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Mathematics for the reformed science A-levels: Implications for science teaching

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Post-16 mathematics remains high on the political agenda in England with attempts to increase the mathematical engagement, confidence and competence of young people being supported by various qualification reforms. This includes *adding* new qualifications under the banner of Core Maths and *embedding* mathematics as mandated percentages in the assessment of science A-levels. Achieving the full aspirations of the *adding* policy will require substantial increases in the number of teachers of mathematics. Successfully delivering the *embedding* policy will require science teachers well-equipped to teach the increased mathematical demands of the reformed science A-levels. This paper explores some of the challenges associated with this *embedding* strategy by drawing on our quantitative analysis of reformed science A-levels, new evidence from chemistry Examiners' Reports and insights from the literature. We discuss curriculum alignment, the need for dialogue between science and mathematics teachers within schools and colleges, as well as implications for teacher professional development.

Keywords: Mathematics; A-level science; teachers; embedding

Introduction

In recent years, mathematics has featured prominently in the UK Government's education policy agenda. Sir Adrian Smith's review of post-16 mathematics (Smith, 2017) highlighted the low uptake of post-16 mathematics in England and the UK more generally (see also Hodgen, Pepper, Sturman & Ruddock, 2010; ACME, 2011; Royal Society, 2008), and drew attention to regional disparities in progression to post-16 mathematics qualifications. The report discussed the shortfall of mathematics teachers in England, identifying this as constraining factor in achieving long-term national goals for the growth towards universal participation in post-16 mathematics. The economic need for a mathematically well-qualified workforce and the expanding need for employees with so called Science, Technology, Engineering and Mathematics (STEM) skills, was a message reinforced further in the UK Government's subsequent Industrial Strategy (Department for Business, Energy & Industrial Strategy, 2017) and has been articulated widely both in the UK (Royal Society, 2014; House of Lords, 2012; Roberts, 2002) and internationally (e.g. in Europe (Gago, 2004), the USA (National Academies, 2007) and Australia (The Australian Industry Group, 2015)).

As a subject, the range and scope of applications of mathematics is diverse and multi-faceted. It underpins much of modern technology and finds widespread application within higher education across disciplines in the natural sciences, engineering, computing, the social sciences and humanities, both within undergraduate and postgraduate study. Yet, evidence shows that there are substantial weaknesses in the levels of awareness and understanding of the prominent role mathematics plays within disciplines in higher education (Hodgen, McAlinden & Tomei, 2014). The reasons behind this observation are complex, and include the failure of universities to adequately signal the mathematical requirements of their degree programmes through their entrance requirements (McAlinden & Noyes, 2018; Hodgen et al., 2014) as well as deep-rooted negative cultural attitudes to mathematics as a subject (Smith, 2017).

In England, recent qualification reforms have had a strong focus on the mathematical needs for higher education study within disciplines. These reforms have been introduced in a phased way over several years with some of the new qualifications still awaiting their first formal assessment. Elsewhere, as part of an historical case study of England, we have set out the drivers and policy levers that have been instrumental in bringing about these qualification reforms (McAlinden & Noyes, 2018). Mathematics for post-16 study is now being developed in two ways: (i) an *adding* policy seeks to increase uptake of post-16 mathematical study, in part through the introduction of new Core Maths qualifications; (ii) an *embedding* policy mandates mathematical assessment requirements within other disciplines (McAlinden & Noyes, 2017).

In this paper we build on our earlier analysis of the mathematics within reformed science A-levels (McAlinden & Noyes, 2017), and present a preliminary analysis of the information regarding mathematics that can be gleaned from Examiner Reports of the first live assessment of the reformed A-level Chemistry. We analyse the messages within these reports pertinent to achieving the aspirations of the *embedding* policy. Then we proceed to consider the implications for science teachers of implementing both the *adding* and *embedding* strategies, with particular reference to the opportunities and challenges within school and college settings.

Qualifications in England and their reforms

In England the study of mathematics is compulsory for the first five years of secondary education, at which point young people take their General Certificate of Secondary Education (GCSE) qualifications at age 16. If they achieve sufficiently good results in their GCSEs young people can progress to further academic study, which, for the majority, takes the form of 3 or 4 subjects at advanced level (A-level). The A-level qualifications are high stakes national qualifications, taught over two years and administered by a small number of independent awarding organisations. The curriculum is set by the Government's Department for Education, with Ofqual having regulatory authority for implementation in line with statutory requirements.

In 2016/17 the result reporting system for GCSE Mathematics in England changed from alphabetic gradings (A-G and U) to numeric gradings (9-1) (Ofqual, 2015a). The achievement of a 'good' pass in GCSE, equivalent to a grade C or a grade 4 in the new system, is identified as the attainment of Level 2 in mathematics. Level 3 qualifications include A-levels, the Advanced Subsidiary (AS) qualifications (approximately equivalent to half of an A-level) and, in the case of mathematics, the recently introduced Core Maths qualifications. The latter provide a post-16 mathematics route for young people who have passed GCSE Mathematics but are not continuing on to AS/A-level Mathematics.

The reformed A-level Physics, Chemistry and Biology now contain statutory minimum percentages for the assessment of disciplinary-relevant mathematical content at Level 2 or above (Department for Education, 2014). These qualifications, along with the new GCSE Mathematics were assessed for the first time in the summer of 2017. First teaching of the new A-level Mathematics was deferred until the following September to facilitate more coherent progression through the mathematics qualifications. However, given the role of GCSE Mathematics as an implicit prerequisite for A-level science study, there is a less obvious misalignment between the timeframes for the introduction of these qualifications. This is an area to which we will return in more detail later.

Embedding mathematics in reformed A-level science assessments

Information available prior to the first assessment point

The reference point for the current study is our earlier analysis of the sample assessment materials (SAMs) for the reformed Biology, Chemistry and Physics A-levels, across three awarding organisations (McAlinden & Noyes, 2017). This work was carried out before the first live assessments of these qualifications. These SAMs will have been a key resource used by teachers in developing curriculum and preparing students for the qualifications, having been previously subjected to scrutiny by the qualifications regulator, Ofqual, to ensure that they gave an accurate indicator of the assessment of the qualifications. Building on the approach of Noyes, Drake, Wake and Murphy (2010) in the Evaluating Mathematics Pathways Project, we undertook a quantitative analysis of the mathematics within the SAMs and investigated a range of areas including: (i) the mark allocations for mathematical work; (ii) the nature of the assessed mathematical content (e.g. numerical, graphical, algebraic etc); (iii) the level of mathematics (whether at GCSE or above); (iv) the mathematical processing skills required (e.g. representing, procedural analysis, reasoning, interpreting etc); (v) the practical or theoretical nature of the tasks in which mathematics arose; (vi) the mathematical complexity; and (vii) the extent of the mathematical embedding within the science subject.

The mathematics in A-level Chemistry SAMs

The results of our earlier analysis of the A-level Chemistry SAMs found that the marks for mathematical content in the SAMs met the 20% statutory requirements (Department for Education, 2014). Based on our findings we developed the following synoptic mathematical portrait of the mathematics within the SAMs of the reformed A-level Chemistry.

> The mathematics within the qualifications is deeply embedded and so is not easily accessible without knowledge of chemistry. The mathematical work is predominantly procedural with most marks coming from questions requiring decisions to be made. The majority of the mathematics requires only standard level GCSE Mathematics although the complexity of calculations is greater than what would be expected at GCSE. It is predominantly numerical, with smaller amounts of algebra and graphical work also being required. (McAlinden & Noyes, 2017, p.11)

We also developed similar mathematical portraits for A-level Biology and Physics.

Of necessity, the chemistry mathematical portrait is based purely on the SAMs published in advance of first live assessment of the qualifications and not on the actual student learning or the achievement of the learning and assessment objectives. An in-depth understanding of the latter will have had to await a detailed evaluation of the mathematical performance of the first student cohort taking the reformed qualifications. In the absence of such, useful insights can be gleaned from the Examiners' Reports from the various awarding organisations.

Review of Examiners' Reports from first actual assessments

The Examiners' Reports on all of the relevant Chemistry A-levels are not in the public domain yet. However, we obtained the reports for the full suite of examination papers from one awarding organisation, which, in line with our earlier work, we have chosen not to identify. We have analysed these reports by searching for information about the assessed mathematical content. Our key observations are summarised below.

Observation (1): Level-2 nature of the mathematics
The synoptic comments within the reports mention the requirement for greater assessment of mathematics at Level 2 within the qualification. The reports identify that the less successful candidates struggled with the calculations, and lost marks on how they used significant figures. This characteristic was identified in the reports as being prominent in achieving the 20% Level 2 mathematical requirement.

Observation (2): Practising calculations within questions
The Examiners' Reports also identified that candidates needed more practice with the new style of questions and particularly the calculations within them.

Observation (3): Interpretation of solutions within subject context
Another weakness that was identified in candidates' work was the submission of mathematical answers which were clearly impossible from a chemistry perspective.

Observation (4): Question structure
The reports also pointed to the wider use of less structured/scaffolded calculations and that those candidates who were most successful were able to carry out such calculations.

Observation (5): Tackling unfamiliar problems
The inclusion of unfamiliar problems within examination papers, (i.e. of a type not previously seen by candidates), was also highlighted within the reports.

Commentary on findings

Our mathematical portrait for A-level Chemistry (McAlinden & Noyes, 2017) has identified the heavy reliance on GCSE Mathematics content, with only small amounts of post-GCSE material. The latter is an area that was not discussed in the Examiners' Reports. Observation (1) draws attention to the importance placed within the mark schemes on the correct use of significant figures as a factor in achieving the Level 2 mathematical assessment requirements within the qualification. This observation points to a need to ensure that a skewed and disproportionate emphasis is not placed on one particular area of Level 2 mathematical content (e.g. significant figures) at the expense of coverage of other more challenging areas. This is a characteristic that should be kept under review and given due consideration in the setting of future examination papers and their accompanying marking schemes. In this context we note that this point is particularly pertinent to the way in which marks are awarded for partially correct solutions.

Observation (2) relates to the revised question styles and the way in which calculations arise within questions. Our subject portrait for the mathematics within A-level Chemistry has identified a high level of mathematical embedding and as such the ability to access the calculations can, in many cases, be reliant on a grasp of the underlying chemistry. The change of question style is also one area that is likely to have posed challenges for teachers who will have had to adapt their teaching approaches to the new specifications and its mathematical requirements, with fewer sources of the new style of questions.

While mathematical embedding features substantially within A-level Chemistry, observation (3) points to the detachment of the mathematical calculation from the chemistry in question by some candidates. This behaviour can be symptomatic of a decontextualisation of the outcome of a calculation from the underpinning chemistry, and/or a failure to interpret the answer in a meaningful way.

Collectively, observations (3), (4) and (5) can all be linked to the general characteristics of (mathematical) problem solving. The unfamiliarity of questions, the use of unstructured questions and the interpretation of solutions are all characteristics which could be expected to arise within mathematical problem solving (ACME, 2016). This is particularly relevant, given the greater emphasis on problem solving within the reformed GCSE and A-level Mathematics qualifications (Ofqual, 2015b, 2015c). In this context it is worth noting that the student cohort about which the Examiners' Reports were written, will not have taken this reformed GCSE Mathematics qualification, which was also assessed for the first time in 2017.

Discussion

Achieving the long-term aspirations of the two-pronged *adding* and *embedding* policies poses many challenges, not least of which is the need for a highly skilled teaching workforce able to deliver new mathematics qualifications and reformed curricula in other disciplines, each including revised mathematical requirements. Current numbers of mathematics teachers in England are insufficient to meet the needs of the 'maths for all to 18' agenda and there is a recognition that teachers from other quantitative disciplines, with appropriate professional development, will have to be recruited to assist with the teaching of Core Maths (Smith, 2017). Less obvious, but perhaps equally pertinent, is the need for renewed, targeted professional development for teachers in other disciplines, such as chemistry, in which mathematical requirements have increased but have actually been playing a well-established role for many years. Such diversification of the training needs of those involved in the teaching of mathematics in the classroom, in whatever form it may take, represents a shift in the overall mathematics education landscape. Of necessity, this is likely to be accompanied by a broadening of the pool of educators involved in its delivery and a greater emphasis on peer learning between teachers across discipline boundaries within school and college settings.

The sharing of sound mathematical knowledge and pedagogy across disciplines, while highly desirable, is non-trivial and the challenges associated with conducting informative conversations in this domain should not be underestimated. In particular, the Association for Science Education (ASE) has drawn attention to differences in the terminologies used by teachers of sciences and mathematics when referring to mathematical concepts and ideas (ASE, 2016a). For example, a reference to a 'line' in the sciences can be taken to mean a straight line or a curve, while in mathematics these two entities are considered distinct and different (p.2). The acquisition of an awareness of these differences has great potential in enabling teachers to facilitate young people in making more effective connections between their different subjects of study.

In secondary school education in England, mathematics and the sciences are traditionally taught separately as distinct, standalone subjects. Consequently, young people will either need to have met mathematical concepts and techniques before they arise in science classrooms, or the teaching of these topics will have to take place within the sciences. From the ages of 11-16 young people in England will be working towards the compulsory GCSE Mathematics qualification. As such, opportunities do exist for curriculum alignment within schools to ensure that the mathematics is taught first within mathematics lessons before it is required within science classes. There is also scope for mathematics and science teachers to work together in planning curriculum delivery in order to assist young people in making connections across the boundaries between their mathematics and science subjects. Examples of such collaborative practice have been identified in recent ASE (2016b) work.

The scenario at A-level is somewhat different. The successful achievement of the aims of the *embedding* policy are inextricably linked to progress towards the *adding* policy. At present there is still no statutory requirement that young people embarking on science A-levels will be studying for a parallel Level 3 mathematics qualification, although there are substantial benefits from so doing (McAlinden & Noyes, 2017). In particular, A-level science classes are very likely to contain some young people studying Level 3 mathematics, along with others who are not. (This is particularly relevant for chemistry and biology, but perhaps less so for physics.) For the latter group of young people, the role of the teacher of Level 3 mathematical content will, of necessity, default to the science teacher. If such teaching is to go beyond purely procedural approaches, the science teacher will also need to have a sound understanding of the mathematics in question, as well as the requisite pedagogic knowledge to teach it effectively. The extent to which science teachers will have had opportunities to acquire and develop this expertise is open to question.

While recognising the importance of the context of the English qualification system in our discussion, it is also constructive to consider if relevant insights can be acquired from experiences in other international contexts. More specifically, some of the likely challenges for teachers that accompany implementation of the *embedding* policy are not dissimilar from those observed in studies of interdisciplinary curricula and teaching across mathematics and science in the USA. For example, in a study in middle schools Burghardt, Lauckhardt, Kennedy, Hech and McHugh (2015) reported a "significant increase in mathematical content scores" for young people who experienced a "mathematics-infused science" curriculum, in which mathematics was taught within science as well as in mathematics (p. 204). However, the authors did note that variability both in the implementation of the mathematics-infusion and in teacher effectiveness, were limitations of their study. They also postulated that some science teachers may have been better placed to reinforce mathematics within a science context, rather than to introduce the mathematical content to young people for the first time. The latter point resonates with the work of Weinberg and Sample McMeeking (2017) who investigated the barriers and enablers to integrated science and mathematics teaching in high schools, again in the USA. They concluded that one aspect that contributed to a lack of success of interdisciplinary teaching approaches was what they referred to as "interdisciplinary pedagogical content knowledge". They identified that the teachers in their study "… expressed some level of discomfort in knowing how to teach interdisciplinary content" (p. 211). With the increased mathematical emphasis within A-levels across subjects this is an area likely to become increasingly important in the future.

Our discussion of the opportunities for dialogue between science and mathematics teachers would be incomplete without some mention of the challenges presented by the timeframes for qualification reform implementation. Specifically, the simultaneous introduction of the reformed GCSE Mathematics and the reformed science A-levels will have complicated such teacher conversations. Queries from science teachers about the mathematical backgrounds of young people on the reformed science A-levels will have required mathematics teachers to respond with reference to the pre-reformed GCSE Mathematics, rather than the curriculum they were in the process of teaching. Furthermore, the greater use of unfamiliar problems identified by the A-level Chemistry Examiners' Reports, could perhaps have been better supported if young people had a background of the reformed GCSE Mathematics, with its stronger emphasis on problem solving (Ofqual, 2015b). Such complications to cross-disciplinary dialogue are neither constructive nor desirable and are symptomatic of a lack of coherence in the overall qualifications reform process.

Conclusion

Our analysis of the A-level science SAMs has demonstrated clear benefits to young people in continuing with post-16 mathematics study in terms of their preparation for A-level sciences (McAlinden & Noyes, 2017). Indeed, such is also the case for many other A-level subjects (e.g. geography, economics, psychology). Constructive conversations between mathematics teachers and science teachers (and conversations between mathematics teachers and those in other quantitative subjects) about mathematical curricula and pedagogy can contribute much towards enhancing the effectiveness of delivery of the *embedding* policy. The consequent increase in awareness of curriculum interdependencies within schools and colleges also has great potential to foster better signalling from teachers to young people regarding the usefulness and value of taking post-16 mathematics qualifications alongside A-levels in the sciences and other quantitative subjects. Such small steps should be encouraged and strongly reinforced by powerful messages from policy influencers, employers and higher education about the long-term value of post-16 mathematics study (McAlinden & Noyes, 2018; Smith, 2017).

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