Information Security based on Temporal Order and Ergodic Matrix

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DECLARATION

I certify that this work has not been accepted in substance for any degree, and is not currently submitted for any degree other than that of Doctor of Philosophy being studied at the University of Greenwich. I also declare that this work is the result of my own investigations except where otherwise identified by references and that I have not plagiarized the work of others.

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PUBLICATIONS

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ABSTRACT

This thesis proposes some information security systems to aid network temporal security applications with multivariate quadratic polynomial equations, image cryptography and image hiding.

In the first chapter, some general terms of temporal logic, multivariate quadratic equations (*MQ*) problems and image cryptography/hiding are introduced. In particular, explanations of the need for them and research motivations are given, i.e., a formal characterization of time-series, an alternative scheme of *MQ* systems, a hybrid-key based image encryption and authentication system and a DWT-SVD (Discrete Wavelet Transform and Singular Value Decomposition) based image hiding system.

This is followed by a literature review of temporal basis, ergodic matrix, cryptography and information hiding. After these tools are introduced, they are used to show how they can be applied in our research.

The main part of this thesis is about using ergodic matrix and temporal logic in cryptography and hiding information. Specifically, it can be described as follows:

A formal characterization of time-series has been presented for both complete and incomplete situations, where the time-series are formalized as a triple (*ts*, *R*, Dur) which denote the temporal order of time-elements, the temporal relationship between time-elements and the temporal duration of each time-element, respectively.

A cryptosystem based on *MQ* is proposed. The security of many recently proposed cryptosystems is mainly based on the difficulty of solving large *MQ* systems. Apart from UOV schemes with proper parameter values, the basic types of these schemes can be broken down without great difficulty. Moreover, there are some shortages lying in some of these examined schemes. Therefore, a bisectional multivariate quadratic equation (*BMQE*) system over a finite field of degree q is proposed. The *BMQE* system is analysed by Kipnis and Shamir's relinearization and fixing-variables method. It is shown

that if the number of the equations is larger or equal to twice the number of the variables, and q^n is large enough, the system is complicated enough to prevent attacks from some existing attacking schemes.

A hybrid-key and ergodic-matrix based image encryption/authentication scheme has been proposed in this work. Because the existing traditional cryptosystems, such as RSA, DES, IDEA, SAFER and FEAL, are not ideal for image encryption for their slow speed and not removing the correlations of the adjacent pixels effectively. Another reason is that the chaos-based cryptosystems, which have been extensively used since last two decades, almost rely on symmetric cryptography. The experimental results, statistical analysis and sensitivity-based tests confirm that, compared to the existing chaos-based image cryptosystems, the proposed scheme provides more secure way for image encryption and transmission.

However, the visible encrypted image will easily arouse suspicion. Therefore, a hybrid digital watermarking scheme based on DWT-SVD and ergodic matrix is introduced. Compared to other watermarking schemes, the proposed scheme has shown both significant improvement in perceptibility and robustness under various types of image processing attacks, such as JPEG compression, median filtering, average filtering, histogram equalization, rotation, cropping, Gaussian noise, speckle noise, salt-pepper noise. In general, the proposed method is a useful tool for ownership identification and copyright protection.

Finally, two applications based on temporal issues were studied. This is because in real life, when two or more parties communicate, they probably send a series of messages, or they want to embed multiple watermarks for themselves. Therefore, we apply a formal characterization of time-series to cryptography (esp. encryption) and steganography (esp. watermarking). Consequently, a scheme for temporal ordered image encryption and a temporal ordered dynamic multiple digital watermarking model is introduced.

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ABREVEATIONS

AES	Advanced Encryption Standard
AI	Artificial Intelligence
BMQE	Bisectional Multivariate Quadratic Equation
COV	Covariance
CTL	Computation Tree Logic
CTS	Complete Time-Series
ℂWT	Dual-Tree Complex Wavelet Transform
DAG	Directed Acyclic Graph
DCT	Discrete Cosine Transform
DES	Data Encryption Standard
DFT	Discrete Fourier Transform
DI	Dividing Instant
DP	Digital Product
DR	Dixon Resultants
DSA	Digital Signature Algorithm
DWT	Discrete Wavelet Transform
ECC	Elliptic Curve Cryptography
EIP	Extended Isomorphism of Polynomials
EM	Ergodic Matrix
EMs	Ergodic Matrices
GTS	General Time-Series
HFE	Hidden Field Equations
iff	If and only if
ITL	Interval Temporal Logic
JPEG	Joint Photographic Experts Group
LFP	Least Fixed Point
ℓIC	ℓ- Invertible Cycles
LSB	Least Significant Bit
LTL	Linear-Time Propositional Temporal Logic

MIA	Matsumoto-Imai Scheme A
MPKCs	MQ-based Public Key Cryptography Schemes
MQ	Multivariate Quadratic
NaN	Not a Number
NC	Normalized Cross-correlation
NP	Nondeterministic Polynomial time
NPCR	Number of Pixels Change Rate
PMI	Perturbed Matsumoto-Imai cryptosystem
PSNR	Peak Signal to Noise Ratio
SoRS	SVD on the Revised Singular Value
SoW	SVD on The Four Shares of The Watermark
STS	Stepwise Triangular Systems
SVD	Singular Value Decomposition
UACI	Unified Average Changing Intensity
UOV	Unbalanced Oil and Vinegar scheme

CHAPTER 1 INTRODUCTION

Owing to the prompting developments of information technology and the rapid growth of computer networks, large amounts of data can be exchanged electronically. Therefore information security becomes an important issue in data storage and transmission. Nowadays, security systems can be classified into more specific as encryption information (Cryptography) or hiding information (Steganography) or a combination between them.

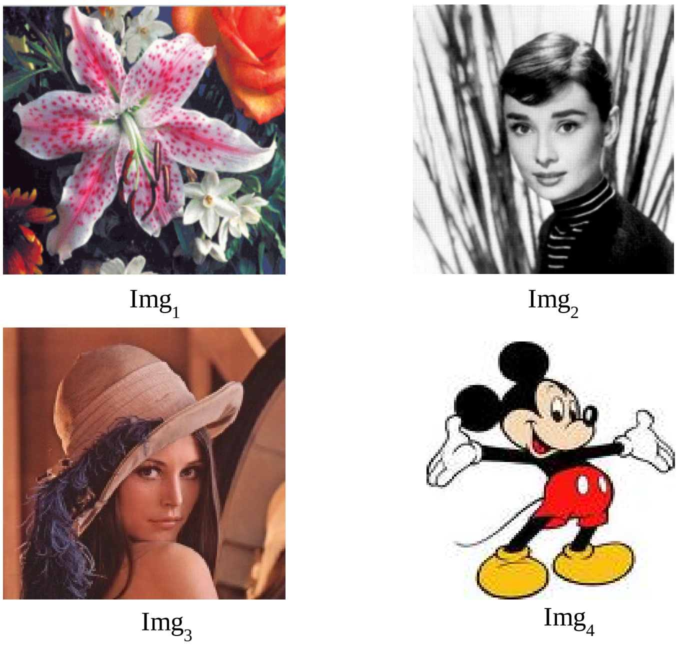
While cryptography hides the contents of the message from an attacker but not the existence of the message, steganography can hide the message both in contents and the existence. As a result of this study, the two techniques have been presented and the combination between these techniques has been discussed.

With plenty of event-based data originating from the distributed systems in an improved computer network, a critical challenge is how to correlate these events across observation. Therefore, by combining information security technologies and temporal theory, we provide two logical frameworks in which systems considering temporal order issues, together with cryptography and steganography, respectively. This is particularly common and important in security scenarios, where one wants to send the other ones messages in different orders or over time.

Section 1.1 Motivations: Security Based on Temporal Order

The term *temporal order* refers to the sequences of events in time. Let’s say, a temporal order of Figure 1.1 is Img_1, Img_2, Img_3 and Img_4.

The *temporal ordered information security* includes temporal ordered information cryptography and information hiding. It is a mechanism that uses formal characterization of time-series in the information, such as texts and images that need to be encrypted or hidden. Therefore, the cryptography and hiding results vary when the order for the sequence of the information is changed.



Img_1 Img_2 Img_3 Img_4

Figure 1.1 A Series of Images

Section 1.1.1 Formal Characterization of Time-series

Generally speaking, temporal aspects of data play an important role in computer science and information systems. In particular, time-series are important patterns and have various applications in economics, finance, environmental and medicine, etc., and have attracted a lot of researchers' interests [CHA1975, KEN1976, BD1986 and DIG1990].

However, in most of proposed formalisms, the fundamental time theories based on which time-series are formed up are usually not explicitly specified. Time-series are simply expressed as index in the form of t_1, t_2, $\ldots .t_n$, where formal characterizations with respect to the temporal basis are neglected, leaving some critical issues unaddressed. For instance:

Question 1.1.1: What a sort of objects do these t_1, t_2, ….and t_n belong to? In other word, are they time points, time intervals, or simply some absolute values from the real numbers, integers, or the clock?

Question 1.1.2: What are the temporal order relationships between these t_1, t_2, ….and t_n? Are they simply well-ordered as the natural numbers, or they may be relatively ordered by means of relations such as “Before”, “Meets”, “During”, and so on [AJ1984, MK1994]?

Therefore, one of the research goals is to formalize the characterization of time-series then use it in temporal ordered information cryptography and information hiding to solve the two questions above.

Section 1.1.2 *BMQE* System

BMQE is a system using bisectional multivariate quadratic equations. Ever since 1994, Peter Shor, the American professor of applied mathematics at MIT proved that quantum computers could calculate logarithm and that their speed far exceeds the existent ones, public key cryptography has undergone much evolution. It is claimed that if a quantum computer was built, RSA, DSA, Elliptic curves, hyperelliptic curves class groups etc., could not be used to resist cryptanalysis [MJ1987].

Nowadays, most cryptographers are quite interesting in finding a substitutable scheme based on the problem of solving Multivariate Quadratic equations (*MQ*-problem) over finite fields [CB2002, CB2005, JD2005 and JDS2006]. This can be traced back to 1980s. Since then there are a few famous schemes, which can be classified into five basic categories in [KPG1999, BWP2005, CAB2006, Pat1995, BCD2008, DSW2008, DW2008 and HBH2006], such as Unbalanced Oil and Vinegar scheme (UOV), Stepwise Triangular Systems (STS), Matsumoto-Imai Scheme (MIA), Hidden Field Equations (HFE) and ℓ- Invertible Cycles (ℓIC).

The advantages of the *MQ*-based public key cryptography schemes (MPKCs) are mainly reflected in their fast speed of encryption (or signature verification) and resistance of quantum computer attacks, such as Shor’s algorithm [Sho1997]. This algorithm factorises much faster than any classical counterpart [Wat2008]: When running on a decent quantum computer, it could break all known public key encryption systems without any difficulties. Nonetheless, there are some shortages lying in MPKCs, such as:

(1) Public-key size is large (it seems it is inevitable, but it is not a big issue as public keys do not need to be updated very often),

(2) Decryption (or signature) algorithms are complicated. Besides, compared to ECC (Elliptic Curve Cryptography) and RSA, no other public-key schemes based on *MQ* problems are safer or more efficient [Wol2005], and

(3) For some schemes, encryption and signature sometimes cannot be taken into account simultaneously. To improve the efficiency of signature is often at the expense of efficiency of decryption (e.g. UOV, TTS signature scheme).

Therefore, we have the question below:

Question 1.2.1: Is there any new *MQ*-based system not only yields an NP-complete problem, but also overcomes the deficiencies in previous *MQ* systems?

In this thesis, based on ergodic matrix in [YSZ2004, ZHJ2005 and PZZ2006], we propose *BMQE* system over finite fields, which will answer Question 1.2.1.

Section 1.1.3 Temporal Ordered Image Cryptography

Cryptography, the science of encompassing the principles and methods of encryption and decryption, plays a central role on many aspects of our daily lives. Images are one kind of plaintext; they are widely used in several processes. For that reason, the secure transmission of confidential digital images over public channels is a common interest in both research and application fields [YWL+2010]. Over decades, some techniques on image encryption have been introduced, such as algorithms based on phase encoding with a fringe pattern in [MR2006], on pseudo random sequences in [RMP2006], on random vectors in [KGK2008], on block-based transformations in [MOH2009], on advanced hill ciphers in [AP2009] and on chaotic systems in [Fri1998, CM2004, MC2004, ZLW2005, GHG2005, GBB+2009, YWL+2010 and YWL2010].

One commonality across in these algorithms is that they encrypt images independently, which means if the probability of an eavesdropper working out one image is *p*, under the same circumstances, i.e., encrypt with the same bits of keys and the same algorithm, the probability *p* stays the same while encrypting other images. In reality however, for instance, when Alice communicates with Bob, she might want to send a sequence of images with different security demands; and in addition, the temporal order in which the sequence of images are sent out may be changed. These will lead to the following questions respectively:

Question 1.3.1: How can Alice encrypt different important-level images without changing the keys?

Question 1.3.2: How can Alice get different encryption results if the temporal order of the images is changed?

While the first question is typical for conventional encryption problems, the second has to deal with the temporal issues involved, which have been neglected in most existing approaches. Therefore, we propose a novel idea of using formal characterization of time-series in temporal ordered image encryption to solve the two questions above.

Section 1.1.4 Temporal Ordered Image Hiding

Information hiding techniques weren't received much attention from the research community and from industry than cryptography until 1996, when the first academic conference on the subject was organized [PAK1999]. The main driving force is concern over copyright; as audio, video, software and other works in digital form, the ease of making copies of these products may lead to large-scale plagiarism.

Digital watermarking is one of the hotspots in information hiding. With digital watermarking technologies applying in more and more digital products, the functions of digital watermarking are different, such as fragile watermarking and robust watermarking. For the reason that the watermarks have various functions and they display at various stages, a new technology is introduced to embed watermarks in one digital product, which is multiple digital watermarking. The technology for multiple digital watermarking is for verifying the different copyrights of multiple authors when they embed their watermark in different stages, such as the product releases, sales and applications. Then there will be issues regarding temporal logic. For example, given *A* is the author of the image *P* produced at time t_1, *B* is a new author wanting to modify *P* at time t_2 (here $t_2>t_1$), the following questions need to be solved:

Question 1.4.1: How can *B* verify the product is actually from *A*?

Question 1.4.2: Besides *B*, if there are other authors such as *C*, *D*, *E*, …, want to join in and modify the product, how do they design the watermark?

Question 1.4.3: Since *A* and *B* are the only two parties considered in a communication. It is not necessary that the new watermark is embedded by both of them, so who will take the responsibility of embedding the watermark? *A* or *B*?

Question 1.4.4: How to design and embed a watermark if *B* modifies the images from multiple authors and integrates them into one image?

While the first two questions need to be solved by digital image watermarking technologies; the last two have to deal with the temporal issues.

Section 1.2 Objectives: Information Security Using Temporal Theory

The objectives of this thesis are to try to apply temporal theory and ergodic matrix in information security and to resolve the questions asked in the previous section. More specifically, this thesis tries to accomplish the following five tightly associated goals:

(1) A formal characterization of time-series: Based on the point- and interval-based theory, we shall present a formal characterization of time-series to describe the objects of time elements. The framework will support both absolute and relative temporal knowledge given either in a complete or in an incomplete form, and therefore is general enough to subsume information security, especially

encryption and watermarking, in the area of temporal based information security.

(2) An *MQ* polynomial equations cryptosystem: It is claimed that most of the cryptosystems (RSA, DSA, Elliptic curves, hyperelliptic curves class groups etc.) could not be used to resist cryptanalysis if a quantum computer was built. [Wol2005 and BBD2009]. Therefore, more cryptosystems need to be explored. It is said *MQ*-based system is one of the prominent alternative cryptosystems for resisting quantum attacks. But of the existing *MQ*-based cryptosystems, apart from UOV schemes with proper parameter values, the basic types of these *MQ*-schemes are considered to be insecure in [PGC1998, KS1999, GC2000, FJ2003, Nic2004, FGS2005, DSY2006, BG2006, DGS2007, DSW2008 and FLP2008,]. Therefore, we design a cryptosystem relies on bisectional multivariate quadratic equations. It will be a system based on NP-hard problem and be able to resist some of the attacks, such as relinearization attack and fixing-variables attack.

(3) A hybrid-key based image encryption and authentication system: The chaos-based cryptosystems, which have been extensively used since last two decades, are almost all based on symmetric cryptography and are lack of authentication. To remedy the imperfections, we shall propose a hybrid-key based image cryptosystem. This system shall not only have fine encryption results but also have fast performance speed so that it can fit for network transmission.

(4) A DWT-SVD based image hiding system: Copyright protection of intellectual properties is necessary for preventing illegal copying and content integrity verification. To achieve this requirement, a hybrid digital image watermarking scheme based on discrete wavelet transform (DWT) and singular value decomposition (SVD) is proposed. This system shall improve the existing algorithms from points of view of the PSNR (Peak Signal-to-noise Ratio) and NC (Normalized Correlation).

(5) Applications of the formal characterization of time-series in the proposed information cryptography and information hiding: time-series have been neglected in most information security applications. So far, they are only applied in protocols. But in information security, especially in cryptography and steganography, temporal order of the information shall be considered. Therefore, we propose two temporal based scheme in image cryptography and image hiding, respectively.

Section 1.3 Outlines of the Main Contributions

This dissertation contributes to the area of Artificial Intelligence and Information Security science. Specifically, it introduces novel thinking and techniques to the fields of temporal logic, quantum-based cryptography, and image security research in general. The primary objectives of this dissertation are that:

(1) Based on the typed point based time-elements and time-series, the formal characterization of time-series was consummated with respect to the two temporal factors including temporal order and temporal duration.

(2) *BMQE* system: Most of currently popular public-key cryptosystems either rely on the integer factorization problem or discrete logarithm problem, which means the “crypto-eggs” are in one basket – too dangerous. Moreover, particular techniques for factorization and solving discrete logarithm improve continually [Chr2005]. *MQ*-based system is said to be one of the prominent alternative cryptosystems for resisting quantum attacks. But of the existing *MQ*-based cryptosystems, apart from UOV schemes with proper parameter values, the basic types of these *MQ*-schemes are considered to be insecure. Therefore, we designed a new *MQ*-equation-based system, which is called bisectional multivariate quadratic equation system (*BMQE*), to resist the existing algebraic attack. We also analysed that fixing variables and relinearization cannot be used to solve this system if these parameters are properly set.

(3) A hybrid-key image cryptosystem: The chaos-based cryptosystems, which have

been extensively used since last two decades, are almost all based on symmetric cryptography. Symmetric cryptography is much faster than asymmetric ciphers, but the requirements for key exchange make them hard to use. To remedy this imperfection, a hybrid-key based image encryption and authentication scheme is proposed. Particularly, ergodic matrices are utilized not only as public keys throughout the encryption/decryption process, but also as essential parameters in the confusion and diffusion stages.

(4) A hybrid image watermarking system based on ergodic matrix and DWT-SVD: As the popularity of digital media is growing, the copyright protection of intellectual properties has become a necessity for prevention of illegal duplication and verification. Therefore, a hybrid digital image watermarking scheme based on DWT and SVD is proposed. To increase the security of the watermark, ergodic matrix is applied in the encryption stage of a watermark before it is embedded into a carrier image.

(5) Schemes based on temporal order for image cryptography and image hiding:

a) A temporal-based image cryptosystem: In reality, when Alice communicates with Bob, she might want to send a sequence of images with different security demands; in addition, to be more secure, she might also wish to get different results if the temporal order in which the sequence of images are changed. But the existing encryption schemes haven't considered this issue. Thus the new model we designed will be a novel model which can encrypt images with various importances without changing the keys and get varied encryption results if the temporal order of the images is changed.

b) A temporal-based image hiding system: Take dynamic multiple digital watermarking for an example, a digital image may be modified by some other authors when it is created. Given the new image may be modified by various authors, from various images and at different time, the watermark embedded in the new image shall change to specify the differences. Thus the model we designed will solve the problem caused by multiple authors modifying the image and protect every author's identity.

Section 1.4 Thesis Structure

CHAPTER 2 provides a comprehensive review of temporal basis, especially the representation of time-series. Then followed by ergodic matrix, this is a mathematical tool we use throughout the information security process. Least but not last, the history and some related concepts of information cryptography and information hiding are given.

CHAPTER 3 proposes a bisectional multivariate quadratic equation system based on ergodic matrix over a finite field with q elements. The system is proven to be NP-complete. The complexity of the system is determined by the number of the variables, of the equations and of the elements of $\mathbb{F}^q$, which is denoted as n, m, and q, respectively. Analysis shows that, if the number of the equations is larger or equal to twice the number of the variables, and q^n is large enough, the system is complicated enough to prevent attacks from relinearization attack and fixing variable attack.

CHAPTER 4 introduces a hybrid-key based image encryption and authentication scheme. Particularly, ergodic matrices are utilized not only as public keys throughout the encryption/decryption process, but also as essential parameters in the confusion and diffusion stages. The experimental results, statistical analysis and sensitivity-based tests confirm that, compared to the existing chaos-based cryptosystems, the proposed image encryption scheme provides more secure way for image encryption and transmission.

CHAPTER 5 proposes a DWT&SVD-based image hiding system for copyright protection as well as information hiding. The security of the proposed scheme is increased by applying ergodic matrix on the watermark to be embedded. We also demonstrate the correlation between the embedded and the extracted watermark with experimental results. One of the major advantages of the proposed scheme is the robustness of the technique on a wide set of attacks. Analysis and experimental results show much improved performance of the proposed method in comparison with the existing DWT&SVD-based algorithms. Experimental results confirm that the proposed scheme provides good quality of watermarked images.

In CHAPTER 6, two temporal based applications for image cryptography and image hiding, particularly with the formal characterization of time-series, have been provided.

In CHAPTER 7, a summary of the current research in this thesis is outlined and the concluding recommendations on the outcome of the research are proposed. Then the suggested future work is presented.

CHAPTER 2 LITERATURE REVIEW

Aiming at the five objectives of this research work, the detailed review of related work will be presented in this chapter.

Section 2.1 Temporal Basis

In the domain of artificial intelligence, especially on the issue of what sorts of objects shall be taken as the time elements, there has been a longstanding debate. On the one hand, points are used in both theoretical and practical modelling of temporal phenomena. On the other hand, intervals are also needed for representing temporal phenomena that take up time with positive duration. Therefore, problems may occur when one conflates different views of temporal structure and questions whether some certain types of temporal propositions can be validly and meaningfully associated with different time elements.

From the point of view of ontological approach, there are three known choices as for the objects that may be taken as the time primitive:

(1) Points, i.e., instants of time with no duration;

(2) Intervals, i.e., periods of time with positive duration;

(3) Both points and intervals, which lead to different time structures.

Section 2.1.1 Point-based Systems

Time points are intuitive and convenient to associate punctual events, such as “The power was automatically switched on at 8:00 pm” and “The velocity of the rocket that had been launched into the air became zero when it reached the apex” etc., these are temporal phenomena which can be meaningfully ascribed only to time points rather than durative intervals.

A typical point-based time structure is an order pair (P, ≤), where P is a set of points, and $\leq$ is a relation that orders *P*. For particular applications, the characteristics of a point-based system may be specified in great detail, such as bounded/unbounded, dense/discrete and linearly/non-linearly ordered, etc. Three obvious models are the real-numbers time ($\boldsymbol{R}$, $\leq$), rational-numbers time ($\boldsymbol{Q}$, $\leq$) and integer-numbers time ($\boldsymbol{Z}$, $\leq$), where $\leq$ denotes the usual partial order relation.

In point-based systems, intervals may be defined as derived temporal objects, either as sets of points in [Dre1982 and Ben1983], or as orderings of points in [Bru1972, Sho1987, Lad1987, HS1991 and Lad1992]. Nevertheless, some researchers argued that defining intervals as objects derived from points may lead to the so-called *Dividing Instant Problem* [Ben1983, All1983, Gal1990 and MK2003], which discussed about the contradictions of changes taking place in time. Allen cites the example of a light that has been off, and becomes on after it is switched on, and asks the question as to exactly what happens at the intermediate instant between the two successive states of the light being off and on at the switching point [All1983].

Intuitively, we can assume the two states, i.e., "The light was off" and "The light was on" holds true throughout two successive point-based intervals, say $<p_1, p>$ and $<p, p_2>$, respectively; then the question becomes as "Was the light off or on at point *p*?" This, in terms of the *open* or *closed* nature of the involved point-based intervals, turns out to be the question of which of the two successive intervals, i.e., $<p_1, p>$ and $<p, p_2>$, is closed/open at the dividing point *p*? Practically, there are four possible cases:

(a) The light was off rather than on at *p*;

(b) The light was on rather than off at *p*;

(c) The light was both off and on at *p*;

(d) The light was neither off nor was it on at *p*.

While both (c) and (d) are illogical, since the former claim violates the *Law of Contradiction* and the latter violates the *Law of Excluded Third* [Ben1983], the choice between (a) and (b) must be arbitrary and artificial. In fact, since we have no better reason, from the point of view of philosophy, for saying that the light was off than for saying that it was on at the dividing-instant, such an arbitrary approach has been criticized as indefensible and hence unsatisfactory [Ben1983, All1983, Gal1990 and Vil1994].

Section 2.1.2 Interval-based Systems

The point-based structure of time has been challenged by many researchers who believe that time intervals are more suited for the expression of commonsense temporal knowledge, especially in the domain of linguistics and AI. Therefore, intervals shall be treated as the temporal primitive, where points may be constructed with a subsidiary status, for instance, as *maximal nests* of intervals (a problem of finding size of maximal nested intervals set) that share a common intersection [Whi1929], or as *meeting places* of intervals [AH1985, HA1987, AH1989 and Gal1990].

Allen's interval calculus [All1983] is a typical example of the interval-based approach. It posits a pair (***I***, ***R***), where ***I*** is a set of intervals and ***R*** is a set of binary relations over ***I***. Allen introduces thirteen temporal relations, including "Meets", "Met by", "Equal", "Before", "After", "Overlaps", "Overlapped by", "Starts", "Starts by", "During", "Contains", "Finishes" and "Finished by". The intuitive meaning of Meets(i_1, i_2) is that interval i_1 is one of the immediate predecessors of interval i_2. Later, in [AH1985 and AH1989], the "Meets" relation is formally characterized as primitive, and the other 12 binary relations can be derived from it.

Allen claims in his papers [All1981, All1983 and All1984] that, an interval-based approach avoids the annoying question of whether or not a given point is part of, or a member of a given interval. Allen's contention is that nothing can be true at a point, for a point is not an entity at which things happen or are true. Therefore, this point of view can successfully overcome/bypass puzzles like the *DI* Problem. However, as Galton [Gal1990] shows in his critical examination of Allen's interval logic [All1984], a theory of time based only on intervals is insufficient for reasoning correctly about continuous change. In fact, many commonsense situations suggest the need for the inclusion of time points in the temporal ontology as an entity different from intervals. For instance, it is intuitive and convenient to say that instantaneous events, e.g. "The court was adjourned at 4:00pm", occur at time points rather than intervals (no matter how small they are).

To characterize the times that some “instant-like” events occupy, Allen and Hayes [AH1989] introduce the idea of *very short intervals*, called *moments*. A moment is simply a non-decomposable time interval. The important distinction between moments and points is: although being non-decomposable, moments are defined by having extent and by means of having distinct start and end points, while extent-less points may be implicitly defined in terms of the “Meets” relation, together with “START” and “END” functions [AH1985 and HA1987]. Relating to the “Meets” relation, another obvious difference between points and moments is that moments can meet other real intervals (but by definition, moments cannot meet other moments), and hence stand between them, while points are not treated as primitive objects and cannot meet anything.

Section 2.1.3 Point&interval-based Theory

In order to overcome the limitations of an interval-based approach while retaining its convenience of expression, a third approach has been introduced in [MK1994], which addresses both points and intervals as temporal primitives on an equal footing: points do not have to be defined as limits of intervals and intervals do not have to be constructed out of points.

Actually, similar to Allen and Hayes’ approach [AH1989], the primitive relation “Meets” (denoting the immediate predecessor relation) can be defined over the set of time elements which consists of both intervals and points. In terms of the single “Meets” relation, there are in total 30 relations over time elements which can be classified into the following groups:

- point–point:

  {Equal, Before, After}

which relate points to points;

- point–interval:

  {Before, After, Meets, Met by, Starts, During, Finishes}

which relate points to intervals;

- interval–point:

  {Before, After, Meets, Met by, Started by, Contains, Finished by}

which relate intervals to points;

- interval–interval:

  {Equal, Before, After, Meets, Met by, Overlaps, Overlapped by, Starts, Started by, During, Contains, Finishes, Finished by}

which relate intervals to intervals.

In addition, in the theory based on both points and intervals as primitives, although there are no definitions about the ending points for intervals, the formalism allows the expression of the “open” and “closed” nature of intervals, which can be formally defined as:

- Interval *i* is *left-open* if and only if there is a point *p* such that Meets*(p, i)*;
- Interval *i* is *right-open* if and only if there is a point *p* such that Meets*(i, p)*;
- Interval *i* is left-closed if and only if there is a point *p* and an interval *i’* such that Meets*(i’, i)* ∧ Meets*(i’, p)*;
- Interval *i* is *right-closed* if and only if there is a point *p* and an interval *i’* such that Meets*(i, i’)* ∧ Meets*(p, i’).*

The above interpretation about the “open” and “closed” nature of primitive intervals is in fact consistent with the conventional meaning of the open and closed nature of intervals of real numbers. For instance, interval (2, 5] which does not include number 2 itself is “left-open”, since (point) 2 is an immediate predecessor of interval (2, 5]. Similarly, (2, 5] which does include number 5 is “right-closed”, since both point 5 and interval (2, 5] are immediate predecessors of interval (5, 8).

However, it is important to note that the distinction between the assertion “point p Meets interval *i*”, i.e., Meets(*p*, *i*), and the assertion “point *p* Starts interval *i*”, i.e., Starts(*p*, *i*), is extremely subtle. First of all, they are mutually exclusive, that is, if Meets(*p*, *i*) holds then Starts(*p*, *i*) will not hold. In fact, by definition, Meets(*p*, *i*) states that point p is one of the immediate predecessors of interval i, that is, while point p is "earlier" than i, there are no other time elements standing between p and i; and Starts(p, i) states that point p is a starting-part (actually, in this case, the left-ending point) of interval i. Therefore, Meets(p, i) implies interval i does not include point p, while Starts(p, i) implies interval i does include point p. In other words, Meets(p, i) implies interval i is left-open at point p, while Starts(p, i) implies interval i is left-closed at point p. In addition, from Meets(p, i), we can form a new interval as the ordered union of p and i, i.e., $p \oplus i$. It is worth pointing out that interval i and interval $p \oplus i$ are two different intervals, though Dur($p \oplus i$) = Dur(p) + Dur(i) = Dur(i) since Dur(p) = 0. Furthermore, we have an important relationship here, i.e.:

$$\text{Meets}(p, i) \Leftrightarrow \text{Starts}(p, p \oplus i).$$

In other words, interval i is left-open at point p if and only if interval $p \oplus i$ is left-closed at point p. Because in the "Meets(p, i)" case, point p is not included in i, while $p \oplus i$ includes p. And the point p can only be denoted as [p, p]. Similar discussions apply to the relationship between Finished-by(i, p) and Meets(i, p).

As mentioned earlier, since the time theory adopted here takes both points and intervals as primitive on an equal footing, points do not have to be defined as the limits of intervals and likewise intervals do not have to be constructed out of points either. In fact, intervals may meet each other without any points standing between them, falling within them, or bounding them. For instance, we may just have the following temporal knowledge:

$$\text{Meets}(i_1, i_2) \wedge \text{Meets}(i_1, i_3) \wedge \text{Meets}(i_2, i_4)$$

That is, interval i_1 "Meets" both intervals i_2 and i_3, and interval i_2 "Meets" interval i_4, without knowledge about any point at all.

However, the time theory does allow for cases where points may stand between, fall within, or start/finish intervals. This kind of knowledge can be expressed in terms of relations such as "Meets", "Starts", "During", etc. For example:

$$\text{Meets}(i_5, p) \wedge \text{Meets}(p, i_6) \wedge \text{Starts}(p, i_7)$$

That is, interval i_5 “Meets” point p, which in turn “Meets” interval i_6 and “Starts” interval i_7. Based on these, if we denote the ordered union of the three adjacent time elements i_5, p and i_6 as i, i.e., $i = i_5 \oplus p \oplus i_6$, then we have During($p$, i). In other words, point p may be viewed as “fall within” interval i (i.e., p is an inner point of interval i). In addition, we can infer that, interval i_5 is right-open at p, i_6 is left-open at p, and interval i_7 is left-closed at p.

Section 2.1.4 The Notion of Time-series

Time-series is a chronological series of observations made. In accordance with different phenomena or problems studied, one can get all kinds of time-series. For example, some economists observe fluctuations in the price index, a meteorologist study the rainfall in some location, Electrical Engineers study electronic receiver's internal noise. All of them will observe a string of data measured by some unit of measurement. The natural order is the chronological order of appearance by the order in time-series. The typical essential characteristic of time-series is the dependency between adjacent observations. This dependence has great practical significance. Time-series analysis is addressed in the techniques of this dependence analysis. The new method of prediction of time-series data not only provides effective prediction method for time-series data produced from the national economy, agriculture, biology, meteorology, hydrology and other fields, but also enables researchers to exercise math skills and programming techniques.

In order to analyse time-series, the formalism is required. However, in most of proposed formalisms, the fundamental time theories based on which time-series are formed up are usually not explicitly specified. Time-series are simply expressed as lists in the form of t_1, t_2, ….t_n, or as sequences of collection of observations, and so on, where formal characterizations with respect to the temporal basis are neglected, leaving some critical issues unaddressed. For example,

- What a sort of objects do these t_1, t_2, … and t_n belong to? In other word, what sorts of objects shall be taken as the time primitive? Are they time points, time intervals, or simply some absolute values from the real numbers, integers, or the clock?

- What are the temporal order relationships between these t_1, t_2, … and t_n, and/or between the sequence of collections? Are they simply well-ordered as the natural numbers, or they may be relatively ordered by means of relations such as "Before", "Meets", "During", and so on?

- What are the associations between time-series/state-sequences and non-temporal data that represent various states of the world in discourse?

Section 2.2 Ergodic Matrix

Ergodic matrix (*EM*) was introduced by Zhao et al. [YSZ2004 and ZMD+2010]. It is constructed based on finite fields and has so many good features. As a result, we introduce *EM* and apply it into information security systems.

Section 2.2.1 Finite Fields

Ergodic matrix is based on finite fields. Finite fields also called Galois field, they are significant in many areas of mathematics and computer science, including Galois theory, algebraic geometry, Quantum error correction, coding theory and cryptography [CRSS1998]. Finite fields are also a very basic building block for *EM* and *MQ*-based cryptosystems.

To understand finite fields, three concepts shall be firstly introduced: *groups, rings,* and *fields*.

A ***group*** is a set of elements, $\mathbb{G}$, together with an operation • that combines any pair of elements *a* and *b*, such that:

(1) $a \bullet b \in \mathbb{G}, \forall a,b \in \mathbb{G}$;

(2) Neutral: $\forall a \in \mathbb{G}, \exists 1 \in \mathbb{G}$, such that $1 \bullet a = a \bullet 1 = a$;

(3) Inverse: $\forall a \in \mathbb{G}, \exists a^{-1} \in \mathbb{G}$ such that $a \bullet a^{-1} = 1$;

(4) Associativity: $\forall a,b,c \in \mathbb{G}$, $(a \bullet b) \bullet c = a \bullet (b \bullet c)$.

A ***ring*** is a set of elements, $\mathbb{R}$, together with two operations + and •. The former operation is like addition, the latter is like multiplication. A ring has the following properties:

(1) The elements of the ring, together with the addition operation, form a group;

(2) Addition is commutative: $\forall a,b \in \mathbb{R}$, $a + b = b + a$;

(3) The multiplication operation is associative: $\forall a,b,c \in \mathbb{R}$, $(a \cdot b) \cdot c = a \cdot (b \cdot c)$;

(4) The distributive law holds: $\forall a,b,c \in \mathbb{R}$, $a \cdot (b+c) = (a \cdot b)+(a \cdot c)$ holds.

A ***field*** $\mathbb{F}$ is a commutative ring which contains a multiplicative inverse for every nonzero element. Denote $\mathbb{F}$ as a set of $q \in \mathbb{N}$ elements with the operations, addition +: $\mathbb{F} \times \mathbb{F} \to \mathbb{F}$ and multiplication •: $\mathbb{F} \times \mathbb{F} \to \mathbb{F}$. We call $(\mathbb{F}, +, \bullet)$ a field if it has the following properties:

(1) $(\mathbb{F}, +)$ is an Abelian group with additive identity denoted by 0:

a) Associativity: $\forall a,b,c \in \mathbb{F}$, we have $((a + b) + c) = (a + (b + c))$;
b) Additive neutral: $\forall a \in \mathbb{F}$, we have $a + 0 = a$;
c) Additive inverse: $\forall a \in \mathbb{F}$, $\exists (-a) \in \mathbb{F}$ such that $a + (-a) = 0$;
d) Commutativity: $\forall a,b \in \mathbb{F}$, $a + b = b + a$ holds.

(2) $(\mathbb{F} \backslash \{0\}, \bullet)$ is an Abelian group with multiplicative identity denoted by 1:

a) Associativity: $\forall a,b,c \in \mathbb{F}$, we have $((a \bullet b) \bullet c) = (a \bullet (b \bullet c))$;
b) Multiplicative neutral: $\forall a \in \mathbb{F}$, we have $a \bullet 1 = a$;
c) Multiplicative inverse: $\forall a \in \mathbb{F}$, $\exists a^{-1} \in \mathbb{F}$ such that $a \bullet a^{-1} = 1$;
d) Commutativity: $\forall a,b \in \mathbb{F}$, $a \bullet b = b \bullet a$ holds;

(3) The distributive law holds: $(a+ b) \bullet c = a \bullet c + b \bullet c$, for $\forall a,b,c \in \mathbb{F}$.

If $\mathbb{F}$ is finite, then the field is said to be finite.

Definition 2.1 Let p be a prime number. The integers modulo p, consisting of the integers $\{0, 1, 2, \cdots, p\text{-}1\}$ with addition and multiplication performed modulo p, is a finite field of order p. We shall denote this field by $\mathbb{F}^p$ and call it a prime field.

Definition 2.2 Finite fields of order 2^m are called binary fields. A method to construct $\mathbb{F}^{2^m}$ is to use a polynomial basis representation. Here, the elements of $\mathbb{F}^{2^m}$ are the binary polynomials of degree no more than m-1, and the coefficients of the polynomials are in the field $\mathbb{F}^2 = \{0, 1\}$:

$$\mathbb{F}^{2^m} = \{a_{m-1}x^{m-1} + a_{m-2}x^{m-2} + \cdots + a_2x^2 + a_1x^1 + a_0\}, \text{ here } a_i \in \{0,1\}.$$

The polynomial basis representation for binary fields can be generalized to all extension fields in:

Definition 2.3 Given p is a prime and m≥2. $\mathbb{F}^p[x]$ denotes the set of all polynomials in the variable x over $\mathbb{F}^p$ with degree n. Let f(x), the reduction polynomial, be an irreducible polynomial of degree m in $\mathbb{F}^p[x]$. Moreover, we define the set $\mathbb{F}^{p^m}$=$\mathbb{F}^p[x]/f(x)$ as equivalence classes of polynomials modulo f(x). Then ($\mathbb{F}^{p^m}$, +, •) is a polynomial field and also a degree n extension of the ground field $\mathbb{F}^p$:

$$\mathbb{F}^{p^m}=\{a_{m-1}x^{m-1}+a_{m-2}x^{m-2}+\cdots+a_2x^2+a_1x^1+a_0\}, \text{ here } a_i \in \mathbb{F}^p$$

Addition "+" is the usual addition of polynomials, with coefficient arithmetic performed in $\mathbb{F}^p$. Multiplication of field elements is performed modulo the irreducible polynomial f (x).

It is worth noting that Definition 2.2 and Definition 2.3 comply with the field axioms from Definition 2.1. Furthermore, all fields are either prime field or extension field. In particular, the following needs to be stressed:

Lemma 2.1 Let $\mathbb{F}^q$ be a finite field of field characteristic q, then for $\forall x \in \mathbb{F}^q$, x^q=x holds, this is also called Frobenius automorphism.

This lemma is useful in the context of schemes defined over extension fields and of affine transformations, such as ergodic matrix.

Section 2.2.2 Definitions and Related Theorems of the *EM*

Definition 2.4 Given $Q \in \mathbb{F}^q_{n\times n}$, if any non-zero column vector $v \in \mathbb{F}^q_{n\times 1} \backslash \{\mathbf{0}\}$, {$Qv$,$Q^2v$, … ,$Q^{q^n-1}$ v} just exhausts $\mathbb{F}^q_{n\times 1} \backslash \{\mathbf{0}\}$, then Q is ***Ergodic Matrix*** over finite field $\mathbb{F}^q$. (Here **0**=[0 0 … 0]$^{\mathrm{T}}$)

Definition 2.5 Given $Q \in \mathbb{F}^q_{n\times n}$, if ⟨$Q$⟩={$Q^x$|$x$ = 1, 2, 3,…}, ⟨Q⟩ is a generating set of Q over $\mathbb{F}^q_{n\times n}$.

The basic idea of the *EM* is briefly described as below:

Let $\mathbb{F}^q_{n\times 1}$ be a set of all n×n matrices over a finite field $\mathbb{F}^q$. A triple, ($\mathbb{F}^q_{n\times n}$, +, ×), forms a 1-ring, here + and × are addition and multiplication over $\mathbb{F}^q$, respectively. We randomly generate two nonsingular matrices $Q_1,Q_2 \in \mathbb{F}^q_{n\times n}$, then:

(1) $(\mathbb{F}^q_{n\times n},\times)$ is a monoid, its identity element is $I_{n\times n}$,

(2) $(\langle Q_1\rangle,\times)$ and $(\langle Q_2\rangle,\times)$ are Abelian groups, their identity elements are also $I_{n\times n}$. Here Q_1, Q_2 are nonsingular and Q_1, $Q_2 \in \mathbb{F}^q_{n\times n}$, and

(3) for any $m_1,m_2 \in \mathbb{F}^q_{n\times n}$, generally $m_1\times m_2 \neq m_2\times m_1$. i.e., the multiplication is not commutative in $\mathbb{F}^q_{n\times n}$.

For any $Q\in \mathbb{F}^q_{n\times n}$, $Q_1\times Q$ does linear transformations to each row of Q and $Q\times Q_2$ does linear transformations to each column of Q. Thus $Q_1\times Q\times Q_2$ distributes each element of Q, This process can be repeated several times, e.g. $Q_1^s\times Q\times Q_2^t$ ($1\leq s\leq|\langle Q_1\rangle|$, $1\leq t\leq|\langle Q_2\rangle|$), so that Q's transformation is much more complex. In order to improve the quality of encryption (or transformation), the generating set $\langle Q_1\rangle$ and $\langle Q_2\rangle$ must be as large as possible. Therefore, the result of Q_1 multiplying a column vector on the left and Q_2 multiplying a row vector on the right will be divergent.

Theorem 2.1 $Q \in \mathbb{F}^q_{n\times n}$ is an *EM*, there are $\varphi(q^n\text{-}1)$ *EM*s in $\langle Q\rangle=\{Q^x|\ x=1,2,3,\dots\}$, these *EM*s are “equivalent to each other”, i.e., two *EM*s are equivalent if and only if they are in the same generating set. Here $\varphi(x)$ is the Euler function.

Theorem 2.2 $Q\in \mathbb{F}^q_{n\times n}$ is an *EM* if and only if the order of Q, under the multiplication, is $(q^n\text{-}1)$.

Zhao has deduced the following lemmas by finite field theory [LH1994, SYOL2002 and ZYZ2004]:

Lemma 2.2 If $m\in \mathbb{F}^q_{n\times n}$ is nonsingular, then m's order is equal to or less than $(q^n\text{-}1)$.

Lemma 2.3 If $Q\in \mathbb{F}^q_{n\times n}$ is an *EM*, then $(F_q[Q],+,\times)$ is a finite field with q^n elements.

Lemma 2.4 If $Q\in \mathbb{F}^q_{n\times n}$ is an *EM*, then Q^{T} (the transpose of Q) must also be an

EM.

Lemma 2.5 If $Q \in \mathbb{F}^q_{n\times n}$ is an *EM*, then $\forall \mathrm{v} \in F_q{}^n \backslash\{\mathbf{0}\}$, $v^{\mathrm{T}}Q$, …, $v^{\mathrm{T}}Q^{q^{n-1}}$ just exhausts $\{v^{\mathrm{T}} | \, v \in \mathbb{F}^q_{n\times n}\} \backslash \{\mathbf{0}^{\mathrm{T}}\}$.

Lemma 2.6 $Q \in \mathbb{F}^q_{n\times n}$ is an *EM*, then there are $\varphi(q^n$-1) ergodic matrices in $\langle Q\rangle$ and these matrices are *equivalent* to each other.

From the theorems and lemmas, any ergodic matrix over finite field $\mathbb{F}^q$ has a largest order and a largest generating set. Besides, the result of a non-zero column vector multiplied by an *EM* on the left side (or a non-zero row vector multiplied on the right side) is substantially distributed.

Section 2.2.3 Security Analysis of a Question Based on *EM*

Question2.1. Given $Q_1, Q_2 \in \mathbb{F}^q_{n\times n}$ are ergodic matrices, knowing that $A,B \in \mathbb{F}^q_{n\times n}$, it is hard to find $Q_1{}^x \in \langle Q_1\rangle$, $Q_2{}^y \in \langle Q_2\rangle$ such that $B=Q_1{}^xAQ_2{}^y$.

Suppose the attacker Eve knows A,B and the relation $B=Q_1{}^xAQ_2{}^y$, for deducing $Q_1{}^x$ and $Q_2{}^y$, she may attacks mainly by:

(1) Brute-force attack

In this attack, for any $Q_1{}^s \in \langle Q_1\rangle$ and $Q_2{}^s \in \langle Q_2\rangle$, Eve computes $C' = Q_1{}^sPQ_2{}^t$, such that $Q_1{}^x=Q_1{}^s$, $Q_2{}^y=Q_2{}^t$.

From Definition 2.4, we know that $|\langle Q_1\rangle|=|\langle Q_2\rangle|=q^n$-1. In order to resist brute-force attack, *n* must be large enough. Generally speaking, 256-bit encryption is enough to resist normal attacks. For example, AES has a fixed block size of 128 bits, and a key size of 128, 192, or 256 bits. While the Rijndael specification *per se* is specified with block and key sizes with a minimum of 128 and a maximum of 256 bits. If stronger encryption is required, *n* can be up to 1024-bit (or even more).

(2) Assumption-based attack

The attacker assumes $Q_1{}^x=Q_1{}^s \in \langle Q_1\rangle$, if P is invertible, from $C= Q_1{}^xPQ_2{}^y=Q_1{}^sPX$ he deduces $X=P^{-1}Q_1{}^{-s}C$. If $X \in \langle Q_2\rangle$, then he can deduce $Q_1{}^x=Q_1{}^s$, $Q_2{}^y= X$.

For assumption-based attacks, since Q_1 and Q_2 are private in normal cases, P is not necessary a full rank matrix, so it is hard to deduce Q_1 and Q_2. Considering a worse case, namely, where P is invertible, since there is no commutative laws for the multiplication of matrix, the success of attack must rely on $X\in\langle Q_2\rangle$, which means the success of attack has to rely on luck.

(3) Equation attack

Eve arbitrarily chooses $a_1,a_2,\ldots,a_m\in\langle Q_1\rangle$ and $b_1,b_2,\ldots,b_m\in\langle Q_2\rangle$. She sets up the simultaneous equations as follows:

$$\begin{cases} B_1=Q_1{}^x A_1 Q_2{}^y \\ B_2=Q_1{}^x A_2 Q_2{}^y \\ \ldots \\ B_m=Q_1{}^x A_m Q_2{}^y \end{cases}$$

(Here $\mathrm{A}_k=a_k\mathrm{A}b_k$, $\mathrm{B}_k=a_k\mathrm{B}b_k$ are known, as it has been specified in *Question 2.1*)

Thus Eve may deduce $Q_1{}^x$ and $Q_2{}^y$.

To analyse simultaneous equations attack, let us take the matrices over $\mathbb{F}_{2\times2}^q$ as an example for discussion:

Because Eve knows $Q_1,Q_2,A,B\in\mathbb{F}_{2\times2}^q$ and $B=\boldsymbol{X}A\boldsymbol{Y}$ ($\boldsymbol{X}\in\langle Q_1\rangle,\boldsymbol{Y}\in\langle Q_2\rangle$), she may set up the following equations from $B=\boldsymbol{X}A\boldsymbol{Y}$:

$$\begin{cases} a_{11}x_{11}y_{11}+a_{12}x_{11}y_{21}+a_{21}x_{12}y_{11}+a_{22}x_{12}y_{21}=b_{11} \\ a_{11}x_{11}y_{12}+a_{12}x_{11}y_{21}+a_{21}x_{12}y_{12}+a_{22}x_{12}y_{22}=b_{12} \\ a_{11}x_{21}y_{11}+a_{12}x_{21}y_{21}+a_{21}x_{22}y_{11}+a_{22}x_{22}y_{21}=b_{21} \\ a_{11}x_{21}y_{12}+a_{12}x_{21}y_{22}+a_{21}x_{22}y_{12}+a_{22}x_{22}y_{22}=b_{22} \end{cases} \quad (2.1)$$

Then she selects $(Q_1^{a_1},Q_2^{b_1})$, $(Q_1^{a_2},Q_2^{b_2})$, $(Q_1^{a_3},Q_2^{b_3})$, $(Q_1^{a_4},Q_2^{b_4})\in\langle Q_1\rangle\times\langle Q_2\rangle$, and from $B=\boldsymbol{X}A\boldsymbol{Y}$ she gets:

$$\begin{cases} B_1=XA_1Y \\ B_2=XA_2Y \\ B_3=XA_3Y \\ B_4=XA_4Y \end{cases} \left(A_i=Q_1^{a_i}AQ_2^{b_i}\ ,\ B_i=Q_1^{a_i}BQ_2^{b_i} \right) \qquad (2.2)$$

Thereafter Eve lets:

$$T=\begin{bmatrix} \mathrm{A}_1[1,1] & \mathrm{A}_1[1,2] & \mathrm{A}_1[2,1] & \mathrm{A}_1[2,2] \\ \mathrm{A}_2[1,1] & \mathrm{A}_2[1,2] & \mathrm{A}_2[2,1] & \mathrm{A}_2[2,2] \\ \mathrm{A}_3[1,1] & \mathrm{A}_3[1,2] & \mathrm{A}_3[2,1] & \mathrm{A}_3[2,2] \\ \mathrm{A}_4[1,1] & \mathrm{A}_4[1,2] & \mathrm{A}_4[2,1] & \mathrm{A}_4[2,2] \end{bmatrix}$$

From (2.1) and (2.2), Eve builds the equations (2.3):

$$T\begin{bmatrix} x_{11}y_{11} \\ x_{11}y_{21} \\ x_{12}y_{11} \\ x_{12}y_{21} \end{bmatrix} = \begin{bmatrix} B_1[1,1] \\ B_2[1,1] \\ B_3[1,1] \\ B_4[1,1] \end{bmatrix} \qquad T\begin{bmatrix} x_{11}y_{12} \\ x_{11}y_{22} \\ x_{12}y_{12} \\ x_{12}y_{22} \end{bmatrix} = \begin{bmatrix} B_1[1,2] \\ B_2[1,2] \\ B_3[1,2] \\ B_4[1,2] \end{bmatrix}$$

$$T\begin{bmatrix} x_{21}y_{11} \\ x_{21}y_{21} \\ x_{22}y_{11} \\ x_{22}y_{21} \end{bmatrix} = \begin{bmatrix} B_1[2,1] \\ B_2[2,1] \\ B_3[2,1] \\ B_4[2,1] \end{bmatrix} \qquad T\begin{bmatrix} x_{21}y_{12} \\ x_{21}y_{22} \\ x_{22}y_{12} \\ x_{22}y_{22} \end{bmatrix} = \begin{bmatrix} B_1[2,2] \\ B_2[2,2] \\ B_3[2,2] \\ B_4[2,2] \end{bmatrix} \tag{2.3}$$

If all the matrices (${Q_1}^{ai}$, ${Q_2}^{bi}$) that the attacker has selected happen to make the matrix $T \in \mathbb{F}_{4\times4}^{q}$ invertible, *X* and *Y* thus can be deduced by equation (2.3).

However, this algorithm is equivalent to solving $2n^2$ unknown quantities by n^2 equations. That is to say, the solution is non-unique. In this case, the attacker has to decipher by assumption – it is quite difficult.

Consider an ideal circumstance where Q_1 and Q_2 are known, *T* is a full rank matrix, *X* and *Y* are exactly the solutions of equation (2.1), then the computational complexity is n^4. But in practice, Q_1 and Q_2 are taken as private keys. Plus, we can add some elements to make *T* a singular matrix, so that it is hard for the attackers to deduce the plaintext *P*. Therefore, it is feasible to use *EM* to encipher. In order to effectively resist a known-plaintext attack, we can select *EM*s of different orders to multiply *P* on the left and right sides simultaneously. Hence *P* is fully distributed and harder to be deduced.

Section 2.3 Cryptography

Back in 1976, Diffie and Hellman proclaimed [DH1976]:

We stand today on the brink of a revolution in cryptography.

And now the revolution has been undergoing for more than three decades. During those past years we have seen an explosion of research in cryptography and in cryptanalysis.

Section 2.3.1 History of Cryptography and Cryptanalysis

Cryptology is the science and art of secure communication. It can be categorized into cryptography and cryptanalysis. Cryptography is the act or art of writing in secret characters while cryptanalysis is the analysis and deciphering of secret writings [Sta2010].

Section 2.3.1.1 Cryptography

The history of cryptography can be roughly classified into three phases, i.e., early cryptography, cryptography during the two world wars and modern cryptography:

- **Early Cryptography**

Strictly speaking, cryptography starts with the origin of language writing. People have tried to conceal information in written form and the remaining stone inscriptions and papyruses show that many ancient civilisations such as the Egyptians, Greeks and Romans all have contributions to the development of cryptographic systems. One of the earliest known forms of cryptography is Atbash which is a simple substitution cipher for the Hebrew alphabet. It consists in substituting the first letter for the last, the second for one before last, et cetera. Another example is around 400 BC, The Greeks and Spartans employed a cipher tool called a “scytale” to send secret communications during military campaigns. The scytale consisted of a cylinder with a strip of parchment wound around it on which is inscribed with a message. Once the parchment is unwrapped it will appear an inexplicable set of letters. It has the advantage of being fast and not easy to mistakes. However, it is easily broken.

The earliest article on the cipher subject was written by a Greek, Aeneas Tacticus, as part of a work entitled *On the Defence of Fortifications*. Polybius, another Greek, later devised a device of encoding letters into pairs of symbols, the device is known as the *Polybius square* and is for fractionating plaintext characters so that they can be represented by a smaller set of symbols. Other than the Greeks there are similar examples of primitive transposition and substitution ciphers used by other civilisations including the Romans.

European cryptography can be traced back to the Middle Ages during which it was developed by the Papal and Italian city states. The earliest ciphers involved only vowel substitution. In 1379 the first European manual on cryptography appeared. It is a collection of ciphers written by Gabriele de Lavinde. The manual describes nomenclator cryptosystems, which combine a codebook with homophonic substitution. Until the 19th century, nomenclators were the mainstay of diplomatic communications in the trading states of southern Europe and the Catholic Church. In 1466 Italian polymath, Leon Battista Alberti, invented the polyalphabetic cipher (a cipher based on substitution, using multiple substitution alphabets) and machine-assisted encryption using the first cipher disk in *Trattati in cifra*. In 1792 French inventor Claude Chappe demonstrated a semaphore system. The system is the first practical telecommunications system and it eventually traverses all of France.

By 1860 large codes were commonly used for diplomatic communications and cipher systems had infrequently applied even though they were prevailed for military communications. During the U.S. Civil War the Federal Army widely used transposition ciphers. The Confederate Army mostly used the Vigenère cipher, a simple form of polyalphabetic substitution, and on occasional monoalphabetic substitution (each alphabetic character is mapped to a unique alphabetic character).

The popularity of cryptography was not limited to those who used it for military and diplomatic intelligence. The increasing popularity of cryptography in the 16th and 17th centuries is clearly verified by the proliferation of books on the subject.

- **Cryptography During The Two World Wars**

In 1914, World War I began, marking a time of intense military use of cryptography. Also in this year, Major Joseph O. Mauborgne publishes the first recorded solution of the Playfair cipher. The technique encrypts pairs of letters, rather than single letters as in the simple substitution cipher and fairly more complex Vigenère cipher systems then in use. A year later, Germany changes its cipher method to complicated substitution ciphers, using twenty-four possible encryption alphabets, or combinations of them. These ciphers become progressively more complicated. In 1917, Gilbert S Vernam invented a practical polyalphabetic cipher machine which uses a key and is totally random and non-repeating, often called a one-time pad. This is the only provably secure cipher. But it is impractical under most circumstances since all parties must have a long and equal key, presenting a logistical nightmare for everyday use.

By World War II, mechanical and electromechanical technology was in wide use, coming together with the needs of telegraphy and radio to bring about a revolution in cryptodevices – the rotor cipher machine. It is an electro-mechanical stream cipher device used for encrypting and decrypting secret messages. Rotor machines were the cryptographic state-of-the-art and prevailed almost 50 years (1920s to 1970s). The most commonly discussed is the German Enigma machine, which was invented by German engineer Arthur Scherbius at the end of World War I [Sin1999]. The early models were used commercially from the early 1920s, and adopted by military and government services of several countries – most notably by Nazi Germany before and during World War II [Lor2011].

In 1930 the Japanese's first rotor machine RED was put into practice by the Japanese Foreign Office. However, benefit from the experience of cryptanalysing the ciphers produced by the Hebern rotor machines, the U.S. Army Signal Intelligence Service team of cryptanalysts successfully cryptanalysed the RED ciphers. The Japanese government therefore set out to develop the best cryptomachines possible. In 1939 the Japanese introduced a new cipher machine named PURPLE which uses telephone stepping switches rather than rotors. The United States introduced SIGABA cipher machine during World War II. The machine was similar to the Enigma in basic theory. However, unlike Enigma's three rotors, the SIGABA included fifteen, and did not use a reflecting rotor [SP1999 and SC2007].

- **Modern Cryptography**

After World War II cryptology has become far more mathematical. Owe it to the wide availability of computers and the Internet as communications medium, cryptography has brought effectively into common use by anyone other than national governments or large enterprises.

The era of modern cryptography actually begins with Claude Shannon, the father of mathematical cryptography. His treatise on communication theory of cryptosystems and a book on the mathematical theory of communication, in addition to the other works on information and communication theory, established a solid basis for cryptology.

The mid-1970s saw two major public advances. One is DES (Data Encryption Standard), a previously predominant algorithm for the encryption of electronic data, it greatly influences the advancement of modern cryptography [Tuc1997]. The other is the publication of the paper *New Directions in Cryptography* by Diffie and Hellman [DH1976]. It introduced a whole new method of distributing cryptographic keys, thus it dramatically changed the way cryptosystems might work.

In 1982, the Nobel prize-winner Richard Feynman, a physicist, thought up the idea of a “quantum computer” which uses the effects of quantum mechanics to its advantage [Deu1985]. For some time, the notion of a quantum computer was mainly a theoretical interest only, but recent developments have bought the idea to scientists’ attention. In fact a quantum computer capable of performing Shor’s algorithm [Sho1997 and BBD2009] would be fast enough to break most of the current cryptosystems in a matter of seconds. With the motivation provided by this algorithm, the topic of quantum computing has gathered momentum and researchers around the world are racing to be the first to create a practical quantum computer.

Section 2.3.1.2 Cryptanalysis

Cryptanalysis is the art of cracking cryptographic security systems, and gaining the contents of encrypted messages without the cryptographic keys. Successful cryptanalysis has undeniably impacted on the history. Sir Harry Hinsley, official historian of British Intelligence in World War II, made a brief comment on Ultra (a designation for breaking high-level encrypted enemy communications), saying that it shortened the war “by not less than two years and probably by four years” [Hin1993].

In cryptography, Kerckhoffs’s principle is generally accepted: A cryptosystem shall be secure enough that everything about the system, except the key, is public knowledge [Ker1883]. Based on the principle, the attack models for cryptography can be divided into four classes:

- Ciphertext only attack: The attacker possesses a string of ciphertext
- Known-plaintext attack: The attacker possesses a string of plaintext and the corresponding ciphertext
- Chosen-plaintext attack: The attacker has obtained temporary access to the encryption machinery. Hence he can choose arbitrary plaintexts to be encrypted and obtain the corresponding ciphertexts.
- Chosen-ciphertext attack: The attacker has obtained temporary access to the decryption machinery. Hence he can gather information, at least in part, by choosing a ciphertext and obtaining the corresponding plaintext under an unknown key.

Section 2.3.2 Symmetric Cryptography vs. Asymmetric Cryptography

Basic techniques of encrypting messages include the symmetric and asymmetric cryptography. The former is the older technique and the latter was developed to make up for its deficiencies. However, both techniques have advantages and disadvantages which are outlined in this section.

Section 2.3.2.1 Symmetric/Secret Key Cryptography

Symmetric key encryption is also known as secret-key encryption. In this type of message encryption, both sender and recipient share the same cryptographic key for both encryption and decryption of messages. Symmetric cryptography can be divided into stream ciphers (encrypt the bits of a message one at a time) and block ciphers (take a number of bits and encrypt them as a single unit) [DK2007]. An example of symmetric key encryption system is AES.

Advantages

- Simple: Algorithms based on this method are easy to carry out. All users need to do is specify and share the secret key before they communicate.
- Fast: symmetric encryption is able to use shorter keys. Thus it is normally 100 times faster than asymmetric encryption, and even 1,000 to 10,000 times faster when using special encryption hardware.

- More secure when using the same key lengths: to achieve the same given level of security, asymmetric encryption must use longer keys, which creates longer messages and slows down the encryption process.

Disadvantages

- Too many keys: A new shared key has to be issued for communication with each different party. Thus it creates a problem with managing and ensuring the security of all the keys.

- Authentication of message cannot be guaranteed: Since both sender and recipient use the same key, messages cannot be verified whether they are from a particular user. This may be a problem if more than one pairs of parties use the same key.

- Secure channel is needed for key exchange: Sharing the secret key in the beginning creates a problem. It has to be exchanged in a way that ensures it remains secret. Because when the private key is sent there is always a risk of expose to unauthorized parties.

Section 2.3.2.2 Asymmetric/Public Key Cryptography

Symmetric cryptography requires too many session keys. For this reason, it is difficult for symmetric cryptosystems in wide use. In addition, the distribution of keys remains the biggest challenge in the use of cryptosystems of this kind [DH1976].

Asymmetric (also called public-key) cryptography is noticed as the most significant new development in cryptography in the last 30-40years [Jor2004]. It is a technique employed by many cryptographic algorithms and cryptosystems. Its feature lies in the asymmetric key algorithm, where the key used to encrypt a plaintext is different from the key used to decrypt it. Each user holds a pair of cryptographic keys – public keys and private keys [Ano1998 and MVO1996]. The public keys can be distributed whilst the private key is kept secret. Plaintexts are encrypted with the recipient’s public key and can be uniquely decrypted with the private key held by the recipient. An example of asymmetric key encryption system is RSA.

Advantages

- Simpler key management: This technique solves the problem of key distributing. Everyone publishes their public keys and private keys are kept secret. Besides, if the key is lost or there is an error, the recipient can request the key again without a secure channel.

- Trusted identification: Asymmetric cryptography allows the use of digital signatures which enables the recipient of a message to verify that the message is truly from a particular sender.

- Scalable communications: Cryptographic keys are not necessarily given before communication, and a secure channel is not necessarily needed. Instead, a public key is available to anyone anytime. The parties can communicate without risking the security of anyone else.

Disadvantages

- Slow: Due to the asymmetric cryptography’s long key lengths and the complexity of the encryption algorithms, asymmetric cryptography is slow compared to symmetric cryptography. Not practicable for use in decrypting bulk messages.

- Public keys must be authenticated: No one can be entirely sure that a public key belongs to the person it specifies, thus everyone must verify that their public keys belong to them.

- Common security compromise is feasible: If an attacker determines a person’s private key, his or her entire messages can be read.

It is hard to say which one is used more extensively based on the discussion above. As a result, in practical applications we can make use of the advantages of both these two algorithms, using symmetric algorithm to encrypt files and asymmetric algorithm to encrypt the keys of the encrypted document key.

Section 2.3.3 Multivariate Quadratic Polynomials

Ever since 1994, Peter Shor, the American professor of applied mathematics at MIT proved that quantum computers could calculate logarithm and that their speed far exceeds the existent ones, public key cryptography has undergone much evolution.

It is claimed that if a quantum computer in the range of 1000 bits was built, RSA, DSA, Elliptic curves, hyperelliptic curves class groups etc., could not resist cryptanalysis [Chr2005]. Because a working quantum computer would be capable of millions of calculations at once, therefore, it is able to crack any computer code on Earth [Wau2012]. But fortunately, there are a few more public key cryptosystems, such as hash-based, code-based, lattice-based and *MQ*-systems [Chr2005]. The last one, i.e., *MQ*-systems, have been a research hotspot of the new generation of public key cryptography in recent years.

The *MQ*-system was introduced by Matsumoto and Imai, who published their paper in 1988 [MI1988]. An *MQ*-system which is in *n* variables defined over a finite field $\mathbb{F}^q$ has polynomial vectors $P(x)$ of degree 2 of the form (Figure 2.1) [Wol2005] with the coefficients γ_{ijk}, β_{ij} and $\alpha_i \in \mathbb{F}^q$[ACOH2007]. These coefficients can be called quadratic (γ_{ijk}), linear (β_{ij}), and constant (α_i), respectively. By convention, the polynomial vector is written as $P(x) = (p_1(x),\ldots,p_m(x))$ and moreover $P(x) \in MQ(\mathbb{F}^n, \mathbb{F}^m)$. Christopher states in his paper [Wol2005] that, these vectors play an important role for the construction of public key schemes based on the *MQ*-problems over finite fields.

$$
\begin{array}{rcl}
p_1(x_1,\ldots,x_n) & := & \displaystyle\sum_{1\le j\le k\le n} \gamma_{1,j,k} x_j x_k + \sum_{j=1}^{n} \beta_{1,j} x_j + \alpha_1 \\
 & \vdots & \\
p_i(x_1,\ldots,x_n) & := & \displaystyle\sum_{1\le j\le k\le n} \gamma_{i,j,k} x_j x_k + \sum_{j=1}^{n} \beta_{i,j} x_j + \alpha_i \\
 & \vdots & \\
p_m(x_1,\ldots,x_n) & := & \displaystyle\sum_{1\le j\le k\le n} \gamma_{m,j,k} x_j x_k + \sum_{j=1}^{n} \beta_{m,j} x_j + \alpha_m
\end{array}
$$

Figure 2.1 The Form of Polynomials when the Degree Is 2

Some efforts have been made to test *MQ*'s security since 1980s. Thus there are a few famous schemes, which can be classified into five basic categories in [Pat1998, KPG1999, BWP2005, CAB2006, HBH2006, BCD2008, DSW2008 and DW2008]:

- Unbalanced Oil and Vinegar scheme (UOV): Let $\mathbb{F}$ be a finite field and $n,m \in \mathbb{N}$ with $m<n$ and coefficients $\alpha'_i, \beta'_{i,j}, \gamma'_{i,j,k} \in \mathbb{F}$. We say that the polynomials below are central equations in UOV-shape:

$$p_i(x'_1, \dots, x'_n) = \sum_{j=1}^{n-m} \sum_{k=1}^{n} \gamma'_{i,j,k}\, x'_j x'_k + \sum_{k=1}^{n} \beta'_{i,j} x'_j + \alpha'_i$$

The UOV scheme can only be used for signature schemes as the "vinegar" variables v=x'_i $(1 \le i \le n-m)$ and the "oil" variables o=x'_i $(n-m \le i \le n)$ satisfy $v \geq 2o$, then it is suitable for a secure construction [Wol2005].

- Stepwise Triangular Systems (STS): These systems are also defined over $\mathbb{F}$ and use a special structure for the central mapping [Wol2005]:

$$step\ 1 \begin{cases} p'_1(x'_1, \dots, x'_r) \\ \vdots \\ p'_r(x'_1, \dots, x'_r) \end{cases}$$

$$\vdots$$

$$step\ l \begin{cases} p'_{(l-1)r+1}\left(x'_1, \dots, x'_r, \dots, x'_{(l-1)r+1}, \dots, x'_{lr}\right) \\ \vdots \\ p'_{lr}\ (x'_1, \dots, x'_r \dots, x'_{(l-1)r+1}, \dots, x'_{lr}) \end{cases}$$

$$\vdots$$

$$step\ L \begin{cases} p'_{(L-1)r+1}\left(x'_1, \dots, x'_r, \dots, x'_{(l-1)r+1}, \dots, x'_{lr}, \dots, x'_{n-r+1}, \dots, x_n\ \right) \\ \vdots \\ p'_{Lr}\ (x'_1, \dots, x'_r \dots, x'_{(l-1)r+1}, \dots, x'_{lr} \dots, x'_{n-r+1}, \dots, x_n\ ) \end{cases}$$

- Matsumoto-Imai Scheme A (MIA): It uses two different finite fields: a ground field $\mathbb{F}$ and an extension field $\mathbb{E}$. Let $\mathbb{F}$ have q elements, $\phi: \mathbb{E} \longrightarrow \mathbb{F}^n$ the canonical bijection between the extension field $\mathbb{E}$ and the corresponding vector space $\mathbb{F}^n$. Additionally, let $\lambda \in \mathbb{N}$ be an integer such that the greatest common divisor between $q^n - 1$ and $q^\lambda + 1$. Then the following central equation is of MIA-shape:

$$P'(X') = (X')^{q^\lambda+1}, \text{ here } X' \in \mathbb{E}$$

MIA is insecure due to a very efficient attack by Patarin [Pat1995].

- Hidden Field Equations (HFE): After breaking MIA, Patarin generalised the under lying trapdoor to HFE, which aims at the central equations and uses a univariate polynomial instead of a univariate monomial [Wol2005]. We can see the following equation in HFE is from the basic idea of MIA:

$$P'(X') = \sum_{\substack{0\le i,j\le d \\ q^i+q^j\le d}} C'_{i,j} X'^{q^i+q^j} + \sum_{\substack{0\le k\le d \\ q^k\le d}} B'_k X'^{q^k} + A', \text{ here } C'_{i,j},\ B'_k,\ A' \in \mathbb{E}$$

The basic HFE scheme is broken by the MinRank-problem which is demonstrated in [KS1999, WP2005a and WP2005b]. It shows that HFE allow many equivalent keys and therefore, waste memory.

- ℓ-Invertible Cycles (ℓIC): Denote the extension field $\mathbb{E}$=$GF(q^k)$, Q = $|\mathbb{E}|$=q^k and m=n=ℓk for the number of variables and equations over $\mathbb{F}$, respectively. the irreducible polynomial used in the computations in $\mathbb{E}$ is $\pi(t) \in \mathbb{F}(t)$. Let $S,T \in Aff^1(\mathbb{F}^n)$ be two invertible affine mappings and the vector V=$(\lambda_1, \ldots, \lambda_\ell) \in \{0, \ldots, k\text{-}1\}^\ell$. Then we have the following mapping:

$$P\text{=}\ \mathbb{E}^\ell \rightarrow\ \mathbb{E}^\ell: (A_1, \ldots, A_\ell) \rightarrow (A_1^{q^{\lambda_1}} A_2, \ldots, A_{\ell-1}^{q^{\lambda_{\ell-1}}} A_\ell,\ A_\ell^{q^{\lambda_\ell}} A_1)$$

Identifying the corresponding coefficients in $\mathbb{F}^n$ and $\mathbb{E}^\ell$, we get a canonical bijection

$$\phi: \mathbb{F}^n \to \mathbb{E}^{\ell}: (x_1, \dots, x_n) \to (x_1' + x_2' t + \cdots + x_k' t^{k-1}, \dots, x_{n-k+1}' + x_{n-k+2}' t + x_n' t^{k-1})$$

Denote the inverse of ϕ as ϕ^{-1}, and we have the polynomials of ℓIC-shape:

$$P(X)\text{:}\ \mathbb{F}^n \to \mathbb{F}^m: T \circ \phi^{-1} \circ P \circ \phi \circ S.$$

These schemes have the same structure, that is, one or more invertible central maps are between two affine maps.

$$\text{Input } \rho\text{: } x \in \mathbb{F}^n \xrightarrow{S \in \mathbb{F}^n} u \xrightarrow{P(x) \in MQ(\mathbb{F}^n, \mathbb{F}^m)} v \xrightarrow{T \in \mathbb{F}^m} \text{ output: } y \in \mathbb{F}^m$$

Where $P(x)$ is what we discussed above. It is an invertible central map to allow the decryption or signing of messages. $S(x) = S_L x + S_C$ *and* $T(x) = T_L x + T_C$ represent two affine transformations. ($\mathbb{F}^n$, ρ=[$\rho_1(x)$, …,$\rho_m(x)$]) are public keys. (S, P, T) is a triple to compose the secrete keys of an *MQ*-system. In addition, the key-size of these schemes can be computed using the following formula [CB2005]:

$$\tau(n) = \begin{cases} 1 + n + \dfrac{n(n-1)}{2} = 1 + \dfrac{n(n+1)}{2}, & \text{if } q = 2 \\ 1 + n + \dfrac{n(n+1)}{2} = 1 + \dfrac{n(n+3)}{2}, & \text{otherwise} \end{cases}$$

The differences between these schemes are mainly in the construction of the central map $P(x)$. The difficulty of attacking the schemes relies on how to solve *MQ*-systems $y_1=\rho_1(x)$, …, $y_m=\rho_m(x)$ and to revert ρ into three parts: $\rho = S \circ P \circ T$. Solving *MQ*-systems has been proven in general to be NP-hard, while the problem of reverting ρ into three parts, which is what-so-called Extended Isomorphism of Polynomials (EIP) problem, is not NP-hard, so it becomes the focus of attacks. Most of the *MQ* cryptosystems are cracked because of the EIP problem.

Apart from UOV schemes with proper parameter values, the basic types of these schemes which are based on *MQ* problems are considered to be insecure in [PGC1998, KS1999, GC2000, FJ2003, Nic2004, FGS2005, DSY2006, BG2006, DGS2007, DSW2008 and FLP2008]. The reason is mainly that the scheme design of public key cryptography must not only be able to resist various attacks, but also make $P^{-1}(x)$ easy to calculate, moreover, the size of public keys $\rho_1(x)$, …,$\rho_m(x)$ cannot be too large. However, it is difficult to take all of these requirements into account when one tries to design an asymmetric cryptography scheme. Therefore, the basic types of the systems are easy to be attacked. As a result, revised *MQ*-based schemes have been proposed, including HFEv-, MIAi+, UOV/, STS (UOV), (ICi+), etc, as in [CB2002, DGS+2005, DG2005 and DS2006].

Section 2.4 Information Hiding

Whereas cryptography protects the contents of a message, information hiding can be said to protect both messages and communicating parties.

In computer science, Information hiding is another main method to hide messages in such a way that no one, apart from the sender and intended recipient, suspects the existence of the message. It is a technology that uses anonymity, watermarking, covert channel and steganography.

Until the early nineties, information hiding techniques had not received very much attention from the research community and from industry than cryptography, but this changed fast. The first academic conference on the subject was held in Cambridge University in 1996. Since then, many other conferences focused on information hiding as well as watermarking. The latest international workshop on information hiding was held in May 2012. New conferences and journals on the subject have continued to flourish.

Section 2.4.1 History of Information Hiding

The idea of “secret communicating” is as old as communication itself. In this section, we give a brief review of the historical development of information hiding techniques from its early beginning to nowadays techniques such as digital steganography, watermarking, cover channel, anonymity, etc.

While information hiding techniques have received a remarkable attention recently, its early application was messy. Before some communication tools such as horses, mail and phones, messages were sent on foot. If one wanted to hide a message, he had to have the messenger memorize it, or hide it on the messenger. The first recorded uses of steganography can be traced back to 440 BC when the Greek historian Herodotus mentions two examples of steganography. One is the famous Greek tyrant Histiaeus, while in prison, sent a message to his son-in-law by shaving the head of a slave to tattoo a message on his scalp. Histiaeus then waited until the hair grew back on slave's head prior to sending him off to his son-in-law. See Figure 2.2 [Har2011].

Figure 2.2 Image of Herodotus (Left) and Hiding Information on the Scalp (Right)

Another example is that of a soldier named Demaratus. He sent a warning about a forthcoming invasion into Greece by writing on a wax-covered tablet. He first removed the wax from the tablet, wrote the secret message on the underlying wood, then recovered the tablet with wax to make it look like a blank tablet and finally sent the document without being perceived.

Invisible ink is also a popular method of steganography. It is applied to a writing surface with a specialty purpose stylus, toothpick, fountain pen, or even a finger dipped in the liquid. Once dry, the written surface will appear blank as there is nothing special on the surface [Kat1999]. Ancient Romans used to write using invisible inks based on readily available materials such as fruit juices, urine and milk. When heated, the invisible inks would grow dark, and become readable. Later chemically affected sympathetic inks were developed. Modern invisible inks fluoresce under ultraviolet light and are used as anti-counterfeit devices. For instance, the word "VOID" is printed on checks and other official documents in an ink that appears under the strong ultraviolet light.

The monk Johannes Trithemius, considered one of the founders of modern cryptography, had ingenuity in spades. His most famous work is *Steganographia,* a book of three volumes, written in 1499 [Ree1998]. This book describes an extensive system for concealing secret messages within innocuous texts. On its surface, the book seems to be a magical text, and the initial reaction in the 16th century was so strong that *Steganographia* was only circulated privately until publication in 1606. But in 1998, Jim Reeds of AT&T Labs deciphered mysterious codes in volume III, showing that Trithemius's work contains hidden cipher messages within what is seemingly a magic work.

The earliest actual book on steganography was called *Steganographia* written by Gaspari Schott in 1665. Although most of the ideas came from Trithemius, it was a start.

Further development in the field occurred in the 19th century, Auguste Kerchoffs published two journal articles on *La Cryptographie Militaire* [Aug1883 and Fab2010], in which he stated six design principles for military ciphers. Although this work was mainly about cryptography, it describes some principles that are worth keeping in mind when designing a new steganographic system.

But it was during the twentieth century that steganography truly flowered [Tha2008]. An example of this comes from early in the century, during the Boer War. The British Lord Robert Baden-Powell was employed to mark the positions of Boer artillery bases, to ensure he was not suspected by the Boers. If he was caught, he would mark his maps into drawings of butterflies. Look as if innocent to a casual observer, certain marks on the wings were in fact the positions of the enemy military installations.

During World War II, null ciphers (plaintext) were used to hide secret messages. The null cipher appeared to be innocent message about ordinary occurrences, thus it would not arouse suspicion, and would not be intercepted. For example, the following message was sent by German spy during World War II [Tha2008].

> *Apparently neutral's protest is thoroughly discounted and ignored. Isman hard hit. Blockade issue affects pretext for embargo on by-products, ejecting suets and vegetable oils.*

The message seems to be innocent, but if we take the second letter in each word it will reveal the following secret message.

Pershing sails from NY June 1.

With the advent of photography, microfilm was created as a way to store a large amount of information in a tiny space. In the two world wars, the Germans used "microdots" to hide information (see Figure 2.3), a technique which J. Edgar Hoover called "the enemy's masterpiece of espionage." Microdots were typically small, approximately less than the size of the period produced by a typewriter. A secret message was photographed, reduced to microdots, and then embedded in an innocuous cover message, magazine, or newspaper. This was reflective and thus detectable by viewing against glancing light. Alternative techniques included storing microdots in slits cut into the edge of post cards.

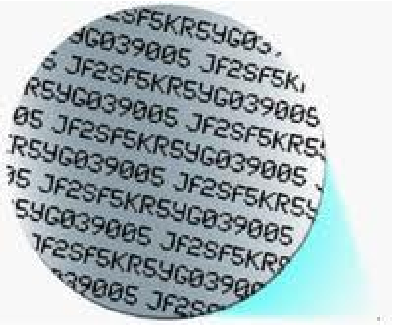


Figure 2.3 Enlarge View of a Microdot

Linguistic steganography (Figure 2.4 [CC2012]), a whole other branch of steganography, is a set of methods and techniques that permit the hiding of any digital information within texts based on some linguistic knowledge. To hide the very fact of hiding, the resulting text shall not only remain inconspicuous, i.e., appear to be ordinary text, with fonts, orthography, lexicon, morphology, syntax, and word order outwardly corresponding to its meaning, but also conserve grammatical correctness and semantic cohesion. In World War I, for example, German spies used fake orders for cigars to represent different types of British warships – cruisers and destroyers. Thus 500 cigars needed in Portsmouth meant that five cruisers were in Portsmouth.

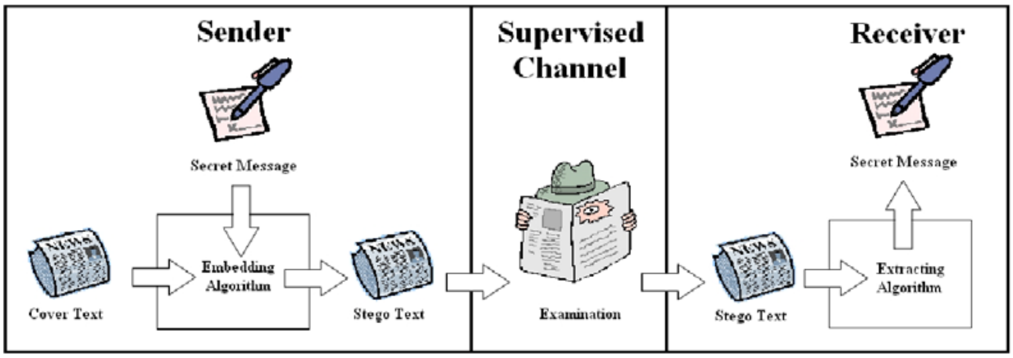


Figure 2.4 Linguistic Steganography Framework

Digital watermarking is the process of embedding information into a digital signal when the users want to verify the authenticity or the identity of the owners. This concept will be further discussed in Section 2.4.2.

A covert channel is a branch of security concerned with hiding information in some cover medium. The term, originated in 1973 by Lampson is defined as "(channels) not intended for information transfer at all, such as the service program's effect on system load" [Lam1973]. It is applied especially in operating systems and networks. The channel is typically used by untrustworthy programs to leak information to their owner while performing service for another program.

Anonymity is finding ways to hide identity information of a person. Anonymity is needed when making online voting or to hide access to some web pages, or to hide sender.

With the ever-growing expansion use of computer and network, information hiding has been given a marvelous boost. We are sure to see a great expansion of steganographic techniques in the coming years.

Section 2.4.2 Steganography vs. Watermarking

Steganography and watermarking bring a variety of techniques on how to hide important information, in an imperceptible and/or irremovable way, in multimedia such as audio, video, and pictures. They are main parts of the fast developing area of information hiding. Technically differences between steganography and watermarking are both subtle and essential. Both techniques belong to the category of information hiding, but the objectives and insertions for these techniques are just opposite.

(1) Importance of carrier data and embedded data

In steganography, the carrier data is not important. It mostly works as an alteration from the most important information in embedded data. While in watermarking, the important information is in the carrier data. The embedded data is placed to protect the carrier data.

(2) Capacity of embedded data

Steganography tools usually hide quite a large amount of information whereas watermarking tools embed less information in an image or sounds.

(3) Different main goals

The main goal of steganography is to hide a message *M* in some audio or video carrier *C*, to gain *C*', practically indistinguishable from *C*, such that an eavesdropper cannot **perceive or detect** the presence of *M* in *C*'. Steganography methods usually do not need to provide strong security against removing or modification of the hidden message.

The main goal of watermarking is to hide *M* in some audio or video carrier *C*, to gain *C*', practically indistinguishable from *C*, such that an eavesdropper cannot **remove or replace** *M* in *C*'. Watermarking methods need to be very robust to attempts to remove or modify a hidden message.

Generally speaking, steganography hides a message in one-to-one communications, while watermarking hides a message in one-to-many communications.

(4) Different applications:

Steganography is applied in secure secret communications where cryptographic encryption methods are not available, or where strong cryptography is impossible. It can also be applied in other cases, such as in military applications, when the communication of two parties is of great importance; or in the health care, especially medical imaging systems.

Watermarking is mainly applied to provide proof of ownership of digital data such as video, music, films, software and images, by embedding copyright statements into digital products. More specific, it can be applied in but not limited in:

- Automatic monitoring and tracking of copy-write material on WEB: for example, a robot searches the Web for marked material and thereby identifies potential illegal issues.

- Copy control: to prevent and control copy. For example, in a closed system where the multimedia data needs special hardware for copying and/or viewing, a digital watermark shall be embedded to indicate how much copies are permitted. Whenever a copy is made the watermark can be modified by the hardware and if the number of the copies is more than the permitted number the hardware would stop working.

- Fingerprinting: when the multimedia content of an owner would like to duplicate and distribute unauthorized, by embedding a distinct fingerprint in each copy of the data. If one day, unauthorized copies of the data are found, then the origin of the copy can be determined by retrieving the fingerprint [SK2012].

- ID card security: information in a passport or ID can be included in the owner’s photo that appears on the ID. By extracting the inserted information and comparing it to the text provided by the card holder, the ID card can be verified.

Section 2.4.3 Data Hiding Technologies in Images

Almost any text, audio, image and any other media that can be encoded into a bit stream can be hidden in a digital image. Straight message embedding can be done by simply encoding each bit of pixel in the image. More complex encoding can be done by embedding the message only in complex (or “noisy”) areas of the image that will not attract much attention. The message may also be distributed randomly all over the cover image. There are a lot of good algorithms which can be divided into spatial domain and transform domain algorithm [Kat2012].

Section 2.4.3.1 Spatial Domain Algorithms

Spatial domain watermarking algorithms embed watermark directly in pixels of carrier image by modifying the lower order bits of the pixels with that of the watermark (to represent an invisible watermark) or adding some fixed intensity value to the pixel values of the picture (to represent a visual watermark) [KKB2006]. The main advantage of the spatial domain algorithms is the low calculation complexity when compared to any techniques requiring domain transforms. This type of watermarking assures a high invisibility but shows low robustness against several attacks.

- **Least significant bit (LSB) algorithm**

This is probably the most famous image steganography algorithm. It embeds information with the least significant bits, which are selected randomly, in a graphical image file. It can ensure the embedded watermark is invisible. But the algorithm has poor robustness, and it is extremely vulnerable to attacks, such as filtering, image quantization, lossy compression, and geometric distortion [Kat2012].

Hiding information in different bits of an image will cause different effects. Take the 256-by-256 Lena 256-bit gray image (see Figure 2.5, the left image) for example, then we hide the 256-by-256 Greenwich 2-bit logo (see Figure 2.5, the right image) in one bit of Lena image each time, the results are shown in Figure 2.6.

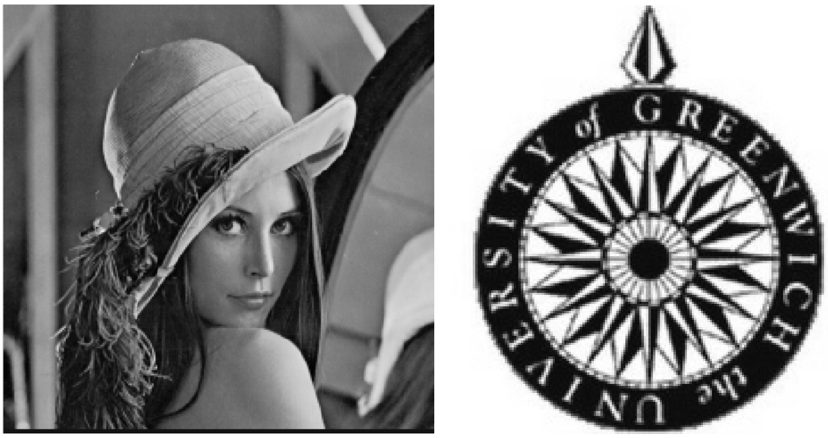
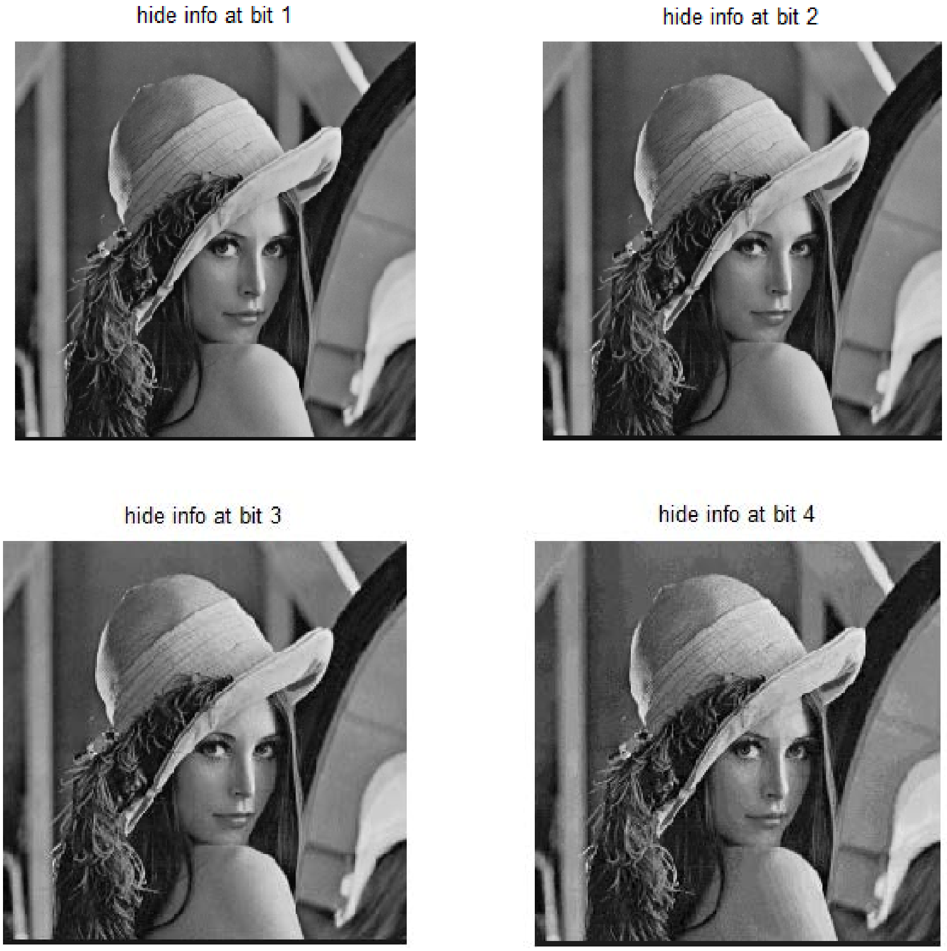


Figure 2.5 Carrier image (Left) and the Watermark (Right)



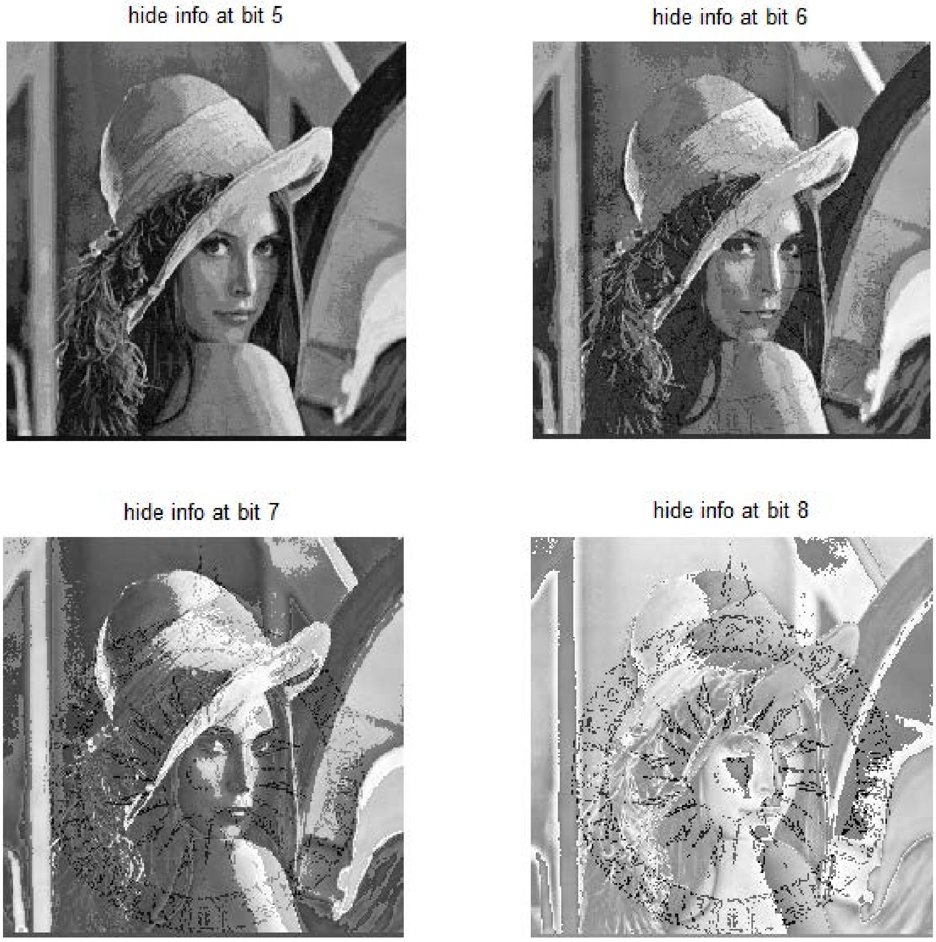


Figure 2.6 Logo Hidden in Different Bits of Lena Image

We can see from Figure 2.6 that hiding watermark in the three least significant bits of a carrier image will not arouse suspicion.

- **Patchwork algorithm**

This algorithm uses the statistical features of pixels to insert the information into the brightness values of pixel. It can resist lossy compression coding and malicious attacks [YK2003]. It chooses random pairs of pixel from the carrier image and forms two sets. Then it modifies the pixel values of each set in various ways. For example, arbitrarily chooses two sets A={a_i} and B={b_i}, where A and B have the same quantity of the elements. A constant *d* is added to all pixel values of A, while *d* is subtracted from B values as long as the patchwork algorithm is additive. However, the amount of embedded information is limited. Most of patchwork algorithms only hide 1 bit of the information, in order to embed more watermark information; we can segment the image, and then implement the embedding operation each image block.

- **Texture mapping coding algorithm**

This algorithm hides the watermark in the texture part of the original image. The algorithm are strongly resistant to attacks for a variety of deformation, but only suitable for areas with a large number of arbitrary texture images, and cannot be done automatically [Kat2012].

Section 2.4.3.2 Transform Domain Algorithms

Spatial domain is sensitive towards several attacks and that is why now-a-days most of the researchers are applying different transform domains for their watermarking algorithms. Transform domain algorithm is a method of hiding data in the transform domain of image and is robust to attacks such as lossy compression, de-noising, sharpening, etc. There are some common used transform domain methods, such as discrete cosine transform (DCT), discrete Fourier transform (DFT), discrete wavelet transform (DWT) and others. In this section, we only introduce three frequently used transforms, which are:

- **Discrete Fourier transform (DFT)**

DFT is a specific kind of discrete transform, used in Fourier analysis. It transforms one function into another, which is called the frequency domain representation of the original function. The DFT approach has some advantages when compared to the spatial domain methods. First, it is translation invariant and rotation resistant, which translates to strong robustness to geometric attacks [PMA2011]. On the other hand, according to Raja et al. in [RCVP2005], fast Fourier transform (FFT) methods introduce rounding errors, which can lead to loss of quality and errors in watermark extraction. So this technique is not suitable for hidden communication.

- **Discrete cosine transform (DCT)**

DCT expresses a sequence of finite data points in terms of a sum of cosine functions dithering at different frequencies [Hab2012]. DCTs are important to numerous applications in science and engineering, from lossy compression of images and audio where small high-frequency components can be castoff, to spectral methods for the numerical solution of partial differential equations. The application of cosine instead of sine functions mainly lies in: for compression, cosine functions are much more efficient, fewer functions are needed to estimate a typical signal.

In particular, a DCT is a Fourier-related transform similar to DFT, but it uses only real numbers. DCTs are equivalent to DFTs of roughly twice the length, operating on real data with even symmetry (while DFT of a real and even function is real and even).

The DCT approach is very robust to JPEG compression, as JPEG compression itself uses DCT. However, DCT methods are fragile to strong geometric distortions.

- **Discrete wavelet transform (DWT)**

Fourier transform is not a good tool. It gives no direct information about when an oscillation occurred. Short-time Fourier transform has equal time interval, thus high-frequency bursts occur are hard to detect.

Wavelets can keep track of time and frequency information. They can be used to “zoom in” on the short bursts, or to “zoom out” to detect long, slow oscillations [AP2003].

DWT transforms a discrete time signal to a discrete wavelet representation. Functionally, it is very much like the DFT, in that the transforming function is orthogonal, a signal passed twice through the transformation is unchanged, and the input signal is assumed to be a set of discrete-time samples. A key advantage it has over DFT is its temporal resolution: it captures both frequency and location.

Wavelet transform decomposes a signal into a set of basis functions. These basis functions are called wavelets. Wavelets are obtained from a single prototype wavelet $y(t)$ called mother wavelet by dilations and shifting:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi(\frac{t-b}{a})$$

where *a* is the scaling parameter and *b* is the shifting parameter.

Haar is the first DWT which was formulated by the Hungarian mathematician Alfréd Haar [Haa1910]. In mathematics, the Haar wavelet is a sequence of rescaled “square-shaped” functions which together form a wavelet basis. It is also the simplest of the wavelet transforms. Its wavelet analysis is similar to Fourier analysis in that it allows a target function over an interval to be represented in terms of an orthonormal function basis. The technical disadvantage of the Haar wavelet is that it is not continuous, and thus not differentiable. However, this property can be an advantage for it to analyse signals with sudden transitions [LT1999].

Daubechies wavelet is the most commonly used set of discrete wavelet transforms. It was invented by the Belgian mathematician Ingrid Daubechies in 1988. This wavelet is based on the use of recurrence relations to generate progressively finer discrete samplings of an implicit wavelet filter function; each resolution is twice that of the previous scale. The Haar wavelet is a special case of Daubechies wavelet. Therefore, interest in this field has exploded since then, and many variations of Daubechies' original wavelets were developed [HA1992].

The Dual-Tree Complex Wavelet Transform (ℂWT) is relatively recent enhancement to DWT. It first introduced by Kingsbury in 1998 [Kin1998]. Like the idea of positive/negative post-filtering of real subband signals, the idea behind the dual-tree approach is quite simple. The dual-tree ℂWT employs two real DWTs; the first DWT gives the real part of the transform while the second DWT gives the imaginary part [SBK2005]. The ℂWT benefits from the vast computational, practical and theoretical resources that have been developed for the standard DWT. For example, software and hardware developed for implementation of the real DWT can be used directly for the ℂWT.

- **Singular Value Decomposition (SVD)**

SVD is an effective numerical analysis tool used to analyse matrices. In SVD transformation, a matrix can be factorized into three matrices. From the view point of linear algebra, an image is an array of non-negative scalar entries that can be regarded as a matrix. Given an image matrix A is an $n \times n$ matrix with rank=r, $r \leq n$, then SVD of A is defined as

$$A = USV^{\mathrm{T}} = [u_1, u_2, \ldots, u_n] \begin{bmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma_n \end{bmatrix} [v_1, v_2, \ldots, v_n]^{\mathrm{T}}.$$

Where U and V are $n \times n$ real orthogonal matrices, and S is an $n \times n$ real diagonal matrix with singular value σ_i which satisfies [AP1976 and CSC2001]:

$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r \geq \sigma_{r+1} = \cdots = \sigma_n = 0$$

The SVD components also satisfy $UU^{\mathrm{T}} = I_{n\times n}$ and $VV^{\mathrm{T}} = I_{n\times n}$.

The reasons that SVD is applied in digital watermarking are, which we can see from the A's decomposition equation, that

(1) Singular values S all appear as positive data in a diagonal position, while all other off-diagonal elements are zeros. If we insert a watermarking value in an off-diagonal position, the embedded image is modified by an addition [OSG2009].

(2) The singular values are quite stable, i.e. the singular values will not change considerably if the image is added some noise.

(3) The singular values are unchangeable to geometric distortion such as mapping, rotation, zoom in/out, shifting.

In conclusion, SVD has so many good properties and it is robust to lots of attacks on digital watermarking, therefore, in the last few years several watermarking algorithms have been proposed based on this technique.

Section 2.5 Conclusions

In this chapter, we give a comprehensive review of representation of the three temporal systems. We also point out the fundamental time theories based on which time-series and sequences are formed up are usually not explicitly specified.

Then ergodic matrix is introduced. From the theorems and lemmas, any ergodic matrix over finite field $\mathbb{F}^q$ has a largest order and a largest generating set. Therefore, it is suitable in information cryptography.

Cryptography is the act or art of writing in secret characters, it has been undergoing for more than three decades. Multivariate quadratic polynomial schemes are a research hotspot of the new generation of public key cryptography. However, these schemes have been proven to be insecure except UOV with proper parameters.

Information hiding is another main method to hide messages in such a way that no one, apart from the sender and intended recipient, suspects the existence of the message. It is a technology that uses anonymity, watermarking, covert channel and steganography.

CHAPTER 3 *BMQE* SYSTEM

The advantages of the *MQ*-based public key cryptography schemes (MPKCs) are mainly reflected in their fast speed of encryption (or signature verification) and resistance of quantum attacks. As pointed in Section 2.3.3, at present, the existing schemes have been proven to be insecure except UOV Apart from UOV with well-chosen parameters, for instance, $UOV+LL'$ in [GP2006]. Unfortunately, UOV cannot be used to construct a secure encryption scheme [Wol2005]. Therefore, we need to design a new scheme to fix this problem.

Based on ergodic matrix in [YSZ2004, ZHJ2005 and PZZ2006], we propose a new *MQ* equations system, i.e., *BMQE* system, over finite fields, which will yield a NP-complete problem.

Section 3.1 Important Characteristics of *EM* for *BMQE* System

As mentioned in the introduction, there are so many fine features of ergodic matrix that makes it favorable in cryptography. But how many ergodic matrices over $\mathbb{F}^q$? Are there enough ergodic matrices to form up a large key space? Therefore, in this section, we shall discuss more features of *EM*.

Denote the number of the ergodic matrix over finite field $\mathbb{F}_{n\times n}^{q}$ as *Num_EM*(q, *n*). If the equivalent ergodic matrices are treated as the same, then denote the number of the non-equivalent ergodic matrices over $\mathbb{F}_{n\times \mathrm{n}}^{q}$ as *Num_DEM*(*q*, *n*).

Zhao stated in his paper [ZHJ2005 and ZMD+2010] that *Num_EM*(*q*, *n*) = *Num_DEM*(*q*,*n*)•φ($q^n - 1$). Table 3.1 is the statistical results of the count for *EM* over some finite fields.

Table 3.1 Statistical Results of the Count for *EM* over Some Finite Fields

$\mathbb{F}^q$	n	$Num\_EM(\mathbb{F}^q, n)$	$Num\_DEM(\mathbb{F}^q, n)$	$q^n - 1$	$\varphi(q^n - 1)$	$\prod_{i=0}^{n-1} q^n - q^i$
$\mathbb{F}^2$	2	2	1	3	2	6
	3	48	8	7	6	168
	4	2688	336	15	8	20160
	5	1935360	64512	31	30	9999360
$\mathbb{F}^3$	2	12	3	8	4	48
	3	1728	144	26	12	11232
	4	2426112	75816	80	32	24261120
$\mathbb{F}^5$	2	80	10	24	8	480
	3	240000	4000	124	60	1488000
$\mathbb{F}^7$	2	336	21	48	16	2016
	3	3556224	32928	342	108	33784128
$\mathbb{F}^{31}$	2	119040	465	960	256	892800

From this table we have the following conjectures [YSZ2004 and ZMD+2010]:

Conjecture 3.1.

$$Num\_DEM(q,n) = \frac{\prod_{i=0}^{n-1}(q^n - q^i)}{n(q^n - 1)} = \frac{(q^n - q)\cdots(q^n - q^{n-1})}{n}.$$

Conjecture 3.2.

$$Num\_EM(q,n) = \frac{(q^n - q)\cdots(q^n - q^{n-1})}{n}\cdot\varphi(q^n - 1).$$

With this number of ergodic matrices, it is not hard to design a public key cryptosystem with large key spaces.

Section 3.2 *BMQE* Based on Ergodic Matrix

After the analysis of ergodic matrix, we shall introduce a scheme called *BMQE* problem, which is actually NP-complete and different from all of the existing *MQ* problems.

Section 3.2.1 *BMQE* Problem

From the definition of *EM* (cf Definition 2.4 in Section 2.2.2), given Q_1,$Q_2 \in \mathbb{F}^q_{n\times n}$, we take any non-zero matrix in the spanning set of Q_1,Q_2 as an n^2-vector, and randomly pick two basis B$_1$=(Q$_1^{a_1}$, Q$_1^{a_2}$….,Q$_1^{a_n}$), B$_2$=(Q$_2^{b_1}$, Q$_2^{b_2}$….,Q$_2^{b_n}$) for Q_1, Q_2 over finite field $\mathbb{F}^q$, respectively. Then there exist exclusive tuples (x_1, x_2, …,x_n) and (y_1, y_2, …, y_n) ∈ $\mathbb{F}^q_n\backslash\{0\}$ such that:

$$Q_1^x = \sum_{i=1}^{n} x_i Q_1^{a_i}, Q_2^y = \sum_{j=1}^{n} y_j Q_2^{b_j}$$

Then we have:

$$T = Q_1^x m Q_2^y = \sum_{i=1}^{n} \sum_{j=1}^{n} (x_i y_j) Q_1^{a_i} m Q_2^{b_j}$$

Linearize the $n\times n$ matrix T and $Q_1^{a_i} m Q_2^{b_j}$ into n^2-vectors (e.g. $t_{i,j}$ ∈ T ↔ $t'_{(i-1)\times n+j,1} \in T'$). Hence there is a system of m equations in 2n variables over a finite field $\mathbb{F}^q$. The terms in these equations are 2 degrees, each consists of x and y. We call a system with this format *BMQE* system, based on which we propose our *BMQE* problem as below:

***BMQE* problem**: Given a multivariate system *MS* over any finite field $\mathbb{F}^q$ has m equations in 2n variables, and each equation has the format as follows:

$$\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}^{(k)} x_i y_j = b_k,$$

where $a_{ij}^{(k)}$, $b_k \in \mathbb{F}^q$ are known values, k = 1, 2, …, m.

How to deduce *MS*'s solution such that x, $y \in \mathbb{F}^q_n$?

It is obvious that the *BMQE* problem is a special case of multivariate quadric problems. The differences are that:

(1) *MS* is composed of x_i and y_j, where i=1, 2, …,n and j=1, 2, …, n;

(2) Each equation of *MS* only has terms with degree 2;

(3) Each term in each equation of *MS* is chosen from $\langle x\rangle$ and $\langle y\rangle$, where $\langle x\rangle=\{x_i \mid i=1, 2, \dots n\}$ and $\langle y\rangle=\{y_j \mid j=1, 2, \dots n\}$.

Therefore, the *BMQE* system in 2n variables has n^2 terms of degree 2, whilst *MQ* equations in 2n variables has $2n^2 + n$ terms of degree two and 2n terms of degree one.

Moreover, *MQ* equations over $\mathbb{F}^q$ may have exclusive non-solution solution if $q \leq$ 2. But for q>2, if *MS* has non-zero solution $(x_1,x_2,\dots x_n)$, $(y_1, y_2, \dots, y_n)$ $\in \mathbb{F}_n^q \wedge$(x, y)≠(0, 0), then for $\forall c \in \mathbb{F}^q \backslash \{0\}$, ($cx$, $c^{-1}y$) must also be a solution to *MS*.

Section 3.2.2 *BMQE* is NP-Complete

To prove whether some problem is NP-complete is first to prove that it is in NP, and then to reduce some known NP-hard problem to it [Coo1971].

MQ problem over $\mathbb{F}_n^q$ has been proven to be NP-complete. In last section we can see that *BMQE* is a special case of *MQ*, thus it is in NP. Next we will prove that the following well-known NP-hard problem is reducible to the *BMQE* problem:

3-colouring of a 4-regular graph: Given an undirected graph G = (V; E) of n vertices, each adjacent to 4 vertices, can the vertices of the graph be coloured in 3 colour, so that no two adjacent vertices have the same colour? [Gon1982]

Theorem 3.1: the *BMQE* problem over any finite field $\mathbb{F}^q$ is NP-complete.

Proof. To prove this theorem, we must clearly show that:

(a) Given an arbitrary instance of the initial problem define a specific instance of problem *BMQE* in polynomial time;

(b) The initial problem has a solution if and only if the constructed instance has a solution.

Therefore, we define an instance of *BMQE* problem over $\mathbb{F}^2$ with $2n$ equations:

For each edge (v_i, v_j), $1 \le i < j \le n$, define the equation over $\mathbb{F}^2$

$$x_i y_j + x_j y_i = 1 \qquad (3.1)$$

The decision version: does there exist a solution (a collection of values x_i, y_i for $1 \le i \le n$, which satisfies all $2n$ equations).

First we shall prove the constructed instance has a solution if and only if the 3-colouring problem has a solution:

Suppose that the colouring problem has a solution, and *a*, *b* and *c* denote the three colours.

Associate each vertex v_i of *G* with a pair (x_i, y_i) in the following way:

(1) If v_i is coloured in colour *a*, then $x_i = 0$ and $y_i = 1$;

(2) If v_i is coloured in colour *b*, then $x_i = 1$ and $y_i = 0$;

(3) If v_i is coloured in colour *c*, then $x_i = 1$ and $y_i = 1$.

We have that if vertex v_i is coloured in colour *a*, and vertex v_j is coloured in colour *b*, then the corresponding equation becomes $0 \bullet 0 + 1 \bullet 1 = 1$. Similarly, if vertex v_i is coloured in colour *a*, and vertex v_j is coloured in colour *c*, then the corresponding equation becomes $0 \bullet 1 + 1 \bullet 1 = 1$. Finally, if vertex v_i is coloured in colour *b*, and vertex v_j is coloured in colour *c*, then the corresponding equation becomes $1 \bullet 1 + 1 \bullet 0 = 1$. Thus, problem *BMQE* has a solution.

Now assume that pairs (x_i, y_i) form a solution of problem of *BMQE*:

Clearly, $(x_i, y_i) \neq (0, 0)$; Otherwise any equation of the form (3.1) that involves x_i and y_i does not hold. This means that in the solution, a pair (x_i, y_i) can be only of the form (0, 1), (1, 0) and (1, 1), which define the three colours for the colouring problem, as outlined above. Besides, for any equation of the form (1) we must have that $(x_i, y_i) \neq (x_j, y_j)$, since over $\mathbb{F}^2$ $x_i y_j + x_j y_i = 1 + 1 = 0$. This means that the adjacent vertices do not get the same colour. Thus, a solution to problem *BMQE* defines a solution to the 3-colouring problem.

Since the reduction of the 3-colouring problem to problem *BMQE* requires at most $O(n)$ time, and the former problem is NP-hard, it follows that the latter problem is NP-hard.

There is no need to look at extensions: it is clear that problem *BMQE* over $\mathbb{F}^q$ for $q>2$ is no easier than that over $\mathbb{F}^2$.

Therefore, Theorem 3.1 is proved.

The complexity status of problem *BMQE* over $\mathbb{F}^2$ with $2n$ variables and less than $2n$ equations remains open.

Section 3.3 Cryptanalysis of *BMQE*

The *BMQE* problem has been proven NP-complete, which means it is both NP problem and NP-hard problem. However, this does not guarantee all bisectional multivariate quadratic equations are difficult enough to be unsolvable by polynomial-time algorithms. By analysis, the robustness of the *BMQE* is actually determined by *q*, *n* and *m*, where *q* is the number of a given finite field $\mathbb{F}^q$, *n* and *m* are the number of variables and equations, respectively.

Section 3.3.1 Methods Used in Solving *MQ*-problems

As mentioned above, *BMQE* is a special case of *MQ* problems, which means the methods used in solving *MQ* equations can also be used in *BMQE*. So far, these methods mainly are as follows:

- Linearization [MJ1987]: the success of linearization depends on $m = n(n+1)/2$, such condition limits its application in the attacks of *MQ* equations.

- Relinearization [KS1999]: due to the high requirements for the number of equations of Linearization, Kipnis and Shamir proposed relinearization and used it to attack both the HFE scheme and the Dragon scheme. The basic idea is to rewrite a system of εm^2 homogeneous quadratic equations in the *m* variables $x_1,\ldots x_m$, as a new system of εm^2 linear equations in the $m^2/2$ new variables $y_{ij}=x_ix_j$, thus find the solution of y_{ij} by Gaussian elimination.

- Gröbner bases [CKPS2000]: This is generally considered to be an efficient algorithm for solving multivariate equations of higher degrees. The idea of the attack is to solve a given ciphertext y by the equation $P(x)$-y = 0 for the plaintext x by forming a Gröbner bases of the ideal ring generated by the polynomials $P(x)$-y.

- XL [Cou2002]: XL is designed to solve multivariate systems that are solved by Gröbner bases attack. This attack attempts to discover the indeterminate of the plaintext x=(x_1, x_2, …,x_n) one at a time, and calls itself on the new and simplified system on each success. So at this level, it is similar to the techniques of the Gröbner bases method. Ding and Schmidt have also noted the fact that the XL algorithm is merely a slower implementation of Gröbner bases attack.

- Dixon resultants [TF2005]: This method uses a necessary condition for finding a common affine zero x of a system of equations P in order to solve $P(x)$-y = 0, given a ciphertext y. Early results indicate that this method is faster than Faugère's F_4 method of calculation, but there are no published comparisons to Faugère's faster F_5 algorithm.

- MinRank-based attacks [GC2000]: The goal of the attack is to learn the private key (T, P', S), where T and S are functions, P' is a multivariate quadratic polynomials. Starting with T, once it is known, the attacker continues to uncover the quadratic coefficients of P' and S. Once these three parts of the private key are known, by using Gaussian elimination, the linear and constant coefficients of P' can be discovered.

- Differential cryptanalysis [FGS2005]: This is introduced to attack the PMI (Perturbed Matsumoto-Imai cryptosystem) *MQ*-based encryption scheme. Pierre Alain Fouque et al. also use it to attack the MIC* cryptosystem. The idea of this attack is to use differentials of quadratic functions P to get bilinear functions $\mathrm{L}_{P,k}(x) = \mathrm{B}_P(x, k)$. The goal is to find clever ways to take these differentials and to manipulate the bilinear maps for generating a bilinear system in terms of the plaintext x and the ciphertext y.

The first five attacks solve an *MQ* problem without using any information apart from the public key itself. Hence, they do not benefit from an attacker having knowledge of the specific *MQ*-based encryption scheme. The last two attacks, namely MinRank-based attacks and differential cryptanalysis require the use of such information, and are consequently less general, therefore not universally applicable [Fel2005].

In what follows, we will crypto-analyse *BMQE*-problems by fixing-variable method and relinearization. Other methods, such as Gröbner bases, Dixon resultants and MinRank-based attacks are what we need to do in the future work.

Section 3.3.2 Fixing Variables

To find out the relation between *q*, *m* and *n*, we proposed a new approach called "fixing variables". This approach is based on the idea of eliminating variables in equation systems, which is also the key idea of those existing attacks such as linearization [MJ1987], relinearization, Gröbner bases [CKPS2000], XL [Cou2002] and DR [TF2005]. However, on one hand, as pointed out in [FGS2005], the method of linearization only succeed when $m = n(n+1)/2$. On the other hand, relinearization, Gröbner bases, XL and DR are designed to solve equation systems with polynomials containing just one tuple of *n* variables, rather than a pair of such tuples.

Lots of the experiment results show that with the increase of $(m\text{-}n)$, the complexity of solving *MQ* problem reduces [MJ1987, TF2005, FGS2005 and WP1994]. The growth trend varies from exponential, sub-exponential to polynomial. If $m \approx n$, it is barely possible to solve *MQ* equation. But if *q* is small, then we can fix *r* variables such that $m > (n\text{-}r)$. If an *MQ*-problem with *m* equations and $(n\text{-}r)$ variables can be solved, then it takes at most q^r times to work out the solution. The following of this section shows how the fixing-variable method attacks *BMQE* system and a conclusion will be drawn at the end.

According to *BMQE* problem, let an equation (3.2) be denoted as follows:

$$\begin{cases} p_1(x,y) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}^{(1)} x_i y_j = b_1 \\ \vdots \\ p_m(x,y) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}^{(m)} x_i y_j = b_m \end{cases} \tag{3.2}$$

Denote the value space of $(p_1, p_2, \ldots, p_m)$ as $Spc = \{\ p_1(x, y), \ldots, p_m(x, y) \mid x, y \in$

$\mathbb{F}_n^q\}$.

For any x, $y \in \mathbb{F}_n^q$, let $x \otimes y = (x_1 y_1, \ldots, x_i y_i, \ldots, x_n y_n) \in \mathbb{F}_{n^2}^q$, then $(p_1, p_2, \ldots, p_m)$ is exclusively decided by $x \otimes y$. It is obvious that (x, y) generates q^{2n} values, thus the results of $x \otimes y$ include a zero and $(q^n-1)^2/(q-1)$ non-zeros. Hence, we have: $|Spc| \leq Min(q^m, (q^n-1)^2/(q-1) + 1)$.

And if $n>1$, $q^{2n-1} < (q^n-1)^2/(q-1) + 1 < q^{2n}$ (with n growing, there is barely no differences between $(q^n-1)^2 + 1$ and q^{2n}), consequently we have:

$$|Spc| \leq \begin{cases} \dfrac{(q^n-1)^2}{q-1} + 1 \ (m \geq 2n) \\ q^m (m < 2n) \end{cases} \qquad (3.3)$$

When $\{p_1(x, y), \ldots, p_m(x, y)\}$ is determined, there are several cases of solutions to Spc:

(1) if $(b_1, b_2, \ldots, b_m) = 0$, then Spc at least has $(2q^n-1)$ solutions with the form $(x=0, y=0) \vee (x \neq 0, y=0) \vee (x=0, y \neq 0)$.

(2) if $(b_1, b_2, \ldots, b_m) \neq 0 \wedge (b_1, b_2, \ldots, b_m) \notin Spc$, equation (3.2) has no solutions.

(3) if $(b_1, b_2, \ldots, b_m) \in Spc \setminus \{0\}$, then equation (3.2) has at least $(q-1)$ equivalent solutions $(x, y) \in (\mathbb{F}_n^q \setminus \{0\})^2$.

If $(b_1, b_2, \ldots, b_m) \in Spc \setminus \{0\}$, higher order correlation attack can be used in solving equation (3.2). For there is a mutual relation between x and y, fixing either of them is enough. And there are two methods, fixing whole or fixing part. The former means to fix all the elements in x, while the latter means to fix a part elements $x_{i_1}, x_{i_2}, \ldots x_{i_t} (1 \leq t < n)$ of x.

Let us take an example of fixing the whole elements of x. The steps are as follows:

(1) Randomly fix $x = (\alpha_1, \alpha_2, \ldots, \alpha_n) \neq 0$

(2) Replace x in equation (3.2) with $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ and we get a linear equation (3.4) with n unknowns $y = (y_1, y_2, \ldots, y_n)$:

$$\begin{cases} p_1(\alpha, y) = b_1 \\ \quad\vdots \\ p_m(\alpha, y) = b_m \end{cases} \quad (3.4)$$

(3) Equation (3.4) has a solution $y = \beta = (\beta_1, \beta_2, \ldots, \beta_n)$, otherwise go to step (1).

(4) $(x, y) = (\alpha, \beta)$ is a solution to equation (3.2).

Obviously, the success of fixing-variable attack is proportional to the solutions of equation (3.2). In addition, the solutions increase with the number of equations diminishing. In particular, when $m = n$, the number of the solutions to equation (3.2) approximates $(q^n\text{-}1)$, which means the probability that one guesses the right solution is nearly 100%. Therefore, if n is fixed and m is too small, it is quite easy to solve equation (3.2).

Similarly, for any $(b_1, b_2, \ldots, b_m) \in Spc \setminus\{0\}$, if equation (3.2) has $(q\text{-}1)$ solutions and $m \geq 2n$, the probability falls down to $(q\text{-}1)/(q^n\text{-}1)^2 < q^{-2n}$ (refer to equation (3.3)). Consequently, we have a theorem:

Theorem 3.2: Randomly create a bisectional multivariate quadratic equation system ES of m equations in $2n$ variables over $\mathbb{F}_n^q$, if ES satisfies $m \geq 2n \wedge |Spc\setminus\{0\}| = (q^n\text{-}1)^2/(q\text{-}1)$ and q^n is large enough, the method of fixing variables cannot solve ES.

Section 3.3.3 Relinearization

Kipnis and Shamir claimed in their paper [KS1999] that relinearization is a good idea because a system of εn^2 equations with n variables can be solved in expected polynomial time for any fixed $\varepsilon > 0$.

Section 3.3.3.1 A Toy Example of Relinearization

A toy example of 5 random quadratic equations in 3 variables modulo 7:

$$\begin{cases} 3x_1x_1 + 5x_1x_2 + 5x_1x_3 + 2x_2x_2 + 6x_2x_3 + 4x_3x_3 = 5 \\ 6x_1x_1 + 1x_1x_2 + 4x_1x_3 + 4x_2x_2 + 5x_2x_3 + 1x_3x_3 = 6 \\ 5x_1x_1 + 2x_1x_2 + 6x_1x_3 + 2x_2x_2 + 3x_2x_3 + 2x_3x_3 = 5 \\ 2x_1x_1 + 0x_1x_2 + 1x_1x_3 + 6x_2x_2 + 5x_2x_3 + 5x_3x_3 = 0 \\ 4x_1x_1 + 6x_1x_2 + 2x_1x_3 + 5x_2x_2 + 1x_2x_3 + 4x_3x_3 = 0 \end{cases}$$

Replace each $x_i x_j$ by y_{ij}, then the equations above can be written in the following form:

$$\begin{bmatrix} 3 & 5 & 5 & 2 & 6 & 4 \\ 6 & 1 & 4 & 4 & 5 & 1 \\ 5 & 2 & 6 & 2 & 3 & 2 \\ 2 & 0 & 1 & 6 & 5 & 5 \\ 4 & 6 & 2 & 5 & 1 & 4 \end{bmatrix} \begin{bmatrix} y_{11} \\ y_{12} \\ y_{13} \\ y_{22} \\ y_{23} \\ y_{33} \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 5 \\ 0 \\ 0 \end{bmatrix}$$

It is easy to solve this system by Gaussian elimination to obtain a single parameter family containing 7 possible solutions:

$$y_{11}=2+5z, y_{12}=z, y_{13}=3+2z, y_{22}=6+4z, y_{23}=6+z, y_{33}=5+3z$$

But not all the solutions solve the original quadratic system. To filter out the parasitic solutions, we need to impose some additional constraints. Since $y_{11}y_{23}= y_{12}\, y_{13}$, $y_{12}\, y_{23}= y_{13}\, y_{22}$, $y_{12}\, y_{33}= y_{13}\, y_{23}$, we substitute y_{ij} by $x_i x_j$, therefore:

$$(2+5z)(6+z)=z(3+2z),\ z(6+4z) =(3+2z)\ (6+4z),\ z(5+3z) =(3+2z)\ (6+z)$$

These equations can be simplified to:

$$3z^2+z+5=0,\ 0z^2+4z+4=0,\ 1z^2+4z+3=0.$$

The relinearization step introduces two new variables: $z_1=z$, and $z_2=z^2$. Take them as unrelated variables, we can work out the unique solution: $z_1=6$, $z_2=1$. Working backwards we find that $y_{11}=4$, $y_{22}=2$, $y_{33}=2$, thus $x_1=\pm2$ or $x_1=\pm5$, $x_2=\pm3$ or $x_2=\pm4$, $x_3=\pm3$ or $x_3=\pm4$. Finally, we use the values $y_{12}=6$, $y_{13}=1$ and $y_{23}=5$ to obtain $x_1=2$, $x_2=3$, $x_3=4$ and $x_1=5$, $x_2=4$, $x_3=3$.

Section 3.3.3.2 The 1st Degree Relinearization

Replace $x_i y_j$ by z_{ij}, hence there is a new multivariate system NMS_1 of $M_1=m$ equations in $N_1=n^2$ variables:

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \mathbb{a}_{ij}^{(k)} z_{ij} = \mathbb{b}_k\ (\boldsymbol{NMS_1})$$

If $M_1 \geq N_1$, then one can work out z_{ij}, and from $x_i y_j = z_{ij}$, it is easy to deduce MS's solution $(x_1, x_2, \ldots, x_n)$, $(y_1, y_2, \ldots, y_n) \in \mathbb{F}_q^n$. Otherwise, i.e., $M_1 < N_1$, let $r = N_1$

-M_1=n^2 - m, then the generic solution of NMS_1 can be represented by:

$$Z=[z_{11},z_{12},\ldots,z_{nn}]^T=Z_0+\alpha_1 Z_1+\alpha_2 Z_2+\ldots+\alpha_r Z_r \quad (3.5)$$

Where $Z_0,Z_1,\ldots,Z_r \in \mathbb{F}_q^{n^2}$ is fixed and $\alpha_2,\ldots,\alpha_r \in \mathbb{F}_q$ is undetermined. Therefore, equation (3.5) has q^r possible solutions, but most of them are parasitic solutions. Especially when MS has q-1 solutions, NMS_1 has a unique non parasitic solution. If MS has solutions, NMS_1 must have non parasitic solutions, and from any NMS_1's non parasitic solution, one can deduce q-1 solutions of MS. However, if q^r is quite large, Then the number of NMS_1's parasitic solutions are numerous and it is impossible to compute the solutions of MS by brute-force attack on NMS_1.

Hence, the key to solving MS is to work out NMS_1's non parasitic solutions Z, which is equivalent to picking out the exact value from q^r possible values of $(\alpha_1,\alpha_2,\ldots,\alpha_r)$.

Section 3.3.3.3 The 2nd Degree Relinearization

Let $Z_k=[s_{11}^k,\ldots,s_{1n}^k,s_{21}^k,\ldots,s_{2n}^k,\ldots,s_{n1}^k,\ldots,s_{nn}^k]^T$. Then from equation (3.5), each element z_{ij} of Z can be represented by:

$$z_{ij}=s_{ij}^0+s_{ij}^1\alpha_1+s_{ij}^2\alpha_2+\ldots+s_{ij}^r\alpha_r$$

To filter out the non parasitic solutions, we impose the additional constraints:

$$z_{ab}z_{cd}=(x_a y_b)(x_c y_d)=(x_a y_d)(x_c y_d)=z_{ad}z_{cb}.$$

Hence we get:

$$s_{ab}^0 s_{cd}^0+(s_{ab}^0 s_{cd}^1+s_{ab}^1 s_{cd}^0)\alpha_1+\cdots+(s_{ab}^0 s_{cd}^r+s_{ab}^r s_{cd}^0)\alpha_r+s_{ab}^1 s_{cd}^1\alpha_1^2+(s_{ab}^1 s_{cd}^2+s_{ab}^2 s_{cd}^2)\alpha_1\alpha_2+\cdots+(s_{ab}^i s_{cd}^j+s_{ab}^j s_{cd}^i)\alpha_i\alpha_j+\cdots+s_{ab}^r s_{cd}^r\alpha_r^2 = s_{ad}^0 s_{cb}^0+(s_{ad}^0 s_{cb}^1+s_{ad}^1 s_{cb}^0)\alpha_1+\cdots+(s_{ad}^0 s_{cb}^r+s_{ad}^r s_{cb}^0)\alpha_r+s_{ad}^1 s_{cb}^1\alpha_1^2+(s_{ad}^1 s_{cb}^2+s_{ad}^2 s_{cb}^2)\alpha_1\alpha_2+\cdots+(s_{ad}^i s_{cb}^j+s_{ad}^j s_{cb}^i)\alpha_i\alpha_j+\cdots+s_{ad}^r s_{cb}^r\alpha_r^2 \quad (3.6)$$

Equation (3.6) can be simplified to a quadratic equation of $(\alpha_1,\alpha_2,\ldots,\alpha_r)$:

$$\sum_{1\le i\le j\le r}\mathbb{a}_{ij}(\alpha_i\alpha_j)+\sum_{1\le k\le r}\mathbb{b}_i\alpha_k=\mathbb{c}\;(\boldsymbol{NMS_2})$$

Biquadratic polynomials of the form $x_a y_b x_a y_d$ or $x_a y_b x_c y_b$ do not generate new polynomials. Hence, there are $C_n^2 \times C_n^2$ biquadratic polynomials of the form $x_a y_b x_c y_d$, each generates one quadratic polynomial. Taking every term $\alpha_i \alpha_j$ and α_k in equation (3.6) as unrelated new variables, we then get a multivariate system NMS_2 of M_2 equations in N_2 variables, where:

$$M_2 = C_n^2 \times C_n^2 = \frac{n^2(n-1)^2}{4}, \quad N_2 = \frac{r(r+1)}{2} + r = \frac{r(r+3)}{2}.$$

If M_2 equations are linearly independent and $M_2 \geq N_2$, then one can solve NMS_2. Working backwards one can solve *MS*. In order to resist the relinearization attack, *n* and *r* at least satisfies $n^2(n\text{-}1)^2/4 < r(r+3)/2$. However, this is not our goal; we need a further analysis to resist stronger attack. If $M_2 < N_2$, there is the 3rd degree relinearization as follows:

Section 3.3.3.4 The 3rd Degree Relinearization

Like what we do in step (2), the additional constraints are imposed as follows:

$$z_{ab}z_{cd}z_{ef} = z_{ab}z_{cf}z_{ed} = z_{ad}z_{cb}z_{ef} = z_{ad}z_{cf}z_{eb} = z_{af}z_{cb}z_{ed} = z_{af}z_{cd}z_{eb} \quad (3.7)$$

where *a,b,c,d,e,f* =1,2,…,*n*. Then there will be:

- $C_n^3 \times C_n^3$ polynomials with the form $(x_a y_b x_c y_d x_e y_f)$, each generates 5 cubic equations, as is shown in equation (3.7);

- $2C_n^2 \times C_n^3$ polynomials with the form $(x_a y_b x_a y_d x_e y_f)$, each generates 2 cubic equations;

- $C_n^3 \times 2C_n^2$ polynomials with the form $(x_a y_b x_c y_b x_e y_f)$, each generates 2 cubic equations;

- $2C_n^2 \times 2C_n^2$ polynomials with the form $(x_a y_b x_a y_b x_e y_f)$, each generates 1 cubic equations;

- polynomials with the form $(x_a y_b x_a y_b x_a y_b)$, each does not generate any cubic equations;

Each equation can be presented as:

$$\sum_{1\le i\le j\le k\le r} \mathbb{a}_{ijk}\left(\alpha_i\alpha_j\alpha_k\right) + \sum_{1\le i\le j\le r} \mathbb{b}_{ij}\left(\alpha_i\alpha_j\right) + \sum_{1\le i\le r} \mathbb{c}_i\alpha_i = \mathbb{d}\ (\boldsymbol{NMS_3})$$

Take each term $\alpha_i\alpha_j\alpha_k$, $\alpha_i\alpha_j$ and α_i in equation (NMS_3) as unrelated new variables, we then get a multivariate system NMS_3 of M_3 equations in N_3 variables, where:

$$M_3 = 5\times C_n^3\times C_n^3 + 2\times 2\times 2C_n^2\times C_n^3 + 2C_n^2\times 2C_n^2 = \frac{n^2(n-1)^2(5n^2+4n+8)}{36}$$

$$N_3 = \frac{r(r+1)}{2} + \frac{(r-1)r}{2} + \frac{(r-2)(r-1)}{2} + \cdots + \frac{1\times 2}{2} + N_2 = \frac{r(r+1)(r+2)}{2\times 3} + N_2 = \frac{r(r^2+6r+11)}{6}.$$

If $M_3 \ge N_3$, then one can eventually solve MS. Otherwise one has to do the 4th degree relinearization.

Section 3.3.3.5 The k^th^ (k≥4) Degree Relinearization

There are 25forms of 8-degree polynomials of the form ($x_a y_b x_c y_d x_e y_f x_g y_h$). As we do in the 3rd degree relinearization, one can get a multivariate system NMS_4 of M_4 equations in N_4 variables, where:

$$\sum_{1\le i\le j\le k\le l\le r} \mathbb{a}_{ijk}\left(\alpha_i\alpha_j\alpha_k\alpha_l\right) + \sum_{1\le i\le j\le k\le r} \mathbb{b}_{ij}\left(\alpha_i\alpha_j\alpha_k\right) + \sum_{1\le i\le j\le r} \mathbb{c}_i\left(\alpha_i\alpha_j\right) + \sum_{1\le i\le r} \mathbb{d}_i\alpha_i = \mathbb{e}\ (\boldsymbol{NMS_4})$$

$$M_4 = \frac{n^2(n-1)^2(23n^4+34n^3+131n^2+84n+108)}{576}$$

$$N_4 = \frac{r(r+1)(r+2)}{6} + \frac{(r-1)r(r+1)}{6} + \frac{(r-2)(r-1)r}{6} + \cdots + \frac{1\times 2\times 3}{6} + N_3 = \frac{r(r+1)(r+2)(r+3)}{6\times 4} + N_3 = \frac{r(r^3+10r^2+35r+50)}{24}$$

If $M_4 < N_4$, further relinearization is needed. But M_4 and N_4 increase rapidly with the growth of k, which makes NMS_k unsolvable in practice. The result of M_k ($2\le k\le 5$) under the different values of n is shown in Table 3.2; the rate of growth between M_k and $M_{k\text{-}1}$ is shown in Figure 3.1.

Table 3.2 Results of M_k under the Different Values of n

$\boldsymbol{n}$	$\boldsymbol{M_2}$	$\boldsymbol{M_3}$	$\boldsymbol{M_4}$	$\boldsymbol{M_5}$
25	90000	32330000	5999817500	8.21E+13
50	1500625	2118882500	1545766801250	1.30E+17
75	7700625	24327985625	39723535451250	8.63E+18
100	24502500	137235780000	397369039876875	1.64E+20
125	60062500	524766062500	2370674427859370	1.59E+21
150	124880625	1569444192500	10199850706164400	1.01E+22
175	231800625	3962065060625	35023995611635600	4.82E+22
200	396010000	8835775120000	101965031306508000	1.86E+23
225	635040000	17924568480000	261691236800280000	6.11E+23
250	968765625	33746197062500	608056566504406000	1.77E+24
275	1419405625	59809494820625	1303657612829950000	4.62E+24
300	2011522500	100846116020000	2615395737242270000	1.11E+25
325	2772022500	163066687582500	4962377560690700000	2.48E+25
350	3730155625	254441375492500	8978732670835150000	5.23E+25
375	4917515625	385004865265625	15594173069995300000	1.05E+26
400	6368040000	567185756480000	26135364554252000000	2E+26
425	8118010000	816160371370000	42451425880635300000	3.68E+26
450	10206050625	1150230977482500	67067117245834500000	6.54E+26
475	12673130625	1591228424395620	103367525266375000000	1.13E+27
500	15562562500	2164939194500000	155818297316703000000	1.88E+27

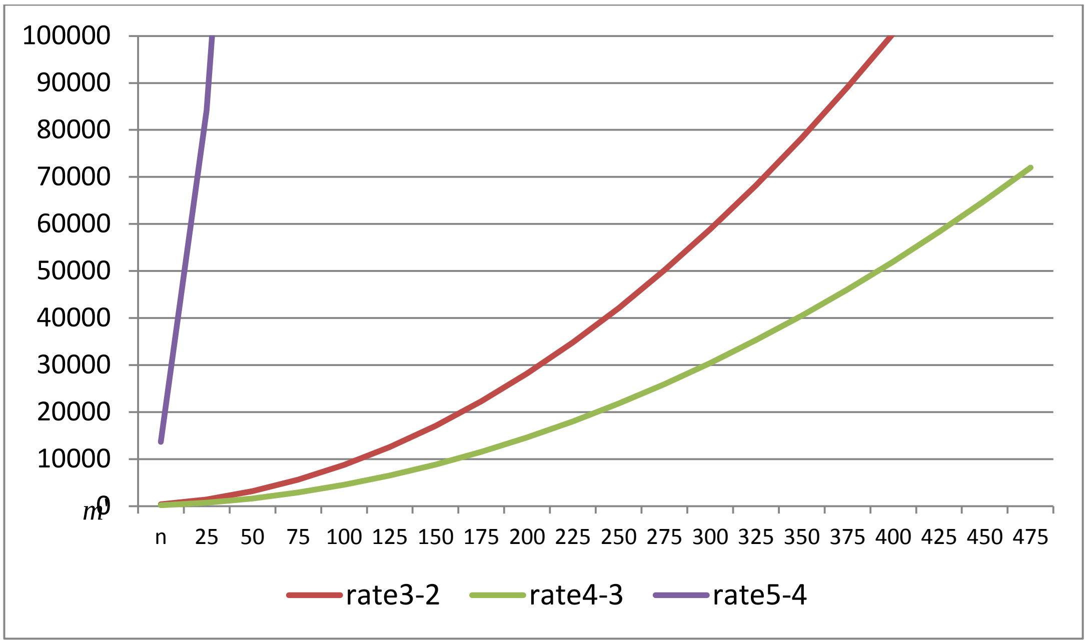


Figure 3.1 Rate of Growth between M_k and $M_{k\text{-}1}$

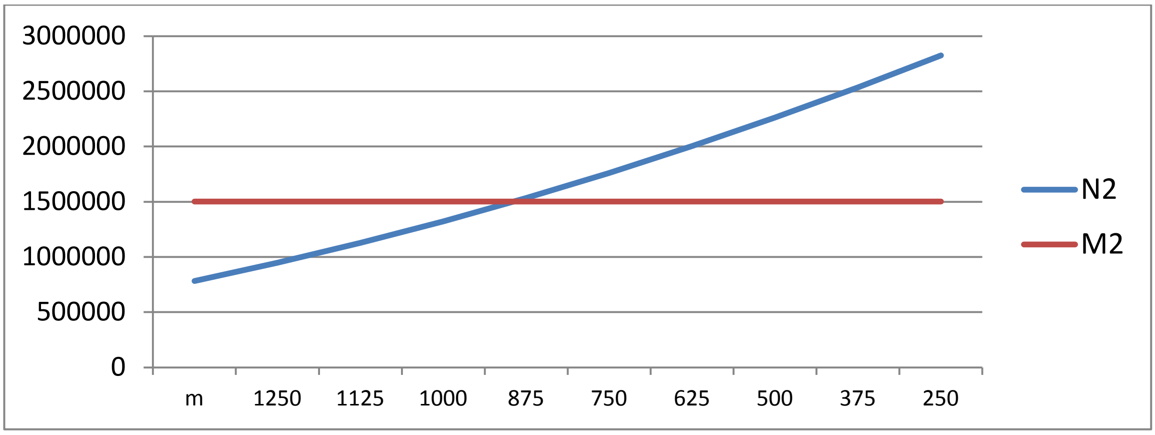
In Figure 3.1, *m*' in the figure denotes the difference of *m* between M_k and $M_{k\text{-}1}$. Rate3-2 represents (M_3 –M_2)/ M_2, the rest can be deduced from this. Figure 3.1 indicates that the growth rate increases with *k*. This means the number of the new equations grows much faster compared to that generated at the last degree relinearization.

The result of N_k under the different values of *n* and *m* is shown in Table 3.3. Figure 3.2 shows the value of M_k and N_k given different value of *m*, and when *n*=50.

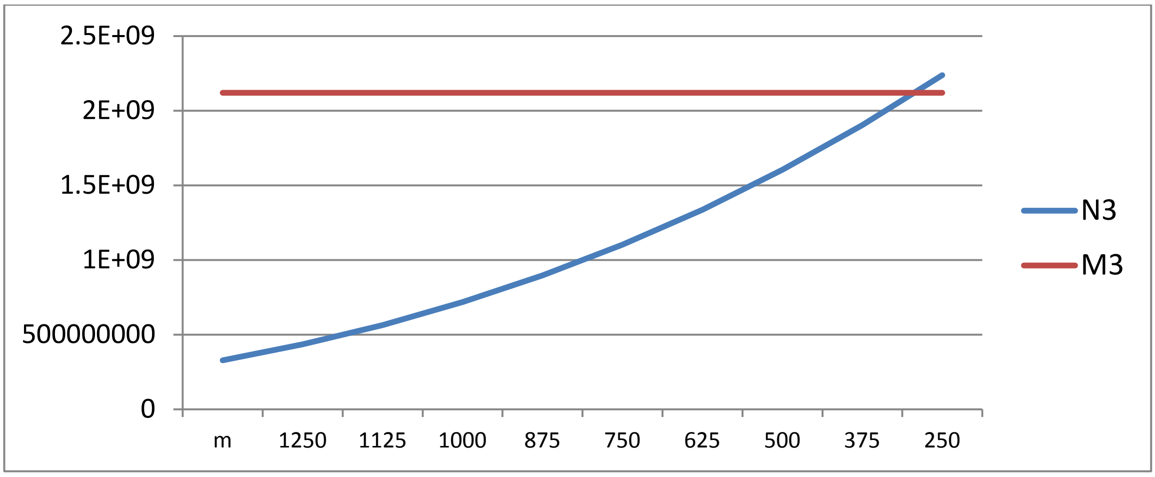
Table 3.3 Values of N_k under The Different Values of *n* and *m*

n	***m***	***r*** **(=*n*²-*m*)**	$N_1(=n^2)$	N_2	N_3	N_4	N_5
50	1250	1250	2500	12507500	20858342500	26093750007294	1.306E+20
	1125	1375	2500	10131750	15207758250	17123906256565	6.943E+19
	1000	1500	2500	8006000	10682674000	10693333339169	3.426E+19
	875	1625	2500	6130250	7158089750	6270468755106	1.538E+19
	750	1750	2500	4504500	4509005500	3386250004377	6.105E+18
100	5000	5000	10000	12507500	20858342500	26093750007294	1.306E+20
	4500	5500	10000	10131750	15207758250	17123906256565	6.943E+19

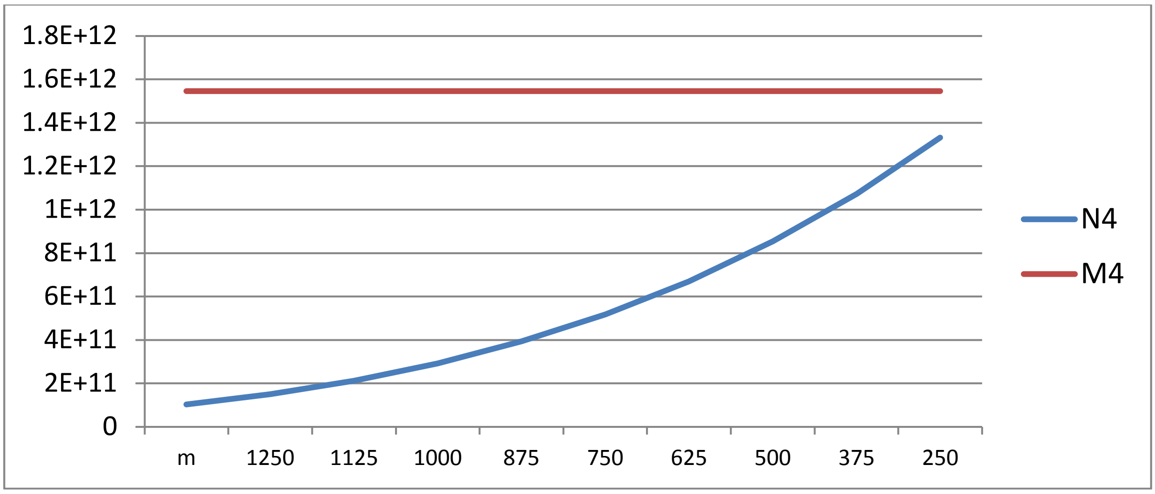
	4000	6000	10000	8006000	10682674000	10693333339169	3.426E+19
	3500	6500	10000	6130250	7158089750	6270468755106	1.538E+19
	3000	7000	10000	4504500	4509005500	3386250004377	6.105E+18
150	11250	11250	22500	63298125	237431270625	668012695328908	1.692E+22
	10125	12375	22500	51273000	173097651375	438326378188596	8.992E+21
	9000	13500	22500	40513500	121581016500	273678750013127	4.436E+21
	7875	14625	22500	31019625	81457537875	160450894786877	1.991E+21
	6750	15750	22500	22791375	51303387375	86625703134846	7.900E+20
200	20000	20000	40000	200030000	1333733370000	6670000000029170	5.337E+23
	18000	22000	40000	162027000	972324033000	4376430000026250	2.837E+23
	16000	24000	40000	128024000	682922696000	2732373333356670	1.399E+23
	14000	26000	40000	98021000	457529359000	1601810000020420	6.281E+22
	12000	28000	40000	72018000	288144022000	864720000017502	2.491E+22



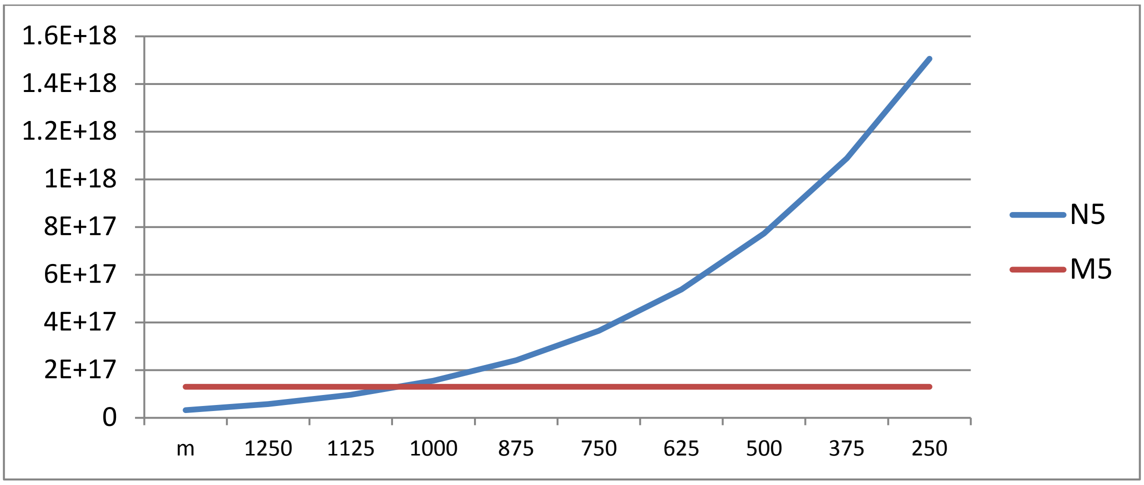
(1) n=50, N_2 and M_2 Given Different m



(2) n=50, N_3 and M_3 Given Different m



(3) n=50, N_4 and M_4 Given Different m



(4) n=50, N_5 and M_5 Given Different m

Figure 3.2 Value of M_k and N_k ($2 \leq k \leq 5$) Given Different Value of m, and when n=50

From what we have discussed above, it is obvious that with further relinearization,

(1) $\sum_{1\le i_1\le i_2\le\cdots\le i_k\le r} \mathbb{a}_{i_1 i_2 \dots i_k}\left(\alpha_{i_1}\alpha_{i_2}\dots\alpha_{i_k}\right)+\cdots+\sum_{1\le i_1\le i_2\le r} \mathbb{b}_{i_1 i_2}\left(\alpha_{i_1}\alpha_{i_2}\right)+\sum_{1\le i_1\le r} \mathbb{c}_{i_1}\alpha_{i_1} = \mathbb{d}\,(\boldsymbol{NMS_k})$

(2) Apart from the 1st degree, the result of relinearization with the new equations M_k grows by more than cn^2($c\approx k!$) times with the last degree. M_k is closely related to n, and it increases rapidly in relation to k;

(3) $N_k = \frac{(r+k-2)!}{(k-1)!(r-1)!} + \frac{(r+k-1)!}{(k-1)!(r-1)!} + \cdots \frac{r!}{1!(r-1)!} + N_{k-1} = \frac{(r+k-1)!}{k!(r-1)!} + N_{k-1} = \sum_{i=1}^{k} C_{r+i-1}^{i}$, the number of the new variables are related to n and m; and monotonically increase by n^2-m;

(4) If $k\ge 3$, $M_k = \sum_{2\le i\le j\le k} t_{ij} C_n^i \times C_n^j = \left(P_k^k - 1\right)C_n^k \times C_n^k + \left(C_k^2 P_{k-2}^{k-2} - 1\right)(k-1)C_n^{k-1} \times C_n^k + \cdots + t_{22} C_n^2 \times C_n^2$. Specially, when n is very large, $M_k \approx \left(P_k^k - 1\right)C_n^k \times C_n^k + \left(C_k^2 P_{k-2}^{k-2} - 1\right)(k-1)C_n^{k-1} \times C_n^k = (k!-1)C_n^k \times C_n^k + (0.5k!-1)(k-1)C_n^{k-1} \times C_n^k$;

(5) Given n, we can always find a cross-over point of N_k and M_k. It means with m descending, there always exists an m such that $M_k<N_k$.

These five conclusions decide whether *MS* is solvable by the k-th degree relinearization. For example, when n= 50, 100, 150, 200, the value of m shall satisfy the condition shown in Table 3.4 such that $M_k<N_k$.

Table 3.4 Values of m in the k-th Degree Relinearization such that $M_k<N_k$

n	***m*_2nd**	***m*_3rd**	***m*_4th**	***m*_5th**
50	≤769	≤168	≤34	≤921
100	≤3001	≤629	≤120	≤4806
150	≤6697	≤1384	≤259	≤12172
200	≤11858	≤2425	≤450	≤23222

A conclusion can be drawn from Table 3.4 that given n= 50, 100, 150, 200, if m≤(34=0.68n), m≤(120=1.2n), m≤(259=1.73n), m≤(450=2.25n), respectively, then one cannot solve the multivariate system *MS* by the 5th degree relinearization.

It is a remarkable fact that many of the new equations generated by any degree of relinearization are linearly dependent [CKPS2000]. This fact actually makes M_k smaller than the theoretical value. Let the number of independent equation of NMS_k be M_{k_indpt}, then if $M_{k\_indpt}<N_k$, even if $M_k \geq N_k$, NMS_k will be unsolvable.

Moreover, the space overhead of coefficients of NMS_k is $N_k^2 log_2 q$ bits. Hence, when N_k is large enough, 2^{64} for example, even if M_k is greater or equal to N_k, NMS_k is still unsolvable whether in time or space. Because theoretically speaking, a 64-qubit computer is 2^{64} times faster than a 64-bit PC. With this speed, a 400-bit length integer can be factorized in one year by a quantum computer (compared to the fastest giant computer in nowadays, the factorization time is 1 billion years).

Therefore, given n and $k\geq 2$, if there exists an r' such that $(r=n^2-m)\geq r'$ and N_k is large enough to make NMS_k unsolvable in practice, we have $(M_2<N_2 \wedge M_3<N_3 \wedge \ldots \wedge M_{k-1}<N_{k-1}) \wedge (N_k<N_{k+1}<\ldots)$. This means one cannot solve NMS_k by any degree of relinearization. Whether other methods, such as XL, can solve this system is what we need to research in the future. Even though $(M_2<N_2 \wedge M_3<N_3 \wedge \ldots \wedge M_{k-1}<N_{k-1})$, if $r_t = N_t - M_t$ $(2\leq t\leq k\text{-}1)$, then one may relinearize NMS_t many times so that each variable of NMS_t can be presented by a multivariate system in r_t variables $\beta_1, \beta_2, \ldots, \beta_{r_t}$ (similar to equation (3.5)):

$$v_i=\alpha_{i_1}\alpha_{i_2}\ldots\alpha_{i_s} = c_0 + c_1\beta_1 + c_2\beta_2 + \cdots + c_{r_t}\beta_{r_t} \quad (1\leq s\leq t, \quad 1\leq i\leq N_t),$$

where $2\leq t \leq k\text{-}1$ and $h\geq 2$.

Then one can relinearize NMS_t by $h(h\geq 2)$ times to get $NMS_t^{(h)}$ until it is solvable, thus one can work backwards to deduce MS. It is easy to compute the number of variables in $NMS_t^{(h)}$:

$$N_t^{(h)} = \frac{(r_t+h)!}{h!(r_t-1)!} + \frac{(r_t+h-1)!}{(h-1)!(r-1)!} + \cdots \frac{r_t!}{1!(r_t-1)!} = \sum_{i=1}^{h} C_{r_t+i-1}^{i} \geq N_t^{(2)} = r_t(r_t+3)/2.$$

Hence, only $2\leq t \leq k\text{-}1$, $h\geq 2$, if $d_t = N_t - M_t$ makes $N_t^{(h)}$ large enough can we be sure that one cannot solve MS by relinearization.

From what has been discussed, we have the following theorem:

Theorem 3.3: Randomly generate a bisectional multivariate quadratic system *MS* of *m* equations in $2n$ variables over a finite field $\mathbb{F}^q$, if the degree of relinearization k is greater than 2 and there exists $r' \leq r = n^2 - m$ such that:

(1) $M_{k\text{-}1} < N_{k\text{-}1}$;

(2) N_k is large enough, e.g. greater than 2^{64};

(3) $N_t^{(2)} = d_t(d_t + 3)/2(2 \leq t \leq k\text{-}1$ and $h \geq 2)$ is large enough.

Then *MS* cannot be solved by relinearization.

In fact, Theorem 3.3 is a conservative assessment to the relation between *m*, *n* and *q*. This is because the number of the linear dependent equations of NMS_t is practically smaller than M_t, while the number of the new variables N_k is fixed given a definite *n*.

It is also important to note that, the above equations of M_k and N_k does not consider the case of $q \leq k$. For example, if q=2, N_k is $\sum_{i=1}^{k} C_r^i$ rather than $\sum_{i=1}^{k} C_{r+i-1}^i$, because x^2 equals to *x* over finite field $\mathbb{F}^2$. Therefore, the equations of N_k and M_k we mention above only apply to the case of $q \geq 5$.

Section 3.4 Conclusions

In this chapter, we firstly summarized that with a huge number of ergodic matrices over a certain finite field $\mathbb{F}^q$, that makes it favorable in cryptography. Thus, combined with *EM,* we propose a multivariate equation system over a finite field $\mathbb{F}^q$.

The complexity analysis shows that the proposed system is NP-hard for *MQ* problem attackers. To strengthen the complexity of the system, we utilize Kipnis & Shamir’s relinearization method to analyse the number of the variables 2*n*, together with the number of the equations *m* and the number of the degree *q* of $\mathbb{F}^q$. The theorem given at section 3.3 shows that fixing-variable method cannot be used to solve this system if $m \geq 2n \wedge |Spc \backslash \{0\}| = (q^n - 1)^2 / (q - 1)$ and q^n is large enough.

BMQE is also robust enough to resist relinearization attack if it satisfies the following conditions: The degree of relinearization $k > 2$ and there exists $r' \leq r = n^2 - m$

such that: (1)$M_{k\text{-}1}<N_{k\text{-}1}$; (2) N_k is large enough; (3)$N_t^{(2)} = d_t(d_t + 3)/2$ (2≤ t ≤k-1 and h≥2) is large enough.

CHAPTER 4 IMAGE CRYPTOGRAPHY

By and large, there are two main schemes that can be used to protect digital images:

(1) Information hiding, it is a technology that uses anonymity, watermarking, covert channel and steganography;

(2) Cryptography, it includes traditional encryption and other such as chaotic encryption [ZLW2005 and Fri1998].

Image cryptography is a vital technology in image security; it has various applications in telemedicine, military image database and multimedia systems, etc. The existing image cryptosystems are almost based on chaos theory, it is ideal for image encryption in some aspects, but the drawbacks of small key space and weak security are obvious. Therefore, in this chapter, we shall introduce a new scheme for image cryptography.

Section 4.1 Review of Image Cryptography

Image cryptography is a technology that takes the transmitting image as a plaintext, and encrypts it with session keys and cryptography algorithms.

Section 4.1.1 Characteristics of Image Cryptography

A still image can be taken as a bivariate continuous function in a plane:

$$I=f(x,y)\ ,\ 0\leq x\leq Lx;\ 0\leq y\leq Ly$$

$f(x,y)$ represents the greyscale value of the point for any point (x, y) in the plane, it corresponds to the brightness and its value is bounded. Consequently, the function $I=f(x,y)$ is limited. When an image is digitalized, $I=f(x,y)$ can be taken as a matrix. The row and the column of a matrix element is the coordinate of the corresponding image pixel that shows on a computer screen, and the element value is the pixel greyscale value.

A digital image has its own unique properties. Therefore, the conventional cryptosystems are not ideal for image cryptography [OS2005]. What makes image cryptography different from other kinds of data cryptography is that:

(1) Image data has high volume and high redundancy, this feature makes an encrypted image fragile to resist various attacks of cryptanalysis. For the high volume data, an attacker can get enough ciphered samples to do the statistic analysis; for high redundancy, the adjacent pixels may have approximate values. The conventional cryptosystems cannot find a good solution to these problems.

(2) The image size is generally much greater than that of text. It results in conventional cryptosystems taking much more time to encrypt images directly. Moreover, most of the digital images are stored in a two-dimensional array, thus conventional cryptosystems have to transfer the image data into binary data stream – it reduces the encryption efficiency. For a real-time image process, if the encryption algorithm runs at a very low speed, even if it performs high security, it will have no practical importance.

(3) The image data has high correlation among adjacent pixels. Consequently, it is rather difficult for the conventional cryptosystems to shuffle and diffuse image data effectively. Shannon pointed out that a good encryption algorithm shall satisfy $E(P/C)=E(P)$, where P denotes plaintext and C denotes ciphertext and E denotes the encryption function. This is to say, the encrypted message shall be random enough not to reveal any plaintext.

(4) A digital image is not as sensitive as a text – it allows a certain degree of distortion if a person’s vision cannot perceive it. In many cases, it is acceptable even the distortion is perceived by human being. Generally speaking, the security of an image is determined by actual application. Apart from special cases such as medical application and military, the value of an image is quite low. Therefore, it is not necessary to encrypt all kinds of images with high security standard.

In view of the above analysis, we need to design an image cryptography scheme to accommodate the characteristics of digital image.

Section 4.1.2 Image Cryptography Algorithms

In this section, we will have a detailed review of some cryptography algorithms proposed by other researchers.

Section 4.1.2.1 A Technique for Image Encryption using Digital Signatures

Aloka Sinha and Kehar Singh [SS2003] propose a new technique to encrypt an image for secure image transmission. The digital signature is used to encrypt the message by adding it, bit-wise, to the encoded original image. This technique works well with images of all sizes. It provides three layers of security. In the first layer, an error control code is used which is based on the size of the input image and determined in real-time. It is very difficult to obtain the original image without the knowledge of the specific error control code. In the second layer, the dimension of the image changes due to the added redundancy. In the third layer, the digital signature is added to the encoded image in a specific manner. Because of the three-layer security, this information can be protected to make the system more secure. At the receiver end, the digital signature can be used to verify the authenticity of the transmitted image.

Section 4.1.2.2 Lossless Image Compression and Encryption Using SCAN

S.S. Maniccam and N.G. Bourbakis [MB2001] propose a methodology performing both lossless compression and encryption of binary and greyscale images. The compression and encryption schemes are based on SCAN patterns or space-filling curves generated by a two-dimensional spatial-accessing methodology called SCAN.

The distinct advantage of simultaneous lossless compression and strong encryption makes the methodology very useful in applications such as multimedia applications, medical imaging, and military applications. The drawback of the algorithm is that compression-encryption takes long time.

Section 4.1.2.3 A New Encryption Algorithm for Image Cryptosystems

Chin-Chen Chang, Min-Shian Hwang, and Tung-Shou Chen [CHC2001] designed an efficient cryptosystem based on VQ (vector quantization) for images. VQ is an efficient approach to low bit-rate image compression. Its major advantage is the simple hardware structure, especially the decoder. In the method of Chang et al., VQ diffuses and confuses the codebook and encrypts the parameters of the codebook using a cryptosystem.

According to the results of their security analyses, for example, the key size is 2^{56}, which means if the illegal users employ a 1000 MIPS computer (a computer features superscalar processing and pipelined access of interleaved secondary cache) to conjecture the private key *K*, the computational load is over 500 years. With this key size, the algorithm not only compresses image data but also is safe from brute force, known-plain image, chosen-plain image jigsaw puzzle, and neighbor attacks.

But this paper was published in 2001 and the computer speed is now much faster than it was ten years ago. Besides, the proposed scheme encrypts information by DES – as we stated in the beginning of this chapter that these traditional encryption algorithms have drawbacks of small key space and low speed, it is not ideal for image cryptography.

Section 4.1.2.4 A New Chaotic Image Encryption Algorithm

Chaos-based cryptosystems usually have higher speeds and lower costs. Moreover, these systems are sensitive to initial conditions and control parameters. These optimistic characters make them suitable for image encryption. In this respect, during the past decade a great number of chaotic systems have been proposed. For example, Chen et al. used a 3D baker map [MC2004] and a 3D cat map [CM2004] in the permutation process. Guan et al. employed a 2D cat map for substitution and the diffusion of Chen’s chaotic system for masking the pixel values [GHG2005]. Jiri Giesl et al. used the chaotic maps of Peter de Jong’s attractor (an attractor written by Peter De Jong, where $x_{n+1} = \sin(a\bullet y_n) - \cos(b\bullet x_n)$ and $y_{n+1} = \sin(c\bullet x_n) - \cos(d\bullet y_n)$) to improve the chaos image encryption speed [GBB+2009]. Yang et al. introduced a keyed hash function to generate a 128-bit hash value so that the scheme could be used to encrypt and authenticate [YWL+2010].

Section 4.1.2.5 Colour Image Encryption Using Double Random Phase Encoding

Shuqun Zhang and Mohammad A. Karim [ZK1999] propose a method to encrypt single-channel colour images using existing double-phase cryptosystems for greyscale images. An RGB format image is converted to their indexed image formats before it is encrypted using atypical optical security system. In the encoding subsystem, image is encoded to stationary white noise with two random phase masks, i.e., the input plane and the Fourier plane. While decrypting, the colour image is converted back to the RGB format. The proposed single-channel colour image encryption method is more compact and robust than the multi-channel methods. As only one channel is needed to encrypt colour images, it reduces the complexity and increases the reliability of the corresponding optical colour image cryptosystems.

Section 4.2 Hybrid-key Based Image Cryptography

According to the author's knowledge, before the paper "Ergodic matrix and hybrid-key based image cryptosystem" was published, the chaos-based cryptosystems, which have been extensively used since last two decades, are based on symmetric cryptography and are lack of authentication [ZK1999, CHC2001, MB2001, SS2003, MC2004, GHG2005, GBB+2009 and YWL+2010]. To remedy the imperfections, a hybrid-key based image cryptography and authentication scheme is proposed.

Section 4.2.1 Encryption/Decryption and Authentication Process

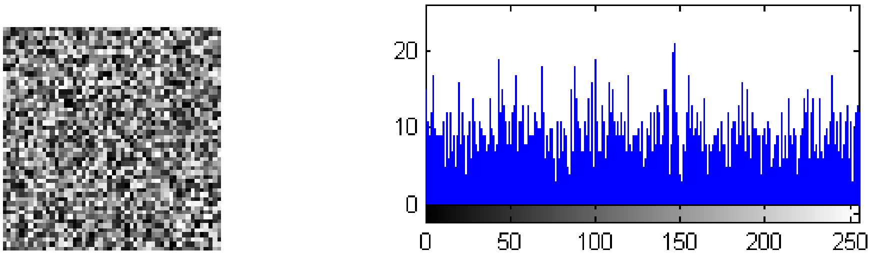
The deficiencies of the existing chaos-based cryptosystems can be summarized as below:

(1) The communication session is based on symmetric cryptography.

(2) Lack of authentication, which means it is difficult for the recipient to confirm that the cipher image is exactly sent by the one he wants to communicate.

As we have addressed in CHAPTER 2 that there are advantages and disadvantages in symmetric and asymmetric algorithms. As a result, in practical applications we can make use of the advantages of these two algorithms, using symmetric algorithm to encrypt files and asymmetric algorithm to encrypt the keys of the encrypted document key (or it may be called *session key*), which is a hybrid cryptosystem. This provides a better way to solve the computing speed issues and the key distribution and management issues.

From the ergodic matrix theorems mentioned in CHAPTER 2, over finite field $\mathbb{F}^q$, all $n\times n$ ergodic matrices have the same order and their generating sets have the same size, which are larger than that of any other $n\times n$ non-ergodic matrices. Take a random ergodic matrix $Q \in \mathbb{F}^{256}_{50\times 50}$ as an example, then the image of the matrix and the histogram is shown in Figure 4.1.



(1) Image of a Random 50×50 Ergodic Matrix (2) Histogram of a Random Ergodic Matrix

Figure 4.1 Image of a Random Ergodic Matrix $Q \in \mathbb{F}^{256}_{50\times 50}$ and the Corresponding Histogram

This figure implies that an *EM* is fairly uniformly distributed, thus it can be used to encrypt an image.

The communication process is composed of encryption, decryption and authentication.

In consideration of the basic need of cryptology, the ciphertext shall have close connection to the key. There are two ways to realize this requirement [IA2010]: One is to utilize a good key generation mechanism, another is to thoroughly mix the key with the plaintext in the encryption process.

Therefore, to realize the hybrid-based cryptosystem which satisfies the requirement, we carry out the process as below:

(1) Alice and Bob respectively take $(x_{\mathrm{a}},y_{\mathrm{a}}=\boldsymbol{f}(x_{\mathrm{a}},k_{\mathrm{a}}))$ and $(x_{\mathrm{b}},y_{\mathrm{b}}=\boldsymbol{f}(x_{\mathrm{b}},k_{\mathrm{b}}))$ as public keys, k_{a} and k_{b} as private keys. Here $x_{\mathrm{a}}=Q_{\mathrm{a}}$, $y_{\mathrm{a}}=\boldsymbol{f}(x_{\mathrm{a}},k_{\mathrm{a}})=Q_{\mathrm{a}}^{ka}$; $x_{\mathrm{b}}=Q_{\mathrm{b}}$, $y_{\mathrm{b}}=\boldsymbol{f}(x_{\mathrm{b}},k_{\mathrm{b}})=Q_{\mathrm{b}}^{kb}$.

(2) When Alice wishes to communicate with Bob, she generates a random key $k_x \in \boldsymbol{K}$ and computes $K_1=\boldsymbol{f}(x_{\mathrm{b}},k_x)=Q_{\mathrm{b}}^{kx}$, $K_{\mathrm{a}}=\boldsymbol{f}(y_{\mathrm{b}},k_x) = Q_{\mathrm{b}}^{kbkx}$. Then she gets CID_{a} by encrypting her identity ID_{a} with K_1. For example, she encodes her identity information into an n-order matrix, then XOR with K_1, or gets CID_{a} by $K_1 \times ID_{\mathrm{a}}=Q_{\mathrm{b}}^{kx}ID_{\mathrm{a}}$. After this, she sends $(CID_{\mathrm{a}}, K_{\mathrm{a}})$ to Bob.

(3) Bob uses his own key k_{b} to deduce $K_1= \boldsymbol{f}(K_{\mathrm{a}},-k_{\mathrm{b}}) = (Q_{\mathrm{b}}^{kbkx})\ Q_{\mathrm{b}}^{-kb} = Q_{\mathrm{b}}^{kx}$ and decipher ID_{a} by XOR CID_{a} with Q_{b}^{kx}, or by the equation $ID_{\mathrm{a}}=K_1^{-1}\times CID_{\mathrm{a}}$, thus he gets Alice's public key $(x_{\mathrm{a}},y_{\mathrm{a}})$ and makes sure CID_{a} is actually from Alice.

(4) Bob generates a random key $k_y \in \boldsymbol{K}$, and computes $K_2=\boldsymbol{f}(x_{\mathrm{a}},k_y)=Q_{\mathrm{a}}^{ky}$, $K_{\mathrm{b}}= \boldsymbol{f}(y_{\mathrm{a}},k_{\mathrm{y}})=Q_{\mathrm{a}}^{kaky}$, $CK_{\mathrm{b}}=\boldsymbol{f}(K_1, K_{\mathrm{b}})= Q_{\mathrm{b}}^{kx}Q_{\mathrm{a}}^{kaky}$. Then he sends CK_{b} to Alice.

(5) Alice decrypts K_{b} and K_2 by $K_{\mathrm{b}}= \boldsymbol{f}^{-1}(CK_{\mathrm{b}}, K_1)$, $K_2= \boldsymbol{f}^{-1}(K_{\mathrm{b}}, k_{\mathrm{a}})$. Thus K_1 and K_2 can be used as session keys between Alice and Bob.

(6) If Alice wants to secretly send an image to Bob, she may encrypt the image by one of the session keys. Take K_1 for example, then she gets the cipher-image Cimg by Cimg=$\boldsymbol{f}(K_1$, image) = $\boldsymbol{f}$ (Q_{b}^{kx}, image).

(7) Bob deciphers Cimg and gets the original image Dimg by Dimg = $\boldsymbol{f}^{-1}$(K_1, image) = $\boldsymbol{f}^{-1}(Q_{\mathrm{b}}^{kx}$, image).

Section 4.2.2 Image Confusion

The confusion (also called discretization, permutation) stage shuffles the pixels in the image. It is an important technique used in image cryptography and information hiding. Scientists have been conducting investigations in this field for a long time. Typical approaches to confusion include Arnold permutation [RE1992], Hilbert permutation [LCLH2003], Kolmogorov flows [Jos2000], baker map [MTS1999], Knight’s tour problem [Par1996 and Par1997], standard map [LSW2005], etc.

These existing techniques imply that a fine algorithm for confusion shall follow the principles as below:

(1) Transform T must be 1-1 map. This is the primate premise for confusion. Only by following this rule can we make sure each pixel of cipher image will not lose its original information, so that it can be completely decrypted.

(2) T must disorganize the correlated positions of the plain-image as much as possible, such that the cipher-image cannot be directly seen by the human eyes or be guessed the information of the original image.

(3) T can rapidly disorganize the correlated positions of the plain-image. This is an important measure of the efficiency of a confusion algorithm.

(4) The existing cryptosystems and the fast speed of computers make it difficult for any cipher-image to resist the brute-force attack. For the security of the encryption algorithm, key space is needed to be as large as possible.

As a result, based on the discretization property of ergodic matrix, we propose a confusion scheme that goes by the following steps:

First, given Q_{b}^{kx} is the key to encrypt the image, if the size of Q_{b}^{kx} is smaller than that of the image, Alice calculates the power of Q_{b} by the following algorithm:

① H and W is the height and width of the image respectively;

② N is the number of the elements of the key Q_{b}^{kx};

③ n rounds the elements of H×W/N to the nearest integers greater than or equal to H×W/N;

④ ArrayB stores the result of n matrices that Alice calculates;

⑤ for i=1 to n

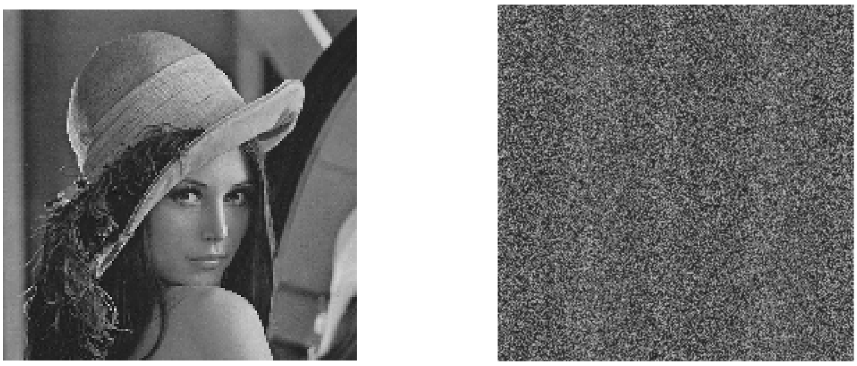
⑥ the i-th row of ArrayB stores the result of Power(Q_{b}^{kx}, Q_{b}^{kx}(i));

⑦ end

⑧ sort(ArrayB) returns an array of indices after sorting ArrayB.

Denote the result of sort(ArrayB) as IX, then Alice discretizes the image according to IX. For example, if IX(i)=37, the i-th pixel of the image is put to the 37th position.

The experimental result for this discretization algorithm is shown in Figure 4.2.



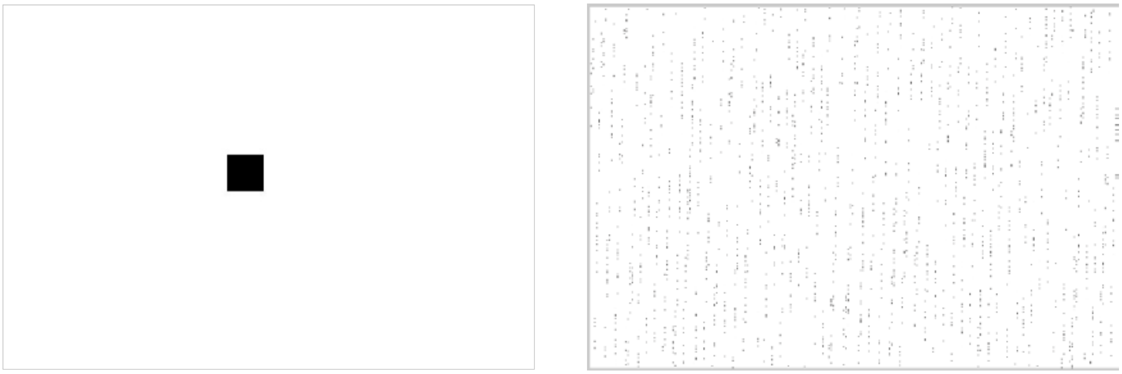
(1) Original Lena Image (2) Permuted Image

Figure 4.2 Comparison of Lena Images after 1-round of Permutation

Another experiment is carried out to prove the discretization algorithm can equally distribute the pixels.

In this experiment, a 50-by-50 black block is in a 713-by-488 white image. After one-round of permutation, the black block has distributed evenly in the whole image. The experimental result is shown in Figure 4.3.

It is easy to prove that this discretization is 1-1 map, which satisfies the 1st principle above. Furthermore, the permuted image disorganizes the plain-image as much as possible so that it is hard to recognize, which satisfies the 2nd principle.



(1) Original Black-block Image (2) Permuted Black-block Image

Figure 4.3 Comparison of Black-block Images after 1-round of Permutation

Section 4.2.3 Image Diffusion

Shannon suggested employing diffusion and confusion in the cryptosystem [Sha1949]. The diffusion stage is necessary because an attacker can break the system by comparing a pair of plain-image and cipher-image to discover useful information. For the purpose of diffusion, an explicit function which uses the ergodic matrix and the "XOR plus mod" [Par1996] operation will spread out the influence of a single pixel, which is from the plain-image, over many cipher image pixels. This is detailed below.

Respectively choose the first element from Q_{b}^{kx} and the plain-image as initial value.

Q_{b}^{kx} is designed as $M(k)$ and is XOR-ed with the values of currently operated pixel (from the plain-image) and previously operated pixel (from the cipher-image), according to formula (4.1):

$$C(\mathrm{k}) = M(k) \oplus \{[I(k) + M(k)]\mathrm{mod}\, CLevel\} \oplus C(k\text{-}1) \qquad (4.1)$$

Where $I(k)$ is the currently operated pixel, $CLevel$ is the colour level ($CLevel$=256 in our experiment) of the image and $C(k\text{-}1)$ is the previously output pixel of the cipher-image. Bob may inverse the transform of the above formula as formula (4.2).

$$I(k) = \{[\ M(k) \oplus C(k) \oplus C(k\text{-}1) + CLevel - I(k)\ \}\mathrm{mod}\, CLevel \qquad (4.2)$$

We can see that the corner pixel $C(1)$ (namely the position is (1,1) in the image) is not diffused at all under this algorithm. Besides, one pixel change image may not massively alter the cipher-image, especially the change is in the last pixel (namely the utmost right-bottom pixel). Hence another diffusion stage from the last pixel in the image is needed.

The experimental result for this discretization algorithm is shown in Figure 4.4.

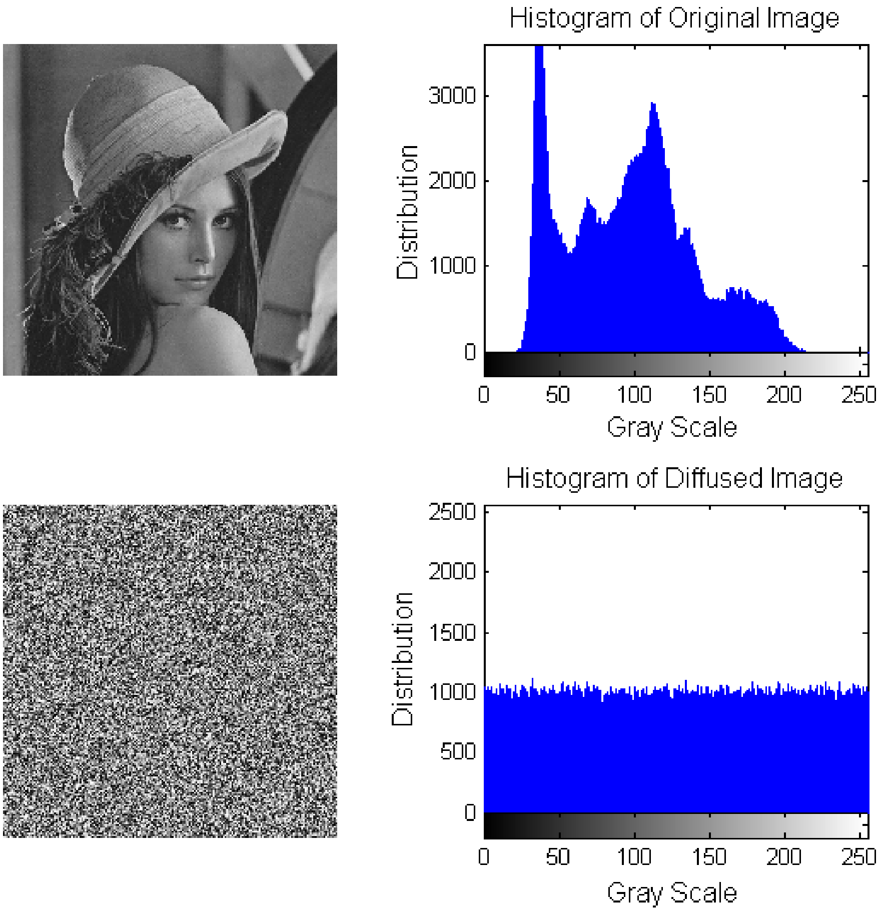


Figure 4.4 Comparison of Images after 1-round of Diffusion

Section 4.3 Security Analysis

After encryption, compared to the original image, the ciphered image shall have various differences:

(1) The position of pixels is changed by confusion. Confusion is an important technology in image cryptography;

(2) The value of pixels is changed by diffusion. From the point of view of information theory, this technology increases the information entropy of the image to a maximum value. While from the point of view of statistics, it makes the histogram of the ciphered image fairly uniform such that the encrypted image characters are hard to achieve;

(3) The correlation among the adjacent pixels is reduced.

All tests in this chapter are conducted on the 512×512 Lena image with 8-bit greyscale. The diffusion round is 2, whilst the confusion round is 1.

Section 4.3.1 Key Space Analysis

The key space of any cryptosystem shall be satisfactorily large enough to resist brute-force attack. In this proposed public-key based image encryption algorithm, key (x_a,y_a=$\boldsymbol{f}$(x_a,k_a) and (x_b,y_b=$\boldsymbol{f}$(x_b,k_b)) are solely used to employ for encryption and decryption. Hence the key space primarily lies on the size of ergodic matrix (meaning unclear). For an n×n ergodic matrix over finite field $\mathbb{F}^q$, the number of non-equivalent matrices is calculated by formula (4.3) [ZPWY2007]:

$$\prod_{i=0}^{n-1}(q^n - q^i)/n(q^n - 1) = (q^n - q)(q^n - q^2)\cdots(q^n - q^{n-1})/n \qquad (4.3)$$

In the experiments we utilize a 50×50 ergodic matrix over finite field $\mathbb{F}^{256}$, thus the key space is $\prod_{i=0}^{50-1}(256^{50} - 256^i)/(50 \times (256^{50} - 1))$≈$3.08\times10^{5898}$. This is quite large and it is sufficient for practical use, thus it can resist brute-force attack whether in time or in space.

The reason for the immense key space in the proposed scheme is that the session keys are n×n matrices, of which the range of each element is [0, 255].

From **Lemma** 2.1, any ergodic matrix $Q \in \mathbb{F}_{n\times n}^{q}$ can be denoted by an *n*-vector over $\mathbb{F}^{q}$. Thus the size of the matrix $Q \in \mathbb{F}_{50\times 50}^{256}$ used in the experiment can be reduced to 50 bytes from 2500 bytes.

Section 4.3.2 Statistical Analysis

Shannon once said in his masterpiece [Sha1949] that it is possible to solve many kinds of ciphers by statistical analysis.

Section 4.3.2.1 Histogram Analysis

Figure 4.4 shows that after 1-round diffusion, the histogram is fairly uniform and does not reveal any statistical information of the plain-image.

Section 4.3.2.2 Correlation of Two Adjacent Pixels

Correlation coefficient factor demonstrates to what extent the proposed encryption algorithm strongly resists statistical attacks. Therefore, encrypted image must be completely different from the original one.

The high correlation of adjacent pixels is fragile to resist statistical cryptanalysis. As a result, a secure encryption scheme, which can eliminate the correlation between adjacent image pixels, is needed. Hence, to calculate the correlation of two adjacent pixels, formula (4.4) is carried out:

$$cov(x,y) = \frac{\sum_{i=1}^{R}(x_i - E(x)) \times (y_i - E(y))}{\sqrt{\sum_{i=1}^{R}(x_i - E(x))^2} \times \sqrt{\sum_{i=1}^{R}(y_i - E(y))^2}} \quad (4.4)$$

where *x* and *y* are greyscale values of two adjacent pixels in the image, *E*(*x*) = $\frac{1}{R}\sum_{i=1}^{R} x$, R is the number of pairs of the adjacent pixels selected in the test. This formulate indicates $-1 \le cov(x, y) \le 1$, and with *cov*(*x*, *y*) getting closer to 0, the two adjacent pixels has less correlations.

The experiment was divided into 10 groups, each group has 10,000 pairs of pixels, i.e., *R*=10,000. The correlation distributions of two adjacent pixels in the cipher-image are tested respectively in horizontal, vertical and diagonal. Table 4.1 shows the results of the correlation coefficients of our proposed algorithm:

Table 4.1 Correlation Coefficients of Two Adjacent Pixels in the Cipher-image

Horizontal	Vertical	Diagonal
-0.00563	0.0033113	-0.0049351
0.00021	0.015407	0.009473
-0.01433	0.0071656	0.0011877
-0.00089	-0.00016373	-0.0082897
-0.0018	-0.00017429	0.016918
-0.00207	-0.0085298	0.0018506
-0.00388	-0.0046942	-0.0053316
0.010665	-0.0033411	0.01446
0.013546	0.00012736	0.0040661
0.011906	0.010477	-0.010537

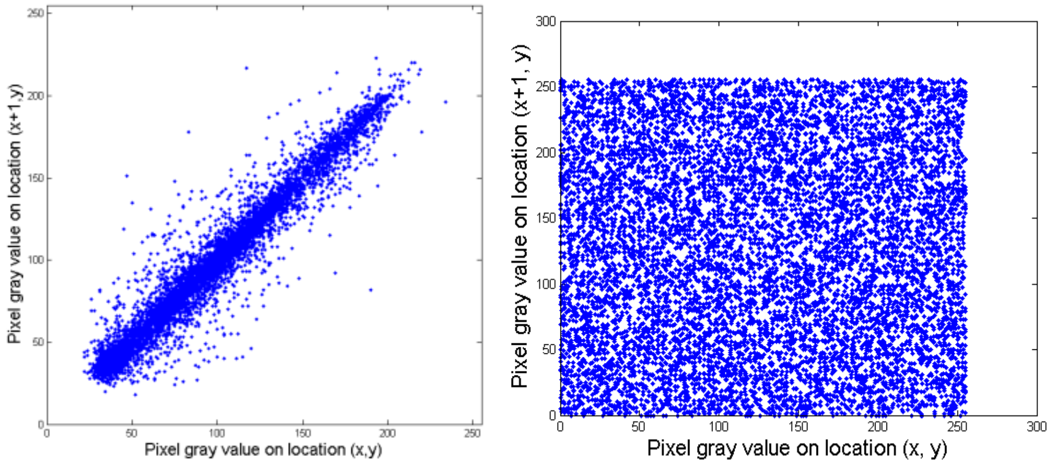
The mean correlation coefficients for our algorithm and the comparisons with other algorithms are listed in Table 4.2.

Table 4.2 Correlation Coefficients Compared in Two Algorithms

	Plain-image	Cipher-image (Proposed, mean value)	Cipher-image (Yang et al. [YWL+2010])	Cipher-image (Ye G. [Ye2010])	Arnold method ([Ye2010])
Horizontal	0.98024213	0.006493	0.002097	-0.0134	0.0787
Vertical	0.97533157	0.005339	-0.016187	0.0012	-0.0793
Diagonal	0.96573878	0.007705	0.017805	0.0398	-0.0633
Average	0.97377083	0.006512	0.01203	0.0181	0.0738

Both experiments utilized 512×512 Lena image with 256 greyscales. It is easy to see that the result of our algorithm is much closer to 0 in general. This indicates that our algorithm has effectively removed the correlation of adjacent pixels in the plain-image, thus it is better for image confusion and diffusion.

Test results for correlation of adjacent pixels are shown in Figure 4.5:



(1) Plain-image (2) Cipher-image

Figure 4.5 Correlation of Two Horizontally Adjacent Pixels in (1) and (2)

Results imply that it is very difficult to deduce secret key from cipher-image when it is attacked by known-plaintext attacks or chosen-plaintext attacks.

Section 4.3.2.3 Entropy Analysis

Entropy is a scalar value representing the entropy of a greyscale image. It is a statistical measure of randomness that can be used to characterize the texture of the input image. Entropy of an image is defined as:

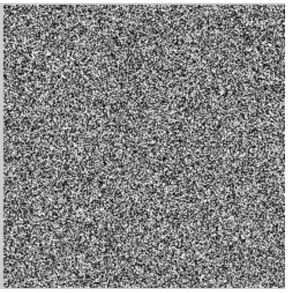
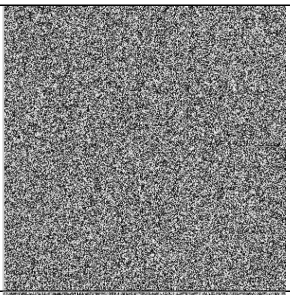
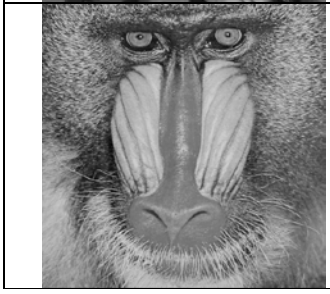
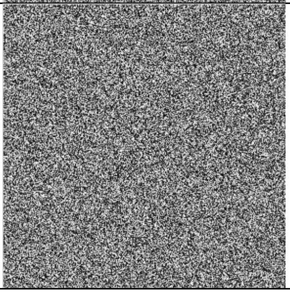
$$E=\sum_{i=0}^{n} p(x_i) log_2 p(x_i) \qquad (4.5)$$

where *p* contains the histogram counts returned from the function *imhist* in MATLAB.

The ideal value of entropy of a cipher-image shall be 8. If it is less than this value, there will be some certain predictability that threatens the security.

Table 4.3 lists the mean entropy values obtained for different original image and the ciphered ones. The obtained results are much closed to the theoretical value. This means that information leakage after 1-round permutation and 2-round diffusion is so tiny that it can be neglected.

Table 4.3 Entropy Values for Different Original Image and the Ciphered Ones

Original image	Cipher-image	Entropy of the Original-image	Entropy of the Cipher-image
		7.4767	7.9994
		7.6517	7.9992
		7.3579	7.9994

Section 4.3.3 Sensitivity-based Attack

An algorithm for encrypting an image shall be robust enough to resist sensitivity-based attack. This means the cryptosystem shall have high key sensitivity and plaintext sensitivity [LSW2005]. Further, a tiny change, even a single pixel being modified by one bit, in the key or in the original image, shall cause a great difference in the cipher-image. These properties make it difficult for diverse sensitivity-based (chosen plaintext, or differential) attacks to break the system.

Section 4.3.3.1 Plain-image Sensitivity

The plain-image sensitivity of the cryptosystem is largely affected by the keys that Alice and Bob are using. Here, two common measures are used to test the system: NPCR (Number of Pixels Change Rate) and UACI (Unified Average Changing Intensity). They are defined as formula (4.6) [CM2004]:

$$\begin{cases} \text{NPCR} = \dfrac{\sum_{k=1}^{P} D_k(pixelValue(i,j))}{H \times W} \times 100\% \\ \text{UACI} = \dfrac{\sum_{k=1}^{P} |C_1^k(i,j) - C_2^k(i,j)|}{H \times W \times 255} \times 100\% \end{cases} \qquad (4.6)$$

where H and W represent the height and the width of encrypted image. D_k(pixelValue(i,j)) is determined by the following rule: if the pixel value of $C_1(i,j)=C_2(i,j)$ then D(pixelValue(i,j))=0; otherwise D(pixelValue(i,j))=1.

NPCR measures the average intensity of different pixel numbers between two cipher-images with only one or more pixels changed. While UACI measures the percentage of pixel value differences between two cipher-images.

Experiments have been carried out on the influence of one-pixel change on a 256 greyscale Lena image of size 512×512, on the proposed cryptosystem. For higher accuracy, we randomly chose four ergodic matrices and divided the experiment into 10 groups, in each of which was arbitrary chosen 1,000 pixels (of different positions, the corresponding pixel value only increased by 1) from the cipher-image.

The values of NPCR and UACI for our algorithm are listed in Table 4.4 and Table 4.5.

Table 4.4 10 Groups of NPCR of the Proposed Algorithm

	***EM*1**	***EM*2**	***EM*3**	***EM*4**	**Average**
NPCR1	0.99614	0.99613	0.99612	0.99612	
NPCR2	0.99609	0.99609	0.99610	0.99609	
NPCR3	0.99609	0.99609	0.99611	0.99609	
NPCR4	0.99608	0.99612	0.99611	0.99609	

NPCR5	0.99608	0.99609	0.99609	0.99607	
NPCR6	0.99609	0.99608	0.99608	0.99610	
NPCR7	0.99610	0.99611	0.99608	0.99610	
NPCR8	0.99609	0.99608	0.99609	0.99612	
NPCR9	0.99610	0.99610	0.99610	0.99610	
NPCR10	0.99611	0.99611	0.99610	0.99609	
Min	0.99608	0.99608	0.99608	0.99607	0.9960775
Max	0.99614	0.99613	0.99612	0.99612	0.9961275

Table 4.5 10 Groups of UACI of the Proposed Algorithm

	***EM*1**	***EM*2**	***EM*3**	***EM*4**	**Average**
UACI1	0.33608	0.33614	0.33439	0.33412	
UACI2	0.33450	0.33454	0.33486	0.33453	
UACI3	0.33451	0.33463	0.33489	0.33454	
UACI4	0.33446	0.33465	0.33488	0.33457	
UACI5	0.33451	0.33462	0.33482	0.33455	
UACI6	0.33454	0.33461	0.3348	0.33460	
UACI7	0.33453	0.33467	0.33482	0.33455	
UACI8	0.33447	0.33463	0.33482	0.33455	
UACI9	0.33459	0.33459	0.33485	0.33450	
UACI10	0.33454	0.33460	0.33485	0.33450	
Min	0.33446	0.33454	0.33439	0.33412	0.3343775
Max	0.33608	0.33614	0.33489	0.3346	0.3354275

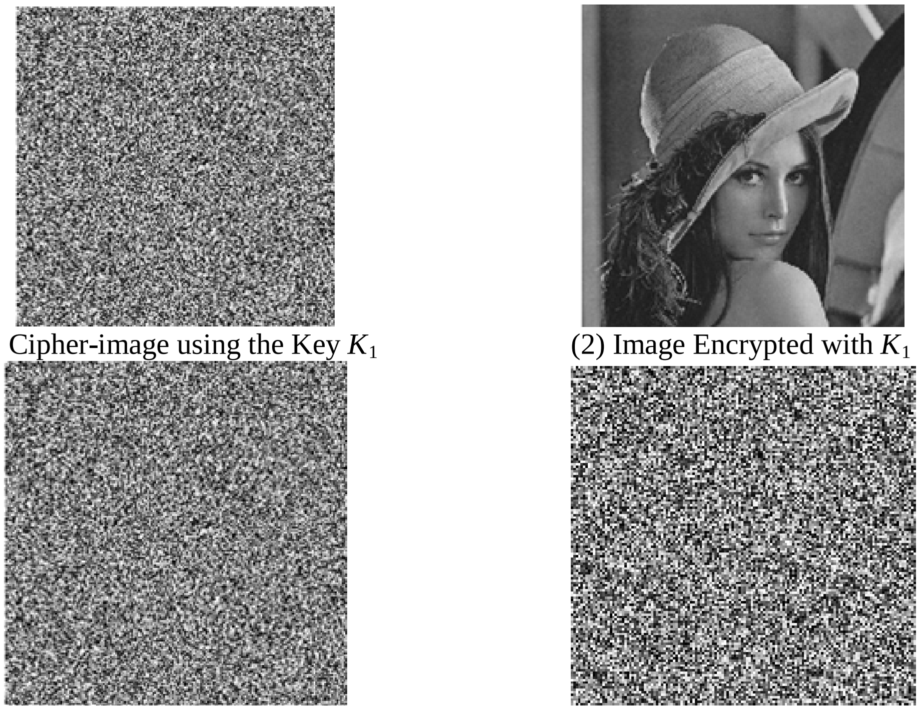
We can see from Table 4.4 and Table 4.5 the values of NPCR and of UACI is similar, respectively. It means the choice of ergodic matrix nearly has effect on the result. For more accurate results of comparisons to Yang et al.’s scheme, we calculate the average minimum and maximum of NPCR and UACI. Experimental results show that the average of the **lowest** NPCR and UACI is 0.9960775 and 0.3343775, respectively; the average of the **highest** NPCR and UACI is 0.9961275 and 0.3354275, respectively.

The NPCR and UACI of the proposed Yang et al.'s cryptosystems is 0.996185 and 0.334795, respectively [YWL+2010].

Results show that the average performance of the proposed scheme is similar to that of the scheme introduced by Yang et al. [YWL+2010]. However, our experiment was tested on the slight change of different pixels and a large number of cases (10×1,000), while Yang et al. tested their system with only one case. Thus our results are more reliable.

Section 4.3.3.2 Key Sensitivity

The key sensitivity of our proposed scheme benefits from the matrix key K_1. To evaluate, the key value is increased by 1 at a random position. The results are depicted in Figure 4.6, which shows that even a difference as small as one value incremented by 1, will result in an incorrectly decrypted image.



(1) Cipher-image using the Key K_1

(2) Image Encrypted with K_1

(3) Cipher-image using the Left-top Value of Matrix Key Increased by 1

(4) Image Encrypted with K_1

Figure 4.6 Key Sensitivity Test and the Results

It is difficult to compare the cipher-image by merely observing these images. So for comparison, NPCR and UACI are calculated between the two cipher-images using a different key which is increased by 1 at a random position.

For this calculation, we use the same formula of NPCR and UACI given in Section 4.3.3.1, except that in this test, the values of corresponding pixels in the two cipher-images are compared. The result is shown in Table 4.6.

Table 4.6 10 Groups of NPCR and UACI of Different Keys

NPCR	0.99586	0.99604	0.99608	0.99607	0.99613
	0.99612	0.99613	0.99615	0.99608	0.99607
UACI	0.33447	0.33457	0.33458	0.33446	0.33449
	0.33455	0.33453	0.33460	0.33451	0.33453

We can get from Table 4.6 that the average values of NPCR and UACI of two cipher-images is 0.996073 and 0.334529, respectively. It means only one element of the session key is increased by 1, a significant difference will occur in the cipher-image. To have more precise results, another test, which calculates the correlation between the corresponding pixels between the two cipher-images, is implemented. The correlation value is calculated by formula (4.4) in Section 4.3.2.2, except that in this test, it is the values of corresponding pixels in the two cipher-images are compared. Results are shown in Table 4.7.

Table 4.7 Correlation Coefficients with Key ${Q_b}^{k_x}$ Increased by 1 at Different Positions

position	left-top (1,1)	right-top (1,50)	left-bottom (50,1)	right-bottom (50,50)	centre (25,1)
Correlation Coefficient	-0.00217286	0.004266592	0.0026746637	-0.000910659	-0.00149085

Results show that, for example, the cipher-image by the original key K_1 has 99.78% of difference (this is better than the result from the algorithm proposed by Ismail et al.

[20], which was 99.59%) from the one encrypted by the key $K_1(1,1)+1$ in terms of pixel greyscale values, although there is only one tiny difference in the two keys.

Section 4.4 Performance Evaluation

Apart from security consideration, other issues, such as the performance speed, of an image cryptosystem also play a significant role. The performance of an algorithm is affected by many conditions such as the programmer's skill, what kind of programme language he is using, the performance of the computer he performs tests on, how many bits the operation system and the software have.

All the experiments in this thesis were carried out with MATLAB programming language as it is strong and readable in the handling of images and matrices. The implementation was done on a personal computer with a 3.20 GHz Core2Duo processor and 2 GB main memory, running with the Windows XP 32-bit operation system.

The encryption procedure consists of three main stages: Calculation of the public key Q_b, confusion and diffusion. Table 4.8 shows the CPU running time when the image was encrypted with the key Q_b^{kx} in MATLAB.

Table 4.8 Running Time (*ms*) of CPU with the Key K_1 in MATLAB

k_x(of Q_b^{kx})	Confusion stage	Diffusion stage	Calculation of Q_b^{kx}
1	15.625	242.375	0.022
100	15.625	234.750	15.625
10,000	15.625	242.375	15.625
1000,000	15.625	223.875	31.250
100,000,000	15.625	242.375	46.875
10,000,000,000	15.625	215.625	62.500

1,000,000,000,000	15.625	250.000	62.500
100,000,000,000,000	15.625	242.375	78.125

Experiment results indicate that it takes on average 200 to 400 milliseconds for the encryption and decryption stage respectively. It is obvious that the speed of confusion stage and the calculation of the power of Q_b is quite fast while the diffusion stage consumes so much time. However, in fact, the diffusion stage is pretty much the same as that proposed by Yang et al. [YWL+2010] and Ismail et al. [IA2010]. The reason for the difference is that the use of the bitwise XOR operation results in a longer delay in MATLAB than in low level languages. For example, a loop that uses the operation and repeats 512×512 times takes less than 1 millisecond to complete in C language, whereas the same loop in MATLAB takes 1.5 seconds.

To compare, we also test in C language. Table 4.9 shows the CPU running time while using C to realize our algorithm.

Table 4.9 Running Time (*ms*) of CPU with the Key K_1 in C

k_x(of Q_b^{kx})	**Confusion stage**	**Diffusion stage**	**Calculation of Q_b^{kx}**
1	1.969	2.047	0.0266
100	1.984	2.063	0.0313
10,000	1.969	2.031	0.0312
1000,000	1.953	2.047	0.0328
100,000,000	1.922	2.063	0.0328
10,000,000,000	1.953	2.063	0.0343
1,000,000,000,000	1.954	2.031	0.0360

100,000,000,000,000	1.937	2.047	0.0500
256^{50}-1	1.945	2.041	1787.6

Table 4.9 shows the running time of confusion and diffusion is quite stable (it is 1.9 to 2.0 milliseconds and 2.0 to 2.1 milliseconds, respectively). The running time of the power of a 50×50 matrix is even much faster than that of confusion and diffusion. Even the matrix Q_b to the power of 256^{50}-1, which we thought will consume much time, only takes 1.7876 seconds.

Therefore, compared to the algorithms proposed by Yang and Ismail in 2010, according to the performance evaluation, the sensitivity and statistical analysis, our proposed algorithm is more suitable for higher security purposes and also fits for network transmission.

Section 4.5 Conclusions

A hybrid-key based image encryption and authentication scheme is proposed in this chapter. Ergodic matrix, which is almost uniformly distributed, plays a central role in the encryption/decryption process. It is demonstrated that an ergodic matrix $Q \in \mathbb{F}_{50\times 50}^{256}$ can be employed to completely shuffle and diffuse the original image and has an immense key space of at least 3.08×10^{5898}. With this key space, it is robust enough for the cryptosystem to resist the brute-force attack. Normally, 1-round diffusion and 1-round permutation is enough for the encryption, but it is vulnerable to differential attack, which, however, is ineffective if a tiny change in the original image (or session key) will cause a great difference in the cipher-image. Because of this, we have applied one more round of diffusion. It is shown that, in the 2-round diffusion and 1-round permutation scheme, either a single pixel is modified by only one bit in the original image, or only one element of the session key is increased by 1, a significant difference will occur in the cipher-image.

Compared with the existing chaotic cryptosystems, the experimental tests demonstrate more optimistic results: the change rate in the number of pixels and unified average changing intensity are both higher, whilst the correlation coefficient is lower. Furthermore, the proposed algorithm takes on average 200 to 400 milliseconds for the encryption and decryption stage respectively, this performance evaluation shows that this algorithm is suitable for network transmission.

CHAPTER 5 IMAGE HIDING

We can see from the last chapter that the proposed algorithm of image cryptography has so many advantages, for example, it increases the information entropy of the image to a maximum value, makes the histogram of the ciphered image fairly uniform such that the encrypted image characters are hard to achieve, and reduces the correlation among the adjacent pixels. However, the visible encrypted messages, no matter how unbreakable, will arouse suspicion. This technology only hides the contents of the message from an attacker, rather than the existence of the message. That is to say, it is quite easy for an attacker to know that the message transmitting between Alice and Bob is important and is encrypted. Hence he will intercept the message, study it and try to deduce the message or the session keys used by Alice and Bob.

Therefore, in this chapter, we shall introduce another method to hide the very existence of the message in the communicating data.

Section 5.1 DWT-SVD Algorithms by Other Researchers

Our algorithm uses the technologies of SVD and DWT. Emir and Ahmet have drawn a conclusion in their paper [GE2004] that SVD is a very convenient tool for watermarking in the DWT domain.In this section, we shall have a review of some DWT-SVD algorithms proposed by other researchers.

Section 5.1.1 Embedding Data in All Frequencies

Emir Ganic and Ahmet M. Eskicioglu present a scheme [GE2004] to decompose the cover image at one level to generate four frequency subbands. Then apply SVD on each subband and the visual watermark, respectively. After that, modify the singular values of the cover image in each subband with the singular values of the visual watermark by a linear function. Thus the four sets of modified DWT coefficients are obtained. At the last stage, apply the inverse DWT using the four sets of modified DWT coefficients to produce the watermarked cover image. Modification in all frequencies allows the development of a watermarking scheme that is robust to a wide range of attacks.

But embedding data in all frequencies will result in massive computing time. Moreover, in order to extract the watermark, the communication process has to carry much of the information of the watermark and the carrier image.

Section 5.1.2 Embedding Data into the YUV Colour Space of the Image

Yin et al. propose a novel watermarking scheme [YLLQ2007] of embedding scrambling watermark into the green component of the colour image based on DWT-SVD. The green component is applied by DWT at *n* levels and then decomposed into four subbands, viz LL*n*, HL*n*, LH*n* and HH*n*. For each subband, different methods are adopted to watermark embedding. For LL*n*, embed pseudo-random after spreading according to the energy. Decompose the three subbands LH*n*, HL*n*, HH*n* with SVD. Modify the singular values of the three subbands with the singular values of the watermark by a linear function, and then the embedding procedure is accomplished. The retrieving watermark algorithm and the blind detecting algorithm both are designed according to the embedding scheme.

Gunjal and Mali modify Yin’s algorithm in [GM2012]. They apply DWT-SVD to all YUV (Y stands for the luminance component (the brightness) and U and V are the chrominance (colour) components) colour spaces. The algorithm is strongly secured including many security levels in watermark embedding and robust to different attacks.

However, applying SVD to the three subbands of all YUV colour spaces resulting carrying much information of watermarking in the communication process. This will occupy lots of bandwidth.

Section 5.1.3 DWT-DCT-SVD Based Watermarking

Navas et al. propose a method of non-blind transform domain watermarking based on DWT-DCT-SVD [NAL+2008]. They first apply DWT to the host image then perform DCT followed by SVD to the chosen subband (which denoted as *B*). After that, the same procedure is performed on the watermark. Let us denote the watermark after SVD as *S*, and then we modify the singular values of *B* using singular values of *S*.

This method of watermarking is found to be robust and the watermark was recovered after various attacks. Though the retrieved watermark was distorted in some cases, it was found to be discernible with reasonable accuracy. But the author doesn’t show in the paper of how “reasonable” the accuracy is.

However, the disadvantage of this algorithm is that the watermarking embedding and the extracting is time consuming because the zigzag scanning, which is for mapping the coefficients into four quadrants based on the frequency, takes up lots of time.

Section 5.1.4 DWT-SVD Based Image Watermarking Using Visual Cryptography

Kambel et al. propose a novel idea of splitting the watermark into two shares in [KMAS2012]. However, only the one share acts as a watermark while the other one acts as the secret key. Therefore, the other share is the key to reconstruct the watermark. In this case, the image authentication is very easy and fast just by superimposing the key share over the decrypted watermark image. The robustness of the technique is justified by giving analysis of the effect of attacks and still we are able to get good visual quality of the embedded watermark.

The deficiency lies in this algorithm is that it embeds watermark in the highest subbands, it will make the scheme least resistant to attacks.

More DWT-SVD based algorithms can be found in [LSF2011, DKSA2012, and KSK2012].

Section 5.2 Our Algorithms

Our algorithms are based on the algorithms proposed by Kambel et al. and Kumar et al. [KSK2012], for their papers are quite up-to-date (published in 2012) and experimental results have shown both the significant improvement in perceptibility and the robustness under possible attacks.

Section 5.2.1 Design Idea

Robustness and transparency is always a pair of contradiction in all transform domain watermarking schemes. If the watermark is embedded in the low subbands, which contain most perceptually significant components, the scheme will be robust to attacks but the watermark may be difficult to hide. On the other hand, if the watermark is embedded in the high subbands, which contain perceptually insignificant components, it will be easier to hide the watermark but the scheme may be least resistant to attacks [GE2004].

Therefore, in our research, we will design two schemes to hide the watermark in the high-high subbands and low-high subbands, to compare the performances, respectively.

Deguillaume et al. mention in their paper [DCOP1999] that in order to be an efficient part of a reliable and secure image copyright protection system, a watermarking algorithm must fulfill the following requirements:

- The watermark has to be secure: it must be robust enough to prevent an unauthorized detection or removal of the watermark;
- The watermark approach shall be possible to detect and decode the watermark without the help of the carrier image;
- The watermark has to be perceptually invisible and shall not degrade the image quality;
- The watermark detection must be reliable, with no false detection and if possible

no false rejection;

- The watermark has to resist manipulations such as photometric transformations (e.g. filtering or luminance correction), geometric transformations (e.g. rotation, scaling, cropping) and other nuisances such as blur, noise, lossy compression.

- The watermark has to be resistant to cryptographic attacks and other intentional watermark destruction attacks such as the jitter attack, mosaic attack, Stirmark, or statistical averaging attack.

Therefore, we will design an algorithm to fulfill the requirements above.

Section 5.2.2 Algorithm SoW: SVD on the Four Shares of The Watermark

This algorithm is based on the scheme proposed in [KMAS2012]. Kamble et al. split the watermark into two shares, applied 1-level DWT and performed SVD on the HH subbands. While in our algorithm, the watermark is split into four shares, 2-level DWT is applied SVD is performed on the HL and LH subbands. The reason of the watermark being split into four shares in our algorithm is that, once the watermark undergoes serious damage, we still can reconstruct the watermark by comparing the corresponding pixels of the four shares.

Algorithm SoW performs SVD on the four shares of the watermark.

Section 5.2.2.1 Communication Procedure

1. Key agreements: Alice and Bob agree on a session key Q_{ab}.
2. Alice encrypts the watermark to enhance the security.
3. Alice embeds the watermark into a carrier image.
4. Alice sends Bob the watermarked image, the encrypted U$_k$,V$_k$ of the SVD results of the four shadowed subbands in Figure 5.3, and the encrypted wU$_k$, wV$_k$ of the SVD results of the four parts of the watermark. This procedure will be described in more detail below.

5. Bob extracts the watermark from the watermarked image and uses the session key to decrypt the extracted watermark. This procedure will also be described in more detail below.

6. Bob verifies the image is from Alice.

Section 5.2.2.2 Embedding Procedure

1. Watermark pre-process

(1) Divide the watermark W into four parts:

$$W' = W_1+W_2+W_3+W_4$$

where W_1 is made of odd rows and odd columns from the original watermark, W_2 is of odd rows and even columns, W_3 is of even rows and odd columns and W_4 is of even rows and even columns.

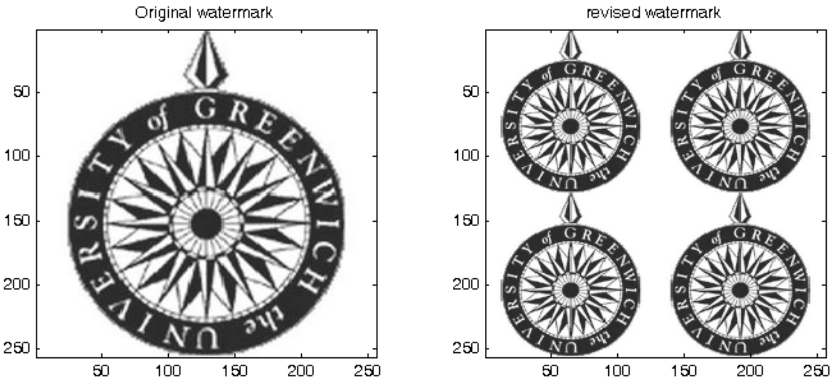


Figure 5.1 Original Watermark and the Revised One

In the right image of Figure 5.1, the upper-left, upper-right, lower-left, lower-right is denoted as W_1, W_2, W_3, W_4, respectively.

(2) The watermark is now encrypted to increase the security of the scheme. For this we apply the method, which is mentioned in Section 4.2.2, on the watermark. First we shall resize Q_{ab} to the same size of W'. Then permute the revised watermark W' with the session key by the algorithm introduced in Section 4.2.2. Figure 5.2 shows the image revised by the original one and the corresponding encrypted image.

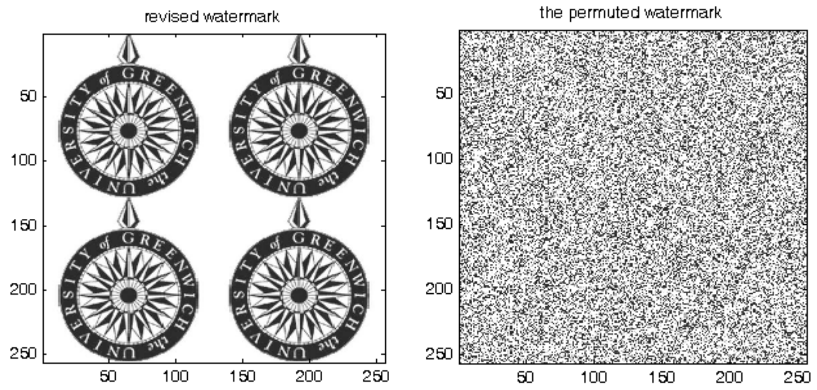


Figure 5.2 Revised Image and the Corresponding Encrypted One

(3) Apply SVD on W_1, W_2, W_3, W_4. Namely we have the formula below:

$$W_k = \mathrm{wU}_k \times \mathrm{wS}_k \times \mathrm{wV}_k^{\mathrm{T}}, \quad (k=1,2,3,4)$$

2. Watermark embedding algorithm

(1) Alice picks a carrier image *I* and applies one-level two-dimensional DWT to decompose *I* into four subbands, i.e., LL1, LH1, HL1 and HH1. Then further next one-level DWT to decompose the LH1 and HL1 subbands into four subbands, respectively. It gives eight subbands LH1_LL2, LH1_LH2, LH1_ HL2, LH1_HH2, HL1_LL2, HL1_LH2, HL1_ HL2, HL1_HH2. The four subbands LH1_LH2, LH1_ HL2, HL1_LL2, HL1_LH2 are selected for the embedding of watermark. See the four shadowed cells in Figure 5.3.

LL1		LH1_LL2	LH1_LH2
		LH1_HL2	LH1_HH2
HL1_LL2	HL1_LH2	HH1	
HL1_HL2	HL1_HH2		

Figure 5.3 Seclected Subbands for SoW

(2) Apply SVD on the selected four subbands of the carrier image:

$$\mathrm{SVD}(\mathrm{A}_k) = [\mathrm{U}_k, \mathrm{S}_k, \mathrm{V}_k^{\mathrm{T}}], \quad (k=1,2,3,4)$$

Where, *k* represents one of four subbands.

(3) SVD on the four parts of watermark

$$\mathrm{SVD}(W_k) = [\mathrm{wU}_k, \mathrm{wS}_k, \mathrm{wV}_k^{\mathrm{T}}], \quad (k=1,2,3,4)$$

Where, *k* represents one of four parts of the watermark.

(4) Modify the singular values in the four selected subbands with singular value matrices of each part of the watermark image:

$$\mathrm{S}_k' = \mathrm{S}_k + \alpha\times\mathrm{wS}_k$$

Here α (0 <α< 1) is the embedding strength, it decides the embedding proportion.

(5) Apply the inverse SVD, i.e.

$$\mathrm{A}_k' = \mathrm{U}_k \times \mathrm{S}_k{}'\times \mathrm{V}_k^{\mathrm{T}}, \quad (k=1,2,3,4)$$

(6) Perform the two-level inverse DWT by combining the subbands with the modified ones to get the watermarked image.

The embedding technique of SoW is shown in Figure 5.5.

Section 5.2.2.3 Extracting Procedure

The extracting procedure is exactly the reverse of the embedding technique, it is shown in Figure 5.6.

(1) Perform two-level Haar DWT to decompose the watermarked image I_w into four subbands: LL1, LH1, HL1 and HH1. Further obtain LH1_LH2, LH1_ HL2, HL1_LL2, HL1_LH2 subbands performing one-level DWT on LH1 and HL1 subbands.

(2) Apply SVD on the LH1_LH2, LH1_ HL2, HL1_LL2, HL1_LH2 subbands of the watermarked image, in order to get S_{wk} , i.e.,

$$\mathrm{SVD}(\mathrm{A}_{wk}) = [\mathrm{U}_{wk}, \mathrm{S}_{wk}, \mathrm{V}_{wk}^{\mathrm{T}}], \quad (k=1,2,3,4)$$

(3) Use the encrypted S_k sent by Alice to get the singular value of each part of the watermark.

$$\mathrm{wS}_k = (\mathrm{wS}_k{}'- \mathrm{S}_{wk}) / \alpha, \quad (k=1,2,3,4)$$

(4) Apply inverse SVD of the watermark with the encrypted wU_k, wS_k sent by Alice, i.e., $\mathrm{W}_k' = \mathrm{wU}_k \times \mathrm{wS}_k \times \mathrm{wS}_k, \quad (k=1,2,3,4)$

(5) Joint the results of W_k' generating in step (4) to obtain the embedded watermark:

$$W = W_1' + W_2' +W_3'+ W_4'$$

Section 5.2.3 Algorithm SoRS: SVD on the Revised Singular Value S_k

Compared to SoW, the differences in this algorithm are that:

(1) All the subbands of 2-level DWT performed on the low subband are selected for embedding watermark, and

(2) SVD is performed on the singular value revised by the watermark and the carrier image.

Section 5.2.3.1 Communication Procedure

The communication procedure is almost the same as the previous algorithm, except these two differences:

(3) The watermark is inserted in one low-low subband and three low-high subbands;

(4) At the fourth step, Alice send Bob the watermarked image, singular value S_k of the carrier image and the encrypted unitary matrices wU_k,wV_k of the four revised shadowed subbands.

Section 5.2.3.2 Embedding Procedure

1. Watermark pre-process

(1) Divide the watermark W into four parts, and then encrypted by the session key Q_{ab}. This is the same as the previous algorithm:

$$W' = W_1+W_2+W_3+W_4$$

2. Watermark embedding algorithm

(1) Alice picks a carrier image I and applies one-level two-dimensional DWT to decompose I into four subbands, i.e., LL1, LH1, HL1 and HH1. Then further next one-level DWT to decompose the LL1 subbands into four subbands, respectively. It gives four subbands LL1_LL2, LL1_LH2, LL1_ HL2 and LL1_HH2. These four subbands are selected for the embedding of watermark. See the four shadowed cells in Figure 5.4.

(2) Apply SVD on the selected four subbands of the carrier image:

$$\mathrm{SVD}(\mathrm{A}_k) = [\mathrm{U}_k, \mathrm{S}_k, \mathrm{V}_k^{\mathrm{T}}], \quad (k=1,2,3,4)$$

Where, *k* presents one of the four subbands.

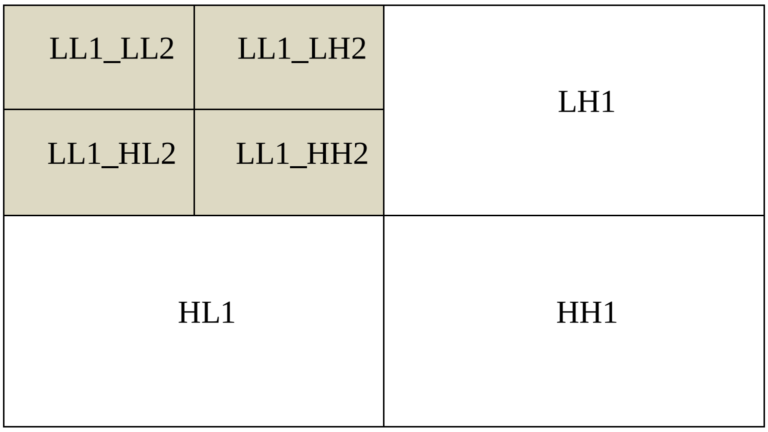


Figure 5.4 Seclected Subbands for SoRS

(3) Modify the singular values in the four selected subband with each part of the watermark image:

$$S_k' = S_k + \alpha\times W_k, \ (k=1,2,3,4,\ 0<\alpha<1)$$

Here α is the embedding strength, it decides the embedding proportion.

(4) Apply SVD to S_k':

$$SVD(S_k') = [wU_k', wS_k', wV_k'], \quad (k=1,2,3,4)$$

(5) Apply inverse SVD and obtain the four sets of modified DWT coefficients:

$$A_k = U_k \times wS_k' \times V_k^T, \quad (k=1,2,3,4)$$

(6) Perform the two-level inverse DWT by combining the subbands with the modified ones to get the watermarked image.

The embedding technique of SoRS is shown in Figure 5.7.

Section 5.2.3.3 Extracting Procedure

The extracting procedure is exactly the reverse of the embedding technique.

(1) Perform two-level Haar DWT to decompose the watermarked image I_w into four subbands: LL1, LH1, HL1 and HH1. Further obtain LL1_LL2, LL1_ HL2, LL1_LH2 and LL1_HH2 subbands by performing one level DWT on LH1 and HL1 subband.

(2) Apply SVD on the four subbands of the watermarked image, in order to get S_{wk}:

$$SVD(A_{wk}) = [U_{wk}, S_{wk}, V_{wk}^T], \quad (k=1,2,3,4)$$

(3) Use wU_k, $\mathrm{wV}_k^{\mathrm{T}}$ to recover the singular values of the four selected subbands:

$$\mathrm{wS}_k' = \mathrm{wU}_k \times \mathrm{S}_{wk} \times \mathrm{wV}_k^{\mathrm{T}}, \quad (k=1,2,3,4)$$

(4) Extract the watermark image from each subband:

$$W_k'=(\mathrm{wS}_k'- \mathrm{S}_{wk}) / \alpha, \quad (k=1,2,3,4)$$

Here α is same in embedding procedure.

(5) Joint the results of W_k' generated in step (4) to obtain the embedded watermark:

$$W= W_1' + W_2' +W_3'+ W_4'$$

The extracting technique of SoRS is shown in Figure 5.8.

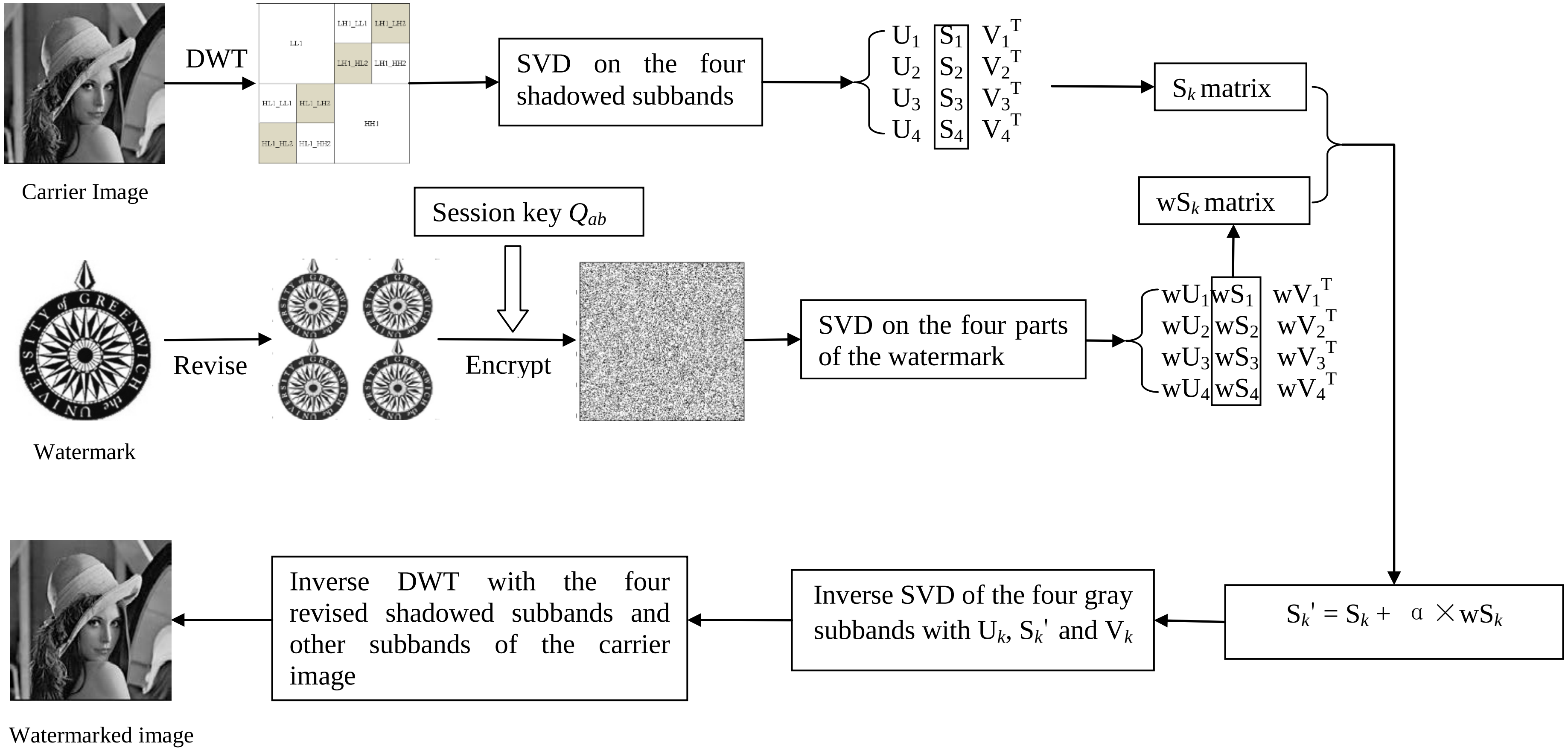


Figure 5.5 Embedding Technique using SoW Algorithm

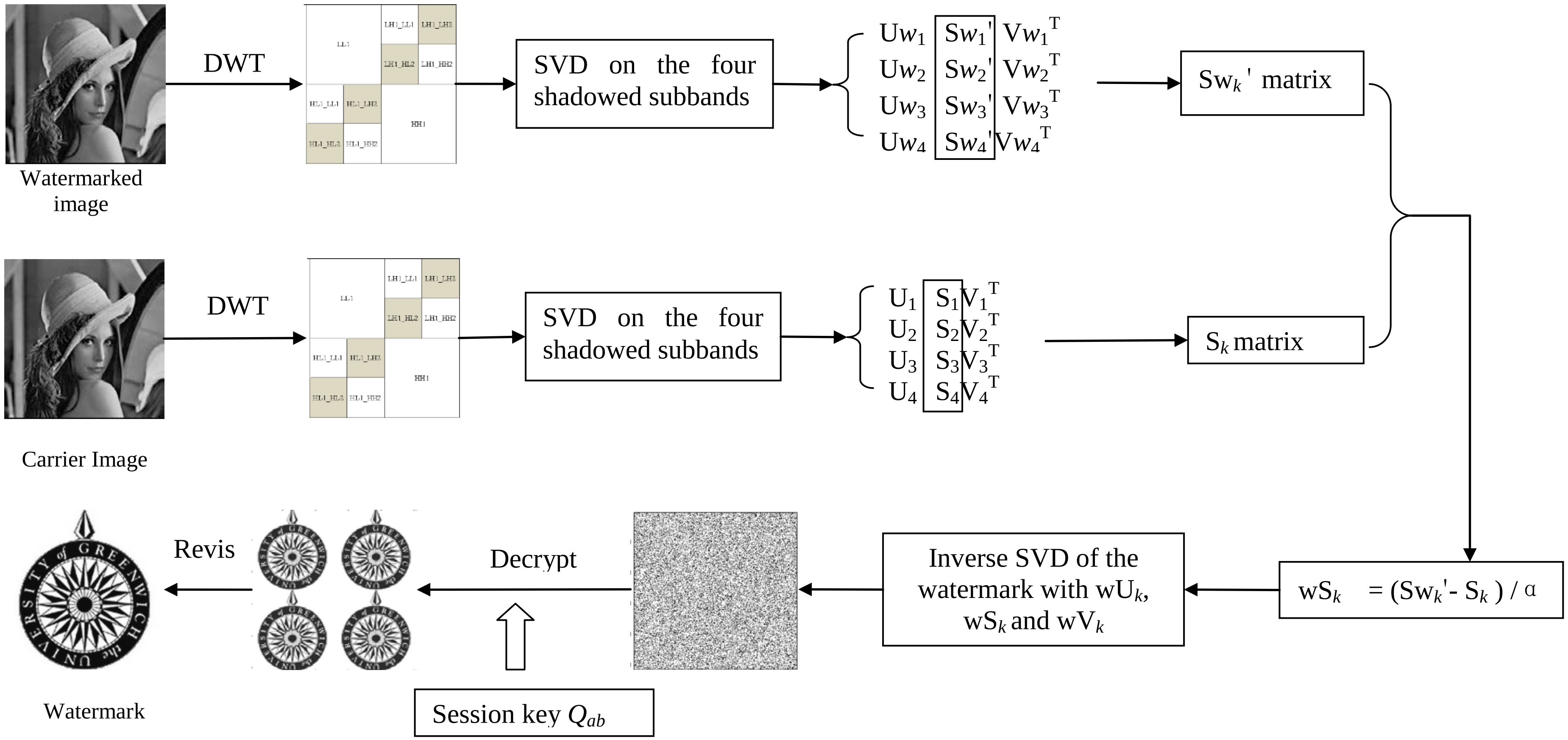


Figure 5.6 Extracting Technique using SoW Algorithm

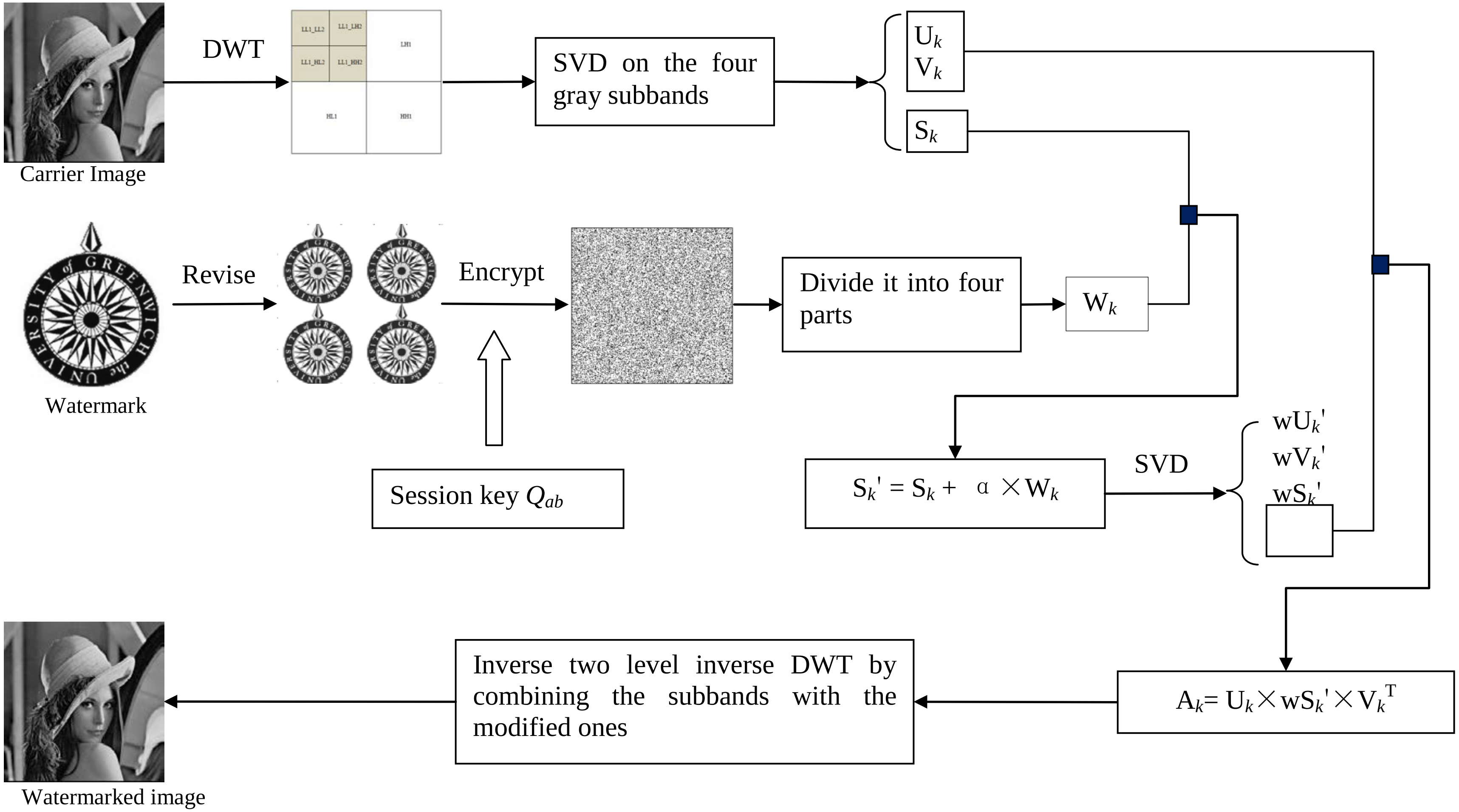

Carrier Image
DWT
LL1_LL2
LL1_LH2
LL1_HL2
LL1_HH2
LH1
HL1
HH1
SVD on the four gray subbands
U_k
V_k
S_k
Watermark
Revise
Encrypt
Session key Q_{ab}
Divide it into four parts
W_k
$S_k' = S_k + \alpha \times W_k$
SVD
wU_k'
wV_k'
wS_k'
$A_k = U_k \times wS_k' \times V_k^T$
Inverse two level inverse DWT by combining the subbands with the modified ones
Watermarked image

Figure 5.7 Embedding Technique using SoRS Algorithm

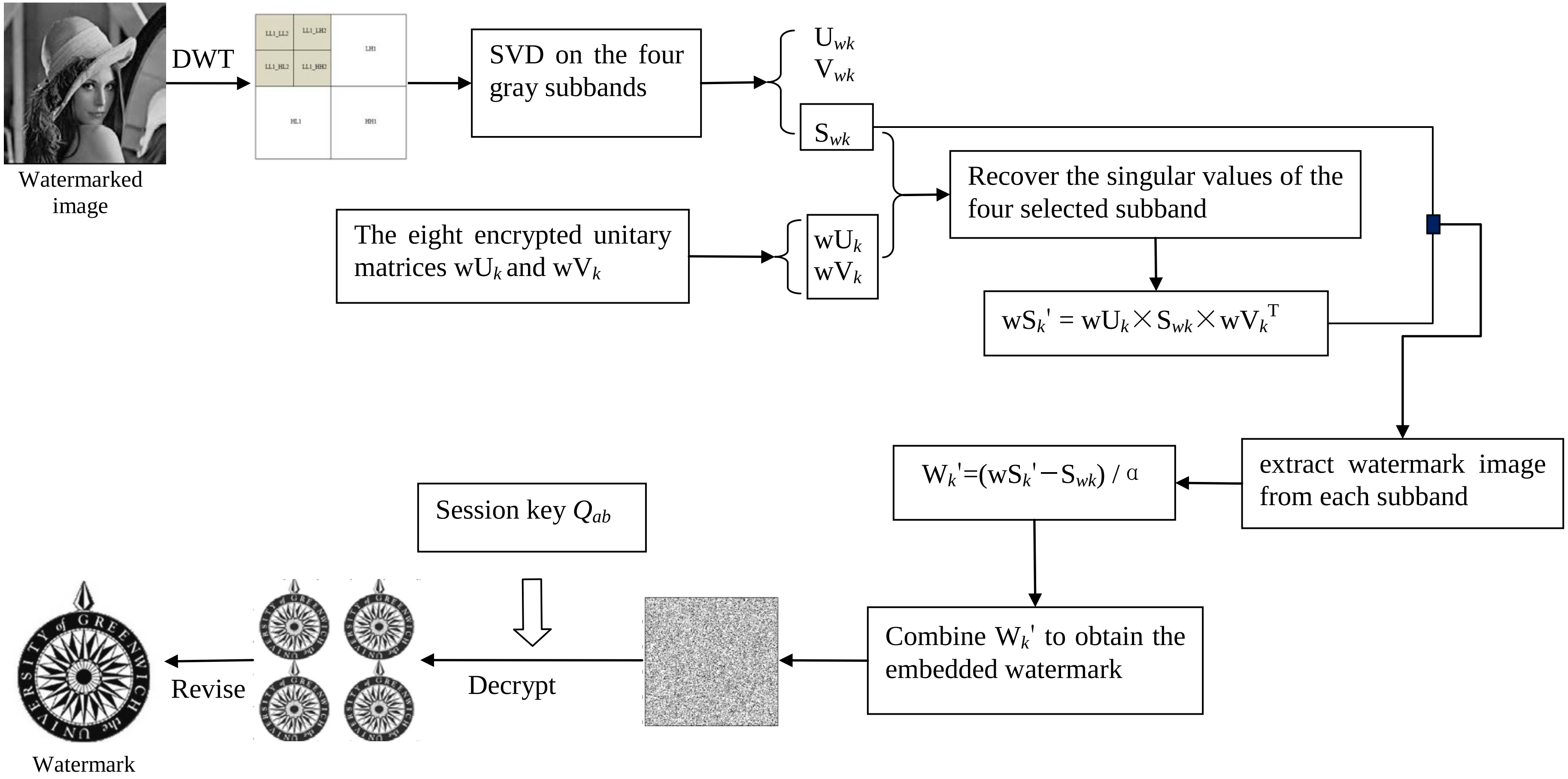

Watermarked image
DWT
LL1_LL2
LL1_LH2
LL1_HL2
LL1_HH2
LH1
HL1
HH1
SVD on the four gray subbands
U_{wk}
V_{wk}
S_{wk}
Recover the singular values of the four selected subband
The eight encrypted unitary matrices wU_k and wV_k
wU_k
wV_k
$wS_k' = wU_k \times S_{wk} \times wV_k^T$
extract watermark image from each subband
$W_k'=(wS_k'-S_{wk}) / \alpha$
Session key Q_{ab}
Combine W_k' to obtain the embedded watermark
Decrypt
Revise
Watermark

Figure 5.8 Extracting Technique using SoRS Algorithm

Section 5.3 Experimental Results

Aura proposes that greyscale images are the best cover images [Aur1996]. Therefore, all of the experiments of information hiding are tested on grey images of size 512-by-512 (as the cover image) and of size 256-by-256 (as the watermark).

Section 5.3.1 Evaluation Criterion in System Performance

Two important evaluation criterion of digital watermarking are transparency and robustness. The peak signal to noise ratio (PSNR) is applied to evaluate the transparency, which shows the quality of the watermarked image. PSNR is calculated by the following equation:

$$\mathrm{PSNR} = 10 \times \mathrm{Log}_{10} \frac{255 \times 255}{\frac{1}{\mathrm{I_H} \times \mathrm{I_W} \sum_{x=1}^{\mathrm{I_H}} \sum_{y=1}^{\mathrm{I_W}} [f(x,y) - g(x,y)]^2}} \mathrm{dB}$$

Here $\mathrm{I_H}$ and $\mathrm{I_W}$ is the height and width of an image *I*, respectively; $f(x,y)$ is the pixel value of *I* and $g(x,y)$ is the pixel value of another image *I′*. The higher value of PSNR, the less difference is between *I* and *I′*.

Another evaluation method is the normalized cross-correlation (NC) between the original watermark and the watermark extracted from the watermarked image. NC is calculated by the following equation:

$$\mathrm{NC} = \frac{1}{\mathrm{W_H} \times \mathrm{W_W}} \sum_{i=1}^{\mathrm{W_H}} \sum_{i=1}^{\mathrm{W_W}} \mathrm{W}(i,j) \times \mathrm{W}'(i,j)$$

Here $\mathrm{W_H}$ and $\mathrm{W_W}$ is the height and width of the watermark, respectively; *W* is the original watermark and *W′* is the extracted watermark. The higher value of NC indicates the better quality of the extracted watermark.

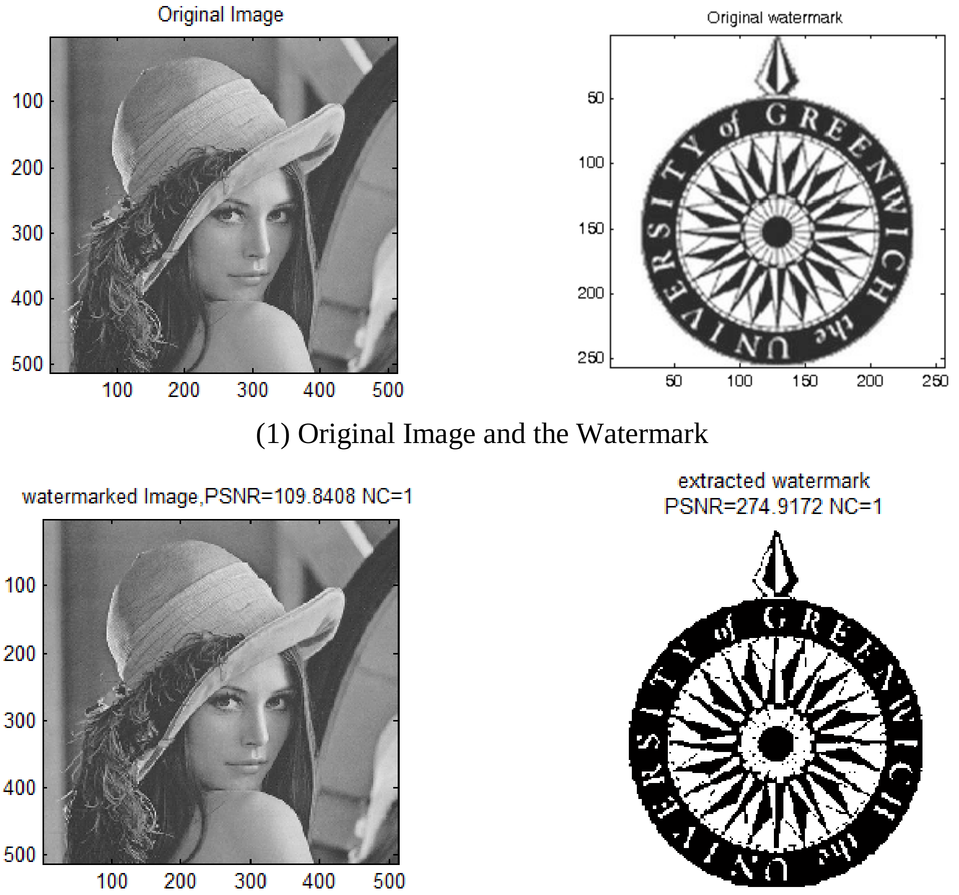
Section 5.3.2 SoW vs. SoRS

The main difference between SoW and SoRS is the SVD’s application stage. In [KMAS2012], Kamble et al. apply SVD on one share of the watermark; in [KSK2012], Kumar et al. perform three levels of DWT and apply SVD on the modified DWT coefficients.

Therefore, in order to compare the algorithms, we design two algorithms based on [KMAS2012 and KSK2012].

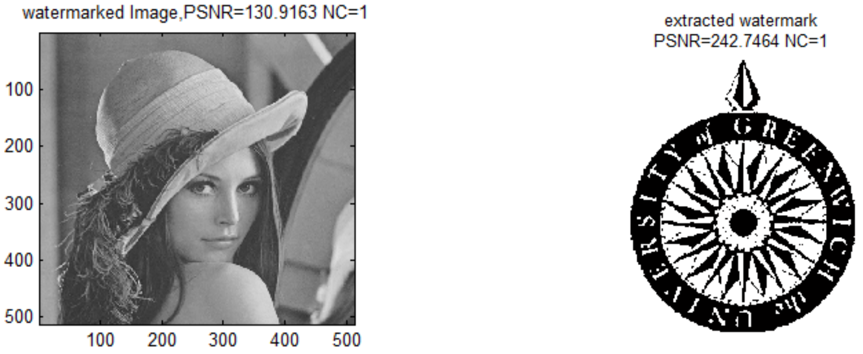
Section 5.3.2.1 Comparing PSNR between the Carrier Image and the Watermarked Image

Figure 5.9 shows the experimental results of the watermarked image and the extracted watermark performed by SoW and SoRS under the embedding strength 0.002.



(1) Original Image and the Watermark

(2) Watermarked Image and the Extracted Watermark Performed by SoW



(3) Watermarked Image and the Extracted Watermark Performed by SoRS

Figure 5.9 Watermarked Image and the Extracted Watermarked by the Two Algorithms

Figure 5.9 indicates that the PSNR and the NC of the watermarks extracted by SoW and SoRS are of not much difference. But the PSNR value of the watermarked image between these two algorithms is of great difference. The conclusion can be drawn from Table 5.1.

Table 5.1 Comparisons of PSNR between SoW and SoRS under Various Embedding Strength α

α / PSNR	0.002	0.20	0.35	0.50	0.65	0.80	0.999
SoW	109.8408	89.8408	64.98	61.882	59.6031	57.7996	55.8701
SoRS	130.9163	90.7894	85.8049	82.582	80.1838	78.2597	76.1678
Difference	19.187%	1.056%	32.048%	33.451%	34.53%	35.398%	36.33%

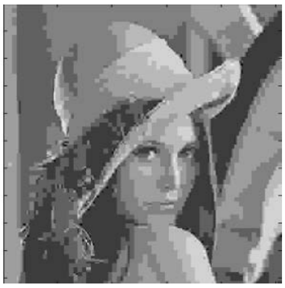
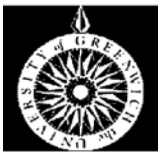
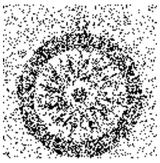
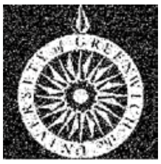
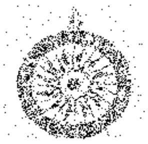
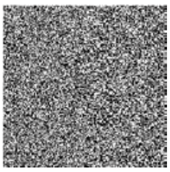
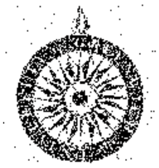
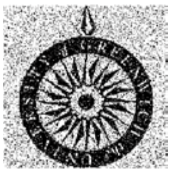
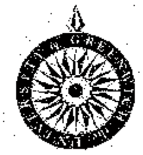
Table 5.1 shows that, in all of the test cases, the PSNR of the watermarked image performed by SoRS is higher than that of SoW. With very high embedding strength, 0.999 for example, SoRS gets 36.33% higher value of PSNR. As a result, for a better quality of transparency of the watermarked image, SoRS is preferable.

Section 5.3.2.2 Comparing PSNR and NC between the Original Watermark and the Extracted Watermark upon Various Attacks

- **JPEG compression attack:**

The JPEG compression algorithm is designed to compress image files created using the Joint Photographic Experts Group (JPEG) standard.

Table 5.2 PSNR and NC between the Original Watermark and the Extracted Watermark upon JPEG Compression Attack

Attacked Image	Extracted Watermark		PSNR		NC	
	SoW	SoRS	SoW	SoRS	SoW	SoRS
 1			48.1308	55.6145	0.0000	0.8777
 40			48.6733	61.2184	0.1835	0.9652
 80			51.6574	63.9353	0.6387	0.9810
 100			53.4655	70.0117	0.7593	0.9952

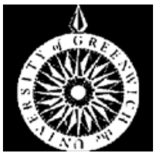
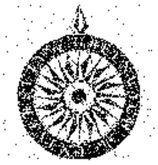
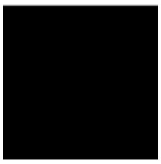
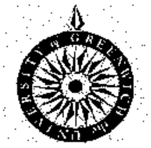
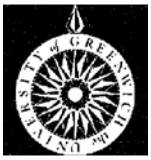
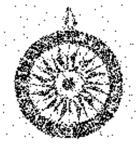
From Table 5.2 we see that the NC of SoRS is higher than that of SoW. SoW shows better visual perceptibility upon strong JPEG compression attack, but with the quality factor of the attack getting higher, SoW cannot extract the watermark properly, especially when the compression quality factor is 80.

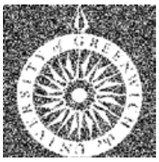
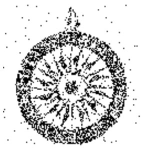
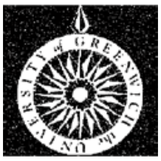
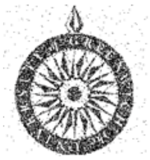
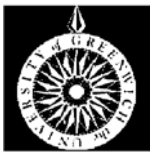
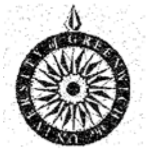
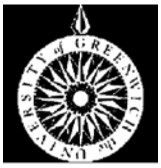
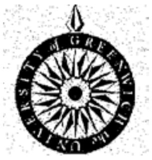
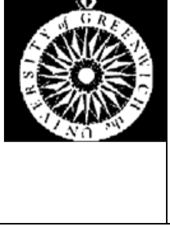
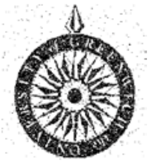
- **Filter attacks:**

Digital filter is a system that performs mathematical operations on a signal to reduce or enhance certain aspects of that signal.

Various filtering attacks, including Gaussian low-pass filter, average filter and median filter, are performed to test the robustness of these two proposed algorithms.

Table 5.3 PSNR and NC between the Original Watermark and the Extracted Watermark upon Filtering Attack

Attack Types	Attacked Image	Extracted Watermark		PSNR		NC	
		SoW	SoRS	SoW	SoRS	SoW	SoRS
Gaussian Low-pass Filter	 3,0.5			48.1308	62.6053	0.0000	0.9742
	 3,0.1			49.8408	70.9174	NaN	0.9961
	 3,0.8			48.1404	58.8778	0.0047	0.9421

	 10,0.5			49.4063	59.5672	0.4069	0.9500
Average Filter	 3			48.2746	57.712	0.0663	0.9262
	 10			48.1308	57.5607	0.0000	0.9238
	 50			48.1308	53.2816	0.0000	0.7409
Median Filter				48.1308	58.2699	0.0000	0.9342

In Table 5.3, NaN stands for *Not a Number*. It indicates the extracted watermark is nothing like the original one.

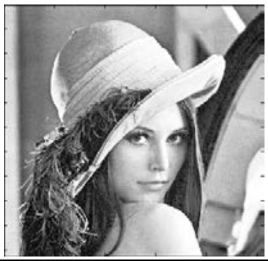
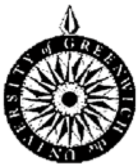
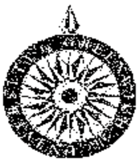
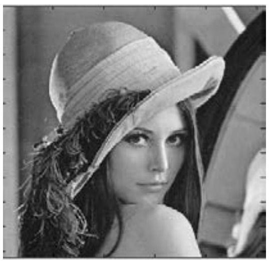
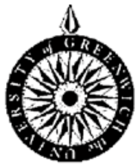
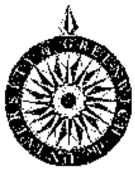
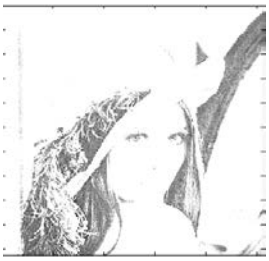
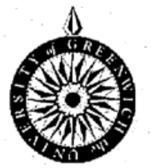
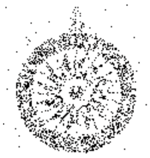
From this table we can see that the performance of SoRS is better than that of SoW whether in PSNR or NC. Most of the extracted watermarks performed by SoW are negative images, but the visual perceptibility is even better than those performed by SoRS.

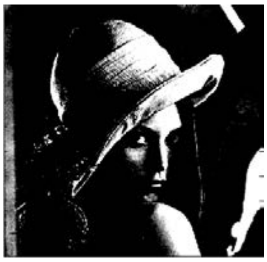
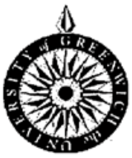
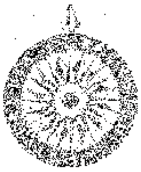
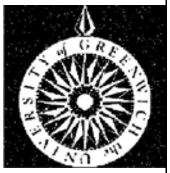
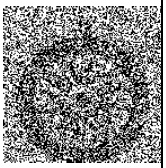
In a nutshell, SoRS shows better robustness than SoW against filter attacks.

- **Contrast adjustment attacks:**

Contrast is determined by the difference in the colour and brightness of the image.

Table 5.4 PSNR and NC between the Original Watermark and the Extracted Watermark upon Contrast Adjustment Attacks

Attack Types	Attacked Image	Extracted Watermark		PSNR		NC	
		SoW	SoRS	SoW	SoRS	SoW	SoRS
Histogram Equalization				Inf	61.6018	1.0000	0.9683
Contrast Increasing	 0.1-0.9			Inf	69.8908	1.0000	0.9951
	 0.1-0.2			81.2441	55.4150	0.9996	0.8842

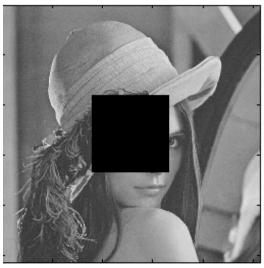
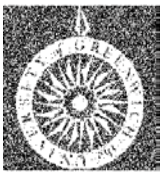
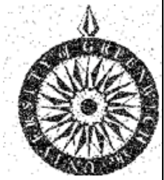
	 0.5-0.6			89.3059	57.5508	0.9999	0.9247
	 0.8-0.9			48.1746	52.4435	0.0211	0.6842

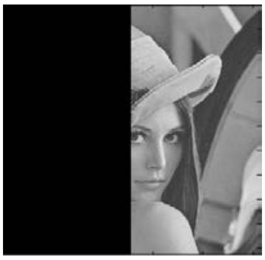
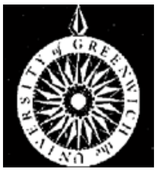
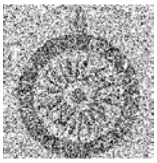
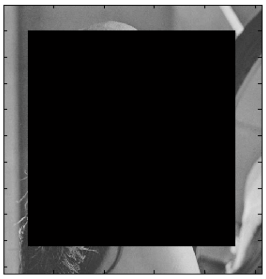
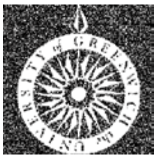
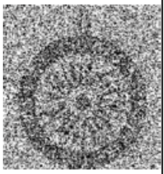
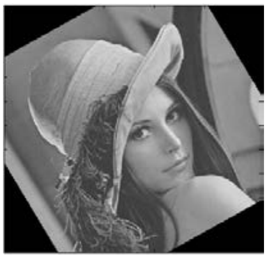
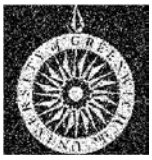
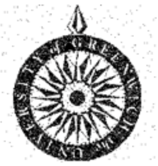
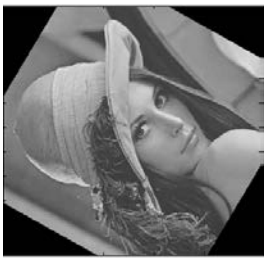
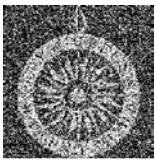
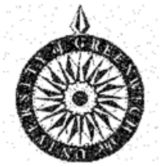
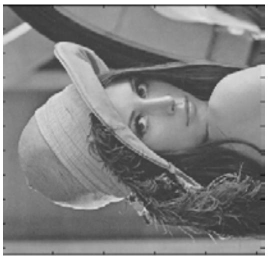
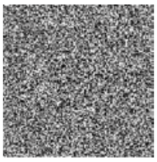
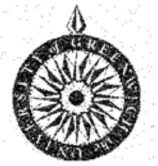
Inf in this table represents infinite value, it means the extracted watermark is nearly the same as the original one. In this test, SoW beats SoRS. Even though the NC of SoW is much lower than that of SoRS when the contrast parameter is 0.8-0.9, from the visual perception aspect, the extracted watermark performed on SoW is easier to perceive.

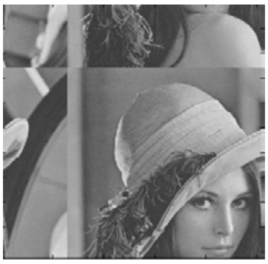
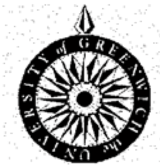
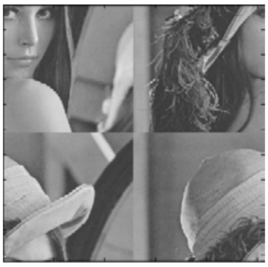
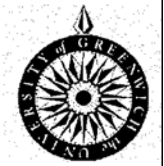
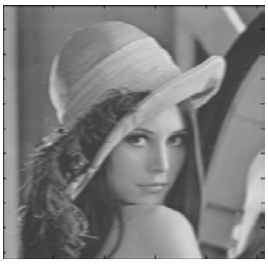
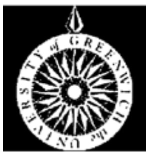
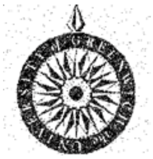
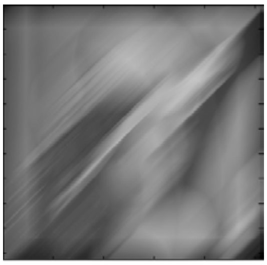
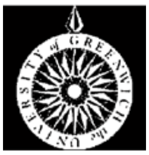
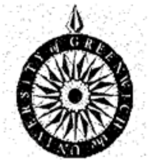
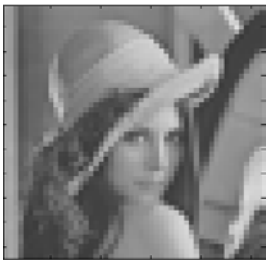
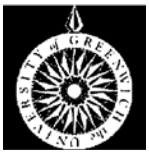
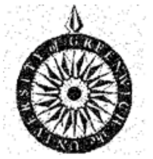
- **Geometrical transform attacks:**

Geometric transformation of images is a class of operations that alter the spatial relationships between the pixels.

Table 5.5 PSNR and NC between the Original Watermark and the Extracted Watermark upon Geometrical Transform Attacks

Attack Types	Attacked Image	Extracted Watermark		PSNR		NC	
		SoW	SoRS	SoW	SoRS	SoW	SoRS
Cropping	 30%			49.3715	58.6718	0.3966	0.9396

	 50%			48.1323	53.1162	0.0007	0.7607
	 80%			48.9106	52.6142	0.2859	0.7134
Rotation	 30			48.6866	64.6969	0.1668	0.9840
	 60			49.6073	63.8976	0.3586	0.9808
	 90			51.1404	62.8536	0.5807	0.9756

Circular Shift	 128,128			49.8408	70.9174	NaN	0.9961
	 256,256			49.8408	57.712	NaN	0.9961
Motion Blurred	 9,0			48.1308	57.0157	0.0000	0.9568
	 128,45			48.1308	62.0078	0.0000	0.9929
Scale	 0.1			48.1308	56.0903	0.0000	0.9774

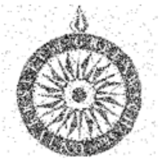
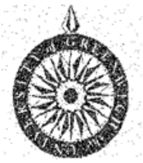
	 0.5			48.1308	59.2653	0.0000	0.9030
	 10			48.1308	64.5725	0.0000	0.9442

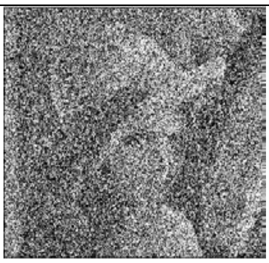
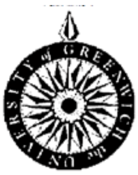
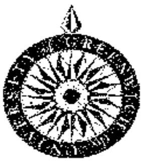
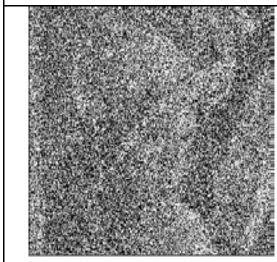
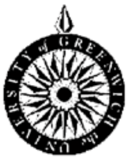
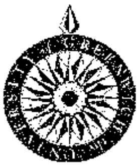
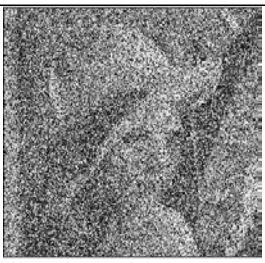
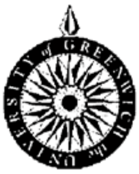
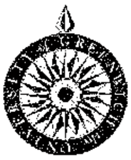
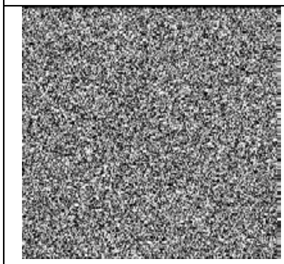
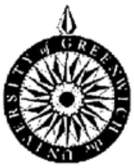
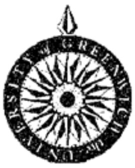
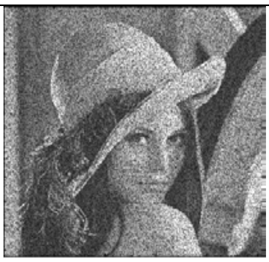
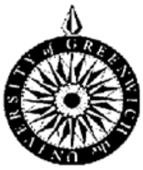
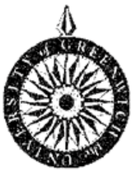
Table 5.5 indicates RoRS performs nicely against most of the geometrical distortion attacks. But SoW performs better than SoRS in cropping attack, even the NC is nearly zero, the extracted watermark performed on SoW is easier to perceive.

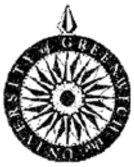
- **Noise addition attacks:**

Image noise is random variation of brightness or colour information in images. Gaussian noise is a standard model of amplifier noise, salt-and-pepper noise will have dark pixels in bright regions and bright pixels in dark regions, while speckle noise in conventional radar results from random fluctuations in the return signal from an object that is no bigger than a single image-processing element. It increases the mean grey level of a local area [TM2009].

Experimental results from Table 5.6 show the strong robustness of these two proposed algorithms against noise addition attacks. But SoW slightly beats SoRS.

Table 5.6 PSNR and NC between the Original Watermark and the Extracted Watermark upon Noise Addition Attacks

Attack Types	Attacked Image	Extracted Watermark		PSNR		NC	
		SoW	SoRS	SoW	SoRS	SoW	SoRS
Gaussian Noise	 0.5			Inf	67.9833	1.0000	0.9924
	 0.99			Inf	68.2884	1.0000	0.9929
Salt-pepper Noise	 0.5			Inf	68.6018	1.0000	0.9934
	 0.99			Inf	68.4210	1.0000	0.9931
Speckle Noise	 0.2			Inf	68.1531	1.0000	0.9927

	 0.5			Inf	68.4494	1.0000	0.9932
	 0.8			Inf	68.1665	1.0000	0.9927

For better comparison, we draw one graph for PNSR and another one for NC. We replace *Inf* with 100 and *NaN* with -1.

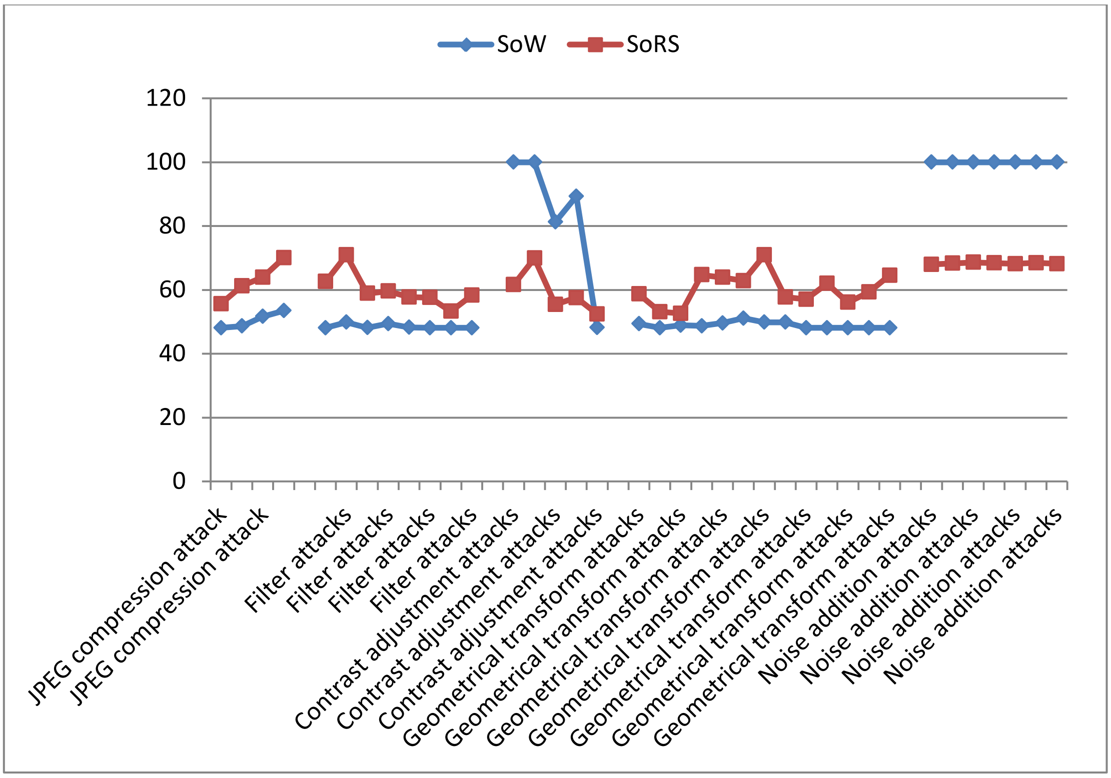


Figure 5.10 PSNR of the Watermark Extracted from SoW and SoRS

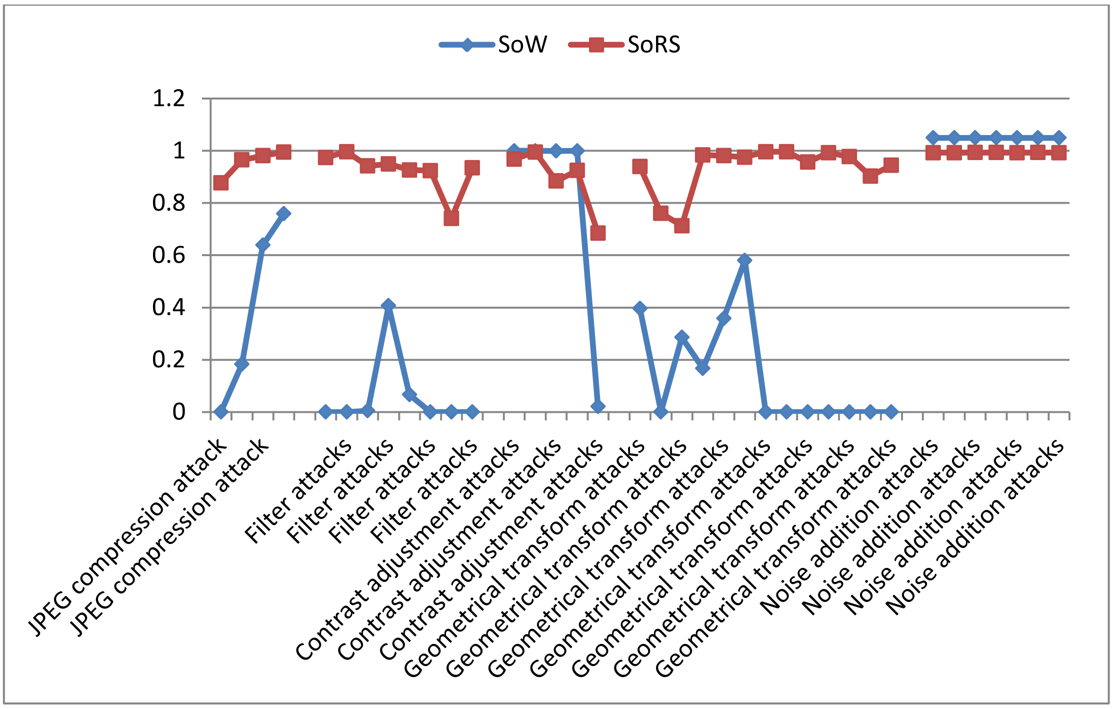


Figure 5.11 NC of the Watermark Extracted from SoW and SoRS

From Figure 5.10 and Figure 5.11, we see that compared to SoW, SoRS performs better and shows more stability in most cases, even though SoW performs much better than SoRS when extracting watermark from the image after noise (Gaussian noise, salt-pepper noise, speckle noise) attacks. This discovery is an interesting and abnormal one. We tried to find the reasons behind the experiments, only to found out it has to be explained in mathematical way (because SVD is about the transform of matrix), but is not actually what we focus on in the research, so we just show the experiments.

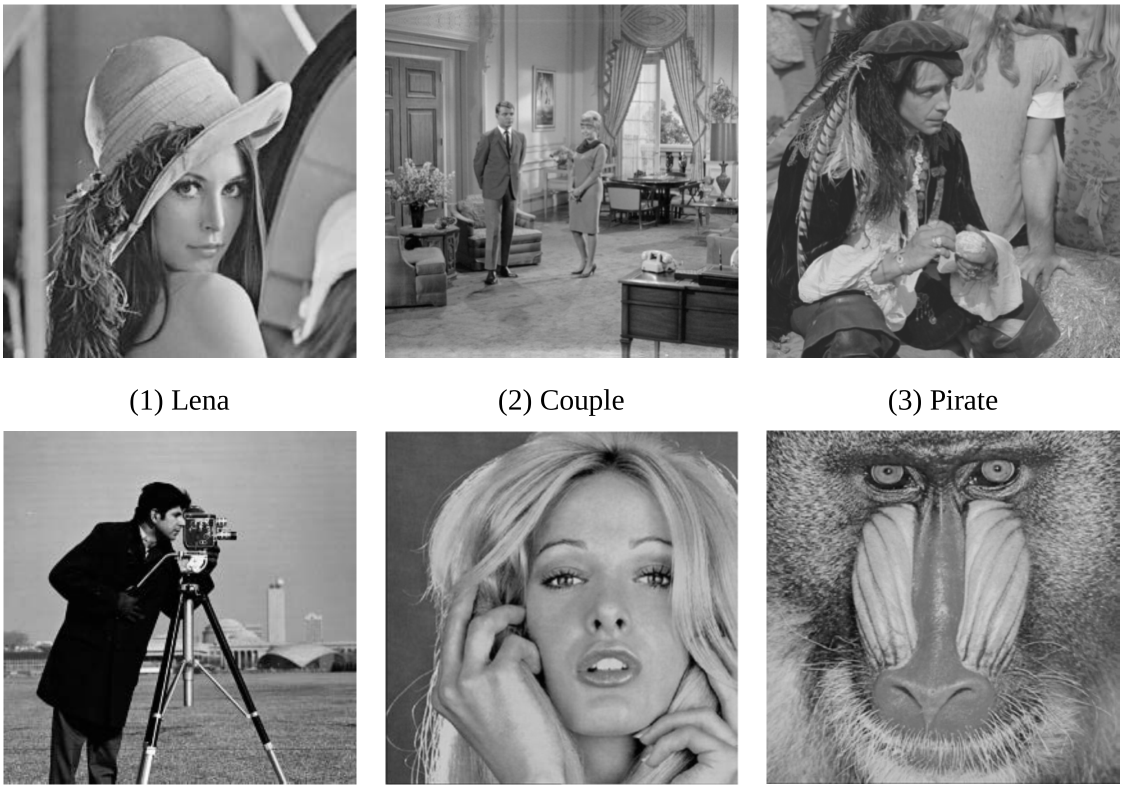
Therefore, in general, SoRS has better results of the extracted watermark because it performs more stable against most of the attacks. Hence for a long-term and general application, SoRS is more suitable.

Section 5.3.3 SoRS vs. Other Algorithms

In order to authenticate the performance of the proposed technique, SoRS is compared with other competing algorithms.

Section 5.3.3.1 SoRS vs. KMAS

We evaluated the proposed watermarking scheme, by testing on several 512-by-512 still images, see Figure 5.12. The watermark used for test can be seen in Figure 5.13.



(1) Lena (2) Couple (3) Pirate

(4) Cameraman (5)Blonde (6) Mandrill

Figure 5.12 Standard Carrier Images

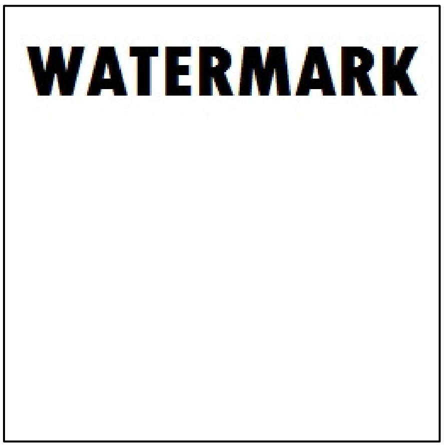


Figure 5.13 Watermark used in the Experiments

In Kamble et al.’s algorithm [KMAS2012], the PSNR, which obtained between carrier image and watermarked image for all standard test images, has 109.5099 as the minimum value and 109.5165 as the maximum value.

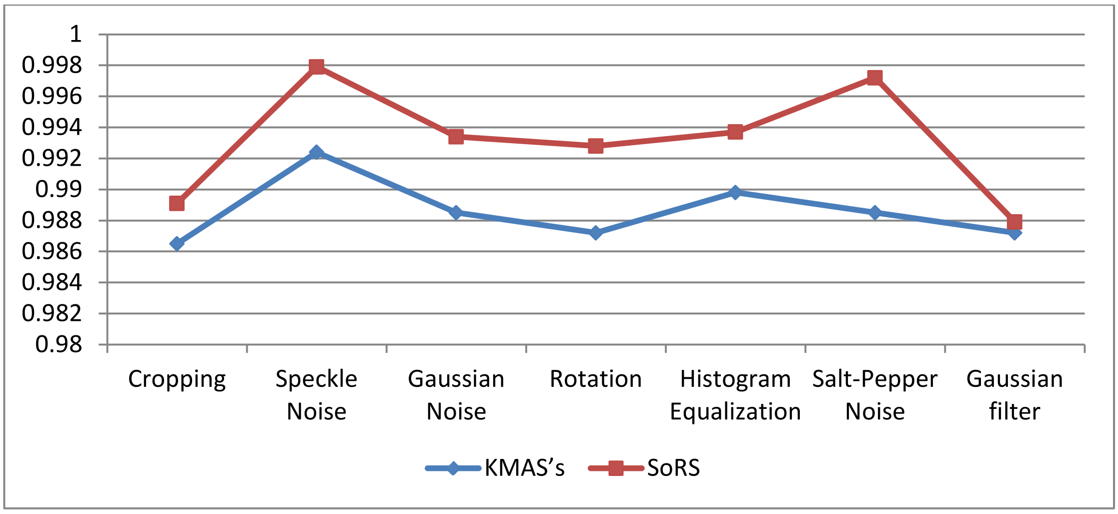
But the value of PSNR is decided by the embedding strength α. By experiment, PSNR of the proposed algorithm can reach 73.9 if α=0.999999 and 235.4 if α=0.00000001. Thus it is meaningless to compare PSNR obtained between KMAS’s algorithm and the proposed one.

Therefore, the performance of the algorithms can be compared by NC between embedded/extracted watermarks. In this case, the embedding strength of the proposed algorithm α=0.002 with the average PSNR value around 129 on all six standard test images.

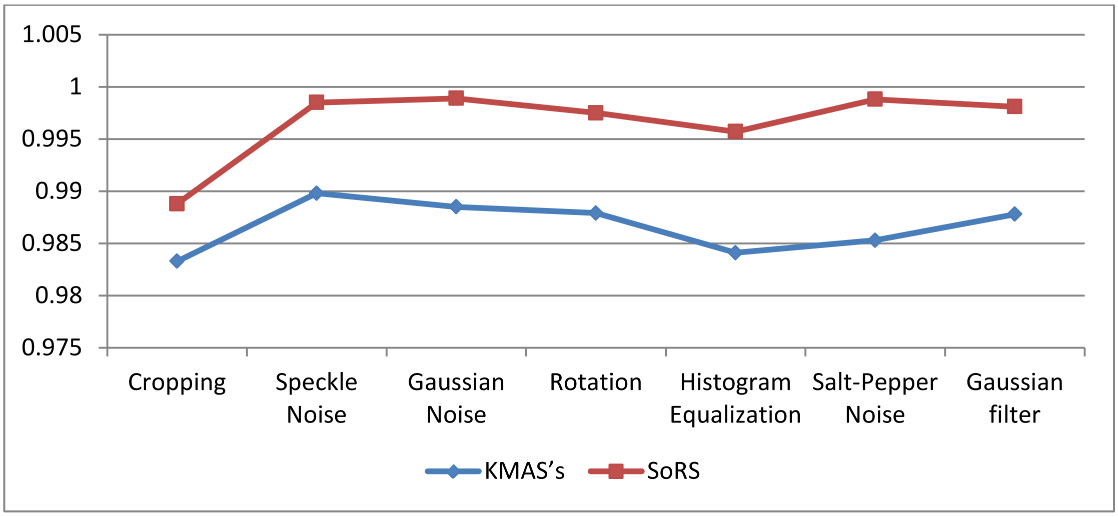
Table 5.7 and Figure 5.14 shows the proposed algorithm is more robust against different attacks.

Table 5.7 Comparisons of NC between Embedded and Extracted Watermark Performed by KMAL's Algorithm and SoRS

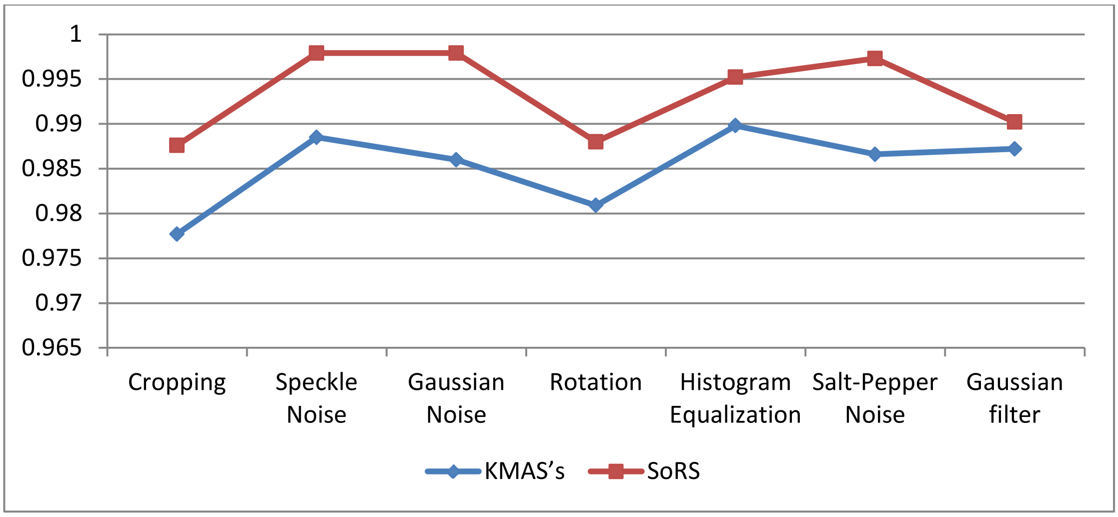
Image Attacks	Lena		Couple		Pirate		Cameraman		Blonde		Mandrill	
	KMAS's	SoRS	KMAS's	SoRS	KMAS's	SoRS	KMAS's	SoRS	KMAS's	SoRS	KMAS's	SoRS
Cropping	0.9865	0.9891	0.9833	0.9888	0.9777	0.9876	0.9865	0.9905	0.9869	0.9918	0.9914	0.9909
Speckle Noise	0.9924	0.9979	0.9898	0.9985	0.9885	0.9979	0.9911	0.9990	0.9894	0.9981	0.9874	0.9977
Gaussian Noise	0.9885	0.9934	0.9885	0.9989	0.9860	0.9979	0.9885	0.9992	0.9864	0.9987	0.9859	0.9980
Rotation	0.9872	0.9928	0.9879	0.9975	0.9809	0.9880	0.9847	0.9976	0.9834	0.9845	0.9891	0.9921
Histogram Equalization	0.9898	0.9937	0.9841	0.9957	0.9898	0.9952	0.9879	0.9981	0.9864	0.9936	0.9874	0.9942
Salt-Pepper Noise	0.9885	0.9972	0.9853	0.9988	0.9866	0.9973	0.9879	0.9991	0.9889	0.9984	0.9884	0.9983
Gaussian filter	0.9872	0.9879	0.9878	0.9981	0.9872	0.9902	0.9866	0.9932	0.9859	0.9925	0.9909	0.9914
Resize	0.9885	0.98594	0.9872	0.9851	0.9872	0.9814	0.9885	0.9907	0.9929	0.9826	0.9934	0.9833
MotionBlurred	0.9815	0.9877	0.8177	0.9834	0.9838	0.9864	0.9140	0.9899	0.9864	0.9984	0.8980	0.9876



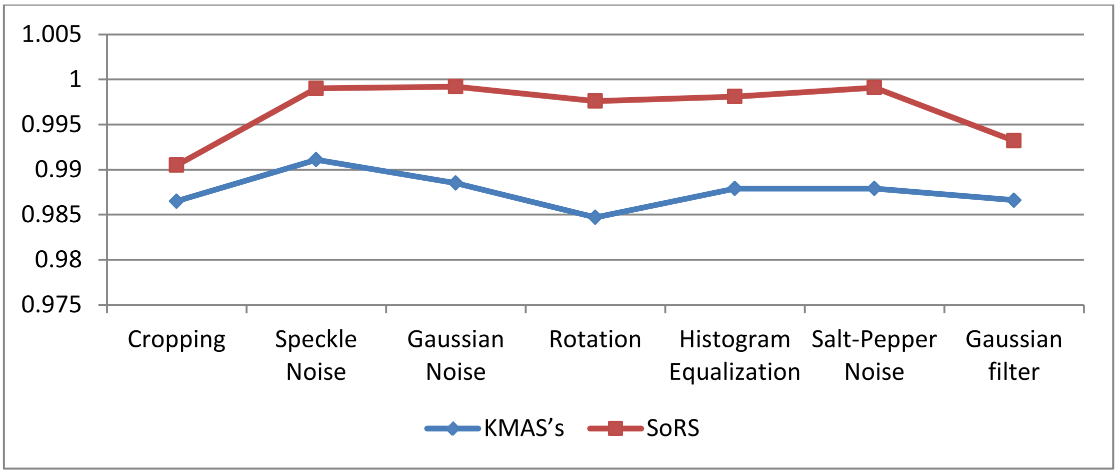
(1) Lena



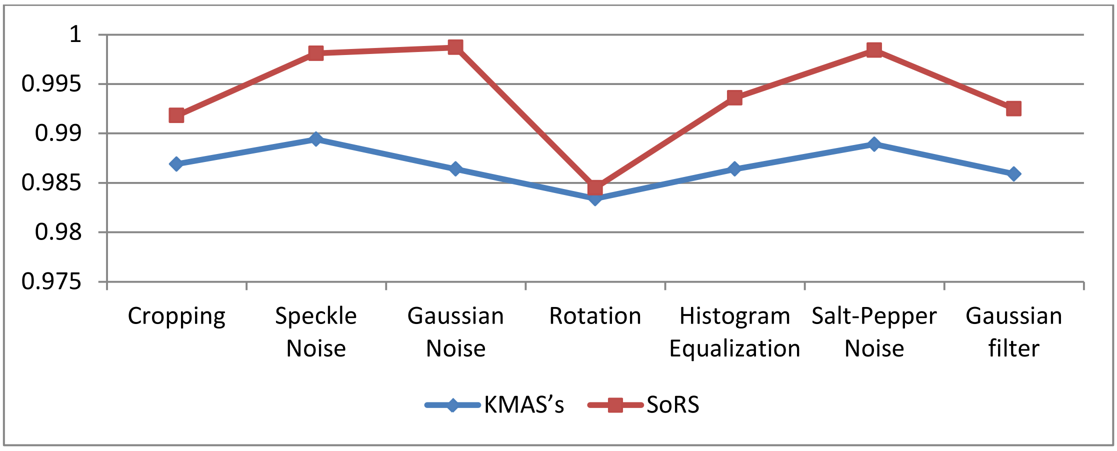
(2) Couple



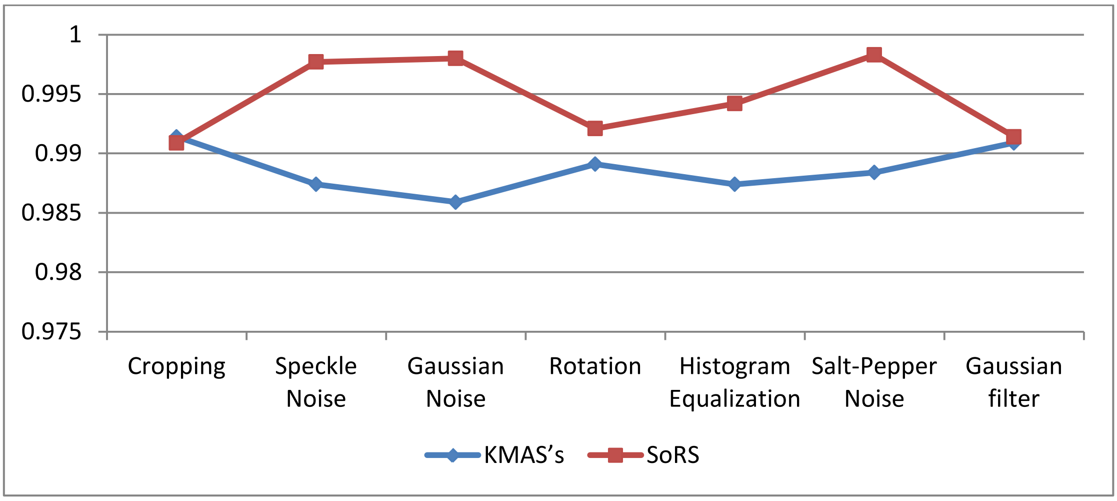
(3) Pirate



(4) Cameraman



(5) Blonde



(6) Mandrill

Figure 5.14 Comparison of NC between KMAS's Algorithm and SoRS Performed on Different Standard Carrier Images

Section 5.3.3.2 Compare the SoRS to the Other Algorithms

We also evaluated the proposed watermarking scheme, by comparing to [WL2004, LLN2006, LWH+2009 and HHDT2010]. See Table 5.8 and Figure 5.15.

Table 5.8 Compare the Robustness of the Proposed Method with [WL2004, LLN2006, LWH+2009 and HHDT2010]

Attack/NC	Wang et al.[WL2004] (PSNR=38.2dB)	Li et al.[LLN2006] (PSNR=40.6dB)	Lin et al.[LWH+2009] (PSNR=40.6dB)	Hajizadeh et al.[HHDT2010] (PSNR=40.6dB)	Proposed Method (PSNR=76.8767dB)
Median Filter(3×3)	0.51	0.35	0.9	0.86	0.92214
Median Filter(5×5)	NA	NA	0.53	0.52	0.89818
JPEG(QF=10)	NA	0.15	0.34	0.49	0.90442
JPEG(QF=20)	NA	0.34	0.67	0.77	0.91345
JPEG(QF=30)	0.15	0.52	0.82	0.91	0.92378
JPEG(QF=50)	0.26	0.52	0.96	0.99	0.93611
JPEG(QF=70)	0.57	0.63	0.97	1	0.95756
JPEG(QF=90)	1	0.78	0.99	1	0.98013
Rotation(0.25°)	0.37	0.46	0.59	0.74	0.93515
Rotation(-0.25°)	0.32	0.5	0.6	0.71	0.93926
Cropping(1/4)	NA	0.61	0.66	0.73	0.89689
Scaling(256×256)	NA	0.35	0.88	0.58	0.49684
Average Filter(3×3)	NA	NA	0.95	0.88	0.87376
Average Filter(7×7)	NA	NA	0.47	0.44	0.85786
Histogram Equalization	NA	NA	0.79	0.89	0.93649

From Table 5.8 and Figure 5.15, we can see the robustness of watermarked image against the applied attacks in the proposed method is higher than the algorithm proposed by Wang et al. [WL2004], Li et al. [LLN2006], Lin et al. [LWH+2009] and Hajizadeh et al. [HHDT2010], in most cases, except that in the scaling attack, SoRS performs a little bit worse than [LWH+2009 and HHDT2010]. The NC value is 43.5% and 14.3% lower than these two algorithms, respectively.

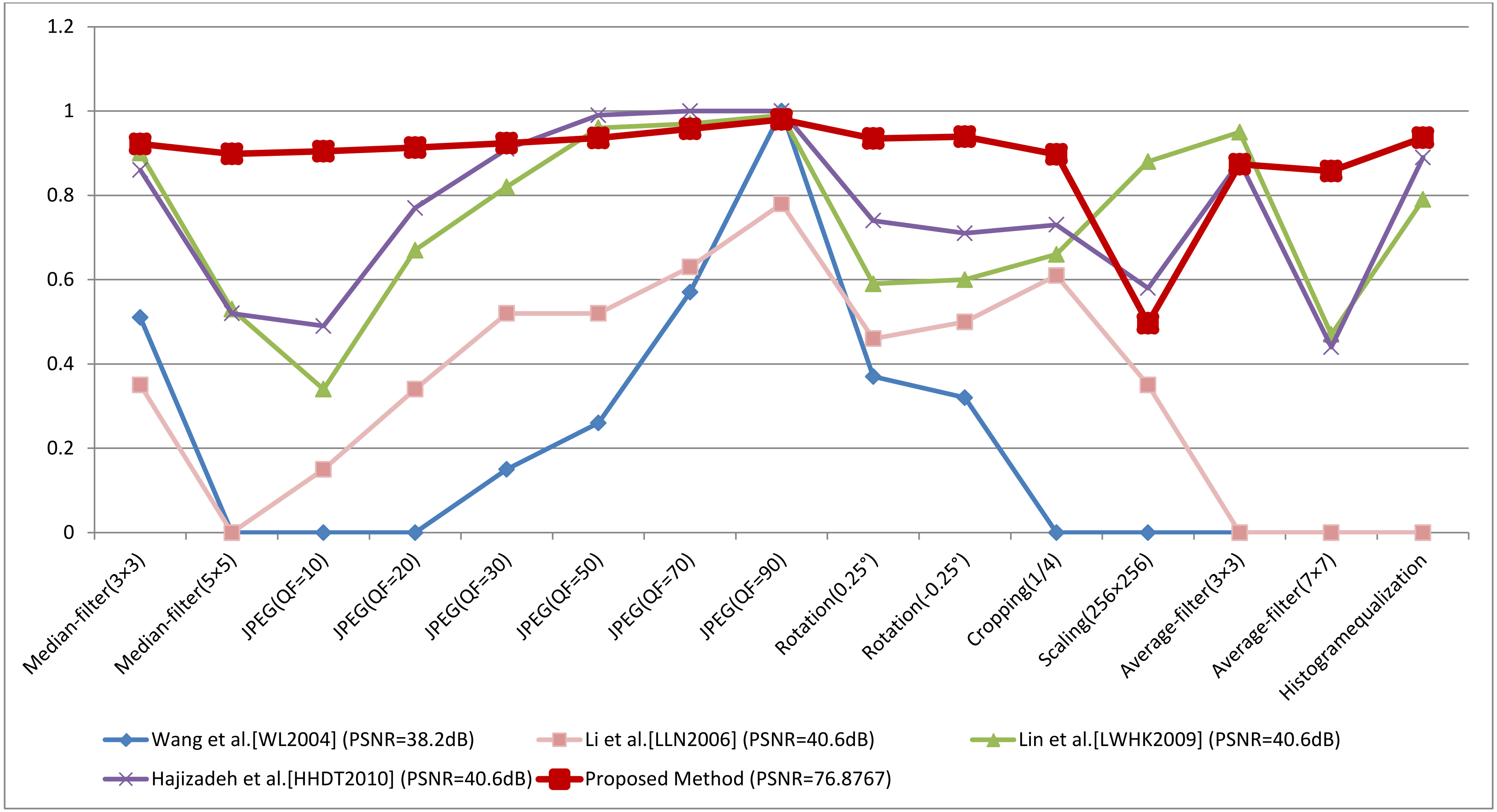


Figure 5.15 Comparing the Robustness of the Proposed Method with [WL2004, LLN2006, LWH+2009 and HHDT2010]

For more comparison of the robustness of the watermark against JPEG compression, we also compare the proposed algorithm with Hajizadeh et al. [HHDT2010] and Byun et al. [BLK2005]. From Table 5.9 and Figure 5.16, the proposed algorithm has the most robustness to against strong JPEG attack with quality factors 10, 20 and 30. And the NC of the proposed algorithm is quite stable compared with the competing methods mentioned in this chapter.

Table 5.9 Compare the NC of the Proposed Method with Hajizadeh et al. [HHDT2010] and Byun et al. [BLK2005] after JPEG Compression

QF	Byun [BLK2005] (PSNR=41.95)	Hajizadeh [HHDT2010] (PSNR=40.6dB)	Proposed Method (PSNR=76.8767)
10	0.49	0.49	0.93
20	0.6	0.77	0.94
30	0.77	0.91	0.95
40	0.81	0.99	0.95
50	0.83	0.99	0.96
60	0.9	1	0.97
70	0.94	1	0.98
80	0.97	1	0.98
90	0.99	1	0.99
100	1	1	0.99

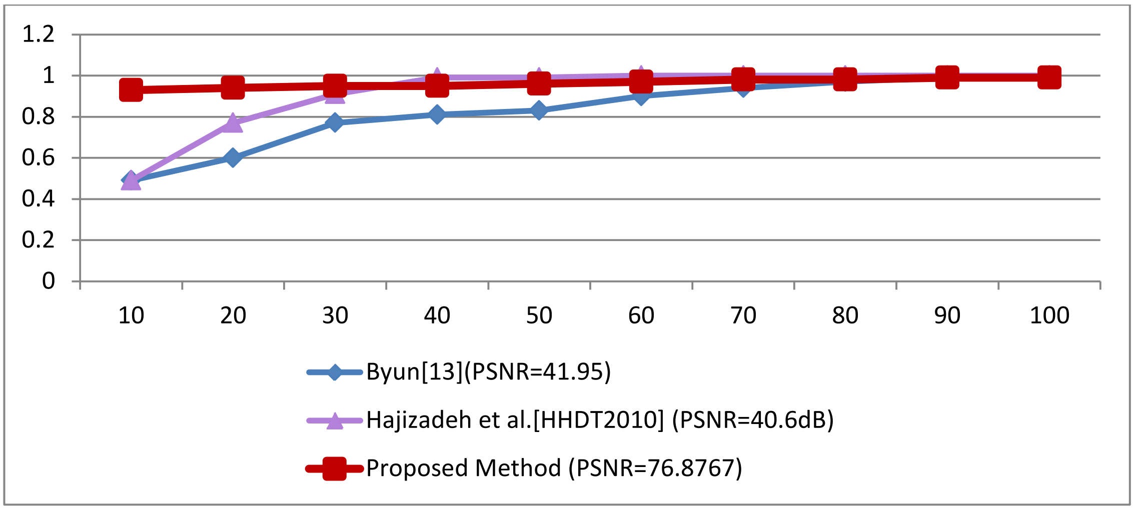


Figure 5.16 Compare the NC of the Proposed Method with Hajizadeh et al. [HHDT2010] and Byun et al. [BLK2005] after JPEG Compression

Section 5.4 Conclusions

In this chapter, we have proposed a hybrid digital watermarking scheme based on DWT-SVD and ergodic matrix. The watermark has been divided into four shares and embedded in the singular values of the carrier image's second-level DWT.

Experimental results show that, in general, performing SVD on the singular value revised by the watermark and the carrier image (SoRS) is better than on the singular value of the watermark (SoW). And the experimental results also show that embedding watermark in the high frequency subbands has more robustness over low frequency subbands.

Compared to other watermarking schemes, SoRS has also shown both the significant improvement in perceptibility and the robustness under various types of image processing attacks. Even though this algorithm is weak against scaling attacks, it extracts watermarks of higher quality than other algorithms under many attacks such as JPEG compression, filtering, histogram equalization, rotation, cropping and noise. As a whole, the proposed method is a useful tool for ownership identification and copyright protection.

CHAPTER 6 TEMPORAL APPLICATIONS

Temporal Logic has been existing for more than a half century, it has so many applications in the variety fields, but in information security field, it has only been applied in protocol so far. However, actually, there is such a relation between temporal logic and information encryption/hiding.

Section 6.1 Formal Characterization of Time-series

Section 6.1.1 Why Choosing Temporal

The term Temporal Logic is a special branch of modal logic. It is largely used to describe systems of symbolism and rules for representing, and reasoning about, propositions qualified in terms of time. It is also more specifically refer to Tense Logic, a modal logic-based system of temporal logic introduced in the 1960s by Arthur Prior and subsequently developed further by logicians and computer scientists, particularly Amir Pnueli [DEJ+2007], he suggested using Linear-Time Propositional Temporal Logic (LTL) to reason about concurrent programs. Temporal logics include interval temporal logic (ITL) and μ calculus. ITL offers flexible specification and proof techniques for reasoning about properties involving safety, liveness and projected time [MWH2002]. Modal μ calculus is an extension of propositional modal logic with a least fixed point (LFP, the least fixed point is less than or equal to all other fixed points according to some partial order) operator μ. It can be described and verified properties of labelled transition systems [LH2007].

Temporal logic can be applied to/in:

- Natural language. Prior, the founding father of temporal logic, lists the precursors of Tense Logic in [MBZ2008]. One of them is Hans Reichenbach. He analysed the tenses of English and put forward that the function of each tense is to specify the temporal relationships amongst a set of three times related to the utterance, specifically *S* (the speech time), *R* (the reference time) and *E* (the event time). Prior points out that Reichenbach's analyses are insufficient to explain the full range of tense usage in natural language. Consequently much work has been done to refine the analysis of tenses and other temporal expressions in language such as the temporal prepositions and connectives ("before", "after", "since", "during", "until"), using the many varieties of temporal logic.

- Artificial intelligence. It is often associated with the work of Allen, which is concerned with how to find a general framework for all the temporal representations required by AI programs [All1984]. A useful survey of issues involving time and temporal reasoning in AI is Galton [MK1994], and a comprehensive coverage of the area is summarized by Fisher et al. [Bee1992] in their handbook in 2005. Much of the work on temporal reasoning in AI has been closely related to the notorious frame problem, which is concerned with formalising the logic of actions and events in such a way that many inferences of this kind are indefinitely made available without our having to encode them all unambiguously.

- Computing science. Temporal logic is given an intensive application in this specific area by Pnueli in [DEJ+2007]. It is concentrated on the specification and verification of programs, especially during the multi-tasking of applications concurrently being processed by the computer. In order to make sure that such a program behaves correctly, it is essential to specify the way in which the actions of the various processors are interrelated. The relative timing of the actions must be cautiously coordinated so that integrity of the information shared among the processors can be surely maintained. Non-determinism is an essential concern in computer science applications. There are two important systems for temporal logic namely, CTL or Computation Tree Logic and the more expressive CTL System.

It is easy to notice that, there is no such a relation between temporal logic and cryptography/hiding even though temporal logic has so many applications in the variety fields. However, in a communication, one party may possibly send a series of information to the other at different time or in different order.

Since temporal logic, especially temporal order theory, deals with time series data, we shall apply it in cryptography and information hiding. From the results of CHAPTER 4 and of CHAPTER 5, we take image security as a special case, to analyse how to realize this goal.

Section 6.1.2 Formalize the Characterization of Time-series

As mentioned in the Section 1.1.1, in most literature in the domain of data mining, the fundamental time theories based on which time-series and sequences are formed up are usually not explicitly specified. However, no matter what a time theory or model is chosen, a temporal order representing the “Immediately Before” relationship is needed for defining time-series.

In a system based solely on intervals as primitive like that of Allen’s interval temporal theory [All1984], or a system based on both points and intervals like that of Ma and Knight [MK1994], such an immediately before relation can be directly expressed by the “Meets” relation.

N.B. The intuitive meaning of Meets(t_1, t_2) is that, on the one hand, t_1 and t_2 don’t overlap each other (i.e., they don’t have any part in common, not even a point); on the other hand, there is not any other time object standing between them.

As shown in [MH2006], the two different approaches to the treatment of intervals, i.e., taking intervals as primitive or as derived objects constructed out of primitive points, are actually reducible to logically equivalent expressions under some requisite interpretations. In fact, in a system based solely on points as primitive, say (P, ≤), as the derived objects, an interval can be defined as a typed (left-open & right-open, left-closed & right-open, left-open & right-closed, left-closed & right-closed) subset of the set of primitive points, which must be in one of the following four forms:

$(p_1, p_2) = \{p \mid p \in P \wedge p_1 < p < p_2\}$

$[p_1, p_2) = \{p \mid p \in P \wedge p_1 \leq p < p_2\}$

$(p_1, p_2] = \{p \mid p \in P \wedge p_1 < p \leq p_2\}$
$[p_1, p_2] = \{p \mid p \in P \wedge p_1 \leq p \leq p_2\}$

where $p_1 < p_2 \Leftrightarrow p \leq p_2 \wedge \neg(p_1 = p_2)$

Without confusion, an interval with the special form of $[p, p]$ is simply taken as identical to point p.

In the same manner as the approach that treats intervals as primitive, an immediately before relation, “Meets”, can be defined over typed-point-based intervals as below:

$$\mathrm{Meets}(t_1, t_2) \Leftrightarrow \exists p_1, p, p_2 \in P(t_1 = (p_1, p) \wedge t_2 = [p, p_2)$$
$$\vee t_1 = [p_1, p) \wedge t_2 = [p, p_2)) \vee t_1 = (p_1, p) \wedge t_2 = [p, p_2]$$
$$\vee t_1 = [p_1, p) \wedge t_2 = [p, p_2] \vee t_1 = (p_1, p] \wedge t_2 = (p, p_2)$$
$$\vee t_1 = [p_1, p] \wedge t_2 = (p, p_2) \vee t_1 = (p_1, p] \wedge t_2 = (p, p_2]$$
$$\vee t_1 = [p_1, p] \wedge t_2 = (p, p_2])$$

Analogously, the 13 relations introduced by Allen for primitive intervals [All1984], including “Equal”, “Before”, “After”, “Meets”, “Met-by”, “Overlaps”, “Overlapped-by”, “Starts”, “Starts-by”, “During”, “Contains”, “Finishes” and “Finished-by”, can also be defined in terms of the above single relation “Meets”.

From the definition, for any two typed-point-based intervals t_1 and t_2 such that Meets(t_1, t_2), there is a unique time interval corresponds to the ordered union of t_1 and $t_{2.}$. By axiomatization, this also applies to systems that based solely on intervals as primitive or based on both points and intervals as primitive. Therefore, in the case Meets(t_1, t_2), the unique ordered union of t_1 and t_2 can be denoted as $t_1 \oplus t_2$.

In a time model based solely on points as primitive, the collection of *time-elements* is defined as the minimal set closed under the following rules:

- Each primitive point is a time-element;
- Each derived typed-point-based interval is a time-element;

- If t_1 and t_2 are two time-elements and Meets(t_1, t_2), then the unique ordered union, $t_1 \oplus t_2$ is a time-element.

Similarly, in a time model based on both points and intervals as primitive, the collection of *time-elements* is defined as the minimal set closed under the following rules:

- Each primitive point and/or interval is a time-element;
- If t_1 and t_2 are time-elements and Meets(t_1, t_2), then the unique ordered union, $t_1 \oplus t_2$ is a time-element.

Within a temporal framework based on a chosen time theory (either point-based, interval-based, or point&interval-based), a time-series *ts* is defined as a vector of time-elements temporally ordered one after another. Formally, a general time-series (GTS) is a triple (*ts*, *R*, Dur) which is defined in terms of the following schema:

GTS 6.1) $ts = [t_1, \ldots, t_n]$

GTS 6.2) R=Meets(t_j, t_{j+1}) $\vee$ Before(t_j, t_{j+1}), for all $j = 1, \ldots, n$-1

GTS 6.3) Dur(t_k) = d_k, for some k where $1 \leq k \leq n$ and d_i is a non-negative real number.

where Before(t_1, t_2) $\Leftrightarrow \exists$t(Meets(t_1, t) $\wedge$ Meets(t, t_2)).

Generally speaking, a time-series may be incomplete in various ways. For example, if the relation between t_j and t_{j+1} is “Before” rather than “Meets”, it means that the knowledge about the time-element(s) between t_j and t_{j+1} is not available. In addition, if Dur(t_k) = d_k is missing for some k, it means that duration knowledge as for time-element t_k is unknown. Correspondingly, a complete time-series (CTS) is defined in terms of the schema as below:

CTS 6.1) $ts = [t_1, \ldots, t_n]$

CTS 6.2) Meets(t_j, t_{j+1}), for all $j = 1, \ldots, n$-1

CTS 6.3) Dur(t_i) = d_i, for all $i = 1, \ldots, n$, where d_i is a non-negative real number.

Section 6.2 Temporal-based Image Cryptosystem

Given Alice sends Bob a sequence of images Img$_1$, Img$_2$, …, Img$_n$, next time she might send the same sequence with different order such as Img$_2$, Img$_1$, …, Img$_n$. What she can do if the importance of each image changes? How can she change the encryption results with diverse order of the images to avoid suspicion? Nevertheless, according to the author's knowledge, such consideration hasn't been included in state-of-the-art cryptosystems, much less in image cryptosystems.

As a result, this section tries to answer the following two questions:

Question6.2.1: How can Alice encrypt different important-level images without changing the keys?

Question6.2.2: How can Alice get different encryption results if the order of the images is changed?

Section 6.2.1 Encryption-decryption Process Using Temporal Logic

To solve two questions listed above, we shall apply temporal logic in our cryptosystem. The basic thoughts are as follows:

(1) The temporal-based image series is defined as a vector of time-elements temporally ordered one after another.

(2) Take each image as a node, draw the directed graph G.

(3) The k-th cipher image is utilized to encrypt its tail-node image (i.e. the $k+1$-th image).

(4) To ensure the k-th image can be decrypted, each tail-node image can have one and only one head-node image.

Given the cipher keys K_a, K_b and K_c, K_a are all matrices of size n×n. Alice is the sender and Bob is the receiver. Then for Alice, she carries out the encryption process as below:

① Load image series [Img_1, Img_2, …, Img_n]. The order of the series can be set by the important level of the images. The image with the lowest importance is the first node, while the image with the highest importance is the last node;

② Draw a directed graph of the image series, and set the corresponding adjacency matrix A;

③ E(Img_1, K_a) returns the cipher-image Img_1 encrypted by the key K_a, only the first image is encrypted by K_a;

④ If [Img_{k+1}, …, Img_{k+m}] are the tail-nodes of Img_k, then diffuse each image of tail-nodes by the key K_a, namely [E(Img_{k+1}, K_a), …, E(Img_{k+m}, K_a)]. Therefore, the histogram of the images (even the pure-colour images) is more equalized. This step is to enhance the encryption effects;

⑤ Encrypt each tail-nodes by its head-node, i.e., [E(E(Img_{k+1}, K_a), Img_k), …, E(E(Img_{k+m}, K_a), Img_k)];

⑥ Go to ④ unless all the images are encrypted;

⑦ Permute the cipher image series S=[E(Img_1, K_a), E(E(Img_{k+1}, K_a), Img_k), …, E(E(Img_{k+m}, K_a), Img_k), …] by K_b, i.e., P(S, K_b). And encrypt the adjacency matrix A by K_c, i.e.,E(A, K_c);

⑧ Send the encrypted adjacency matrix A' and the permuted series to Bob.

When Bob receives the series and A', he decrypts the images by the following steps:

① Deduce A by K_a, i.e., D(E(A', K_c), K_c) = A;

② Get the original order of the series by K_b, i.e., DP(P(S, K_b), K_b) = S;

③ Decrypt the first image by K_a, i.e., D(E(Img_1, K_a), K_a) = Img_1;

④ Decrypt the tail-nodes images by its head-node image Img_k, i.e.,D(E(E(Img_{k+1}, K_a), Img_k)) = E(Img_{k+1}, K_a);

⑤ Decrypt the image k+1 by the key K_a, i.e., D(E(Img_{k+1}, K_a), K_a) = Img_{k+1};

⑥ Go to ④ unless all the images are decrypted.

Section 6.2.2 Expand the Key Size

If the size of K_a is smaller than that of the image, Alice expands K_a according to the first encrypted image by the following algorithm:

① H and W is respectively the height and width of the first encrypted image;

② N is the number of the elements of K_a;

③ R rounds the elements of H×W/N to the nearest integers greater than or equal to H×W/N;

④ ArrayB stores the result of R matrices that Alice calculates;

⑤ for i=1 to R

⑥ the i-th row of ArrayB stores the result of Power(K_a,K_a(i));

⑦ endfor

Then ArrayB is the actually what Alice uses to encrypt the image.

Section 6.2.3 Adjust the Images

In an temporal-based image system, each image in the series [Img_1, Img_2, …, Img_n], for example Img_k, could be a “key” to encrypt the next image Img_{k+1}. The encryption algorithm can be the same as what we described in Section 4.2 when Img_1 is encrypted by the session key K_a.

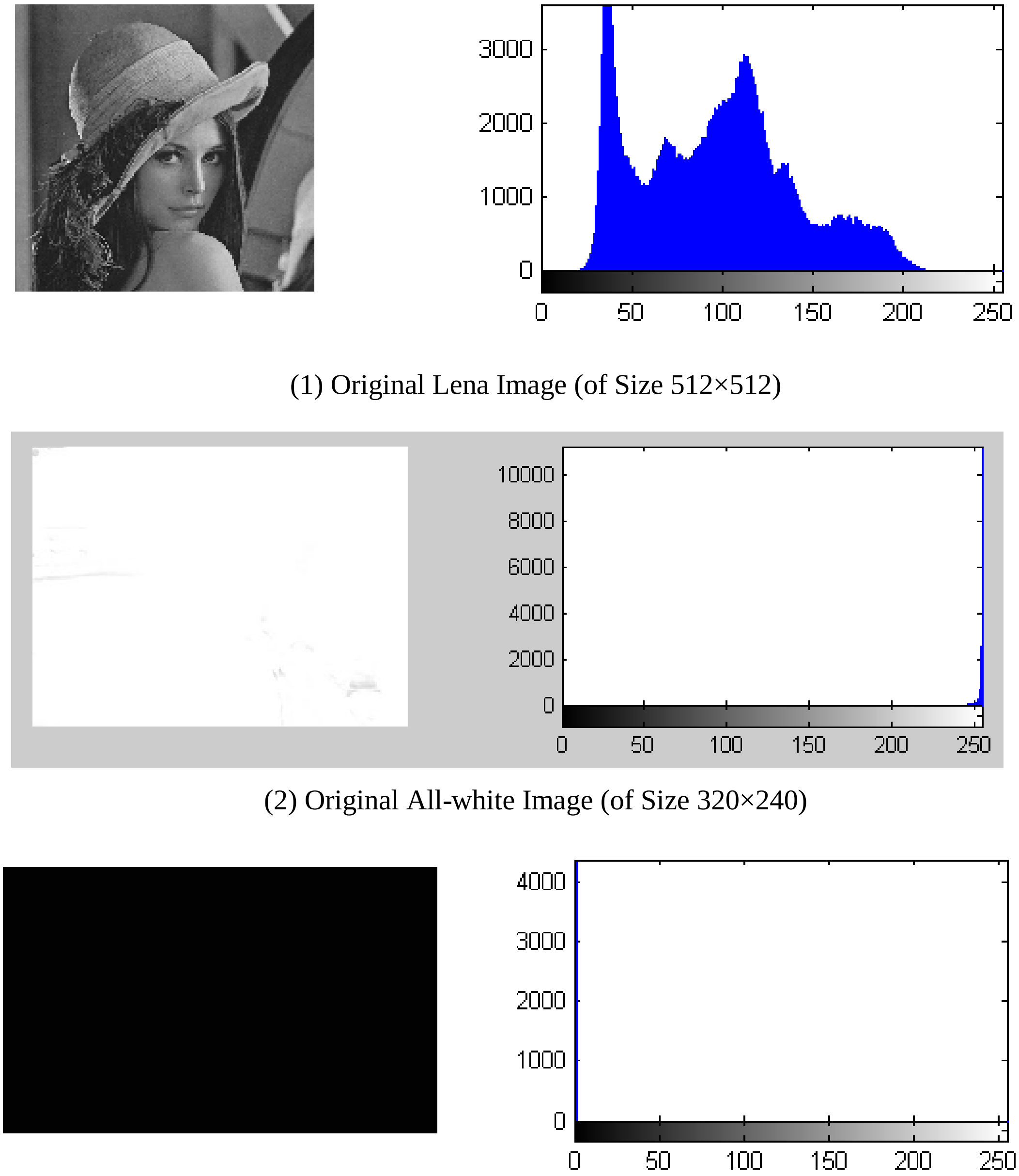
Noted that the size of the images are not necessarily equal to each other, we need to adjust the images so that it can be used to encrypt the next one(s). Such algorithm can be described as follows:

```
①Ceil($A$) rounds the elements of $A$ to the nearest integers greater than or equal to $A$;
  Sqrt($A$) returns the square root of $A$;
  Size($A$) returns the size of the matrix $A$;
②$n$ = size(Img$_k$) – size(Img$_{k+1}$);
③if $n$> 0, delete the last $n$ elements from Img$_k$;
④else if $n \leq 0$ do the following:
⑤        m=ceil(sqrt(ceil(size(Img$_{k+1}$)/size(Img$_k$))));
⑥for $k$=1:$m$
⑦form a new matrix Img$_k$' such that the weight and the height of it equals to
   size(Img$_{k+1}$);
⑧end for
⑨delete the last $n$ elements from Img$_k$' such $n$ = size(Img$_k$') – size(Img$_{k+1}$);
```

Section 6.2.4 Security Analysis

A good encryption system shall resist all kinds of known attacks, including ciphertext-only attack, know-plaintext attack, statistical attack, and various brute-force attacks. Thus in what follows, we will analyse on the proposed image encryption scheme, including the most important ones such as key space analysis and statistical analysis, which demonstrates the satisfactory security of the proposed scheme.

Before security analysis, three original images, which are utilized in the experiments, are provided in Figure 6.1.



(1) Original Lena Image (of Size 512×512)

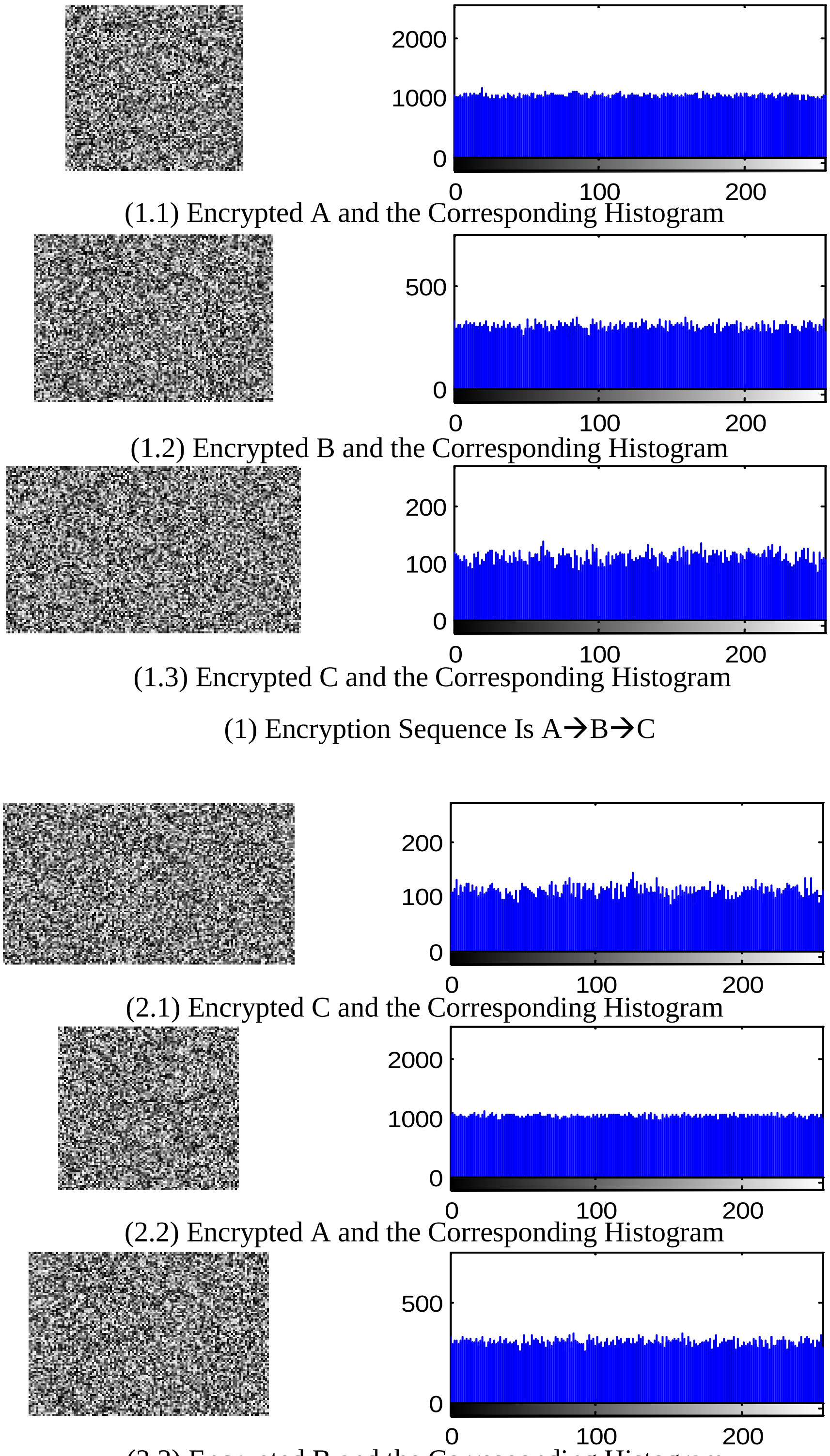
(2) Original All-white Image (of Size 320×240)

(3) Original All-black Image (of Size 214×131)

Figure 6.1 Three Images and Their Corresponding Histograms

We randomly pick three images: Lena image, all-white image and all-black image. Let these images be denoted as A, B, C, respectively. We apply the encryption algorithm proposed in [Bru1972] and the temporal-based encryption algorithm in this cryptosystem.

Then the encryption result can be seen from Figure 6.2.



(1.1) Encrypted A and the Corresponding Histogram

(1.2) Encrypted B and the Corresponding Histogram

(1.3) Encrypted C and the Corresponding Histogram

(1) Encryption Sequence Is A→B→C

(2.1) Encrypted C and the Corresponding Histogram

(2.2) Encrypted A and the Corresponding Histogram

(2.3) Encrypted B and the Corresponding Histogram

(2) Encryption Sequence Is C→A→B

Figure 6.2 Encryption Results with Different Images Sequence

We can see from Figure 6.2 that the cipher images disorganize the plain-images as much as possible so that they are hard to recognize. And the histogram is fairly uniform and does not reveal any statistical information of the corresponding plain-image.

Section 6.2.4.1 Key Space Analysis

Aimed at Problem 6.2.1, the key space of a good temporal-based image encryption system shall not only be large enough to make brute-force attacks infeasible, but also can encrypt images of different importance without changing the keys.

The algorithms used to encrypt the images can be referred to [Bru1972, Fri1998, CM2004, MC2004, ZLW2005, GHG2005, AP2009, Moh2009, GBB+2009 and YWL+2010]. Suppose the secret key K_a has *spc* different combinations, then for the encryption systems in these algorithms, the key space is always the same if each image is encrypted independently. However, in a temporal-based image encryption system, a head-node image can be taken as a key for the tail-node images, namely a tail-node image can be encrypted by its head-node image. Therefore, the probability is different because of working out the images not only relies on the session keys but also on the head-node image. Even an eavesdropper knows the key K_a, it is still hard for him to get the exact order of the images without K_b and K_c.

Therefore, in the proposed temporal-based image encryption system, the key space is $K_a \times K_b \times K_c$.

Section6.2.4.2 Statistical Analysis

Statistical analysis is shown, in this section, by a test on the histograms of the enciphered images and the cipher image, and on the correlations of adjacent pixels in the two ciphered images encrypted from the same original image with the same key while in different sequences (i.e. the head-node is different).

(1) Histogram of the plain-image and the cipher-image

Figure 6.2 shows that the histogram of each image is fairly uniform and does not reveal any statistical information of the original image.

(2) Correlation of two adjacent pixels

The experiment was divided into 10 groups, each group has 10,000 pairs of pixels, i.e., *R*=10,000. The correlation distributions of two adjacent pixels in the cipher-image are tested randomly in horizontal, vertical and diagonal. Table 6.1 shows the results of the correlation coefficients of the cipher images using different images as their head-note:

Table 6.1 Correlation Coefficients of Two Adjacent Pixels in the Cipher-image

B → A	C → A	A → B	C → B	A → C	B → C
0.007334	-0.01572	-0.01689	-0.00813	0.015068	-0.01082
0.007585	-0.00642	-0.00408	-0.00315	-0.01026	0.008444
-0.01344	0.010856	-0.01093	-0.00747	0.006084	0.003443
-0.00615	0.006587	-0.00548	0.00245	-0.00153	0.014232
0.003614	0.008215	-0.01892	0.011897	-0.00812	-0.00719
0.014298	0.005353	-0.009	-0.00049	0.003064	0.009228
0.00893	0.009849	0.006174	0.010911	-0.00302	-0.00264
7.18E-05	-0.00621	-0.00674	0.011962	0.018025	0.011177
0.001893	0.004651	0.015908	-0.00019	-0.0067	3.17E-03
0.012082	-0.01394	-0.02577	-0.00468	0.003624	0.01623

Here the denotation A → C means the correlation coefficients of C using E(A, K_a) as a key. It is the same to the rest of the denotations.

We also test on various images to compare the results, see Figure 6.3 and Table 6.2. Image (1) of Figure 6.3 is the original image and (2)-(8) are the images as keys to encrypt (1). Table 6.2 shows that with different images and the same key K_a, the correlation coefficients of two adjacent pixels in the cipher-image diverse.

(1) Lena (2) Blonde (3) Hill (4) Man

(5) Couple (6) House (7) Pentagon (8) Peppers

Figure 6.3 Original Image (Image (1)) and the Images used as Keys (Image (2)-(8))

Table 6.2 Correlation Coefficients of Two Adjacent Pixels in the Cipher-image

	Blonde	Hill	Man	Couple	House	Pentagon	Peppers
Correlation values	0.0011	0.0108	-0.0069	-0.0018	0.0313	0.0011	-0.0033

It is easy to see that the all the results of correlation coefficients are quite close to 0. This indicates that our scheme has effectively removed the correlation of adjacent pixels in the plain-image.

Section6.2.4.3 Sensitivity-based Attack

The temporal-based image encryption algorithm shall be robust enough to resist sensitivity-based attack. This means if the order of the image series changes, it will cause a great difference in the cipher-images, or else the temporal theory would be utterly meaningless in this thesis.

The sensitivity of the cryptosystem is largely infected not by the keys that Alice and Bob are holding (read [YWL+2010] for reference), but also by the order of the image series. Here, two common measures are used to test the system: NPCR and UACI.

For more evidence, we used the same formula (4.4) given in Section 4.3.2.2, except that in this test, we compare the values of corresponding pixels in the two cipher-images.

The values of NPCR, UACI and COV (Covariance) for the cipher-image using different images as keys are listed in Table 6.3.

Table 6.3 NPCR and UACI of the Cipher-image using Different Images as Keys

Encryption Sequence	NPCR	UACI	COV
B → A and C → A	0.996025	0.334778	-0.00055453
A → B and C → B	0.996068	0.334648	-0.0017352
A → C and B → C	0.995898	0.336168	-0.0057025

This table indicates that in a temporal ordered cryptosystem, the results of the ciphered images are quite different even they are from the same image and encrypted by the same key.

Section 6.3 Temporal-based Image Hiding

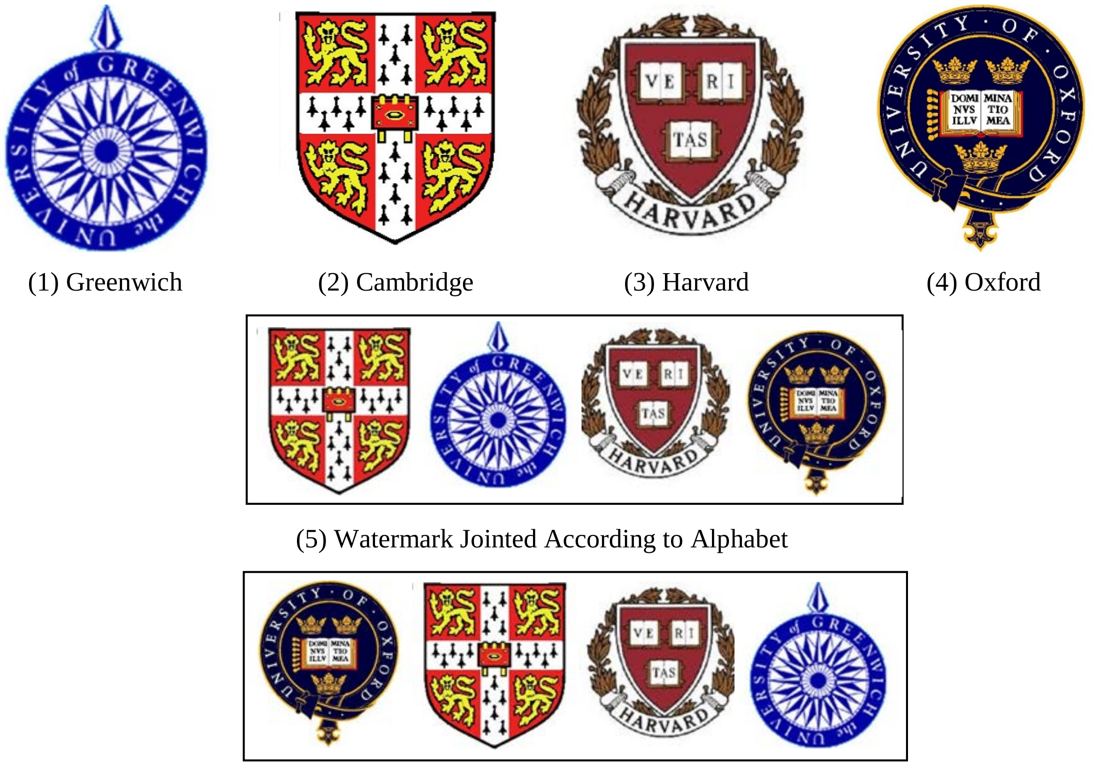
With more applications of information hiding, especially watermarking, various techniques are introduced, such as fragile watermarking and robust watermarking. For the reason that different watermarking techniques showing at different stages with different purposes, therefore, a new technique, called multiple digital watermarking, is introduced by other scientists [GPK2005, Lee2009 and HHLL2009].

The multiple digital watermarking techniques are for fixing the problems of the copyright certification of a digital product produced by multiple authors when it is released, sold and used. Compared to single watermarking techniques, embedding multiple watermarks is more complicated. It requires the watermark signals shall not interfere with each other through the embedding and the detecting procedure. Therefore, the balance between watermark transparency and robustness is a tough issue.

Multiple digital watermarking can be classified into two categories, i.e., static multiple digital watermarking and dynamic multiple digital watermarking.

Section 6.3.1 Static Multiple Digital Watermarking with Temporal Logic

Static multiple digital watermarking techniques are simple. Various watermarks can be jointed into one larger watermark according to some certain rules. For example, if all the authors create the product simultaneously, multiple watermarks can be sorted according to the contributions of the authors. See Figure 6.4 for an example.



(1) Greenwich (2) Cambridge (3) Harvard (4) Oxford

(5) Watermark Jointed According to Alphabet

(6) Watermark Jointed According to University Founding year

Figure 6.4 Multiple Watermarks and the Joint Watermark

Considered the limited capacity of the carrier image, multiple watermarks shall not be simply jointed by putting them together. Other issues, such as image compression or re-encoding images to decrease the image size, shall be considered. But this is not what we shall discuss in this thesis, for we only need to focus on the techniques of watermark embedding and extracting.

Once multiple watermarks are integrated into one, the watermarking scheme proposed in CHAPTER 5 can be used in static multiple digital watermarking system.

Section 6.3.2 Dynamic Multiple Digital Watermarking Considering Temporal Logic

Although lots of work has been done in the field of watermarking [BYK1998 and TWZ2006], little specific work is done in the area of "**dynamic** multiple digital watermarking". By the time we are writing this thesis, we have only found one paper (which is written by Tang et al. [TWZ2006]), is on this topic. Other papers discussing "dynamic multiple watermarking" actually reflect in the embedding/extracting process [GPK2005, TX2008, Lee2009, HHLL2009 and ZYLC2012], rather than in the collaborating digital products considering temporal issue in an open network. How to design watermarks created by different authors at different time? – This is what we need to discuss in this section.

Section 6.3.2.1 How Problems Occur

In a procedure of designing a digital product, especially of collaborating on a product through network protocol, there are often multiple authors. However, not every author designs the product at the same time - this makes it difficult for one taking into account at the start of a product design that:

(1) Who will collaborate on the product design?

(2) What is the number of the authors?

In the highly developed network of today, it is very common that multiple authors cooperating on a digital product even they don't know each other. With the improvement of the work, the number of the authors may also be increasing because such authors are not a team; they work independently and as an individual. As a result, in order to protect the copyrights of the authors, each identity of the author shall be designed into one watermark and be embedded into the digital products.

It is easy to notice that, at a certain time, even when the digital product has been released on the network, all the validated watermarks have been embedded in the product by then. But it is not possible to predict in advance whether other watermarks, or which watermark, will be embedded. This is a common problem of dynamic multiple digital watermarking: We cannot predict how many watermarks will be embedded in a digital product.

Therefore, with temporal logic theory, we set parameters as follows:

(1) Denote the digital product as *DP*;

(2) The number of the collaborative authors is N_i at time T_i;

(3) Denote the collaborative authors as $A_1^i, A_2^i, \dots, A_{N_i}^i$ at time T_i;

(4) Consequently, the number of the watermarks is also N_i. Denote the watermark $W_1^i, W_2^i, \dots, W_{N_i}^i$ for the authors $A_1^i, A_2^i, \dots, A_{N_i}^i$, respectively;

(5) The version of the product *DP* released at time T_i is denoted as DP_i; and

(6) The single joint watermark for co-authors $A_1^i, A_2^i, \dots, A_{N_i}^i$ is denoted as W^i.

With these parameters, it is obvious that, every time an author takes part in the product design, one more watermark will need to be embedded in the product. But since every product has its volume limitation, we cannot embed countless number of watermarks over time. As a result, Question 6.3.1 will occur when we design a multiple digital watermarking technique:

Question6.3.1: How to overcome the contradiction between the limitation of the image volume and the countless number of watermarks trying to be embedded in?

According to what has been analysed in CHAPTER 5, we decide to solve the problem by embedding only one watermark regardless how many watermarks there will be during the product design process.

However, other questions will occur that force us to face them:

Question6.3.2: Who will map the multiple watermarks to a single watermark? And who will embed it into the image?

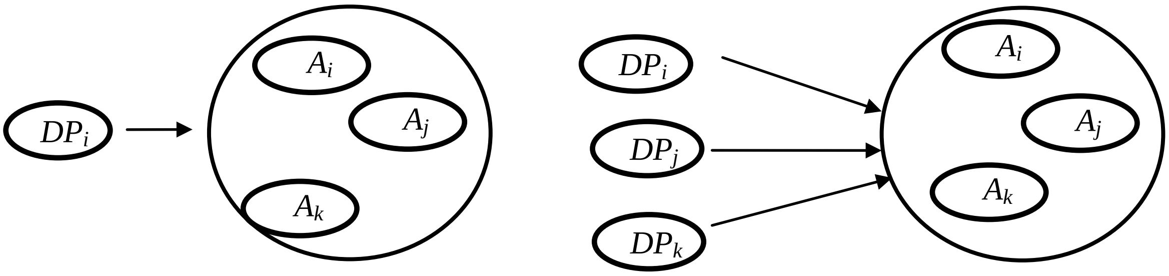
Question6.3.3: How to design such single watermark so that it contains the information of the previous authors as well as the author(s) who will participate in the product?

Question6.3.4: How to design such single watermark without revealing the plain information of the authors to those who are going to design the watermark?

Question6.3.5: How to ensure each author that his own watermark has been embedded into the image?

Tang proposed their method in [TWZ2006], but there are some problems lie in it:

Problem 6.3.1: Tang hasn't considered the situation that the current authors may collaborate on the digital product based on multiple previous products. i.e., Tang's model can be ascribed by Figure 6.5 (1), while ours can be ascribed by Figure 6.5 (2).



(1) Tang's Model (2) Our Model

Figure 6.5 Model Comparision between Tang's Method and Ours

Problem 6.3.2: The digital product may be produced based on multiple products (different versions of the same product are regarded as different product as well), thus, how to generate the watermark for the new product? Who is responsible for embedding the watermark?

Therefore, combine with ergodic-matrix based zero-knowledge protocol, we propose a scheme to realize dynamic multiple digital watermarking technique to improve Tang's method, as well as to solve the problems we proposed.

Section 6.3.2.2 Basic Idea of the Proposed Scheme

At the time T_i when the watermark W^i is about to be inserted in the product $DP_{i\text{-}1}$, the watermark W^{i-1}, which has been embedded in $DP_{i\text{-}1}$, shall be able to authenticate the copyright of all the previous authors. A collaborated digital product work, considers temporal issues (i.e., which work comes first and which comes next), can be drawn into a directed graph as the form shown in Figure 6.6:

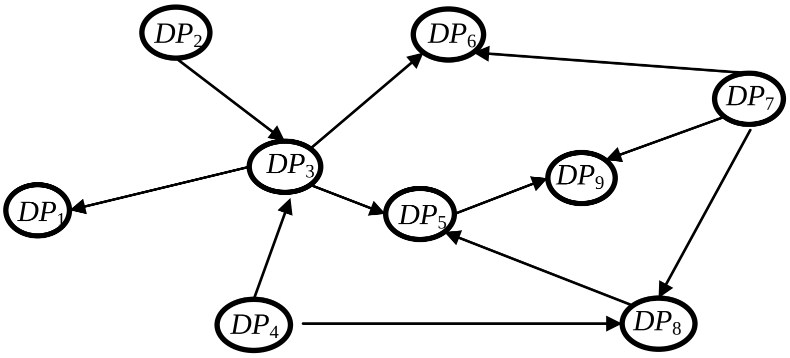


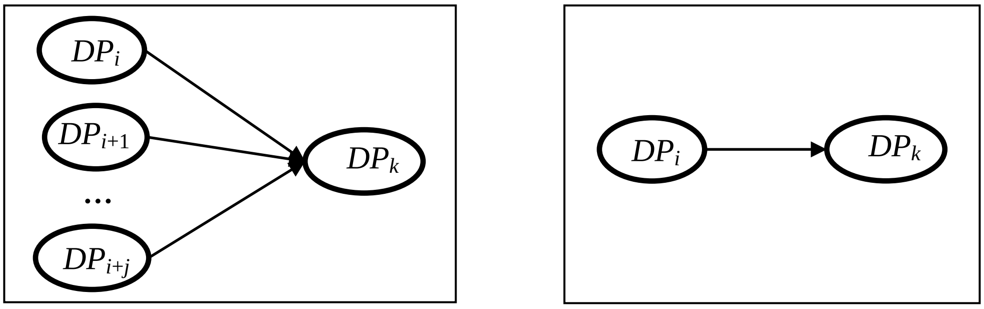
Figure 6.6 Directed Graph Describing a Collaborated Digital Product Work

The graph is acyclic and unweighted for two reasons:

(1) A new version of the product is generated based on the previous one(s);

(2) Authentication of the product's authors is the main concern, thus the interval time of the two products will not affect the watermarks.

Therefore, the graph shall be a DAG (directed acyclic graph) without weight. Such graph, if only from the view of a new version of *DP*, can be classified into two scenarios, as shown in Figure 6.7:



(1) One New *DP* Is based on Multiple *DP*s (2) One New *DP* Is based on a Single *DP*

Figure 6.7 Two Scenarios of a Collaborated *DP* Work

For Problem 6.3.1, there's only one watermark to embed in the product regardless how many authors are collaborating. Considering the host image's volume and the algorithm complexity, we shall firstly map the multiple watermarks to a single one and then embed it into the host image. Therefore, from the view of the authors who are going to design a new product, there is only one scenario, which is (2) in Figure 6.7. Thus, what we need to concern about is to prevent malicious authors cheating in the watermark. This can be solved by a multi-secret sharing authentication scheme proposed by Wang et al. in [WQ2006]: In the (*t*, *n*) secret sharing scheme, a dealer splits a secret into *n* shares and sends a share to each of *n* participants. The secret message can be recovered by no less than *t* members by using a publicly specified algorithm. .

After the watermark design, we can apply authentication protocols to identify the ownership of the previous version product and embed watermark with the safe duplex computing protocol [TWZ2006].

For Problem 6.3.2 (see (1) in Figure 6.7 for reference), the new product is based on the existing products, and each existing product is independent from each other. As a result, it is not appropriate for any of the authors of the existing products to embed the watermark. Therefore, we decide to let the author of the new product to do this work. In this case, there will be various watermarks generated from the new author and any previous author. Hence, we convert the problem to how to integrate static multiple watermarks into one.

The identification procedure which Bob identifies whether Alice is the true author is realized by zero-knowledge protocol. The procedure of embedding/extracting watermark can be referred to the algorithms proposed in CHAPTER 5.

Section 6.3.2.3 Identify the Product's Ownership by Zero-Knowledge Protocol Based on Ergodic Matrix

Zero-knowledge protocol is a method for one party to prove to another that a statement (usually a mathematical one) is true, without revealing anything other than the genuineness of the statement [BFS1988].

Given Alice is the author of DP_1 at time T_1, Bob is the author at time T_2 ($T_2 > T_1$). Before Bob decides to work on DP_1, he needs to identify whether DP_1 is the genuine product. In other words, he needs to know if Alice is truly the author of DP_1. Because Bob doesn't want to be a partner of pirates – he has to take up the responsibility of piracy and may have to face prosecution, and also, he might not get paid for the product design.

On the other hand, Alice wishes to protect her copyright through the identification procedure, so that Bob will not be able to cheat when producing the joint watermark (if Bob does, Alice still holds the fact that Bob admits she is the true author and then sues him). But in a certain case, such as visual watermark, Alice doesn't want to reveal too much detail of her own watermark W_a. Once the watermark is public, anyone can easily remove the watermark from the host image.

For all the above reasons, we introduce zero-knowledge protocol to prove Alice is the author of DP_1.

(1) Zero-knowledge Protocol Scheme 1:

Registration Process

①Alice sends the personal identification ID_a to Bob for the request of registration;

②Bob searches the registration database. If ID_a exists, he returns a message to

Alice regarding the registration failure. Otherwise he picks an ergodic matrix Q over finite field $\mathbb{F}^{q}$ and sends it to Alice;

③Alice computes PWD_a=Q^a (a is Alice's private key). Then she sends (ID_a, PWD_a) to Bob.

④Bob sets up a new record for Alice in the registration database, and save (ID_a, PWD_a, Q). Thus Alice successfully registers.

Identification Process

①If Alice needs to be identified by Bob, she sends ID_a to Bob.

②Bob check records of Alice's ID in the registration database. If it exists, he will pick an integer s and sends Q^s to Alice.

③Alice computes PWD_a' = $(Q^s)^a$ and sends PWD_a' to Bob.

④Bob then computes PWD_a'' = $(PWD_a)^s$= $(Q^a)^s$ and compares PWD_a' with PWD_a'', if they matches, Alice is successfully identified.

Security Analysis

Ciphertext attack: The attacker intercepts PWD_a and Q, and he wants to get Alice's private key a with PWD_a = Q^a. But since the order of $Q(Q\in\mathbb{F}_{n\times n}^{q})$ is q^n-1, which means the private keys a and s are in the range from 0 to q^n-1. Therefore, if q and n are properly set, a and s can be fairly large so that to get a from Q^a is as hard as breaking the discrete logarithm problem.

Disguise attack: The attacker intercepts Alice's identification ID_a, he then disguises as Alice. But since a is Alice's private key and the key space is quite large, he cannot deduce PWD_a, thus he is unable to be identified by Bob.

Resending attack: The attacker intercepts all the information exchanged in an identification process. He then resends the outdated information in order to disguise himself as Alice or Bob. But this kind of attack is based on query/response, the identification information changes each time when Alice and Bob communicate. Therefore, the attack fails when the attacker resend $PWD_a' = (Q^s)^a$. Because Bob's private key s is a random number, it changes in each identification process. Thus the attack will be found when Bob compares PWD_a' with PWD_a''.

Replacement attack: This is the most threatening attack. The attacker intercepts (ID_a, PWD_a) when Alice is registering. He then replace Alice's information with his (ID_{att}, PWD_{att}), and logs in as Alice. Thus he can tamper with or intercept the content of the communication between Alice and Bob.

Replacement attack can succeed because Alice's ID is public, which gives the attacker a good opportunity to replace Alice's legal information in registration database. To prevent this attack we propose the other scheme.

(2) Zero-knowledge Protocol Scheme 2–Advanced Version:

In this advanced version, Alice will encrypt her ID with a private key to prevent the replacement attack.

Registration Process

①Alice requests identification with Bob. Bob then picks an ergodic matrix Q over finite field $\mathbb{F}^q$ and sends it to Alice;

②Alice encrypts her information by computing $ID_{aa}=Q^{aa}$ and $PWD_a = Q^a$, then he sends (ID_{aa}, PWD_a) to Bob.

③Bob searches the registration database. If ID_{aa} exists, he will return a message to Alice regarding the failed registration. Otherwise he will create a new record for Alice in the registration database, and will store (ID_{aa}, PWD_a, Q). Thus Alice successfully registers.

Identification Process

①If Alice needs to be identified by Bob, she sends ID_{aa} to Bob.

②Bob checks Alice's ID record in the registration database. If it exists, he will randomly pick an integer s and sends Q^s to Alice.

③Alice computes $ID_{aa}' = (Q^s)^{aa}$ and sends ID_{aa}' to Bob.

④Bob then computes $ID_{aa}''=(ID_{aa})^s= (Q^{aa})^s$ and compares ID_{aa}' with ID_{aa}''.

⑤If they match, Bob will further identify Alice's password PWD_a. This process can refer to Scheme 1.

It is obvious that Scheme 2 is more secure than Scheme 1.Because compared to Scheme 1, Scheme 2 needs two identifications, i.e., the ID and the password of Alice. To enhance security, Bob can send different ergodic matrices Q to Alice.

Moreover, Scheme 2 solves the security problem in Scheme 1. This scheme can resist replacement attack, because in this scheme, Alice encrypts her ID by $ID_{aa}=Q^{aa}$, rather than putting it in public without any security protection.

Section 6.3.2.4 Joint Watermark Generation

Joint watermark can be generated by a trusted institution, and can also be generated by the communication parties. The former technique is easier to realize: To achieve the joint watermark, Alice and Bob send their watermarks to a trusted institution, then the institution produces a watermark according to the features of Alice and Bob's watermarks, such as by the formula (6.1):

$$\begin{cases} W_c(i,j) = 1, & if\ W_a(i,j) \geq W_b(i,j) \\ W_c(i,j) = 0, & if\ W_a(i,j) < W_b(i,j) \end{cases} \quad (6.1\ )$$

here $W_a(i,j)$, $W_b(i,j)$, $W_c(i,j)$ is the pixel value of Alice's watermark, of Bob's watermark and of the joint watermark, respectively.

If it is not involved with a third party, plus Alice and Bob do not want to reveal their own watermark, how do they generate a joint watermark?

Yao's protocol

In [TWZ2006], Tang et al. argues that the joint watermark can be produced with Yao's protocol [Yao1982]. Yao's protocol is described as follows:

For definiteness, suppose Alice holds an integer i and Bob holds j, where $1 < i, j < 10$. Let M be the set of all n-bit nonnegative integers, and Q_n be the set of all bijections (i.e.,1-to-1 onto functions) from M to M. Let E_a be the public key of Alice, it is generated by picking a random element from Q_n. The protocol proceeds as follows:

①Bob picks a random n-bit integer Int, and computes $EInt = E_a(Int)$ with Alice's public key;

②Bob sends Alice the number $EInt - j + 1$;

③Alice computes the following 10 values of $y_u = D_a(EInt - j + u)$ for u=1, 2, …, 10.

④Alice generates a random prime p of n/2 bits, then computes the values $z_u = y_u$ (mod p) for all u; if $|z_u - z_v| \geq 2$ for all $u \neq v$, stop; otherwise generates another random prime and repeat the process until all z_u differ by at least 2;

⑤Alice sends the prime p and the following 10 numbers to Bob: $z_1, z_2, \ldots, z_i, z_i + 1, z_{i+1} + 1, z_{i+2} + 1, \ldots, z_{10} + 1$;

⑥Bob checks the j-th number (not counting p) sent from Alice, and decides that $i \geq j$ if $z_j = x \bmod p$, otherwise $i < j$.

⑦Bob tells Alice the conclusion.

What the greatest part about this protocol is that it enables two parties to properly decide who holds the bigger number without knowing each other's detailed information. But the biggest drawback is that Bob may refuse to tell Alice the conclusion or may even lie to her.

In Tang et al.'s scheme, they argue their proposed scheme doesn't need to worry about this issue. Because Bob's refusing to tell or lying about the conclusion to Alice is simply for the purpose of infringing Alice's ownership. However, Bob has confirmed Alice's copyright before the joint watermark is generated, thus he has no need to do so.

As a result, based on Yao's protocol, Tang's scheme [TWZ2006] and ergodic matrix, we propose a hybrid protocol to produce the joint watermark.

The proposed hybrid scheme

The proposed scheme can be described as follows:

- Generation of the session keys. This can be referred to Section 4.2.1. Alice and Bob use K_1 and K_2 as their session keys.

- Alice and Bob encrypt their watermarks by K_1 and K_2, hence they get the encrypted watermark W_a' and W_b', respectively.

- W_a' and W_b' are taken respectively as *i* and *j*, with Yao's protocol and formula (6.1), the joint watermark W_{ab} is generated.

Section 6.3.2.5 the Temporal Model for Watermark Embedding

No matter how many authors collaborate on a digital product, from the view of the new authors, there're only two scenarios, see Figure 6.7. Since the watermarks of multiple digital products can be integrated into one with static multiple digital watermarking technique, the scenarios can be simplified into one, i.e., the scenario involves only two watermarks: the watermark of the previous authors and the watermark of the new authors.

Given $(A_1^i, A_2^i, \dots, A_{N_i}^i)$, $(A_1^{i+1}, A_2^{i+1}, \dots, A_{N_{i+1}}^{i+1})$,…,$(A_1^j, A_2^j, \dots, A_{N_j}^j)$ are the authors of DP_i, DP_{i+1},…, DP_j at time T_i,T_{i+1}, …, T_j, respectively. Here $i,j \in \mathbb{N}$ and $i<j$. All of the watermarks have been properly embedded in the products by time T_{i+j}.

Later, at time T_k, authors $(A_1^k, A_2^k, \dots, A_{N_k}^k)$ wish to design DP_k based on DP_i, DP_{i+1},…, DP_j, in this case, the multiple watermarks, W_{ik},$W_{(i+1)k}$,…, W_{jk} (designed by the watermarks between $(A_1^i, A_2^i, \dots, A_{N_i}^i)$ and $(A_1^k, A_2^k, \dots, A_{N_k}^k)$, $(A_1^{i+1}, A_2^{i+1}, \dots, A_{N_{i+1}}^{i+1})$ and $(A_1^k, A_2^k, \dots, A_{N_k}^k)$,…, $(A_1^j, A_2^j, \dots, A_{N_j}^j)$ and $(A_1^k, A_2^k, \dots, A_{N_k}^k)$, respectively), shall be integrated into one (denoted as W_{ijk}) before they are embedded in DP_k.

Our model is described as follows:

(1) $(A_1^k, A_2^k, \dots, A_{N_k}^k)$ verifies if $(A_1^i, A_2^i, \dots, A_{N_i}^i)$,$(A_1^{i+1}, A_2^{i+1}, \dots, A_{N_{i+1}}^{i+1})$,…, $(A_1^j, A_2^j, \dots, A_{N_j}^j)$ are the true authors of the digital products, but they don't need to verify every author in $(A_1^i, A_2^i, \dots, A_{N_i}^i)$,$(A_1^{i+1}, A_2^{i+1}, \dots, A_{N_{i+1}}^{i+1})$,…, $(A_1^j, A_2^j, \dots, A_{N_j}^j)$, because in Section 6.3.2.2 we have presume that the secret message can only be recovered by no less than t members. In addition, when there are lots of co-authors of a product it will be a huge work to verify all the authors. The number of verification is N_i+N_{i+1}+…+N_k. We presume the $(A_1^k, A_2^k, \dots, A_{N_k}^k)$ only need to verify one author of the digital product, let's say such author A^i is the "representative" of $(A_1^i, A_2^i, \dots, A_{N_i}^i)$. With the ergodic matrix-based zero-knowledge protocol introduced in Section 6.3.2.3. Then each representative author of $(A_1^i, A_2^i, \dots, A_{N_i}^i)$,$(A_1^{i+1}, A_2^{i+1}, \dots, A_{N_{i+1}}^{i+1})$,…, $(A_1^j, A_2^j, \dots, A_{N_j}^j)$ is verified by $(A_1^k, A_2^k, \dots, A_{N_k}^k)$;

(2) Generate multiple watermarks W_{ik},$W_{(i+1)k}$,…, W_{jk} by Yao's protocol;

(3) Generate the joint watermark W_{ijk} from W_{ik},$W_{(i+1)k}$,…, W_{jk};

(4) A^{N_k} is responsible for embedding the watermark W_{ijk} into DP_k. In Tang's paper [TWZ2006], they explain that since the authors A_{Nk} have admitted $(A_1^i, A_2^i, \dots, A_{N_i}^i)$, $(A_1^{i+1}, A_2^{i+1}, \dots, A_{N_{i+1}}^{i+1})$, …,$(A_1^j, A_2^j, \dots, A_{N_j}^j)$ are the authors of DP_i, DP_{i+1}, …, DP_j, they cannot take possession of or make use of the digital product exclusively for themselves.

(5) Extract W_{ijk} when it is needed.

Section 6.3.2.6 Security Analysis

The security of the proposed model for dynamic multiple digital watermarking lies in these aspects: The zero-knowledge protocol, the encrypted watermarks and the joint watermark.

The security of zero-knowledge protocol based on ergodic matrix has been analysed in Section 6.3.2.3.

The security of the encrypted watermark depends on the session keys used by Alice and Bob. This has been analysed in Section 4.3.1.

The security of the joint watermark mainly lies in Yao's protocol, analysis can be found in [Yao1982]. The biggest issue lies in this protocol is Bob might refuse to tell or lie about the conclusion. But since he has confirmed Alice's copyright before the joint watermark is generated, thus he has no need to do so.

Section 6.4 Conclusions

In this chapter, based on ergodic matrix, we have applied a formal characterization of time-series into information cryptography (esp. encryption) and hiding (esp. watermarking).

We have introduced the computational technique and scheme for temporal ordered image encryption. The security analysis and experimental results show that it is useful in encrypting images of different important level without changing the keys. And different encryption results can be achieved if the temporal order of the images is changed.

We also introduce a static and a dynamic multiple digital watermarking model. The security analysis shows that, the method of mapping multiple watermarks into a single one solves the problem of volume limitation and overlapping of multiple watermarks in a multi-digital-watermarking system.

CHAPTER 7 CONCLUSIONS AND FUTURE WORK

Section 7.1 Conclusions

In this thesis, we have accomplished four goals: first, to design a novel multivariate quadratic equation cryptosystem; second, to build a hybrid-key based image cryptosystem; third, to propose a DWT-SVD based image hiding scheme; last but not least, two temporal based frameworks for image cryptography and image hiding, particularly with the formal characterization of time-series, have been investigated and explored.

The evolution of representation of time-series and the conventional similarity measurements have been reviewed in detail. The questions have been pointed out as the motivation of this dissertation: the general framework with the formal characterization of time-series.

A formal characterization of time-series has been presented for both complete and incomplete situations, where the time-series is formalized as a triple (*ts*, *R,* Dur) which denotes the temporal order of time-elements, the temporal relationship between time-elements and the temporal duration of each time-element respectively.

An *MQ*-based cryptosystem, which is called *BMQE* system is proposed. We firstly summarized that there are so many fine features of ergodic matrix that makes it favorable in cryptography. Thus, combined with ergodic matrix, we propose such a system over a finite field $\mathbb{F}^q$. The complexity analysis shows that the proposed system is NP-hard for *MQ* problem attackers. And if the parameters (i.e., the number of the variables *n*, together with the number of the equations *m* and the number of the degree *q* of $\mathbb{F}^q$) are properly set, relinearization and fixing variables cannot be used to solve this system.

A hybrid-key and ergodic-matrix based image encryption/authentication scheme has been proposed as we have found some nice features of ergodic matrix while investigating *BMQE* system. It is demonstrated that an ergodic matrix $Q \in \mathbb{F}_{50\times 50}^{256}$ can be employed to completely shuffle and diffuse the original image and has an immense key space of at least 3.08×10^{5898}. With this key space, it is robust enough for the cryptosystem to resist the brute-force attack. Moreover, compared with the existing chaotic cryptosystems, the proposed system demonstrates more optimistic results: The change rate in the number of pixels and unified average changing intensity are both higher, whilst the correlation coefficient is lower. Furthermore, the proposed algorithm takes on average 200 to 400 milliseconds for the encryption and decryption stage respectively, which means it is suitable for network transmission.

But the visible encrypted image will easily arouse suspicion. Therefore, we investigate information hiding techniques and introduce a hybrid digital watermarking scheme based on DWT-SVD and ergodic-matrix. The information can be designed as an image and be divided into four shares, then embedded in the singular values of the carrier image’s second level DWT. Experimental results show that, compared to other watermarking schemes, the proposed scheme has shown both the significant improvement in perceptibility and the robustness under various types of image processing attacks, such as JPEG compression, median filtering, average filtering, histogram equalization, rotation, cropping, Gaussian noise, speckle noise, salt-pepper noise. In general, the proposed method is a good useful tool for ownership identification and copyright protection.

In a real life, when two or more parties communicate, they probably send a series of images, whether the main communication content lies in the image itself or in the copyright of the image. Therefore, image cryptography and image hiding involve temporal issues. Moreover, temporal logic has been neglected in information security except in network protocol. As a result, we apply a formal characterization of time-series into information cryptography (esp. encryption) and hiding (esp. watermarking). Consequently, a scheme for temporal ordered image encryption and a temporal ordered dynamic multiple digital watermarking model is introduced.

All in all, we believe that temporal logic and ergodic matrix are very practical tools to build information security systems in an open network environment. Though the systems we proposed haven't been actually used in a real world, we are confident that they will draw more scientists' attention and provide valuable references to them.

Section 7.2 Future Work Discussion

- **Multivariate Quadratic public key system**

We have proposed a cryptosystem based on Multivariate Quadratic, but a public key system is yet to explore. How to use *BMQE* system to encrypt a message? How to apply it in digital signature? These are the problem we need to solve.

In addition, more algebraic methods need to analyse the robustness of *BMQE*. Such as Gröbner bases, XL, Dixon resultants, MinRank-based attacks, differential cryptanalysis, etc.

- **Image cryptosystem**

Unlike texts, images have some intrinsic features such as bulk data capacity and high redundancy. Therefore, how to decipher images not "exact" the same to the original ones since images have bulk data capacity and high redundancy, is what we concern in the future work. In fact, without specific requirements, some reduction in the size and the colour quality of an image, or even changing some pixels in the decipher image, will not affect human perception of the images. For this reason, our future work will focus on how to encrypt an image such that the deciphered one is not exact the same to the original one, taking into account that human perception of the images shall not be affected very much. If this is achieved, the size of the images can be reduced, thus the transmission speed improves via network.

- **Image Hiding**

1. *Watermark strategies for quantum images*

The quantum computer and quantum network attracts people’s interests to study quantum information hiding. But until now, not many image hiding research regarding quantum issues so far [ZGL+2012].

2. *The best trade-off between the amount of transmitting information and the preferable results*

In image hiding, the communication procedure has to involved with four singular values of size 1/4M-by-1/4N (given the size of the carrier image is M-by-N) and eight unitary matrices of size 1/4m-by-1/4n (given the size of the watermark is m-by-n). These will cause two problems:

1) to encrypt these matrices is really a time-consuming work, and

2) to transmit these matrices takes up a great deal of bandwidth.

Compared to other algorithms, our experimental results is better not only in PSNR but also in NC, this comes at the expense of the transmitting a large bulk of the information of the carrier image and the watermark. But in most cases, the purpose of watermarking is to verify the copyright. Therefore, the value of PSNR is around 20~40 (see ref. [BLK2005, BYK1998, DCOP1999, DKSA2012, GE2004, GM2012, GPK2005, HHDT2010 and Jos2000]) and the NC is around 0.8 (according to the experimental results), the watermarked image is hard to detect and the extracting watermark is still recognisable.

Therefore, to find the best trade-off between the amount of the transmitting information and the preferable PSNR and NC will be our future work to be conducted.

3. *Investigation on blind watermarking method*

Digital watermarking can be divided into visual watermarking and blind watermarking according to the detection process. Visual watermarking needs the original data in the test course, it has stronger robustness, but its application is limited. Blind watermarking does not need original data, which has wide application field, but requires a more sophisticated watermark technology.

- **Temporal logic**

How to apply more temporal logic theory in image encryption and image hiding system, since the relations between images are not just “Before”, “After”, “Meet”, etc, they may be taken into account the duration availability.

- **A temporal- and quantum-based security system**

In a nutshell, no matter how we improve *MQ*-based system, image cryptosystem, image hiding system and or temporal logic, our biggest goal is to implement a temporal- and quantum-based security system, which uses *BMQE* system as the basic cryptosystem to encrypt messages and hide them in some carrier media for watermarking or steganography. For more practical use (which has been discussed in CHAPTER 6), temporal logic is considered.

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