

1 **Seismic Protection of Structures with Supplemental Rotational Inertia**

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7 **ABSTRACT**

8 In this paper we investigate the alternative strategy of suppressing ground-induced vibrations with supplemental
9 rotational inertia. The proposed concept employs a rack-pinion-flywheel system that its resisting force is proportional to
10 the relative acceleration between the vibrating mass and the support of the flywheels. This arrangement, known in the
11 mechanical networks literature as the “inertor”, complements the traditional supplemental damping and stiffness
12 strategies used for the seismic protection of structures. The paper shows that the seismic protection of structures with
13 supplemental rotational inertia has some unique advantages; in particular in suppressing the spectral displacements of
14 long period structures –a function that is not efficiently achieved with large values of supplemental damping. The paper
15 shows that this happens at the expense of transferring appreciable forces at the support of the flywheels and proceeds by
16 examining to what extent the finite stiffness and damping of the support of the flywheels affects the dynamics of the
17 system. The proposed concept may be attractive for the seismic protection of bridges given that the rack-pinion-
18 flywheel system strategy can accommodate large displacements.

20 **INTRODUCTION**

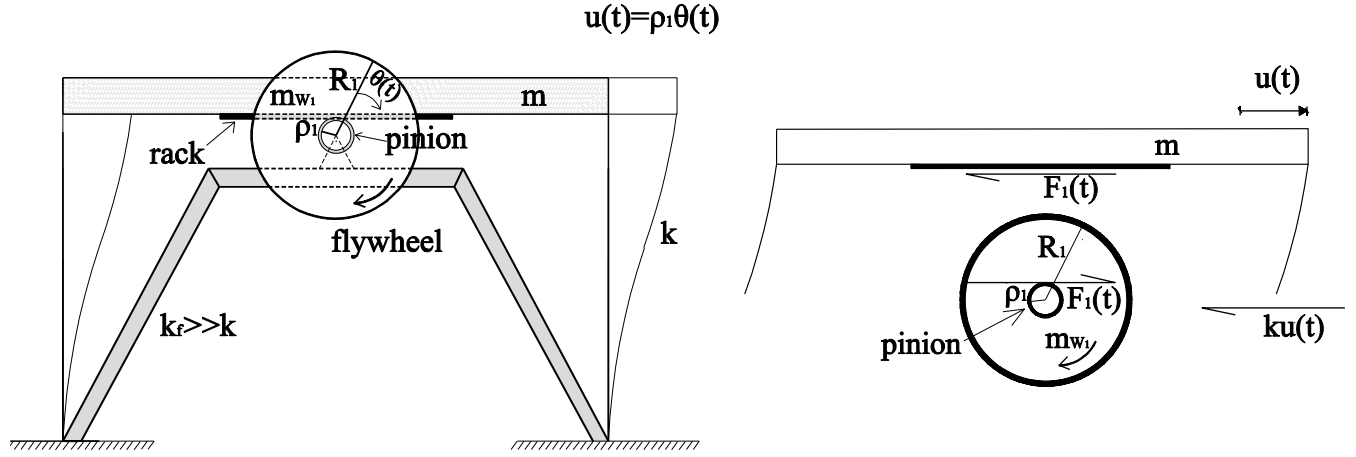
21 Most common civil engineering structures are framing systems or shear-beam type structures which when subjected to
22 lateral inertial loading the dominant motion of their masses (floors in buildings or bridge decks) is a linear translation.
23 In such structural systems the seismic induced displacements are primarily controlled with elasticity, damping and

24 strength (Clough and Penzien 1975; Kelly 1997; Constantinou et al. 1998; Naeim and Kelly 1999; Chopra 2000;
25 Makris and Chang 2000a, 2000b; Black et al. 2004; Symans et al. 2008). A notable exception to this kind of response is
26 the seismic response of the free-standing slender column, which upon uplifting it enters rocking motion (Kirkpatrick
27 1927; Housner 1963). It is because of this rotational motion that most of the seismic resistance of the free-standing
28 rocking column originates from the difficulty to mobilize its rotational inertia (Makris 2014a, 2014b, Makris and
29 Kampas 2016 and references reported therein) –a quantity that is proportional to the square of the column size.

30 The main motivation of this paper is to examine whether the unique advantages for seismic protection that result from
31 the mobilization of rotational inertia can be implemented in traditional framing systems where the dominant motion of
32 their masses is translation (no rotations). The paper proposes the use of supplemental flywheels which are engaged in
33 motion through a rack-and-pinion system (Patton 1980; PTDA 2014; among others). The proposed concepts may also
34 apply to the seismic protection of bridges given the more-than-a-century-long experience of power transmission
35 technology in movable bridges (Engineering News 1913; Hahin 1998; Movable Bridge Engineering 2014; among
36 others). The advantage of a set of flywheels which are engaged in motion through a rack-and-pinion system is that the
37 supplemental inertia is proportional only to the relative acceleration between the structure and the support of the
38 flywheels. This mechanical arrangement that is coined “the inerter” has been proposed in the context of synthesis of
39 mechanical networks in an effort to achieve a completely analogous correspondence between mechanical and electrical
40 circuits (Smith 2002; Papageorgiou and Smith 2005). More recently the concept of a two-terminal device, that its output
41 force is proportional to the relative acceleration has been proposed to enhance the performance of tuned mass dampers
42 (Marian and Giaralis 2013, 2014; Giaralis and Taflanidis 2015). Furthermore, the proposed rack-pinion-flywheel can
43 accommodate large, translational displacements which may challenge the implementation of other seismic protection
44 devices such as fluid or metallic dampers.

45 **REDUCTION OF VIBRATIONS WITH SUPPLEMENTAL ROTATIONAL INERTIA**

46 Figure 1(left) depicts a single-degree-of-freedom structure with stiffness k and mass, m . A stiff chevron frame
47 supports a flywheel with radius, R_1 and mass m_{w1} which can rotate about an axis O. First we consider the case of a
48 very stiff chevron frame that its deformation is negligible to the translational displacement, $u(t)$, of the SDOF
49 structure. Concentric to the flywheel there is an attached pinion with radius, ρ_1 , engaged to a linear rack connected to



50 **Fig. 1.** Left: A single-degree-of-freedom structure with mass, m , and stiffness, k , with supplemental rotational inertia
 51 from a flywheel with radius, R , supported on a chevron frame with stiffness, k_f , that is much larger than k ; Right:
 52 Free-body diagram of the vibrating mass, m , when engaged to the pinion of the flywheel shown below.
 53 the bottom of the vibrating mass, m , of the SDOF. With this arrangement when the mass m undergoes a positive
 54 displacement, $u(t)$, the flywheel is subjected to a clockwise rotation, $\theta_1(t)$. Given that there is no slipping between
 55 the rack and the pinion,

56
$$\theta_1(t) = \frac{u(t)}{\rho_1} \quad (1)$$

57 Figure 1(right) shows the free-body diagrams of the vibrating mass, m , and the rotating pinion-flywheel system. For a
 58 positive displacement, $u(t)$, to the right, the internal force, F_1 , at the rack-pinion interface opposes the motion (to the
 59 left). Accordingly, dynamic equilibrium of the vibrating mass when subjected to a ground acceleration, $\ddot{u}_g(t)$, gives

60
$$m[\ddot{u}(t) + \ddot{u}_g(t)] = -ku(t) - F_1(t) \quad (2)$$

61 where the internal force $F_1(t)$ needs to satisfy the moment equilibrium of the flywheel about point O

62
$$I_{w1} \ddot{\theta}_1(t) = F_1(t) \rho_1 \quad (3)$$

63 In equation (3), $I_{w1} = (1/2)m_{w1}R_1^2$, is the moment of inertia of the flywheel about point O. Substitution of equation
 64 (3) into (2) in association with equation (1) gives

65
$$\left(1 + \frac{1}{2} \frac{m_{w1}}{m} \frac{R_1^2}{\rho_1^2}\right) \ddot{u}(t) + \omega_o^2 u(t) = -\ddot{u}_g(t) \quad (4)$$

66 where, $\omega_o = \sqrt{k/m}$, is the natural frequency of the structure when the pinion-flywheel is disengaged. Upon dividing
 67 with the acceleration coefficient, equation (4) gives,

68
$$\ddot{u}(t) + \underbrace{\frac{\omega_o^2}{1 + \frac{1}{2} \frac{m_{w1}}{m} \frac{R_1^2}{\rho_1^2}}}_{\text{Lengthening of the period}} u(t) = - \underbrace{\frac{1}{1 + \frac{1}{2} \frac{m_{w1}}{m} \frac{R_1^2}{\rho_1^2}}}_{\text{Suppression of the input ground motion}} \ddot{u}_g(t) \quad (5)$$

69

70 Equation (5) indicates that the engagement of the flywheel in a rotational motion lengthens the vibration period of the
 71 structure and most importantly it suppresses the level of ground shaking given that the denominator in the right hand
 72 side is always larger than unity.

73 **AMPLIFICATION OF THE ROTATIONAL INERTIA EFFECT**

74 Equation (5) dictates that when the SDOF with stiffness, k , and mass, m , is equipped with a single flywheel with
 75 radius, R_1 , and mass, m_{w1} , together with a pinion with radius, ρ_1 , the input ground acceleration is divided by the
 76 term

77
$$1 + \sigma = 1 + \frac{1}{2} \frac{m_{w1}}{m} \frac{R_1^2}{\rho_1^2} \quad (6)$$

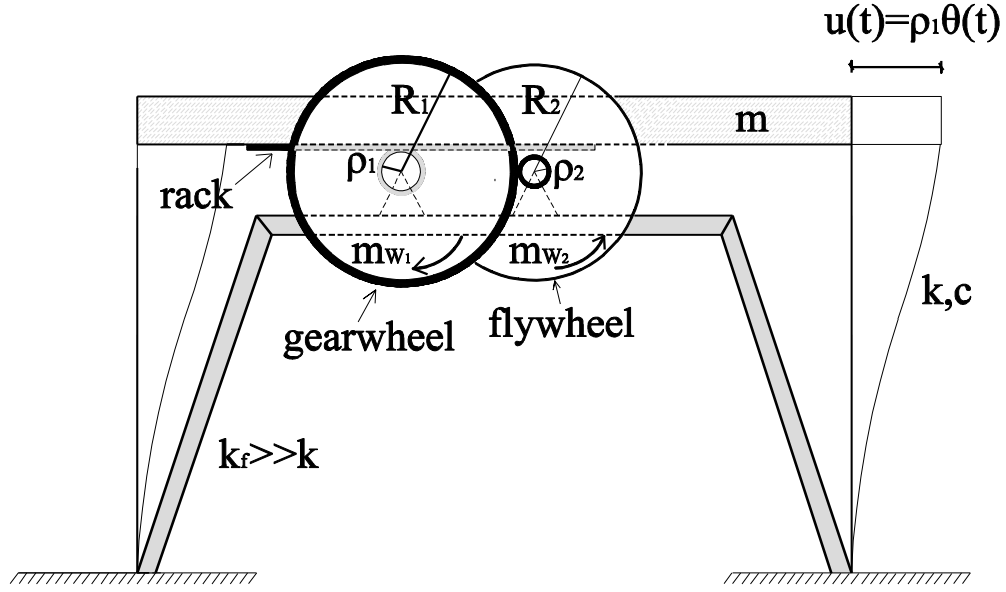
78 While the denominator $= 1 + \sigma$ is always larger than unity and the radius of the flywheel, R_1 , may be as large as ten

79 (10) times the radius of the pinion, ρ_1 , (or even larger, $\frac{R_1^2}{\rho_1^2} > 100$), the suppression coefficient $= \frac{1}{2} \frac{m_w}{m} \frac{R_1^2}{\rho_1^2}$ may

80 remain small given that the mass of the flywheel, m_{w1} , is appreciably smaller than the mass of the structure, m .

81 Nevertheless, the effect of supplementing framing structures with rotational inertial may be amplified by installing two

82 (or more) flywheels in series where the first flywheel is a gear-wheel as shown in Figure 2.



83

84

Fig. 2. More than one flywheels in series that amplify the effect of supplemental rotational inertia.

85

For the dynamic analysis of a system shown in Fig. 2 one can certainly proceed with a direct formulation as was done in

86

the previous section for a single flywheel. Nevertheless, given that this proposed concept may involve several flywheels

87

we proceed with a variational formulation where there is no need to calculate the internal forces, F_{j+1} , between the

88

flywheel j and the pinion of the flywheel, $j + 1$. Application of Lagrange's equation to the system shown in Fig. 4

89

gives

90

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{u}} \right) - \frac{\partial \mathcal{L}}{\partial u} = - \frac{dW}{du} \quad (7)$$

91

where, $\mathcal{L} = T - V$, is the Langrangian function (difference between the kinetic energy, T , and the potential energy, V

92

, of the system) and W is the work done by the external field forces. During an admissible translation δu , the variation

93

of the work, $\delta W = m \ddot{u}_g(t) \delta u$ and given that $\delta W = \frac{dW}{du} \delta u$, one obtains

94

$$\frac{dW}{du} = m \ddot{u}_g(t) \quad (8)$$

95

For the two-wheel SDOF system shown in Figure 2, the kinetic energy is

96
$$T = \frac{1}{2} m \dot{u}^2(t) + \frac{1}{2} I_{w1} \dot{\theta}_1^2(t) + \frac{1}{2} I_{w2} \dot{\theta}_2^2(t) \quad (9)$$

97 where $\theta_1(t)$ is given by equation (1) and $\theta_2(t)$ satisfies compatibility of the displacements between the gearwheel and
 98 the pinion of the second flywheel.

99
$$\theta_2(t) = \theta_1(t) \frac{R_1}{\rho_2} \quad (10)$$

100 Substitution of equations (1) and (10) into equation (9), together with that $I_{w1} = \frac{1}{2} m_{w1} R_1^2$ and $I_{w2} = \frac{1}{2} m_{w2} R_2^2$,

101 gives the expression of the kinetic energy only in terms of the velocity of the SDOF.

102
$$T = \frac{1}{2} m \dot{u}^2(t) \left(1 + \frac{1}{2} \frac{m_{w1}}{m} \frac{R_1^2}{\rho_1^2} + \frac{1}{2} \frac{m_{w2}}{m} \frac{R_1^2 R_2^2}{\rho_1^2 \rho_2^2} \right) \quad (11)$$

103 The potential energy of the SDOF is merely,

104
$$V = \frac{1}{2} k u^2(t) \quad (12)$$

105 and the Lagrangian function of the SDOF shown in Figure 2 assumes the expression:

106
$$\mathcal{L} = \frac{1}{2} m \dot{u}^2(t) \left(1 + \frac{1}{2} \frac{m_{w1}}{m} \frac{R_1^2}{\rho_1^2} + \frac{1}{2} \frac{m_{w2}}{m} \frac{R_1^2 R_2^2}{\rho_1^2 \rho_2^2} \right) - \frac{1}{2} k u^2(t) \quad (13)$$

107 Substitution of equations (13) and (8) into Lagrange equation (7) gives the equation of motion of the SDOF structure
 108 with a two flywheel rotational inertia system as shown in Figure 2.

109
$$\left(1 + \frac{1}{2} \frac{m_{w1}}{m} \frac{R_1^2}{\rho_1^2} + \frac{1}{2} \frac{m_{w2}}{m} \frac{R_1^2 R_2^2}{\rho_1^2 \rho_2^2} \right) \ddot{u}(t) + \omega_0^2 u(t) = -\ddot{u}_g(t) \quad (14)$$

110 where, $\omega_0 = \sqrt{k/m}$, is again the natural frequency of the SDOF structure when is disengaged from the flywheel
 111 system. Accordingly, the equation of motion of the SDOF structure assumes the form

$$112 \quad \ddot{u}(t) + \frac{\omega_0^2}{1+\sigma} u(t) = -\frac{1}{1+\sigma} \ddot{u}_g(t) \quad (15)$$

113 where for the case of a two-flywheel rotational inertia system, the suppression coefficient, σ , is

$$114 \quad \sigma = \frac{1}{2} \frac{m_{w1}}{m} \frac{R_1^2}{\rho_1^2} + \frac{1}{2} \frac{m_{w2}}{m} \frac{R_1^2 R_2^2}{\rho_1^2 \rho_2^2} \quad (16)$$

115 For a ratio $R_2 / \rho_2 \approx 10$, the second term in equation (16) is two orders of magnitude larger than the first term. When
 116 n flywheels are installed in series, the rotation of the n^{th} flywheel is

$$117 \quad \theta_n = \theta_1 \frac{R_1}{\rho_2} \frac{R_2}{\rho_3} \dots \frac{R_{n-1}}{\rho_n} = \theta_1 \prod_{j=2}^n \frac{R_{j-1}}{\rho_j} \quad (17)$$

118 By virtue of equation (17), the kinetic energy of the n^{th} flywheel is

$$119 \quad T_n = \frac{1}{2} I_{wn} \theta_n^2(t) = \frac{1}{2} \frac{m_w}{2} R_n^2 \theta_1^2 \prod_{j=2}^n \left(\frac{R_{j-1}}{\rho_j} \right)^2; \quad (18)$$

120 and upon replacing, $\theta_1(t)$ with $u(t) / \rho_1$, equation (18) gives

$$121 \quad T_n = \frac{1}{2} \frac{m_w}{2} \frac{R_1^2 R_2^2 \dots R_n^2}{\rho_1^2 \rho_2^2 \dots \rho_n^2} \dot{u}^2(t) \quad (19)$$

122 Consequently, the equation of motion of the SDOF structure equipped with a n -flywheel rotational inertia system is
 123 given again by equation (15); where now the suppression coefficient, σ , is given by

$$124 \quad \sigma = \frac{1}{2} \frac{m_{w1}}{m} \frac{R_1^2}{\rho_1^2} + \frac{1}{2} \frac{m_{w2}}{m} \frac{R_1^2 R_2^2}{\rho_1^2 \rho_2^2} + \dots + \frac{1}{2} \frac{m_{wn}}{m} \frac{R_1^2 R_2^2 \dots R_n^2}{\rho_1^2 \rho_2^2 \dots \rho_n^2} \quad (20)$$

125 For a ratio $R_j / \rho_j \approx 10$, each term in equation (20) is two order of magnitude larger than the previous term, therefore
 126 for any number, n , of flywheels selected, the suppression coefficient is merely governed by the last term of equation
 127 (20).

$$128 \quad \sigma \approx \frac{1}{2} \frac{m_{wn}}{m} \frac{R_1^2 R_2^2 \dots R_n^2}{\rho_1^2 \rho_2^2 \dots \rho_n^2} \quad (21)$$

129 Accordingly, regardless how small is the ratio m_{wn} / m , the suppression coefficient σ can assume any desired value
 130 with the sufficient size and number of flywheels. Figure 3 (left) shows a schematic of the mass-spring-inerter system
 131 described by equation (15). It is a linear system that does not dissipate any energy—the vibration suppression happens by
 132 transferring kinetic energy from the vibrating mass to the rotating flywheel.

133 **FORCE TRANSFERRED TO THE CHEVRON FRAME**

134 The force transferred to the chevron frame, F_1 , is an internal force, which can be recovered from the final form of the
 135 equation of motion given by (15) in association with the original dynamic equilibrium equation (2) of the vibrating
 136 mass that is expressed as

$$137 \quad m\ddot{u}(t) + F_1(t) + ku(t) = -m\ddot{u}_g(t) \quad (22)$$

138 Equation (15) is expressed as

$$139 \quad m\ddot{u}(t) + \sigma m\ddot{u}(t) + ku(t) = -m\ddot{u}_g(t) \quad (23)$$

140 Upon subtracting equation (23) from (22) one obtains the internal force transferred to a stiff chevron frame

$$141 \quad F_1(t) = \sigma m\ddot{u}(t) = M_R \ddot{u}(t) \quad (24)$$

142 where σ is the suppression coefficient given by equation (20) or (21). The quantity $M_R = \sigma m$ is an additional
 143 apparent mass in the system (namely the rotational mass = M_R) which is due to the rotational inertia of the flywheels,
 144 Equation (24) offers the force transferred to a stiff chevron frame that its deformation is negligible compared to the

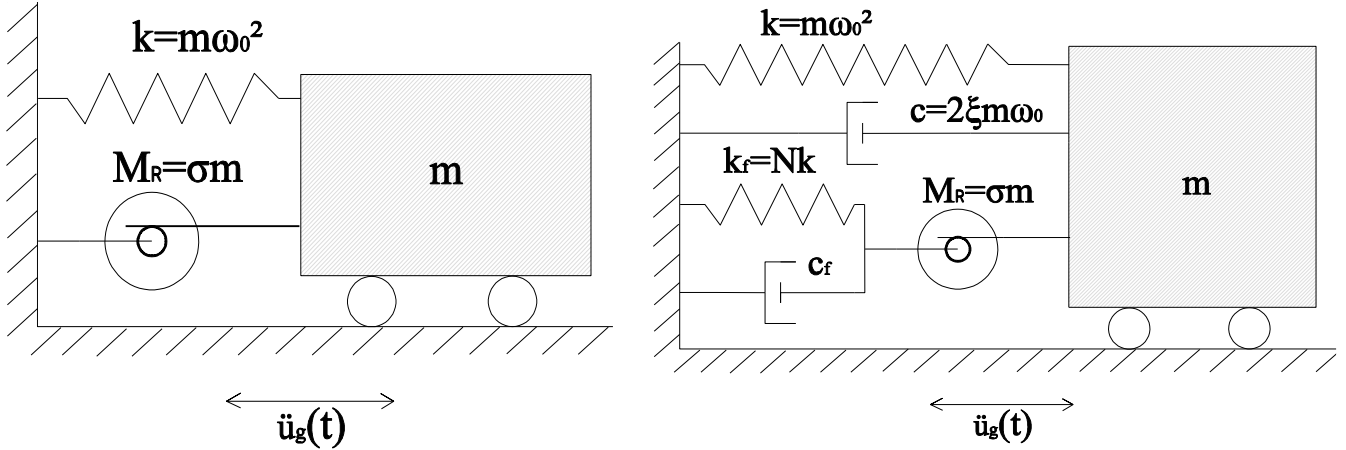


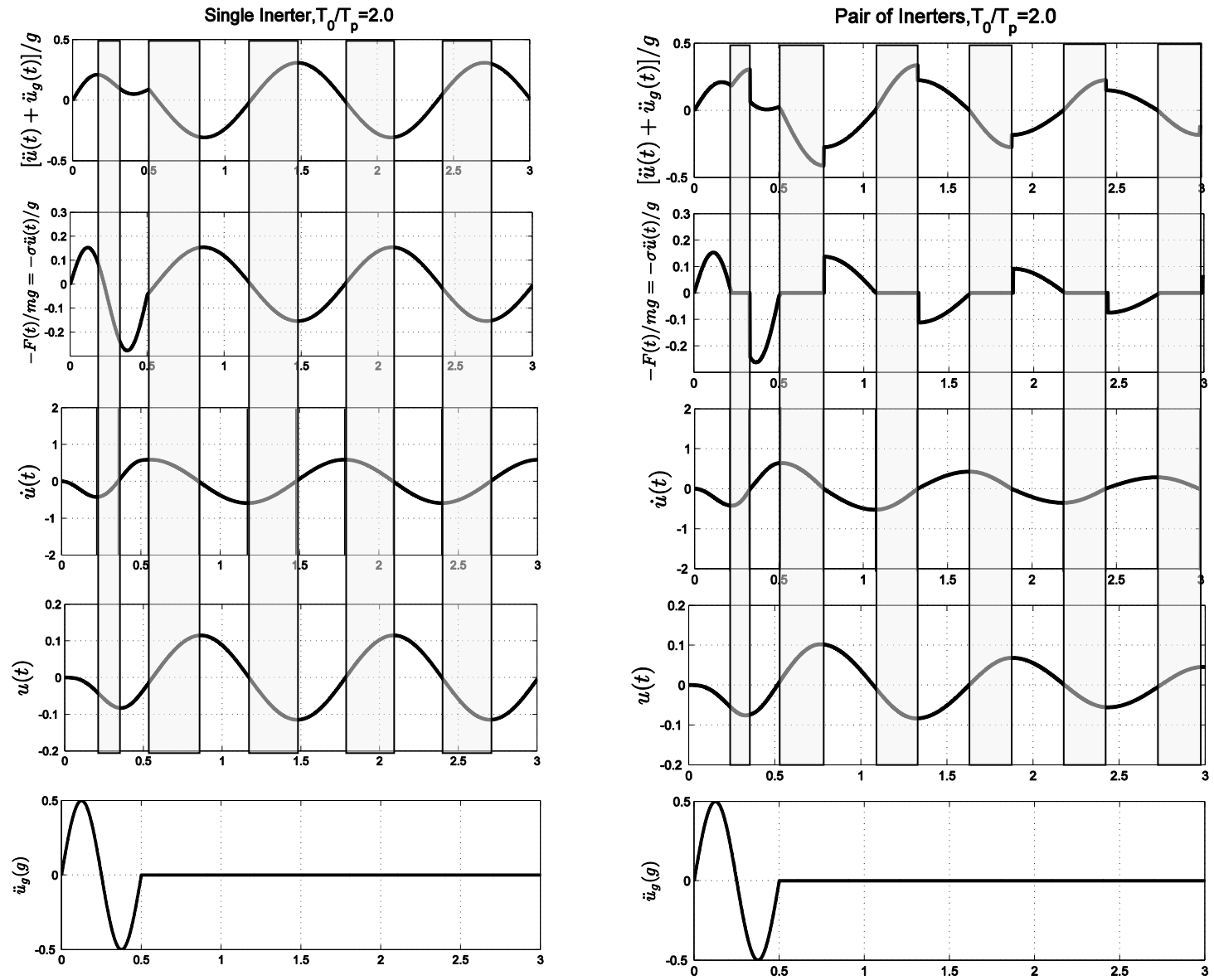
Fig. 3. Left: Schematic of a mass-spring-inerter single-degree-of-freedom system: the inerto-elastic oscillator; Right: The inerto-visco-elastic oscillator.

145 displacement of the structure, $u(t)$, and it indicates that in this case, $F_1(t)$, is proportional to the relative acceleration
 146 of the SDOF- system. The case of a “flexible” chevron frame is treated in a following section.

147 **THE NEED FOR TWO PARALLEL INERTIA SYSTEMS AND THE OPPORTUNITY FOR ENERGY**
 148 **HARVESTING**

149 In the previous sections we introduced the concept of supplemental rotational inertia for the seismic protection of
 150 traditional framing systems and it was shown that the proposed concept is physically realizable with the arrangement of
 151 more than one flywheel in series so that the amplification coefficient given by equation (20) becomes sufficiently large.

152 In the ideal case where energy is not dissipated through friction or other energy dissipation mechanism, equation (15)
 153 describes an undamped system (see Figure 3-left) in which part of the ground induced energy is transferred in to the
 154 flywheels. Figure 4 (left) plots the relative displacement, velocity, force transferred to the chevron frame and absolute
 155 acceleration of the inerto-elastic oscillator show in Figure 3 (left) with $T_0 = 1.0s$ when subjected to a one-sine
 156 acceleration with acceleration amplitude $a_p = 0.5g$ and pulse duration $T_p = 0.5s$. The shaded stripes in Figure 4
 157 correspond to the segments where the magnitude of the relative velocity of the oscillator described by equation (15)
 158 reduces on its way to reach a peak displacement. During this interval, the flywheels have built angular momentum and
 159 now as the translating mass tends to move slower the flywheels may drive the mass; therefore, inducing deformations –
 160 a situation that is not desirable.



161 **Fig. 4.** Response of an inerto-elastic oscillator with an infinite stiff chevron frame. Left: Single inerter which may induce deformations; Right: Pair of inerters that
 162 can resist only the motion as described by equations (27) and (28). The force from the inerter only opposes the motion.

163 One challenge with the proposed concept is that the rotating flywheels should only resist the motion of the structure
164 without inducing any deformations. This is feasible if the pinion of the first gearwheel that is engaged to the rack is
165 unable to drive the rack and only the motion of the translating rack can drive the pinion-gearwheel. This is similar to the
166 motion of a bicycle where the cyclist can drive the wheel through the pedals; yet, when the bicycle is rolling the pedals
167 may remain idle. Without loss of generality, let assume that upon initiation of motion the structure moves to the left,
168 therefore the front gearwheel rotates counterclockwise and the force on the mass from the gearwheel is to the right
169 (positive). As the mass keeps moving to the left it will slow down and at the instant where the gearwheel will tend to
170 drive he mass due to its angular momentum the force transmission need to become idle. With the proposed
171 arrangement, upon the structure has reached its first maximum displacement and the motion reverses to the right (
172 $\dot{u}(t) < 0$); the front gearwheel keeps rotating freely counterclockwise without inducing any force to the structure.
173 When the structure starts moving to the right ($\dot{u}(t) > 0$) a second, parallel rotational inertia system (the back
174 flywheels) is needed to oppose the motion, and during the course of this motion the first gearwheel of the back system
175 that is engaged to the rack rotates clockwise. The sequential engagement of the two parallel rotational inertial systems
176 that can only resist the motion is expressed mathematically:

$$177 \quad \frac{F_1(t)}{m} = \sigma \ddot{u}(t) \quad \text{when } \text{sgn} \left[\frac{\ddot{u}(t)}{\dot{u}(t)} \right] > 0 \quad (25a)$$

$$178 \quad \text{and} \quad \frac{F_1(t)}{m} = 0 \quad \text{when } \text{sgn} \left[\frac{\ddot{u}(t)}{\dot{u}(t)} \right] < 0 \quad (25b)$$

179 Accordingly, for the two parallel rotational inertia systems that only resist the motion of the structure, the equation of
180 motion (15) is modified to

$$181 \quad (1 + \delta\sigma)\ddot{u}(t) + \omega_0^2 u(t) = -\ddot{u}_g(t) \quad (26)$$

$$182 \quad \text{in which} \quad \delta = \begin{cases} 1, & \text{when } \text{sgn} \left[\frac{\ddot{u}(t)}{\dot{u}(t)} \right] > 0 \\ 0, & \text{when } \text{sgn} \left[\frac{\ddot{u}(t)}{\dot{u}(t)} \right] < 0 \end{cases} \quad (27)$$

183 Clearly, with the two parallel front and back rotational inertia systems the flywheels only resist the motion of the
 184 structure and do not induce any energy. Nevertheless, during the time-period where one of the flywheel systems is
 185 rotating idle its rotation needs to decelerate appreciably so that when it is again engaged into motion, to be capable to
 186 resist the motion through its rotational inertia. One possibility for decelerating the flywheels when rotating idle is to
 187 append to their axis an induction generator. With this arrangement, part of the earthquake induced energy will be
 188 converted into electricity that may be very much needed at that time.

189 Figure 4 (right) plots the same response quantities as these presented on Figure 4(left); however now, the rotating
 190 flywheels only resist the motion of the structure (when the flywheels rotate idle the transmitting force is zero and in this
 191 way throughout the response history the force from the flywheels and the velocity have always opposite signs). In this
 192 case the response of the SDOF system described with equations (26) and (27) is decaying with time since part of the
 193 seismic induced energy has been harvested through the rotation of the flywheels.

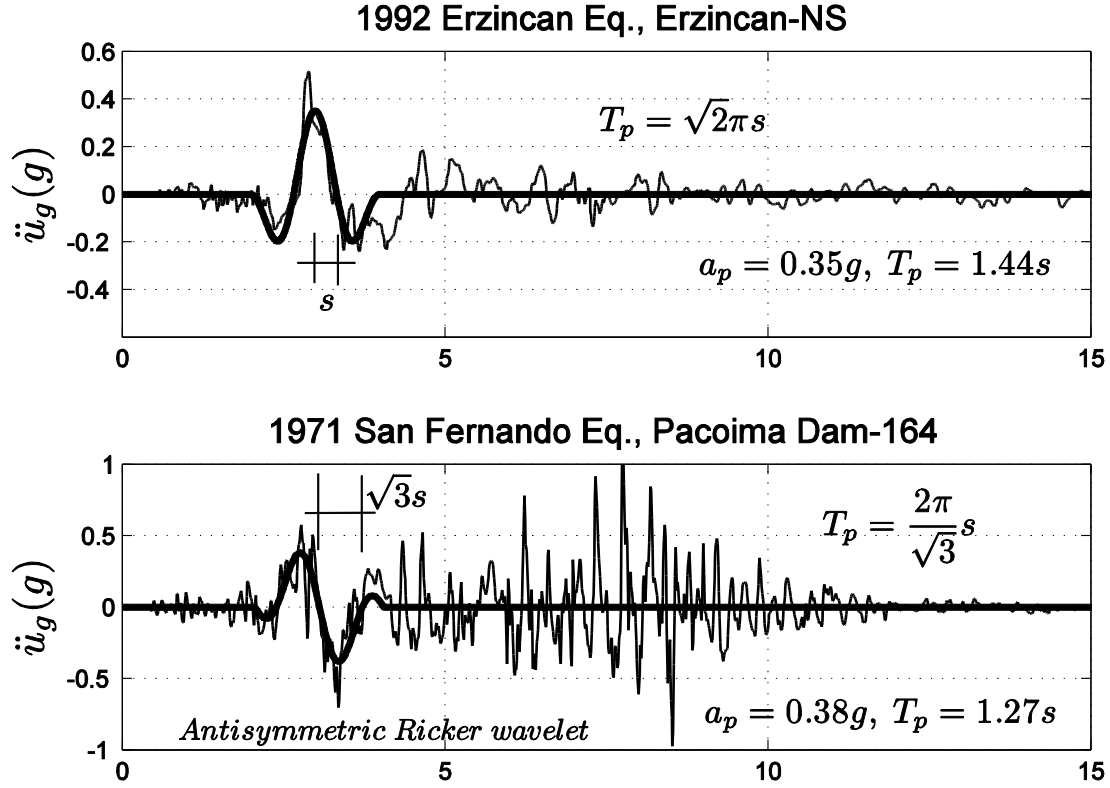
194 **RESPONSE SPECTRA OF A MASS WITH A SPRING AND INERTER IN PARALLEL (“INDEFINITELY”**
 195 **STIFF CHEVRON FRAME)**

196 The seismic response of the undamped SDOF structure with supplemental rotational inertia as described by equations
 197 (15) or (26) and (27) is compared with the seismic response of the linear damped oscillator (Clough and Penzien 1975;
 198 Chopra 2000; Symans et al. 2008)

199
$$\ddot{u}(t) + 2\xi_d \omega_0 \dot{u}(t) + \omega_0^2 u(t) = -\ddot{u}_g(t) \quad (28)$$

200 The values of the suppression coefficient, σ , appearing in equation (15) and the values of the damping ratio, ξ_d ,
 201 appearing in equation (28) control the reduction of the response of the two systems; yet, they are also responsible for
 202 the force that is transferred to the chevron frame. Accordingly, together with the response spectra associated with the
 203 two oscillators described by equation (15) or (27) and (28) we present the peak value of the force transferred to the
 204 chevron frame, $F_1(t)$, from equations (24) or (25) and $F_d(t) = 2\xi_d m \omega_0 \dot{u}(t)$, in equation (28).

205 In an effort to illustrate some of the advantages and challenges when suppressing vibrations with supplemental
 206 rotational inertia we first present response spectra to pulse excitations (Veletsos et al. 1965; Bertero et al. 1978; Hall et
 207 al. 1965; Alavi and Krawinkler 2001; Mavroeidis and Papageorgiou 2003; Vassiliou and Makris 2011; among others).



208 **Fig. 5.** Top: Nort-South component of the acceleration time history recorded during the 1992 Erzincan, Turkey
 209 earthquake together with a symmetric Ricker wavelet. Bottom: fault-normal component of the acceleration time-
 210 history recorded during the 1971 San Fernando earthquake, together with an antisymmetric Ricker wavelet.

211 As an example, the heavy dark line in Figure 5(top) that approximates the long-period acceleration pulse of the NS
 212 component of the 1992 Erzincan, Turkey, record is a scaled expression of the second derivative of the Gaussian
 213 distribution, $e^{-t^2/2}$, known in the seismological literature as the symmetric Ricker wavelet (Ricker 1943; Ricker
 214 1944; see also Makris and Vassiliou 2013; Garini et al. 2014)

215
$$\ddot{u}_g(t) = a_p \left(1 - \frac{2\pi^2 t^2}{T_p^2} \right) e^{-\frac{1}{2} \frac{2\pi^2 t^2}{T_p^2}} \quad (29)$$

216 The value of the $T_p = \frac{2\pi}{\omega_p}$ is the period that maximizes the Fourier spectrum of the symmetric Ricker wavelet;

217 therefore, $T_p = \sqrt{2\pi}s$, where s is the time from the peak pulse acceleration to the first zero crossing that follows,

218 a_p is the acceleration amplitude of the pulse. Similarly, the heavy dark line in Figure 5(bottom), which

219 approximates the long-period acceleration pulse of the Pacoima Dam motion recorded during the 1971 San
220 Fernando, California, earthquake is a scaled expression of the third derivative of the Gaussian distribution , $e^{-t^2/2}$,

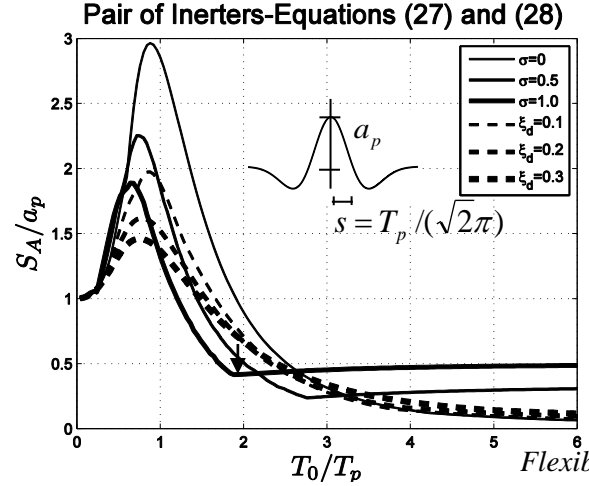
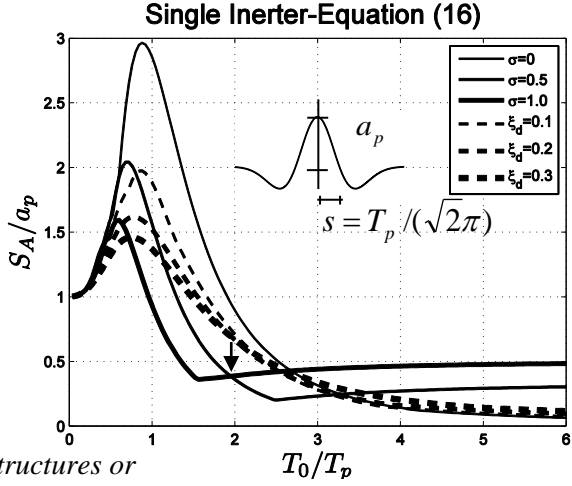
$$221 \quad \ddot{u}_g(t) = a_p \left(\frac{2\pi^2 t^2}{T_p^2} - 3 \right) t e^{-\frac{1}{2} \frac{\pi^2 t^2}{T_p^2}} \quad (30)$$

222 Figure 6 plots total acceleration, relative to the ground displacement and transferred force spectra (with a sufficiently
223 stiff chevron frame) of the SDOF system described with equations (15) or (26) and (27). For the frame with
224 supplemental rotational inertia described by equation (15) or (26) (solid lines) values of $\sigma = 0.5$ and $\sigma = 1.0$
225 have been used; whereas, for the linearly damped oscillator described with equation (28) (dashed lines) values of
226 $\xi_d = 0.1, 0.2$ and 0.3 have been used.

227 The first observation is that supplemental rotational inertia is effective in suppressing appreciably the peak
228 displacement response for moderately long to long -period structures (say $T_0/T_p > 1.5$). Spectral accelerations are
229 suppressed within the range of moderately long periods (say $1.5s < T_0/T_p < 3.0s$). For instance, for
230 $T_0/T_p = 2$ and $\sigma = 1$ both spectral accelerations and spectral displacement are appreciably lower than the
231 corresponding spectra of a heavily damped, linear oscillator with $\xi = 30\%$. However, the corresponding forces
232 transferred to the chevron frame are appreciably higher and in practical applications the chevron frame may
233 experience some non-negligible deformations.

234 When comparing the left plots in Figure 6 which are for the single inerter described with equation (15) (that may
235 induce displacements into the structure) with the right plots in Figure 6 which are for a pair of inerters described
236 with equations (26) and (27) (that can only resist the motion of the structure) we make the following observation.
237 For the case where a very stiff chevron frame is used; when a pair of inerters is employed, spectral displacement are
238 lower for most of the spectrum; while, the forces transferred to the chevron frame are smaller only up to values of
239 $T_0/T_p = 2$. In this frequency range the spectral acceleration from the single inerter system are slightly lower.

240



Stiff structures or long period pulses

Flexible structures or short period pulses

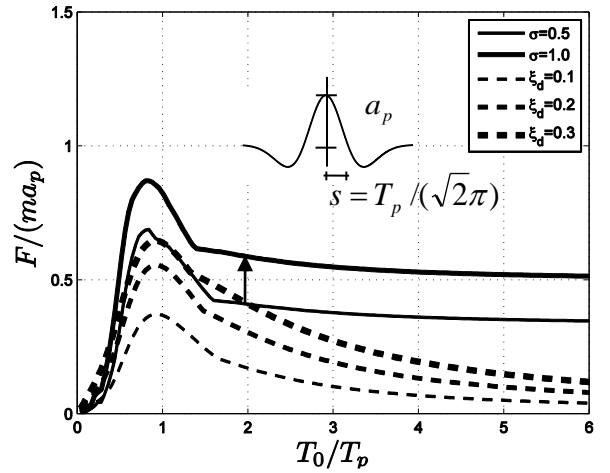
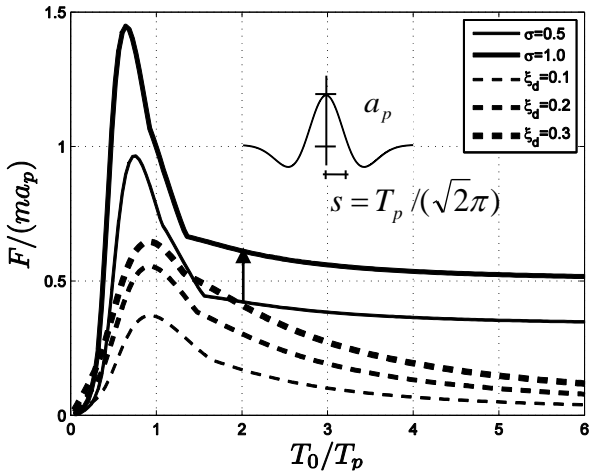
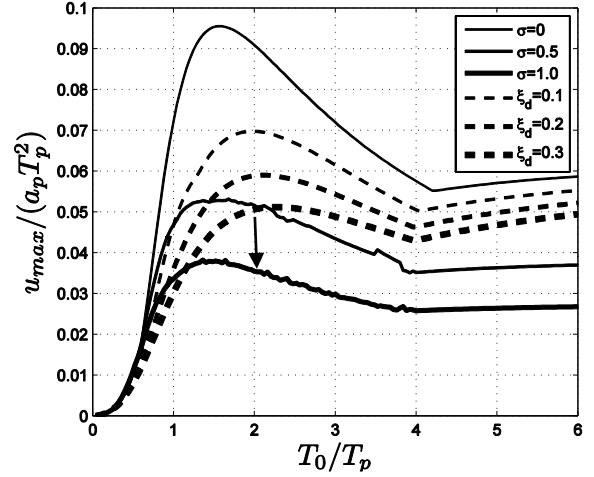
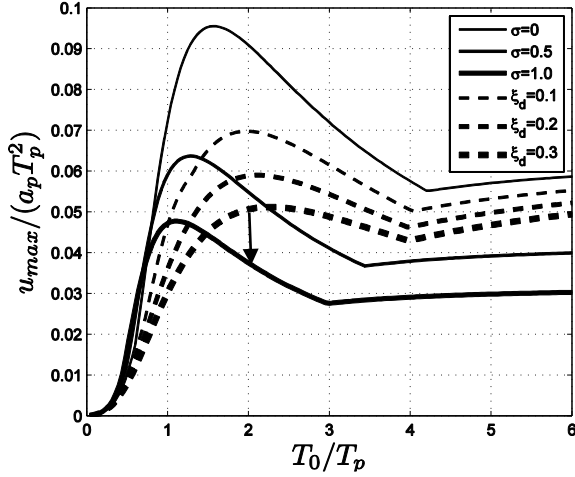


Fig. 6. Total acceleration, relative to the ground displacement and transferred force spectra of a mass-spring-inerter oscillator (solid lines: $\sigma = 0.5, 1$) and a linearly damped oscillator (dashed lines: $\xi_d = 0.1, 0.2$ and 0.3) when subjected to a symmetric Ricker pulse. Left: Single inerter which may drive occasionally the structure; Right: Pair of inerters which can only resist the motion of the structure.

241 **RESPONSE SPECTRA WITH A CHEVRON FRAME WITH FINITE STIFFNESS AND DAMPING**

242 We consider now the case where the support of the rotational inertia system (chevron frame) as shown in Figures 1
 243 and 2 has a finite stiffness, k_f . In the case of bridges this support may be the stiff end abutments of the bridge, as
 244 shown in Figure 7, which when pushed against the backfill soil may mobilize appreciable damping, c_f . The
 245 proposed seismic protection strategy with supplemental rotational inertia with a rack-pinion-flywheel system at each
 246 end-abutment may be attractive for bridges which are flexible in the longitudinal direction; yet, the motion of the
 247 deck is restrained along the transverse direction at the end abutments (Makris et al. 2010). This restriction is nearly
 248 imperative in railway bridges in order to avoid misalignment of the rails at the deck-abutment joints during
 249 transverse shaking; while, it is also common in highway bridges (Kampas and Makris 2012).

250 Due to its finite stiffness as the rack engages with the pinion, the support of the rotational inertia systems (chevron
 251 frame) deforms by the displacement, $u_f(t)$; therefore, the displacement of the mass of the SDOF structure is

252
$$u(t) = u_f(t) + \rho_1 \theta_1(t) \quad (31)$$

253 and upon differentiating two times equation (31) gives

254
 255
$$\ddot{u}_f(t) = \ddot{u}(t) - \rho_1 \ddot{\theta}_1(t) \quad (32)$$

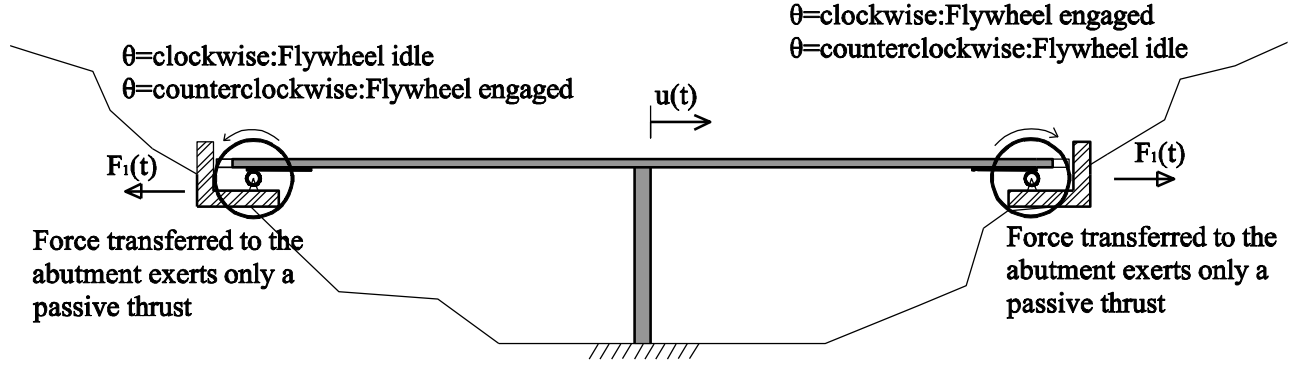
256 The lateral displacement of the chevron frame, $\ddot{u}_f(t)$, is related to the internal force, $F_1(t)$, via the constitutive
 257 law:

258
$$F_1(t) = k_f u_f(t) + c_f \dot{u}_f(t) \quad (33)$$

259 which upon differentiating once gives:

260
$$\ddot{u}_f(t) = \frac{1}{c_f} \dot{F}_1(t) - \frac{k_f}{c_f} \dot{u}_f(t) \quad (34)$$

261 At the same time, the internal force, $F_1(t)$, that acts on the pinion of the gearwheel is given by



262

263 **Fig. 7.** Seismic protection of a bridge along the longitudinal direction with a rack-pinion-flywheel system that exerts
 264 only passive thrust on each end-abutment.

265
$$F_1(t) = \sigma \rho_1 m \ddot{\theta}_1(t) = M_R \rho_1 \ddot{\theta}(t) \quad (35)$$

266 where $\sigma m = M_R$ is the “rotational mass” of the inerter. By equating the right hand sides of the compatibility
 267 equation (32) and the constitutive equation (34) together with the help of (35) one obtains

268
$$\frac{1}{\sigma} \frac{F_1(t)}{m} + \frac{1}{c_f} \dot{F}_1(t) - \frac{k_f}{c_f} \dot{u}_f(t) = \ddot{u}(t) \quad (36)$$

269 In order to eliminate the internal variable, $\dot{u}_f(t)$, in the left-hand-side of equation (36) we differentiate once more
 270 and replace the term $\ddot{u}_f(t)$ with equation (32),

271
$$\frac{1}{\sigma} \frac{\dot{F}_1(t)}{m} + \frac{1}{c_f} \ddot{F}_1(t) + \frac{1}{\sigma} \frac{k_f}{c_f} \frac{F_1(t)}{m} = \frac{k_f}{c_f} \ddot{u}(t) + \ddot{u}(t) \quad (37)$$

272 By introducing the relaxation time of the chevron frame, $\lambda_f = \frac{c_f}{k_f} = \frac{\xi_f}{\pi} T_f$, where $T_f = 2\pi \sqrt{\frac{m}{k_f}}$, is the period

273 of the system when “locked” to the chevron frame; equation (37) becomes

274
$$\frac{1}{\sigma} \left(\frac{F_1(t)}{m} + \lambda_f \frac{\dot{F}_1(t)}{m} + \tau^2 \frac{\ddot{F}_1(t)}{m} \right) = \ddot{u}(t) + \lambda_f \ddot{u}(t) \quad (38)$$

275 in which the parameter

276
$$\tau = \sqrt{\frac{\sigma m}{k_f}} = \sqrt{\frac{M_R}{k_f}} \quad (39)$$

277 is defined as the retardation time of the system.

278 Equation (38) indicates that in the case where a SDOF structure is protected with a supplemental rotational inertia
279 system that is supported on a chevron frame with finite stiffness, k_f , and damping,

280 $c_f = 2\xi_f m \omega_f$ ($\omega_f = \sqrt{k_f / m}$), the internal force, $F_1(t)$, satisfies the differential equation (38) rather than the
281 algebraic equation (24) that is for an infinite stiff frame. In the special case where the damping of the chevron frame
282 is neglected, $c_f = 0$; therefore, $\lambda_f = 0$, equation (38) reduces to

283
$$\frac{F_1(t)}{m} + \tau^2 \frac{\ddot{F}_1(t)}{m} = \sigma \ddot{i}(t) \quad (40)$$

284 In this special case, there is no need to involve the third derivative of the ground displacement in the time-domain
285 analysis. In order to obtain practical values of the retardation time, τ , let assume that the chevron frame is N times
286 stiffer than the SDOF structure; that is, $k_f = Nk = Nm\omega_0^2$. Accordingly, the normalized retardation time to the
287 undamped natural frequency of the structure $T_0 = 2\pi / \omega_0$ is given by

288
$$\frac{\tau}{T_0} = \frac{1}{2\pi} \sqrt{\frac{\sigma}{N}} \quad (41)$$

289 For a chevron frame that is 20 to 100 times stiffer than the SDOF structure and for values of the suppression
290 coefficient $\sigma = 0.5$ and 1, the range of practical values for τ/T_0 is $0.01 \leq \tau/T_0 \leq 0.04$.

291 When $k_f = Nk$, the “locked” period of the system $T_f = 2\pi \sqrt{\frac{m}{Nk}} = \frac{1}{\sqrt{N}} T_0$. Accordingly, the normalized

292 relaxation time of the system to the undamped natural frequency of the structure, $T_0 = 2\pi / \omega_0$ is given by

293
$$\frac{\lambda_f}{T_0} = \frac{\xi_f}{\pi} \frac{1}{\sqrt{N}} \quad (42)$$

294 Equation (42) indicates that the range of practical values for λ_f / T_0 is $0.01 < \lambda_f / T_0 < 0.001$.

295 In this analysis we are also adding a small amount of viscous damping in the SDOF structure (say $\xi = 2\%$).

296 Accordingly, the equation of motion of the damped SDOF structure with supplemental rotational inertia shown in

297 Figure 3 (right) is

298
$$\ddot{u}(t) + 2\xi\omega_0\dot{u}(t) + \omega_0^2 u(t) + \frac{F_1(t)}{m} = -\ddot{u}_g(t) \quad (43)$$

299 in which the internal force $F_1(t)/m$ is given by equation (38).

300 The response of the system with supplemental rotational inertia shown in Figure 3(right) is compared with the

301 response of a heavily damped SDOF structure where the seismic protection is achieved with supplemental viscous

302 damping with damping constant, $C_d = 2\xi_d m \omega_0$. In this case, the chevron frame also deforms and it can be shown

303 by following a similar analysis (Constantinou et al. 1998) that the damping force, F_d , that develops in the

304 supplemental damper satisfies a Maxwell equation

305
$$\frac{F_d(t)}{m} + \lambda \frac{\dot{F}_d(t)}{m} = 2\xi_d \omega_0 \dot{u}(t) \quad (44)$$

306 where $\lambda = C_d / k_f$, is known as the relaxation time of the system. For a chevron frame that is N times stiffer than

307 the SDOF structure, $k_f = Nk$, the normalized relaxation time to the undamped natural frequency of the structure,

308 $T_0 = 2\pi / \omega_0$, is given by

309
$$\frac{\lambda}{T_0} = \frac{1}{\pi} \frac{\xi_d}{N} \quad (45)$$

310 For a chevron frame that is 20 to 100 times stiffer than the SDOF structure and for values of $0.1 \leq \xi_d \leq 0.3$ the

311 range of practical values of λ/T_0 is $0.0005 \leq \lambda/T_0 \leq 0.005$.

312 When the chevron frame that supports the supplemental damper with damping constant, $C_d = 2\xi_d m \omega_0$ is as

313 strong as the chevron frame that supports the supplemental inerter with inertial mass, $M_R = \sigma m$, the relaxation

314 time, λ , appearing in equation (45) is related to the retardation time, τ , appearing in equation (41) via the equation

$$315 \quad \lambda = \frac{2\xi_d \omega_0}{\sigma} \tau^2 \quad (46)$$

316 The solution of the system of differential equations given by (43) and (38) is computed numerically via a state-space

317 formulation. The state vector of the system is

$$318 \quad \{y(t)\} = \begin{Bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \\ y_4(t) \end{Bmatrix} = \begin{Bmatrix} u(t) \\ \dot{u}(t) \\ F_1(t)/m \\ \dot{F}_1(t)/m \end{Bmatrix} \quad (47)$$

319 and the time-derivative of the state vector, $\{\dot{y}(t)\}$, is expressed in terms of the state variables as

$$320 \quad \{\dot{y}(t)\} = \begin{Bmatrix} y_2(t) \\ -\ddot{u}_g(t) - 2\xi\omega_0 y_2(t) - \omega_0^2 y_1(t) - y_3(t) \\ y_4(t) \\ \frac{\sigma\lambda_f}{\tau} \{-\ddot{u}_g(t) - 2\xi\omega_0[-\ddot{u}_g(t) - 2\xi\omega_0 y_2(t) - \omega_0^2 y_1(t) - y_3(t)] - \omega_0^2 y_2(t) - y_4(t)\} \\ + \frac{\sigma}{\tau^2} [-\ddot{u}_g(t) - 2\xi\omega_0 y_2(t) - \omega_0^2 y_1(t) - y_3(t)] - \frac{1}{\tau^2} y_3(t) - \frac{\lambda_f}{\tau^2} y_4(t) \end{Bmatrix} \quad (48)$$

321 The numerical integration of equations (48) is performed with standard ODE solvers available in MATLAB (2002).

322 Alternatively, given that equations (38) and (43) are linear differential equations, the response history of the SDOF-

323 system can be computed with the Fourier transform,

324
$$u(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \mathcal{U}(\omega) e^{i\omega t} d\omega \quad (49)$$

325 where

326
$$\mathcal{U}(\omega) = - \frac{1 - \omega^2 \tau^2 + i\omega \lambda_f}{(1 - \omega^2 \tau^2 + i\omega \lambda_f)(\omega_0^2 - \omega^2 + 2i\xi\omega_0\omega) - \sigma\omega^2(1 + i\omega \lambda_f)} \ddot{\mathcal{U}}_g(\omega) \quad (50)$$

327 When a single rotational inertia system is used, the frequency domain solution offered by equations (49) and (50) is
 328 attractive since it does not involve the differentiation of the ground acceleration. The agreement of the time-domain
 329 solution expressed with equations (38) and (43) and the frequency domain solution given by equations (49) and (50)
 330 has been confirmed during the course of this study.

331 The response of the SDOF structure with supplemental damping with constant $C_d = 2\xi_d m\omega_0$ is also computed
 332 with equation (49); where now

333
$$\mathcal{U}(\omega) = - \frac{1}{\omega_0^2 - \omega^2 + 2i\xi\omega_0\omega + \frac{2i\xi_d\omega_0\omega}{1 + i\omega\lambda}} \ddot{\mathcal{U}}_g(\omega) \quad (51)$$

334 When the two parallel rotational inertia system are employed which can only resist the motion of the structure
 335 without inducing any deformations (the pinion of the gearwheel that is engaged to the rack is unable to drive the
 336 rack and only the motion of the translating rack can drive the pinion), the term $F_1(t)/m$ appearing in equation (43)

337 is given by equation (38) when $\text{sgn}\left[\frac{\ddot{u}(t)}{\dot{u}(t)}\right] > 0$ and by

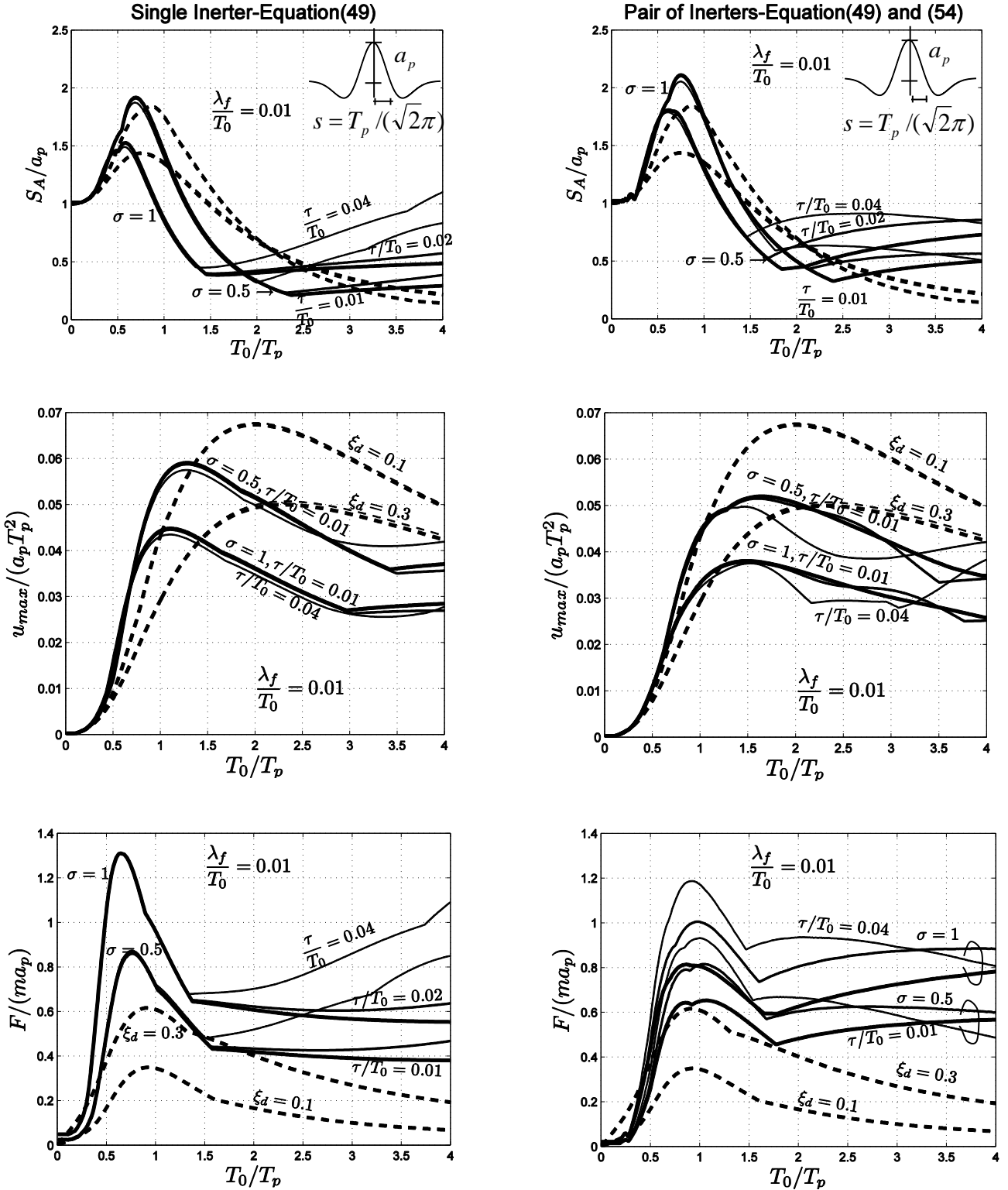
338
$$\frac{F_1(t)}{m} = 0 \quad \text{when } \text{sgn}\left[\frac{\ddot{u}(t)}{\dot{u}(t)}\right] < 0 \quad (52)$$

339 In this case the equation of motion of our SDOF structure becomes piece-wise linear and only a time-domain
 340 solution is feasible. The state vector of the system is given by equation (47) when $\text{sgn}[\ddot{u}(t)/\dot{u}(t)] > 0$ and by

341
$$\{y(t)\} = \begin{Bmatrix} y_1(t) \\ y_2(t) \end{Bmatrix} = \begin{Bmatrix} u(t) \\ \dot{u}(t) \end{Bmatrix} \quad \text{when } \text{sgn} \begin{bmatrix} \ddot{u}(t) \\ \dot{u}(t) \end{bmatrix} < 0 \quad (53)$$

342 Figure 8 plots total acceleration, relative to the ground displacement and transferred force spectra of the SDOF
 343 system described with equations (48) or with equations (48) and (53) for $\sigma = 0.5$ and $\sigma = 1.0$, $\lambda_f / T_0 = 0.01$
 344 and three different values of the dimensionless retardation time $\tau / T_0 = 0.01, 0.02$ and 0.04 when the SDOF
 345 system is excited by a symmetric Ricker wavelet described with equation (29). The time derivative of the ground
 346 acceleration appearing in the fourth component of the time-derivative of the state vector given by equation (48) is
 347 offered by equation (30) –that is the antisymmetric Ricker pulse. For the linearly damped oscillator where the
 348 damping force from the supplemental dampers that resist on the chevron frame is given by equation (44), values of
 349 $\xi_d = 0.1$ and 0.3 have been used.

350 The first observation (as in the case of the infinite stiff chevron frame) is that the supplemental rotational inertia is
 351 effective in suppressing appreciably the peak displacement response for moderately long to long-period structures
 352 (say $T_0 / T_p > 1.5$). Spectral accelerations are suppressed within the range of moderately long periods (say
 353 $1.5 < T_0 / T_p < 3.0$); nevertheless, this is true only for small values of the retardation time ($\tau / T_0 < 0.02$ –stiff
 354 chevron frames). When the chevron frame is less stiff spectral accelerations increase appreciably for longer-period
 355 structures. When comparing the left plots in Figure 8 which are for a single rotational inertia system described
 356 continuously with equation (47) or with equations (49) and (50) (that may induce displacements into the structure)
 357 with the right plots in Figure 8 which are for a pair (front and back) of rotational inertia systems described by
 358 equations (48) and (53) (that can only resist the motion of the structure) we make the following observation. The
 359 pair of rotational inertia systems control the spectral accelerations and forces transferred for large values of
 360 T_0 / T_p ; however, the response is sensitive to the retardation time, τ (finite stiffness of the system). It is concluded
 361 that the strategy of suppressing vibrations with supplemental rotational inertia is attractive when the support of the
 362 rotational inertia system is stiff ($\tau / T_0 < 0.02$).



363 **Fig. 8.** Total acceleration, relative to the ground displacement and transferred force spectra of a mass-spring-inerter
 364 oscillator (solid lines: $\sigma = 0.5, 1$ and three values of retardation times: $\tau/T_0 = 0.01, 0.02$ and 0.04) and a linearly
 365 damped oscillator (dashed lines: $\xi_d = 0.1$ and 0.3) when subjected to a symmetric Ricker pulse. Left: Single inerter
 366 which may drive occasionally the structure; Right: Pair of inerters which can only resist the motion of the structure.

367

CONCLUSIONS

368 In this paper we investigated the potential advantages of the alternative strategy of suppressing ground-induced
369 vibrations with supplemental rotational inertia. The proposed concept employs a rack-pinion-flywheel system that
370 its resisting force is proportional to the relative acceleration between the vibrating mass and the support of the
371 flywheels. The paper shows that the seismic protection of structures with supplemental rotational inertia has the
372 unique advantage of suppressing the spectral displacements of long period structures –a function that is not
373 efficiently achieved even with large values of supplemental damping. Furthermore, the proposed strategy can
374 accommodate large relative displacements without suffering from the issues of viscous heating and potential leaking
375 that challenge the implementation of fluid dampers. At the same time, the paper shows that the forces transferred to
376 the support of the rotational inertia system are appreciable and that the use of stiff supports is recommended (
377 $\tau/T_0 < 0.02$).

378 This paper examines the dynamic response of a SDOF structure when two parallel rotational inertia systems are
379 installed so that they can only resist the motion of the structure without inducing any deformation. This can be
380 achieved if the pinion of each of the gearwheels of the two parallel rotational inertia systems that is engaged to the
381 rack is unable to drive the rack and only the motion of the translating rack can drive the pinion-gearwheel. This
382 arrangement reduces further the spectral displacements; whereas, the results for the forces transferred to the support
383 of the gearwheels are mixed.

384 Finally, the proposed concept where reduction of vibrations is achieved with supplemental rotational inertia so that
385 the resisting force is proportional to the relative acceleration introduces the subject of inerto-visco-elasticity.

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459

460 **FIGURE CAPTIONS**

Fig. 1. Left: A single-degree-of-freedom structure with mass, m , and stiffness, k , with supplemental rotational inertia from a flywheel with radius, R , supported on a chevron frame with stiffness, k_f , that is much larger than k ; Right: Free-body diagram of the vibrating mass, m , when engaged to the pinion of the flywheel shown below.

Fig. 2. More than one flywheels in series that amplify the effect of supplemental rotational inertia.

Fig. 3. Left: Schematic of a mass-spring-inerter single-degree-of-freedom system: the inerto-elastic oscillator; Right: The inerto-visco-elastic oscillator.

Fig. 4. Response of an inerto-elastic oscillator with an infinite stiff chevron frame. Left: Single inerter which may induce deformations; Right: Pair of inerters that can resist only the motion as described by equations (27) and (28). The force from the inerter only opposes the motion.

Fig. 5. Top: Nort-South component of the acceleration time history recorded during the 1992 Erzincan, Turkey earthquake together with a symmetric Ricker wavelet. Bottom: fault-normal component of the acceleration time-history recorded during the 1971 San Fernando earthquake, together with an antisymmetric Ricker wavelet.

Fig. 6. Total acceleration, relative to the ground displacement and transferred force spectra of a mass-spring-inerter oscillator (solid lines: $\sigma = 0.5, 1$) and a linearly damped oscillator (dashed lines: $\xi_d = 0.1, 0.2$ and 0.3) when subjected to a symmetric Ricker pulse. Left: Single inerter which may drive occasionally the structure; Right: Pair of inerters which can only resist the motion of the structure.

Fig. 7. Seismic protection of a bridge along the longitudinal direction with a rack-pinion-flywheel system that exerts only passive thrust on each end-abutment.

Fig. 8. Total acceleration, relative to the ground displacement and transferred force spectra of a mass-spring-inerter oscillator (solid lines: $\sigma = 0.5, 1$ and three values of retardation times: $\tau / T_0 = 0.01, 0.02$ and 0.04) and a linearly damped oscillator (dashed lines: $\xi_d = 0.1$ and 0.3) when subjected to a symmetric Ricker pulse. Left: Single inerter which may drive occasionally the structure; Right: Pair of inerters which can only resist the motion of the structure.