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**Highlights**

- A macrodynamic model with Minskyan insights is presented.
- Firms' and banks' desired margins of safety change endogenously.
- A higher sensitivity of the desired margins of safety to the investment cycle is conducive to instability.
- The relationship between investment and leverage cycles is explored.
- The stabilising role of fiscal policy is emphasised.

Accepted Manuscript

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3 **Margins of safety and instability**  
4 **in a macrodynamic model with Minskyan insights**  
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9 **Abstract**

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11 This paper develops a stock-flow consistent macrodynamic model in which firms' and  
12 banks' desired margins of safety play a central role in macroeconomic performance.  
13 The model incorporates an active banking sector and pays particular attention to the  
14 leverage of both firms and banks. It is shown that the endogenous change in the  
15 desired margins of safety of firms and banks is likely to transform an otherwise stable  
16 debt-burdened economy into an unstable one. The endogeneity of the desired margins  
17 of safety can also produce, under certain conditions, investment and leverage cycles  
18 during which investment and leverage move both in the same and in the opposite  
19 direction. Furthermore, the paper investigates the potential stabilising role of fiscal  
20 policy. It is indicated that fiscal policy can reduce the destabilising forces in the  
21 macroeconomy when government expenditures adjust adequately to variations in the  
22 divergence between the actual and the desired margins of safety.  
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27 *Keywords:* Margins of safety; instability; leverage ratios; Minskyan macroeconomic  
28 analysis  
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31 *JEL Classifications:* E12; E32; E44; E62  
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## Margins of safety and instability

### in a macrodynamic model with Minskyan insights

#### 1. Introduction

The financial crisis that hit the world economy in 2007-8 has brought to the fore the crucial role of economic agents' desired margins of safety in the emergence of financial fragility and macroeconomic instability. The prolonged period of stable and high growth witnessed by many developed countries during the last decades, in conjunction with the absence of important financial episodes, boosted the euphoria of economic agents inducing them to accept lower margins of safety. This provided the ground for increasing financial fragility, which was not confined to the production sector, but was also remarkably associated with the banking sector. The growing financial fragility rendered the macro systems prone to instability and crisis.

The financial crisis has also put at the centre of the stage the potential stabilising role of fiscal policy. Scholars who draw on Minsky's macroeconomic analysis have pointed out that fiscal policy is a major vehicle for ensuring the stability of the macroeconomic system when private consumption and investment are weak (see e.g. Papadimitriou and Wray, 1998; Tymoigne, 2009). It has been argued that government expenditures can place a floor to incomes and economic activity, reducing the possibility of financial breakdown. Although expansionary fiscal policy was initially used by many governments as a response to the crisis (see Arestis and Sawyer, 2010), concerns about fiscal deficits and rising public indebtedness quickly produced a change in attitude toward the implementation of austerity measures.

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2 The purpose of this paper is to formalise some theoretical aspects of the above-  
3 mentioned developments and considerations within a macrodynamic model with  
4 Minskyan insights. The paper draws on the extensive literature that has modelled  
5 various dimensions of Minsky's (1975, 1982, 2008) macroeconomic analysis.<sup>1</sup> The  
6 contribution of the paper, compared to this literature, lies on the explicit examination  
7 of the following two issues within a stock-flow consistent framework.<sup>2</sup>

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19 First, the constructed model allows the desired margins of safety of firms and banks to  
20 change endogenously during the investment cycle. Although the role of economic  
21 agents' desired margins of safety is critical to Minsky's analysis for the emergence of  
22 financial fragility and instability,<sup>3</sup> the formal literature has so far paid little attention to  
23 the distinction between the actual and the desired margins of safety.<sup>4</sup> Most  
24 importantly, this literature has not sufficiently analysed the endogenous character of  
25 these margins of safety and the exact mechanisms through which the change in the  
26 desired margins of safety is conducive to macroeconomic instability.<sup>5</sup> The current  
27 paper shows both analytically and via simulations the destabilising role of endogenous  
28 movements in the desired margins of safety. In our framework the margins of safety of  
29 firms and banks are captured by their leverage ratios.<sup>6</sup>

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<sup>1</sup> See e.g. Ryoo (2010) and the references therein.

<sup>2</sup> For the stock-flow consistent approach to macro modelling see Godley and Lavoie (2007).

<sup>3</sup> See e.g. Kregel (1997), Tymoigne (2009) and Vercelli (2011).

<sup>4</sup> For some exceptions see Dafermos (2012), Le Heron (2008, 2011, 2012, 2013) and Le Heron and Mouakil (2008).

<sup>5</sup> Some recent attempts to endogenise the desired margins of safety can be found in Le Heron (2011, 2013) where the conventional leverage ratio is a function of the state of confidence or the growth rate. Ryoo (2010) has investigated some macro effects of the endogenous change in the desired margins of safety. However, in his model the desired margins of safety are basically driven by households' behaviour in the stock market and not by the endogenous changes in the euphoria of firms and banks during the investment cycle, as is the case in this paper.

<sup>6</sup> As Minsky (2008, p. 266) points out, 'increased leverage by banks and ordinary firms decreases the margins of safety'.

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2 The analysis of this paper focuses on the case of a debt-burdened regime. In our debt-  
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5 burdened regime the capacity utilisation and the investment rate are both negatively  
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7 affected by the leverage of firms.<sup>7</sup> Nishi (2012) argues that in the Minskyan analytical  
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9 framework the debt-burdened regime corresponds to the downturn phases, when the  
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11 leverage ratio affects negatively investment, while the debt-led regime is consistent  
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13 with the boom phase, in which leverage and capital accumulation both increase. This  
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15 paper indicates that the incorporation of endogenous desired margins of safety in an  
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17 economy characterised by a debt-burdened regime can produce cycles during which  
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19 investment and leverage move both in the same and in the opposite direction. This  
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21 implies that the Minskyan boom and downturn phases can be reproduced without  
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23 being necessary to switch from a debt-burdened to a debt-led regime. Furthermore, the  
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25 paper shows that the endogeneity of the desired margins of safety can generate  
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27 instability in an otherwise stable debt-burdened economy.  
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37 Second, the model of this paper examines the extent to which fiscal policy is capable  
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39 of preventing in a debt-burdened economy the instability that stems from the  
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41 endogenous changes in firms' and banks' desired margins of safety. In particular, it  
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43 sets forth a fiscal rule according to which the government expenditures increase  
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45 (decrease) when the desired margins of safety tend to rise (fall) relative to the actual  
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47 ones. Numerical simulations show that this rule has a stabilising role which is broadly  
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49 in line with Minsky's arguments about the capacity of the government to reduce  
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51 destabilising forces in the macro system. Although the stabilising effects of fiscal  
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53 policy have been examined within similar frameworks (see e.g. Charpe et al., 2011,  
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59 <sup>7</sup> For the distinction between the debt-burdened and debt-led regimes see Hein (2013), Nishi (2012) and  
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61 Sasaki and Fujita (2012).  
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1 ch. 9; Keen, 1995; Yoshida and Asada, 2007), our model provides a new perspective  
2 on this issue by linking fiscal policy with the desired margins of safety and the  
3 leverage of firms and banks.  
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9 Importantly, the above-mentioned issues are examined within a framework that  
10 incorporates an active banking sector. Following various recent contributions in macro  
11 modelling (see e.g. Charpe and Flaschel, 2013; Dafermos, 2012; Le Heron, 2008,  
12 2011, 2012, 2013; Le Heron and Mouakil, 2008; Ryoo, 2013b), it is assumed that  
13 banks impose credit rationing when they provide loans to firms. In our setup, the  
14 degree of credit rationing depends upon the financial position of both firms and banks.  
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26 The paper is organised as follows. Section 2 sets out the structure of the model.  
27 Section 3 presents the main properties, the dynamic equations and the steady state of  
28 the macro system. Section 4 explores analytically and via simulations the destabilising  
29 effects of the endogenous changes in the desired margins of safety of firms and banks.  
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## 2. Structure of the model

The economy of the model is composed of households, firms, banks, the central bank and the government. Table 1 displays the balance sheet matrix. Table 2 depicts the transactions matrix. Households receive wage income, interest income and the

1 distributed profits of firms and banks.<sup>8</sup> They keep their wealth only in the form of  
 2 bank deposits. They do not take out loans from banks. Firms finance their investment  
 3 expenditures using loans and retained profits. Banks provide loans to firms, hold  
 4 treasury bills and high-powered money; their liabilities comprise household deposits  
 5 and advances from the central bank. Banks' undistributed profits are used to build  
 6 capital. Central bank holds treasury bills and advances on the asset side of its balance  
 7 sheet and high-powered money on the liability side. Its profits are distributed to the  
 8 government. Government issues treasury bills to finance its expenditures.<sup>9</sup> Inflation is  
 9 assumed away and the level of prices is set, for simplicity, equal to unity. There is  
 10 only one type of product which can be used for both consumption and investment  
 11 purposes.  
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39 Eq. (1) gives the disposable income of households ( $Y_d$ ):  
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$$44 Y_d = W + i_d D + PF_d + PB_d \quad (1)$$

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 49 where  $W$  is the wage bill,  $i_d$  is the interest rate on deposits,  $D$  is the amount of  
 50 deposits,  $PF_d$  denotes the distributed profits of firms and  $PB_d$  denotes the distributed  
 51 profits of banks.  
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57 <sup>8</sup> Households are the owners of firms and banks. To avoid complications, it is assumed that firms and  
 58 banks do not issue shares.

59 <sup>9</sup> For simplicity, there are no taxes in the model. Thus, fiscal policy is implemented via changes only in  
 60 the government expenditures.  
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2 The wage bill of households is written as:  
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$$6 \quad W = s_w Y \quad (2)$$

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12 where  $s_w$  is the income share of wages and  $Y$  is the level of output.  
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17 Households' consumption ( $C$ ) depends on their disposable income and deposits:  
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$$22 \quad C = c_1 Y_d + c_2 D \quad (3)$$

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28 where  $0 < c_2 < c_1 < 1$ .  
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33 The change in deposits is determined by the following equation:  
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$$38 \quad \dot{D} = Y_d - C \quad (4)$$

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43 Eq. (5) shows the profits of firms ( $PF$ ):  
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$$48 \quad PF = Y - W - i_l L \quad (5)$$

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54 where  $i_l$  is the lending interest rate and  $L$  is the amount of firms' loans.  
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The undistributed profits of firms ( $PF_u$ ) are determined as a proportion ( $s_f$ ) of their total profits:

$$PF_u = s_f PF \quad (6)$$

Eq. (7) gives the distributed profits of firms ( $PF_d$ ):

$$PF_d = PF - PF_u \quad (7)$$

In the formulation of investment expenditures, the distinction between the desired investment of firms ( $I^d$ ) and the effective one ( $I$ ) is adopted (Dafermos, 2012; Le Heron and Mouakil, 2008). The effective investment is equal to the desired one minus the amount of new loans that are credit rationed by banks ( $NL^{cr}$ ). In particular, it holds that:

$$I = I^d - NL^{cr} \quad (8)$$

From Eq. (8) it is straightforward that credit rationing exerts a negative impact on effective investment. The desired investment scaled by capital stock ( $g^d$ ) is given by:

$$g^d = \frac{I^d}{K} = \delta_0 + \delta_1 u - \delta_2 (lf - lf^T) \quad (9)$$

1 where  $\delta_0, \delta_1, \delta_2 > 0$ ,  $K$  is the capital stock,  $\delta_0$  denotes the ‘animal spirits’ of  
 2 entrepreneurs and  $u$  is the rate of capacity utilisation. The utilisation rate is written as  
 3  $u = Y/(vK)$ , where  $v$  is the exogenously given full-capacity output-to-capital ratio.  
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 5 Eq. (9) shows that the desired investment rate is affected by endogenous changes in  
 6 capacity utilisation and in the leverage ratio relative to the target one.<sup>10</sup> It is postulated  
 7 that the leverage ratio (i.e. the loans to capital ratio,  $lf = L/K$ ) is used by firms as a  
 8 proxy for their actual margins of safety: a high (low) leverage ratio implies low (high)  
 9 margins of safety. The desired margins of safety are reflected in the value of firms’  
 10 target leverage ratio ( $lf^T$ ). Eq. (9) suggests that the lower the actual leverage ratio  
 11 relative to the target one, the higher the investment rate (and vice versa).<sup>11</sup> This  
 12 formulation is broadly in line with Minsky’s (2008) emphasis on the role of leverage  
 13 and desired margins of safety in the capital accumulation process (see, e.g., Minsky,  
 14 2008, p. 209).

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 35 It is important to point out that our formulation does not imply that a rise in the target  
 36 leverage ratio of firms always leads to a higher actual leverage ratio. The induced  
 37 increase in desired investment, which tends to make  $lf$  higher, might be  
 38 overcompensated by the increase in undistributed profits (due to higher economic  
 39 activity) and the rise in capital stock (due to higher investment), both of which tend to  
 40 reduce  $lf$ . If this happens, a ‘paradox of debt’ occurs: although firms try to increase  
 41 their leverage ratio by increasing investment they end up with a lower leverage ratio.<sup>12</sup>

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 54 <sup>10</sup> Obviously, capital accumulation may also rely on other variables, such as the rate of profit, the  
 55 interest rate or the Tobin’s  $q$ . In this paper, we use a simple specification to focus on the effects of  
 56 firms’ margins of safety.

57 <sup>11</sup> For some similar formulations that capture the impact of desired and actual margins of safety on  
 58 investment see Dafermos (2012) and Le Heron (2008, 2011, 2013).

59 <sup>12</sup> For a detailed discussion of the ‘paradox of debt’ in formal models see Hein (2007, 2013), Lavoie  
 60 (1995) and Ryoo (2013a).

1 Interestingly, the overall result on  $lf$  is also affected by the credit rationing behaviour  
 2 of banks.  
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 7 The change in loans ( $\dot{L}$ ) is given by the following formula:  
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$$9 \quad \dot{L} = NL^d - NL^{cr} - repL \quad (10)$$

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 12 where  $NL^d$  stands for the demanded amount of new loans and  $rep$  for the loan  
 13 repayment ratio. Since the amount of credit rationed loans are always a fraction of  
 14 demanded loans it invariably holds that  $NL^{cr} < NL^d$ .  
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 28 The demanded amount of new loans are determined as follows:  
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$$30 \quad NL^d = I^d - PF_u + repL \quad (11)$$

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 38 The amount of new loans that are credit rationed, scaled by capital stock, are given by  
 39 the following formula:  
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$$42 \quad \frac{NL^{cr}}{K} = b_0 + b_1 lf + b_2 (lb - lb^T) \quad (12)$$

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 52 where  $b_0, b_1, b_2 > 0$ . The term  $b_0$  captures exogenous factors that affect credit  
 53 rationing (such as the ‘animal spirits’ of banks, the degree of securitisation etc.). The  
 54 second term illustrates that a higher leverage of firms reduces the willingness of banks  
 55 to provide credit: when the leverage of firms increases banks conceive the risk of  
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borrowers' default to increase.<sup>13</sup> Eq. (12) also suggests that the bank leverage plays a crucial role in the determination of credit availability. The leverage of banks ( $lb$ ) is given by their assets-to-capital ratio:

$$lb = \frac{lf + b_b + hpm}{lf + b_b + hpm - d - a} \quad (13)$$

where  $b_b = B_b/K$  is the banks' treasury bills ( $B_b$ )-to-capital ratio,  $hpm = HPM/K$  is the high-powered money ( $HPM$ )-to-capital ratio,  $d = D/K$  is the deposits-to-capital ratio and  $a = A/K$  is the advances ( $A$ )-to-capital ratio. Note that according to the balance sheet matrix (see Table 1) the bank capital ( $K_b$ ) is equal to  $L + B_b + HPM - D - A$ . Minsky (2008, ch. 10) emphasises the importance of banks' leverage in the processes that lead the macroeconomy toward higher financial fragility. In Minsky's analysis, the inducement of banks to increase their leverage as a means to heighten the return on equity is one of the principal factors that increase the supply of financing by banks. In our framework, a higher bank leverage increases, *ceteris paribus*, banks' concerns about their own financial position. Thus, credit rationing is positively affected by bank leverage. However, any rise in the target bank leverage ratio ( $lb^T$ ), which as will be shown below changes endogenously during the investment cycle, decreases credit rationing. This implies that, in broad line with Minsky's arguments, any inducement of banks to accept higher leverage ratios pushes up the accumulation of firm debt.<sup>14</sup>

<sup>13</sup> See Le Heron and Mouakil (2008) for a similar assumption.

<sup>14</sup> Charpe and Flaschel (2013) use a similar formulation in which credit rationing is connected with banks' net wealth. Ryoo (2013b), who also relies on Minsky's framework, postulates a positive effect of bank leverage on credit availability.

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Eqs. (8)-(12) suggest that the undistributed profits of firms have both first-round and second-round effects on the leverage of firms. The first-round effects stem from the fact that higher retained profits reduce, *ceteris paribus*, firms' demand for new loans driving down their leverage. This fall in leverage produces, however, some second-round feedback effects because it boosts the desired investment of firms and decreases credit rationing. These second-round effects tend to increase both the numerator and the denominator in the leverage ratio with the overall result being ambiguous.

Banks' profits ( $PB$ ) are given by:

$$PB = i_l L + i_b B_b - i_d D - i_a A \quad (14)$$

where  $i_b$  is the interest rate on treasury bills and  $i_a$  is the interest rate on advances;  $i_a$  is determined by the central bank. For simplicity, it is assumed that  $i_b = i_a$ .

Banks retain a proportion ( $s_b$ ) of their profits:

$$PB_u = s_b PB \quad (15)$$

The distributed profits of banks ( $PB_d$ ) are equal to:

$$PB_d = PB - PB_u \quad (16)$$

The interest rates on deposits and loans are determined as follows:

$$i_d = h_d i_a \quad (17)$$

$$i_l = h_l i_a \quad (18)$$

where  $h_d < 1$  is the mark-down and  $h_l > 1$  is the mark-up over the interest rate on advances. Note that  $h_d$  and  $h_l$  are exogenously given in our analysis.

Banks hold reserves, which are a fixed proportion ( $h_1$ ) of deposits:

$$HPM = h_1 D \quad (19)$$

Banks also hold treasury bills as a fixed proportion ( $h_2$ ) of deposits:

$$B_b = h_2 D \quad (20)$$

The advances act as a residual in the balance sheet of banks :<sup>15</sup>

$$\dot{A} = H\dot{P}M + \dot{B}_b + \dot{L} - \dot{D} - PB_u \quad (21)$$

The change in government's treasury bills ( $B$ ) is determined by its budget constraint:

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<sup>15</sup> Note that  $\dot{K}_b = PB_u$ .

$$\dot{B} = GOV + i_b B - PCB \quad (22)$$

where  $PCB$  denotes the profits of the central bank (recall that these profits are distributed to the government) and  $GOV = govK$  denotes the government expenditures.

The profits of the central bank are equal to the sum of the interest on treasury bills ( $B_{cb}$ ) and the interest on advances:

$$PCB = i_b B_{cb} + i_a A \quad (23)$$

The treasury bills held by the central bank are given by Eq. (24):

$$B_{cb} = HPM - A \quad (24)$$

Eq. (25) gives the output of the economy:

$$Y = C + I + GOV \quad (25)$$

Note that the redundant equation of the model is:

$$B_{cb-red} = B - B_b \quad (26)$$



1 This equation should be verified in our simulations so as to ensure that the model is  
 2 stock-flow consistent.  
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 7 Having presented the main structure of the model, we are now in a position to describe  
 8 the law of motion of the target leverage ratios (desired margins of safety) of firms and  
 9 banks. As shown above, the target leverage ratios play a central role in the behaviour  
 10 of the macroeconomy since they influence the investment and lending decisions.  
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 19 The law of motion of firms' target leverage ratio is captured by the following formula:  
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$$21 \dot{l}^T = \xi_1(g - g_n) + \xi_2(l_0^T - l^T) \quad (27)$$

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 25 where  $\xi_1, \xi_2 > 0$ . Eq. (27) suggests that the change in the target leverage ratio of firms  
 26 relies on the difference between the effective investment rate ( $g = I/K$ ) and what is  
 27 conceived as a normal rate of investment ( $g_n$ ), which is used as a reference point.  
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When the rate of effective investment in the economy is higher than  $g_n$ , there is a rise in the euphoric expectations of firms, since the economy appears to perform much better than what is normally expected. With everything else given, this leads firms to relax their desired margins of safety or, equivalently stated, to increase their target leverage ratio: what before was conceived as a risky project may now be evaluated as a safe investment due to the general good performance of the economy. The parameter  $\xi_1$  reflects the sensitivity of firms' target leverage ratio to differences between the effective and the normal investment rate. The higher this parameter the more prone the expectations of firms to the investment cycle.

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2 The second term in Eq. (27) implies that firms do not allow their target leverage ratio  
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4 to deviate significantly from a reference value ( $lf_0^T$ ). When the target leverage ratio  
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6 increases (decreases) relative to the reference value, firms are prompted to reduce  
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8 (increase) their target leverage ratio.  
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14 Minsky (2008, p. 255) points out that in an environment of favourable expectations,  
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16 the higher willingness of firms to invest is accompanied by a higher willingness of  
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18 bankers to finance investment projects: '[b]ecause bankers live in the same  
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20 expectational climate as businessmen, profit-seeking bankers will find ways of  
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22 accommodating their customers; this behavior by bankers reinforces the  
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24 disequilibrating pressures'. In order to capture this Minskyan idea we allow the target  
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26 leverage ratio of banks to co-move with the target leverage ratio of firms:  
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$$lb^T = \phi lf^T \quad (28)$$

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39 where  $\phi$  is a positive parameter. Eqs. (27)-(28) imply that both firms' and banks'  
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41 desired margins of safety change during the investment cycle. When, for instance, the  
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43 effective investment rate is higher than the normal one, not only firms increase their  
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45 target leverage ratio, placing upward pressures on investment, but also banks become  
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47 more willing to target a higher leverage ratio and increase thereby credit availability.  
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52 The reason is that the expansionary environment improves the repayment history of  
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borrowers and, hence, banks become less concerned about the repercussions of an increase in their own leverage ratios.<sup>16</sup>

Overall, Eqs. (27) and (28) are consistent with Minsky's (2008, p. 209) argument that '[a] history of success will tend to diminish the margin of safety that business and bankers require... a history of failure will do the opposite'. It will be shown below that this endogenous change in the desired margins of safety of both firms and banks is likely to transform an otherwise stable debt-burdened economy into an unstable one.

As mentioned in the introduction, one of the additional purposes of this paper is to examine whether fiscal policy can play a stabilising role in our macrodynamic system.

In an economy in which the desired margins of safety change endogenously, this stabilising role could be attained if the government expenditures adjust adequately to variations in the divergence between the actual and the desired margins of safety. The fiscal rule described in Eq. (29) captures this idea:

$$g\dot{ov} = e_1 [(lf - lf^T) - (lf_0 - lf_o^T)] + e_2 [(lb - lb^T) - (lb_0 - lb_o^T)] + e_3 (gov_r - gov) \quad (29)$$

Note that  $e_1, e_2, e_3 > 0$ . Eq. (29) states that, other things equal, the government expenditures-to-capital ratio increases (decreases) when the difference between the actual and the target leverage ratio of firms and banks becomes higher (lower) than their difference in the steady state. The economic intuition of this rule is the following: when the actual leverage ratios are much higher than the target leverage ratios there is

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<sup>16</sup> For the endogenous change in the desired margins of safety of banks during the economic cycle see also Kregel (1997) and Tymoigne (2009). Moreover, for macro models in which the endogenous changes in the lender's risk play a crucial role in the credit rationing procedure see Le Heron (2011, 2013).

1 a tendency for investment to decrease since firms and banks are less willing to  
 2 participate in new debt contracts; this produces contractionary forces in the economy.  
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 4 By increasing its expenditures the government can counteract these forces, stabilising  
 5 economic activity and thereby the leverage ratios. The same stabilising role can be  
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 7 played when government expenditures are driven down in response to a decline in the  
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 9 difference between the actual and the target leverage ratios.  
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17 The third term in Eq. (29) has been introduced to capture the fact that the government  
 18 attempts to avoid excessive expenditures;  $gov_r$  is a reference value. When  
 19  $gov > gov_r$ , the government expenditures-to-capital ratio tends to decrease, and vice  
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### 3. The 5D macroeconomic system

The equilibrium in the product market is brought about by changes in the rate of  
 capacity utilisation.<sup>17</sup> We insert Eqs. (3) and (8) into (25) and divide through by capital  
 stock. Making the necessary substitutions and solving for the equilibrium rate of  
 capacity utilisation ( $u^*$ ) we obtain:

$$u^* = \frac{\delta_0 - b_0 + (\delta_2 + b_2\phi)lf^T + gov + (c_1\Gamma_1 + c_2)d + (c_1\Gamma_2 - \delta_2 - b_1)lf - c_1\Gamma_3a - b_2lb}{\Delta} \quad (30)$$

<sup>17</sup> In the current paper the rate of capacity utilisation is endogenously determined both in the short run and the long run. For the debate over the long-run endogeneity of capacity utilisation see Hein et al. (2012) and Skott (2012).

where  $\Gamma_1 = i_d + (1-s_b)i_a h_2 - (1-s_b)i_d > 0$ ,  $\Gamma_2 = (1-s_b)i_l - (1-s_f)i_l$ ,  $\Gamma_3 = (1-s_b)i_a > 0$  and  $\Delta = v - c_1 v (1-s_f(1-s_w)) - \delta_1$ . The product market equilibrium requires that the denominator of (30) be positive (i.e.  $\Delta > 0$ ). We also assume that the numerator in Eq. (30) is positive to obtain a positive  $u^*$ .

Substituting Eq. (30) into Eq. (8) we get the equilibrium rate of effective investment

( $g^*$ ):

$$g^* = \delta_0 - b_0 + (\delta_2 + b_2 \phi) l f^T + \delta_1 u^* - (\delta_2 + b_1) l f - b_2 l b \quad (31)$$

Differentiating Eqs. (30) and (31) with respect to  $l f$ ,  $d$ ,  $a$ ,  $l f^T$  and  $gov$  yields:<sup>18</sup>

$$\partial u^* / \partial l f = u_{l f}^* = \frac{c_1 \Gamma_2 - \delta_2 - b_1 - b_2 l b_{l f}}{\Delta} \quad (32)$$

$$\partial u^* / \partial d = u_d^* = \frac{c_1 \Gamma_1 + c_2 - b_2 l b_d}{\Delta} \quad (33)$$

$$\partial u^* / \partial a = u_a^* = \frac{-c_1 \Gamma_3 - b_2 l b_a}{\Delta} < 0 \quad (34)$$

$$\partial u^* / \partial l f^T = u_{l f^T}^* = \frac{\delta_2 + b_2 \phi}{\Delta} > 0 \quad (35)$$

$$\partial u^* / \partial gov = u_{gov}^* = \frac{1}{\Delta} > 0 \quad (36)$$

$$\partial g^* / \partial l f = g_{l f}^* = \delta_1 u_{l f}^* - \delta_2 - b_1 - b_2 l b_{l f} \quad (37)$$

$$\partial g^* / \partial d = g_d^* = \delta_1 u_d^* - b_2 l b_d \quad (38)$$

$$\partial g^* / \partial a = g_a^* = \delta_1 u_a^* - b_2 l b_a < 0 \quad (39)$$

<sup>18</sup> It can be easily shown that the economic activity in the model is wage-led (i.e.  $\partial u^* / \partial s_w > 0$ ).

$$\partial g^* / \partial l f^T = g_{l f^T}^* = \delta_2 + b_2 \phi + \delta_1 u_{l f^T}^* > 0 \quad (40)$$

$$\partial g^* / \partial g o v = g_{g o v}^* = \delta_1 u_{g o v}^* > 0 \quad (41)$$

where<sup>19</sup>

$$\partial l b / \partial l f = l b_{l f} = l b^2 \frac{-a - d}{(l f + (h_1 + h_2) d)^2} < 0 \quad (42)$$

$$\partial l b / \partial d = l b_d = l b^2 \frac{l f - (h_1 + h_2) a}{(l f + (h_1 + h_2) d)^2} \quad (43)$$

$$\partial l b / \partial a = l b_a = \frac{l b^2}{l f + (h_1 + h_2) d} > 0 \quad (44)$$

The impact of firms' leverage on capacity utilisation and effective investment cannot be unambiguously determined (see Eqs. (32) and (37)). In the model there are three unfavourable and two favourable effects of a higher firms' leverage on economic activity (see Table 3). An increase in the leverage of firms tends to depress investment due to the direct adverse impact on desired investment and credit rationing. Moreover, it places downward pressures on consumption because it affects negatively firms' distributed profits. These are the unfavourable effects. The favourable effects are associated with the expansionary impact of banks' distributed profits on consumption as well as with the inverse link between the leverage of firms and the leverage of banks (see Eq. (42)); the latter implies that, other things equal, when the firm leverage increases (decreases) the bank leverage falls (rises), increasing (reducing) thereby credit availability.

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<sup>19</sup> Scaling Eqs. (19) and (20) by capital stock and substituting into (13), yields:  
 $l b = [l f + (h_1 + h_2) d] / [l f - (1 - h_1 - h_2) d - a]$ .

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7 As mentioned at the outset, this paper focuses on the case of a debt-burdened regime  
8 in which, according to the definition adopted, the partial derivatives of capacity  
9 utilisation and effective investment with respect to the leverage of firms are both  
10 negative. This is ensured by assuming that  $\delta_2$ ,  $s_b$  and  $b_1$  are sufficiently large and  $s_f$   
11 and  $b_2$  are sufficiently small (and, hence,  $\Gamma_2$  is small) so as for the negative effects of  
12 the firm leverage on aggregate demand to outweigh the positive ones; this implies that  
13 (32) and (37) are postulated to be negative.  
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27 Table 3 shows that an increase in the deposits-to-capital ratio on economic activity has  
28 both favourable and unfavourable effects on economic activity. Therefore, the sign of  
29 Eqs. (33) and (38) is ambiguous. On the one hand, a rise in  $d$  tends to boost  
30 consumption via the wealth effect and the induced increase in the interest income of  
31 households. On the other hand, a higher  $d$  increases, *ceteris paribus*, the leverage of  
32 banks and hence credit rationing (throughout the paper we adopt the plausible  
33 assumption that  $h_1$  and  $h_2$  are sufficiently small so as for  $lb_d > 0$ ; see Eq. (43)).  
34 Moreover, there is an ambiguous impact on consumption from the distributed profits  
35 of banks: a higher  $d$  increases the interest paid by banks on deposits but it also  
36 increases the interest received on treasury bills (recall that treasury bills are a  
37 proportion of deposits).  
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57 Eqs. (34) and (39) show that an increase in the advances-to-capital ratio produces  
58 unambiguously a decrease in capacity utilisation and effective investment rate: a rise  
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in advances-to-capital ratio leads, other things equal, to more liabilities and to a higher bank leverage (see Eq. (42)), enhancing thereby credit rationing; it also reduces the distributed profits of banks with negative effects on consumption. Eqs. (35) and (40) show that a higher target leverage ratio of firms increases the rate of capacity utilisation and the effective investment rate; the same holds for the target leverage ratio of banks which is a linear function of  $lf^T$  (see Eq. (28)). Lastly, Eqs. (36) and (41) show that, when government expenditures-to-capital ratio increases,  $u^*$  and  $g^*$  become higher.

For the purposes of our analysis, attention is confined to the system of the five dynamic equations for the leverage of firms ( $\dot{lf}$ ), the deposits-to-capital ratio ( $\dot{d}$ ), the advances-to-capital ratio ( $\dot{a}$ ), the target leverage of firms ( $\dot{lf}^T$ ), and the government expenditures-to-capital ratio ( $\dot{gov}$ ).<sup>20</sup> It is assumed that in the dynamic evolution of the system the equilibrium values of  $u$  and  $g$  are always attained. We have that:

$$\dot{lf} = \left( \frac{\dot{L}}{K} \right) = \delta_0 - b_0 + (\delta_2 + b_2\phi)lf^T + [\delta_1 - s_f(1-s_w)v]u^* + (s_f i_l - \delta_2 - b_1 - g^*)lf - b_2 lb \quad (45)$$

$$\dot{d} = \left( \frac{\dot{D}}{K} \right) = (1-c_1)[1-s_f(1-s_w)]vu^* + [(1-c_1)\Gamma_1 - c_2 - g^*]d + (1-c_1)\Gamma_2 lf - (1-c_1)\Gamma_3 a \quad (46)$$

$$\dot{a} = \left( \frac{\dot{A}}{K} \right) = lf - (1-h_1-h_2)\dot{d} - (s_b i_l - g^*)lf - [\Gamma_4 + (1-h_1-h_2)g^*]d + (s_b i_a - g^*)a \quad (47)$$

$$\dot{lf}^T = \xi_1(g - g_n) + \xi_2(lf_0^T - lf^T) \quad (48)$$

$$\dot{gov} = e_1[(lf - lf^T) - (lf_0 - lf_0^T)] + e_2[(lb - lb^T) - (lb_0 - lb_0^T)] + e_3(gov_r - gov) \quad (49)$$

<sup>20</sup> Note that this 5D system is independent of the treasury bills held by the commercial banks, the central bank and the government. The treasury bills are determined as a residual, without having feedback effects on the 5D system (see Eqs. (20), (22) and (24)).



$$lb^T = \phi f^T \quad (50)$$

The steady-state values of the variables are estimated by setting the above differential equations equal to zero.<sup>21</sup> The unique steady state of the system denoted by a subscript 0 is the following:

$$g_0 = g_n, gov_0 = gov_r, lf_0 = \frac{s_f(1-s_w)v u_0 - g_n}{s_f i_l - g_n}, lb_0 = lb_0^T = \frac{lf_0 + (h_1 + h_2)d_0}{lf_0 - (1-h_1-h_2)d_0 - a_0},$$

$$d_0 = \frac{(1-c_1)(\varepsilon_0 + \varepsilon_2 lf_0)}{g_n + c_2 - (1-c_1)\varepsilon_1}, a_0 = \frac{(s_b i_l - g_n)lf_0 + [\Gamma_4 + (1-h_1-h_2)]d_0}{s_b i_a - g_n},$$

$$u_0 = \frac{g_n + gov_r + (c_1 \Gamma_1 + c_2)d_0 + c_1 \Gamma_2 lf_0 - c_1 \Gamma_3 a_0}{\Delta_0} \text{ and}$$

$$lf_0^T = \frac{g_n - (\delta_0 - b_0 + \delta_1 u_0 - (\delta_2 + b_1)lf_0 - b_2 lb_r)}{\delta_2 + b_2 \phi}$$

where  $\Gamma_4 = s_b(i_a h_2 - i_d)$ ,  $\Delta_0 = v - c_1 v [1 - s_f(1-s_w)]$ ,  $\varepsilon_0 = [(1-s_f(1-s_w))v(g_n + gov_r)]/\Delta_0$ ,

$$\varepsilon_1 = [\Gamma_1 v + (1-s_f(1-s_w))c_2 v]/\Delta_0 - [\Gamma_3 v(\Gamma_4 + (1-h_1-h_2)g_n)]/[(s_b i_a - g_n)\Delta_0] \text{ and}$$

$$\varepsilon_2 = [v\Gamma_2]/\Delta_0 - [v\Gamma_3(s_b i_l - g_n)]/[(s_b i_a - g_n)\Delta_0].$$

<sup>21</sup> In the mathematical analysis and the simulation exercises presented in section 4 it is assumed that  $g_n > (\delta_0 - b_0 + \delta_1 u_0 - (\delta_2 + b_1)lf_0 - b_2 lb_0)$ ,  $s_b i_a < g_n$ ,  $(s_b i_l - g_n)lf_0 + (\Gamma_4 + (1-h_1-h_2))d_0 < 0$ ,  $s_f i_l > g_n$  and  $s_f(1-s_w)v u_0 > g_n$ . These conditions ensure that the values of the variables at the steady state are always positive.

#### 4. Instability, cycles and the stabilising role of fiscal policy

##### 4.1 The macro system with exogenous desired margins of safety and government expenditures

We initially focus on the 3D subsystem given by the laws of motion for  $lf$ ,  $d$  and  $a$ ;  $lf^T$  and  $gov$  are kept at their steady-state values. The interactions between the endogenous variables in this subsystem are quite complex. As described in section 3,  $lf$ ,  $d$  and  $a$  affect the investment rate and the capacity utilisation rate. Simultaneously, any change in investment and capacity utilisation influences  $lf$ ,  $d$  and  $a$  through various channels. This implies that the three endogenous variables are all interconnected in a complex way.

It is worth mentioning briefly the channels through which investment and capacity utilisation influence  $lf$ ,  $d$  and  $a$ . A common effect of investment on the loans-to-capital ratio, the deposits-to-capital ratio and the advances-to-capital ratio is the impact on the denominator of these ratios through the resulting changes in capital stock. Remarkably, the higher these ratios the more important the impact of capital stock variations.

Regarding the law of motion of  $lf$ , an increase in capacity utilisation exerts counteracting effects on new loans (and therefore on the numerator of the leverage ratio). On the one hand, there is a tendency of new loans to increase since desired investment is positively affected by a higher capacity utilisation rate. On the other hand, new loans tend to decline because higher economic activity increases the sales

1 of firms and, thus, their undistributed profits. The deposits-to-capital ratio is positively  
 2 influenced by a rise in capacity utilisation and investment: higher economic activity  
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 4 tends to increase the income of households and, therefore, their saving and deposits.  
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 6 The advances-to-capital ratio is not directly affected by economic activity; however,  
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 8 the balance sheet of banks implies that there are indirect effects through the change in  
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 10 loans and deposits.  
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17 The stability properties of the 3D subsystem are summarised in Proposition 1.  
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22 **Proposition 1.** *Consider the 3D subsystem of Eqs. (45)-(47). Suppose that economic*  
 23 *activity is debt-burdened (i.e.  $\delta_2$ ,  $s_b$  and  $b_1$  are sufficiently large and  $s_f$ ,  $b_2$  and  $\Gamma_2$*   
 24 *are sufficiently small). If  $lf_0$ ,  $d_0$ ,  $a_0$ ,  $\Gamma_1$  and  $\Gamma_3$  are sufficiently small, the steady state*  
 25 *of the 3D subsystem is locally stable (see Appendix A for the proof).*  
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35 The economic rationale behind Proposition 1 can be explained as follows. Sufficiently  
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 37 low values of  $s_f$  and  $lf_0$  ensure that any increase (decrease) in investment and  
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 39 capacity utilisation translates into a higher (lower)  $lf$ : the new loans created by the  
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 41 inducement of firms to invest more (less) outweigh the increase (decline) in  
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 43 undistributed profits and the increase (decrease) in capital stock. Therefore, the  
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 45 existence of a debt-burdened regime in conjunction with a low  $lf_0$  ensures a stabilising  
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 47 relationship between the investment rate and the leverage of firms: a rise in  $lf$  reduces  
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 49 investment, lower investment decreases  $lf$  and the decline in  $lf$  brings the investment  
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 51 rate back to its steady-state value (and vice versa). Moreover, sufficiently low values  
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 53 of  $d_0$  and  $a_0$ ,  $\Gamma_1$  and  $\Gamma_3$  ensure that there is a similar stabilising relationship between  
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economic activity, the deposits-to-capital ratio and the advances-to-capital ratio. Recall that  $\Gamma_1$  and  $\Gamma_3$  are related with the impact of  $d$  and  $a$  on capacity utilisation: the lower they are the lower this impact. Hence, if the conditions described in Proposition 1 are satisfied, the system becomes overall stable.

#### 4.2 Making the desired margins of safety endogenous

We now turn to analyse the stability properties of the subsystem in which the target leverage ratios change endogenously. This is the 4D subsystem consisting of Eqs. (45)-(48);  $gov$  is kept at its steady-state value. Its stability properties are described in Proposition 2.

**Proposition 2.** *Consider the 4D subsystem of Eqs. (45)-(48). Suppose that the conditions described in Proposition 1 hold (i.e. the 3D subsystem is stable). Suppose also that the Conditions (51)-(54) hold.*

$$g_{lf^r} < \frac{a_1^{(3)} + \xi_2}{\xi_1} \quad (51)$$

$$g_{lf^r} < \frac{\Omega}{a_1^{(3)}} \quad (52)$$

$$g_{lf^r} < \frac{-(\Omega_1 + \Omega_2 + \Omega_3)}{a_2^{(3)}} \quad (53)$$

$$g_{lf^r} < \frac{\Psi}{a_3^{(3)}} \quad (54)$$

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*Then, the steady state of the 4D subsystem is locally stable, unstable or exhibits a limit cycle depending on the value of  $\xi_1$  (the sensitivity of target leverage ratios to the investment cycle). In particular, it holds that:*

*(I) The system is locally stable for sufficiently small values of  $\xi_1$ .*

*(II) The system is locally unstable for sufficiently high values of  $\xi_1$ .*

*(III) There is a parameter value  $\xi_1^b$  at which a simple Hopf bifurcation occurs and the subsystem exhibits a limit cycle.*

*(See Appendix B for the proof).*

The endogenous change in the target leverage ratios can generate destabilising forces in an otherwise stable system in which economic activity is debt-burdened. The reason is briefly the following: As the effective investment rate increases (decreases) relative to the normal rate, the target leverage ratios become higher (lower) (see Eqs. (48) and (50)). Consequently, the negative stabilising effect of the leverage of firms and banks on desired investment and credit availability becomes less (more) strong due to the higher (lower) euphoria of firms and banks and the decline (increase) in perceived risk.

Proposition 2 suggests that the stability of the 4D subsystem is guaranteed only if the sensitivity of the target leverage ratios to the investment cycle is below a critical value, as well as if the partial derivative of effective investment with respect to the firms' target leverage ratio is not high enough (see Conditions (51)-(54)). These conditions ensure that the destabilising forces of increasing euphoria and lower perceived risk are not sufficiently large. If these conditions are not met instability emerges.

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In order to analyse in greater detail the destabilising effects of endogenous alterations in the target leverage ratio we have conducted some simulations using the parameter values reported in Appendix C.<sup>22</sup> In the simulation analysis  $\xi_1$  has been used as the critical parameter for the stability properties of the subsystem.<sup>23</sup> Moreover, the underlying 3D subsystem described in section 4.1 is always stable.

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Fig. 1 shows the effects of an increasing  $\xi_1$  on the stability of the subsystem, in the aftermath of an exogenous rise in the target leverage ratios. It can be readily seen that, as the sensitivity of the target leverage ratios to the investment cycle rises, the subsystem gradually turns from stability to instability.<sup>24</sup>

<Insert Fig. 1 here>

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In order to understand the underlying mechanisms, it is useful first to outline the case in which  $\xi_1 = 0$ . In this case an exogenous rise in the target leverage ratios leads to higher desired investment and greater credit availability. The resulting higher effective investment increases firm and bank leverage (in our simulations it also leads to a higher level of deposits and advances relative to capital stock). Since economic activity is debt-burdened, the increasing firm leverage generates lower investment which, in turn, brings loans, deposits and advances to their steady-state values.

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On the other hand, when the target leverage ratios are endogenous, an exogenous increase in these targets does not only increase effective investment and new loans, but

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<sup>22</sup> The Matlab codes for the simulation exercises are available upon request.

<sup>23</sup> The simulation exercises presented in Fig. 1 as well as in Fig. 4 have been inspired by Chiarella et al. (2012).

<sup>24</sup> The system turns from stability to instability at  $\xi_1 = 0.659$ .

1 also positively affects, via higher accumulation, the euphoria of firms and banks. This  
2 euphoria combined with the lower perceived risk tends to further increase loan  
3 accumulation. If  $\xi_1$  is high enough, this new second-round effect is likely to produce  
4 an excessive increase in the leverage of firms and banks, giving rise to a destabilising  
5 mechanism. The inverse mechanisms are at work when the effective investment rate  
6 falls short of the normal one. Overall, the higher the value of  $\xi_1$  the stronger the  
7 destabilising forces, as Fig. 1 illustrates.  
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20 Proposition 2 suggests that there is a critical value for  $\xi_1$  at which the destabilising  
21 forces exactly offset the stabilising ones, producing a limit cycle. Fig. 2 illustrates the  
22 trajectories of the main variables of the 4D subsystem in our simulations when a limit  
23 cycle emerges. Fig. 3 shows the relationship between the leverage ratio of firms and  
24 the effective investment rate under the case of a limit cycle.  
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45 The cyclical behaviour of the economy can be described as follows. Initially, the  
46 effective investment rate is driven up, following the exogenous increase in the target  
47 leverage ratios; the economy is located at point A in Fig. 3. Since the effective  
48 investment rate becomes higher than the normal one (the latter is equal to 0.04 in our  
49 simulations), a second-round endogenous increase in the target leverage ratio occurs.  
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greater willingness of both firms and banks to undertake more risky projects. The rise in the firm leverage produces in our simulations an increase in bank leverage.

The higher leverage of both firms and banks has negative feedback effects on the effective investment rate. Eventually, this rate falls short of the normal one (point B in Fig. 3), generating a fall in the target leverage ratio of firms and banks. As a consequence, the leverage of firms and banks start falling. When these variables reach a sufficiently low value (point C in Fig. 3), the effective investment rate starts increasing. Yet, the economy continues to experience a fall in  $l_f$  and  $l_b$  for some periods: the pessimism of economic agents keeps rising and the effective investment rate is still low to cause a sufficient increase in new loans. When the effective investment rate passes the  $g_n$  threshold (point D in Fig. 3), the euphoric expectations begin to dominate again, producing a rise in the leverage ratio of firms and banks. Simultaneously, the effective investment rate continues to increase until the leverage ratios of firms and banks become high enough to cause a fall in the effective investment rate. When this happens, a new cycle begins.

Interestingly enough, during the cycles investment and leverage move both in the same and in the opposite direction. In particular, during the investment boom periods, in which the investment rate is high and growing, the leverage ratios also increase; in the investment bust periods the leverage ratios decline. This movement of leverage and investment towards the same direction is caused by the endogenous change in the desired margins of safety that weakens the debt-burdened effect. However, there are also phases in which the effective investment rate moves inversely with the leverage ratios of firms and banks. In particular, when the effective investment rate starts rising



1 (declining), the leverage ratios continue to fall (increase) until the effective investment  
2 rate becomes high (low) enough to trigger a rise (decline) in the target leverage ratios.  
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4 It becomes thereby clear that the relationship between leverage and effective  
5 investment rate crucially relies on the way that the desired margins of safety change  
6 during the investment cycle.  
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### 11 12 13 14 **4.3 The role of fiscal policy** 15

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18 We now turn to investigate whether fiscal policy can reduce the destabilising forces  
19 generated by the endogenous changes in the desired margins of safety. The  
20 government expenditures-to-capital ratio is allowed to change endogenously according  
21 to the fiscal rule described in Eq. (49). We examine whether, for identical parameter  
22 values as in Fig. 1 and for the same range of values for  $\xi_1$ , the 5D system is  
23 characterised by higher stability. Fig. 4 indicates that this is indeed the case: the rise in  
24 the sensitivity of target leverage ratios to the investment cycle does not increase the  
25 fluctuation of the macroeconomic variables, as it is the case in Fig. 1.  
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46 The economic interpretation is the following. In the 4D subsystem in which  $gov$  is  
47 exogenous the rise in the target leverage ratio of firms and banks leads to an economic  
48 expansion that produces second-round destabilising forces in the system due to the  
49 positive impact of investment on target leverage ratios. In the 5D system the fiscal rule  
50 mitigates these second-round forces. By generating a reduction in  $gov$  as a response to  
51 the rise in the target leverage ratios, the induced increase in the investment rate is less  
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1 strong and, hence, the increase in the target leverage ratios is less significant.  
2 Moreover, in the periods in which the expectations deteriorate and the target leverage  
3 ratios decline relative to the actual ones the fiscal rule causes a rise in  $gov$  preventing  
4 a significant reduction in economic activity. Consequently, the fiscal rule put forward  
5 in this paper dampens the large oscillations in the macroeconomic variables, which are  
6 fuelled by the rise in  $\xi_1$ , rendering the macro system more stable.  
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## 17 5. Conclusions

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22 This paper presented a stock-flow consistent macrodynamic model in which firms'  
23 and banks' desired margins of safety play a central role in the behaviour of the  
24 macroeconomy. The model incorporates an active commercial banking sector,  
25 allowing us to pay particular attention to the evolution of the leverage of both firms  
26 and banks during the investment cycle. Dynamic analysis illustrated that a higher  
27 sensitivity of firms' and banks' desired margins of safety to the investment cycle  
28 makes the macro system more prone to instability. Therefore, the euphoria and low  
29 perceived risk of both firms and banks during an investment boom and the excessively  
30 high desired margins of safety during an investment bust can be important sources of  
31 instability. Moreover, simulation analysis showed that leverage and investment can  
32 move both in the same and in the opposite direction during the cycles without being  
33 necessary to turn from a debt-burdened regime to a debt-led one.  
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52 The paper also analysed the stabilising role of fiscal policy in an economy in which  
53 desired margins of safety change endogenously. The paper put forward a fiscal rule  
54 that produces a rise (decline) in government expenditures when firms and banks have  
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1 excessively high (low) desired margins of safety. Simulation analysis indicated that  
2 this rule has stabilising effects. Therefore, a fiscal policy that responds adequately to  
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4 the endogenous changes in the desired margins of safety appears to be essential for the  
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6 stability of the macroeconomic system.  
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**References**

- 1  
2  
3  
4  
5 Arestis, P., Sawyer, M., 2010. The return of fiscal policy. *Journal of Post Keynesian*  
6  
7 *Economics* 32 (3), 327-346.  
8  
9  
10 Asada, T., Yoshida, H., 2003. Coefficient criterion for four-dimensional Hopf  
11  
12 bifurcations: a complete mathematical characterization and applications to  
13  
14 economic dynamics. *Chaos, Solitons and Fractals* 18 (3), 525-536.  
15  
16  
17 Charpe, M., Flaschel, P., 2013. Workers' debt, default and the diversity of financial  
18  
19 fragilities. *Structural Change and Economic Dynamics* 27 (C), 48-65.  
20  
21  
22 Charpe, M., Chiarella, C., Flaschel, P., Semmler, W., 2011. *Financial Assets, Debt and*  
23  
24 *Liquidity Crises*. Cambridge University Press, New York.  
25  
26  
27 Chiarella, C., Flaschel, P., Hartmann, F., Proano, C., 2012. Stock market booms,  
28  
29 endogenous credit creation and the implications of broad and narrow banking for  
30  
31 macroeconomic stability. *Journal of Economic Behavior and Organization* 83 (3),  
32  
33 410-423.  
34  
35  
36 Dafermos, Y., 2012. Liquidity preference, uncertainty, and recession in a stock-flow  
37  
38 consistent model. *Journal of Post Keynesian Economics* 34 (4), 749-776.  
39  
40  
41 Gandolfo, G., 2010. *Economic Dynamics*. Springer, New York.  
42  
43  
44 Godley, W., Lavoie, M., 2007. *Monetary Economics: An Integrated Approach to*  
45  
46 *Credit, Money, Production and Wealth*. Palgrave Macmillan, Basingstoke, UK.  
47  
48  
49 Hein, E., 2007. Interest rate, debt, distribution and capital accumulation in a Post-  
50  
51 Kaleckian model. *Metroeconomica* 58 (2), 310-339.  
52  
53  
54 Hein, E., 2013. On the importance of the retention ratio in a Kaleckian distribution and  
55  
56 growth model with debt accumulation-A comment on Sasaki and Fujita (2012).  
57  
58 *Metroeconomica* 64 (1), 186-196.  
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59  
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61  
62  
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64  
65
- Hein, E., Lavoie, M., van Treeck, T., 2012. Harrodian instability and the ‘normal rate’ of capacity utilization in Kaleckian models of distribution and growth—a survey. *Metroeconomica* 63 (1), 139-169.
- Keen, S., 1995. Finance and economic breakdown: modeling Minsky’s ‘financial instability hypothesis’. *Journal of Post Keynesian Economics* 17 (4), 607-635.
- Kregel, J., 1997. Margins of safety and weight of the argument in generating financial fragility. *Journal of Economic Issues* 31 (2), 543-548.
- Lavoie, M., 1995. Interest rates in Post-Keynesian models of growth and distribution. *Metroeconomica* 46 (2), 146-177.
- Le Heron, E., 2008. Fiscal and monetary policies in a Keynesian stock-flow consistent model. In: Creel, J., Sawyer, M. (Eds.), *Current Thinking on Fiscal Policy*. Palgrave Macmillan, London.
- Le Heron, E., 2011. Confidence and financial crisis in a post-Keynesian stock flow consistent model. *Intervention. European Journal of Economics and Economic Policies: Intervention* 8 (2), 361-388.
- Le Heron, E., 2012. A debate with Wynne Godley on the neutrality of fiscal policy. In: Papadimitriou, D. B., Zezza, G. (Eds.), *Contributions in Stock-Flow Modeling: Essays in Honor of Wynne Godley*. Palgrave Macmillan, London.
- Le Heron, E., 2013. Macro-foundations of micro and micro-foundations of macro: Income distribution, increasing risks and household behaviours. Limerick SFC Workshops.
- Le Heron, E., Mouakil, T., 2008. A Post-Keynesian stock-flow consistent model for dynamic analysis of monetary policy shock on banking behaviour. *Metroeconomica* 59 (3), 405-440.
- Minsky, H. P., 1975. *John Maynard Keynes*. Columbia University Press, New York.

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62  
63  
64  
65
- Minsky, H. P., 1982. *Inflation, Recession and Economic Policy*. M.E. Sharpe, New York.
- Minsky, H. P., 2008 [1986]. *Stabilizing an Unstable Economy*. Mc Graw Hill, New York.
- Nishi, H., 2012. A dynamic analysis of debt-led and debt-burdened growth regimes with Minskian financial structure. *Metroeconomica* 63 (4), 634-660.
- Papadimitriou, D. B., Wray, R., 1998. The economic contributions of Hyman Minsky: varieties of capitalism and institutional reform. *Review of Political Economy* 10 (2), 199-225.
- Ryoo, S., 2010. Long waves and short cycles in a model of endogenous financial fragility. *Journal of Economic Behaviour and Organization* 74 (3), 163-186.
- Ryoo, S., 2013a. The paradox of debt and Minsky's financial instability hypothesis. *Metroeconomica* 64 (1), 1-24.
- Ryoo, S., 2013b. Bank profitability, leverage and financial instability: a Minsky-Harrod model. *Cambridge Journal of Economics* 37 (5). 1127-1160.
- Sasaki, H., Fujita, S., 2012. The importance of the retention ratio in a Kaleckian model with debt accumulation. *Metroeconomica* 63 (3), 417-428.
- Skott, P., 2012. Theoretical and empirical shortcomings of the Kaleckian investment function. *Metroeconomica* 63 (1), 109-138.
- Tymoigne, E., 2009. *Central Banking, Asset Prices and Financial Fragility*. Routledge, London.
- Vercelli, A., 2011. A perspective on Minsky moments: revisiting the core of the financial instability hypothesis. *Review of Political Economy* 23 (1), 49-67.

1 Yoshida, H., Asada, T., 2007. Dynamic analysis of policy lag in a Keynes-Goodwin  
2 model: stability, instability, cycles and chaos. Journal of Economic Behavior and  
3 Organization 62 (3), 441-469.  
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## Appendix A. Proof of Proposition 1

The Jacobian matrix of the 3D subsystem ( $J_{3D}$ ) consisting of Eqs. (45)-(47) evaluated at the steady state is written as:

$$J_{3D} = \begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix}$$

where

$$J_{11} = \partial \dot{f} / \partial f = H_1 - b_1 \left( 1 - lf_0 + \frac{X_0}{\Delta} \right)$$

$$J_{12} = \partial \dot{f} / \partial d = X_0 u_d - (1 - lf_0) b_2 l b_d$$

$$J_{13} = \partial \dot{f} / \partial a = X_0 u_a - (1 - lf_0) b_2 l b_a$$

$$J_{21} = \partial \dot{d} / \partial f = H_2 - b_1 \left( \frac{Z_0}{\Delta} - d_0 \right)$$

$$J_{22} = \partial \dot{d} / \partial d = Z_0 u_d + (1 - c_1) \Gamma_1 - c_2 - g_n + b_2 l b_d d_0$$

$$J_{23} = \partial \dot{d} / \partial a = Z_0 u_a - (1 - c_1) \Gamma_3 + d_0 b_2 l b_a$$

$$J_{31} = \partial \dot{a} / \partial f = H_3 - b_1 \left( \frac{\Gamma_5 - a_0 \delta_1}{\Delta} + 1 - a_0 \right)$$

$$J_{32} = \partial \dot{a} / \partial d = (\Gamma_5 - a_0 \delta_1) u_d - b_2 l b_d (1 - a_0) - \Gamma_4 - (1 - h_1 - h_2) [(1 - c_1) \Gamma_1 - c_2]$$

$$J_{33} = \partial \dot{a} / \partial a = (\Gamma_5 - a_0 \delta_1) u_a - b_2 l b_a (1 - a_0) + s_b i_a - g_n + (1 - h_1 - h_2) (1 - c_1) \Gamma_3$$

We have that  $X_0 = \delta_1 (1 - lf_0) - s_f (1 - s_w) v$ ,  $Z_0 = (1 - c_1) [1 - s_f (1 - s_w)] v - d_0 \delta_1$ ,



$$\Gamma_5 = \delta_1 - s_f(1-s_w)v - (1-h_1-h_2)(1-c_1)[1-s_f(1-s_w)]v,$$

$$H_1 = s_f i_l - g_n - (\delta_2 + b_2 l b_f) \left( 1 - l f_0 + \frac{X_0}{\Delta} \right) + \frac{X_0 c_1 \Gamma_2}{\Delta},$$

$$H_2 = -(\delta_2 + b_2 l b_f) \left( \frac{Z_0}{\Delta} - d_0 \right) + c_1 \Gamma_2 + \frac{Z_0}{\Delta} (1-c_1) \Gamma_2 \text{ and}$$

$$H_3 = s_f i_l - s_b i_l - (1-h_1-h_2)(1-c_1) \Gamma_2 - (\delta_2 + b_2 l b_f) \left( \frac{\Gamma_5 - a_0 \delta_1}{\Delta} + 1 - a_0 \right) + \frac{\Gamma_5 - a_0 \delta_1}{\Delta} c_1 \Gamma_2.$$

The Routh-Hurwitz necessary and sufficient conditions for the stability of the 3D subsystem require that the coefficients  $a_1^{(3)}, a_2^{(3)}, a_3^{(3)}, b^{(3)}$  be all positive in the steady state (see Gandolfo, 2010). These coefficients are as follows:

$$a_1^{(3)} = -Tr(J_{3D}) = \Theta_1 + \left( 1 - l f_0 + \frac{X_0}{\Delta} \right) b_1 \quad (\text{A.1})$$

$$a_2^{(3)} = \begin{vmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{vmatrix} + \begin{vmatrix} J_{11} & J_{13} \\ J_{31} & J_{33} \end{vmatrix} + \begin{vmatrix} J_{22} & J_{23} \\ J_{32} & J_{33} \end{vmatrix} = \Theta_2 + \Theta_3 b_1 \quad (\text{A.2})$$

$$a_3^{(3)} = -Det(J_{3D}) = \Theta_4 + \Theta_5 b_1 \quad (\text{A.3})$$

$$b^{(3)} = a_1^{(3)} a_2^{(3)} - a_3^{(3)} = \Theta_3 \left( 1 - l f_0 + \frac{X_0}{\Delta} \right) b_1^2 + \left( \Theta_1 \Theta_3 + \Theta_2 \left( 1 - l f_0 + \frac{X_0}{\Delta} \right) - \Theta_5 \right) b_1 + \Theta_1 \Theta_2 - \Theta_4 \quad (\text{A.4})$$

where  $\Theta_1 = -H_1 - J_{22} - J_{33}$ ,  $\Theta_2 = H_1(J_{22} + J_{33}) - H_3 J_{13} + J_{22} J_{33} - H_2 J_{12} - J_{32} J_{23}$ ,

$$\Theta_3 = -\left( 1 - l f_0 + \frac{X_0}{\Delta} \right) (J_{22} + J_{33}) + \left( \frac{Z_0}{\Delta} - d_0 \right) J_{12} + \left( \frac{\Gamma_5 - a_0 \delta_1}{\Delta} + 1 - a_0 \right) J_{13},$$

$$\Theta_4 = -H_1(J_{22} J_{33} - J_{32} J_{23}) + H_2(J_{12} J_{33} - J_{13} J_{32}) + H_3(J_{13} J_{22} - J_{12} J_{23}) \text{ and}$$

$$\Theta_5 = \left(1 - lf_0 + \frac{X_0}{\Delta}\right)(J_{22}J_{33} - J_{32}J_{23}) - \left(\frac{Z_0}{\Delta} - d_0\right)(J_{12}J_{33} - J_{13}J_{32}) - \left(\frac{\Gamma_5 - \delta_1 a_0}{\Delta} + 1 - a_0\right)(J_{13}J_{22} - J_{12}J_{23}).$$

We have  $a_1^{(3)} > 0$  since  $J_{11}, J_{22}, J_{33} < 0$ . In particular,  $J_{11} < 0$  due to the assumptions that  $lf_0$ ,  $b_2$ ,  $s_f$ , and  $\Gamma_2$  are sufficiently small;  $J_{22} < 0$  because of the assumptions that  $lb_d > 0$  and that  $d_0$ ,  $\Gamma_1$ ,  $b_2$  are sufficiently small;  $J_{33} < 0$  due to the assumptions that  $s_b i_a < g_n$  (see footnote 21) and that  $a_0$ ,  $\Gamma_3$  are sufficiently small.

It holds that  $a_2^{(3)} > 0$  since  $\Theta_2, \Theta_3 > 0$ . We have  $\Theta_2 > 0$  because the terms  $H_1(J_{22} + J_{33})$ ,  $J_{22}J_{33}$  are positive and adequately large. In particular,  $H_1(J_{22} + J_{33})$  is positive because  $J_{22}, J_{33} < 0$  (see above) and  $H_1 < 0$  due to a sufficiently small  $lf_0$ ;  $J_{22}J_{33}$  is positive because  $J_{22}, J_{33} < 0$  (see above). The terms  $H_1(J_{22} + J_{33})$ ,  $J_{22}J_{33}$  are sufficiently large because of the assumption that  $a_0$  is low enough. Moreover,

$\Theta_3 > 0$  since the term  $-\left(1 - lf_0 + \frac{X_0}{\Delta}\right)(J_{22} + J_{33})$  is positive and adequately large.

This term is positive because  $J_{22}, J_{33} < 0$  (see above) and  $lf_0$  is sufficiently small.

Additionally, the term  $-\left(1 - lf_0 + \frac{X_0}{\Delta}\right)(J_{22} + J_{33})$  is adequately large due to a sufficiently small  $a_0$ .

We have  $a_3^{(3)} > 0$  because  $\Theta_4, \Theta_5 > 0$ . In particular,  $\Theta_4 > 0$  because the term  $-H_1 J_{22} J_{33}$  is positive and adequately large; it is positive since  $J_{22}, J_{33}, H_1 < 0$  (see above) and it is adequately large due to a sufficiently small  $a_0$ . Moreover,  $\Theta_5 > 0$  because the term

$\left(1 - lf_0 + \frac{X_0}{\Delta}\right)J_{22}J_{33}$  is positive and adequately large; it is positive since  $J_{22}, J_{33} < 0$  (see

1 above) and due to a sufficiently small  $lf_0$ ; it is adequately large due to a sufficiently  
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 3 small  $a_0$ .

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 7 Finally,  $b^{(3)} > 0$  because a sufficiently small  $d_0$  and a sufficiently small  $lf_0$  ensure that

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 11  $\Theta_1\Theta_2 - \Theta_4 > 0$  and  $\Theta_1\Theta_3 + \Theta_2\left(1 - lf_0 + \frac{X_0}{\Delta}\right) - \Theta_5 > 0$ .  
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## Appendix B. Proof of Proposition 2

The Jacobian matrix of the 4D subsystem ( $J_{4D}$ ) consisting of Eqs. (45)-(48) evaluated at the steady state is written as:

$$J_{4D} = \begin{pmatrix} J_{11} & J_{12} & J_{13} & J_{14} \\ J_{21} & J_{22} & J_{23} & J_{24} \\ J_{31} & J_{32} & J_{33} & J_{34} \\ J_{41} & J_{42} & J_{43} & J_{44} \end{pmatrix}$$

where

$$J_{14} = \partial \dot{f} / \partial l f^T = (\delta_2 + b_2 \phi) \left( 1 - l f_0 + \frac{X_0}{\Delta} \right)$$

$$J_{24} = \partial \dot{d} / \partial l f^T = (\delta_2 + b_2 \phi) \left( \frac{Z_0}{\Delta} - d_0 \right)$$

$$J_{34} = \partial \dot{a} / \partial l f^T = (\delta_2 + b_2 \phi) \left( 1 - a_0 + \frac{\Gamma_5 - a_0 \delta_1}{\Delta} \right)$$

$$J_{41} = \partial \dot{f}^T / \partial l f = \xi_1 g_{lf}$$

$$J_{42} = \partial \dot{f}^T / \partial d = \xi_1 g_d$$

$$J_{43} = \partial \dot{f}^T / \partial a = \xi_1 g_a < 0$$

$$J_{44} = \partial \dot{f}^T / \partial l f^T = \xi_1 g_{lf^T} - \xi_2$$

The rest entries of the Jacobian matrix are reported in Appendix A.

The conditions of Proposition 1 suggest that  $J_{14}, J_{24}, J_{34} > 0$  and  $J_{41} < 0$ . In particular, a sufficient low value of  $lf_0$  implies that  $J_{14} > 0$ ; a sufficient low value of  $d_0$  suggests that  $J_{24} > 0$ ; a sufficient low value of  $a_0$  implies that  $J_{34} > 0$ ; the existence of debt-burdened regime suggests that  $J_{41} < 0$ .

The Routh-Hurwitz necessary and sufficient conditions for the stability of the 4D subsystem require that the coefficients  $a_1^{(4)}, a_2^{(4)}, a_3^{(4)}, a_4^{(4)}, b^{(4)}$  be all positive in the steady state (see Gandolfo, 2010). These coefficients are written as follows:

$$a_1^{(4)} = -Tr(J_{4D}) = a_1^{(3)} + \xi_2 - \xi_1 g_{lf^T} \quad (\text{B.1})$$

$$\begin{aligned} a_2^{(4)} &= \begin{vmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{vmatrix} + \begin{vmatrix} J_{11} & J_{13} \\ J_{31} & J_{33} \end{vmatrix} + \begin{vmatrix} J_{22} & J_{23} \\ J_{32} & J_{33} \end{vmatrix} + \begin{vmatrix} J_{11} & J_{14} \\ J_{41} & J_{44} \end{vmatrix} + \begin{vmatrix} J_{22} & J_{24} \\ J_{42} & J_{44} \end{vmatrix} + \begin{vmatrix} J_{33} & J_{34} \\ J_{43} & J_{44} \end{vmatrix} = \\ &= a_2^{(3)} + \xi_2 a_1^{(3)} - \xi_1 (a_1^{(3)} g_{lf^T} - \Omega) \end{aligned} \quad (\text{B.2})$$

$$\begin{aligned} a_3^{(4)} &= - \begin{vmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{vmatrix} - \begin{vmatrix} J_{22} & J_{23} & J_{24} \\ J_{32} & J_{33} & J_{34} \\ J_{42} & J_{43} & J_{44} \end{vmatrix} - \begin{vmatrix} J_{11} & J_{13} & J_{14} \\ J_{31} & J_{33} & J_{34} \\ J_{41} & J_{43} & J_{44} \end{vmatrix} - \begin{vmatrix} J_{11} & J_{12} & J_{14} \\ J_{21} & J_{22} & J_{24} \\ J_{41} & J_{42} & J_{44} \end{vmatrix} = \\ &= a_3^{(3)} + \xi_2 a_2^{(3)} - \xi_1 (g_{lf^T} a_2^{(3)} + \Omega_1 + \Omega_2 + \Omega_3) \end{aligned} \quad (\text{B.3})$$

$$a_4^{(4)} = Det(J_{4D}) = \xi_2 a_3^{(3)} - \xi_1 (a_3^{(3)} g_{lf^T} - \Psi) \quad (\text{B.4})$$

$$b^{(4)} = a_1^{(4)} a_2^{(4)} a_3^{(4)} - (a_1^{(4)})^2 a_4^{(4)} - (a_3^{(4)})^2 = \Psi_0 + \Psi_1 \xi + \Psi_2 \xi^2 + \Psi_3 \xi^3 \quad (\text{B.5})$$

where  $\Omega = -J_{14}g_{lf} - J_{34}g_a - J_{24}g_d$ ,  $\Omega_1 = g_{lf}(J_{12}J_{24} - J_{14}J_{22} + J_{13}J_{34} - J_{14}J_{33})$ ,

$\Omega_2 = g_d(-J_{24}J_{11} + J_{14}J_{21} + J_{23}J_{34} - J_{24}J_{33})$ ,  $\Omega_3 = g_a(-J_{11}J_{34} + J_{14}J_{31} - J_{22}J_{34} + J_{24}J_{32})$ ,

$\Psi = g_a[J_{11}(-J_{22}J_{34} + J_{24}J_{32}) - J_{12}(-J_{21}J_{34} + J_{24}J_{31}) - J_{14}(J_{21}J_{32} - J_{22}J_{31})]$   
 $+ g_d[J_{11}(J_{23}J_{34} - J_{24}J_{32}) + J_{13}(-J_{21}J_{34} + J_{24}J_{31}) - J_{14}(-J_{21}J_{33} + J_{23}J_{31})]$   
 $+ g_{lf}[-J_{12}(J_{23}J_{34} - J_{24}J_{33}) + J_{13}(J_{22}J_{34} - J_{24}J_{32}) - J_{14}(J_{22}J_{33} - J_{23}J_{32})]$ ,

$\Psi_0 = (a_3^{(3)} + \xi_2 a_2^{(3)})b^{(3)} + (a_1^{(3)} + \xi_2)\xi_2^2 b^{(3)} > 0$  and

$\Psi_3 = -(g_{lf^T} + \Omega_1 + \Omega_2 + \Omega_3)g_{lf^T}(a_1^{(3)}g_{lf^T} - \Omega) + g_{lf^T}^2(a_3^{(3)}g_{lf^T} - \Psi) < 0$ .

Note that  $\Psi_1$  and  $\Psi_2$  are independent of  $\xi_1$ .

**Proof of 2 (I).** Differentiating Eqs. (B.1)-(B.4) with respect to  $\xi_1$ , yields:

$$\partial a_1^{(4)} / \partial \xi_1 = -g_{lf^T} < 0 \quad (\text{B.6})$$

$$\partial a_2^{(4)} / \partial \xi_1 = -(a_1^{(3)}g_{lf^T} - \Omega) \quad (\text{B.7})$$

$$\partial a_3^{(4)} / \partial \xi_1 = -(g_{lf^T}a_2^{(3)} + \Omega_1 + \Omega_2 + \Omega_3) \quad (\text{B.8})$$

$$\partial a_4^{(4)} / \partial \xi_1 = -(a_3^{(3)}g_{lf^T} - \Psi) \quad (\text{B.9})$$

Eq. (B.6) implies that  $a_1^{(4)}$  is a decreasing function of  $\xi_1$ ; recall that  $g_{lf^T} > 0$  (see Eq.

(40)). The coefficient  $a_1^{(4)}$  becomes equal to zero for  $\xi_1 = \xi_1^{a_1} = (a_1^{(3)} + \xi_2) / g_{lf^T}$ ; note

that  $\xi_1^{a_1} > 0$  because  $a_1^{(3)} > 0$  (see Appendix A). Therefore,  $a_1^{(4)} > 0$  if  $\xi_1 < \xi_1^{a_1}$  and

$a_1^{(4)} < 0$  if  $\xi_1 > \xi_1^{a_1}$ . Moreover, since  $a_2^{(3)}, a_3^{(3)} > 0$  (see Appendix A) and  $\xi_2 > 0$ , the

coefficients  $a_2^{(4)}, a_3^{(4)}$  and  $a_4^{(4)}$  are all positive under the Conditions (52), (53) and (54).

By setting Eq. (B.5) equal to zero we obtain:

$$b^{(4)} = \Psi_3 \xi_1^3 + \Psi_2 \xi_1^2 + \Psi_1 \xi_1 + \Psi_0 = 0 \quad (\text{B.10})$$

At  $\xi_1 = 0$  we have  $b^{(4)} = \Psi_0 > 0$ . At  $\xi_1 = \xi_1^{a_1}$ , we have  $b^{(4)} = -\left(a_3^{(4)}\right)^2 < 0$ . Therefore, due to continuity, we obtain that for sufficiently positive low values of  $\xi_1$  all of the Routh-Hurwitz conditions are satisfied (i.e.  $a_1^{(4)}, a_2^{(4)}, a_3^{(4)}, a_4^{(4)}, b^{(4)} > 0$ ) and the system is thereby stable.

**Proof of 2 (II).** For sufficiently high values of  $\xi_1$  we have  $b^{(4)} < 0$  and, therefore, one of the Routh-Hurwitz conditions is violated. This implies that the system is unstable.

**Proof of 2 (III).** At  $\xi_1 = 0$  we have  $b^{(4)} > 0$  and at  $\xi_1 = \xi_1^{a_1}$  we have  $b^{(4)} < 0$ . Hence, the cubic equation  $b^{(4)}(\xi_1) = 0$  has at least one solution,  $\xi_1^b$ , such that  $0 < \xi_1^b < \xi_1^{a_1}$  with the property that  $b^{(4)} \neq 0$  for all  $\xi_1$  near but not equal to  $\xi_1^b$ . Furthermore, at  $\xi_1 = \xi_1^b$  we have  $a_1^{(4)}, a_2^{(4)}, a_3^{(4)}, a_4^{(4)} > 0$ . According to Asada and Yoshida (2003), these properties are sufficient for the existence of a simple Hopf bifurcation at  $\xi_1 = \xi_1^b$ .

## Appendix C. Parameter values in simulations

$\delta_o$	0.02	$b_2$	0.01	$h_2$	0.75
$\delta_1$	0.1	$c_1$	0.7	$\xi_2$	0.5
$\delta_2$	0.5	$c_2$	0.1	$e_1$	0.8
$s_f$	0.6	$h_l$	4	$e_2$	0.05
$s_w$	0.6	$h_d$	0.5	$e_3$	20
$v$	0.25	$i_b$	0.02	$\varphi$	15
$b_o$	0.01	$s_b$	0.3	$gov_r$	0.005
$b_1$	0.5	$h_1$	0.05	$g_n$	0.04



**Table 1**

Balance sheet matrix.

	Households	Firms	Commercial banks	Government	Central bank	Total
Deposits	$+D$		$-D$			0
Loans		$-L$	$+L$			0
Treasury bills			$+B_b$	$-B$	$+B_{cb}$	0
High-powered money			$+HPM$		$-HPM$	0
Advances			$-A$		$+A$	0
Capital		$+K$				$+K$
Total (net worth)	$+D$	$+V_f$	$+K_b$	$-B$	0	$+K$

**Table 2**

Transactions matrix.

	Households	Firms		Commercial banks		Government	Central bank	Total
		Current	Capital	Current	Capital			
Investment		$+I$	$-I$					0
Consumption	$-C$	$+C$						0
Government expenditures		$+GOV$				$-GOV$		0
Wage bill	$+W$	$-W$						0
Interest on loans		$-i_l L$		$+i_l L$				0
Interest on treasury bills				$+i_b B_b$		$-i_b B$	$+i_b B_{cb}$	0
Interest on deposits	$+i_d D$			$-i_d D$				0
Interest on advances				$-i_a A$			$+i_a A$	0
Commercial banks' profits	$+PB_d$			$-PB$	$+PB_u$			0
Central bank's profits						$+PCB$	$-PCB$	0
Firms' profits	$+PF_d$	$-PF$	$+PF_u$					0
Change in deposits	$-\dot{D}$				$+\dot{D}$			0
Change in loans			$+\dot{L}$		$-\dot{L}$			0
Change in treasury bills					$-\dot{B}_b$	$+\dot{B}$	$-\dot{B}_{cb}$	0
Change in advances					$+\dot{A}$		$-\dot{A}$	0
Change in high-powered money					$-\dot{HPM}$		$+\dot{HPM}$	0
Total	0	0	0	0	0	0	0	0

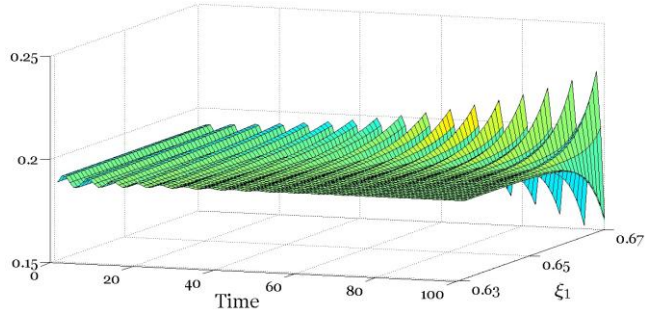
**Table 3**

Effects of firms' leverage ratio and deposits-to-capital ratio on economic activity.

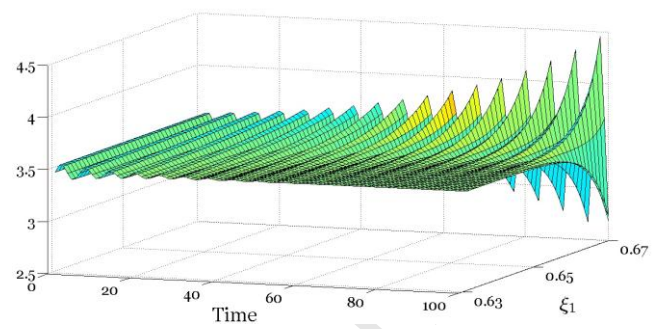
Effect	Parameter(s) that capture the effect
<b>Effects of firms' leverage ratio on economic activity</b>	
Direct negative effect on desired investment	$\delta_2$
Direct negative effect on credit availability	$b_1$
Indirect negative effect on consumption via the distributed profits of firms	$c_1(1-s_f)i_1$
Indirect positive effect on consumption via the distributed profits of banks	$c_1(1-s_b)i_1$
Indirect positive effect on credit availability via the leverage of banks	$b_2$
<b>Effects of deposits-to-capital ratio on economic activity</b>	
Indirect positive or negative effect on consumption via the distributed profits of banks	$c_1(1-s_b)(i_a h_2 - i_d)$
Direct positive effect on consumption via wealth	$c_2$
Direct positive effect on consumption via interest payments	$c_1 i_d$
Indirect negative effect on credit availability via the leverage of banks	$b_2$

**Fig. 1.** Dynamic adjustments of the 4D subsystem to a 1% increase in the target leverage ratios for varying values of target leverage ratios' sensitivity to the investment cycle ( $\xi_1$ ).

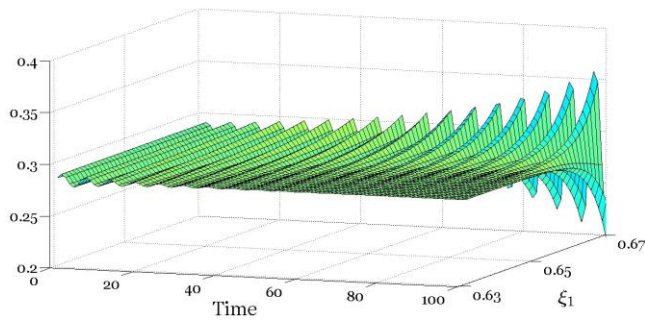
(a) Firms' leverage ratio ( $lf$ )



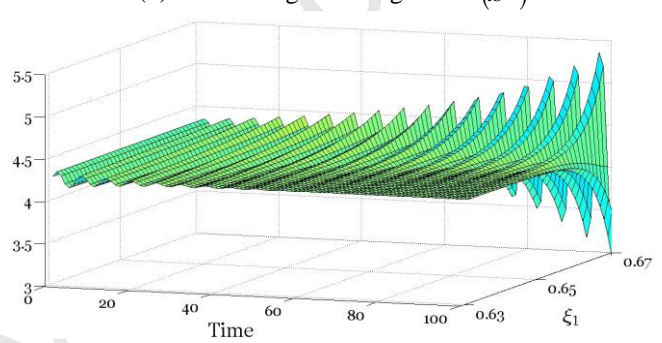
(b) Banks' leverage ratio ( $lb$ )



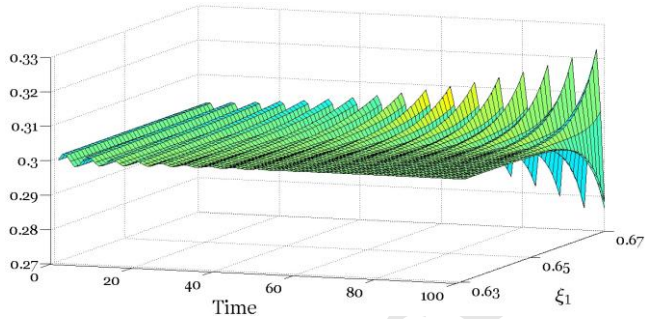
(c) Firms' target leverage ratio ( $lf^T$ )



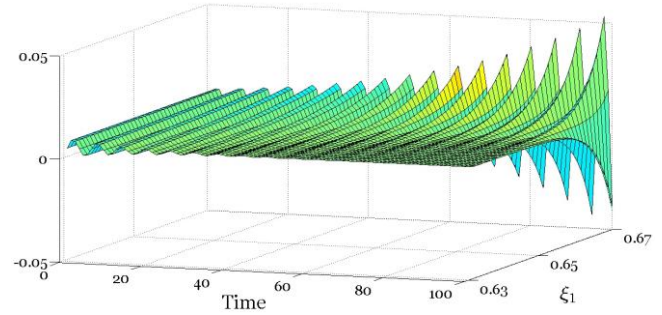
(d) Banks' target leverage ratio ( $lb^T$ )



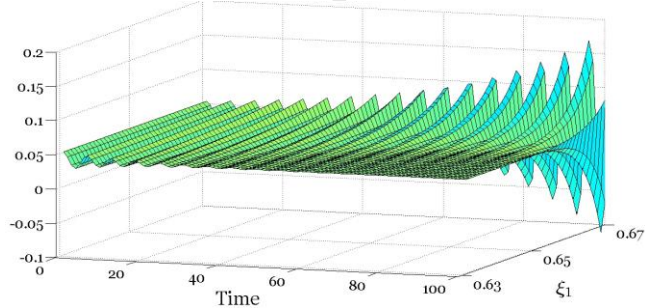
(e) Deposits-to-capital ratio ( $d$ )



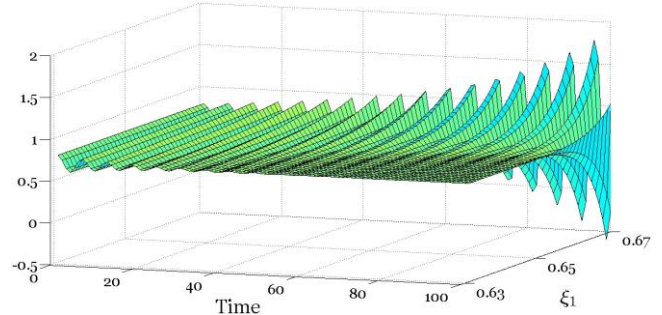
(f) Advances-to-capital ratio ( $a$ )

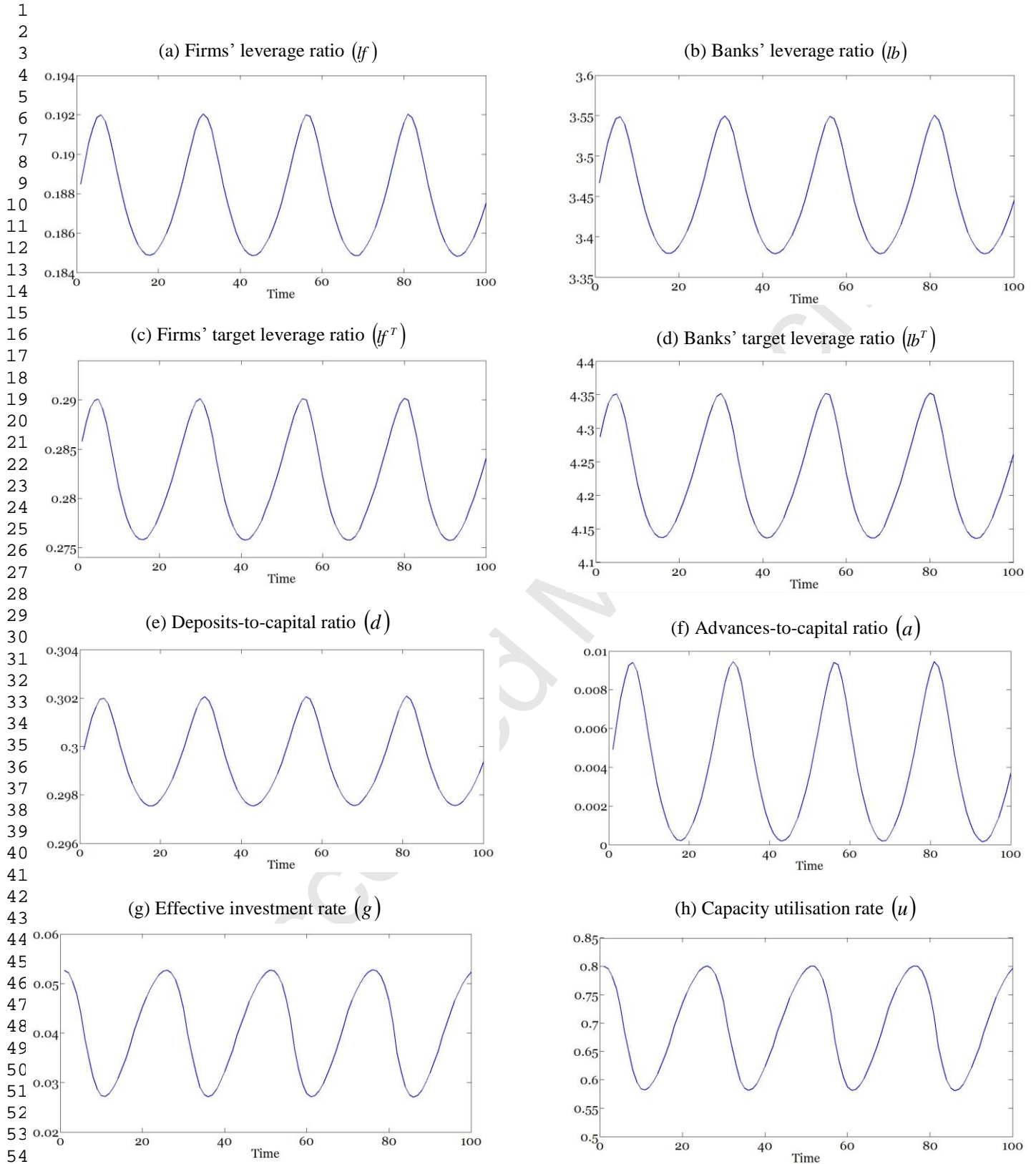


(g) Effective investment rate ( $g$ )

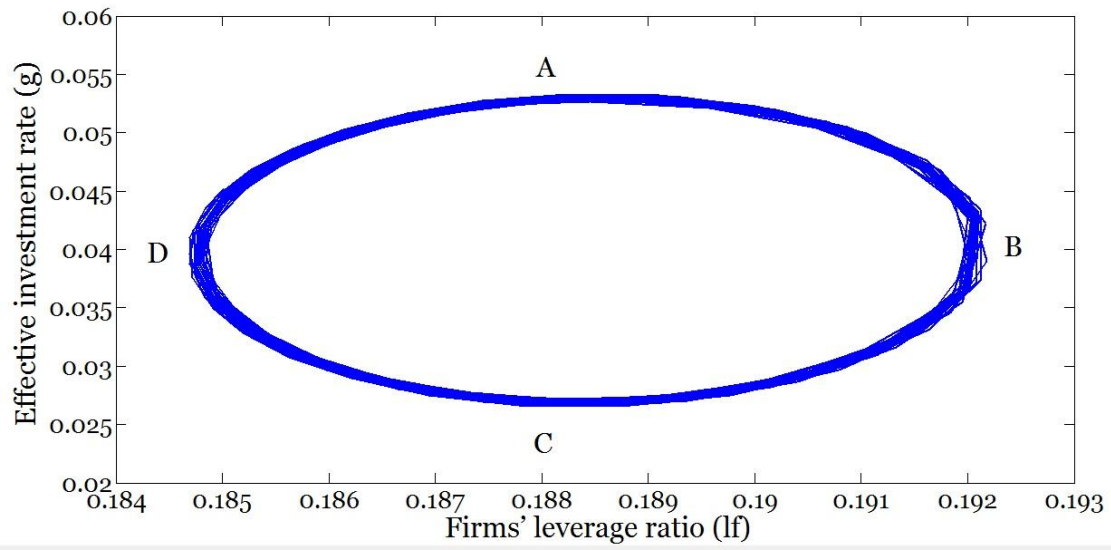


(h) Capacity utilisation rate ( $u$ )



**Fig. 2.** Dynamic trajectories under the case of a limit cycle in the 4D subsystem.

**Fig. 3.** Relationship between firms' leverage ratio and effective investment rate under the case of a limit cycle in the 4D subsystem.

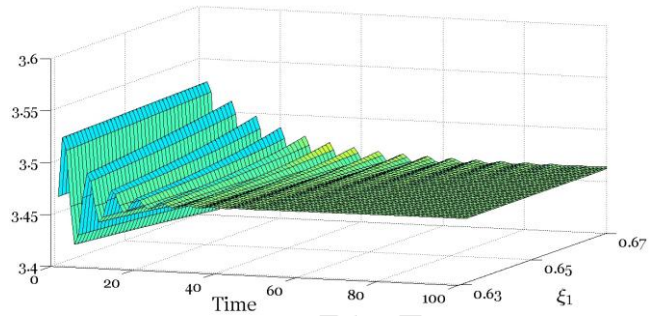
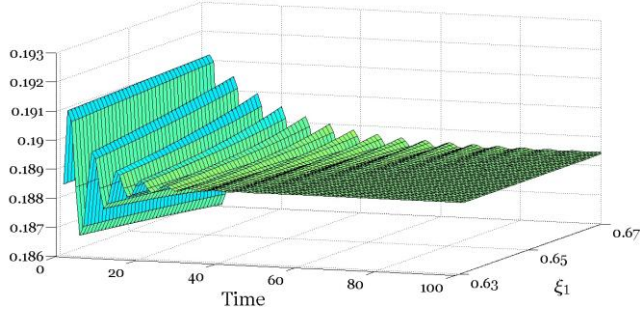




**Fig. 4.** Dynamic adjustments of the 5D system to a 1% increase in the target leverage ratios for varying values of target leverage ratios' sensitivity to the investment cycle ( $\xi_1$ ).

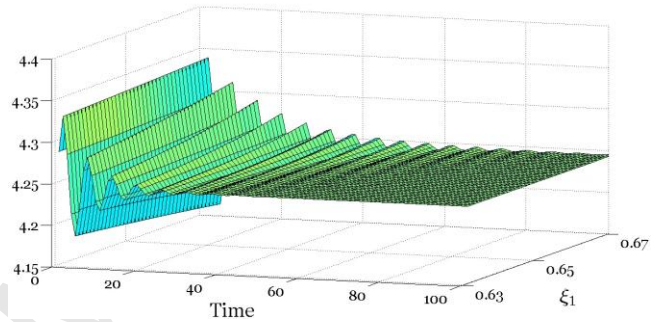
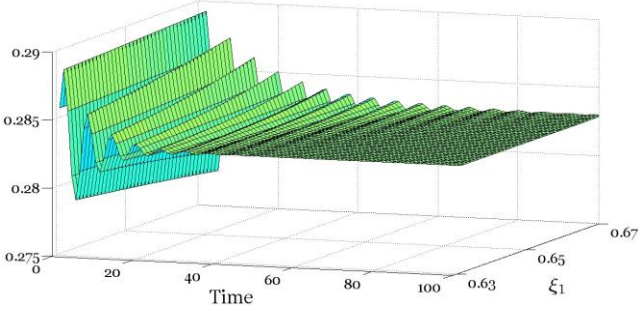
(a) Firms' leverage ratio ( $lf$ )

(b) Banks' leverage ratio ( $lb$ )



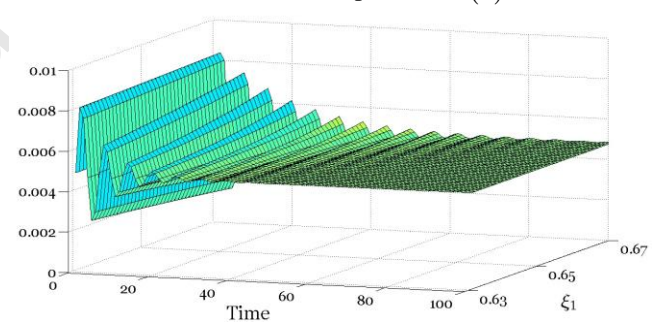
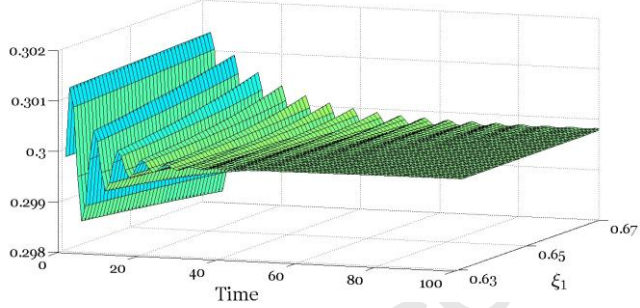
(c) Firms' target leverage ratio ( $lf^T$ )

(d) Banks' target leverage ratio ( $lb^T$ )



(e) Deposits-to-capital ratio ( $d$ )

(f) Advances-to-capital ratio ( $a$ )



(g) Effective investment rate ( $g$ )

(h) Government expenditures-to-capital ratio ( $gov$ )

