Mathematics in the disciplines at the transition to university

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Abstract

Mathematics education is highly valued in advanced economies due to its role in developing skilled workforces, economic resilience and social wellbeing. However, university academics across disciplines regularly bemoan undergraduate students' under-preparedness for the mathematical and quantitative demands of undergraduate degree programmes. In this paper we consider this issue. We begin with a research synthesis of relevant literatures on mathematics within university study in the natural sciences (physics, chemistry and biology) and social sciences (economics, geography, psychology) that highlights the international nature of this problem. We then develop an historical national case study of how mathematics for pre-university study in England has evolved, culminating in a recent policy move which mandates the assessment of mathematics within disciplines. Finally, by integrating these two distinct perspectives we discuss wider issues relating to mathematics for the transition to higher education.

Keywords: mathematics, science, university, transition, qualifications

1. Background

Governments and education ministries across the world recognise the importance of having a mathematically well-educated populace (Gago, 2004; National Academies, 2007; Kounine et al., 2008; Vorderman et al., 2011). Arguably, the strongest driver behind this political interest is the compelling evidence of the link between mathematics qualifications, earnings potential and economic productivity (Adkins & Noyes, 2016; Vignoles et al., 2011).

A second driver of this political interest in mathematics education is the increasingly influential global comparisons such as the Trends in International Mathematics and Science Study (TIMSS) and the Organisation for Economic Cooperation and Development's Programme for International Student Assessment (PISA) and Programme for the International Assessment of Adult Competencies (PIAAC). These assessments produce rankings of school systems and adult competences, making possible analyses of the relationships between the (mathematical) performances of nations and their economic productivity, levels of inequality and social wellbeing.

A third driver, which brings us closer to the concerns of this paper, is the *science* lobby and its concern for mathematically well-educated school leavers who can sustain and grow the science base through advanced study and employment in scientific fields, whether in the Science, Technology, Engineering and Mathematics (STEM) heartlands or in other mathematically demanding disciplines (e.g. economics). This driver is entwined with the previous two, although this link is not always made explicit. Modern societies require, and benefit from, engagement with mathematical methods and data analysis in a range of settings, from the models embedded in regulating financial markets and monitoring educational systems to increasingly complex applications of statistics in a range of social, political, creative, scientific and medical fields (Deloitte, 2012).

Given the value of mathematics education - economically, comparatively and for employers – this paper reconsiders the challenge of students developing and transferring relevant and usable mathematics into a range of university disciplines. In particular, we investigate mathematical discontinuities at the school-university interface in order to discuss important issues regarding curriculum, qualifications reform, knowledge transfer and disciplinary expectations.

The mathematical preparedness of young people in the move to university is a concern in England, more widely within the UK (Royal Society, 2011) and indeed internationally. This so called 'mathematics problem' (Howson et al., 1995; Hawkes & Savage, 1999) is sufficiently general to warrant analysis. However, failure to acknowledge the peculiarities of context can result in a homogenised literature that is of little relevance anywhere. For this reason, we commit a substantial part of this paper to a case study of policy and practice relating to pre-university mathematical education in England.

This paper integrates two complementary perspectives on this problem of mathematical transitions into the disciplines. Firstly, the international research literature on mathematics within other university disciplines is reviewed, with a particular emphasis on transition. The current literature is fragmented with little synthesis elsewhere and our integrative analysis highlights the international nature of the problem, across disciplines and nations. Secondly, the school-university interface is considered through the development of an historical case study of mathematics for pre-university study in England. We consider what has happened in schools, in national examination systems, policy reforms and the changing influences and interventions of higher education. Finally, the paper concludes by integrating these two perspectives into a more holistic discussion of the current mathematics education landscape for the transition to university.

2. Mathematics transitions into the disciplines

For several decades, the mathematical preparedness of students entering higher education in various countries has been discussed in the literature. Research examines transitions into mathematics degrees (for example, Brandell et al., 2008; Clark & Lovric, 2009; Thomas & Klymchuk, 2012) but this is not the focus herein. Rather, this overview discusses mathematics within a) the natural sciences of physics, chemistry and biology, and b) three social[1] sciences: economics, geography and psychology.

2.1 *Mathematics in science at the school-university interface*

Many researchers point to a problematic gap between school mathematics and university applications of mathematics within the sciences (Heck & Van Gastel, 2006; Tai et al., 2005; Groen et al., 2015). Such studies support a general consensus that success in undergraduate science is built upon 'two pillars' (Sadler & Tai, 2007): the level of mathematics and discipline-specific science knowledge. Small-scale studies of early undergraduate performance within the sciences have been reported from several countries including Australia (e.g. Rylands & Coady, 2009), New Zealand (Comer et al., 2011), the Netherlands (Heck & van Gastel, 2006) and the USA (e.g. Tai, Sadler & Loehr, 2005). There is a dearth of research relating school mathematics qualifications to final science degree outcomes, though a recent study of a full national cohort in

[1] There are some issues with calling both psychology and geography 'social sciences' but they are sufficiently so, and different from the natural sciences that we have organised the section in this way.

England (Adkins & Noyes, 2018) showed that advanced mathematics did not predict success in biology and chemistry degrees.

The Australian education system shares a common problem with England; because the study of mathematics to age 18 is not compulsory, many students opt out of the subject at 16 even though they might later progress to mathematically demanding undergraduate programmes (Hughes & Rubenstein, 2006). A study at the University of Sydney (Nicholas et al., 2015) suggests that whilst higher levels of mathematical study are in general terms beneficial, higher performance at a lower level of mathematics (e.g. at age 16) is also an important predictor of success, particularly for chemistry. Nicholas *et al.* conclude that it is important to consider the underlying mathematical ability of students as well as the level of mathematics studied; failure to do so would be to overlook potentially strong science candidates. Elsewhere in Australia, at Wollongong, Armstrong *et al.* (2014) conclude that high school mathematics rather than chemistry qualifications are the best predictor of general chemistry performance. In the United States, Spencer (1996) reported a single-site study which found that performance on the mathematics Scholastic Aptitude/Assessment Test (SAT) is a good predictor of general chemistry attainment. Tai *et al*. (2005) also commented on “the striking role of preparation in advanced mathematics on college chemistry success” and other US studies have come to the same conclusion (e.g. Donovan & Wheland, 2009). These chemistry studies are normally of ‘general’, introductory and first year modules. Notably different, and somewhat at odds with these findings, is Brogt *et al.'s* (2011) single-site study in New Zealand which reports no association between school mathematics preparation and first year undergraduate outcomes in biology.

While the quantitative aspects of undergraduate chemistry and biology require the application of mathematical techniques, the extent of the reliance of physics on mathematics is even more pronounced. Many years ago Rutter (1994) pointed to evidence in England of the links between performance in A level Physics and A level Mathematics, while Barham (2012) has presented a 35-year analysis that shows declining performance in both of these qualifications. In a more recent 3-site study in the UK, the Institute of Physics (IOP, 2011) found that those who had studied the highest level of advanced mathematics in school were better prepared for the early mathematical demands of their physics degree programme. Subsequently Bowyer and Darlington (2016) reported further on the views of undergraduate physics students' perceptions of the advanced mathematics they had studied. Other studies have considered the transition in undergraduate physics from different perspectives. For example, a study at Loughborough University (Symonds et al., 2010) found that physics undergraduates who were mathematically less well prepared when entering university had less confident attitudes towards the mathematics they encountered at university, even when they had been given additional support at the beginning of their university studies.

There seems little doubt in science disciplinary communities that the effective application of mathematical and statistical techniques is necessary for good science learning (e.g. Hoban et al., 2013). That said, the extant literature does not fully support these assumptions for biology and chemistry and whether school mathematics qualifications are good preparation is moot. Indeed, Nicoll and Francisco's (2001) analysis of physical chemistry performance concludes that neither students nor tutors were able to identify the correlates of success. In England, and elsewhere, there is a general deficit view of the mathematics that students bring with them to undergraduate studies in science (SCORE, 2012; IOP, 2011; Koenig, 2011; Shallcross & Yates, 2014). For some, that is due to them having opted out of mathematics from 16-18 (c.f. Australia). There is also a problem in terms of the mathematical content studied and in students' capacity to apply mathematics in unfamiliar contexts.

2.2 *Mathematics in selected social sciences at the school-university interface*

The literature on the place of mathematics in undergraduate social sciences is less well developed and is thinned further when looking at studies of the relationship between school preparation in mathematics and disciplinary applications at university. Recent reports from the Higher Education Academy relating to the transition into geography (Souch et al., 2014), economics (Dawson, 2014) and psychology (Field, 2014) offer helpful recent literature summaries. The British Academy (2012, 2015) has been advocating a step change in quantitative literacy. In parallel there has been a major investment in quantitative skills training in social sciences through the Q-step initiative (http://www.nuffieldfoundation.org/q-step).

Economics is arguably the most mathematically demanding of these three disciplines and there is matched interest in the literature about the importance of mathematics in the transition to undergraduate economics degrees. Arnold and Straten's (2012) study of the influence of mathematics and motivation concludes that 'a deficient math preparation bodes ill for first-year success…Differences in preparatory education account for most of the variation in study success across economic students' (p. 45). They also conclude that strong intrinsic motivation can mitigate the effects of 'inadequate math education'. In the USA, Anderson *et al's* (1994) early work reported calculus and overall grade point average (GPA) as predictors of success. Ten years later Ballard and Johnson (2004) aimed to tease out more precisely what elements of school mathematics predicted success in particular undergraduate economics modules. They found that the problem was more complex than whether or not students had completed calculus and that ‘mastery of extremely basic quantitative skills is among the most important factors for success in introductory microeconomics’ (p. 21). Mallik and Lodewijks (2010) in Australia came to the same broad conclusion that school mathematics and economics performance predict early success. However, these studies are all single-site and, as in the science studies above, there are researchers who do not find the same results particularly when focused on other aspects of the curriculum (e.g. Cohn et al., 1998).

The psychology literature on the influence of prior mathematics learning is rather thin and where it appears is more concerned with statistics. In one study, Mulhern and Wylie (2006) tested a large UK sample of psychology undergraduates to assess six areas of mathematical thinking relevant for statistical applications in psychology. They found, as with Ballard and Johnson (2004) in economics, that students ‘exhibited marked deficiencies in many aspects of mathematical thinking’ (p 119) and that there was some gender differentiation with girls performing less well than boys.

In geography and psychology there is much discussion of mathematics anxiety (for example, this study of second year psychology students in Spain: Núñez-Peña et al., 2013). Chapman (2010) writes of one local approach to dealing with this anxiety in an undergraduate geography programme in England. He describes the shock experienced by students when they re-encounter mathematics at university and how this combines with a reported reduction in the mathematics skills of new undergraduates. This, and the study by Folkard (2004), are small-scale qualitative studies that investigate the problem of undergraduate students arriving with insufficient mathematics but also with significant anxieties about the applications of mathematics and statistics within the subject. Geography has gone through many turns to and from quantitative applications but there continues, in the UK at least, to be a lack of awareness of the mathematical demands of many parts of modern geography (Souch et al., 2014).

There are similarities between this literature and that in the natural sciences. In subjects like chemistry and economics, there are many single-site studies that model how the level of school mathematics achieved predicts success in undergraduate programmes. However, in both cases there are other studies that disagree. There is a tendency to see school mathematics education from a deficit position, e.g. ‘inadequate math education’ (Arnold & Straten, 2012). The Higher Education Academy study across a selection of disciplines also highlighted marked differences in the expectations of lecturers and students about the mathematical demands of degree programmes (Hodgen et al., 2014). The research literature on mathematics preparation for university is thin and does not yet deal with the increased quantification in society, the linked data explosion across the sciences and the new forms of analysis and representation that demand better quantitative literacies. That said, the growing need for these skills is increasingly evident within graduate employment, interdisciplinary research as well as in new and emerging applied scientific fields.

Having taken care to attend to the national contexts of these studies we now turn our attention to a more in-depth discussion of the English context in order to understand the relationship between pre-university mathematics education, the growing calls from academic disciplines for better mathematics preparation in schools and the broader concerns for greater quantitative literacy.

3. The case of pre-university curriculum and assessment reform in England

3.1 *Pre 2000: 'the mathematics problem' emerges*

Since 1951, advanced or A level awards have been the main university entrance qualifications taken by young people at age 18 in England. Although the qualifications have evolved in style, and the numbers taking them have increased dramatically over the last 66 years, they have remained remarkably resilient to change, albeit with some questioning the maintenance of standards over time (Jones et al., 2016). In the early years, these qualifications were designed for those in selective grammar schools and were set by several small, regional matriculation boards, often linked to local universities.

In 1988, with The Education Reform Act (HMSO, 1988), and the introduction of the new General Certificate of Secondary Education or GCSE for 16-year-olds, there began a slow move to a merger of these smaller examination boards and by 2000 there were three main Awarding Organisations acting nationally and in competition with one another. This process also saw the gradual decline of university/academic involvement in these qualifications.

The A level curriculum is quite a narrow one with students normally specialising in three subjects. For the majority of 16-18 year olds this selection has not included mathematics. As a result, universities consider the mathematics (and English) GCSE qualifications at age 16 with a grade C or above in English and mathematics normally a condition of entry to university. However, for some new undergraduates this means that they may not have studied mathematics for two years with their skills having atrophied (ACME, 2011a).

The 1990s saw the start of a significant growth in university participation and A level take-up. The 1992 Further and Higher Education Act[2] was followed by a substantial expansion of the UK university sector (see, for example Bathmaker, 2003; Bolton, 2012). The 1990s also saw ongoing relative decline in the proportion of young people on academic pathways studying advanced mathematics (Royal Society, 2008). This combination of higher education growth and decline in school mathematics created a perfect storm and resulted in much discussion of 'the mathematics problem' (Hawkes & Savage, 1999; Howson et al., 1995), which has since been identified as a long-standing problem. These reports, and others that followed in their wake (Smith, 2004; Roberts, 2002), highlighted the gulf that had opened up between the mathematical demands of higher education programmes and students' preparedness for those courses.

3.2 *The noughties: 'mathematics counts'*

At the turn of the century, changes that resulted from a major review of the 14-19 curriculum (Dearing, 1996) were implemented. The new modular A levels and the introduction of a demanding Advanced Subsidiary (AS) qualification had a sharp and negative impact upon post-16 mathematics participation (Royal Society, 2008). This sharp drop catalysed the mathematics community into action and pushed advanced mathematics right up the educational policy agenda where it has remained ever since. The Roberts (2002) report 'SET for Success' envisioned STEM education that would 'lead to exciting, challenging and rewarding experiences for all pupils' (p 7). Two years later Sir Adrian Smith's report *Mathematics Counts* (2004), set out a policy agenda that proposed major curriculum and assessment reforms.

[2] http://www.legislation.gov.uk/ukpga/1992I/13/resources

One piece of econometric analysis that had been commissioned at the time of the Dearing Review (Dolton & Vignoles, 2002) was gradually becoming more influential through a network of policy brokers and would gain full potency at the end of the decade with the election of a new Conservative-led coalition government (see Noyes & Adkins, 2016a, for an analysis of how this research became common knowledge). Dolton and Vignoles' analysis of historic national surveys showed that there was a wage return to A level Mathematics for people in their early 30s and the original study has since been updated with similar findings (Adkins & Noyes, 2016).

The curriculum reforms at the turn of the century also produced an alternative applied mathematics pathway: Use of Mathematics (Hutcheson et al., 2011). This modular curriculum was much more applied in nature but never became a mainstream alternative pathway to A level Mathematics (Noyes et al., 2010). Although it had the potential to offer valuable learning experiences to young people (Noyes et al., 2011), particularly those who might proceed to undergraduate studies that require applications of mathematics, it was devalued following a critical and well-publicised report (REFORM, 2009). The idea of an alternative, applied mathematics route was not lost, however, as we show below.

By the end of the noughties, influential business organisations were calling for all young people in England to study mathematics to 18 and the scene was set for another step change in the focus on 14-19 mathematics: 'Government needs to ensure that all young people, regardless of what route they choose, study some form of maths or numeracy education after 16' (CBI, 2009, Recommendation 22, p.46). Meanwhile, in the universities, student numbers continued to rise quickly throughout the first decade of the new century (Bolton, 2012; UUK, 2011).

3.3 *From 2010: Reshaping the qualification landscape*

Following the election in 2010 of a Conservative-led coalition government, the white paper: *The Importance of Teaching* (DfE, 2010) articulated a commitment to increasing the minimum age at which young people could leave compulsory education to 18, by the year 2015. It also gave notification of major reforms to national qualifications with the expressed intention that universities and learned bodies would be involved in the reform process. This activity was accompanied by a succession of reports on 14-19 mathematics education as well as on mathematics in the disciplines, both within school and in the transition to university.

The Nuffield Foundation's report *Is the UK an outlier?* (Hodgen et al., 2010) provided clear evidence of the low levels (less than 20%) of participation in pre-university advanced mathematics in England. The poor international comparison was a political deal breaker, particularly when combined with the emergence of Dolton and Vignoles' (2002) work into policy discourse and calls for more mathematics for all school learners from employers. Also influential at the time was the work of the Royal Society's Advisory Committee on Mathematics Education (ACME) which highlighted that the low levels of participation should be cause for national concern (ACME, 2011a). The work identified a substantial difference between the numbers of young people actually studying mathematics post-GCSE and those who would potentially benefit from so doing. In parallel work ACME (2011b) also highlighted the dearth of valued post-16 mathematics pathways and issued a timely reminder that the mathematical needs of learners, higher education and employers all needed to be taken into consideration in future developments. The Conservative party commissioned the 'Vorderman Report' (Vorderman et al., 2011) which raised the stakes further so that the then Secretary of State for Education (Gove, 2011), announced the government's goal that 'within a decade the vast majority of pupils are studying maths right through to the age of 18'. Subsequently, in 2012 he went further and set out his intention for ambitious reforms aimed at producing qualifications, across the disciplines, which met the needs of the nation's research intensive universities. This marked a turning point in the role of universities in influencing A level curricula and assessments, or so it seemed. These two policy directions are considered in turn.

3.3.1 *Mathematics for the disciplines (adding mathematics)*

Running in parallel with A level reform, the aspiration of achieving 'maths for all to 18' required new qualification pathways for around one third of each national cohort (>200,000 students) who have achieved the requisite GCSE Mathematics grade C at age 16, but then choose to opt out of mathematical study. The Government's approach to this issue centred on a new qualification: Core Maths. Commissioned by the Department for Education, an ACME-convened expert panel developed 'guidelines' for Core Maths (Browne et al., 2013) and recommended a qualification to consolidate and build on the content of GCSE Mathematics, with a strong emphasis on problem solving. It would include mathematical applications and statistics but not calculus and would support the needs of students requiring mathematics in other subject areas or employment. As such the qualification was distinctive from A level mathematics which continued to provide the main pathway to post-16 mathematics study. In 2014 six new qualifications were introduced under the collective name of Core Maths and these were first assessed in 2016.

3.3.2 *Mathematics within disciplines (embedding mathematics)*

In 2012, the mathematical content of A levels in a range of other disciplines became the subject of renewed interest with the publication of two influential reports. The Science Community Representing Education (SCORE) focussed on mathematics within the sciences

(SCORE, 2012) while the Nuffield Foundation considered the quantitative demands of business studies, computing, economics, geography, psychology and sociology (Nuffield, 2012). In parallel studies, using similar methodologies, the 2010 A level examinations were analysed. Both studies found differences in the mathematical experiences of young people taking the same A level qualifications offered by the different awarding organisations. In addition, the SCORE report identified weaknesses in the nature of the mathematical work within science assessments and called for greater coherence across mathematics and science qualifications.

A pivotal role in the overall reform process was the Government-commissioned independent review of A level subjects (Smith, 2013; Smith, 2014). Drawing on the aforementioned SCORE and Nuffield reports and other evidence, including from stakeholders in higher education, the review proposed timelines for the introduction of reformed A levels and made recommendations on changes to mathematical requirements within a range of subjects. Subsequently, Department for Education subject criteria and regulations included more detail on mathematical requirements along with prescribed percentages for mathematical content within science A level assessments (DfE, 2014). The same review considered A level Mathematics sufficiently important to warrant more substantial changes and the work was devolved to a new university-led organisation, the A level Content Advisory Board (ALCAB).

3.4 *Mathematical interventions within universities*

This case study of England would not be complete without some mention of the reactionary measures to the 'mathematics problem' taken within higher education. Cognisant of the issues and the challenges arising from the diverse preparatory experiences of students, universities in England (and elsewhere in the UK) have had to put in place provision to support students at module, programme and institutional levels. Practice varies across institutions, as does awareness of the options available, but some commonly used mechanisms include diagnostics, modules to support mathematics and embedding mathematics within the curriculum, the provision of supplementary resources offered through a variety of mechanisms and online platforms, and mathematics support (Hodgen et al., 2014). In this context, one area which has seen considerable growth in the last ten years is the provision and uptake of institutional mathematics (and statistics) support (Perkin et al., 2013). This is often located centrally within institutions and provides opportunities for students to obtain help with mathematics from tutors who frequently are not directly involved in their undergraduate programmes. However, the effective use of mathematics support does rely heavily on student self-referral and tutor-referral mechanisms, which are predicated on an awareness of, and understanding of, the benefits of the provision.

4. Discussion

There is little doubt amongst a wide variety of researchers and commentators that young people leaving school and university are increasingly required to have the competence and confidence to apply mathematics, statistics and new modes of data analysis in many contexts. However, the international research evidence is far from conclusive regarding the extent to which pre-university mathematics qualifications work to this end. This raises questions about: 1) the mathematical knowledge, skills and understanding needed to bridge the gap between school and undergraduate disciplines; 2) the limits of curriculum continuity between school curricula and diverse mathematical applications and cultures in disciplines; and 3) the management of academics' expectations and post-transition support for mathematically-engaged learners.

The metaphor of 'gap' between the mathematics skills acquired in school and those required in undergraduate study is unhelpful. It suggests something one-dimensional that can be closed and overlooks the structural disconnect between mathematics in schools (with a small number of programmes) and in universities (with multiple contexts of application) and ignores the situatedness of knowledge (Lave & Wenger, 1991) and the challenge of developing expertise in mathematical problem solving and modelling (Kaiser, 2014). Falling back on deficit 'gap' discourses fails to address the real and complex challenges of preparing young people for the increasingly quantitative demands of study and work. That said, improving the mathematical confidence and competence of people entering (and leaving) higher education remains a high priority in many countries.

The case study presented herein has shown how reforms of pre-university mathematics in England are being tackled in two ways: 1) an 'adding maths' policy aims for the vast majority of young people to continue with their study of mathematics to 18; and 2) an 'embedding maths' approach mandates mathematics in reformed subject assessment (McAlinden & Noyes, 2017). Of necessity, the uptake of A level Mathematics and the new Core Maths are integral to the success of this two-pronged policy approach. However, there remain many questions regarding which mathematical route will provide the most effective support for the individual needs of students engaged in the study of other disciplines, both at A level and at university (Hodgen et al, 2014; McAlinden & Noyes, 2017).

Given that the first public examination of Core Maths took place in the summer of 2016, it is too soon to evaluate its long-term success. However, there were a number of substantial weaknesses in the communication of its introduction that are worthy of note as they provide constructive insights into pitfalls associated with introducing new qualifications. While six Core Maths qualifications were introduced, none of them actually carried the name Core Maths in its title. Furthermore, there was little concerted effort by Government to bring the qualification to the attention of universities in a timely way. In addition, following the 2014 Government reshuffle, and the accompanying changes at the Department for Education, there was a notable reduction in the championing of Core Maths, despite the fact that overall Government policy in the area remained the same. The successful uptake of Core Maths will ultimately depend on whether it is valued by employers and to what extent universities will recommend/require it. Its long-term value in preparing the majority of students for university and employment remains to be seen and it seems that officials might have underestimated the challenge in securing such a dramatic step change in school mathematics participation.

The English policy approach of *adding* and *embedding* mathematics has considerable merits, albeit with some chance of falling short of the hoped for outcomes. This is largely due to underestimating the sheer complexity and level of challenge of procuring such a societal shift in expectations, *viz.* that all young people continue their study of mathematics to age 18. We have examined the embedding side of this elsewhere through an analysis of the mathematical content of the reformed A level qualifications (McAlinden & Noyes, 2017). This new approach is a significant shift and will require professional development for teachers and conversations between teachers in different subjects to improve curriculum alignment.

The policy of *adding* new qualification pathways aims to enable all young people to study mathematics throughout their school career and up to the transition to university. However, encouraging this move without compulsion is a challenge, particularly given recent evidence that 80% of young people do not support this idea (Noyes & Adkins, 2016b). The competition between different mathematics pathways is also a potential problem in England and in other countries (e.g. New Zealand). This reflects longstanding debates about educational priorities and quantitative literacy (Steen, 1990). The studies at the outset of this paper reflect this tension between privileging higher level, advanced mathematics (e.g. Tai, et al. 2005) and confident, sophisticated application of basic mathematics (e.g. Nicholas et al. 2015), though these are not independent.

Whether the current reforms in England are likely to work or not there is always going to be a limit to the continuity between a national school curriculum and more diverse, locally implemented, university curricula. Our analysis shows that much of the policy attention to this problem has hitherto been in schools. Autonomous university departments have generally been reluctant to require students to have studied a particular form of mathematics for fear of limiting their pool of applicants in what has been an expanding and increasingly competitive student market. Some academics recognise and accept that mathematical support and 'on course' mathematics education will be required for new undergraduates. It is also evident that many academics do not have up-to-date knowledge of the mathematical demands of pre-university qualifications, or realistic expectations of the skills of students with particular pre-university qualification grades (Koenig, 2011; Lee, 2016). In some disciplines there is also a further disconnect between students' expectations of the mathematical demands of their university programmes and that of lecturers (Hodgen et al, 2014). It is problematic when academics avoid requiring particular forms of prior mathematical learning and then bemoan the fact that their students do not have that learning.

There is some way to go in understanding this complex relationship between school and university applications of mathematics. This paper has set out this complexity in detail in one national context and this no doubt resonates with other systems, at least in part, as evidenced by the current literature. With the increasing level of international student mobility and a culture of ever-more global comparisons, this problem is perhaps also becoming more international in nature.

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