Which service is better on a linear travel corridor: park & ride or on-demand public bus?

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Abstract

This paper develops an analytical model to support the decision-making for selection of a public transport service (PTS) provision between park & ride and on-demand public bus (ODPB). The objective of the model is to maximise the total social welfare, which includes consumer surplus and operator's net profit. The model is solved by a heuristic solution procedure and tested on an idealized linear travel corridor. The case study considers the effects from population density, density distribution, size of residential area, P&R station location, distance from the residential area to centre business area (CBD), as well as the changes of residential area layout and population growth. Results show that P&R fits for low population density area while ODPB is more suitable for high population density area. Population distribution type has little influence on the services' social welfare. ODPB is a preferable service for the city which does not have advanced metro network. The investment time for building ODPB service in the planning horizon is discussed at last with consideration of the development of residential area.

Keywords

Public transport, service selection, park and ride, on-demand public bus, social welfare

1. Introduction

In presence of increasing vehicle ownership and limited land resources, many major cities are facing more and more urban transportation problems like traffic congestion and vehicle emission. Empirical studies have clearly shown that the development of public transport is one of the most efficient ways to alleviate these traffic problems in urban cities (Song, 2013). However, only providing more public transport services does not necessarily lead to higher public transport usage. To attract more users to give up private transport, researchers and transport planners have realised that it is important to offer diversified public transport services to cater for various travel demands from different users (Idris et al., 2015; Qu and Wang, 2015; Clark et al., 2016). Indeed, public transport system has offered multiple modes of PTS in different levels of capacities, such as mass rapid transit (MRT), light rapid transit (LRT) and bus rapid transit (BRT). Besides, public transport system provides complementary PTS modes to ensure multimodal trips, such as Park and Ride (P&R), bicycle sharing, car sharing and car pool.

Among these innovative services, P&R is undoubtedly one of the most widely known schemes, which has been adopted and successfully operated by many cities in Europe, America and Australia. P&R has been commonly recognized as an attractive travel mode for long distance commuting, which can contribute significantly to ease traffic jams. However, not all P&R schemes are completely successful. Some P&R schemes failed or did not achieve expected effects, especially in Asia like China and Singapore. These schemes could not promote the mode switching rate from purely private mode to P&R service. On the contrary, the emergent on-demand public bus (ODPB) service does attract the favour of potential P&R consumers. From 2013 to 2017, there are more than 100 cities in at least 8 countries launched this service. It targets on the similar type of user group with P&R. ODPB is designed for specific user group which has origin, destination in close proximity, as well as similar expected departure and arrival time. Like P&R, ODPB ensures low user cost, low energy consumption, less travel time and relatively high travel comfort service. This indeed encourages private car users to use public transport services. A recent study showed that P&R and ODPB share several similarities in function and could replace each other in certain scenarios (Zhang et al, 2017). They also found that P&R and ODPB have its own advantages to attract private vehicle travellers under different conditions. Subsequently, some questions naturally arise for decision makers in transport planning on this issue: what are the reasons behind the fact that the ODPB is more successful than P&R in Asia? What are the major factors that significantly affect the commuter's service preference on a macroscopic level, such as urban city structure and population density? What is the suitable and favourable condition to develop P&R or ODPB, respectively? How do the operators determine the optimal operation strategies to ensure financially sustainable PTS while satisfying the requirements from the users simultaneously? No previous research has ever clearly answered these questions, and this study aims to fill in this research gap by explicitly addressing this service selection problem between P&R and ODPB. An analytical approach with proposed model formulation and solution algorithm is presented. This study would provide a modelling framework to the transportation planners so that the model results would provide policy guidelines to the decision makers when planning PTS in urban area.

Having the above-mentioned motivations, we organize the rest of the paper as follows: Section 2 gives a brief overview of the development of P&R and ODPB, as well as the current service selection research outcomes. The basic consideration pertaining to modelling development is described in Section 3. Section 4 is the proposed model and Section 5 is the solution algorithm. Section 6 illustrates the model through a case study on an ideal corridor. Conclusions and discussions are drawn in Section 7.

2. Literature Review

2.1. P&R development

P&R scheme was originated in the 1930 as ad hoc parking along public transport routes in the rural area of large cities (Noel, 1988). With years of development, this scheme has been gradually improved and applied in many western and US cities due to its competitive advantages, such as low user cost, low energy consumption, less travel time, relatively high travel comfort, etc. The core idea of P&R scheme is to encourage more private car users to use public transport services, especially in city centre, by providing transfer facilities. The expected outcome is to increase the mode shift rate from lower occupancy modes to higher occupancy modes.

In the literature, there are four major types of studies regarding P&R scheme planning and operation. First, one of the core focuses is how to select the optimal P&R facility locations. An ideal P&R facility location could best meet the needs of the surrounding community (without any fatal flaws) while attaining ridership demand characteristics that provide acceptable cost-benefit performance ratios (Spillar, 1997). However, due to various local constraints (e.g. lack of land resource, environmental limitation, inadequate alternatives), trade-offs among a set of goals must be balanced. Researchers have proposed various mathematical or simulation-based methods to determine the optimal P&R facility location (Du and Wang, 2014; Farhan and Murray, 2008; Li et al., 2007; Liu et al., 2009; Song et al, 2017; Wang et al., 2004). Second, another major group of studies investigate the optimal parking capacity and pricing setting issues. It is usually described as a network design problem where the capacity and fare are two design variables in the model (Bagloee et al., 2012; García and Marín, 2002; Habib et al., 2013; Wang et al., 2014). Third, how to accurately forecast the P&R demand is also a hot topic in this area. Initially, demand for P&R parking facility was predicted by using a technique that identified the draw (catchment) area for each parking facility and estimated demand without considering capacity constraints (Hendricks and Outwater, 1994). This simple analysis method could not be applied in the scheme with high user demand. The common method now has combined the behaviour survey together to estimate shifts in demand for parking on the basis of existing capacity and user fees (Hamer, 2010; Hole, 2004; Mingardo, 2013; Qin et al., 2013). Fourth, many researchers analyse the P&R system performance from different aspects, such as the integration with other modes (Duncan and Cook, 2014; Meek et al., 2008), profit evaluation (Meek et al., 2011; Hounsell et al., 2011) and experience review (Lam et al., 2001; Martens et al., 2007; Stieffenhofer et al., 2016).

2.2. ODPB development

ODPB is designed for specific traveller group that requires high-quality service, especially the commuters with long travel distance. From the demand side, travellers in the same ODPB service area have origin and destination points in close proximity, as well as similar expected departure and arrival time. Consequently, from the supply side, ODPB provides high-speed service with none or few midway stops along the route, and runs mostly on the bus lane or expressway to ensure the punctuality immune from traffic congestion effects. Meanwhile, various amenities are offered in the bus vehicle, e.g. reserved seat, Wi-Fi, newspaper, entertainment, water and tissue, which further enhance the ODPB service quality. The planning and operation process of a new ODPB service starts from gathering travel demand through online platform (e.g. website and smartphone), followed by designing the route and schedule, and releasing the service information online. Once there are sufficient subscribers, this particular ODPB service begins its operation. Compared to conventional bus and metro service, ODPB has the advantages of personalization, flexibility, attractiveness, reliability, speediness, and transfer-free (Liu and Ceder 2015); while compared to auto and taxi, ODPB possesses a unique superiority of cost saving. It is envisioned that ODPB service, as an alternative mode to both auto and conventional public transport mode, will complement the existing multimodal transport system and improve the urban mobility significantly.

Due to its emerging status, the research on ODPB is quite limited. Tsubouchi et al. (2009) programed the vehicle-choosing algorithm and routing algorithm to design route and schedule for the vehicle chosen to serve the request. Tsubouchi et al. (2010) developed a cloud computing system from cost viewpoint and the result of field tests shows that the system is valid for different city types. Raymond et al. (2011) built a location recommendation system that predicts data would help riders during the reservation process and help target potential riders when buses are idle. Kameda et al. (2011) assumed the on-demand bus use electrical vehicles and used taxi probe data to optimize the placement of charging stations in a service area. Hamilton and Sankaranarayanan (2013) built an intelligent agent based RFID system for bus scheduling and ticketing. Liu et al. (2016) applied a methodological analysis framework to quantify operational performance measures that enable the comparison of the different travel modes. Cao and Wang (2017) proposed an optimization method of passenger assignment for ODPB, which guarantees benefits to passengers by balancing the elements of travel time, waiting time, delay, and economic cost.

In terms of operational characteristic, ODPB belongs to a special demand-responsive public transport service (DRT). As compared to traditional DRT, ODPB is more for long distance commuting trips, which requires large-capacity vehicle during specific hours. Traditional DRT may operate anytime for any type of service, where the capacity is usually smaller. Some DRT research outcomes could be simply utilized by ODPB's study, such as performance evaluation and preference analysis. Horn (2002a) proposed a public transport system performance evaluation framework with particular attention to DRT's performance in multimodal transport network. Horn (2002b) tested this framework with considering DRT fleet scheduling and dispatching issues. Horn (2004) also described journey-planning procedures designed for use in a traveller information system covering DRT services, where alternative fare structures were considered. Edwards and Watkins (2013) compared the fixed-route transport system and the DRT by using passenger survey data, published transit schedules, and optimal routing techniques. Results showed that DRT can be used to improve transit service levels in low-demand areas. Davison et al. (2014) conducted a survey to examine and assess the design and performance of DRT schemes in Britain. Results showed that funding or the commercial potential for DRT, continued to be the factor which required the most attention from practitioners and policy-makers. Furuhata et al. (2015) determined properties of cost-sharing mechanisms that make DRT systems attractive to both operators and passengers, namely proportional online cost sharing, and examined this mechanism in theory and computational experiments. Amirgholy and Gonzales (2016) proposed an analytical model to approximate the agency's operating cost for running a DRT system, where dynamic demand is considered. This study compared several optimization strategies using a numerical example. Jokinen (2016) presented a model addressing welfare optimal policies, and unique cost structures of DRT, where three optimal pricing structures were discussed.

2.3. Service selection

There are various public transport modes that have been successfully operated in urban cities, like metro, light rail transit (LRT), bus rapid transit (BRT), tram, bus, and streetcars. In presence of emerging technologies, many novel public transport services also attract public's attention, such as carpool, autonomous feeder, P&R, ODPB, bike-sharing. When more than one mode could satisfy the transport demand, transport authorities will compare different alternatives and select one for practical implementation. How to select the suitable service under certain condition has been discussed through different aspects. The common approach is to compare the cost or operational efficiency (Smith, 1973; Allport, 1981; Stutsman, 2002; Stutsman, 2002; Bruun, 2005; Jara-Díaz and Gschwender, 2009; Tirachini et al., 2010a; Casello, 2014). Recently, some researchers started to explore this issue in detail with optimized parameters of transit lines. Parajuli and Wirasinghe (2001) proposed a decision analytic model which considered the users', operators' and community requirements roughly equally and had identical level of comfort, convenience and other nonquantifiable attributes of performance measures. Tirachini et al. (2010b) compared analytically and numerically the optimised performance of different urban public transport modes including bus, LRT, BRT and metro in terms of three objectives: total cost minimisation, profit maximisation, and welfare maximisation. Chen et al. (2015) and Li et al. (2015) aimed to maximize the social welfare of the transit system during the service selection by determining the optimal combination of transit line length, number of stations, station location (or spacing), headway, and fare. Moccia and Laporte (2016) considered optimal stop spacing, train length, crowding cost, and multiple periods in their service selection model, and tested this model in an illustrative example. Barabino and Di Francesco (2016) developed a framework for the involvement of all stakeholders in the characterisation, measurement, and management of the stages of quality monitoring, which was jointly analyzed at different planning levels. Sun et al. (2017) explored how the selection of public service can be optimized over a planning horizon. A series of static models, namely total cost minimization models for uncoordinated bus and rail operations, rail only, bus only, and coordinated bus and rail operations, were formulated and solved for a commuter corridor.

However, these previous studies mostly focused on operational strategies, like station spacing, fare and headway which can be implemented in all cities, rather than strategic factors from macroscopic level, such as population density, distribution of the demand density, residential area size and so on which are decided by the cities themselves. Also, the previous studies concerned with the comparison among existing and traditional public transport service, which cannot be applied to consider emerging public transport service modes. ODPB is new transport service. There is no other research in studying the competition and interaction between P&R and ODPB in academics and practice. This study aims to fill in this research gap and answering the question on which service is to be selected considering the macroscopic factors in the urban area.

3. Basic Consideration

3.1. List of symbols

L_1	length of the residential area (km)
L_2	length from the border of residential area to the P&R metro station (km)

L_3	length of metro from the P&R metro station to CBD (km)
W	width of the residential area (km)
$g(x,y)$	population density in the residential area (person/km^2)
$p(x,y)$	potential passenger density at location (x,y) (person/km^2)
ϕ	average number of trips to city centre per person per day (trips/person/day)
τ	peak hour factor
$q(x,y)$	actual passenger demand at location (x,y) (person/km^2)
$f_1(x,y)$	access time (hour)
$f_2(x,y)$	waiting time (hour)
$f_3(x,y)$	in-vehicle time (hour)
$f_4(x,y)$	fare ($)
μ_1	sensitivity parameters for access time
μ_2	sensitivity parameters for waiting time
μ_3	sensitivity parameters for in-vehicle time
μ_4	sensitivity parameters for fare
$t_{parking}$	parking time of P&R (hour)
$t_{transfer}$	transfer time of P&R (hour)
$t_{waiting}$	waiting time of P&R (hour)
H_{metro}	head way of metro (hour)
v_{auto}	speed of auto (km/hour)
v_{metro}	speed of metro (km/hour)
f_{auto}	auto cost per distance ($/km)
$f_{parking}$	parking fee ($)
f_{metro}	metro ticket fee ($)
Q^{PR}	total passengers of P&R (passengers)
Q^{O}	total passengers of ODPB (passengers)
c^{ODPB}	capacity of one ODPB line (passengers)
SW	social welfare ($/hour)
C	consumer surplus ($/hour)
O	operator's net profit ($/hour)
R	total operating revenue ($/hour)
Ct	total cost ($/hour)
M^{PR}	average cost per P&R passenger ($/hour)
M^{O}	average cost per ODPB line ($/hour)
l	growth rate of *L1* (%)
w	growth rate of *W* (%)
g	growth rate of population (%)

3.2. Assumptions

Several basic assumptions are made before the modelling as follows:

A1: A linear urban transportation corridor that connects the city centre and suburb (satellite city) is considered. This assumption has been widely adopted by previous studies (Liu et al., 2009; Qu et al., 2014; Chen et al., 2015; Meng et al., 2016).

A2: Passengers will choose either P&R service or ODPB service.

A3: All the ODPB passengers are charged with the same fare regardless of the distance of their trips.

A4: The demand is considered as elastic, which is influenced by adjustments in disutility cost.

A5: The social welfare and operator's net profit of non-ODPB or non-P&R passengers are zero.

A6: All the routes in the residential area are projected along the horizontal and/or vertical direction. Only one road connects the residential area and CBD which is along the corridor. Vehicles start to collect the passengers in the residential area and enter the corridor at the same point. The station of P&R is single which locates on the terminal of the metro.

A7: The parking and transfer time of all P&R passengers are regarded with the same value.

A8: The access time of all ODPB passengers are the same.

A9: The number of parking space is equal to the number of P&R passengers.

A10: Each bus serves one band of the residential area, as shown in Figure 1.

3.3. Passenger demand

Consider a linear travel corridor of length L (=L_{1+} L_{2+} L_3) connecting city centre and suburb residential area, as shown in Figure 1. w and L_1 are the width and length of the residential area, L_2 is the length from the border of residential area to the nearest metro station. L_3 is the length of metro from the P&R metro station to CBD.

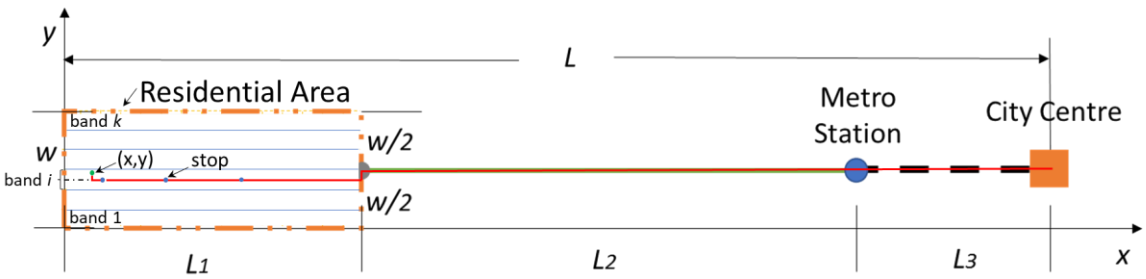

Figure 1 A Linear Corridor

Let $g(x,y)$ be the population density in the residential area. Then the potential demand density in peak hour can be defined by

$$p(x,y) = \phi\tau g(x,y), \forall x \in [0, L_1], y \in [0, W] \tag{1}$$

where ϕ is the average number of trips per person to city centre per day, τ is the peak hour factor. The distribution of the demand density is the same with that of population.

According to A4, sensitivity parameters are considered to model the effects of demand elasticity. Three time cost components, including access time, waiting time and in-vehicle time, and one fare component are considered as the factors that lead to different passenger perceptibility (Chang and Schonfeld, 1991; Li et al, 2012). As was done in many previous research works (Anas, 1982; Li et al, 2012), the actual passenger demand is represented as

$$q(x,y)=p(x,y)\exp\left(\sum_{n=1}^{4}u_n f_n(x,y)\right),\forall x\in[0,L_1],y\in[0,W] \tag{2}$$

where $f_1(x,y)$, $f_2(x,y)$, $f_3(x,y)$and $f_4(x,y)$ are access time, waiting time, in-vehicle time and fare respectively; μ_1 , μ_2 , μ_3 and μ_4 are the sensitivity parameters for access time, waiting time, in-vehicle time and fare respectively. The increment of access time, waiting time, in-vehicle time and fare will decrease the actual demand, the sensitivity parameters are all nonpositive. If the sensitivity parameter equals to zero, the factor has no effects on the travel demand.

3.3.1. For P&R

The whole trip time includes in-vehicle time (IVT) and out-vehicle time (OVT). For P&R passengers, the main OVT are the access time (park cars and walk to the metro platform) and waiting time. According to A8, the detail OVT cost of P&R passengers can be given by

$$f_1^{PR}(x,y)=t_{parking}+t_{transfer},\forall x\in[0,L_1],y\in[0,W] \tag{3}$$

$$f_2^{PR}(x,y)=t_{waiting}=\frac{H_{metro}}{2},\forall x\in[0,L_1],y\in[0,W] \tag{4}$$

where H_{metro} is the metro service headway, and we assume that the average waiting time is equal to half of the headway.

The P&R passengers' in-vehicle travelling time from the origin to destination includes three parts: the in-vehicle travel time by car ($\text{driving distance}/v_{auto}$) and the in-vehicle travel time by metro (L_3/v_{metro}). The driving distance for each passenger is different, according to A6 and Figure 1, one can obtain the passengers' in-vehicle time at different locations as follows

$$f_3^{PR}(x,y)=\begin{cases}\dfrac{L_1+L_2+\dfrac{W}{2}-x-y}{v_{auto}}+\dfrac{L_3}{v_{metro}},\forall 0\le y\le\dfrac{W}{2}\\ \dfrac{L_1+L_2-\dfrac{W}{2}-x+y}{v_{auto}}+\dfrac{L_3}{v_{metro}},\forall\dfrac{W}{2}<y\le W\end{cases},\forall x\in[0,L_1],y\in[0,W] \tag{5}$$

The monetary cost of P&R passengers includes three parts: vehicle cost, parking fee and metro ticket. The vehicle cost is mainly composed of fuel cost and vehicle depreciation which is distance base. Parking fee and metro ticket is fixed. So, the total cost is defined as

$$f_4^{PR}(x,y)=\begin{cases}\left(L_1+L_2+\frac{W}{2}-x-y\right)f_{auto}+f_{parking}+f_{metro},\forall 0\le y\le\frac{W}{2}\\ \left(L_1+L_2-\frac{W}{2}-x+y\right)f_{auto}+f_{parking}+f_{metro},\forall\frac{W}{2}<y\le W\end{cases},\forall\begin{cases}x\in[0,L_1]\\ y\in[0,W]\end{cases} \tag{6}$$

where f_{auto} is the vehicle cost per unit distance.

The total P&R passenger demand can be calculated by

$$Q^{PR}=\int_0^W\int_0^{L_1}q(x,y)dxdy,\forall x\in[0,L_1],y\in[0,W] \tag{7}$$

Substituting Eqs.(1) –(6) into (7), one can obtain

$$Q^{PR}=\phi\tau\int_0^{W/2}\int_0^{L_1}g(x,y)e^{\lambda_1+\lambda_2\left(\frac{W}{2}-x-y\right)}dxdy+\phi\tau\int_{W/2}^{W}\int_0^{L_1}g(x,y)e^{\lambda_1+\lambda_2\left(-\frac{W}{2}-x+y\right)}dxdy,\forall\begin{cases}x\in[0,L_1]\\ y\in[0,W]\end{cases} \tag{8}$$

where

$$\lambda_1=\mu_1\left(t_{parking}+t_{transfer}\right)+\mu_2\frac{H_{metro}}{2}+\lambda_2\left(L_1+L_2\right)+\frac{\mu_3}{v_{metro}}L_3+\mu_4\left(f_{parking}+f_{metro}\right) \tag{9}$$

$$\lambda_2=\mu_4 f_{auto}+\frac{\mu_3}{v_{auto}} \tag{10}$$

3.3.2. For ODPB

Similarly, according to A6, A10 and Figure 1, all routes are along horizontal and/or vertical direction. The ODPB passengers' access time is the time cost from their location (x, y) to nearest ODPB stop (on line *i*), which can be expressed as follows

$$f_1^O(x,y,i)=\begin{cases}\frac{y-\frac{2i-1}{2k}W}{v_{access}}+t_{access,x},\forall\frac{2i-1}{2k}W\le y\le\frac{i}{k}W\\ \frac{\frac{2i-1}{2k}W-y}{v_{access}}+t_{access,x},\forall\frac{i-1}{k}W\le y<\frac{2i-1}{2k}W\end{cases},\forall\begin{cases}x\in[0,L_1]\\ y\in[0,W]\end{cases},i=1,2,\cdots,k \tag{11}$$

where v_{access} is the passengers' average speed from the location (x,y) to nearest ODPB line, $t_{access,x}$ is the average time cost from passengers' projected points on ODPB line to nearest stop, k is the number of ODPB service lines.

Passengers' in-vehicle time is calculated by ODPB trip distance and average speed, as follows

$$f_3^O(x,y,i)=\begin{cases}\frac{L_1+L_2+L_3-x+\frac{2i-1}{2k}W-\frac{W}{2}}{v_O},\forall 2i\ge k+1\\ \frac{L_1+L_2+L_3-x+\frac{W}{2}-\frac{2i-1}{2k}W}{v_O},\forall 2i<k+1\end{cases},\forall\begin{cases}x\in[0,L_1]\\ y\in[0,W]\end{cases},i=1,2,\cdots,k \tag{12}$$

ODPB ticket price is fixed and can be given by

$$f_4^O(x,y,i)=f_O,\forall x\in[0,L_1],y\in[0,W],i=1,2,\cdots,k \tag{13}$$

According to A10, one bus serves one band, total passenger demand of each band should not excess the bus capacity. Meanwhile, minimum bus constraint is also considered. Thus, *k* satisfies the condition of

$$(k-1)c^O<Q^O\le kc^O \qquad \forall k=1,2,\cdots,N \tag{14}$$

where c^O is the capacity of one ODPB service line.

Thus, the ODPB total demand can be calculated by

$$Q^O=\sum_{i=1}^{k}\int_0^W\int_0^{L_1}p(x,y)\exp\left(\sum_{n=1}^{4}\mu_n f_n^O(x,y,i)\right)dxdy,\forall x\in[0,L_1],y\in[0,W],i=1,2,\cdots,k \tag{15}$$

3.4. Social welfare

As shown in Figure 2, consumer surplus is calculated by analysing the difference between what consumers are willing and able to pay for a good or service relative to its market price, it is the welfare of consumer. Operator's net profit is defined as the difference between the amount the operator is willing to supply goods for and the actual amount received by him when he makes the trade. It is the welfare of operator. So, the social welfare is defined as the sum of the consumer surplus (noted by C) and the operator's net profit (noted by O), that is

$$SW=C+O \tag{16}$$

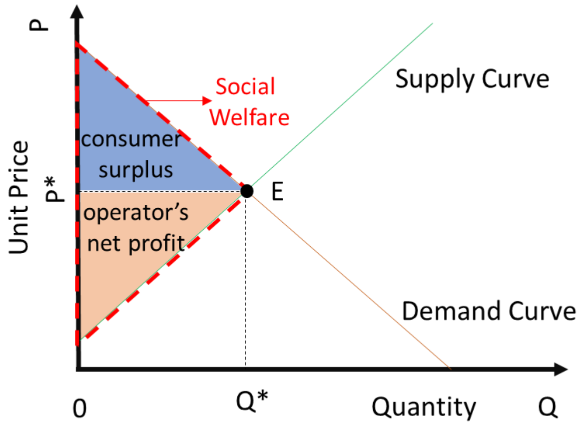

Figure 2 A Supply-demand Diagram

3.4.1. Consumer surplus

With the derived demand density function, one can express the fare as a function of demand density. The total social benefit can be obtained by integrating the inverted function over the demand. Then the consumer surplus can be derived as cost savings, i.e., the users are willing to pay minus the price that the users actually pay.

3.4.1.1. For P&R

Based on the discussion above, the inverse of the demand density function of P&R can be expressed as

$$q^{PR^{-1}}(x,y)=\frac{1}{\mu_4}\left(\ln\frac{q^{PR}(x,y)}{p(x,y)}-\sum_{n=1}^{3}\left(\mu_n f_n(x,y)\right)\right),\forall x\in[0,L_1],y\in[0,W] \tag{17}$$

The consumer surplus of P&R originating at location (x,y) can be formulated as

$$C^{PR}(x,y)=\int_{0}^{q^{PR}(x,y)}\frac{1}{\mu_4}\left(\ln\frac{z}{p(x,y)}-\sum_{n=1}^{3}\left(\mu_n f_n(x,y)\right)\right)dz-q^{PR}(x,y)\,f_4^{PR}(x,y),\forall\begin{cases}x\in[0,L_1]\\y\in[0,W]\end{cases} \tag{18}$$

According to the Eqs.(8)-(10) and (18), the consumer surplus for P&R can be calculated by

$$C^{PR}=-\frac{\phi\tau}{\mu_4}\left(\int_0^{W/2}\int_0^{L_1}g(x,y)e^{\lambda_1+\lambda_2\left(\frac{W}{2}-x-y\right)}dxdy+\int_{W/2}^{W}\int_0^{L_1}g(x,y)e^{\lambda_1+\lambda_2\left(-\frac{W}{2}-x+y\right)}dxdy,\right) \tag{19}$$

3.4.1.2. For ODPB

In a similar way, we can get the inverse of the demand density function for ODPB as

$$q^{O^{-1}}(x,y,i)=\frac{1}{\mu_4}\left(\ln\frac{q^{O}(x,y,i)}{p(x,y,i)}-\sum_{n=1}^{3}\left(\mu_n f_n(x,y,i)\right)\right),\forall x\in[0,L_1],y\in[0,W],i=1,2,\cdots,k \tag{20}$$

Then, the ODPB consumer surplus originating at location (x,y) and boarding on line *i* can be expressed as

$$C^{O}(x,y,i)=-\frac{1}{\mu_4}p(x,y)\exp\left(\sum_{n=1}^{4}\left(\mu_n f_n^{O}(x,y,i)\right)\right),\forall x\in[0,L_1],y\in[0,W],i=1,2,\cdots,k \tag{21}$$

Thus, according to the Eqs.(8)-(10) and (21), the total consumer surplus of ODPB can be calculated by

$$C^{O}=-\frac{1}{\mu_4}\sum_{i=1}^{k}\int_0^{W}\int_0^{L_1}p(x,y)\exp\left(\sum_{n=1}^{4}\left(\mu_n f_n^{O}(x,y,i)\right)\right)dxdy,\forall i=1,2,\cdots,k \tag{22}$$

3.4.2. Operator's net profit

The net profit of operator can be defined as the total operating revenue (noted by R) minus the total cost (noted by Ct), that is

$$O=R-Ct \tag{23}$$

3.4.2.1 For P&R

The total revenue is the sum of parking fee and metro price paid by all the P&R passengers, that is

$$R^{PR}=Q^{PR}\left(f_{parking}+f_{metro}\right) \tag{24}$$

The cost mainly includes metro operation cost, parking construction and management fees, while the metro line and service are already existing base on the assumptions, so metro operation cost is neglected here, and we regard the cost is related to the passenger demand, which is calculated by

$$Ct^{PR} = Q^{PR} M^{PR} \tag{25}$$

where M^{PR} is the average cost per passenger.

3.4.2.2 For ODPB

Similarly, we can get the total revenue and cost of ODPB

$$R^{O} = Q^{O} f_{O} \tag{26}$$

$$Ct^{O} = kM^{O} \tag{27}$$

where M^{O} is the cost of one ODPB line.

4. Service selection model

Substituting Eqs.(8)-(10), (19) and (23)-(25), one can obtain the social welfare of P&R as follows

$$SW^{PR} = \left(f_{parking} + f_{metro} - \frac{1}{\mu_4} - M^{PR} \right) Q^{PR}, \quad \forall x \in [0, L_1], y \in [0, W] \tag{28}$$

The social welfare maximization model of P&R service can be formulated as

$$\max SW^{PR}\left(L_1, L_2, L_3, W, \mathrm{f}_{\mathrm{parking}}, g(x,y)\right)$$
$$= \phi\tau \left(f_{parking} + f_{metro} - \frac{1}{\mu_4} - M^{PR} \right) \left(\int_0^{W/2} \int_0^{L_1} g(x,y) e^{\lambda_1 + \lambda_2 \left(\frac{W}{2} - x - y\right)} dxdy + \int_{W/2}^{W} \int_0^{L_1} g(x,y) e^{\lambda_1 + \lambda_2 \left(-\frac{W}{2} - x + y\right)} dxdy \right)$$

(29)

s.t. $$\lambda_1 = \mu_1 \left(t_{parking} + t_{transfer} \right) + \mu_2 \frac{H_{metro}}{2} + \lambda_2 \left(L_1 + L_2 \right) + \frac{\mu_3}{v_{metro}} L_3 + \mu_4 \left(f_{parking} + f_{metro} \right) \tag{9}$$

$$\lambda_2 = \mu_4 f_{auto} + \frac{\mu_3}{v_{auto}} \tag{10}$$

$$f_{parking} \geq 0$$

In a similar way, substituting Eqs.(16),(22),(23),(26) and (27), one can obtain the social welfare of ODPB as follows

$$SW^{O} = \left(f_{O} - \frac{1}{\mu_4} \right) Q^{O} - kM^{O}, \forall x \in [0, L_1], y \in [0, W], k = 1, 2, \cdots, N \tag{30}$$

The social welfare maximization model of ODPB service can be formulated as

$$\max SW^{O}\left(L_1, L_2, L_3, W, \mathrm{f}_{\mathrm{O}}, g(x,y)\right)$$
$$= \left(f_{O} - \frac{1}{\mu_4} \right) \sum_{i=1}^{k} \int_0^{W} \int_0^{L_1} \phi\tau g(x,y,i) \exp\left(\sum_{n=1}^{4} \left(\mu_n f_n^{O}(x,y,i) \right) \right) dxdy - kM^{O} \tag{31}$$

s.t. $$(k-1)c^{O} < Q^{O} \leq kc^{O}$$

$$i = 1, 2, \cdots, k$$
$$k = 1, 2, \cdots, N$$
$$f_O \geq 0$$

The authority usually chooses the service which can bring higher social welfare. Therefore, the proposed service selection model satisfies

$$\begin{aligned} & SW_{m*}\left(L_1, L_2, L_3, W, f_{m*}, g\left(x, y\right)\right) \\ & = \max\left\{SW_{m1}\left(L_1, L_2, L_3, W, f_{m1}, g\left(x, y\right)\right), SW_{m2}\left(L_1, L_2, L_3, W, f_{m2}, g\left(x, y\right)\right)\right\} \end{aligned} \tag{32}$$

where SW is social welfare, $m1, m2$ are the alternative transit modes. This model also works when only consumer surplus or operator's net profit is considered.

As only two transit modes are considered in this study, the mode $m1$ is more attractive if and only if it satisfies

$$SW_{m1}\left(L_1^*, L_2^*, L_3^*, W^*, f_{m1}^*, g^*\left(x, y\right)\right) \geq SW_{m2}\left(L_1^*, L_2^*, L_3^*, W^*, f_{m2}^*, g^*\left(x, y\right)\right) \tag{33}$$

Other than finding the favourable public transport mode in different scenarios, one can also use the model to acquire more policy implications. For example, from Eq. (33), it is easy to find the critical values of different parameters, which satisfies

$$SW_{m1}\left(L_1^*, L_2^*, L_3^*, W^*, f_{m1}^*, g^*\left(x, y\right)\right) = SW_{m2}\left(L_1^*, L_2^*, L_3^*, W^*, f_{m2}^*, g^*\left(x, y\right)\right) \tag{34}$$

In this case, if the authority has the basic data and growth rate, they can directly predict the future construction and investment plan. Assuming that the growth rate of $L_1, W, g\left(x, y\right)$ per year are l, w, g respectively. Then we can define the simple growth rate models of the parameters for year *t* as follows

$$L_1\left(t\right) = L_1\left(0\right)\left(1 + l\right)^t \tag{35}$$

$$W\left(t\right) = W\left(0\right)\left(1 + w\right)^t \tag{36}$$

$$g\left(t\right) = g\left(0\right)\left(1 + g\right)^t \tag{37}$$

With the growth of these parameters, the SW of each mode will also change. If they have the critical value (Eq.(34)) in the future, the SW lines will intersect with each other. The time of the intersection point of SW lines will be the best investing time for new service mode. Thus, we can calculate the investing time as follows

$$t^* = \left\{ t \left| \begin{aligned} & SW_{m1}\left(L_1^*\left(t\right), L_2^*\left(t\right), L_3^*, W^*\left(t\right), f_{m1}^*, g^*\left(t, x, y\right)\right) \\ & = SW_{m2}\left(L_1^*\left(t\right), L_2^*\left(t\right), L_3^*, W^*\left(t\right), f_{m2}^*, g^*\left(t, x, y\right)\right) \end{aligned} \right. \right\} \tag{38}$$

5. Solution Algorithm

If all the parameters ($L_1, L_2, L_3, W, g\left(x, y\right), f_{parking}, f_O$) are considered as unknown variables, and tour objective is to search the global optimal value of the function, then it is a proper approach to develop a heuristic algorithm. However, in practice, the density, distance and residential area size are usually known parameters, the optimal solutions can be obtained by utilizing the information of partial derivatives of objective function. In this case, the computational efficiency and accuracy of the heuristic algorithm is not an issue.

5.1. For P&R

Firstly, we take the first order partial derivative of the P&R objective function with respect to parking fee, and set the partial derivatives equal to zero. Then we have

$$\frac{\partial SW^{PR}}{\partial f_{parking}} = \mu_4\left(f_{parking} + f_{metro} - M^{PR}\right)\int_0^W\int_0^{L_1} p(x,y)\exp\left(\sum_{n=1}^{4}\left(\mu_n f_n^{PR}(x,y)\right)\right)dxdy = 0 \tag{39}$$

where the sensitivity parameter of fare μ_{fare} is always negative under the elastic assumption. If the population density is non-zero, the integral item is also positive. We can get the extreme value as follows

$$f_{parking} = M^{PR} - f_{metro} \tag{40}$$

Taking second order partial derivative of the P&R objective function with respect to parking fee, we can obtain

$$\frac{\partial^2 SW^{PR}}{\partial f_{parking}{}^2} = \left(1 + \mu_4\left(f_{parking} + f_{metro} - M^{PR}\right)\right)\mu_4\int_0^W\int_0^{L_1} p(x,y)\exp\left(\sum_{n=1}^{4}\left(\mu_n f_n^{PR}(x,y)\right)\right)dxdy \tag{41}$$

Base on the elastic assumption, if $f_{parking} = M^{PR} - f_{metro}$, the partial derivatives will always be negative, so the extreme value is the maximum value , i.e., the optimal solution of the model. Meanwhile, from Eq.(34), we can also find that the optimal parking fee is independent of the other variables ($L_1, L_2, L_3, W, g(x,y)$).

Similarly, we can prove the optimal parking fee for operator's net profit maximization satisfies

$$f_{parking}^{*} = M^{PR} - f_{metro} - \frac{1}{\mu_4} \tag{42}$$

5.2. For ODPB

The passengers demand is elastic. According to Eqs.(14) and (15), we can notice that if the ODPB ticket price f_O decreases, the total demand Q^O will increase, then more buses are needed, *k* will increase. It means lower price will lead to larger *k*, the maximum *k* exists when $f_O = 0$. As *k* is integer, the ODPB objective function can be regarded as a piecewise function. For any section, the partial derivative of function could be obtained as follows

$$\frac{\partial SW^{O}}{\partial f_O} = \mu_4 f_O \sum_{i=1}^{k}\int_0^W\int_0^{L_1} p(x,y)\exp\left(\sum_{n=1}^{4}\left(\mu_n f_n^{O}(x,y,i)\right)\right)dxdy \tag{43}$$

where the ticket and the integral item are always positive, the sensitivity parameter of fare is always negative $\mu_4 < 0$, so the partial derivative is negative. It means the ODPB social welfare objective function is a decreasing and convergent function in each section. Thus, the optimal ODPB price is the maximum value of all the sections. It satisfies the following equation

$$SW^O = \max\left\{SW^O\left(f_O^{\square}\right), SW^O\left(f_O = 0\right)\right\} \tag{44}$$

where $f_O^{\square}$ and $k^{\square}$ are bound value, satisfy

$$Q^O\left(f_O^{\square}\right) = kc^O, \forall k = 1,2,\cdots,k^* - 1 \tag{45}$$

$$Q^O\left(f_O = 0\right) = k^{\square} c^O \tag{46}$$

Following the same logic, we obtain the partial derivative function for operator's net profit in the followings

$$\frac{\partial O^O}{\partial f_O} = \left(1 + \mu_4 f_O\right)\sum_{i=1}^{k}\int_0^W\int_0^{L_1} p\left(x,y\right)\exp\left(\sum_{n=1}^{4}\left(\mu_n f_n^O\left(x,y,i\right)\right)\right)dxdy \tag{47}$$

If $1+\mu_4 f_O > 0$, the function O^O is increasing in each section; otherwise the function O^O is decreasing in each section. In either case, the maximum is at the boundary. Therefore, the optimal ODPB price for operator's net profit maximization model satisfies the following equation

$$O^O = \max\left\{O^O\left(f_O^{\square 1}\right), O^O\left(f_O^{\square 2}\right), O^O\left(-\frac{1}{\mu_4}\right)\right\} \tag{48}$$

where $f_O^{\square 1}$ and $f_O^{\square 2}$ are bound value, satisfy

$$Q^O\left(f_O^{\square 1}\right) = kc^O, \forall 1 + \mu_4 f_O > 0, k = 1,2,\cdots,N \tag{49}$$

$$Q^O\left(f_O^{\square 2}\right) = \left(k-1\right)c^O, \forall 1 + \mu_4 f_O < 0, k = 1,2,\cdots,N \tag{50}$$

5.3. Investing time

The procedure of searching the investing time in term of Eq. (38) is listed as follows:

Step 1: Set the initial values ($L_1, W, g\left(x,y\right), l, w, g, t$) and the iterative step$\square t$;

Step 2: Solve the social welfare or the profit maximization models for ODPB and P&R, and compare the social welfare or the profit result and calculate the difference;

Step 3: Update the value of $L_1, W, g\left(x,y\right), t$ by Eqs.(35)-(37), and redo Step 2;

Step 4: If the difference becomes larger, then stop (no critical point). Otherwise, repeat Step 3 until the difference changes its plus-minus.

6. Case study

Consider the corridor in Figure 1 with two alterative services P&R and ODPB, wherein parameters are set as follows:

The parameters relevant to the size of the residential area and corridor are set to be: L_1=2, L_2=20, L_3=3, W=1; the settings of other parameters are referred to some transport statistical reports and previous studies (Beijing Traffic Management Bureau, 2016; Chen et al., 2015; Meng et al., 2017): $\phi = 1$, $\tau = 0.1$, $t_{parking} = 1/30$, $t_{transfer} = 1/60$, $\mu_1 = -3.6$, $\mu_2 = -2.3$, $\mu_3 = -1.6$, $\mu_4 = -0.025$, $H_{metro} = 1/30$, $v_{auto} = 54$, $v_{metro} = 43.2$, $v_O = 30$, $v_{access} = 7.2$, $f_{auto} = 1$,

$f_{metro} = 5$, $c^O = 100$, $M^{PR} = 30$, $M^O = 1000$.

6.1. Influence of population distribution

Despite that uniform population density was commonly used in many previous research due to its simplicity, it is not very realistic. In this study, aiming to investigate the influence of different types of population distribution onto the proposed model, we consider two types of classical non-uniform distribution patterns (Berry et al., 1963; Alperovich et al., 1994):

a) Type a: The density is high in the middle of residential area and reduces exponentially toward the city boundary, which can be expressed as

$$g(x,y) = g_0 e^{\varphi\left[\left(\frac{L_1}{2}\right)^2 + \left(\frac{W}{2}\right)^2 - \left(\frac{L_1}{2} - x\right)^2 - \left(\frac{W}{2} - y\right)^2\right]} \tag{51}$$

where φ is the sensitivity parameter, if it is equal to zero, the density is uniform.

b) Type b: The density is highest at the city centre and goes down linearly towards the city boundary, which can be expressed as

$$g(x,y) = g_0\left(1 + \frac{x}{L_1}\psi\right) \tag{52}$$

where ψ is the sensitivity parameter, if it is equal to zero, the density is uniform.

For comparison purpose, the uniform population density is named as type c. Figure 3 describes the general patterns of the three types of population distribution.

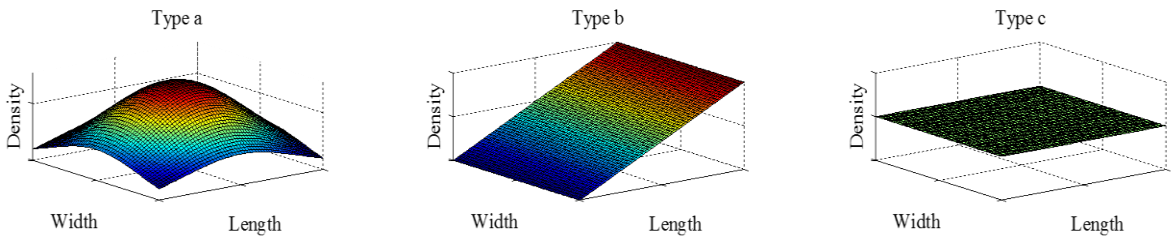

Figure 3 Three types of population distribution

For a fixed size and layout corridor, we assume that the highest density is three times higher than that lowest and the total population of different distribution is identical. Then we can calculate the maximum social welfare and operator's net profit that can be achieved in presence of P&R and ODPB respectively. The numerical results are shown in Table 2.

Table 2 Social welfare and operator's net profit of different modes for different density distribution

		Social Welfare				Operator's net profit			
Mode		P&R		ODPB		P&R		ODPB	
Distribution		a	c	b	c	a	c	b	c
Density	1	1565	1564	1464	1459	489	489	-1020	-1022
	2	3130	3129	2834	2826	978	977	-40	-44
	3	4695	4693	4456	4448	1467	1466	940	934

	4	6260	6258	7127	7111	1955	1955	1939	1930
	5	7825	7822	9253	9228	2444	2444	1949	1939
	6	9390	9387	11144	11111	2933	2932	2939	2927
	7	10955	10951	13611	13577	3422	3421	3929	3915
	8	12520	12516	15597	15556	3911	3910	4963	4945
	9	14085	14080	17464	17414	4400	4398	4965	4947
	10	15650	15645	19993	19943	4889	4887	5962	5941
	11	17214	17209	21929	21870	5377	5376	6958	6935
	12	18779	18774	23784	23718	5866	5865	8000	7974

From Table 2, it can be seen that if the total population is the same, the population distribution type has little influence on the social welfare and operator's net profit. Therefore, the following tests shall use uniform population density to analyse the model characteristics instead of other distribution patterns.

6.2. Influence of population density

Figures 4 and 5 demonstrate the relationship between population density and social welfare and operator's net profit respectively. As is well known, high level of social welfare or operator's net profit are the major factors that determine whether the transport facility project could be eventually be implemented in practice. As the population density increases, both the social welfare and operator's net profit of ODPB and P&R will increase. Specifically, both increasing rates of the ODPB are quicker than P&R. As a result, the social welfare and operator's profit curves of ODPB intersect with those of P&R at points A and B, respectively. The population density at point A is 3240 persons/km^2, while the population density at point B is 6010 persons/km^2. It indicates that, if we only consider the social welfare, when the population density is higher than 3240 persons/km^2, the ODPB is a better choice. If we take the operator's profit into account, the lowest requirement of population density is 6010 persons/km^2 if ODPB is preferred. These results are very indicative to transport planners when they need to choose between the service of ODPB and P&R. Basically, only when population density reaches certain minimum requirement, will the ODPB be preferred. One can also observe that, as the population density goes up, the social welfare of ODPB is increasing more rapidly than operator's profit, which is not uncommon for public transport service. Therefore, if the project is invested by government, the required population density in favour of ODPB is lower. When private sectors are involved into the project investment and the operator's net profit has to be considered, higher population density is required.

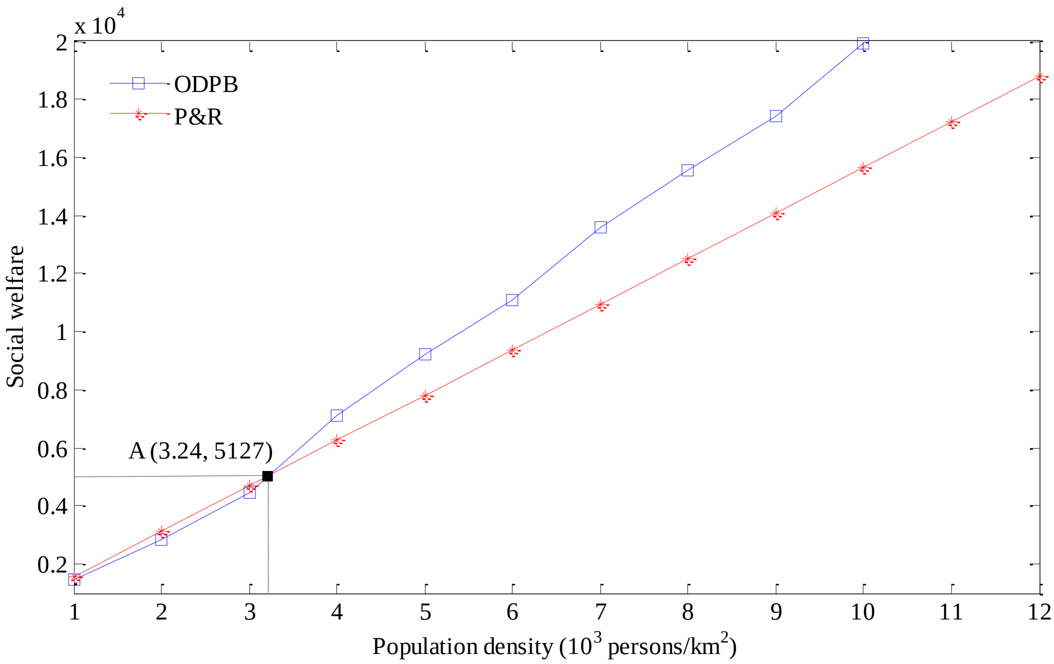

Figure 4 The relationship between population density and social welfare

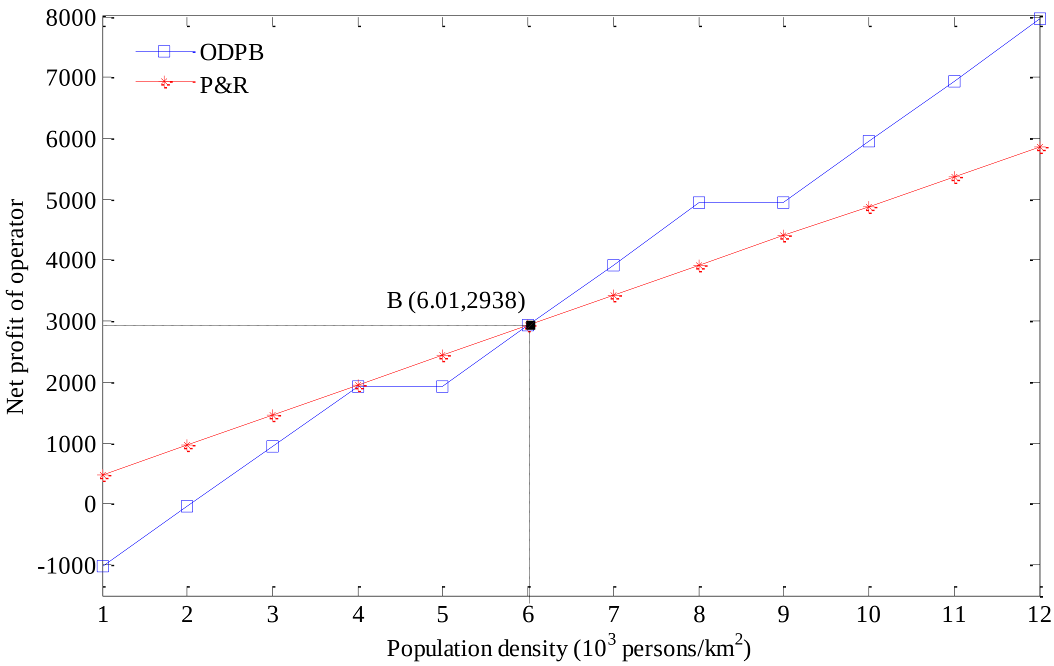

Figure 5 The relationship between population density and operator's net profit

As mentioned above, P&R is usually popular in Europe, Australia and America, while almost no successful case in Asia. The results of Figures 4 and 5 provide the explanation. P&R mode has advantage only if the population density is low, which features well the cities in western countries. However, it is way different from Asian cities, like Beijing and Singapore, which has very high urban population density. Chen et al. (2015) found that LRT is preferable than BRT if the population density reached to 10400 persons/km^2. Together with the results from Figures 4 and 5, it can be concluded that in this case, the ODPB service is a smart choice when the population density is above 3240 persons/km^2 and below 10400 persons/km^2.

6.3. Influence of residential area size and layout

The size and layout of residential area will also affect travellers' choice behaviour. Figure 6 indicates the preferable transit mode for different residential area size. It is obvious that when the population density is fixed, increasing residential area means larger total population size. In this case, ODPB is at a position with evident advantage.

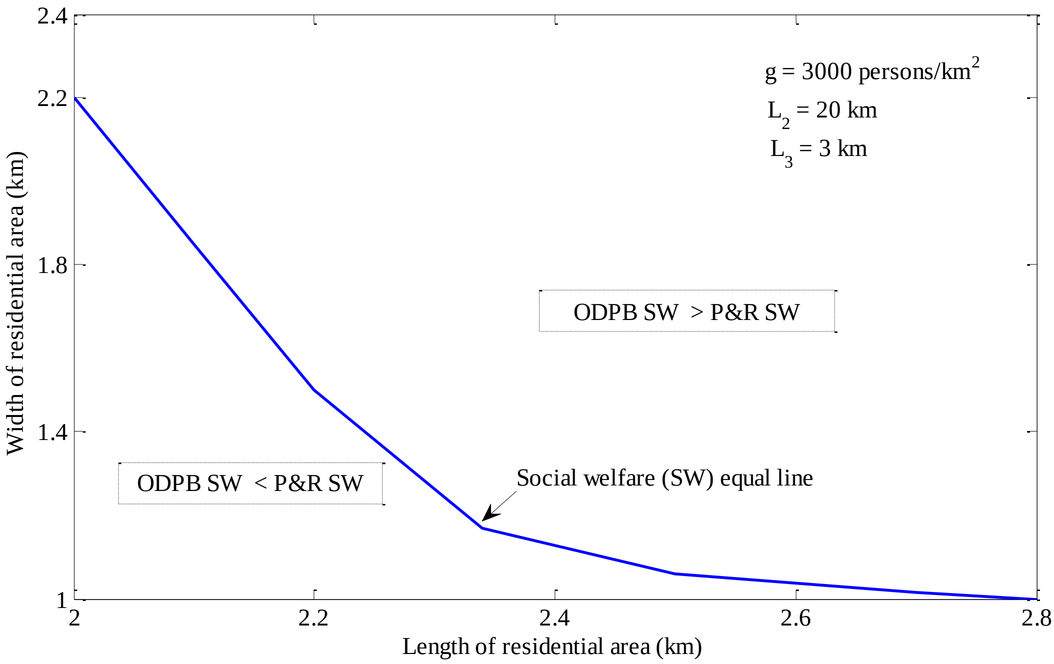

Figure 6 The advantage transit mode for different residential area size

6.4. Influence of P&R facility location social welfare

In the literature, the optimal location of P&R facility has been investigated by many researchers (e.g., Du and Wang, 2014). Indeed, the determination of P&R facility location also in practice needs to take into account many factors, such as land use planning. Therefore, it is necessary to test the influence of corridor distance and P&R facility location onto the model results. Figures 7 and 8 showed the influence of P&R location onto social welfare under high and low population density conditions. In principle, short corridor is not suitable for operating P&R and ODPB services. As compared with other transport modes, these two services could not reduce the travel cost significantly. From Figure 7, one can note that when the distance from residential area to P&R station is relatively shorter, P&R service performs better in terms of leading to higher level of social welfare. Nevertheless, this finding may have little practical value, because when the distance from residential are to P&R station is sufficiently short, most of the travellers would directly use metro to complete their trips rather than using P&R. However, if the distance from P&R station to CBD is long, the advantage of P&R service will shrink, which is partly due to the fixed metro fare assumption. In reality, many of developing countries with high population density are lack of complete metro system. OPDB, which does not need high investment cost, is an ideal service to support the transition period before a complete transport system is built up. Figure 8 depicted that P&R remains the advantage than

ODPB service when the population density is low, which further supported the findings in section 6.2.

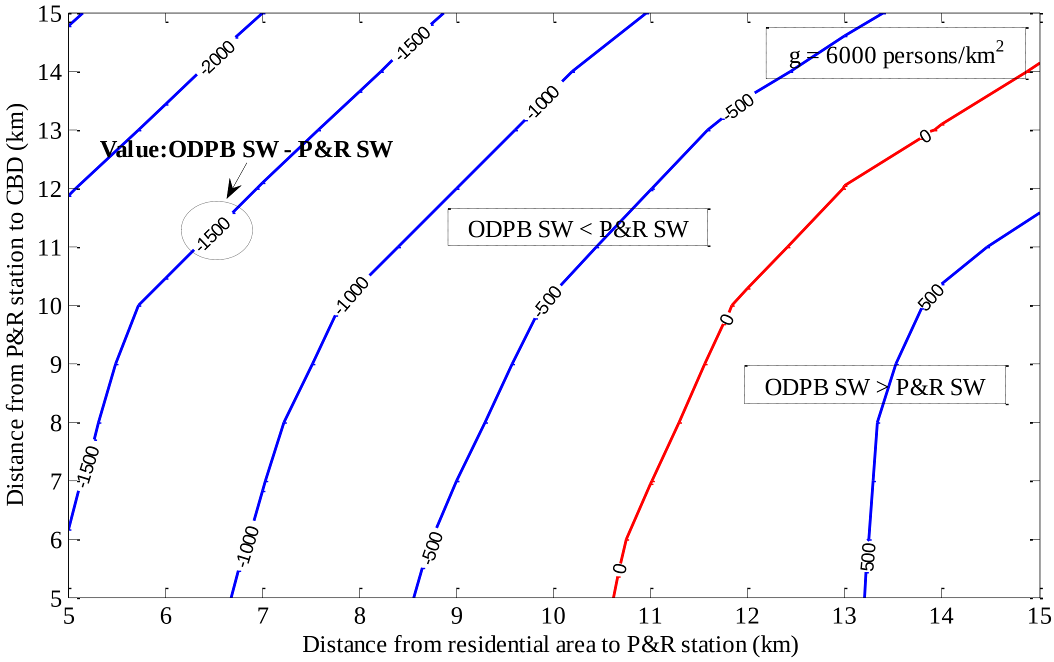

Figure 7 The influence of P&R station facility on social welfare for high population density

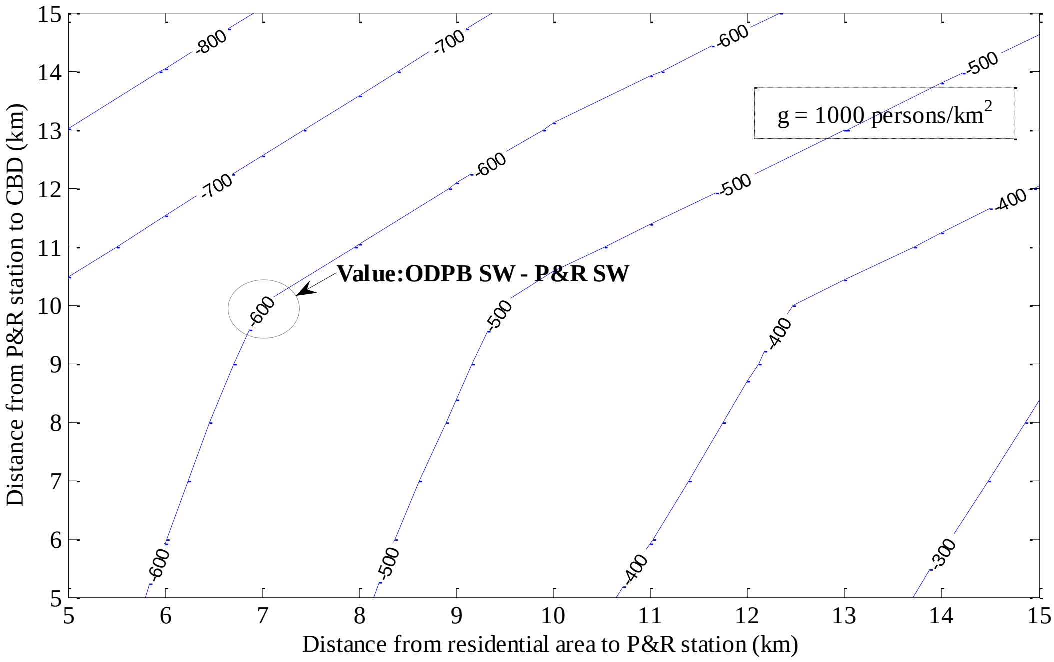

Figure 8 The influence of P&R facility location on social welfare for low population density

6.5. Investment time

From previous analysis, we can conclude that both the residential area and the of total population size have significant effects on the attractiveness of ODPB and P&R services to travellers. Therefore, one intrinsic question to be addressed by the policy makers is that, when is the best time to invest and provide these two services in the planning horizon. The effect of the changes from some major factors on the service selection are analysed as follows.

a) The case when population density increases but residential area size remains unchanged

This situation can be explained based on the results from Figures 4 and 5. According to the crossing points A and B, we can obtain the population density threshold value depending on the objective of the service selection. If social welfare is set as the primary objective for service selection, then point A will be the critical population density threshold value. When the population density is larger than the value at point A (3240 persons/km^2), ODPB is more suitable than P&R for investment. If the operator's net profit is set as the goal during service selection, then point B will be the critical population density threshold point. When the population density is larger than the value at point B (6010 persons/km^2), ODPB is more favourable for investment. Figure 9 shows a case wherein we assume the population density is 3000 persons/km^2 and the annual growth rate is 5%. It can be seen that two years later the population density will reach the critical population density threshold point for introducing ODPB if the project will be operated by government. If both social welfare and operator's profit are considered, the proper investment time for ODPB will be 14 years later. If the density continues increasing, then LRT or metro will be the best choice in next phase according to Chen et al. (2016)'s results.

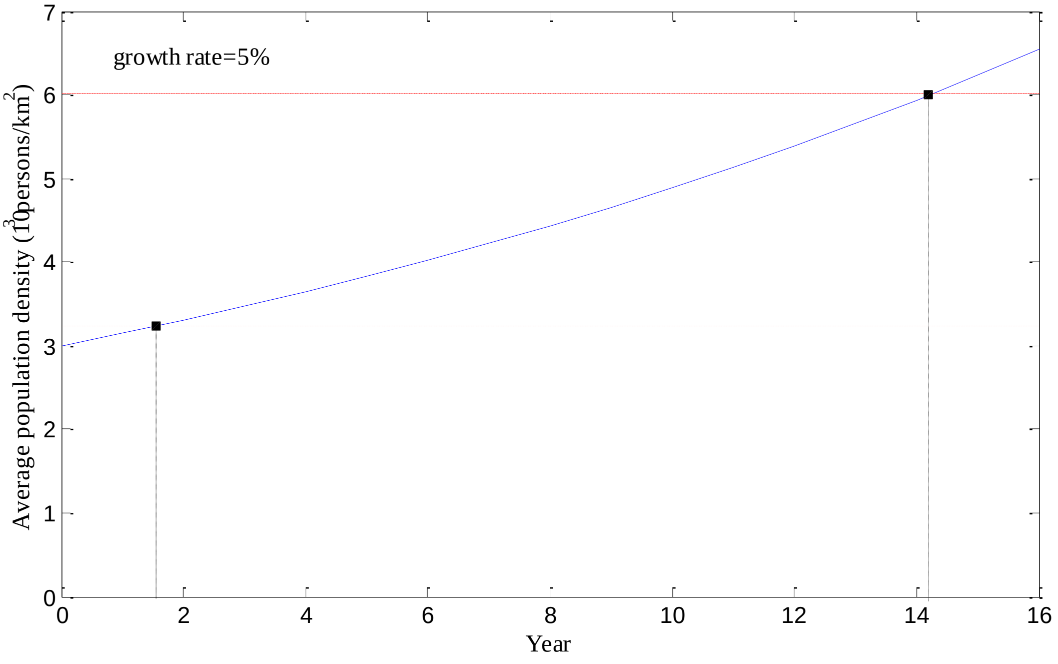

Figure 9 Estimation of ODPB investment time from the view of social welfare and profit

b) The case when residential area size increases but population density remains unchanged

This situation can be explained based on the results from Figure 6. The figure shows the social welfare critical curves for ODPB and P&R services. If the residential area size is 2km * 1km, we use the point (2.8, 0) as an example. It means that when the expansion is only along the corridor and the rate is 0.1 km/year, 8 (= (2.8-2)/0.1) years later the ODPB service will be more favourable. Additionally, if the operator's net profit is the objective in this service selection, the investment time of new service is longer, as is similar to the previous examples.

7. Conclusion

In this paper, a service selection model is proposed to assist the policy makers in selecting one transport service from the two alternative choices: P&R and ODPB. To the authors' best knowledge, the selection between these two services has not been discussed before. The model aims to achieve maximum social welfare or operator's net profit. The optimal solution is obtained through a heuristic procedure.

A set of numerical results have been given to illustrate the model's effectiveness. The influence from several macroscopic factors have been discussed, such as the level of population density, distribution of population density, size of residential area, distance between the residential area to centre business area, P&R station location, the length of metro line, as well as the changes of residential area layout and population density growth.

The model results indicate that, if the total population is stable in a fixed residential area, the population distribution type has little influence on the social welfare and operator's net profit. Meanwhile, the level of population density, trip distance and other factors considered in this model significantly affect the service selection results. P&R fits for low population density area, while ODPB is more suitable for high population density area. This finding also explains why P&R has been successful in European cities than Asian cities. Achieving positive social welfare is easier than achieving positive operator's net profit as the required urban density is lower. This is one of the reasons why most of the public services are launched and operated by government. ODPB can bring more social welfare than P&R for long distance trip, especially when the total length of metro lines is short. Due to the low cost of investment, ODPB is a smart choice for the cities in developing countries with insufficient infrastructure systems to serve the long trip passengers. At last, the investment time of the service provision in the planning horizon is analysed.

The proposed model could provide guidelines for government agencies and operators in their service selection process. The model can be also extended to the other service selection issues, considering different objectives. However, the model also has many limitations due to the simplified model assumptions. Future work could investigate the service selection issues in a multimodal public transport system where the conventional public transport services are also considered.

References

Allport, R. J. (1981). The costing of bus, light rail transit and metro public transport systems. *Traffic Engineering & Control*, *22*, 633-639.

Alperovich, G., & Deutsch, J. (1994). Joint estimation of population density functions and the location of the central business district. *Journal of Urban Economics*, 36(3), 239-248.

Amirgholy, M., & Gonzales, E. J. (2016). Demand responsive transit systems with time-dependent demand: user equilibrium, system optimum, and management strategy. *Transportation Research Part B: Methodological*, *92*, 234-252.

Anas, A. (1982). Residential location markets and urban transportation. *Economic Theory, Econometrics, and Policy Analysis with Discrete Choice Models. New York: Academic Press*.

Bagloee, S., Asadi, M., & Richardson, L. (2012). Methodology for parking modeling and pricing in traffic impact studies. *Transportation Research Record: Journal of the Transportation Research Board*, (2319), 1-12.

Barabino, B., & Di Francesco, M. (2016). Characterizing, measuring, and managing transit service quality. *Journal of Advanced Transportation*, *50*(5), 818-840.

Beijing Traffic Management Bureau. (2016). ''Beijing Transport Annual Report''. *Available from: (http://www.bjjtgl.gov.cn/)*

Berry, B. J., Simmons, J. W., & Tennant, R. J. (1963). Urban population densities: structure and change. *Geographical Review*, 389-405.

Bruun, E. (2005). Bus rapid transit and light rail: Comparing operating costs with a parametric cost model. *Transportation Research Record: Journal of the Transportation Research Board*, (1927), 11-21.

Casello, J. M., Lewis, G. M., Yeung, K., & Santiago-Rodríguez, D. (2014). A transit technology selection model. Journal of Public Transportation, 17(4), 5.

Cao, Y., & Wang, J. (2017). An Optimization Method of Passenger Assignment for Customized Bus. *Mathematical Problems in Engineering*, 2017.

Chang, S. K., & Schonfeld, P. M. (1991). Multiple period optimization of bus transit systems. *Transportation Research Part B: Methodological*, 25(6), 453-478.

Chen, Y. J., Li, Z. C., & Lam, W. H. (2015). Modeling transit technology selection in a linear transportation corridor. *Journal of Advanced Transportation*, *49*(1), 48-72.

Clark, B., Chatterjee, K., & Melia, S. (2016). Changes to commute mode: The role of life events, spatial context and environmental attitude. *Transportation Research Part A: Policy and Practice*, *89*, 89-105.

Davison, L., Enoch, M., Ryley, T., Quddus, M., & Wang, C. (2014). A survey of demand responsive transport in Great Britain. *Transport Policy*, *31*, 47-54.

Du, B., & Wang, D. Z. (2014). Continuum modeling of park-and-ride services considering travel time reliability and heterogeneous commuters–A linear complementarity system approach. *Transportation Research Part E: Logistics and Transportation Review*, *71*, 58-81.

Duncan, M., & Cook, D. (2014). Is the provision of park-and-ride facilities at light rail stations an effective approach to reducing vehicle kilometers traveled in a US context?. *Transportation Research Part A: Policy and Practice*, *66*, 65-74.

Edwards, D., & Watkins, K. (2013). Comparing fixed-route and demand-responsive feeder transit systems in real-world settings. *Transportation Research Record: Journal of the Transportation Research Board*, (2352), 128-135.

Farhan, B., & Murray, A. T. (2008). Siting park-and-ride facilities using a multi-objective spatial optimization model. *Computers & Operations Research*, *35*(2), 445-456.

Furuhata, M., Daniel, K., Koenig, S., Ordonez, F., Dessouky, M., Brunet, M. E., Cohen, L. & Wang, X. (2015). Online cost-sharing mechanism design for demand-responsive transport systems. *IEEE Transactions on Intelligent Transportation Systems*, *16*(2), 692-707.

García, R., & Marín, A. (2002). Parking capacity and pricing in park'n ride trips: a continuous equilibrium network design problem. *Annals of operations research*, *116*(1-4), 153-178.

Habib, K., Mahmoud, M., & Coleman, J. (2013). Effect of parking charges at transit stations on park-and-ride mode choice: lessons learned from stated preference survey in Greater Vancouver, Canada. *Transportation Research Record: Journal of the Transportation Research Board*, (2351), 163-170.

Hamer, P. (2010). Analysing the effectiveness of park and ride as a generator of public transport mode shift. *Road & Transport Research: A Journal of Australian and New Zealand Research and Practice*, *19*(1), 51.

Hamilton, P., & Sankaranarayanan, S. (2013). Intelligent agent based RFID system for on demand bus scheduling and ticketing. *International Journal of Future Computer and Communication*, 2(5), 399.

Hendricks, S., & Outwater, M. (1998). Demand forecasting model for park-and-ride lots in King County, Washington. *Transportation Research Record: Journal of the Transportation Research Board*, (1623), 80-87.

Hole, A. R. (2004). Forecasting the demand for an employee Park and Ride service using commuters' stated choices. *Transport Policy*, *11*(4), 355-362.

Horn, M. E. T. (2002). Multi-modal and demand-responsive passenger transport systems: a modelling framework with embedded control systems. *Transportation Research Part A: Policy and Practice*, *36*(2), 167-188.

Horn, M. E. T. (2002). Fleet scheduling and dispatching for demand-responsive passenger services. *Transportation Research Part C: Emerging Technologies*, *10*(1), 35-63.

Horn, M. E. T. (2004). Procedures for planning multi-leg journeys with fixed-route and demand-responsive passenger transport services. *Transportation Research Part C: Emerging Technologies*, *12*(1), 33-55.

Hounsell, N., Shrestha, B., & Piao, J. (2011). Enhancing Park and Ride with access control: A case study of Southampton. *Transport Policy*, *18*(1), 194-203.

Idris, A. O., Habib, K. M. N., & Shalaby, A. (2015). An investigation on the performances of mode shift models in transit ridership forecasting. *Transportation Research Part A: Policy and Practice*, *78*, 551-565.

Jara-Díaz, S. R., & Gschwender, A. (2009). The effect of financial constraints on the optimal design of public transport services. *Transportation*, *36*(1), 65-75.

Jokinen, J. P. (2016). On the Welfare Optimal Policies in Demand Responsive Transportation and Shared Taxi Services. *Journal of Transport Economics and Policy (JTEP)*, *50*(1), 39-55.

Kameda, H., & Mukai, N. (2011). Optimization of charging station placement by using taxi probe data for on-demand electrical bus system. *Knowledge-Based and Intelligent Information and Engineering Systems*, 606-615.

Lam, W. H., Holyoak, N. M., & Lo, H. P. (2001). How park-and-ride schemes can be successful in Eastern Asia. *Journal of Urban Planning and Development*, *127*(2), 63-78.

Li, Z. C., Guo, Q. W., Lam, W. H., & Wong, S. C. (2015). Transit technology investment and selection under urban population volatility: A real option perspective. *Transportation Research Part B: Methodological*, *78*, 318-340.

Li, Z. C., Lam, W. H., Wong, S. C., & Sumalee, A. (2012). Design of a rail transit line for profit maximization in a linear transportation corridor. *Transportation Research Part E: Logistics and Transportation Review*, 48(1), 50-70.

Li, Z. C., Lam, W., Wong, S., Zhu, D. L., & Huang, H. J. (2007). Modeling park-and-ride services in a multimodal transport network with elastic demand. *Transportation Research Record: Journal of the Transportation Research Board*, (1994), 101-109.

Liu, T., & Ceder, A. A. (2015). Analysis of a new public-transport-service concept: Customized bus in China. *Transport Policy*, *39*, 63-76.

Liu, T., Ceder, A., Bologna, R., & Cabantous, B. (2016). Commuting by Customized Bus: A Comparative Analysis with Private Car and Conventional Public Transport in Two Cities. *Journal of Public Transportation*, *19*(2), 55-74.

Liu, T. L., Huang, H. J., Yang, H., & Zhang, X. (2009). Continuum modeling of park-and-ride services in a linear monocentric city with deterministic mode choice. *Transportation Research Part B: Methodological*, *43*(6), 692-707.

Martens, K. (2007). Promoting bike-and-ride: The Dutch experience. *Transportation Research Part A: Policy and Practice*, *41*(4), 326-338.

Meek, S., Ison, S., & Enoch, M. (2008). Role of bus-based park and ride in the UK: a temporal and evaluative review. *Transport reviews*, *28*(6), 781-803.

Meek, S., Ison, S., & Enoch, M. (2011). Evaluating alternative concepts of bus-based park and ride. *Transport Policy*, *18*(2), 456-467.

Meng, M., Shao, C.F., Wong, Y.D., & Zhang, J. (2016). Multi-modal traffic assignment with traffic emission effects. Proceedings of ICE - Engineering Sustainability, 169 (3), 114-122.

Meng, M., Mahardhika, H., & Rau, A. (2017). Passenger travel time perception in public transport trip. Proceeding of Transportation Research Board (TRB) 96th Annual Meeting, Washington, D.C, America, 17-03956.

Mingardo, G. (2013). Transport and environmental effects of rail-based Park and Ride: evidence from the Netherlands. *Journal of Transport Geography*, *30*, 7-16.

Moccia, L., & Laporte, G. (2016). Improved models for technology choice in a transit corridor with fixed demand. *Transportation Research Part B: Methodological*, *83*, 245-270.

Noel, E. C. (1988). Park-and-ride: alive, well, and expanding in the United States. *Journal of Urban Planning and Development*, *114*(1), 2-13.

Parajuli, P. M., & Wirasinghe, S. C. (2001). A line haul transit technology selection model. *Transportation planning and technology*, *24*(4), 271-308.

Qin, H., Guan, H., & Wu, Y. J. (2013). Analysis of park-and-ride decision behavior based on Decision Field Theory. *Transportation research part F: traffic psychology and behaviour*, *18*, 199-212.

Qu, X., Oh, E., Weng, J., & Jin, S. (2014). Bus travel time reliability analysis: a case study. In *Proceedings of the Institution of Civil Engineers: Transport*, *167*(3), 178-184.

Qu, X., & Wang, S. (2015). Long-Distance-Commuter (LDC) Lane: A New Concept for Freeway Traffic Management. *Computer-Aided Civil and Infrastructure Engineering*, *30*(10), 815-823.

Raymond, R., Sugiura, T., & Tsubouchi, K. (2011). Location recommendation based on location history and spatio-temporal correlations for an on-demand bus system. *In Proceedings of the 19th ACM SIGSPATIAL International Conference on Advances in Geographic Information Systems (pp. 377-380). ACM*.

Smith, E. (1973). An economic comparison of urban railways and express bus services. Journal of Transport Economics and Policy, 20-31.

Song, Z. (2013). Transition to a transit city: case of Beijing. *Transportation Research Record: Journal of the Transportation Research Board*, (2394), 38-44.

Song, Z., He, Y., & Zhang, L. (2017). Integrated planning of park-and-ride facilities and transit service. *Transportation Research Part C: Emerging Technologies*, 74, 182-195.

Spillar, R. J. (1997). Park-and-ride planning and design guidelines. *Parsons Brinckerhoff Inc., New York.*

Sun, Y., Guo, Q., Schonfeld, P., & Li, Z. (2017). Evolution of public transit modes in a commuter corridor. *Transportation Research Part C: Emerging Technologies*, *75*, 84-102.

Stieffenhofer, K. E., Barton, M., & Gayah, V. (2016). Assessing Park-and-Ride Efficiency and User Reactions to Parking Management Strategies. *Journal of Public Transportation*, *19*(4), 75-92.

Stutsman, J. (2002). Bus Rapid Transit or Light Rail Transit—How to Decide?: Los Angeles Case Study. *Transportation Research Record: Journal of the Transportation Research Board*, (1793), 55-61.

Tirachini, A., Hensher, D. A., & Jara-Díaz, S. R. (2010a). Comparing operator and users costs of light rail, heavy rail and bus rapid transit over a radial public transport network. *Research in transportation economics*, *29*(1), 231-242.

Tirachini, A., Hensher, D. A., & Jara-Díaz, S. R. (2010b). Restating modal investment priority with an improved model for public transport analysis. *Transportation Research Part E: Logistics and Transportation Review*, *46*(6), 1148-1168.

Tsubouchi, K., Hiekata, K., & Yamato, H. (2009, April). Scheduling algorithm for on-demand bus system. *In Information Technology: New Generations, 2009. ITNG'09. Sixth International Conference on* (pp. 189-194). IEEE.

Tsubouchi, K., Yamato, H., & Hiekata, K. (2010). Innovative on-demand bus system in Japan. *IET Intelligent Transport Systems*, 4(4), 270-279.

Wang, G., Gao, Z., Xu, M., & Sun, H. (2014). Joint link-based credit charging and road capacity improvement in continuous network design problem. *Transportation Research Part A: Policy and Practice*, *67*, 1-14.

Wang, J. Y., Yang, H., & Lindsey, R. (2004). Locating and pricing park-and-ride facilities in a linear monocentric city with deterministic mode choice. *Transportation Research Part B: Methodological*, *38*(8), 709-731.

Zhang, J., Wang, D. Z., & Meng, M. (2017). Analyzing Customized Bus Service on a Multimodal Travel Corridor: An Analytical Modeling Approach. Journal of Transportation Engineering, Part A: Systems, 143(11), 04017057.