

Stability Analysis and Stabilization of an Amplifier with Nonlinearity Compensation

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The rapid development of wireless communication system has led to pressing need for highly linear amplifier design and implementation as amplifier is one of the most critical devices in transmit and receive chain. In recent years, a number of new and emerging amplifier linearization techniques using special distortion correction have been receiving more and more attention due to their significant advantages. One of the techniques is the negative impedance compensation. The design results showed that high gain, good linearity and wider bandwidth can be achieved by using the novel compensation method. However, as some kind of feedback can be introduced when applying the negative impedance compensation, the stability of the whole system should be investigated carefully. In this paper, stability analysis and stabilization have been performed for the amplifier with negative impedance compensation. The simulation results showed that stable circuit behaviour can be achieved by using the capacitive compensation in the negative impedance circuit.

Keywords: amplifier; linearization; negative impedance compensation; stability; stabilization

1. Introduction

The applications of a number of advanced modulation techniques have increased the capacity and speed of modern wireless and mobile communication systems significantly. The wide dynamic signals used in these modulation techniques need highly linear transmit and receive system architecture. As amplifier is key component in both transmit and receive systems, its linearity becomes more and more important. The traditional amplifier linearization techniques (Kenington 2000), such as predistortion, feedforward and Cartesian feedback, are not applicable to all systems, for example, handset (Kim et al. 2000). Some new techniques

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3 based on special distortion cancellation and compensation such as digital feedback (Woo et
4 al. 2007), special transistor combination and arrangement (Kim et al. 2000; Yum et al. 2006;
5 Kang et al. 2006 and Nakatsugawa 2001), signal-injection (Cheng et al. 2003 and Mizusawa
6 et al. 2005), compensation using auxiliary amplifier (Liang et al. 2005, Zhang 2011; Wu et al.
7 2007, Wu et al. 2010 and Ali et al. 2014) have been reported.

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11 In implementation of RF and microwave amplifier linearization stability can be a great
12 concern because the degree of linearity improvement may be limited by the accuracy and
13 stability of the circuitry with linearization (Breed 2010). In above mentioned new
14 linearization methods, because auxiliary transistors or amplifiers are used to compensate the
15 nonlinear distortion generated by the main amplifier, some kind of feedback could be
16 introduced. Therefore, the stability of the amplifiers with compensation should be considered
17 carefully. However, stability analysis and stabilization of the amplifiers with linearization is
18 quite challenging due to high complexity of the circuitry. Several researchers have reported
19 their work on the stability analysis of RF amplifier with traditional envelope feedback and
20 Cartesian feedback (Kheirkhahi et al. 2011 and Dowson et al. 2004). In this paper, the
21 stability of the amplifier with the negative impedance compensation reported in (Wu et al.
22 2007, Wu et al. 2010 and Ali et al. 2014) is investigated by using effective stability analysis
23 methods such as the Routh-Hurwitz criterion (Surhone et al. 2010 and Jagan 2008). The
24 certain region to guarantee the stability of the amplifier can be determined by using additional
25 capacitance compensation, which provides improved flexibility for the amplifier design.
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38 **2. Negative impedance compensation**

39 **2.1. Principle**

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44 In order to compensate the non-ideal characteristics of amplifiers, a novel amplifier design
45 method based on the negative impedance compensation has been reported in (Wu et al. 2007,
46 Wu et al. 2010 and Ali et al. 2014). The principle of the method is described below.
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50 Usually, distortion generated by a nonlinear amplifier can be represented either at the
51 input or at the output, which leads to two distortion representation methods: the input-referred
52 method and output referred method. Consequently, amplifier linearization techniques can be
53 categorized into the input-referred and output-referred technique. The negative impedance
54 compensation is a type of input-referred method.
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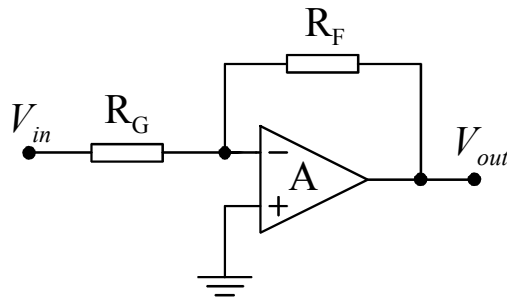


Figure 1. A classic feedback amplifier configuration

Consider the negative feedback amplifier shown in Figure 1. If the amplifier is assumed ideal in all respects, the input impedance $Z_i = \infty$, the output impedance $Z_o = 0$, the open-loop gain $A = \infty$ and the input current $i_i = 0$. Then, the closed-loop gain, G , can be written as,

$$G = \frac{V_o}{V_{in}} = -\frac{R_F}{R_G} \quad (1)$$

or,

$$-\frac{V_o}{R_F} = \frac{V_{in}}{R_G} \quad (2)$$

For a practical amplifier where the non-ideal open-loop gain, A , is finite or nonlinear and $Z_i \neq \infty$, $Z_o = 0$, $A \neq \infty$, $i_i \neq 0$, a straightforward circuit analysis for Figure 1 shows that,

$$-\frac{V_o}{R_F} - \frac{V_o}{A} \left(\frac{1}{Z_i} + \frac{1}{R_G} + \frac{1}{R_F} \right) = \frac{V_{in}}{R_G} \quad (3)$$

Comparing Equation (3) with (2) it can be seen that there is an additional V_o term in Equation (3). Actually, this term can be regarded as a distortion current, i_d , caused by the non-ideal behaviour of the amplifier, given by

$$i_d = -\frac{V_o}{A} \left(\frac{1}{Z_i} + \frac{1}{R_G} + \frac{1}{R_F} \right) \quad (4)$$

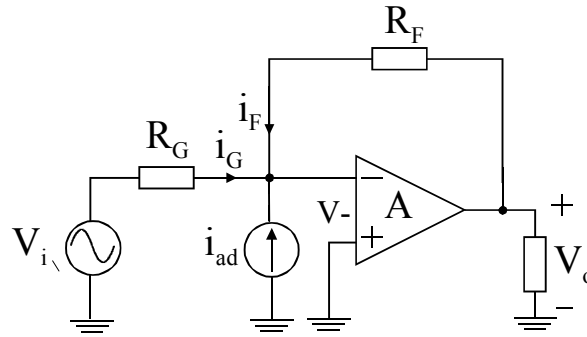


Figure 2. Negative feedback amplifier of Figure 1 with distortion correction at the input

As can be seen, if an anti-distortion current i_{ad} is injected into the inverting terminal, then the input current to the inverting terminal, i_i , is given by,

$$i_i = i_G + i_F + i_{ad} \quad (5)$$

Use of Equation (3) gives

$$-\frac{V_o}{R_F} - \frac{V_o}{A} \left(\frac{1}{Z_i} + \frac{1}{R_G} + \frac{1}{R_F} \right) = \frac{V_{in}}{R_G} + i_{ad} \quad (6)$$

Clearly, if i_{ad} is chosen so that

$$i_{ad} = -\frac{V_o}{A} \left(\frac{1}{Z_i} + \frac{1}{R_G} + \frac{1}{R_F} \right) = V_- \left(\frac{1}{Z_i} + \frac{1}{R_G} + \frac{1}{R_F} \right) \quad (7)$$

where V_- is the voltage at the inverting input terminal, then the distortion current, i_d , will be cancelled.

For most amplifiers, R_F , R_G are such that R_F , $R_G \ll |Z_i|$, in which case Equation (7) simplifies to

$$i_{ad} \approx V_- \left(\frac{1}{R_G} + \frac{1}{R_F} \right) = \frac{V_-}{R_G // R_F} \quad (8)$$

Equation (8) reveals that injecting a current $i_{ad} = V_- / (R_G // R_F)$ into the inverting terminal is equivalent to connecting a negative resistance to the terminal as shown in Figure 3. The negative resistance is given by

$$R_n = -(R_G // R_F) \quad (9)$$

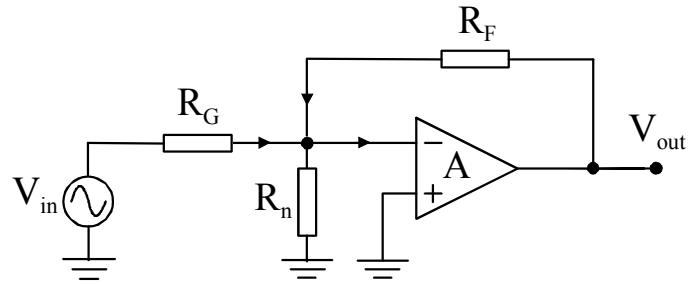


Figure 3. Amplifier of Figure 1 with negative resistance compensation

2.2. Circuit implementation

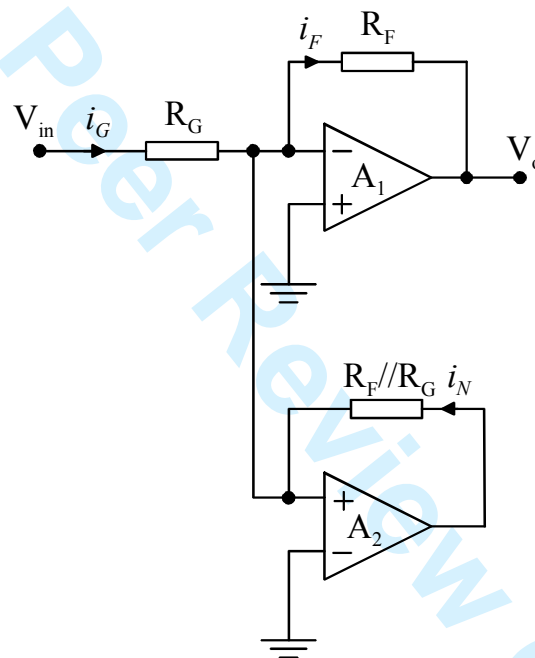


Figure 4. Amplifier with negative resistance compensation

Figure 4 shows a general circuit for negative resistance compensation, A_1 being the main amplifier and A_2 the complementary amplifier. In this circuit, assuming the input resistance for both amplifiers are high enough to be neglected, the current equation at the inverting input terminal is given by

$$i_F = i_G + i_N \quad (10)$$

If the gain of A_2 is set to 2 and $R_N = R_F/R_G$ then i_N equals i_{ad} of Equation (8) and the distortion is theoretically almost cancelled out.

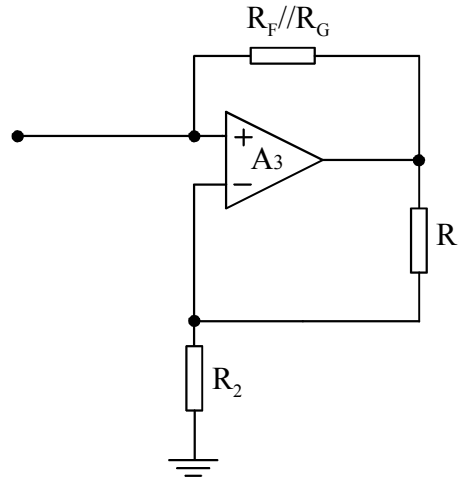


Figure 5. An implementation of negative resistance

Negative resistance can be implemented, in practice, using the special amplifier topology shown in Figure 5, where the amplifier A_2 in Figure 4 is realized by a non-inverting configuration with $R_1=R_2$ to have a gain of 2. Assuming the amplifier has a high open-loop input resistance and high open-loop gain the voltage between the inverting and noninverting terminals is negligibly small, the input impedance of the circuit can be obtained as

$$R_{in} = R_n = -R_F // R_G \quad (11)$$

In practice the negative resistance is slightly reduced because the input resistance of the amplifier is finite. The closed-loop gain of the negative resistance circuit is only +6dB, and when used to linearise the amplifier of Figure 1 it is connected to the ‘virtual-ground’ inverting input of the amplifier. The negative resistance circuit only sees very small input signals, and the output signal is also very small, hence it is possible to assume that any distortion effects that the second amplifier may introduce are minimal. Also, it should be noted that this negative resistance realisation using a combination of positive and negative feedback is stable when connected to the low impedance node of the inverting terminal of the main amplifier. This fact is further validation of the appropriateness of the choice of topology to realise R_n .

3. Stability analysis and stabilization of amplifier with the negative impedance compensation

3.1. Analysis of stability for amplifier with compensation

In practical applications of the negative impedance compensation method, stability could be an important issue due to the auxiliary amplifier. Therefore, it is crucial to carefully analyze the impact of the additional negative element on the stability of the amplifier.

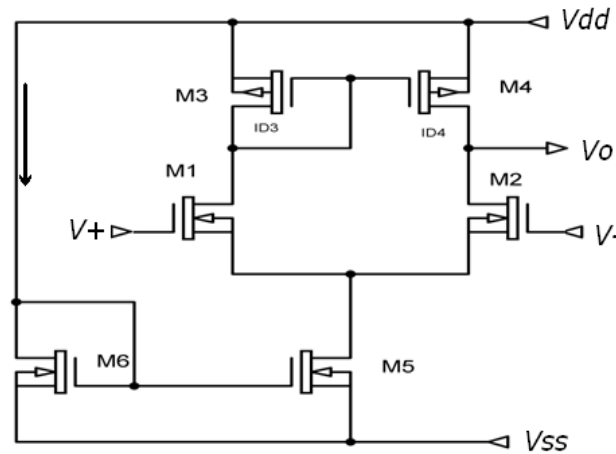


Figure 6. CMOS differential amplifier

As mentioned in Section 2, the negative impedance compensation technique can be implemented by connecting the negative resistance circuit shown in Figure 5 to the main amplifier shown in Figure 1. The amplifier A_1 in the main amplifier and the amplifier A_3 in the negative resistance circuit may be implemented not only by an operational amplifier but also by a differential amplifier etc. Therefore, in this paper, the CMOS differential amplifier shown in Figure 6 as described in (Allen et al. 2011) has been used to simulate the amplifiers, where the open loop gain has been characterised as 40dB (i.e. $A_o=100$) and the unity-gain bandwidth as 300MHz.

As an example to test the stability of an amplifier with the negative impedance method, the main amplifier in Figure 1 has been designed to have a closed-loop gain of -4. The values of the components to realise the gain are chosen as $R_F=2k\Omega$, $R_G=500\Omega$. For the negative resistance circuit shown in Figure 5, the two resistors, R_1 and R_2 , have been considered as $R_1=R_2=10k\Omega$. Also, according to the negative impedance compensation theory and using Equation (9) in (Wu et al. 2007), the value of the compensation component, R_N , can be calculated as $R_N=390\Omega$. The stability can be tested by injecting step signal at the input and

observing the output. In this case, a -0.1V step signal with $0.1\mu\text{s}$ delay was injected at the input. As shown in Figure 7, the example amplifier delivers an output with excessive overshoot (up to 0.614V) and ringing (till $0.75\mu\text{s}$).

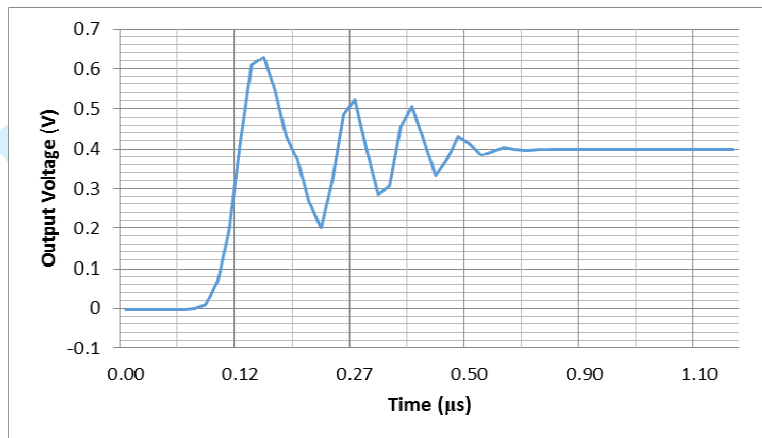


Figure 7. Simulation result of the step response for a linearised amplifier

The simulation result in Figure 7 shows that the amplifier with the proposed negative impedance compensation may have stability problem for some values of its parameters. In this paper, Routh–Hurwitz stability criterion is used to carry out stability analysis and find the stable region for the parameters of the amplifier with the compensation.

3.2. Stability analysis and stabilization

In order to examine the stability of the amplifier with the negative impedance compensation, the Routh-Hurwitz criterion can be used.

As a very popular and useful stability analysis method, the Routh–Hurwitz criterion allows us to compute the number of roots of the characteristic equation in the right half-plane without actually computing the values of the roots. Thus, stability can be determined without the added computational burden of determining characteristic root locations. This gives us a design method for determining values of certain system parameters that will lead to closed-loop stability (Dorf et al. 2014). At present, many stability analysis software tools based on the Routh–Hurwitz method are being used (Vo et al. 2016).

For a transfer function with following general form

$$T(s) = \frac{N(s)}{D(s)} \quad (12)$$

If the denominator of the transfer function is a fourth-order polynomial

$$D(s) = a_0 + a_1s + a_2s^2 + a_3s^3 + a_4s^4 \quad (13)$$

According to the Routh-Hurwitz criterion all the coefficients must satisfy following conditions if the system is stable

$$a_3 > 0 \quad (14a)$$

$$a_3a_2 > a_4a_1 \quad (14b)$$

$$a_3a_2a_1 > a_4a_1^2 + a_3^2a_0 \quad (14c)$$

Stability analysis using above approach has been performed for the amplifier in Figure 4 and 5. In order to stabilize the amplifier, $R_F//R_G$ (R_N), R_1 and R_2 in Figure 5 are replaced by the conductance-capacitance parallel structures:

$$Y_A = 1/R_N + j\omega C_A = G_A + j\omega C_A, \quad Y_B = 1/R_1 + j\omega C_B = G_B + j\omega C_B \text{ and } Y_C = 1/R_2 + j\omega C_C = G_C + j\omega C_C,$$

where C_A , C_B and C_C are chosen as adjustable parameters to determine the stability region.

Using the one-pole model of op-amp, the transfer function of the amplifier in Figure 4 can be derived as

$$\frac{v_{out}}{v_{in}} = -\frac{A_m \omega_{T1} \omega_{T2} G_B + A_m \omega_{T1} (\omega_{T2} C_B + G_B + G_C) s + A_m \omega_{T1} (C_B + C_C) s^2}{as^4 + bs^3 + cs^2 + ds + e} = -\frac{N'(s)}{D'(s)} \quad (15)$$

where $a = R_F C_A C_C + R_F C_A C_B$

$$b = A_{m1} C_B + A_{m1} C_C + R_F G_A C_C + R_F G_A C_B - R_F C_A C_C \omega_{T2} + R_F C_A G_C + R_F C_A G_B$$

$$c = \omega_{T1} C_B + \omega_{T1} C_C + A_{m1} \omega_{T2} C_B + A_{m1} G_B + A_{m1} G_C - R_F G_A C_C \omega_{T2} + R_F G_A G_C + R_F G_A G_B - R_F C_A G_C \omega_{T2}$$

$$d = \omega_{T1} \omega_{T2} C_B + \omega_{T1} G_B + \omega_{T1} G_C + A_{m1} \omega_{T2} G_B - R_F G_A G_C \omega_{T2}$$

$$e = \omega_{T1} \omega_{T2} G_B$$

$$A_m = R_F / R_G$$

$$A_{m1} = A_m + 1$$

As can be seen, $D'(s)$, the denominator of Equation (15), can be used as a fourth-order characteristic polynomial for stability analysis based on the Routh-Hurwitz criteria. Then, by choosing C_A , C_B and C_C as adjustable parameters in the coefficients of $D'(s)$ and using

Equation (14) the stable region of the amplifier with the negative impedance compensation can be determined by a group of compensating capacitors.

To demonstrate the above method, stability analysis and stabilization has been performed for the example in Section 3.1. For the closed-loop gain of $-4 A_m$ and A_{m1} in Equation (15) are equal to 4 and 5, respectively. The unity-gain bandwidth $\omega_{T1}=\omega_{T2}=2\pi f_T=1.885\times 10^9$ rad/s. As described above, the amplifier can be stabilised using parallel capacitance C_A , C_B and C_C . Using MATLAB, separate stability regions have been generated using the stability criterion in Equation (14a, b, and c) as shown in Figure 8, 9 and 10. To illustrate the global relationship of C_A , C_B and C_C in the stability analysis, a 3-dimentional plot was drawn as shown in Figure 11. The shape of the plot provides clear information about the stable region. From the figures, it can be identified that when $C_B = C_C = 0$, the amplifier is unstable, which has proved the result in the example in Section 3.1.

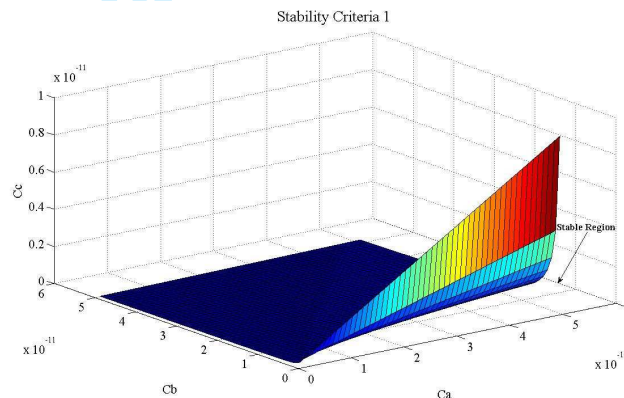


Figure 8. Stable region for the linearised amplifier for $a_3 > 0$

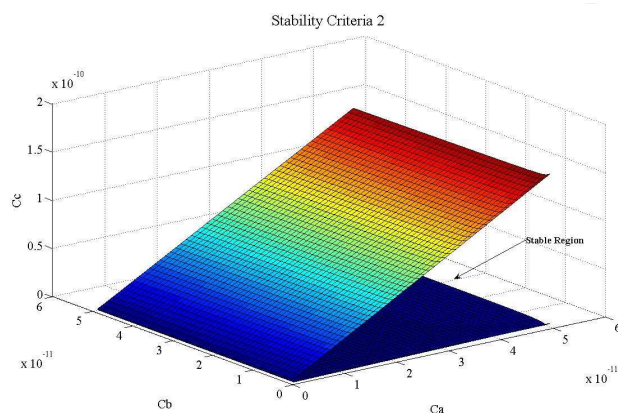


Figure 9. Stable region for the linearised amplifier for $a_3a_2 > a_4a_1$

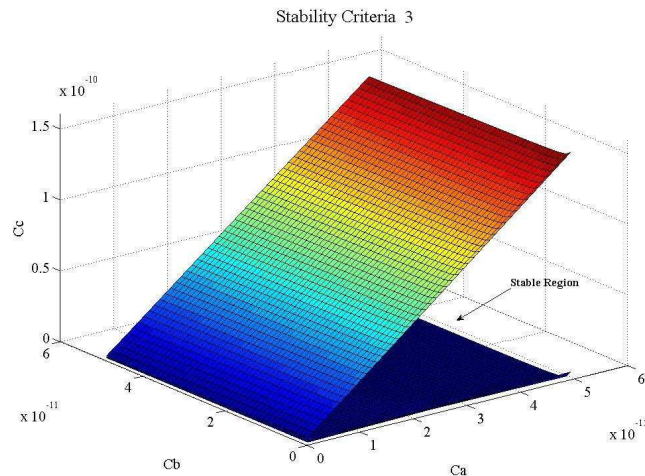


Figure 10. Stable region for the linearised amplifier for $a_3 a_2 a_1 > a_4 a_1^2 + a_3^2 a_0$

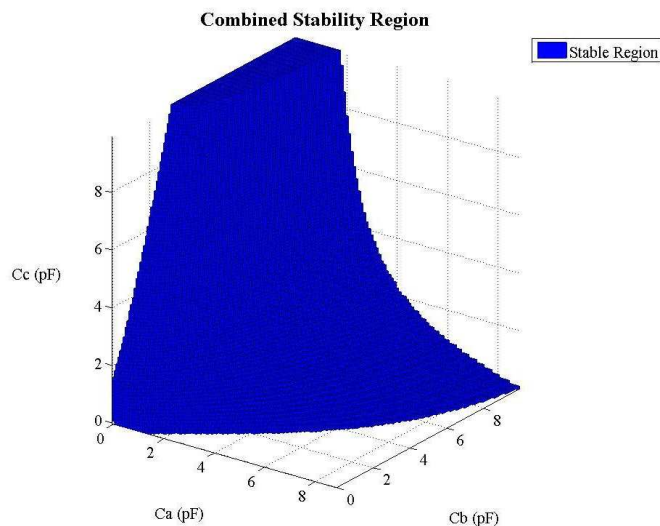


Figure 11. Combined stability region

Figure 11 reveals that the stability of the amplifier can be guaranteed by choosing a group of C_A , C_B and C_C in the stable region. In the example with the CMOS differential amplifier in Section 3.1, in order to stabilize the amplifier with compensation, the circuit has been modified with the conductance-capacitance parallel structures including the capacitors C_A , C_B and C_C where the main and auxiliary amplifiers still remain the same as Figure 4. Figure 12 shows the simulation result with $C_A=3.3$ pF, $C_B=3.3$ pF and $C_C=2.8$ pF. As can be seen from the figure, the stability has been significantly improved compared with the result shown in Figure 7.

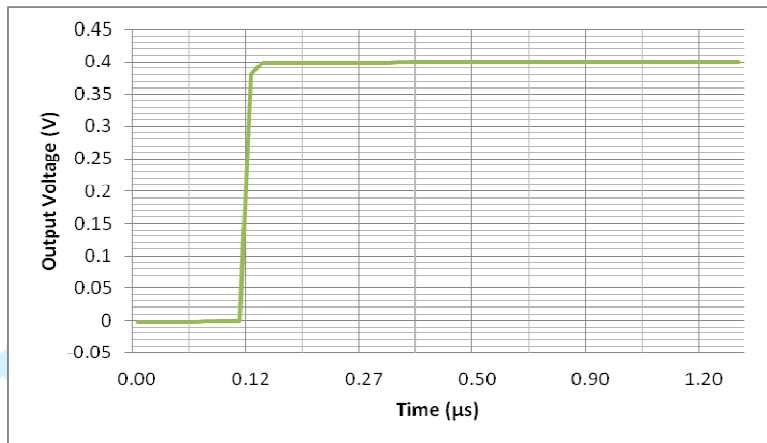


Figure 12. Simulation result of the step response for the amplifier with stabilization

The stability analysis using above method has also been performed for the highly linear amplifier designed by the author in (Wu et al. 2007). The amplifier was implemented with BFR520 9GHz wideband transistor (<http://www.nxp.com>). For example, the SPICE simulation result showed that the boundary of C_B for a stable region is 9pF when $C_A=8$ pF and $C_C=0$ pF. Normally, the stable region for a practical circuit could be smaller than that in Figure 11 due to the use of high level transistor models.

In summary, the stability region of an amplifier with the special nonlinearity compensation could be obtained by the following steps:

- (1) For a designed amplifier with nonlinearity compensation, identify the values of the relevant parameters such as the gain and bandwidth for both the main and the auxiliary amplifier as well as the resistors.
- (2) Based on above parameters, determine a predictive stable region specified by C_A , C_B and C_C by using the simulation tool presented in the paper.
- (3) Test and verify the predictive region by using a specific circuit simulation software such as SPICE with high-level device models.

4. Conclusion

In this paper, stability analysis and stabilization for the amplifier with the negative impedance compensation have been performed. As can be seen, the stable region of the amplifier with the compensation can be determined by a group of compensating capacitors. The simulation results show that the stability of the amplifier can be improved significantly.

The research is ongoing. In order to improve the accuracy, reliability and practicability of the proposed method, the future work will include

- (1) Add the algorithm for analyzing both gain margin and phase margin to the design tool.
- (2) Develop more effective function for determining suitable values and ranges of the compensating capacitors to meet required stability criteria. Such the function could be realized by considering different ranges, values and analytical expressions of R_N , R_I and R_2 as well as practical amplifier parameters based on more complicated device models.
- (3) Build an interface between the stability analysis tool and the existing circuit simulation software such as SPICE etc. so that the result of the stabilization can be evaluated using standard technology.

The design, implementation and testing of the amplifier with the negative impedance compensation is in progress. It is expected that the stability analysis and stabilization will be carried out in the process to examine the validity of the proposed method practically.

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