Applications of Probability in Engineering

Note: Using pen and paper helps you learn better

Dr. Mehdi Baghdadi

Monday, March 14, 2016

While waiting for others to arrive:

Question: Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random.

What is the probability that the ticket drawn has a number which is **a multiple of 3 and 5**?

A. 1/2 B. 1/20 C. 8/15 D. 9/20

Time: 3 minutes, Difficulty Level: Average

Review:

Question: Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random.

What is the probability that the ticket drawn has a number which is **a multiple of 3 and 5**?

A. 1/2 B. 1/20 C. 8/15 D. 9/20

Time: 3 minutes, Difficulty Level: Average

Solution:

1	11
2	12
3	13
4	14
5	15
6	16
7	17
8	18
9	19
10	20

Solution:

1	11
2	12
3	13
4	14
5	15
6	16
7	17
8	18
9	19
10	20

Multiplication of 3

Solution:

1	11
2	12
3	13
4	14
5	15
6	16
7	17
8	18
9	19
10	20

Multiplication of 3

3 6 9

15 12 18

P(3) = 6/20

Solution:

1 2 3 4 5 6 7 8 9 10

11 12 13 14 15 16 17 18 19 20

Multiplication of 3

3 6 9

15 12 18

P(3) = 6/20

Multiplication of 5

Solution:

1 11
2 12
3 13
4 14
5 15
6 16
7 17
8 18
9 19
10 20

Multiplication of 3

3 6 9
15 12 18

P(3) = 6/20

Multiplication of 5

5 15
10 20

P(5) = 4/20

Solution:

1	11
2	12
3	13
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Multiplication of 3

3 6 9

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$P(3) = 6/20$

Multiplication of 5

5 15

10 20

$P(5) = 4/20$

Finally

$P\{A \text{ AND } B\} = \{A \cap B\} / \{\Omega\}$

Solution:

1 11
2 12
3 13
4 14
5 15
6 16
7 17
8 18
9 19
10 20

Multiplication of 3

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15 12 18

P(3) = 6/20

Multiplication of 5

5 15
10 20

P(5) = 4/20

Finally

P{A AND B} = {A ∩ B} / {Ω}

15

Solution:

1 2 3 4 5 6 7 8 9 10

11 12 13 14 15 16 17 18 19 20

Multiplication of 3

3 6 9

15 12 18

P(3) = 6/20

Multiplication of 5

5 15

10 20

P(5) = 4/20

Finally

P{A AND B} = {A ∩ B} / {Ω}

15

P(3 and 5) = 1/20

Review:

Question: Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random.

What is the probability that the ticket drawn has a number which is **a multiple of 3 OR 5**?

A. 1/2 B. 1/20 C. 8/15 D. 9/20

Time: 3 minutes, Difficulty Level: Average

Review:

Question: Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random.

What is the probability that the ticket drawn has a number which is **a multiple of 3 OR 5**?

A. 1/2

B. 1/20

C. 8/15

D. 9/20

Time: 3 minutes, Difficulty Level: Average

Solution:

1 11
2 12
3 13
4 14
5 15
6 16
7 17
8 18
9 19
10 20

Multiplication of 3

3 6 9

15 12 18

P(3) = 6/20

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5 15

10 20

P(5) = 4/20

Solution:

1 2 3 4 5 6 7 8 9 10

11 12 13 14 15 16 17 18 19 20

Multiplication of 3

3 6 9

15 12 18

P(3) = 6/20

Multiplication of 5

5 15

10 20

P(5) = 4/20

Finally

P{A OR B} =P(A) + P(B) - P(A and B)

Solution:

1	11
2	12
3	13
4	14
5	15
6	16
7	17
8	18
9	19
10	20

Multiplication of 3

3 6 9

15 12 18

P(3) = 6/20

Multiplication of 5

5 15

10 20

P(5) = 4/20

Finally

P{A OR B} =P(A) + P(B) - P(A and B)

3 6 9 5 15

15 12 18 10 20

Solution:

1 2 3 4 5 6 7 8 9 10

11 12 13 14 15 16 17 18 19 20

Multiplication of 3

3 6 9

15 12 18

P(3) = 6/20

Multiplication of 5

5 15

10 20

P(5) = 4/20

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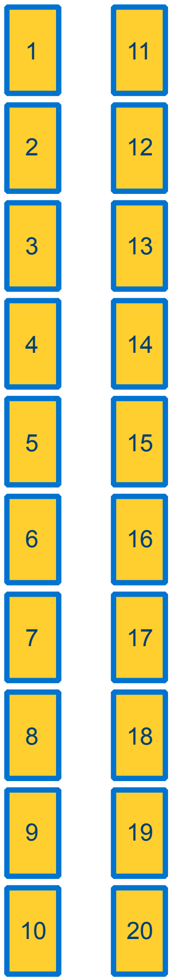
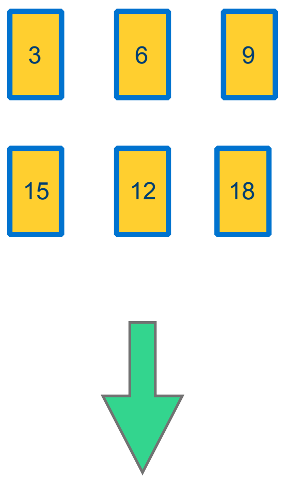
3 6 9 5 15

15 12 18 10 20

Solution:

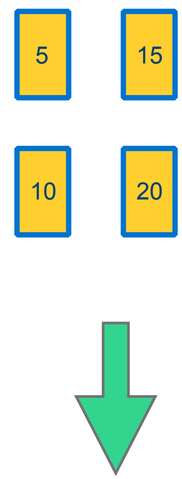


Multiplication of 3



P(3) = 6/20

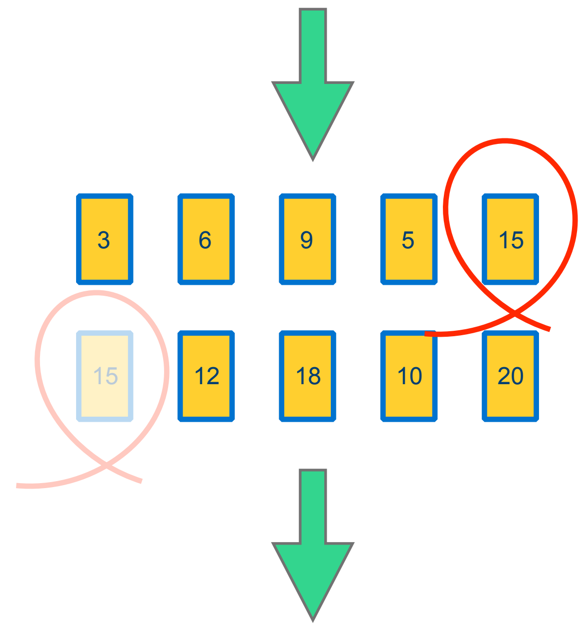
Multiplication of 5



P(5) = 4/20

Finally

P{A OR B} =P(A) + P(B) - P(A and B)



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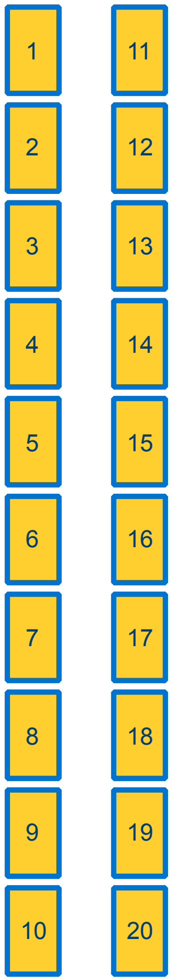
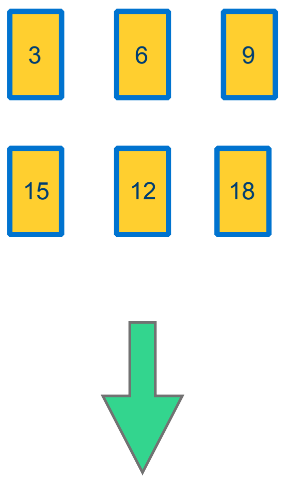
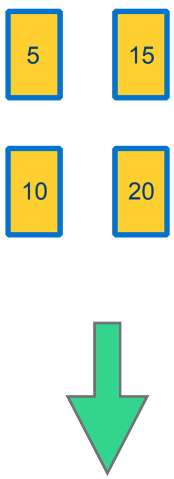


Multiplication of 3



$P(3) = 6/20$

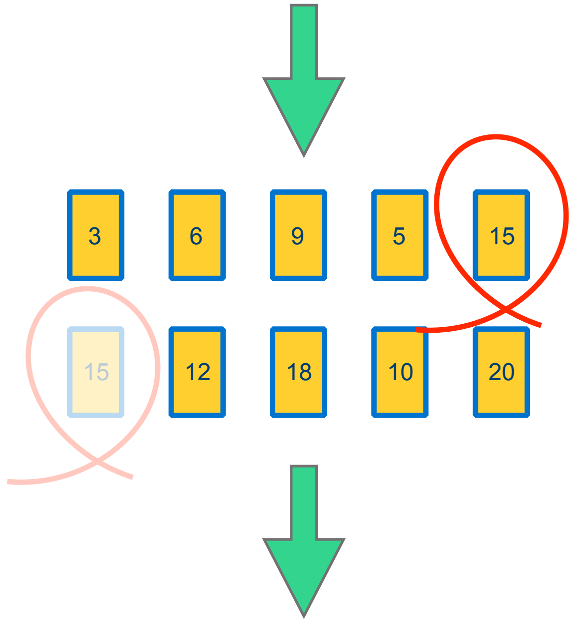
Multiplication of 5



$P(5) = 4/20$

Finally

P{A OR B} =P(A) + P(B) - P(A and B)



P{3 OR 5} =P(3) + P(5) - P(3 and 5)

P(3 or 5) = 6/20 + 4/20 - 1/20
=9/20

Conditional Probability

Independent and Dependent Events:

Independent Event: Each event is **not affected** by any other events.

Conditional Probability

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Example: Tossing a coin:

50% chance for Tail.

What it did in the past will not affect the current toss and its probability.

50% chance for Head.

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Independent and Dependent Events:

Dependent Event: Each event can be **affected** by previous events.

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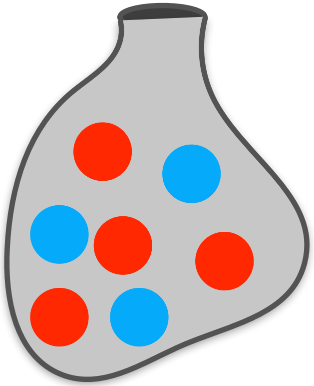
Example: 3 blue and 4 red marbles are in the bag. What is the chance of getting two blue marbles?



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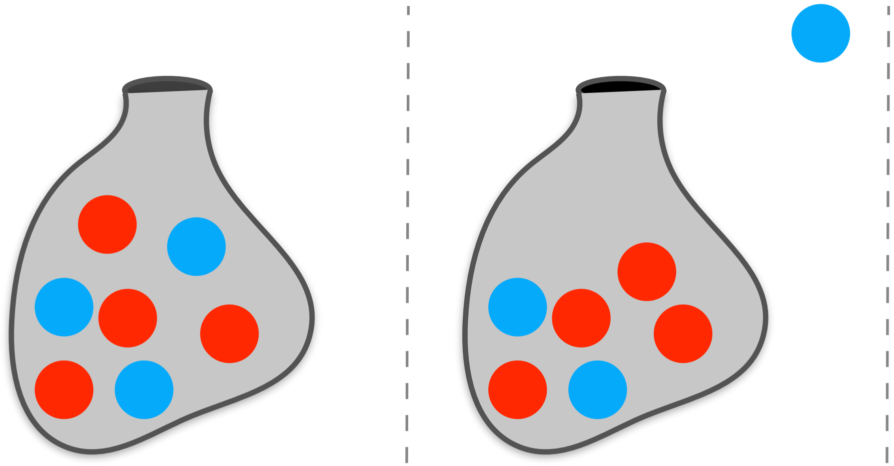

P(first blue) = 3/7

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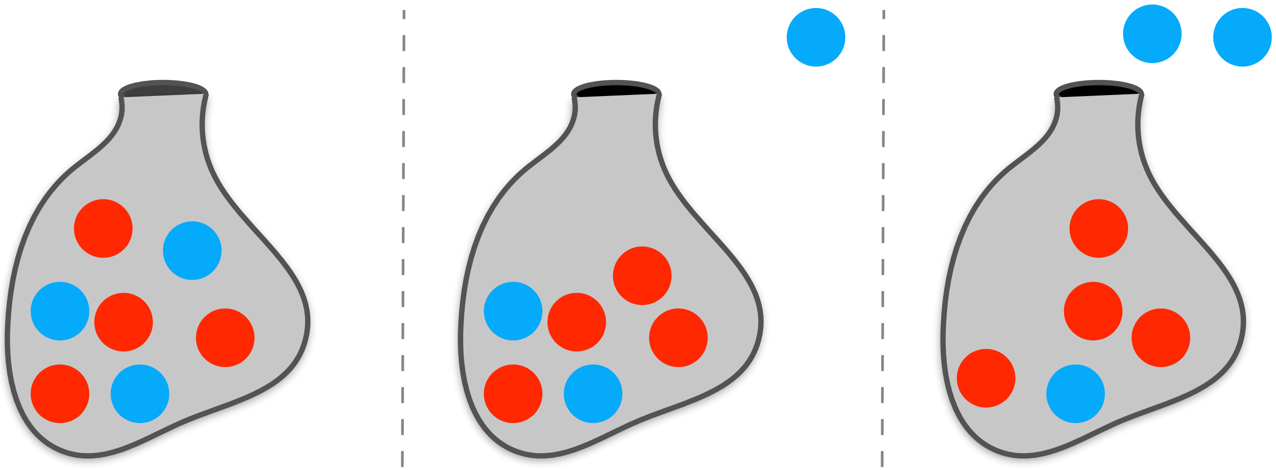

P(first blue) = 3/7

P(Second blue) = 2/6

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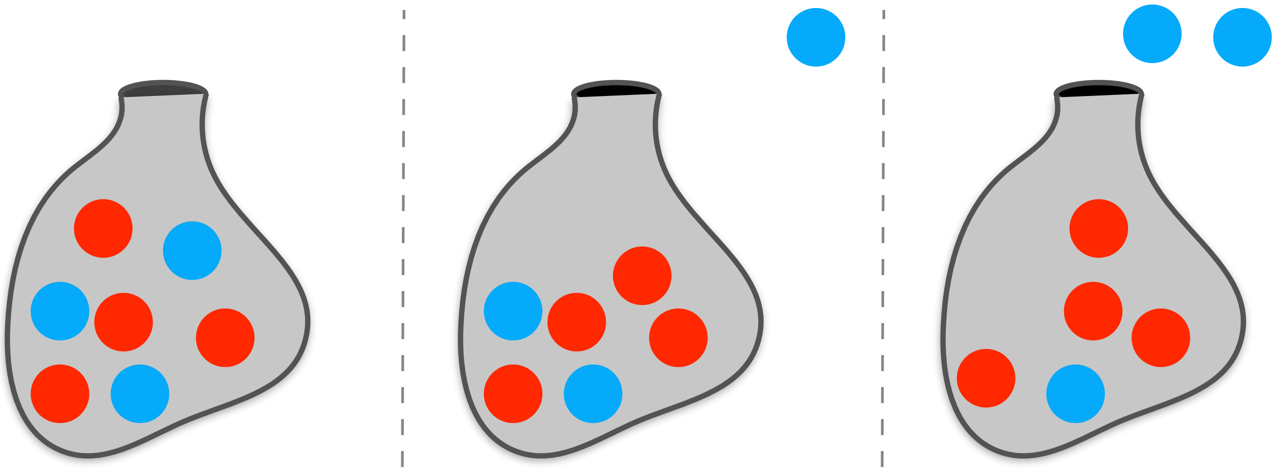

P(first blue) = 3/7

P(Second blue) = 2/6

P(both blue) =

3/7 x 2/6 = 6/42

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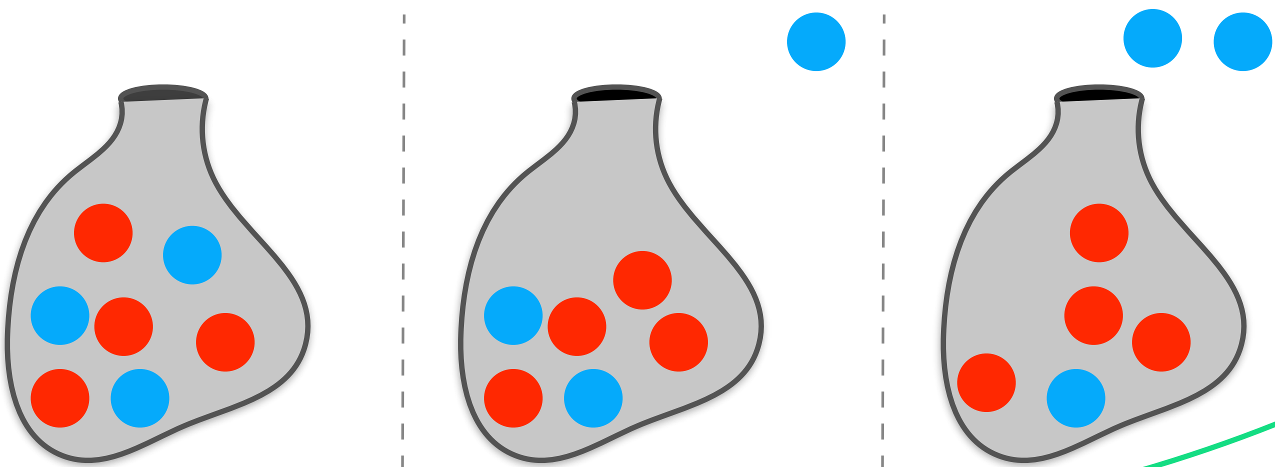

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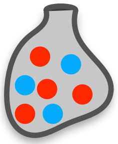

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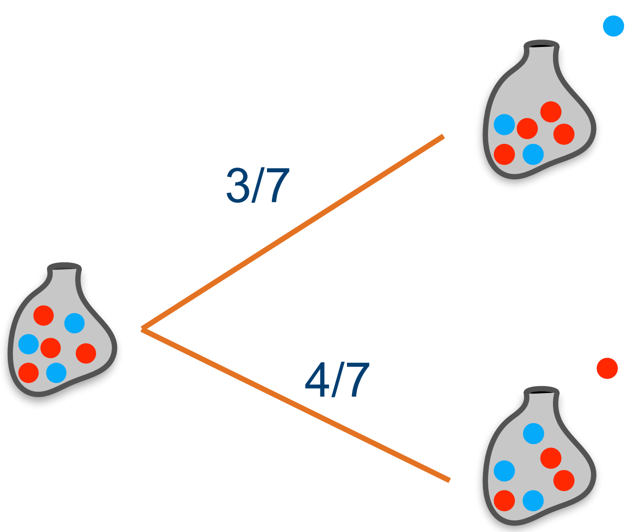

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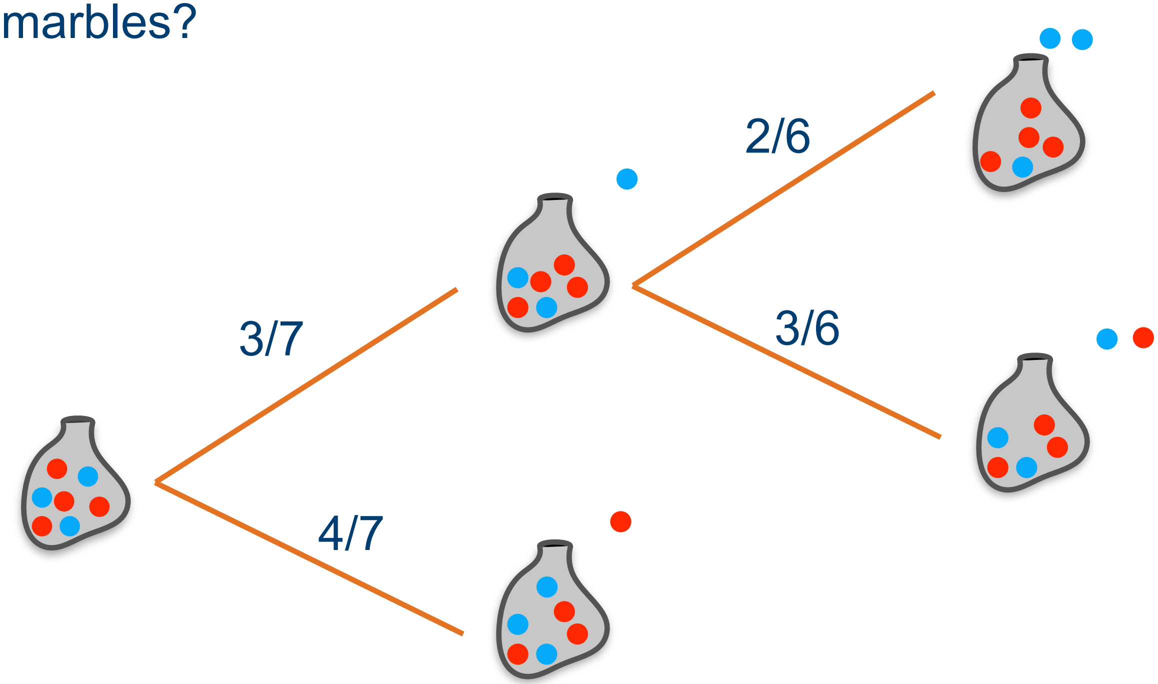

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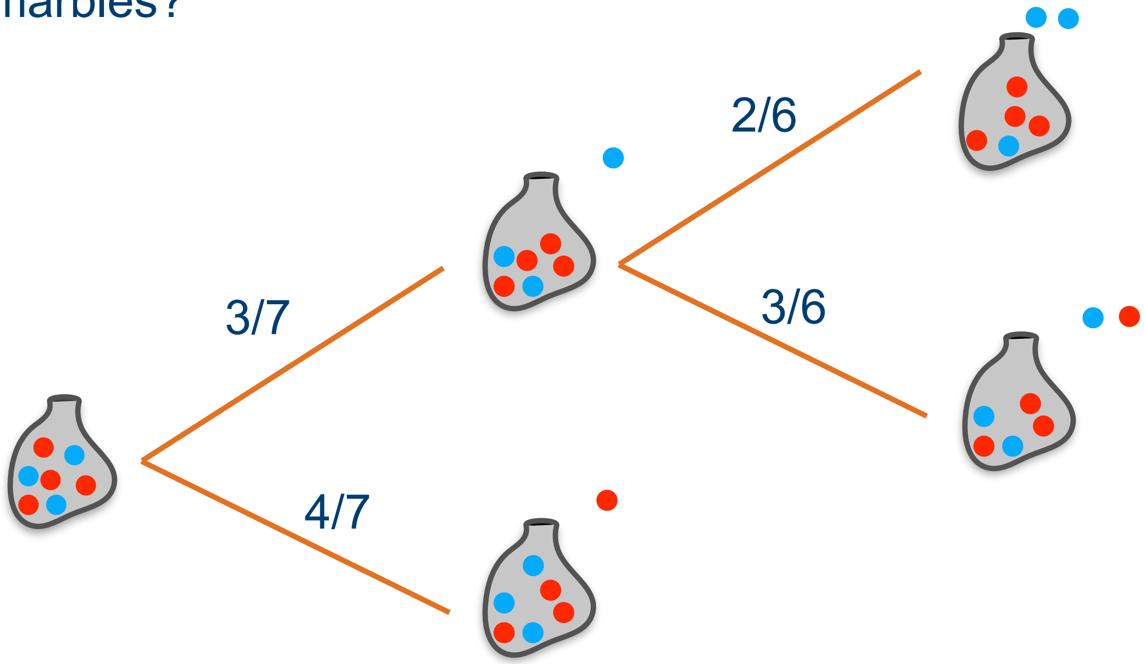

P(two blues) = 3/7 x 2/6 = 6/42

Conditional Probability

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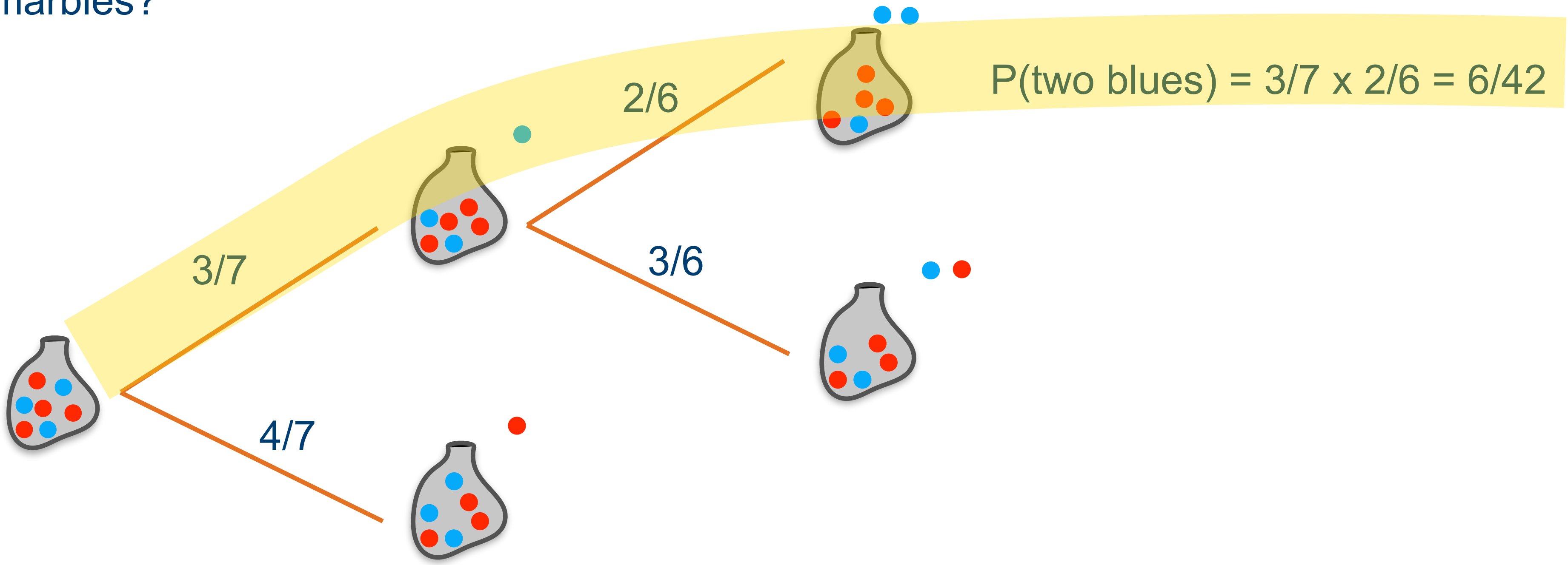

Conditional Probability

Dependent Events:

Mathematically, we can define **dependent probability** as:

$$P(A \text{ AND } B) = P(A) \times P(B \mid A)$$

Conditional Probability

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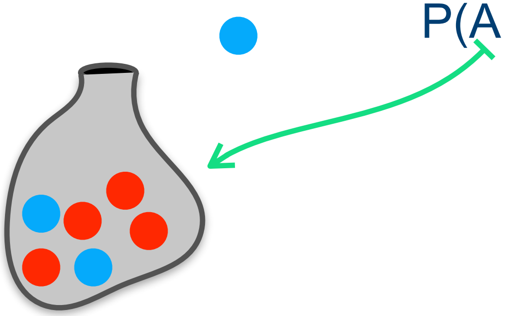

P(A AND B)= P(A) x P(B | A)

Conditional Probability

Dependent Events:

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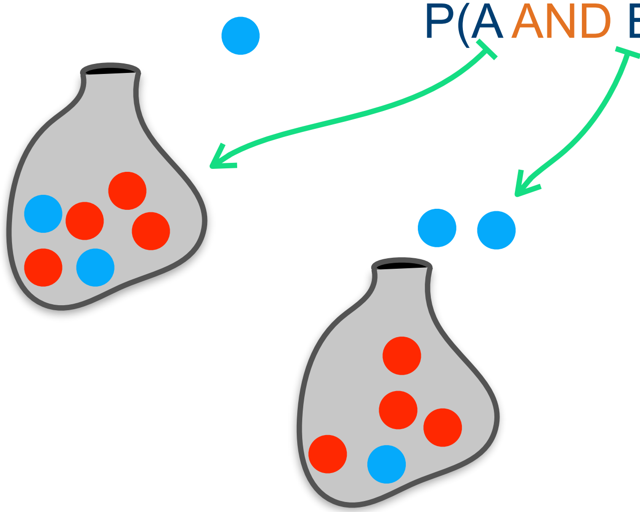

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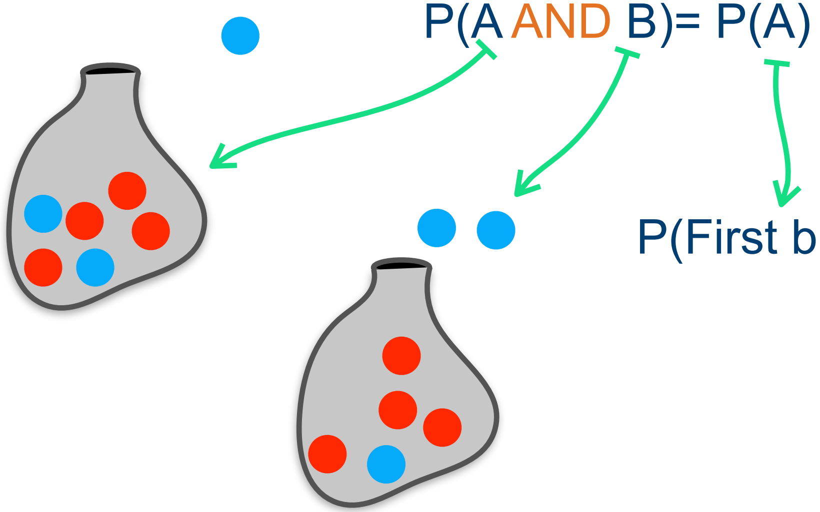


P(First blue) = 3/7

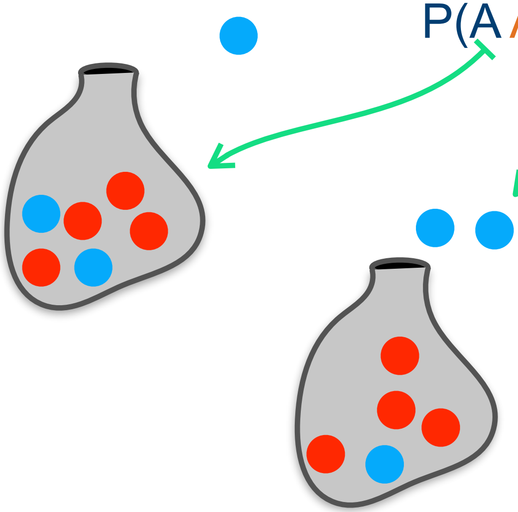
Conditional Probability

Dependent Events:

Mathematically, we can define **dependent probability** as:

Given that

P(A AND B)= P(A) x P(B | A)



P(First blue) = 3/7

P(Second blue **given that** first one is blue) = 2/6

Conditional Probability

Question: You decide to tell your fortune by drawing two cards from a standard deck of 52 cards.

What is the probability of drawing two cards of the **spades** in a row?

The cards are not replaced in the deck.

A. 1/17 B. 12/51 C. 13/51 D. 12/52

Time: 4 minutes, Difficulty Level: A bit difficult

Conditional Probability

Question: You decide to tell your fortune by drawing two cards from a standard deck of 52 cards.

What is the probability of drawing two cards of the **spades** in a row?

The cards are not replaced in the deck.

A. 1/17

B. 12/51

C. 13/51

D. 12/52

Time: 4 minutes, Difficulty Level: A bit difficult

Conditional Probability

Question: You decide to tell your fortune by drawing two cards from a standard deck of 52 cards.

What is the probability of drawing two cards of the **spades** in a row?

The cards are not replaced in the deck.

$$P(A \text{ AND } B) = P(A) \times P(B \mid A)$$

Conditional Probability

Question: You decide to tell your fortune by drawing two cards from a standard deck of 52 cards.

What is the probability of drawing two cards of the **spades** in a row?

The cards are not replaced in the deck.

P(A AND B)= P(A) x P(B | A)

P(Second card:Spades **given that** first card: Spades) = 12/51

First card is from Spades

Second card is Spades

P(First card:Spades) = 13/52

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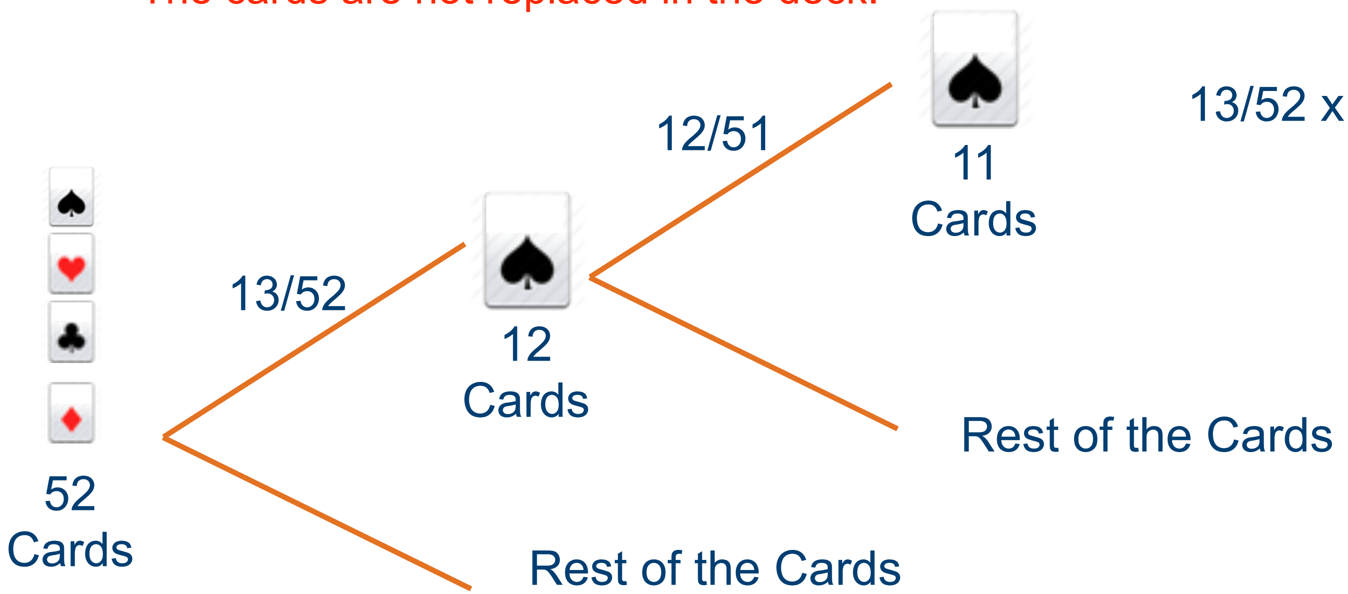


13/52 x 12/51 = 1/17

Conditional Probability

Question: You decide to tell your fortune by drawing two cards from a standard deck of 52 cards.

What is the probability of drawing two cards of the **same suite** in a row? The cards are not replaced in the deck.

A. 1/17

B. 12/51

C. 13/51

D. 12/52

Time: 4 minutes, Difficulty Level: tricky

Conditional Probability

Question: You decide to tell your fortune by drawing two cards from a standard deck of 52 cards.

What is the probability of drawing two cards of the **same suite** in a row? The cards are not replaced in the deck.

A. 1/17 B. 12/51 C. 13/51 D. 12/52

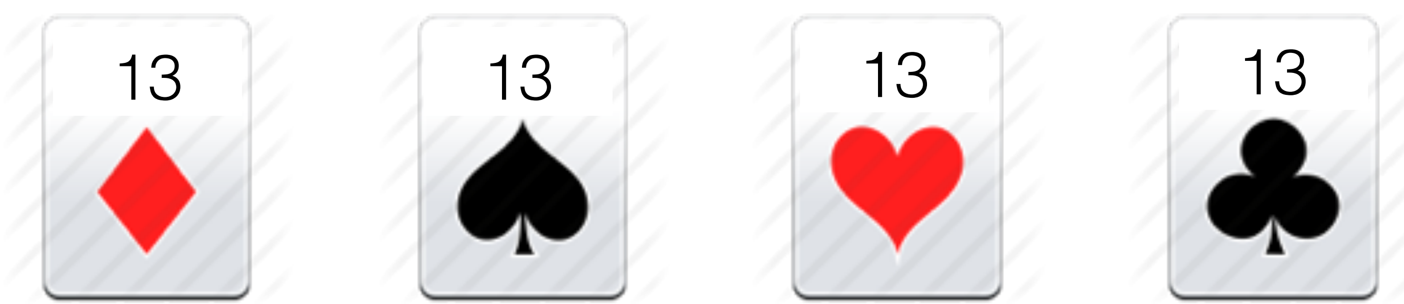
Time: 4 minutes, Difficulty Level: tricky

Conditional Probability

Question: You decide to tell your fortune by drawing two cards from a standard deck of 52 cards.

What is the probability of drawing two cards of the **same suite** in a row? The cards are not replaced in the deck.

The first card could be either from:



Conditional Probability

Question: You decide to tell your fortune by drawing two cards from a standard deck of 52 cards.

What is the probability of drawing two cards of the **same suite** in a row? The cards are not replaced in the deck.

P(both the same suit) = P(1st:♥) x P(2nd:♥ | 1st:♥) + P(1st:♦) x P(2nd:♦ | 1st:♦)
+ P(1st:♣) x P(2nd:♣ | 1st:♣) + P(1st:♠) x P(2nd:♠ | 1st:♠)

Conditional Probability

Question: You decide to tell your fortune by drawing two cards from a standard deck of 52 cards.

What is the probability of drawing two cards of the **same suite** in a row? The cards are not replaced in the deck.

13/52 12/51 13/52 12/51

$$P(\text{both the same suit}) = P(1^{st}:\heartsuit) \times P(2^{nd}:\heartsuit \mid 1^{st}:\heartsuit) + P(1^{st}:\diamondsuit) \times P(2^{nd}:\diamondsuit \mid 1^{st}:\diamondsuit)$$

$$+ P(1^{st}:\clubsuit) \times P(2^{nd}:\clubsuit \mid 1^{st}:\clubsuit) + P(1^{st}:\spadesuit) \times P(2^{nd}:\spadesuit \mid 1^{st}:\spadesuit)$$

13/52 12/51 13/52 12/51

Conditional Probability

Question: You decide to tell your fortune by drawing two cards from a standard deck of 52 cards.

What is the probability of drawing two cards of the **same suite** in a row? The cards are not replaced in the deck.

No matter what the first card is, the second card should be the same as the first card.

E.g., the second card should be heart if the first one is heart.

12/51

Conditional Probability

$$P(A \text{ AND } B) = P(A) \times P(B \mid A)$$

Conditional Probability

$$P(A \text{ AND } B) = P(A) \times P(B \mid A)$$

↓

$$P(B \mid A) = \frac{P(A \text{ AND } B)}{P(A)}$$

Conditional Probability

Question: The probability that it is **Monday** **and** **that a student is absent is 0.03**. Since there are 5 school days in a week, the probability that it is Monday is 0.2. What is the probability that a student is absent given that today is Monday?

Conditional Probability

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A: It is Monday
B: The students is Absent

Conditional Probability

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What is the probability that a student is absent given that today is Monday?

P(B | A)

A: It is Monday
B: The students is Absent

Conditional Probability

P(A AND B)

Question: The probability that it is **Monday and that a student is absent is 0.03**. Since there are 5 school days in a week, the probability that it is Monday is 0.2.

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Conditional Probability

P(A AND B)

Question: The probability that it is **Monday and that a student is absent is 0.03**. Since there are 5 school days in a week, the probability that it is Monday is 0.2.

P(A)

What is the probability that a student is absent given that today is Monday?

P(B | A)

A: It is Monday
B: The students is Absent

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Conditional Probability

P(A AND B)

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P(B | A)

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P(A AND B)= P(A) x P(B | A)
P(A)

$$P(B \mid A) = \frac{P(A \text{ AND } B)}{P(A)}$$

Conditional Probability

P(A AND B)

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P(B | A)

A: It is Monday
B: The students is Absent

P(absent | Monday) = 0.03/0.2 = 0.15

P(A AND B)= P(A) x P(B | A)

P(A)

$$P(B \mid A) = \frac{P(A \text{ AND } B)}{P(A)}$$

Expected Value:

Expected value is exactly what you might think it means intuitively: the **return you can expect for some kind of action**, like how many questions you might get right if you guess on a multiple choice test.

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Example: For example, if you take a 20 question multiple-choice test with A,B,C,D as the answers, and you guess all “A”.

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You can expect to get 25% right **(5 out of 20)**.

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The expected value gives us the expected long term average of measurements.

Expected Value = Expected Average

Binomial Expected Value:

Binomial Expected Value, the outcome is between two options, e.g., True or False.

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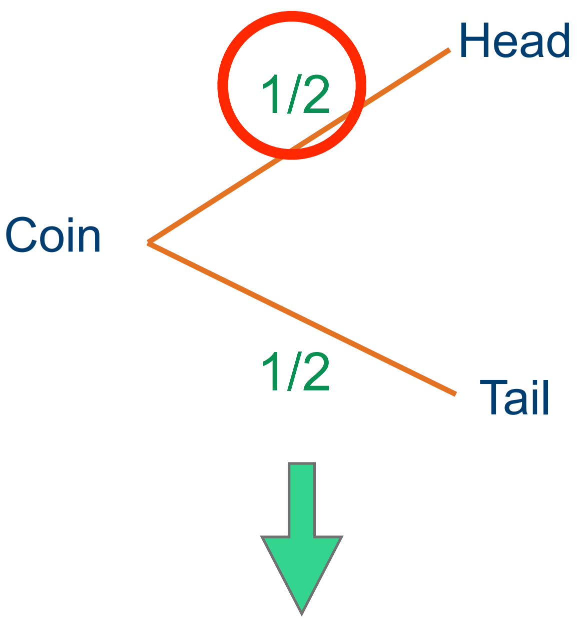
Binomial Expected Value, the outcome is between two options, e.g., True or False.

Example: We toss a coin 10 times. How many Heads you could expect?

Binomial Expected Value:

Binomial Expected Value, the outcome is between two options, e.g., True or False.

Example: We toss a coin 10 times. How many Heads you could expect?



1/2 x 10 = 5

Binomial Expected Value:

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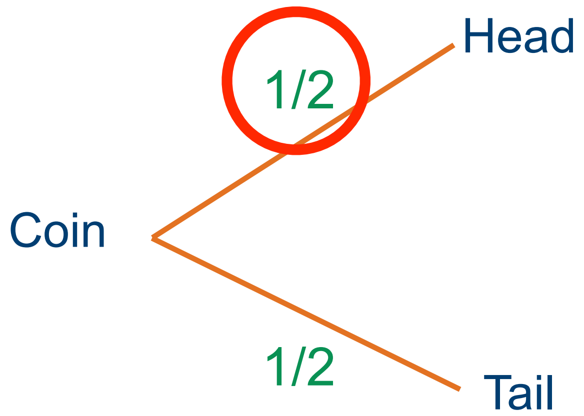
Example: We toss a coin 10 times. How many Heads you could expect?



1/2 x 10 = 5

Binomial Expected Value

$$E(x) = P(x) * X$$

Probability of Head

Number of Trials

Expected Value:

If there are more than two outcomes:

OUTCOME	X_1	X_2	X_3	…	X_n
PROBABILITY	P_1	P_2	P_3	…	P_n

$$E(x) = x_1 P_1 + x_2 P_2 + x_3 P_3 + \cdots + x_n p_n$$

$$E(x) = \sum_{i=1}^{n} x_i P_i$$

Challenge: Shall we play or …. not?

The expected value gives us the expected long term average of measurements.

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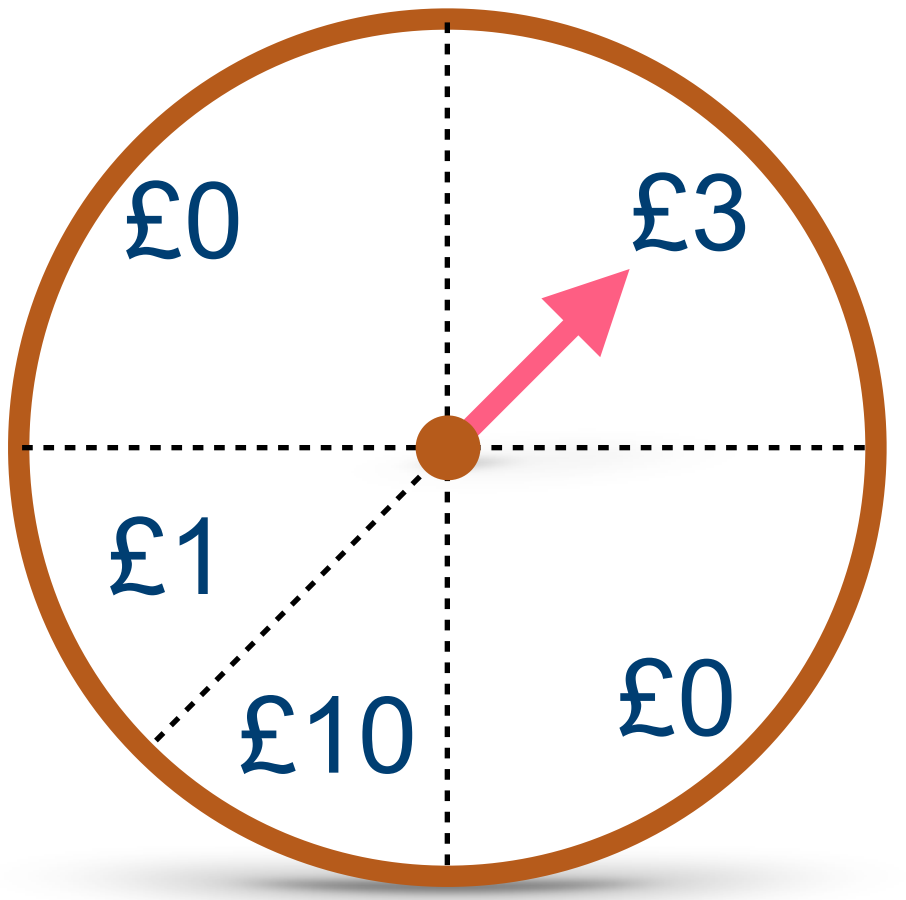
The expected value gives us the expected long term average of measurements.

This game costs you £1 per game.

You will not receive your money back no matter you win or lose.

Shall we play?



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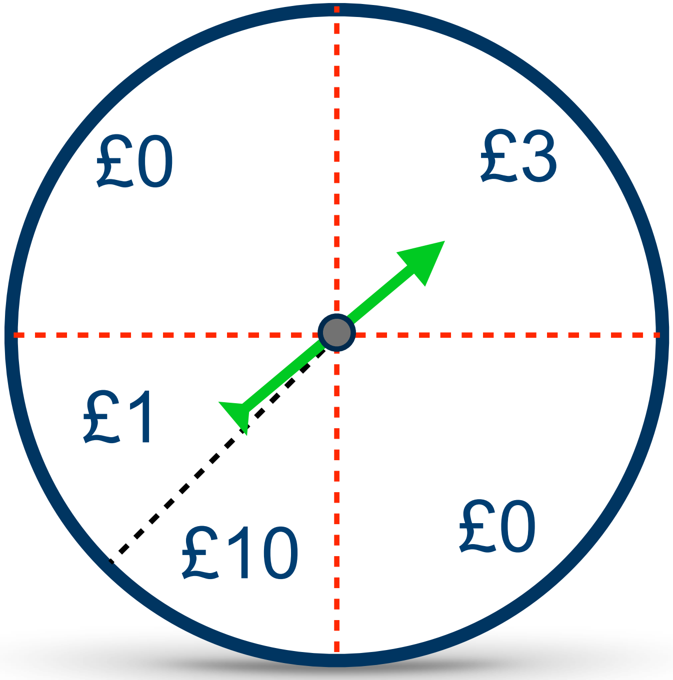

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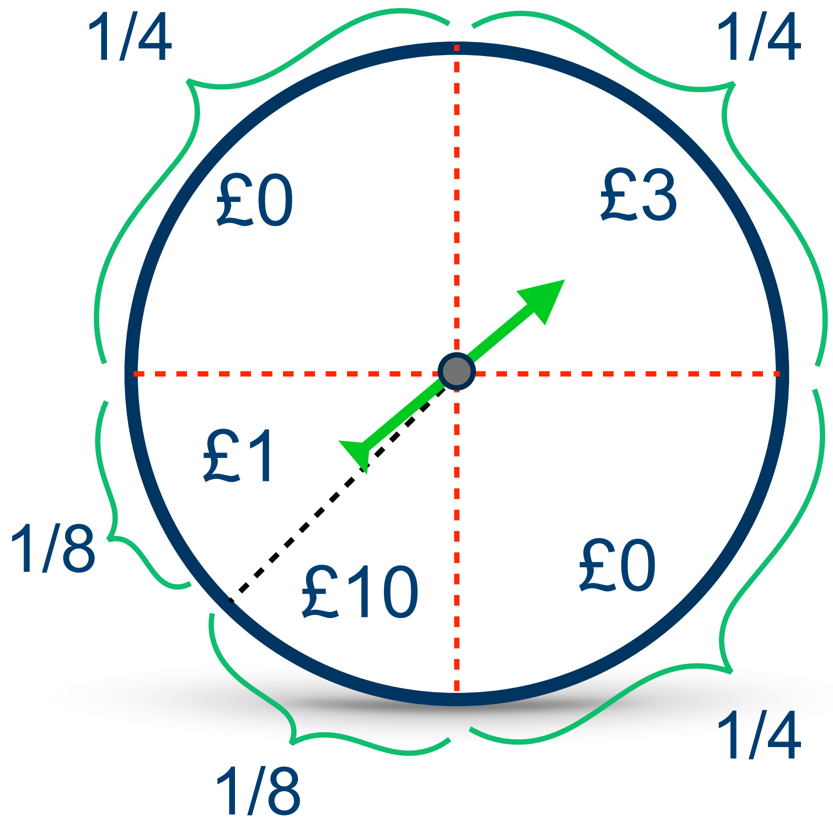
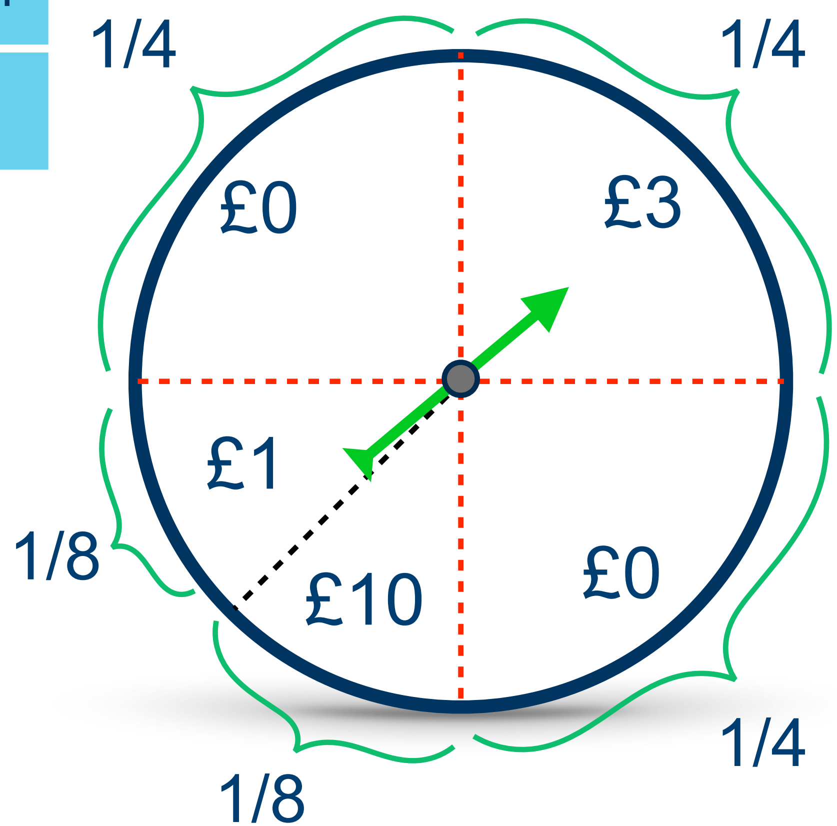


Challenge: Shall we play or …. not?

OUTCOME	£0	£3	£0	£10	£1	-£1
PROBABILITY	1/4	1/4	1/4	1/8	1/8	1



Challenge: Shall we play or …. not?

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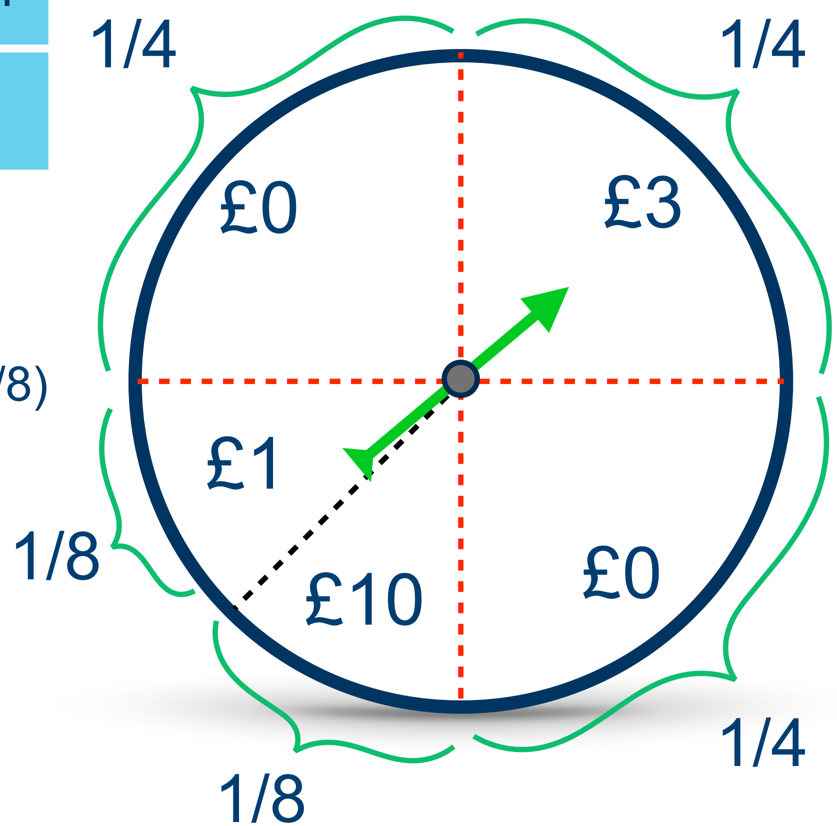

Your expectation is:

E = -1£1(1) + £0(1/2) + £1(1/8) + £2(1/4) + £10(1/8)

E= £7/8

Challenge: Shall we play or …. not?

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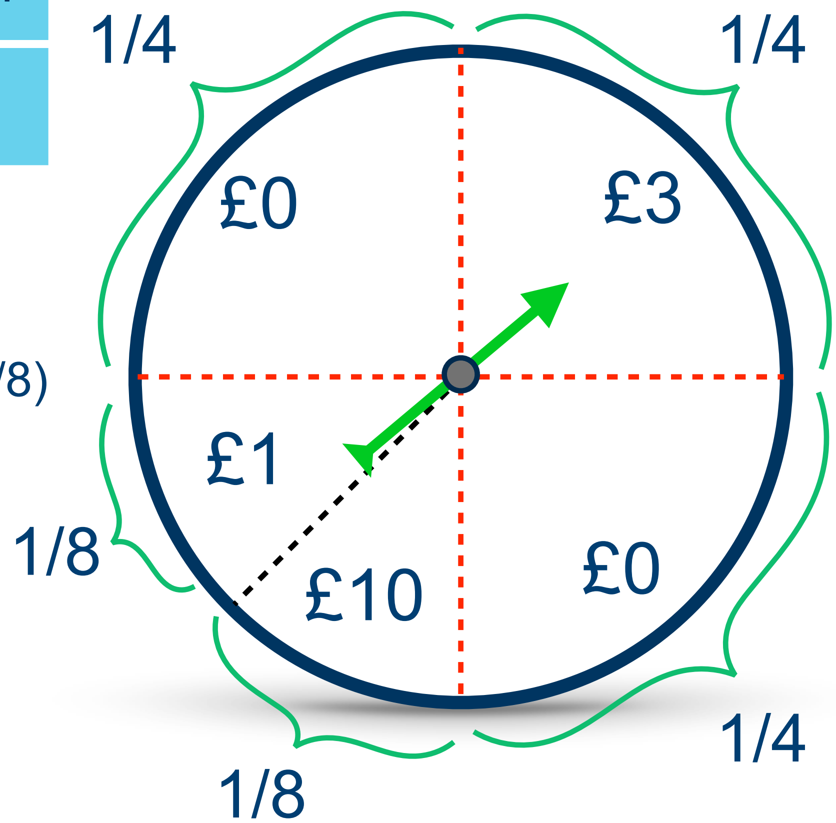

Your expectation is:

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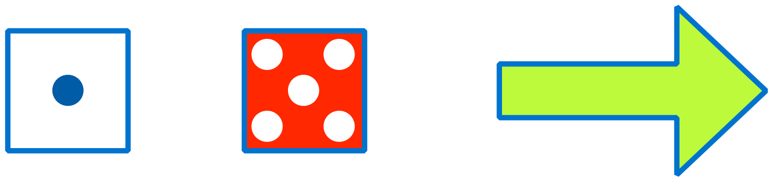
E= £7/8

You can expect to win £0.875 ON AVERAGE per game.
For instance: 100 games, you win £87.5.

Probability density function (PDF)

Example: We roll Two Dice, and we sum up the shown numbers.

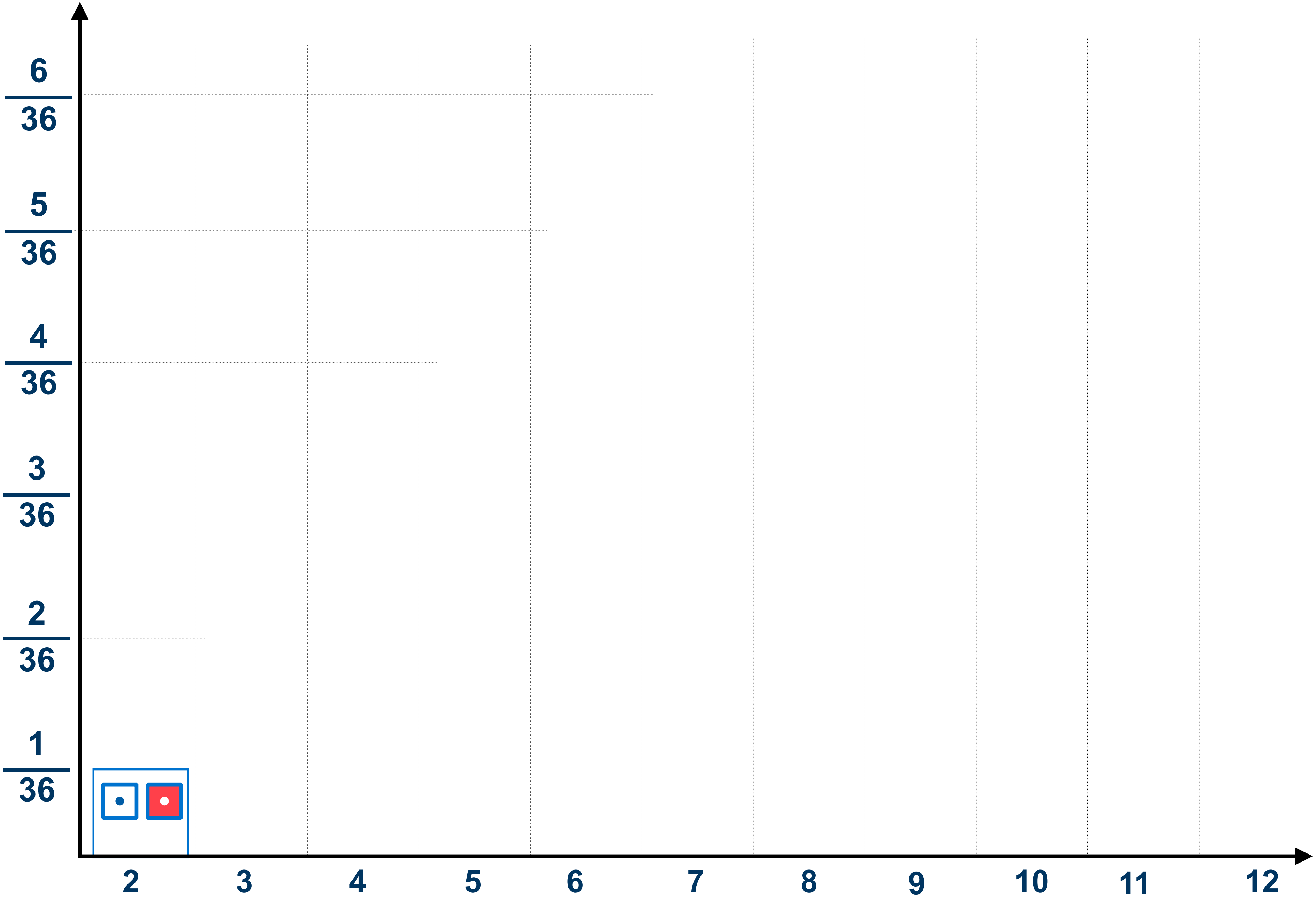
For instance:



5+1 = 6

P(1,5) = 1/36

What is the probability that sum of two numbers is 5?



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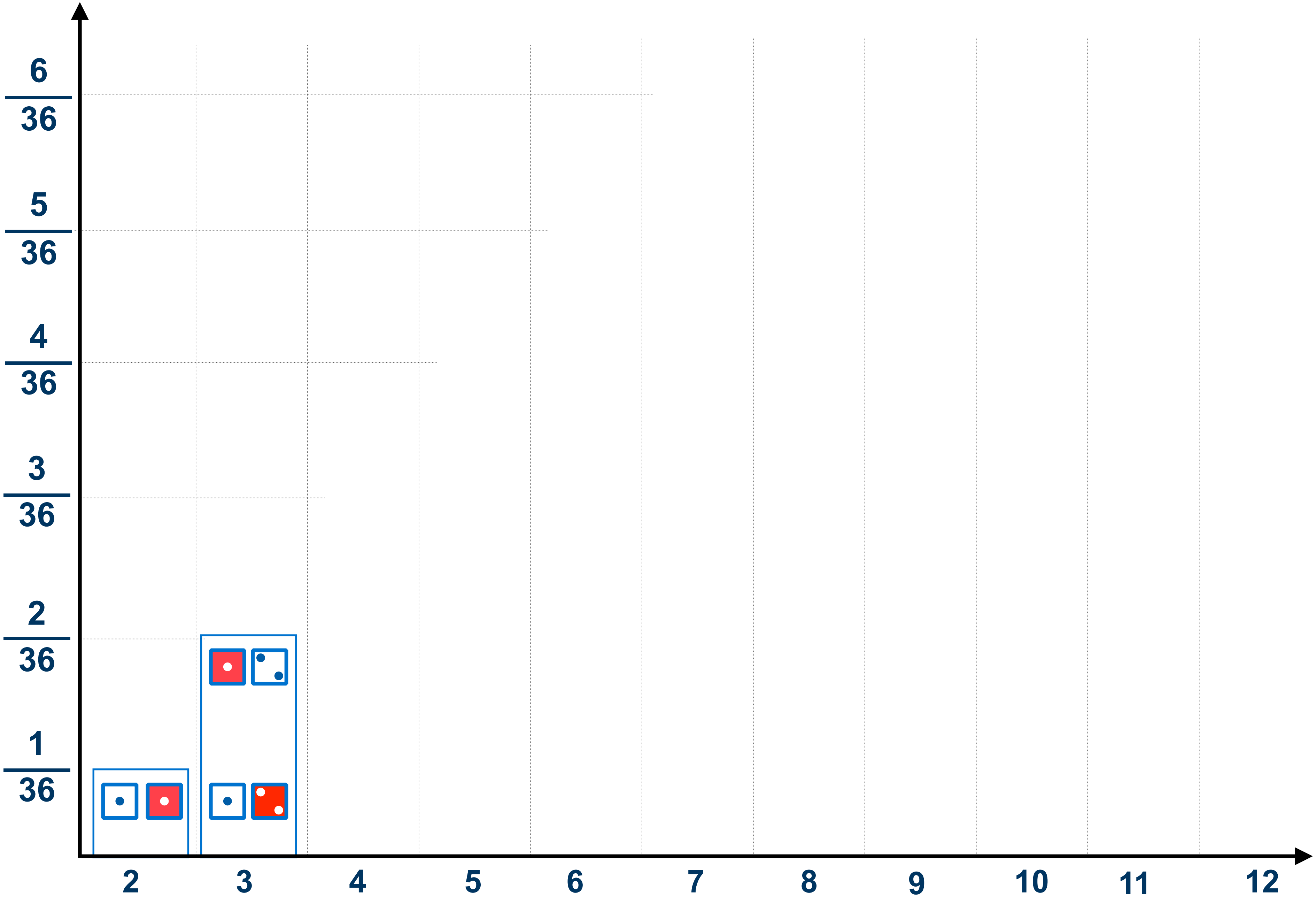

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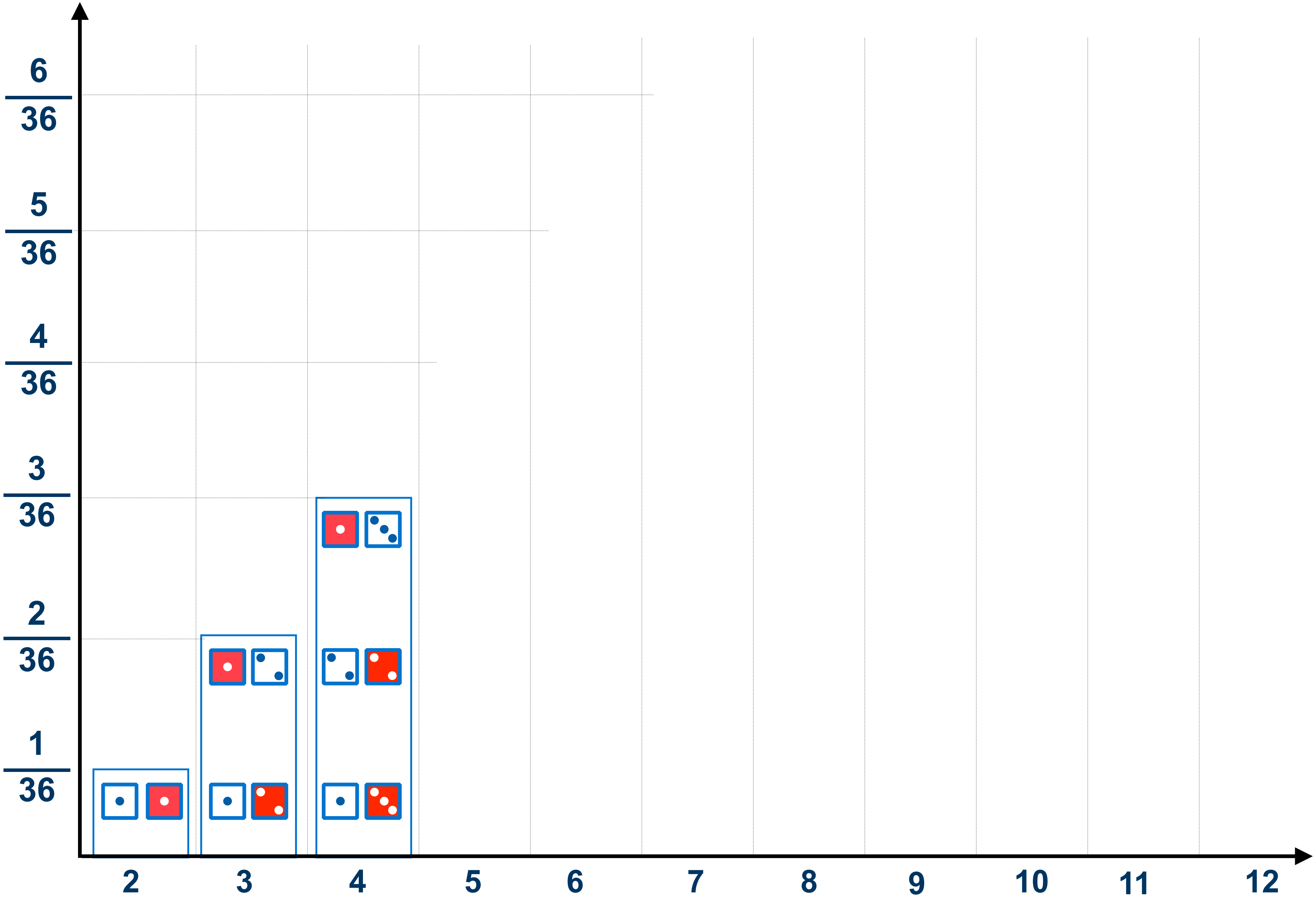
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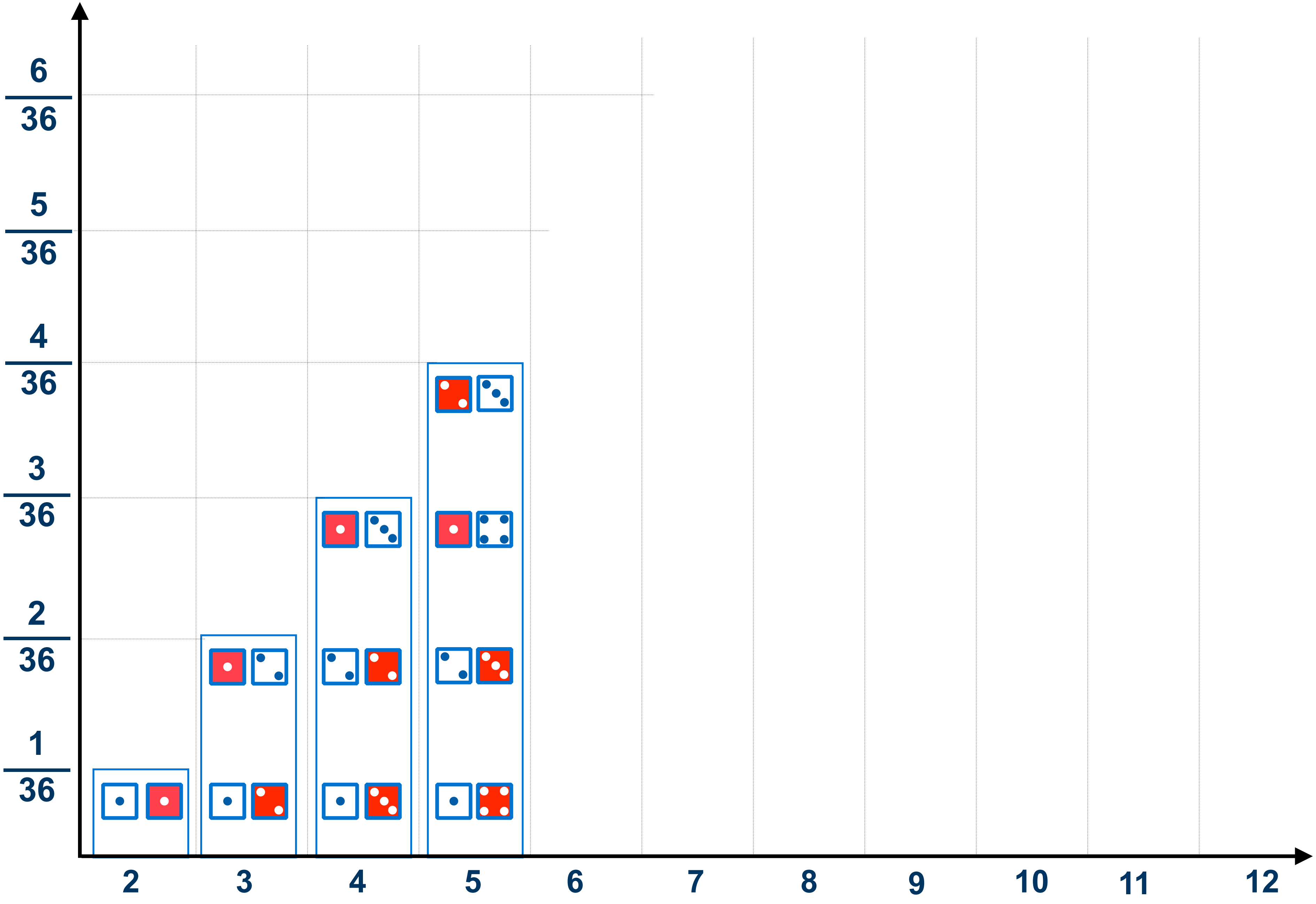

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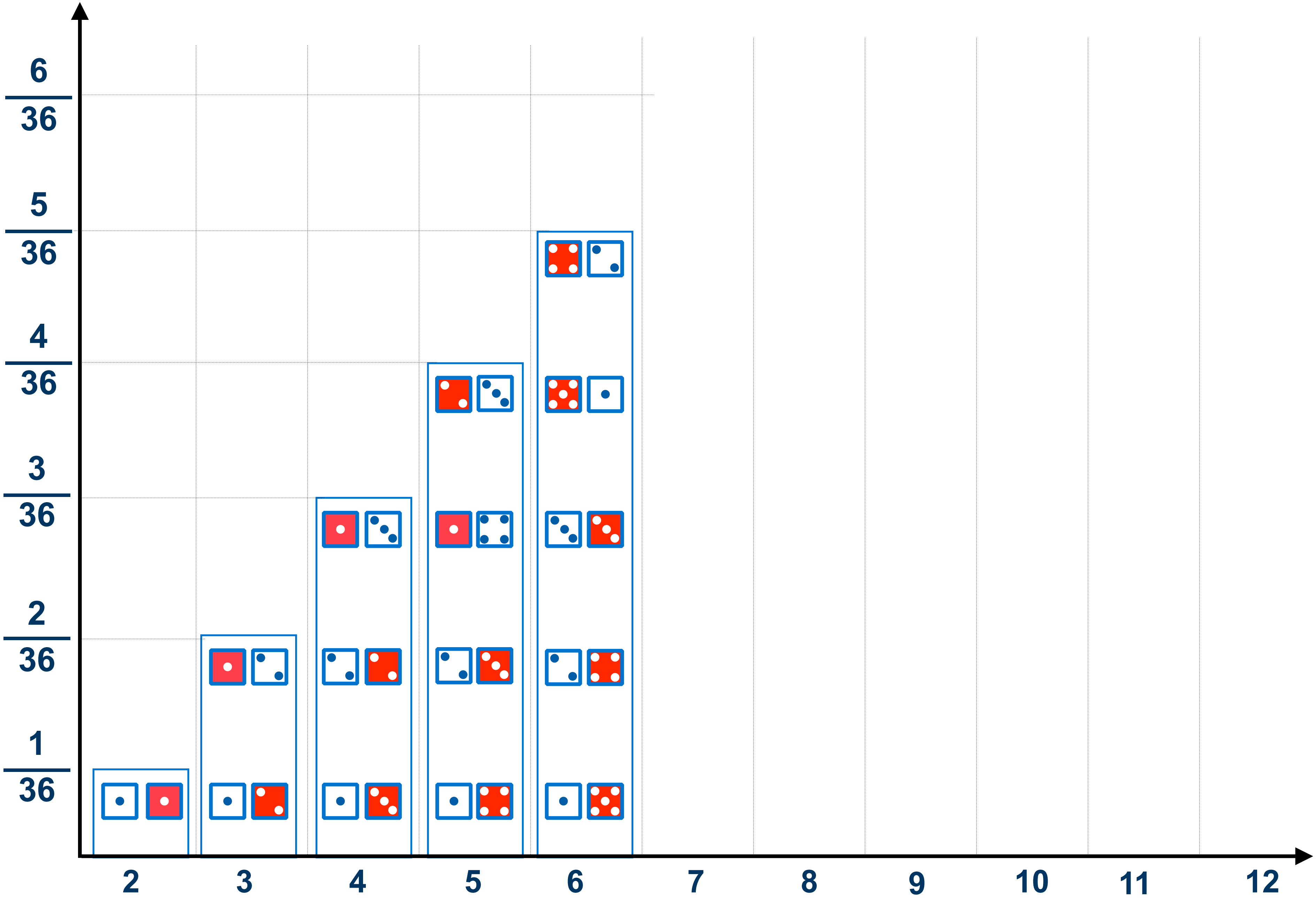

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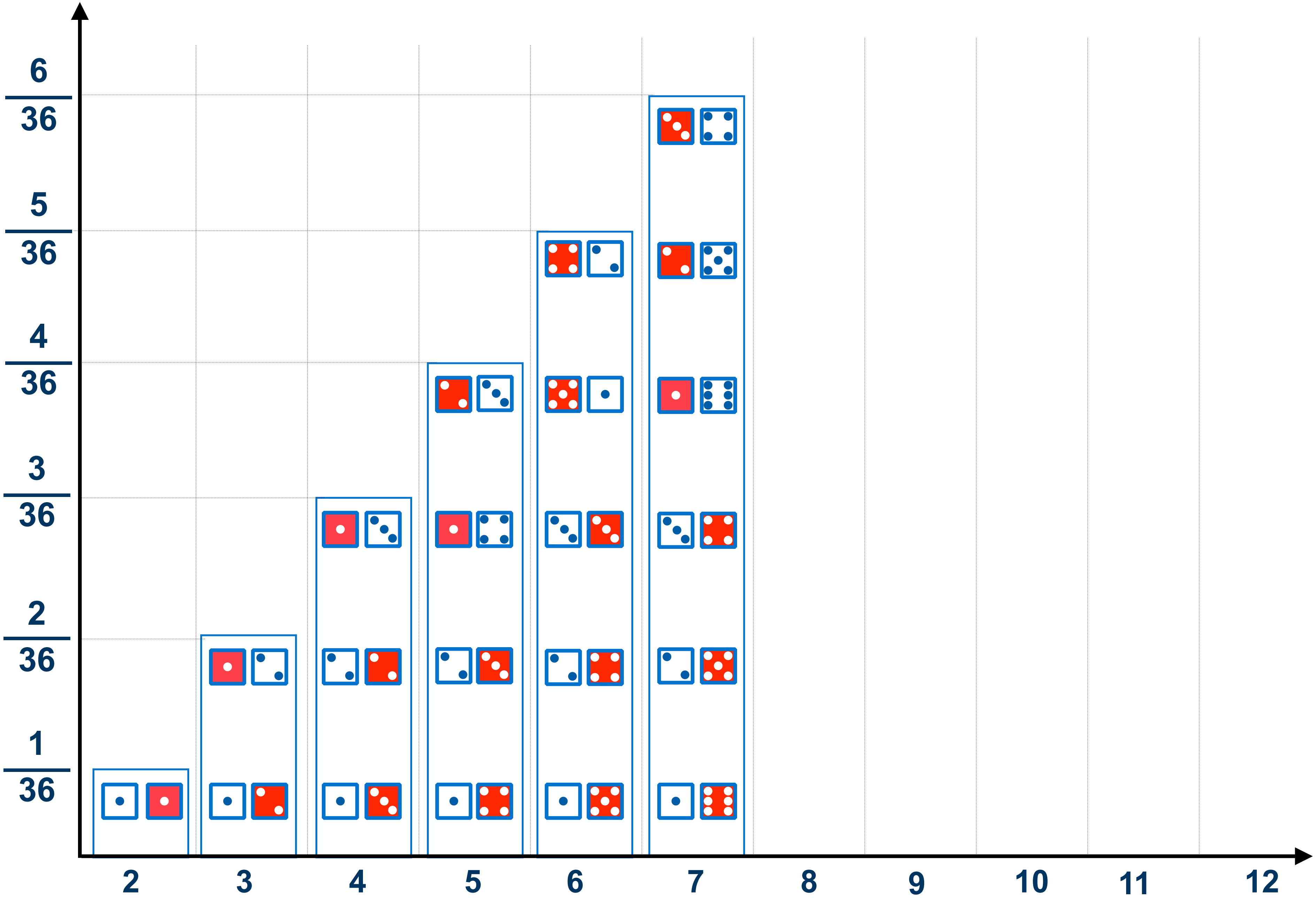

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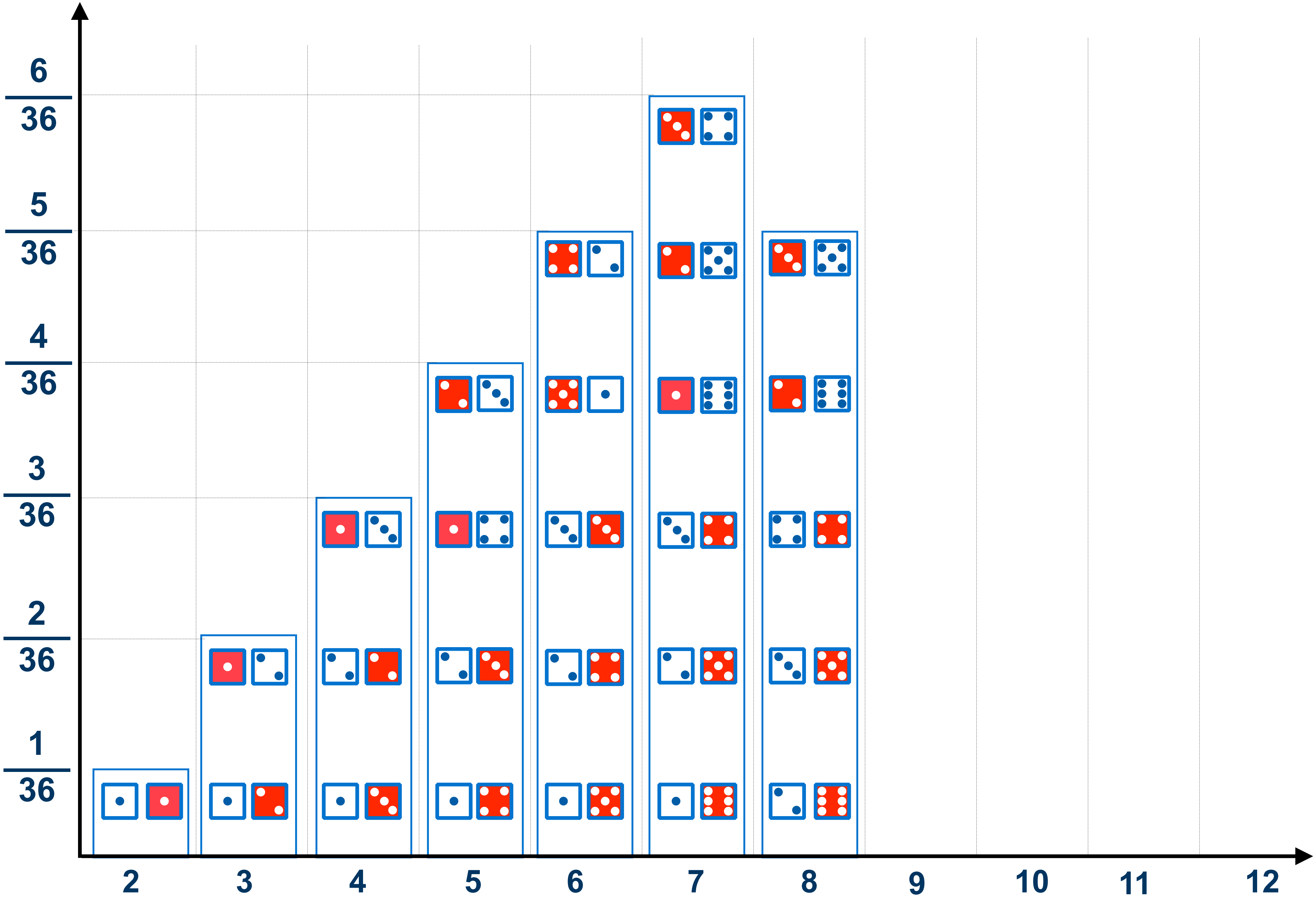

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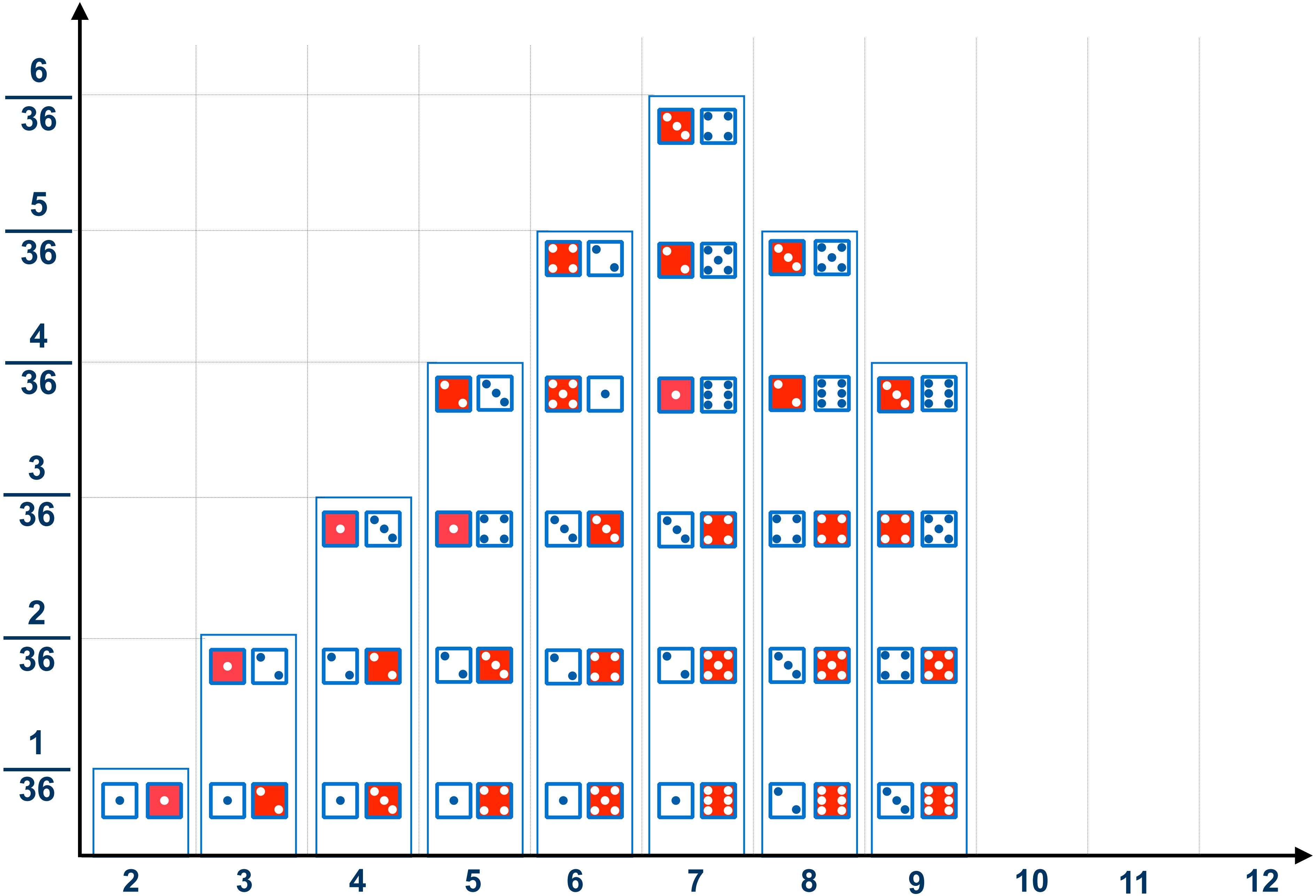

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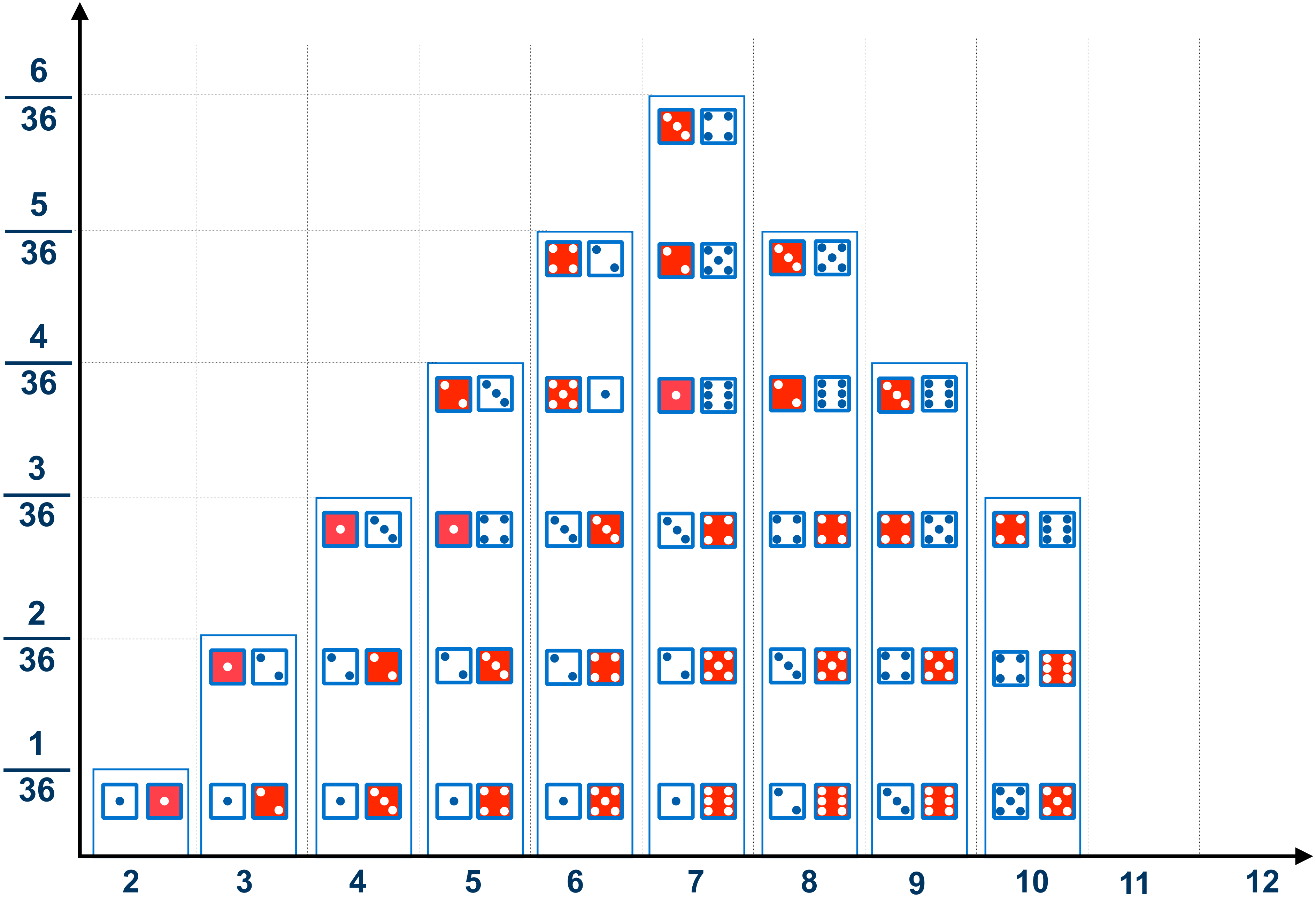

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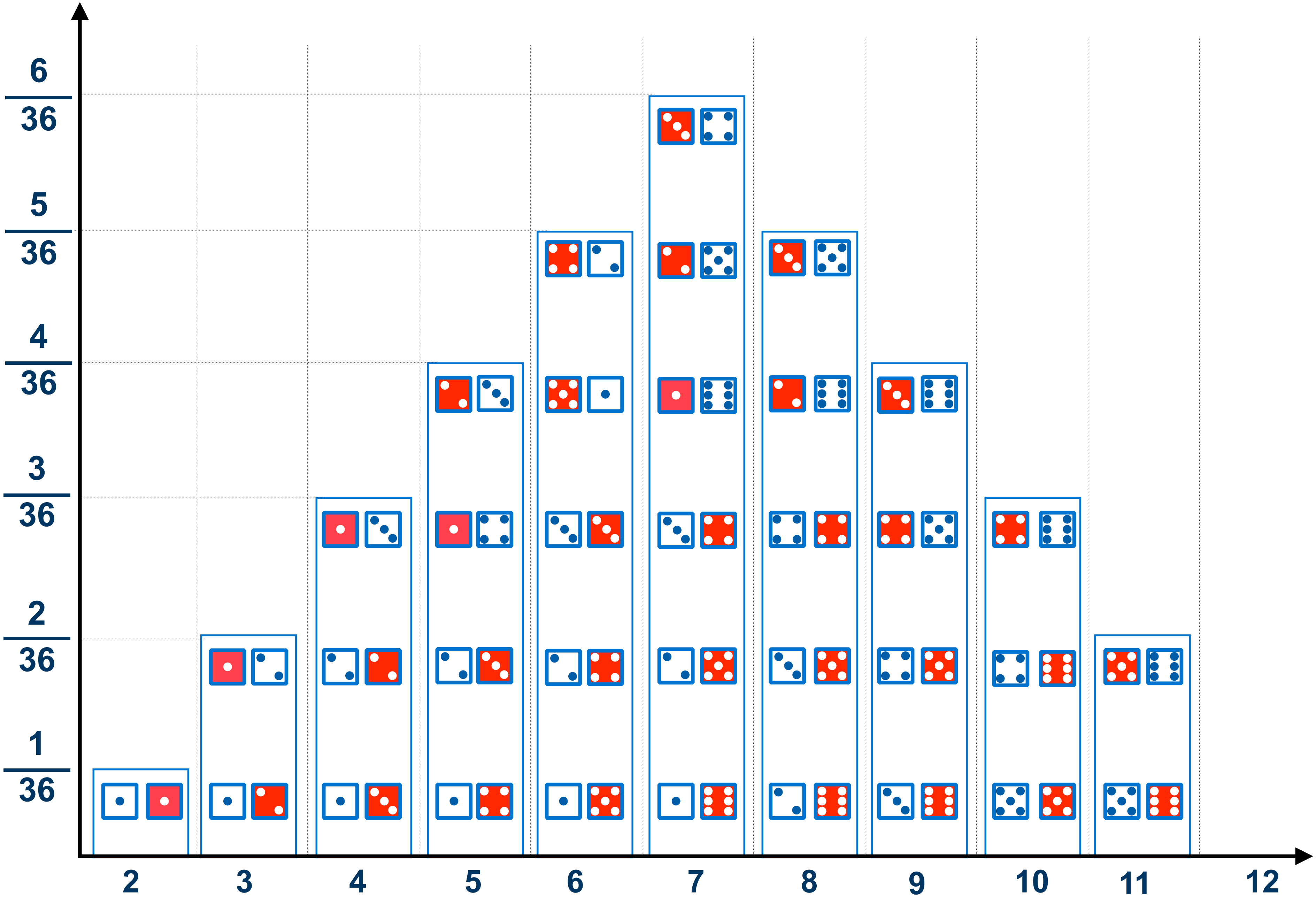

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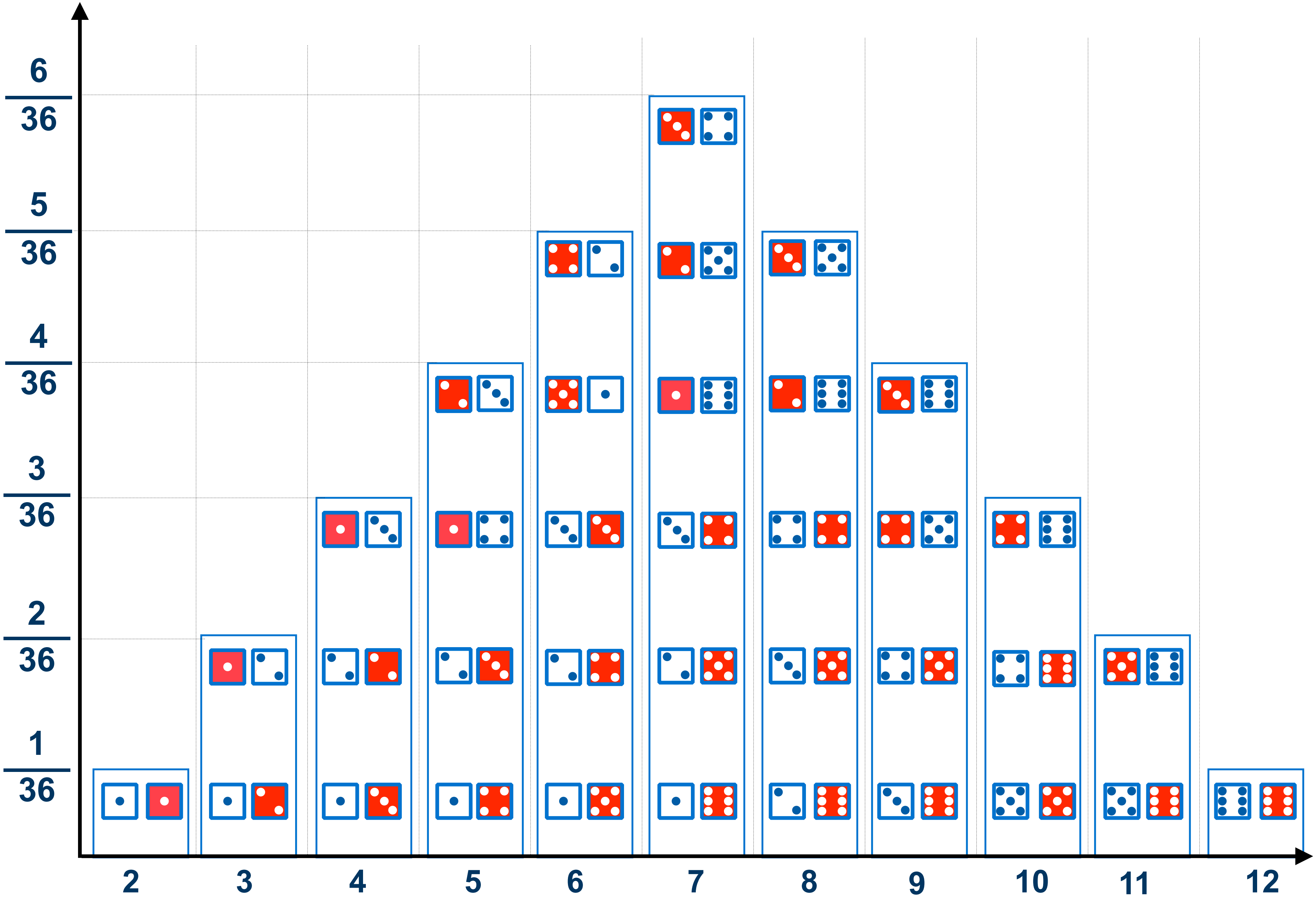

6/36
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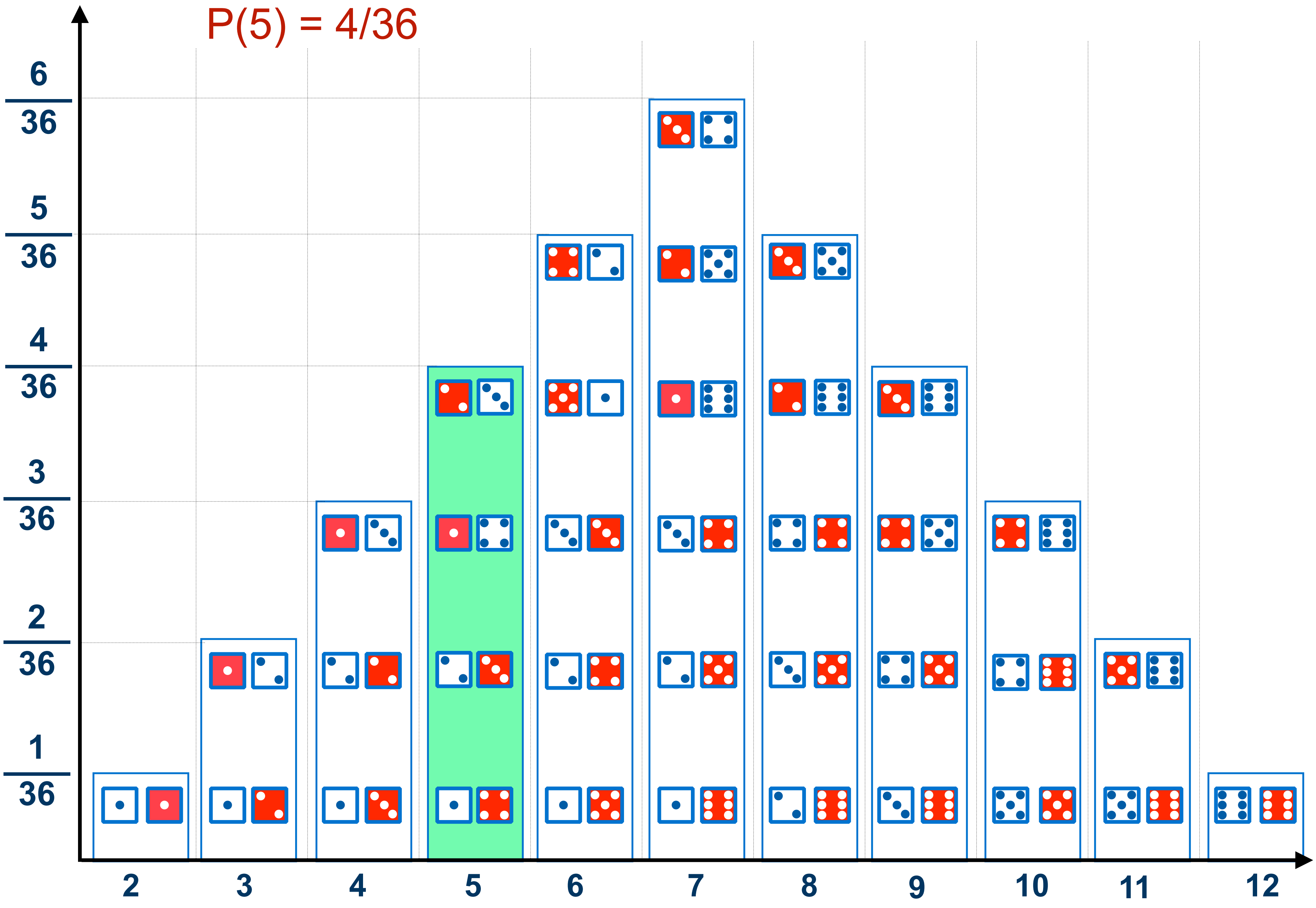
P(5) = 4/36
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4/36
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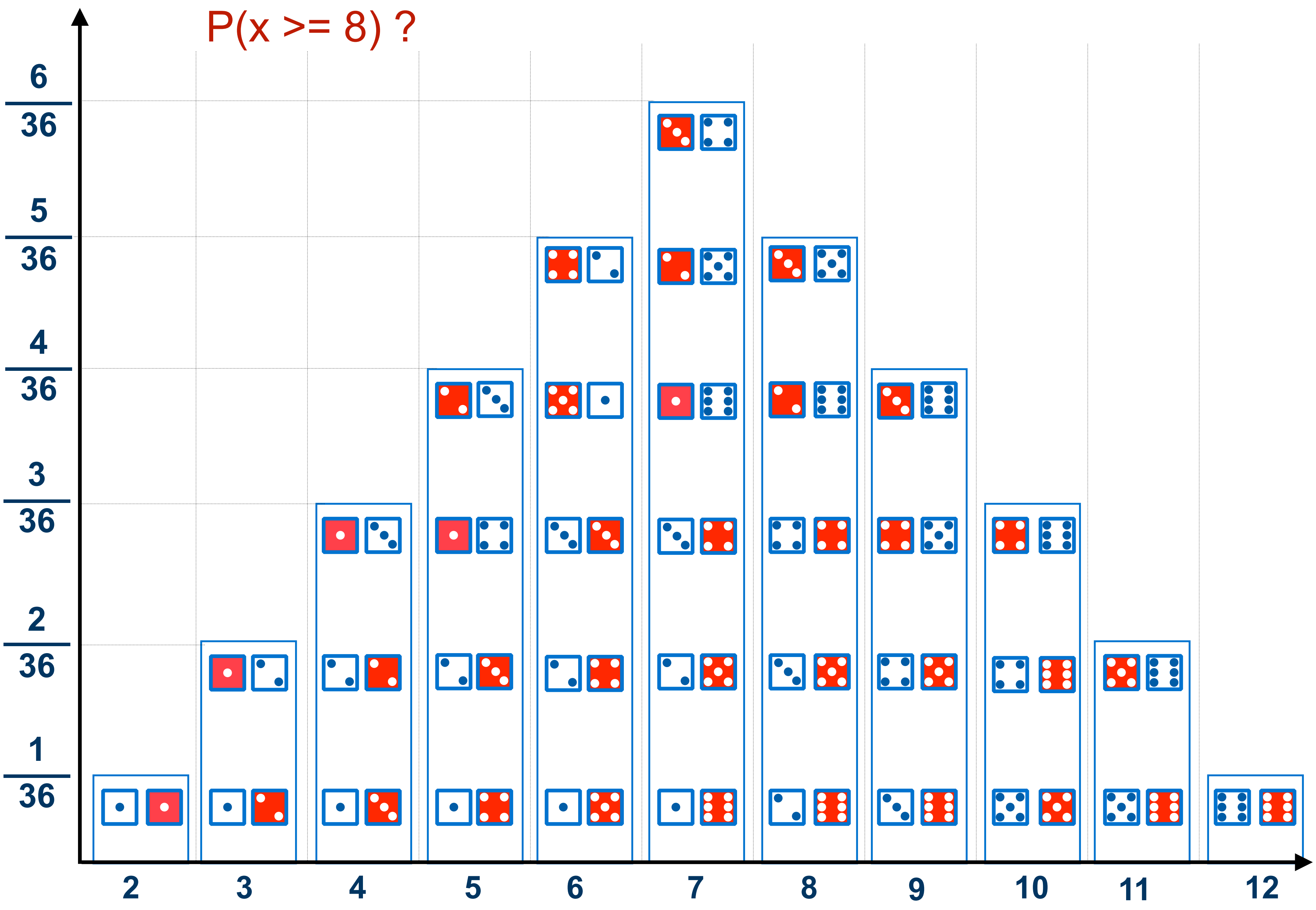

P(x >= 8) ?
6/36
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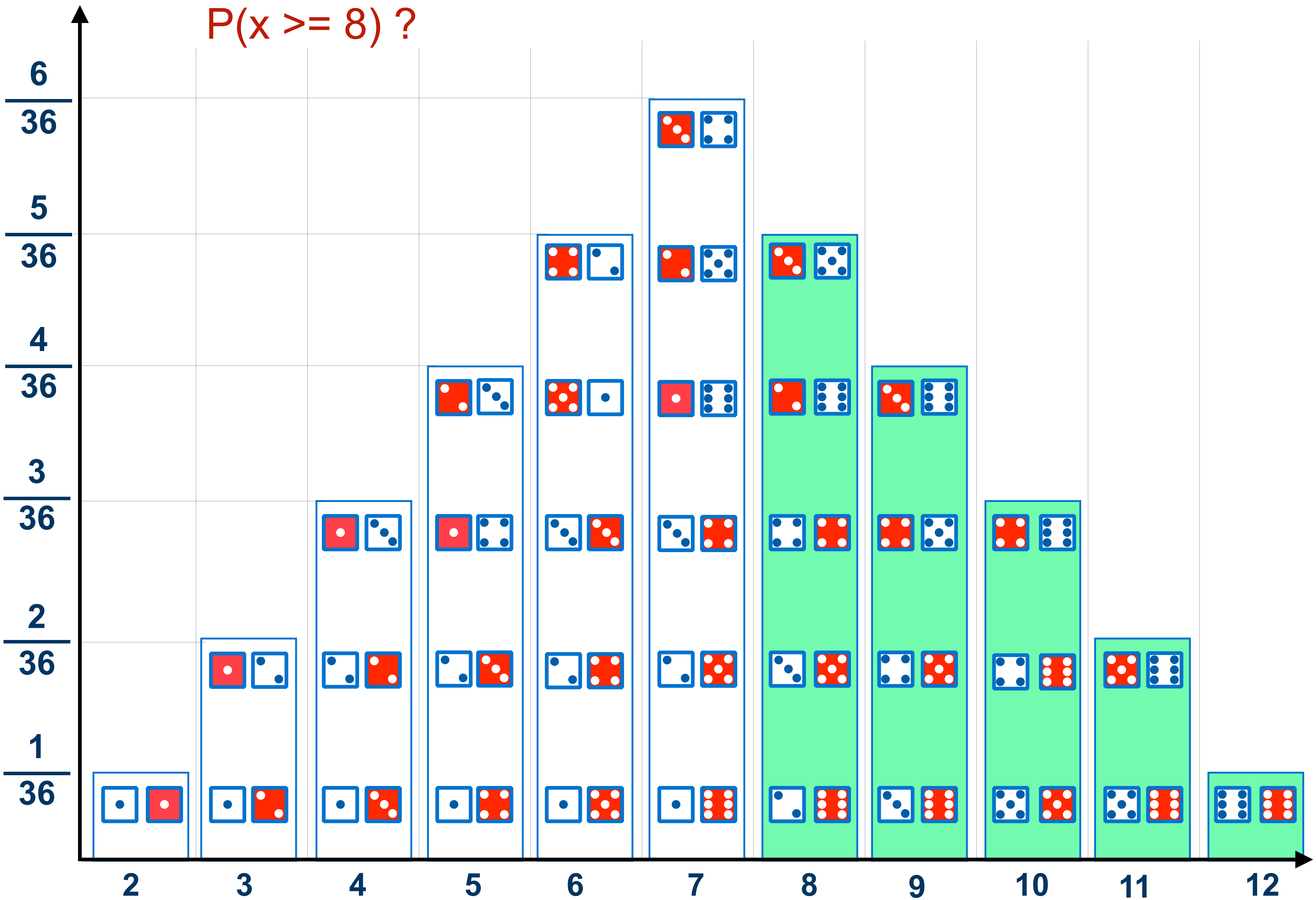

P(x >= 8) ?
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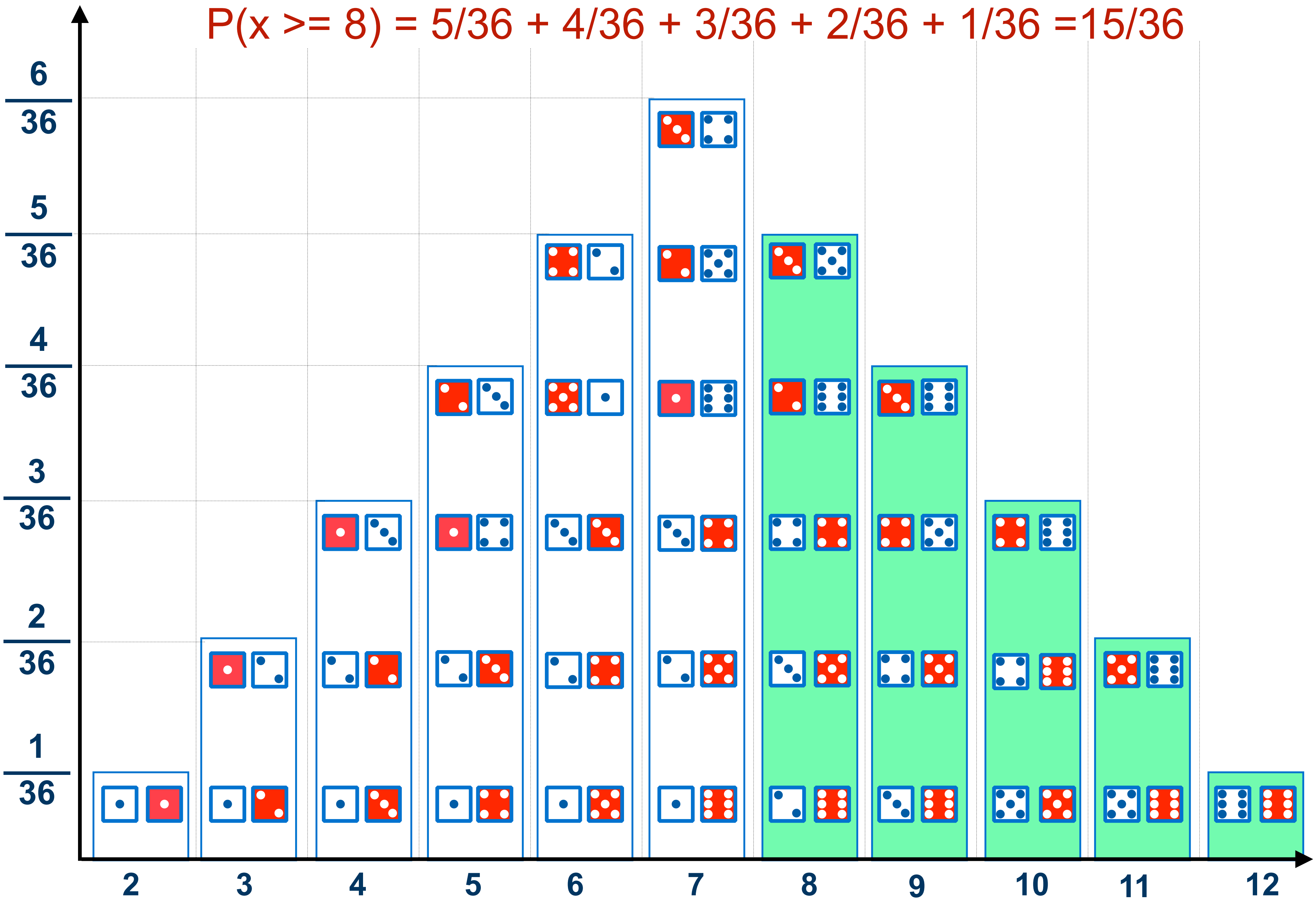

P(x >= 8) = 5/36 + 4/36 + 3/36 + 2/36 + 1/36 =15/36
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4/36
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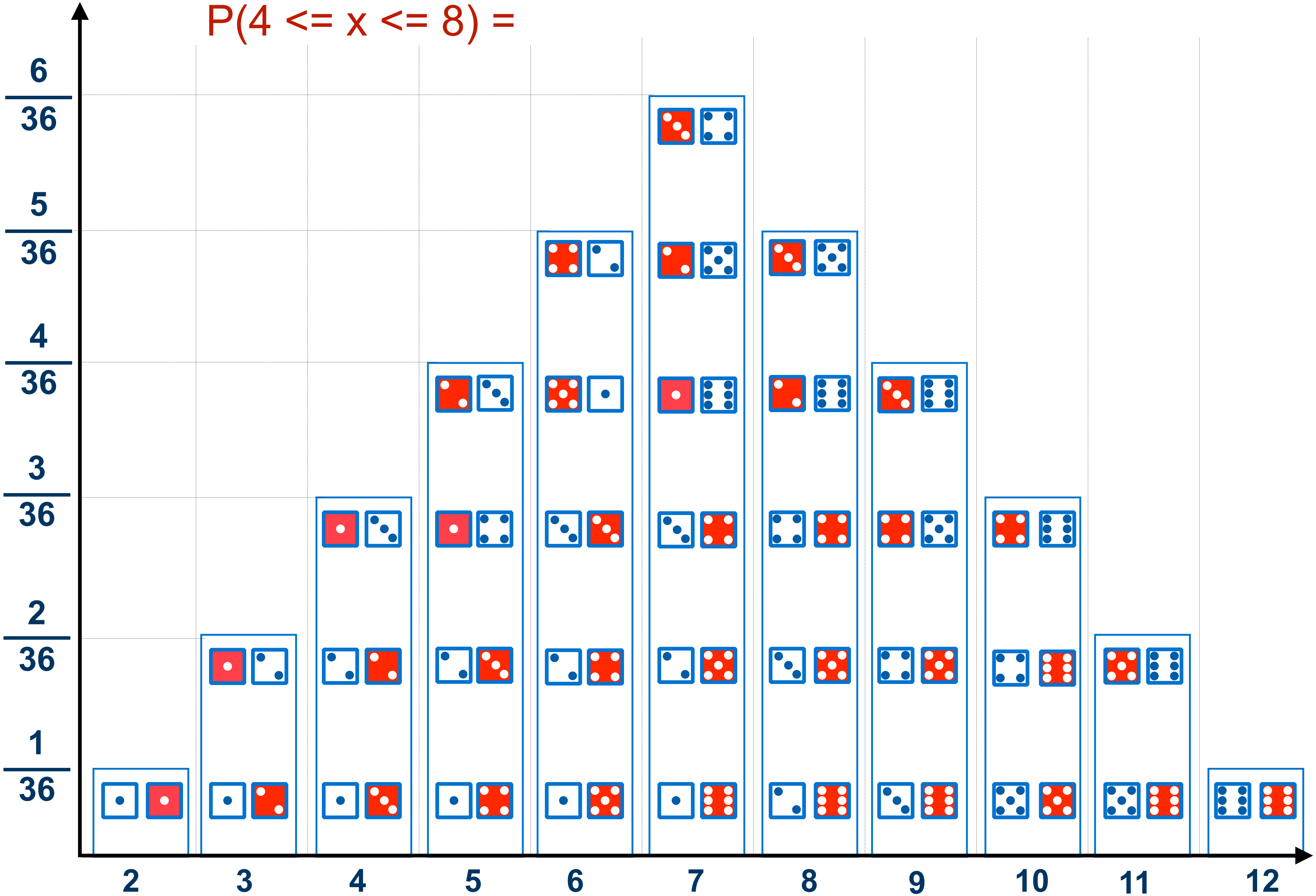
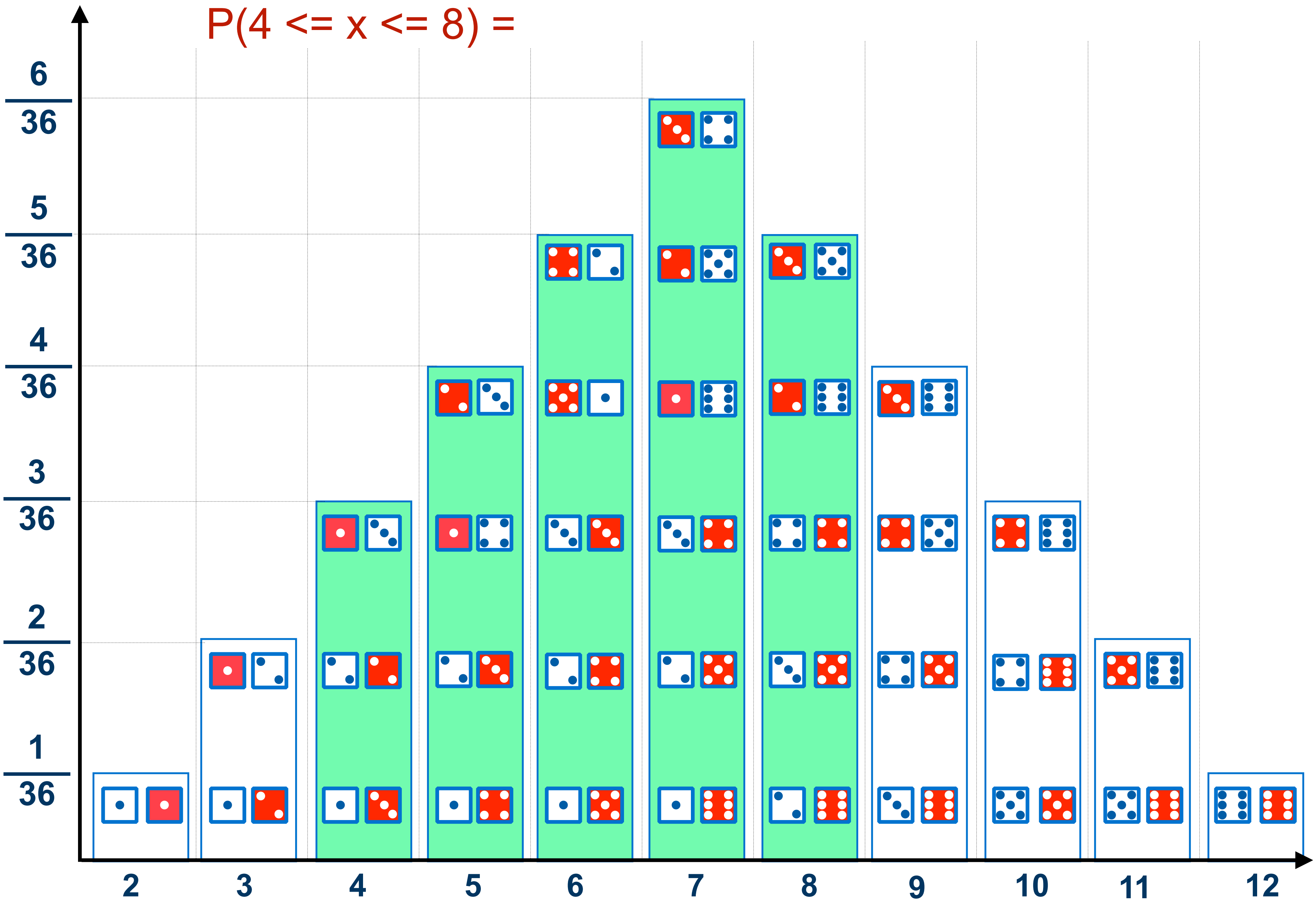
P(4 <= x <= 8) =





P(4 <= x <= 8) =
6/36
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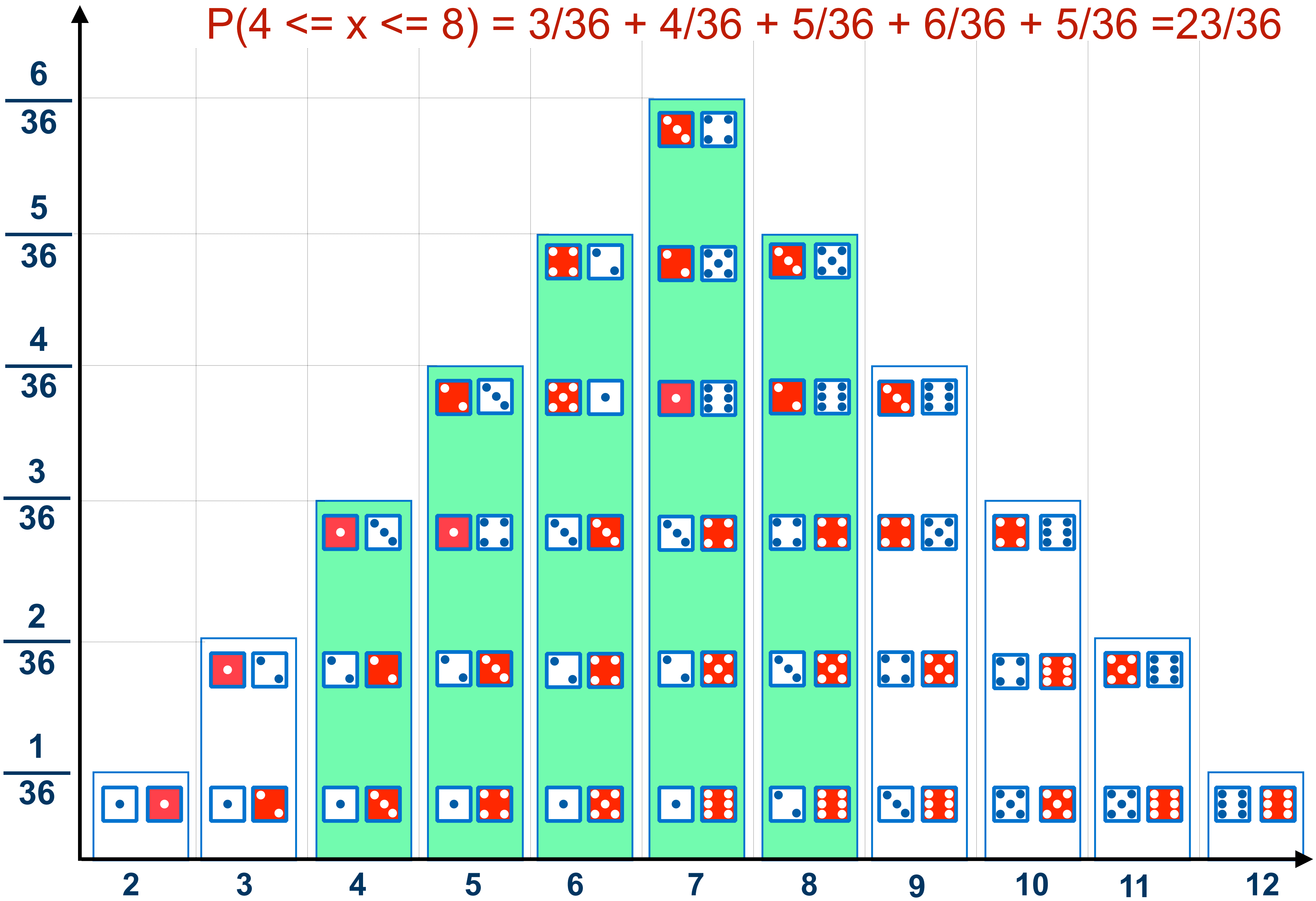

P(4 <= x <= 8) = 3/36 + 4/36 + 5/36 + 6/36 + 5/36 =23/36
6/36
5/36
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Example

The probability distribution function of a random variable, x, is given as:

X	2	2.5	3	3.5	4	4.5
P(X)	0.07	0.36	0.21	0.19	0.10	0.07

Calculate:

(i) $P(x = 3.5)$

(ii) $P(x \geq 3.5)$

(iii) $P(x < 4)$

(iv) $P(x > 3.5)$

(v) $P(x \leq 3.9)$

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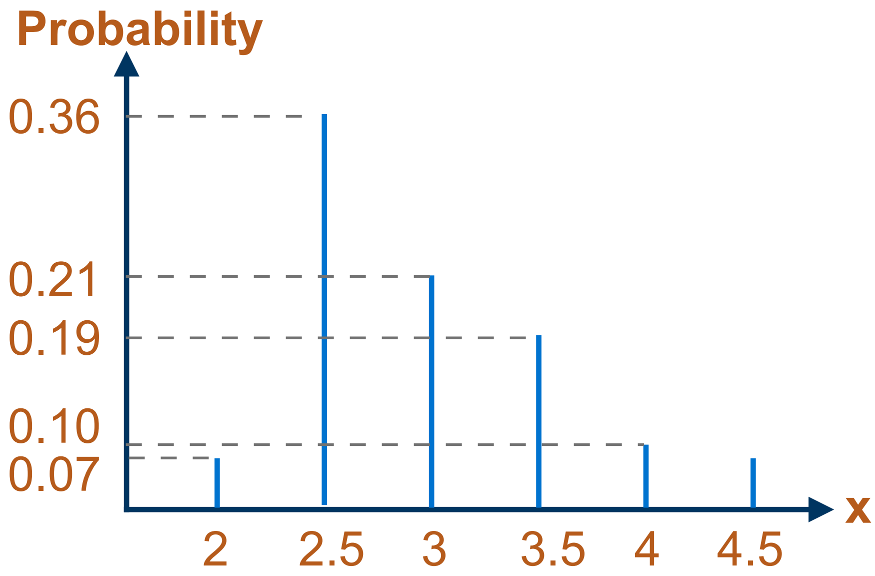

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(i) $P(x = 3.5)$ → $= 0.19$

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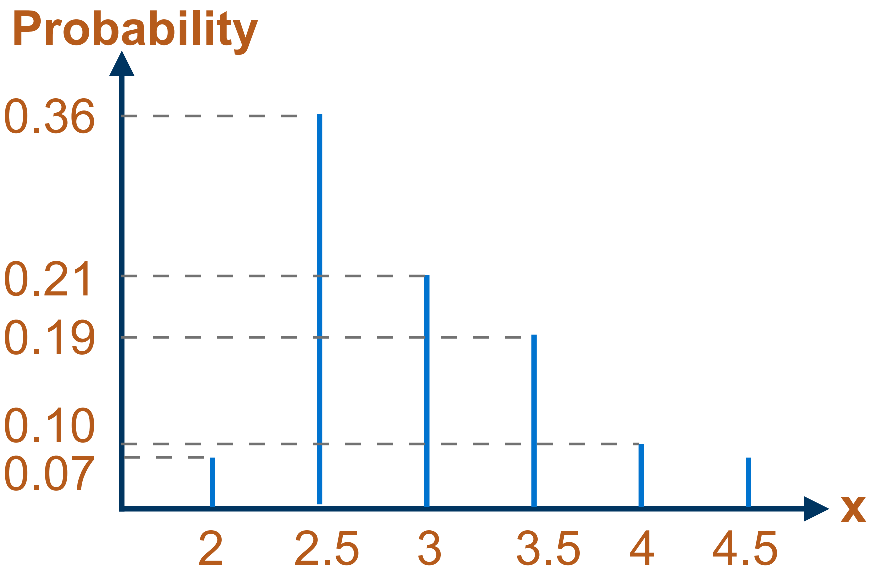

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(ii) $P(x \geq 3.5)$ → $= 0.19 + 0.1 + 0.07$

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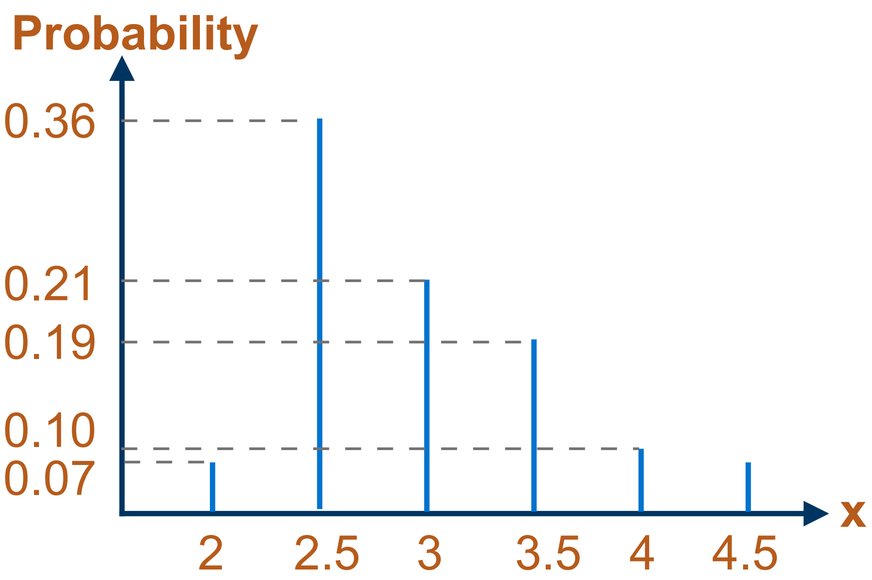

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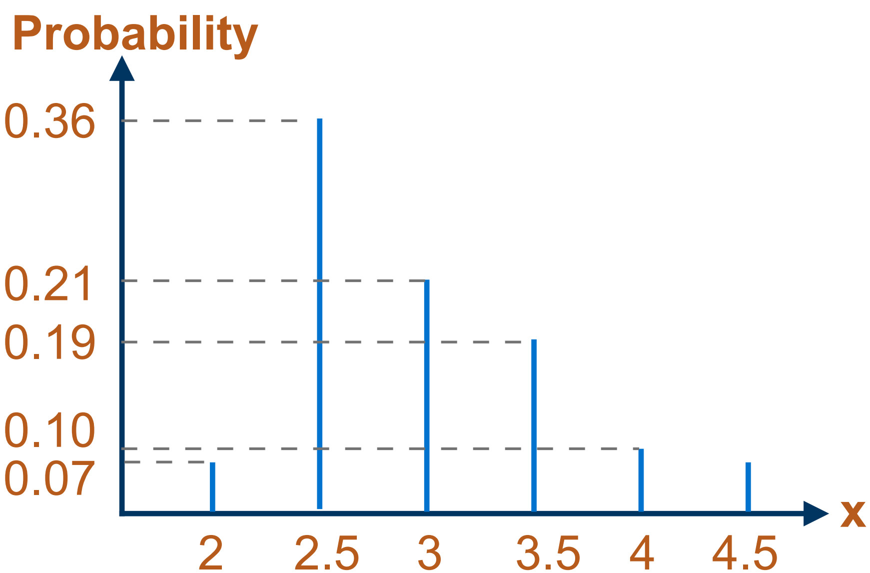

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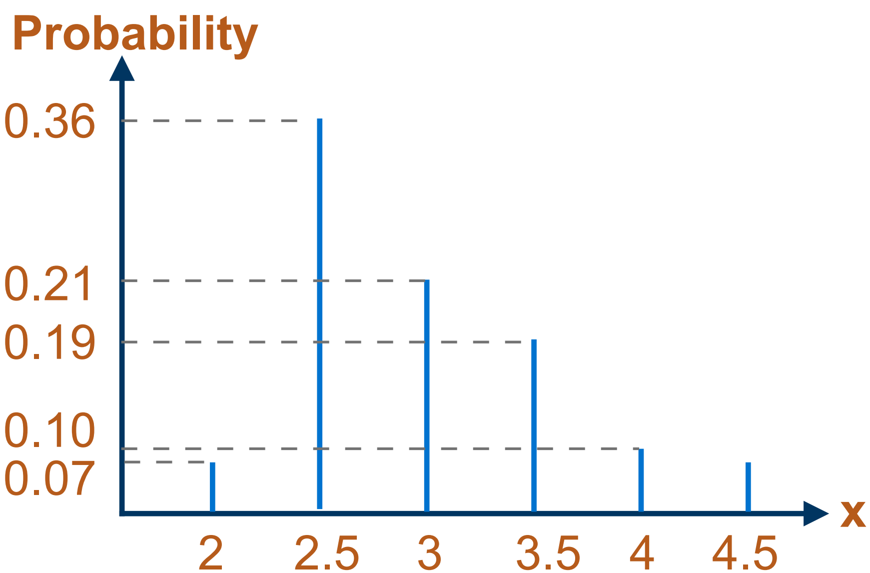

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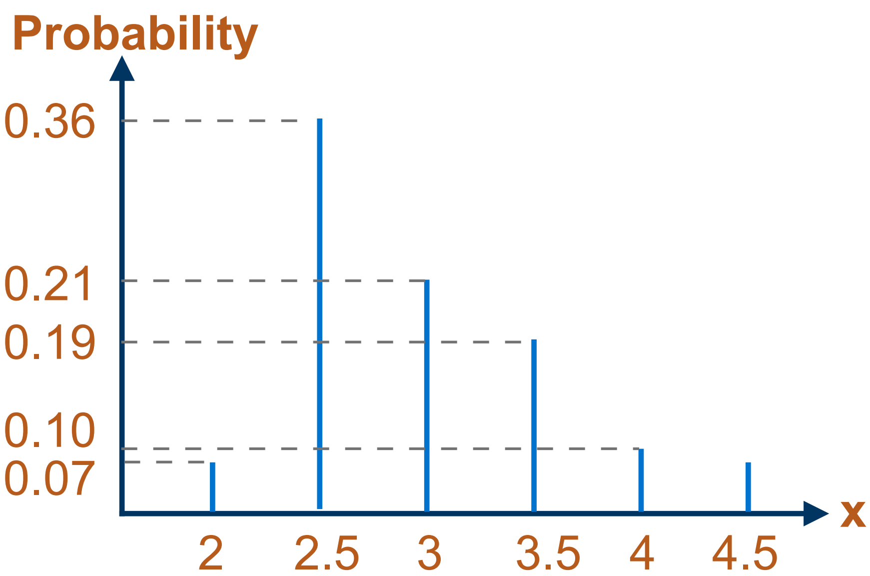

Standard Deviation (SD)

Matlab syntax: std(A)

Standard deviation (σ) is a measure of the **spread out** of a set of data from its **Mean**.

The more spread apart the data, the higher the deviation.

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Average

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$$

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Why 2?

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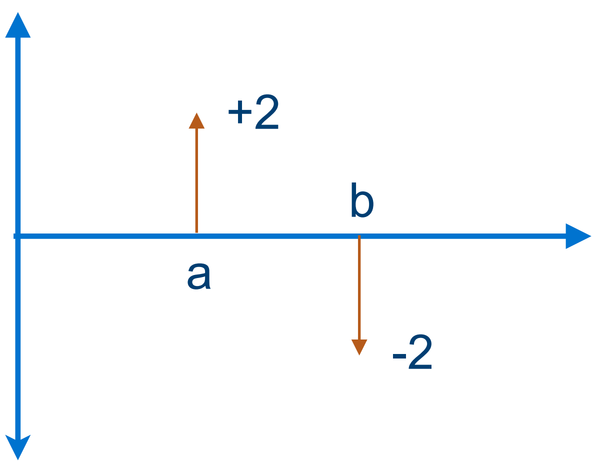
Why 2?

Average = (2 + -2) /2 = 0



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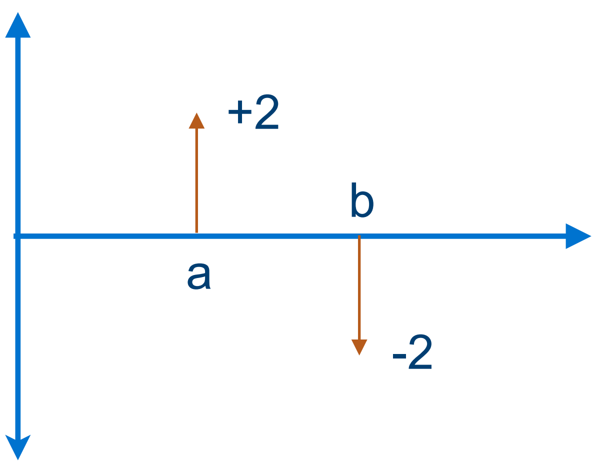


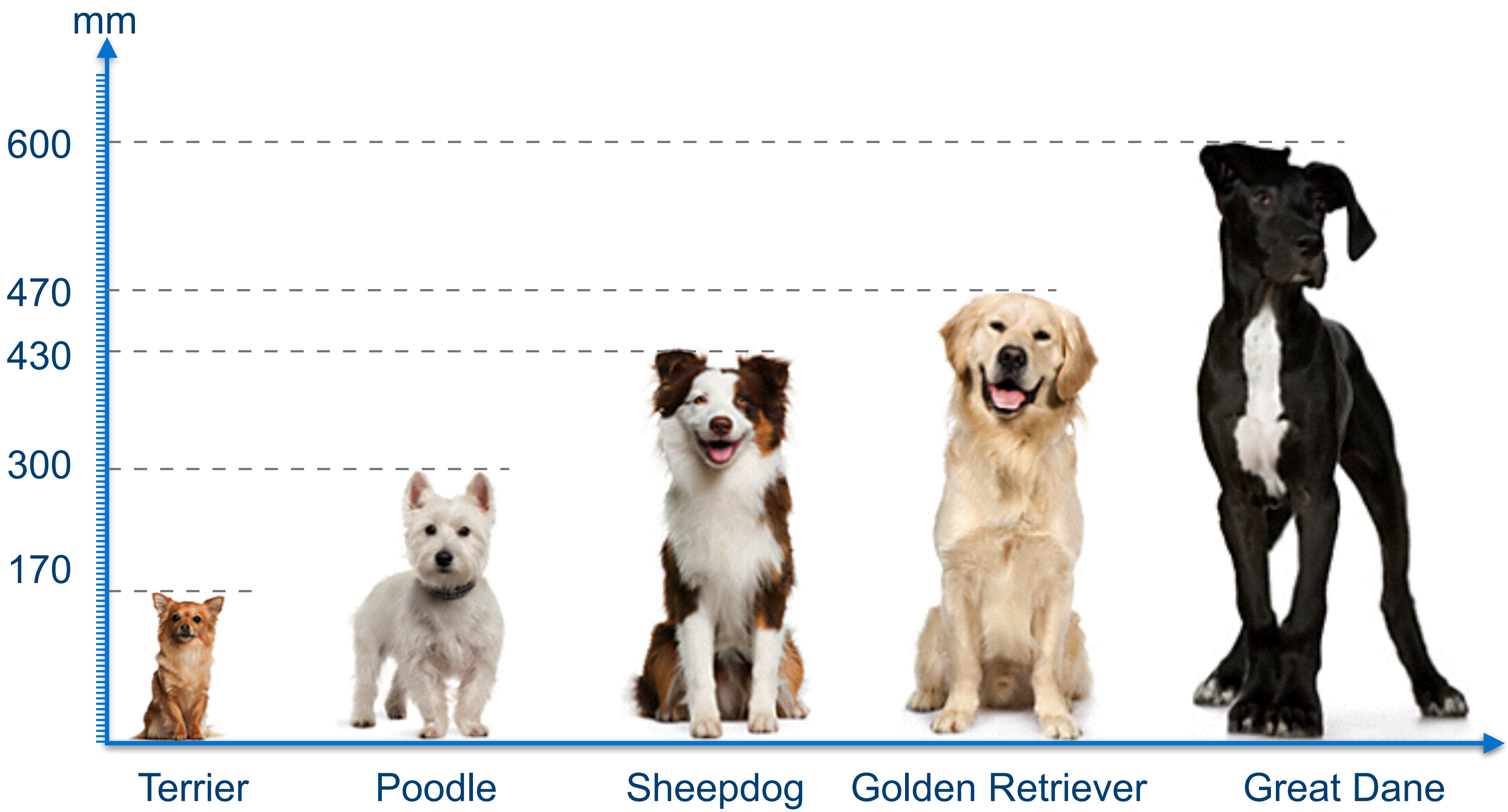
Average = (2 + -2) /2 = 0

$$\sigma = \sqrt{\frac{(2-0)+(-2-0)}{2}} = 0$$

$$\sigma = \sqrt{\frac{(2-0)^2+(-2-0)^2}{2}} = 4$$

Standard Deviation (SD)

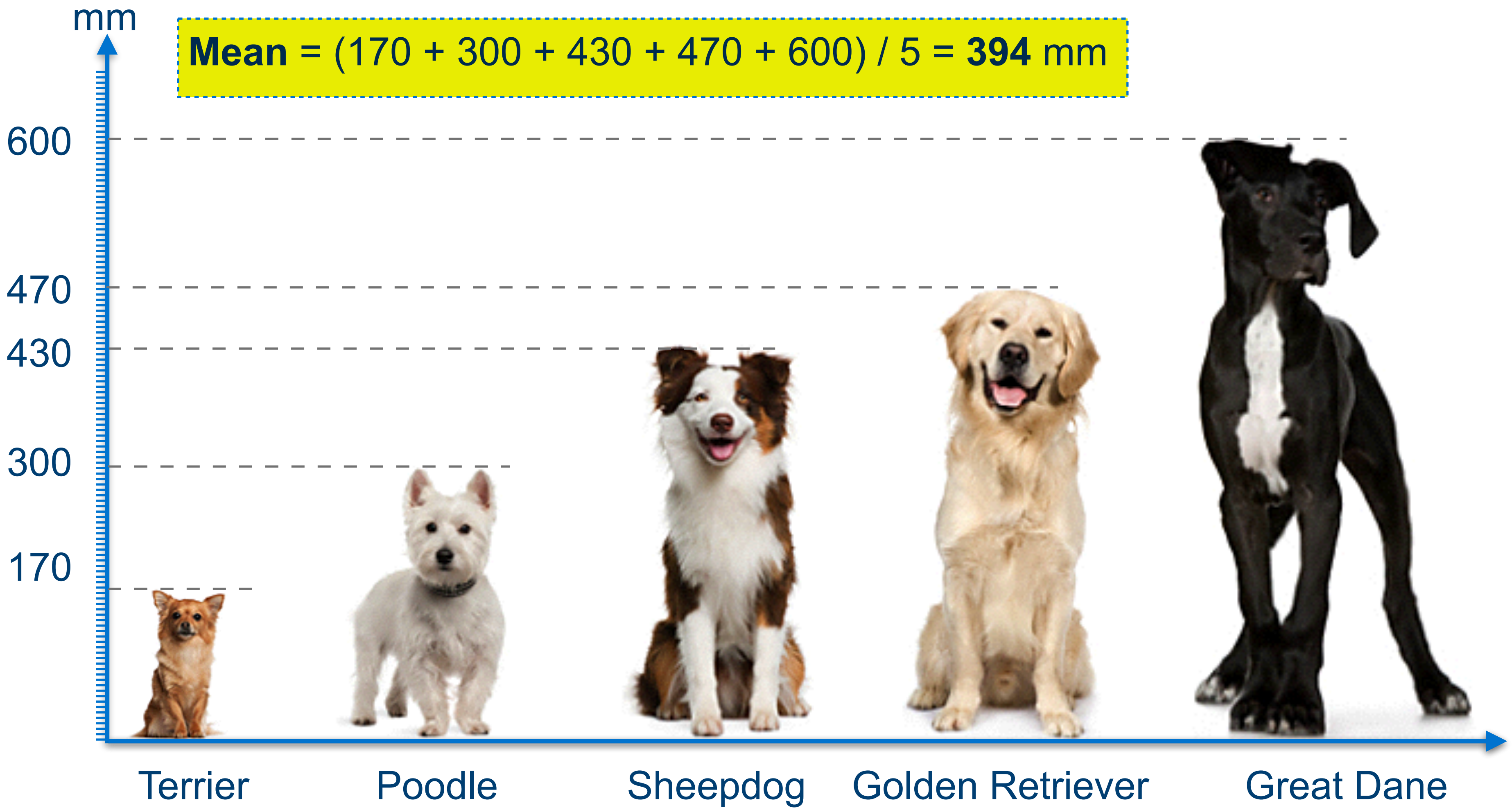
Matlab syntax: std(A)



Standard Deviation (SD)

Matlab syntax: std(A)

Mean = (170 + 300 + 430 + 470 + 600) / 5 = **394** mm



Standard Deviation (SD)

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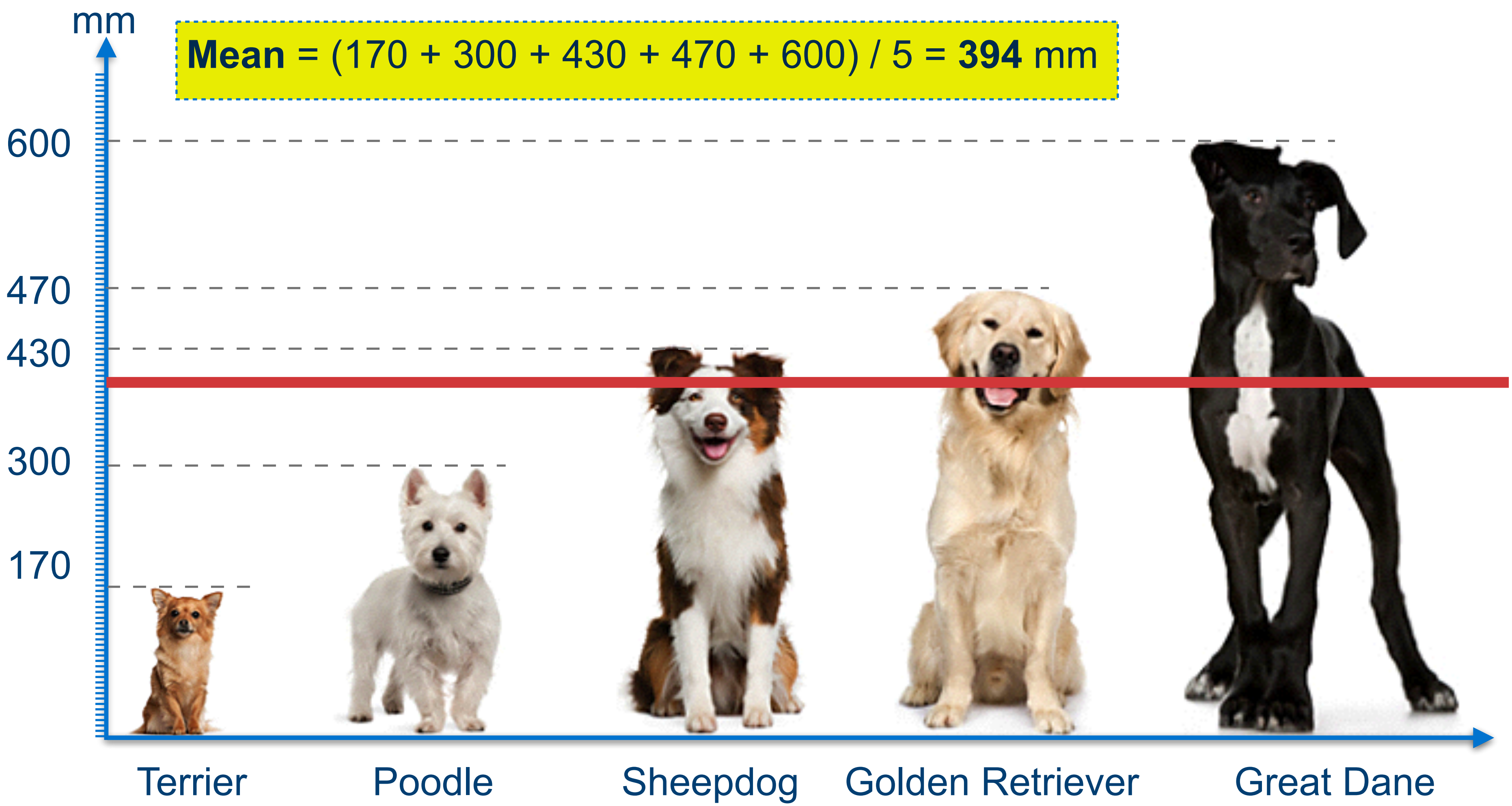

Standard Deviation (SD)

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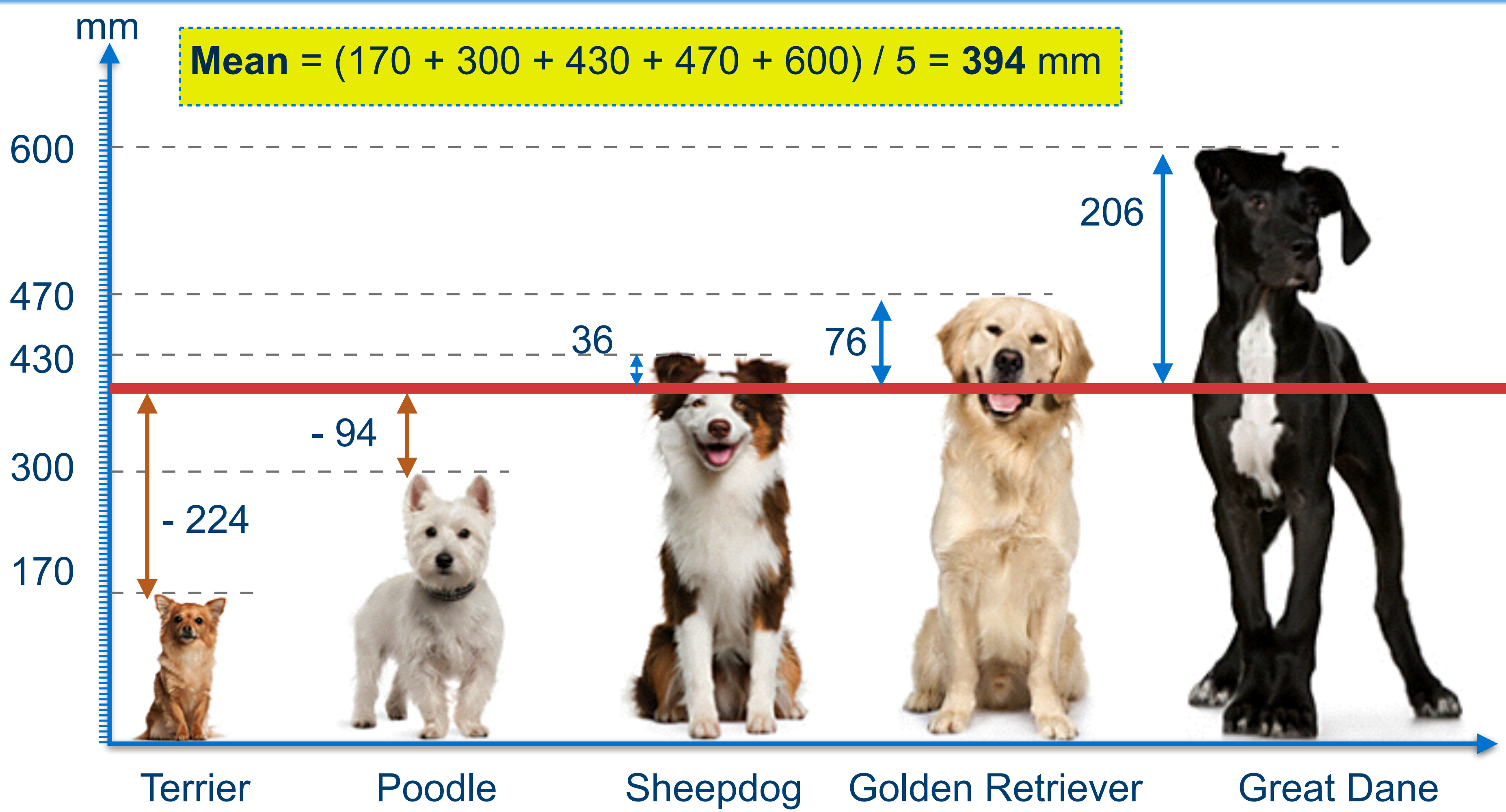
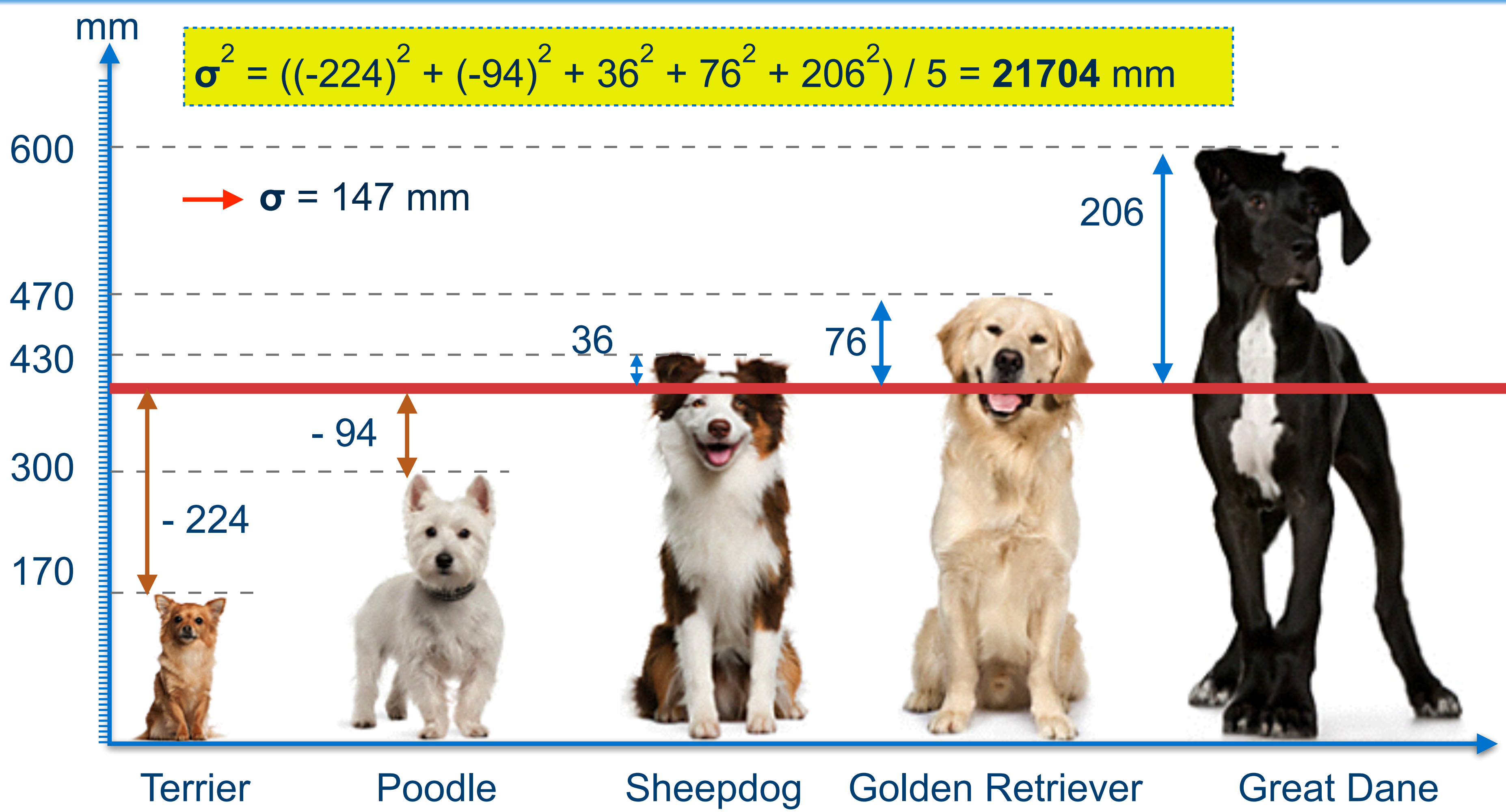


Standard Deviation (SD)

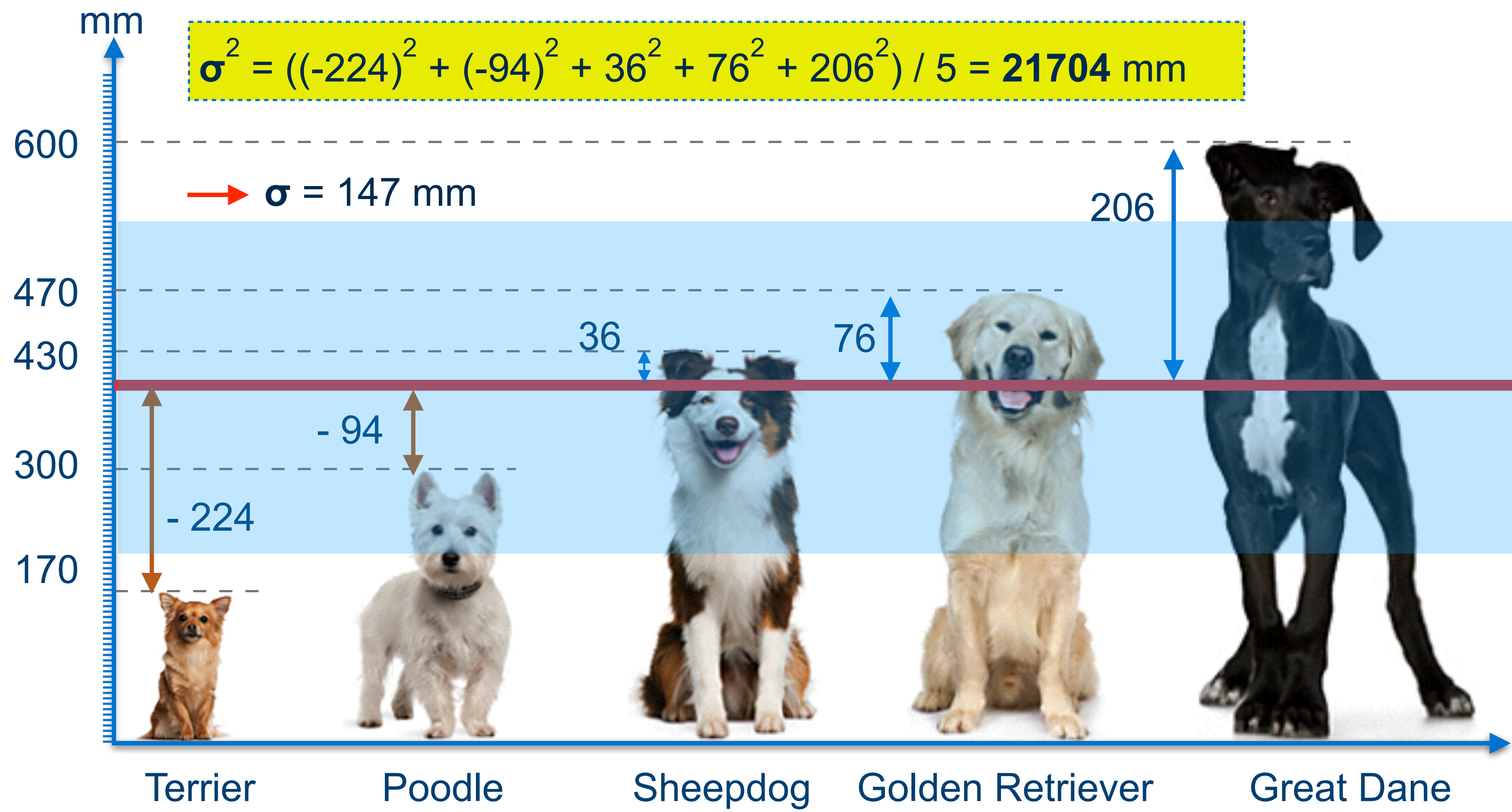
Matlab syntax: std(A)

$\sigma^2 = ((-224)^2 + (-94)^2 + 36^2 + 76^2 + 206^2) / 5 = \mathbf{21704}$ mm



Standard Deviation (SD)

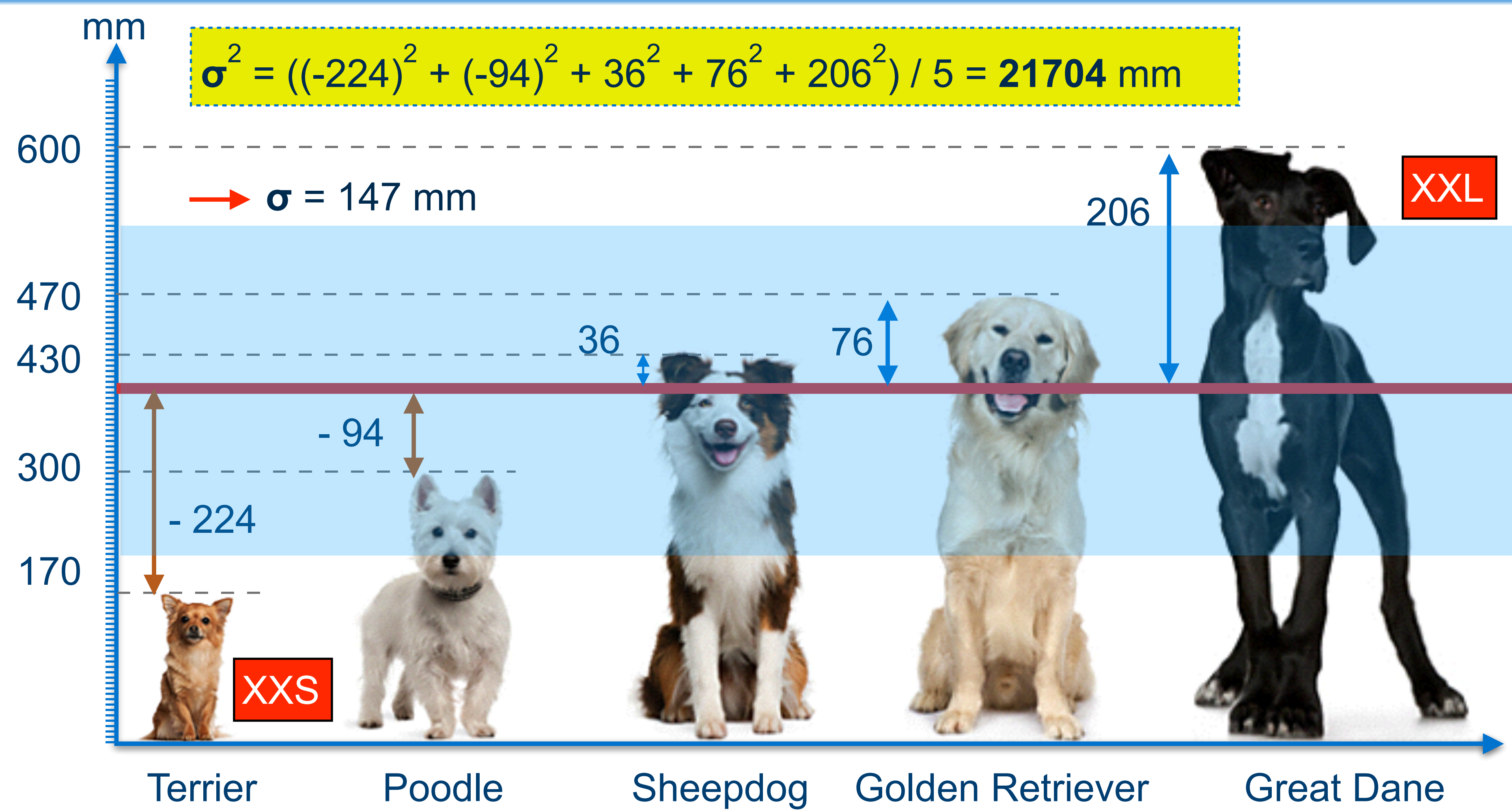
Matlab syntax: std(A)



Standard Deviation (SD)

Matlab syntax: std(A)

$$\sigma^2 = ((-224)^2 + (-94)^2 + 36^2 + 76^2 + 206^2) / 5 = \mathbf{21704} \text{ mm}$$



Standard Deviation Application: Machine Learning

Image Processing:

In Image Processing, you train your system with some initial Data and then the system:

1- Calculates the Standard deviation of the new inputs

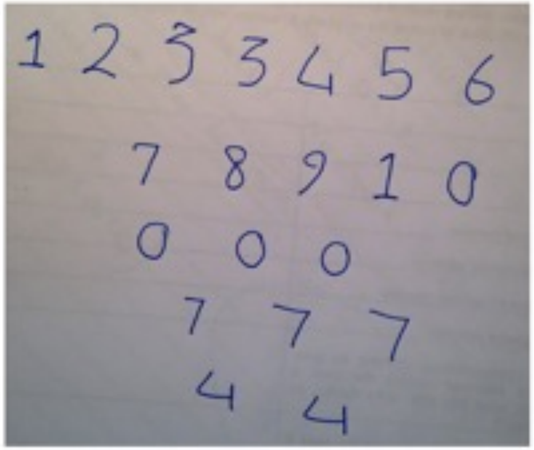
2- Compares it with the data it has into the system.

3- Maps the data to find the closet data that it has in its database.

Example: Face recognition, Handwriting

Lets do some Machine Learning:

Can computer recognise my hand-writing?



My Hand Writing

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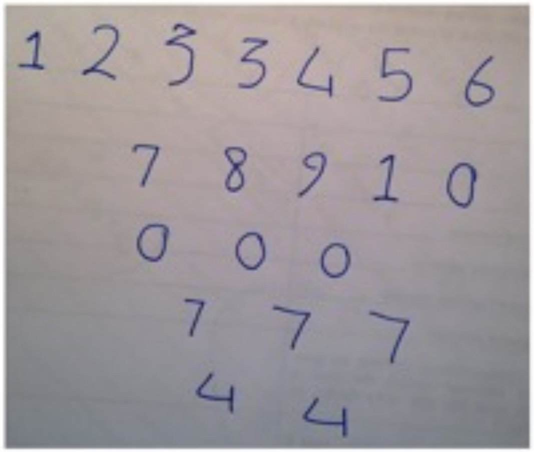

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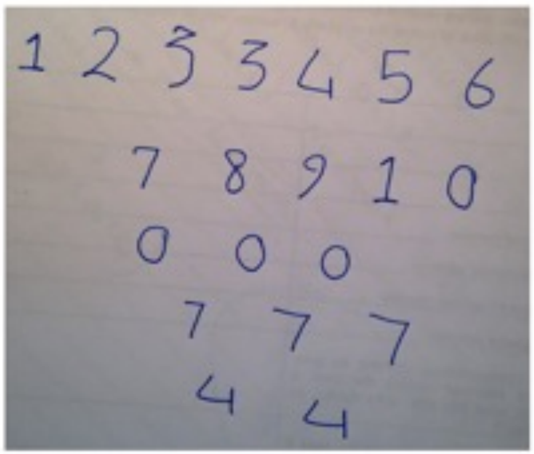

My Hand Writing

Machine-Learning's recognition

The Machine Learning Algorithm compares the Standard Deviation of each Character by its Training Data to distinguish the characters.

Standard Deviation in Probability

In probability, the Standard deviation is similar to the mathematic equation, however, the **average value** will be replaced by the **Expected Value**.

$$\bar{x} \Longrightarrow E(x)$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}$$

$$\sigma = \sqrt{\sum_{i=1}^{n} p_i (x_i - E[x])^2}$$

Example

A software company tested a new product of theirs and found that the number of errors per 100 CDs of the new software had the following probability distribution:

X	2	3	4	5	6
P(X)	0.01	0.25	0.4	0.3	0.04

Find the Standard Deviation of X:

Example

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$$E(x) = [(2 \times 0.01) + (3 \times 0.25) + (4 \times 0.4) + (5 \times 0.3) + (6 \times 0.04)] = 4.11$$

Example

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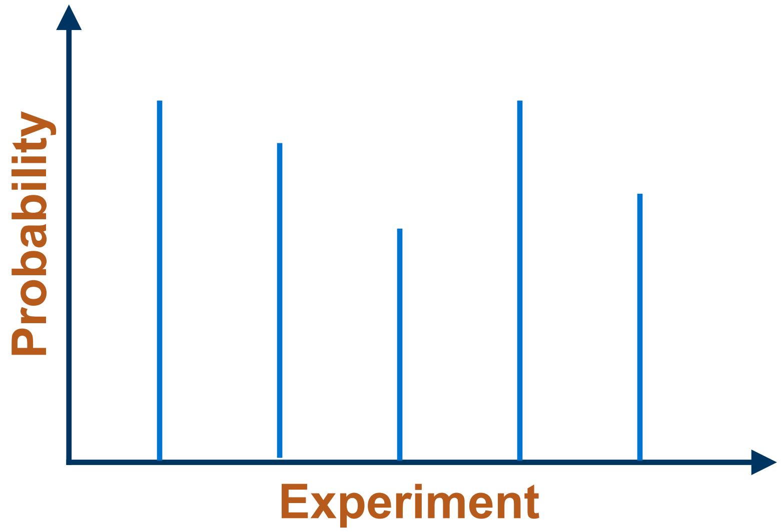
$$\sigma = \sqrt{(2 - 4.11)^2(0.01) + (3 - 4.11)^2(0.25) + (4 - 4.11)^2(0.4) + (5 - 4.11)^2(0.3) + (6 - 4.11)^2(0.04)}$$

$$\sigma = \sqrt{0.74} = 0.86$$

Continuous Random Variables

Many practical random variables are modelled as Continuous:

1- Speed of a car
2- Measurement Error
3- Electricity Consumption



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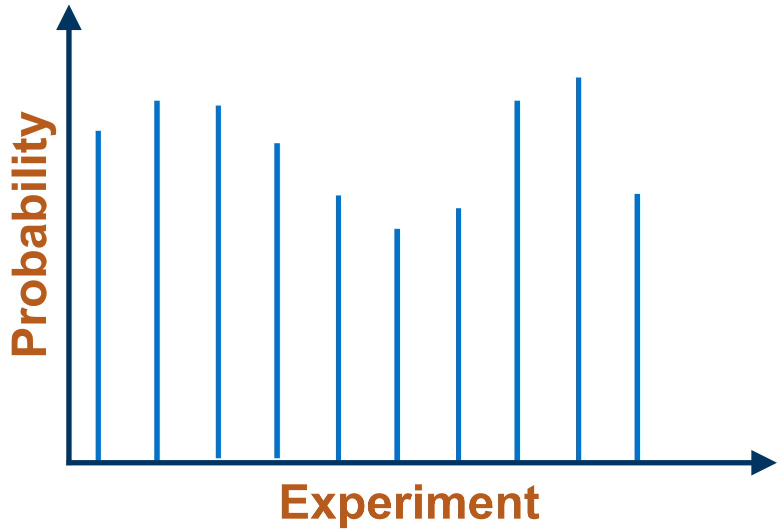

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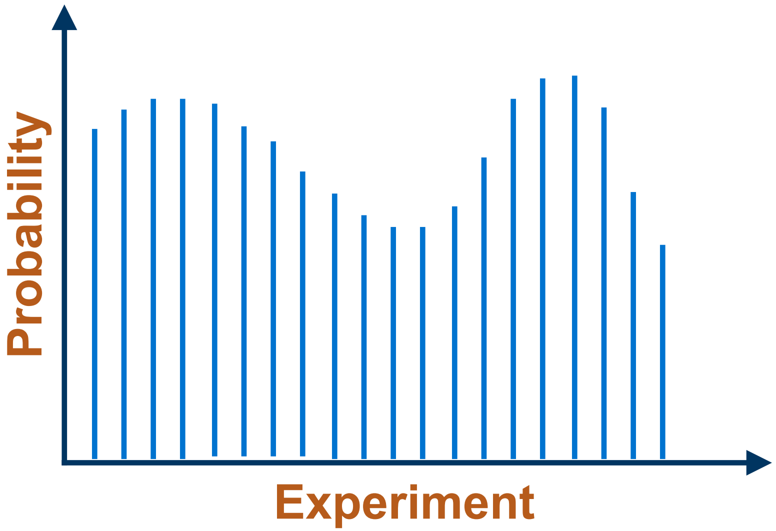

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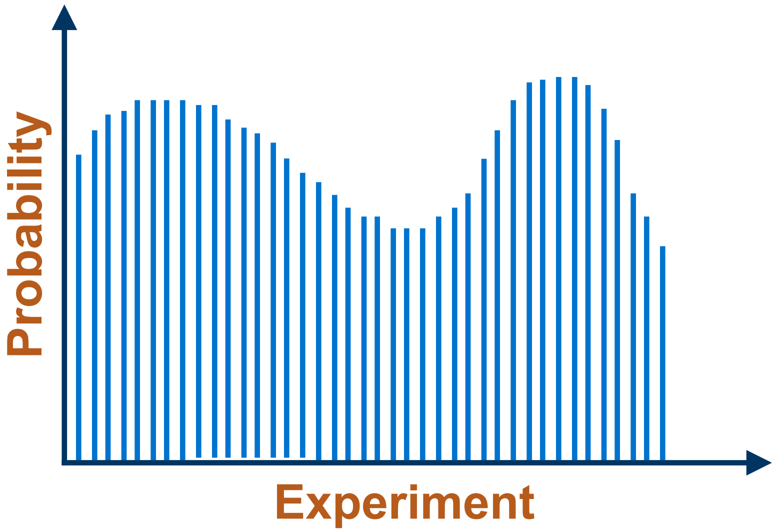

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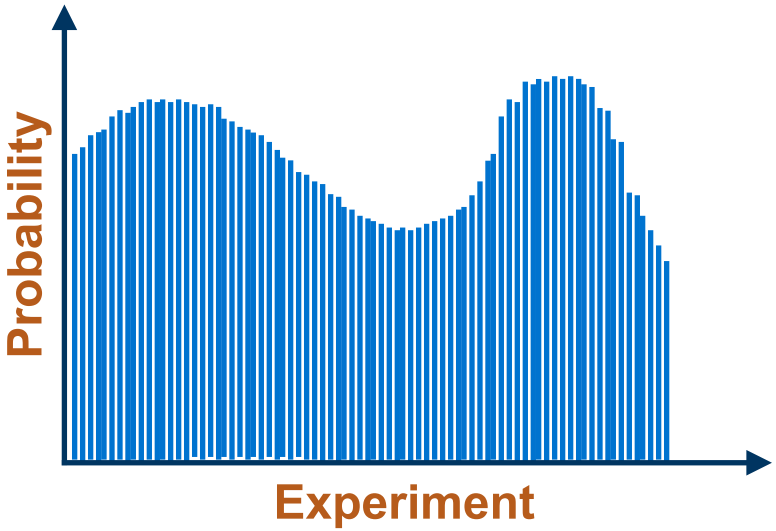

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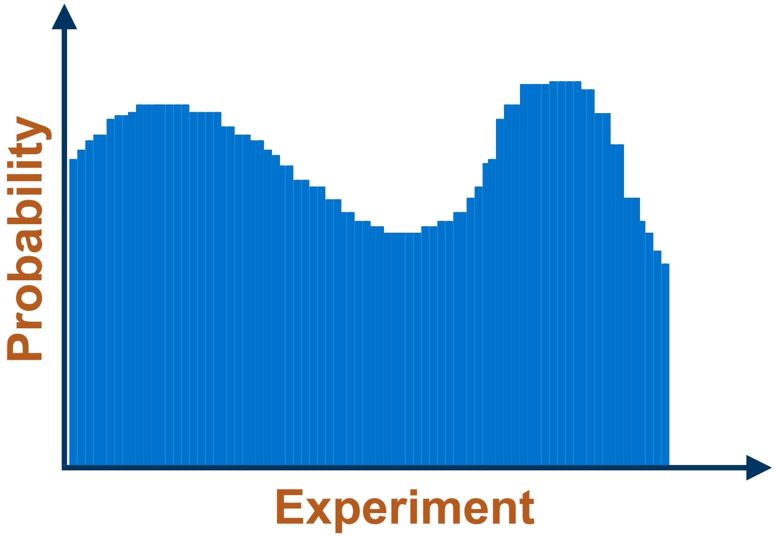

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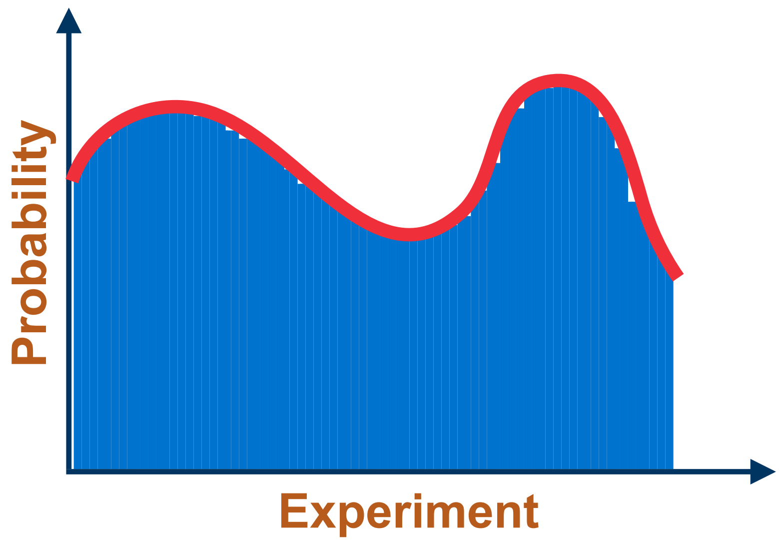

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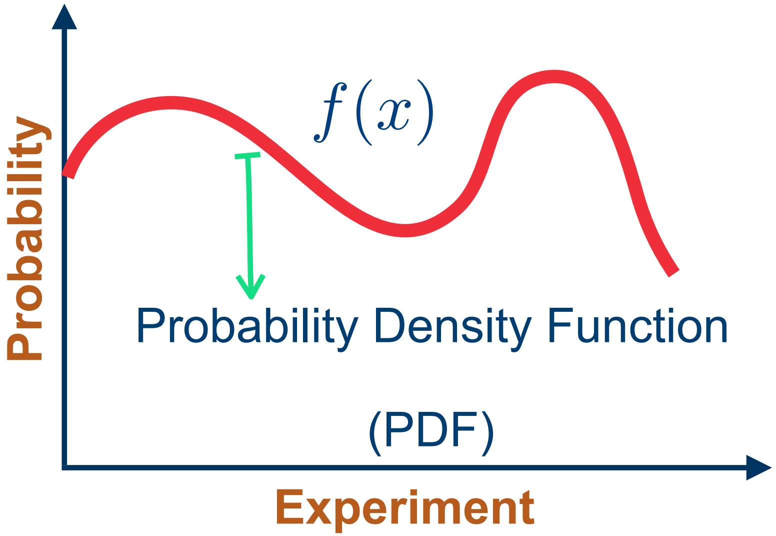
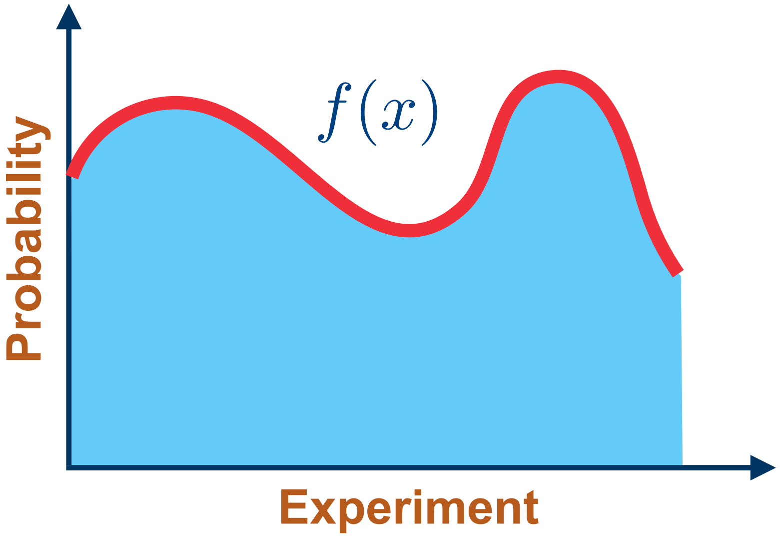


Continuous Random Variables

Probability function:

$$\int_{-\infty}^{\infty} f(x)dx = 1$$



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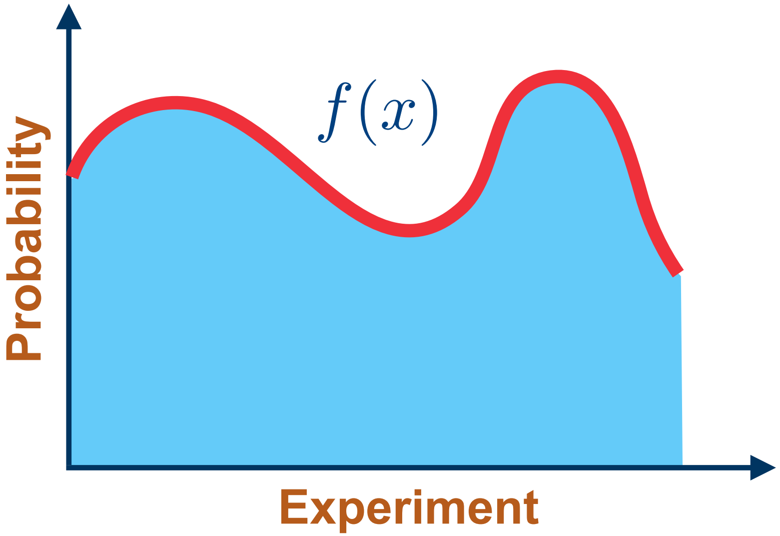

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$$E(x) = \sum_{i=1}^{n} x_i P_i$$

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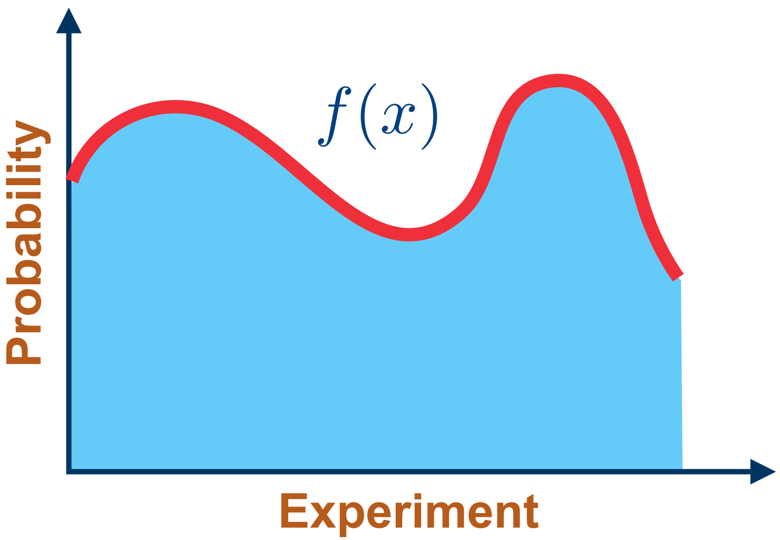

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$$E(x) = \sum_{i=1}^{n} x_i P_i$$

$$\mu = E[x] = \int_{-\infty}^{+\infty} x\, f(x)\, dx$$

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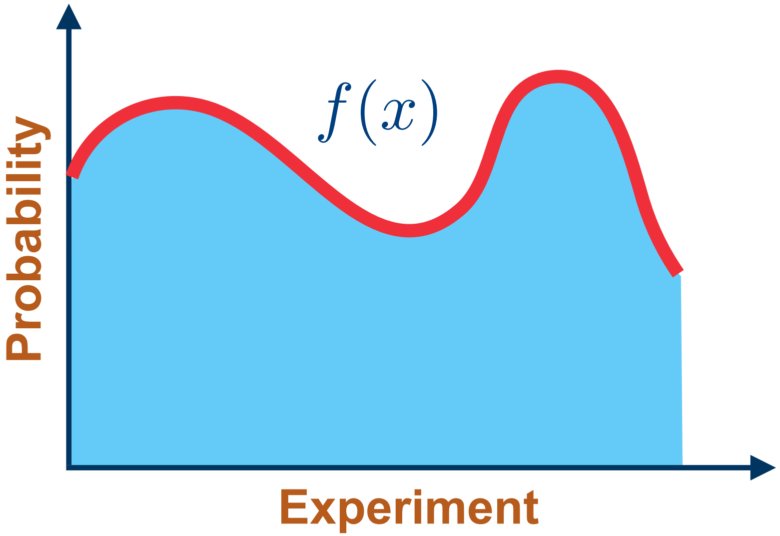

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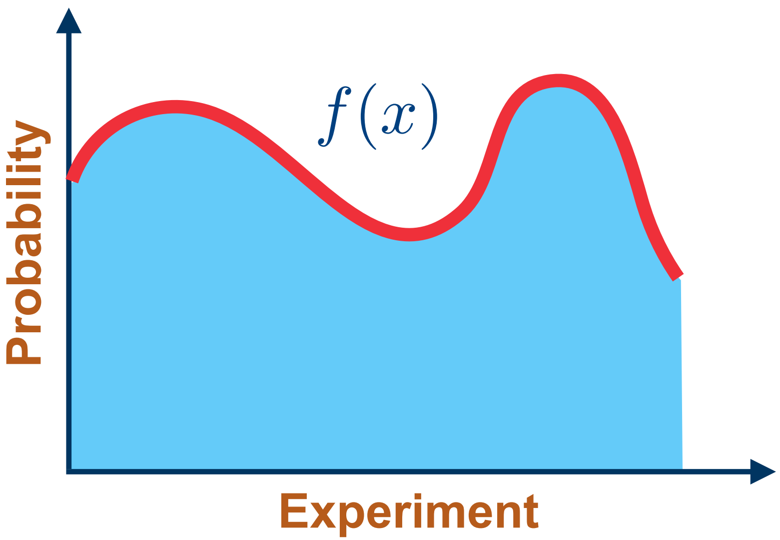

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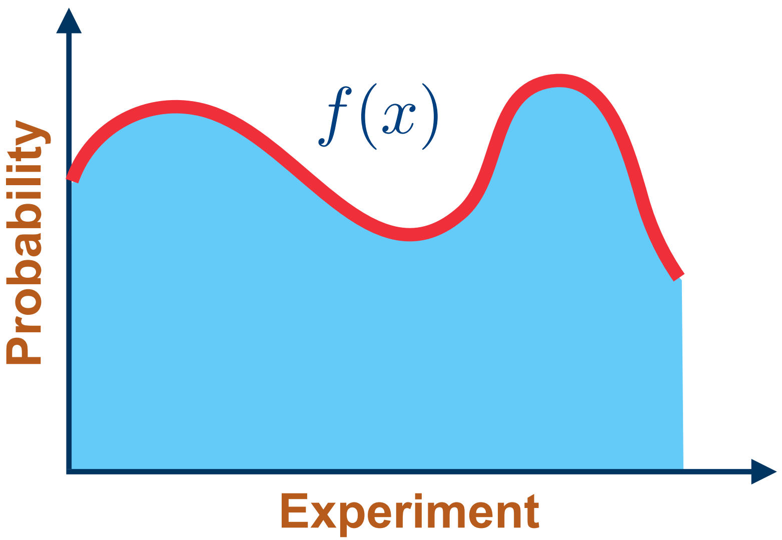

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Standard Deviation:

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OR

$$\sigma = \sqrt{E(x^2) - E(x)^2}$$

Example:

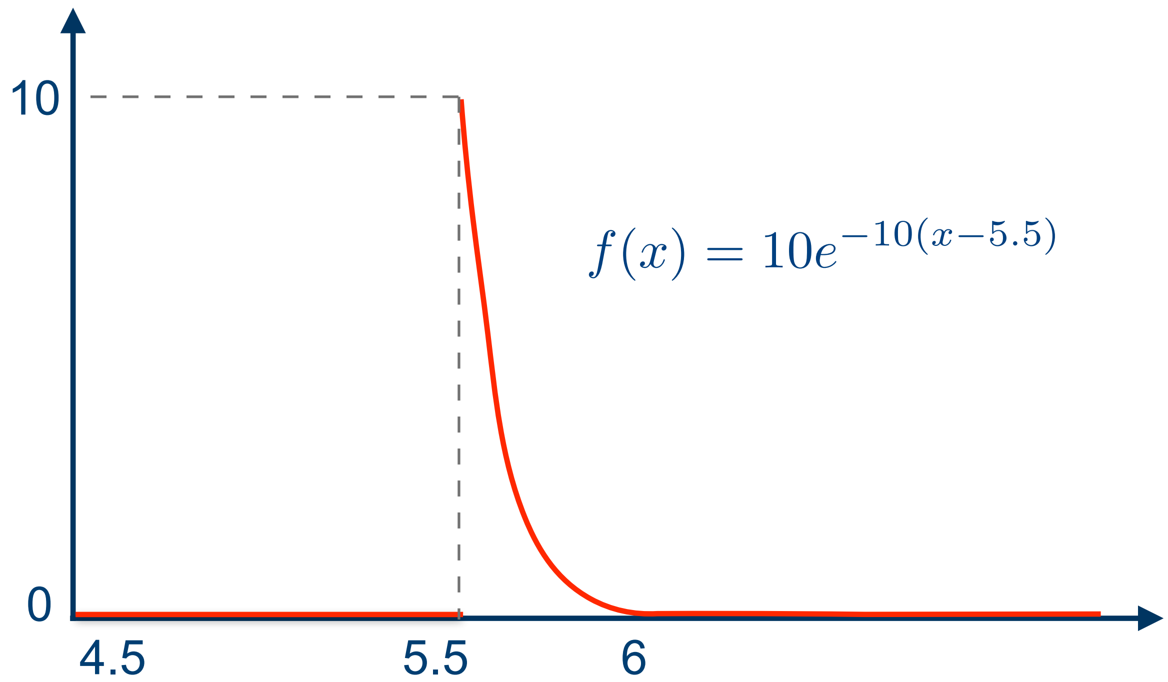
Let X denote the width of metal pipes from an automated production line. If X has the probability density function **$f(x)=0$** for $x<5.5$, **$f(x)=10e^{-10(x-5.5)}$** for $x \geq 5.5$.

Determine:

(i) $P(X < 5.7)$

(ii) $P(X > 6)$

(iii) $P(5.6 < X \leq 6)$



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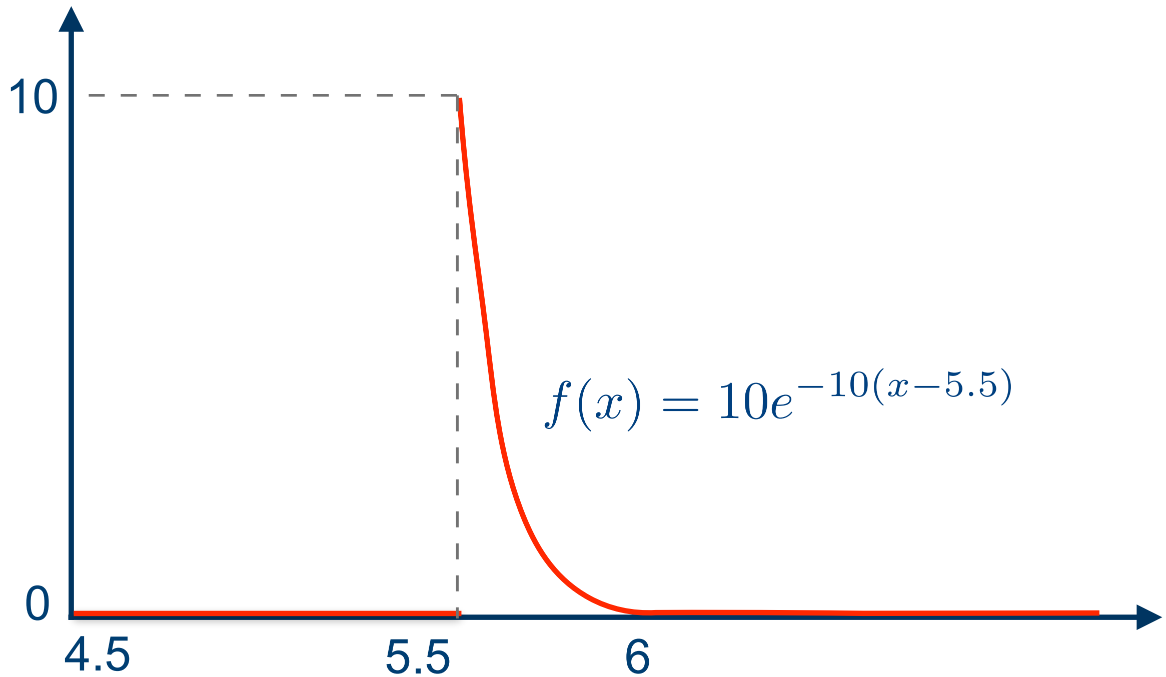

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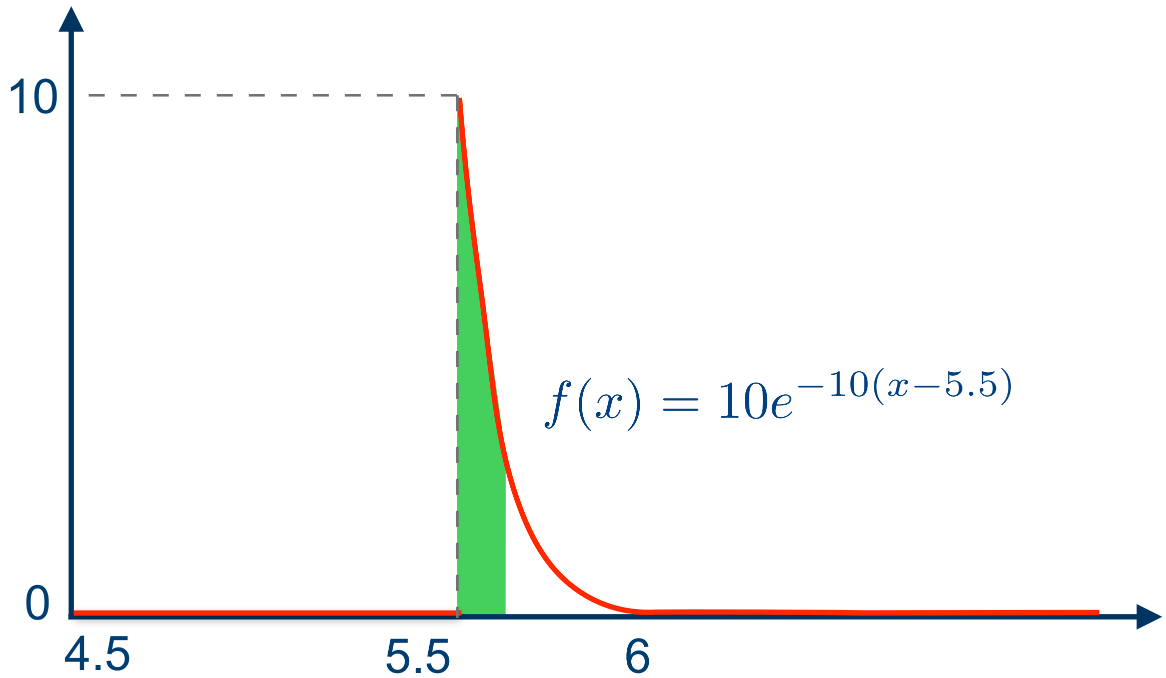

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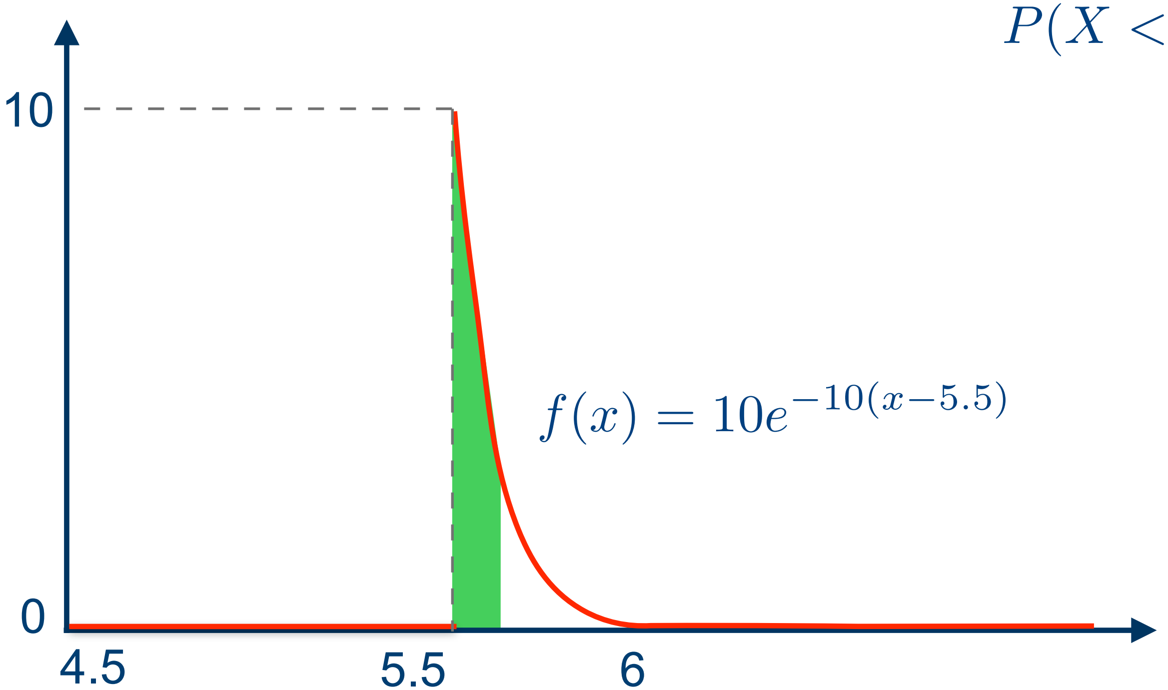

$$P(X < 5.7) = \int_{5.5}^{5.7} 10e^{-10(x-5.5)}dx$$

$$= -e^{-10(x-5.5)}|_{5.5}^{5.7}$$

$$= 1 - e^{-2} = 0.865$$

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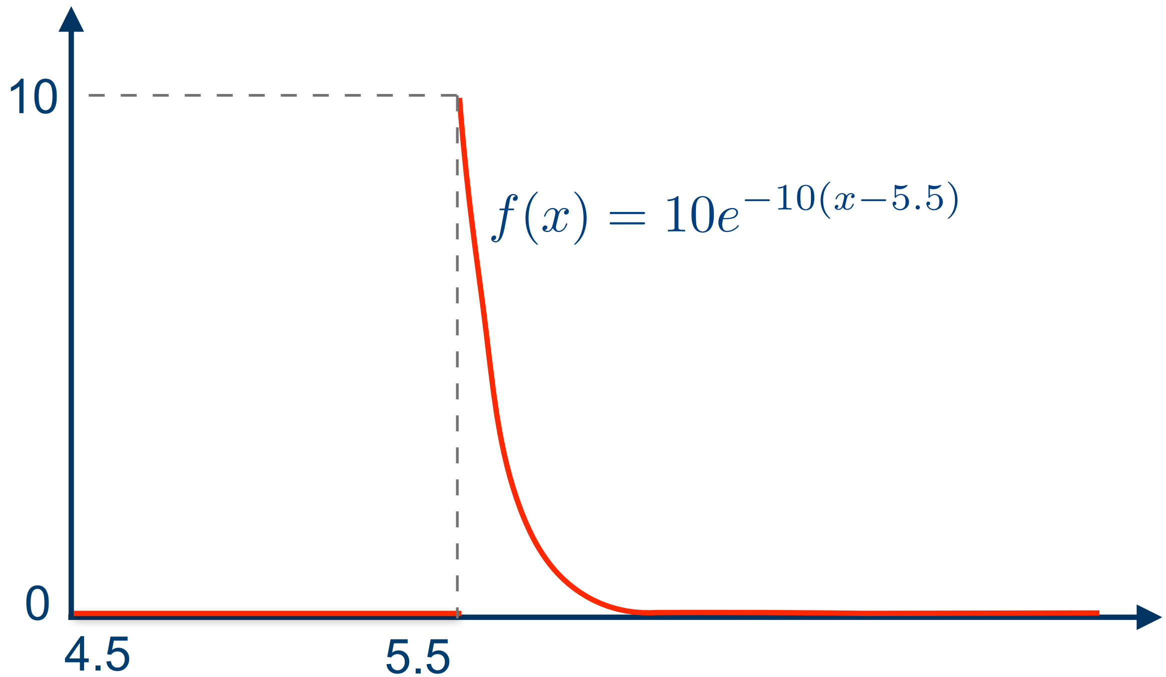

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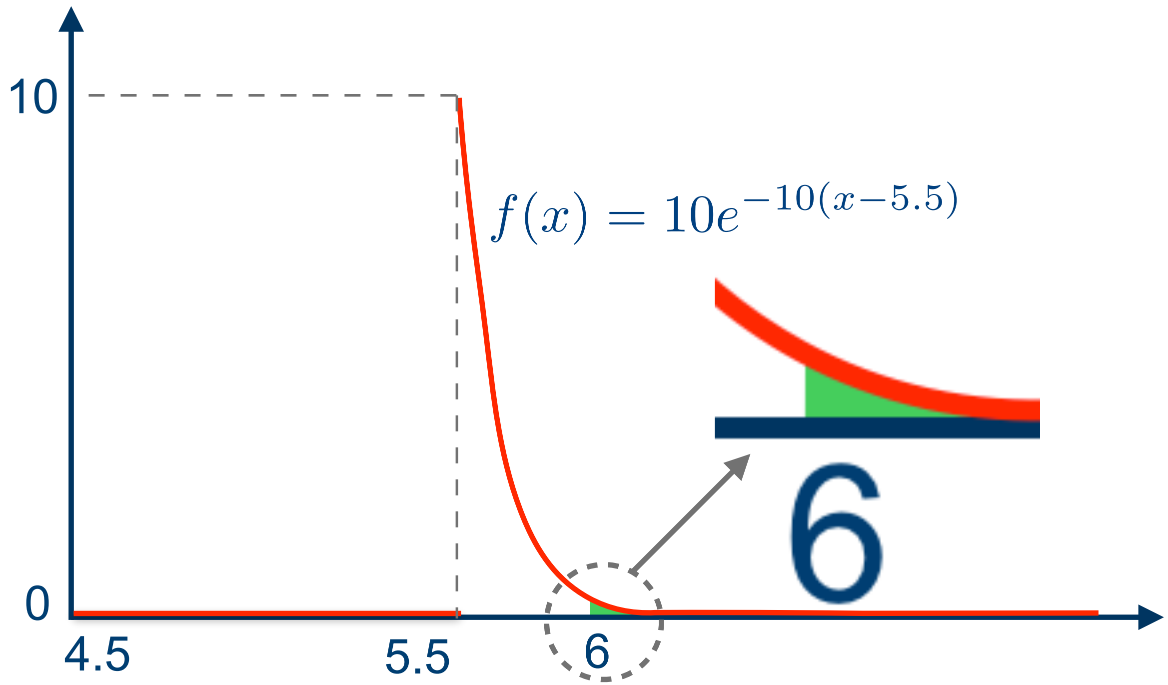

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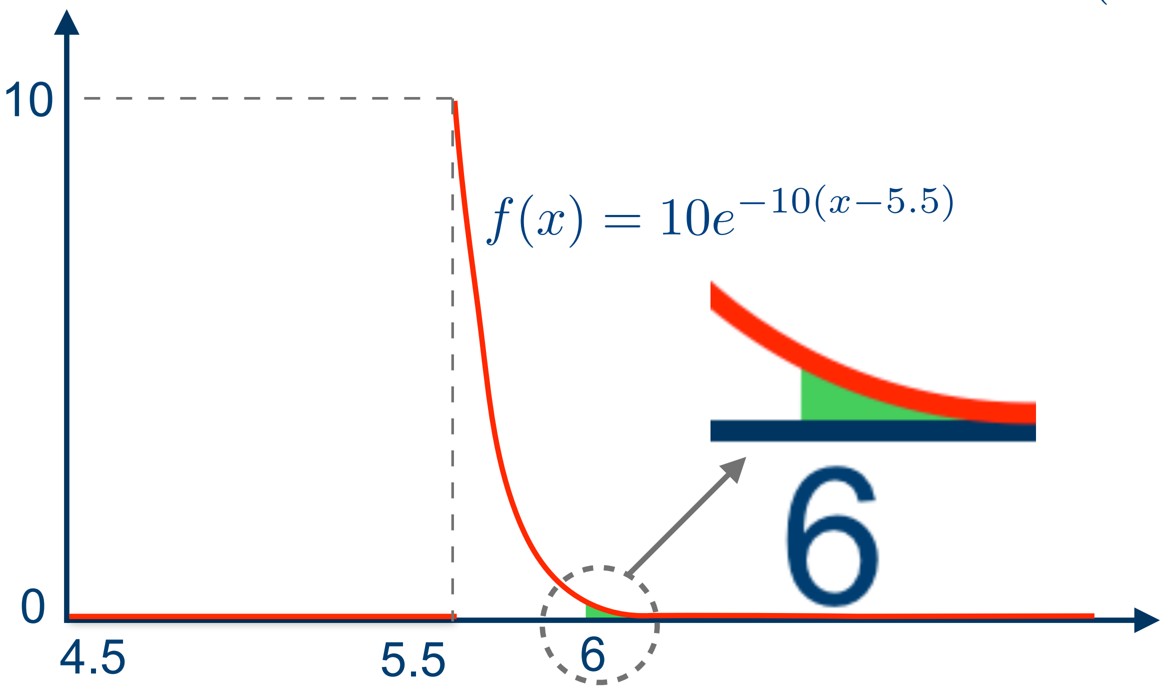

$$P(X > 6) = \int_6^{+\infty} 10e^{-10(x-5.5)}dx$$

$$= -e^{-10(x-5.5)}|_6^{\infty}$$

$$= e^{-5} = 0.007$$

Example:

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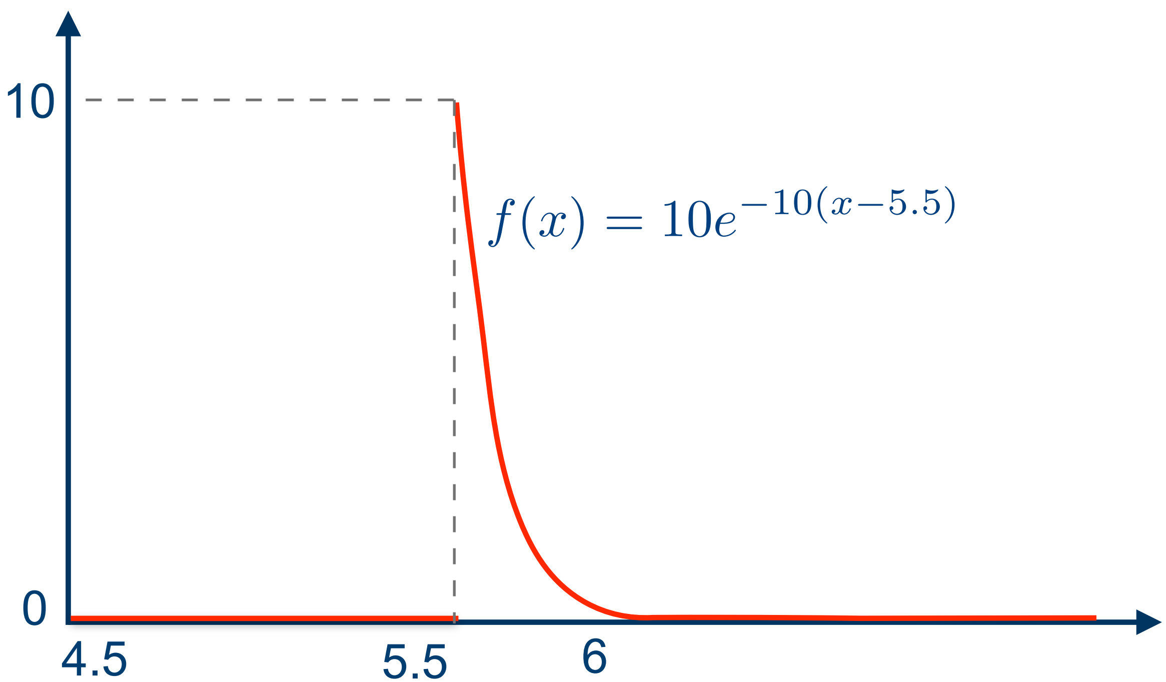

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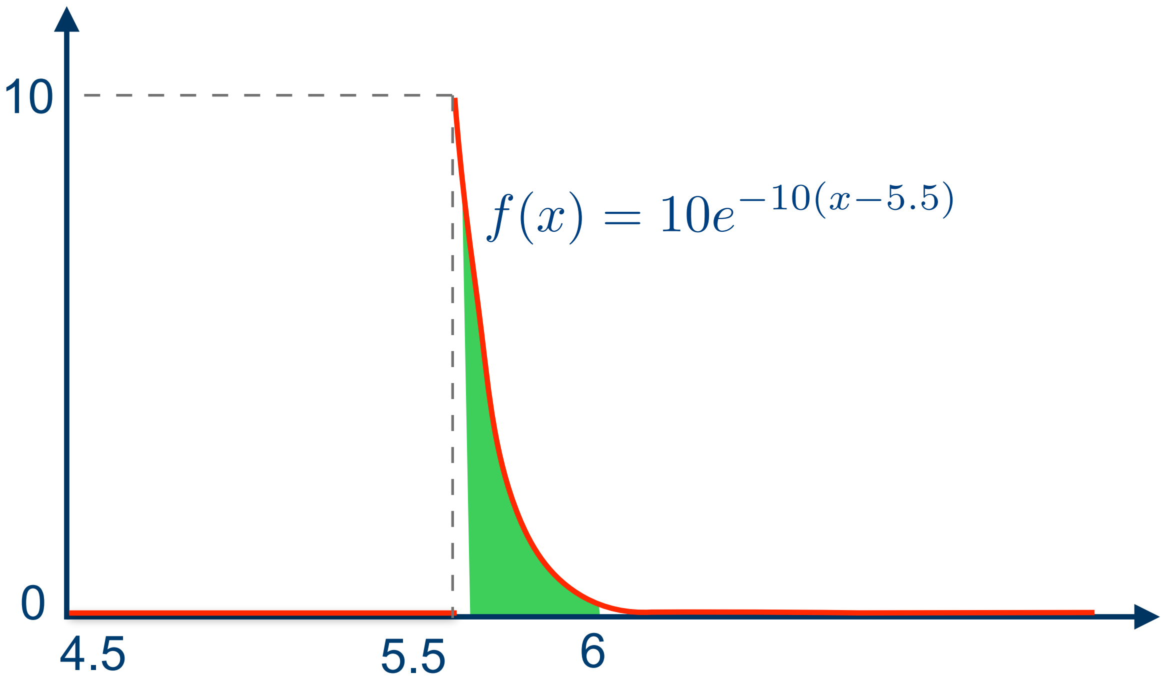
$$= e^{-1} - e^{-5} = 0.361$$



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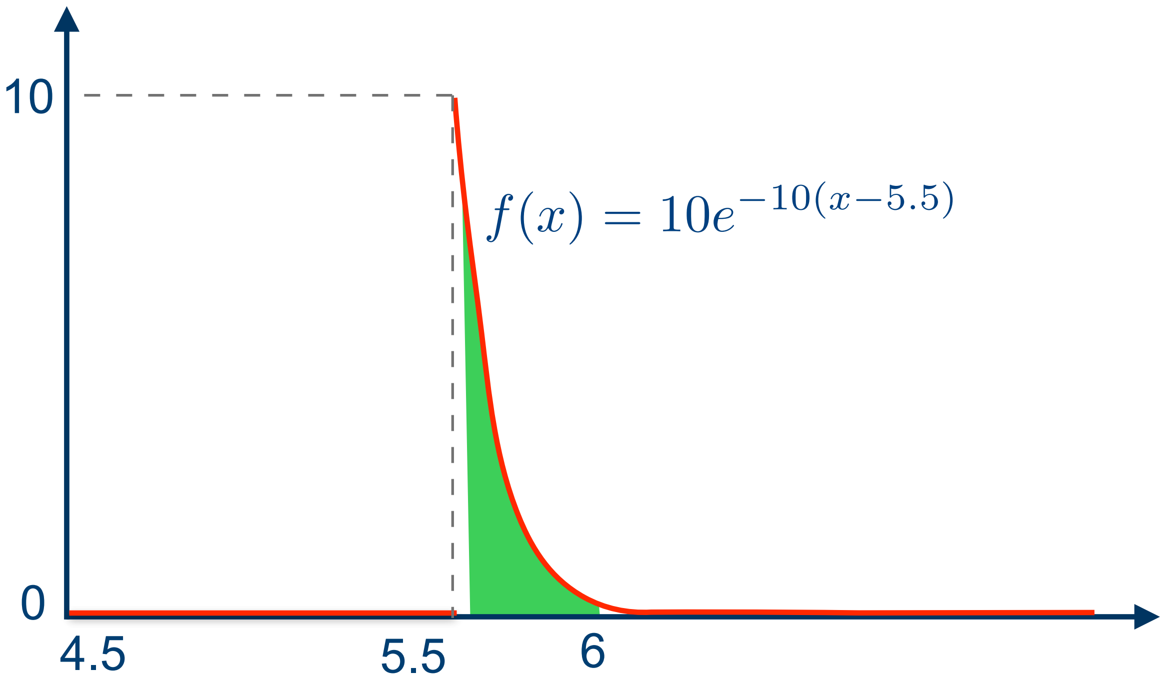

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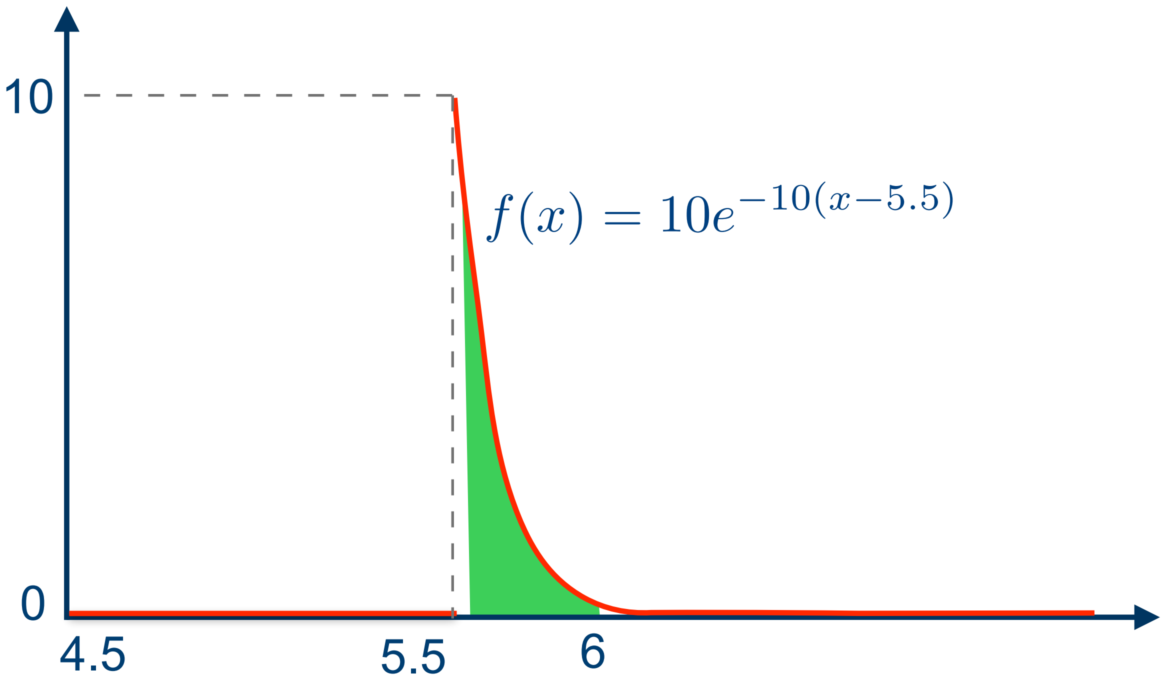

Example: Expected Value

A random variable has a PDF given by:

$$f(x) = \frac{1}{2\sqrt{x}} \qquad 1 \leq x \leq 4$$

Calculate the expected value of x:

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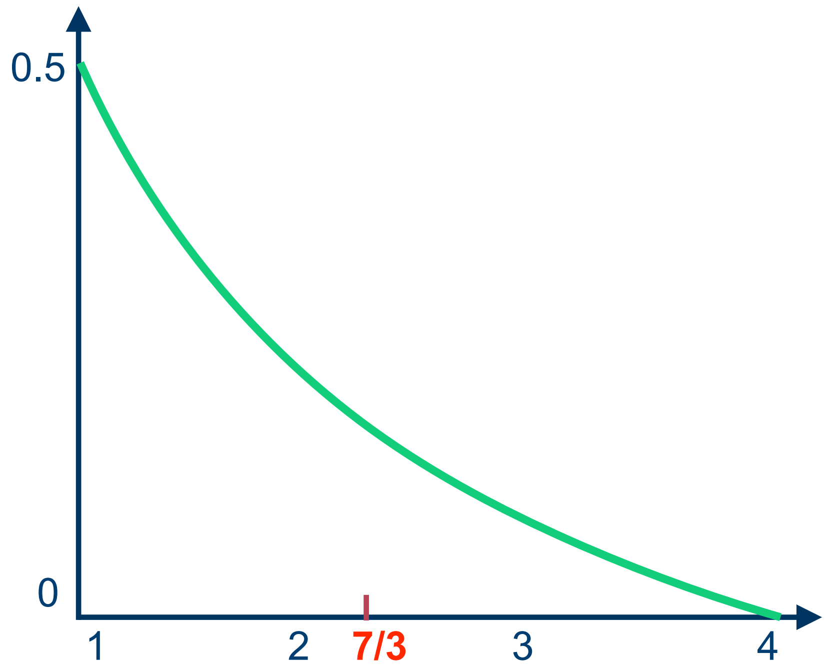

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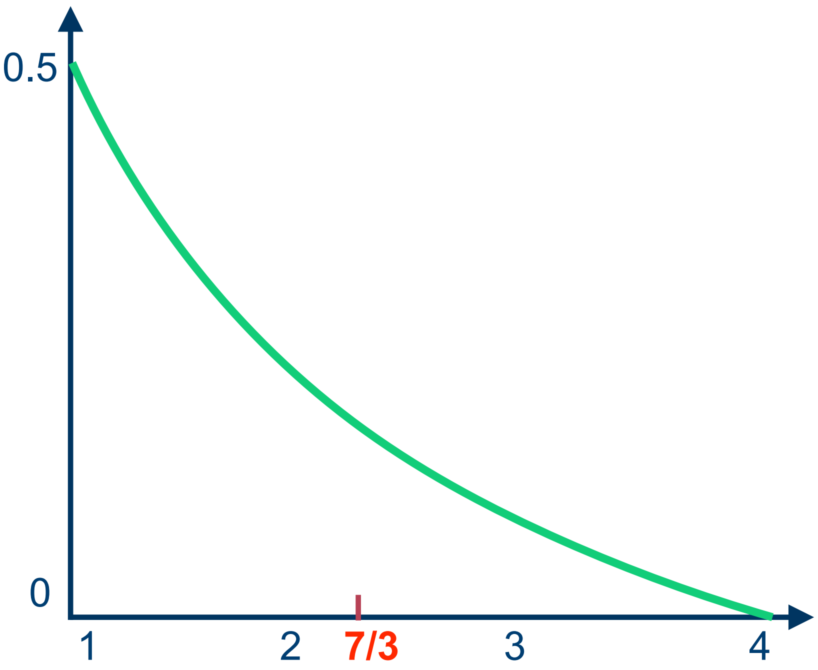

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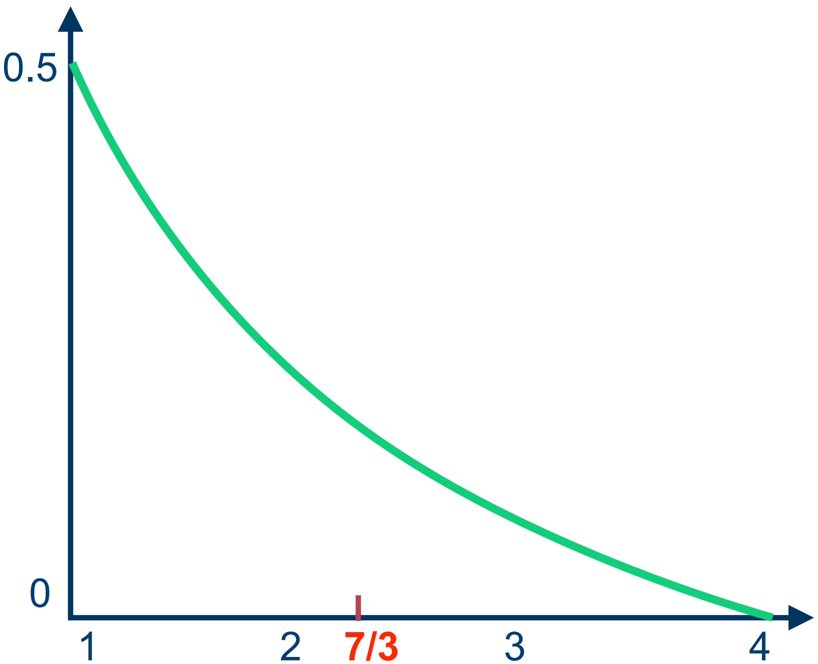
$$= \int_1^4 x\frac{1}{2\sqrt{x}}dx = \int_1^4 \frac{1}{2}\sqrt{x}dx$$

$$= \frac{\sqrt{x^3}}{3}|_1^4 = \frac{1}{3}(8-1) = \frac{7}{3}$$



Example:

A charity group raises funds by collecting waste paper. The collected materials will contain an amount, X, of other materials such as plastic bags and rubber bands. X may be regarded as a random variable with probability density function:

$$f(x) = \begin{cases} f(x) = k(x-1)(4-x) & 1 < x < 4 \\ 0 & Otherwise \end{cases}$$

(i) Show that K=2/9

(ii) Find the Expected Value and Standard Deviation of X.

(iii) Find the Probabilitv of X that exceeds 3.5

Example:

$$f(x) = \begin{cases} f(x) = k(x-1)(4-x) & 1 < x < 4 \\ 0 & Otherwise \end{cases}$$

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$$\int_{-\infty}^{\infty} f(x)dx = 1$$

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Example:

$$f(x) = \begin{cases} f(x) = k(x-1)(4-x) & 1 < x < 4 \\ 0 & Otherwise \end{cases}$$

(i) Show that K=2/9

$$\int_{-\infty}^{\infty} f(x)dx = 1 \quad \Rightarrow$$

$$\int_1^4 k\,(x-1)\,(4-x)\,dx = 1$$

$$k\int_1^4 (x-1)\,(4-x)\,dx = 1$$

$$k\left[\frac{-x^3}{3} + \frac{-5x^2}{2} - 4x\right]_1^4 = 1$$

$$k\left[\frac{8}{3} - \left(-\frac{11}{6}\right)\right] = 1$$

$$4.5k = 1$$

$$k = \frac{2}{9}$$

Example:

$$f(x) = \begin{cases} f(x) = k(x-1)(4-x) & 1 < x < 4 \\ 0 & Otherwise \end{cases}$$

(ii) Find the Expected Value and Standard Deviation of X.

$$\mu = E[x] = \int_{-\infty}^{+\infty} x f(x)\, dx$$

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$$\mu = E[x] = \int_{-\infty}^{+\infty} x\,f(x)\,dx$$

$$\Rightarrow$$

$$E[x] = \int_1^4 \frac{2}{9} x(x-1)(4-x)\,dx$$

$$= \frac{2}{9}\left(\int_1^4 x(x-1)(4-x)\right)dx$$

$$= \frac{2}{9}\left[\frac{-x^4}{4} + \frac{5x^3}{3} - \frac{4x^2}{2}\right]_1^4$$

$$= \frac{2}{9}\left(\frac{32}{3} - \left(-\frac{7}{12}\right)\right)$$

$$= 2.5$$

Example:

$$f(x) = \begin{cases} f(x) = k(x-1)(4-x) & 1 < x < 4 \\ 0 & Otherwise \end{cases}$$

(ii) Find the Expected Value and Standard Deviation of X.

$$\sigma = \sqrt{\int_a^b (x - E[x])^2 f(x)}$$

$$\sigma = \sqrt{\int_1^4 \left(x - \frac{2}{9}x(x-1)(4-x)\right)^2 \frac{2}{9}(x-1)(4-x)\,dx}$$

Not easy to solve

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(ii) Find the Expected Value and Standard Deviation of X.

$$E\left(x^2\right) = \int_1^4 \frac{2}{9}x^2\left(x-1\right)\left(4-x\right)dx$$

$$= \frac{2}{9}\left[-\frac{x^5}{5} + \frac{5x^4}{4} - \frac{4x^3}{x}\right]_1^4$$

$$= \frac{2}{9}\left(\frac{448}{15} - \left(-\frac{17}{60}\right)\right)$$

$$= 6.7$$

$$\sigma = \sqrt{E(x^2) - E(x)^2}$$

Example:

$$f(x) = \begin{cases} f(x) = k(x-1)(4-x) & 1 < x < 4 \\ 0 & Otherwise \end{cases}$$

(ii) Find the Expected Value and Standard Deviation of X.

$$E\left(x^2\right) = \int_1^4 \frac{2}{9} x^2 \left(x-1\right)\left(4-x\right) dx$$

$$= \frac{2}{9}\left[-\frac{x^5}{5} + \frac{5x^4}{4} - \frac{4x^3}{x}\right]_1^4$$

$$= \frac{2}{9}\left(\frac{448}{15} - \left(-\frac{17}{60}\right)\right)$$

$$= 6.7$$

$$\sigma = \sqrt{E(x^2) - E(x)^2}$$

$$E(x) = 2.5$$

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$$\sigma = \sqrt{6.7 - 2.5^2} = 0.671$$

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$$f(x) = \begin{cases} f(x) = k(x-1)(4-x) & 1 < x < 4 \\ 0 & Otherwise \end{cases}$$

(iii) Find the Probability of X that exceeds 3.5

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$$f(x) = \begin{cases} f(x) = k(x-1)(4-x) & 1 < x < 4 \\ 0 & Otherwise \end{cases}$$

(iii) Find the Probability of X that exceeds 3.5

$$\begin{aligned} P(x > 3.5) &= \int_{3.5}^{4} \frac{2}{9}(x-1)(4-x)\,dx \\ &= \frac{2}{9}\left[-\frac{x^3}{3} + \frac{5x^2}{2} - 4x\right]_{3.5}^{4} \\ &= \frac{2}{9}\left(\frac{8}{3} - \frac{7}{3}\right) \\ &= 0.0741 \end{aligned}$$