FINITE-VOLUME 2D RICHARDS EQUATION MODEL OF STORMWATER INFILTRATION AND FOCUSED RECHARGE, WITH AN APPLICATION TO AN EXPERIMENTAL RAIN GARDEN

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ABSTRACT

Source control is the fundamental principle behind sustainable management of stormwater. Rain gardens are an infiltration practice that provides volume and water quality control, recharge, and multiple landscape, ecological and economic potential benefits. The fulfillment of these objectives requires understanding their behavior during water input events as well as long term, and tools for their design. A model based on Richards equation coupled to a surface water balance was developed, using a 2D finite volume Fortran code which allows alternating upper boundary conditions, including ponding, which is not present in available 2D Richards models. Also, the model (R2D) can simulate heterogeneous soil -layered or more complex geometries- to estimate infiltration and recharge. The algorithm is conservative, showing good performance for several cases (less than 0.1% error); being an advantage compared to most available finite difference and finite element methods. Performance comparisons to known models and to experimental data from a bioretention cell are presented. This experimental rain garden receives roof water to its surface depression planted with native species in an organic-rich root zone soil layer, underlain by a high conductivity lower layer that, while providing inter-event storage, percolates water readily. R2D simulated well the matrix flow, soil water distribution, as well as deep percolation (potential recharge) for a natural rainfall event in the controlled experimental lysimeter. R2D now validated is available for investigating different rain garden configurations, and also for benchmarking more operational models for bioretention cells or rain garden hydrologic design purposes.

Keywords: Richards equation; recharge; finite-volume; rain garden; bioretention; urban stormwater; lysimeter; TDR

1 INTRODUCTION

Presently, an important drive in urban stormwater management is to apply alternative approaches (BMPs) that provide multiple benefits: technical, economical, ecological, recreational, aesthetical, and others (Ferguson, 1990; Huhn & Stecker, 1997; Fischer *et al.*, 2003). There is concern that traditional stormwater practices have been unsuccessful in diminishing impacts on altered hydrology by the rapid increase of urban growth, specifically due to diminished groundwater recharge and increased pumping, that combined have lowered groundwater levels in cities, particularly in fast growing suburban areas (Alley *et al.*, 2002).

Infiltration practices provide an attractive option, as they can be applied at the source of stormwater generation, can reduce runoff volume and therefore erosion and contaminant transport (Ferguson, 1990). They are particularly effective for dealing with small and medium rainfall events, which may account for most of annual volume (e.g. in Madison, Wisconsin, USA, 90% of the annual rainfall is accounted for by events with intensities of 2.5 cm/h or less, and in Santiago, Chile, 90% by events of 1.8 cm/h or less). Crucially, in particular for lower income countries, infiltration practices are a decentralized approach that can reduce the need for large stormwater structures downstream. Particularly, if infiltration is focused, it can enhance groundwater recharge, as in the case of rain gardens and bioretention areas. Rain gardens are low-cost to install, can be visibly pleasing, and foster social involvement, as well transfer maintenance to private owners, potentially decreasing costs and risk.

A rain garden is a vegetated depression with engineered soil (Figure 1) that receives stormwater from a much larger area impervious surface such a roof or a parking lot (1:10 to 1:20 ratio). Plants, besides providing aesthetic and potentially ecological value, enhance soil bio-turbation, potentially maintaining or even increasing infiltration capacity in time via macropore formation. Soil layering includes a permeable organic root zone that enhances water retention for plant survival in dry periods, as well as trapping pollutants, and a permeable layer below that permits water storage between events for enhancing recharge.

For modeling the matrix flow of water in the unsaturated zone of the soil, the common state of the art is the use of Richards Equation and its solution (Richards 1931; Miller et al. 1998; van Dam & Feddes 2000, Celia et al. 1990; Kavetski et al. 2001, Miller et al. 2006), albeit with recent correction suggestions such as fractional derivatives (Pachepsky et al. 2003) and representative elementary volume considerations (Roth 2008).

The Richards Equation with a sink term follows from the combination of a soil water mass balance and Darcy-Buckingham's law, (assuming one-phase incompressible flow through the soil matrix with no air resistance nor thermal effects), can be written in general as:

$$C(h)\frac{\partial h}{\partial t} = -\nabla \bullet (-K(h)\nabla H) + S_w \tag{1}$$

where *C(h)* is the moisture capacity function; H(*h,z)* is the total head (matrix head, or capillary suction, *h*, plus gravitational head, *z*); *t* is time; *K(h)* is soil hydraulic conductivity; and S_w is a sink (usually plant transpiration). As seen, this is a nonlinear second order parabolic-type partial differential equation.

The rain garden is designed to pond and then overflow, so this particular physical behavior needs to be included in the soil surface boundary condition, which is still uncommon in readily available models. For example, two popular Richards Equation models, UNSAT-H (Fayer 2000) and HYDRUS (Simunek et al. 1998) lack this capability or is limited.

In past work, a 1D Richards equation model, was developed (RECHARGE) so as to study rain garden water budgets in the short and long term (Dussaillant 2002; Dussaillant et al. 2004), as well as used to benchmark simpler, operational models for design purposes (e.g. Dussaillant et al. 2005b). One further step needed was to expand to a 2D model, to investigate more flexible garden configurations, for example terracing, which may bring added recharge augmenting performance, as well as landscaping secondary benefits.

However, 2D Richards equation based models are few (e.g. Simunek et al. 1999; Tocci et al. 1998; Farthing et al. 2003; Lee et al. 2004), particularly those that can handle a surface water based coupled boundary condition and a layered soil, aspects which enhance numerical problems compared to the 1D case (Dussaillant 2002; Aravena 2006), making very fine spatial and/or temporal discretizations necessary for finite difference or finite element schemes. Regarding the numerical method, we only found one using the conservative finite volume approach, that of Manzini & Ferraris (2004), but without the switching top BBCC -combined with a surface water balance- plus unsteady rainfall and layered soil that was required for the rain garden situation.

Our general project goal is to study and model rain garden hydrological behavior to ultimately provide design criteria for their implementation in specific climate, soil and urban settings. This research specific objective was to develop a bi-dimensional numerical modeling tool to simulate different rain garden configurations. The approach included developing a finite-volume model for simulations, based on Richards equation coupled with the rain garden surface water balance, and validated with literature cases and a rainfall event in an experimental lysimeter setup.

2 METHODS

A Richards-equation based finite volume model was developed, validated with literature results and compared with data gathered in our experimental rain garden lysimeter setup (Aravena 2006).

In integral form, Richards equation (1) can be written in generalized form as:

$$\int_{\forall} C(h)\frac{\partial h}{\partial t}\,d\forall - \int_{\forall} \nabla\cdot\left(K(h)\nabla H\right)d\forall + \int_{\forall} S_w d\forall = 0 \qquad (2)$$

where $\forall$ is volume; H is total hydraulic head, composed of h (matrix head) and z (gravitational head); and t is time.

2.1 FINITE-VOLUME MODEL

The basic idea of a finite volume scheme is that the domain is divided using a grid of small control volumes where the PDE is integrated, obtaining conservative equations for mass, momentum and energy, both for the control volumes as well as domain borders (Versteeg & Malalasekera, 1995).

If a control volume, Ω, with boundary conditions $\partial\Omega$ and normal unit vector, $\hat{n}$, the integral form of Richards 2D equation, using Green´s theorem, can be written in integral form as:

$$\int_{\Omega} C\frac{\partial H}{\partial t}\,d\Omega - \int_{\partial\Omega}\left(K\ \nabla H\right)\cdot\hat{\mathbf{n}}\,ds + \int_{\Omega} S_w\,d\Omega = 0 \qquad \text{in } (x,z)\in\Omega,\quad \text{for } t>0 \qquad (3)$$

where x is horizontal position. Boundary conditions $\partial\Omega$ can be of the form:

$$\begin{cases} K\,\nabla H = q_{borde}(t) \\ \text{ó} \\ H = H_{borde}(t) \end{cases} \qquad \text{if } (x,z)\in\partial\Omega,\quad \text{for } t>0 \qquad (4)$$

where q is water flux.

Considering a second-order scheme,

$$\int_{\Omega} g(\cdot)\, d\Omega = g_i \cdot \Delta\Omega_i \tag{5}$$

where g_i is the value of the function evaluated in the centroid of the control volume, thus we obtain:

$$C_i \cdot \left.\frac{\partial H}{\partial t}\right|_i \cdot \Delta\Omega_i + \sum_{\partial\Omega_i} \underbrace{-K\ \nabla H}_{f(H)} \cdot \hat{\mathbf{n}} \cdot \partial\Omega_i + S_{wi} \cdot \Delta\Omega_i = 0 \tag{6}$$

where:

$$f(H) = -K(H)\nabla H \tag{7}$$

Note that the control volume terms are evaluated in the corresponding centroid, while the flux terms are calculated in the borders; therefore, the sum will have as many terms as the control volume has borders.

Using a Crank-Nicolson type averaging (Aravena 2006) for estimating the gradient between adjacent control volumes, in two dimensions, the hydraulic gradient is:

$$\nabla H^n_{i-\frac{1}{2},j} = (1-\delta)\left(\frac{H^n_{i,j} - H^n_{i-1,j}}{\Delta z}\right) + \delta\left(\frac{H^{n+1}_{i,j} - H^{n+1}_{i-1,j}}{\Delta z}\right) \tag{8a}$$

$$\nabla H^n_{i,j-\frac{1}{2}} = (1-\delta)\left(\frac{H^n_{i,j} - H^n_{i,j-1}}{\Delta x}\right) + \delta\left(\frac{H^{n+1}_{i,j} - H^{n+1}_{i,j-1}}{\Delta x}\right) \tag{8b}$$

where i is the horizontal position x (column) counter and j is the vertical position z (row) counter, from the bottom of the profile (z is vertical position measured upwards); and δ is the Crank-Nicholson scheme weight.

Using:

$$F_{i-\frac{1}{2}}(H) = \int_{\partial\Omega_i} f(H)\, ds \qquad \text{for } \partial\Omega_i = \text{control surface i} \tag{9}$$

$$F_{i-\frac{1}{2}}(H) = \int_{\partial\Omega_{i-\frac{1}{2}}} -K(H)\ \nabla H \cdot \hat{\mathbf{n}}\, ds \tag{10}$$

And assuming that in the frontier of the control volume, $\partial\Omega_{i-\frac{1}{2}}$, hydraulic conductivity can be calculated with the arithmetic mean of the adjacent nodes:

$$K(H) \approx \frac{K_i + K_{i\pm 1}}{2} = K_{i\pm\frac{1}{2}} \tag{11}$$

then,

$$F_{i-\frac{1}{2}}(H) = -K_{i-\frac{1}{2}} \cdot \nabla H \cdot \partial\Omega_{i-\frac{1}{2}} \tag{12}$$

And we can define the integral flux term, in time, across the control volume, as:

$$\overline{F}_{i-\frac{1}{2}}^{n+\frac{1}{2}} = \frac{1}{\Delta t}\int_{t_n}^{t_{n+1}} F_{i-\frac{1}{2}}\left(H\right)dt \tag{13}$$

where $H = H\left(t, x, y, z\right)$

Also, in the control surface it can be assumed that $H_{i-\frac{1}{2}}^{n} \approx H\left(x_{i-\frac{1}{2}}, y_{i-\frac{1}{2}}, z_{i-\frac{1}{2}}, t_n\right)$; then, $\nabla H \approx \nabla H_{i-\frac{1}{2}}^{n}$

And thus finally:

$$\overline{F}_{i-\frac{1}{2}}^{n+\frac{1}{2}} = \partial\Omega_{i-\frac{1}{2}} \cdot K_{i-\frac{1}{2}}^{n+\frac{1}{2}} \cdot \nabla H_{i-\frac{1}{2}}^{n+\frac{1}{2}} \tag{14}$$

Therefore:

$$C_{i,j} \cdot H_{i,j}^{n+1} = C_{i,j} \cdot H_{i,j}^{n} - \frac{\Delta t}{\forall c_i}\left(F_{i+\frac{1}{2},j}^{n} - F_{i-\frac{1}{2},j}^{n} + F_{i,j+\frac{1}{2}}^{n} - F_{i,j-\frac{1}{2}}^{n}\right) + \Delta t \cdot S_{wi,j}^{n} \tag{15}$$

Expanding terms presented above, and grouping according to the notation P: (j, i); N: (j+1, i); S: (j-1, i); E: (j, i+1); and W:(j, i-1) . Then the linear system is of the form:

$$\begin{aligned}&\left(C_{j,i} + \frac{\Delta t}{\Delta x \cdot \Delta z} \cdot a_P\right) \cdot H_{j,i}^{n+1} + \frac{\Delta t}{\Delta x \cdot \Delta z} \cdot \left(a_N \cdot H_{j+1,i}^{n+1} + a_S \cdot H_{j-1,i}^{n+1} + a_E \cdot H_{j,i+1}^{n+1} + a_W \cdot H_{j,i-1}^{n+1}\right)\ldots \\ &= C_{j,i} \cdot H_{j,i}^{n} + \frac{\Delta t}{\Delta x \cdot \Delta z} \cdot b_1 + \Delta t \cdot S_{w\,j,i}^{n}\end{aligned} \tag{16}$$

with:

$$a_P = \delta\left(\frac{K_{j,i-\frac{1}{2}} \cdot \Delta z}{2 \cdot \Delta x} + \frac{K_{j,i+\frac{1}{2}} \cdot \Delta z}{2 \cdot \Delta x} + \frac{K_{j-\frac{1}{2},i} \cdot \Delta x}{2 \cdot \Delta z} + \frac{K_{j+\frac{1}{2},i} \cdot \Delta x}{2 \cdot \Delta z}\right) \tag{17a}$$

$$a_N = -\delta\left(\frac{K_{j+\frac{1}{2},i} \cdot \Delta x}{2 \cdot \Delta z}\right) \tag{17b}$$

$$a_S = -\delta\left(\frac{K_{j-\frac{1}{2},i} \cdot \Delta x}{2 \cdot \Delta z}\right) \tag{17c}$$

$$a_E = -\delta\left(\frac{K_{j,i+\frac{1}{2}} \cdot \Delta z}{2 \cdot \Delta x}\right) \tag{17d}$$

$$a_W = -\delta\left(\frac{K_{j,i-\frac{1}{2}} \cdot \Delta z}{2 \cdot \Delta x}\right) \tag{17e}$$

$$b_1 = \left(C^n_{i,j} - \left(\frac{1-\delta}{\delta}\right)\left(a_N + a_S + a_E + a_W\right)\right) \cdot H^n_{i,j}$$

$$+\left(\frac{1-\delta}{\delta}\right)\left(a_N \cdot H^n_{i,j+1} + a_S \cdot H^n_{i,j-1} + a_E \cdot H^n_{i+1,j} + a_W \cdot H^n_{i-1,j}\right) \tag{17f}$$

$$+\overline{S}_w \cdot \Delta x \cdot \Delta z$$

$$K_{j\pm\frac{1}{2},i} = \frac{K_{j,i} + K_{j\pm1,i}}{2} \tag{17g}$$

$$K_{j,i\pm\frac{1}{2}} = \frac{K_{j,i} + K_{j,i\pm1}}{2} \tag{17h}$$

Soil water properties are approximated with van Genuchten and Mualem functions (van Genuchten, 1980; Mualem 1976) which, assuming no hysteresis, can be expressed as:

$$\theta(h) = \frac{\theta_{sat} - \theta_{res}}{\left[1 + \left(\alpha|h|\right)^n\right]^m} + \theta_{res}\,, \tag{18}$$

$$K(h) = K_{sat}\,\frac{\left\{1 - \left(\alpha|h|\right)^{n-1}\left[1 + \left(\alpha|h|\right)^n\right]^{-m}\right\}^2}{\left[1 + \left(\alpha|h|\right)^n\right]^{m/2}}\,, \tag{19}$$

$$C(h) = \frac{d\theta}{dh} = \frac{\alpha m\left(\theta_{sat} - \theta_{res}\right)}{1-m}\,\frac{\left\{1 - \left[1 + \left(\alpha|h|\right)^n\right]^{-1}\right\}^m}{\left[1 + \left(\alpha|h|\right)^n\right]^m}\,, \tag{20}$$

where θ_{sat} is the saturation volumetric water content, θ_{res} is the residual water content, K_{sat} is the saturated hydraulic conductivity, and m and n are the van Genuchten parameters (where $m=n+1$).

In R2D the soil surface upper boundary (Aravena 2006) can fluctuate between the following conditions:

(i) Dirichlet: when water begins to pond at the soil surface i.e. variable head in time,

$$H^k_{J,i} = h^k_s + z_{J,i} \tag{21}$$

where h^k_s is the ponded depth at time k and $z_{J,i}$ is the height of the soil surface node, respectively

(ii) Neumann: this condition occurs before ponding and after pond drainage i.e. flux is equal to water input to the soil surface

$$q^{k+1}_o = -K\left[\frac{\partial h}{\partial z} + 1\right] \tag{22}$$

$$q_o^{k+1} = -K_{J-\frac{1}{2},i}^{k+1}\left[\frac{H_{J,i}^{k+1} - H_{J-1,i}^{k+1}}{\Delta z}\right] \tag{23}$$

where $q_0^{k+1} = r + q_{in}$, is known from the current rainfall rate, r, and surface water inflow q_{in}. Therefore,

$$H_{J,i}^{k+1} = H_{J-1,i}^{k+1} + \Delta z \cdot \frac{q_o^{k+1}}{K_{J-\frac{1}{2},i}^{k+1}} \tag{24}$$

Note that the flux equation is highly nonlinear and needs to be solved iteratively. The bisection method was used to solve (17) in each upper node of the domain, given the discontinuities in the derivative of the van Genuchten soil equations, which prevented the use of other faster methods e.g. Newton (Aravena 2006).

The top boundary condition is given by the coupling of Richards equation to a simple surface water balance of the type:

$$A\frac{dh_s}{dt} = r + q_{in} - Q_{INFILTRATION} - Q_{RUNOFF}, \tag{25}$$

where A is the infiltrating area, h_s is the surface water ponded depth, $Q_{INFILTRATION}$ is the infiltrating flux into the soil (estimated as the surface node fluxes from Richards equation solution in the previous iteration) and Q_{RUNOFF} is the overspill from the infiltrating depression (any ponding water that exceeds the depression storage h_d). We assume that rain and runon are uniformly distributed over the infiltrating area (Dussaillant at al. 2004).

For the lower boundary condition, the user can select from Dirichlet, flux and free drainage (unit gradient); the latter was used herein.

Linearizing using a Picard type iteration (Celia et al. 1990), results in a penta-diagonal system which is solved using a known scheme for such systems, the preconditioned bi-conjugate gradient method (Numerical Recipes, 1992; Aravena 2006).

Thus, in our model R2D, the Richards equation is coupled with surface water balance as top boundary condition, is conservative, bi-dimensional, and can model conditions as non-homogeneous water input and surface irregularities e.g. terracing.

2.2 EXPERIMENTAL RAIN GARDEN

A rain garden inside a lysimeter experimental facility was installed and connected to the 116 m^2 roof of the Hydraulic and Environmental Engineering Dept. at the P. Universidad Católica San Joaquín campus. The lysimeter is an impermeable structure built 2.1 m deep, 2 m wide and 3 m long (6 m^2), and divided in 2 sections: 60 cm of transparent acrylic walls in the top, and 1.5 m deep concrete walls. The soil has two layers: 1.5 m of sand in the bottom and 50 cm of a mixture of sand (50%) and compost (50%) for root zone upper soil. To minimize preferential flow between the soil and the lysimeter walls, clay rings were installed to redirect flow towards the interior of the soil (Dussaillant et al. 2005a). There is 10 cm deep surface depression storage in the acrylic section.

Native plant species were planted in 2004. Sensors installed include Time Domain Reflectometry (TDR) probes in vertical profiles for moisture (Figure 2), pressure transducers for surface flow heads, a triangular weir and transducer for water inflow from roof, and a tipping bucket for lysimeter bottom drainage. Soil hydraulic properties were measured by permeameters, tensioinfiltrometer and pressure plates. More detailed information of the lysimeter setup can be found in Dussaillant et al. (2005a,b) and Aravena (2006).

3 RESULTS AND DISCUSSION

The R2D model was validated with both literature data and experimental data from the lysimeter setup in Santiago, Chile.

3.1 MODEL VALIDATION WITH LITERATURE DATA

As a first step, and following the approach we took in a previous 1D model (Dussaillant et al. 2004; Dussaillant et al. 2005a), we validated our 2D Richards model with literature cases that would be similar to situations a rain garden system would encounter (Aravena 2006).

First, R2D was applied with a sharp gradient given by a sudden increase in hydraulic head at the soil surface, given by Celia et al. (1990), for a soil with characteristics given in Table 1. The soil column boundary conditions are Dirichlet type: h_{top} = -75 cm; h_{bottom} = -1000 cm. And the initial condition of h = -1000 cm in the whole column. For the base case of a hundred node discretization in space, and Δt = 10 s, for a simulation tome of 24 hours, R2D gave good results compared to Celia et al. (1990) and a comparison with Hydrus (Simunek et al. 1999), as shown in Figure 3a. Minor differences can be attributed to the different methods: finite differences in the case of Celia et al. (1990), and finite elements in the case of Hydrus, as opposed to R2D's finite volume approach.

Second, since rain garden soil is layered, we followed the Pan & Wierenga (1995) layered soil case, with two types of Berino soil, a top sandy layer (0 to 60 cm depth), an intermediate clayey layer (between 60 and 90 cm depth) and a bottom sandy layer (90 to 100 cm depth), under a constant flux upper boundary condition of $q_{top} = 1.25\,\mathrm{cm\,h^{-1}}$ and a no flow bottom boundary condition. Soil van Genuchten parameters are summarized in Table 2. The soil initial condition is h = -1000 cm in the whole column. For the base case of a hundred node discretization in space, and Δt = 0.001 h, for a simulation tome of 5 hours, R2D gave good results compared to Pan & Wierenga (1990) and a comparison with Hydrus (Figure 3b), again with good agreement, although R2D gave better results than Hydrus due to lower mass balance errors (see below, and Aravena 2006).

Finally, we successfully repeated the Celia et al. (1990) classic case but using a 2D flux situation (Figure 3c) just for illustration of the bi-dimensional case.

In all validation runs, R2D mass balance errors were substantially lower (< 0.1%) than the errors by using a commercially available software package, HYDRUS-2D (Figure 4).

3.2 MODEL VALIDATION WITH EXPERIMENTAL LYSIMETER DATA

Several controlled rainfall experimental runs were performed in the lysimeter, which R2D could simulate very well without calibration (results not shown). We also had the opportunity of a natural rain event in October 2006, to validate R2D for a controlled yet more real situation. Figure 5 presents the hyetograph of the event; using the period from 15:35 October 12th till 23:15 of the following day: 59.3 mm total direct precipitation; i.e. almost 21 times this amount is what the rain garden received given the roof-garden area ratio).

The event was simulated with R2D, given an approximate linearization of the initial soil moisture condition (shown in Figure 5). Setup parameters are summarized in Table 3.

Figure 5 shows that the R2D simulation did rather well, for moisture contents as measured by TDRs, as well as bottom drainage (Aravena 2006), which is a relevant performance variable for rain gardens, since maximizing potential recharge is one of the key benefits of these practices.

4 CONCLUSIONS

A bi-dimensional, finite-volume, Richards equation based model with coupled top boundary condition was developed successfully (R2D; Aravena 2006), that allows for representing rain garden short term water fluxes (and long term water balance). R2D can model heterogeneous soils and surfaces, as well as alternating boundary conditions and non-homogeneous water inputs. Its conservative algorithm has very low mass balance errors – an improvement versus other models available, as shown with the validation runs performed for cases of interest given the bioretention cell (rain garden) configuration, of high soil surface head gradients, and layered soils.

R2D was successfully compared to a lysimeter controlled experiment with a natural rainfall event, without calibrating soil hydraulic parameters. The model represented temporal variability satisfactorily, and approximated well the soil water content dynamics for the upper root zone organic layer as well as the bottom sandy storage zone (also for controlled water input experiments, not shown here). More importantly for our purposes, R2D reproduced rain garden deep percolation (potential recharge) well for the test case, and the tests with natural rains will be continued, once the La Niña current dry spell is over in Chile.

Ongoing work with R2D includes: numerical simulations for different rain garden configurations e.g. terracing; benchmark operational models based on Green-Ampt equation (e.g. Dussaillant et al. 2005b) to our 1D (Dussaillant et al. 2004) or 2D (present paper) Richards-based models; and to work on model modifications to include other processes e.g. macropore flows.

We plan to develop design curves as a function of different climates and soils (Dussaillant et al. 2005b). A practical next step is to construct and monitor rain gardens in urbanizations across Chile. More generally, it is deemed necessary to upscale the analysis to development and watershed scale, to include drainage and spatial considerations, and verify effective impact on basin recharge, for which different modeling strategies may be necessary.

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Notation

The following symbols are used in this paper:

A	=	infiltrating area
$C(h)$	=	soil moisture capacity function ($\partial\theta/\partial h$)
h	=	suction head
h_d	=	maximum depression depth
h_s	=	surface ponding depth
d	=	surface water ponded depth
F	=	integral flux term
g_i	=	value of the function evaluated in the centroid of the control volume i
H	=	total head ($H=h+z$)
i,j	=	spatial counters
k	=	temporal counter
$K(h)$	=	unsaturated hydraulic conductivity
K_{sat}	=	saturated hydraulic conductivity
K_{ss}	=	saturated hydraulic conductivity of the lower subsoil
m, n	=	van Genuchten parameters
q	=	water flux
q_{in}	=	runon input of water to infiltrating area
$Q_{INFILTRATION}$	=	infiltration of water into soil
Q_{RUNOFF}	=	runoff as overspill from infiltrating area
r	=	direct rainfall input to infiltrating area
S_w	=	soil moisture sink (plant transpiration rate)
t	=	time
z	=	vertical position
α	=	van Genuchten parameter
Ω	=	control volume

$\partial\Omega$	=	boundary conditions for the control volume Ω
δ	=	weight for the Crank-Nicolson scheme
θ	=	soil volumetric moisture content
θ_{res}	=	residual water content
θ_{sat}	=	saturation water content
$\forall$	=	volume

FIGURE CAPTIONS (FIGURES ATTACHED AS TIF FILES):

Figure 1: Conceptual diagram of a Rain Garden and associated water fluxes

Figure 2: Experimental rain garden, installed inside a lysimeter configuration: a) general view and plant growth; b) setup scheme for profile of TDR sensors

Figure 3: Validation runs for R2D: a) Celia et al. 1990 (pressure); b) Pan & Wierenga 1995 layered soil (water content); c) Celia et al. 1990 modified to a 2D flux case (pressure).

Figure 4: Mass balance errors of R2D compared with HYDRUS for different bottom boundary conditions: a) no flux; b) constant flux; c) free drainage.

Figure 5: Natural rain event on the experimental rain garden, comparison of data versus R2D model (data in circles, model simulations in thin continuous line): rainfall event and initial soil moisture profile (top); two TDR location data versus R2D model results (middle, left for a root zone node, right for a storage zone node); lysimeter drainage flux and cumulative volume measured vs. modeled (bottom).

Table 1

Table 1: van Genuchten soil parameters for the Celia et al (1990) case

Parameters	**New Mexico Soil**
θ_s	0.368
θ_r	0.102
$\alpha_v\left(cm^{-1}\right)$	0.0355
n	2.00
$K_s\left(cm\ s^{-1}\right)$	0.00922

Source: (Celia *et al.*, 1990)

Table 2: van Genuchten soil parameters for the Berino type soil, Pan & Wierenga (1995) case

Parameters	**Berino sand**	**Berino clay**
θ_s	0.3658	0.4686
θ_r	0.0286	0.1060
$\alpha_v\left(cm^{-1}\right)$	0.0280	0.0104
n	2.2390	1.3954
$K_s\left(cm\ h^{-1}\right)$	22.54	0.5458

Source: (Pan y Wierenga, 1995)

Table 3
Click here to download Table: R2Dtable3.doc

Table 3: Experimental rain garden soil hydraulic parameters

Parameters	Bottom sand	Upper sand & compost mix
θ_s	0.3623	0.5363
θ_r	0.0383	0.2457
$\alpha_v \left(cm^{-1}\right)$	0.3571	0.2419
n	2.5621	2.1165
$K_s \left(cm\ h^{-1}\right)$	90.47	36.04

Source: (Aravena, 2006)

Figure 1

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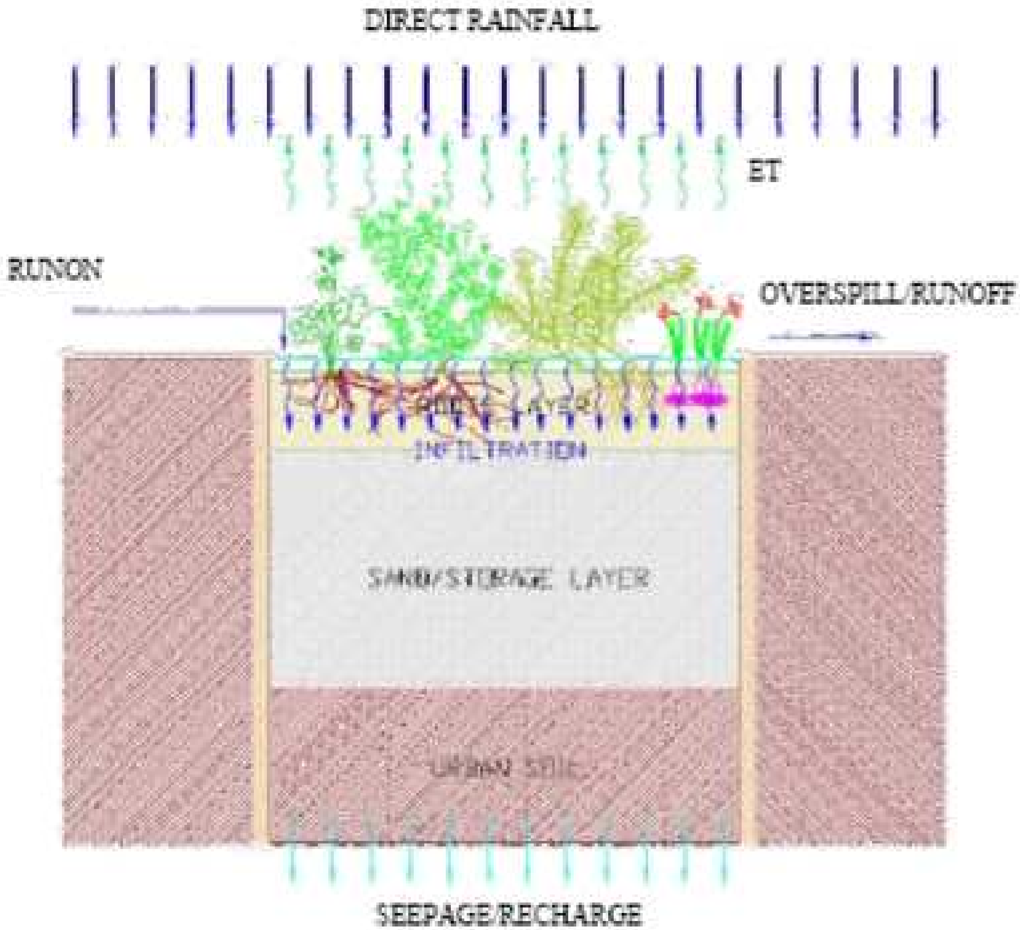

Figure 2
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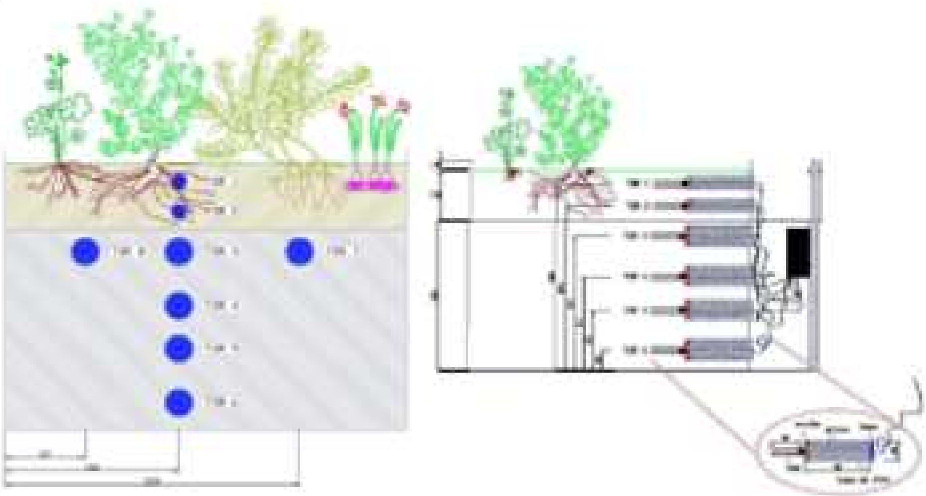
a)

b)



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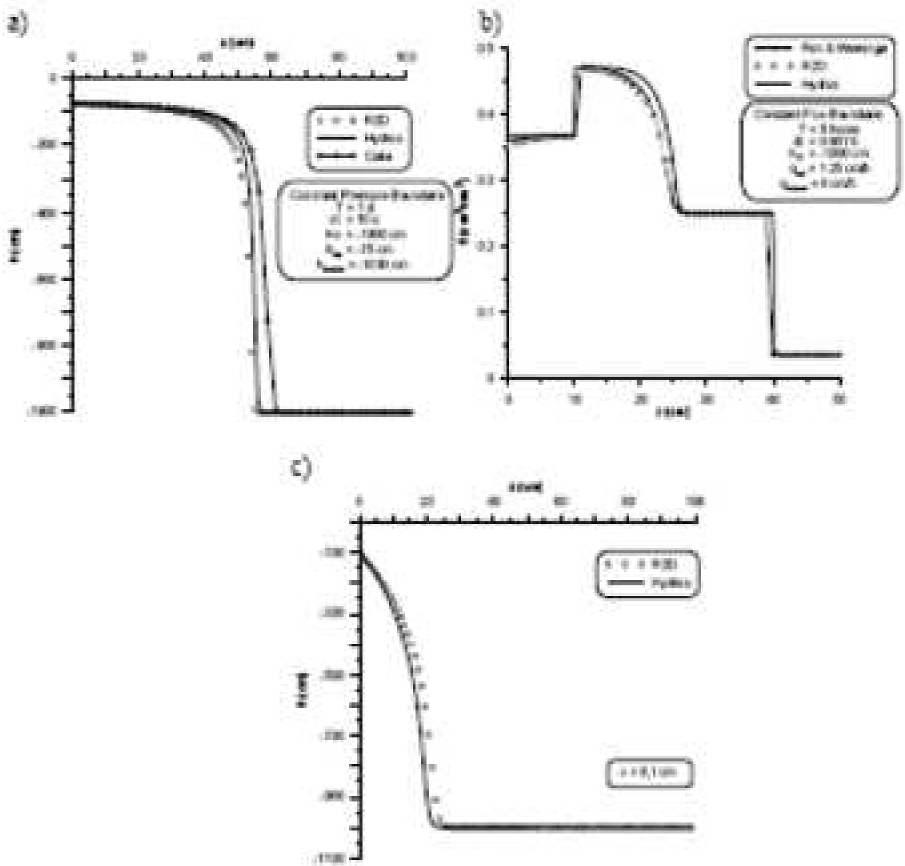

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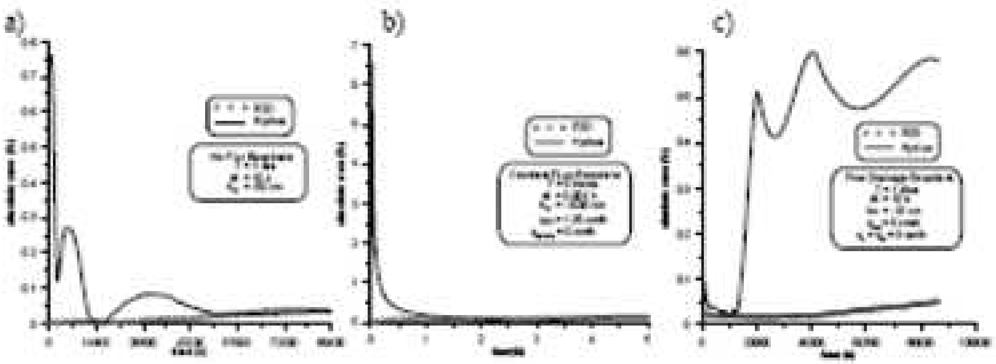

Figure 5

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