



UNIVERSITY  
*of*  
GREENWICH

# Applications of Probability in Engineering

## Review + Normal Distribution

Dr. Mehdi Baghdadi

Monday, April 04, 2016

# Let's review

**Question:** In a recovery control system, the probability that the system recovers from the one fault is 0.64 and the probability that the system recovers from two faults is 0.51. What is the probability that the system recovers from the second fault given that it covered from the first fault successfully?

A. 0.797

B. 0.741

C. 0.45

D. 0.54

Time: 3 minutes, Difficulty Level: Average

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**Question:** In a hospital, 35% of those with high blood pressure have had strokes and 20% of those without high blood pressure have had strokes. If 40% of the patients have high blood pressure, what percent of the patients have had strokes?

A. 0.13

B. 0.26

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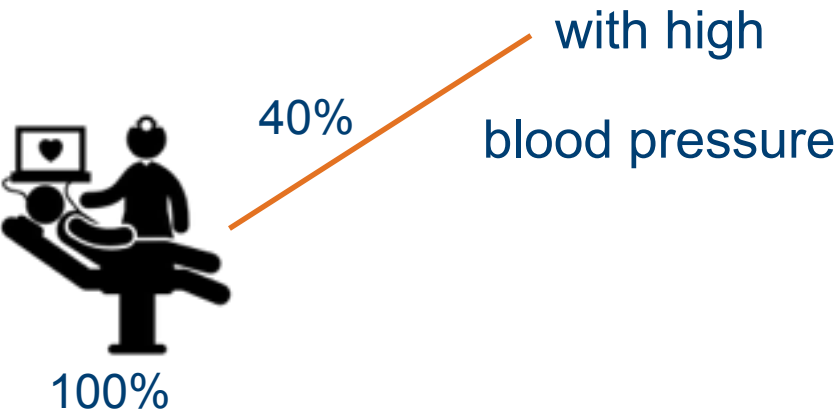
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100%

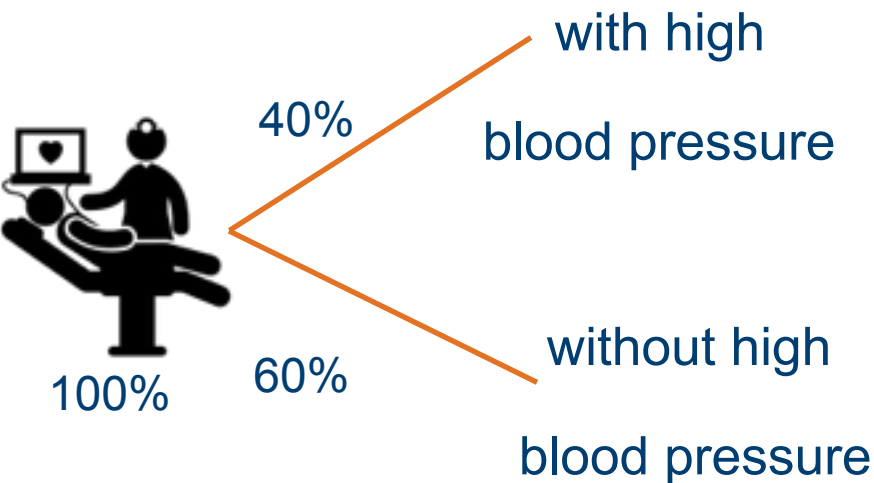
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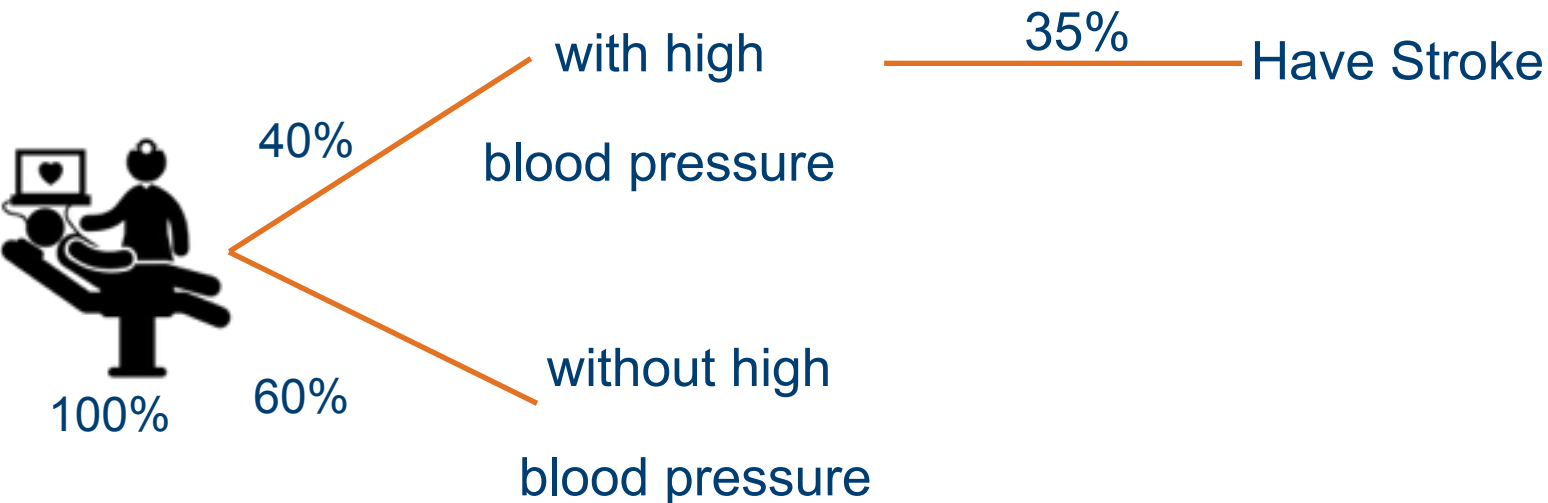
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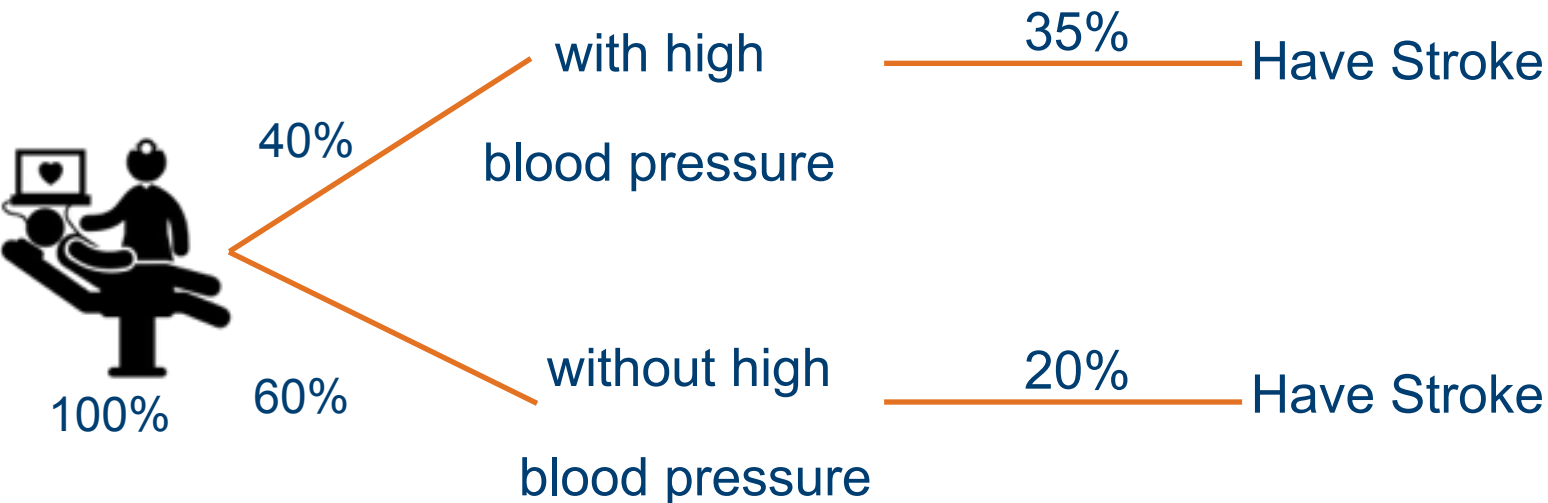
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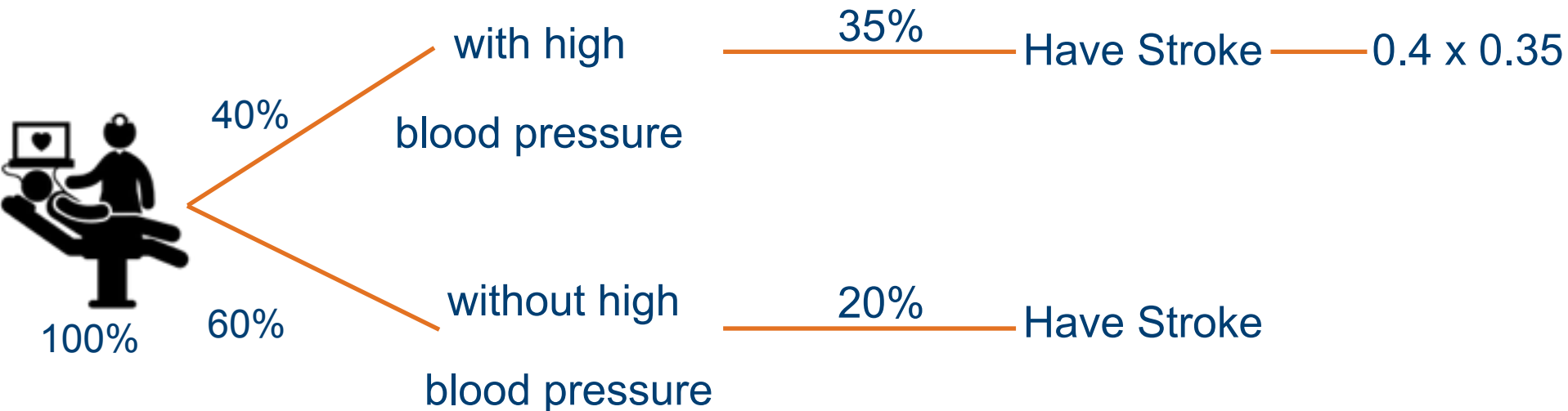
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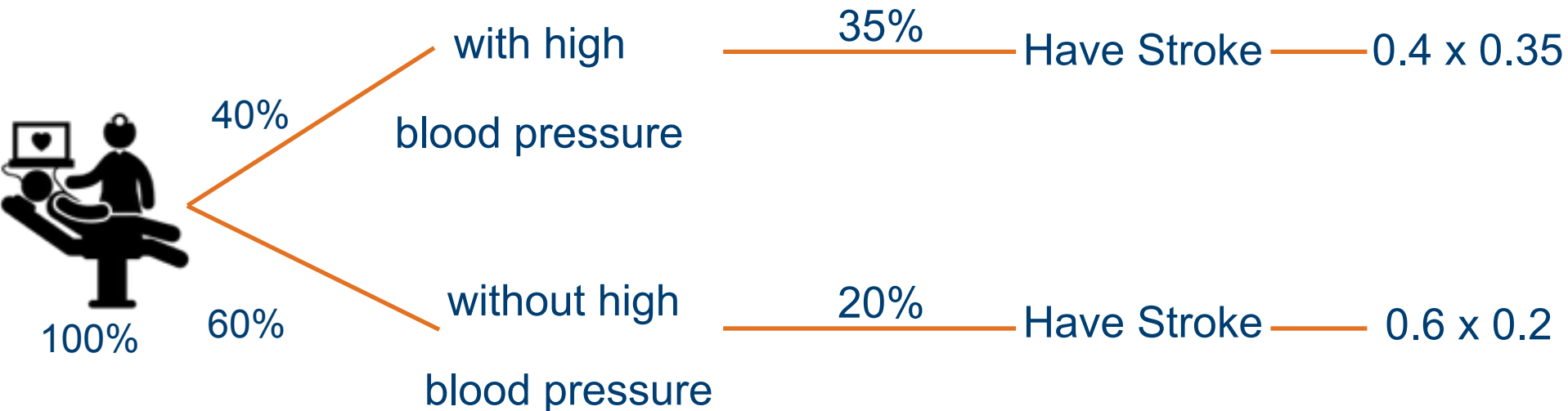
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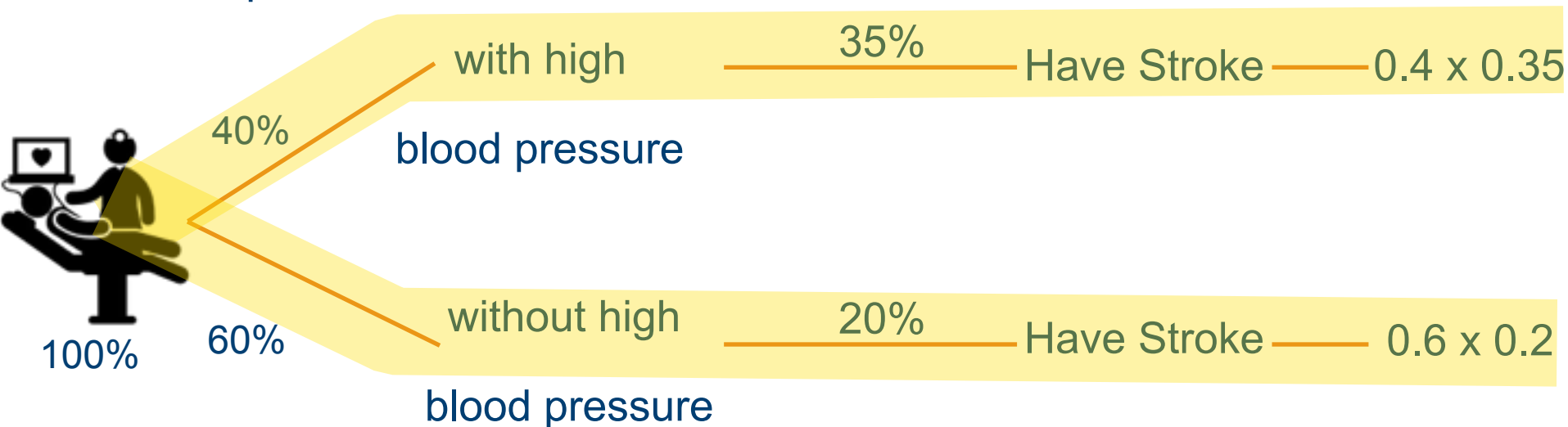
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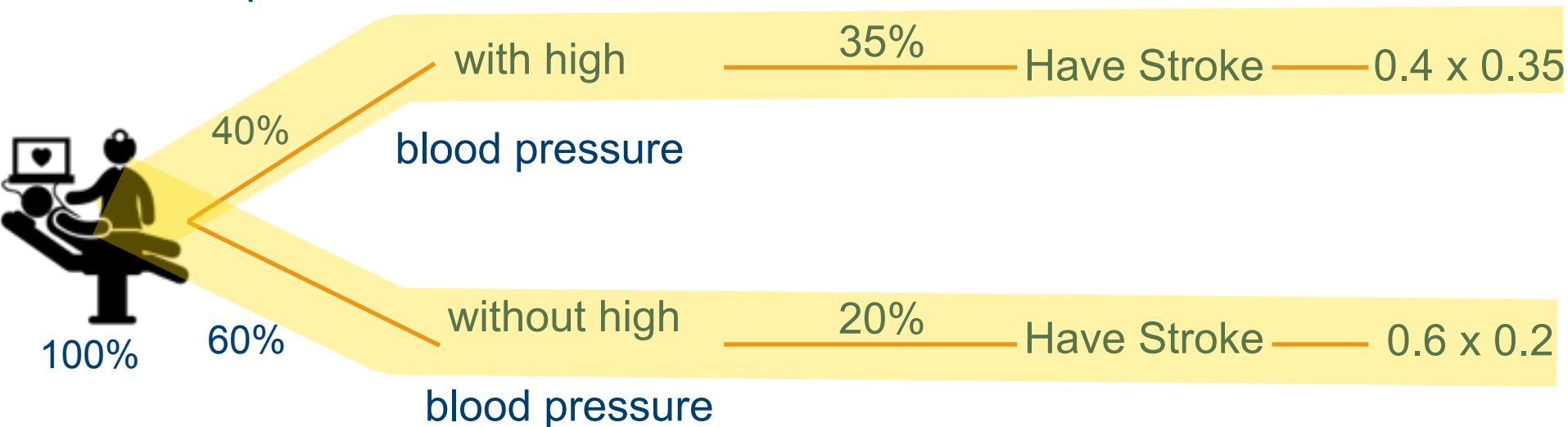
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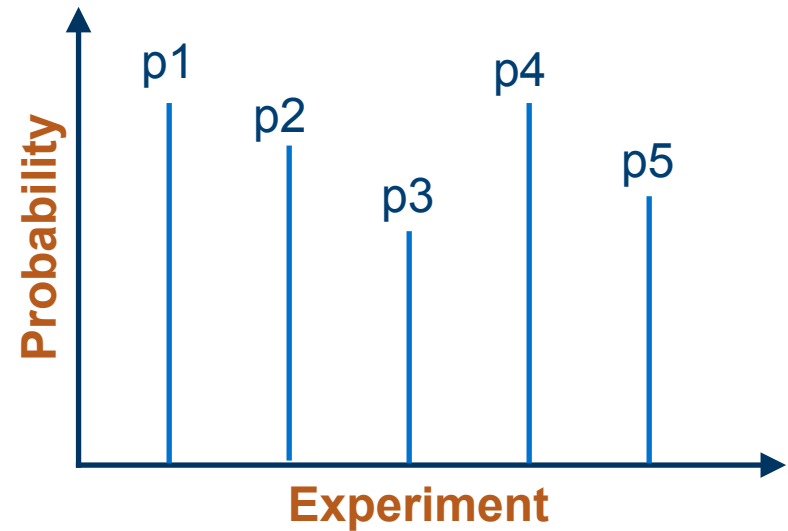
$$P(\text{Patient had Stroke}) = 0.4 \times 0.35 + 0.6 \times 0.2 = 0.26$$

# What we need to know for this session

- **Sum of all the probabilities for every random experiment:**
- **Expected Value:**
- **Standard Deviation:**
- **Integral:**

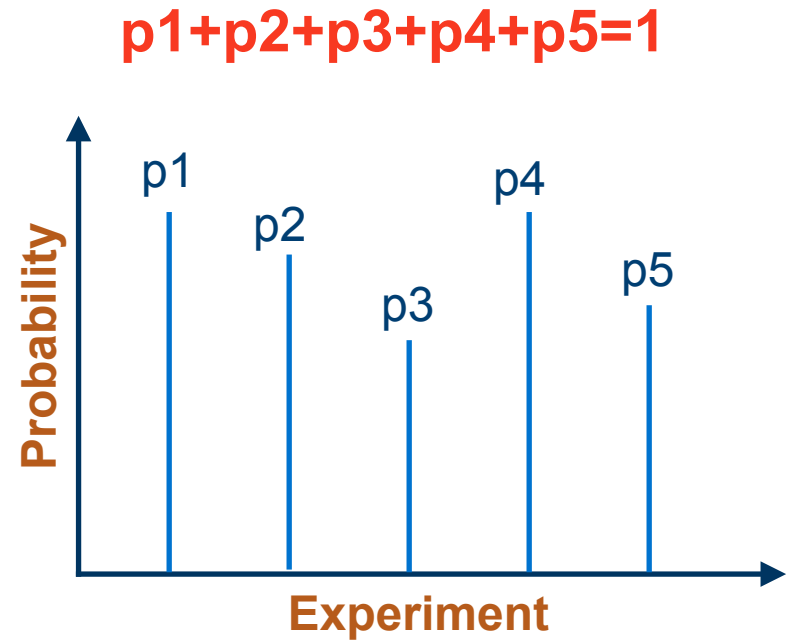
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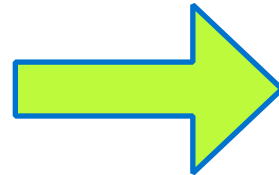
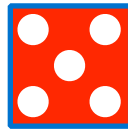
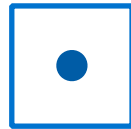
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# Probability density function (PDF)

**Example:** We roll Two Dice, and we sum up the shown numbers.

For instance:

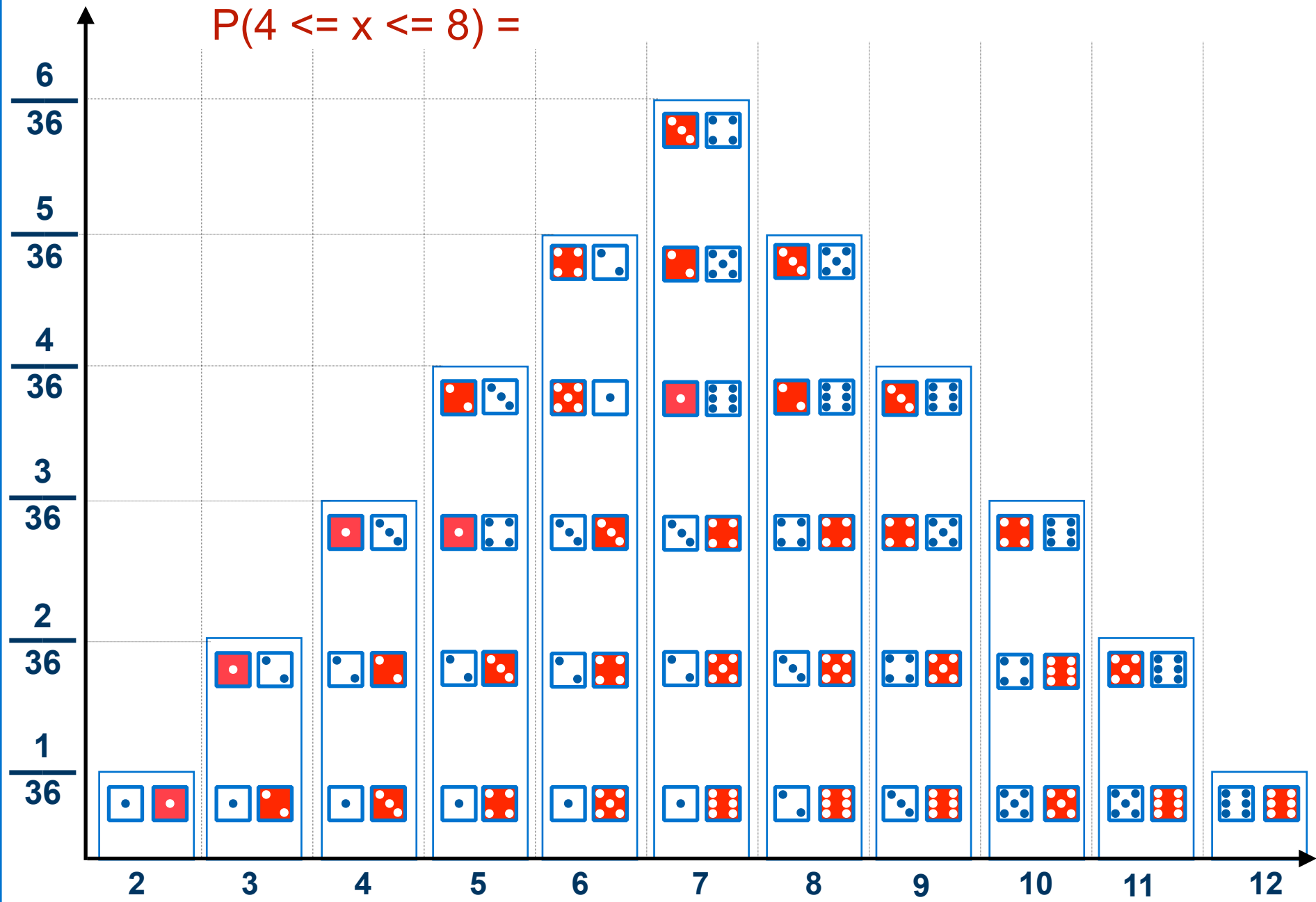


$$5+1 = 6$$

$$P(1,5) = 1/36$$

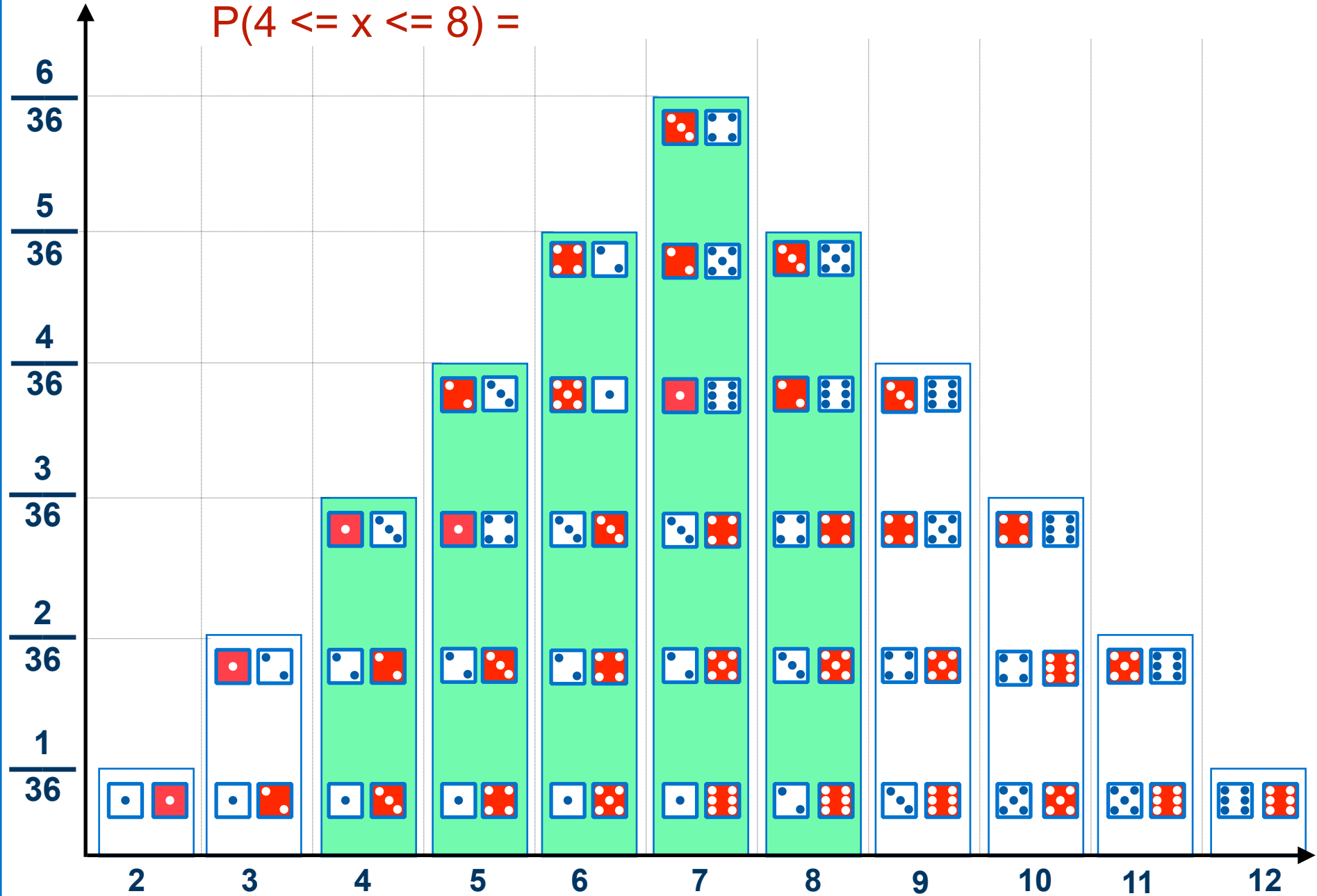
What is the probability that sum of two numbers is bigger than 4 and smaller than 8?

$$P(4 \leq x \leq 8) =$$

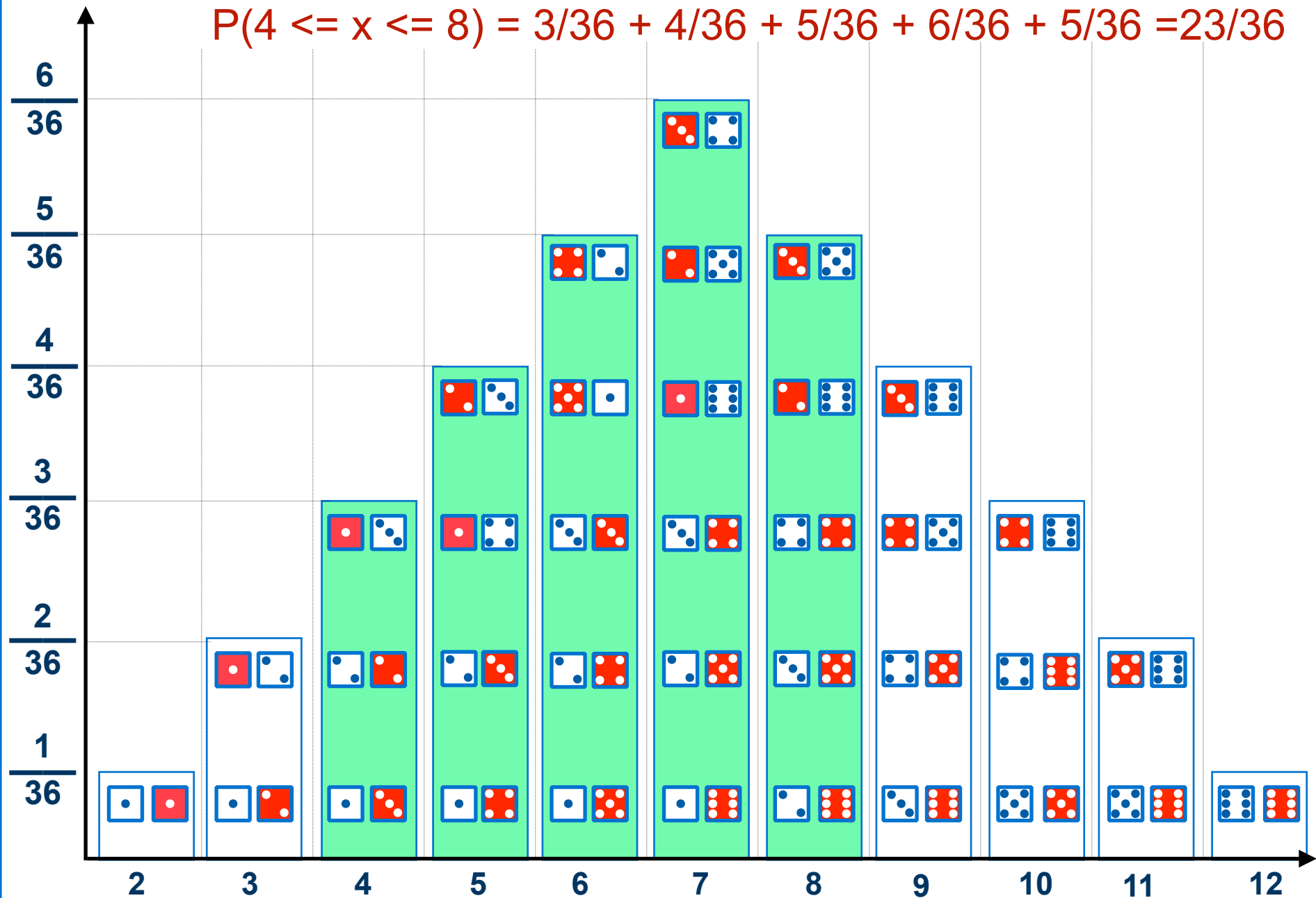




$$P(4 \leq x \leq 8) =$$



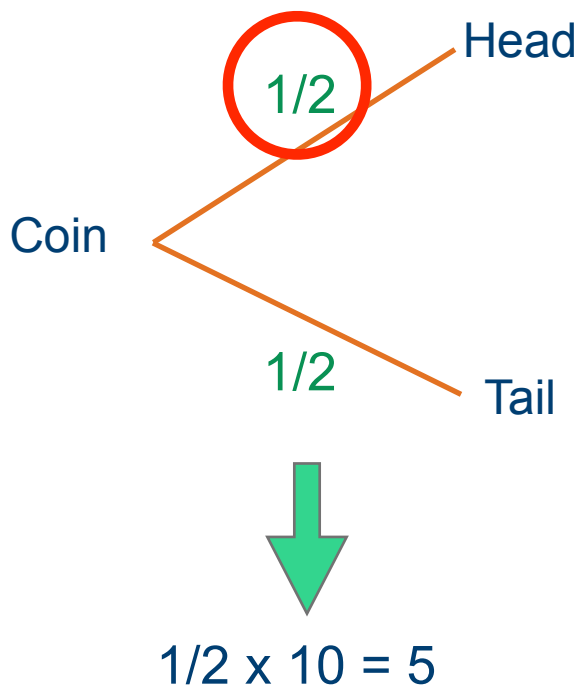
$$P(4 \leq x \leq 8) = \frac{3}{36} + \frac{4}{36} + \frac{5}{36} + \frac{6}{36} + \frac{5}{36} = \frac{23}{36}$$



# Expected Value:

Binomial Expected Value, the outcome is between two options, e.g., True or False.

**Example:** We toss a coin 10 times. How many Heads you could expect?



Binomial Expected Value

$$E(x) = P(x) * X$$

Probability of  
Head

Number of  
Trials

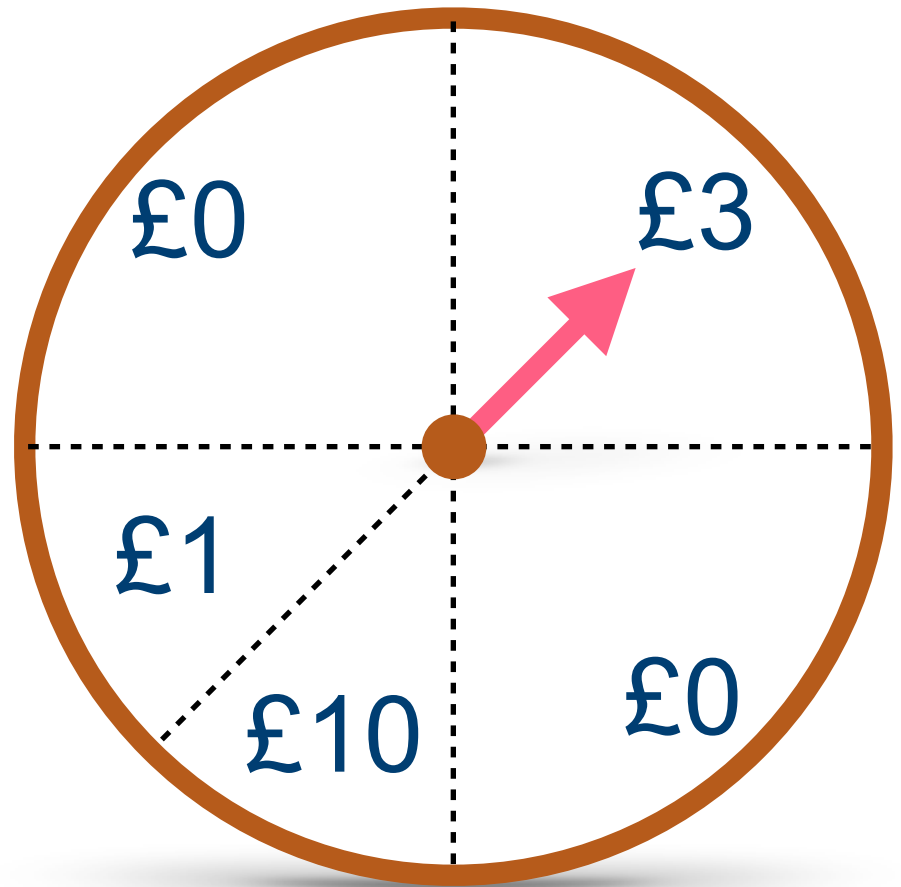
# Challenge: Shall we play or .... not?

The expected value gives us the expected **long term average** of measurements.

This game costs you £1 per game.

You will not receive your money back no matter you win or lose.

**Shall we play?**



# Challenge: Shall we play or .... not?

OUTCOME	£0	£3	£0	£10	£1	-£1
PROBABILITY	1/4	1/4	1/4	1/8	1/8	1

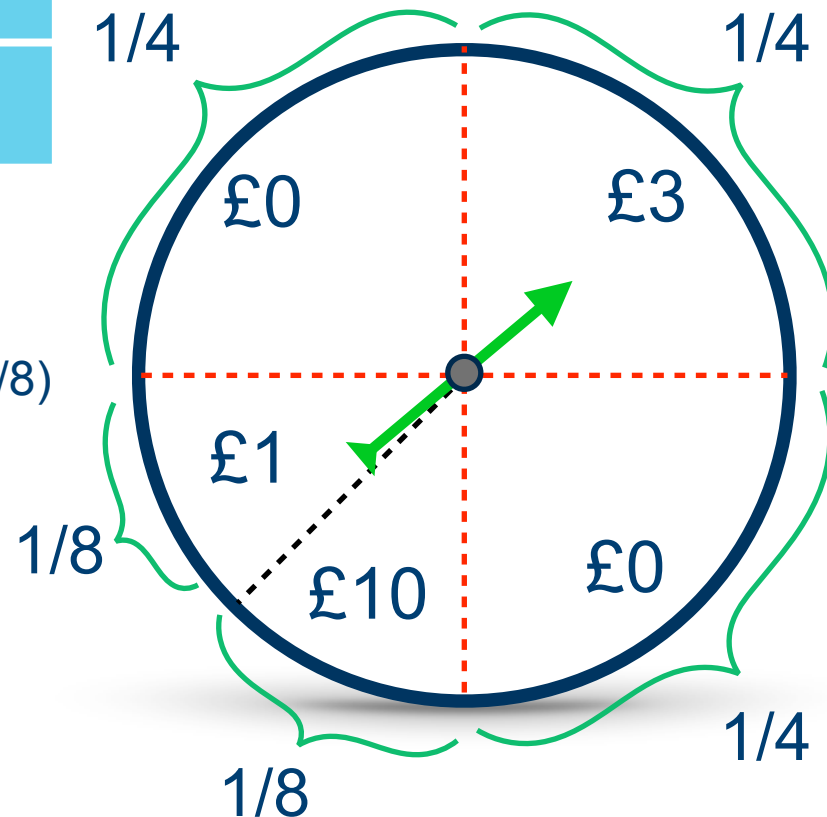
Your expectation is:

$$E = -1\text{£}1(1) + \text{£}0(1/2) + \text{£}1(1/8) + \text{£}2(1/4) + \text{£}10(1/8)$$

$$E = \text{£}7/8$$

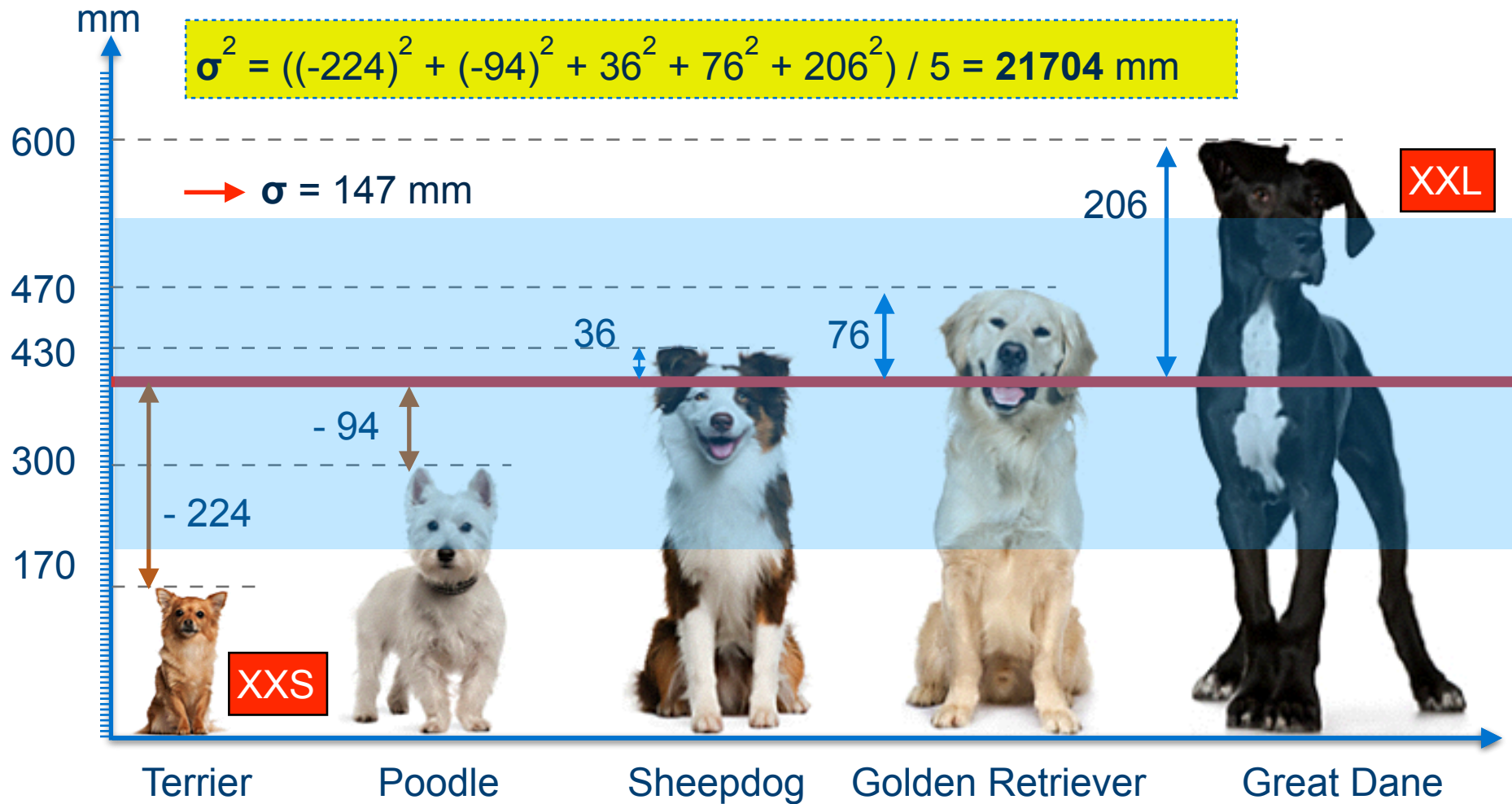
You can expect to win **£0.875 ON AVERAGE** per game.

For instance: 100 games, you win £87.5.



# Standard Deviation (SD)

Matlab syntax: std(A)



# Example

<b>X</b>	2	3	4	5	6
<b>P(X)</b>	0.01	0.25	0.4	0.3	0.04

Find the Standard Deviation of X:

$$\sigma = \sqrt{\sum_{i=1}^n p_i (x_i - E[x])^2}$$

$$E(x) = [(2 \times 0.01) + (3 \times 0.25) + (4 \times 0.4) + (5 \times 0.3) + (6 \times 0.04)] = 4.11$$

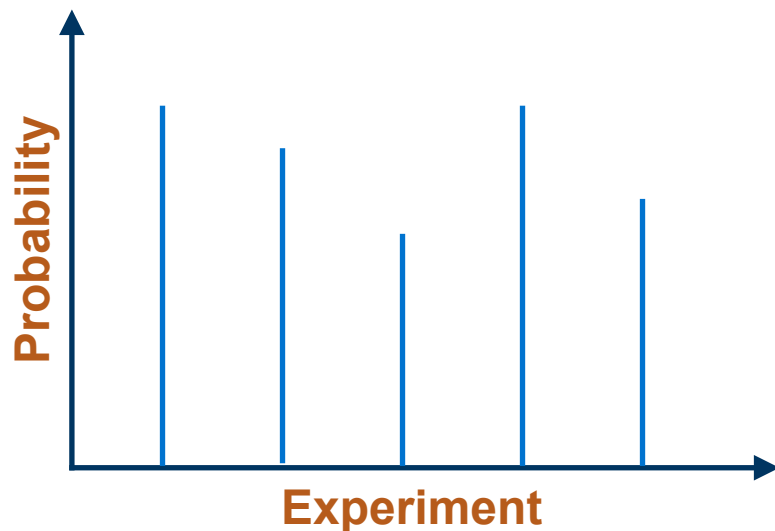
$$\sigma = \sqrt{(2 - 4.11)^2(0.01) + (3 - 4.11)^2(0.25) + (4 - 4.11)^2(0.4) + (5 - 4.11)^2(0.3) + (6 - 4.11)^2(0.04)}$$

$$\sigma = \sqrt{0.74} = 0.86$$

# Continuous Random Variables

Many practical random variables are modelled as **Continuous**:

- 1- Speed of a car
- 2- Measurement Error
- 3- Electricity Consumption

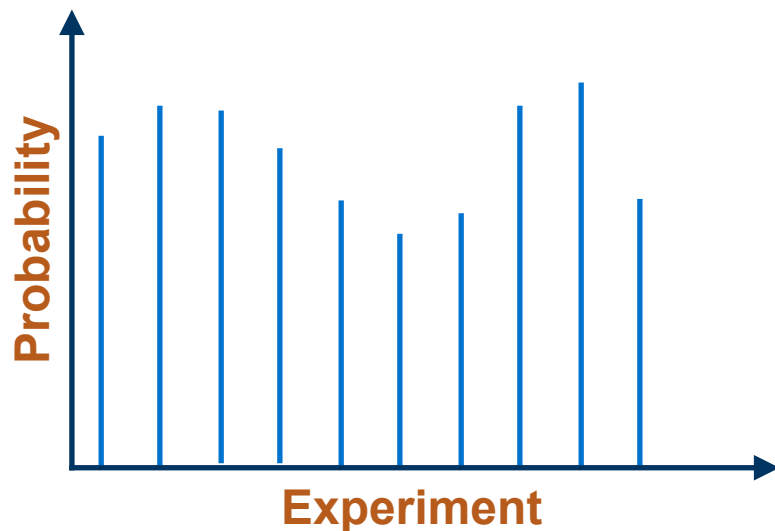




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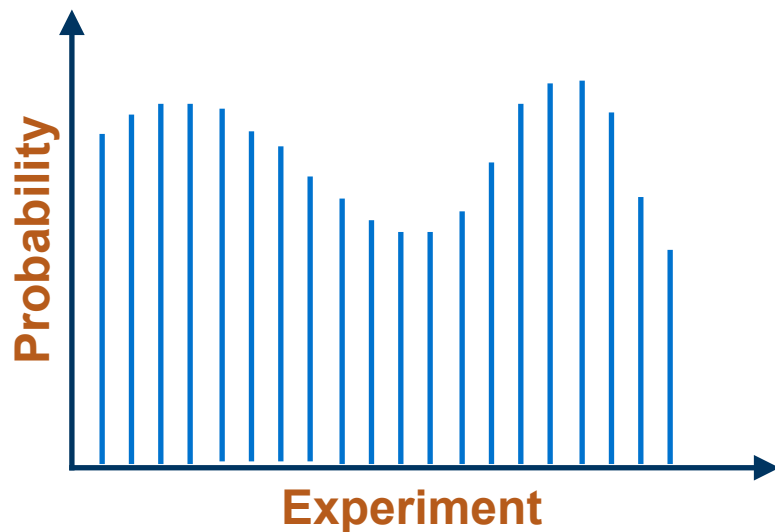
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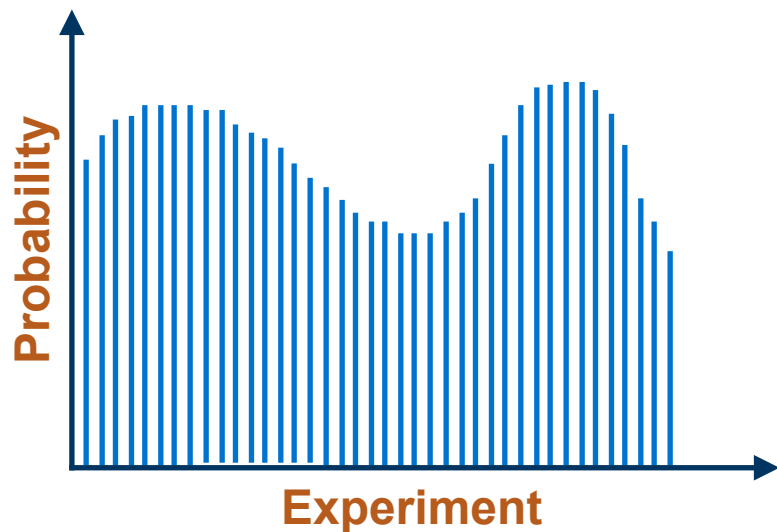
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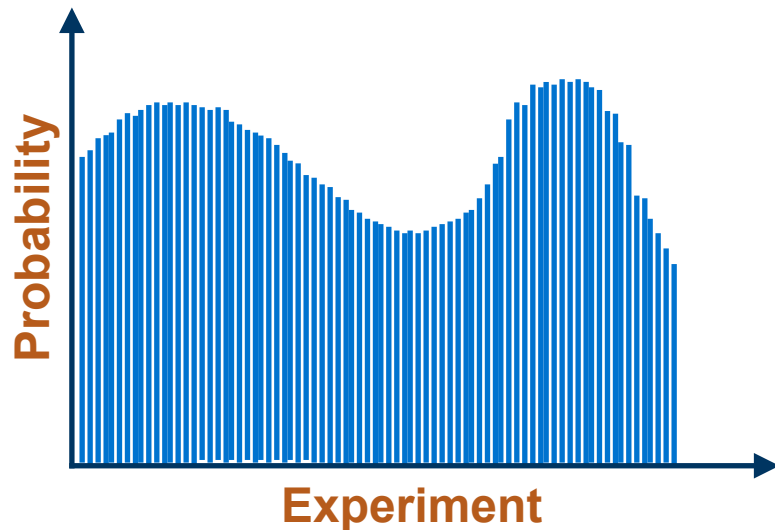
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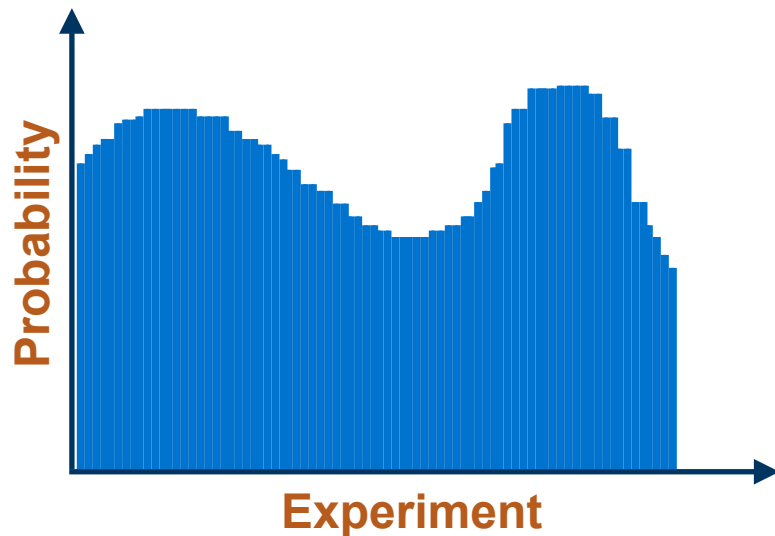
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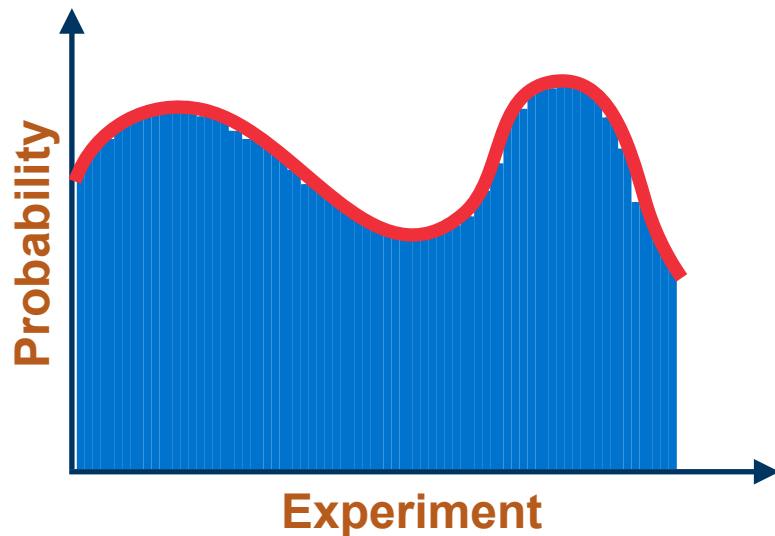
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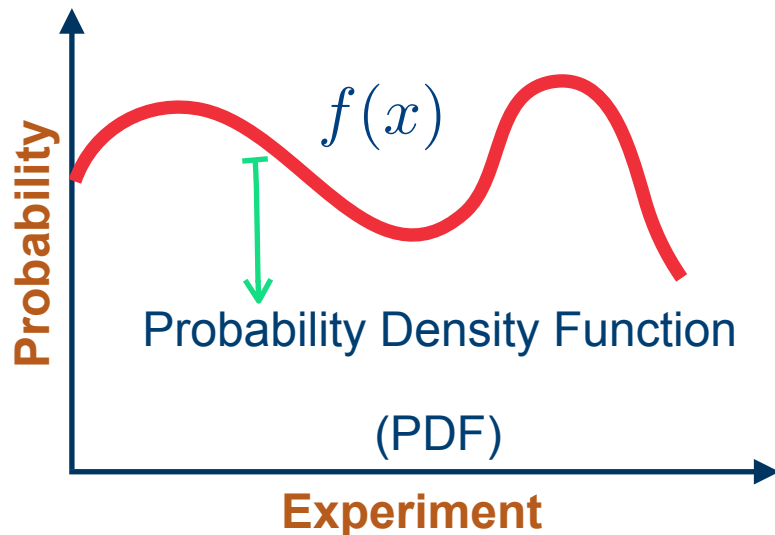
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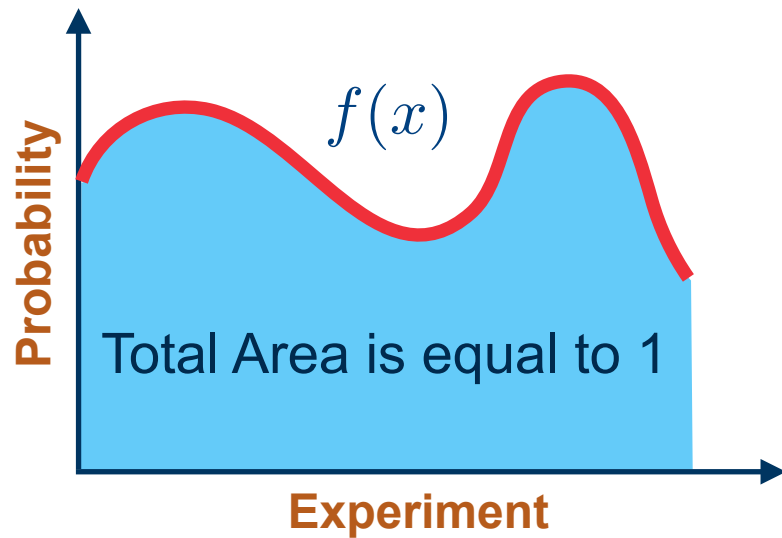
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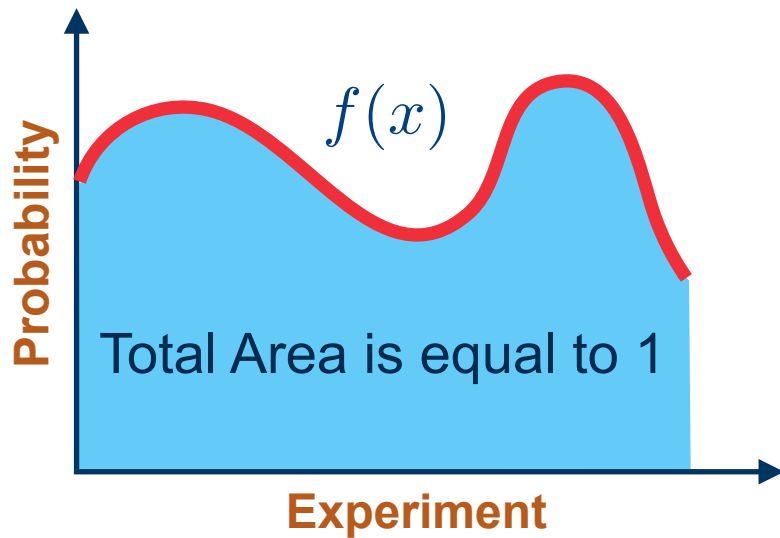
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$f(x)$  :Probability Density Function



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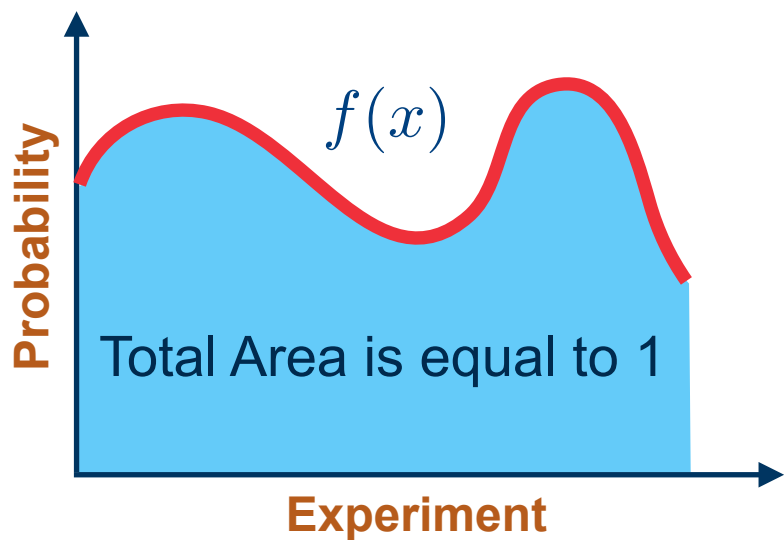


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**Rule 1: Probability Density function (PDE):**

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

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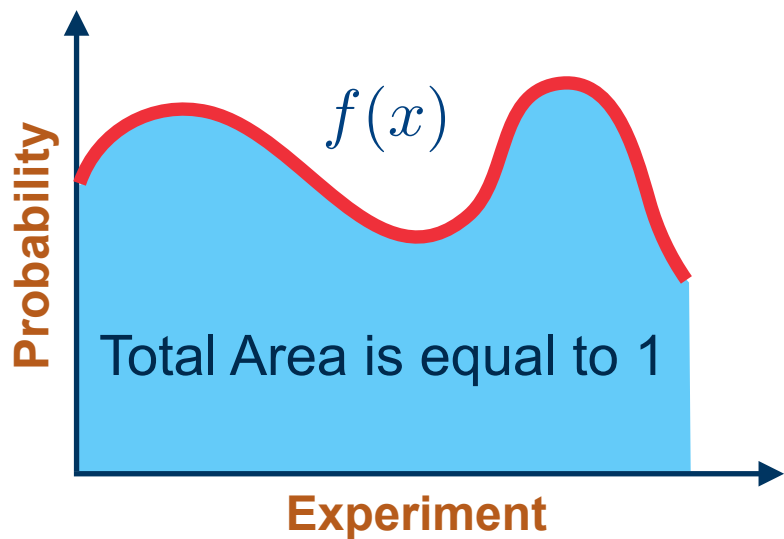
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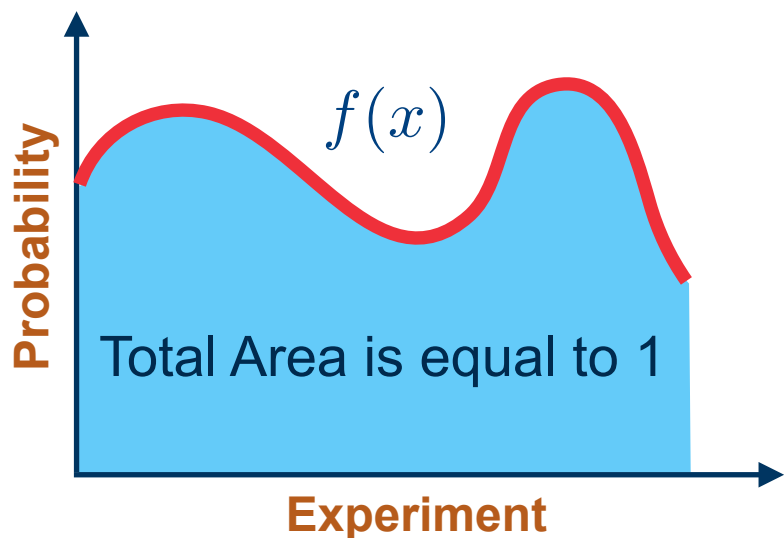
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$$\sigma = \sqrt{E(x^2) - E(x)^2}$$

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$$\begin{aligned}\int \frac{12}{\sqrt[3]{x^2}} dx &= \int \frac{12}{x^{\frac{2}{3}}} dx = \int 12x^{-\frac{2}{3}} dx \\ &= 12 \frac{x^{-\frac{2}{3}+1}}{-\frac{2}{3}+1}\end{aligned}$$



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$$= 12 \frac{x^{-\frac{2}{3}+1}}{-\frac{2}{3}+1} = 12 \frac{x^{\frac{1}{3}}}{\frac{1}{3}} = 36 \sqrt[3]{x}$$

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$$\int e^{ax} dx$$

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$$\int e^{ax} dx = \frac{e^{ax}}{a}$$

$$\int_1^4 e^{\sqrt{2}x} dx = \left[ \frac{e^{\sqrt{2}x}}{\sqrt{2}} \right]_1^4 = \frac{1}{\sqrt{2}} \left( e^{\sqrt{2} \times 4} - e^{\sqrt{2} \times 1} \right)$$

# Question

The lifetime of an electronic component (in thousands of hours) is a continuous random variable with the probability density function given by:

$$f(x) = Ae^{-\frac{x}{2}} \quad \text{for } x \geq 0$$

and zero otherwise:

Calculate the A:



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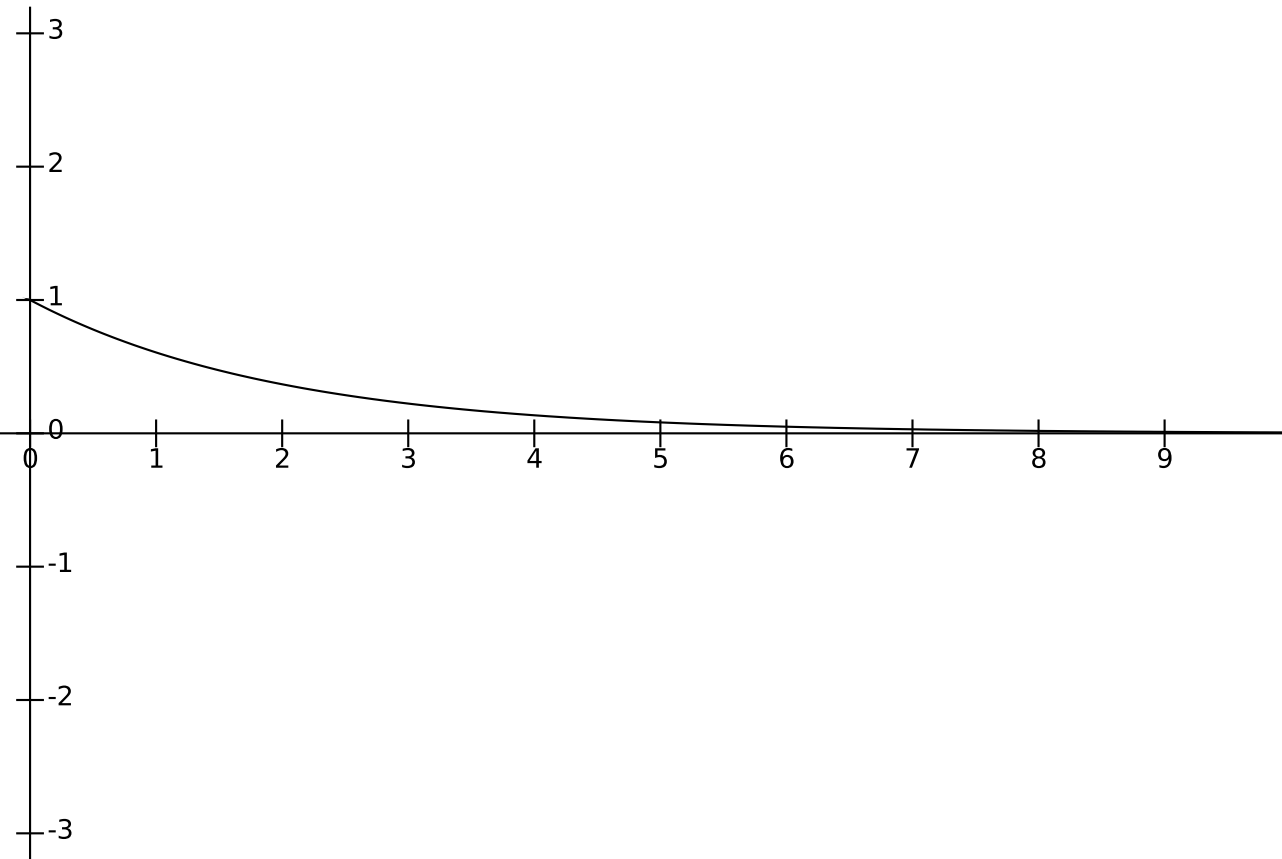
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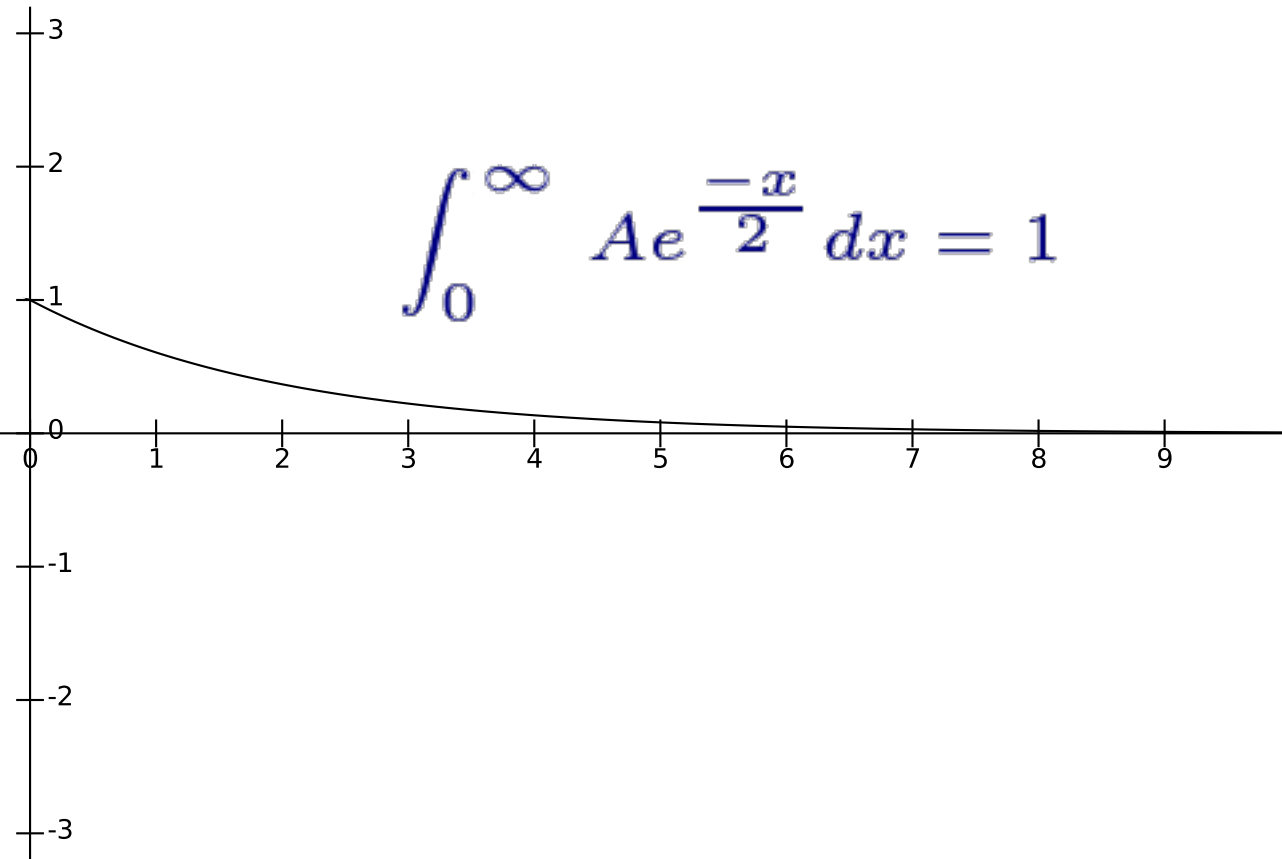


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$$\int_0^{\infty} Ae^{-\frac{x}{2}} dx = 1$$

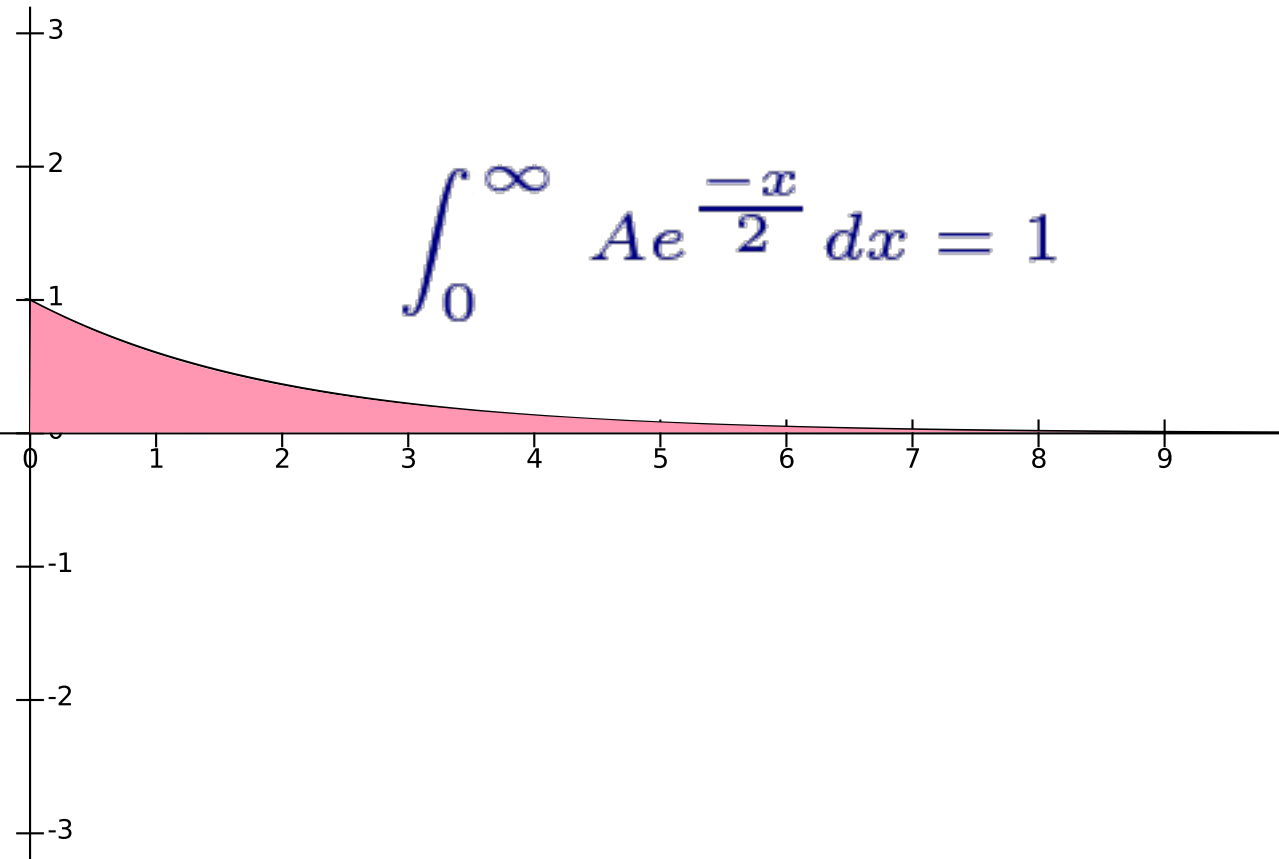


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$$\int_{-\infty}^{\infty} f(x) dx = 1$$

$$\int_0^{\infty} Ae^{-\frac{x}{2}} dx = 1$$

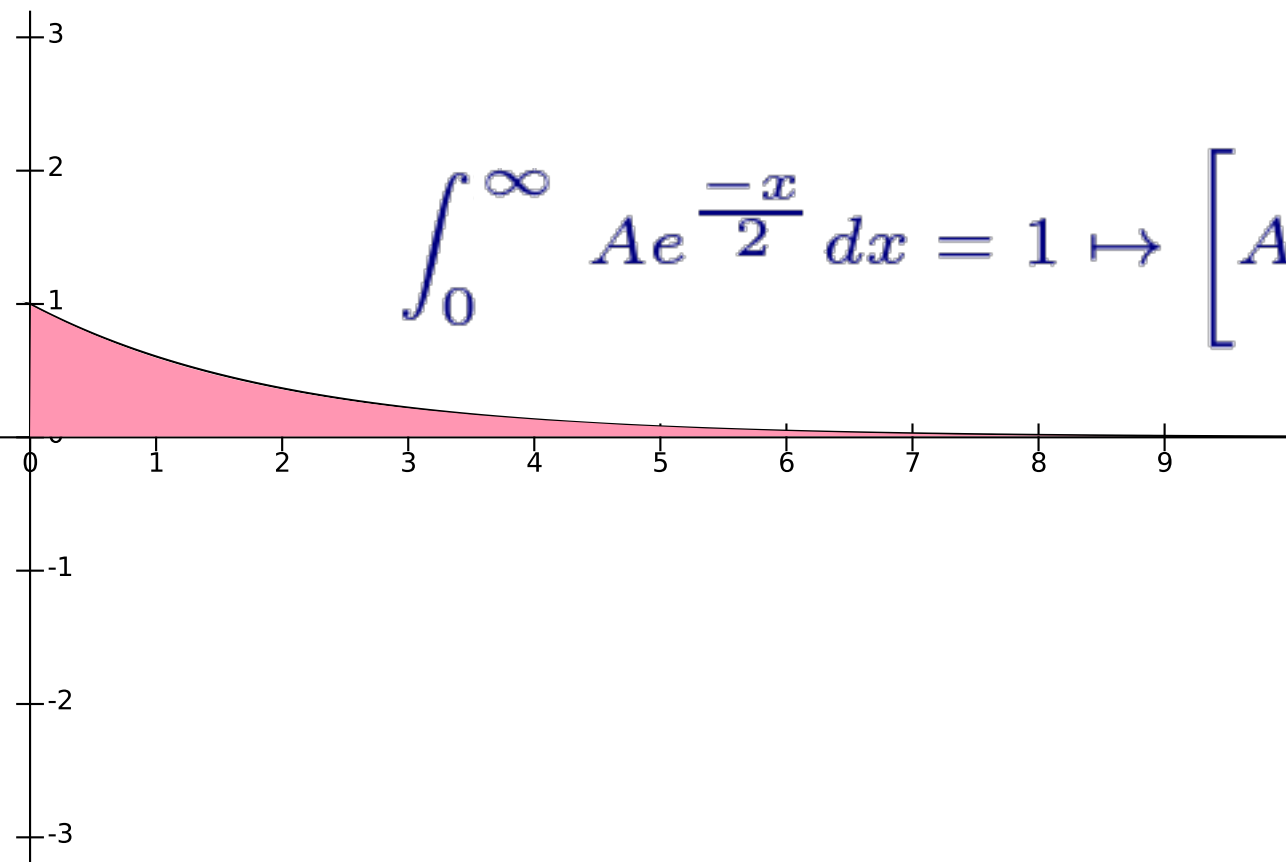


# Question

$$f(x) = Ae^{-\frac{x}{2}} \quad \text{for } x \geq 0$$

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

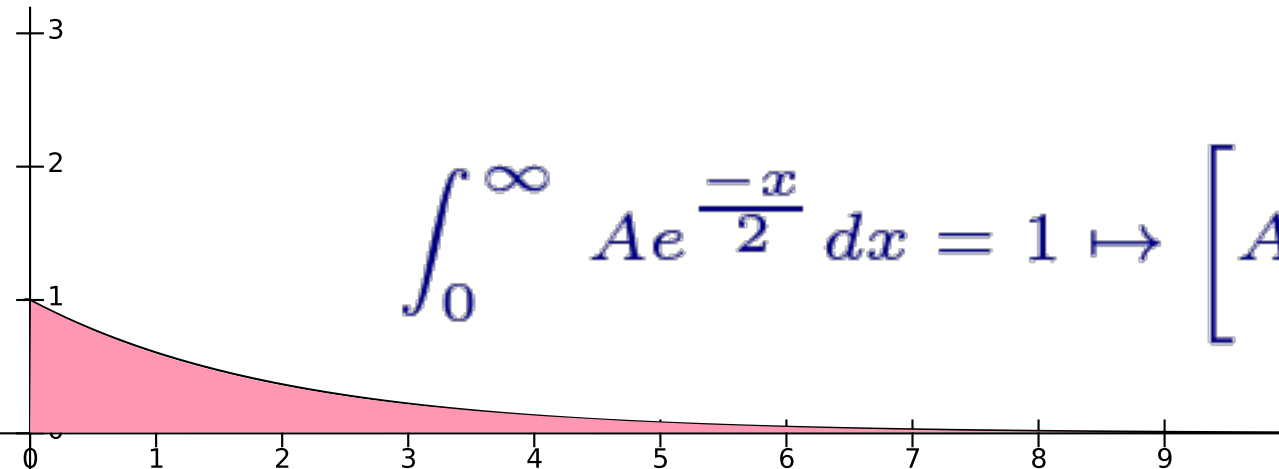
$$\int_0^{\infty} Ae^{-\frac{x}{2}} dx = 1 \mapsto \left[ A \frac{1}{\frac{-1}{2}} e^{-\frac{x}{2}} \right]_0^{\infty} = 1$$



# Question

$$f(x) = Ae^{-\frac{x}{2}} \quad \text{for } x \geq 0$$

$$\int_{-\infty}^{\infty} f(x) dx = 1$$



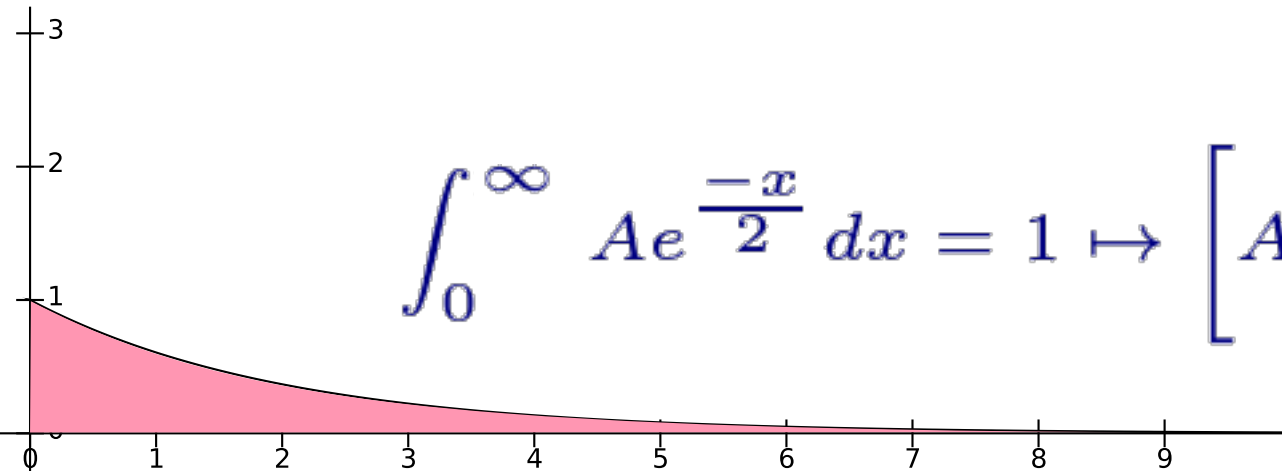
$$\int_0^{\infty} Ae^{\frac{-x}{2}} dx = 1 \mapsto \left[ A \frac{1}{\frac{-1}{2}} e^{\frac{-x}{2}} \right]_0^{\infty} = 1$$

$$\left[ -2Ae^{\frac{-x}{2}} \right]_0^{\infty}$$

# Question

$$f(x) = Ae^{-\frac{x}{2}} \quad \text{for } x \geq 0$$

$$\int_{-\infty}^{\infty} f(x) dx = 1$$



$$\int_0^{\infty} Ae^{-\frac{x}{2}} dx = 1 \mapsto \left[ A \frac{1}{\frac{-1}{2}} e^{-\frac{x}{2}} \right]_0^{\infty} = 1$$

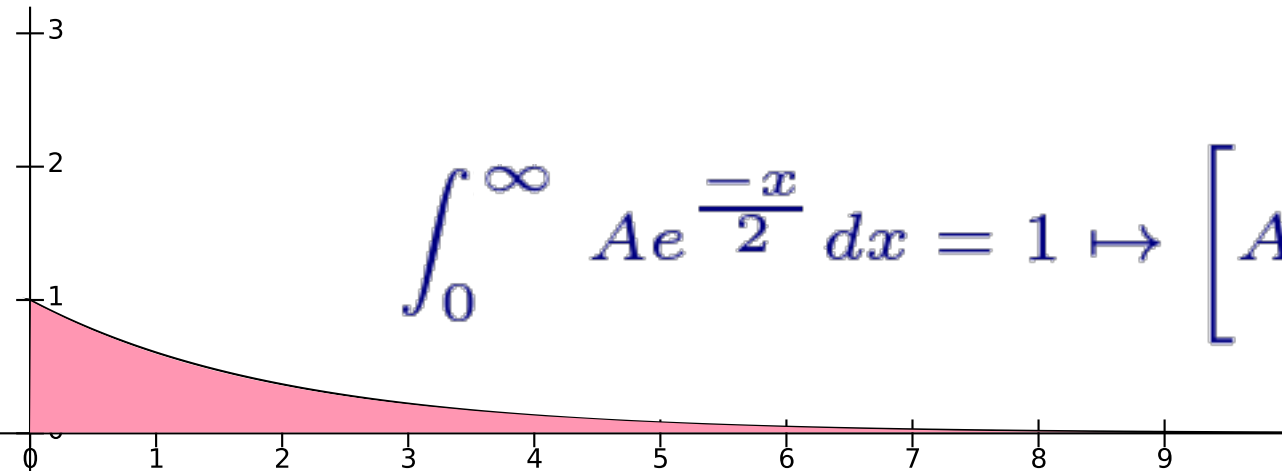
$$\left[ -2Ae^{-\frac{x}{2}} \right]_0^{\infty} = -2A \left( e^{-\infty} - e^0 \right) = 1$$



# Question

$$f(x) = Ae^{-\frac{x}{2}} \quad \text{for } x \geq 0$$

$$\int_{-\infty}^{\infty} f(x) dx = 1$$



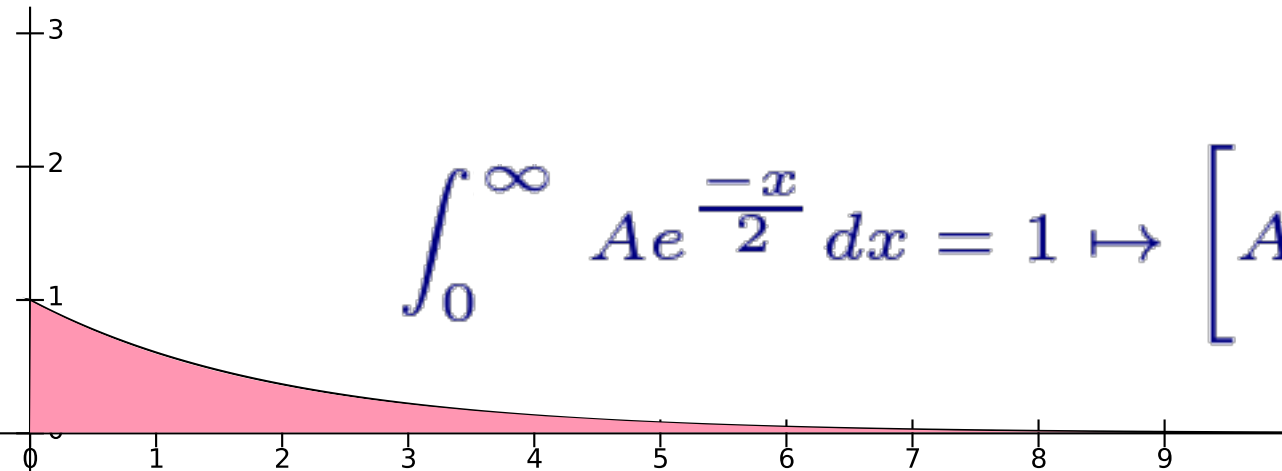
$$\int_0^{\infty} Ae^{-\frac{x}{2}} dx = 1 \mapsto \left[ A \frac{1}{\frac{-1}{2}} e^{-\frac{x}{2}} \right]_0^{\infty} = 1$$

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# Question

$$f(x) = Ae^{-\frac{x}{2}} \quad \text{for } x \geq 0$$

$$\int_{-\infty}^{\infty} f(x) dx = 1$$



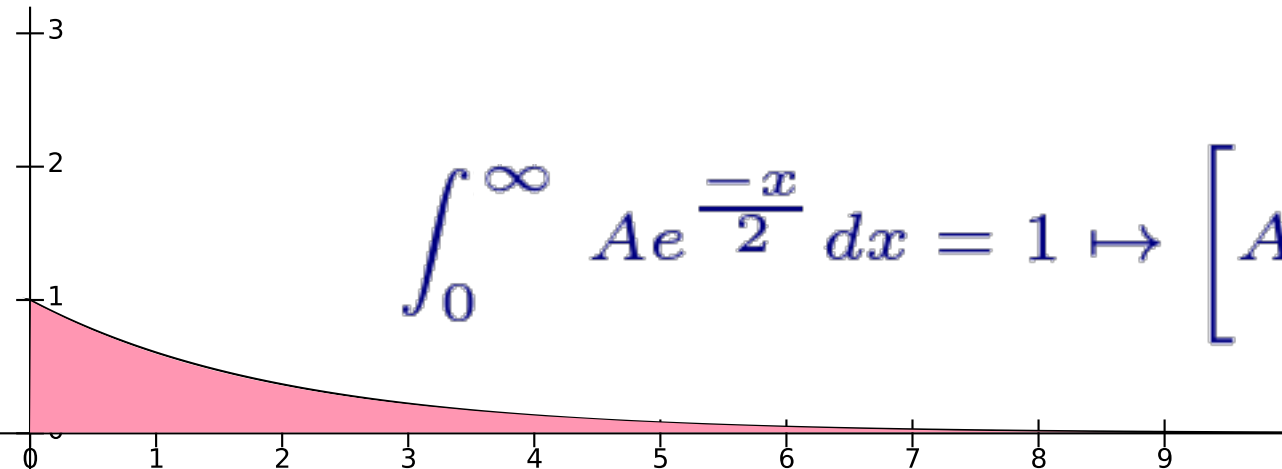
$$\int_0^{\infty} Ae^{-\frac{x}{2}} dx = 1 \mapsto \left[ A \frac{1}{\frac{-1}{2}} e^{-\frac{x}{2}} \right]_0^{\infty} = 1$$

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# Question

$$f(x) = Ae^{-\frac{x}{2}} \quad \text{for } x \geq 0$$

$$\int_{-\infty}^{\infty} f(x) dx = 1$$



$$\int_0^{\infty} Ae^{-\frac{x}{2}} dx = 1 \mapsto \left[ A \frac{1}{\frac{-1}{2}} e^{-\frac{x}{2}} \right]_0^{\infty} = 1$$

$$\left[ -2Ae^{-\frac{x}{2}} \right]_0^{\infty} = -2A \left( \cancel{e^{-\infty}}_0 - \cancel{e^0}_1 \right) = 1$$

$$A = \frac{1}{2}$$

# Question

The lifetime of an electronic component (in thousands of hours) is a continuous random variable with the probability density function given by:

$$f(x) = \frac{1}{2}e^{-\frac{x}{2}} \quad \text{for } x \geq 0$$

and zero otherwise:

What portion of the components last longer than 4000 hours?

# Question

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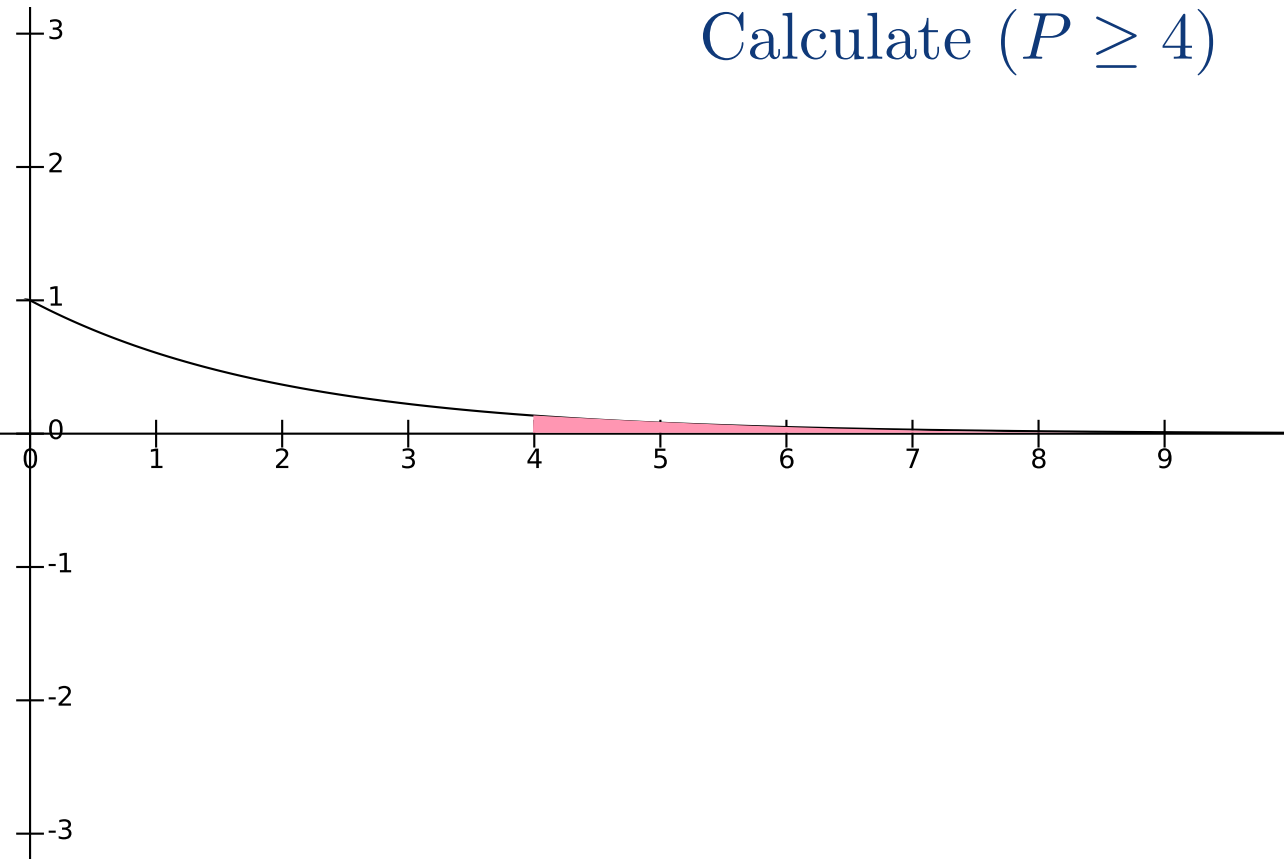
and zero otherwise:

What portion of the components last longer than **4000** hours?

Calculate  $(P \geq 4)$

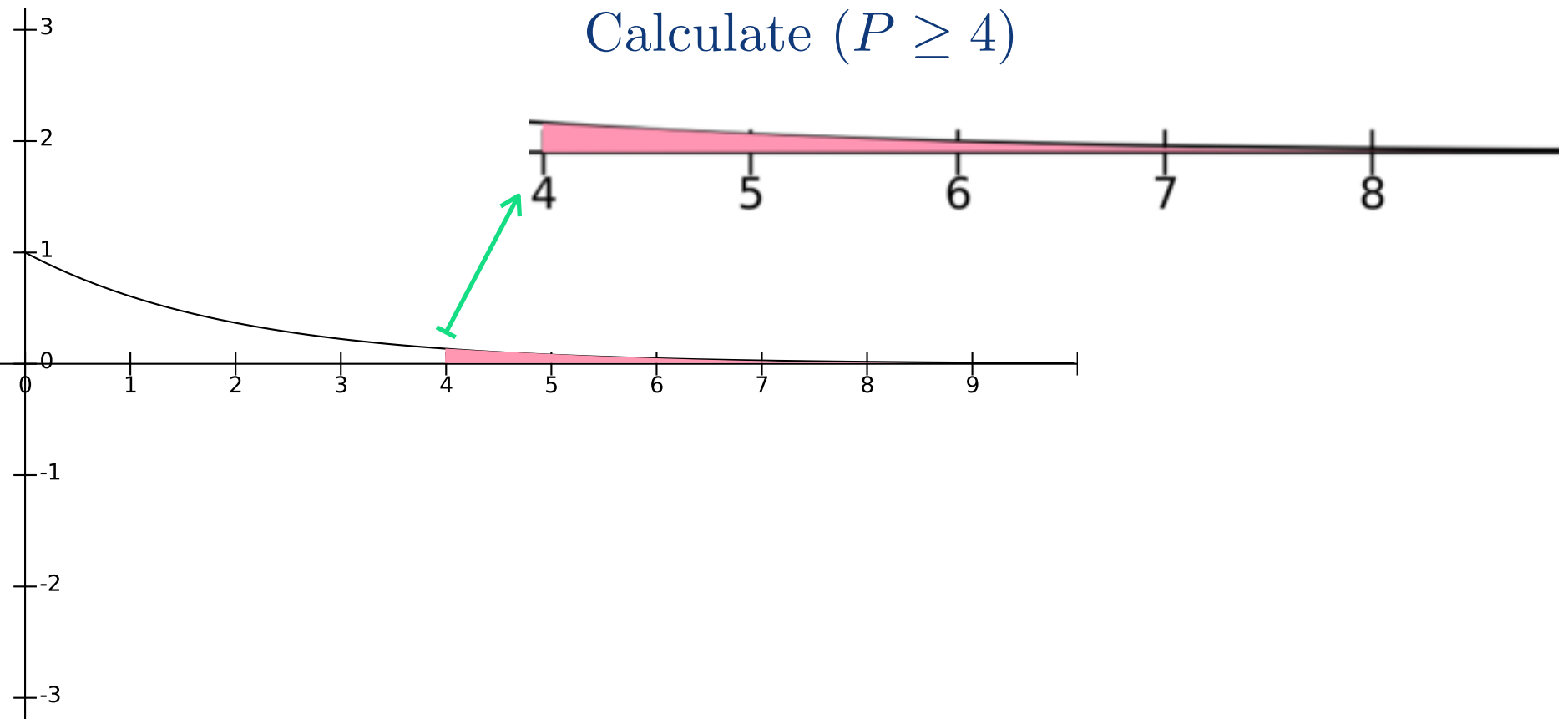
# Question

Calculate  $(P \geq 4)$



# Question

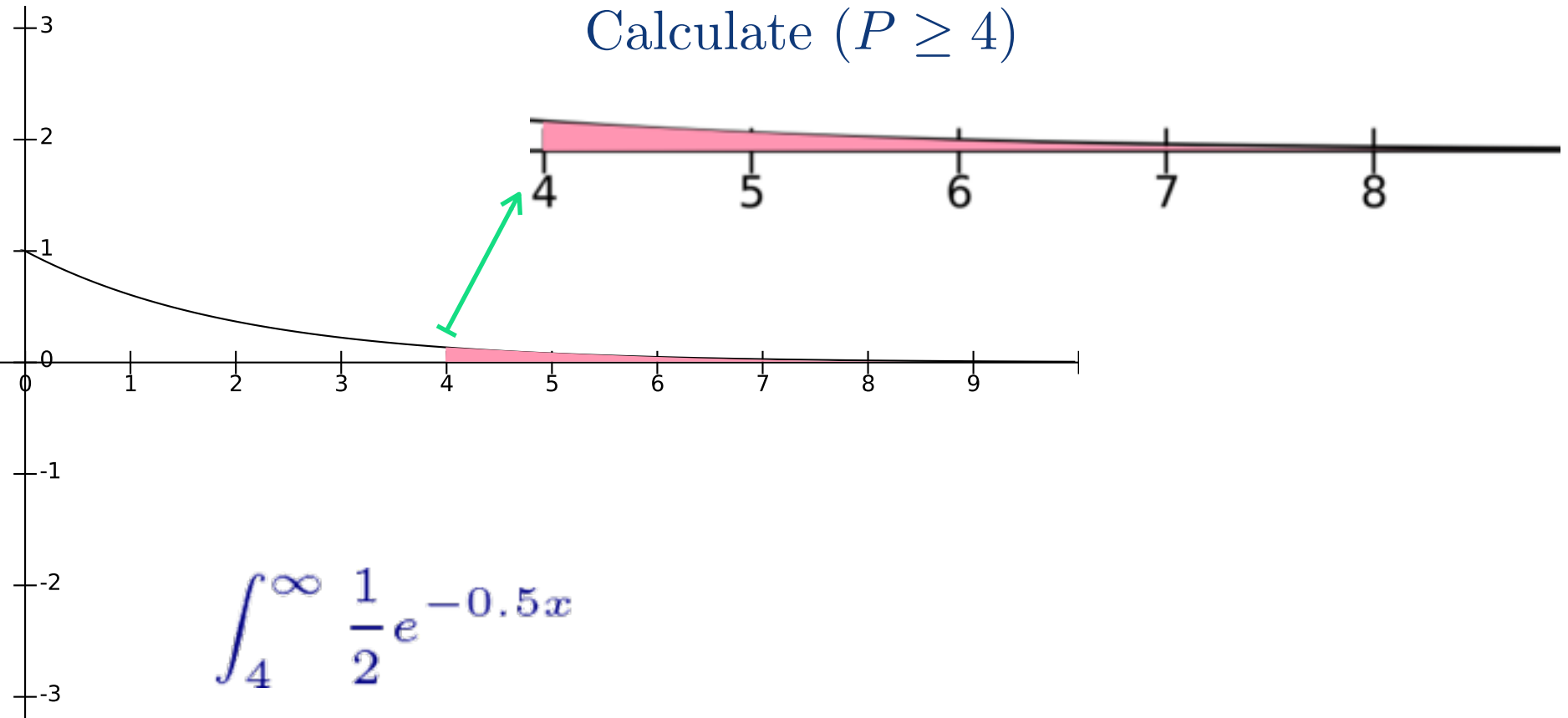
Calculate  $(P \geq 4)$





# Question

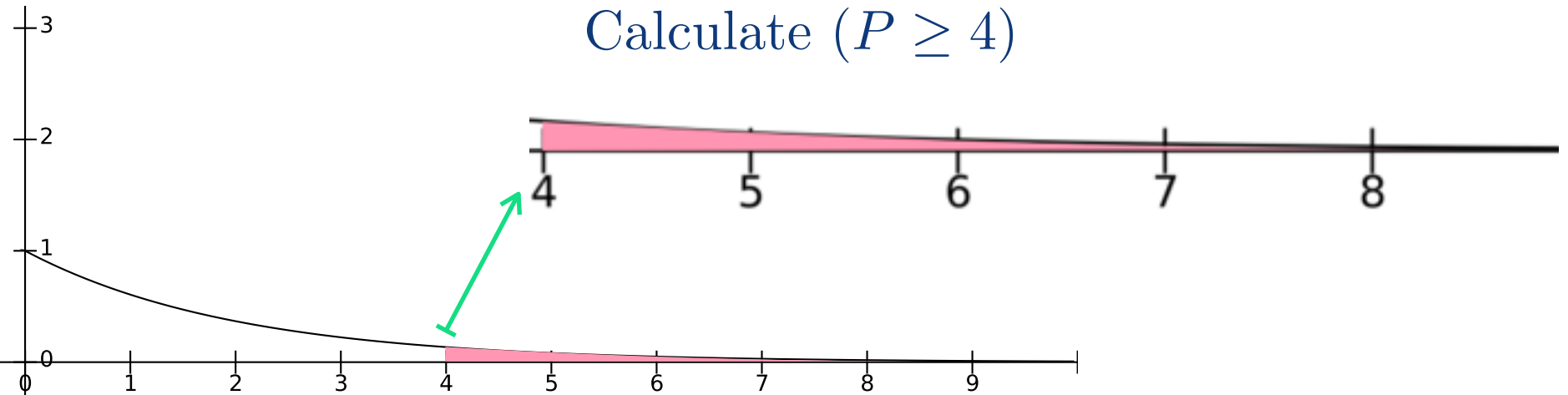
Calculate  $(P \geq 4)$



$$\int_4^{\infty} \frac{1}{2} e^{-0.5x}$$

# Question

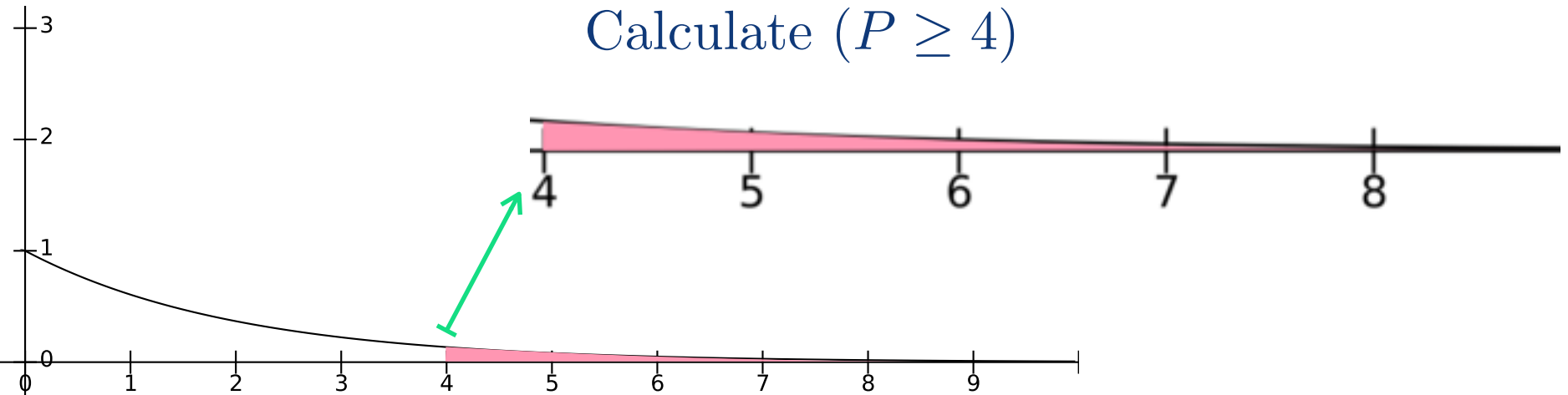
Calculate  $(P \geq 4)$



$$\int_4^{\infty} \frac{1}{2} e^{-0.5x} = \left[ -e^{-0.5x} \right]_4^{\infty}$$

# Question

Calculate  $(P \geq 4)$



$$\int_4^{\infty} \frac{1}{2} e^{-0.5x} = \left[ -e^{-0.5x} \right]_4^{\infty} = \frac{1}{2} e^{-2} = 0.067$$

# Example:

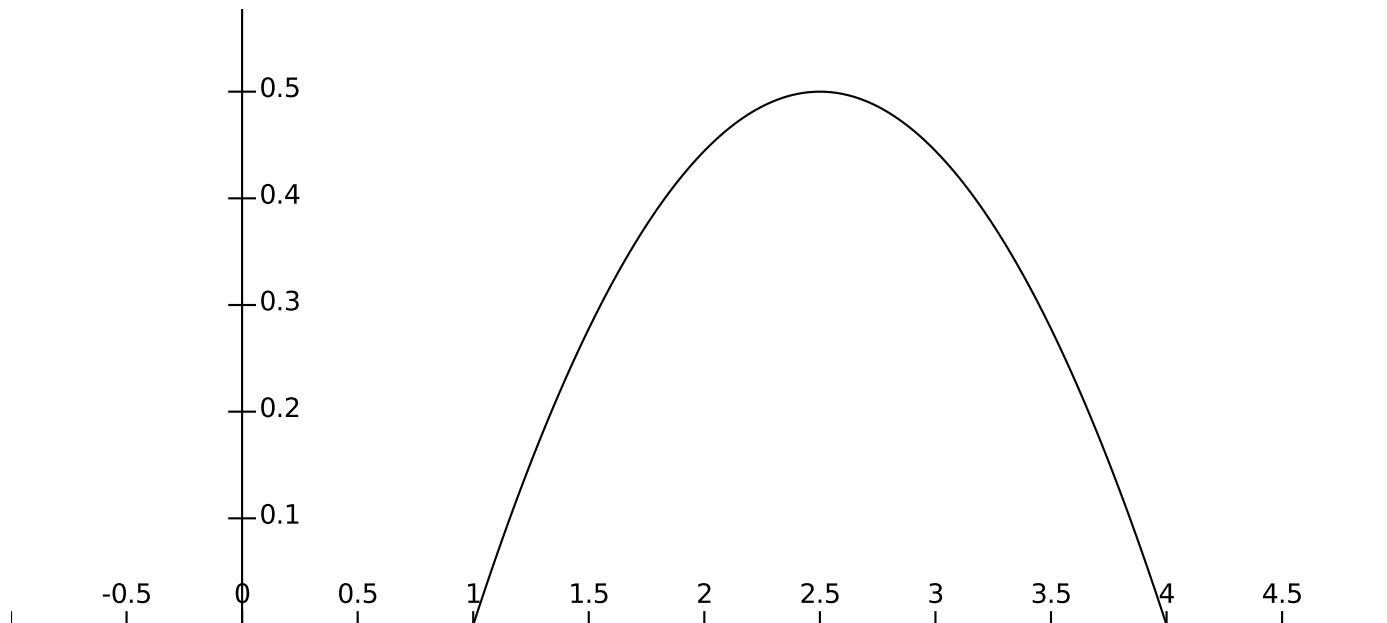
A charity group raises funds by collecting waste paper. The collected materials will contain an amount,  $X$ , of other materials such as plastic bags and rubber bands.  $X$  may be regarded as a random variable with probability density function:

$$f(x) = \begin{cases} f(x) = k(x - 1)(4 - x) & 1 < x < 4 \\ 0 & \text{Otherwise} \end{cases}$$

- (i) Show that  $K=2/9$
- (ii) Find the Expected Value and Standard Deviation of  $X$ .
- (iii) Find the Probability of  $X$  that exceeds 3.5

# Example:

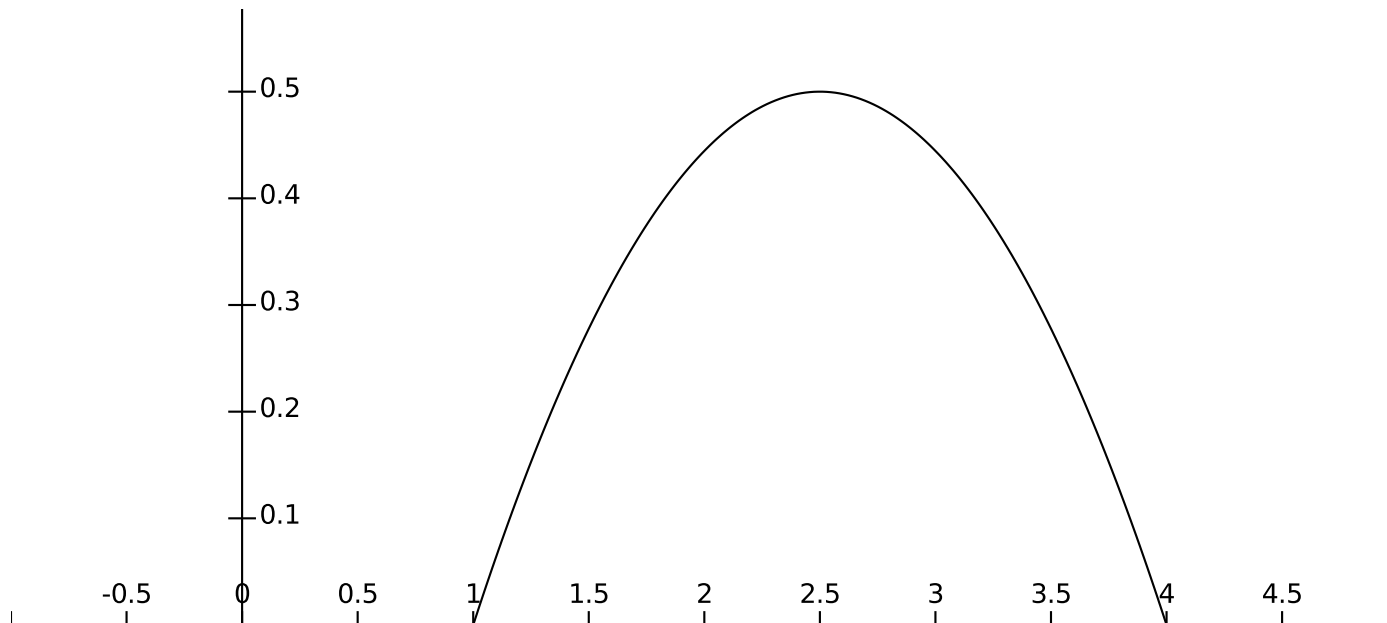
$$f(x) = \begin{cases} f(x) = k(x-1)(4-x) & 1 < x < 4 \\ 0 & \text{Otherwise} \end{cases}$$



# Example:

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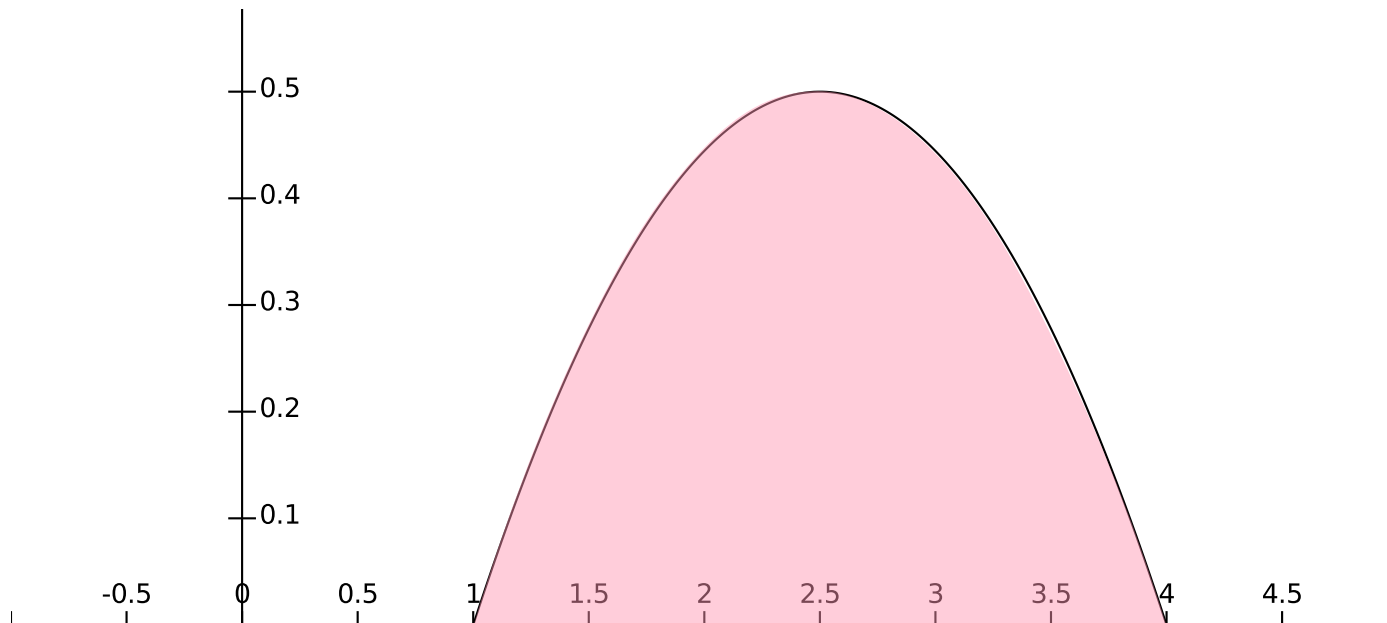
(i) Show that  $K=2/9$



# Example:

$$f(x) = \begin{cases} f(x) = k(x-1)(4-x) & 1 < x < 4 \\ 0 & \text{Otherwise} \end{cases}$$

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$$\int_{-\infty}^{\infty} f(x) dx = 1$$

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$$\int_1^4 k(x-1)(4-x) dx = 1$$

$$\int_{-\infty}^{\infty} f(x) dx = 1 \quad \rightarrow$$

# Example:

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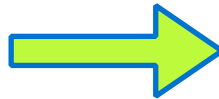
$$k \int_1^4 (x-1)(4-x) dx = 1$$

# Example:

$$f(x) = \begin{cases} f(x) = k(x-1)(4-x) & 1 < x < 4 \\ 0 & \text{Otherwise} \end{cases}$$

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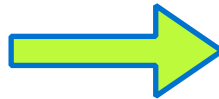
$$k \left[ \frac{-x^3}{3} + \frac{-5x^2}{2} - 4x \right]_1^4 = 1$$

# Example:

$$f(x) = \begin{cases} f(x) = k(x-1)(4-x) & 1 < x < 4 \\ 0 & \text{Otherwise} \end{cases}$$

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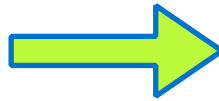
$$k \left[ \frac{8}{3} - \left( -\frac{11}{6} \right) \right] = 1$$

# Example:

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$$k \left[ \frac{8}{3} - \left( -\frac{11}{6} \right) \right] = 1$$

$$4.5k = 1$$

$$k = \frac{2}{9}$$

# Example:

$$f(x) = \begin{cases} f(x) = k(x - 1)(4 - x) & 1 < x < 4 \\ 0 & \textit{Otherwise} \end{cases}$$

**(ii)** Find the Expected Value and Standard Deviation of X.

# Example:

$$f(x) = \begin{cases} f(x) = k(x-1)(4-x) & 1 < x < 4 \\ 0 & \text{Otherwise} \end{cases}$$

(ii) Find the Expected Value and Standard Deviation of X.

$$\mu = E[x] = \int_{-\infty}^{+\infty} x f(x) dx$$



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# Example:

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$$\begin{aligned} E[x] &= \int_1^4 \frac{2}{9} x (x-1)(4-x) dx \\ &= \frac{2}{9} \left( \int_1^4 x (x-1)(4-x) dx \right) \\ &= \frac{2}{9} \left[ \frac{-x^4}{4} + \frac{5x^3}{3} - \frac{4x^2}{2} \right]_1^4 \\ &= \frac{2}{9} \left( \frac{32}{3} - \left( -\frac{7}{12} \right) \right) \end{aligned}$$

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# Example:

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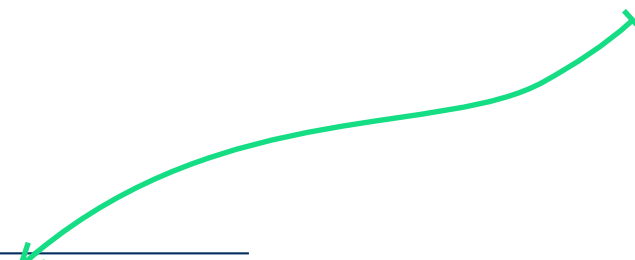
$$\sigma = \sqrt{E(x^2) - E(x)^2}$$

# Example:


$$f(x) = \begin{cases} f(x) = k(x-1)(4-x) & 1 < x < 4 \\ 0 & \text{Otherwise} \end{cases}$$

(ii) Find the Expected Value and Standard Deviation of X.

$$E(x^2) = \int_1^4 \frac{2}{9} x^2 (x-1)(4-x) dx$$



$$\sigma = \sqrt{E(x^2) - E(x)^2}$$





# Example:

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$$= \frac{2}{9} \left( \frac{448}{15} - \left( -\frac{17}{60} \right) \right)$$
$$= 6.7$$

# Example:

$$f(x) = \begin{cases} f(x) = k(x-1)(4-x) & 1 < x < 4 \\ 0 & \text{Otherwise} \end{cases}$$

(ii) Find the Expected Value and Standard Deviation of X.

$$\begin{aligned} E(x^2) &= \int_1^4 \frac{2}{9} x^2 (x-1)(4-x) dx \\ &= \frac{2}{9} \left[ -\frac{x^5}{5} + \frac{5x^4}{4} - \frac{4x^3}{x} \right]_1^4 \\ &= \frac{2}{9} \left( \frac{448}{15} - \left( -\frac{17}{60} \right) \right) \\ &= 6.7 \end{aligned}$$
$$\sigma = \sqrt{E(x^2) - E(x)^2}$$
$$E(x) = 2.5$$

# Example:

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(ii) Find the Expected Value and Standard Deviation of X.

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$$E(x^2) = \int_1^4 \frac{2}{9} x^2 (x-1)(4-x) dx$$
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$$= 6.7$$
$$E(x) = 2.5$$

# Example:

$$f(x) = \begin{cases} f(x) = k(x-1)(4-x) & 1 < x < 4 \\ 0 & \text{Otherwise} \end{cases}$$

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$\sigma = \sqrt{E(x^2) - E(x)^2}$

$$E(x) = 2.5$$

$$\sigma = \sqrt{6.7 - 2.5^2} = 0.671$$

# Example:

$$f(x) = \begin{cases} f(x) = k(x - 1)(4 - x) & 1 < x < 4 \\ 0 & \textit{Otherwise} \end{cases}$$

**(iii)** Find the Probability of X that exceeds 3.5

# Example:

$$f(x) = \begin{cases} f(x) = k(x-1)(4-x) & 1 < x < 4 \\ 0 & \text{Otherwise} \end{cases}$$

**(iii)** Find the Probability of X that exceeds 3.5

$$P(x > 3.5) = \int_{3.5}^4 \frac{2}{9} (x-1)(4-x) dx$$

# Example:

$$f(x) = \begin{cases} f(x) = k(x-1)(4-x) & 1 < x < 4 \\ 0 & \text{Otherwise} \end{cases}$$

**(iii)** Find the Probability of X that exceeds 3.5

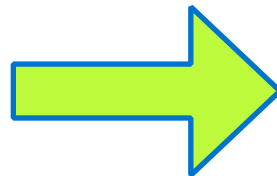
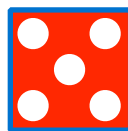
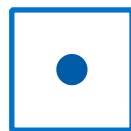
$$\begin{aligned} P(x > 3.5) &= \int_{3.5}^4 \frac{2}{9} (x-1)(4-x) dx \\ &= \frac{2}{9} \left[ -\frac{x^3}{3} + \frac{5x^2}{2} - 4x \right]_{3.5}^4 \\ &= \frac{2}{9} \left( \frac{8}{3} - \frac{7}{3} \right) \\ &= 0.0741 \end{aligned}$$



# Normal Distribution

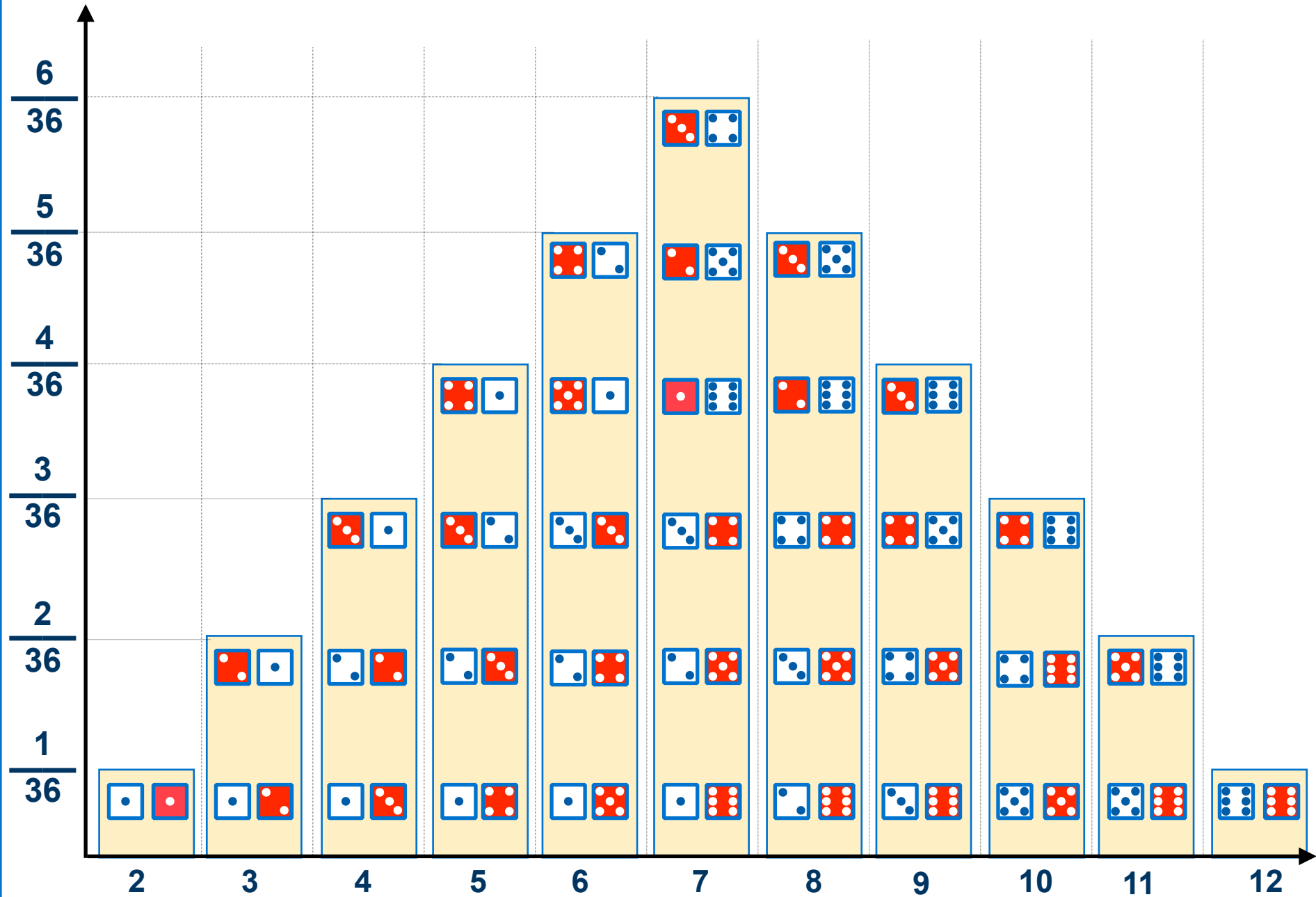
**Example:** We roll Two Dice, and we sum up the shown numbers.

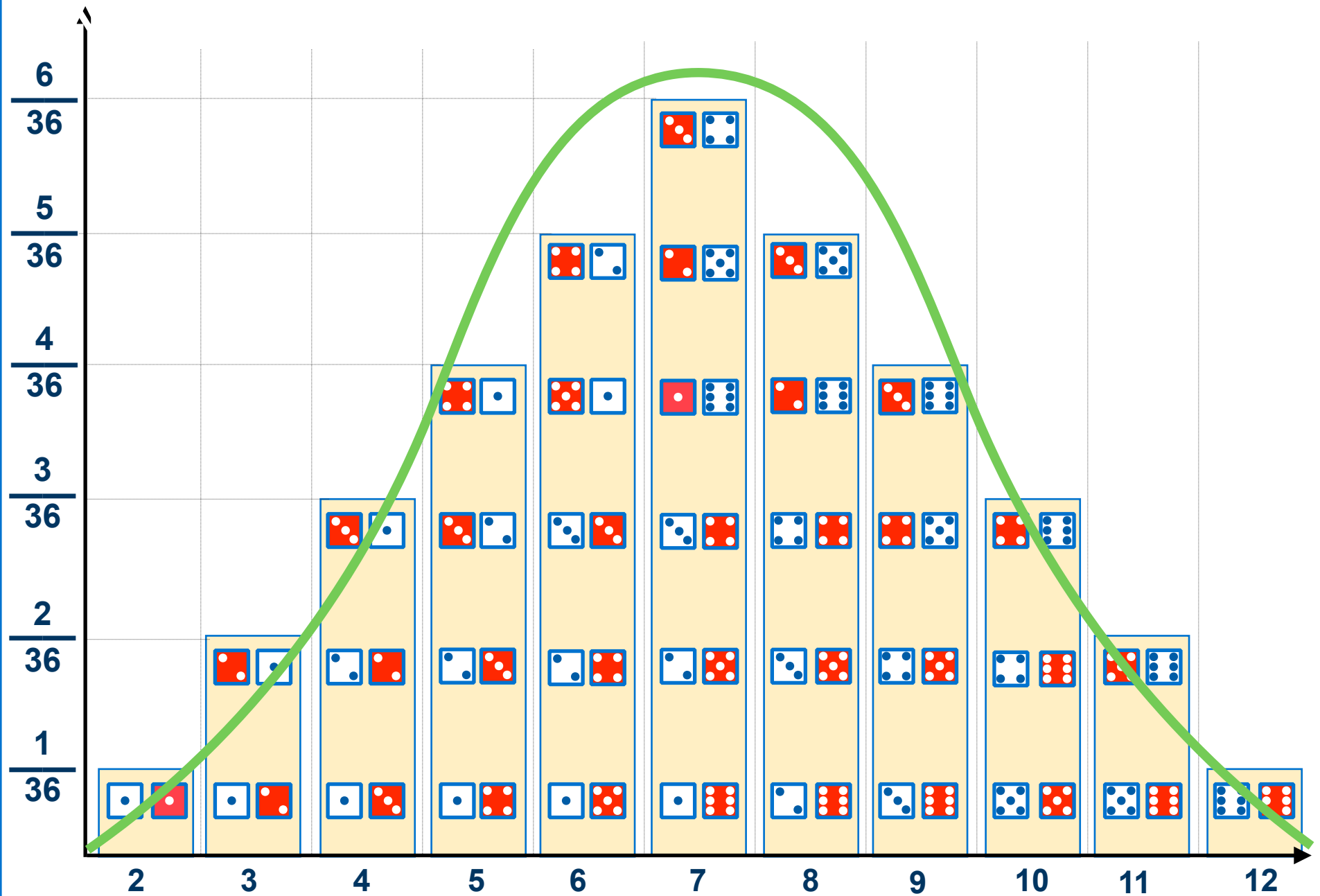
For instance:

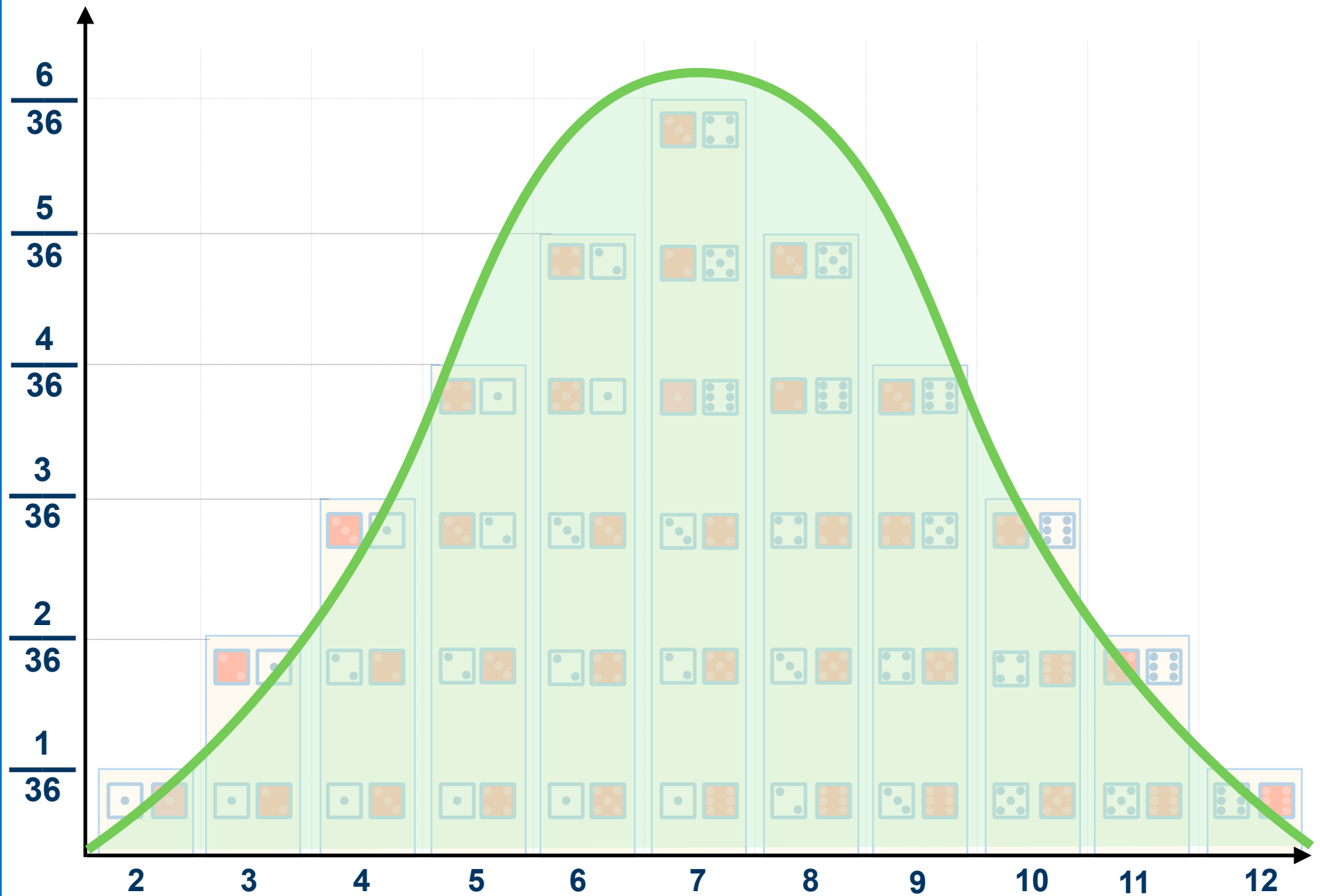


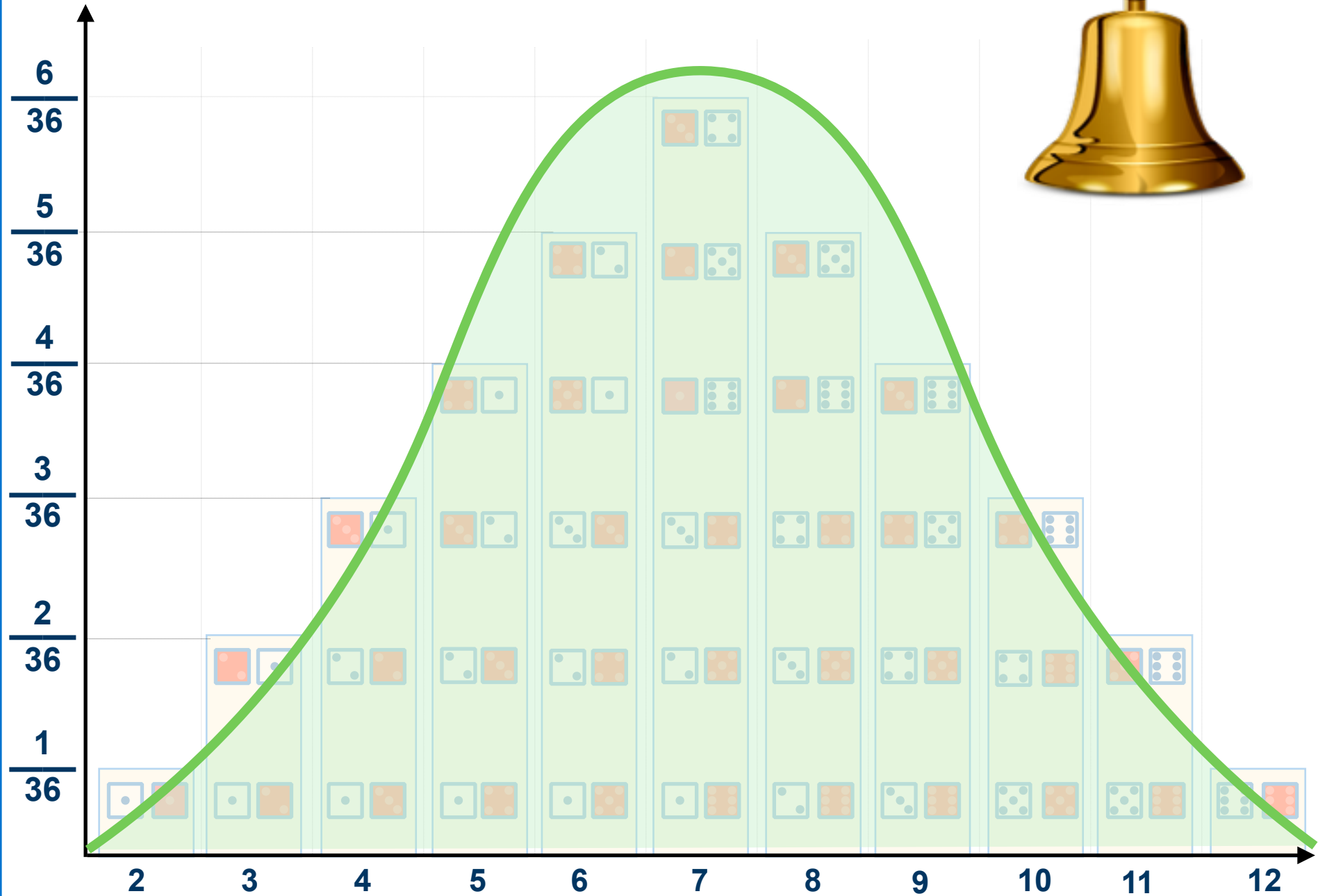
$$5+1 = 6$$

$$P(1,5) = 1/36$$

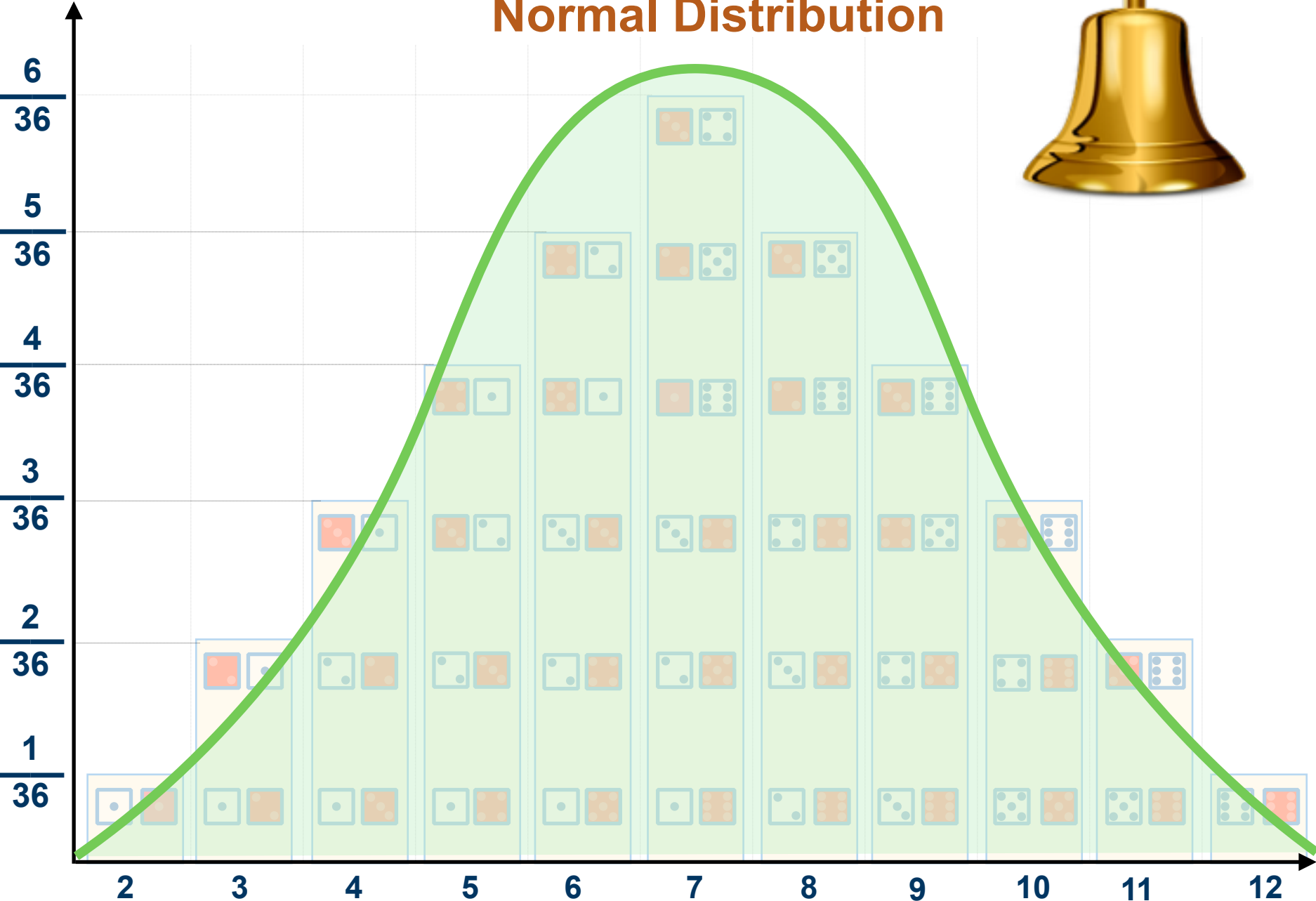








# Normal Distribution



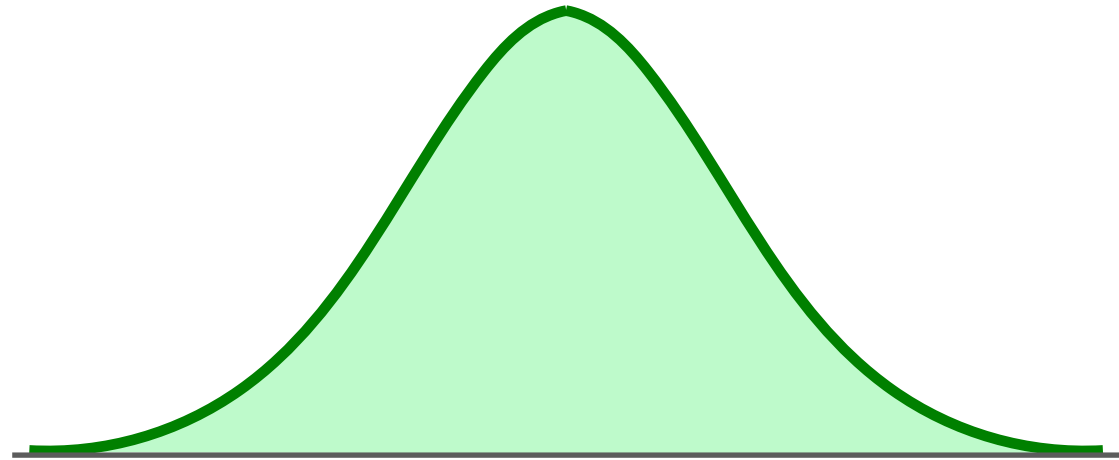
# Normal Distribution

## Characteristics of a Normal Distribution

# Normal Distribution

## Characteristics of a Normal Distribution

1- It's curve looks like a bell





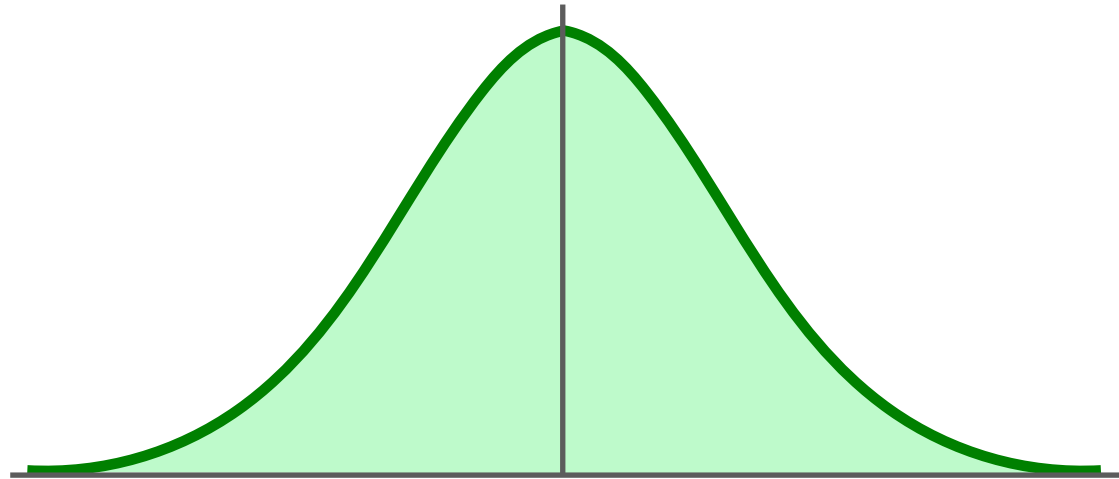
# Normal Distribution

## Characteristics of a Normal Distribution

1- It's curve looks like a bell



2- Symmetric



# Normal Distribution

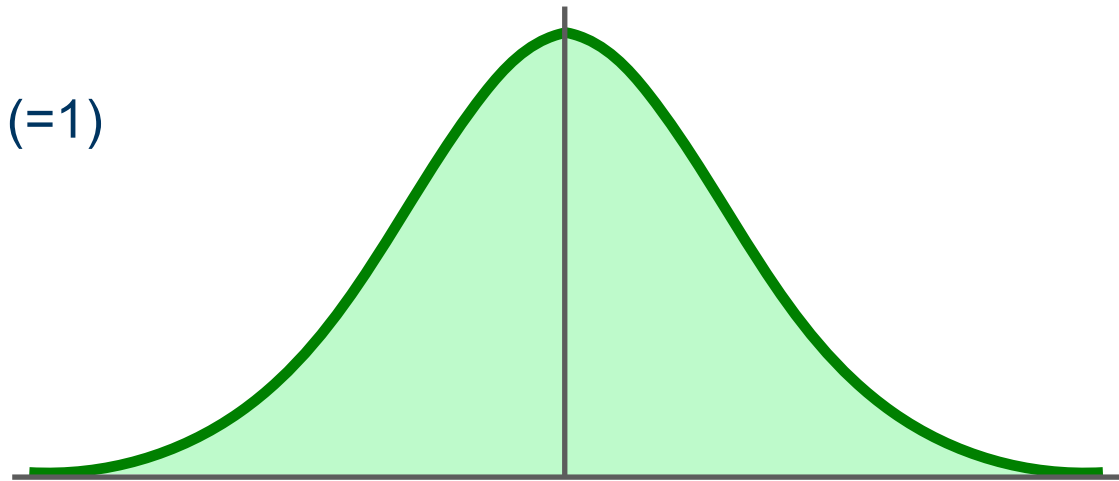
## Characteristics of a Normal Distribution

1- It's curve looks like a bell



2- Symmetric

3- Total Area is equal to ONE (=1)



# Normal Distribution

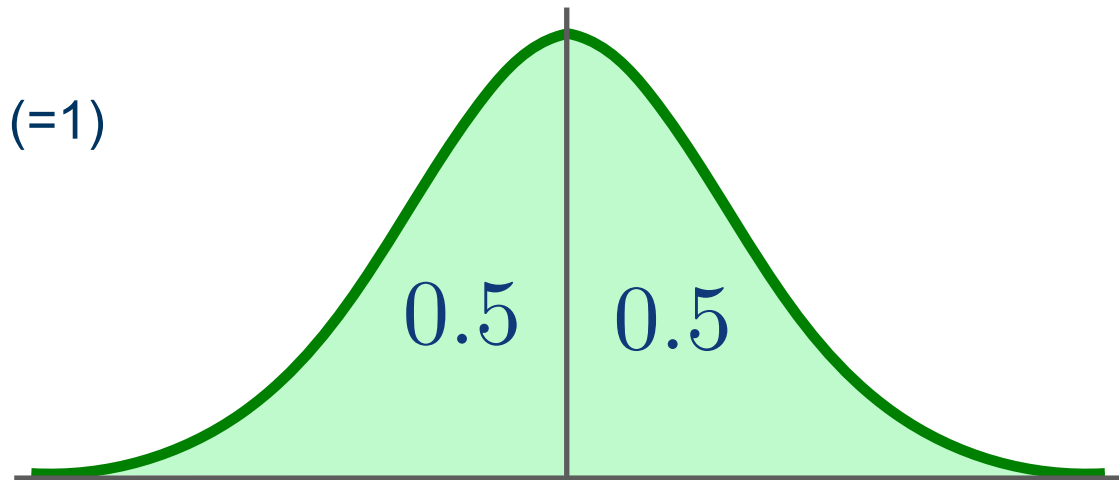
## Characteristics of a Normal Distribution

1- It's curve looks like a bell



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3- Total Area is equal to ONE (=1)



# Normal Distribution

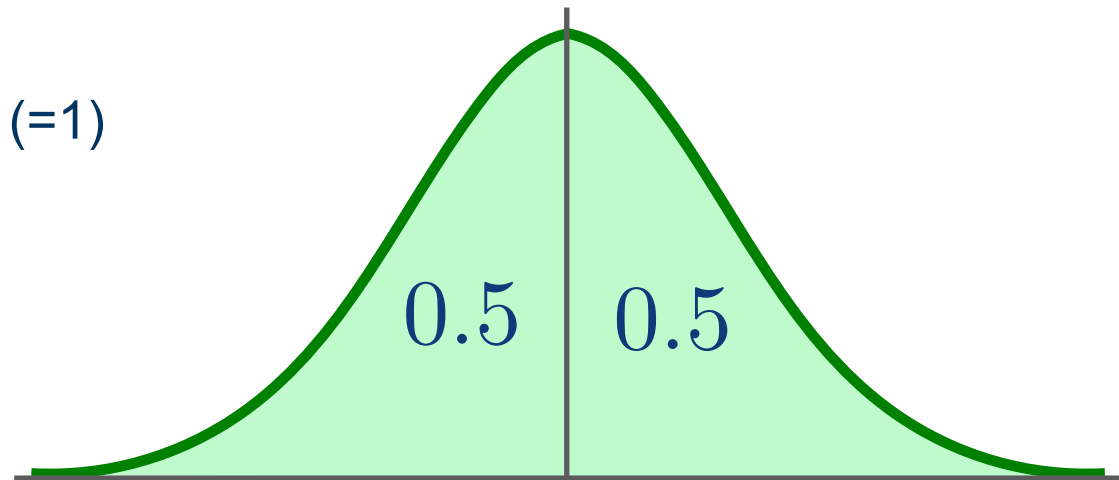
## Characteristics of a Normal Distribution

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2- Symmetric

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# Normal Distribution

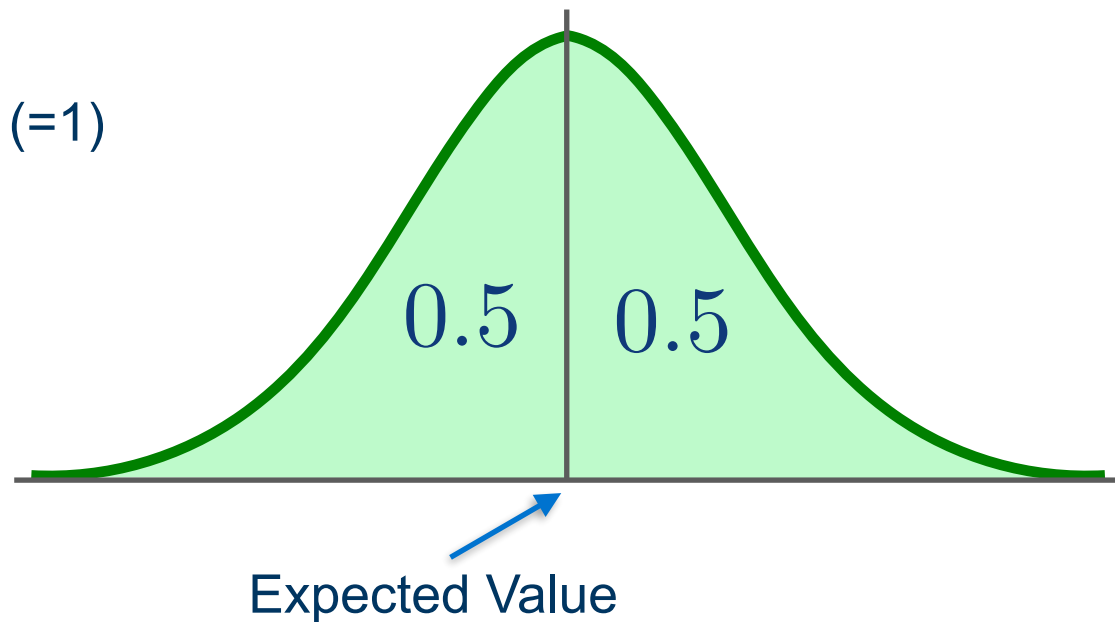
## Characteristics of a Normal Distribution

1- It's curve looks like a bell



2- Symmetric

3- Total Area is equal to ONE (=1)



# Normal Distribution

## Characteristics of a Normal Distribution

1- It's curve looks like a bell

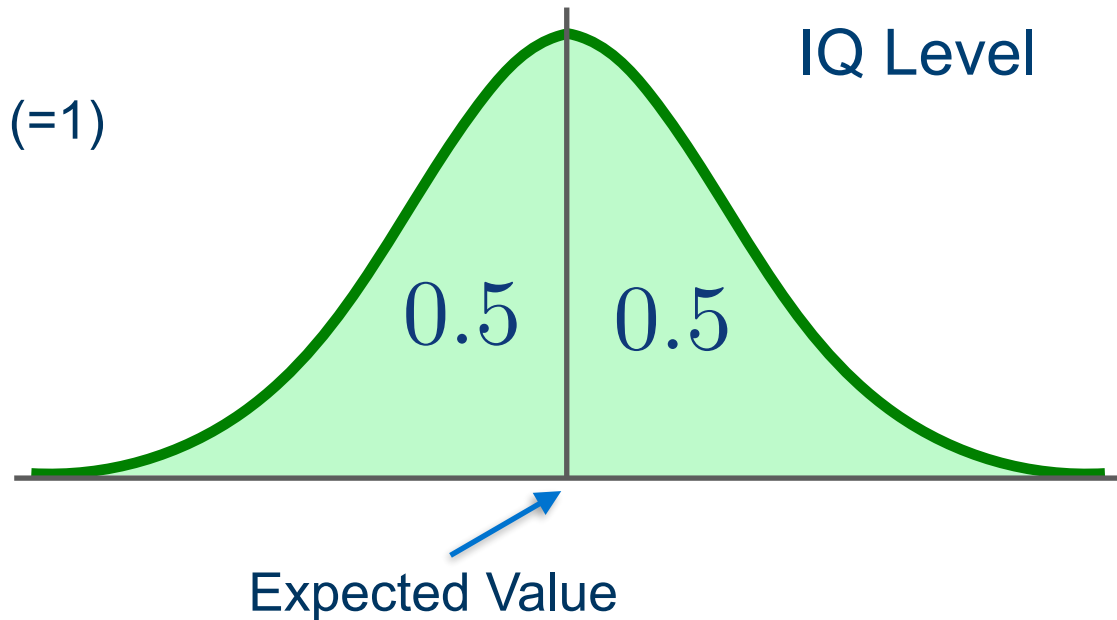


2- Symmetric

3- Total Area is equal to ONE (=1)

**Example:**

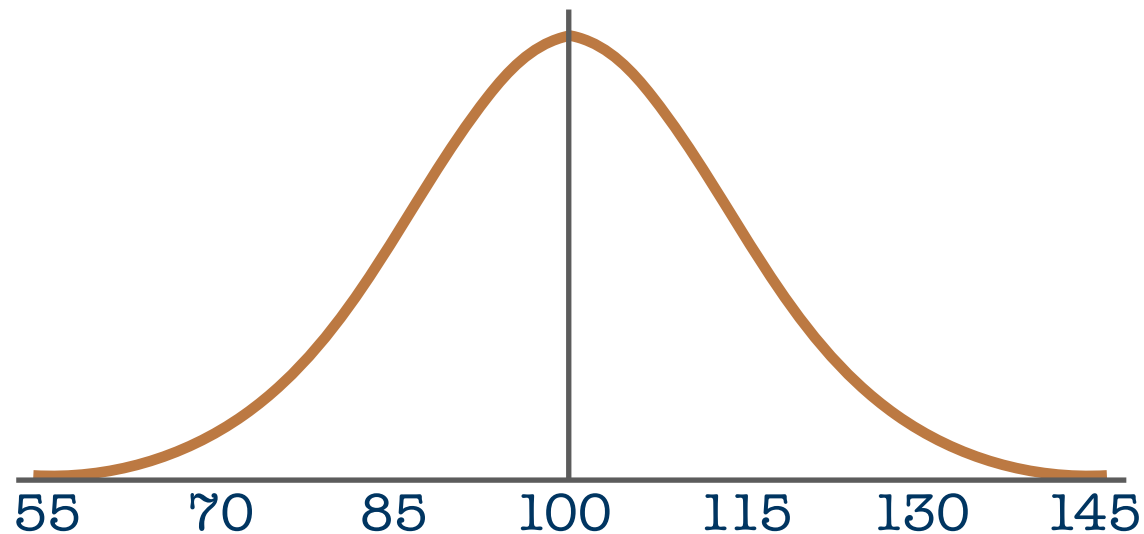
**IQ Level**



# Normal Distribution

**Question:** Suppose scores on an IQ test are normally distributed as shown.

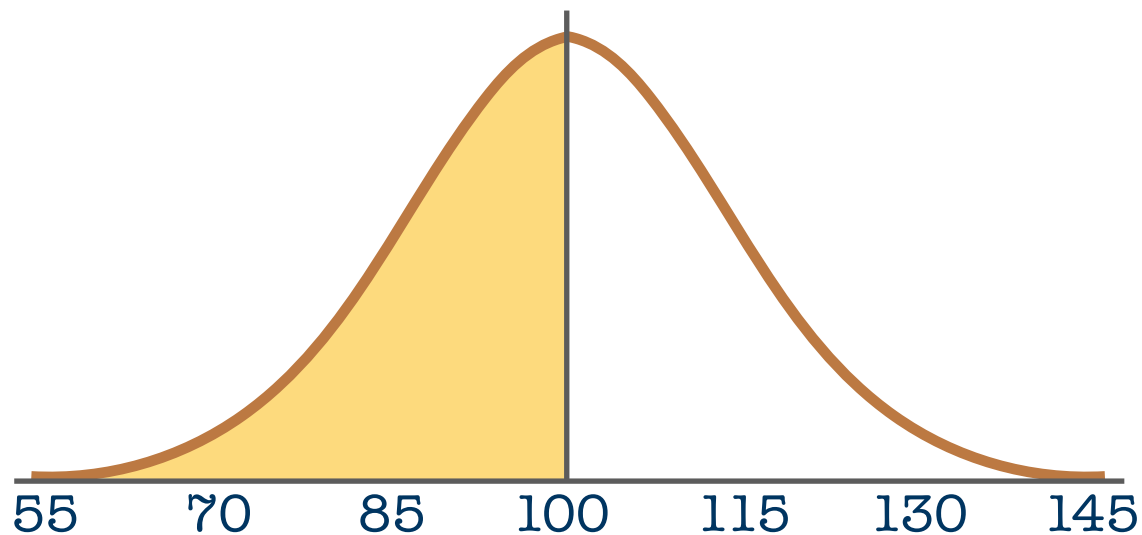
What is the probability that a person who takes the test will score below 100?



# Normal Distribution

**Question:** Suppose scores on an IQ test are normally distributed as shown.

What is the probability that a person who takes the test will score below 100?



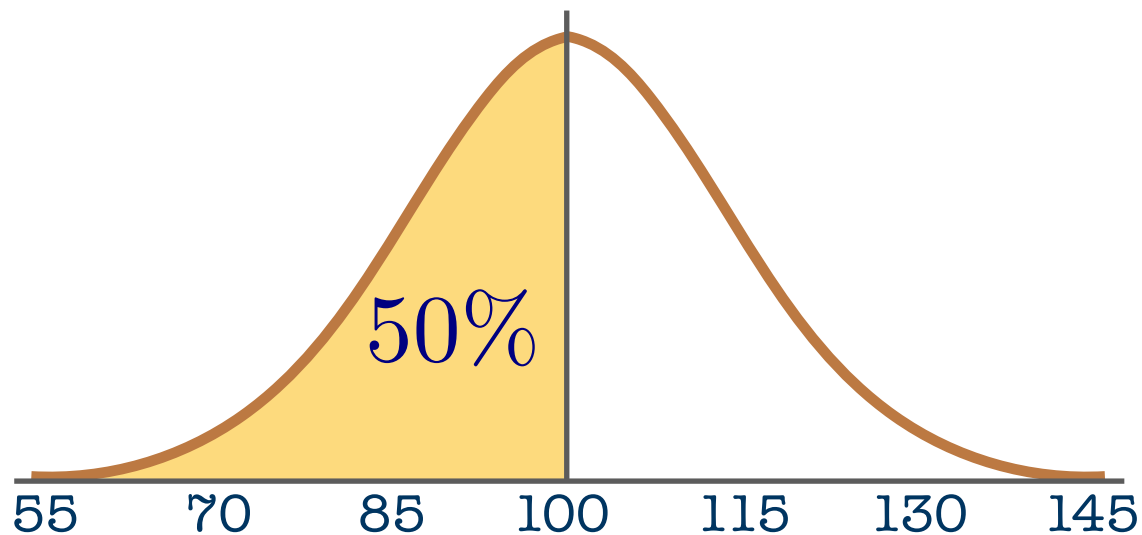


# Normal Distribution

**Question:** Suppose scores on an IQ test are normally distributed as shown.

What is the probability that a person who takes the test will score below 100?

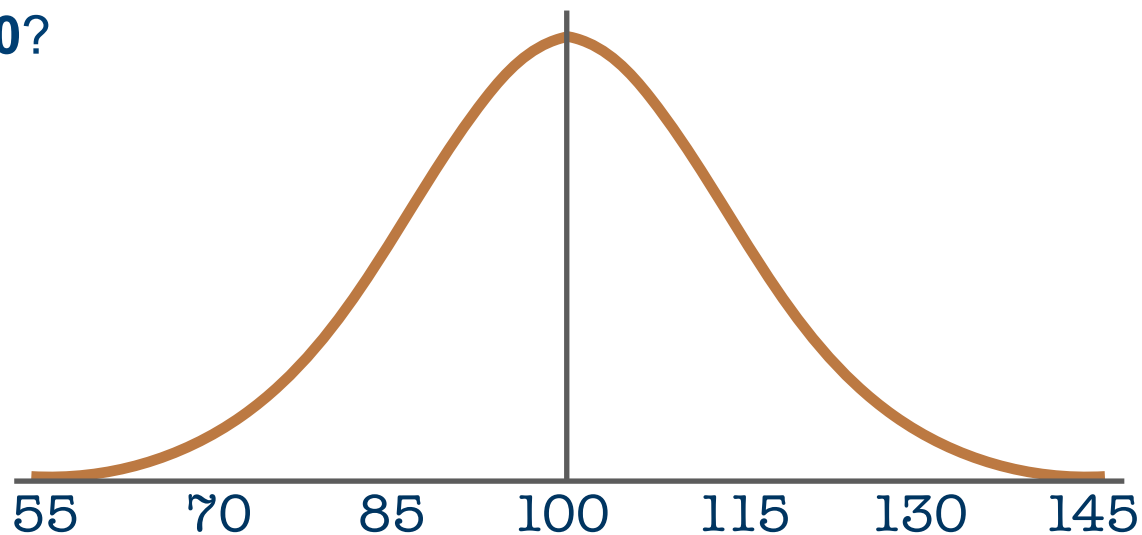
The answer is 50%.



# Normal Distribution

**Question:** Suppose scores on an IQ test are normally distributed as shown.

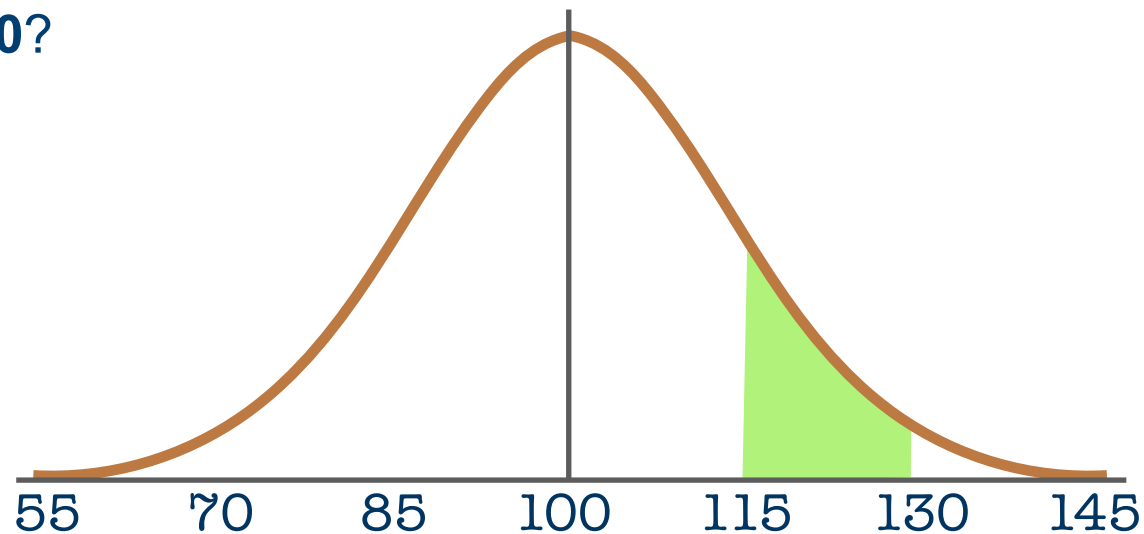
What is the probability that a person who takes the test will score **above 115 and below 130**?



# Normal Distribution

**Question:** Suppose scores on an IQ test are normally distributed as shown.

What is the probability that a person who takes the test will score **above 115 and below 130**?



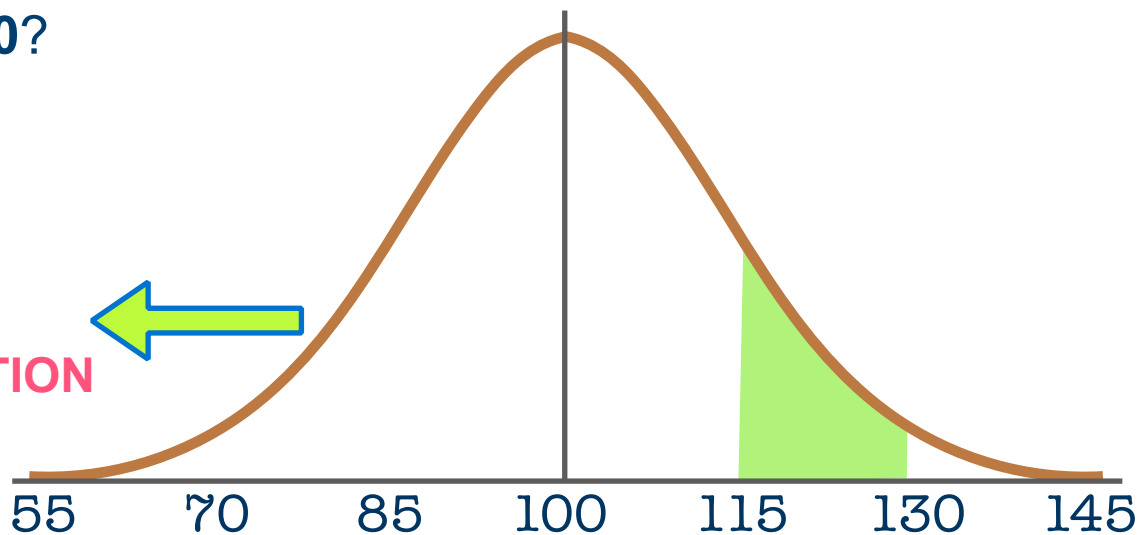
# Normal Distribution

**Question:** Suppose scores on an IQ test are normally distributed as shown.

What is the probability that a person who takes the test will score **above 115 and below 130**?

It is easier to use

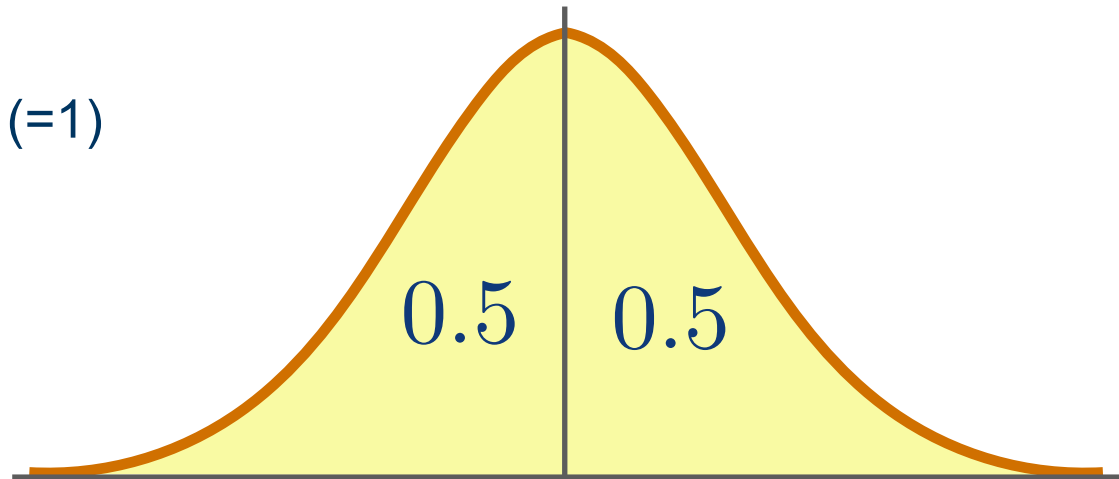
**STANDARD NORMAL DISTRIBUTION**



# “Standard” Normal Distribution

Standard Normal Distribution has:

- 1- It's curve looks like a bell
- 2- Symmetric
- 3- Total Area is equal to ONE (=1)

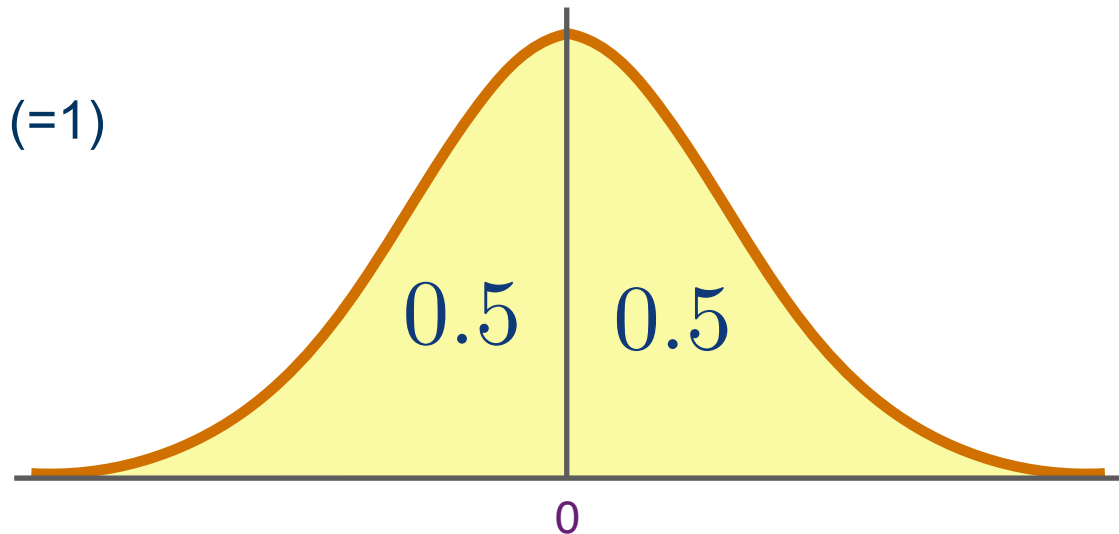


# “Standard” Normal Distribution

Standard Normal Distribution has:

- 1- It's curve looks like a bell
- 2- Symmetric
- 3- Total Area is equal to ONE (=1)
- 4- Mean ( $\mu$ ) = 0

$$\mu = 0$$



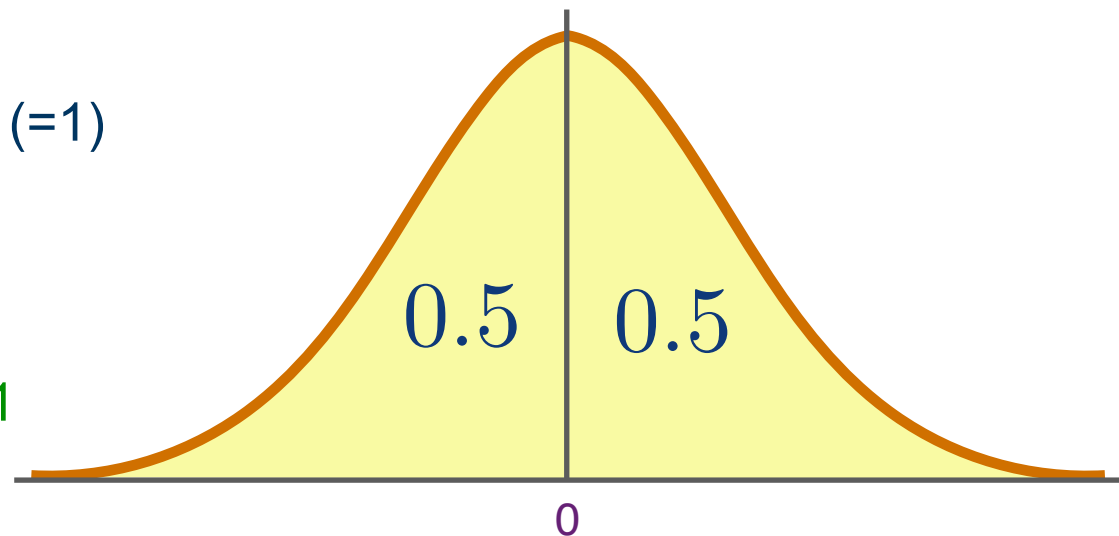
# “Standard” Normal Distribution

Standard Normal Distribution has:

- 1- It's curve looks like a bell
- 2- Symmetric
- 3- Total Area is equal to ONE (=1)
- 4- Mean ( $\mu$ ) = 0
- 5- Standard Deviation ( $\sigma$ ) is 1

$$\mu = 0$$

$$\sigma = 1$$



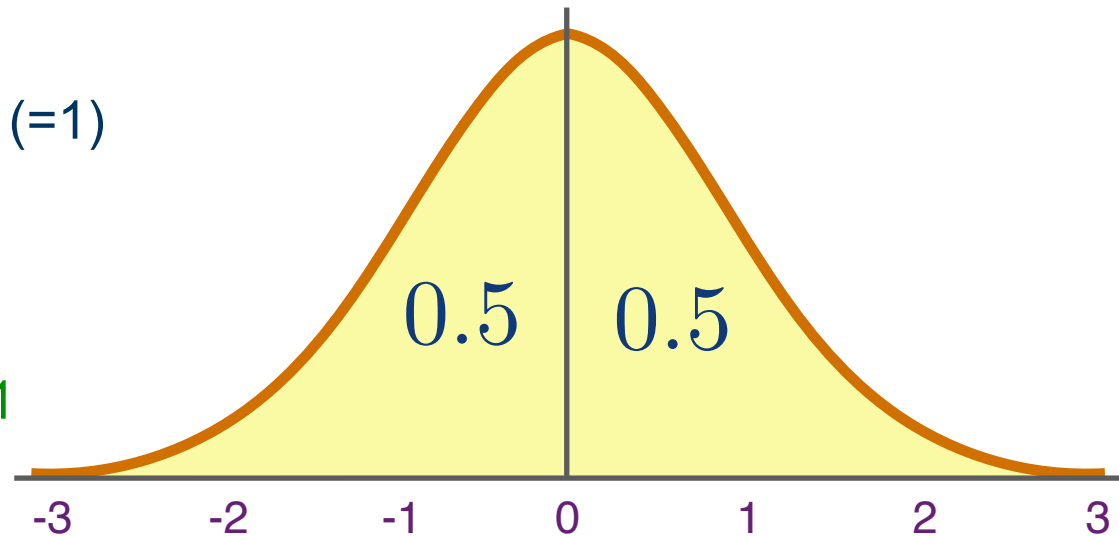
# “Standard” Normal Distribution

Standard Normal Distribution has:

- 1- It's curve looks like a bell
- 2- Symmetric
- 3- Total Area is equal to ONE (=1)
- 4- Mean ( $\mu$ ) = 0
- 5- Standard Deviation ( $\sigma$ ) is 1

$$\mu = 0$$

$$\sigma = 1$$



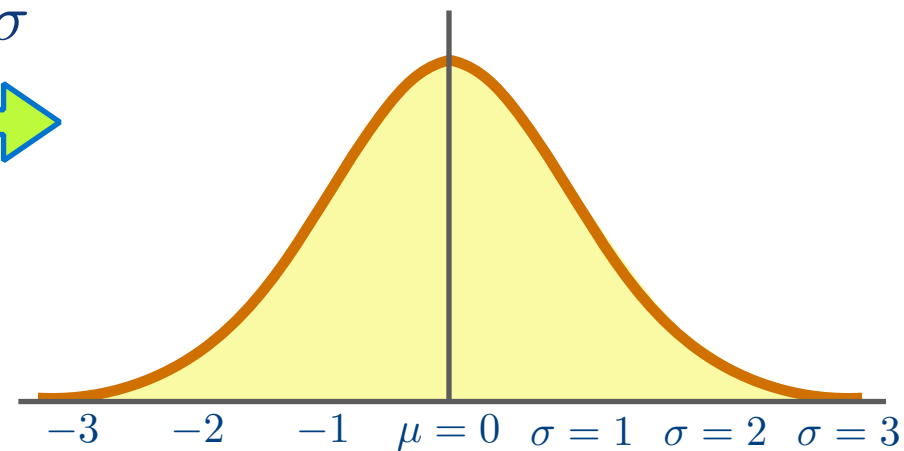
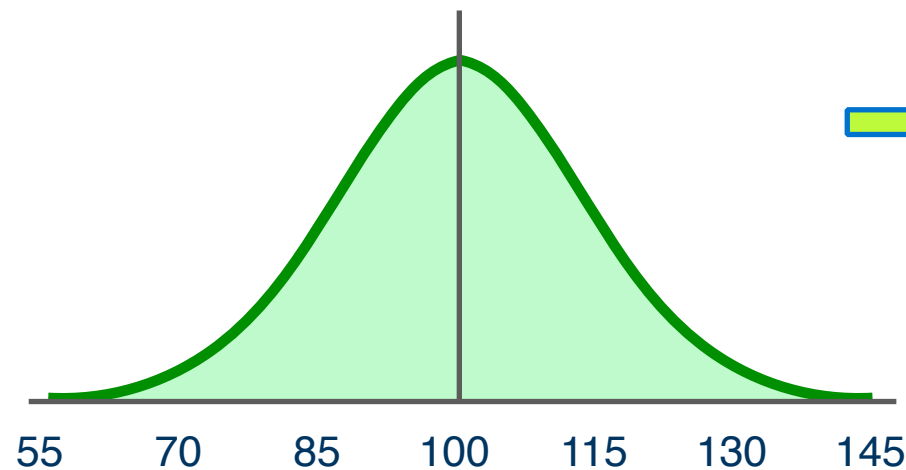


# Standard Normal Distribution

Normal Distribution

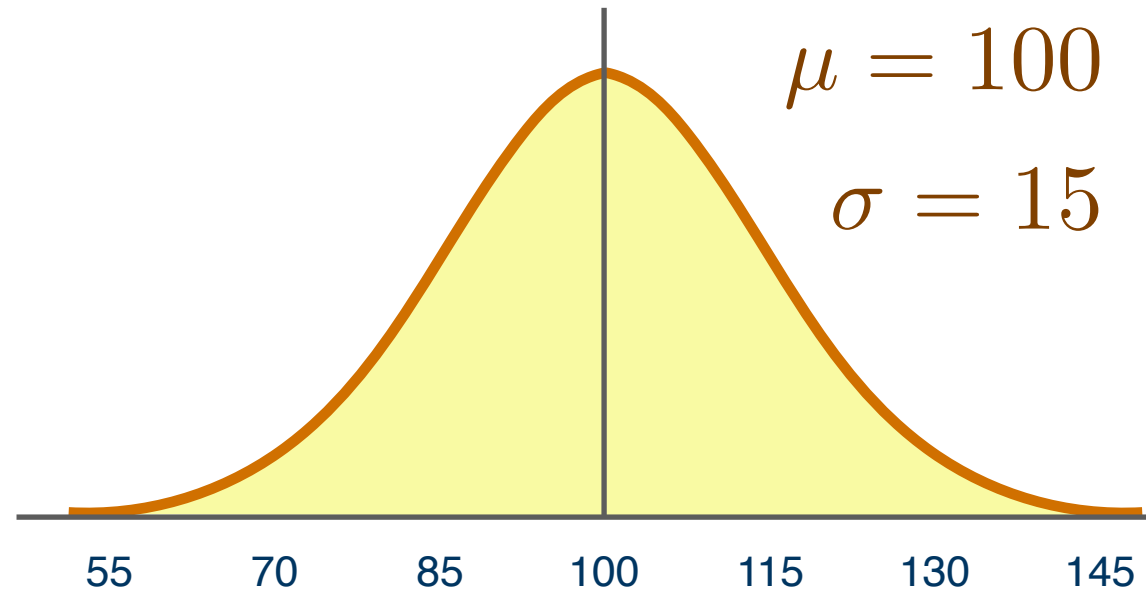
Standard Normal Distribution

$$Z = \frac{X - \mu}{\sigma}$$



# “Standard” Normal Distribution

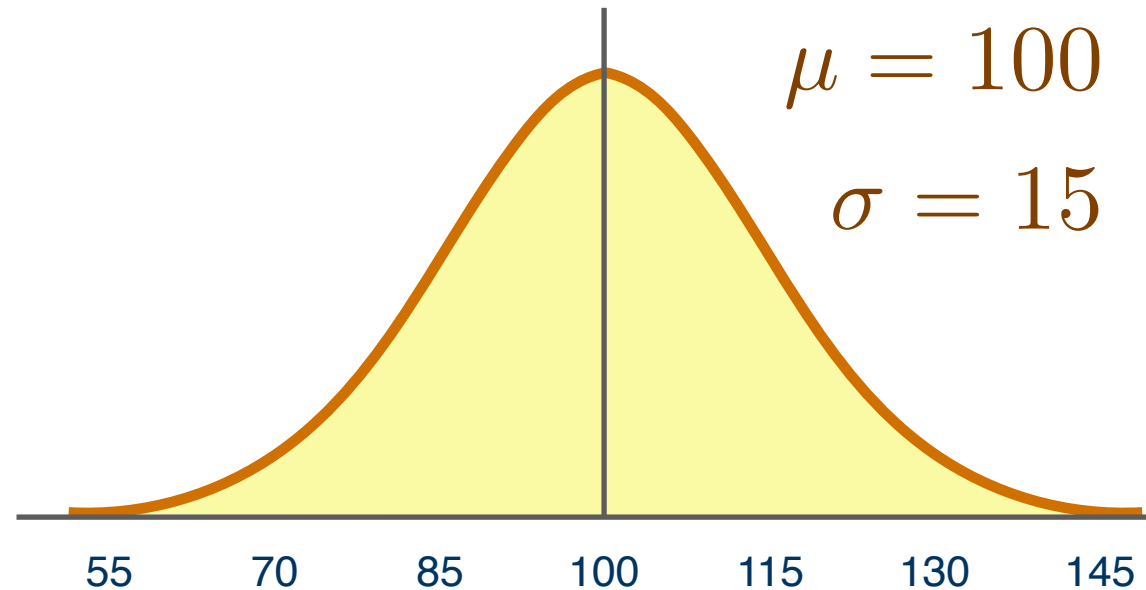
Converting Normal Distribution to Standard Normal Distribution



# “Standard” Normal Distribution

Converting Normal Distribution to Standard Normal Distribution

$$Z = \frac{X - \mu}{\sigma}$$

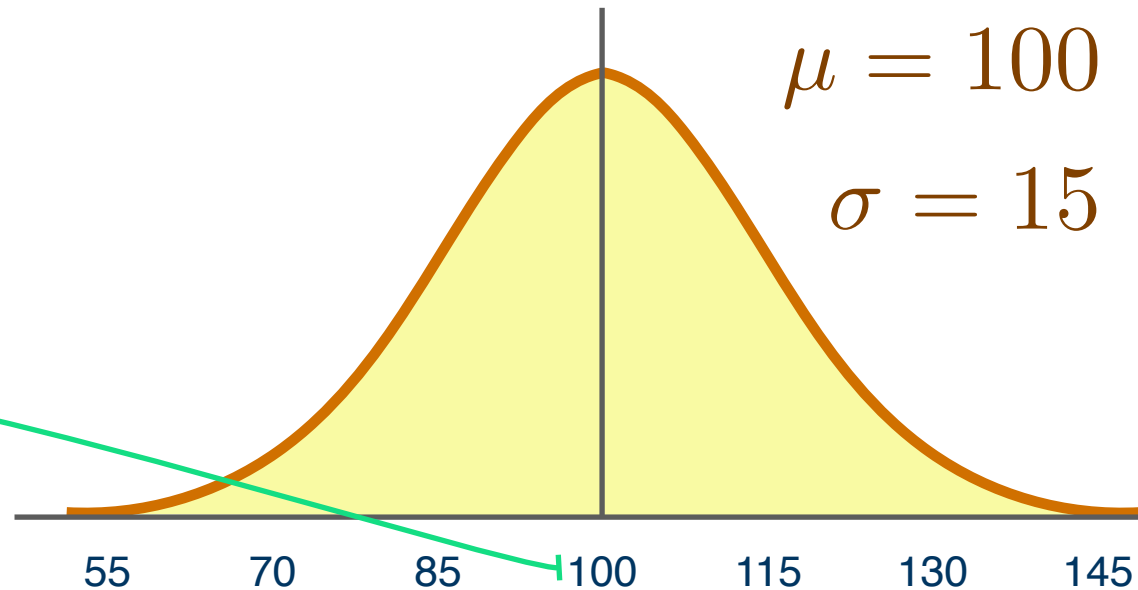


# “Standard” Normal Distribution

## Converting Normal Distribution to Standard Normal Distribution

$$Z = \frac{X - \mu}{\sigma}$$

$$\frac{100 - 100}{15} = 0$$



# “Standard” Normal Distribution

## Converting Normal Distribution to Standard Normal Distribution

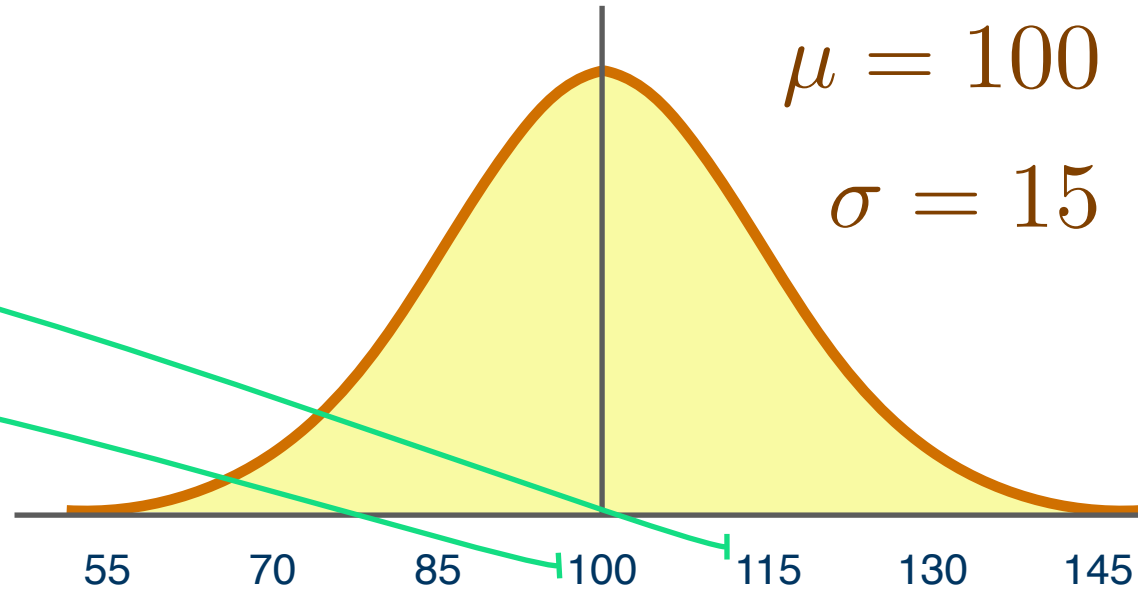
$$Z = \frac{X - \mu}{\sigma}$$

$$\frac{115 - 100}{15} = 1$$

$$\frac{100 - 100}{15} = 0$$

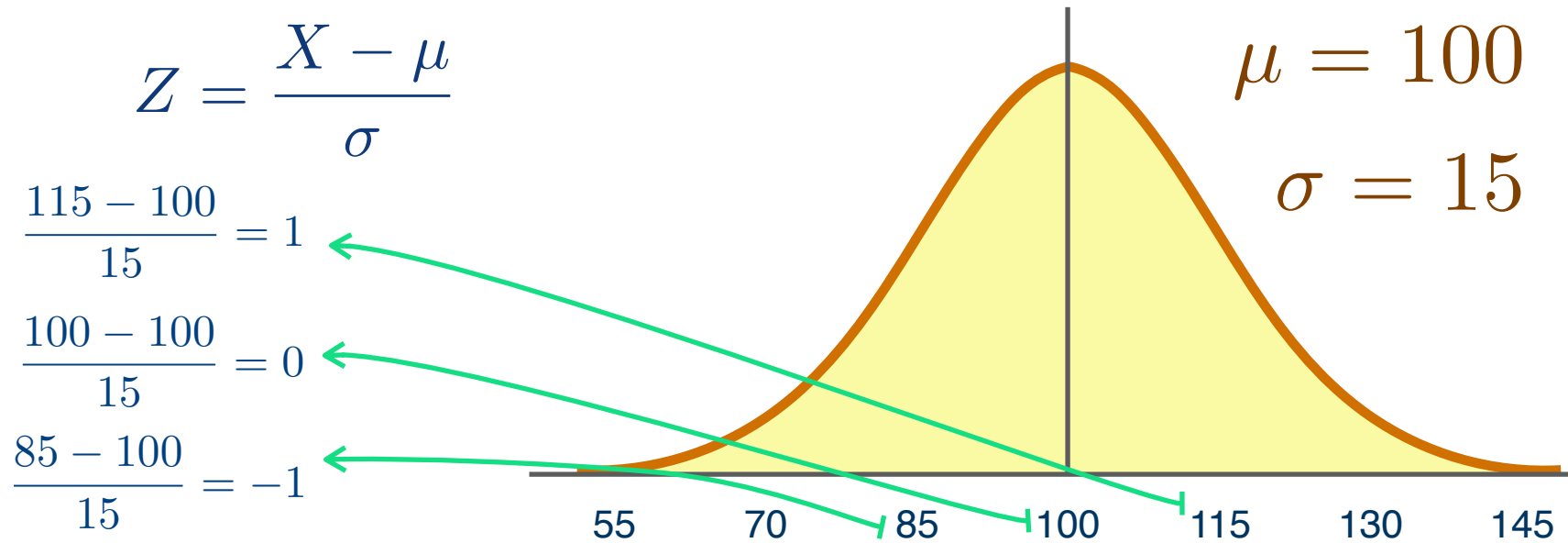
$$\mu = 100$$

$$\sigma = 15$$



# “Standard” Normal Distribution

## Converting Normal Distribution to Standard Normal Distribution



# “Standard” Normal Distribution

## Converting Normal Distribution to Standard Normal Distribution

$$Z = \frac{X - \mu}{\sigma}$$

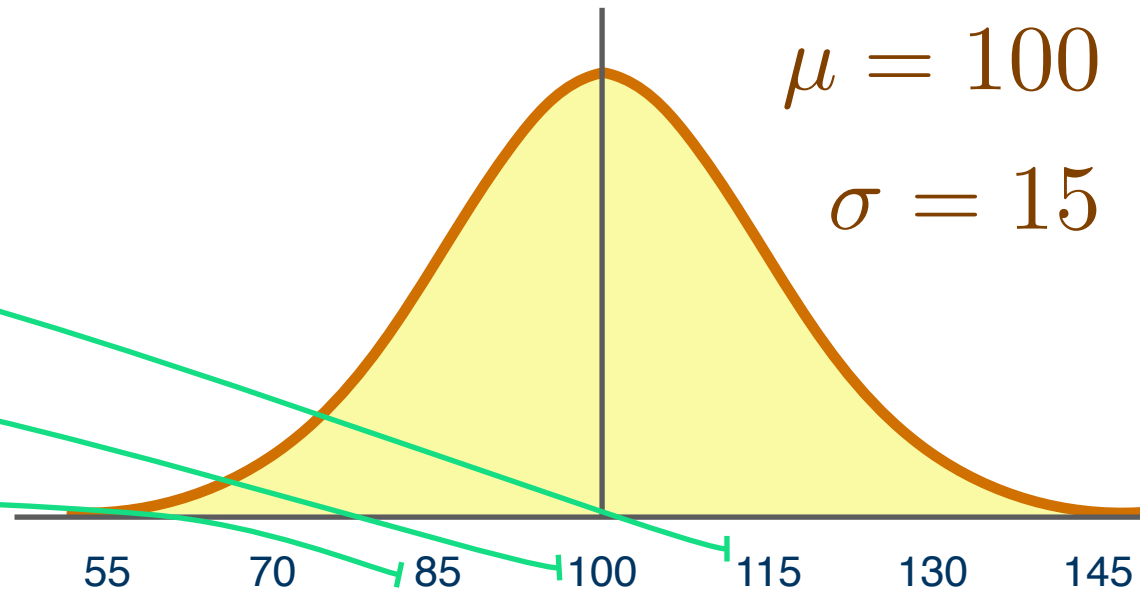
$$\frac{115 - 100}{15} = 1$$

$$\frac{100 - 100}{15} = 0$$

$$\frac{85 - 100}{15} = -1$$

$$\mu = 100$$

$$\sigma = 15$$



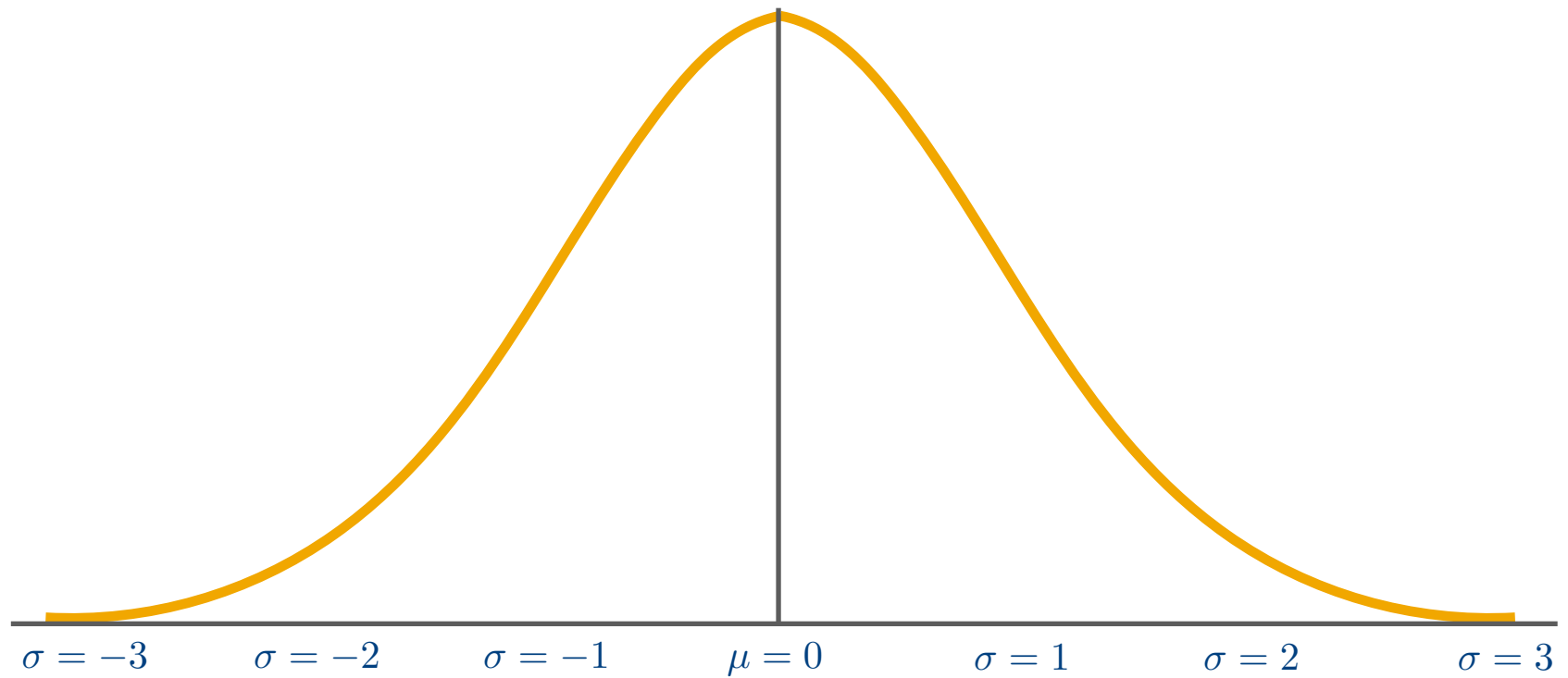
Subtract each value from mean

-45      -30      -15      0      15      30      45

Then divide by Standard Deviation

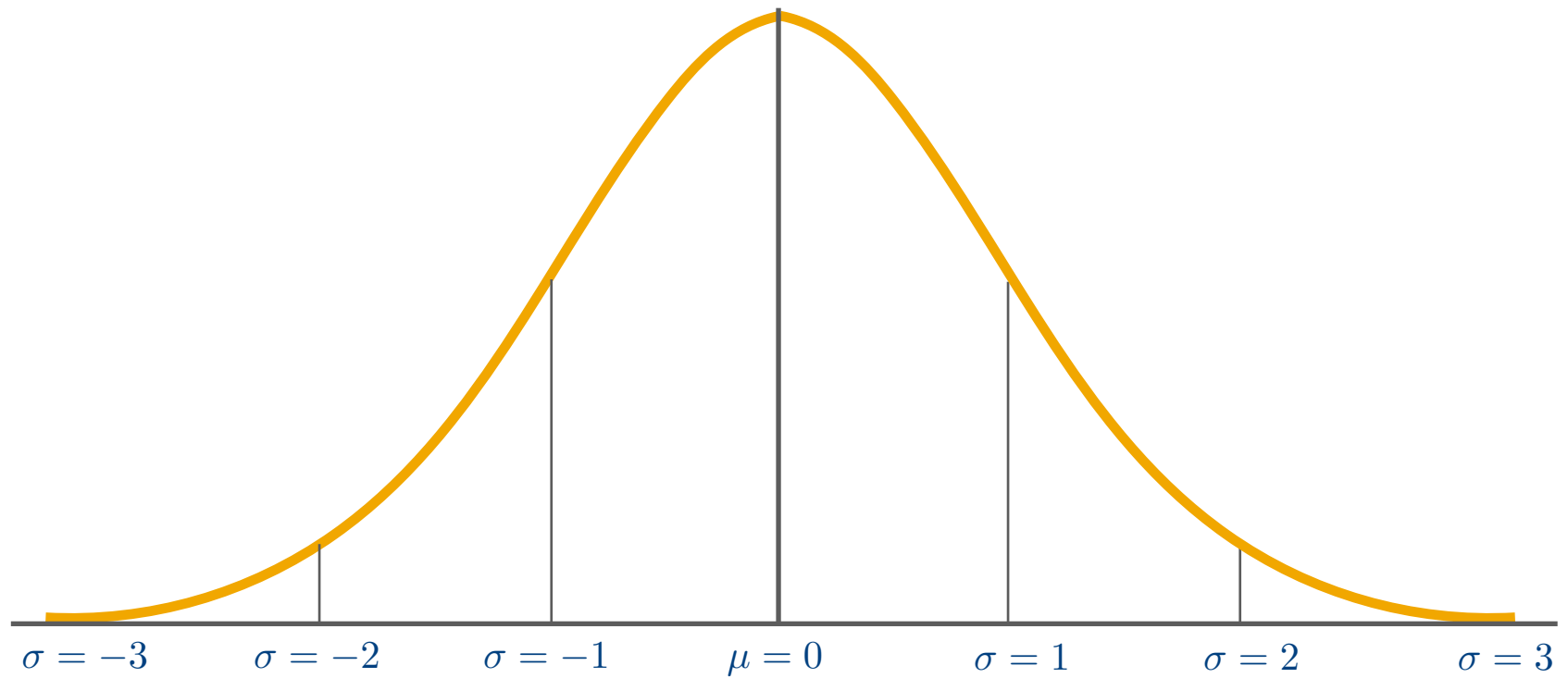
-3      -2      -1      0      1      2      3

# Standard Normal Distribution

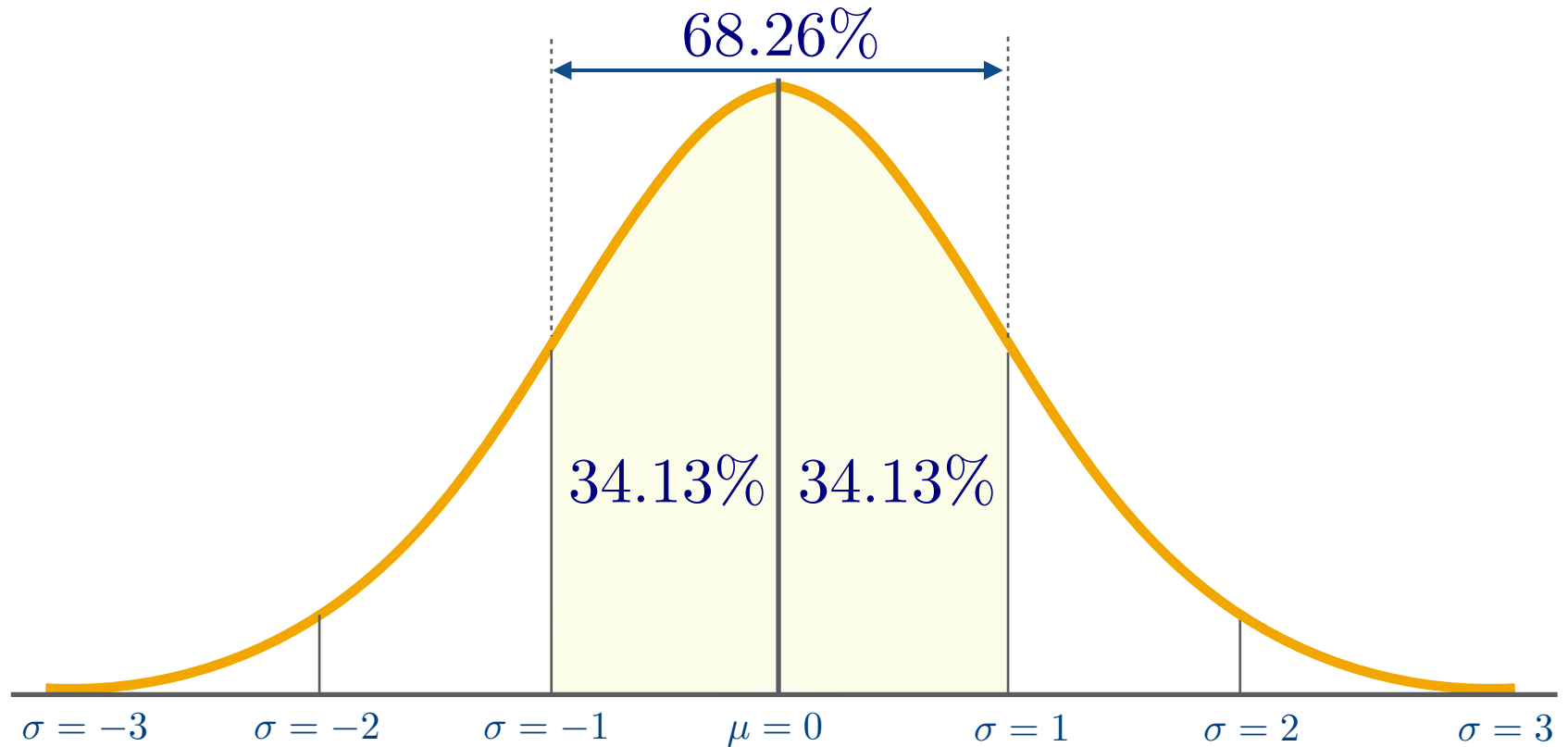




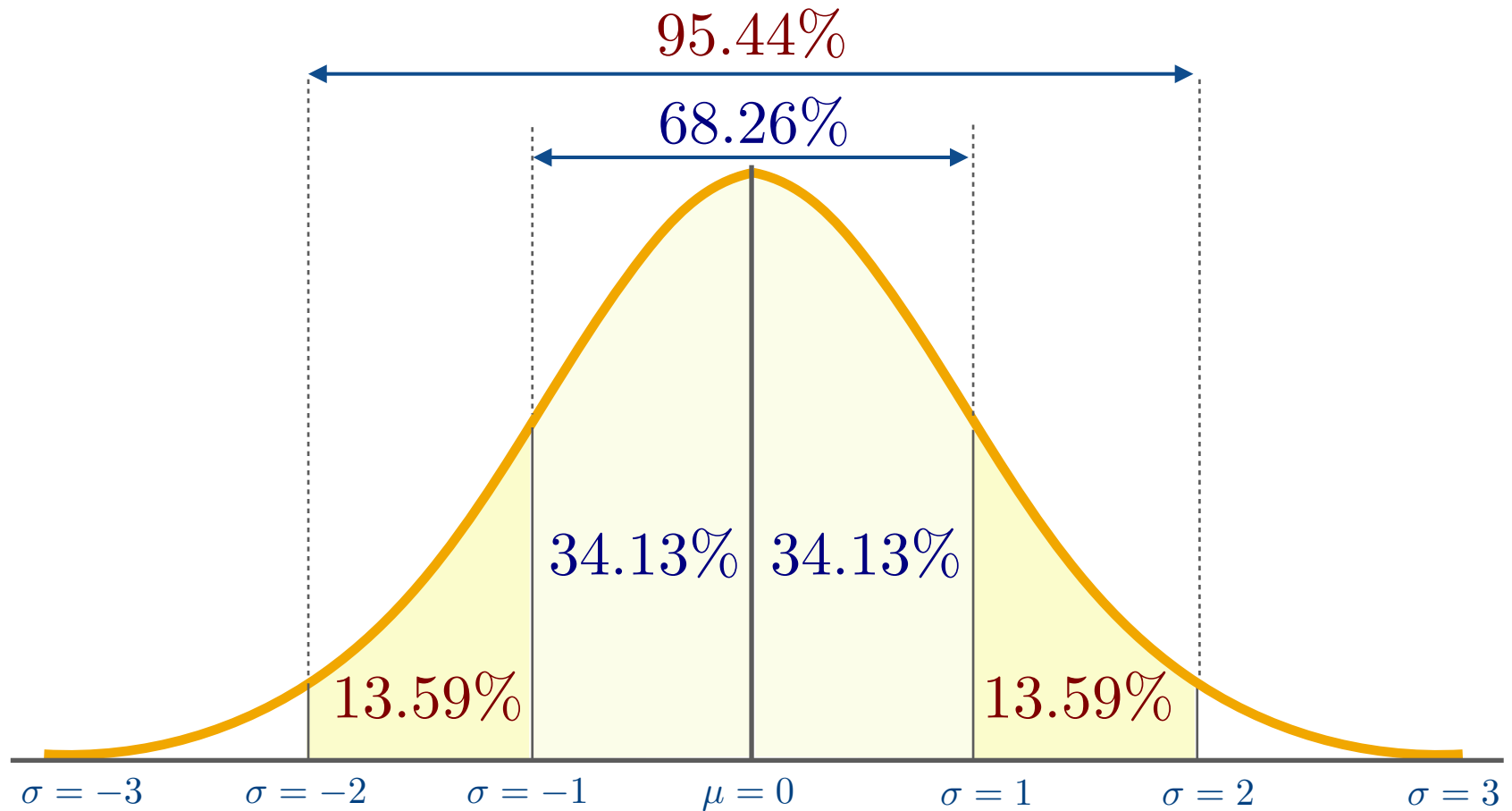
# Standard Normal Distribution



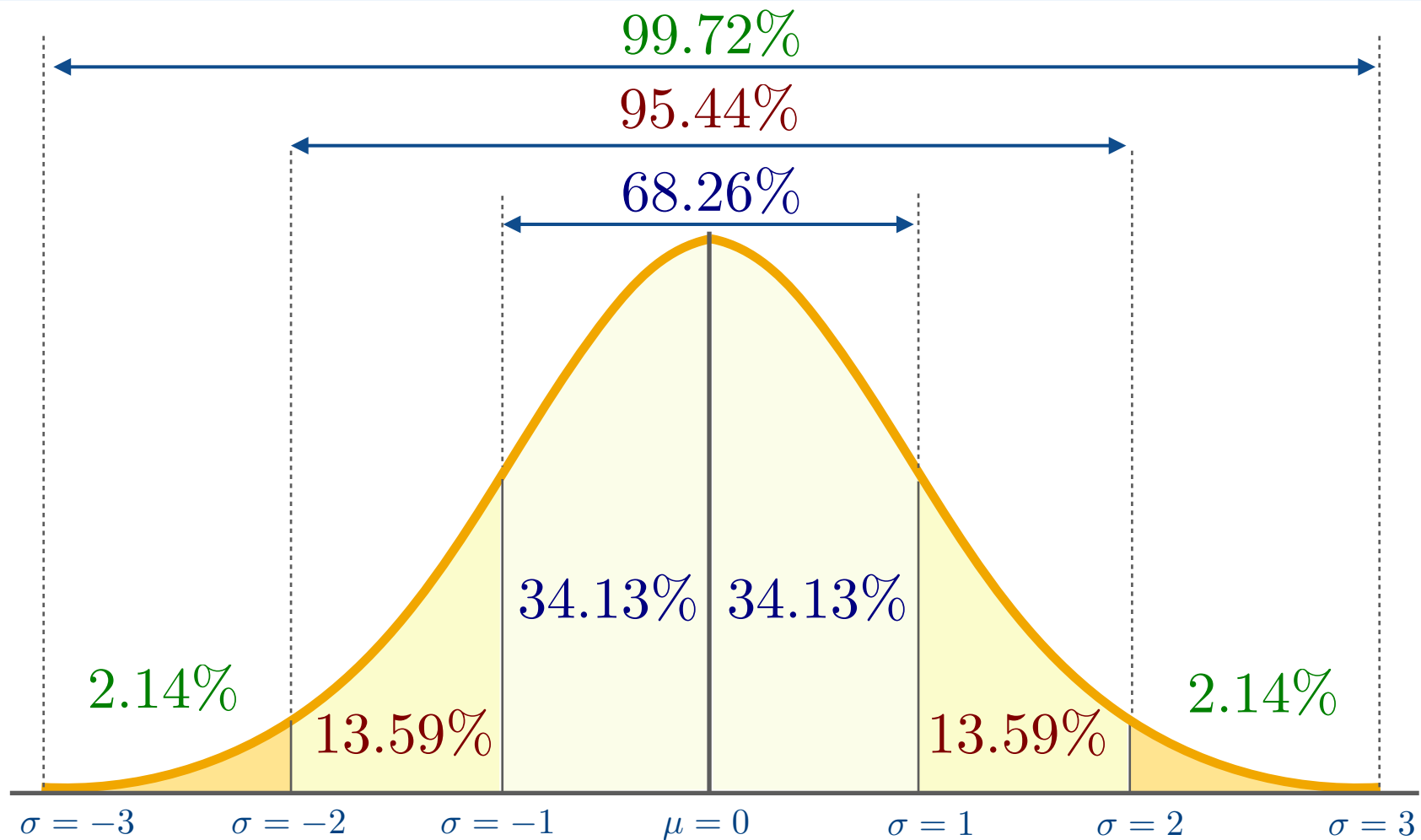
# Standard Normal Distribution



# Standard Normal Distribution



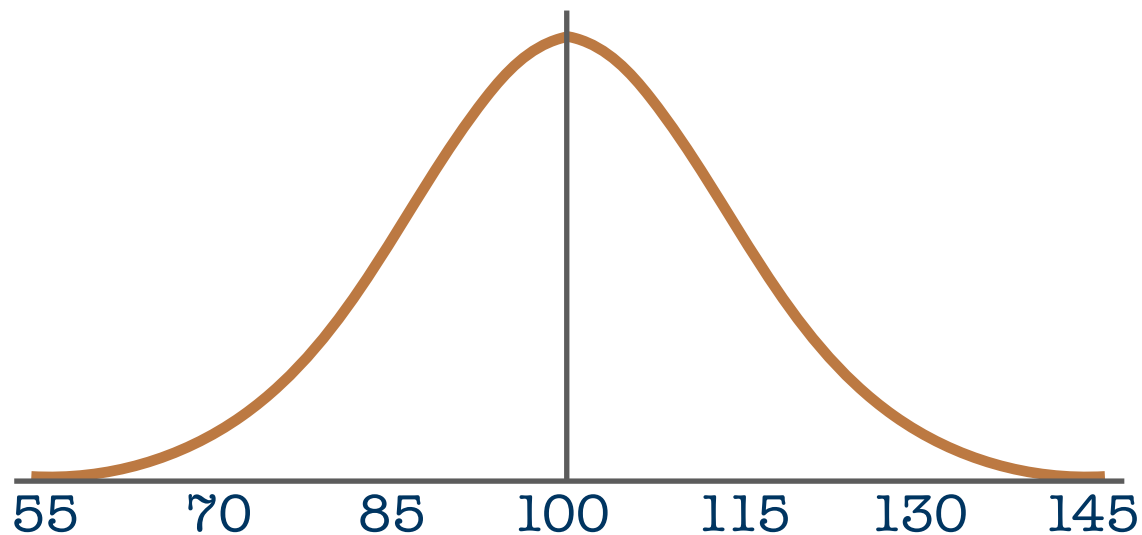
# Standard Normal Distribution



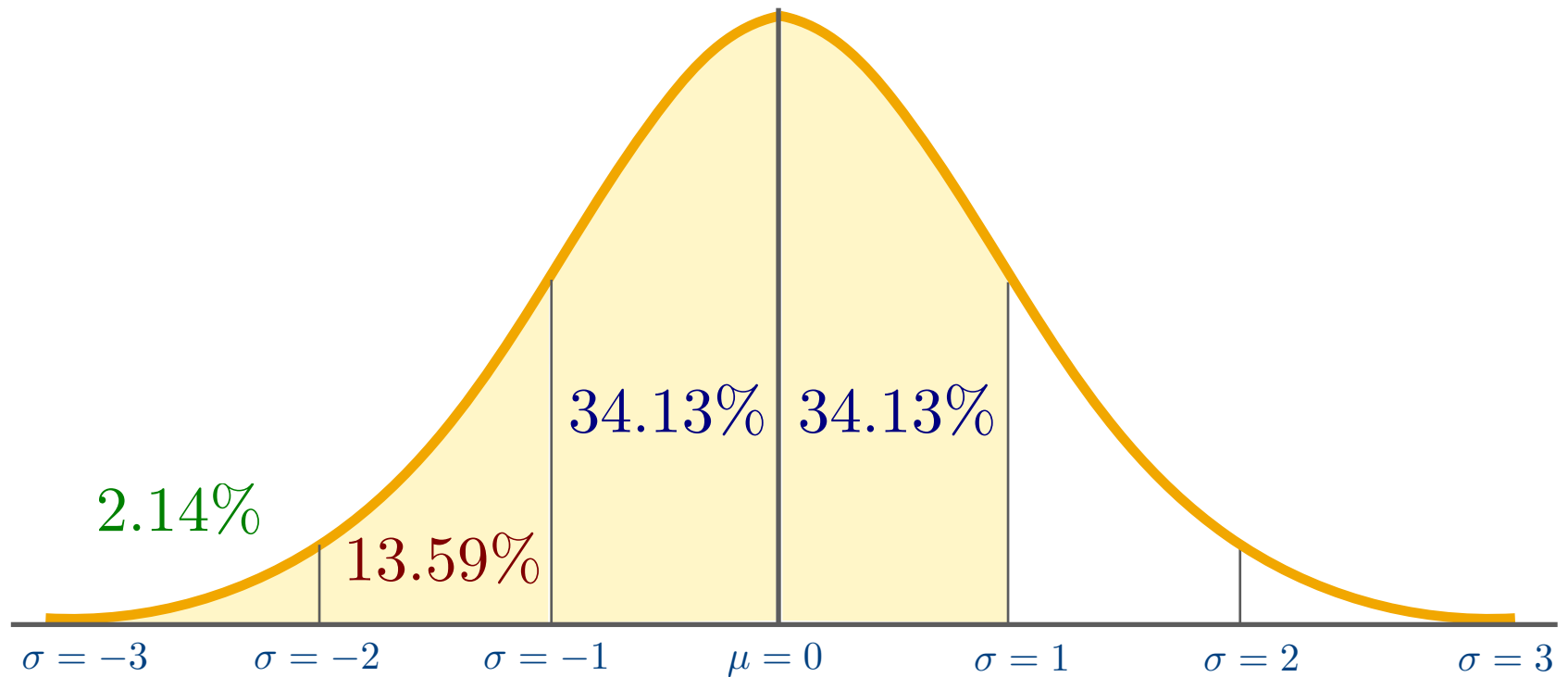
# Normal Distribution

**Question:** Suppose scores on an IQ test are normally distributed as shown.

What is the probability that a person who takes the test will score **BELOW** 115?



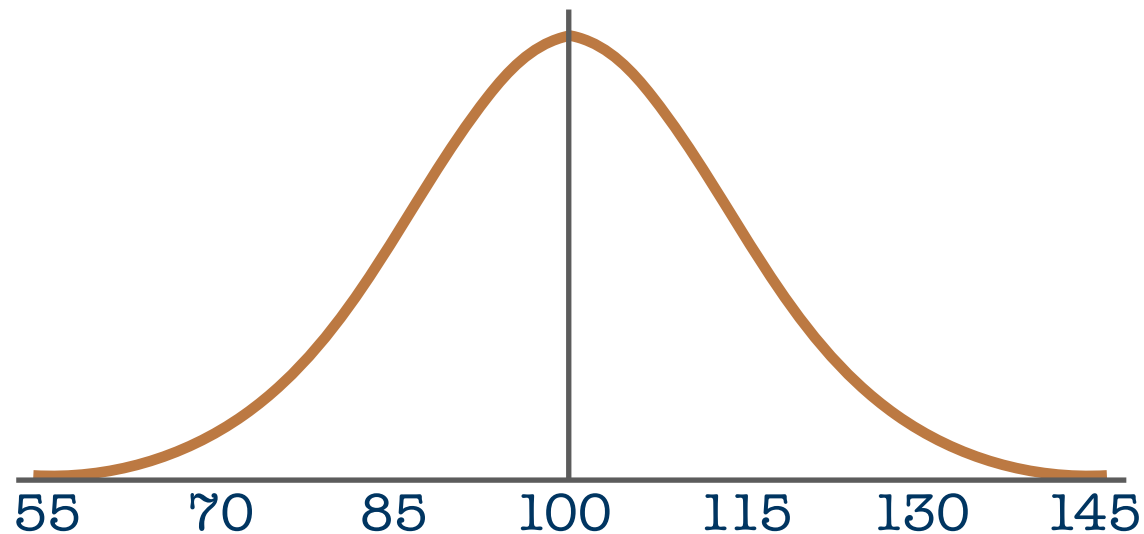
# Standard Normal Distribution



# Normal Distribution

**Question:** Suppose scores on an IQ test are normally distributed as shown.

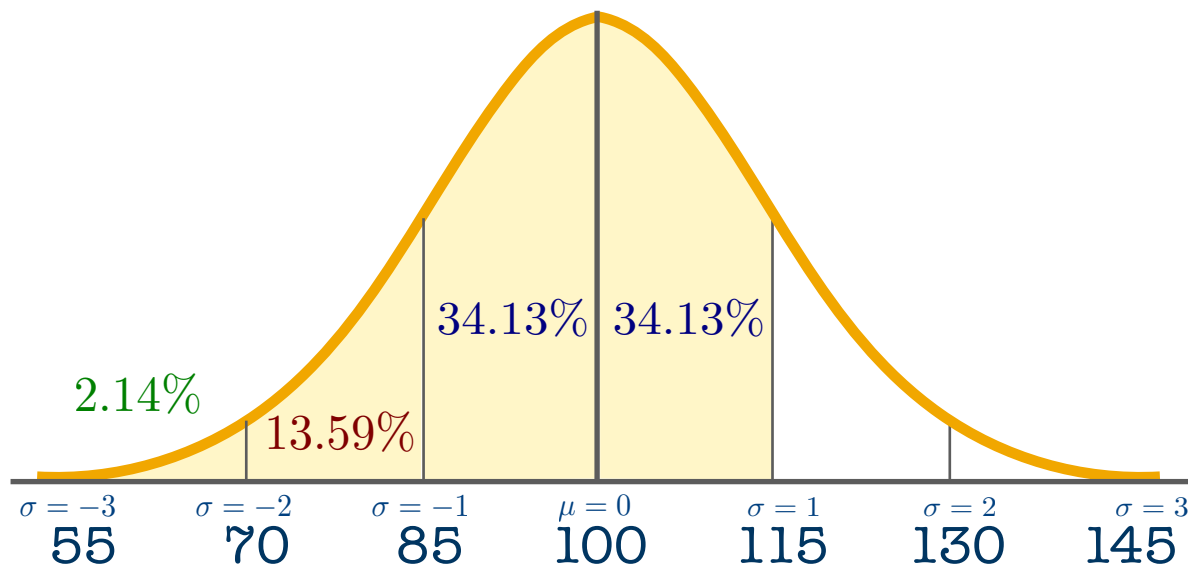
What is the probability that a person who takes the test will score **BELOW** 120?



# Normal Distribution

**Question:** Suppose scores on an IQ test are normally distributed as shown.

What is the probability that a person who takes the test will score **BELOW** 120?





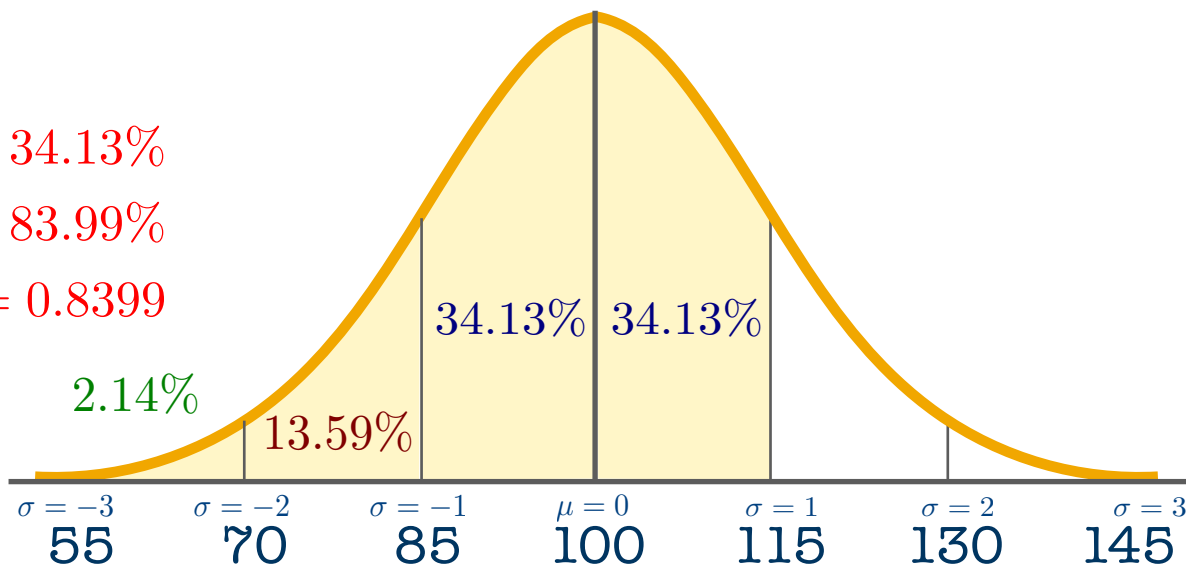
# Normal Distribution

**Question:** Suppose scores on an IQ test are normally distributed as shown.

What is the probability that a person who takes the test will score **BELOW** 120?

$$P(IQ \leq 120) =$$

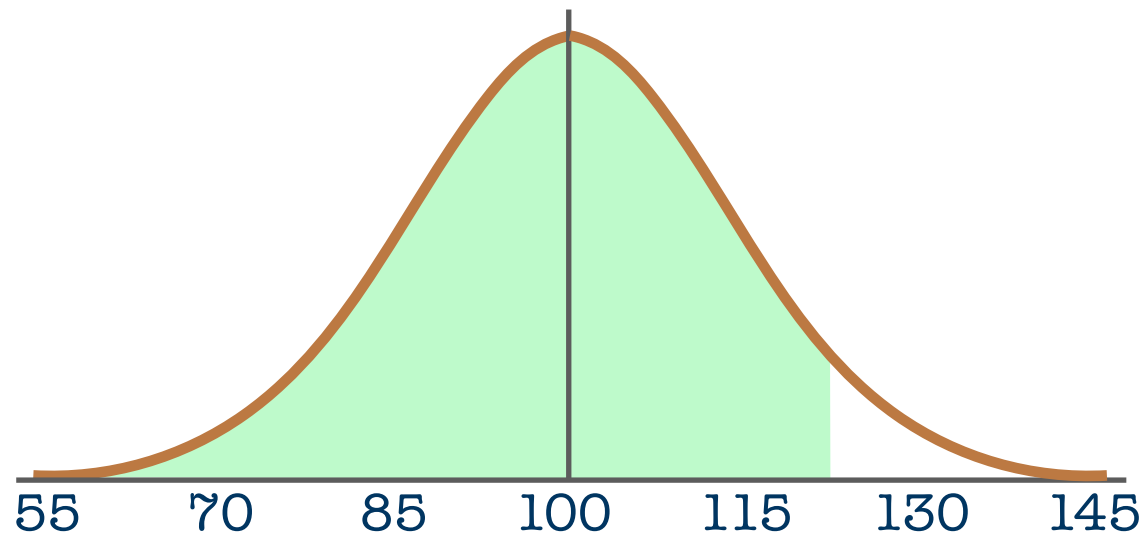
$$\begin{aligned} &2.14\% + 13.59\% + 34.13\% + 34.13\% \\ &= 83.99\% \\ &= 0.8399 \end{aligned}$$



# Normal Distribution

**Question:** Suppose scores on an IQ test are normally distributed as shown.

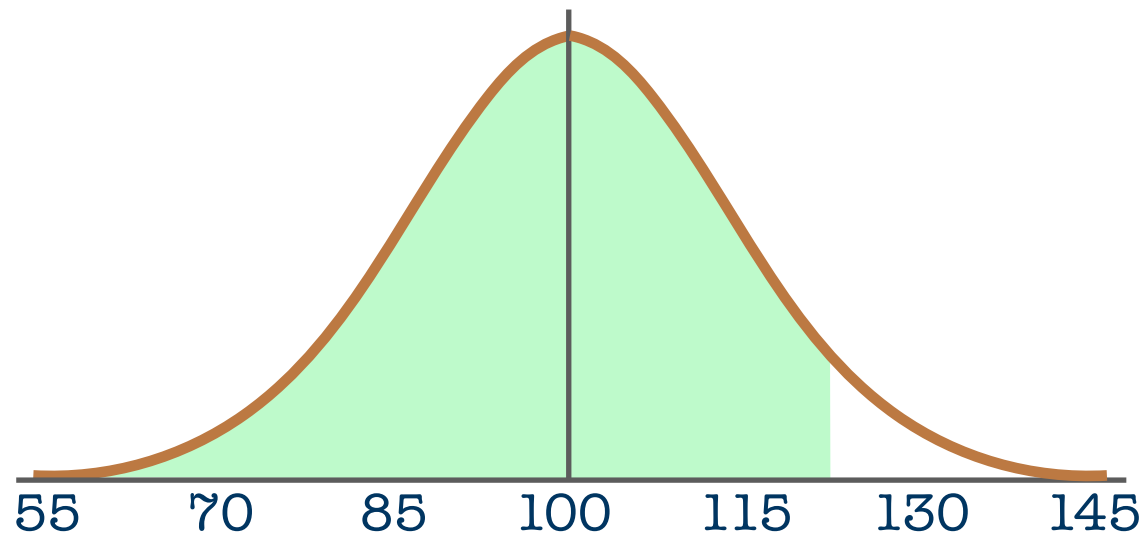
What is the probability that a person who takes the test will score **BELOW** 120?



# Normal Distribution

**Question:** Suppose scores on an IQ test are normally distributed as shown.

What is the probability that a person who takes the test will score **BELOW** 120?



Using Z SCORE TABLE

# Normal Distribution

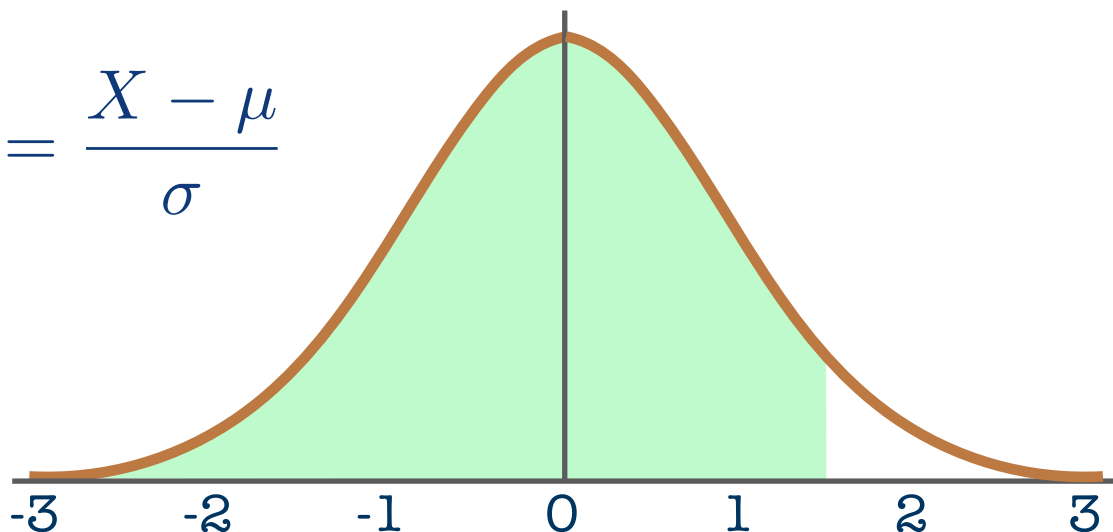
**Question:** Suppose scores on an IQ test are normally distributed as shown.

What is the probability that a person who takes the test will score **BELOW** 120?

↓

$$z = \frac{120 - 100}{15} = 1.33$$

$$Z = \frac{X - \mu}{\sigma}$$



Using Z SCORE TABLE

# Z Score Table

**Z Score Table:** Table entry for  $z$  is the area under the standard normal curve to the left of  $z$ .

# Z Score Table

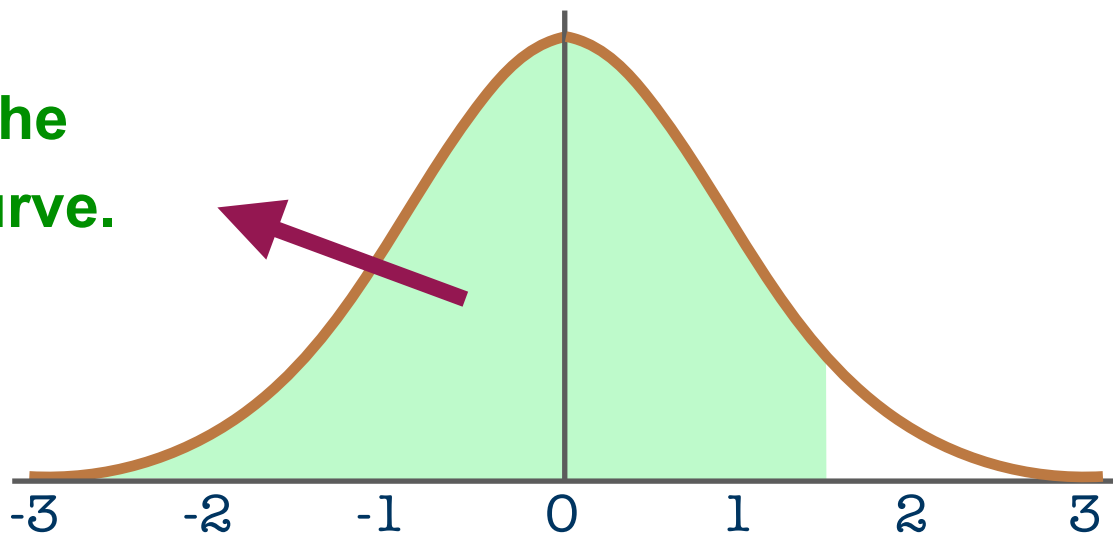
**Z Score Table:** Table entry for  $z$  is the area under the standard normal curve to the left of  $z$ .

$$z = \frac{120 - 100}{15} = 1.33$$

The probability is the surface under the curve.



Look at the Z Score TABLE



# Z Score Table (Part of the whole Table)

<b>Z</b>	<b>.00</b>	<b>.01</b>	<b>.02</b>	<b>.03</b>	<b>.04</b>	<b>.05</b>	<b>.06</b>	<b>.07</b>	<b>.08</b>	<b>.09</b>
<b>0.0</b>	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
<b>0.1</b>	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
<b>0.2</b>	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
<b>0.3</b>	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
<b>0.4</b>	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
<b>0.5</b>	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
<b>0.6</b>	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
<b>0.7</b>	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
<b>0.8</b>	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
<b>0.9</b>	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
<b>1.0</b>	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
<b>1.1</b>	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
<b>1.2</b>	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
<b>1.3</b>	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
<b>1.4</b>	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
<b>1.5</b>	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
<b>1.6</b>	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
<b>1.7</b>	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
<b>1.8</b>	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
<b>1.9</b>	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670

# Z Score Table (Part of the whole Table)

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670



# Z Score Table (Part of the whole Table)

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670

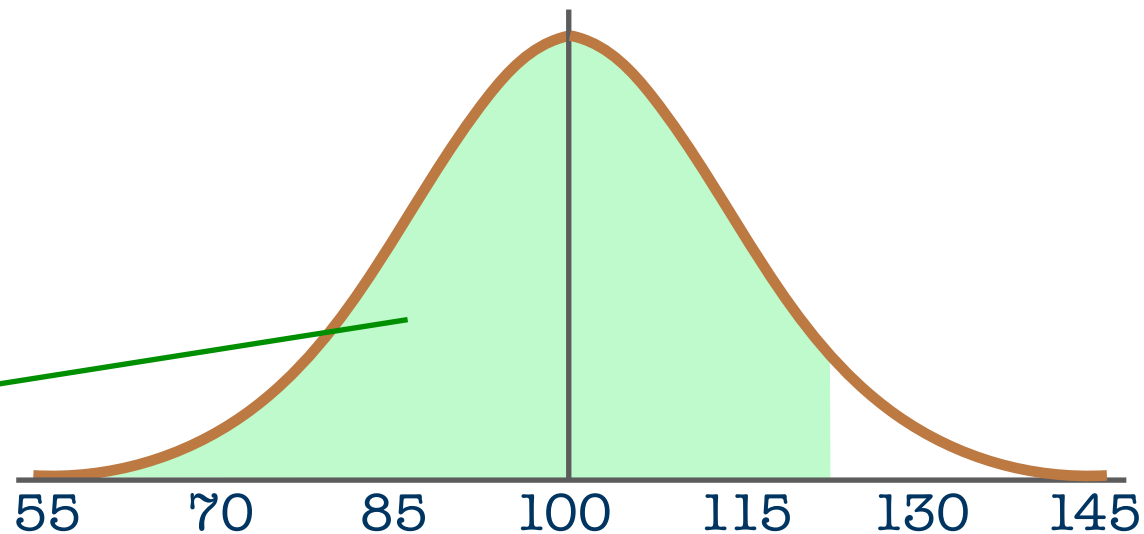
# Z Score Table

**Question:** Suppose scores on an IQ test are normally distributed as shown.

What is the probability that a person who takes the test will score **BELOW** 120?

$$z = \frac{120 - 100}{15} = 1.33$$

$$P(IQ \leq 120) = 0.90824$$



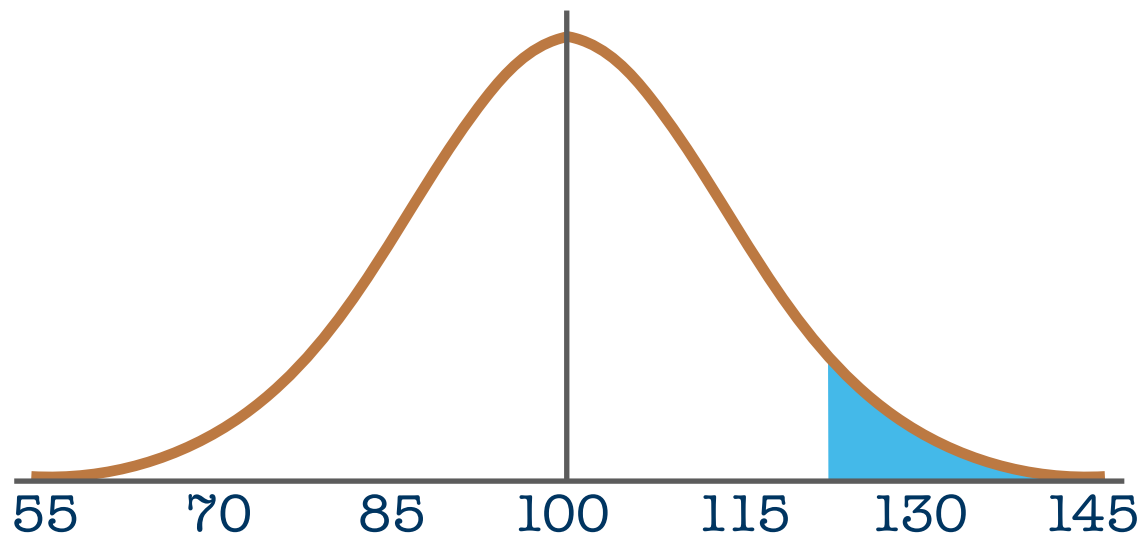
# Normal Distribution

**Question:** Suppose scores on an IQ test are normally distributed as shown.

What is the probability that a person who takes the test will score above 120?



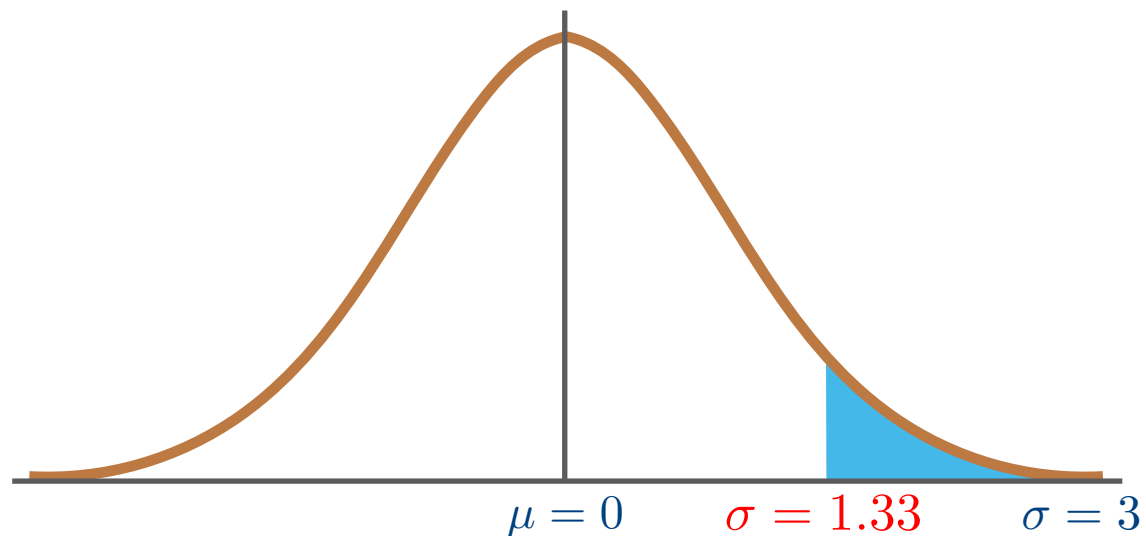
$$z = \frac{120 - 100}{15} = 1.33$$



# Normal Distribution

**Question:** Suppose scores on an IQ test are normally distributed as shown.

What is the probability that a person who takes the test will score above 120?



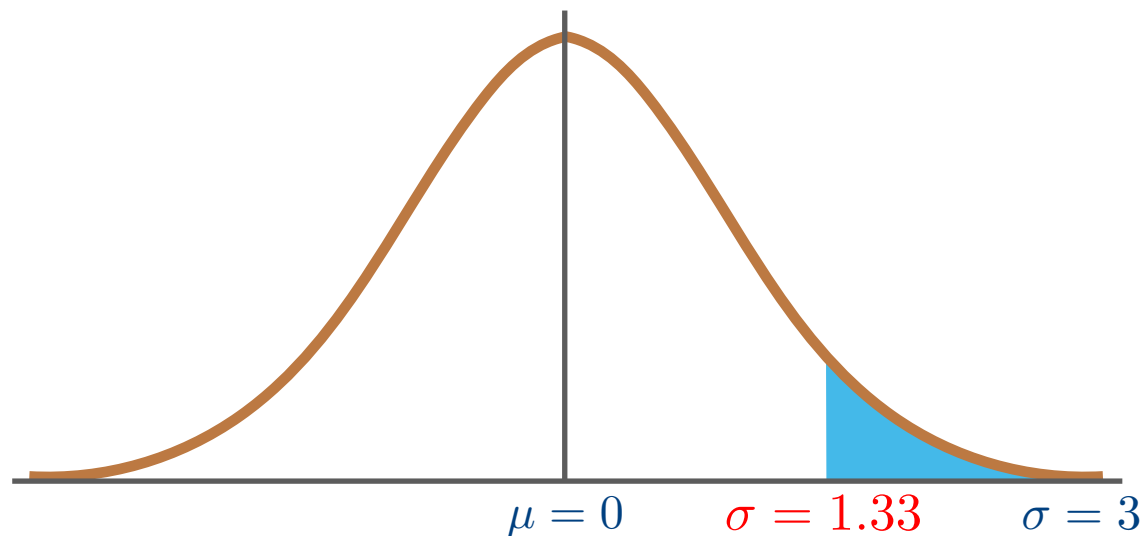
# Normal Distribution

**Question:** Suppose scores on an IQ test are normally distributed as shown.

What is the probability that a person who takes the test will score above 120?



$$z = \frac{120 - 100}{15} = 1.33$$



# Normal Distribution

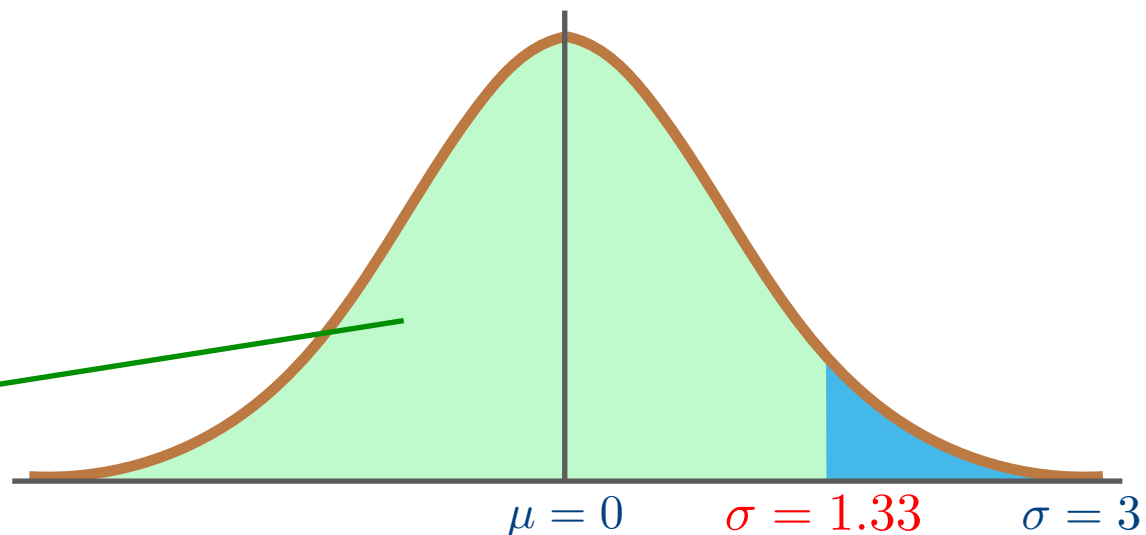
**Question:** Suppose scores on an IQ test are normally distributed as shown.

What is the probability that a person who takes the test will score above 120?



$$z = \frac{120 - 100}{15} = 1.33$$

$$P(IQ \leq 120) = 0.90824$$



# Normal Distribution

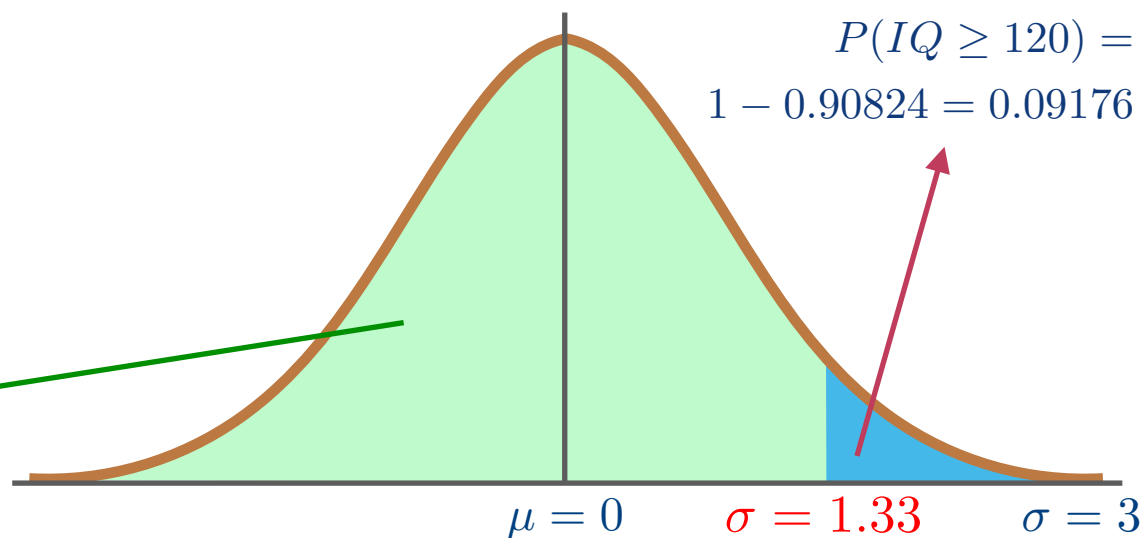
**Question:** Suppose scores on an IQ test are normally distributed as shown.

What is the probability that a person who takes the test will score above 120?

$$z = \frac{120 - 100}{15} = 1.33$$

$$P(IQ \leq 120) = 0.90824$$

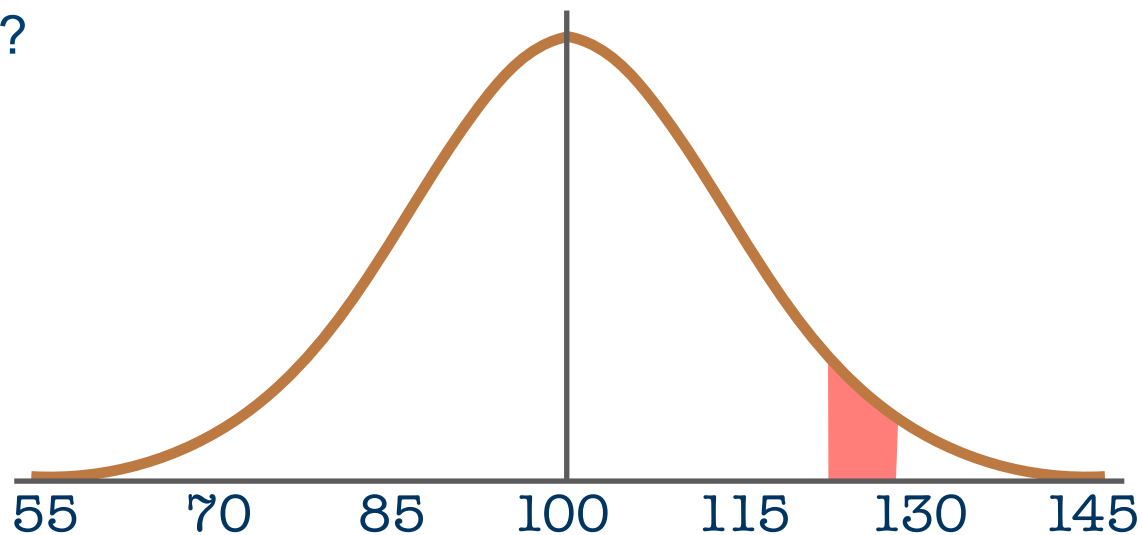
$$P(IQ \geq 120) = 1 - 0.90824 = 0.09176$$



# Normal Distribution

**Question:** Suppose scores on an IQ test are normally distributed as shown.

What is the probability that a person who takes the test will score above 120 and below 128?



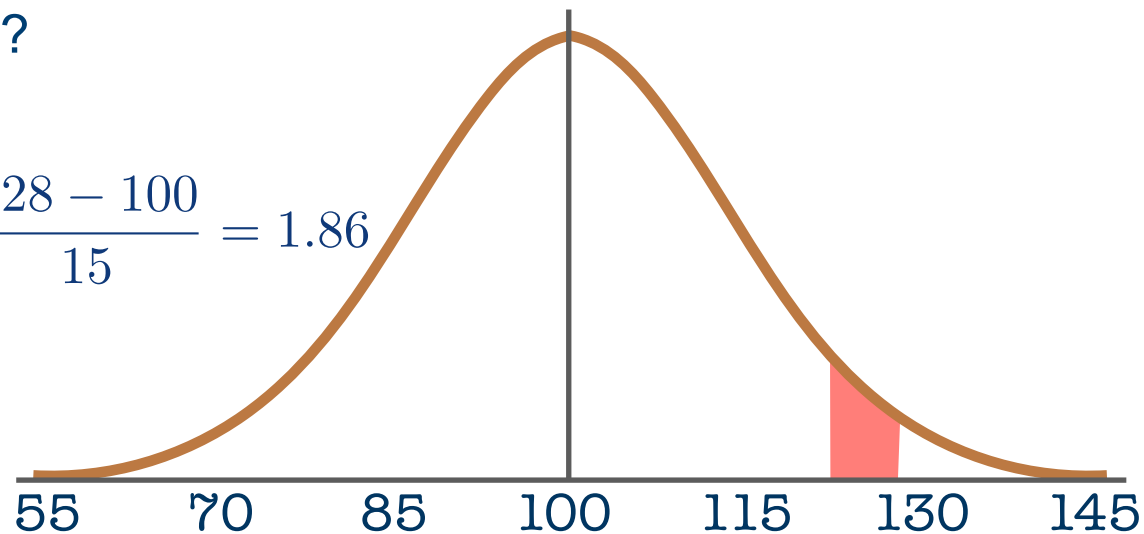


# Normal Distribution

**Question:** Suppose scores on an IQ test are normally distributed as shown.

What is the probability that a person who takes the test will score above 120 and below 128?

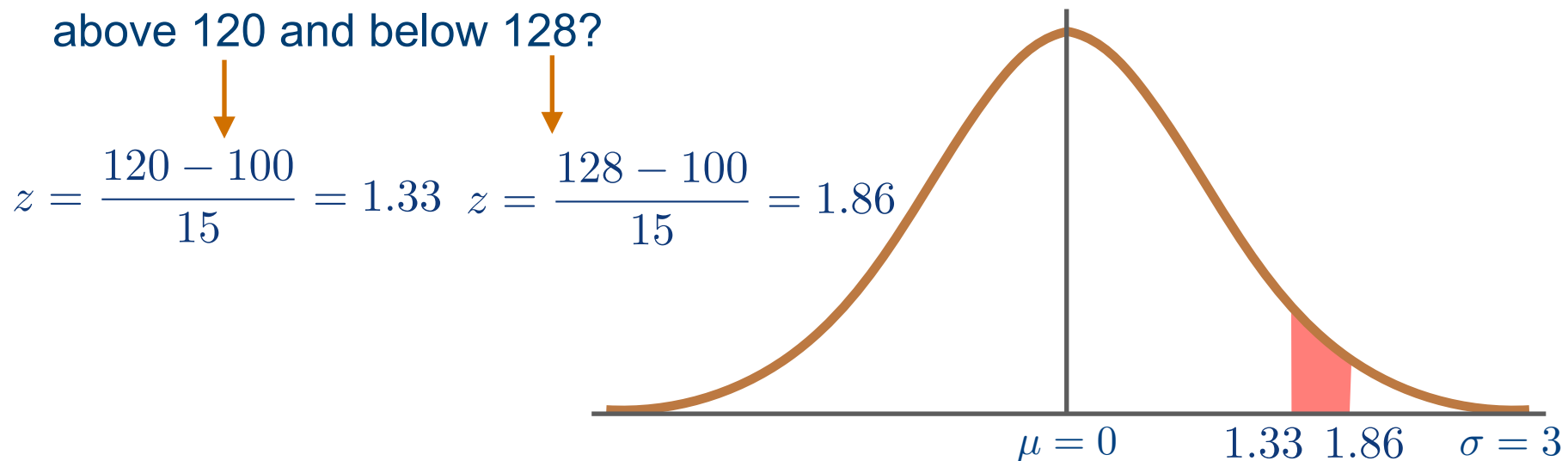
$$z = \frac{120 - 100}{15} = 1.33 \quad z = \frac{128 - 100}{15} = 1.86$$



# Normal Distribution

**Question:** Suppose scores on an IQ test are normally distributed as shown.

What is the probability that a person who takes the test will score above 120 and below 128?

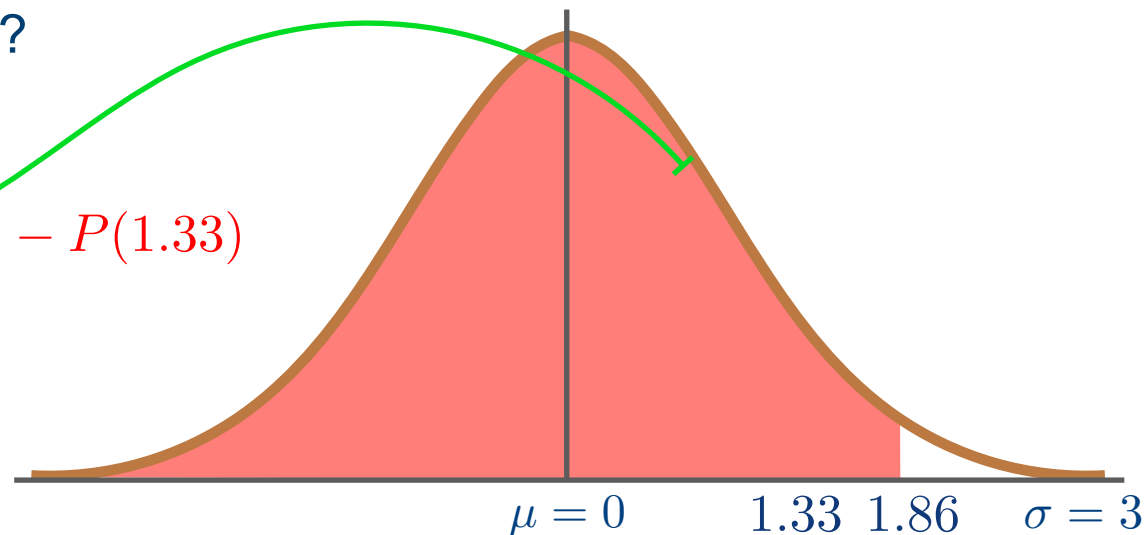


# Normal Distribution

**Question:** Suppose scores on an IQ test are normally distributed as shown.

What is the probability that a person who takes the test will score above 120 and below 128?

$$P(120 \leq IQ \leq 128) = P(1.86) - P(1.33)$$

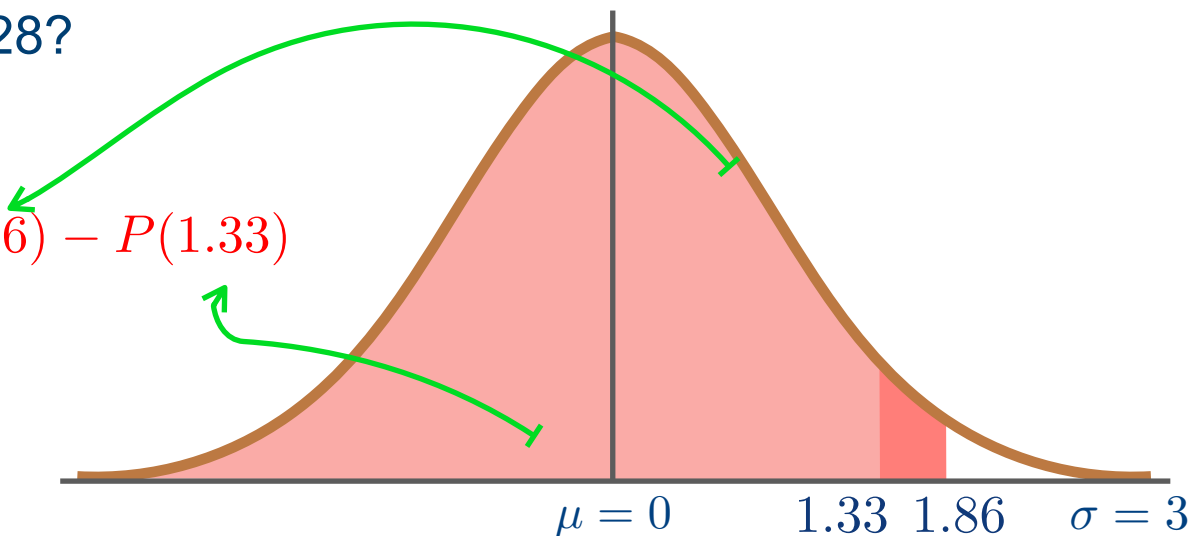


# Normal Distribution

**Question:** Suppose scores on an IQ test are normally distributed as shown.

What is the probability that a person who takes the test will score above 120 and below 128?

$$P(120 \leq IQ \leq 128) = P(1.86) - P(1.33)$$



# Z Score Table (Part of the whole Table)

<b>Z</b>	<b>.00</b>	<b>.01</b>	<b>.02</b>	<b>.03</b>	<b>.04</b>	<b>.05</b>	<b>.06</b>	<b>.07</b>	<b>.08</b>	<b>.09</b>
<b>0.0</b>	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
<b>0.1</b>	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
<b>0.2</b>	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
<b>0.3</b>	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
<b>0.4</b>	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
<b>0.5</b>	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
<b>0.6</b>	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
<b>0.7</b>	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
<b>0.8</b>	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
<b>0.9</b>	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
<b>1.0</b>	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
<b>1.1</b>	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
<b>1.2</b>	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
<b>1.3</b>	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
<b>1.4</b>	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
<b>1.5</b>	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
<b>1.6</b>	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
<b>1.7</b>	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
<b>1.8</b>	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
<b>1.9</b>	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670

# Z Score Table (Part of the whole Table)

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670



# Z Score Table (Part of the whole Table)

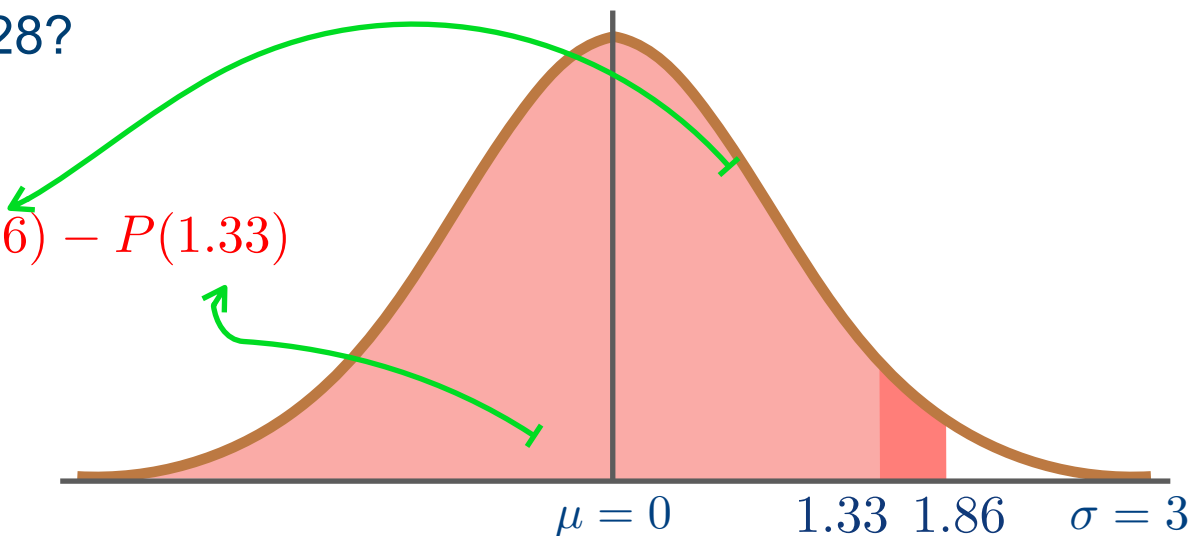
Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
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0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
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1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
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1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670

# Normal Distribution

**Question:** Suppose scores on an IQ test are normally distributed as shown.

What is the probability that a person who takes the test will score above 120 and below 128?

$$P(120 \leq IQ \leq 128) = P(1.86) - P(1.33)$$





# Normal Distribution

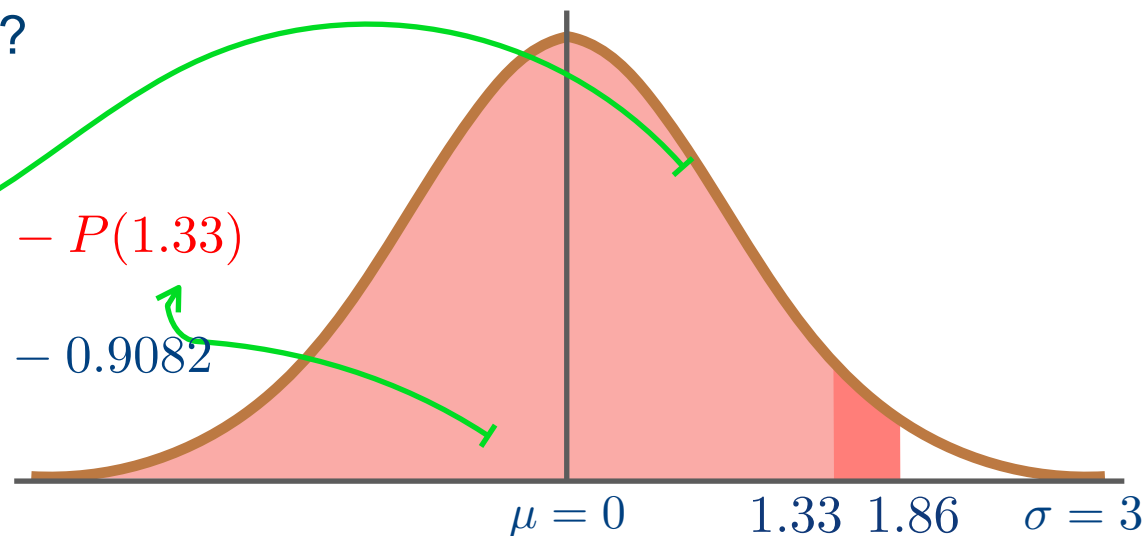
**Question:** Suppose scores on an IQ test are normally distributed as shown.

What is the probability that a person who takes the test will score above 120 and below 128?

$$P(120 \leq IQ \leq 128) = P(1.86) - P(1.33)$$

$$P(120 \leq IQ \leq 128) = 0.96856 - 0.9082$$

$$P(120 \leq IQ \leq 128) = 0.0603$$

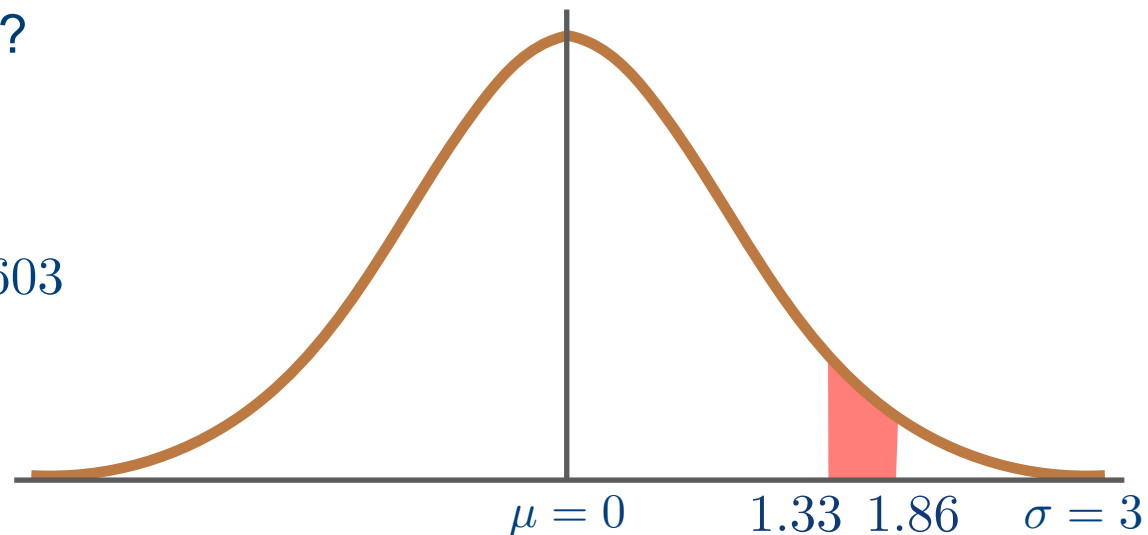


# Normal Distribution

**Question:** Suppose scores on an IQ test are normally distributed as shown.

What is the probability that a person who takes the test will score above 120 and below 128?

$$P(120 \leq IQ \leq 128) = 0.0603$$



# Normal Distribution

**Question:** The lifetime of a battery is normally distributed with a mean life of 40 hours and a standard deviation of 1.2 hours.

Find the probability that a randomly selected battery lasts longer than 42 hours.

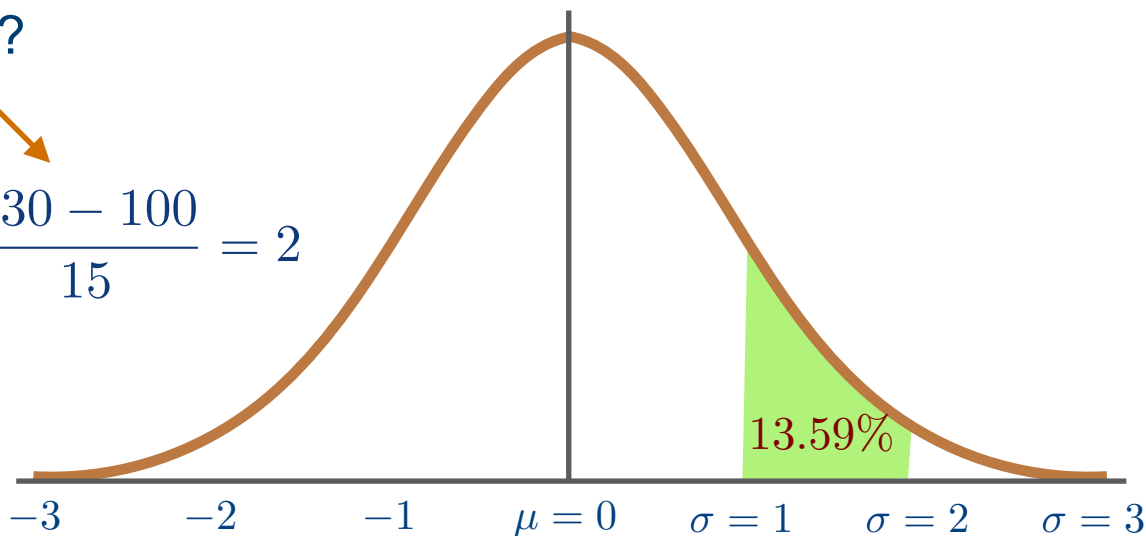
# Normal Distribution

**Question:** Suppose scores on an IQ test are normally distributed as shown.

What is the probability that a person who takes the test will score above 115 and below 130?

$$z = \frac{115 - 100}{15} = 1$$

$$z = \frac{130 - 100}{15} = 2$$



The answer is 13.59%.

# Normal Distribution

**Question:** The lifetime of a battery is normally distributed with a mean life of 40 hours and a standard deviation of 1.2 hours.

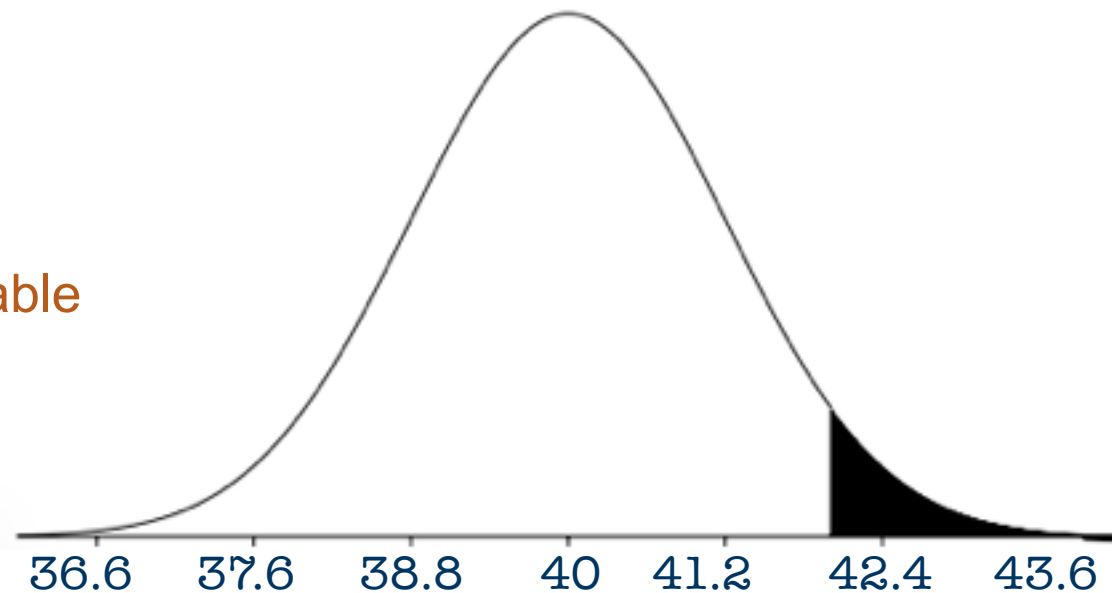
Find the probability that a randomly selected battery lasts longer than 42 hours.

# Normal Distribution

**Question:** The lifetime of a battery is normally distributed with a mean life of 40 hours and a standard deviation of 1.2 hours.

Find the probability that a randomly selected battery lasts longer than 42 hours.

- 1- Convert to Z value of 42
- 2-  $P(\text{BL} < 42)$  from Z Score Table
- 3-  $P(\text{BL} > 42) = 1 - P(\text{BL} < 42)$



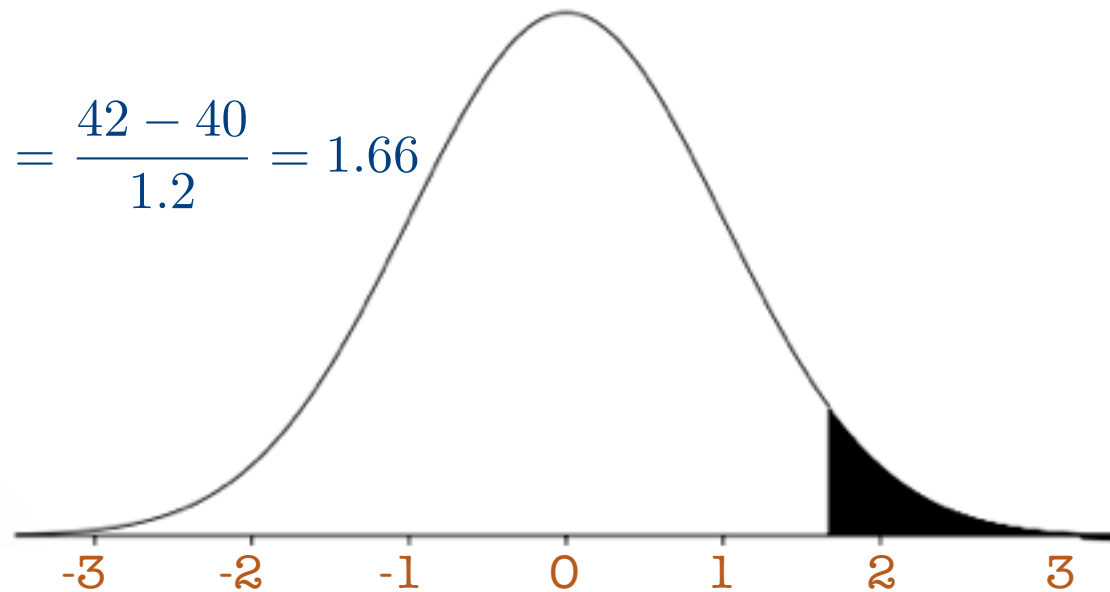
Battery Life = BL

# Normal Distribution

**Question:** The lifetime of a battery is normally distributed with a mean life of 40 hours and a standard deviation of 1.2 hours.

Find the probability that a randomly selected battery lasts longer than 42 hours.

1- Convert to Z value of 42  $Z = \frac{42 - 40}{1.2} = 1.66$



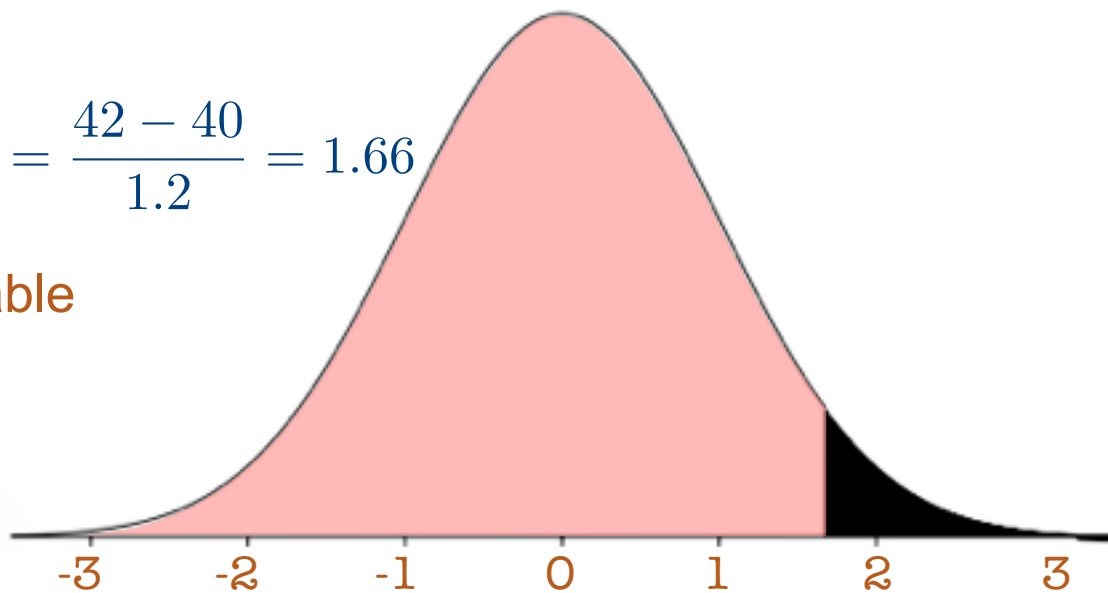
# Normal Distribution

**Question:** The lifetime of a battery is normally distributed with a mean life of 40 hours and a standard deviation of 1.2 hours.

Find the probability that a randomly selected battery lasts longer than 42 hours.

1- Convert to Z value of 42  $Z = \frac{42 - 40}{1.2} = 1.66$

2- P(BL < 42) from Z Score Table





# Z Score Table

<b>Z</b>	<b>.00</b>	<b>.01</b>	<b>.02</b>	<b>.03</b>	<b>.04</b>	<b>.05</b>	<b>.06</b>	<b>.07</b>	<b>.08</b>	<b>.09</b>
<b>0.0</b>	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
<b>0.1</b>	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
<b>0.2</b>	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
<b>0.3</b>	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
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<b>0.8</b>	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
<b>0.9</b>	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
<b>1.0</b>	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
<b>1.1</b>	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
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<b>1.6</b>	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
<b>1.7</b>	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
<b>1.8</b>	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
<b>1.9</b>	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670

# Z Score Table

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
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0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
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1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
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1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670

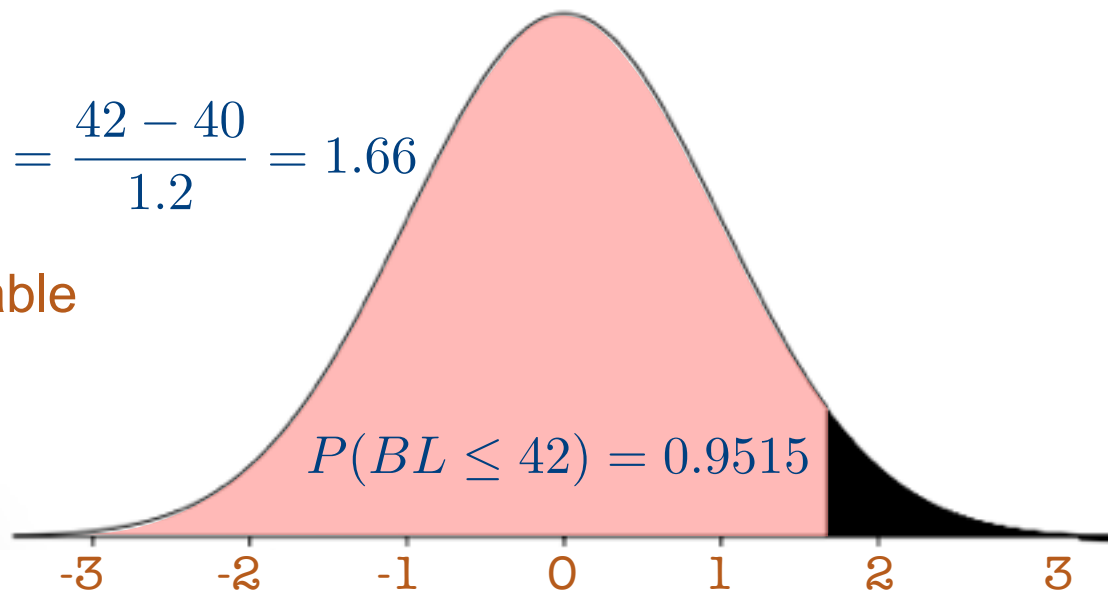
# Normal Distribution

**Question:** The lifetime of a battery is normally distributed with a mean life of 40 hours and a standard deviation of 1.2 hours.

Find the probability that a randomly selected battery lasts longer than 42 hours.

1- Convert to Z value of 42  $Z = \frac{42 - 40}{1.2} = 1.66$

2-  $P(BL < 42)$  from Z Score Table



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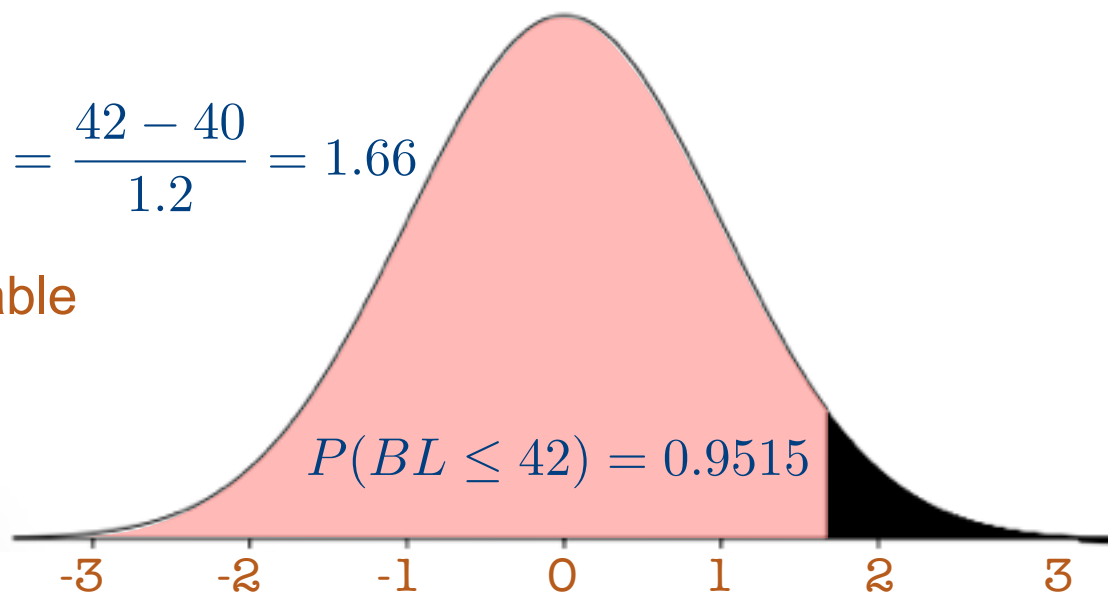
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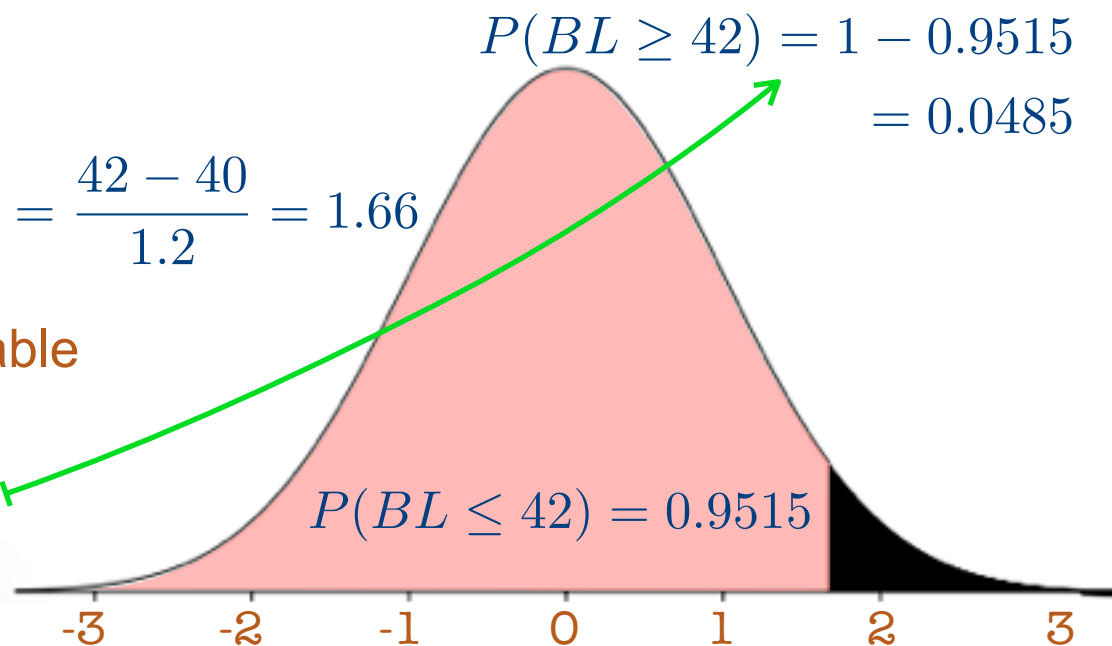
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Battery Life = BL

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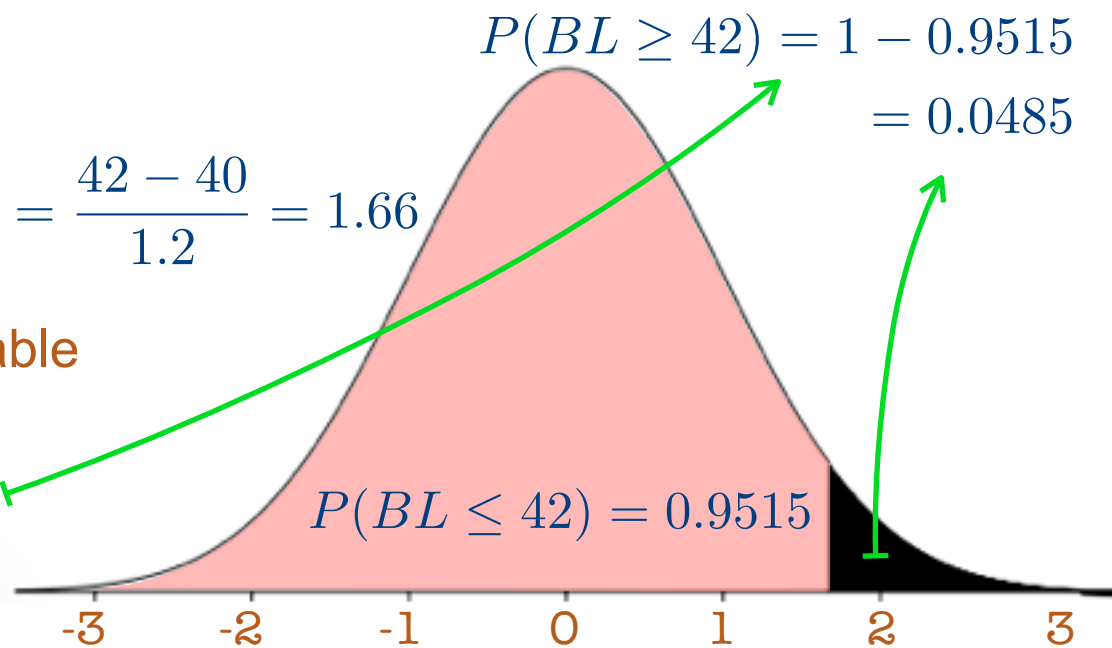
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# Homework: Normal Distribution

**Question:** The time taken to assemble a car in a certain plant is a random variable having a normal distribution of 20 hours and a standard deviation of 2 hours.

What is the probability that a car can be assembled at this plant in a period of time

- a) less than 19.5 hours?
- b) between 20 and 22 hours?



# Normal Distribution for a group

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Find the probability that this pack of batteries lasts longer than 405 hours.



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Mean value multiplied by number of batteries

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# Z Score Table

<b>z</b>	<b>0.00</b>	<b>0.01</b>	<b>0.02</b>	<b>0.03</b>	<b>0.04</b>	<b>0.05</b>	<b>0.06</b>	<b>0.07</b>	<b>0.08</b>	<b>0.09</b>
<b>0.0</b>	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
<b>0.1</b>	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
<b>0.2</b>	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
<b>0.3</b>	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
<b>0.4</b>	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
<b>0.5</b>	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
<b>0.6</b>	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
<b>0.7</b>	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
<b>0.8</b>	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
<b>0.9</b>	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
<b>1.0</b>	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
<b>1.1</b>	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
<b>1.2</b>	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
<b>1.3</b>	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
<b>1.4</b>	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
<b>1.5</b>	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
<b>1.6</b>	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545

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$$P(BL \geq 405) = 0.0934$$