Relational event models for longitudinal network data with an application to interhospital patient transfers

Duy Vu[ab*], **Alessandro Lomi**[a], **Daniele Mascia**[c], **Francesca Pallotti**[d]

The main objective of this paper is to introduce and illustrate relational event models, a new class of statistical models for the analysis of time-stamped data with complex temporal and relational dependencies. We outline the main differences between recently proposed relational event models and more conventional network models based on the graph-theoretic formalism typically adopted in empirical studies of social networks. Our main contribution involves the definition and implementation of a marked point process extension of currently available models. According to this approach, the sequence of events of interest is decomposed into two components: (a) event time, and (b) event destination. This decomposition transforms the problem of selection of event destination in relational event models into a conditional multinomial logistic regression problem. The main advantages of this formulation are the possibility of controlling for the effect of event-specific data and a significant reduction in the estimation time of currently available relational event models. We demonstrate the empirical value of the model in an analysis of interhospital patient transfer within a regional community of health care organizations. We conclude with a discussion of how the models we presented help to overcome some the limitations of statistical models for networks that are currently available.

Keywords: Social network analysis; relational event models; inter-organizational relations; interhospital patient transfers

1. Introduction

Interest in statistical models for the analysis of longitudinal network data has increased considerably in recent years [1, 2, 3]. In medicine, for example, innovative approaches to the statistical analysis of longitudinal networks has helped to reframe as network problems a wide variety of problems traditionally understood almost exclusively in terms of individual attitudes, behaviors, or decisions [4, 5]. The interest in longitudinal analysis of network data is not only stimulating fundamental innovation in statistical modeling, but it is also gradually changing the way in which we think of social

[a] *Social Network Analysis Research (SONAR) Center, University of Italian Switzerland at Lugano (Switzerland)*
[b] *Department of Mathematics and Statistics, University of Melbourne (Australia)*
[c] *Department of Public Health and Healthcare Management, Catholic University of Rome (Italy)*
[d] *Centre for Business Network Analysis, University of Greenwick (UK)*
[*] *Correspondence to: Duy Vu, Department of Mathematics and Statistics, University of Melbourne, Australia.*

networks as bases for policy intervention [6], and - more importantly - as objects of theoretical interest [7]. Despite the increasing methodological and theoretical interest in the dynamic analysis of social networks [3], relatively little work has been done on continuous-time models for large-scale dynamic networks [8]. Recently derived relational event models represent one of the most promising directions for developing such models [9].

Relational event models are based on a representation of network data that departs in at least four ways from that imposed by the more widely adopted - but also more restrictive formalism of graph theory [7]. First, relational event data consists of sequences of events encoding interactions between actors represented conventionally as nodes. Edges are not simply present, absent, or characterized in terms of an aggregate weight. Rather, edges are defined in terms of observed sequences of discrete events that need not be discretized into network ties. Second, edges are not recorded at discrete time intervals of fixed length, but as time-stamped event sequences that are continuously observed. Third, edges are defined in terms of event recurrences, rather than state transitions [10]. Unlike the more established stochastic actor-oriented models (SAOMs) [11], relational events models do not conceive network ties as states with an intrinsic tendency to endure over time [2]. Fourth, and finally, relational event models do not assume that the observed event network is the outcome of a Markov process such that the current state of the network determines probabilistically its further evolution: the time frame in which network effects actually operate on behavior is considered more like an empirical problem than like an assumption.

Against the backdrop of this general discussion, the main methodological contribution of this paper involves the implementation of a marked point process extension of currently available relational event models [9]. The main advantage of the new approach is the possibility to control for effects associated with event-specific data. The proposed model extension accomplishes this task by decomposing events sequences into two parts: (a) event times which can be modelled by counting processes, and (b) event destinations (or the "marks") which can be modelled via discrete choice functions. As far as we are aware, this paper presents the first empirical application of this model.

We demonstrate the empirical merits of the present extension of the relational event model in the context of data that we have collected on the network of interhospital patient transfer events observed within a small regional community. Health care organizations may interact in a variety of ways involving a mixture of collaborative resource exchanges and competitive interdependences [12]. Interhospital patient transfers - events generated by decisions of medical staff in one hospital (sender) to move the patient to another hospital (receiver) - have frequently been interpreted as an example of how the embeddedness in networks of resource exchange relations affects the capacity of hospitals to deliver health care services [13, 14, 15, 16], and to collaborate for the benefit of patients [17].

We organize the rest of this paper as follows. In Section 2, we present our new relational event model based on marked point processes. In section 3, we introduce the empirical setting, describe the data, and define a suite of covariates and network statistics which may assist in modeling the complex dynamics of the interhospital patient transfer event network. Section 4 reports the empirical estimates. Section 5 concludes the paper by discussing the contribution of relational event models to the analysis of network data, and by outlining potential directions for future research.

2. Relational event models

Building on models for the analysis of event histories [18], the basic idea behind relational event models involves modeling time-to-event data as the outcome of a multivariate point process. A number of recent extensions for the relational event framework have been proposed to address specific issues raised by the modelling of time-stamped data with complex temporal and spatial dependencies. For example, multivariate point processes have been used to derive a model for multicast events, i.e., events with one sender but multiple receivers - and study the asymptotic properties of its maximum partial likelihood [19]. Relational event models with additive intensity functions allowing for time-varying network effects have also been considered [8]. Other variants with diminishing effects (decay) of events are currently being developed [20].

In this paper, we contribute to this rapidly expanding line of contemporary research by considering the relational event framework from the marked point process perspective. This will allow us to control for critical covariate effects on multiple types of events that may connect the same nodes in a dynamic event network. One innovative aspect of our prosed model is its being less computationally intensive than previous approaches that have attempted to deal with the same issue [9, 8].

2.1. The analysis of relational event networks

A point process is a stochastic process whose realization is an ordered list of points on the time line where events occur, denoted by $E = \{t_1, t_2, \ldots, t_m\}$ where $t_1 < t_2 < \ldots < t_m \in \mathbb{R}^+$. Marked point processes extend this concept further by allowing other important information to be stored along with each event time [21]. For example, in the case of patient transfer data that we discuss later in the paper, a marked point process on each hospital i can be used to characterize a time-ordered set of transfer events from i, denoted by $E_i = \{(t_e, j_e)\}$ where $t_e \in \mathbb{R}^+$ and $j_e \in \{1, \ldots, n\}$ is the time and destination of the eth event. We assume that the total number of hospitals is n and they are indexed from 1 to n.

The marked point process of relational events originating from hospital i in turn can be modelled by a conditional intensity function $\lambda_i\big(t, j|\mathbf{H}_{t^-}\big)$ where $\mathbf{H}_{t^-}$ denotes the history of all transfer events right before time t. In the illustrative case that we develop later in the paper, history also includes information on the medical specialties from and towards which a patient is transferred denoted by $m_s(e)$ and $m_r(e)$, respectively (where the subscript s stands for sender and r for receiver). Such information is already available before the transfer time and can affect the choice of transfer destination. We further assume a separable form for the marked point intensity function [22]:

$$\lambda_i\big(t, j|\mathbf{H}_{t^-}\big) \quad = \quad \lambda_i\big(t|\mathbf{H}_{t^-}\big) \times p_i\big(j|\mathbf{H}_{t^-}\big),$$

where the transfer time t is modelled by the conditional intensity function $\lambda_i\big(t|\mathbf{H}_{t^-}\big)$ and the mark j is modelled by conditional mark probabilities $p_i\big(j|\mathbf{H}_{t^-}\big)$.

Intuitively, this separable form assumes that the relational process of interest may be divided into two steps: the first involves a decision to initiate - or "send" a specific relational event. In the example we develop in the empirical part of the paper, the decision concerns transferring a patient from a sender to a receiver hospital. The second step involves the choice of destination - or "receiver" of the event. In our illustration, this second step involves the choice of one among the eligible hospitals with the required knowledge and resources as signaled by the presence of the medical specialty $m_r(e)$ for which the patient is being transferred. A number of exogenous and endogenous factors can also contribute to this decision process. For example, hospitals prefer to transfer their patients to other hospitals in the same administrative unit or to established partners with which they have collaborated in the past. The former (location) is an example of an exogenous factor - a factor whose effect does not depend on network structure. The latter (event repetition) is an example of an endogenous factor - a factor whose effect depends on the self-organizing tendencies of the network. Sections 3.3 and 3.4 will discuss in detail how these (exogenous) covariate and (endogenous) network effects may be incorporated in relational event models.

For our current purposes, a particularly important feature of patient transfer data is that transfer events can be classified into within- and between- specialty. A within-specialty events involve a patient transferred across different hospitals but within the same medical specialties (patients for whom, in other words, $m_s(e) = m_r(e)$). A between-specialty event is recorded whenever a patient is transferred across hospitals but between different medical specialties, i.e. $m_s(e) \neq m_r(e)$. To compare hospital collaboration networks between these different event sequences, we will assume two distinct models for each event type. Each model has a different baseline intensity function and separate vectors of covariate and network effects.

Within-specialty model. For the sequence of within-specialty events, the conditional intensity function $\lambda_i^w(t|\mathbf{H}_{t^-})$ for transfer times from hospital i can be defined in the Cox proportional hazard form [23]:

$$\lambda_i^w(t|\mathbf{H}_{t^-}) \quad = \quad \lambda_0^w(t) \exp\left[\boldsymbol{\theta}_w^\top \mathbf{s}(t, i)\right], \tag{1}$$

where the superscript w stands for the within-specialty setting, $\mathbf{s}(t,i)$ is the vector of covariate and network statistics for the sending hospital i, $\boldsymbol{\theta}_w$ and $\lambda_0^w(t)$ are sending effects and the baseline rate of the transfer time model for within-specialty events, respectively. To control for the effect of erratic changes in the baseline rate during the observation period, we also assume a non-parametric form for $\lambda_0^w(t)$, a flexible and widely used approach in survival and event history analysis [24]. Other parametric choices for $\lambda_0^w(t)$ are also possible, though the resulting statistical inference will be less tractable in the presence of time-dependent network statistics $\mathbf{s}(t,i)$ [25].

The mark j or the transfer destination can be considered as a conditional multinomial logistic regression problem [26] where potential choices are limited to hospitals having the specialty for which the patient is being transferred, i.e. $m_r(e)$. In other words, the probability that an eligible hospital j is chosen as the transfer destination is defined by:

$$p_i^w(j|\mathbf{H}_{t^-}) = \frac{f_i^w(j|\mathbf{H}_{t^-})}{\sum\limits_{k\in D(m_r(e))} f_i^w(k|\mathbf{H}_{t^-})}, \tag{2}$$

where $D\big(m_r(e)\big)$ is the set of hospitals at time t having the specialty $m_r(e)$ and $f_i^w(j|\mathbf{H}_{t^-})$ is individual choice function. The sending hospital i must be excluded from the potential set $D\big(m_r(e)\big)$ since we are only considering interhospital patient transfer events. Each individual choice function in turn is given by:

$$f_i^w(j|\mathbf{H}_{t^-}) = \exp\left[\boldsymbol{\beta}_w^\top \mathbf{s}(t,i,j)\right], \tag{3}$$

where $\mathbf{s}(t,i,j)$ is the vector of covariate and network statistics between i and j, and $\boldsymbol{\beta}_w$ are corresponding effects of the transfer destination model for within-specialty events.

Between-specialty model. Similar to within-specialty events, the conditional intensity function for the transfer times of between-specialty events is given by:

$$\lambda_i^b(t|\mathbf{H}_{t^-}) = \lambda_0^b(t)\exp\left[\boldsymbol{\theta}_b^\top \mathbf{s}(t,i)\right],$$

where $\boldsymbol{\theta}_b$ and $\lambda_{b,0}(t)$ are sending effects and the baseline rate of the tranfer time model for between-specialty events. Similar network statistics $\mathbf{s}(t,i)$ can be used for both within and between-specialty models; however, different baseline intensities, i.e. $\lambda_0^w(t)$ and $\lambda_0^b(t)$, and network effects, i.e. $\boldsymbol{\theta}_w$ and $\boldsymbol{\theta}_b$, are assumed.

Similarly, the mark j for between-specialty events is also modelled as a conditional multinomial logistic regression problem where potential choices are limited to hospitals having the specialty for which the patient is being transferred. The individual choice function for between-specialty events has the same form as the individual choice function for within-specialty events, but with a different set of network effects $\boldsymbol{\beta}_b$:

$$f_i^b(j|\mathbf{H}_{t^-}) = \exp\left[\boldsymbol{\beta}_b^\top \mathbf{s}(t,i,j)\right],$$

Aggregate model. To demonstrate the relevance of our separability assumption and the predictive ability of network statistic, we also consider the aggregate setting where the model is fitted to all patient transfer events. A likelihood ratio test is then used to compare the goodness of fit between this aggregate model and the above combination of two different within and between-speciality models.

2.2. Model estimation, comparison, and interpretation

For parsimony, but without loss of generality, we limit our discussion to estimation procedures only for within-specialty events. Similar procedures are applied to the between-specialty model. Firstly, since the non-parametric form of the baseline intensity $\lambda_0^w(t)$ is assumed, parameters $\boldsymbol{\theta}_w$ can be estimated by maximizing the partial likelihood of the form

[24]:

$$PL(\boldsymbol{\theta}_w) = \prod_{e \in E_w} \frac{\exp\left[\boldsymbol{\theta}_w^\top \mathbf{s}(t_e, i_e)\right]}{\sum_{i=1}^{n} \exp\left[\boldsymbol{\theta}_w^\top \mathbf{s}(t_e, i)\right]}, \tag{4}$$

where E_w is the sequence of within-specialty events, i_e is the sending hospital of transfer event e. The covariance matrix of the maximum partial likelihood estimate $\hat{\boldsymbol{\theta}}_w$ is computed as the inverse of the negative Hessian matrix of the last iteration. Large-sample results based on the partial likelihood can be found in [24, 18] where no assumption about the independence of individual point processes is made. A recent theoretical justification of this partial likelihood approach especifically for network data has been discussed thoroughly in [19]. In practice, we can generate a nested case-control data set of network statistics $\mathbf{s}(t, i)$ over event times, and then rely on conditional logistic regression models to obtain $\hat{\boldsymbol{\theta}}_w$ and its standard errors. We discuss in details about this implementation in the appendix B.

Secondly, parameters $\boldsymbol{\beta}_w$ of the transfer destination model may be estimated by maximizing the likelihood of discrete choice probabilities of all within-specialty transfer events given by:

$$L(\boldsymbol{\beta}_w) = \prod_{e \in E_w} \frac{\exp\left[\boldsymbol{\beta}_w^\top \mathbf{s}(t_e, i_e, j_e)\right]}{\sum_{j=1}^{n} \exp\left[\boldsymbol{\beta}_w^\top \mathbf{s}(t_e, i_e, j)\right]}. \tag{5}$$

In practice, similar to the estimation of $\hat{\boldsymbol{\theta}}_w$, we can generate a nested case-control data set of network statistics $\mathbf{s}(t, i, j)$ over event times, and then employ a discrete-choice modelling library to obtain $\hat{\boldsymbol{\beta}}_w$ and its standard errors.

Without considering the computation cost of network statistics $\mathbf{s}(t, i)$ and $\mathbf{s}(t, i, j)$ which can vary significantly by modelling choices, the time complexity of both estimation procedures for transfer time and destination models is linear in the number of nodes n and the number of events $|E|$, i.e. $O(n \times |E|)$. For the original relational event framework [9] where a single intensity function is used to model both transfer times and destination choices, however, time complexity of the partial likelihood estimation procedure is quadratic in the number of nodes n, i.e. $O(n^2 \times |E|)$. Consequently, besides the ability to control for covariate data on events, computational gain is another advantage of our proposed framework, which is sustantially important when the network size in terms of nodes n is large. For those readers who are interested in the scalability of the relational event framework, two other promising approaches that have been explored are online inference [8] and nested case-control sampling [27]. They have been applied successfully to the analysis of networks with ten thousands of nodes and events. The state of the art exponential random graph models and stochastic actor-oriented models can not handle such large size of network data. Finally, conditional logistic regression can be slow when the number of predictors is large. In this case, regularization terms such as lasso penalty [28] can be used not only to introduce sparsification but also reduce the estimation time.

To verify the separability assumption and demonstrate the predictive ability of different sets of network statistics, we carry yearly rolling prediction experiments which are the most appropriate out-of-sample evaluation for timestamped event data [25, 8]. At the beginning of every year of the observation period, we estimate model coefficients using all historical events up to that time point and then employ this learned model to make prediction about hospital destinations for all events in the next year. This yearly rolling evaluation process is repeated until all test events are considered. To compare different models on this prediction task, we employ recall criterion which is defined as the percentage of the correct hospitals that are found in the sorted likelihood lists from positions 1 to K:

$$\text{Recall} = \frac{\sum_{(t,i,j) \in E} \mathbf{I}[j \in Top(t, i, K)]}{|E|} \tag{6}$$

where K is the cut-point, E is the set of test events and $Top(t, i, K)$ is the top-K list of potential hospital destinations for the transfer event from i ranked based on the intensity function $\lambda_i\big(t, j|\mathbf{H}_{t^-}\big)$. However, to compare network effects across three speciality settings, i.e. an in-sample evaluation, we use Akaike Information Criterion with a correction for finite sample sizes (AICc) [29] defined as follows:

$$\text{AICc} = -2 \times \log PL(\hat{\boldsymbol{\theta}}) + 2 \times p + 2 \times \frac{(p+1)(p+2)}{m \times n - p - 2}, \tag{7}$$

where $PL(\boldsymbol{\theta})$ is the partial likelihood (4), $\hat{\boldsymbol{\theta}}$ is the maximum likelihood estimate of the coefficient vector $\boldsymbol{\theta}$, p is the number of variables, and $m = |E|$ is the total number of transfer events. Models with smaller AICc values are preferred.

Regarding model interpretation, a positive estimate of parameter θ typically indicates that events are more likely to occur with higher values of the corresponding covariate or network statistic. A negative estimate means that events are less likely to occur with higher values of the corresponding covariate. Specific interpretations for each covariate or network statistics will be discussed in greater detail in Sections 3.3 and 3.4. Finally, to allow for the relative comparison between effect sizes of covariates and network statistics, we also apply a standardization step, i.e. we transform all variables so that their means and variances are 0 and 1, respectively.

2.3. Related work

Compared to the original relational event model [9], the main novelty of our separable sender intensity and receiver choice model is that it allows a more detailed representation of the data-generating process where event-specific information is available. Moreover, the weighted relational event model in [30] considers the separation between events weights and dyads rather than the separation between senders and receiver effects. Our model adds more flexibility to the relational event framework by elaborating that different aspects of event generation processes can actually be modelled by different probabilistic components: full parametric survival models can be used in modelling event times or sender effects while different discrete-choice and regression models can be used to model receiver and weight effects, respectively. This separability in turn allows us to control for effects associated with event-specific data.

Another distinctive feature of this separable model is its scalability. Under this intensity decomposition approach, the model estimation time can be reduced from quadratic to linear in the number of network nodes. In combination with the nested case-control sampling method [27], this model may be usefully applied to the analysis of large collaboration networks. Although this scalability feature can not be illustrated fully in our relatively small application data set, it is critical in promoting wider applications of the relational event framework. Network data as continuous-time processes have been considered long ago time in statistical social network literature [31, 32]; however, computationally intensive estimation procedures have limited their applications until recently.

3. Empirical illustration: Interhospital patient transfer

In medicine, methodological interest in developing statistical models for the analysis of social networks follows but at the same time helps to frame - the recurrent observation that if individuals are connected so is their health [33]. Recently, this argument has been transposed and extended to the analysis of relations between health care organizations: if health care organizations are connected, so must be the outcomes of their activities and the quality of the services they render [34, 35, 17]. In the illustrative analysis that we present later in the paper, hospitals are represented as nodes in an event network. Patient transfer events defined edges in a network of interdependent event streams. In the empirical part of the paper we focus our attention on the dynamics of the networks generated by different kinds of interhospital patient transfer events. The case study illustrates how the marked point process approach helps to specify and estimate models that take into account - and distinguish between - different kinds of events.

3.1. Interhospital patient transfer as a network process

Given increasingly scarce healthcare resources and highly differentiated hospitals, interhospital transfer is an essential part of the care of many patients. This is particularly the case for critically ill patients [36]. Pressure on resources is making patient transfer and sharing between partner hospitals increasingly relevant also for the treatment of elective patients.

Building on recent studies on interhospital patient mobility, we are interested in how sequences of interhospital patients transfer events give rise to an interorganizational network whose statistical properties may be examined empirically [35]. Patient transfer requires that partner hospitals commit resources to joint infrastructural investments in support of relational coordination [13, 37]. For this reason, patient transfer relations are typically interpreted as a reliable signal of collaboration between sending and receiving hospitals [15, 12]. Interhospital patient transfer is a dyadic relation established by hospitals in a highly decentralized fashion. In the analysis that we report in Section 4, we focus on the emergent properties of the network resulting from the accumulation of these apparently independent dyadic relations. We limit our analysis to the transfer of elective patients. We exclude emergency transfers because emergency networks are strongly constrained by exogenous factors.

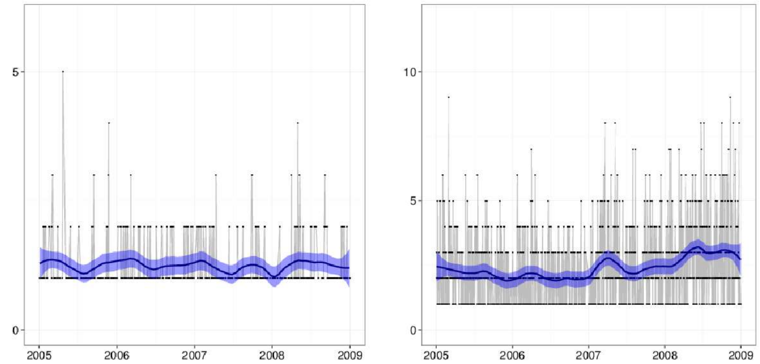
In the empirical analysis we concentrate on differences in the relational mechanisms underlying two different kinds of interhospital patient transfer. The first involves patient that are transferred across hospital and between different clinical specialties. The second involves patients that are transferred between hospitals but within the same clinical specialty. We emphasize this distinction because recent research has shown that different kinds of patient transfer are driven by different partner selection criteria and give rise to different forms of relational coordination between hospitals [17].

3.2. Empirical setting

The data set we analyze contains information on interhospital patient transfer events from 2005 to 2008 between 35 hospitals located in Abruzzo, a small central Italian region with 1,300,000 inhabitants. There are 22 public and 13 accredited private hospitals serving the regional community. Among a total number of 3,462 transfer events, 2,858 events are across clinical specialties, i.e., the specialty in the sender hospital is different from the specialty in the receiving hospital. In other words, the majority of interhospital patient transfer events occur between, rather than within clinical specialties. Daily rates of within and between-specialty events are plotted in Figures 1(a) and 1(b). They varied substantially across the observation period - an observation lending support for the non-parametric form of baseline intensities discussed in Section 2. Patient transfer events only occurred among 35 out of 44 distinct specialties listed by the Agency of Public Health.

In Section 2 we have discussed how the model we propose may be adopted to account for different kinds of events. That discussion is directly relevant here because interhospital transfer events within and between specialties are clearly different. The former kind of event (within specialty transfer) is typically motivated by contingent operational factors related to bed availability, or differences in capacity constraints between the sender and the receiver hospital [17]. The latter kind of event (between specialty interhospital transfer) is typically motivated by difference in competencies and capabilities between the sender and receiver hospital. It is possible that the networks induced by these different kinds of events will be different, and it may be interesting to understand exactly how these differences emerge and affect the dynamics of interhospital patient transfer relations.

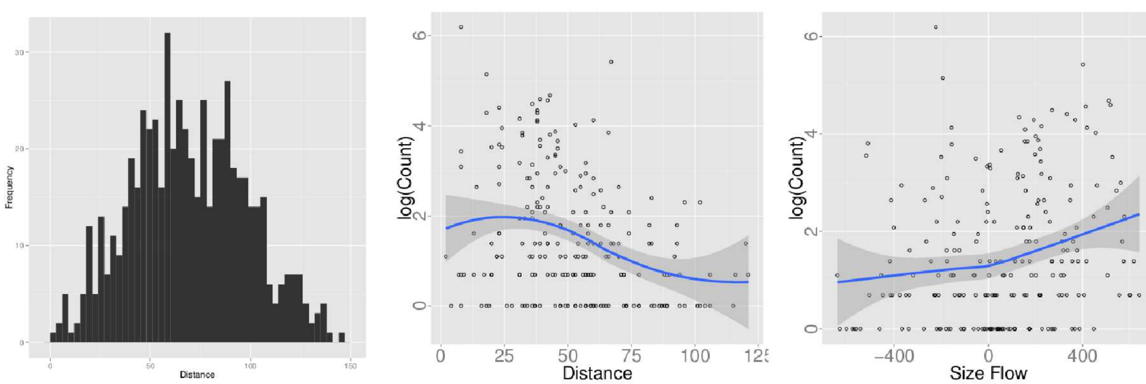
Besides internal differences in medical specialties, other factors like, for example, the geographical location of partner hospitals can also affect the likelihood of observing interhospital patient transfer events. Figure 2(a) shows the histogram of pairwise distances between 35 hospitals. Minimum and maximum distances are 2 and 146 kilometers, respectively. Figure 2(b) shows the correlation plot between these distances and edge event counts. It reveals that hospitals are more likely to establish patient transfer relationships with closer partners. Figures 3(a) and 3(b) overlay the within and between-specialty patient transfer networks on the geographical map of the region. Additional information on hospitals provided by the Regional Agency of Public Health is listed in Table 1. Moreover, there is a negative effect of the matching between institutional categories - as defined by public and private ownership - of sending and receiving hospitals on event counts.



(a) Within-specialty events (b) Between-specialty events

Figure 1. Daily rates of patient transfer events within and between medical specialities.

Transfers are more likely to occur between hospitals in different institutional categories (i.e. 81% of events are from a private to a public hospital or vise versa) - a result possibly due to the division of labor between public and private hospitals in the region that makes collaboration across hospitals in different institutional categories necessary. The correlation plot between directional size flow and event counts in Figure 2(c) provides additional evidence on the presence of hospital-specific covariate effects. For example, patient transfer events are more likely to flow from small to large hospitals (where size is measured by the number of staffed hospital beds).



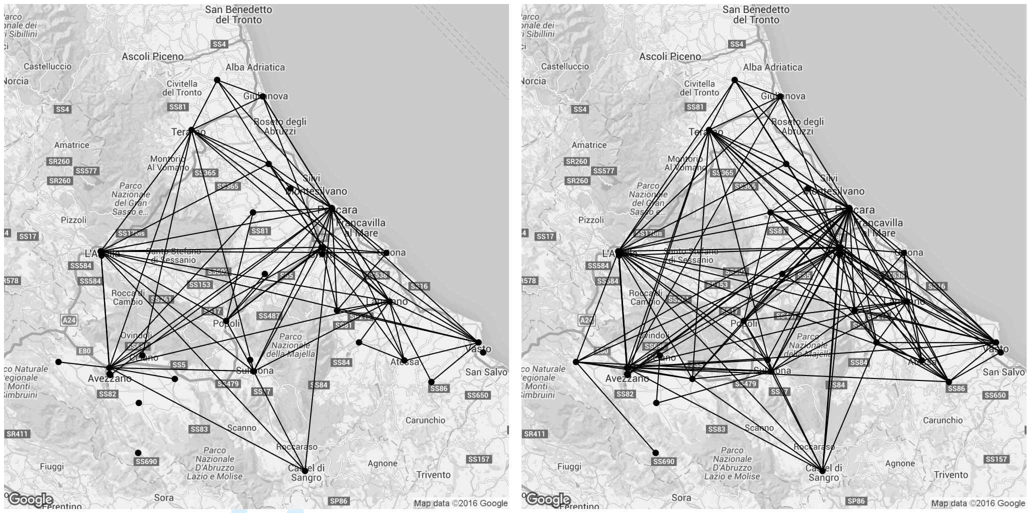
(a) Distance Distribution (b) Distance vs Transfer Counts (c) Hospital size (beds)

Figure 2. Geographical data and correlation plots. (a) Histogram of pairwise distances. (b)(c) Correlation plots between pairwise distances, size flow and transfer event counts on edges.

3.3. Covariates effects

As suggested by Figure 2, covariate data on hospitals such as institutional category (private or public), distance, or size (as measured by the number of staffed beds) could affect the propensity that a hospital is chosen as a transfer destination. To model these - and other - covariate effects, we consider four types of nodal and dyadic covariates which in general can be time-dependent such as hospital size or occupancy rate.

Receiver covariates. A receiver covariate, i.e $c(t, j)$, represents the current covariate value of a potential receiver. They are used to characterize which hospitals are more likely selected as transfer destinations. The positive effect of a receiver



(a) Within-specialty event network

(b) Between-specialty event network

Figure 3. The interhospital patient transfer networks overplaid over the geographical map. Edge width is plotted proportionally to the cumulative numbers of events over edges.

Table 1. Covariate data on hospitals provided by the Agency of Public Health. Longitudinal covariates marked by (*) are updated at the beginning of every year.

Covariate	Unit of Measure	Controls for Differences in
Geographical distance	Kilometers	Distances
Institutional category	Dimensionless category	Organizational form
Local health units (LHUs)	Dimensionless category	Administrative areas
Level of care provided	Dimensionless category	Capacity and technology
Size*	Hospital beds	Hospital size
Occupancy rate*	Proportion of beds occupied	Hospital capacity management
Competition	Dimensionless proportion	Levels of rivalry
Similarity*	Dimensionless proportion	Internal organizational structure

covariate implies that patients are more likely to be transferred to hospitals with high covariate values. A negative effect, on the other hand, means transfer events tend to flow to hospitals with low covariate values. In our analysis, two covariates including size and occupancy rate are used as receiver variables.

Matching covariates. A matching covariate is used to represent the binary similarity between two hospitals in terms of their categorical charateristics. Formally, a matching covariate between two hospitals i and j on a time-dependent categorical covariate c is defined as:

$$s_c(t, i, j) = \mathbf{I}\big[c(t, i) = c(t, j)\big], \tag{8}$$

where $c(t, i)$ and $c(t, j)$ denote the covariate values of hospitals i and j at time t, respectively; and $\mathbf{I}(x)$ is an indicator function that equals to 1 if the statement x is true or 0, otherwise. A matching covariate on institutional category, for example, equals to 1 if two hospitals are both private or public, otherwise it is 0. A negative effect of this covariate means that patients are more likely to be transferred between institutional categories, i.e. from public hospitals to private ones or vise versa. In our analysis, matching variables are used for three covariates including institutional category, local health units (LHUs), and level of care provided.

Undirectional covariates. An undirectional covariate is used to represent some symmetric dyadic charateristics between two hospitals such as their geographical distance or similarity measured by the proportion of overlapping medical specialities that they provide. A negative effect of geographical distance, for example, means that patients are more likely

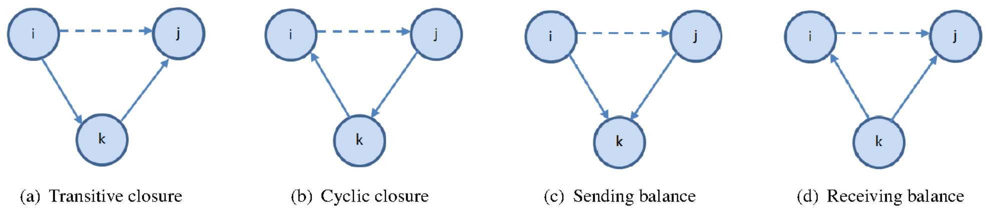


Figure 4. Clustering network statistics for the directed edge from i to j. Dash arrows represent potential patient transfer events.

to be transferred between hospitals in geographically close proximity. Our analysis considers undirectional variables for three covariates including geographical distance, competition, and similarity.

Competition is a dyadic covariate which records niche overlap - the extent to which two hospitals compete for patients in the same zip area code [38]. Other conditions being equal, two hospitals compete to the extent that they attract patients from the same geographical area [12]. Similarity is also a dyadic covariate based on the Jaccard coefficient computed on the two-mode matrix of hospitals by clinical specialties they contain. Structural similarity between two hospitals is high if they contain the same clinical specialties, i.e., if the two hospitals offer the same health care services.

Complementarity. Within the sequence of between-specialty events, we introduce an indicator *Complementarity* which is equal to 1 if the sending hospital does not have the specialty for which the patient is being transferred, and 0 otherwise. A positive effect of this indicator implies that patients in the between-specialty setting are more likely to be transferred from hospitals that do not have the event specialty, rather than from hospitals having that specialty.

3.4. Network effects

Network effects in relational event models can be represented as network statistics which can be divided into four types: nodal, dyadic, assortative , and clustering network statistics. Table 2 lists these network statistics and their interpretations while the appendix A presents their definitions in mathematical details. Unlike models that consider network ties as states associated with a duration, the relational event model which we estimate in the next section of the paper considers relationships between actors as activity event processes and network statistics are computed based on cumulative event counts. The corresponding effects tell how future activities can be predicted from past ones.

4. Analysis

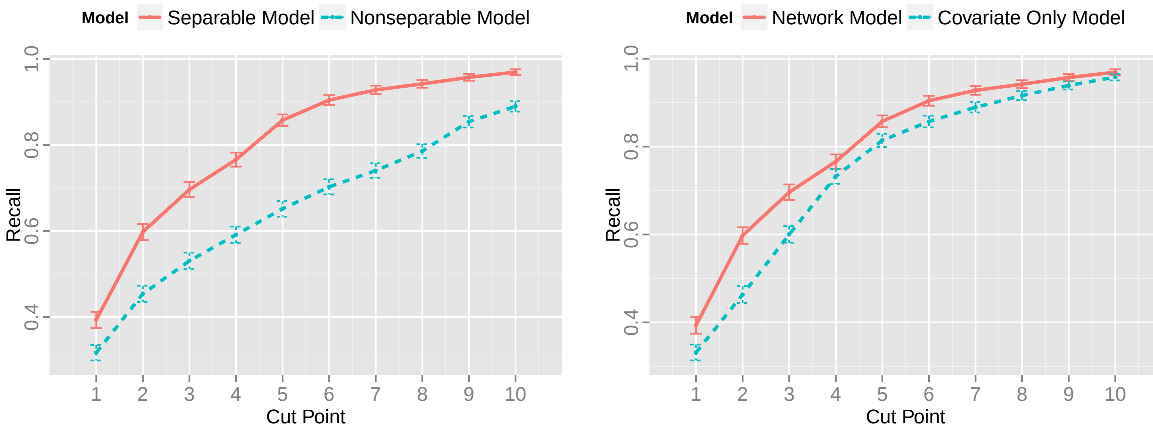
4.1. Motivating questions

Our analysis is designed to answer three main research questions. Our first question concerns the relevance of our separability assumption. To address this question, we will consider two relational event models with and without the separability assumption, and then compare them using a rolling prediction experiment with the recall criterion defined in Section 2.2. Our second question concerns the predictive ability of network statistic introduced in Section 3.4. In other words, we would like to demonstrate that high-order network statistics such as reciprocity, assortative and clustering can further explain for the propensity of transfer event sequences among hospitals in addition to covariate data. The existence of high-order network statistics implies that dependencies between transfer events may go beyond dyadic dependencies. To address this question, we will consider two relational event models with and without network statistics, and compare them using a rolling prediction experiment in a similar setting to the comparison of nonseparability and separability assumptions.

Table 2. Network statistics and interpretations.

Network Statistics	Interpretation
Out degree	A positive estimate of the *out degree* coefficient may be taken as evidence that hospitals with a higher number of established partners are more likely to transfer patients out.
Out intensity	Its positive *out intensity* effect may be interpreted as evidence that hospitals with high patient transfer intensity in the past tend to transfer more patients out.
In degree	A positive effect of *in degree* will provide evidence for preferential attachment phenomenon where hospitals with a larger number of sending partners tend to receive more patients in the future.
In intensity	A positive effect of *in intensity* is also expected to support for the preferential attachment phenomenon, i.e. hospitals currently receiving a large number of patients are more likely to be selected as transfer destinations in future transfers.
Recent sending	A negative coefficient of *recent sending* provides empirical evidence for the recency effect in patient transfer events, i.e. there is an increase in the event likelihood following a recent one.
Recent receiving	A negative estimate of coefficient associated with *recent sending* implies a clustering tendency in the selection of hospital destinations, hospitals recently chosen as transfer destinations are more likely to be selected in the future.
Repetition	A positive effect of *repetition* can be interpreted as a tendency that hospitals prefer to choose previous partners as destinations for their future patient transfers.
Reciprocity	A positive effect of *reciprocity* may be taken as evidence that hospitals tend to reciprocate collaboration ties by transfering patients to their previous partners who have shared patients with them before.
Assortativity by degree	A positive coefficient of *assortativity by degree* would implies that the network is assortative in terms of collaboration degrees, i.e. hospitals with a higher number of established partners are more likely to collaborate with hospitals with a larger number of sending partners.
Assortativity by intensity	A positive effect of *assortativity by intensity* implies that hospitals with high sending intensity are more likely to collaborate with hospitals with high receiving intensity.
Transitive closure	A positive estimate of coefficient associated with the *transitive closure* statistic (Figure 4(a)) may be interpreted as evidence that hospitals prefer to select partners of their partners as transfer destinations.
Cyclic closure	A negative effect of *cyclic closure* (Figure 4(b)) in the presence of a positive *transitive closure* effect, on the other hand, implies a hierarchical clustered collaboration structure.
Sending and receiving balance	Negative effects of these statistics (Figures 4(c) and 4(d)) imply an unbalanced structure in the patient transfer network. In other words, hospitals are less likely to collaborate with each other if they share the same sending or receiving partners. The existence of this unbalanced structure could be attributed to the specialization of hospitals.

Our third question concerns differences in covariate effects and collaboration structures between two different kinds of event sequences: within-specialty and between-specialty patient transfer across hopsitals. For example, we would expect that geographical distance to have a similar effect on both event settings, i.e., patients should be more likely to be transferred between hospitals in geographically close proximity. However, we anticipate some substantial variations in collaboration structures between within-specialty and between-specialty settings. In our comparative analysis, we report parameter estimates of the full relational event model fitted on three event settings, within-speciality, between-speciality, and aggregate event sequences.



(a) Comparison of Nonseparability and Separability Assumptions

(b) Comparison of Covariates and Network Statistics

Figure 5. Predictive comparison experiments between: (a) nonseparability and separability assumptions, and (b) covariates and network statistics. Error bars represents 2 standard errors of recall means on all test events. The higher the recall is, the better out-of-sample prediction performance that a model achieves.

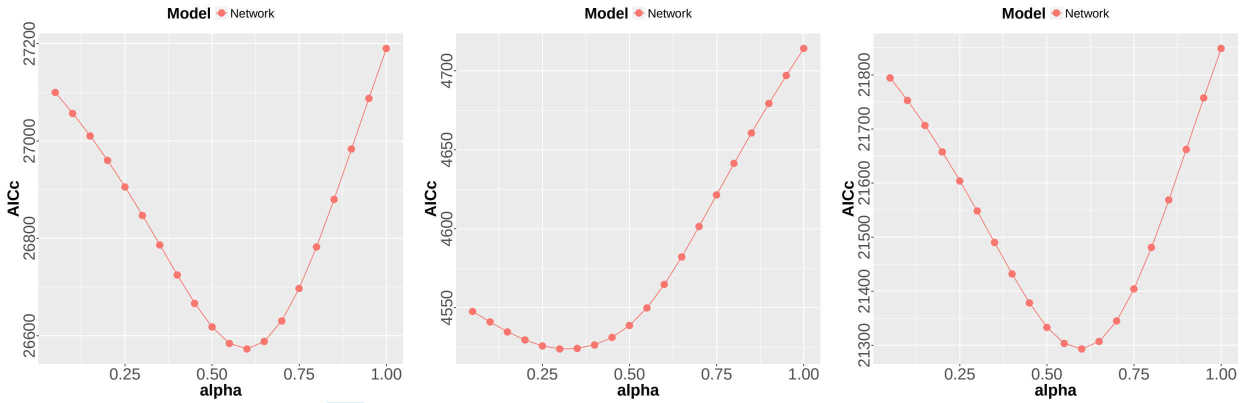
4.2. Results

Separability assumption. We consider the full relational event model of all covariates and network statistics discussed in Section 3.4 with two different assumptions: the nonseparable model where both sender and receiver components are modelled by the same intensity function [9] and our proposed separable model as discussed in Section 2.1. To isolate the effect of the time weighting parameter α in this experiment, we fix $\alpha = 0$, i.e. all events are assigned equal weights 1.0, resulting in count-based network statistics considered previously in [9, 25, 19, 17]. Both models are compared in a yearly rolling prediction experiment where the goal is to correctly predict hospital destinations of 2709 test transfer events during the period from 2006 to 2008. We do not compare these models on the period 2005 since historical events are not yet available to construct network statistics. Moreover, we only focus on the aggregate setting to allow for a large number of test events. Figure 5(a) shows the recall performance of two models. Consistently across all cut points, the separable model achieves better prediction performance than the nonseparable one.

Network statistics. To highlight the predictive ability of our proposed network statistics for interhospital patient transfer data, we compare two nested separable relational event models. The first covariate only model includes all covariates listed in Table 1, the *Complementarity* indicator discussed in Section 3.3, and two temporal statistics, *recent sending* and *recent receiving*. The second model is the full network model that includes all covariates and network statistics discussed in Section 3.4. An evaluation setting similar to the nonseparability and separability assumption experiment is also used for this comparison. Figure 5(b) shows the consistently better prediction performance of the network model over the covariate one.

Model interpretation. Supporting by two comparision results above, we only consider the full network model with the separable assumption in this model interpretation discussion. In particular, since our main goal here is to compare network effects across three event settings, within-speciality, between-speciality, and aggregate event sequences, we will estimate the models on all available data, i.e. consider an in-sample evaluation. To achieve the best model fit, we vary time decay parameter α from 0 to 1.0 with step size 0.05 to search for its optimal value. Different value of α can be used for each network statistics; however, such extreme choice requires a high dimensional grid search to find the optimal set of these time-weighting parameters. We choose to use only one time-weight parameter α for all network statistics to achieve parsimony, especially when our application data set is small.

Figure 6 shows AICcs of the network model across all event sequences. These AICc plots demonstrate the predictive



(a) Aggregate events (b) Within-specialty events (c) Between-specialty events

Figure 6. Model comparison between the network model at different time-weighting parameter values across aggregate, within-specialty, and between-specialty event sequences. Time decay parameter α is varied 0 to 1.0 with step size .05 to search for the optimal value when computing time-weighted network statistics.

improvement when the time-weighted method is used in the computation of network statistics. In all settings, the optimal value of α is far from 0 where all events are assigned equal weights 1.0. Optimal values for time decay parameter α for aggregate, within-specialty, and between-specialty settings are 0.6, 0.3, and 0.6, respectively. A goodness of fit test can also be used to compare two models: the aggregate model (the null hypothesis) versus the combination of two separate within and betwee-speciality models (the alternative hypothesis). The likelihood ratio statistic computed at optimal time decay parameters is 808.2 with 25 degrees of freedom. It supports for our choice of considering two different models for within and between-speciality event sequences. Further model checking results of these selected models using Schoenfeld residuals are presented in the appendix C. The selected models are presented in Table 3 and their interpretations will be discussed next. It is important to note that standard errors of main model parameters $\boldsymbol{\theta}$ and $\boldsymbol{\beta}$ in Table 3 are approximated in the sense that their calculations are carried independently from the time decay paramter α.

Temporal and network effects of the transfer time model in Table 3 are all similar and statistically significant across both within-specialty and between-specialty settings. The positive effects of *out degree* and *out intensity* provide evidence that hospitals who have many established partners or are very active in patient transfer are more likely to continue sending their patients out. The negative effect of *recent sending* implies that transfer events are clustered rather than homogeneoulsy distributed over time. Finally, the positive effect of the *complementarity* indicator implies that patients in the between-specialty setting are more likely to be transferred from hospitals that do not have the medical specialty for which a patient is being transferred, rather than from hospitals having that specialty.

Regarding covariate effects in the transfer destination model, the positive and statistically significant of *size flow* in both within-specialty and between-specialty settings implies that patients are more likely to be transferred to large hospitals in terms of beds. Additionally, the effect of *occupancy rate flow* is statistically significant and negative in both event sequences, which can be interpreted that patients are more likely to be transferred low hospitals in terms of occupancy rates. Moreover, negative and statistically significant effects of *geographical distance* and *institutional category* in both sequences confirm our findings in Section 3, i.e. hospitals prefer to send their patients to nearby ones or ones in different institutional categories. In particular, the hazard ratio of a patient transfer event connecting 2 hospitals in the within-specialty network decreases by an estimated 1.9% for one additional km in distance, while the same estimated value in the between-specialty network is 3.1%. In other words, the preference for establishing patient transfer relationships with closer partners is 1.6 times weaker in the within-specialty network than in the between-specialty network. Besides the geographical preference, hospitals are more likely to establish patient transfer relations with others in their same local health units. The effect of *LHU membership* is statistically significant and positive in both sequences. However, *level of* *care provided* is not associated with the propensity of being chosen as transfer destinations in both event sequences. Both estimated coefficients are not statistically significant.

The within-specialty and between-specialty networks are different on two covariate effects: *competition* and *similarity*. These effects are statistically significant in the between-specialty network, but not in the within-specialty network. Their positive effects provide evidence that in the between-specialty network, highly competing or similar hospitals are more preferred as transfer destinations. The presence of collaborative patient transfer events between competing hospitals is consistent with prior results on collaboration between competitors [12], while the higher likelihood of observing collaborative patient transfer events between similar hospitals is consistent with absorptive capacity arguments according to which similarity facilitates interorganizational communication, learning and knowledge sharing [39].

Regarding collaboration structures between hospitals, positive effects of *repetition* and *reciprocity* across both within-specialty and between-specialty settings provide evidence that hospitals tend to maintain and reciprocate to their established patient transfer relations. Furthermore, the *repetition* and *reciprocity* effects are stronger in the within-specialty network. For instance, the hazard ratio of a patient transfer event connecting 2 hospitals in the within-specialty network increases by an estimated 0.3% per one unit increment in the *repetition* statistic, while the same estimated value in the between-specialty network is 0.01%. In other words, the preference for transfering patients to partners who have collaborated on one transfer in the past is nearly 30 times stronger in the within-specialty network than in the between-specialty network. Moreover, the negative effect of *recent receiving* implies a clustering tendency in the selection of hospital destinations, i.e. hospitals recently chosen as transfer destinations are more likely to be selected in the future. However, other network effects act differently between within-specialty and between-specialty settings. While hospitals' popularity measured by *in degree* and *in intensity* has no effect or negative effect on attracting future tranfers in the within-specialty setting, they are both positive in the between-specialty setting. Additionally, the within-specialty patient transfer network is disassortative (i.e., negative assortativity effects) while the between-specialty patient transfer network is assortative (i.e., positive assortativity effects). Finally, the only positive and statistically significant effect of *transitive closure* in the within-specialty network at level .05 implies a weakly clustered but hierarchical collaboration structure. On the other hand, for the between-specialty patient transfer network, the effect of cyclic closure is positive and statistically significant, while transitive closure is not significant. These effects, associated with negative and significant sending and receiving balance effects imply a clustered but flat and unbalanced collaboration structure among hospitals. Clearly, the different logics underlying the interhospital patient within and between specialties sustain different principles of organziational bonding [40].

5. Discussion and conclusions

In this paper we have introduced relational event models for the analysis of longitudinal network data. The marked point process variant that we have presented improves over available models by allowing comparative analysis of the networks generated by sequences of different micro-relational events.

The model we have presented has four main advantages over alternative analytical frameworks that are currently adopted in the analysis of social and interorganizational networks. The first is that relational events are directly observable and need not be aggregated somewhat arbitrarily into network ties whose stability may depend delicately on dichotomization thresholds. This facilitates the development of models that better reflect the data generating process [7]. The second is that the model scales to event networks of arbitrary size. In combination with nested case-control sampling methods [27], this feature facilitates the analysis of large and very large event networks. The third advantage is that the marked point process model that we have implemented allows identification of different events underlying observed relational patterns. This feature of the model reduced the possibility that observed networks will be constructed out of heterogeneous events [17]. Finally, the identification of a timing and a choice component in the overall relational event process opens the door

Table 3. Estimated effects of the full network model in aggregate, within-specialty, and between-specialty event sequences. Standard errors are in parentheses and ** for $p < .01$ and * for $p < .05$. First four effects are related to transfer times and the rest are related to destination choices.

	Aggregate	Within specialty	Between specialty
Complementarity	0.18^{**}	-	0.41^{**}
	(0.02)	-	(0.03)
Out degree	0.67^{**}	0.55^{**}	0.71^{**}
	(0.02)	(0.06)	(0.03)
Out intensity	0.44^{**}	0.41^{**}	0.43^{**}
	(0.01)	(0.03)	(0.01)
Recent sending	-1.39^{**}	-1.67^{**}	-1.36^{**}
	(0.15)	(0.25)	(0.16)
In degree	0.36^{**}	-0.02	0.22^{**}
	(0.07)	(0.15)	(0.07)
In intensity	0.51^{**}	-0.48^{**}	0.6^{**}
	(0.05)	(0.12)	(0.05)
Recent receiving	-1.05^{**}	-1.23^{**}	-0.94^{**}
	(0.16)	(0.24)	(0.15)
Assortativity by degree	-0.07	-0.23^{**}	0.05
	(0.04)	(0.08)	(0.04)
Assortativity by intensity	0.12^{**}	-0.24^{**}	0.11^{**}
	(0.03)	(0.08)	(0.04)
Reciprocity	0.09^{**}	0.12^{**}	0.08^{**}
	(0.03)	(0.04)	(0.03)
Repetition	0.39^{**}	1.07^{**}	0.34^{**}
	(0.03)	(0.15)	(0.05)
Transitive closure	3×10^{-3}	0.18^{*}	-0.02
	(0.04)	(0.09)	(0.04)
Cyclic closure	0.32^{**}	0.02	0.28^{**}
	(0.05)	(0.13)	(0.06)
Sending Balance	-0.35^{**}	-0.19	-0.23^{**}
	(0.05)	(0.15)	(0.05)
Receiving Balance	-0.28^{**}	-0.07	-0.32^{**}
	(0.04)	(0.06)	(0.05)
Size flow	0.19^{*}	1.22^{**}	0.56^{**}
	(0.09)	(0.18)	(0.1)
Occupancy rate flow	-0.18^{**}	-0.39^{*}	-0.21^{**}
	(0.06)	(0.18)	(0.07)
Geographical distance (Km)	-0.78^{**}	-0.47^{**}	-0.85^{**}
	(0.05)	(0.11)	(0.06)
Institutional category	-0.48^{**}	-1^{*}	-0.55^{**}
	(0.06)	(0.4)	(0.06)
LHU membership	0.70^{**}	0.59^{**}	0.74^{**}
	(0.03)	(0.07)	(0.03)
Level of care provided	-0.09^{**}	-0.07	-0.02
	(0.04)	(0.08)	(0.04)
Competition	0.06^{**}	0.07	0.06^{*}
	(0.02)	(0.05)	(0.03)
Similarity	0.16^{**}	0.05	0.20^{**}
	(0.04)	(0.08)	(0.05)

to a more explicit framing of relational event models as decision models.

The case study that we have examined on interhospital patient transfer provides a useful illustration of how these various features of the models we have proposed may be able to sustain empirical analysis of complex relational data. More specifically, we have shown that the aggregate results mask a considerable level of heterogeneity in logics underlying patient transfer relations between hospitals. We have found that in the within-specialty network hospitals are selected as transfer destinations regardless their levels of competition or similarity with sending hospitals. On the contrary, high levels of competition or similarity between hospitals in the between-specialty network predict a higher likelihood of observing a collaborative patient transfer relation. Moreover, the well-known preferential attachment mechanism [41] also acts in opposite ways across the two network settings. On one hand, popular hospitals in the between-specialty network tend to receive more patients. On the other hand, popular hospitals in the within-specialty network are less likely to be selected as transfer destinations. Another difference between two networks is their assortative mixing patterns [42]. While the between-specialty network is assortative (i.e., highly collaborative hospitals tend to transfer their patients to popular hospitals), the within-specialty network is disassortative (i.e., highly collaborative hospitals are less likely to send their patients to popular hospitals). Finally, clustering structures of two patient transfer networks are significantly different. Collaborations between hospitals in the within-specialty network is weakly clustered but hierarchical, whereas collaborations between hospitals in the between-specialty network is clustered but flat and unbalanced.

Many research questions still need to be addressed in future work to advance the proposed marked point process approach for relational event data. The first problem is to develop a rigorous approach for model checking where observed network statistics can be compared with simulated ones following goodness of fit methods that have been introduced in exponential random graph models [43] and stochastic actor oriented models [44]. Such a goodness of fit procedure will require the estimation not only of covariate and network parameters but also the baseline intensity function so that network event data sets can be simulated. While Breslow's estimator of the baseline intensity function [23] can be obtained by plugging in the partial likelihood estimator of the transfer time model, another more efficient approach is to use a fully parametric model for event times. The second problem is to employ other discrete choice models for event marks (i.e., transfer destinations) such as probit models [26]. A more advanced approach to discrete choice modelling would be generalized additive models [45] which allow for non-linear effects of covariate and network statistics. For example, it is possible that the preferential attachment effect, rather than strictly following a log-linear pattern imposed by the logit function, might saturate when the *in degree* statistic reaches a threshold value. Such a network threshold effect has been considered in modelling diffusion through networks [46].

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Appendix: Relational event models for longitudinal network data with an application to interhospital patient transfers

Duy Vu[ab*], **Alessandro Lomi**[a], **Daniele Mascia**[c], **Francesca Pallotti**[d]

A. Network effects

Network effects in relational event models can be represented as network statistics which can be divided into four types: nodal, dyadic, assortative , and clustering network statistics.

A.1. Nodal network statistics

They are computed based on past transfer events from and to each hospital. They are included in both the transfer event and destination models to estimate structural and temporal effects of past patient transfer activities on future ones. For the analysis of interhospital patient transfer networks, we consider six nodal network statistics including *out degree*, *out intensity*, *in degree*, *in intensity*, *recent sending*, and *recent receiving*.

Out degree is the number of hospitals to which a hospital has shared patients. Formally, it may be defined as:

$$\textit{Out degree}(t, i) = \sum_{k \neq i} \mathbf{I}\big[\mathrm{N}_{ik}(t) > 0\big], \tag{1}$$

where $N_{ik}(t)$ is the number of transfer events from hospital i to hospital k by time t. This nodal network statistic which is visualized in Figure 1(a) is included in the transfer event model to estimate the effect of hospitals' collaboration scope on their propensity to initiate new patient transfer events. A positive estimate of the coefficient associated with *out degree* may be taken as evidence that hospitals with a higher number of established partners are more likely to transfer patients out.

Out intensity. While *out degree* measures the past collaboration activity of a hospital in terms of scope, *out intensity* measures its collaboration activity in terms of intensity (see Figures 1(a) and 1(b)). Particularly, it is defined as the average

[a] *Social Network Analysis Research (SONAR) Center, University of Italian Switzerland at Lugano (Switzerland)*
[b] *Department of Mathematics and Statistics, University of Melbourne (Australia)*
[c] *Department of Public Health and Healthcare Management, Catholic University of Rome (Italy)*
[d] *Centre for Business Network Analysis, University of Greenwick (UK)*
[*] *Correspondence to: Duy Vu, Department of Mathematics and Statistics, University of Melbourne, Australia.*

number of patient transfer events that hospital i has initiated per each of its collaboration ties:

$$\textit{Out intensity}(t,i) = \frac{\sum_{k\neq i}\sum_{e=1}^{\mathrm{N}_{ik}(t)} f(t, T_{ik}^{e}, \alpha)}{\sum_{k\neq i} \mathbf{I}\big[\mathrm{N}_{ik}(t) > 0\big]}, \tag{2}$$

where T_{ik}^{e} is time of transfer event e from hospital i to hospital k. Moreover, each transfer event is assigned a time-dependent weight $f(t, T_{ik}^{e}, \alpha)$ to account for its temporal relevance:

$$f(t, T_{ik}^{e}, \alpha) \quad = \quad \frac{1}{(t - T_{ik}^{e})^{\alpha}}, \tag{3}$$

where $\alpha > 0$ is time-decay parameter which could be optimized by a grid search. Intuitively, this time-weighting scheme [1] effectively imposes a criterion that events close to the current time t are more important than those further in the past. This statistic is included in the transfer event model. Its positive effect may be interpreted as evidence that hospitals with high patient transfer intensity in the past tend to transfer more patients out.

In degree. To explore the effects of popularity on the propensity that hospitals will be chosen as transfer destinations, two nodal network statistics, *in degree* and *in intensity* in Figures 1(c) and 1(d), are considered. Particularly, to measure the popularity scope of hospital j, we define *in degree* as the number of hospitals from which j has received patients:

$$\textit{In degree}(t,j) = \sum_{k\neq j} \mathbf{I}\big[\mathrm{N}_{kj}(t) > 0\big]. \tag{4}$$

A positive effect of *in degree* will provide evidence for preferential attachment phenomenon [2] where hospitals with a larger number of sending partners tend to receive more patients in the future.

In intensity. To measure hospitals' popularity in terms of intensity, we define *in intensity* as the number of transfer events that a hospital has received per each of its sending partners:

$$\textit{In intensity}(t,j) = \frac{\sum_{k\neq j}\sum_{e=1}^{\mathrm{N}_{kj}(t)} f(t, T_{kj}^{e}, \alpha)}{\sum_{k\neq j} \mathbf{I}\big[\mathrm{N}_{kj}(t) > 0\big]}. \tag{5}$$

Each receiving event is also assigned a time-dependent weight $f(t, T_{kj}^{e}, \alpha)$ similar to the definition of *out intensity*. A positive effect of *in intensity* is also expected to support for the preferential attachment phenomenon [2], i.e. hospitals currently receiving a large number of patients are more likely to be selected as transfer destinations in future transfers.

Recent sending. To control for the recent activeness of a hospital in patient transfer, we define *recent sending* as the gap time since its last sending event:

$$\textit{Recent sending}(t,i) \quad = \quad t - \max_{e\in E_i} t_e, \tag{6}$$

where t_e denotes time of transfer event e from hospital i. A negative coefficient of *recent sending* statistic provides empirical evidence for the recency effect in patient transfer events, i.e. there is an increase in the event likelihood following a recent one [3]. This can also be interpreted as evidence that patient transfer events tend to be clustered into groups rather equally distributed over time.

Recent receiving. Similarly to *recent sending*, we define *recent sending* as the gap time since the last receiving event. It is considered to estimate the recency effect in the transfer destination model. A negative estimate of coefficient associated with *recent sending* implies a clustering tendency in the selection of hospital destinations, hospitals recently chosen as

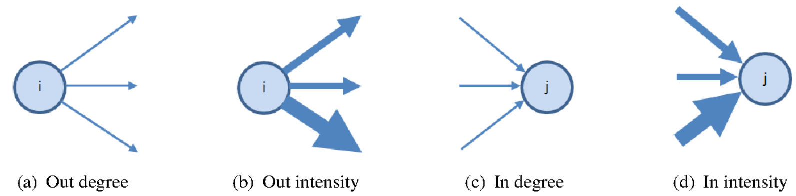


Figure 1. Visual representation of nodal network statistics where sizes of event flows from and to hospitals are considered for intensity statistics.

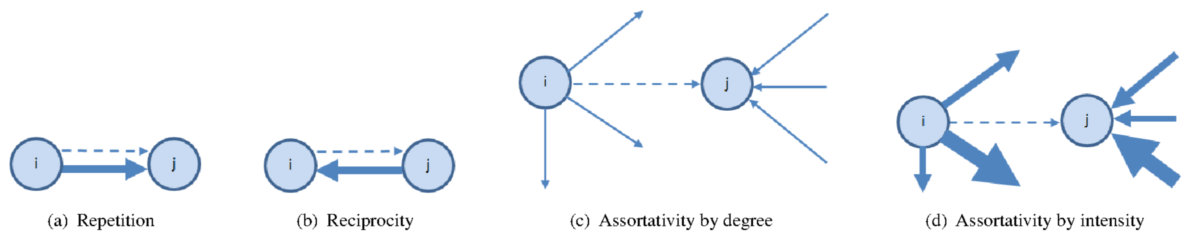


Figure 2. Dyadic and assortativity network statistics for the directed edge from i to j. Dash arrows represent potential patient transfer events.

transfer destinations are more likely to be selected in the future.

A.2. Dyadic network statistics

They are computed based on past patient transfer events between two hospitals. They are considered in our destination choice model to represent temporal and structural effects of previous pairwise collaboration on future transfer events. Two dyadic network statistics, *repetition* and *reciprocity*, are defined below.

Repetition. This statistic measures the propensity that hospital i continues its current transfer tie with hospital j by sending more patients to j (see Figure 2(a)). It is defined as the current number of transfer events from hospital i to hospital j:

$$Repetition(t,i,j) = \sum_{e=1}^{\mathrm{N}_{ij}(t^-)} f(t,T^e_{ij},\alpha). \tag{7}$$

Each event is also assigned a temporal weight $f(t,T^e_{ij},\alpha)$ to model the diminishing importance of transfer events far in the past. A positive effect of *repetition* can be interpreted as a tendency that hospitals prefer to choose previous partners as destinations for their future patient transfers.

Reciprocity. To measure the propensity that hospital i reciprocates the patient transfer collaboration with hospital j, we define *reciprocity* as the current number of transfer events from hospital j to hospital i (see Figure 2(b)):

$$Reciprocity(t,i,j) = \sum_{e=1}^{\mathrm{N}_{ji}(t^-)} f(t,T^e_{ji},\alpha), \tag{8}$$

where each event is also weighted by the temporal function $f(t,T^e_{ji},\alpha)$. A positive effect of *reciprocity* may be taken as evidence that hospitals tend to reciprocate collaboration ties by transfering patients to their previous partners who have shared patients with them before.

A.3. Assortative network statistics

Degree and intensity assortative mixing in our patient transfer network can be interpreted as a tendency whereby events are more likely to occur between hospitals that are similar in collaboration degrees or intensities. Assortative network statistics can also be considered as dyadic network statistics; however, they are defined as interactions between corresponding sending and receiving statistics.

Assortativity by degree. This interaction between *out degree* of sending hospital i and *in degree* of receiving hospital j models the assortativity effect in terms of collaboration scope and can be defined formally as:

$$\textit{Assortativity by degree}(t,i,j) \quad = \quad \textit{out degree(t, i)} \times \textit{in degree}(t,j). \tag{9}$$

A positive coefficient of *assortativity by degree* would implies that the network is assortative in terms of collaboration degrees. More specifically, it can be interpreted as evidence that hospitals with a wide sending scope prefer to transfer their patients to hospitals with a high number of receiving ties (see Figure 2(c)).

Assortativity by intensity. This interaction between *out intensity* of sending hospital i and *in intensity* of receiving hospital j models the assortativity effect in terms of collaboration intensities. A positive effect of *assortativity by intensity* implies that hospitals with high sending intensity are more likely to collaborate with hospitals with high receiving intensity (see Figure 2(d)).

A.4. Clustering network statistics

To model the tendency that patient transfer collaborations are clustered together rather than equally distributed across the network, we propose four new local clustering statistics. Rather than based on 2-path statistics that ignore event counts on edges as previous approaches [4, 5, 6], we measure the strength of these new clustering statistics by combining time-weighted dyadic patient flows through harmonic mean.

Transitive closure. This network statistic measures the total strength of all 2-paths (see Figure 3(a)) from hospital i to hospital j:

$$\textit{Transitive closure}(t,i,j) \quad = \quad \sum_{k \neq i,j} g\big(\mathrm{w}(t,i,k), \mathrm{w}(t,k,j)\big), \tag{10}$$

where the patient flow from hospital i to hospital k is measured by the current number of transfer events from i to k. To model the diminishing importance of events far in the past, similar to the definitions of *repetition* and *reciprocity*, these events are also assigned a temporal weights $f(t, T^e_{ik}, \alpha)$:

$$\mathrm{w}(t,i,k) \quad = \quad \sum_{e=1}^{\mathrm{N}_{ik}(t^-)} f(t, T^e_{ik}, \alpha). \tag{11}$$

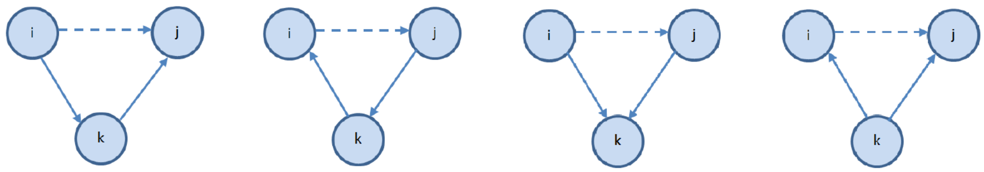
The intensity of the patient flow from i to j through k is then computed as harmonic mean of patient flows from hospital i to hospital k and from hospital k to hospital j. Formally, the harmonic mean of two flow values w_1 and w_2 is given by:

$$g(w_1, w_2) = \frac{2w_1w_2}{w_1 + w_2}. \tag{12}$$

A positive estimate of coefficient associated with the *transitive closure* statistic may be interpreted as evidence that hospitals prefer to select partners of their partners as transfer destinations.

Cyclic closure. This network statistic measures the total strength of all 2-paths to hospital i from hospital j (see Figure 3(b)) and may be defined formally as follows:

$$\textit{Cyclic closure}(t,i,j) = \sum_{k \neq i,j} g\big(\mathrm{w}(t,k,i), \mathrm{w}(t,j,k)\big) \tag{13}$$



(a) Transitive closure (b) Cyclic closure (c) Sending balance (d) Receiving balance

Figure 3. Clustering network statistics for the directed edge from i to j. Dash arrows represent potential patient transfer events.

A positive effect of *cyclic closure* provides evidence for a clustered but flat rather hierarchical collaboration structure among hospitals in patient transfer. A negative effect of *cyclic closure* in the presence of a positive *transitive closure* effect, on the other hand, implies a hierarchical clustered collaboration structure. Finally, negative coefficients of all clustering statistics may be interpreted as evidence of a non-clustered collaboration structure among hospitals.

Sending and receiving balance. These network statistics measure the total strength of all 2-paths that share destinations (or departures) from (or to) hospital i and hospital j:

$$\begin{aligned} \textit{Sending balance}(t,i,j) &= \sum_{k\neq i,j} g\big(\mathrm{w}(t,i,k),\mathrm{w}(t,j,k)\big), \\ \textit{Receiving balance}(t,i,j) &= \sum_{k\neq i,j} g\big(\mathrm{w}(t,k,i),\mathrm{w}(t,k,j)\big). \end{aligned}$$

These statistics are visualized in Figures 3(c) and 3(d). Negative effects of these statistics imply an unbalanced structure in the patient transfer network. In other words, hospitals are less likely to collaborate with each other if they share the same sending or receiving partners. The existence of this unbalanced structure could be attributed to the specialization of hospitals. For example, shared destinations in Figure 3(c) could have some unique specialities that both hospitals i and j do not have. Therefore, they keep sending their new patients for those specialized treatments at k rather to each other.

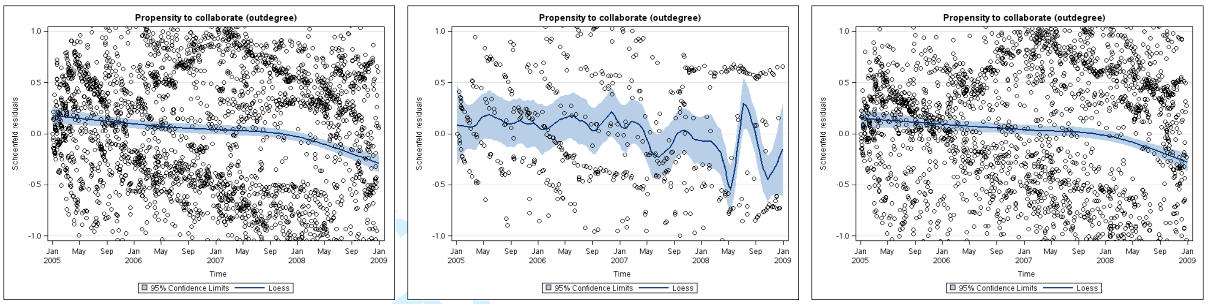
B. Nested case-control implementation for model estimation

This section discusses the nested case-control method that can be efficiently used for the model estimation step. The main idea is that when iterative optimization methods such as a Newton-Raphson algorithm are used to maximize the partial and discrete choice likelihoods, it is computationally expensive to regenerate network statistics for all nodes and edges at event times for every optimization step. However, it is able to exchange more computer memory usage for a faster estimation speed and also take advantage of available software packages for proportional hazard and discrete-choice models. In other words, the method runs through the history of transfer events only one time to generate a nested case-control data set of all covariates and network statistics which can then be used as input data in proportional hazard and discrete-choice modelling libraries. The data generation and estimation procedure can be summarized as follows:

1. Sort transfer events in the chronological order,
2. Initialize the event history $\mathbf{H}_{t-}$ as empty,
3. For each event $e_k \in e_1,\ldots,e_M$:
   (a) For each edge (i,j) in the risk set right before time t_{e_k}, generate all covariates and network statistics for nodes i and j, as well as the directed edge (i,j) and store them in a data row as illustrated in Table 1,
   (b) Add event e_k to the network history $\mathbf{H}_{t-}$ and continue to the next event.

Table 1. The data frame of nested case-control data. The number of data rows for each unique event index is the current number of at-risk edges right before the event time. Each of these data rows corresponds to each at-risk edge and is used to store all current values of its covariates and network statistics. Among these at-risk data rows for a transfer event, only the row on which the event occurs has case indicator value equal to 1.

Event Index	Event Time	Sender ID	Receiver ID	Case Indicator	Statistic 1	…	Statistic P
10	15/07/2005	1	2	0	1.0	…	4.5
10	15/07/2005	1	3	1	4.0	…	2.5
10	15/07/2005	…	…	…	…	…	…
10	15/07/2005	34	35	0	3.0	…	7.2



(a) Aggregate events (b) Within-specialty events (c) Between-specialty events

Figure 4. Schoenfeld residuals for *out degree* effects at aggregate, within-specialty, between-specialty settings.

4. The final data matrix can be used as input for proportional hazard and discrete-choice modelling libraries. In particular, we use the procedure *PHREG* in *SAS* to estimate coefficients for all models dicussed in our case study. The event index of each event is unique; therefore, it can be used a stratification covariate in *PHREG*.

C. Model checking results based on Schoenfeld residuals

One important assumption of our separable relational event model is that covariate and network effects are time-invariant. One visual check for this assumption is to plot Schoenfeld residuals against the observation time [7]. Figures 4, 5, 6, 7, 8, 9, and 10 show Schoenfeld residual plots for some covariate and network statistics of three models selected in Section 4.2: *out degree*, *geographical distance*, *competition*, *similarity*, *repetition*, *transitive closure*, and *cyclic closure*. Only some effects such as *repetition* or *out degree* slightly deviate from this time-invariant assumption on two ends of the observation time which could be due to the lack of data points for the *loess* smoothing method. The small number of events could also explain for the fluctuation of estimated effects in the within-speciality setting.

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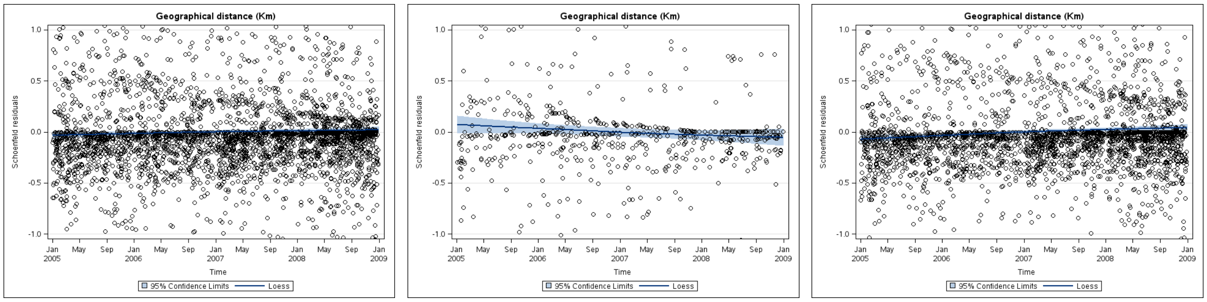
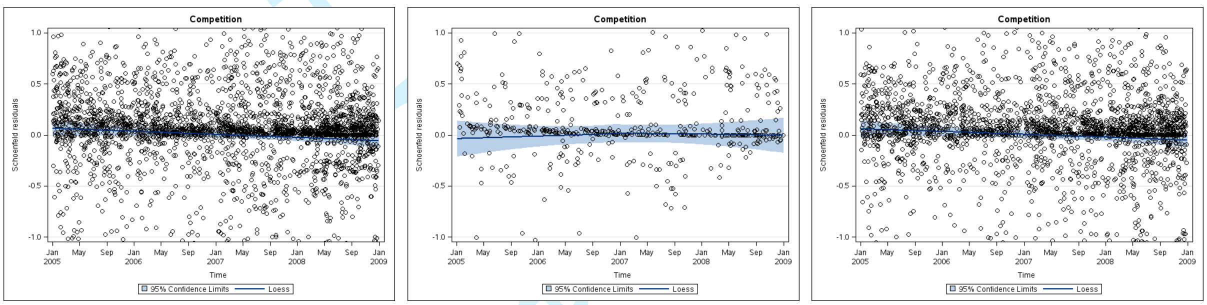


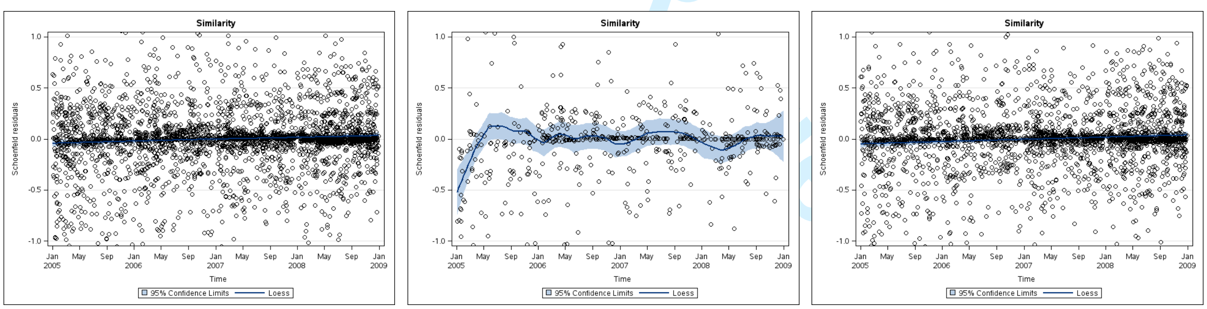
(a) Aggregate events (b) Within-specialty events (c) Between-specialty events

Figure 5. Schoenfeld residuals for *geographical distance* effects at aggregate, within-specialty, between-specialty settings.



(a) Aggregate events (b) Within-specialty events (c) Between-specialty events

Figure 6. Schoenfeld residuals for *competition* effects at aggregate, within-specialty, between-specialty settings.



(a) Aggregate events (b) Within-specialty events (c) Between-specialty events

Figure 7. Schoenfeld residuals for *similarity* effects at aggregate, within-specialty, between-specialty settings.

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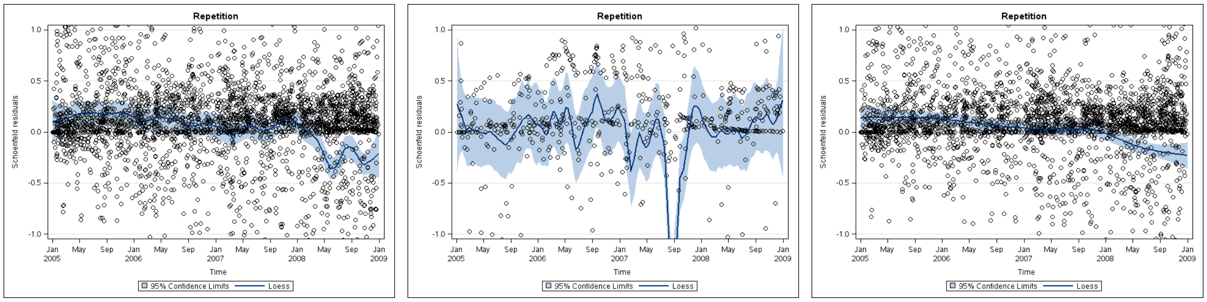
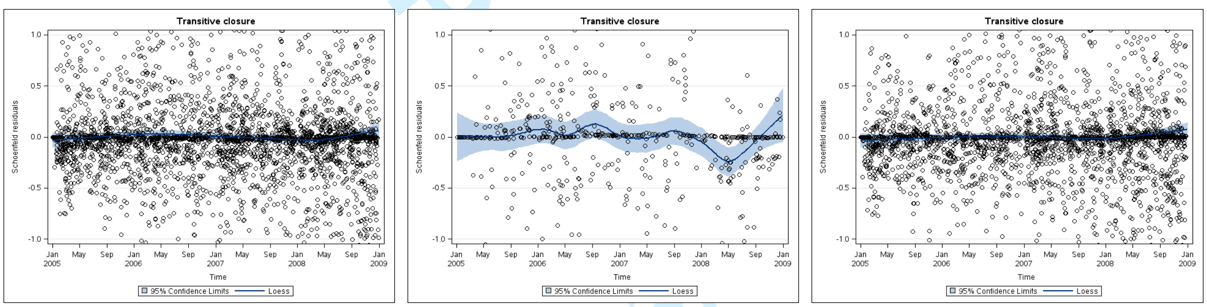


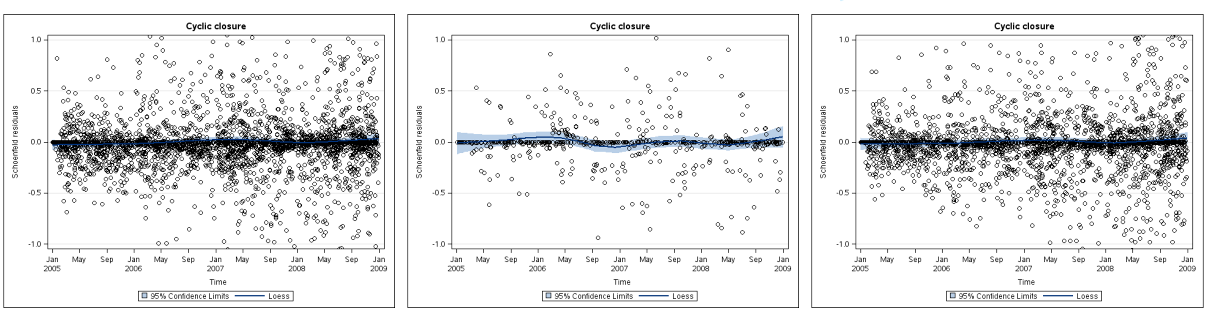
(a) Aggregate events (b) Within-specialty events (c) Between-specialty events

Figure 8. Schoenfeld residuals for *repetition* effects at aggregate, within-specialty, between-specialty settings.



(a) Aggregate events (b) Within-specialty events (c) Between-specialty events

Figure 9. Schoenfeld residuals for *transitive closure* effects at aggregate, within-specialty, between-specialty settings.



(a) Aggregate events (b) Within-specialty events (c) Between-specialty events

Figure 10. Schoenfeld residuals for *cyclic closure* effects at aggregate, within-specialty, between-specialty settings.