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RESEARCH ARTICLE

A framework for models of movement in geographic space

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This paper concerns the theoretical foundations of movement informatics. We discuss general frameworks in which models of spatial movement may be developed. In particular, the paper considers the object-field and Lagrangian-Eulerian dichotomies, and the SNAP/SPAN ontologies of the dynamic world, and classifies the variety of informatic structures according to these frameworks. A major challenge is transitioning between paradigms. Usually data is captured with respect to one paradigm but can usefully be represented in another. We discuss this process in formal terms and then describe experiments that we performed to show feasibility. It emerges that observational granularity plays a crucial role in these transitions.

Keywords: movement; field; object; Eulerian; Lagrangian; granularity

1. Introduction

Movement is a subcategory of change, in which physical entities change their locations in space. However, information about movement is captured and represented in many different ways. Some differences in these representations may be superficial; others reflect fundamental ontological differences in approach. As ever increasing amounts of movement data are generated by an increasing variety of sources, understanding these fundamental structurings of information about movement is important to coordinate progress in movement analysis.

As with any geographic information, information about movement may be structured within the frameworks of field and object models. Similarly, the Eulerian and Lagrangian models, originating from studies in fluid dynamics, are also widely used (often implicitly) as underlying structures for dynamic geographic information. All the above have connections to the influential SNAP and SPAN ontologies that "provide a treatment of dynamic features of what exists in space and in spacetime" (Grenon and Smith 2004). Our aim in this paper is to provide an underlying formal structure for analysis of the different types of movement information, in the context of their connections to these established frameworks, and to consider issues involved in moving between frameworks. Examples of the questions arising are, "For a given domain, what information is lost in moving from the Lagrangian to the Eulerian perspective?" and "How are the object and Lagrangian perspectives related?" The formal analysis developed in this paper allows such questions at least to be posed precisely, if not always to be precisely answered. The formal analysis

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is complemented with some experiments to test our approach and highlight issues that may have remained hidden. One of the emerging results of this work is to highlight the critical role of information granularity in allowing transformation of information between perspectives.

Information about movement is collected according to one or more of the frameworks described in this paper. It is important to not only understand the framework that applies to a particular data collection, but also to be able to transform the information between frameworks. This is not always possible, and this paper contributes to our understanding of the role of granularity in these transformations. To see why such transformations may be important, consider the following:

- **Eulerian to Lagrangian**: In this example, imagine an art gallery or shopping mall wishing to position signage to facilitate movement through the space. The technique used is to determine popular routes that people choose, and ensure that these are well signed. It may be infeasible in practice to track individual movements but quite possible to count flows from one sub-area to another. The challenge here is to transform this flow data in the Eulerian framework to Lagrangian trajectory data.
- **Lagrangian to Eulerian**: In this second example, imagine that on designing a new road layout, car drivers have been polled about routes they are likely to take (Lagrangian framework). It would be important to be able to transform this data so as to have an understanding of hot-spots (Eulerian framework).

There are many cases where such transformations are required, and this paper seeks to understand the mechanisms involved by firstly formally describing the frameworks , and then considering transitions between them.

The paper is structured from the general to the specific. The paper begins first by reviewing the types of movement that arose and their classification. Section 3 introduces the formal foundations that our study is based on. Section 4 describes and formalises movement from the object-field and Eulerian-Lagrangian perspectives, leading in particular to a comparison of the different perspectives. Section 5 places this discussion in the specific context of movement in networks. Section 6 looks at two specific examples of transformations of data about movement in a network: one Lagrangian to Eulerian, the other Eulerian to Lagrangian. The discussion is supported by empirical results from agent-based simulations that demonstrate the critical role of granularity in controlling the accuracy of these transformations. Finally, Section 7 concludes the paper with a discussion of future work.

2. Background

Previous work has investigated the classification of different types of movement that arose. Dodge *et al.* (2008) classified movement patterns as generic (primitive and compound) and behavioural. Generic patterns were further classified as spatial, spatio-temporal, and temporal. This classification has contributed to the development of data mining and visualisation of movement. Andrienko *et al.* (2011) proposed a conceptual framework for movement with atomic spatial, temporal, and object components. The paper mainly focused on an object perspective but their analysis also covered properties of locations (spatial and temporal). There was no explicit discussion of the Eulerian-Lagrangian dichotomy in this paper, nor of dynamic objects *vs.* fields. The paper also discussed several approaches to the transformation of movement information, including trajectory interpolation, division, and resampling.

As argued above, several different perspectives on geo-information are relevant here, including field/object, Eulerian/Lagrangian, and SNAP/SPAN dichotomies. The field-object dichotomy is amongst the most well-established conceptual distinctions made in connection with geographic information (cf. Egenhofer and Frank 1992, Goodchild 1989, Worboys *et al.* 1990, Peuquet 1984, Couclelis 1992, 1982). In short, the field-based model concerns collections of spatial (or spatio-temporal) distributions of phenomena, while the object-based model represents the space (or space-time) as populated by discrete, identifiable objects (Worboys and Duckham 2004).

In one of the most influential attempts to address the need to model dynamic spatial entities, Smith and collaborators developed the dynamic spatial SNAP and SPAN ontologies (Smith and Grenon 2004, Grenon and Smith 2004). SNAP ontologies recognise essentially spatial entities, which have continuous existence in time and preserve their identities through time. SPAN ontologies recognise essentially temporal entities, which unfold themselves through time and exist only in their successive phases. Another relevant approach to modelling change over time includes Galton (2004), who presented different ways to extend field- and object-based approaches to fully four-dimensional *hyperobjects* whose positions were specified as "chunks" of space-time (Galton 2004, p. 19).

Another pair of approaches to deal specifically with movement can be found in classical fluid mechanics. Eulerian and Lagrangian mathematical representations have been used to describe the motion of liquids and gases. In short, a Lagrangian approach to movement focuses on information that is referenced to entities, while the Eulerian approach focuses on information that is referenced to locations (Laube 2014).

All of these different concepts are evident in current research in movement. A commonly encountered representation of movement is as a *trajectory*: a discrete sample of time-space positions of an entity (Buchin *et al.* 2011). The trajectory adopts an essentially object-based, Lagrangian perspective on movement, with its focus on entities and their varying locations. In fact, this perspective underlies the overwhelming majority of research on movement within GI science (see, for example, Gudmundsson *et al.* 2008, Soleymani *et al.* 2014, Laube *et al.* 2005, 2008, Wolfson *et al.* 1998, Van der Weghe *et al.* 2006).

By contrast, others (most frequently outside of GI science) often adopted other perspectives more suited to their data and analyses. Chertock *et al.* (2014), for example, described pedestrian flow using a fundamentally Eulerian, field-based approach. At the finest level of granularity, micro-level pedestrian movement was represented as a succession of chess-like moves through a fixed spatial framework. Coarser-grained macro-level flows were captured with partial differential equations. Other approaches to movement in transportation science frequently adopted an Eulerian, field-based perspective (cf. Bernot *et al.* 2009) with the movement data collected from region-based traffic sensors (Xia and Li 2013, Xia *et al.* 2014). Brillinger (2007) applied the Lagrangian, object-based approach to the flow of play in soccer. In a military context, a Lagrangian-based approach was used for tracking missiles at real-time scale (Wells 1981). Within the GI domain, the 'checkpoint' view, where the times at which movers pass fixed observation sites in a transportation network were recorded, is akin to the Eulerian view (Both *et al.* 2013a).

In movement ecology, the Eulerian approach remains the major approach to study fine-scale movement, for example the externally vectored transport of micro-organisms (Nathan *et al.* 2008, para. 1, p. 19053). In contrast, the Lagrangian approach usually quantifies the movement of individual macro-organisms over larger spatio-temporal scales (Nathan *et al.* 2008, Turchin 1998, Benhamou 2004). A mixed Eulerian-Lagrangian approach has been proposed by Adioui *et al.* (2003) for the alignment of fish schooling behaviour.

In what follows, we investigate more systematically our classification of the types of information about movement that exist, starting with a recapitulation of the fundamental structures underlying spatial information in the following section.

3. Formal Foundations

In Goodchild *et al.* (1999) a *geo-atom* is posited as the fundamental element of geographic information. A geo-atom, in the form defined in Goodchild *et al.* (2007) is a triple $\langle x, V, v(x)\rangle$, where x is a geographic location in space-time, V is the domain of values of the property being measured at specified locations, and $v(x)$ is the value of the property at location x. If we fix the space-time domain to be X and the property type to be V, then the collection of all such atoms will be a subset of $X \times V$, in other words a relation between X and V. We term such a collection an $\langle X, V\rangle$-*collection*.

The space-time domain X may be purely spatial, in the static case, or spatio-temporal in general. The spatial domain might consist of a traditional Cartesian space in two or three dimensions, a road network modelled by a graph, a space of qualitative spatial relationships, or any number of other structures. The temporal domain, which is a component of the spatio-temporal domain, could be a structure made up of time instants or intervals, or a temporal representation showing qualitative relations between temporal intervals (e.g., Allen 1983). A spatio-temporal domain is a mix of spatial and temporal dimensions, and the nature of this mix is discussed later.

An important constraint on the relation between X and V is that the value measured at any location, given above by $v(x)$, is unique. That is, the relation is *functional*. The justification for this constraint is that the properties and characteristics of a spatio-temporal location are uniquely determined by that location. Thus, if we fix the property to be v, then the valuation at spatio-temporal location x is single-valued and uniquely determined by x. For example, suppose that V is a temperature domain. Then the temperature at each spatio-temporal location takes a single, unique value. In those cases where it appears that there are multiple values of one property type at a specific location, then the reality is that the valuation is vectorial and the multiple values are just the components of the valuation vector. For example, wind velocity is a vector composed of the two independent scalar attributes, speed and direction.

We are ready to state this fundamental fact as a principle: Each relation between X and V that expresses a $\langle X, V\rangle$-*collection* of geo-atoms is functional. In other words, an $\langle X, V\rangle$-collection of atoms is a member of the function space $X \to V$.

We should note that this principle, when expressed formally, expresses more information about the valuations. Because, functions are assumed to be total, then every point of the spatio-temporal framework X is assigned a value and no points are left unassigned. Of course, this can be generalised to the partial function case. We can also note because the value domain can consist of complex entities, such as vectors or multi-sets, it is possible to define quite general fields. For example, X might be a set of regions and elements of V might be multi-sets of temperatures, allowing a region to have an associated range of temperatures. However, this takes us some way from Goodchild's original conception of a geo-atom.

4. Movement from Different Perspectives

In this section, we develop the framework above in the context of movement, in order to elucidate distinctions between different perspectives on information about movement. In particular we examine distinctions between field–object, and Eulerian–Lagrangian dichotomies, and find the relation between them. (It is not quite the case that Eulerian is field and Lagrangian is object). Extending the above notation, let S, T, V, and E be the spatial, temporal, value, and entity domains, respectively.

4.1. *Fields and Objects*

A *field* in this context is an assignment of values to locations. For our first example, we might associate with each location, say, a location of a room in a building, the number of people in that room. To give another example, let the locations be doorways between rooms in a building, and the field associates with each doorway a vector of flows into and out of the rooms. Formally, field F is a function, $F : X \to V$, where X is a set of locations and V is a set of values of properties measured at those locations. This is essentially the approach discussed in the previous section.

In the static, atemporal case, X is a set of spatial locations S, often called the *spatial framework* for the field, and $F : S \to V$. In the dynamic case, X becomes a *spatio-temporal framework* with spatial and temporal components. Formally, $X = S \times T$, where S is some purely spatial domain and T is a purely temporal domain. In this case, $F : S \times T \to V$. In the first example above, F might assign the number of people in a particular room at a particular time, for rooms in S and times in T, and similarly for the second example regarding flows.

In what follows we will use the process of *currying* (Abelson *et al.* 1996) to convert functions with more than one argument into sequences of functions, each with a single argument. Currying the function of a cross-product gives us two cases to consider: $F_1 : S \to (T \to V)$ and $F_2 : T \to (S \to V)$. In the first case, F_1 assigns to each spatial location a time series of values. (Each room has an associated time series of occupancy numbers, and each doorway has a time series of flows). We can term this perspective a *spatial checkpoints* approach. In the second case, F_2 assigns to each time a spatial field of values. Thus we have a time-series of spatial fields. In our examples, for each time, we have a known distribution of numbers of people in and flows between rooms. We term F_2 a *temporal checkpoints* approach.

In contrast to fields, which are assignments of values to locations, the object approach models a dynamic application as a collection of static and dynamic entities. Returning to our earlier example of movement in a building, typical objects would be the moving people (dynamic objects) and the elements of the spaces through with they move, such as rooms, corridors, and doorways (mostly static objects).

The *object approach* turns out to be close to taking inverses of the above functions. Rather than going from location to value, we go from values (or, more accurately, aggregations of values that we can consider as wholes with unique identifiers) to their locations and other properties. In the usual way, we call such uniquely identifiable aggregations *objects*, and label the domain of such entities as E. We need to introduce another value domain, W, here. The reason that we have two value domains V and W is that V is a set of values at locations while W is a set of attributes of entities. The distinction may be subtle, but it is important in what follows.

In the classic static case, each such object is assigned a value of an attribute and its spatial location. Formally, we have a function: $G : E \to (W \times S)$

In the dynamic case, each object is assigned a spatio-temporal aggregation of values, that is a subset of triples (s, t, w) where $s \in S, t \in T, w \in W$. This subset can represent any kind of dynamic activity – growth, movement, change of shape, etc. From a global functional perspective, we have a function $G : E \to \wp(S \times T \times W)$, where $\wp$ indicates the powerset operation. The collection of entities under consideration in the dynamic case may be divided into *continuants*, those "entities that have continuous existence and a capacity to endure through time even while undergoing different sorts of changes" (Grenon and Smith 2004, p.139), and *occurrents*, events and changes. Examples of continuants are cities, pedestrians, and vehicles, while examples of occurrents are city growth, pedestrian journeys and vehicle collisions. Grenon and Smith expressed this distinction in their SNAP/SPAN ontology pair, SNAP being temporal development of continuants and SPAN being collections of occurrents. An example of an occurrent being modelled in this way would be a journey being represented as a trajectory, where the trajectory function could either be represented analytically all of a piece, or discretised into a collection of linear sub-trajectories.

The problem with the definition of a dynamic object as an assignment of arbitrary structures (or indeed no structure of space-time-value triples) is that it is too general for the construction of efficient implementations. This leads us to the Eulerian-Lagrangian distinction that is the topic of the next section.

4.2. *Eulerian and Lagrangian Perspectives*

Consider the motion of water on a lake or river, or of a group of people in a busy shopping mall. The moving elements might be parcels of water, or individual people. There are two quite distinct approaches that we can take to model such motion.

In the Eulerian approach, named after the Swiss mathematician Leonhard Euler (1707–1783), one imagines being stationed at a fixed location and observing properties of the motion at that location. Typical quantities that might be of interest include: flow — the amount of water passing through the location per unit time, and velocity — its speed and direction. One then makes these observations at a sample of locations of interest, and from them attempts to construct a global model of the movement.

In the Lagrangian approach, named after the Italian mathematician Joseph-Louis Lagrange (1736–1813), one imagines being a part of the motion (in a boat on the river, or one of the pedestrians in the mall) and observing properties of that individual's motion. The aggregate property of an individual's motion is the trajectory, and this is usually the property of interest. One then makes these observations for a sample of individuals, and as before extrapolates to a global model.

The next step is to more formally define Eulerian and Lagrangian approaches.

4.2.1. The Eulerian perspective

In the Eulerian view, we assume given a location $s \in S$, and a time $t \in T$, and we observe and measure some value $v \in V$ associated with the motion. As above, typical values are a count of entities in a motion of discrete elements, flow, and velocity. We should point out that elements of V are not constrained to be atomic (e.g. single numbers or strings) but can be vectors with multiple components or sets themselves (such as collections of entity identifiers). Formally, the Eulerian perspective is the functional space $(S \times T) \to V$.

As with the field case we can curry the function of a cross-product to give two cases:

$F_1 : S \to (T \to V)$ (*spatial checkpoints*) and $F_2 : T \to (S \to V)$ (*temporal checkpoints*). We can see that this is identical to the field paradigm.

4.2.2. The Lagrangian perspective

In this view, we assume given an entity $e \in E$ that we observe during its motion. At each time $t \in T$, we observe and measure an attribute $\mathbf{Attrib} = \mathbf{Attrib}(e, t)$ of the entity e at time t. In formal terms $\mathbf{Attrib} : (E \times T) \to (W \times S)$, where function **Attrib** has domain the product of entity and temporal domains E and T, respectively, and codomain a product of spatial and non-spatial domains S and W, respectively, where W is the set of values of non-spatial attributes of entities. Examples of attribute types that might have values in W include velocity as well as non-spatial properties of entities. A global model of the motion is obtained by sampling an appropriate collection of entities in E.

The two curried functions in this case are as follows.

Trajectories. Here we keep the entity fixed and allow the time to vary. Formally, $\mathbf{Attrib}_1 : E \to (T \to (W \times S))$. The function space $T \to (W \times S)$ is an association of a unique value of the attribute being observed with the time of observation, and $\mathbf{Attrib}_1 : E \to (T \to (W \times S))$ associates the result with the parent entity. So in this model, each entity has an associated *trajectory*. In the case where there is no non-spatial attribute to consider, the trajectory of an entity is a collection of ordered pairs (t, s) where $t \in T, s \in S$. This is the usual definition of a trajectory. Note that our definition is more general, in that the measurements can include other attributes beyond location, for example, velocity, tiredness (in case of pedestrians). We can term these more general trajectories *augmented trajectories*, to indicate they are more than purely spatio-temporal traces.

Dynamic inventories. Here we keep the time fixed and allow the entity to vary. Formally, $\mathbf{Attrib}_2 : T \to (E \to (W \times S))$. The function space $E \to (W \times S)$ is an association of a unique value of the attribute being observed with its parent entity, and $\mathbf{Attrib}_2 : T \to (E \to (W \times S))$ pins the observation to a specific time. So in this model, there is at each time a snapshot of entities and their attribute values. We call this model a *dynamic inventory*.

4.3. *Rapprochement between perspectives*

4.3.1. Comparisons

We have seen that both object/field and Eulerian/Lagrangian provide dichotomies of approaches that are reflected in their formal descriptions. To assist the reader, we have summarised the formal descriptions in Tables 1 and 2, giving the static and dynamic cases, respectively.

Based on the above descriptions, we now note similarities and dissimilarities between different models, and then demonstrate how, from a formal perspective, these models are in a sense inverses to each other. Table 1 shows the classic field and object models in the atemporal case, and Table 2 lists the four major dynamic models, called spatial checkpoint, temporal checkpoint, trajectory and dynamic inventory, respectively.

An examination of Table 2 allows juxtaposition of field/object and Eule-

Table 1. Summary of the Static Cases

Formal term	Perspective	Description
$S \to V$	Field	Timeless field view
$E \to (S \times W)$	Object	Timeless object view

Table 2. Summary of the Dynamic Cases

Short name	Formal term	Perspective	Description
"Spatial checkpoint"	$S \to (T \to V)$	Field, Eulerian	Spatially distributed time series, spatial checkpoints
"Temporal checkpoint"	$T \to (S \to V)$	Field, Eulerian	Dynamic spatial pattern, temporal checkpoints
"Trajectory"	$E \to (T \to (S \times W)$	Object, Lagrangian	Entity trajectories
"Dynamic inventory"	$T \to (E \to (S \times W))$	Field-Object, Lagrangian	Dynamic entity pattern, temporal checkpoints

rian/Lagrangian perspectives. In the dynamic case, the two Eulerian views, $S \to (T \to V)$ and $T \to (S \to V)$, coincide exactly with the corresponding field views. The Lagrangian viewpoint, $E \to (T \to (S \times W)$ in the third row associates trajectories as complex attributes of entities, and therefore sits squarely in the object camp. The final Lagrangian entry, $T \to (E \to (S \times W))$, is interesting because it has both field and object features. It can be considered as a field, because it is structured as a function from a temporal framework, but its codomain is the static object structure $E \to (S \times W)$, and so it also has object characteristics.

Having related field/object to Eulerian/Lagrangian paradigms, the remaining step is to link this with the SNAP/SPAN dichotomy of real world dynamic entities into sequences of temporal snapshots of continuants (SNAP) or temporally continuous occurrents (SPAN). In the Lagrangian case, the procedure is to follow entities as they evolve through time. In this case it is more appropriate to take these entities as continuants, and the mapping from each entity to a member of the function space $E \to (T \to (S \times W)$ provides for the entities' dynamic behaviours. The entity as it evolves through time is a SNAP entity, however its evolution is SPAN. For example, as a vehicle moves along a road, the vehicle is a dynamic continuant (SNAP) while the movement, captured for example by its trajectory, is a SPAN entity. In the Eulerian case, consider the example of the movement of a crowd of people. The Eulerian perspective $S \to (T \to V)$ represents the movement (SPAN) as a spatially parametrised collection of time series. Each time series might be measuring the temporal variation of the number of people at a location (a collection of SNAP entities). A similar analysis applies in the other Eulerian case. In both cases we see a mix of SNAP and SPAN entities being represented. The conclusion is that there is no easy mapping between Eulerian/Lagrangian to SNAP/SPAN.

4.3.2. Transformations

We now consider how we might move between perspectives. We will provide the general arguments here and provide examples later in the paper. In the static case, it is well known that it is usually straightforward to move from object to field, but often difficult to move from field to object. Consider, for example, image recognition, where the problem is to identify objects within a raster image (field). Usually, more semantic information is required to be successful. The same kinds of arguments extend to the dynamic cases. We consider both intra-relationships between the two Eulerian cases as well as the two Lagrangian cases, as well as inter-relationships between Eulerian and Lagrangian.

Regarding intra-relationships, moving between the two Eulerian cases, $S \to (T \to V)$ and $T \to (S \to V)$ is essentially a matter of transposing the $S \times T$ matrix. Similarly, in the two Lagrangian cases, it is usually a matter of restructuring the information, this time transposing at the $E \times T$ matrix, where each member of E is uniquely identified by its object identifier.

As for inter-relationships, as with the object to field transition, it is usually straightforward to convert from Lagrangian to Eulerian perspectives. Consider the case where we have data about trajectories of vehicles. Provided we have complete and precise information, the data can be converted without loss to an Eulerian count of numbers of vehicles in given locations at given times. In the more difficult case of moving from Eulerian to Lagrangian, the stumbling block is that from broad aggregate measures it is not in practice possible to retrieve individuals. Consider the problem of moving from counts of vehicles at locations through time to trajectories of specific vehicles. Information is almost certain to be lost. However, we shall see later that with finer precision in the Eulerian data, we can make more precise estimates about the Lagrangian structure.

5. Movement in networks

Many applications of movement can be constrained as being spatially restricted to movement in a network. That is, the spatial framework is a graph. In this section we look at some of the extra constraints that a graph framework imposes. The graphs may be directed or undirected and possibly allow loops. In some of the paradigms above, the graph may itself vary with time, either in its nodal structure or in connections between nodes.

To take an example, consider the movement of a collection of individuals constrained by a network. This would be the case with vehicles moving through a road network. We can model the network as a graph $G = \langle N, L \rangle$, of nodes, N, and links (edges), L. The four main perspectives described above specialise as follows (illustrated in Figure 1):

Temporal checkpoints. Associated with each moment in time, the nodes and/or edges of the network are assigned values that provide a snapshot of the movement in the network at the time. These values might be numbers or flow rates of individuals situated at each location. They may also be collections of entity identifiers. (As stated above, the graph itself might evolve through time.) By way of illustration, Figure 1(d) shows counts of moving entities at different places on a graph at a particular time t_3.

Spatial checkpoints. Each location in the network, represented by either a node or edge of the graph, is assigned some time series of values that characterises this movement through time. This value might be a time series of numbers of individuals occupying that node or edge, or a time series of flow rates at that location. For example, Figure 1(c) shows the temporal variation from time t_1 to t_{11} of counts of moving entities at a specific location (edge or node).

Trajectories. Each movement of an individual entity might be modelled as a trajectory through the graph, where a trajectory is an assignment of a unique location and other attributes of the individual to each time within the timespan of that individual's movement. Figure 1(a) illustrates the trajectory of a specific moving entity as a sequence of locations (green stars) on the graph from time t_1 to t_8.

Dynamic inventories. At each moment in time, we have information about each entity at that moment. This information might be its location on a node or edge of the graph, along with other attributes of the entity. Figure 1(b) illustrates the dynamic inventory view of the movement so that at time t_3 we know the locations of all the 17 moving entities (green stars) on the graph.

In all these cases, there may be possible constraints on movement through the graph. We give some examples based on movement of people through the rooms of a building. The directed graph models the building, where nodes represent rooms, and a directed edge connects two rooms if there is a doorway that enables movement from one room to the other. We can associate flows of pedestrians by assigning values to the edges.

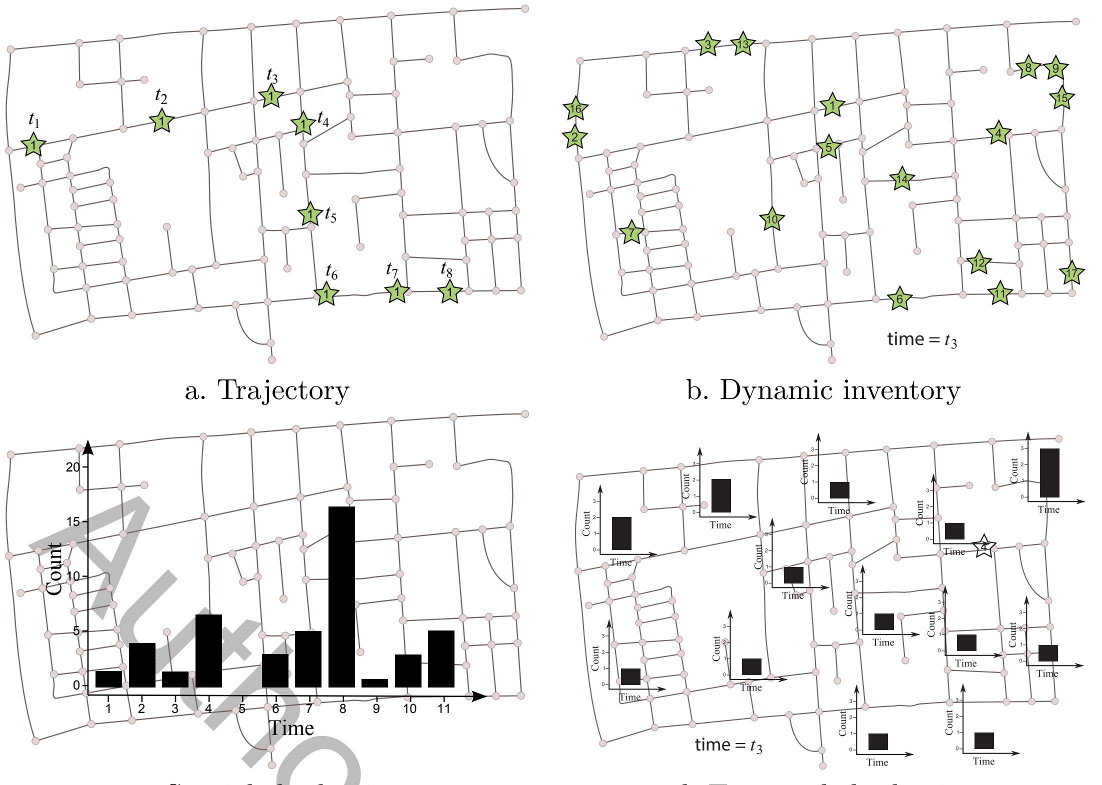
- **Conservation**: In our example, it probably makes sense to assume that no pedestrians are gained or lost in movements between internal rooms. This translates to a constraint that at each time, the directed flows at an internal node sum to zero.
- **Capacity**: We might constrain the model to ensure that the time-varying flows are such that predefined capacities of rooms are adhered to.
- **Speed**: We might constrain the model so that it is not possible for an individual to exceed a given speed, maybe expressed in terms of the number of edges that can be traversed per unit time.

6. Transformations

This section examines in more detail the transformation of data captured within different movement perspectives. As discussed in Section 4.3.2, transformations between different Lagrangian, or between different Eulerian perspectives, are expected to be less challenging than transformations from Lagrangian to Eulerian or vice versa (with Eulerian to Lagrangian typically the more challenging). Thus, our discussion focuses on two specific example transformations: the first Lagrangian to Eulerian; the second Eulerian to Lagrangian. In both cases we take a network as our spatial framework, modelled as a graph $G = \langle N, L \rangle$.

The spatial and temporal granularity of movement data are essential characteristics that fundamentally affect the accuracy of transformations applied to that data. Thus, the analysis in this section focuses primarily on how the accuracy of transformation degrades with coarsening granularity. We also show, however, that it is still possible to achieve perfect transformations given fine enough levels of granularity.

In practical applications, data is usually captured in only one movement perspective, making comparison between perspectives impossible. Consequently, we test our transformations using data generated from simulations, which yield comparable data from



a. Trajectory b. Dynamic inventory

c. Spatial checkpoint d. Temporal checkpoint

Figure 1. Illustrative examples of the four movement perspectives in the context of movement in a graph: a. trajectory, b. dynamic inventory, c. spatial checkpoint, and d. temporal checkpoint.

multiple perspectives.

Following a brief discussion of the spatial and temporal granularity of the movement data (Section 6.1), the simulation environment is described in Section 6.2. Subsequently, the two example transformations are defined and evaluated, in Sections 6.3 and 6.4.

6.1. *Granularity*

The relative role played by spatial versus temporal granularity depends on the functional structure of the movement perspective under consideration. In functions that have space as part of the domain and time as part of the codomain (mapping from space to time, i.e., spatial checkpoints) the spatial granularity of the spatial framework is of overriding importance. We assume for simplicity that given a particular location, time can be measured accurately and precisely. For example, in the case of spatial checkpoints, the spatial density and locations of the checkpoints are of overriding importance to the granularity; it may be assumed that at each checkpoint the times at which values are observed can be accurately and precisely recorded.

In the remaining perspectives (those that map from time to space), however, the temporal granularity plays a starring role. In these cases it is necessary to a priori specify a *frequency* for fixes or snapshots. Trajectories with fixes every second, for example, capture very different information about movement when compared with trajectories with fixes every hour (cf. Laube and Purves 2011). One might argue, by symmetry with the discussion above, that we should then be able to precisely and accurately measure spatial location given a particular time. However, in practice that is rarely the case. The limitations of localisation technologies as well as the inherent scales of movement in geographic space are such that it is often not possible to locate moving objects precisely enough in space to unambiguously describe geographic movement.

6.2. *Simulation environment*

A multi-agent simulation was developed in NetLogo to enable data at different granularities and from different movement perspectives to be generated. The underlying space in which agents moved was a raster grid of 33×33 cells. Mobile agents were able to move freely around this space subject to the constraint that *no two agents were able to occupy the same grid cell at the same time*. The aim was to model at the finest granularity the essentially functional nature of space: that no two objects can occupy exactly the same space. Thus, the spatial environment for the simulations can be thought of as the discretisation of a large room in which agents are able to move about, but not "bump" into each other (if we imagine the room as filled with people, it would need to be just big enough to fit about 1000 standing people in).

Modelling geographic space as a graph always presents two options, depending on whether the spatial "containers" of objects are represented by edges or by nodes. The first case is intuitively akin to a road network, where edges in the graph represent the "containers" of moving objects (e.g., roads) and nodes capture the connections between these edges (i.e., junctions). The alternative is more closely akin to a raster, where nodes represent the "containers" of objects (e.g., the cells of the raster) and edges represent the connectivity between nodes (i.e., adjacency between cells). In our simulations we elected to adopt the latter perspective—nodes represent the raster cells that contain moving objects—for reasons of conceptual simplicity. However, it would equally have been possible, if slightly more confusing, to adopt the alternative perspective (i.e., where raster cells are treated as conduits between cell boundaries). Thus, the spatial framework for our simulations was a grid-based static, undirected graph G with 1089 nodes (33×33) and edges connecting rooks-case neighbours.

There already exists a wide variety of different movement models tailored to specific application domains, such as pedestrian, traffic, and animal movements. However, for simplicity, in our simulations we adopt a classic, abstract movement model from artificial intelligence, based on Braitenberg vehicles (Braitenberg 1986). Each mobile agent ("Braitenberg vehicle") in our simulation senses the immediate environment in front of it and to each side. Vehicles are attracted to empty space (moving towards it) but repelled by occupied space (turning away from it towards unoccupied space, slowing down if surrounded, and even stopping altogether if necessary to avoid a collision). These simple rules lead to a wide diversity of movement patterns, with agents exploring their environment and interacting with other agents to maintain distances, with no collisions. As long as a moderate number of vehicles is chosen (in our case, less than about 300), movement is relatively free, with no chance of gridlock. For more information on Braitenberg vehicles moving in space, see Braitenberg (1986), Both *et al.* (2013b).

Data about the actual movement of the vehicles can then be generated by the simulation in formats reflecting the different movement perspectives as well as different spatial and temporal granularities, including:

- Trajectory data such that $E \rightarrow (T' \rightarrow N)$ where $T' \subseteq T$ is a subset of discrete times reflecting the temporal frequency of trajectory fixes (such as, for example, timesteps $T' = \{2, 5, 8, 11, 14, ...\}$). Trajectory data is output in the form of a database table, with the relation scheme TRAJECTORY($\underline{T'}, \underline{E}, N$) (where T' and E used together to form the compound primary key, although any pair of attributes could be combined to give a primary key).

- Spatial checkpoint data such that $L' \to (T \to \wp E)$, where $L' \subseteq N' \times N'$ and N' is a *partition* of N. In this instance, *subsets* of nodes in N' form granular *zones*, in which the underlying nodes in each grain are indiscernible. Pairs of zones in L' indicate the movement from one zone to another (e.g., $(\{1,2\},\{3,4\}) \in L'$ indicates a crossing of the boundary from the zone made up of nodes $1, 2 \in N$ to the zone comprising nodes $3, 4 \in N$). The codomain of the function is the identities of objects in a zone (a subset of E). Spatial checkpoint data is output as a database table with the relation scheme SPATIAL_CHECKPOINT($\underline{T}, \underline{E}, L'$) (with T and E forming the primary key, and L' comprising the identifiers of the zones from which and to which a movement occurred).
- Temporal checkpoint data such that $T' \to (N \to V)$, where $c \in V$ is the count of vehicles in a node $n \in N$ aggregated over the consecutive timesteps, t_{i-1}–t_i. Temporal checkpoint data is generated as a database table with the relation scheme TEMPORAL_CHECKPOINT($\underline{T'}, V, \underline{N}$) (with T and N forming the primary key).

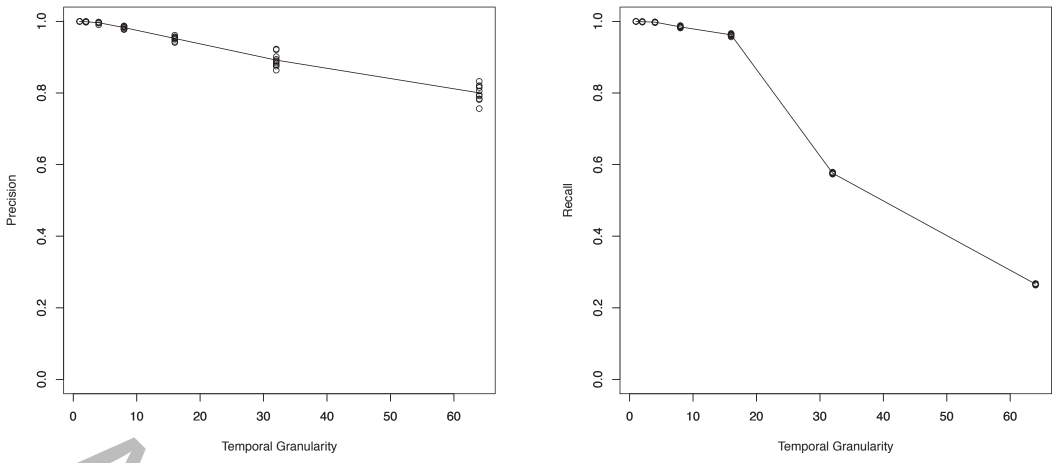
The simulation is entirely configurable as to the specific number of vehicles, temporal granularity of trajectories or temporal checkpoints, and spatial granularity of zones (i.e., the partition of the nodes).

6.3. *Lagrangian to Eulerian transformation*

The transformation of trajectory data to spatial checkpoints is used as our Lagrangian to Eulerian example. As described above, our trajectory data is an assignment of the identifier for each moving entity at granular timesteps to a unique node in the graph, $E \to (T' \to N)$. The spatial checkpoint data is structured as an assignment of pairs of zones (i.e., boundaries between adjacent zones) to a time series of entities passing that checkpoint, $L' \to (T \to \wp E)$. Zones themselves are constructed from pairwise disjoint and possibly singleton subsets of nodes in the graph (i.e., a partition of the nodes in the graph). Our challenge is then to define a transformation of the raw trajectory table generated by the simulation into a table that as closely as possible matches the raw spatial checkpoint data table generated in parallel by the simulation.

Looking for consecutive trajectory fixes that lie in different zones can provide a very close approximation of the time at which the zone boundary was traversed. Formally, given two trajectory fixes of a moving agent a at (t_i, a, v_1), (t_{i+1}, a, v_2), assume that $[v_1] \neq [v_2]$, where $[v]$ is the equivalence class of node v (in the partition N'). If $[v_1]$ and $[v_2]$ are adjacent (i.e., there exists a $(v, v') \in L$ such that $v \in [v_1]$ and $v' \in [v_2]$) then we can reasonably assume that at some time between t_i and t_{i+1}, agent a first left a node in $[v_1]$ and arrived at a node in $[v_2]$. Given arbitrarily fine temporal granularity of trajectory data, t_i and t_{i+1} will be arbitrarily close to the true checkpoint timing.

As trajectory fixes become less frequent, however, not only will imprecision in timing increase, but multiple checkpoints may separate two consecutive trajectory fixes. Consequently, as temporal granularity coarsens, it is expected that for two consecutive trajectory fixes (t_i, a, v_1), (t_{i+1}, a, v_2) zones $[v_1]$ and $[v_2]$ may not be adjacent, and no $(v, v') \in L$ such that $v \in [v_1]$ and $v' \in [v_2]$ may exist (i.e., the agent has moved through multiple zones between trajectory fixes). In such cases, the transformation attempts to identify the most likely trajectory point of exit after t_i from zone $[v_1]$ and the most likely point of entry before t_{i+1} into zone $[v_2]$. We use a simple heuristic that assumes vehicles



a. Precision b. Recall

Figure 2. Precision and recall of trajectory to spatial checkpoint transformation with coarsening trajectory granularity. Straight lines connect average precision/recall for 10 repetitions (data points).

will have taken one of the shortest paths between known trajectory fixes, and included in the transformation those fixes close to the exit and entry points for the two zones along that path. In cases where multiple shortest paths of equal length exist (frequently in a grid), we add all possibilities to the transformed output.

6.3.1. Experimental results

As a result of our transformation, it is possible that the transformed trajectories may miss some checkpoints (i.e., where trajectory fixes are too sparse). It is further possible that transformed trajectories may include some incorrect checkpoints (i.e., where our assumption of shortest path between zones fails, or where multiple possibilities appear in the output). Figure 2 shows the precision and recall of the transformation from trajectory to spatial checkpoint for 10 randomised simulations, each with 200 vehicles run over 500 timesteps. The temporal granularity of the trajectory database was coarsened from fixes every timestep (the smallest simulation time unit) to fixes every 2, 4, 8, 16, 32, and 64 timesteps. The precision and recall were computed by comparing the “raw” spatial checkpoint database table (generated directly from the object movements with fine granularity) with the “output” spatial checkpoint database table (generated indirectly by transformation of the variable-granularity trajectory data). Precision captures the proportion of checkpoints found in the output that are also in the raw data (computed as the number of tuples in the output that are also in the raw data, divided by the number of tuples in the output). Recall captures the proportion of checkpoints in the raw data that are also in the output (computed as the number of tuples in the output that are also in the raw data, divided by the number of tuples in the raw data).

In addition, the temporal latency in the transformation output was computed (Figure 3). These results provide an indication as to how close in time are the estimated checkpoints (contained in the transformed trajectory data) when compared to the raw checkpoint data (generated directly from the simulation). Further discussion of these results is contained in the following subsection.

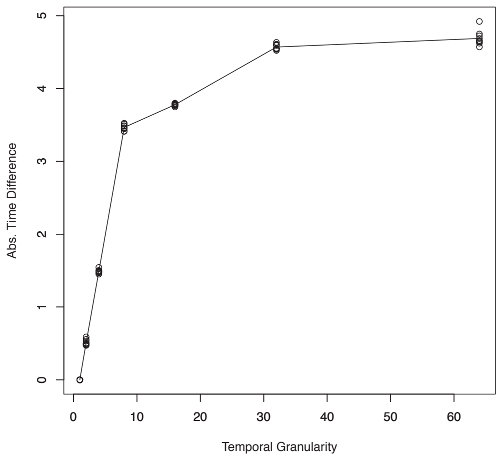


Figure 3. Increasing absolute time difference between raw and transformed checkpoints (latency) with coarsening trajectory granularity. Straight lines connect average time differences.

6.3.2. Discussion

A number of immediate observations about the results in Figures 2 and 3 are possible. First it should be noted that it is, as expected, possible to achieve perfect transformations with no latency in cases where the trajectory granularity is high enough (indicated by the perfect recall and precision, and zero latency at the finest possible temporal granularity). However, precision, recall, and latency all degrade as the temporal granularity of trajectories coarsens. Although this is also to be expected (since we have less information about the movement as temporal granularity coarsens), it is still worthy of note as in many applications we often intuitively feel that trajectories are somehow a more precise form of movement data than checkpoints. The figures highlight that this is not the case: as the temporal granularity of a trajectory coarsens it will inevitably become a less precise representation of movement when compared with a precise spatial checkpoint representation.

It is also clear that recall declines more precipitously than precision with increasing granularity. This is a consequence of our heuristic for identifying only checkpoints close to the boundaries of zones (but not for example along the entire shortest path between zones). As a result, we can have relatively high confidence that the heuristic will infer only valid checkpoints from the trajectory; but will fail to identify increasing numbers of intermediate checkpoints as temporal granularity coarsens.

Finally, it is noticeable that latency (Figure 3) increases at first rapidly and then asymptotically with coarsening temporal granularity. The asymptotic behaviour occurs because of the interaction between spatial and temporal granularity. Beyond a certain point it does not matter how infrequently temporal fixes occur, they will still occur at most a certain maximum distance from the boundary of a zone, even traveling at maximum speed. Thus, at the coarsest temporal granularity, the maximum latency is limited by the spatial size of the zones traversed by the agents.

6.4. *Eulerian to Lagrangian transformation*

Transformation from Eulerian to Lagrangian movement is similarly a process where granularity plays a key role. In this section we examine the example of a transformation from temporal checkpoint data to trajectory data. An especially challenging type of transformation concerns the generation of trajectory data from temporal checkpoint data structured as *counts* of moving objects at zones in the graph, $T' \to (N' \to \mathrm{N})$. Trajectory data is again described by the function $E \to (T' \to N)$, mapping the identifier for a moving entity over time to a unique node in the graph.

Identifying movement from the count data uses a simple heuristic that identifies any changes in counts at adjacent locations in the graph that can unambiguously be attributed to the movement of objects between those locations. For simplicity of explanation, we consider here the simpler case of a temporal checkpoint $T' \to (N \to \mathrm{N})$, i.e., where all locations are precise nodes in the graph. It is straightforward, if more laborious, to generalize this to the case where locations are zones (subsets of nodes) in the graph, $T' \to (N' \to \mathrm{N})$.

More specifically, consider two adjacent nodes in the graph $v_1, v_2 \in N$, where $(v_1, v_2), (v_2, v_1) \in L$, and the sets of neighbors of v_1 and v_2, written $nbr(v_1)$ or $nbr(v_2)$ where $nbr(v) = \{v' \in N | (v, v') \in L\}$. Consider further the following three conditions for two consecutive time steps $t_i, t_{i+1} \in T'$ and two neighboring locations v_1 and v_2:

(1) $count(t_i, v_1) > count(t_{i+1}, v_1)$, where $count(t, v)$ represents the number of objects at location $v \in N$ at time $t \in T'$ recorded in the temporal checkpoint data, i.e., $T' \to (N \to \mathrm{N})$
(2) $\exists v_2 \in nbr(v_1), count(t_{i+1}, v_2) - count(t_i, v_2) = count(t_i, v_1) - count(t_{i+1}, v_1)$
(3) $\forall v' \in (nbr(v_1) \cup nbr(v_2)) - \{v_1, v_2\}, count(t_i, v') = count(t_{i+1}, v')$

Condition 1 above expresses the situation that the count of objects at location v_1 has decreased by some amount c over two consecutive timesteps. Condition 2 expresses the situation that the count of objects at a neighbouring location v_2 has increased by the same amount c as the decrease at v_1. Finally, condition 3 above expresses the situation that none of the other neighbours of v_1 or v_2 have experienced any changes in counts over those consecutive timesteps. If all three conditions hold, it is not strictly *required* that c objects have moved from v_1 at time t_i to v_2 at time t_{i+1}. However, assuming arbitrarily fine temporal and spatial granularity, and conservation (objects do not appear or disappear, see Section 5), the movement of c objects from v_1 to v_2 is certainly the simplest explanation. Consequently, we adopt this heuristic reasoning as the basis for our example Eulerian to Lagrangian transformation (although the experiments in the following section also examine conditions under which this heuristic fails). Figure 4 illustrates this heuristic with a simplified example.

Note, however, even given this heuristic it remains impossible to infer correct object identities (E) from count data alone. Each instance of our inference above will only allow us to create c transformed trajectory tuples of form (t_i, v_1, e') and (t_{i+1}, v_2, e') where e' is some *pseudonym* identifier, linking only *pairs* of consecutive trajectory fixes, but not providing identities to associate longer trajectory sequences. Thus, if at some subsequent time t_{i+2} it happens that we infer c further objects move from v_2 to v_3, these objects may or may not be the same as those previously identified. Thus, by default we assign subsequent pairs of inferred trajectory fixes *different* pseudonym identifiers. In measuring the accuracy of transformation (precision and recall) we only compare transformed trajectories with raw trajectories up to relabelling of pairs of trajectory fixes (i.e., as long as each pair of trajectory fixes with the same pseudonym in the transformed

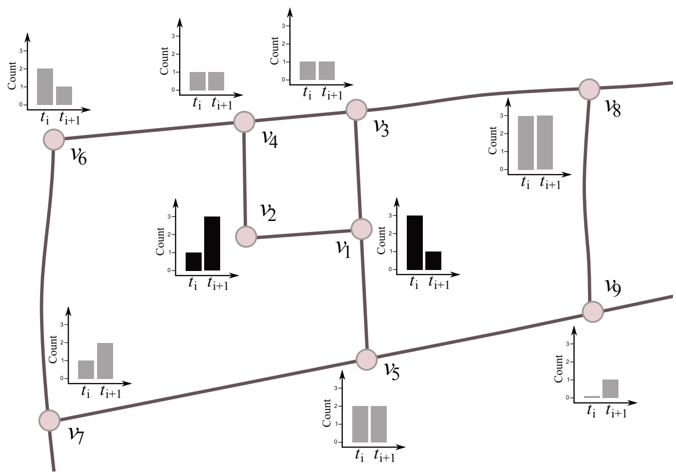


Figure 4. Illustrative example, satisfying the three conditions in Section 6.4 (over consecutive time steps: 1. decrease in counts at v_1; 2. matching increase of counts at v_2; 3. buffer of unchanging counts at other neighbors of v_1 and v_2), leading to the inference that two objects moved from v_1 at time t_i to v_2 at time t_{i+1}.

trajectory data matches a pair of trajectory fixes from *some* agent's trajectory, then that pair of fixes is counted as a correct match).

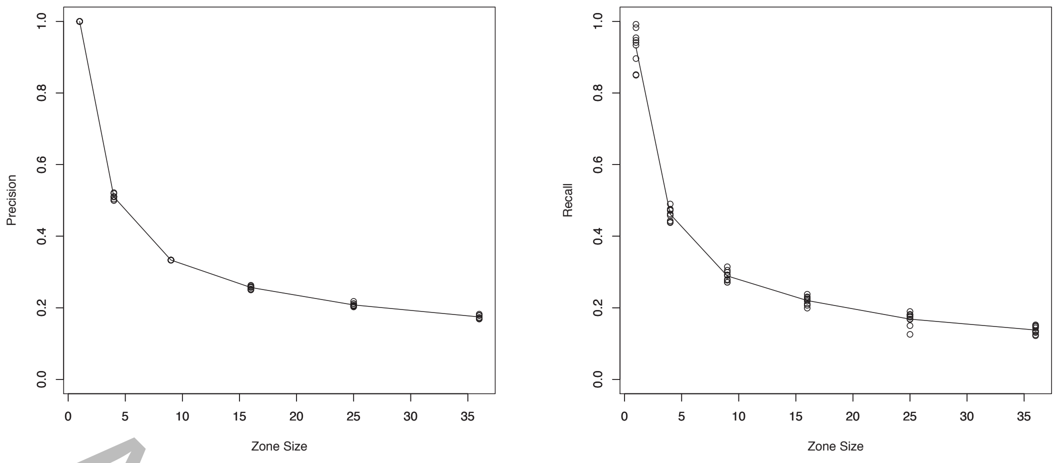
6.4.1. Experimental results

To illustrate the role of granularity on our Eulerian to Lagrangian transformation, we again executed randomised simulations over 500 timesteps, although this time with 50 vehicles in each simulation. In this second experiment the temporal granularity was kept fixed (i.e., it was assumed that the granularity of the temporal checkpoints was the same as the desired granularity of the transformed trajectory data). Instead, the spatial granularity of the zones was varied, with counts applied over larger and larger subsets of nodes in the graph.

Figure 5 shows the precision and recall of the count (temporal checkpoint) to trajectory transformation with varying spatial granularity, 1, 4, 9, 16, 25, and 36 nodes per zone. Again, the precision and recall were computed by comparing the "raw" trajectory table (generated directly from the object movements with fine granularity) with the "output" trajectory database table (generated indirectly by transformation of the counts in the temporal checkpoints).

It is immediately noticeable from Figure 5 that both the precision and recall are almost entirely dependent on the spatial granularity of the temporal checkpoint data. The precision decreases with spatial granularity, because larger spatial zones have a larger number of edges connecting them to neighbouring zones. Consequently, our transformation cannot be as precise about exactly which of these pairs of adjacent nodes to assign a inferred trajectory fix to. (As for the Lagrangian to Eulerian transformation above, our Eulerian to Lagrangian transformation assigns a fix to all possibilities in cases of ambiguity, thus lowering the precision). Similarly, larger zones contain larger numbers of internal nodes, in which trajectory movements of vehicles are unobserved, thus lowering the recall.

However, given these expected granularity effects, the transformation does perform moderately well. Figure 6 shows the recall from Figure 5 adjusted for these granularity effects, by multiplying the measured recall by the number of unobserved nodes in each zone at the different levels of spatial granularity. The result provides a better comparison



a. Precision b. Recall

Figure 5. Precision and recall of temporal checkpoint to trajectory transformation with coarsening spatial granularity. Straight lines connect average precision/recall for 10 repetitions (data points).

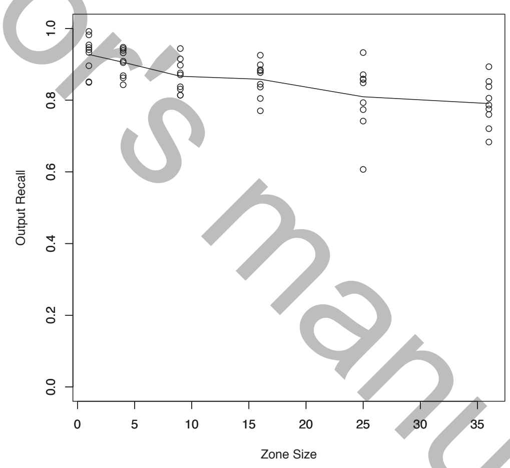


Figure 6. Recall of temporal checkpoint to trajectory transformation with coarsening spatial granularity, normalised for granularity effects. Straight lines connect average precision/recall for 10 repetitions (data points).

of the “real” recall, allowing for the expected effects of changing sizes of unobserved interiors of zones.

It is, however, also evident from Figure 6 that recall is still not perfect, even at the finest spatial granularity—exactly one node per zone. This effect can be attributed to adverse interactions between moving agents. Although unlikely, there are cases when multiple agents may engage in a complex patterns of movement that generate a count pattern that accords with the conditions set out in our simple heuristic above. Figure 7 shows the effect upon recall of increasing numbers of agents (1, 12, 25, 50, 100, 200, 400), based on randomised simulations at the finest granularity (each zone contains one node in the graph). Although the transformation does perform perfectly in the case of

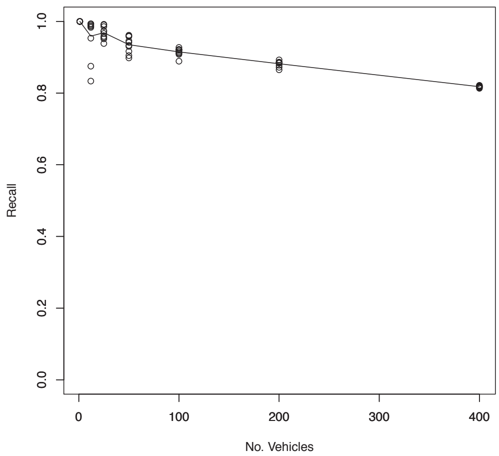
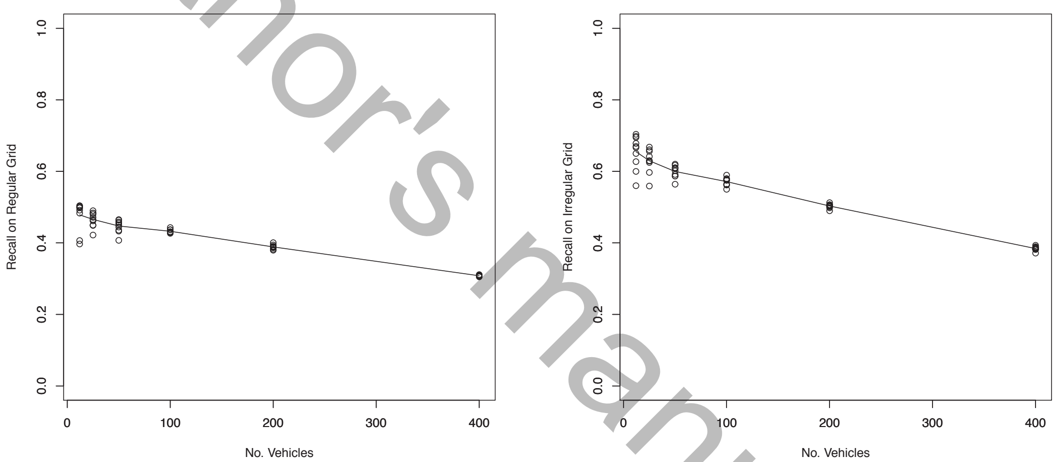


Figure 7. Recall of temporal checkpoint to trajectory transformation with increasing number of simulated agents. Straight lines connect average precision/recall for 10 repetitions (data points).



a. Regular b. Irregular

Figure 8. Recall for regular and irregular spatial granulation, for 50 agents moving through a graph with an average of four nodes per zone. Straight lines connect average precision/recall for 10 repetitions (data points).

only one vehicle, as the number of agents increases, the chance of adverse interactions steadily increases, with a concomitant decrease in precision.

Finally, the structure of the spatial granulation also has an influence over the accuracy of transformation. All the experiments described above used a regular grid-based spatial granulation. Figure 8 shows the results using a randomised granulation, where zones were constructed from dilation of a set of randomly selected seed points. While the average number of nodes per zone (4) is the same for both Figures 8a and 8b, the zones in 8b can vary between 1 and 12 nodes per zone. All other factors being equal, the results show a clear increase in recall with irregular spatial granulations (Figure 8b) when compared with regular spatial granulations (Figure 8a).

6.4.2. Discussion

The results provide an illustration that, provided fine enough granularity information is available, it is still possible to retrieve exact information about trajectories even from anonymised counts based on temporal checkpoints (Figure 5). In such cases, however, only pseudonymity rather than actual identity is retrievable. In addition to granularity, however, the level of interactions between agents provides a confounding factor. The more numerous and more complex interactions that arise from larger numbers of moving agents can lead to the assumptions behind count-based inferences, like the one formalised above, being violated. This in turn reduces transformation accuracy (Figure 7). Finally, the structure of the spatial granulation also has an impact upon transformation accuracy (Figure 8). Highly regular granulations can lead to lower levels of transformation accuracy. In our example transformation, the inhomogeneity of an irregular granulation provides a stronger basis for discerning apart movements, because small, precise neighbourhoods that facilitate stronger inferences are more likely to be in the immediate vicinity of any movements.

7. Discussion and Future Work

This paper presents work that draws on and extends the work of others, in particular Andrienko *et al.* (2011) and Laube (2014), to develop a framework for the modelling of information on movement. We have carefully expressed the field/object and Eulerian/Lagrangian perspectives in formal terms, and have demonstrated sometimes quite subtle distinctions between these two dichotomies. We have also considered how the SNAP/SPAN ontological framework fits into these perspectives.

The formal representations allows us to precisely determine what in general terms is required to move between field and object, and between Eulerian and Lagrangian, as well across these pairs. In other words, we have determined general intra- and inter-relationships. As the work has progressed, it has become clear to us that granularity plays a very important role in these transformations. In a perfect world, all is possible; but imprecise information about such factors as the spatio-temporal locations of entities or which entities are in which locations, leads to further information loss in transitions. The final part of the presentation here is to get a handle experimentally on this information loss in terms of precision (what true information we have lost) and recall (what false information we have gained). The results show clearly the kinds of relationships between change in granularity and precision/recall of information, in several different transition types. It is also noteworthy that the homogeneity of the granular structure also plays a role.

Directions that we are interested to pursue in the future include exploration of the practical implications of these findings. In fact, we have already begun a study of pedestrian movement in East London, and working with data about fish movements in the Murray River, Australia, both rich sources of granular, field-based spatio-temporal information about movement. There is also room for experiments on movement in multi-level indoor spaces. The experimental work so far has been on grid-like graphs, and a further direction is to extend to general graphs, and indeed to spaces where an underlying network spatial structure may not be appropriate.

In terms of extensions to our core framework, there are several directions that need pursuing. One area is provision of "mixed" models that incorporate elements of Eulerian and Lagrangian paradigms in the same model. A further important area to consider is the impact of group behaviour on our structures. Our work up to now assumes no dependencies between the individual movers, such as common or opposite motivations and goals. We intend to extend our framework to encompass a theory of collective movement.

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