_2 = 1$, $b_2 = \Pi_2(a_2)$. And the fitted straight line is

$$\hat{\Pi_2(\alpha)} = \frac{b_2 - b_1}{a_2 - a_1}\alpha + (b_2 - \frac{b_2 - b_1}{a_2 - a_1}a_2) \tag{B-12}$$

Through using gird search and based on the assumption above, the linearity range is $-5.57\% \leq \eta_{2\theta} \leq 0$. So, θ and Π_2 can be treated as linear relationship.

Therefore, the relationship between $\Pi_2(\theta)$ and θ is negative linear.

$$N_{10} = -4\left(c_{ntr} - \delta_m\right)^2 \tag{B-13}$$

$$N_{11} = \left(\begin{array}{l} 4(Hkq_{EV1})(-\alpha\beta + (\alpha - \alpha\beta)c_h^2 + \alpha(\beta - 1)c_h(c_{ntr}(\lambda_h + 1) + (\lambda_h - 1)\delta_m - 2) \\ +\alpha(c_{ntr}(\beta + (\beta - 1)\lambda_h(-\lambda_h\delta_m + \delta_m + 1)) - (\beta - 1)c_{ntr}^2\lambda_h + \delta_m((\beta - 1)\lambda_h - \beta)) \\ -(\alpha - 1)c_l^2 - (\alpha - 1)c_{ntr} \cdot \lambda_l^2\delta_m - (\alpha - 1)\lambda_l((c_{ntr} - 1)c_{ntr} - (c_{ntr} + 1)\delta_m) \\ +(\alpha - 1)c_l(c_{ntr}(\lambda_l + 1) + (\lambda_l - 1)\delta_m - 2) - c_{ntr} + \delta_m + 1) \end{array} \right) \tag{B-14}$$

$$N_{12} = (\alpha - 1)\alpha(\beta - 1)(Hkq_{EV1})^2(-c_l(\lambda_h - 1) + c_h(\lambda_l - 1) + (c_{ntr} - 1)(\lambda_h - \lambda_l))^2 \tag{B-15}$$

$$D_{11} = (Hkq_{EV1})\left(\alpha\beta + \alpha(\beta - 1)(\lambda_h - 2)\lambda_h + (\alpha - 1)\lambda_l^2 - 2(\alpha - 1)\lambda_l - 1\right) \tag{B-16}$$

(2) α

$$\Pi_2(\alpha) = \frac{N_{20} + N_{21}\alpha + N_{22}\alpha^2}{4k(D_{20} + D_{21}\alpha)} \tag{B-17}$$

$$\begin{cases} -4 \le N_{20} \le 20 \\ -24 \le N_{21} \le 21 \\ 0 \le N_{22} \le -2.27 \times 10^{-33} \\ -5 \le D_{20} \le -4 \\ -1 \le D_{21} \le 1 \end{cases} \tag{B-18}$$

We find The first derivative $-5.25/(4k) \le \Pi_2'(\alpha) \le 6.25/(4k)$ and the linearity is $-5.57\% \le \eta_{2\alpha} \le 5.57\%$. Therefore, the relationship between α and Π_2 can be treated as linear.

$$N_{20} = \left(\begin{array}{l} 4(\theta(Hkq_{EV1})c_l^2 - \theta(Hkq_{EV1})c_l(c_{ntr}(\lambda_l + 1) + (\lambda_l - 1)\delta_m - 2) \\ +c_{ntr}(\delta_m(\theta(Hkq_{EV1})(\lambda_l - 1)\lambda_l + 2) - \theta(Hkq_{EV1})(\lambda_l + 1)) \\ +c_{ntr}^2(\theta(Hkq_{EV1})\lambda_l - 1) + \theta(Hkq_{EV1}) - \delta_m(\theta(Hkq_{EV1})(\lambda_l - 1) + \delta_m)) \end{array} \right) \tag{B-19}$$

$$N_{22} = (\beta - 1)\theta^2(Hkq_{EV1})^2(-c_l(\lambda_h - 1) + c_h(\lambda_l - 1) + (c_{ntr} - 1)(\lambda_h - \lambda_l))^2 \tag{B-20}$$

$$D_{20} = \theta(-(Hkq_{EV1})) - \theta(Hkq_{EV1})(\lambda_l - 2)\lambda_l - 4 \tag{B-21}$$

$$D_{21} = \theta(Hkq_{EV1})(\beta + (\beta - 1)(\lambda_h - 2)\lambda_h + (\lambda_l - 2)\lambda_l) \tag{B-22}$$

$$N_{21} =$$
$$\left(\begin{array}{l} \theta(Hkq_{EV1})(-4\beta-(\beta-1)c_h^2\left(\theta(Hkq_{EV1})+\theta(Hkq_{EV1})\left(\lambda_l-2\right)\lambda_l+4\right) \\ +c_l^2\left(-(\beta-1)\theta(Hkq_{EV1})\left(\lambda_h-2\right)\lambda_h-\beta\theta(Hkq_{EV1})+\theta(Hkq_{EV1})-4\right) \\ +2(\beta-1)c_h(\theta(Hkq_{EV1})c_l\left(\lambda_h-1\right)\left(\lambda_l-1\right)+\lambda_h(c_{ntr}(\theta(Hkq_{EV1}) \\ -\theta(Hkq_{EV1})\lambda_l+2)+\theta(Hkq_{EV1})\left(\lambda_l-1\right)+2\delta_m)+\theta(Hkq_{EV1})\left(c_{ntr}-1\right) \\ \cdot\left(\lambda_l-1\right)\lambda_l+2\left(c_{ntr}-\delta_m-2\right))+2\lambda_l((\beta-1)\theta(Hkq_{EV1})\left(c_{ntr}-1\right)^2\lambda_h \\ +2\left(c_{ntr}\left(-c_{ntr}+\delta_m+1\right)+\delta_m\right))+2c_l(c_{ntr}((\beta-1)\theta(Hkq_{EV1})\left(\lambda_h-1\right) \\ \cdot\left(\lambda_h-\lambda_l\right)+2\left(\lambda_l+1\right))+(\beta-1)\theta(-(Hkq_{EV1}))\left(\lambda_h-1\right)\left(\lambda_h-\lambda_l\right) \\ +2\left(\lambda_l-1\right)\delta_m-4)-\beta\theta(Hkq_{EV1})c_{ntr}^2\lambda_h^2+2\beta\theta(Hkq_{EV1})c_{ntr}\lambda_h^2 \\ +\theta(Hkq_{EV1})c_{ntr}^2\lambda_h^2-2\theta(Hkq_{EV1})c_{ntr}\lambda_h^2-4\beta c_{ntr}\lambda_h^2\delta_m+4\beta c_{ntr}\lambda_h\delta_m \\ +4c_{ntr}\lambda_h^2\delta_m-4c_{ntr}\lambda_h\delta_m-4\beta c_{ntr}^2\lambda_h+4\beta c_{ntr}\lambda_h+4c_{ntr}^2\lambda_h-4c_{ntr}\lambda_h \\ +\lambda_l^2\left((\beta-1)\theta(-(Hkq_{EV1}))\left(c_{ntr}-1\right)^2-4c_{ntr}\delta_m\right)+4\beta c_{ntr} \\ -\beta\theta(Hkq_{EV1})\lambda_h^2+\theta(Hkq_{EV1})\lambda_h^2+4\beta\lambda_h\delta_m-4\lambda_h\delta_m-4\beta\delta_m) \end{array} \right) \tag{B-23}$$

(3) $\boldsymbol{\beta}$

$$\Pi_2(\beta) = \frac{N_{30}+N_{31}\beta}{4k(D_{30}+D_{31}\beta)} \tag{B-24}$$

$$\begin{cases} -4 \leq N_{30} \leq 20 \\ -24 \leq N_{31} \leq -1.64\times10^{-7} \\ -5 \leq D_{30} \leq -4 \\ 3.29\times10^{-7} \leq D_{31} \leq 1 \end{cases} \tag{B-25}$$

We find The first derivative $9.22\times10^{-8}/(4k) \leq \Pi_2'(\beta) \leq 6.25/(4k)$ and the linearity is $7.44\times10^{-10} \leq \eta_{2\beta} \leq 5.57\%$. Therefore, the relationship between β and Π_2 can be treated as positive linear.

$$D_{30} = \theta(Hkq_{EV1})\left(-\alpha\lambda_h^2+2\alpha\lambda_h+(\alpha-1)\left(\lambda_l-2\right)\lambda_l\right)+\theta(-(Hkq_{EV1}))-4 \tag{B-26}$$

$$D_{31} = \alpha\theta(Hkq_{EV1})\left(\lambda_h-1\right)^2 \tag{B-27}$$

$$N_{30} = \left(\begin{aligned} & -\alpha^2\theta^2(Hkq_{EV1})^2c_{ntr}^2\lambda_h^2 + 2\alpha^2\theta^2(Hkq_{EV1})^2c_{ntr}\lambda_h^2 + \alpha\theta^2(Hkq_{EV1})^2c_{ntr}^2\lambda_h^2 \\ & -2\alpha\theta^2(Hkq_{EV1})^2c_{ntr}\lambda_h^2 + \alpha\theta(Hkq_{EV1})c_h^2((Hkq_{EV1})(\theta-\alpha\theta) \\ & -(\alpha-1)\theta(Hkq_{EV1})\left(\lambda_l-2\right)\lambda_l+4) - (\alpha-1)\theta(Hkq_{EV1})c_l^2 \\ & \cdot\left(\alpha\theta(Hkq_{EV1})\left(\lambda_h-2\right)\lambda_h + \alpha\theta(Hkq_{EV1})+4\right) + 2\alpha\theta(Hkq_{EV1})c_h \\ & \cdot((\alpha-1)\theta(Hkq_{EV1})c_l\left(\lambda_h-1\right)\left(\lambda_l-1\right) + \lambda_h(c_{ntr}((\alpha-1)\theta(Hkq_{EV1}) \\ & -(\alpha-1)\theta(Hkq_{EV1})\lambda_l-2) + (\alpha-1)\theta(Hkq_{EV1})\left(\lambda_l-1\right)-2\delta_m) \\ & +(\alpha-1)\theta(Hkq_{EV1})\left(c_{ntr}-1\right)\left(\lambda_l-1\right)\lambda_l - 2c_{ntr}+2\delta_m+4) + 2(\alpha-1) \\ & \cdot\theta(Hkq_{EV1})\lambda_l\left(\alpha\theta(Hkq_{EV1})\left(c_{ntr}-1\right)^2\lambda_h + 2\left(c_{ntr}\left(-c_{ntr}+\delta_m+1\right)+\delta_m\right)\right) \\ & +2(\alpha-1)\theta(Hkq_{EV1})c_l(c_{ntr}\left(\alpha\theta(Hkq_{EV1})\left(\lambda_h-1\right)\left(\lambda_h-\lambda_l\right)+2\left(\lambda_l+1\right)\right) \\ & +\alpha\theta(-(Hkq_{EV1}))\left(\lambda_h-1\right)\left(\lambda_h-\lambda_l\right)+2\left(\lambda_l-1\right)\delta_m-4) + 4\alpha\theta(Hkq_{EV1})c_{ntr} \\ & \cdot\lambda_h^2\delta_m - 4\alpha\theta(Hkq_{EV1})c_{ntr}\lambda_h\delta_m + 4\alpha\theta(Hkq_{EV1})c_{ntr}^2\lambda_h - 4\alpha\theta(Hkq_{EV1})c_{ntr}\lambda_h \\ & -(\alpha-1)\theta(Hkq_{EV1})\lambda_l^2\left(\alpha\theta(Hkq_{EV1})\left(c_{ntr}-1\right)^2+4c_{ntr}\delta_m\right) - 4\theta(Hkq_{EV1})c_{ntr} \\ & +8c_{ntr}\delta_m - 4c_{ntr}^2 - \alpha^2\theta^2(Hkq_{EV1})^2\lambda_h^2 + \alpha\theta^2(Hkq_{EV1})^2\lambda_h^2 - 4\alpha\theta(Hkq_{EV1})\lambda_h\delta_m \\ & +4\theta(Hkq_{EV1}) + 4\theta(Hkq_{EV1})\delta_m - 4\delta_m^2 \end{aligned} \right) \tag{B-28}$$

$$N_{31} = \left(\begin{aligned} & \alpha\theta(Hkq_{EV1})(c_h^2\left((\alpha-1)\theta(Hkq_{EV1}) + (\alpha-1)\theta(Hkq_{EV1})\left(\lambda_l-2\right)\lambda_l-4\right) \\ & +(\alpha-1)\theta(Hkq_{EV1})c_l^2\left(\lambda_h-1\right)^2 + 2c_h(-(\alpha-1)\theta(Hkq_{EV1})c_l\left(\lambda_h-1\right) \\ & \cdot\left(\lambda_l-1\right) + \lambda_h((\alpha-1)\theta(Hkq_{EV1})\left(c_{ntr}-1\right)\lambda_l + c_{ntr}((Hkq_{EV1})(\theta-\alpha\theta)+2) \\ & +(\alpha-1)\theta(Hkq_{EV1})+2\delta_m) - (\alpha-1)\theta(Hkq_{EV1})\left(c_{ntr}-1\right)\left(\lambda_l-1\right)\lambda_l \\ & +2\left(c_{ntr}-\delta_m-2\right)) - 2(\alpha-1)\theta(Hkq_{EV1})c_l\left(c_{ntr}-1\right)\left(\lambda_h-1\right)\left(\lambda_h-\lambda_l\right) \\ & -2(\alpha-1)\theta(Hkq_{EV1})\left(c_{ntr}-1\right)^2\lambda_h\lambda_l + \lambda_h(\lambda_h((\alpha-1)\theta(Hkq_{EV1})\left(c_{ntr}-1\right)^2 \\ & -4c_{ntr}\delta_m) + 4\left(c_{ntr}\left(-c_{ntr}+\delta_m+1\right)+\delta_m\right)) + (\alpha-1)\theta(Hkq_{EV1})\left(c_{ntr}-1\right)^2\lambda_l^2 \\ & +4\left(c_{ntr}-\delta_m-1\right)) \end{aligned} \right) \tag{B-29}$$

(4) $\boldsymbol{\lambda_l}$

$$\Pi_2(\lambda_l) = \frac{N_{40} + N_{41}\lambda_l + N_{42}\lambda_l^2}{4k(D_{40} + D_{41}\lambda_l + D_{42}\lambda_l^2)} \tag{B-30}$$

$$\begin{cases} -4 \leq N_{40} \leq 20 \\ -16 \leq N_{41} \leq -5.69 \times 10^{-8} \\ 6.11 \times 10^{-8} \leq N_{42} \leq 4 \\ -5 \leq D_{40} \leq -4 \\ 8.69 \times 10^{-8} \leq D_{41} \leq 2 \\ -1 \leq D_{42} \leq -9.08 \times 10^{-8} \end{cases} \tag{B-31}$$

We find The first derivative $8.68 \times 10^{-8}/(4k) \leq \Pi_2'(\lambda_l) \leq 2.33/(4k)$ and the linearity is $0\% \leq \eta_{2\lambda_l} \leq 24.72\%$.Therefore, the relationship between λ_l and Π_2 can be treated as positive.

$$N_{42} = (\alpha-1)\theta(Hkq_{EV1})\left(\alpha(\beta-1)\theta(Hkq_{EV1})\left(c_h - c_{ntr} + 1\right)^2 - 4c_{ntr}\delta_m\right) \tag{B-32}$$

$$D_{40} = \alpha(\beta-1)\theta(Hkq_{EV1})\left(\lambda_h - 2\right)\lambda_h + \theta(Hkq_{EV1})(\alpha\beta - 1) - 4 \tag{B-33}$$

$$D_{41} = -2(\alpha-1)\theta(Hkq_{EV1}) \tag{B-34}$$

$$D_{42} = (\alpha-1)\theta(Hkq_{EV1}) \tag{B-35}$$

$$N_{41} = \begin{pmatrix} 2(\alpha-1)\theta(Hkq_{EV1})(\alpha(\beta-1)\theta(-(Hkq_{EV1}))c_h^2 + c_l(\alpha(\beta-1)\theta \\ \cdot(Hkq_{EV1})\left(c_{ntr}-1\right)\lambda_h + c_{ntr}(2-\alpha(\beta-1)\theta(Hkq_{EV1})) \\ +\alpha(\beta-1)\theta(Hkq_{EV1}) + 2\delta_m) - \alpha(\beta-1)\theta(Hkq_{EV1})c_h(c_l\left(\lambda_h-1\right) \\ -\left(c_{ntr}-1\right)\left(\lambda_h+1\right)) - \alpha(\beta-1)\theta(Hkq_{EV1})\left(c_{ntr}-1\right)^2\lambda_h \\ +2\left(c_{ntr}\left(-c_{ntr}+\delta_m+1\right)+\delta_m\right)) \end{pmatrix} \tag{B-36}$$

$$N_{40} = \left(\begin{array}{l} \alpha(\beta-1)\theta(Hkq_{EV1})c_h^2((\alpha-1)\theta(Hkq_{EV1})-4)+(\alpha-1)\theta(Hkq_{EV1})c_l^2 \\ \cdot\left(\alpha(\beta-1)\theta(Hkq_{EV1})\left(\lambda_h-2\right)\lambda_h+\alpha(\beta-1)\theta(Hkq_{EV1})-4\right) \\ +2\alpha(\beta-1)\theta(Hkq_{EV1})c_h((\alpha-1)\theta(Hkq_{EV1})c_l\left(\lambda_h-1\right) \\ +c_{ntr}\left(\lambda_h(-\alpha\theta(Hkq_{EV1})+\theta(Hkq_{EV1})+2)+2\right)+(\alpha-1)\theta(Hkq_{EV1})\lambda_h \\ +2\left(\lambda_h-1\right)\delta_m-4)+2(\alpha-1)\theta(Hkq_{EV1})c_l(c_{ntr}(2-\alpha(\beta-1)\theta \\ \cdot(Hkq_{EV1})\left(\lambda_h-1\right)\lambda_h)+\alpha(\beta-1)\theta(Hkq_{EV1})\left(\lambda_h-1\right)\lambda_h-2\delta_m-4) \\ +\alpha(\beta-1)\theta(Hkq_{EV1})\lambda_h^2\left((\alpha-1)\theta(Hkq_{EV1})\left(c_{ntr}-1\right)^2-4c_{ntr}\delta_m\right) \\ -4\alpha(\beta-1)\theta(Hkq_{EV1})\lambda_h\left(\left(c_{ntr}-1\right)c_{ntr}-\left(c_{ntr}+1\right)\delta_m\right) \\ +4\left(c_{ntr}-\delta_m\right)\left(-c_{ntr}+\theta(Hkq_{EV1})(\alpha\beta-1)+\delta_m\right)-4\theta(Hkq_{EV1})(\alpha\beta-1) \end{array} \right) \tag{B-37}$$

(5) $\boldsymbol{\lambda_h}$

$$\Pi_2(\lambda_h) = \frac{N_{50}+N_{51}\lambda_h+N_{52}\lambda_h^2}{4k(D_{50}+D_{51}\lambda_h+D_{52}\lambda_h^2)} \tag{B-38}$$

$$\begin{cases} -4 \leq N_{50} \leq 20 \\ -16 \leq N_{51} \leq -7.66\times10^{-8} \\ 6.92\times10^{-8} \leq N_{52} \leq 4 \\ -5 \leq D_{50} \leq -4 \\ 9.10\times10^{-8} \leq D_{51} \leq 2 \\ -1 \leq D_{52} \leq -8.87\times10^{-8} \end{cases} \tag{B-39}$$

We find The first derivative $9.04\times10^{-8}/(4k) \leq \Pi_2'(\lambda_h) \leq 2.33/(4k)$ and the linearity is $-24.25\% \leq \eta_{2\lambda_h} \leq 24.25\%$. Therefore, the relationship between λ_h and Π_2 can be treated as positive.

$$D_{50} = \theta(Hkq_{EV1})(\alpha\beta-1)+(\alpha-1)\theta(Hkq_{EV1})\lambda_l^2-2(\alpha-1)\theta(Hkq_{EV1})\lambda_l-4 \tag{B-40}$$

$$D_{51} = -2\alpha(\beta-1)\theta(Hkq_{EV1}) \tag{B-41}$$

$$D_{52} = \alpha(\beta-1)\theta(Hkq_{EV1}) \tag{B-42}$$

$$N_{50} = \begin{pmatrix} \alpha(\beta-1)\theta(Hkq_{EV1})c_h^2((\alpha-1)\theta(Hkq_{EV1})+(\alpha-1)\theta(Hkq_{EV1}) \\ \cdot(\lambda_l-2)\lambda_l-4)+2\alpha(\beta-1)\theta(Hkq_{EV1})c_h((\alpha-1)\theta(Hkq_{EV1}) \\ \cdot(\lambda_l-1)(c_l-(c_{ntr}-1)\lambda_l)+2(c_{ntr}-\delta_m-2))+(\alpha-1)\theta(Hkq_{EV1})c_l^2 \\ \cdot(\alpha(\beta-1)\theta(Hkq_{EV1})-4)+(\alpha-1)\theta(Hkq_{EV1})\lambda_l^2(\alpha(\beta-1)\theta(Hkq_{EV1}) \\ \cdot(c_{ntr}-1)^2-4c_{ntr}\delta_m)+2(\alpha-1)\theta(Hkq_{EV1})c_l(c_{ntr}(\lambda_l(2-\alpha(\beta-1) \\ \cdot\theta(Hkq_{EV1}))+2)+\alpha(\beta-1)\theta(Hkq_{EV1})\lambda_l+2(\lambda_l-1)\delta_m-4) \\ -4(\alpha-1)\theta(Hkq_{EV1})\lambda_l((c_{ntr}-1)c_{ntr}-(c_{ntr}+1)\delta_m)+4(c_{ntr}-\delta_m) \\ \cdot(-c_{ntr}+\theta(Hkq_{EV1})(\alpha\beta-1)+\delta_m)-4\theta(Hkq_{EV1})(\alpha\beta-1) \end{pmatrix} \tag{B-43}$$

$$N_{51} = \begin{pmatrix} 2\alpha(\beta-1)\theta(Hkq_{EV1})(c_h((\alpha-1)\theta(Hkq_{EV1})(\lambda_l(-c_l+c_{ntr}-1) \\ +c_l-c_{ntr})+2c_{ntr}+(\alpha-1)\theta(Hkq_{EV1})+2\delta_m)+(\alpha-1)\theta \\ \cdot(-(Hkq_{EV1}))c_l^2+(\alpha-1)\theta(Hkq_{EV1})c_l(c_{ntr}-1)(\lambda_l+1)-(\alpha-1) \\ \cdot\theta(Hkq_{EV1})(c_{ntr}-1)^2\lambda_l+2(c_{ntr}(-c_{ntr}+\delta_m+1)+\delta_m)) \end{pmatrix} \tag{B-44}$$

$$N_{52} = \alpha(\beta-1)\theta(Hkq_{EV1})\left((\alpha-1)\theta(Hkq_{EV1})(c_l-c_{ntr}+1)^2-4c_{ntr}\delta_m\right) \tag{B-45}$$

c_{ntr}

$$\Pi_2(c_{ntr}) = \frac{N_{60}+N_{61}c_{ntr}+N_{62}c_{ntr}^2}{4kD_{60}} \tag{B-46}$$

The equation above shows the relationship between c_{ntr} and Π_2. Expressions from N_{60} to D_{60} are presented from Eq. B-48 to Eq. B-51 in appendix as well. By using gird searching method again, the ranges for each character are

$$\begin{cases} -4 \le N_{60} \le 20 \\ -16 \le N_{61} \le 8 \\ -4 \le N_{62} \le -2.18\times10^{-7} \\ -5 \le D_{60} \le -4 \end{cases} \tag{B-47}$$

We then find that the first derivative $-2/(4k) \le \Pi_2'(c_{ntr}) \le 4/(4k)$ and the linearity is $-6.25\times10^7 \le \eta_{2c_{ntr}} \le 1.15\times10^8$. Back to Eq. B-46 again, the relationship can be quadratic.

$$N_{60} = \left(\begin{array}{l} \alpha(\beta-1)\theta(Hkq_{EV1})c_h^2((\alpha-1)\theta(Hkq_{EV1})+(\alpha-1)\theta(Hkq_{EV1}) \\ \cdot(\lambda_l-2)\lambda_l-4)+(\alpha-1)\theta(Hkq_{EV1})c_l^2(\alpha(\beta-1)\theta(Hkq_{EV1})(\lambda_h-2)\lambda_h \\ +\alpha(\beta-1)\theta(Hkq_{EV1})-4)+2\alpha(\beta-1)\theta(Hkq_{EV1})c_h(-(\alpha-1)\theta \\ \cdot(Hkq_{EV1})c_l(\lambda_h-1)(\lambda_l-1)-(\alpha-1)\theta(Hkq_{EV1})(\lambda_l-1)(\lambda_h-\lambda_l) \\ +2(\lambda_h-1)\delta_m-4)+2(\alpha-1)\theta(Hkq_{EV1})c_l(\alpha(\beta-1)\theta(Hkq_{EV1})(\lambda_h-1) \\ \cdot(\lambda_h-\lambda_l)+2(\lambda_l-1)\delta_m-4)+\alpha^2\beta\theta^2(Hkq_{EV1})^2\lambda_h^2-\alpha^2\theta^2(Hkq_{EV1})^2\lambda_h^2 \\ -\alpha\beta\theta^2(Hkq_{EV1})^2\lambda_h^2+\alpha\theta^2(Hkq_{EV1})^2\lambda_h^2+2(\alpha-1)\theta(Hkq_{EV1})\lambda_l \\ \cdot(2\delta_m-\alpha(\beta-1)\theta(Hkq_{EV1})\lambda_h)+4\alpha\beta\theta(Hkq_{EV1})\lambda_h\delta_m-4\alpha\theta(Hkq_{EV1}) \\ \cdot\lambda_h\delta_m+(\alpha-1)\alpha(\beta-1)\theta^2(Hkq_{EV1})^2\lambda_l^2-4\alpha\beta\theta(Hkq_{EV1})+4\theta(Hkq_{EV1}) \\ -4\alpha\beta\theta(Hkq_{EV1})\delta_m+4\theta(Hkq_{EV1})\delta_m-4\delta_m^2 \end{array} \right) \tag{B-48}$$

$$N_{61} = \left(\begin{array}{l} 2(\theta(Hkq_{EV1})((\alpha-1)c_l(2(\lambda_l+1)-\alpha(\beta-1)\theta(Hkq_{EV1})(\lambda_h-1) \\ \cdot(\lambda_h-\lambda_l))+\alpha(\beta-1)c_h(\lambda_h(-\alpha\theta(Hkq_{EV1})+\theta(Hkq_{EV1})+(\alpha-1)\theta \\ \cdot(Hkq_{EV1})\lambda_l+2)+(\alpha-1)\theta(-(Hkq_{EV1}))(\lambda_l-1)\lambda_l+2)+2(\alpha-1) \\ \cdot\lambda_l(\alpha(\beta-1)\theta(Hkq_{EV1})\lambda_h+\delta_m+1)-\alpha(\beta-1)\lambda_h((\alpha-1)\theta \\ \cdot(Hkq_{EV1})\lambda_h+2(\lambda_h-1)\delta_m-2)-(\alpha-1)\lambda_l^2(\alpha(\beta-1)\theta(Hkq_{EV1}) \\ +2\delta_m))+2\theta(Hkq_{EV1})(\alpha\beta-1)+4\delta_m) \end{array} \right) \tag{B-49}$$

$$N_{62} = \left(\begin{array}{l} \theta(Hkq_{EV1})((\alpha-1)\alpha(\beta-1)\theta(Hkq_{EV1})\lambda_h^2-2\alpha(\beta-1)\lambda_h \\ \cdot((\alpha-1)\theta(Hkq_{EV1})\lambda_l+2)+(\alpha-1)\lambda_l(\alpha(\beta-1)\theta(Hkq_{EV1})\lambda_l-4))-4 \end{array} \right) \tag{B-50}$$

$$D_{60} = \theta(Hkq_{EV1})\left(\alpha(\beta-1)(\lambda_h-2)\lambda_h+(\alpha-1)\lambda_l^2-2(\alpha-1)\lambda_l\right)+\theta(Hkq_{EV1})(\alpha\beta-1)-4 \tag{B-51}$$

(7) c_l

$$\Pi_2(c_l) = \frac{N_{70}+N_{71}c_l+N_{72}c_l^2}{4kD_{70}} \tag{B-52}$$

$$\begin{cases} -4 \leq N_{70} \leq 20 \\ 9.18 \times 10^{-8} \leq N_{71} \leq 12 \\ 8.24 \times 10^{-8} \leq N_{72} \leq 4 \\ -5 \leq D_{70} \leq -4 \end{cases} \tag{B-53}$$

We find The first derivative $-4.24/(4k) \leq \Pi_2'(c_l) \leq -9.06 \times 10^{-8}/(4k)$ and the linearity is $-25\% \leq \eta_{2c_l} \leq -6.29\%$.Therefore, the relationship between c_l and Π_2 has negative relationship.

$$N_{71} = \begin{pmatrix} 2(\alpha-1)\theta(Hkq_{EV1})(-\alpha(\beta-1)\theta(Hkq_{EV1})c_h(\lambda_h-1)(\lambda_l-1) \\ -c_{ntr}(\alpha(\beta-1)\theta(Hkq_{EV1})(\lambda_h-1)(\lambda_h-\lambda_l)-2(\lambda_l+1)) \\ +\alpha(\beta-1)\theta(Hkq_{EV1})(\lambda_h-1)(\lambda_h-\lambda_l)+2(\lambda_l-1)\delta_m-4) \end{pmatrix} \tag{B-54}$$

$$N_{72} = (\alpha-1)\theta(Hkq_{EV1})(\alpha(\beta-1)\theta(Hkq_{EV1})(\lambda_h-2)\lambda_h+\alpha(\beta-1)\theta(Hkq_{EV1})-4) \tag{B-55}$$

$$D_{70} = \theta(Hkq_{EV1})\left(\alpha(\beta-1)(\lambda_h-2)\lambda_h+(\alpha-1)\lambda_l^2-2(\alpha-1)\lambda_l\right)+\theta(Hkq_{EV1})(\alpha\beta-1)-4 \tag{B-56}$$

$$N_{70}=\left(\begin{array}{l}\alpha(\beta-1)\theta(Hkq_{EV1})c_h^2((\alpha-1)\theta(Hkq_{EV1})+(\alpha-1)\theta(Hkq_{EV1})\left(\lambda_l-2\right)\\ \cdot\lambda_l-4)+2\alpha(\beta-1)\theta(Hkq_{EV1})c_h(c_{ntr}(\lambda_h(-\alpha\theta(Hkq_{EV1})+\theta(Hkq_{EV1})\\ +(\alpha-1)\theta(Hkq_{EV1})\lambda_l+2)+(\alpha-1)\theta(-(Hkq_{EV1}))\left(\lambda_l-1\right)\lambda_l+2)\\ -(\alpha-1)\theta(Hkq_{EV1})\left(\lambda_l-1\right)\left(\lambda_h-\lambda_l\right)+2\left(\lambda_h-1\right)\delta_m-4)\\ +2c_{ntr}(2\theta(Hkq_{EV1})\left(\alpha\beta+\alpha(\beta-1)\lambda_h-1\right)+(\alpha-1)\theta(Hkq_{EV1})\\ \cdot\left(2\lambda_l-\alpha(\beta-1)\theta(Hkq_{EV1})\left(\lambda_h-\lambda_l\right)^2\right)+2\delta_m(\theta(Hkq_{EV1})\\ \cdot\left(-\alpha(\beta-1)\left(\lambda_h-1\right)\lambda_h-(\alpha-1)\lambda_l^2+(\alpha-1)\lambda_l\right)+2))+c_{ntr}^2\\ \cdot(\theta(Hkq_{EV1})((\alpha-1)\alpha(\beta-1)\theta(Hkq_{EV1})\lambda_h^2-2\alpha(\beta-1)\lambda_h\\ \cdot\left((\alpha-1)\theta(Hkq_{EV1})\lambda_l+2\right)+(\alpha-1)\lambda_l\left(\alpha(\beta-1)\theta(Hkq_{EV1})\lambda_l-4\right))-4)\\ +\alpha^2\beta\theta^2(Hkq_{EV1})^2\lambda_h^2-\alpha^2\theta^2(Hkq_{EV1})^2\lambda_h^2-\alpha\beta\theta^2(Hkq_{EV1})^2\lambda_h^2\\ +\alpha\theta^2(Hkq_{EV1})^2\lambda_h^2+2(\alpha-1)\theta(Hkq_{EV1})\lambda_l(2\delta_m-\alpha(\beta-1)\theta\\ \cdot(Hkq_{EV1})\lambda_h)+4\alpha\beta\theta(Hkq_{EV1})\lambda_h\delta_m-4\alpha\theta(Hkq_{EV1})\lambda_h\delta_m\\ +(\alpha-1)\alpha(\beta-1)\theta^2(Hkq_{EV1})^2\lambda_l^2-4\alpha\beta\theta(Hkq_{EV1})+4\theta(Hkq_{EV1})\\ -4\alpha\beta\theta(Hkq_{EV1})\delta_m+4\theta(Hkq_{EV1})\delta_m-4\delta_m^2\end{array}\right) \tag{B-57}$$

(8) c_h

$$\Pi_2(c_h)=\frac{N_{80}+N_{81}c_h+N_{82}c_h^2}{4kD_{80}} \tag{B-58}$$

$$\begin{cases}-4\leq N_{80}\leq 20\\ 1.56\times10^{-7}\leq N_{81}\leq 12\\ 6.66\times10^{-8}\leq N_{82}\leq 4\\ -5\leq D_{80}\leq -4\end{cases} \tag{B-59}$$

We find The first derivative $-4.24/(4k)\leq\Pi_2'(c_l)\leq-9.10\times10^{-8}/(4k)$ and the linearity is $-24.91\%\leq\eta_{2c_h}\leq-6.25\%$.Therefore, the relationship between c_h and Π_2 has negative relationship.

$$N_{82}=\alpha(\beta-1)\theta(Hkq_{EV1})\left((\alpha-1)\theta(Hkq_{EV1})+(\alpha-1)\theta(Hkq_{EV1})\left(\lambda_l-2\right)\lambda_l-4\right) \tag{B-60}$$

$$D_{80} = \theta(Hkq_{EV1})\left(\alpha(\beta-1)(\lambda_h-2)\lambda_h + (\alpha-1)\lambda_l^2 - 2(\alpha-1)\lambda_l\right) + \theta(Hkq_{EV1})(\alpha\beta-1) - 4 \tag{B-61}$$

$$N_{81} = \begin{pmatrix} -2\alpha(\beta-1)\theta(Hkq_{EV1})((\alpha-1)\theta(Hkq_{EV1})c_l(\lambda_h-1)(\lambda_l-1) \\ +\lambda_h(c_{ntr}((\alpha-1)\theta(Hkq_{EV1}) - (\alpha-1)\theta(Hkq_{EV1})\lambda_l - 2) \\ +(\alpha-1)\theta(Hkq_{EV1})(\lambda_l-1) - 2\delta_m) + (\alpha-1)\theta(Hkq_{EV1})(c_{ntr}-1) \\ \cdot(\lambda_l-1)\lambda_l - 2c_{ntr} + 2\delta_m + 4) \end{pmatrix} \tag{B-62}$$

$$N_{80} = \begin{pmatrix} (\alpha-1)\theta(Hkq_{EV1})c_l^2(\alpha(\beta-1)\theta(Hkq_{EV1})(\lambda_h-2)\lambda_h + \alpha(\beta-1)\theta \\ \cdot(Hkq_{EV1}) - 4) + 2(\alpha-1)\theta(Hkq_{EV1})c_l(c_{ntr}(2(\lambda_l+1) - \alpha(\beta-1) \\ \cdot\theta(Hkq_{EV1})(\lambda_h-1)(\lambda_h-\lambda_l)) + \alpha(\beta-1)\theta(Hkq_{EV1})(\lambda_h-1)(\lambda_h-\lambda_l) \\ +2(\lambda_l-1)\delta_m - 4) + 2c_{ntr}(2\theta(Hkq_{EV1})(\alpha\beta + \alpha(\beta-1)\lambda_h - 1) \\ +(\alpha-1)\theta(Hkq_{EV1})(2\lambda_l - \alpha(\beta-1)\theta(Hkq_{EV1})(\lambda_h-\lambda_l)^2) \\ +2\delta_m(\theta(Hkq_{EV1})(-\alpha(\beta-1)(\lambda_h-1)\lambda_h - (\alpha-1)\lambda_l^2 + (\alpha-1)\lambda_l) + 2)) \\ +c_{ntr}^2(\theta(Hkq_{EV1})((\alpha-1)\alpha(\beta-1)\theta(Hkq_{EV1})\lambda_h^2 - 2\alpha(\beta-1)\lambda_h \\ \cdot((\alpha-1)\theta(Hkq_{EV1})\lambda_l + 2) + (\alpha-1)\lambda_l(\alpha(\beta-1)\theta(Hkq_{EV1})\lambda_l - 4)) - 4) \\ +\alpha^2\beta\theta^2(Hkq_{EV1})^2\lambda_h^2 - \alpha^2\theta^2(Hkq_{EV1})^2\lambda_h^2 - \alpha\beta\theta^2(Hkq_{EV1})^2\lambda_h^2 \\ +\alpha\theta^2(Hkq_{EV1})^2\lambda_h^2 + 2(\alpha-1)\theta(Hkq_{EV1})\lambda_l(2\delta_m - \alpha(\beta-1)\theta(Hkq_{EV1})\lambda_h) \\ +4\alpha\beta\theta(Hkq_{EV1})\lambda_h\delta_m - 4\alpha\theta(Hkq_{EV1})\lambda_h\delta_m + (\alpha-1)\alpha(\beta-1)\theta^2 \\ \cdot(Hkq_{EV1})^2\lambda_l^2 - 4\alpha\beta\theta(Hkq_{EV1}) + 4\theta(Hkq_{EV1}) - 4\alpha\beta\theta(Hkq_{EV1})\delta_m \\ +4\theta(Hkq_{EV1})\delta_m - 4\delta_m^2 \end{pmatrix} \tag{B-63}$$

B.2.2 Proof for parameters vs total optimal profits in Period 3

(1) $\boldsymbol{\theta}$

$$\Pi_3(\theta) = \frac{N_{90} + N_{91}\theta + N_{92}\theta^2}{4k(-4 + D_{91}\theta)} \tag{B-64}$$

$$\begin{cases} 0 \leq N_{90} \leq 0 \\ 3.80 \times 10^{-8} \leq N_{91} \leq 24 \\ -1.83 \times 10^{-16} \leq N_{92} \leq 7.01 \times 10^{-17} \\ -1 \leq D_{91} \leq -1.35 \times 10^{-7} \end{cases} \tag{B-65}$$

We find The first derivative $-6/(4k) \leq \Pi_3'(\theta) \leq -8.06 \times 10^{-8}$ and the linearity is $-5.57\% \leq \eta_{3\theta} \leq -1.86 \times 10^{-13}$. Therefore, the relationship between θ and Π_3 can be treated as negative linear.

$$N_{90} = -4\left(c_{ntr} - \delta_m\right)^2 \tag{B-66}$$

$$D_{91} = \begin{pmatrix} (Hkq_{EV2})(\alpha(\beta-1)\lambda_h^2 - 2\alpha(\beta-1)\lambda_h + (\alpha-1)\left(\lambda_l - 2\right)\lambda_l \\ -\alpha\beta\lambda_u^2 + 2\alpha\beta\lambda_u) - (Hkq_{EV2}) \end{pmatrix} \tag{B-67}$$

$$N_{91} = \begin{pmatrix} 4(Hkq_{EV2})((\alpha-\alpha\beta)c_h^2 + \alpha(\beta-1)c_h\left(c_{ntr}\left(\lambda_h+1\right) + \left(\lambda_h-1\right)\delta_m - 2\right) \\ -\alpha\beta c_{ntr}\lambda_h^2\delta_m + \alpha\beta c_{ntr}\lambda_h\delta_m + \alpha c_{ntr}\lambda_h^2\delta_m - \alpha c_{ntr}\lambda_h\delta_m - \alpha\beta c_{ntr}^2\lambda_h \\ +\alpha\beta c_{ntr}\lambda_h + \alpha c_{ntr}^2\lambda_h - \alpha c_{ntr}\lambda_h - (\alpha-1)c_l^2 - \alpha c_{ntr}\lambda_l^2\delta_m + \alpha c_{ntr}\lambda_l\delta_m \\ +(\alpha-1)c_l\left(c_{ntr}\left(\lambda_l+1\right) + \left(\lambda_l-1\right)\delta_m - 2\right) + c_{ntr}\lambda_l^2\delta_m - c_{ntr}\lambda_l\delta_m \\ -\alpha c_{ntr}^2\lambda_l + \alpha c_{ntr}\lambda_l + c_{ntr}^2\lambda_l - c_{ntr}\lambda_l + \alpha\beta c_{ntr}\delta_m\lambda_u^2 \\ +\alpha\beta\lambda_u\left(-c_{ntr}\left(c_u+\delta_m+1\right) - \left(c_u+1\right)\delta_m + c_{ntr}^2\right) + \alpha\beta c_u\delta_m - \alpha\beta c_{ntr}c_u \\ -c_{ntr} + \alpha\beta c_u^2 + 2\alpha\beta c_u + \alpha\beta\lambda_h\delta_m - \alpha\lambda_h\delta_m + \alpha\lambda_l\delta_m - \lambda_l\delta_m + \delta_m + 1) \end{pmatrix} \tag{B-68}$$

$$N_{92}=\left(\begin{array}{l}(Hkq_{EV2})^2\alpha(-\alpha c_u^2\beta^2-\alpha c_{ntr}^2\lambda_h^2\beta^2-\alpha c_u^2\lambda_h^2\beta^2-\alpha\lambda_h^2\beta^2+2\alpha c_{ntr}\lambda_h^2\beta^2\\ -2\alpha c_u\lambda_h^2\beta^2+2\alpha c_{ntr}c_u\lambda_h^2\beta^2+2\alpha c_u^2\lambda_h\beta^2+2\alpha c_u\lambda_h\beta^2-2\alpha c_{ntr}c_u\lambda_h\beta^2\\ +c_u^2\beta+2\alpha c_{ntr}^2\lambda_h^2\beta-c_{ntr}^2\lambda_h^2\beta+\alpha c_u^2\lambda_h^2\beta+2\alpha\lambda_h^2\beta-4\alpha c_{ntr}\lambda_h^2\beta\\ +2c_{ntr}\lambda_h^2\beta+2\alpha c_u\lambda_h^2\beta-2\alpha c_{ntr}c_u\lambda_h^2\beta-\lambda_h^2\beta-\alpha c_u^2\lambda_l^2\beta+c_u^2\lambda_l^2\beta\\ -2\alpha c_u\lambda_l^2\beta+2\alpha c_{ntr}c_u\lambda_l^2\beta-2c_{ntr}c_u\lambda_l^2\beta+2c_u\lambda_l^2\beta-(\alpha\beta-1)\\ \cdot(c_{ntr}-1)^2\lambda_u^2\beta-2\alpha c_u^2\lambda_h\beta-2\alpha c_u\lambda_h\beta+2\alpha c_{ntr}c_u\lambda_h\beta+2\alpha c_u^2\lambda_l\beta\\ -2c_u^2\lambda_l\beta+2\alpha c_u\lambda_l\beta-2\alpha c_{ntr}c_u\lambda_l\beta+2c_{ntr}c_u\lambda_l\beta-2c_u\lambda_l\beta-2\alpha c_{ntr}^2\lambda_h\lambda_l\beta\\ +2c_{ntr}^2\lambda_h\lambda_l\beta-2\alpha\lambda_h\lambda_l\beta+4\alpha c_{ntr}\lambda_h\lambda_l\beta-4c_{ntr}\lambda_h\lambda_l\beta+2\lambda_h\lambda_l\beta\\ +2(c_{ntr}-1)((c_{ntr}-1)(\alpha(\beta-1)\lambda_h+(\alpha-1)\lambda_l)+c_u(\alpha\beta+\lambda_l\\ -\alpha((\beta-1)\lambda_h+\lambda_l)-1))\lambda_u\beta-\alpha c_{ntr}^2\lambda_h^2+c_{ntr}^2\lambda_h^2-\alpha\lambda_h^2+2\alpha c_{ntr}\lambda_h^2\\ -2c_{ntr}\lambda_h^2+\lambda_h^2-\alpha c_{ntr}^2\lambda_l^2+c_{ntr}^2\lambda_l^2-\alpha\lambda_l^2+2\alpha c_{ntr}\lambda_l^2-2c_{ntr}\lambda_l^2+\lambda_l^2\\ +2\alpha c_{ntr}^2\lambda_h\lambda_l-2c_{ntr}^2\lambda_h\lambda_l+2\alpha\lambda_h\lambda_l-4\alpha c_{ntr}\lambda_h\lambda_l+4c_{ntr}\lambda_h\lambda_l-2\lambda_h\lambda_l\\ -(\beta-1)c_h^2(\alpha\beta\lambda_u^2-2\alpha\beta\lambda_u+\alpha(\beta-1)-(\alpha-1)(\lambda_l-2)\lambda_l+1)\\ +(\alpha-1)c_l^2(-\beta\lambda_u^2+2\beta\lambda_u+(\beta-1)(\lambda_h-2)\lambda_h-1)+2(\alpha-1)c_l\\ \cdot(\beta c_u(\lambda_l-1)(\lambda_u-1)-(c_{ntr}-1)(-\beta\lambda_u^2+\beta(\lambda_l+1)\lambda_u+(\beta-1)\lambda_h\\ \cdot(\lambda_h-\lambda_l-1)-\lambda_l))+2(\beta-1)c_h(-(\alpha-1)c_l(\lambda_h-1)(\lambda_l-1)+\alpha\beta c_u\\ \cdot(\lambda_h-1)(\lambda_u-1)-(c_{ntr}-1)(-\alpha\beta\lambda_u^2+\alpha\beta\lambda_u+(\alpha-1)(\lambda_l-1)\lambda_l\\ +\lambda_h(-\beta\alpha-\lambda_l\alpha+\beta\lambda_u\alpha+\alpha+\lambda_l-1))))\end{array}\right) \tag{B-69}$$

(2) $\boldsymbol{\alpha}$

$$\Pi_3(\alpha)=\frac{N_{100}+N_{101}\alpha+N_{102}\alpha^2}{4k(D_{100}+D_{101}\alpha)} \tag{B-70}$$

$$\begin{cases}-4\le N_{100}\le 20\\ -19.2\le N_{101}\le 21\\ -3.77\times10^{-16}\le N_{102}\le 1.42\times10^{-16}\\ -5\le D_{100}\le -4\\ -1\le D_{101}\le 1\end{cases} \tag{B-71}$$

We find The first derivative $-5.25/(4k) \leq \Pi_3'(\alpha) \leq 4.76/(4k)$ and the linearity is $-5.41\% \leq \eta_{3\alpha} \leq 5.46\%$. Therefore, the relationship between α and Π_3 can be treated as linear.

$$D_{100} = \theta(-(Hkq_{EV2})) - \theta(Hkq_{EV2})\left(\lambda_l - 2\right)\lambda_l - 4 \tag{B-72}$$

$$D_{101} = \theta(Hkq_{EV2})\left(\left(\beta - 1\right)\left(\lambda_h - 2\right)\lambda_h + \left(\lambda_l - 2\right)\lambda_l - \beta\lambda_u^2 + 2\beta\lambda_u\right) \tag{B-73}$$

$$N_{100} = \left(\begin{array}{l} 4(\theta(Hkq_{EV2})c_l^2 - \theta(Hkq_{EV2})c_l\left(c_{ntr}\left(\lambda_l + 1\right) + \left(\lambda_l - 1\right)\delta_m - 2\right) \\ +c_{ntr}\left(\delta_m\left(\theta(Hkq_{EV2})\left(\lambda_l - 1\right)\lambda_l + 2\right) - \theta(Hkq_{EV2})\left(\lambda_l + 1\right)\right) \\ +c_{ntr}^2\left(\theta(Hkq_{EV2})\lambda_l - 1\right) + \theta(Hkq_{EV2}) - \delta_m(\theta(Hkq_{EV2})\left(\lambda_l - 1\right) \\ +\delta_m)) \end{array}\right) \tag{B-74}$$

$$N_{101} = \left(\begin{array}{l} (Hkq_{EV2})\theta(-(\beta-1)\left((Hkq_{EV2})\theta+(Hkq_{EV2})\left(\lambda_l-2\right)\lambda_l\theta+4\right)c_h^2 \\ +2(\beta-1)(2\left(c_{ntr}-\delta_m-2\right)+(Hkq_{EV2})\theta c_l\left(\lambda_h-1\right)\left(\lambda_l-1\right) \\ +(Hkq_{EV2})\theta\left(c_{ntr}-1\right)\left(\lambda_l-1\right)\lambda_l+\lambda_h(2\delta_m+(Hkq_{EV2})\theta\left(\lambda_l-1\right) \\ +c_{ntr}\left((Hkq_{EV2})\theta-(Hkq_{EV2})\lambda_l\theta+2\right)))c_h+4\beta c_u^2+(Hkq_{EV2})\beta\theta c_u^2 \\ +(Hkq_{EV2})\theta c_{ntr}^2\lambda_h^2-(Hkq_{EV2})\beta\theta c_{ntr}^2\lambda_h^2+(Hkq_{EV2})\theta\lambda_h^2 \\ -(Hkq_{EV2})\beta\theta\lambda_h^2-2(Hkq_{EV2})\theta c_{ntr}\lambda_h^2+2(Hkq_{EV2})\beta\theta c_{ntr}\lambda_h^2 \\ -4\beta c_{ntr}\delta_m\lambda_h^2+4c_{ntr}\delta_m\lambda_h^2+(Hkq_{EV2})\theta c_{ntr}^2\lambda_l^2+(Hkq_{EV2})\beta\theta c_u^2\lambda_l^2 \\ +(Hkq_{EV2})\theta\lambda_l^2-2(Hkq_{EV2})\theta c_{ntr}\lambda_l^2+2(Hkq_{EV2})\beta\theta c_u\lambda_l^2 \\ -2(Hkq_{EV2})\beta\theta c_{ntr}c_u\lambda_l^2-4c_{ntr}\delta_m\lambda_l^2+\beta((Hkq_{EV2})\theta\left(c_{ntr}-1\right)^2 \\ +4c_{ntr}\delta_m)\lambda_u^2+8\beta c_u-4\beta c_{ntr}c_u+4\beta c_u\delta_m-4\beta c_{ntr}^2\lambda_h+4c_{ntr}^2\lambda_h \\ +4\beta c_{ntr}\lambda_h-4c_{ntr}\lambda_h+4\beta\delta_m\lambda_h+4\beta c_{ntr}\delta_m\lambda_h-4c_{ntr}\delta_m\lambda_h \\ -4\delta_m\lambda_h-4c_{ntr}^2\lambda_l-2(Hkq_{EV2})\beta\theta c_u^2\lambda_l+4c_{ntr}\lambda_l-2(Hkq_{EV2}) \\ \cdot\beta\theta c_u\lambda_l+2(Hkq_{EV2})\beta\theta c_{ntr}c_u\lambda_l+4c_{ntr}\delta_m\lambda_l+4\delta_m\lambda_l-2(Hkq_{EV2}) \\ \cdot\theta c_{ntr}^2\lambda_h\lambda_l+2(Hkq_{EV2})\beta\theta c_{ntr}^2\lambda_h\lambda_l-2(Hkq_{EV2})\theta\lambda_h\lambda_l+2(Hkq_{EV2}) \\ \cdot\beta\theta\lambda_h\lambda_l+4(Hkq_{EV2})\theta c_{ntr}\lambda_h\lambda_l-4(Hkq_{EV2})\beta\theta c_{ntr}\lambda_h\lambda_l \\ -2\beta(((Hkq_{EV2})\theta\lambda_l-2)\,c_{ntr}^2+(((Hkq_{EV2})\theta+2)c_u+2\left(\delta_m+1\right) \\ -(Hkq_{EV2})\theta\left(c_u+2\right)\lambda_l)c_{ntr}+2\delta_m+c_u\left(2\delta_m+(Hkq_{EV2})\theta\left(\lambda_l-1\right)\right) \\ +(Hkq_{EV2})\theta\lambda_l)\lambda_u+c_l^2((Hkq_{EV2})\theta+(Hkq_{EV2})(\beta\lambda_u^2-2\beta\lambda_u-(\beta-1) \\ \cdot\left(\lambda_h-2\right)\lambda_h)\theta-4)+2c_l((Hkq_{EV2})\beta\theta\lambda_u^2-(Hkq_{EV2})\beta\theta\left(\lambda_l+1\right)\lambda_u \\ -2\delta_m+2\delta_m\lambda_l-(Hkq_{EV2})\theta\lambda_h\lambda_l+(Hkq_{EV2})\beta\theta\lambda_h\lambda_l+(Hkq_{EV2})\theta \\ \cdot\left(\lambda_l-(\beta-1)\left(\lambda_h-1\right)\lambda_h\right)-(Hkq_{EV2})\beta\theta c_u\left(\lambda_l-1\right)\left(\lambda_u-1\right) \\ +c_{ntr}(2\left(\lambda_l+1\right)+(Hkq_{EV2})\theta(-\beta\lambda_u^2+\beta\left(\lambda_l+1\right)\lambda_u+(\beta-1)\lambda_h \\ \cdot\left(\lambda_h-\lambda_l-1\right)-\lambda_l))-4)) \end{array} \right) \tag{B-75}$$

$$N_{102} = \left(\begin{array}{l} \theta^2(Hkq_{EV2})^2(-2\beta c_{ntr}^2\lambda_h\lambda_l + 4\beta c_{ntr}\lambda_h\lambda_l + 2c_{ntr}^2\lambda_h\lambda_l - 4c_{ntr}\lambda_h\lambda_l \\ +2\beta\left(c_{ntr}-1\right)\lambda_u\left(\left(c_{ntr}-1\right)\left((\beta-1)\lambda_h+\lambda_l\right)+c_u\left(\beta-(\beta-1)\lambda_h-\lambda_l\right)\right) \\ +2c_l(\beta c_u\left(\lambda_l-1\right)\left(\lambda_u-1\right)-\left(c_{ntr}-1\right)\left((\beta-1)\lambda_h\left(\lambda_h-\lambda_l-1\right)-\lambda_l\right. \\ \left.+\beta\left(\lambda_l+1\right)\lambda_u-\beta\lambda_u^2\right))+2(\beta-1)c_h(-c_l\left(\lambda_h-1\right)\left(\lambda_l-1\right)-\left(c_{ntr}-1\right) \\ \cdot\left(\lambda_h\left(-\beta-\lambda_l+\beta\lambda_u+1\right)+\left(\lambda_l-1\right)\lambda_l-\beta\left(\lambda_u-1\right)\lambda_u\right)+\beta c_u\left(\lambda_h-1\right) \\ \cdot\left(\lambda_u-1\right))-(\beta-1)c_h^2\left(\beta-\left(\lambda_l-2\right)\lambda_l+\beta\left(\lambda_u-2\right)\lambda_u-1\right) \\ +c_l^2\left(\left(\beta-1\right)\left(\lambda_h-2\right)\lambda_h-\beta\lambda_u^2+2\beta\lambda_u-1\right)+\beta^2\left(-c_{ntr}^2\right)\lambda_h^2 \\ +2\beta^2c_{ntr}\lambda_h^2+2\beta c_{ntr}^2\lambda_h^2-4\beta c_{ntr}\lambda_h^2-c_{ntr}^2\lambda_h^2+2c_{ntr}\lambda_h^2+2\beta^2c_{ntr}c_u\lambda_h^2 \\ -2\beta^2c_{ntr}c_u\lambda_h-2\beta c_{ntr}c_u\lambda_h^2+2\beta c_{ntr}c_u\lambda_h-\beta^2c_u^2\lambda_h^2-2\beta^2c_u\lambda_h^2 \\ +2\beta^2c_u^2\lambda_h+2\beta^2c_u\lambda_h+\beta c_u^2\lambda_h^2+2\beta c_u\lambda_h^2-2\beta c_u^2\lambda_h-2\beta c_u\lambda_h-c_{ntr}^2\lambda_l^2 \\ +2c_{ntr}\lambda_l^2+2\beta c_{ntr}c_u\lambda_l^2-2\beta c_{ntr}c_u\lambda_l-\beta c_u^2\lambda_l^2-2\beta c_u\lambda_l^2+2\beta c_u^2\lambda_l \\ +2\beta c_u\lambda_l-\beta^2\left(c_{ntr}-1\right)^2\lambda_u^2-\beta^2c_u^2-\beta^2\lambda_h^2+2\beta\lambda_h^2-\lambda_h^2-2\beta\lambda_h\lambda_l \\ +2\lambda_h\lambda_l-\lambda_l^2) \end{array} \right) \tag{B-76}$$

(3) $\boldsymbol{\beta}$

$$\Pi_3(\beta) = \frac{N_{110} + N_{111}\beta + N_{112}\beta^2}{4k(D_{110} + D_{111}\beta)} \tag{B-77}$$

$$\begin{cases} -4 \leq N_{110} \leq 20 \\ -19.2 \leq N_{111} \leq 21 \\ 0 \leq N_{112} \leq -1.10 \times 10^{-31} \\ -5 \leq D_{110} \leq -4 \\ -1 \leq D_{111} \leq 1 \end{cases} \tag{B-78}$$

We find The first derivative $-5.25/(4k) \leq \Pi_3'(\beta) \leq 4.76/(4k)$ and the linearity is $-5.57\% \leq \eta_{3\beta} \leq 5.31\%$. Therefore, the relationship between β and Π_3 can be treated as linear.

$$N_{112} = \alpha^2\theta^2\left(-(Hkq_{EV2})^2\right)\left(\left(c_{ntr}-1\right)\left(\lambda_h-\lambda_u\right)-c_u\left(\lambda_h-1\right)+c_h\left(\lambda_u-1\right)\right)^2 \tag{B-79}$$

$$D_{110} = \left(\begin{array}{l} \alpha\theta(-(Hkq_{EV2}))\lambda_h^2 + 2\alpha\theta(Hkq_{EV2})\lambda_h - \theta(Hkq_{EV2}) \\ +(\alpha-1)\theta(Hkq_{EV2})\left(\lambda_l-2\right)\lambda_l - 4 \end{array} \right) \tag{B-80}$$

$$D_{111} = \alpha\theta(Hkq_{EV2})\lambda_h^2 - 2\alpha\theta(Hkq_{EV2})\lambda_h - \alpha\theta(Hkq_{EV2})\lambda_u^2 + 2\alpha\theta(Hkq_{EV2})\lambda_u \quad \text{(B-81)}$$

$$N_{111} = \left(\begin{array}{l} \alpha\theta(Hkq_{EV2})(-2c_h((\alpha-1)\theta(Hkq_{EV2})c_l\left(\lambda_h-1\right)\left(\lambda_l-1\right) \\ -c_{ntr}(\lambda_h(-2\alpha\theta(Hkq_{EV2})+\theta(Hkq_{EV2})+\theta(Hkq_{EV2})((\alpha-1)\lambda_l \\ +\alpha\lambda_u)+2)+\theta(Hkq_{EV2})\left(-(\alpha-1)\left(\lambda_l-1\right)\lambda_l-\alpha\lambda_u^2+\alpha\lambda_u\right)+2) \\ -\alpha\theta(Hkq_{EV2})c_u\lambda_h+\alpha\theta(Hkq_{EV2})\lambda_u\left(c_u\left(\lambda_h-1\right)+\lambda_h+1\right) \\ +\alpha\theta(Hkq_{EV2})c_u+(1-2\alpha)\theta(Hkq_{EV2})\lambda_h+\alpha\theta(Hkq_{EV2})\lambda_h\lambda_l \\ -\theta(Hkq_{EV2})\lambda_h\lambda_l-2\lambda_h\delta_m-\alpha\theta(Hkq_{EV2})\lambda_l^2+\alpha\theta(Hkq_{EV2})\lambda_l \\ +\theta(Hkq_{EV2})\lambda_l^2-\theta(Hkq_{EV2})\lambda_l-\alpha\theta(Hkq_{EV2})\lambda_u^2+2\delta_m+4) \\ +2c_u(\alpha\theta(-(Hkq_{EV2}))\left(c_{ntr}-1\right)\lambda_h^2+\alpha\theta(Hkq_{EV2})\left(c_{ntr}-1\right)\lambda_h \\ \cdot\left(\lambda_u+1\right)-\lambda_u((\alpha-1)\theta(Hkq_{EV2})\left(c_{ntr}-1\right)\lambda_l+c_{ntr}(\theta(Hkq_{EV2})+2) \\ +\theta(-(Hkq_{EV2}))+2\delta_m)+(\alpha-1)\theta(Hkq_{EV2})\left(c_{ntr}-1\right)\left(\lambda_l-1\right)\lambda_l \\ +(\alpha-1)\theta(Hkq_{EV2})c_l\left(\lambda_l-1\right)\left(\lambda_u-1\right)-2c_{ntr}+2\delta_m+4)+\left(\lambda_h-\lambda_u\right) \\ \cdot(2c_{ntr}(\theta(Hkq_{EV2})\left((1-2\alpha)\lambda_h+2(\alpha-1)\lambda_l+\lambda_u\right)-2\delta_m \\ \cdot\left(\lambda_h+\lambda_u-1\right)+2)-2(\alpha-1)\theta(Hkq_{EV2})c_l\left(c_{ntr}-1\right)\left(\lambda_h-\lambda_l+\lambda_u-1\right) \\ +c_{ntr}^2\left((2\alpha-1)\theta(Hkq_{EV2})\lambda_h-\theta(Hkq_{EV2})\left(2(\alpha-1)\lambda_l+\lambda_u\right)-4\right) \\ +(\alpha-1)\theta(Hkq_{EV2})c_l^2\left(\lambda_h+\lambda_u-2\right)+\theta(Hkq_{EV2})((2\alpha-1)\lambda_h \\ -2(\alpha-1)\lambda_l-\lambda_u)+4\delta_m)+c_h^2((2\alpha-1)\theta(Hkq_{EV2})+\theta(Hkq_{EV2}) \\ \cdot\left((\alpha-1)\left(\lambda_l-2\right)\lambda_l+\alpha\lambda_u^2-2\alpha\lambda_u\right)-4)+c_u^2(\theta(Hkq_{EV2}) \\ \cdot\left(\alpha\lambda_h^2-2\alpha\lambda_h-(\alpha-1)\left(\lambda_l-2\right)\lambda_l\right)+\theta(Hkq_{EV2})+4)) \end{array} \right) \quad \text{(B-82)}$$

$$N_{110} = \left(\begin{array}{l} -\alpha^2\theta^2(Hkq_{EV2})^2c_{ntr}^2\lambda_h^2 + 2\alpha^2\theta^2(Hkq_{EV2})^2c_{ntr}\lambda_h^2 + \alpha\theta^2(Hkq_{EV2})^2 \\ \cdot c_{ntr}^2\lambda_h^2 - 2\alpha\theta^2(Hkq_{EV2})^2c_{ntr}\lambda_h^2 + \alpha\theta(Hkq_{EV2})c_h^2((Hkq_{EV2})(\theta - \alpha\theta) \\ -(\alpha-1)\theta(Hkq_{EV2})\left(\lambda_l - 2\right)\lambda_l + 4) - (\alpha-1)\theta(Hkq_{EV2})c_l^2 \\ \cdot\left(\alpha\theta(Hkq_{EV2})\left(\lambda_h - 2\right)\lambda_h + \alpha\theta(Hkq_{EV2}) + 4\right) + 2\alpha\theta(Hkq_{EV2})c_h \\ \cdot((\alpha-1)\theta(Hkq_{EV2})c_l\left(\lambda_h - 1\right)\left(\lambda_l - 1\right) + \lambda_h(c_{ntr}((\alpha-1)\theta(Hkq_{EV2}) \\ -(\alpha-1)\theta(Hkq_{EV2})\lambda_l - 2) + (\alpha-1)\theta(Hkq_{EV2})\left(\lambda_l - 1\right) - 2\delta_m) \\ +(\alpha-1)\theta(Hkq_{EV2})\left(c_{ntr} - 1\right)\left(\lambda_l - 1\right)\lambda_l - 2c_{ntr} + 2\delta_m + 4) \\ +2(\alpha-1)\theta(Hkq_{EV2})\lambda_l(\alpha\theta(Hkq_{EV2})\left(c_{ntr} - 1\right)^2\lambda_h \\ +2\left(c_{ntr}\left(-c_{ntr} + \delta_m + 1\right) + \delta_m\right)) + 2(\alpha-1)\theta(Hkq_{EV2})c_l \\ \cdot\left(c_{ntr}\left(\alpha\theta(Hkq_{EV2})\left(\lambda_h - 1\right)\left(\lambda_h - \lambda_l\right) + 2\left(\lambda_l + 1\right)\right)\right. \\ \left.+\alpha\theta(-(Hkq_{EV2}))\left(\lambda_h - 1\right)\left(\lambda_h - \lambda_l\right) + 2\left(\lambda_l - 1\right)\delta_m - 4\right) \\ +4\alpha\theta(Hkq_{EV2})c_{ntr}\lambda_h^2\delta_m - 4\alpha\theta(Hkq_{EV2})c_{ntr}\lambda_h\delta_m \\ +4\alpha\theta(Hkq_{EV2})c_{ntr}^2\lambda_h - 4\alpha\theta(Hkq_{EV2})c_{ntr}\lambda_h - (\alpha-1)\theta(Hkq_{EV2}) \\ \cdot\lambda_l^2\left(\alpha\theta(Hkq_{EV2})\left(c_{ntr} - 1\right)^2 + 4c_{ntr}\delta_m\right) - 4\theta(Hkq_{EV2})c_{ntr} \\ +8c_{ntr}\delta_m - 4c_{ntr}^2 - \alpha^2\theta^2(Hkq_{EV2})^2\lambda_h^2 + \alpha\theta^2(Hkq_{EV2})^2\lambda_h^2 \\ -4\alpha\theta(Hkq_{EV2})\lambda_h\delta_m + 4\theta(Hkq_{EV2}) + 4\theta(Hkq_{EV2})\delta_m - 4\delta_m^2 \end{array} \right) \tag{B-83}$$

(4) $\boldsymbol{\lambda_l}$

$$\Pi_3(\lambda_l) = \frac{N_{120} + N_{121}\lambda_l + N_{122}\lambda_l^2}{4k(D_{120} + D_{121}\lambda_l + D_{122}\lambda_l^2)} \tag{B-84}$$

$$\begin{cases} -4 \le N_{120} \le 20 \\ -16 \le N_{121} \le -9.09 \times 10^{-8} \\ 9.10 \times 10^{-8} \le N_{122} \le 4 \\ -5 \le D_{120} \le -4 \\ 9.03 \times 10^{-8} \le D_{121} \le 2 \\ -1 \le D_{122} \le -9.28 \times 10^{-8} \end{cases} \tag{B-85}$$

We find The first derivative $9.08 \times 10^{-8}/(4k) \le \Pi_3'(\lambda_l) \le 2.33/(4k)$ and the linearity is $0.05\% \le \eta_{3\lambda_l} \le 24.99\%$. Therefore, the relationship between λ_l and Π_3 can be treated as positive linear.

$$N_{121} = \left(\begin{array}{l} -2(\alpha-1)\theta(Hkq_{EV2})(\alpha(\beta-1)\theta(Hkq_{EV2})c_h^2 + c_l(c_{ntr}(\alpha\theta(Hkq_{EV2}) \\ \cdot(-\beta\lambda_h + \lambda_h + \beta\lambda_u) + \alpha\theta(-(Hkq_{EV2})) - 2) - \alpha\theta(Hkq_{EV2}) \\ \cdot(-\beta\left(c_u + \lambda_h\right) + \beta\left(c_u + 1\right)\lambda_u + \lambda_h) + \alpha\theta(Hkq_{EV2}) - 2\delta_m) \\ +\alpha(\beta-1)\theta(Hkq_{EV2})c_h\left(c_l\left(\lambda_h - 1\right) - \left(c_{ntr} - 1\right)\left(\lambda_h + 1\right)\right) \\ +\alpha\beta\theta(Hkq_{EV2})c_{ntr}^2\lambda_h - 2\alpha\beta\theta(Hkq_{EV2})c_{ntr}\lambda_h - \alpha\theta(Hkq_{EV2})c_{ntr}^2\lambda_h \\ +2\alpha\theta(Hkq_{EV2})c_{ntr}\lambda_h - \alpha\beta\theta(Hkq_{EV2})\left(c_{ntr} - 1\right)\lambda_u\left(c_{ntr} - c_u - 1\right) \\ +\alpha\beta\theta(Hkq_{EV2})c_{ntr}c_u - \alpha\beta\theta(Hkq_{EV2})c_u^2 - \alpha\beta\theta(Hkq_{EV2})c_u - 2c_{ntr}\delta_m \\ -2\left(c_{ntr} + \delta_m\right) + 2c_{ntr}^2 + \alpha\beta\theta(Hkq_{EV2})\lambda_h - \alpha\theta(Hkq_{EV2})\lambda_h) \end{array} \right) \tag{B-86}$$

$$N_{122} = \left(\begin{array}{l} (\alpha-1)\theta(Hkq_{EV2})(\alpha(\beta-1)\theta(Hkq_{EV2})c_h^2 - 2\alpha(\beta-1)\theta(Hkq_{EV2})c_h \\ \cdot(c_{ntr} - 1) - \alpha\theta(Hkq_{EV2})\left(-2\beta\left(c_{ntr} - 1\right)c_u + \left(c_{ntr} - 1\right)^2 + \beta c_u^2\right) \\ -4c_{ntr}\delta_m) \end{array} \right) \tag{B-87}$$

$$D_{120} = \alpha\theta(Hkq_{EV2})\left(\left(\beta-1\right)\left(\lambda_h - 2\right)\lambda_h - \beta\lambda_u^2 + 2\beta\lambda_u\right) + \theta(-(Hkq_{EV2})) - 4 \tag{B-88}$$

$$D_{121} = -2(\alpha-1)\theta(Hkq_{EV2}) \tag{B-89}$$

$$D_{122} = (\alpha-1)\theta(Hkq_{EV2}) \tag{B-90}$$

$$N_{120} = \left(\begin{array}{l} -(Hkq_{EV2})^2\alpha^2\beta^2c_u^2\theta^2 + (Hkq_{EV2})^2\alpha\beta c_u^2\theta^2 - (Hkq_{EV2})^2\alpha^2\lambda_h^2\theta^2 \\ -(Hkq_{EV2})^2\alpha^2\beta^2\lambda_h^2\theta^2 - (Hkq_{EV2})^2\alpha^2c_{ntr}^2\lambda_h^2\theta^2 - (Hkq_{EV2})^2\alpha^2\beta^2 \\ \cdot c_{ntr}^2\lambda_h^2\theta^2 + (Hkq_{EV2})^2\alpha c_{ntr}^2\lambda_h^2\theta^2 + 2(Hkq_{EV2})^2\alpha^2\beta c_{ntr}^2\lambda_h^2\theta^2 \\ -(Hkq_{EV2})^2\alpha\beta c_{ntr}^2\lambda_h^2\theta^2 - (Hkq_{EV2})^2\alpha^2\beta^2c_u^2\lambda_h^2\theta^2 + (Hkq_{EV2})^2\alpha^2\beta \\ \cdot c_u^2\lambda_h^2\theta^2 + (Hkq_{EV2})^2\alpha\lambda_h^2\theta^2 + 2(Hkq_{EV2})^2\alpha^2\beta\lambda_h^2\theta^2 - (Hkq_{EV2})^2\alpha\beta\lambda_h^2\theta^2 \\ +2(Hkq_{EV2})^2\alpha^2c_{ntr}\lambda_h^2\theta^2 + 2(Hkq_{EV2})^2\alpha^2\beta^2c_{ntr}\lambda_h^2\theta^2 - 2(Hkq_{EV2})^2\alpha \\ \cdot c_{ntr}\lambda_h^2\theta^2 - 4(Hkq_{EV2})^2\alpha^2\beta c_{ntr}\lambda_h^2\theta^2 + 2(Hkq_{EV2})^2\alpha\beta c_{ntr}\lambda_h^2\theta^2 \\ -2(Hkq_{EV2})^2\alpha^2\beta^2c_u\lambda_h^2\theta^2 + 2(Hkq_{EV2})^2\alpha^2\beta c_u\lambda_h^2\theta^2 + 2(Hkq_{EV2})^2\alpha^2 \\ \cdot\beta^2c_{ntr}c_u\lambda_h^2\theta^2 - 2(Hkq_{EV2})^2\alpha^2\beta c_{ntr}c_u\lambda_h^2\theta^2 + 2(Hkq_{EV2})^2\alpha^2\beta^2c_u^2\lambda_h\theta^2 \\ -2(Hkq_{EV2})^2\alpha^2\beta c_u^2\lambda_h\theta^2 + 2(Hkq_{EV2})^2\alpha^2\beta^2c_u\lambda_h\theta^2 - 2(Hkq_{EV2})^2\alpha^2\beta \\ \cdot c_u\lambda_h\theta^2 - 2(Hkq_{EV2})^2\alpha^2\beta^2c_{ntr}c_u\lambda_h\theta^2 + 2(Hkq_{EV2})^2\alpha^2\beta c_{ntr}c_u\lambda_h\theta^2 \\ +4(Hkq_{EV2})\alpha\beta c_u^2\theta + 4(Hkq_{EV2})\alpha c_{ntr}\delta_m\lambda_h^2\theta - 4(Hkq_{EV2})\alpha\beta c_{ntr}\delta_m\lambda_h^2\theta \\ +(Hkq_{EV2})\alpha\beta\left(4c_{ntr}\delta_m - (Hkq_{EV2})(\alpha\beta-1)\theta\left(c_{ntr}-1\right)^2\right)\lambda_u^2\theta + 4(Hkq_{EV2})\theta \\ -4(Hkq_{EV2})c_{ntr}\theta + 8(Hkq_{EV2})\alpha\beta c_u\theta - 4(Hkq_{EV2})\alpha\beta c_{ntr}c_u\theta + 4(Hkq_{EV2}) \\ \cdot\delta_m\theta + 4(Hkq_{EV2})\alpha\beta c_u\delta_m\theta + 4(Hkq_{EV2})\alpha c_{ntr}^2\lambda_h\theta - 4(Hkq_{EV2})\alpha\beta c_{ntr}^2\lambda_h\theta \\ -4(Hkq_{EV2})\alpha c_{ntr}\lambda_h\theta + 4(Hkq_{EV2})\alpha\beta c_{ntr}\lambda_h\theta - 4(Hkq_{EV2})\alpha\delta_m\lambda_h\theta \\ +4(Hkq_{EV2})\alpha\beta\delta_m\lambda_h\theta - 4(Hkq_{EV2})\alpha c_{ntr}\delta_m\lambda_h\theta + 4(Hkq_{EV2})\alpha\beta c_{ntr}\delta_m\lambda_h\theta \\ +2(Hkq_{EV2})\alpha\beta(((Hkq_{EV2})\alpha(\beta-1)\theta\lambda_h + 2)\,c_{ntr}^2 + (((Hkq_{EV2})(\alpha\beta-1)\theta \\ -2)c_u - 2\left(\delta_m+1\right) - (Hkq_{EV2})\alpha(\beta-1)\theta\left(c_u+2\right)\lambda_h)c_{ntr} - 2\delta_m \\ +c_u\left((Hkq_{EV2})(1-\alpha\beta)\theta - 2\delta_m\right) + (Hkq_{EV2})\alpha(\beta-1)\theta\left(c_u+1\right)\lambda_h)\lambda_u\theta \\ -(Hkq_{EV2})\alpha(\beta-1)c_h^2((Hkq_{EV2})(\alpha(\beta-1)+1)\theta + (Hkq_{EV2})\alpha\beta\left(\lambda_u-2\right)\lambda_u\theta \\ +4)\theta - 2(Hkq_{EV2})\alpha(\beta-1)c_h(-(Hkq_{EV2})\alpha\beta\theta\left(c_{ntr}-1\right)\lambda_u^2 + (Hkq_{EV2})\alpha\beta\theta \\ \cdot\left(c_{ntr}-1\right)\left(\lambda_h+1\right)\lambda_u - 2c_{ntr} + 2\delta_m - (Hkq_{EV2})(\alpha-1)\theta c_l\left(\lambda_h-1\right) \\ -\left((Hkq_{EV2})(-\beta\alpha+\alpha-1)\theta + ((Hkq_{EV2})(\alpha(\beta-1)+1)\theta+2)c_{ntr} + 2\delta_m\right)\lambda_h \\ -(Hkq_{EV2})\alpha\beta\theta c_u\left(\lambda_h-1\right)\left(\lambda_u-1\right) + 4)\theta + (Hkq_{EV2})(\alpha-1)c_l^2(-(Hkq_{EV2})\alpha\theta \\ +(Hkq_{EV2})\alpha\left(-\beta\lambda_u^2 + 2\beta\lambda_u + (\beta-1)\left(\lambda_h-2\right)\lambda_h\right)\theta - 4)\theta - 2(Hkq_{EV2})(\alpha-1) \\ \cdot c_l(2\left(\delta_m+2\right) + (Hkq_{EV2})\alpha\theta\left(\beta\lambda_u^2 - \beta\lambda_u - (\beta-1)\left(\lambda_h-1\right)\lambda_h + \beta c_u\left(\lambda_u-1\right)\right)) \\ +c_{ntr}\left((Hkq_{EV2})\alpha\theta\left(-\beta\lambda_u^2 + \beta\lambda_u + (\beta-1)\left(\lambda_h-1\right)\lambda_h\right) - 2)\right)\theta - 4c_{ntr}^2 \\ -4\delta_m^2 + 8c_{ntr}\delta_m \end{array} \right) \tag{B-91}$$

(5) $\boldsymbol{\lambda_h}$

$$\Pi_3(\lambda_h) = \frac{N_{130} + N_{131}\lambda_h + N_{132}\lambda_h^2}{4k(D_{130} + D_{131}\lambda_h + D_{132}\lambda_h^2)} \tag{B-92}$$

$$\begin{cases} -4 \leq N_{130} \leq 20 \\ -16 \leq N_{131} \leq -9.09 \times 10^{-8} \\ 5.20 \times 10^{-8} \leq N_{132} \leq 4 \\ -5 \leq D_{130} \leq -4 \\ 1.49 \times 10^{-7} \leq D_{131} \leq 2 \\ -1 \leq D_{132} \leq -9.29 \times 10^{-8} \end{cases} \tag{B-93}$$

We find The first derivative $9.08 \times 10^{-8}/(4k) \leq \Pi_3'(\lambda_h) \leq 2.33/(4k)$ and the linearity is $-31.14\% \leq \eta_{3\lambda_h} \leq 25.04\%$. Therefore, the relationship between λ_h and Π_3 has positive relationship.

$$N_{131} = \left(\begin{array}{l} -2\alpha(\beta-1)\theta(Hkq_{EV2})(c_h(\theta(Hkq_{EV2})((\alpha-1)c_l(\lambda_l-1)+c_{ntr} \\ \cdot(\alpha(-\beta)+\alpha-\alpha\lambda_l+\lambda_l+\alpha\beta\lambda_u-1)+\alpha(-\beta(c_u+1)\lambda_u+\beta c_u+\lambda_l) \\ -\lambda_l)-2c_{ntr}+\theta(Hkq_{EV2})(\alpha(\beta-1)+1)-2\delta_m)+(\alpha-1)\theta(Hkq_{EV2})c_l^2 \\ -(\alpha-1)\theta(Hkq_{EV2})c_l(c_{ntr}-1)(\lambda_l+1)+\alpha\theta(Hkq_{EV2})c_{ntr}^2\lambda_l-2\alpha\theta \\ \cdot(Hkq_{EV2})c_{ntr}\lambda_l-\theta(Hkq_{EV2})c_{ntr}^2\lambda_l+2\theta(Hkq_{EV2})c_{ntr}\lambda_l-\alpha\beta\theta \\ \cdot(Hkq_{EV2})(c_{ntr}-1)\lambda_u(c_{ntr}-c_u-1)+\alpha\beta\theta(Hkq_{EV2})c_{ntr}c_u \\ -\alpha\beta\theta(Hkq_{EV2})c_u^2-\alpha\beta\theta(Hkq_{EV2})c_u-2c_{ntr}\delta_m-2(c_{ntr}+\delta_m) \\ +2c_{ntr}^2+\alpha\theta(Hkq_{EV2})\lambda_l-\theta(Hkq_{EV2})\lambda_l) \end{array} \right) \tag{B-94}$$

$$D_{130} = \theta(-(Hkq_{EV2})) + \theta(Hkq_{EV2})\left((\alpha-1)(\lambda_l-2)\lambda_l - \alpha\beta\lambda_u^2 + 2\alpha\beta\lambda_u\right) - 4 \tag{B-95}$$

$$D_{131} = -2\alpha(\beta-1)\theta(Hkq_{EV2}) \tag{B-96}$$

$$D_{132} = \alpha(\beta-1)\theta(Hkq_{EV2}) \tag{B-97}$$

$$N_{132} = \left(\begin{array}{l} \alpha(\beta-1)\theta(-(Hkq_{EV2}))((\alpha-1)\theta(-(Hkq_{EV2}))c_l^2+2(\alpha-1)\theta \\ \cdot(Hkq_{EV2})c_l(c_{ntr}-1)+\theta(Hkq_{EV2})((\alpha(\beta-1)+1)(c_{ntr}-1)^2 \\ -2\alpha\beta(c_{ntr}-1)c_u+\alpha\beta c_u^2)+4c_{ntr}\delta_m) \end{array} \right) \tag{B-98}$$

$$N_{130} = \left(\begin{array}{l} (Hkq_{EV2})^2\alpha(-(\alpha(\beta-1)+1)(\beta-1)c_h^2 - (\beta-1)(\alpha\beta\lambda_u^2 - 2\alpha\beta\lambda_u \\ -(\alpha-1)(\lambda_l-2)\lambda_l)c_h^2 + 2(\beta-1)((\alpha-1)c_l(\lambda_l-1) - \alpha\beta c_u(\lambda_u-1) \\ -(c_{ntr}-1)(-\alpha\beta\lambda_u^2 + \alpha\beta\lambda_u + (\alpha-1)(\lambda_l-1)\lambda_l))c_h - (\alpha-1)c_l^2 \\ -\alpha\beta^2c_u^2 + \beta c_u^2 - \alpha c_{ntr}^2\lambda_l^2 + c_{ntr}^2\lambda_l^2 - \alpha\beta c_u^2\lambda_l^2 + \beta c_u^2\lambda_l^2 - \alpha\lambda_l^2 \\ +2\alpha c_{ntr}\lambda_l^2 - 2c_{ntr}\lambda_l^2 - 2\alpha\beta c_u\lambda_l^2 + 2\beta c_u\lambda_l^2 + 2\alpha\beta c_{ntr}c_u\lambda_l^2 \\ -2\beta c_{ntr}c_u\lambda_l^2 + \lambda_l^2 - \beta(\alpha\beta-1)(c_{ntr}-1)^2\lambda_u^2 - 2(\alpha-1)\beta c_l\lambda_u^2 \\ +2\alpha\beta c_u^2\lambda_l - 2\beta c_u^2\lambda_l - 2(\alpha-1)c_l\lambda_l + 2(\alpha-1)c_lc_{ntr}\lambda_l + 2\alpha\beta c_u\lambda_l \\ -2\beta c_u\lambda_l - 2\alpha\beta c_{ntr}c_u\lambda_l + 2\beta c_{ntr}c_u\lambda_l + 2(\alpha-1)\beta c_lc_u(\lambda_l-1) \\ \cdot(\lambda_u-1) - 2\beta(\alpha\beta-1)c_u\lambda_u + 2\beta(\alpha\beta-1)c_{ntr}c_u\lambda_u + 2(\alpha-1)\beta c_{ntr}^2\lambda_l\lambda_u \\ -2(\alpha-1)\beta c_lc_{ntr}\lambda_l\lambda_u + 2(\alpha-1)\beta(c_u+1)\lambda_l\lambda_u - 2(\alpha-1)\beta c_{ntr}(c_u+2) \\ \cdot\lambda_l\lambda_u + 2(\alpha-1)\beta c_l(\lambda_l+1)\lambda_u - (\alpha-1)\beta c_l^2(\lambda_u-2)\lambda_u + 2(\alpha-1)\beta c_l \\ \cdot c_{ntr}(\lambda_u-1)\lambda_u)\theta^2 + 4(Hkq_{EV2})((\alpha-\alpha\beta)c_h^2 + \alpha(\beta-1)(c_{ntr}-\delta_m-2)c_h \\ -(\alpha-1)c_l^2 + \alpha\beta c_u^2 - \alpha c_{ntr}\delta_m\lambda_l^2 + c_{ntr}\delta_m\lambda_l^2 + \alpha\beta c_{ntr}\delta_m\lambda_u^2 - c_{ntr} + 2\alpha\beta c_u \\ -\alpha\beta c_{ntr}c_u + \alpha\beta c_u\delta_m + \delta_m - \alpha c_{ntr}^2\lambda_l + c_{ntr}^2\lambda_l + \alpha c_{ntr}\lambda_l - c_{ntr}\lambda_l + \alpha\delta_m\lambda_l \\ +\alpha c_{ntr}\delta_m\lambda_l - c_{ntr}\delta_m\lambda_l - \delta_m\lambda_l + (\alpha-1)c_l(\delta_m(\lambda_l-1) + c_{ntr}(\lambda_l+1) - 2) \\ +\alpha\beta c_{ntr}^2\lambda_u - \alpha\beta c_{ntr}\lambda_u - \alpha\beta c_{ntr}c_u\lambda_u - \alpha\beta\delta_m\lambda_u \\ -\alpha\beta c_{ntr}\delta_m\lambda_u - \alpha\beta c_u\delta_m\lambda_u + 1)\theta - 4(c_{ntr}-\delta_m)^2 \end{array} \right) \tag{B-99}$$

(6) $\boldsymbol{\lambda_u}$

$$\Pi_3(\lambda_u) = \frac{N_{140} + N_{141}\lambda_u + N_{142}\lambda_u^2}{4k(D_{140} + D_{141}\lambda_u + D_{142}\lambda_u^2)} \tag{B-100}$$

$$\begin{cases} -4 \leq N_{140} \leq 20 \\ -16 \leq N_{141} \leq -9.18 \times 10^{-8} \\ -9.09 \times 10^{-8} \leq N_{142} \leq 4 \\ -5 \leq D_{140} \leq -4 \\ -9.17 \times 10^{-8} \leq D_{141} \leq 2 \\ -1 \leq D_{142} \leq -9.10 \times 10^{-8} \end{cases} \tag{B-101}$$

We find The first derivative $6.01 \times 10^{-8} \leq \Pi_3'(\lambda_u) \leq 2.33/(4k)$ and the linearity is $0.02\% \leq \eta_{3\lambda_u} \leq 24.97\%$. Therefore, the relationship between λ_u and Π_3 cannot be treated as positive linear.

$$N_{142}=\left(\begin{array}{l}\alpha\beta\theta(Hkq_{EV2})(\alpha(\beta-1)\theta(-(Hkq_{EV2}))c_h^2+2\alpha(\beta-1)\theta(Hkq_{EV2})c_h\\ \cdot(c_{ntr}-1)+\theta(Hkq_{EV2})(-(\alpha-1)c_l^2+2(\alpha-1)c_l\,(c_{ntr}-1)\\ -(\alpha\beta-1)\,(c_{ntr}-1)\,^2)+4c_{ntr}\delta_m)\end{array}\right) \tag{B-102}$$

$$D_{140}=\theta(Hkq_{EV2})\left(\alpha(\beta-1)\,(\lambda_h-2)\,\lambda_h+(\alpha-1)\lambda_l^2-2(\alpha-1)\lambda_l\right)+\theta(-(Hkq_{EV2}))-4 \tag{B-103}$$

$$D_{141}=2\alpha\beta\theta(Hkq_{EV2}) \tag{B-104}$$

$$D_{142}=\alpha\beta\theta(-(Hkq_{EV2})) \tag{B-105}$$

$$N_{141}=\left(\begin{array}{l}2\alpha\beta\theta^2(Hkq_{EV2})^2(\alpha(\beta-1)c_h^2+(c_{ntr}-1)\,((c_{ntr}-1)\,(\alpha(\beta-1)\lambda_h\\ +(\alpha-1)\lambda_l)+c_u\,(\alpha\beta-\alpha\,((\beta-1)\lambda_h+\lambda_l)+\lambda_l-1))-\alpha(\beta-1)c_h\\ \cdot(\lambda_h\,(c_{ntr}-c_u-1)+c_{ntr}+c_u-1)+(\alpha-1)c_l^2-(\alpha-1)c_l\\ \cdot(\lambda_l\,(c_{ntr}-c_u-1)+c_{ntr}+c_u-1))+2\alpha\beta\theta(Hkq_{EV2})\\ \cdot(2c_{ntr}\,(c_{ntr}-c_u-1)-2\delta_m\,(c_{ntr}+c_u+1))\end{array}\right) \tag{B-106}$$

$$N_{140} = \left(\begin{array}{l} (Hkq_{EV2})^2\alpha(-\alpha c_u^2\beta^2 - \alpha c_{ntr}^2\lambda_h^2\beta^2 - \alpha c_u^2\lambda_h^2\beta^2 - \alpha\lambda_h^2\beta^2 + 2\alpha c_{ntr}\lambda_h^2\beta^2 \\ -2\alpha c_u\lambda_h^2\beta^2 + 2\alpha c_{ntr}c_u\lambda_h^2\beta^2 + 2\alpha c_u^2\lambda_h\beta^2 + 2\alpha c_u\lambda_h\beta^2 - 2\alpha c_{ntr}c_u\lambda_h\beta^2 \\ +c_u^2\beta + 2\alpha c_{ntr}^2\lambda_h^2\beta - c_{ntr}^2\lambda_h^2\beta + \alpha c_u^2\lambda_h^2\beta + 2\alpha\lambda_h^2\beta - 4\alpha c_{ntr}\lambda_h^2\beta \\ +2c_{ntr}\lambda_h^2\beta + 2\alpha c_u\lambda_h^2\beta - 2\alpha c_{ntr}c_u\lambda_h^2\beta - \lambda_h^2\beta - 2\alpha c_u^2\lambda_h\beta - 2\alpha c_u\lambda_h\beta \\ +2\alpha c_{ntr}c_u\lambda_h\beta - \alpha c_{ntr}^2\lambda_h^2 + c_{ntr}^2\lambda_h^2 - \alpha\lambda_h^2 + 2\alpha c_{ntr}\lambda_h^2 - 2c_{ntr}\lambda_h^2 + \lambda_h^2 \\ -(\alpha-1)\left((c_{ntr}-1)^2 - 2\beta c_u(c_{ntr}-1) + \beta c_u^2\right)\lambda_l^2 + (\alpha-1)c_l^2 \\ \cdot\left((\beta-1)(\lambda_h-2)\lambda_h - 1\right) + 2(\alpha-1)(\beta c_u(-c_{ntr}+c_u+1) - (\beta-1) \\ \cdot(c_{ntr}-1)^2\lambda_h)\lambda_l - (\beta-1)c_h^2\left(\alpha(\beta-1) - (\alpha-1)(\lambda_l-2)\lambda_l + 1\right) \\ +2(\alpha-1)c_l\left((c_{ntr}-1)(\lambda_l - (\beta-1)\lambda_h(\lambda_h-\lambda_l-1)) + c_u(\beta-\beta\lambda_l)\right) \\ +2(\beta-1)c_h(-\alpha\beta c_u(\lambda_h-1) - (\alpha-1)c_l(\lambda_l-1)(\lambda_h-1) + (c_{ntr}-1) \\ \cdot(\lambda_h(\alpha(\beta-1) + (\alpha-1)\lambda_l + 1) - (\alpha-1)(\lambda_l-1)\lambda_l)))\theta^2 + 4(Hkq_{EV2}) \\ \cdot((\alpha-\alpha\beta)c_h^2 + \alpha(\beta-1)(\delta_m(\lambda_h-1) + c_{ntr}(\lambda_h+1) - 2)c_h \\ -(\alpha-1)c_l^2 - (\alpha-1)c_{ntr}\delta_m\lambda_l^2 - c_{ntr} + \delta_m + \alpha(\beta c_u^2 + \beta(-c_{ntr}+\delta_m+2)c_u \\ +(\beta-1)\lambda_h(\delta_m + c_{ntr}(-c_{ntr}+\delta_m-\delta_m\lambda_h+1))) - (\alpha-1)((c_{ntr}-1)c_{ntr} \\ -(c_{ntr}+1)\delta_m)\lambda_l + (\alpha-1)c_l(\delta_m(\lambda_l-1) + c_{ntr}(\lambda_l+1) - 2) + 1)\theta \\ -4(c_{ntr}-\delta_m)^2 \end{array} \right) \tag{B-107}$$

(7) c_{ntr}

$$\Pi_3(c_{ntr}) = \frac{N_{150} + N_{151}c_{ntr} + N_{152}c_{ntr}^2}{4kD_{150}} \tag{B-108}$$

$$\begin{cases} -4 \leq N_{150} \leq 20 \\ -16 \leq N_{151} \leq 8 \\ -4 \leq N_{152} \leq -3.34\times10^{-7} \\ -5 \leq D_{150} \leq -4 \end{cases} \tag{B-109}$$

We find The first derivative $-2/(4k) \leq \Pi_3'(c_{ntr}) \leq 4/(4k)$ and the linearity is $-1.24\times10^7 \leq \eta_{3c_{ntr}} \leq 1.82\times10^7$. Therefore, the relationship between c_n and Π_3 cannot be treated as linear.

$$D_{150} = \begin{pmatrix} \theta(Hkq_{EV2})(\alpha(\beta-1)\lambda_h^2 - 2\alpha(\beta-1)\lambda_h + (\alpha-1)(\lambda_l-2)\lambda_l \\ -\alpha\beta\lambda_u^2 + 2\alpha\beta\lambda_u) + \theta(-(Hkq_{EV2})) - 4 \end{pmatrix} \quad \text{(B-110)}$$

$$N_{152} = \begin{pmatrix} \theta(-(Hkq_{EV2}))(\alpha(\beta-1)\theta(Hkq_{EV2})(\alpha(\beta-1)+1)\lambda_h^2 - 2\alpha(\beta-1)\lambda_h \\ \cdot((\alpha-1)\theta(-(Hkq_{EV2}))\lambda_l + \alpha\beta\theta(Hkq_{EV2})\lambda_u - 2) + (\alpha-1)\lambda_l \\ \cdot(\alpha\theta(Hkq_{EV2})\lambda_l + 4) - 2\alpha\beta\lambda_u((\alpha-1)\theta(Hkq_{EV2})\lambda_l + 2) \\ +\alpha\beta\theta(Hkq_{EV2})(\alpha\beta-1)\lambda_u^2) - 4 \end{pmatrix} \quad \text{(B-111)}$$

$$N_{150} = \begin{pmatrix} (Hkq_{EV2})^2\alpha(-\alpha c_u^2\beta^2 - \alpha c_u^2\lambda_h^2\beta^2 - \alpha\lambda_h^2\beta^2 - 2\alpha c_u\lambda_h^2\beta^2 + 2\alpha c_u^2\lambda_h\beta^2 \\ +2\alpha c_u\lambda_h\beta^2 + c_u^2\beta + \alpha c_u^2\lambda_h^2\beta + 2\alpha\lambda_h^2\beta + 2\alpha c_u\lambda_h^2\beta - \lambda_h^2\beta - \alpha c_u^2\lambda_l^2\beta \\ +c_u^2\lambda_l^2\beta - 2\alpha c_u\lambda_l^2\beta + 2c_u\lambda_l^2\beta + (1-\alpha\beta)\lambda_u^2\beta - 2\alpha c_u^2\lambda_h\beta - 2\alpha c_u\lambda_h\beta \\ +2\alpha c_u^2\lambda_l\beta - 2c_u^2\lambda_l\beta + 2\alpha c_u\lambda_l\beta - 2c_u\lambda_l\beta - 2\alpha\lambda_h\lambda_l\beta + 2\lambda_h\lambda_l\beta \\ +2(\alpha(\beta-1)\lambda_h + (\alpha-1)\lambda_l + c_u(-\alpha\beta + \alpha(\beta-1)\lambda_h + (\alpha-1)\lambda_l + 1)) \\ \cdot\lambda_u\beta - \alpha\lambda_h^2 + \lambda_h^2 - \alpha\lambda_l^2 + \lambda_l^2 + 2\alpha\lambda_h\lambda_l - 2\lambda_h\lambda_l - (\beta-1)c_h^2(\alpha\beta\lambda_u^2 \\ -2\alpha\beta\lambda_u + \alpha(\beta-1) - (\alpha-1)(\lambda_l-2)\lambda_l + 1) + (\alpha-1)c_l^2(-\beta\lambda_u^2 + 2\beta\lambda_u \\ +(\beta-1)(\lambda_h-2)\lambda_h - 1) + 2(\alpha-1)c_l(-\beta\lambda_u^2 + \beta(\lambda_l+1)\lambda_u + (\beta-1)\lambda_h \\ \cdot(\lambda_h - \lambda_l - 1) - \lambda_l + \beta c_u(\lambda_l-1)(\lambda_u-1)) + 2(\beta-1)c_h(\alpha\lambda_l^2 - \lambda_l^2 - \alpha\lambda_l \\ -\alpha\lambda_h\lambda_l + \lambda_h\lambda_l + \lambda_l - \alpha\beta\lambda_u^2 + \alpha\lambda_h - \alpha\beta\lambda_h - \lambda_h - (\alpha-1)c_l(\lambda_h-1) \\ \cdot(\lambda_l-1) + \alpha\beta c_u(\lambda_h-1)(\lambda_u-1) + \alpha\beta(\lambda_h+1)\lambda_u))\theta^2 + 4(Hkq_{EV2}) \\ \cdot((\alpha-\alpha\beta)c_h^2 + \alpha(\beta-1)(\delta_m(\lambda_h-1)-2)c_h - (\alpha-1)c_l^2 + \alpha\beta c_u^2 + 2\alpha\beta c_u \\ +\alpha\beta c_u\delta_m + \delta_m - \alpha\delta_m\lambda_h + \alpha\beta\delta_m\lambda_h + (\alpha-1)c_l(\delta_m(\lambda_l-1)-2) + \alpha\delta_m\lambda_l \\ -\delta_m\lambda_l - \alpha\beta(c_u+1)\delta_m\lambda_u + 1)\theta - 4\delta_m^2 \end{pmatrix} \quad \text{(B-112)}$$

$$N_{151} = \begin{pmatrix} 2\alpha\theta^2(Hkq_{EV2})^2(\beta\lambda_u(c_u\left(\alpha\beta - \alpha\left((\beta-1)\lambda_h + \lambda_l\right) + \lambda_l - 1\right) \\ -2\alpha(\beta-1)\lambda_h - 2(\alpha-1)\lambda_l) - (\alpha-1)c_l((\beta-1)\lambda_h\left(\lambda_h - \lambda_l - 1\right) \\ -\lambda_l + \beta\left(\lambda_l+1\right)\lambda_u - \beta\lambda_u^2) - (\beta-1)c_h(\lambda_h(\alpha(-\beta) + \alpha - \alpha\lambda_l + \lambda_l \\ +\alpha\beta\lambda_u - 1) + (\alpha-1)\left(\lambda_l - 1\right)\lambda_l - \alpha\beta\lambda_u^2 + \alpha\beta\lambda_u) + \alpha\beta^2 c_u\lambda_h^2 \\ -\alpha\beta^2 c_u\lambda_h - \alpha\beta c_u\lambda_h^2 + \alpha\beta c_u\lambda_h + \alpha\beta c_u\lambda_l^2 - \alpha\beta c_u\lambda_l - \beta c_u\lambda_l^2 + \beta c_u\lambda_l \\ +\alpha\beta^2\lambda_h^2 - 2\alpha\beta\lambda_h^2 + \alpha\lambda_h^2 + \beta\lambda_h^2 - \lambda_h^2 + 2\alpha\beta\lambda_h\lambda_l - 2\alpha\lambda_h\lambda_l - 2\beta\lambda_h\lambda_l \\ +2\lambda_h\lambda_l + \alpha\lambda_l^2 - \lambda_l^2 + \beta(\alpha\beta-1)\lambda_u^2) + 4\theta(Hkq_{EV2})(\alpha(\beta-1)c_h\left(\lambda_h+1\right) \\ +(\alpha-1)c_l\left(\lambda_l+1\right) - \alpha\beta\lambda_u\left(c_u + \delta_m + 1\right) - \alpha\beta c_u + \alpha\beta\lambda_h - \alpha\lambda_h \\ -\alpha\beta\lambda_h^2\delta_m + \alpha\beta\lambda_h\delta_m + \alpha\lambda_h^2\delta_m - \alpha\lambda_h\delta_m + \alpha\lambda_l - \lambda_l - \alpha\lambda_l^2\delta_m + \alpha\lambda_l\delta_m \\ +\lambda_l^2\delta_m - \lambda_l\delta_m + \alpha\beta\delta_m\lambda_u^2 - 1) + 8\delta_m \end{pmatrix} \tag{B-113}$$

(8) c_l

$$\Pi_3(c_l) = \frac{N_{160} + N_{161}c_l + N_{162}c_l^2}{4kD_{160}} \tag{B-114}$$

$$\begin{cases} -4 \leq N_{160} \leq 20 \\ 9.08 \times 10^{-8} \leq N_{161} \leq 12 \\ 8.28 \times 10^{-8} \leq N_{162} \leq 4 \\ -5 \leq D_{160} \leq -4 \end{cases} \tag{B-115}$$

We find The first derivative $-4.23/(4k) \leq \Pi_3'(c_l) \leq -9.13 \times 10^{-8}/(4k)$ and the linearity is $-24.99\% \leq \eta_{3c_l} \leq -6.25\%$. Therefore, the relationship between c_l and Π_3 cannot be treated as negative linear.

$$N_{161} = \begin{pmatrix} 2(\alpha-1)\alpha\theta^2(Hkq_{EV2})^2(-(\beta-1)c_h\left(\lambda_h-1\right)\left(\lambda_l-1\right) - \left(c_{ntr}-1\right) \\ \cdot\left((\beta-1)\lambda_h\left(\lambda_h - \lambda_l - 1\right) - \lambda_l + \beta\left(\lambda_l+1\right)\lambda_u - \beta\lambda_u^2\right) \\ +\beta c_u\left(\lambda_l-1\right)\left(\lambda_u-1\right)) + 4(\alpha-1)\theta(Hkq_{EV2})(c_{ntr}\left(\lambda_l+1\right) \\ +\left(\lambda_l-1\right)\delta_m - 2) \end{pmatrix} \tag{B-116}$$

$$N_{162} = \begin{pmatrix} (\alpha-1)\theta(Hkq_{EV2})(\alpha\theta(Hkq_{EV2})\left((\beta-1)\left(\lambda_h-2\right)\lambda_h - \beta\lambda_u^2 + 2\beta\lambda_u\right) \\ +\alpha\theta(-(Hkq_{EV2})) - 4) \end{pmatrix} \tag{B-117}$$

$$D_{160}=\left(\begin{array}{l}\theta(Hkq_{EV2})\left(\alpha(\beta-1)\lambda_h^2-2\alpha(\beta-1)\lambda_h+(\alpha-1)\left(\lambda_l-2\right)\lambda_l-\alpha\beta\lambda_u^2+2\alpha\beta\lambda_u\right)\\+\theta(-(Hkq_{EV2}))-4\end{array}\right) \tag{B-118}$$

$$N_{160}=\left(\begin{array}{l}\alpha\theta^2(Hkq_{EV2})^2(2(\beta-1)c_h(\alpha\beta c_u\left(\lambda_h-1\right)\left(\lambda_u-1\right)-\left(c_{ntr}-1\right)\\\cdot(\lambda_h\left(\alpha(-\beta)+\alpha-\alpha\lambda_l+\lambda_l+\alpha\beta\lambda_u-1\right)+(\alpha-1)\left(\lambda_l-1\right)\lambda_l-\alpha\beta\lambda_u^2\\+\alpha\beta\lambda_u t))+2\beta\left(c_{ntr}-1\right)c_u(\alpha(\beta-1)\lambda_h^2-\alpha(\beta-1)\lambda_h\left(\lambda_u+1\right)+(\alpha-1)\\\cdot\left(\lambda_l-1\right)\lambda_l+\lambda_u\left(\alpha\beta-\alpha\lambda_l+\lambda_l-1\right))-\left(c_{ntr}-1\right)^2((\beta-1)(\alpha(\beta-1)\\+1)\lambda_h^2-2(\beta-1)\lambda_h\left(-\alpha\lambda_l+\lambda_l+\alpha\beta\lambda_u\right)+(\alpha-1)\lambda_l^2-2(\alpha-1)\beta\lambda_l\lambda_u\\+\beta(\alpha\beta-1)\lambda_u^2)-(\beta-1)c_h^2(\alpha(\beta-1)-(\alpha-1)\left(\lambda_l-2\right)\lambda_l+\alpha\beta\lambda_u^2\\-2\alpha\beta\lambda_u+1)+\beta c_u^2(-\alpha\beta+(\alpha-\alpha\beta)\lambda_h^2+2\alpha(\beta-1)\lambda_h-(\alpha-1)\left(\lambda_l-2\right)\\\cdot\lambda_l+1))+4\theta(Hkq_{EV2})((\alpha-\alpha\beta)c_h^2-c_{ntr}(\alpha\beta c_u\left(\lambda_u+1\right)+\alpha(\beta-1)\lambda_h^2\delta_m\\-\alpha(\beta-1)\lambda_h\left(\delta_m+1\right)+(\alpha-1)\lambda_l\left(\left(\lambda_l-1\right)\delta_m-1\right)-\alpha\beta\delta_m\lambda_u^2\\+\alpha\beta\left(\delta_m+1\right)\lambda_u+1)+c_{ntr}^2\left((\alpha-\alpha\beta)\lambda_h-(\alpha-1)\lambda_l+\alpha\beta\lambda_u\right)+\alpha(\beta-1)\\\cdot c_h\left(c_{ntr}\left(\lambda_h+1\right)+\left(\lambda_h-1\right)\delta_m-2\right)-\alpha\beta c_u\delta_m\lambda_u+\alpha\beta c_u\delta_m+\alpha\beta c_u^2\\+2\alpha\beta c_u+\alpha\beta\lambda_h\delta_m-\alpha\lambda_h\delta_m+\alpha\lambda_l\delta_m-\lambda_l\delta_m+\delta_m-\alpha\beta\delta_m\lambda_u+1)\\-4\left(c_{ntr}-\delta_m\right)^2\end{array}\right) \tag{B-119}$$

(9) c_h

$$\Pi_3(c_h)=\frac{N_{170}+N_{171}c_h+N_{172}c_h^2}{4kD_{170}} \tag{B-120}$$

$$\begin{cases}-4\le N_{170}\le 20\\8.57\times10^{-8}\le N_{171}\le 12\\6.89\times10^{-8}\le N_{172}\le 4\\-5\le D_{170}\le -4\end{cases} \tag{B-121}$$

We find The first derivative $-4.23/(4k)\le \Pi_3'(c_l)\le -8.43\times10^{-8}/(4k)$ and the linearity is $-24.91\%\le \eta_{3c_h}\le -6.26\%$. Therefore, the relationship between c_l and Π_3 cannot be treated as negative linear.

$$N_{161} = \left(\begin{array}{l} 2(\alpha-1)\alpha\theta^2(Hkq_{EV2})^2(-(\beta-1)c_h(\lambda_h-1)(\lambda_l-1)-(c_{ntr}-1) \\ \cdot((\beta-1)\lambda_h(\lambda_h-\lambda_l-1)-\lambda_l+\beta(\lambda_l+1)\lambda_u-\beta\lambda_u^2) \\ +\beta c_u(\lambda_l-1)(\lambda_u-1))+4(\alpha-1)\theta(Hkq_{EV2})(c_{ntr}(\lambda_l+1) \\ +(\lambda_l-1)\delta_m-2) \end{array} \right) \tag{B-122}$$

$$N_{162} = \left(\begin{array}{l} (\alpha-1)\theta(Hkq_{EV2})(\alpha\theta(Hkq_{EV2})((\beta-1)(\lambda_h-2)\lambda_h-\beta\lambda_u^2+2\beta\lambda_u) \\ +\alpha\theta(-(Hkq_{EV2}))-4) \end{array} \right) \tag{B-123}$$

$$D_{160} = \left(\begin{array}{l} \theta(Hkq_{EV2})(\alpha(\beta-1)\lambda_h^2-2\alpha(\beta-1)\lambda_h+(\alpha-1)(\lambda_l-2)\lambda_l-\alpha\beta\lambda_u^2+2\alpha\beta\lambda_u) \\ +\theta(-(Hkq_{EV2}))-4 \end{array} \right) \tag{B-124}$$

$$N_{170} = \left(\begin{array}{l} \alpha\theta^2(Hkq_{EV2})^2(2(\alpha-1)c_l(\beta c_u(\lambda_l-1)(\lambda_u-1)-(c_{ntr}-1) \\ \cdot((\beta-1)\lambda_h(\lambda_h-\lambda_l-1)-\lambda_l+\beta(\lambda_l+1)\lambda_u-\beta\lambda_u^2))+2\beta \\ \cdot(c_{ntr}-1)c_u(\alpha(\beta-1)\lambda_h^2-\alpha(\beta-1)\lambda_h(\lambda_u+1)+(\alpha-1)(\lambda_l-1)\lambda_l \\ +\lambda_u(\alpha\beta-\alpha\lambda_l+\lambda_l-1))-(c_{ntr}-1)^2((\beta-1)(\alpha(\beta-1)+1)\lambda_h^2 \\ -2(\beta-1)\lambda_h(-\alpha\lambda_l+\lambda_l+\alpha\beta\lambda_u)+(\alpha-1)\lambda_l^2-2(\alpha-1)\beta\lambda_l\lambda_u \\ +\beta(\alpha\beta-1)\lambda_u^2)+(\alpha-1)c_l^2((\beta-1)(\lambda_h-2)\lambda_h-\beta\lambda_u^2+2\beta\lambda_u-1) \\ +\beta c_u^2(-\alpha\beta+(\alpha-\alpha\beta)\lambda_h^2+2\alpha(\beta-1)\lambda_h-(\alpha-1)(\lambda_l-2)\lambda_l+1)) \\ +4\theta(Hkq_{EV2})(-c_{ntr}(\alpha\beta c_u(\lambda_u+1)+\alpha(\beta-1)\lambda_h^2\delta_m-\alpha(\beta-1)\lambda_h \\ \cdot(\delta_m+1)+(\alpha-1)\lambda_l((\lambda_l-1)\delta_m-1)-\alpha\beta\delta_m\lambda_u^2+\alpha\beta(\delta_m+1)\lambda_u+1) \\ +c_{ntr}^2((\alpha-\alpha\beta)\lambda_h-(\alpha-1)\lambda_l+\alpha\beta\lambda_u)-(\alpha-1)c_l^2+(\alpha-1)c_l \\ \cdot(c_{ntr}(\lambda_l+1)+(\lambda_l-1)\delta_m-2)-\alpha\beta c_u\delta_m\lambda_u+\alpha\beta c_u\delta_m+\alpha\beta c_u^2 \\ +2\alpha\beta c_u+\alpha\beta\lambda_h\delta_m-\alpha\lambda_h\delta_m+\alpha\lambda_l\delta_m-\lambda_l\delta_m+\delta_m-\alpha\beta\delta_m\lambda_u+1) \\ -4(c_{ntr}-\delta_m)^2 \end{array} \right) \tag{B-125}$$

(10) c_u

$$\Pi_3(c_u) = \frac{N_{180}+N_{181}c_u+N_{182}c_u^2}{4kD_{180}} \tag{B-126}$$

$$\begin{cases} -4 \leq N_{180} \leq 20 \\ 1.82 \times 10^{-8} \leq N_{181} \leq 12 \\ 9.09 \times 10^{-8} \leq N_{182} \leq 4 \\ -5 \leq D_{180} \leq -4 \end{cases} \tag{B-127}$$

We find The first derivative $-4.24/(4k) \leq \Pi_3'(c_u) \leq -9.08 \times 10^{-8}/(4k)$ and the linearity is $-33.33\% \leq \eta_{3c_u} \leq -6.29\%$. Therefore, the relationship between c_u and Π_3 cannot be treated as negative linear.

$$N_{180} = \left(\begin{array}{l} \alpha\theta^2(Hkq_{EV2})^2(-2(\beta-1)c_h((\alpha-1)c_l(\lambda_h-1)(\lambda_l-1)+(c_{ntr}-1) \\ \cdot(\lambda_h(\alpha(-\beta)+\alpha-\alpha\lambda_l+\lambda_l+\alpha\beta\lambda_u-1)+(\alpha-1)(\lambda_l-1)\lambda_l \\ -\alpha\beta\lambda_u^2+\alpha\beta\lambda_u))-2(\alpha-1)c_l(c_{ntr}-1)((\beta-1)\lambda_h(\lambda_h-\lambda_l-1)-\lambda_l \\ +\beta(\lambda_l+1)\lambda_u-\beta\lambda_u^2)-(c_{ntr}-1)^2((\beta-1)(\alpha(\beta-1)+1)\lambda_h^2 \\ -2(\beta-1)\lambda_h(-\alpha\lambda_l+\lambda_l+\alpha\beta\lambda_u)+(\alpha-1)\lambda_l^2-2(\alpha-1)\beta\lambda_l\lambda_u \\ +\beta(\alpha\beta-1)\lambda_u^2)-(\beta-1)c_h^2(\alpha(\beta-1)-(\alpha-1)(\lambda_l-2)\lambda_l+\alpha\beta\lambda_u^2 \\ -2\alpha\beta\lambda_u+1)+(\alpha-1)c_l^2((\beta-1)(\lambda_h-2)\lambda_h-\beta\lambda_u^2+2\beta\lambda_u-1)) \\ +4\theta(Hkq_{EV2})((\alpha-\alpha\beta)c_h^2+\alpha(\beta-1)c_h(c_{ntr}(\lambda_h+1)+(\lambda_h-1)\delta_m \\ -2)-\alpha\beta c_{ntr}\lambda_h^2\delta_m+\alpha\beta c_{ntr}\lambda_h\delta_m+\alpha c_{ntr}\lambda_h^2\delta_m-\alpha c_{ntr}\lambda_h\delta_m-\alpha\beta c_{ntr}^2\lambda_h \\ +\alpha\beta c_{ntr}\lambda_h+\alpha c_{ntr}^2\lambda_h-\alpha c_{ntr}\lambda_h-(\alpha-1)c_l^2-\alpha c_{ntr}\lambda_l^2\delta_m+\alpha c_{ntr}\lambda_l\delta_m \\ +(\alpha-1)c_l(c_{ntr}(\lambda_l+1)+(\lambda_l-1)\delta_m-2)+c_{ntr}\lambda_l^2\delta_m-c_{ntr}\lambda_l\delta_m \\ -\alpha c_{ntr}^2\lambda_l+\alpha c_{ntr}\lambda_l+c_{ntr}^2\lambda_l-c_{ntr}\lambda_l+\alpha\beta c_{ntr}\delta_m\lambda_u^2 \\ +\alpha\beta\lambda_u((c_{ntr}-1)c_{ntr}-(c_{ntr}+1)\delta_m)-c_{ntr}+\alpha\beta\lambda_h\delta_m-\alpha\lambda_h\delta_m \\ +\alpha\lambda_l\delta_m-\lambda_l\delta_m+\delta_m+1)-4(c_{ntr}-\delta_m)^2 \end{array} \right) \tag{B-128}$$

$$N_{181} = \left(\begin{array}{l} 2\alpha\beta\theta^2(Hkq_{EV2})^2((c_{ntr}-1)(\alpha(\beta-1)\lambda_h^2-\alpha(\beta-1)\lambda_h(\lambda_u+1) \\ +(\alpha-1)(\lambda_l-1)\lambda_l+\lambda_u(\alpha\beta-\alpha\lambda_l+\lambda_l-1))+\alpha(\beta-1)c_h(\lambda_h-1) \\ \cdot(\lambda_u-1)+(\alpha-1)c_l(\lambda_l-1)(\lambda_u-1))-4\alpha\beta\theta(Hkq_{EV2}) \\ \cdot(c_{ntr}(\lambda_u+1)+\delta_m(\lambda_u-1)-2) \end{array} \right) \tag{B-129}$$

$$N_{182} = \left(\begin{array}{l} \alpha\beta\theta(Hkq_{EV2})(\theta(Hkq_{EV2})(-\alpha(\beta-1)\left(\lambda_h-2\right)\lambda_h-(\alpha-1)\lambda_l^2 \\ +2(\alpha-1)\lambda_l)+(Hkq_{EV2})(\theta-\alpha\beta\theta)+4) \end{array} \right) \tag{B-130}$$

$$D_{180} = \left(\begin{array}{l} \theta(Hkq_{EV2})(\alpha(\beta-1)\lambda_h^2-2\alpha(\beta-1)\lambda_h+(\alpha-1)\left(\lambda_l-2\right)\lambda_l \\ -\alpha\beta\lambda_u^2+2\alpha\beta\lambda_u)+\theta(-(Hkq_{EV2}))-4 \end{array} \right) \tag{B-131}$$