OPTIMAL FINANCIAL POLICIES IN AN OPEN ECONOMY:
THE U.K. CASE

BY

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The object of this study is to examine the 'monetary instrument' problem, at both theoretical and empirical levels, using a framework in which 'domestic' and 'external' monetary policy are analysed concurrently. Our theoretical analysis generalises and extends some of the propositions on the stabilising properties of alternative financial policies in the case of a small open economy, subject to both internal and external shocks.

An econometric model of the U.K. economy is built to test these propositions. To get our results, we make use of an optimal control framework which employs an objective function depicting the desires of the policy makers, to yield optimal paths for the target variables as well as the policy variables. Most of the results are of the open-loop deterministic type, although we also approximate a closed-loop stochastic system by perturbing the system with certain shocks and optimizing again.

Among the pegging regimens considered, the one involving targets for foreign reserves and the monetary aggregate seems to be preferable. However, the analysis also reveals that the policy makers should not adhere to the optimal rule, but should allow the paths of the intermediate targets to alter in response to new information as it becomes available. Since the quantitative results are model specific, the study should be regarded as demonstrating a methodology for the design of policy, rather than as offering actual policy guidance.
To my mother
and my late father
I wish to express my deepest appreciation to my supervisor, Dr. P. Arestis, for his valuable guidance and helpful criticism during the whole period of this study. Without his invaluable help, this work would not have been possible.

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1.1 Introduction

In this era of monetary targets, it is very likely, in an open economy like the U.K., that there could be a conflict between the achievement of the monetary targets and exchange rate considerations. For instance, high interest rates bring about capital inflows which may imply that the money supply target can only be achieved if the exchange rate is allowed to fluctuate, with adverse effects on competitiveness. Thus as the Governor of the Bank of England remarked in his speech at the Lord Mayor's dinner in October 1981, while "a monetary target provides a necessary discipline... in the short run our actions need to be guided by a range of considerations... To give weight to exchange rate considerations would on some occasions, have meant loosening our monetary control. But in present circumstances pursuit of the two objectives has been complementary and mutually reinforcing." (BEQB, 1981, p.546). Nevertheless, it is perhaps true to say that when there has been a sharp conflict between the money supply and exchange rate objectives, the Bank seems to have given priority to the money supply objective (Savage, 1979). The important point, however, is that the above quotation shows quite clearly that domestic and external aspects of monetary policy are inherently interlinked and should not therefore be analyzed separately. Although this seems to be an obvious state of affairs, its implications for policy making in general and for the instrument choice problem in particular, do not seem to be fully appreciated.

The purpose of this study is to show formally why, and how, the monetary instrument choice problem (1) can be investigated within a framework in which 'domestic' and 'external' (2) monetary policy are analyzed concurrently. This is done at a theoretical level in chapter 2, and chapter 4 provides an empirical analysis based on this framework of monetary policy. For instance, we analyze the relative merits of a regimen involving an exchange rate peg together with an interest rate peg, with one involving a foreign
reserves peg together with a monetary aggregate peg. This implies that the money supply, the interest rate, the exchange rate as well as foreign reserves, can all be considered as proximate or intermediate targets. Thus monetary policy can take the form of a two-stage process which involves decisions at two levels, one choosing the target time paths of some intermediate target variables, and the other manipulating the true instruments (e.g. open market operations) in order to achieve those target paths for the intermediate variables. The alternative to the two-stage or intermediate target strategy, is of course, a single-stage strategy where given target time paths for the ultimate target variables (e.g. inflation, unemployment, balance of payments) time paths for the 'true' instruments are calculated in a single-stage process without having recourse to any intermediate target. The issue of two-stage versus single-stage strategies, or strategy choice, or the procedure problem, has not been given much attention in the literature (notable exceptions being Niehans, 1978, and Bryant 1980). We therefore, also briefly analyze this problem as well.

Different assumptions can be made about the periodicity of decision taking in a two-stage intermediate target strategy for monetary policy. The decisions about varying the actual instruments - in the lower stage of the two-stage process - are usually assumed to be made almost continuously; that is as new information for the intermediate target variables is obtained, it is immediately processed and the actual instruments are altered accordingly. What about the intermediate targets themselves? At one end of the spectrum, it is assumed that once a rule has been obtained for determining the paths of the intermediate targets, that rule is not changed as new information become available. For example, one such simple rule might be to let the money stock (the intermediate target) grow at a specified rate à la Friedman (1969) and that rule is rarely, if ever, changed over time. Relaxing that constraint a little bit could imply respecifying that rule every say six months to one year. The U.K. could be placed in that category with its announced annual monetary target. Towards the other end of the spectrum, the path for the intermediate target is recalculated
regularly, say every quarter, as new information becomes available. For example, in the United States the target paths for the intermediate target variable(s) are reviewed at the meetings of the Open Market Committee. If the paths for the intermediate targets are recalculated as data for any of the endogenous variables become available, then for all practical purposes the two-stage strategy will be indistinguishable from a single-stage strategy.

If the setting of the intermediate target is not rigidly maintained on a predetermined path, but is discretionally adjusted from one period to another, then the choice of the intermediate target itself may be of less importance. As Bryant (1980) rightly notes "instrument choice as opposed to instrument variation is not a trivial problem under discretionary instrument adaptation. But instrument choice cannot be an overriding concern in its own right unless policy makers wish to follow instrument rules." (p.343)

This conclusion is confirmed in our empirical analysis in chapter 4, which extends the analysis of the pegging regimens in chapter 2 to their discretionary counterparts, using the model of the U.K. economy developed in chapter 3, within an optimal control framework. Before we move on to the theoretical analysis of chapter 2 we need to review the theoretical background of the existing literature. This is what the rest of this chapter is all about, with the next section introducing us to the subject and section 1.3 providing a brief review of existing studies.

1.2 Instrument Choice in an Open Economy

In this section we will show how Poole's (1970) analysis of the monetary instrument problem in a closed economy can be translated to an open economy to yield similar conclusions. If capital is perfectly mobile, and we abstract from exchange rate expectations, it is well known that the authorities in a 'small' open economy, cannot fix its interest rate at a level which is different from world interest rates (see, for example, Mundell, 1968). This means that in such an economy, there is now a choice between the money supply and the exchange rate. Poole's (1970) analysis can be reformulated so that it can be conducted in such a setting, with the aid of the following IS and LM equations:
(1) \( Y_t = a_1 S_t + a_2^1 Q_t + U_t \); \( a_1 > 0 \),

and

(2) \( M_t = b_1 Y_t + b_2^1 Q_t + b_2 S_t + V_t \); \( b_1, b_2 > 0 \),

where \( Y_t \) = income, \( S_t \) = the exchange rate, \( M_t \) = money stock, \( Q_t \) = vector of values of relevant exogenous variables.

The two disturbances \( u \) and \( v \), have zero means and finite variances \( (\sigma_u^2 \text{ and } \sigma_v^2) \) and covariance \( \sigma_{u,v} (= p \sigma_u \sigma_v \text{, where } p \text{ is the correlation coefficient between } u \text{ and } v) \). The authorities' expected loss function is simply assumed to be

(3) \( E(L) = \mathbb{E} ((Y_t - Y^*_t)^2) \)

where \( \mathbb{E} \) is the mathematical expectation operator, and, \( Y^* \) is the target level of income. The expected loss for the exchange rate policy can be shown to be given by (6)

(4) \( E(L)_{S=S^*} = \mathbb{E} ((u_t)^2) = \sigma_u^2 \)

and that for the money supply policy by

(5) \( E(L)_{M=M^*} = (b_2 + a_1 b_1)^{-2} \left( b_2^2 \sigma_u^2 + a_1^2 \sigma_v^2 - 2p b_1 a_1 \sigma_u \sigma_v \right) \)

To find out which policy gives the smallest expected loss, the ratio of the losses in (4) and (5) are taken to obtain

(6) \[
\frac{E(L)_{M=M^*}}{E(L)_{S=S^*}} = \frac{b_2^2 + a_1^2 (\sigma_v^2 / \sigma_u^2) - 2p a_1 b_2 (\sigma_v / \sigma_u)}{(b_2 + a_1 b_1)^2}
\]

If the ratio given by (6) is greater than 1, then it is preferable to hold \( S = S^* \), while if it is less than 1 then the preferable policy is to hold \( M = M^* \). Thus, as in Poole's analysis for a closed economy the important point is that the superiority of either policy depends on the values of the parameters of the system as well as the variance-covariance structure of the disturbances. Again, in the extreme case where there are only real disturbances, a money supply strategy is the preferable one to follow \((b_2^2 \sigma_u^2 / (b_2 + a_1 b_1)^2 < \sigma_u^2\) given the expected signs of the coefficients) whilst if there are only monetary disturbances an exchange rate policy is the superior one \((a_1^2 \sigma_v^2 / (b_2 + a_1 b_1)^2 < \sigma_v^2\).

The above analysis, can in fact be extended in a number of ways.
For instance, in a closed economy, the point has been made that observations on the monetary aggregate as well as the interest rate, are important because they provide useful information about the source of random disturbances in the economy (see for example, Kareken, Muench and Wallace, 1973 and Friedman, 1975, 1977). Using equations (1) and (2) and the assumptions above, we can extend this information approach to an open economy to obtain a trade off between the money supply and the exchange rate. We further assume, that the money supply and the exchange rate are observable, whilst income is not in the short term.

We first obtain the reduced form for \(M_t\) by eliminating \(Y_t\) from (1) and (2) to get

\[
M_t = (b_1 a_1 + b_1) Q_t + (b_1 a_1 + b_2) S_t + Z_t
\]

This means that we can now observe a composite disturbance \((Z_t = b_1 U_t + V_t)\) although \(U_t\) and \(V_t\) are not known since \(M_t\), \(S_t\) and \(Q_t\) are all observable. We now assume, that the monetary authorities adjust both \(M_t\) and \(S_t\) in response to the composite disturbance \(Z_t\) according to the following policy rule:

\[
S_t = \theta (b_1 U_t + V_t)
\]

where \(\theta\) is a constant. This implies that the resulting income is

\[
Y_t = a_2 Q_t + (1 + a_1 b_1 \theta) U_t + a_1 \theta V_t
\]

which gives a variance of

\[
\sigma^2_y = (1 + a_1 b_1 \theta)^2 \sigma_u^2 + a_1^2 \theta^2 \sigma_v^2 + 2 a_1 \theta (1 + a_1 b_1 \theta) \sigma_{u,v}
\]

We must choose the value of \(\theta\) to minimize (10) to obtain the optimal exploitation of the information on \(Z_t\). Thus, differentiating (10) with respect to \(\theta\) and setting it equal to zero, we get

\[
\frac{\delta \sigma^2_y}{\delta \theta} = 0
\]

which gives an optimal value for \(\theta\) of

\[
\hat{\theta} = \frac{-b_1 \sigma_u^2 + \sigma_{u,v}}{a_1 (b_1 \sigma_u^2 + \sigma_v^2 + 2 b_1 \sigma_{u,v})}
\]

The optimal trade-off in the adjustment of \(M_t\) and \(S_t\) can be easily obtained by first substituting the policy rule (8) into (7), and
then substituting for the value of in the resulting equation. The minimized variance of income is obtained by simply substituting for \( \theta \) into (10) to get

\[
\sigma_y^2 = \frac{\sigma_u^2 \sigma_v^2 (1-p^2)}{b_1^2 \sigma_u^2 + \sigma_v^2 + 2b_1 p \sigma_u \sigma_v}
\]

which is similar to the expression as in equation (17) for the 'combination' policy in Poole (1970) although the approach is different.\(^{(8)}\) Ignoring the correlation between \( U_t \) and \( V_t \) (i.e. \( p = 0 \)) we can easily show that the loss in this policy is never greater than that in either of the two pure policies. The loss from this policy is not greater than that in the exchange rate policy if

\[
\sigma_u^2 > \frac{\sigma_u^2 \sigma_v^2}{(b_1^2 \sigma_u^2 + \sigma_v^2)}
\]

which is trivial. The corresponding condition for the money supply policy is

\[
\frac{b_2^2 \sigma_u^2 + a_1^2 \sigma_v^2}{(b_2 + a_1 b_1)^2} > \frac{\sigma_u^2 \sigma_v^2}{(b_1^2 \sigma_u^2 + \sigma_v^2)}
\]

which after manipulation, reduces to

\[
(b_1 b_2 \sigma_u^2 - a_1 \sigma_v^2)^2 > 0
\]

which always hold. We should perhaps note though, that the information requirements of the 'combination' policy, are more stringent than either of the two pure policies.\(^{(9)}\) Another point to mention is that, if as is generally the case, data on the exchange rate is available almost continuously, but that on the money supply with a one-month lag, and that on income with a one-quarter lag, then a combination policy cannot be implemented, as it is not possible to maintain a fixed relationship between money and the exchange rate. Thus there are practical difficulties in implementing such a policy, for this reason we do not analyze it any further in either our theoretical or empirical work.

There are a number of problems with the kind of analysis we have conducted so far in this section. First of all, the assumption that the authorities intervene on the foreign exchange market only
for income stabilization purposes, is rather restrictive. We know, for example, that they are also concerned about the effect of the exchange rate on prices. In fact, the literature on the monetary instrument problem (apart from Craine and Havenner, 1978, 1981) and the few papers which make the assumption of rational expectations which we refer to below\(^{10}\) seems to be confined to models where prices are fixed. In our analysis, in the next chapter, we drop this assumption so that our objective function incorporates income, price and external stability - the latter is of course also important in an open economy. Thus our analysis can be seen as a further extension of Poole's and subsequent analyses.

The second problem is the assumption of perfect capital mobility, which is again rather restrictive. There is no doubt that, even in a small open economy, the authorities can, theoretically at least, control at least one rate of interest. Thus, our own model in chapter 2 assumes that capital is only imperfectly mobile.

The third, and perhaps the main problem with our analysis above, is that by simply assuming an exchange rate target, it discards the point that once we are dealing with an open economy, the issue of optimal foreign exchange market intervention becomes important and should be analyzed concurrently with the question of instrument choice.\(^{11}\) In fact, in an open economy, we cannot analyze the consequences of any domestic monetary action for stabilization purposes without specifying whether we are operating under a fixed, flexible or managed floating exchange rate system. This latter issue of optimal foreign exchange market intervention, which is basically about determining which exchange rate policy will best stabilize an economy perturbed by various disturbances, cannot also be analyzed without referring to the domestic monetary procedures in operation. However, apart from a few exceptions, notably the paper by Henderson (1979) and the book by Bryant (1980), these two issues have been discussed separately in the literature.

To understand why we should analyze these two problems concurrently consider the following examples.\(^{12}\) Suppose the monetary authorities are operating a fixed foreign exchange reserve regime (foreign reserves management), and they envisage an open market sale of foreign exchange. The ensuing transactions will result in a decrease in domestic money accompanied by changes in the exchange
rate and asset yields. Suppose the authorities are pursuing a money supply target, they will need to engage into domestic open market purchases in order to keep the money supply at its previous level. If instead, the authorities are operating an interest rate target, the decrease in the supply of domestic money and the increase in the supply of foreign money will call forth a domestic open market operation in order to keep the interest rate on target. There is, however, no a priori reason why the two domestic open market operations should be equivalent in the two cases. Thus, we must specify the domestic operating regimes in order to define and analyze the consequences of an exchange market intervention action.

We must also specify the external operating regime in order to assess the consequences of a domestic monetary policy action. Suppose the authorities are pursuing a money supply target and they contemplate a domestic open market purchase. If they are operating a regime of fixed foreign exchange reserves, they will allow interest rates to fall and the exchange rate to depreciate, and nothing more need to be said. However, under a regime of exchange rate peg an exchange market intervention is necessary to keep the exchange rate at its par value. This latter action will produce a fall in the money supply, so that further open market operations are needed to bring the money supply back to its desired path. This will again affect the exchange rate and so the sequence is repeated and if it finally settles down, the resulting fall in interest rates will probably be bigger than in the fixed foreign exchange reserves case. Strictly speaking then, one cannot even define a domestic open market operation without specifying the external operating regime. Thus the issue of the optimal monetary instrument and that of the optimal foreign exchange market intervention, cannot be analyzed in isolation. This point is recognized by the contributors to the monetary instrument problem in an open economy (except it seems Turnovsky, 1978); however, none of them provide a formal and comprehensive analysis based on this approach; Sparks (1979) views the exchange rate and some kind of monetary aggregate as alternative targets only, while Henderson (1979) provides a graphical analysis of only two of the four possible pegging regimens within that framework (see chapter 2). Bryant (1980) on the other hand, introduces us to the various
pegging regimens as well as their discretionary counterparts, but does not provide a comparison of these alternative regimens. Thus our analysis in chapter 4 takes off where Bryant's analysis ends and examine empirically the regimens suggested by him, while in chapter 2 we provide a theoretical comparison of the four pegging regimens using the techniques of optimal control. In the rest of this chapter, we briefly review the papers mentioned above and attempt to relate them to our own work.

1.3 A review of the Literature

As we noted in the last section, three important points have to be taken into account in analyzing the instrument choice problem in an open economy. Firstly, we should consider, in addition to income stability, both price and external stability; secondly we should allow for the imperfect mobility of capital, and thirdly we cannot determine 'domestic' and 'external' monetary policy independently of one another. Turnovsky (1978) does not even recognize the importance of this last point, and only partly deals with the first point. He examines the instrument choice problem within an optimal control framework for an economy where both domestic and foreign price levels are assumed constant and capital is imperfectly mobile. He assumes a fixed exchange rate (which is set at unity for simplicity), and thus considers the added objective of external stability to that of income stability. We will only briefly discuss Turnovsky's results, as our own model (in chapter 2) is an extension of his to allow for a flexible exchange rate and to endogenize prices as well. (13)

Turnovsky looks at three alternative monetary 'instruments', the domestic monetary base, the domestic rate of interest and the total domestic money supply. However, the monetary base and the stock of money are not parallel 'instruments' because a monetary base policy involves a single-stage strategy whilst a money supply policy involves a two-stage strategy of conducting monetary policy (see Friedman, 1975 and Bryant, 1980). Thus, a comparison of these two policies is, in fact, a study of single-stage versus two-stage strategies of conducting monetary policy and therefore does not shed any light on the instrument choice problem.
Turnovsky finds that Poole's results for a closed economy are still valid for an open economy. Thus, if there is instability in domestic output, the interest rate is the worst instrument as far as internal (output) stability is concerned. He finds that when capital is highly mobile, a money supply policy is preferred to a monetary base policy, while the reverse is true for low capital mobility. As far as external stability (in terms of the variance of foreign reserves) is concerned, the domestic monetary base is always the best instrument. In the case of domestic monetary uncertainty, the interest rate policy is the best one and the money supply the worst one to follow. Turnovsky also considers international monetary uncertainty (random shifts in capital flows) and finds that domestically the money supply and the interest rate are equally successful in coping with the disturbances and are both superior to the monetary base policy. Externally, however, it is the latter which is again the best policy. There is, therefore, once more a conflict between the two objectives and this is also true for the last type of disturbance examined which is in the balance of trade. Here, domestically the money supply is the best policy, but the worse externally. Thus, the relative importance of the two objectives in the loss function becomes very important in determining the optimal instrument. However, as we make clear in our analysis in chapter 2, such a conflict arises in the determination of the optimal monetary policy strategy rather than in that of instrument choice, that is, they are due to failure on his part to distinguish between a single-stage strategy and a two-stage strategy of conducting monetary policy.

The Turnovsky paper assumes a fixed exchange rate. Sparks (1979), however, allows for both fixed and flexible exchange rates. This study uses a modification of Mundell's (1961) model of stabilization policy under flexible exchange rates to analyze the question of instrument choice in an open economy. Sparks, though, assumes that the sole ultimate target of policy is the stabilization of income and does not consider external or price stability. In Sparks' model, the following equation describes equilibrium in the foreign exchange market so that the current account deficit is matched by a capital account surplus:

\[ d_1 (r + (1-f)q) + U_k = d_2 y - d_3 q - U_c \]
where \( r \) is the interest rate, \( q \) is the exchange rate, \( Y \) is income and \( U_c \) and \( U_k \) are the current account and capital account error terms respectively. Thus following Dornbush (1976), capital flows are assumed to depend on the interest rate plus the expected rate of appreciation of the exchange rate; \( f \) is the elasticity of expectations coefficient and relates the current to the expected exchange rate.

Poole's IS equation is replaced by the following equation to take into consideration the effect of the current account on income:

\[
Y = a - a_1 r + b_3 q + U_y + h U_c
\]

where \( h \) is the expenditure multiplier and \( U_y \) is an error term in the goods market. The monetary sector, following Friedman (1975) is composed of a money demand equation which is similar to Poole's

\[
M = b_1 r + b_2 Y + u_M
\]

and a money supply equation

\[
M = c + c_1 H + u_M
\]

where \( H \) is the stock of bank reserves, and \( u_M \) are error terms. Sparks assumes that the authorities can exercise control over \( r, q \) or \( H \). In this framework, if \( q \) is controlled, then he assumes that \( r \) will adjust to eliminate imbalances in the foreign exchange market though capital flows. If \( q \) is allowed to move freely, then either \( r \) or \( H \) can be controlled independantly of the balance of payments. Therefore if \( r \) is controlled, (14) and (15) are solved to give the error in \( Y \)

\[
e(Y/r) = (f_3 - hd_3 d_2)^{-1} (f_3 U_y - hd_3 u_k + d_1 (1-f) h U_c)
\]

Thus as in the closed economy discussed by Poole, the choice of the interest rate result in insulation of \( Y \) from disturbances in the monetary sector. However, now the effect of random changes in domestic demand is amplified by the induced changes in the current account, as for instance an exogenous increase in \( U_y \) stimulates imports bringing about a depreciation of the exchange rate which in turn increase exports. Because Sparks takes prices as fixed, however he does not take the argument any further. In fact, if prices are endogenously determined as they are in our model in chapter 2, there is a further effect on output as the exchange rate depreciation...
brings about a rise in prices. Under control of the exchange rate, Sparks again obtains the error in Y from (14) and (15) and it is this time given by

\[ (19) \quad e(Y/q) = (d_1 + a_1 d_2)^{-1} (d_1 U_y + a_1 U_k + (a_1 + h d_1) U_c) \]

Thus control of the exchange rate also results in insulation of Y from disturbances in the monetary sector. \(^{(14)}\) Comparing interest rate to exchange rate control, Sparks finds that control of the latter is better the larger is \(d_2\), that is the more sensitive the current account is to changes in income. Also, under exchange rate control there is a stabilizing feedback as disturbances in the goods market are damped by changes in the interest rate. This stabilization is greater the smaller the interest elasticity of capital flows (represented by \(d_1\)). Sparks states that in general, a combination policy is superior to either of these two pure strategies unless there is perfect capital mobility \((d_1 = \infty)\) and inelastic exchange rate expectations \((f < 1)\), when the two are equivalent and lead to errors in Y of

\[ (20) \quad e(Y/r) = e(Y/q) = U_y + h U_c \]

However, as we saw earlier there are a number of practical problems involved in the operation of a combination policy. The reduced form for reserves control is complicated and is not presented by Sparks, but he considers the case of perfect capital mobility which gives an error in Y of

\[ (20) \quad e(Y/H) = b_2^{-1} (U_{MD} - U_{MS}) \]

Thus in this case there is complete insulation from disturbances in the goods market. Random changes in aggregate demand are fully offset by changes in the current account brought about by changes in the exchange rate. \(^{(15)}\) However, reserves control also permits monetary disturbances to destabilize income.

Although Sparks recognizes the link between domestic and external monetary policy, he only views the domestic and external monetary instruments as alternative but not as complementary ones. Henderson (1979) does just that. He uses a diagramatic framework to investigate an 'aggregates constant policy' and a 'rates constant policy'. By an 'aggregates constant' policy, Henderson means one where the money supply as well as foreign exchange reserves are kept constant.
at some chosen values and by a 'rates constant policy', he means one where the interest rate and the exchange rate are kept constant at some selected values. The financial assets held by domestic residents are assumed to be home money and home and foreign securities which are strict gross substitutes. Foreigners are not supposed to hold home money.

In figure 1 below, \( \overline{XX} \) shows the values of the interest rate \( (r) \) and output \( (Y) \) consistent with equilibrium in the home goods market.

![Equilibrium in the market for home money is given by the \( M_0M_0 \) schedule, while \( B_0B_0 \) is the equilibrium schedule for the home security which is held by both home and foreign residents. \( \overline{XX} \), \( M_0M_0 \) and \( B_0B_0 \) intersect at the full employment level of output \( (Y_f) \). Henderson assumes that in the short run the prices of the home and foreign goods (in their respective currencies) are fixed and also that the foreign interest rate and foreign output are kept constant by the foreign authorities. Henderson invites us first of all, to consider the effects of random shifts in the \( XX \) schedule due to say changes in saving behaviour or to changes in preferences between the two goods at home or abroad. Suppose for instance, an increase in demand for the home good causes the \( XX \) schedule to shift outwards to \( XX' \). The increase in income will cause an excess demand for home money and an excess supply of home securities. If the authorities pursue an 'aggregates constant policy' they will allow the interest rate...
to go up and the exchange rate to appreciate to remove the
disequilibria. Here, Henderson simply assumes that the exchange
rate actually appreciates. As we will see in chapter 2, this will
only happen, in such a context, if the capital flow effect dominates
the trade effect adjusted for price increases. (Henderson, though
takes prices as fixed). Henderson further assumes, that an
appreciating home currency raises excess supply for the home good,
home money as well as home security. This implies that $X_2X_2$,
$M_0M_0$ and $B_0B_0$ schedules move together until we have an intersection
in the shaded area 'abc'. Thus, if the XX schedule shifts between
$X_1X_1$ and $X_2X_2$, we have the output level between $Y_1$ and $Y_2$. Suppose
instead we have a 'rates constant policy'. As the exchange rate
is not allowed to change, the XX schedule remains at $X_2X_2$. To
prevent the interest rate from going up, the authorities undertake
an open market purchase of home securities with home money shifting
both the MM and BB schedules to the right. A sale of home securities
in exchange for foreign securities is also needed. This intervention
policy is required because when income increases not only does the
demand for home securities fall but that for foreign securities as
well, which means that the increase in demand for home money is
greater than the decrease in demand for home securities. Only
after these two operations will the MM and BB schedules intersect
at point d. So, with shifts in the XX schedule between $X_1X_1$ and
$X_2X_2$ output levels between $Y_1$ and $Y_2$ result. Henderson concludes
that if only random shocks in the XX schedule are present, an
'aggregates constant policy' will be preferable to a 'rates constant
policy' as it leads to less variation in output.

Henderson goes on next to consider the case where there are shifts
in the BB schedule only, due to say changes in preferences between
the two securities either at home or abroad. As can be seen
from figure 2, with stochastic shifts between $B_1B_1$ and $B_2B_2$, the
output level will be between $Y_1$ and $Y_2$ under an 'aggregates constant
policy' but at $Y_f$ under a 'rates constant policy' so that the latter
is the preferable one this time.

One important conclusion can be drawn from Henderson's analysis
when the economy is perturbed by all types of shocks considered
above. If we normalize the three equilibrium relations on income
with equal variances for the normalized disturbances, Henderson
tells us that an 'aggregates constant policy' may or may not be preferable to a 'rates constant policy', whereas in the closed economy analysis of Poole under similar assumptions a money supply policy is better than an interest rate policy. According to Henderson, an 'aggregates constant policy' will be preferable to a 'rates constant policy' the higher the degree of substitutability between home and foreign securities.

We should perhaps note that the comparison in Henderson (1979) is restricted to two regimens only, with one involving a pair of 'price' and the other pair of 'quantities.' However, a fuller treatment of the subject requires the examination of two more pegging regimens each involving a 'price' and 'quantity', that is an exchange rate and money supply policy and a foreign reserves and interest rate policy. This is done in chapter 2 where we analyze the relative merits of four different financial regimens (each involving a pair of intermediate 'domestic' and 'external' targets) for stabilization purposes. Thus our analysis can be considered as providing a general framework which embodies the work of Poole (1970), Henderson (1979), Turnovsky (1978) and others as special cases.

Another problem with Henderson's analysis is that it deals with income stability only. As we have seen, in an open economy external stability is also important and as our analysis in chapter 2 shows, consideration of price stability adds an extra dimension as it were to the problem at hand. Once prices are endogenized though, there is the problem of price expectation to
deal with. However, Craine and Havenner (1981) have shown that within the linear-quadratic framework that are commonly used "the basic forces affecting the instrument choice decision can be analyzed without an explicit specification for price expectations since the distribution of the error terms in a linear model is independent of the predetermined variables" (p.219). To illustrate their point, consider the following reduced form linear model:

\[ (22) \quad Z_t = B\hat{Z}_t + CX_t + e_t \]

where \( Z \) = vector of endogenous variables,
\( \hat{Z} \) = vector of expectations of current endogenous variables formed at beginning of period,
\( X \) = vector of predetermined variables,
\( e \) = vector of reduced-form errors.

If expectations are rational, so that the expectations of the private sector are the conditional mean vector

\[ (23) \quad \hat{Z}_t = E(Z_t/\Omega_{t-1}) = E_{t-1}Z_t \]

where \( \Omega_{t-1} \) is the information set at time \( t-1 \), then (22) and (23) can be solved simultaneously to give

\[ (24) \quad E_{t-1}Z_t = Z_{\text{rational}}^{t/t-1} = (I-B)^{-1}CX_t = \hat{Z}_t \]

If expectations are adaptive, that is

\[ (25) \quad \hat{Z}_t = \lambda Z_{t-1} + (1-\lambda)\hat{Z}_{t-1} \]

then (22) and (25) are solved recursively to give

\[ (26) \quad z_{\text{adaptive}}^{t/t-1} = B^{\infty} \left( \sum_{j=0}^{\infty} (I-\lambda)^j \lambda Z_{t-1-j} \right) + CX_t \]

Thus the two conditional mean vectors are different, but the conditional covariance are the same and is given by

\[ (27) \quad E(Z_t - z_{\text{adaptive}}^{t/t-1})(Z_t - z_{\text{adaptive}}^{t/t-1})' = E(Z_t - z_{\text{rational}}^{t/t-1})(Z_t - z_{\text{rational}}^{t/t-1}) = E(e_t e_t') \]

This is because in a linear model the distribution of the reduced form errors is independent of the exogenous variables. Therefore, defining the variables as deviations from their conditional means allows us to avoid the problem associated with the specification of expectations without any loss of generality. Thus in our own model in the next chapter, we do not model expectations explicitly.
However, there are a few studies on the instrument choice problem which do specify a scheme for the formation of expectations, in particular that of rational expectations à la Muth (1961). In the next section, we therefore look briefly at the meaning and implications of rational expectations, and attempt to spell out our reasons (apart from the one mentioned above) for not using it in our analysis.

1.4 **Rational Expectations**

The basic idea behind the rational expectations hypothesis, is that each economic agent makes optimal use of all the information available to it given the constraints that it faces and its preferences as well as its model of how the economy works. However, the assumptions behind recent theoretical research can be quite controversial, as this hypothesis "usually gets translated into the requirement that expectations are in the model at hand formed in a way that is stochastically consistent with the behaviour of the realized values of the variables in question. (McCallum, 1980, p.717). (16) In other words, as Muth (1961) has argued, rational economic agents have expectations that are unbiased estimates of the actual stochastic process in question. If the expectations were different from the mean value of the true process, the rational economic agent would observe that the expectation were systematically in error and would correct its expectations accordingly. Thus, any errors in expectations will be random and have zero means. The appeal of this hypothesis is that any other expectations scheme will consistently yield systematic expectation errors so that economic agents will ultimately abandon the scheme (see, for example Minford, 1978).

If there are costs in acquiring information, so that expectations adjust only gradually, then during this time of adjustment expectations are biased. However, it is assumed that if the stochastic process changes in one period then economic agents learn of it by the beginning of the next period, so that expectations become unbiased once more (see, for example, Lucas, 1975). The length of time that corresponds to 'the period' is unfortunately not explicitly defined by the proponents of this hypothesis; implicitly it is the amount of time needed for complete learning to take place.
One of the reasons why we do not use rational expectations in our model in chapter 2 is because of the strong information assumptions made in these models (see, for example, Friedman, 1979). It seems that economic agents are not only assumed to be able to analyze the effects of any policy, and monetary policy in particular but are also able to decipher the actual monetary rule pursued by the authorities. The problem here is that economic agents do not necessarily agree with each other regarding the nature and probabilities of various future shocks. Since information is almost always incomplete, economic agents are assumed to form their expectations as if they knew the correct model of how the economy works. It is generally accepted that sometimes people may behave as if a certain abstract economic relationship existed, although they would not describe their behaviour in this way. For instance, people do not talk about indifference curves in describing their consumer choices, although their behaviour could very well be formulated in that way. However, this is not the same thing as saying that people form their expectations as if they had in their own mind a correct model of how the economy works. "In almost all cases, their own description of the way they form their expectations is not one which bears any resemblance to a formulation of this kind. Even if it were the case that economists were agreed on the nature of the correct model of the working of the economy, it would seem extreme to argue that people behave as if they knew it" (Mayes, 1982, p.56). Economic agents do not just have to know this correct model "they must believe in rational expectations theory itself for it to work!" (Buiter, 1977, p.4).

Thus, everybody is a monetarist and draws the same conclusions given the same information, or if there are differences these will average out. However, given that different economic agents react differently, it seems rather strong to simply assume that the average will be the monetarist one (Haberler, 1980, p.833)

Even if we were to use rational expectations, which is one essential component of New Classical Economics, by itself it would not lead to the strong proposition of the stochastic neutrality theorem of Sargent and Wallace (1975) and Sargent (1979). As emphasised on a number of occasions by Tobin (see, for example, Tobin, 1980 a, b) and as Sargent (1979) himself recognises, this proposition depends on this joint assumption of rational expectations and a model
which has neoclassical properties, in particular the assumption of continuous market clearing embodied in the natural rate hypothesis. Sargent and Wallace (1975) and Sargent (1979) have shown that rational expectations is only a necessary condition for this proposition to hold, while Karakitsos and Rustem (1981) have shown formally that the natural rate hypothesis is a sufficient condition for the neutrality proposition to hold. The market clearing assumption is as the word says an assumption; it is not justified by any evidence and allows no room for orders not filled, stocks not sold, trade made at false prices which are phenomena which certainly occur in real life. It is certainly true that prices are set by identifiable agents and are changed only at discrete intervals. (Tobin, 1980 b, p.788)

The study of the monetary instrument problem under rational expectations has been confined to the closed economy case. One of the main conclusions that emerge from the Sargent and Wallace type of models, is that under an interest rate rule the price level is indeterminate. However, the variance of the price level is finite and determinate; so if we are interested in a comparison of the conditional variances of prices in the money and interest rate strategies of conducting monetary policy, a clear criterion exists (Turnovsky, 1980, p 40). If the authorities are concerned with minimizing the expected value of output squared at time t conditional on information at t-1, then the choice of instrument will affect this expected value, which is also influenced by where the disturbances impinge on the system (Dickinson, Driscoll and Ford, 1980). Thus although both kinds of policy are neutral in the Sargent and Wallace type of models, a choice between the interest rate and the money supply is still relevant, as the chosen policy will condition the shocks or the instability perturbing the system. Also a number of studies have shown that active monetary policy can be effective if some of the conditions of rational expectations models are dropped, (see, for example, Phelps and Taylor, 1977, Fischer, 1977, Shiller, 1978, Persson, 1979, Woglom, 1979, Dickinson, Driscoll and Ford, 1980 and Turnovsky 1980 for a closed economy). For an open economy Wirick (1981) has shown that the policy ineffectiveness proposition does not hold.
because the supply function would include, not only price surprises but also the terms of trade. Thus unless we accept all the assumptions of rational expectations models its contributions to the literature on the monetary instrument problem is rather minimal. This explains why we do not analyze its consequences for the analysis is this study.

Before we conclude this introductory chapter, we should perhaps briefly comment on the form that the rest of this study takes. This chapter has revealed that, in the existing literature on the instrument choice problem in an open economy, there is no formal and comprehensive analysis based on the concept that 'domestic' and 'external' monetary policy ought to be analyzed concurrently. The aim of this study, is to provide such an analysis at both the theoretical and empirical levels. The analysis also considers the added objectives of price and external stability to that of income stability, which is the usual objective assumed in the literature. The study is organised as follows:

Chapter 2 provides the theoretical analysis based on the approach mentioned above. Thus, we examine which pegging regimen (involving a pair of intermediate targets for 'domestic' and 'external' monetary policy) best stabilizes an economy perturbed by various domestic and foreign stochastic disturbances. The question of strategy choice is also briefly examined within the same framework. A number of simplifying assumptions are made in order to keep the analysis tractable; most of these assumptions are, however, relaxed in the empirical exercises (in chapter 4). The optimization framework within which these exercises are conducted is described in appendix B, and basically involves the minimisation of an objective function (described in chapter 4) subject to the constraint imposed by the model employed by the policy makers.

Chapter 3 deals with the construction and estimation of an econometric model of the U.K. economy on which these control exercises are based. The theoretical background to the various equations in the model is discussed together with an assessment of the simulation properties of the model.
Chapter 4 then provides the empirical results of the optimization exercises for the pegging regimens of chapter 2, as well as their discretionary counterparts. Thus this chapter takes off as it were where Bryant (1980) leaves the scene, and provides an empirical examination of the various regimens proposed there. The overall conclusion seems to be that among the pegging regimens, one involving a monetary aggregate and foreign reserves peg fares best; however, it is preferable not to adhere rigidly to the optimal rule, but to allow the paths of the intermediate targets to respond to newly available information.

Finally, chapter 5 presents a summary as well as some concluding comments based on the whole study.
NOTES

(1) The monetary instrument problem was first analyzed formally by Poole (1970) for a closed economy. Among the earlier developments of this literature, are Poole and Lieberman (1972), Holbrook and Shapiro (1970), Kareken (1970), Sargent (1971), Moore (1972) and Waud (1972). More recent extensions involve the applications of the techniques of optimal control theory as in Pindyck and Roberts (1974), Turnovsky (1975), Le Roy and Waud (1977), Campbell (1979) and Craine and Havenner (1977, 1981). The other contributors (for the closed economy) include LeRoy and Lindsey (1978) and Driscoll and Ford (1979). It must be made clear at the outset that, in common with the literature, we will throughout this study, refer to the choice between 'money' and interest rates as the monetary instrument or the instrument choice problem, but of course these variables are not regarded as instruments on which the authorities have a firm control but rather as intermediate or proximate targets.

(2) 'External' monetary policy is taken here to refer to the question of optimal foreign exchange market intervention. Among the recent contributors to this literature, are Turnovsky (1976), Fischer (1977), Boyer (1978), Flood (1979), Roper and Turnovsky (1980 a), Weber (1981) and Marston (1982). For an analysis of this problem under rational expectations, see for example Buiter (1977) and Chan (1982).

(3) See Friedman (1975, 1977) for a detailed analysis of an intermediate target strategy; see also Brunner and Meltzer (1967).


(5) This conclusion is also reached by Garbade (1975) and Craine and Havenner (1977). It should be noted though, that instrument choice per se becomes more important as less of the relevant information about the economy is assumed to be available to the authorities.

(6) Under the assumptions above, 'certainty-equivalence' prevails so that the setting that minimizes $E(L)$ is obtained by
simply taking the expected value of the reduced form $Y_t$, setting $E(Y_t) = Y_t^*$ and solving for the value of $S_t^*$ or $M_t^*$.

(7) Poole’s approach has been directed towards the question of whether to use reserves or the interest rate to achieve an intermediate money supply target. This has been examined by, for example, Pierce and Thompson (1972), Friedman (1975, 1977), Parkin (1978), Sivesind and Hurley (1980) and Axilrod and Lindsey (1981). We do not, however, concern ourselves with this problem in this study.

(8) This equivalence between Poole’s combination policy and the policy implied by the kareken, Muench and Wallace (1973) approach for a closed economy, is also demonstrated using the Kalman filter by LeRoy and Waud (1977). Expression (13) is also obtained by Roper and Turnovsky (1980 b, equation (12)) who examine the stabilization of an optimum monetary aggregate within the same IS-LM framework.

(9) Also it is not clear from Poole’s analysis how money and the interest rate could be related deterministically “(see his equation (15)) when all the equations in the model contain random errors.

(10) All these papers, however, deal with closed economies.

(11) The money supply and the exchange rate have also been viewed as alternative intermediate targets (see, for example, Artis and Currie, 1981 and Curre and Karakitsos, 1982).

(12) For a more thorough analysis, see Bryant (1980).

(13) For further details, see chapter 2.

(14) This result is also obtained by Boyer (1978).

(15) It is by the same mechanism that fiscal policy becomes ineffective in the simple Munell-Flemming model, see Mundell (1961, 1968).
(16) McCallum (1980) provides a very useful survey of models and countermodels in the literature. See also the book edited by Lucas and Sargent (1981) which brings together some of the main papers on rational expectations.

(17) The difficult problem of how people form their forecasts when they do not know the true model is still unresolved (see, for example, DeCaino 1979 and Friedman, 1979).

(18) Even in these models, there may be the problem of non-existent or multiple equilibria (see, for example, Taylor, 1977 and McCafferty and Driscoll, 1980). If expectations can be fulfilled along a number of paths besides the one that returns to equilibrium the question is why should people choose the path that is stable. That stable path would prevail only if people know the equilibrium and believe the system will return to it. Without belief that government policy will aim for equilibrium, people have little reason to assume that the equilibrium path is stable.

(19) If adaptive instead of rational expectations are assumed in the Sargent and Wallace type of models then the neutrality proposition does not hold.

(20) The contributions to this literature are Sargent and Wallace (1975), Sargent (1979), Turnovsky (1980) and Dickinson, Driscoll and Ford (1980).
CHAPTER 2

OPTIMAL FINANCIAL POLICIES IN AN OPEN ECONOMY

2.1 Introduction

As we noted in chapter 1, in an open economy the issue of the optimal monetary instrument and that of the optimal foreign exchange market intervention for stabilization purposes, have to be analyzed concurrently, and this is what we propose to do here using the techniques of optimal control theory.

Our aim is to analyze the outcomes of alternative financial regimens within the context of optimal stabilization policies. The financial regimens we examine, involve the pegging of a pair of 'domestic' and 'external' targets. The optimization involves minimizing an objective function incorporating a weighted sum of the variances of output, prices and foreign reserves, subject to a linear dynamic model with additive autoregressive errors.

In order to test the hypothesis that a real balance effect as well as a monetarist price structure tend to favour a money supply policy, we will work with two specifications of a simple model, with model A having some form of Phillips Curve, and model B a monetarist price structure as well as the inclusion of a real balance effect in its absorption function. Both our models are recursive in the manner described by Bryant (1980, p.267), so that we are justified in dichotomising the decision process in two stages. The model can thus be divided into as it were two sub-models, in one of which the intermediate target variables (the money supply \( M_t \), the interest rate \( R_t \), the exchange rate \( E_t \) and foreign reserves \( F_t \)) are treated as though they were exogenous variables. Given values for the ultimate targets, as reflected in the objective function, as well as forecasts for the exogenous variables and expected values of the noises in the system, the upper sub-model can be solved for the policy 'instruments' including the intermediate target variables. The remaining sub-model can then be used to calculate the appropriate levels for the true monetary instruments.
We will also briefly examine the strategy choice question, and the single-stage strategy we put forward, is one where the domestic component of the monetary base \((D_t)\) is used as a direct policy instrument in order to affect the ultimate targets. Because the second stage of the two-stage process is undertaken without any reference to the loss function, comparison of the expected loss in the first stage of the two-stage strategy with that in the single-stage strategy, gives us a clear idea of the relative merits of the two alternative strategies. We need, of course, to assume that the same loss function and the same model are being used in the two cases.

We will proceed in the following order. In the next section we describe model A. (as well as the alternative specification of the price and absorption equations of model B) and then use it in section 2.3 to analyze the various regimens. Section 2.4 presents a comparison of the welfare costs associated with the regimens with some concluding comments in section 2.5.

2.2 The Models

Our basic model will be an extended and modified version of the linear dynamic IS-LM model in Turnovsky (1978); the optimal control techniques used in this chapter are also similar to those in Turnovsky (1978) although the analysis is more complicated in our case as we deal with 'domestic' and 'external' monetary policy simultaneously. Following Turnovsky, we analyse a small open economy with linear behavioural relationships and additive random disturbances. Our absorption function (for model A) as well as the monetary sector are similar to his, but the other behavioural equations are quite different. For instance, it is necessary for our purpose to bring in the exchange rate explicitly.\(^{(5)}\) Also, the price level is endogenized so that price stability becomes an additional objective to those of income stability and external stability considered by Turnovsky. The authorities objective is thus assumed to be that of minimizing

\[
J = \mathbb{E} \left\{ \sum_{t=1}^{T} \left[ w_1 (Y_t - \bar{Y})^2 + w_2 (\Delta P_t - \Delta \bar{P})^2 + w_3 (F_t - \bar{F})^2 \right] \right\}
\]

\((w_i > 0 , i = 1, 2, 3)\)
subject to the model described below. $\bar{y}$, $\Delta \bar{p}$ and $\bar{f}$ are the target values of income, rate of change prices and foreign exchange reserves respectively, and the $w_{i}$'s are the weights attached to each objective. The authorities are thus assumed to minimize the expectation of the $T$ period sum of squared deviations of the objectives from their target values. Although there are objections to such an objective function, which are for instance that it exhibits 'satiation' meaning that it reaches a maximum or a minimum at $\bar{y}$, $\Delta \bar{p}$ and $\bar{f}$, and it is 'symmetric' implying that it assigns the same cost to a positive or a negative deviation, it is still descriptively realistic.

We shall first introduce the following notation for the lag polynomial:

$$h(L) = h_{0} L + h_{1} L^2 + \ldots \quad \text{and} \quad h'(L) = h_{1} L + h_{2} L^2 + \ldots \quad \text{and} \quad h(L) - h_{0} \text{ where } L \text{ is the lag operator, i.e. } L^{n} q_{t} = q_{t-n} (n=1,2,\ldots)$$

We write the equation for the goods market equilibrium as

$$Y_{t} = A_{t} + G_{t} + X_{t} - Z_{t}$$

Real absorption, $A_{t}$, is assumed to be given as a distributed lag function or real income, $Y_{t}$, tax receipts $T_{t}$ (assumed exogenous) and the domestic rate of interest $R_{t}$ thus:

$$A_{t} = a(L)Y_{t} - t(L)T_{t} + p(L)R_{t} + U_{lt} \quad , \quad 0 < a_{0} < 1$$

$$t_{i} > 0$$

$$\rho_{i} < 0 \quad , i=0,1,\ldots$$

The restrictions on the coefficients need no comments except that as Turnovsky points out we can only say unambiguously that $a_{0} > 0$, as the other $a_{i}$'s will depend upon acceleration effects. $U_{lt}$ is an additive random disturbance. Notice here that we are abstracting from any wealth effects, this is because as Turnovsky (1975,1977) has shown, for consistency with the underlying budget constraint of the economy, wealth effects should appear with a one period lag in discrete time models. This implies that omitting wealth effects will not affect our results.
In model B, though, we include a real-balance effect in the absorption function which we write as

\[ a_t = a(L)Y_t - \rho(L)R_t + g(L)M_t + U_{1t} \quad (g_i > 0, i = 0, 1, \ldots) \]

This is done in order to assess the role, if any, of the real balance effect on the monetary instrument problem.

Going back to model A, the import function is expressed as

\[ Z_t = z(L)Y_t + \varepsilon(L)E_t + \mu(L)\Delta P_t + U_{2t} \quad z_i > 0 \]

Imports, \( Z_t \), is thus assumed to be dependent on the past levels of domestic income, the exchange rate \( E_t \), the domestic rate of change of prices proxied here by \( \Delta P_t \), as well as a stochastic disturbance \( U_{2t} \), so that we are ignoring the effects of changes in foreign prices which we take to be fixed.

We formulate the price equation for model A as

\[ P_t = \Omega(L)\Delta Y_t + w(L)\Delta W_t + \lambda(L)\Delta E_t + \nu_t \quad \Omega_i > 0 \]

This says that the rate of change of domestic prices proxied by \( \Delta P_t \) is a function of demand pressures \( \Delta Y_t \) in the domestic goods market proxied by \( \Delta Y_t \), changes in costs proxied by \( \Delta W_t \), as well as competitiveness of domestic products proxied by \( \Delta E_t \). This last effect can be rationalized by assuming that as the domestic price of foreign goods rise due to a depreciation of the exchange rate (an increase in \( E_t \)), domestic producers can increase their prices without suffering a fall in demand, thus \( \lambda_i \) will be positive.

The rate of change of domestic money wages, proxied by \( \Delta W_t \) is assumed to be given by the following Phillips curve relationship:
This says that $\Delta W_t$ depends on the rate of unemployment $U_t$ as well as a distributed lag of $\Delta P_t$.

We endogenise unemployment by the simple approximation:

(7) $U_t = \xi(L) \Delta Y_t + \nu_3t$, $\xi_i < 0$, $i=0,1,...$

Substituting equations (6) and (7) into equation (5) we get

(8) $\Delta P_t = \left[ \Omega(L) + \omega(L) \cdot \beta(L) \cdot \xi(L) \right] \Delta Y_t + \left[ \omega(L) \cdot \chi(L) \right] \Delta P_t + \lambda(L) \Delta E_t + \nu_t$

where the random disturbance $\nu_t$ is a linear combination of $\nu_1t$, $\nu_2t$ and $\nu_3t$.

Making the following substitutions:

$\gamma(L) \equiv \Omega(L) + \omega(L) \cdot \beta(L) \cdot \xi(L)$,

$\pi(L) \equiv \omega(L) \cdot \chi(L)$,

and $\Gamma'(L) Y_t \equiv (\gamma_1 - \gamma_o) Y_{t-1} + (\gamma_2 - \gamma_1) Y_{t-2} + \ldots$

$\Lambda'(L) E_t \equiv (\lambda_1 - \lambda_o) E_{t-1} + (\lambda_2 - \lambda_1) E_{t-2} + \ldots$

We can write equation (8) as

(9) $(1 - \pi(L)) \Delta P_t = \gamma(L) \Delta Y_t + \lambda(L) \Delta E_t + \nu_t$

or as

(10) $(1 - \pi(L)) \Delta P_t = \gamma_o Y_t + \Gamma'(L) Y_t + \lambda_o E_t + \Lambda'(L) E_t + \nu_t$

$\gamma_o > 0$

$\pi_o > 0$

$\lambda_o > 0$
In model B, we have a simple monetarist specification of the price equation thus:

\[(5) \Delta P_t = p(L)\Delta M_t + c(L)\Delta E_t + \nu_t\]

which says simply that the rate of change of prices is a distributed lag function of the rate of change of the money stock, proxied here by $\Delta M_t$, and the rate of change of the exchange rate, proxied by $\Delta E_t$.

Using the following definitions:

\[x'(L)M_t = (p_1 - P_0)M_{t-1} + (p_2 - p_1)M_{t-2} + \ldots, \text{and}\]
\[\Omega'(L)E_t = (c_1 - c_0)E_{t-1} + (c_2 - c_1)E_{t-2} + \ldots,\]
we can write (5) as

\[(10') \Delta P_t = p_0 M_t + x'(L)M_t + c_0 E_t + \Omega'(L)E_t + \nu_t,\]

where $p_0 > 0$

and $c_0 > 0$

We turn now to the monetary sector. Our demand for money function is the standard one found in the literature and is expressed as

\[(11) M^D_t = m(L)Y_t + \delta(L)R_t + \eta_{1t}\]

where $\eta_{1t}$ is a random disturbance. The domestic component of the monetary base, $D_t$, together with the volume of foreign reserves, $F_t$, make up the total monetary base $H_t$:

\[(12) H_t = F_t + D_t\]

Following Turnovsky (1978), we assume that we have a fractional reserve banking system with a required reserve ratio given by $1/\theta$. If we abstract from coins in circulation we can specify the supply of money $M_t$ as

\[(13) M_t = \theta(F_t + D_t) + \eta_{2t}\]

where $\eta_{2t}$ is a stochastic disturbance. This equation represents the lower sub-model where given the optimal path for $M_t$, the path for the instrument $D_t$ can be calculated in the second stage of the two-stage strategy. We could here similarly relate the
market rate of interest $R_t$ to an instrument such as the discount rate. Of course, it is only in a two-stage strategy that the model is broken down into two sub-models; in a single stage strategy, the model is analyzed in its entirety.

The balance of payments, $\text{BOP}_t$, is made up of the sum of the trade balance and capital flows.

\begin{equation}
\text{BOP}_t \equiv \Delta F_t = X_t - Z_t + K_t
\end{equation}

We postulate capital flows to be simply a function of the domestic rate of interest, with the foreign rate of interest assumed to be fixed exogeously:

\begin{equation}
K_t = k \cdot R_t + \phi_{1t} \quad k > 0
\end{equation}

where $\phi_{1t}$ is a random disturbance.

Combining equations (14), (15) and (4) with exports, $X_t$, exogenously given, we get

\begin{equation}
F_t - F_{t-1} = X_t - z(L)Y_t - \varepsilon(L)E_t - \mu(L)\Delta P_t + k \cdot R_t + \phi_t
\end{equation}

The additive stochastic disturbance, $\phi_t$, includes shifts in capital flows ($\phi_{1t}$) as well as in imports and exports.

We now introduce the government budget restraint

\begin{equation}
\Delta D_t + \frac{\Delta B_t}{R_t} = P_t \cdot (G_t - T_t) + B_{t-1}
\end{equation}

which states that the authorities can finance their deficit $P_t(G_t - T_t)$ together with interest owned on outstanding debt, $B_{t-1}$, by issuing more government bonds, which we assume are perpetuities, or by increasing the domestic component of the monetary base $D_t$. This constraint in fact determines the amount of open market operations required to keep the policy variables at their targeted values.

We are now in a position to summarize our models in terms of the following equations:
Equation (18) is simply the IS curve obtained by substituting equations (3) and (4) into equation (2). The disturbance \( U_t \) includes \( (U_{1t} + U_{2t}) \) plus the stochastic components of exports. Money market equilibrium is given by equation (20) where the disturbance \( \eta_t \) is given by \( (\eta_{1t} - \eta_{2t}) \). Equations (19) and (19') are just the price equations for model A and B respectively and equation (21), the balance of payments equation.

All the additive stochastic disturbances \( (U_t, \eta_t, \nu_t \) and \( \phi_t \) are assumed to follow the general autoregressive process:

\[
(22) \ x_t = \zeta'(L)x_t + e_{xt} \quad \quad (x_t = u_t, \ eta_t, \ nu_t, \ phi_t)
\]

with \( e_{xt} \) being independently distributed over time with mean zero and variance \( \sigma^2_x \). The disturbances will thus have conditional finite variance of \( \sigma^2_u, \sigma^2_\eta, \sigma^2_\nu \) and \( \sigma^2_\phi \) respectively.

We have assumed that the authorities have three targets \( \tilde{Y}, \tilde{\Delta P} \) and \( \tilde{F} \) and that the random disturbances are additive. Our loss function (equation (1)) also implies that control of the instruments is assumed to be costless. In this setting, it is possible for the authorities to achieve their objective by using three instruments only. We will analyze a system of flexible exchange rates characterized by \( \Delta F_t = 0 \) as well as one of fixed exchange rates characterized by \( \Delta E_t = 0 \). Under a fixed exchange rate system, the 'instruments' available to the authorities are:
(i) Government expenditure, $G_t$;  
(ii) the exchange rate, $E_t$; and  
(iii) $D_t$, $R_t$ or $M_t$ depending on which monetary instrument is used as well as whether we are using a single-stage or a two-stage strategy. (see below)

Under a flexible exchange rate system, the balance of payments is in equilibrium so that only (i) and (iii) above are needed.

The model has five endogenous variables, $Y_t$, $\Delta P_t$, $F_t$ or $E_t$, $M_t$ and $R_t$ which are jointly determined and their solutions can be written as:

\[
Y_t = Y(D_t, G_t, E_t or F_t, Q_t, x_{1t})
\]

\[
P_t = P(D_t, G_t, E_t or F_t, Q_t, x_{2t})
\]

\[
F_t = F(D_t, G_t, E_t or F_t, Q_t, x_{3t})
\]

\[
M_t = M(D_t, G_t, E_t or F_t, Q_t, x_{4t})
\]

\[
R_t = R(D_t, G_t, E_t or F_t, Q_t, x_{5t})
\]

where $Q_t$ stands for all exogenous and predetermined variables and $x_{1t}, \ldots, x_{5t}$ are linear combinations of the random disturbances $\nu_t$, $\nu_t$, $\nu_t$, $\nu_t$ and $\nu_t$. In order to control $R_t$ or $M_t$ exactly, we must assume that the authorities know $x_{4t}$ and $x_{5t}$ but not $x_{1t}$, $x_{2t}$ and $x_{3t}$ when making their policy decisions. If the latter three disturbances were known at the same time, the policy actions could have been directed straight at $Y_t$, $\Delta P_t$ and $F_t$ without any need for proximate monetary targets as is indeed the case in regimen (v). When we use $M_t$ or $R_t$ as instruments below, we are assuming that the authorities can use open market operations to offset the stochastic disturbances in $x_{4t}$ and $x_{5t}$.

We will be concerned with the following regimens:

(I) Fixed Exchange Rate, Interest Rate policy;
Although we consider three monetary instruments, namely $R_t$, $M_t$, and $D_t$, we do not consider all three as parallel variables. Under a two-stage strategy, we consider the use of $R_t$ against $M_t$ but only compare these against $D_t$ in examining the issue of two-stage versus single-stage strategy. Thus we examine the monetary instrument problem with the aid of regimens (I) to (IV), and then compare these four regimens with regimen (V) in order to shed some light on the question of strategy choice. Thus we will investigate which of these regimens (I) to (V) will best stabilize an economy perturbed by various domestic and foreign stochastic disturbances. For these regimens to be attainable, we need of course to assume that the authorities have the power to neutralize the monetary consequences of imbalances in the balance of payments including international capital flows. The latter implies that we are making the assumption of imperfect mobility of capital. We will examine the five regimens in turn so that the next section begins with an analysis of regimen (I).

**NOTATION**

- $A_t = \text{real absorption (private domestic expenditure)}$
- $B_t = \text{domestic bonds}$
- $BOP_t = \text{balance of payments}$
- $D_t = \text{domestic component of monetary base}$
- $E_t = \text{exchange rate, defined as price of foreign exchange in terms of domestic currency}$
- $F_t = \text{foreign exchange reserves}$
- $G_t = \text{real government expenditure}$
- $H_t = \text{total monetary base}$
- $K_t = \text{capital flows}$
\[ M^D_t = \text{demand for money} \]
\[ M_t = \text{stock of money} \]
\[ \Delta P_t = \text{proxy for domestic rate of change of prices} \]
\[ R_t = \text{domestic rate of interest} \]
\[ T_t = \text{real domestic taxes (assumed to be exogenous)} \]
\[ U_t = \text{rate of unemployment} \]
\[ \Delta W_t = \text{proxy for domestic rate of change of wages} \]
\[ X_t = \text{exports (assumed to be exogenous)} \]
\[ Y_t = \text{real domestic income} \]
\[ Z_t = \text{real imports} \]

2.3.1 Regimen (I): Fixed exchange rate, Interest rate Policy

Under a system of fixed exchange rate, the exchange rate \( E_t \) becomes an exogenous policy instrument which is kept unchanged over some time periods by the judicious use of exchange market intervention actions. In this case, the path of foreign reserves \( F_t \) is determined endogenously.

Given acquiescence from the foreign central bank and enough foreign reserves, it is technically possible for the domestic central bank to precisely peg the exchange rate. The procedure for an interest rate peg is basically similar to that for an exchange rate peg, this time by the use of domestic open market operations. In this case, the money supply \( M_t \) becomes endogenously determined. Once more, it is within the technical power of the central bank to peg the interest rate precisely. Obviously there are many practical problems, and even if it is feasible, the central bank must be willing to allow the money supply and foreign reserves to fluctuate without limit. This in itself may turn out to be intolerable. However, in this chapter, we will abstract from these difficulties\(^{15}\), and assume that the authorities can peg both \( E_t \) and \( R_t \) at their desired levels so that we can regard them as 'direct policy variables' or instruments.
Using the notation defined earlier, we can rewrite equation (18) as

\[(23) \quad (1-a_o+z_o)Y_t = \left[a'(L) - z'(L)\right]Y_t - t(L)T_t + \rho_o R_t + \rho'(L)R_t - \epsilon_o E_t \]
\[-\epsilon'(L)E_t - \mu_o \Delta \rho_t - \mu'(L) \Delta P_t + x_t + G_t + U_t\]

and equation (19) as

\[(24) \quad (1-n_o) \Delta P_t = \pi'(L) \Delta P_t + \gamma_o Y_t + \gamma'(L)Y_t + \lambda_o E_t + \lambda'(L)E_t + \nu_t\]

The third reduced form equation representing the balance of payments (eq. (21)) becomes

\[(25) \quad F_t - F_{t-1} = X_t - z_o Y_t - z '(L)Y_t - \epsilon E_t - \epsilon'(L)E_t - \mu_o \Delta \rho_t \]
\[\quad - \mu'(L) \Delta P_t + k R_t + \phi_t\]

The optimisation problem can now be formulated as to choose \(R_t, G_t\), and \(E_t\) so as to minimize

\[
E \left\{ \sum_{t=1}^{T} \left[ w_1 (Y_t - \bar{Y})^2 + w_2 (\Delta P_t - \bar{\Delta P})^2 + w_3 (F_t - \bar{F})^2 \right] \right\}
\]

subject to equations (23), (24) and (25).

In the literature on control theory, it is well known that "as long as (a) all coefficients are deterministic, (b) there are as many instruments as targets, and (c) control is costless", the optimal policy is to choose \(R_t, G_t\), and \(E_t\) "so as to completely destroy the autoregressive structure of the system conditional on information available at time \(t\)". (Turnovsky, 1978, p.140). Setting \(Y_t = \bar{Y}, \Delta P_t = \bar{\Delta P}, F_t = \bar{F}, U_t = \bar{U}_t, V_t = \bar{V}_t, \eta_t = \bar{\eta}_t\) and \(\phi_t = \bar{\phi}_t\) in equations (23), (24) and (25) we get:

\[
\begin{bmatrix}
\rho_o & 1 & -\epsilon_o \\
0 & 0 & \lambda_o \\
k & 0 & -\epsilon_o \\
\end{bmatrix}
\begin{bmatrix}
R_t \\
G_t \\
E_t \\
\end{bmatrix}
= 
\begin{bmatrix}
\end{bmatrix}
\]
This gives us the optimal policies in terms of $\bar{Y}$, $\Delta \bar{P}$, $\bar{F}$, the expected values of the random disturbances, as well as the lags in the system. We now substitute the optimal policies given by (26) into equations (23), (24) and (25) to derive the deviations of the targets $Y_t$, $AP_t$ and $F_t$ about their respective desired values $\bar{Y}$, $\Delta \bar{P}$ and $\bar{F}$. This gives us the following equations, where we have dropped the time subscript '0' on the first period parameters.

(27) \((1-a+z)(Y_t-\bar{Y}) + \mu (\Delta P_t - \Delta \bar{P}) = (U_t - \bar{U})\)

(28) \((1-\pi)(\Delta P_t - \Delta \bar{P}) - \gamma(Y_t - \bar{Y}) = (\nu_t - \bar{\nu})\)

(29) \((F_t - \bar{F}) + z(Y_t - \bar{Y}) + \mu (\Delta P_t - \Delta \bar{P}) = (\phi_t - \bar{\phi})\)

Multiplying equation (27) by \((1-\pi)\) and equation (28) by \(\mu\), we get

(30) \((1-a+z)(1-\pi)(Y_t-\bar{Y}) + (1-\pi) \mu (\Delta P_t - \Delta \bar{P}) = (1-\pi)(U_t - \bar{U})\)

(31) \((1-\pi) \mu(\Delta P_t - \Delta \bar{P}) - \mu \gamma(Y_t - \bar{Y}) = \mu(\nu_t - \bar{\nu})\)

We now subtract equation (31) from equation (30) to get

(32) \[ (1-a+z)(1-\pi) + \mu \gamma(Y_t - \bar{Y}) = (1-\pi)(U_t - \bar{U}) - \mu(\nu_t - \bar{\nu}) \]

so that

(33) \(Y_t - \bar{Y} = \frac{1}{(1-\pi)(1-a+z+\mu \gamma)} \left[ (1-\pi)(U_t - \bar{U}) - \mu(\nu_t - \bar{\nu}) \right] \)
The resulting path for \( (\Delta P_t - \Delta \bar{P}) \) can then be written as

\[
(34) \quad \Delta P_t - \Delta \bar{P} = (\nu_t - \bar{\nu}_t) + \frac{\gamma}{(1-\pi)[1-a+z+\mu_Y]}(1-\pi)(U_t - \bar{U}_t) - \mu(\nu_t - \bar{\nu}_t)
\]

which after manipulation yields

\[
(35) \quad \Delta P_t - \Delta \bar{P} = \frac{\gamma}{(1-\pi)(1-a+z+\mu_Y)}(U_t - \bar{U}_t) + \frac{(1-a+z)}{(1-\pi)(1-a+z+\mu_Y)}(\nu_t - \bar{\nu}_t)
\]

Subtracting equation (29) from equation (27), we get

\[
(36) \quad (1-a)(Y_t - \bar{Y}) - (F_t - \bar{F}) = (U_t - \bar{U}_t) - (\phi_t - \bar{\phi}_t)
\]

We can thus write the resulting path for \( (F_t - \bar{F}) \) as

\[
(37) \quad (F_t - \bar{F}) = (\phi_t - \bar{\phi}_t) - (U_t - \bar{U}_t) + \frac{(1-a)}{(1-\pi)(1-a+z+\mu_Y)}[(1-\pi)(U_t - \bar{U}_t) - (\nu_t - \bar{\nu}_t)]
\]

which we rearrange to give

\[
(38) \quad (F_t - \bar{F}) = (\phi_t - \bar{\phi}_t) - \frac{\mu(1-a)}{(1-\pi)(1-a+z+\mu_Y)}(\nu_t - \bar{\nu}_t) - \frac{(z + \mu_Y \frac{1-\pi}{1-\pi})}{(1-a+z+\mu_Y)}(U_t - \bar{U}_t)
\]

As we can see from equations (33), (35) and (38) \( Y_t, \Delta P_t \) and \( F_t \) will fluctuate about \( \bar{Y}, \Delta \bar{P} \) and \( \bar{F} \) respectfully in each period. Thus \( E(Y_t - \bar{Y})^2, E(\Delta P_t - \Delta \bar{P})^2 \) and \( E(F_t - \bar{F})^2 \) will all be constant in each period, so that we can write the welfare costs of having a fixed exchange rate and an interest rate policy (regimen (I)) as

\[
(39) \quad T[w_1 \sigma_y^2 + w_2 \sigma_p^2 + w_3 \sigma_F^2]
\]

where \( \sigma_y^2, \sigma_p^2 \) and \( \sigma_F^2 \) are the per unit conditional variances of income, foreign exchange reserves and the change in prices respectively. We delay the calculations of these variances until we have analyzed the other regimens.
2.3.2 Regimen (II): Fixed Exchange Rate, Money Supply Policy.

In this sub-section we keep the assumption of a fixed exchange rate system, but assume that domestic monetary policy takes the form of a money stock target. Here, domestic open market operations are used to keep the money supply $M_t$ on target so that the interest rate $R_t$ is determined endogenously. So we have to eliminate $R_t$ from the system using (20) so that the quasi-reduced form for $Y_t$ can now be written (after some manipulation) as

$$\frac{\rho_o}{\delta_o} M_t + G_t - \varepsilon_o E_t = \left[1 - a_o + z_o + m_o \rho_o \right] Y_t - \left[ a'(L) - z'(L) - \rho_o m'(L) \right] Y_t$$

$$+ t(L) T_t + \left[ - \frac{\rho_o}{\delta_o} \phi(L) - \rho'(L) \right] R_t + \varepsilon'(L) E_t$$

$$+ \mu_o \Delta P_t + \mu'(L) \Delta P_t - X_t - U_t + \frac{\rho_o}{\delta_o} \eta_t$$

The quasi-reduced form for prices is still given by (24) and the balance of payments equation after substituting for $R_t$ becomes

$$\frac{k}{\delta_o} M_t - \varepsilon_o E_t = F_t - F_{-1} - X_t + \left[ z_o + k m_o \right] Y_t + \varepsilon'(L) E_t$$

$$+ \left[ z'(L) + k m'(L) \right] Y_t + \mu_o \Delta P_t + \mu'(L) \Delta P_t$$

$$+ \frac{k}{\delta_o} \phi(L) R_t - \phi_t + \frac{k}{\delta_o} \eta_t$$

In this regimen, the optimization problem is to choose $M_t$, $G_t$ and $E_t$ so as to minimize the loss function (equation (1)) subject to (40), (41) and (24). The resulting paths for $(Y_t - \bar{Y})$, $(\Delta P_t - \Delta \bar{P})$ and $(F_t - \bar{F})$ are given by the following equations:

$$Y_t - \bar{Y} = \frac{1}{(1-\pi)} \left[ (1-\pi)(U_t - \bar{U}) + (1-\pi) \theta(\eta_t - \bar{\eta}) - \mu(\bar{v}_t - \bar{\bar{v}}_t) \right]$$

$$\Delta P_t - \Delta \bar{P} = \frac{\gamma}{(1-\pi)} \left[ (U_t - \bar{U}) - \theta(\eta_t - \bar{\eta}_t) \right] + \frac{1 - a + z + \theta}{(1-\pi)} (\bar{v}_t - \bar{\bar{v}}_t)$$

(43) $(\Delta P_t - \Delta \bar{P}) = \frac{\gamma}{(1-\pi)} \left[ (U_t - \bar{U}) - \theta(\eta_t - \bar{\eta}_t) \right] + \frac{1 - a + z + \theta}{(1-\pi)} (\bar{v}_t - \bar{\bar{v}}_t)$
and

\[ (44) \quad (F_t - F) = (\phi_t - \phi_t) - \left[ z + k + \frac{\mu Y}{1 - \pi} \right] (U_t - U_t) - \mu_0 \left[ 1 - a + \frac{(\rho - k)}{\delta} \right] (\nu_t - \nu_t) \]

\[ + \left[ \frac{(\rho(z + \mu Y) - k(1 - a) + \frac{\mu Y}{1 - \pi})}{\delta} \right] (\eta_t - \eta_t) \]

where \( C = [1 - a + z + \frac{m_0(\rho - k)}{\delta}] \)

As in the previous subsection (42), (43), and (44) tell us that \( Y_t, \Delta P_t \) and \( F_t \) will fluctuate about their desired values in each period.

2.3.3 Regimen (III): Flexible Exchange Rate, Money Supply Policy

We turn now to the analysis of a flexible exchange rate system and in this section we keep the assumption of a money stock target as the specification of domestic monetary policy. The flexible exchange rate system is characterised by

\[ (45) \quad F_t - F_{t-1} = 0 \]

which states simply that the balance of payments is equal to zero. The exchange rate becomes endogenous and will adjust so as to clear the foreign exchange market. Thus from equation (21), we can write

\[ (46) \quad -\varepsilon_0 E_t = -X_t + z Y_t + z'(L)Y_t + \varepsilon'(L)E_t + \mu_0 \Delta P_t + \mu'(L)\Delta P_t \]

\[ -kR_t = \phi_t \]

Using (46) and eliminating \( R_t \) in the same way as in the last subsection, the quasi-reduced form for \( Y_t \) becomes

\[ (47) \quad G_t + \frac{(\rho_0 - k)M_t}{\delta_0} = \left[ 1 - a_0 + \frac{m_0(\rho_0 - k)}{\delta_0} \right] Y_t + \left[ (\rho_0 - k)m'(L) - a'(L) \right] Y_t \]

\[ + \left[ (\rho_0 - k)\delta'(L) - \rho'(L) \right] R_t + t(L)T_t + \phi_t + \frac{(\rho_0 - k)\gamma_t}{\delta_0} \]

The quasi reduced form for prices is now much more complicated and can be written as
Note that since we now have a system of fixed foreign exchange reserves, we are left with only two targets namely income and price stability. Hence we need only two instruments which in this regimen are $G_t$ and $M_t$. So the optimization problem becomes simply to choose $G_t$ and $M_t$ so as to minimize the loss function subject to equations (47) and (48). The resulting paths for income and prices are given by

$$ \text{(49)} \quad (Y_t - \bar{Y}) = \frac{1}{Q} \left[ (U_t - \bar{U}_t) - (\phi_t - \bar{\phi}_t) - (\rho-k) (\eta_t - \bar{\eta}_t) \right] $$

and

$$ \text{(50)} \quad (\Delta P_t - \bar{P}) = \frac{\varepsilon}{W} \left( \Delta Y_t - \bar{Y}_t \right) - \lambda \left( z + k \frac{\Delta Y_t}{\lambda} \right) \left( U_t - \bar{U}_t \right) 
+ \frac{\lambda}{W} \left( \frac{1-a+z+ \frac{m \rho-k}{\lambda}}{\lambda} \right) \left( \phi_t - \bar{\phi}_t \right) 
+ \frac{\lambda}{W} \left[ \frac{\rho(z- \frac{\varepsilon}{\lambda}) - k \left( 1-a+z- \frac{\varepsilon}{\lambda} \right)}{\frac{\lambda}{\delta}} \right] \left( \eta_t - \bar{\eta}_t \right) $$

where

$$ Q \equiv \left[ 1-a+m(\rho-k) \right] $$

and

$$ W \equiv \mu \lambda + \varepsilon (1-\pi) $$

From equations (49) and (50) we can calculate $\sigma^2_Y$ and $\sigma^2_P$ to get the welfare costs associated with this regimen.

2.3.4 **Regimen(IV): Flexible Exchange Rate, Interest Rate Policy.**

In this sub-section we examine yet another possible pegging
regimen, namely that of foreign reserves and the interest rate. The quasi-reduced forms for $Y_t$ and $P_t$ are given by

\[(51) \quad (\rho - k)R_t + G_t = (1 - \alpha_o)Y_t - a'(L)Y_t + t(L)T_t - \rho'(L)R_t - U_t + \phi_t \]

and

\[(52) \quad k R_t = \left[ u_0 + \frac{\xi_0 (1 - \pi_o)}{\lambda_o} \right] \Delta P_t + \left[ u'(L) - \frac{\xi_0 \pi'(L)}{\lambda_o} \right] \Delta P_t - X_t + \left[ z_0 - \frac{\xi_0 Y_0}{\lambda_o} \right] Y_t + \left[ \delta_0 - \frac{\xi_0 \pi Y_0}{\lambda_o} \right] E_t - \frac{\xi_0 \pi}{\lambda_o} \nu_t - \phi_t \]

The optimization problem is now to choose $R_t$ and $G_t$ so as to minimize the loss function subject to equations (51) and (52). The resulting optimal path for income is given by

\[(53) \quad (Y_t - \bar{Y}) = \frac{1}{1 - \alpha} \left[ (U_t - \bar{U}) - (\phi_t - \bar{\phi}) \right] \]

and that for prices by

\[(54) \quad (\Delta P_t - \Delta \bar{P}) = \frac{\xi (\nu_t - \bar{\nu}) + (\xi Y - z\lambda)}{1 - \alpha} \left[ (U_t - \bar{U}) + \frac{\lambda (1 + \alpha - \xi Y / \lambda)}{(1 - \alpha)\lambda} (\phi_t - \bar{\phi}) \right] \]

where once more

\[W = \mu Y + \xi (1 - \pi)\]

Thus $Y_t$ and $\Delta P_t$ will fluctuate about $\bar{Y}$ and $\Delta \bar{P}$ in each period so that $\sigma^2_Y$ and $\sigma^2_P$ can be obtained from (53) and (54).

2.3.5 Regimen (V): Fixed Exchange Rate, Monetary Base Policy

Up to now we have assumed a two-stage strategy for conducting monetary policy. We turn now to a regimen that employs a single-stage strategy. We will only consider the case of a fixed exchange rate as in our simple models, a monetary base policy under a flexible exchange rate system. In this case both $R_t$ and $M_t$ are endogenously determined so that the money market equilibrium condition is now written as

\[(55) \quad \theta (F_t + D_t) = m(L) Y_t + \delta(L) R_t + \eta_t \]
Using (55), and substituting for $R_t$ the goods market equilibrium condition can be rewritten as

\begin{align*}
(56) \quad & [1-a_0 + z_0 + m_0 \frac{\rho_0}{\sigma_0}] Y_t = [a'(L)-z'(L)- \frac{\rho_0}{\sigma_0} m'(L)] Y_t - t(L) T_t \\
& + \left[ \frac{\rho'(L)-\rho_0}{\sigma_0} \delta'(L) \right] R_t - \varepsilon_0 E_t - \varepsilon'(L) E_t \\
& - \mu_0 \Delta P_t - \mu'(L) \Delta P_t + X_t + G_t + \theta \frac{\rho_0}{\sigma_0} F_t \\
& + \theta \frac{\rho_0}{\sigma_0} D_t + U_t - \frac{\rho_0}{\sigma_0} \eta_t
\end{align*}

The price equation is the same as in sub-section 2.3.1 and 2.3.2 and is given by equation (24), and the balance of payments equation can now be written as

\begin{align*}
(51) \quad & \frac{k \theta}{\sigma_0} D_t - \varepsilon_0 E_t = (1- \frac{k \theta}{\sigma_0}) F_t - F_{t-1} - X_t + [z_0 + \frac{k}{\sigma_0} m_0] Y_t \\
& + \left[ \frac{z'(L)}{\sigma_0} + \frac{k}{\sigma_0} m'(L) \right] Y_t + \varepsilon'(L) E_t + \mu_0 \Delta P_t \\
& + \mu'(L) \Delta P_t + \frac{k}{\sigma_0} \delta'(L) R_t - \phi_t + \frac{k}{\sigma_0} \eta_t
\end{align*}

The optimization problem is therefore to choose $D_t, G_t$ and $E_t$ so as to minimize the loss function subject to equations (56) and (24). The resulting paths for income, price, and foreign reserves are as follows:

\begin{align*}
(58) \quad (Y_t - \bar{Y}) = & \frac{1}{\text{DEN}} \left\{ -\mu \left[ 1+\theta(\rho-k) \right] (\nu_t - \bar{\nu}_t) + (1- \frac{k \theta}{\sigma_0}) (1-\pi)(U_t - \bar{U}_t) \\
& + \frac{\theta \rho(1-\pi)(\phi_t - \bar{\phi}_t) - \mu(1-\pi) \rho(\eta_t - \bar{\eta}_t)}{\sigma_0} \right\}
\end{align*}

\begin{align*}
(59) \quad (\Delta P_t - \Delta P) = & \frac{1}{\text{DEN}} \left\{ \gamma \left(1- \frac{k \theta}{\sigma_0} \right) (U_t - \bar{U}_t) + \frac{\gamma \theta \rho (\phi_t - \bar{\phi}_t) - \gamma \rho (\eta_t - \bar{\eta}_t)}{\sigma_0} \\
& + \left[ (1-a + z + m \rho) + \theta z (\rho-k) - k \theta (1-a) \right] (\nu_t - \bar{\nu}_t) \right\}
\end{align*}

and

\begin{align*}
(60) \quad (F_t - \bar{F}) = & \frac{1}{\text{DEN}} \left\{ -\left[ \frac{z + m k + \mu y}{\sigma_0} \right](1-\pi)(U_t - \bar{U}_t) + (1-\pi)C (\phi_t - \bar{\phi}_t) \\
& + (1-\pi) \left[ (\rho-k)^{(\mu y + z) - k(1-a)} (\eta_t - \bar{\eta}_t) \\
& - \mu(1-a + m (\rho-k) (\nu_t - \bar{\nu}_t) \right\}
\end{align*}
where

$$\text{DEN} \equiv (1 - \pi) \left[ \frac{\theta (\rho - k)}{\delta} (z + \mu y) + \frac{(C - k\theta (1 - a))}{\delta} \right]$$

and as before

$$C \equiv \left[ \frac{1 - a + z + m \rho + \mu y}{\delta} \right] \frac{1}{1 - \pi}$$

From equations (58), (59) and (60) we can calculate the conditional covariances \( \sigma_y^2 \), \( \sigma_p^2 \) and \( \sigma_F^2 \) to get the welfare costs of this regimen.
2.4.1 Choosing The Optimal Regimen.

Before we go on to compare the welfare costs of the different regimens, it will be instructive to examine the effects of the various instruments on the targets. The results of model A. are in table 1.A. and those for model B. in table 1.B. We note at the outset that for model B. we need to impose the restriction that \([1-a_o-g_o m_o] > 0\) so as not to get perverse results- a quite plausible assumption given the low value of \(g_o\) commonly observed.

As expected in both models an increase in government expenditure, \(G_t\), will increase output under any of the five regimens. In model A. it will also unambiguously increase prices under a fixed exchange rate whereas the effect under a flexible exchange rate is indeterminate. This is because in the latter case the effect on the exchange rate, \(E_t\), is ambiguous. This can be explained in the following way: an increase in \(G_t\) brings about an increase in imports, \(Z_t\), which will tend to bring \(E_t\) up (depreciation). However, the increase in \(G_t\), through an increase in the transactions demand for money also puts upward pressure on the domestic rate of interest, \(R_t\), which will cause an incipient capital inflow which in turn will tend to bring \(E_t\) down. Since \(\Delta P_t\) and \(E_t\) are directly related, the net effect on \(E_t\) and hence prices is thus indeterminate. In so far as the price equation in model B. has an \(E_t\) argument, the same comments apply in the latter as well. Note, however, that the inclusion of the money supply in this equation implies that under a fixed exchange rate and a money supply policy (regimen (II)), \(G_t\) has no effect on \(\Delta P_t\) as expected. Under a fixed exchange rate and monetary base policy, an increase in \(G_t\) has two opposing effects on \(\Delta P_t\). The ensuing increase in imports brings the level of foreign reserves, \(F_t\), down. Since \(F_t\) is a component of total
### TABLE 1A

<table>
<thead>
<tr>
<th>REGIMEN</th>
<th>$\delta Y_e/\delta G_e$</th>
<th>$\delta Y_e/\delta R_e$</th>
<th>$\delta Y_e/\delta M_e$</th>
<th>$\delta Y_e/\delta D_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I)</td>
<td>$\frac{1}{A} &gt; 0$</td>
<td>$-\frac{\epsilon_e - (\mu_e \lambda_e)/1 - \pi_e}{A} &gt; 0$</td>
<td>$\frac{\rho_e}{A} &lt; 0$</td>
<td>$-$</td>
</tr>
<tr>
<td>(II)</td>
<td>$\frac{1}{C} &gt; 0$</td>
<td>$-\frac{\epsilon_e - (\mu_e \lambda_e)/1 - \pi_e}{C} &gt; 0$</td>
<td>$-$</td>
<td>$\frac{\rho_e}{\delta_e} &gt; 0$</td>
</tr>
<tr>
<td>(III)</td>
<td>$\frac{1}{Q} &gt; 0$</td>
<td>$-$</td>
<td>$-$</td>
<td>$\frac{(\rho_e - k)/\delta_e}{Q} &gt; 0$</td>
</tr>
<tr>
<td>(IV)</td>
<td>$\frac{1}{(1-a_e)} &gt; 0$</td>
<td>$-$</td>
<td>$\frac{(\rho_e - k)}{(1-a_e)} &lt; 0$</td>
<td>$-$</td>
</tr>
<tr>
<td>(V)</td>
<td>$\frac{(1-(\kappa\theta/\delta_e)(1-\pi_e))}{(DEN)} &gt; 0$</td>
<td>$-\left[ \epsilon_e (1-\pi_e) + \mu_e \lambda_e \right]/1 + \frac{\rho_e}{(1-a_e)} &gt; 0$</td>
<td>$-$</td>
<td>$\frac{(1-\pi_e)\theta\rho_e/\delta_e}{(DEN)} &gt; 0$</td>
</tr>
</tbody>
</table>

* iff $| \epsilon_e + (\mu_e \lambda_e)/(1-\pi_e) | < 0$.

$A \equiv [1-a_e + z_e + (\mu_e \gamma_e)/(1-\pi_e)] > 0$ , $C \equiv [1-a_e + z_e + m_e \rho_e/\delta_e + (\mu_e \gamma_e)/(1-\pi_e)] > 0$

$Q \equiv [1-a_e + m_e (\rho_e - k)/\delta_e] > 0$ , $\text{(DEN)} \equiv (1-\pi_e)\left[ \theta (\rho_e - k)/(1/\delta_e) + z_e + (\mu_e \gamma_e)/(1-\pi_e) + c - k\theta(1-a_e)/\delta_e \right] > 0$
<table>
<thead>
<tr>
<th>REGIMEN</th>
<th>(\delta (P_t)/\delta S_t)</th>
<th>(\delta (P_t)/\delta R_t)</th>
<th>(\delta (P_t)/\delta M_t)</th>
<th>(\delta (P_t)/\delta D_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I)</td>
<td>(\frac{\lambda_a z_a - \varepsilon_a Y_a}{(1 - \pi_a)} &gt; 0)</td>
<td>(\frac{\rho_a Y_a}{(1 - \pi_a)} &lt; 0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(II)</td>
<td>(\frac{\lambda_a (1 - a_o + z_o + m_o \rho_o / \delta_o)}{(1 - \pi_a)} &gt; 0)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(III)</td>
<td>(- \frac{\varepsilon_o Y_o - \lambda_o z_o}{(1 - \pi_o)} &gt; 0)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(IV)</td>
<td>(- \frac{\varepsilon_o Y_o (1 - k \delta_o / \delta_o)}{(1 - \pi_o)} &gt; 0)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(V)</td>
<td>(- \frac{\varepsilon_o Y_o (1 - k \theta / \delta_o)}{(1 - \pi_o)} &gt; 0)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>REGIMEN</th>
<th>(\delta F_t/\delta S_t)</th>
<th>(\delta F_t/\delta R_t)</th>
<th>(\delta F_t/\delta M_t)</th>
<th>(\delta F_t/\delta D_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I)</td>
<td>(- \frac{\varepsilon_o Y_o (1 - \pi_o)}{A} &lt; 0)</td>
<td>(\frac{a_o + m_o (\rho_o - k) / \delta_o}{(1 - \pi_a)} \geq 0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(II)</td>
<td>(- \frac{\varepsilon_o \mu_o Y_o (1 - \pi_o)}{C} \geq 0)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(V)</td>
<td>(- \frac{\varepsilon_o (1 - \pi_o) + \mu_o \lambda_o}{(1 - \pi_o)} \geq 0)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**TABLE 1B**

<table>
<thead>
<tr>
<th>REGIMEN</th>
<th>$\frac{\delta Y_L}{\delta G_L}$</th>
<th>$\frac{\delta Y_L}{\delta E_L}$</th>
<th>$\frac{\delta Y_L}{\delta R_L}$</th>
<th>$\frac{\delta Y_L}{\delta M_L}$</th>
<th>$\frac{\delta Y_L}{\delta D_L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I)</td>
<td>$\frac{1}{J} &gt; 0^1$</td>
<td>$-\frac{(e_0 +e_0 c_o)}{J}$ &gt; 0$^{1/2}$</td>
<td>$\frac{\rho_0 +g_0 \delta_e -\mu_0 p_0 \delta_e}{J}$ &gt; 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(II)</td>
<td>$\frac{1}{N} &gt; 0$</td>
<td>$-\frac{(e_0 +e_0 c_o)}{N}$ &gt; 0$^2$</td>
<td></td>
<td>$\frac{(g_0 +(\rho_0 /\delta_e) -\mu_0 p_0)}{N}$ &gt; 0</td>
<td></td>
</tr>
<tr>
<td>(III)</td>
<td>$\frac{1}{S} &gt; 0$</td>
<td></td>
<td></td>
<td>$\frac{[g_0 +(\rho_0 -k)/\delta_e]}{S}$ &gt; 0</td>
<td></td>
</tr>
<tr>
<td>(IV)</td>
<td>$\frac{1}{[1-a_0 -g_0 m_o]} &gt; 0^3$</td>
<td></td>
<td></td>
<td>$\frac{(\rho_0 -k+g_0 \delta_e)}{[1-a_0 -g_0 m_o]} &lt; 0^3$</td>
<td></td>
</tr>
<tr>
<td>(V)</td>
<td>$\frac{[1-(k\theta/\delta_e)+\mu_0 p_0 \theta]}{(DENR)} &gt; 0^4$</td>
<td>$\frac{[-(e_0 +e_0 c_o)(1+\theta)(g_0 +\theta)}{(\rho_0 -k)/\delta_e}]}{(DENR)} &gt; 0^1$</td>
<td></td>
<td></td>
<td>$\frac{\theta[(g_0 +(\rho_0 /\delta_e) -\mu_0 p_0)]}{(DENR)}$ &gt; 0</td>
</tr>
</tbody>
</table>

$^1$If \( V = [1-a_0 +z_0 -g_0 m_0] > 0 \); $^2$If \( (e_0 +e_0 c_o) < 0 \); $^3$If \( B = [1-a_0 -g_0 m_0] > 0 \).

\[ J = [1-a_0 +z_0 -g_0 m_0 +\mu_0 p_0 m_o] > 0^1; \quad N = [1-a_0 +z_0 +(m_0 \rho_0 )/\delta_e] > 0; \quad S = [1-a_0 +m_0 (\rho_0 -k)/\delta_e] > 0; \]

\[ (DENR) = [N+\mu_0 p_0 \theta(1-a_0 +m_0 (\rho_0 -k)/\delta_e)-(k\theta V/\delta_e)+z_0 \theta(g_0 +(\rho_0 /\delta_e))] > 0^1. \]
<table>
<thead>
<tr>
<th>REGIMEN</th>
<th>$\delta (AP_t)/\delta G_t$</th>
<th>$\delta (AP_t)/\delta E_t$</th>
<th>$\delta (AP_t)/\delta R_t$</th>
<th>$\delta (AP_t)/\delta M_t$</th>
<th>$\delta (AP_t)/\delta D_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I)</td>
<td>$p_0 \cdot \frac{m_e}{j} &gt; 0^1$</td>
<td>$((1-a_e + z_e - g_m) c_e)^{-1}$</td>
<td>$((1-a_e + z_e - g_m) c_e)^{-1}$</td>
<td>$p_0 \cdot \frac{e_0}{j} &gt; 0^1$</td>
<td>$p_0 \cdot \frac{e_0}{j} &gt; 0^1$</td>
</tr>
<tr>
<td>(II)</td>
<td>$0$</td>
<td>$c_e &gt; 0$</td>
<td>$0$</td>
<td>$p_0 &gt; 0$</td>
<td>$0$</td>
</tr>
<tr>
<td>(III)</td>
<td>$-\left[ z_e + (k_m / \delta_e) \right]$</td>
<td>$\left[ \mu_0 + (\varepsilon_0 / \delta_e) \right] S &gt; 0$</td>
<td>$\mu_0 + (\varepsilon_0 / \delta_e) \left[ g_e + (\rho_e - k) / \delta_e \right]$</td>
<td>$\mu_0 + (\varepsilon_0 / \delta_e) \left[ g_e + (\rho_e - k) / \delta_e \right]$</td>
<td>$\mu_0 + (\varepsilon_0 / \delta_e) \left[ g_e + (\rho_e - k) / \delta_e \right]$</td>
</tr>
<tr>
<td>(IV)</td>
<td>$\frac{\varepsilon_0 \cdot p_0 \cdot m_e - c_e \cdot \varepsilon_0}{(\varepsilon_0 + \mu_0) (1 - a_e - g_m)} &gt; 0^2$, $s$</td>
<td>$\mu_0 + (\varepsilon_0 / \delta_e) \left[ \frac{(\varepsilon_0 + \mu_0) \left[ g_e + (\rho_e - k) / \delta_e \right]}{(\varepsilon_0 + \mu_0) \left[ g_e + (\rho_e - k) / \delta_e \right]} \right]$</td>
<td>$\mu_0 + (\varepsilon_0 / \delta_e) \left[ g_e + (\rho_e - k) / \delta_e \right]$</td>
<td>$\mu_0 + (\varepsilon_0 / \delta_e) \left[ g_e + (\rho_e - k) / \delta_e \right]$</td>
<td>$\mu_0 + (\varepsilon_0 / \delta_e) \left[ g_e + (\rho_e - k) / \delta_e \right]$</td>
</tr>
<tr>
<td>(V)</td>
<td>$\frac{-p_0 \cdot \theta \left( z_e + (k_m / \delta_e) \right)}{(DENR)}$</td>
<td>$\left[ z_e - \left( \theta \cdot 0 \right) \right]$</td>
<td>$\left[ z_e - \left( \theta \cdot 0 \right) \right]$</td>
<td>$p_0 \cdot \frac{\theta \cdot 0}{(DENR)} &gt; 0^1$</td>
<td>$p_0 \cdot \frac{\theta \cdot 0}{(DENR)} &gt; 0^1$</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>REGIMEN</th>
<th>$\delta F_t/\delta G_t$</th>
<th>$\delta E_t/\delta E_t$</th>
<th>$\delta F_t/\delta R_t$</th>
<th>$\delta F_t/\delta M_t$</th>
<th>$\delta F_t/\delta D_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I)</td>
<td>$\frac{- (z_e + \mu_0 \cdot p_0 \cdot m_e)}{j} &lt; 0^1$</td>
<td>$\frac{(1-a_e - g_m) - (\varepsilon_0 - \mu_0) c_e}{j}$</td>
<td>$\frac{-(\rho_e + g_e \cdot \delta_e - k)}{j}$</td>
<td>$\frac{-(\rho_e + g_e \cdot \delta_e - k)}{j}$</td>
<td>$\frac{-(\rho_e + g_e \cdot \delta_e - k)}{j}$</td>
</tr>
<tr>
<td>(II)</td>
<td>$\frac{- (z_e + (k_m / \delta_e))}{(DENR)} \geq 0$</td>
<td>$\frac{-(\varepsilon_0 + \mu_0) c_e}{(DENR)} &gt; 0^2$</td>
<td>$\frac{-(\rho_e + g_e / \delta_e - \mu_0 p_0)}{(DENR)} &gt; 0^2$</td>
<td>$\frac{-(\rho_e + g_e / \delta_e - \mu_0 p_0)}{(DENR)} &gt; 0^2$</td>
<td>$\frac{-(\rho_e + g_e / \delta_e - \mu_0 p_0)}{(DENR)} &gt; 0^2$</td>
</tr>
<tr>
<td>(V)</td>
<td>$\frac{- (z_e + (k_m / \delta_e))}{(DENR)} \geq 0$</td>
<td>$\frac{-(\varepsilon_0 + \mu_0) c_e}{(DENR)} &gt; 0^1$</td>
<td>$\frac{-(\rho_e + g_e / \delta_e - \mu_0 p_0)}{(DENR)} &gt; 0^1$</td>
<td>$\frac{-(\rho_e + g_e / \delta_e - \mu_0 p_0)}{(DENR)} &gt; 0^1$</td>
<td>$\frac{-(\rho_e + g_e / \delta_e - \mu_0 p_0)}{(DENR)} &gt; 0^1$</td>
</tr>
</tbody>
</table>
base money, this implies that $M_t$ and hence $\Delta P_t$ decreases. The higher $R_t$, on the other hand implies that there is an opposing effect on $F_t$ and hence on $\Delta P_t$. Thus the net effect on $\Delta P_t$ depends on what happens to $F_t$. Similarly under a fixed exchange rate, the effect of an increase in $G_t$ on the level of foreign exchange reserves $F_t$ in both models is indeterminate (except of course for the interest rate policy of regimen (I) where only one of the effects mentioned above is present), depending amongst other things on the degree of capital mobility. With a low degree of capital mobility ($k \to 0$), the effect on imports will be dominant so that $F_t$ falls, whereas with capital highly mobile ($k \to \infty$), the capital inflow will more than offset the import effect increasing the level of $F_t$.

In both models, it is of no surprise that a devaluation will increase domestic prices as this would follow naturally from our price equation. Its effect on $Y_t$ and $F_t$ will depend on the sign of $(\xi_0 + \mu_0 \lambda_0 / 1 - \pi_0)$ in model A and $(\xi_0 + \mu_0 c_0)$ in model B. When $E_t$ increases, imports go down but prices increase and the latter will in turn bring imports up again. If we assume that the overall effect of a devaluation is to decrease imports, that is we assume that $(\xi_0 + \mu_0 \lambda_0 / 1 - \pi_0)$ and $(\xi_0 + \mu_0 c_0)$ are both negative, then its effects on $Y_t$ and $F_t$ are as expected.

In model A, an expansionary monetary policy taking the form of an increase in the money supply $M_t$, or the domestic component of the monetary base $D_t$, or a decrease in the rate of interest $R_t$ will increase output and decrease the level of foreign reserves as expected. It will also unambiguously increase prices, if we assume (under a flexible exchange rate) that $(\xi_0 + \mu_0 \lambda_0 / 1 - \pi_0)$ is negative as above. Similarly in model B, if we assume that the exchange rate does not actually appreciate (i.e. $\xi_0 + c_0 \mu_0 < 0$), there is a definite positive effect on prices in all five regimens. The effects on $Y_t$ and $F_t$ are ambiguous under a fixed exchange rate system depending upon the relative signs of $|\rho_0 + \sigma_0 \delta_0|$ and $|\mu_0 P_0 \delta_0|$. This is to be expected as $(\rho_0 + \sigma_0 \delta_0)$ tells us how absorption $A_t$, and hence income increases due to an expansionary monetary policy whilst $\mu_0 P_0 \delta_0$ gives us the contractionary effect on output as higher prices increase imports. This implies that
the effect on the current account and hence on the level of foreign exchange reserves is also indeterminate.

We turn now to the analysis of the welfare costs associated with the five regimens in both models A. and B. Because these expressions are quite complicated, we will only consider initially, the various disturbances in turn. Of course this implies that we are ignoring any correlations between the various stochastic disturbances. Tables 2A. and 2B show the variances of output \( \sigma_y^2 \), prices \( \sigma_p^2 \) and foreign reserves \( \sigma_F^2 \) in the different regimens when the following disturbances appear in turn:

(a) Domestic demand disturbance \( (U_t) \),
(b) Domestic monetary disturbance \( (\eta_t) \),
(c) Domestic price disturbance \( (\nu_t) \),
(d) Foreign monetary disturbance \( (\phi_t) \).

What we want to find out is, which of the five regimens will best annihilate the effects of the disturbances on the three target variables \( Y_t \), \( \Delta P_t \) and \( F_t \) and whether the same results hold for both models A. and B. which differ only in the specification of their price and absorption equations. We will discuss model A. first and then look at the differences, if any, for model B and in particular of the importance of the real balance effect in the monetary instrument problem.

(a) **Domestic demand disturbances \( (U_t) \)**

An unexpected increase in domestic demand \( (U_t) \) will create an excess demand in the goods market. The ensuing increase in income will be accompanied by an increase in the transactions demand for money which will put upward pressure on the interest rate. Under regimen (I) both the exchange rate \( E_t \) and the interest rate \( R_t \) are pegged, and so cannot move to eliminate this excess demand in the goods market. What would happen then, is that prices and output will increase and imports will tend to increase too, leading to a fall in the trade balance and a decrease in the level of reserves.
### Table 2A

<table>
<thead>
<tr>
<th>REGIMEN</th>
<th>$\sigma^2_e$</th>
<th>$\sigma^2_p$</th>
<th>$\sigma^2_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I)</td>
<td>$\frac{\sigma^2_u}{[1-a+z+(\mu/1-\pi)]^2}$</td>
<td>$\frac{\gamma^2 \sigma^2_u}{(1-\pi)^2[1-a+z+(\mu/1-\pi)]^2}$</td>
<td>$\frac{[z+(\mu/1-\pi)]^2 \sigma^2_u}{[1-a+z+(\mu/1-\pi)]^2}$</td>
</tr>
<tr>
<td>(II)</td>
<td>$\frac{\sigma^2_u}{[1-a+z+(mp/\delta)+(\mu/1-\pi)]^2}$</td>
<td>$\frac{\gamma^2 \sigma^2_u}{(1-\pi)^2[1-a+z+(mp/\delta)+(\mu/1-\pi)]^2}$</td>
<td>$\frac{[z+(km/\delta)+(\mu/1-\pi)]^2 \sigma^2_u}{[1-a+z+(m/\delta)+(\mu/1-\pi)]^2}$</td>
</tr>
<tr>
<td>(III)</td>
<td>$\frac{\sigma^2_u}{[1-a+m(p-k)/\delta]^2}$</td>
<td>$\frac{\gamma^2 \sigma^2_u}{(1-\pi)^2[1-a+m(p-k)/\delta]^2}$</td>
<td>$\frac{[\lambda^2+(\mu/1-\pi)]^2 \sigma^2_u}{[1-a+m(p-k)/\delta]^2}$</td>
</tr>
<tr>
<td>(IV)</td>
<td>$\frac{\sigma^2_u}{[1-a]^2}$</td>
<td>$\frac{\gamma^2 \sigma^2_u}{[1-a]^2}$</td>
<td>$\frac{[\mu+\epsilon(1-\pi)]^2 \sigma^2_u}{[1-a]^2}$</td>
</tr>
<tr>
<td>(V)</td>
<td>$\frac{(1-\pi)^2[1-(k/\delta)]^2 \sigma^2_u}{(DEN)^2}$</td>
<td>$\frac{\gamma^2 \sigma^2_u}{(DEN)^2}$</td>
<td>$\frac{(1-\pi)^2[1-(k/\delta)]^2 \sigma^2_u}{(DEN)^2}$</td>
</tr>
</tbody>
</table>

### Denominators

- $DEN \equiv (1-\pi)[\theta(p-k)(z+\mu/1-\pi)/\delta+\kappa(z\theta/1-a)/\delta]$
- $C \equiv [1-a+z+(mp/\delta)+(\mu/1-\pi)]^2$
<table>
<thead>
<tr>
<th>REGIMEN</th>
<th>$\sigma_Y^2$</th>
<th>$\sigma_P^2$</th>
<th>$\sigma_\phi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I)</td>
<td>$\frac{\mu^2 \sigma_Y^2}{(1-\eta)^2 (1-\alpha z + (\mu\gamma/1-\eta))^2}$</td>
<td>$\frac{(1-\alpha z)^2 \sigma_Y^2}{(1-\eta)^2 (1-\alpha z + (\mu\gamma/1-\eta))^2}$</td>
<td>$\frac{\mu^2 (1-\alpha)^2 \sigma_Y^2}{(1-\eta)^2 (1-\alpha z + (\mu\gamma/1-\eta))^2}$</td>
</tr>
<tr>
<td>(II)</td>
<td>$\frac{\mu^2 \sigma_Y^2}{(1-\eta)^2 (1-\alpha z + (mp/\delta) + (\mu\gamma/1-\eta))^2}$</td>
<td>$\frac{(1-\alpha z + (mp/\delta))^2 \sigma_Y^2}{(1-\eta)^2 (1-\alpha z + (mp/\delta) + (\mu\gamma/1-\eta))^2}$</td>
<td>$\frac{\mu^2 (1-\alpha m(p-k)/\delta)^2 \sigma_Y^2}{(1-\alpha z + (mp/\delta) + (\mu\gamma/1-\eta))^2}$</td>
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<td>$\frac{\epsilon^2 \sigma_Y^2}{(\mu\lambda + \epsilon (1-\eta))^2}$</td>
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<td>(IV)</td>
<td>$\frac{\epsilon^2 \sigma_Y^2}{(\mu\lambda + \epsilon (1-\eta))^2}$</td>
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<td>(V)</td>
<td>$\frac{\mu^2 [1+\theta(p-k)/\delta]^2 \sigma_Y^2}{(\text{DEN})^2}$</td>
<td>$\frac{((1-\alpha z + (mp/\delta) + \theta z(p-k)/\delta - k\theta(1-a)/\delta)^2)/(\text{DEN})^2}{(\text{DEN})^2}$</td>
<td>$\frac{\mu^2 [1-\alpha m(p-k)/\delta]^2 \sigma_Y^2}{(1-\alpha z + (mp/\delta) + (\mu\gamma/1-\eta))^2}$</td>
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<tr>
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<td>$\frac{\lambda^2 [1-a z + (mp/\delta) - (\epsilon \gamma/\lambda)]^2 \sigma_\phi^2}{[\mu \lambda + \epsilon (1-\eta)]^2 [1-a z + (mp/\delta) /\delta]^2}$</td>
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<td>$\frac{\lambda^2 [1-a z - (\epsilon \gamma/\lambda)]^2 \sigma_\phi^2}{[\mu \lambda + \epsilon (1-\eta)]^2 (1-a)^2}$</td>
<td>-</td>
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<tr>
<td>(V)</td>
<td>$\frac{\rho^2 \theta^2 (1-\eta)^2 \sigma_\phi^2}{\delta^2 (\text{DEN})^2}$</td>
<td>$\frac{\rho^2 \theta^2 \sigma_\phi^2}{\delta^2 (\text{DEN})^2}$</td>
<td>$\frac{(1-\eta)^2 [1-a z + (mp/\delta) + (\mu\gamma/1-\eta)]^2 \sigma_\phi^2}{(\text{DEN})^2}$</td>
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\text{DEN} \equiv (1-\eta)[\theta(p-k)(z+\mu\gamma/1-\eta)/\delta + (c-k\theta(1-a)/\delta)] \\
\text{C} \equiv [1-a z + (mp/\delta) + (\mu\gamma/1-\eta)]
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<td>$\frac{(z+km)^2 \sigma^2 u}{[1-a+gm+\mu p]^2}$</td>
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<td>$\frac{\sigma^2 u}{[1-a+gm+\mu p]^2}$</td>
<td>$\frac{\sigma^2 (z+km)^2 \sigma^2 u}{[1-a+gm+\mu p]^2}$</td>
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<tr>
<td>(IV)</td>
<td>$\frac{\sigma^2 u}{[1-a+gm+\mu p]^2}$</td>
<td>$\frac{(\epsilon+\mu)^2 \sigma^2 u}{[1-a+gm+\mu p]^2}$</td>
<td>$\frac{(z+km)^2 \sigma^2 u}{[1-a+gm+\mu p]^2}$</td>
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<td>(V)</td>
<td>$\frac{\sigma^2 u}{[1-a+gm+\mu p]^2}$</td>
<td>$\frac{\sigma^2 (z+km)^2 \sigma^2 u}{[1-a+gm+\mu p]^2}$</td>
<td>$\frac{(z+km)^2 \sigma^2 u}{[1-a+gm+\mu p]^2}$</td>
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**TABLE 2b**

- **Domestic Demand Disturbance**
- **Domestic Monetary Disturbance**

$S \equiv [1-a+gm+\mu p/\delta]; (\text{DENR}) \equiv [N+\mu \theta[1-a+gm+\mu p/\delta]-[k0/\delta]+\theta(q+\mu/\delta)]; N \equiv [1-a+gm+\mu p/\delta]; V \equiv [1-a+gm].$
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<td>$\frac{\mu^2 \sigma^2_Y}{[1-\alpha z + \varepsilon \pi m \delta]^2}$</td>
<td>$\sigma^2_Y$</td>
<td>$\frac{\mu^2 [1-\alpha z (p-k) / \delta]^2 \sigma^2_Y}{[1-\alpha z + \varepsilon \pi m \delta]^2}$</td>
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<td>(III)</td>
<td>0</td>
<td>$\frac{\varepsilon^2 \sigma^2_Y}{(\varepsilon \pi + \varepsilon)^2}$</td>
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<tr>
<td>(IV)</td>
<td>0</td>
<td>$\frac{\varepsilon^2 \sigma^2_Y}{(\varepsilon \pi + \varepsilon)^2}$</td>
<td>—</td>
</tr>
<tr>
<td>(V)</td>
<td>$\frac{\mu^2 [1+\theta (g+(p-k) / \delta)]^2 \sigma^2_Y}{(DNR)^2}$</td>
<td>$\frac{[N-(k (1-\alpha gm) / \delta) + \varepsilon \theta (g+p \delta)]^2 \sigma^2_Y}{(DNR)^2}$</td>
<td>$\frac{\mu^2 [1-\alpha z (p-k) / \delta]^2 \sigma^2_Y}{(DNR)^2}$</td>
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<table>
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<tr>
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<td>$\sigma^2_\phi$</td>
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<tr>
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<td>$\sigma^2_\phi$</td>
</tr>
<tr>
<td>(III)</td>
<td>$\frac{\sigma^2_\phi}{[1-\alpha z (p-k) / \delta]^2}$</td>
<td>$\frac{\sigma^2_\phi [1-\alpha z + \varepsilon \pi m / \delta]^2 \sigma^2_\phi}{(\varepsilon \pi + \varepsilon)^2 [1-\alpha z (p-k) / \delta]^2}$</td>
<td>—</td>
</tr>
<tr>
<td>(IV)</td>
<td>$\frac{\sigma^2_\phi}{[1-a gm]^2}$</td>
<td>$\frac{[c (1-\alpha z gm) - \varepsilon \pi m]^2 \sigma^2_\phi}{(\varepsilon \pi + \varepsilon)^2 [1-\alpha gm]^2}$</td>
<td>—</td>
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<tr>
<td>(V)</td>
<td>$\frac{\theta^2 (g+(p/\delta)-u p]^2 \sigma^2_\phi}{(DNR)^2}$</td>
<td>$\frac{\rho^2 \theta^2 [1-\alpha z + \varepsilon \pi m / \delta]^2 \sigma^2_\phi}{(DNR)^2}$</td>
<td>$\frac{[1-\alpha z + \varepsilon \pi m / \delta]^2 \sigma^2_\phi}{(DNR)^2}$</td>
</tr>
</tbody>
</table>

$s \equiv [1-\alpha z (p-k) / \delta] ; (DNR) \equiv [N+\varepsilon \theta [1-\alpha z (p-k) / \delta] - k \theta \varepsilon / \delta] + \varepsilon \theta (g+p \delta) ; n \equiv [1-\alpha z + \varepsilon \pi m / \delta] ; v \equiv [1-\alpha gm]$. 

Note: The table continues with similar entries for other conditions.
If the interest rate is allowed to adjust upwards whilst the exchange rate is still kept fixed, as in regimens (II) and (V) this will dampen the initial increase in output and prices. The higher $R_t$ will also result in an incipient capital inflow which will offset, at least partially, the initial decline in $F_t$. The main difference with model B. is that with regimen (II), the variance of prices ($\sigma^2_P$) is equal to zero. This is of course because given the specification of the price equation, only changes in $M_t$ and $E_t$ will affect $\Delta P_t$. Under a flexible exchange rate (regimens (III) and (IV)), the initial trade deficit—caused by the increase in imports—will result in a depreciation of the exchange rate which will further increase the excess demand in the goods market. Under a money supply policy (regimen (III)), the increase in $R_t$ will partially offset the expansion, whereas under an interest rate policy (regimen (IV)) this effect will be non-existent. In model B. the inclusion of the real balance effect in the absorption function makes things worse for regimen (IV), as the money stock is now free to adjust and to feed through to affect $Y_t$ and $\Delta P_t$. We should also note that in both models if capital is highly mobile ($k \to \infty$), the increased capital flows due to the higher $R_t$, may more than offset the trade effect resulting in an appreciation of the exchange rate, which will mitigate the initial expansion in income. This will make regimen (III) very desirable.

As far as the problem of optimal foreign exchange market intervention is concerned, our results are in accordance with those found, for example, by Sti (1979) and Turnovsky (1976), namely that with low capital mobility ($k \to 0$), a fixed exchange rate system is preferable to a flexible one, with the converse being true for high capital mobility ($k \to \infty$). Note, however, that even if capital is highly mobile, we find that in both our models that under an interest rate policy a fixed exchange rate system (regimen (I)) is better than a flexible exchange rate system (regimen (IV)) in terms of providing a lower variance of output about its desired value when there are only domestic demand disturbances. This shows quite clearly how the monetary instrument problem and the question of optimal foreign exchange market intervention are interlinked.
As far as domestic demand disturbances \((U_t)\) are concerned, the main conclusion seems to be that the preferred policy in terms of price, output and external stability is that of controlling some form of monetary aggregate whatever the external operating regime.\(^{(23)}\) Our results thus seem to confirm those previously obtained by Poole (1970) in that in the case of domestic demand disturbance \((U_t)\) the money supply policy is superior to the interest rate policy as far as income stability is concerned.\(^{(24)}\) Moreover, this seems to hold under both fixed and flexible exchange rates. However, using Model B. we can only unambiguously confirm Poole's results in the case of a flexible exchange rate system. Under a fixed exchange rate, the rankings depend on the relative size of \(\frac{\phi_O}{\delta}\) and \((\mu p - g)\). We saw earlier that an unexpected increase in domestic demand \((U_t)\) will put an upward pressure on the interest rate as people try to alleviate their excess demand for money. Under an interest rate peg regimen (I), the authorities have to buy the bonds which people want to sell, thus increasing the stock of money which will feed through to income via the real balance effect in the absorption function, and to prices through our monetarist specification of the price function. Imports will in turn increase, bringing about a contractionary effect on output. This has to be weighed against the contractionary effect in regimen (II) when \(R_t\) is flexible. Because the real balance effect amplify the initial increase in income due to a rise in \(U_t\), it works, as expected, in favour of a money supply policy. The rankings of the regimens, though, still depend upon the relative size of the parameters given above. So, in a setting where prices are endogenous, the reliability of even Poole's simple rules, depend upon the specification of the model.

Our results, in as far as they can be compared, are in broad agreement with those of Turnovsky (1978), except that in the presence of domestic demand disturbances, Turnovsky finds that there may be a conflict in the choice of the monetary instrument for internal and external objectives. This conflict arises only because Turnovsky takes the domestic component of the monetary base \(D_t\) and the money supply \(M_t\) as parallel instruments;
as we saw earlier $D_t$ and $M_t$ are not instruments in the same sense. They can only be compared under different assumptions about strategy choice, namely two-stage versus single-stage strategies.

It would be interesting to compare our results with those of Henderson (1979) while keeping in mind that he keeps prices fixed in his paper. Henderson compares an 'aggregates constant policy', that is a flexible exchange rate and a money supply policy (our regimen (III)), with a 'rates constant policy', that is a fixed exchange rate and an interest rate policy (our regimen (I)). Shifts in the XX schedule in Henderson's paper is equivalent to that in our domestic demand disturbance $U_t$. To get Henderson's results that an 'aggregates constant policy' leads to less variation in output than a 'rates constant policy', we need 

$$
\frac{\{z_t \alpha V \cdot \pi_t \}}{\delta r} \frac{m(t-k)}{\delta} \text{ to be greater than } \{z-t g m + t p m \} \text{ in model B.}
$$

that is, the capital flow effect (together with the real balance effect in model B.) to dominate the trade effect adjusted for price increase, so that there is an actual appreciation of the exchange rate, which is the implicit assumption made by Henderson.

(b) **Domestic monetary disturbance** ($\eta_t$)

Again we discuss the results for model A first. An unexpected increase in the demand for money will put an upward pressure on the interest rate as people sell bonds in an attempt to increase their cash balances. This will have a contractionary effect on output. However, under an interest rate policy (regimen (I) and (IV)) the authorities are necessarily a willing buyer of these bonds at the market price thus preventing the rise in $R_t$ and hence the fall in output. This also imply that the disturbance will have no effect on prices or foreign exchange reserves. Under any form of monetary aggregate peg (regimen (II), (III) and (V)), however, the rise in $R_t$ will affect output prices and foreign exchange reserves (25) and under a flexible exchange rate the appreciation due to capital inflows will also be contractionary. Thus regimens (I) and (IV) will be the preferred ones in this case.

We thus seem to confirm Poole's (1970) result that in the case of domestic monetary disturbances, an interest rate policy is
superior to a money supply policy as far as income stability is concerned, as well as Henderson's result that as far as domestic monetary disturbances are concerned (shifts in the MM and BB schedules in his paper), a 'rates constant policy' (regimen (I)) is preferable to an 'aggregate constant policy' (regimen (III)). However, this is not unambiguously so for model B. Let us look first at Henderson's result. We saw that an unexpected increase in the demand for money will put upward pressure on the domestic rate of interest but that under an interest rate policy (regimen (I)), the authorities will buy the bonds being sold by the public thus preventing the rise in the interest rate. This will increase the amount of money in circulation, and in model B, this implies that both absorption and prices will increase. The increase in prices will in turn stimulate imports thus bringing about a contractionary effect on output so that the net effect on output is ambiguous. This must now be compared with the contractionary effect on output in regimen(III) where $R_t$ is allowed to float upwards and the exchange rate appreciate due to capital inflows. For the same sort of reasons, we cannot unambiguously confirm the simple result of Poole (1970) for either a fixed or a flexible exchange rate system. This is due to both the real balance effect in the absorption function as well as the monetarist specification of the price equation.

As far as the question of optimal foreign exchange market intervention is concerned, our results are only in broad agreement with the 'monetary variability' rule of thumb advocated by Johnson (26) and by Kenen (1969); a 'weak' version of which advances that increases in the variance of domestic disturbances would tend to favour a fixed exchange rate. Our results show that although the specification of the model is very important, the crucial factor is the domestic monetary regime in operation. For instance, we see that under an interest rate policy a fixed exchange rate system is indeed preferable to a flexible one in model B., but on an equal parity in model A., whereas under a money supply policy, the ranking depends upon various parameters in the models.
(c) **Domestic Price Disturbance** \((v_t)\)

An unexpected increase in domestic prices will initially reduce domestic demand and increase imports, so that income \(Y_t\) and the level of foreign exchange reserves \(F_t\) fall. In our simple models, the fall in \(Y_t\) will lead to a decrease in the demand for money, and hence to a fall in the rate of interest which will stimulate output and thus offset, at least partially, the initial contraction in income. In model A. under a fixed exchange rate then, a money supply policy (regimen (II)\(1\)) will be preferable to an interest rate policy (regimen (I)) as far as income stability is concerned. Again we find that this is not unambiguously so for model B., where once more the rankings depend upon the relative size of \(\rho_o/\delta_o\) and \(\mu_o p_o - g_o\).

Under a flexible exchange rate, in both models, there is an additional effect in that the direct increase in imports, following the unexpected rise in domestic prices (and the induced fall in the rate of interest when it is not pegged), will lead to a depreciation of the exchange rate which will exactly offset the initial reduction in income.

We assumed earlier that \((\varepsilon_o + \eta_o \lambda_o /1-\pi_o)\) is negative. Keeping this assumption, we find that the regimens which are optimal for income stability (regimens (III) and (IV)), also give rise to a greater degree of price instability (i.e. regimens (I) and (II) give smaller \(\sigma^2_p\)). This feature is in fact, a consequence of our price equation (19). Differentiating it with respect to \(v_t\) we get

\[
(1-\pi_o) \frac{\delta(\Delta P_t)}{\delta v_t} - \gamma_o \frac{\delta v_t}{\delta v_t} - \lambda_o \frac{\delta E_t}{\delta v_t} = 1
\]

Under a fixed exchange rate,\(^{(27)}\) this equation tells us that any policy which increases \(\delta(\Delta P_t)/\delta v_t\) and hence \(\sigma^2_p\) must also increase \(\delta v_t/\delta v_t\). An unexpected increase in \(v_t\) will result in higher prices and lower output under a fixed exchange rate,
so that \( \delta(\Delta p_t) / \delta u_t \) is greater than zero. Thus an increase in \( \delta y_t / \delta u_t \) will make it less negative and hence reduce its variance \( \sigma^2_y \).

Under a flexible exchange rate we saw that \( \delta y_t / \delta u_t \) is equal to zero, but here the depreciation of \( E_t \) will feed back again onto prices increasing its variance. This latter effect applies to model B as well, so that the variance of prices \( \sigma^2_p \) is less in the regimens with a fixed exchange rate with regimen (I) being the preferred one. To see why this is so, recall that an increase in domestic price disturbance will bring forth a downward pressure on the rate of interest through the excess supply of money in the economy. Keeping the interest rate at its par value as regimen (I) requires, implies a reduction in the stock of money which will offset, at least partially, the initial increase in prices. Obviously this effect is absent in regimen (II) where the money stock is kept fixed.

As far as external stability is concerned, in both models it is difficult to compare the variances \( \sigma^2_F \), although in theory, if the interest rate is allowed to fall as in regimens (II) and (V), this should affect \( F_t \) even more adversely than under an interest rate policy (regimen (I)).

d) Foreign monetary disturbance \( (\phi_t) \)

Since we are assuming a policy of complete sterilization, the variances of income and prices under a fixed exchange rate (regimens (I) and (II)) are obviously zero, except under a monetary base policy. This is because the unexpected capital flows will have no effect on the money supply and hence will not affect income and prices. Thus, so far as internal stability is concerned, the authorities will be indifferent as to using a money supply or an interest rate target under a fixed exchange rate and will prefer them to a monetary base policy (i.e. will prefer a two-stage to a single-stage strategy). In the case where the authorities control the domestic component of the monetary base \( D_t \), when the level of foreign exchange reserves \( F_t \) increases due to capital inflows, total money supply increases as
F is a component of total base money. Thus output and prices will be affected. Note, however, that when prices go up, this brings up imports too so that in both models a monetary base policy (regimen (V)) (single-stage strategy) is unambiguously preferable to an interest rate or money-supply policy (regimen (I), (II)) (two-stage strategy) as far as external stability is concerned. This implies that there may be a direct conflict in the choice of the monetary strategy (but not instrument) for internal and external stability. (28)

Under a flexible exchange rate system (regimen (III) and (IV)), the capital inflows will cause an appreciation of the exchange rate and this is contractionary. Note, however, that under a money supply policy (regimen (III)), the interest rate is allowed to decrease thus offsetting partially the initial contraction in output.

By looking at the disturbances in turn, we have been ignoring any correlations that may exist among the disturbances. Although we recognise that this is very restricting, analysing a situation where all four disturbances occur simultaneously is impractical because of the complexity of the expressions that it yields. What we chose to do instead is to look at the situation where the domestic demand disturbances $U_t$ occurs in turn together with domestic monetary disturbances $\eta_t$, domestic price disturbance $V_t$ or foreign monetary disturbances $\phi_t$. That is, we consider the following cases in turn:

(a) $u_t$ and $\eta_t$

(b) $u_t$ and $V_t$

(c) $u_t$ and $\phi_t$.

What we will be concerned with, is to see what effect the covariances of these disturbances have on the variances of income ($\sigma^2_Y$), prices ($\sigma^2_P$) and foreign reserves ($\sigma^2_F$). In Table 3 we present only these effects - instead of the expressions for the variances ($\sigma^2_Y$, $\sigma^2_P$ and $\sigma^2_F$) which are rather cumbersome - given the following assumptions about the covariances:
\[ H_0: \text{cov}(u_t, \eta_t) < 0 \]
\[ H_1: \text{cov}(u_t, \nu_t) < 0 \]
\[ H_2: \text{cov}(u_t, \phi_t) < 0 \]

From budget constraint considerations, it is normally assumed that \( u_t \) and \( \eta_t \) are negatively correlated. Following Poole (1970), this is the assumption implied by the hypothesis \( H_0 \). The hypothesis \( H_1 \) implies that an unexpected increase in domestic prices (\( \nu_t \)) is assumed to have a negative effect on domestic demand for goods (\( u_t \)). The hypothesis \( H_2 \) is quite acceptable under a flexible exchange rate system whereby the capital inflows causes an appreciation of the exchange rate, which is contractionary. However, in our simple models the foreign monetary disturbance \( \phi_t \) does not affect income or prices under a fixed exchange rate except under a monetary base policy and even then the effect would seem to be expansionary.

To give an example of how these effects given in Table 3 have been obtained, we look at the case where \( u_t \) and \( \eta_t \) occur together under regimen (II) in model A. The variance of income is then given by:

\[
\sigma_y^2 = \frac{\sigma_u^2 + (\rho/\delta)^2 \sigma_\eta^2 - 2(\rho/\delta)\sigma_{u,\eta}}{[1-a+z+(m\rho/\delta)+\mu\gamma/(1-\pi)]^2}
\]

where \( \sigma_{u,\eta} \) denotes the covariance between \( u_t \) and \( \eta_t \). Given the assumption that \( \sigma_{u,\eta} \) is negative and the signs of \( \rho \) and \( \delta \), this implies that the covariance term \( -2(\rho/\delta)\sigma_{u,\eta} \) would in fact have a positive effect on \( \sigma_y^2 \). The effects in the other cases are worked out in a similar way, although at times it turns out - especially in the case of \( \sigma_p^2 \) - that this effect is indeterminate.

Recall that without taking covariances into account, the main conclusion in the case of domestic demand disturbances \( u_t \) was that whatever the external operating regime, in model A, a policy of controlling some form of monetary aggregate - be it the money stock or the domestic component of the monetary base
### EFFECT OF COVARIANCE OF DISTURBANCES ON VARIANCES OF TARGET VARIABLES

#### (a) $U_t$ and $\eta_t$, given $H_o^+$

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<th>$\sigma^2_F$</th>
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<td>? (?)</td>
</tr>
<tr>
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<td>- -</td>
</tr>
<tr>
<td>(IV)</td>
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<td>0 (-)</td>
<td>- -</td>
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<tr>
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<td>+ (?)</td>
<td>? (?)</td>
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#### (b) $U_t$ and $\nu_t$, given $H_1$

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<td>+ (+)</td>
<td>- (?)</td>
<td>? (?)</td>
</tr>
</tbody>
</table>

#### (c) $U_t$ and $\phi_t$, given $H_2$

<table>
<thead>
<tr>
<th>REGIMENS</th>
<th>$\sigma^2_y$</th>
<th>$\sigma^2_p$</th>
<th>$\sigma^2_F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I)</td>
<td>0 (0)</td>
<td>0 (0)</td>
<td>+ (+)</td>
</tr>
<tr>
<td>(II)</td>
<td>0 (0)</td>
<td>0 (0)</td>
<td>? (?)</td>
</tr>
<tr>
<td>(III)</td>
<td>+ (+)</td>
<td>? (?)</td>
<td>- -</td>
</tr>
<tr>
<td>(IV)</td>
<td>+ (+)</td>
<td>+ (+)</td>
<td>- -</td>
</tr>
<tr>
<td>(V)</td>
<td>- (-)</td>
<td>- (?)</td>
<td>? (?)</td>
</tr>
</tbody>
</table>

* The effects in model B. are given in brackets.

† + means a positive effect on $\sigma^2_i$ (i=Y,P,F); - means a negative effect; 0 means no effect; and ? means effect is indeterminate.
under a single-stage strategy - was always preferable to an
interest rate policy in terms of internal and external stability.
Note also that this was not unambiguously so in model B under
a fixed exchange rate, depending as it turns out upon the relative
size of certain parameters in the model. Given hypothesis $H_0$
we can see form part (a) of Table 3 that, when we take the
covariance between $u_t$ and $\eta_t$ into account, this diminishes the
superiority of $M_t$ as the dominant monetary instrument, at least as
far as income and price stability are concerned - we cannot
comment on external stability without making specific assumptions
because of the indeterminateness of the covariance effect. This
is because the covariance tends to increase $\sigma^2_y$ and $\sigma^2_p$
under the money supply policy of regimens (II) and (III), whilst
decreasing, or having no effect under the interest rate policy
of regimen (I) and (IV). Recall also that in the presence of
domestic monetary disturbance alone, we saw that at least in
model A. the interest rate policy was unambiguously preferable
to a money supply policy. Taking the covariance between $u_t$
and $\eta_t$ into consideration reinforces that conclusion.

In part (b) of Table 3, we look at the covariance between domestic
demand disturbance $u_t$ and the domestic price disturbance $v_t$. We saw
earlier that at least for model A., ignoring any covariances,
in the presence of $v_t$ a money supply policy (regimen (II) ) is
preferable to an interest rate policy (regimen (I) ) under
a fixed exchange rate as far as income stability is concerned.
The covariance term affects $\sigma^2_y$ by exactly the same amount and
in the same positive direction in both regimens (I) and (II).
Therefore, regimen (II) will continue to be preferable to
regimen (I) in model A. In model B there was no clear cut
superiority, therefore not much more can be said as the covariance
increases $\sigma^2_y$ in both regimens (I) and (II).

Recall that both our models favoured a flexible exchange rate
system in the presence of domestic price disturbances $v_t$ alone
as far as income stability is concerned, but disfavoured it as
far as price stability is concerned. Taking the covariance into
account and given hypothesis $H_1$, $\sigma^2_y$ is not affected under the
flexible exchange rate system of regimen (III) and (IV), but
increases in the other regimens. Also $\sigma^2_p$ decreases in all but
regimen (III) where the effect is indeterminate. Thus the covariance improves the rating of a flexible exchange rate system. Given that the variance of income $\sigma^2_y$ is equal to zero under a flexible exchange rate system in the presence of $\nu_t$ only, when $u_t$ and $\nu_t$ appear together $\sigma^2_y$ will be equal to its value in the presence of $u_t$ only. In this case, we know that a money supply policy is unambiguously preferable to an interest rate policy in both models. Thus the covariance between $u_t$ and $\nu_t$ improves the case for a money supply policy.

We turn next to the case in part (c) of Table 3., where the domestic demand disturbances $u_t$ appears together with the foreign monetary disturbances $\phi_t$. Abstracting from covariances, we noted earlier that as far as income stability is concerned, in the presence of $\phi_t$, there is an unequivocal preference for a money supply policy over an interest rate policy under a flexible exchange rate, while they are equivalent under a fixed exchange rate. Looking at the flexible exchange rate case first, when we take the covariance between $u_t$ and $\phi_t$ into account, we see that given the hypothesis $H_2$ both regimens (III) and (IV) are subjected to an equal positive effect on $\sigma^2_y$. Thus the above conclusion remains valid. Under a fixed exchange rate we know that $\phi_t$ does not have any effect on $\sigma^2_y$ or $\sigma^2_p$ under either an interest rate policy (regimen (I) ) or a money supply policy (regimen (II) ). Therefore when we have both $u_t$ and $\phi_t$, $\sigma^2_y$ and $\sigma^2_p$ will be equal to their respective values in the presence of $u_t$ alone. Thus at least in model A, a money supply policy (regimen (II) ) will be unambiguously preferable to an interest rate policy. So once more we see that taking covariance into account, this time between $u_t$ and $\phi_t$, strengthens the case for a money supply policy.

2.4.2 Strategy Choice

The analysis above has only mentioned, here and there, the question of strategy choice. Here, we look at the problem in more detail.

As we mentioned earlier, if all the stochastic disturbances ($x_{it}$'s) are known to the authorities at the time that they make
their policy decisions, then the policy actions could be directed straight at the ultimate targets without any need for intermediate targets. In this single stage process, the authorities minimise the expected value of the loss function (equation (1)) subject to the model being used given target values for income ($\bar{Y}$), the rate of change of prices ($\Delta \bar{P}$) and foreign exchange reserves ($\bar{F}$) as well as forecasts for all exogenous and predetermined variables ($Q_t$) and the noises ($x_{it}$) to determine the optimal paths for the policy instruments $D_t$, $G_t$ and $E_t$. All available data are used and the whole experiment is repeated each period (e.g. each month) as new data becomes available.

To make the issue of strategy choice more interesting, under a two-stage strategy, we assumed earlier that $x_{4t}$ and $x_{5t}$ are known to the authorities but not $x_{1t}$, $x_{2t}$ and $x_{3t}$ when they make their policy decisions. In this two-stage strategy, however, only the second stage, namely that of altering the true instruments so as to keep the intermediate variable on target is repeated each period. The first stage, that is that of calculating the optimal path for the intermediate target is done less frequently. For example, if the second-stage decisions are taken every month, then the first-stage decision may be taken quarterly or even annually. This implies that the decision procedures involved in a two-stage strategy is not continuous. Thus the two-stage strategy discards some information, albeit temporarily, that the single-stage strategy processes as soon as it becomes available.

Thus the single-stage strategy seems to be preferable to the two-stage strategy in terms of realising a lower loss by virtue of using the available data more efficiently. If as is sometimes assumed, data for the ultimate targets are not received every period but say every three periods (i.e. data for say income being received quarterly instead of monthly) whilst data for the other variables are received every period, the intermediate target strategy discards less information than it does when data on all variables are received every period. The single-stage strategy seems to be still preferable, however, as it uses data for $Q_t$ as it becomes available while these are ignored temporarily in the two-stage strategy.
Thus "contrary to what is often intuitively supposed, therefore, differences in the frequencies of observation of variables do not logically justify the use of a money strategy. Even during periods when ultimate-target variables are unobserved, discretionary instrument adaptation is superior in its use of new data". (Bryant, 1980, p. 290)

Proponents of intermediate target strategy suggest that by announcing their targets in advance, the authorities make their own behaviour less unpredictable; this in turn induce the private sector to make better decisions and hence lead to a better evolution of the economy (see, for example, Fellner, 1976 and Richardson, 1978). Another possible justification for the use of an intermediate money supply target is provided by Poole in the discussion on B. Friedman's 1977 paper. Poole argues that "the case for an intermediate monetary target reflects the desire to prevent money-supply disturbances from existing long enough to feed into income." (p.341). He believes that a money supply disturbance could affect the amount of money held without any initial feedback on the arguments of the demand for money function. This argument is linked to that in NIESR (1982) which runs along the following lines. It is assumed that the monetary authorities have a good idea of the relationships between instrument, intermediate targets and ultimate targets, although no special link between the intermediate target and the ultimate targets are suggested. Also controllability of the intermediate target is assumed not to be a problem. In this setting whether we base policy changes on instruments or intermediate targets is quite arbitrary. However, intermediate targets may still play a role in the period in between policy changes, when the economy may be perturbed by various internal and external disturbances. "By defining policy over this time horizon in terms of some fixed level of a controllable intermediate target rather than the policy instrument itself, we may increase the automatic stabilising properties of the economy. For example, we may help reduce the inflationary consequences of a rise in commodity prices before policy has a chance to react". (NIESR, p.61). We should perhaps note that this justification is only valid in the period before policy has had the time to react to changing circumstances.
The length of this time period is, however, not easily determined.

Bryant (1980) seems to suggest that we should first solve the issue of strategy choice, and then look at the "secondary" issue of instrument choice. (see, for example, p.258). However, our results show that whether a two-stage strategy is superior or not to a single-stage strategy, may depend on what is being actually used as the intermediate target. For instance, from Table 2.A, we can see that in the presence of domestic monetary disturbances, a two-stage strategy utilising an interest rate target is definitely preferable to a single-stage strategy using a monetary base target whatever the external operating regime. (36) Our results unfortunately do not help us give a definite answer to the question of strategy choice. The determining factors seem to be the same as those which are important in the instrument choice problem.

2.5 Concluding Remarks

We have shown formally how the monetary instrument problem could (and should) be analyzed concurrently with the problem of optimal foreign exchange market intervention and how the results depend upon the specification of the model. In this setting where prices are endogenously determined, we cannot unambiguously confirm Poole's (1970) results that as far as income stability is concerned, a money supply policy is superior to an interest rate policy in the case of domestic demand disturbances, with the converse applying for domestic monetary disturbances. We also find that the simple graphical results obtained by Henderson (1979) when comparing a 'rates constant' policy with an 'aggregate constant' policy are not strictly valid once prices are endogenized. This latter result, moreover, does not seem to depend on the specification of the model.

We also examined situations where the domestic demand disturbances appear together with the other disturbances in turn. Given the assumptions made about the covariances between these disturbances, we find that when domestic demand and domestic monetary disturbances appear together, the preference tends to move
towards the interest rate policy. On the other hand, when
domestic demand disturbances appear together either with domestic
price disturbances or foreign monetary disturbances, this
strengthens the case for a money supply policy.

The important point that emerges from our analysis is that there
is no one regimen which dominates the others under all types
of disturbances. To get these results, we have assumed that the
authorities objective is to minimize a weighted sum of the variance
of output, prices and foreign reserves subject to a linear dynamic
model with additive autoregressive errors. The optimal policies
are linear feedback control laws with the three targets fluctuating
about their desired values each period. It should be noted that
the fact that the variances $\sigma_Y^2$, $\sigma_P^2$ and $\sigma_F^2$
depend only on
first period values of the parameters of the model, is a
result of two assumptions, namely non-stochastic parameters and
zero costs associated with the instruments. This last assumption
is dropped in the empirical examination of chapter 4 with,
nevertheless, the assumption of deterministic parameters being
retained. Before we move on to these empirical exercises,
however, we need to describe the econometric model upon which
these exercises are based. This is the purpose of the next chapter.
(1) The work on stabilization policies using optimal control theory seems to have been confined to closed economies (see, for example, Sengupta, 1970; Turnovsky, 1973; and Pindyck, 1973); a notable exception being Turnovsky (1978).

(2) According to Bryant (1980), if the model is 'recursive' with respect to say the money supply, that is, causation runs only from the money supply to the endogenous variables in the model and not vice-versa, then the decision process can be dichotomised into two stages. If causation were to run in both directions, then a two-stage dichotomisation would not take the relevant simultaneity into account. For a more formal analysis, see Aftalion and White (1978).

(3) Although we do not tackle this second problem in our theoretical work, our empirical analysis in chapter 4 does not entail this sort of dichotomisation and deals with the two issues simultaneously. See footnote 7 in chapter 1 for a list of the theoretical analyses of this issue.

(4) The parallel analysis for model B is not presented as it is basically similar to that for model A.

(5) Turnovsky (1978) assumes a fixed exchange rate and is not concerned with the issue of exchange market intervention.

(6) $\Delta P_t$ is used as a proxy for the rate of change of prices.

(7) See, for example, Buiter (1977).

(8) Demand pressures are usually proxied by the deviation of output from its full employment level, so that $Y_t$ could instead be taken to proxy changes in productivity; this would, however, make $\Omega_i < 0$. This will not affect our results so long as $\Omega_i < \omega_i^q \xi_i$.

(9) Similar price equations can be found in Lipsey and Parkin (1970), Turnovsky and Wilson (1973) and Turnovsky and Kasprzak (1974).

(10) A similar price equation can be found in Melitz (1979).
(11) The amount of foreign bonds held domestically is assumed to be negligible, so that the foreign rate of interest is assumed to be determined only by the foreign country's demand and supply of foreign bonds. This assumption is made only for simplifying purposes.

(12) Taxes are assumed to be exogenously given.

(13) In the simple models we are considering, under a flexible exchange rate, a money supply policy is virtually identical to a monetary base policy except for a factor $\theta$ to be explained below.

(14) On this point see B. Friedman (1975), p. 453.

(15) This assumption is quite common in the literature; see the papers mentioned in chapter 1. However, in the empirical examination in chapter 4, we relax these assumptions.

(16) On this proposition, see, for example, Howrey (1967), and Pagan (1975).

(17) We are again abstracting from the problems of controlling the money supply.

(18) The algebra involved are similar to that in the previous section, and is thus not presented.

(19) We are again abstracting from the problems of controlling the monetary base.

(20) The actual calculations are not presented, but are quite straightforward.

(21) This is of course under a money supply policy.

(22) Or, indeed under a monetary base policy, which is not examined explicitly here; see footnote (3).

(23) Externally $D_t$ is the best instrument, implying the superiority of single-stage over two-stage strategy.

(24) Unlike Poole (1970) we do not keep prices fixed.

(25) Of course $F_t$ will change in regimens (II) and (V) only.

(26) This rule was proposed by H. G. Johnson at the Winter 1970 University of Chicago Preliminary Examination in International trade.
(27) This implies that $\frac{\delta E_t}{\delta v_t} = 0$.

(28) This conflict is also found by Turnovsky (1978), but he views it as one of instrument choice instead of strategy choice.

(29) However, they may be positively correlated, see Turnovsky (1980).

(30) It also increases $\sigma_y^2$ and $\sigma_p^2$ under a monetary base policy in model A., although the effect in model B. is indeterminate.

(31) For a lucid exposition on strategy choice, see Bryant (1980).

(32) Recall that the $x_{it}$'s are linear combinations of $U_t$, $\eta_t$, $v_t$ and $\phi_t$.

(33) Of course, the latter applies for a fixed exchange rate system only.

(34) Although $x_{1t}$, $x_{2t}$ and $x_{3t}$ are not known at the time of the policy decisions, they do become known in later periods.

(35) According to Friedman (1975), a two-stage process does not "in general constitute optimal central bank operating procedure." (p.470)

(36) However, under a single-stage strategy that uses an interest rate instrument (like the discount rate), the loss would again be zero, thus making the choice of the instrument a non-trivial matter even under a single-stage strategy.
CHAPTER 3
AN ECONOMETRIC MODEL OF THE UK ECONOMY

3.1 Introduction
The objective of this chapter is to develop an econometric model of the UK economy which attempts to capture the constituent elements of the theoretical model of chapter 2. The model is a quarterly one and is estimated, in common with most models of the UK economy, by ordinary least squares (using the computer programme 'GIVE'). It is based on seasonally adjusted data for the period 1963(i) to 1980(iv), although unavailability of official statistics for the whole period for some variables meant that some equations had to be estimated using data for a shorter period of time (this is made clear in Appendix A, where a complete listing of the model is provided). The model can be described as essentially Keynesian, and seeks to explain the components of aggregate demand with the latter assumed to be equal to real output. It should be noted that, in common with most model builders in the UK, we do not model supply explicitly, although it is recognised that the supply-side relationships (like demand for labour and capital) lie in the background. The kind of system advocated by, for example, Klein (1978) whereby the supply side apparatus of production relationships, factor demand and factor supply of the Leontief model is combined with an elaborated Keynesian model, is beyond the scope of this study. However, we provide an explanation for the price level for aggregate output around which the other prices in the model are built; this is probably adequate (see Klein, 1978, p.2).
Consumption, investment, stockbuilding, imports and exports are determined endogenously, while adjustment to factor cost and government expenditure are treated as exogenous. The main determinants of the components of aggregate demand are real output, the exchange rate, interest rates, real personal disposable income, world trade and relative prices. In the wage-price sectors, prices are largely determined by costs with real wages following a trend in the long run. Although the real and international sectors of the model are similar to those of the theoretical model, the monetary sector is disaggregated into a number of equations which build up an explanation of sterling M3 balances from the portfolio behaviour of the non-bank private sector and the commercial banks, as well as the government budget constraint. This is considered to be more realistic and more revealing to the problem at hand than the simple money demand and supply equations of the theoretical model. We can thus relax the assumption that the money stock can be regarded as a variable that the monetary authorities have a firm fulcrum on,
but recognize instead that they can only influence it in a very indirect manner. Changes in the money stock, however, are assumed to have only an indirect effect on activity and prices, the transmission mechanism being modelled through interest rates and the effective exchange rate; the latter, as well as capital movement, are also modelled explicitly.

All the equations mentioned above are discussed in some detail in sections 3.2 to 3.8 below. Section 3.9 then deals with the simulation properties of the model as a whole. We will only be concerned with historical simulation, since the purpose of building the model is to run some control exercises (in chapter 4) and not for forecasting. This section also discusses the dynamic properties of the model, which is also the concern of section 3.10 using, however, a different technique.

3.2 THE GOODS MARKET

Equation (1) is the standard income identity

\[ QY = QCE + QKP + QDS + QG + QEX - QIM - QAFC \]

where all variables are in 1975 prices, and a bar over a variable name indicates an exogenous item. GDP expressed at factor cost or real output \( QY \) is equal to the sum of the components of aggregate demand, that is, private consumers' expenditure \( QCE \), private investment \( QKP \), changes in stocks \( QDS \), public expenditure on consumption and investment goods \( QG \), export expenditures \( QEX \) less import expenditures \( QIM \), less adjustment to factor cost \( QAFC \) which is indirect taxes minus subsidies. The underlying assumption here is that an increase in aggregate demand at a given price level is actually met by an increase in output supplied.

We need thus to explain five components of expenditure - \( QCE, QKP, QDS, QEX \) and \( QIM \). We look at each of them in turn, starting with consumers' expenditure.

3.2.1 Consumers' expenditure

We estimate an equation for total consumption on goods and services \( QCE \) from which consumer durable expenditure \( QCDE \) is then determined with consumer expenditure on other goods \( QC0 \) projected exogenously. We recognize that it is common nowadays to estimate separate equations for durable and non-durable consumption, but given the fact that in the theoretical model we aggregate consumers' expenditure with private investment to give us a function for total private expenditure or absorption, this was considered acceptable. This should not, however, affect the results in any significant way. Keynesian
theory model consumption as basically a function of personal disposable income with both variables in real terms to avoid the possibility of 'money illusion'. However, such simple equations, as estimated for example, by Davis (1952), are known to predict very badly. In particular, OLS estimation of equations such as that of Davis underpredicted actual consumption, which implied that the cause was not the simultaneity problem as this produces an upward and not a downward bias in the OLS estimate of the marginal propensity to consume. A number of hypotheses have been put forward to overcome this problem. For example, Brown (1952), in his development of Duesenberry's (1949) hypothesis (in which people's behaviour is assumed to change only slowly over time), includes the lagged dependent variable instead of the previous peak of income (as Duesenberry does) to reflect the dependence of current consumption on past behaviour. Thus, his equation is of the form

\[ QCE_t = c_0 + c_1 QYD_t + c_2 QCE_{t-1} + u_t \]

where \( u_t \) is an error term and \( QYD \) is real personal disposable income. However, this equation can also be obtained from Friedman's (1957) permanent income hypothesis if we proxy permanent income by a distributed lag on income. It should be noted, though, that the two alternative hypotheses have quite different implications for policy changes. More recently, some authors (see, for example, Townend, 1976; Bean, 1978 and Davidson et al, 1978) have added liquid assets as a main explanatory variable in an attempt to explain the unprecedented rise in the savings ratio in the UK which could not be easily explained by existing theories. This additional variable could be explained in terms of the life-cycle hypothesis (see, for example, Ando and Modigliani, 1963) but also, perhaps more importantly, to take the effects of inflation into account. Deaton (1977) has suggested instead that both the rate of inflation and the rate of change of the rate of inflation be included in the equation. The rationale for the inclusion of the rate of inflation is well documented (see, for example, Deaton, 1977; Davidson et al, 1978; Howard, 1978 and, more recently, Siegel, 1979). The rate of change variable is included to take account of the fact that people will not react immediately to changes in the rate of inflation as these are normally unanticipated. Davidson et al (1978) preferred the two inflation terms of Deaton to the liquid assets term, while recent work at the LBS (1980) includes both a liquid assets/income ratio term and an inflation term.

Another variable which has been included in UK consumption function is the rate of interest (see, for example, Arestis and Driver, 1980 and Arestis and
Hadjimatheou, 1982a). It is possible to argue in favour of either a negative or a positive sign of the interest rate term (see, for example, Taylor, 1971, p.393 and Arestis and Driver, 1980, p.91) depending upon the relative strength of the substitution and income effects. The usual assumption is that the substitution predominates so that a negative sign is to be expected; that is, higher interest rates result in higher returns on savings which implies that people consume less and save more. There is a further substitution effect in that, as interest rates increase, H.P. as well as mortgage repayments increase, which could result in lower consumer expenditure (Arestis and Driver, 1980).

Our preferred estimated equation is of the form

\[ QCE = C (QYD, RPC, RLA, QCE_{-1}) \quad C_{1,4} > 0 \quad C_{2,3} < 0. \]

Thus real consumers' expenditure is assumed to be determined by real personal disposable income (QYD), the rate of change of consumer prices over a year earlier (RPC)\(^{(1)}\), the local authority rate (RLA) which is the central short rate of interest in the model, and lagged consumers' expenditure \(^{(2)}\). The estimated consumption function is shown as equation (A3) in appendix A. An indication of the goodness of fit is provided by the standard error of the regression of 0.198 and R\(^2\) statistic of 0.985. The value of \(\chi^2\) \(^{(3)}\) is equal to 1.03 which is insignificant at the 5 per cent level of significance, implying the validity of the autoregressive restriction \(^{(3)}\) in the restricted transformed equation and hence the dynamic specification of the equation. All parameters bear the a priori expected sign and their coefficients are significant at the 5 per cent confidence level. We also get a significant negative interest rate effect implying the predominance of the substitution effect mentioned above; this result is quite common for recent studies in the UK (see Arestis and Driver, 1980 and Arestis and Hadjimatheou, 1982a). Thus interest rates provide a mechanism by which monetary policy can affect economic activity in the model. The short-run marginal propensity to consume is 0.28, while the long-run marginal propensity to consume is 0.93 (= 0.2773/(1-0.7014)), which is reasonable. The significance of the inflation variable with a negative sign reflects the depressing effect of price increases on consumers' expenditure.

Because recent empirical estimates of the consumption equation distinguish between durable and non-durable expenditure \(^{(4)}\) (unlike our estimates), it is very difficult to meaningfully compare our results with other estimates. However, our results seem to be plausible. An empirical estimate of an aggregate consumption equation for the UK is in the 1979 version of the National Institute model (they now have separate equations for durable and
non-durable expenditure). Our estimates are not very different from theirs, with a slightly higher short-run marginal propensity to consume but a lower long-run value. We should note that their equation does not have an interest rate term, but instead has a variable reflecting the availability of credit. We also estimated a similar equation (which has current grants from the government as an additional variable) to that of the National Institute, but this did not perform as well as equation (A3) in appendix A either in terms of single equation residuals or in overall simulation of the model.

3.2.2 Fixed Investment

The second component of aggregate expenditure that we look at is fixed investment. Although a lot of work has been done on the theoretical explanation of investment expenditure, there is no generally accepted theory (see, for example, Bridge, 1971; Lund, 1971; Greenberg, 1976; and Mayes, 1981 for a survey on the subject). In considering the problem of investment demand there are two problems to be tackled. First, what determines the optimal or desired capital stock \( K^* \) and, second, what determines the rate at which actual capital stock \( K_t \) adjusts towards the optimal capital stock? In other words, if \( I_t = F(K^*_t, K_t) \) then we must determine both \( K^*_t \) and the form of \( F \).

One of the most widely used hypotheses of investment behaviour is the accelerator principle, where firms attempt to maintain an optimum relationship between the capital stock and output \( Y_t \), thus

\[ K_t = \alpha Y_t \]

Ignoring depreciation, we can therefore write net investment \( (I_t) \) as

\[ I_t = (K_t - K_{t-1}) = \alpha (Y_t - Y_{t-1}) \]

Estimation of such simple accelerator models, however, yields in general quite poor results (Mayes, 1981, p.124). So instead of assuming that the desired capital stock is actually met, it is better to assume that the adjustment is only partial, that is,

\[ I_t = \Delta k_t = \lambda (k^*_t - K_{t-1}) \quad 0 < \lambda < 1 \]

or

\[ K_t = \lambda K^*_t + (1-\lambda) K_{t-1} \]

where \( \lambda \) is the rate of adjustment to optimal capital stock. Thus we can write

\[ (K_t - K_{t-1}) = \lambda (k^*_t - K^*_{t-1}) + (1-\lambda) (K_{t-1} - K_{t-2}) \]

or

\[ I_t = \lambda \Delta k^*_t + (1-\lambda) I_{t-1} . \]

We now assume that
where is the rate of interest which reflects the cost of external funds. The rationale for including is that, in Keynesian theory, it is usually assumed that a favourable investment decision will be taken if the marginal efficiency of capital (internal rate of return) exceeds the marginal cost of getting external funds, which is usually proxied by the rate of interest (Junankar, 1972, p.21). A significant effect for this variable in investment functions has been found by, amongst others, Hines and Catephores (1970).

Thus substituting (8) into (7) we have

\[
(9) \quad I_t = \lambda \beta_1 \Delta r_t + \lambda \beta_2 \Delta Y_{t-1} + (1-\lambda) I_{t-1} + v_t
\]

Our estimated equation rests on this flexible accelerator model adapted to take into account the costs of external funds, but, in addition, we consider another factor, namely, the availability of internal funds. Thus, the larger the internal funds (retentions) available to the firm, the easier it is for it to carry out desirable investment programmes. This variable has been used by, for example, De Leeuw (1962), Evans (1969), Bean (1979) and Arestis and Hadjimatheou (1982b). We estimate an equation of the form

\[
(10) \quad QKP = K (\Delta QY_{-1}, \Delta RCL, \text{IF}_{-1}, QKP_{-1}) K_{1,3,4} \times 0, K_{2} < 0
\]

Thus real private fixed investment (QKP) is assumed to be a function of the change in output (QY), the change in the consol rate (RCL), internal funds of the corporate sector (IF) and lagged investment. The empirical estimate is shown as equation (A6) in appendix A. No significant coefficients could be found on higher lags on \(\Delta QY\) and the lag on IF turned out to be one. The equation is fairly satisfactory with an overall fit as indicated by an \(R^2\) of 0.967 and a standard error of 0.089, and statistically significant coefficients with expected signs. However, the coefficients on both the interest rate term and the change in income term are rather low, and higher lags on \(\Delta QY\) are to be expected (7). The estimated equation gives some support for the capital stock adjustment hypothesis. The implied rate of adjustment \(\lambda\) is equal to 0.1, indicating a very slow realization of desired capital stock. The significance of the interest rate variable is particularly interesting because it is in contrast to common findings in the UK (see, for example, Savage, 1978). The internal funds variable is also relevant in that it confirms Bean's (1979) findings that certain financial factors do contribute to the explanation of investment behaviour. We should perhaps note that it is more usual nowadays, in large models of the UK economy, to disaggregate investment into different categories. For instance, in NIESR (1979) investment is disaggregated into investment in private dwellings, in manufacturing industry, and in other industries (LBS, 1981, is even more disaggregated).
However, we felt that in our case this kind of disaggregation would enlarge the model unnecessarily.

3.2.3 Stockbuilding

The next endogenous component of the GDP identity, stockbuilding ($QDS$), is highly volatile due perhaps to the ease with which it can be adjusted and also to the frequency of unanticipated changes in output and sales. One of the problems facing model builders is that macro data are in general subject to certain imperfections; this problem is particularly acute for inventory investment. This arises because the data are collected from companies and the valuations of stocks prepared by companies' accountants are often based on fairly casual inspection of the stocks on hand. The method of valuation is not clear: it can vary from year to year even in the same company and practices vary wildly across companies. The 'book' value of the stocks is then subjected to a revaluation process by national-income statisticians, based upon their guess of the methods used by accountants and the structure of the inventories." (Hilton, 1976, p.133). Also, these estimates are often subject to substantial revisions. The optimal level of stocks is usually posited to depend upon the expected level of output or sales - which is the dominant factor - the expected interest rate costs and the expected change in the price of stocks held. It is usually very difficult to get any significant interest rate effect empirically. According to Trivedi (1970), this could well be due to the estimation techniques used. He found, using quarterly UK data, that while ordinary least squares estimates did not provide a significant interest rate effect, maximum likelihood estimates did. Price changes could have either a positive or a negative effect on stockbuilding. A negative correlation implies that if there are excess stocks firms try to sell them at lower prices, while a positive correlation is explained by the precautionary or speculative motives of the producers (Evans, 1969). Klein (1974) found a role for price changes, but both Lovell (1961) and Trivedi (1970) failed to do so.

Such a stockbuilding equation depending on the three variables mentioned above could theoretically be derived from the minimisation of a cost function (8) by a firm and then aggregated. The problems that could arise from possible feedbacks from stockbuilding to output or sales is usually ignored on the assumption that the lags in these feedbacks are quite long. Uncertainty implies that the firm considers the expected value of the variables involved. The firm may not react immediately to changes in the expected value of the determinants of stockbuilding. This is because there are usually costs involved in changing the level of stocks and firms.
are reluctant to incur these costs. Thus we would expect some lags to be involved. Inventory investment, like fixed investment, is thus assumed to be determined by an accelerator mechanism and interest rate effects. Thus

\[ Q_{DS} = S(RCL_{-1}, \Delta QY_{-1}) \quad S_{1,2} > 0 \]

As expected, this equation turned out to be very difficult to estimate, as can be seen from equation (A7) in appendix A. No significant role was found for price changes and the lags that were obtained were of the second order for the interest rate variable and first and second order for \( \Delta QY \). All the t-values are rather small and the goodness of fit is quite low with an \( R^2 \) of only 0.23, but taking into account the well known difficulties of estimating stockbuilding, the equation is probably acceptable. However, \( \chi^2 (2) \) is equal to 1.80 which is insignificant at the 5 per cent level, implying that the dynamic specification of the equation is adequate. We actually found that inventories could still be left endogenously determined in the model without significantly affecting the simulating properties of the model as a whole and bank lending (of which it is a major determinant) in particular. Perhaps Budd (1979) is right in saying that "one should be highly suspicious ......... of a well fitting equation for stockbuilding" (p.13).

3.2.4 Exports of goods and services

We are now left to explain only two endogenous components of expenditure, namely, exports and imports. In the theoretical model we have a single equation for the trade balance or net exports. Here, however, we have separate equations for aggregate exports and aggregate imports. The exports equation which we estimate is a conventional one depending on the relative price of exports (RPX), the effective exchange rate (ER), as well as world demand (WD), thus:

\[ Q_{EX} = X(WD, RPX, ER) \quad X_1 > 0, X_2,3 < 0 \]

Here \( RPX \) is defined as the ratio of the domestic price of exports (PX) to the world price of exports (WPX), i.e.

\[ RPX = PX/WPX \]

A major problem in relation to this equation seems to be the lack of uniformity in the measurement of the relevant explanatory variables. For instance, as noted by Arestis and Hadjimatheou (1982b), the world demand variables used in a UK exports equation "should encompass only those countries which constitute potential customers of the UK; it should also be weighted on the basis of UK specialisation in production of exportables."
(p.86). This is because an increase in world demand of a product not produced in the UK will not have any direct effect on UK exports. Whilst this is a very plausible argument, tracing that particular set of data is not an easy task at all. Another recurrent problem is to find a variable to reflect UK price competitiveness. A number of indicators have been used by other researchers (9); we opted for the relative price index simply because the data was readily available.

As can be seen from equation (A8) in appendix A, the equation is fairly satisfactory in terms of goodness of fit, with an $R^2$ of 0.956 and standard error of 0.224, as well as significant t statistics. RPX, as well as its lagged value, turned out to be significant; although the coefficient on the contemporaneous relative price term was estimated with a perverse sign, the sum of the coefficients on the two RPX terms is negative, so that in the long run the expected effect prevails. However, this sum is small in magnitude, indicating that UK price competitiveness has a significant but small effect on the volume of exports. The mean elasticity of exports with respect to world demand is 0.65, which is nearer the lower end of the range of UK exports demand elasticities (10). Meaningful comparisons of our results with other studies is, however, limited because of the level of aggregation we adopted compared to other researchers. For instance, it is more usual in large models of the UK economy to distinguish between (i) exports of manufactured goods, (ii) food and basic materials, and (iii) services (11). However, it was felt that the level of aggregation adopted was adequate for our purposes.

3.2.5 Imports of goods and services

The nature of the aggregate function for imports has not changed very much over the years, with import demand viewed like the demand for any commodity and depending upon the level of real economic activity (real income) and the relative price of imports. In addition, an index of capacity utilisation has been tried by some researchers to reflect the fact that when bottlenecks in domestic supply develop, imports have to increase to meet domestic demand (see, for example, Rees and Layard, 1971). Exports are also sometimes included as an argument to account for the import content (for example, raw materials) of commodity exports (see, for example, Coghlan, 1979, 1981). However, we could not find significant effects for these additional variables. We thus write the equation as

\[
QIM = Z(QY, (P/PM), Z_{1,2}) > 0
\]

Thus real imports ($QIM$) depend on the ratio of the domestic price level ($P$) to the price of imports in domestic currency ($PM$) and the level of real
income. Note that although the exchange rate does not appear in the equation, its effect is taken into account by the relative price term \((P/PM)\), i.e. \(PM\) is defined as

\[ PM = \frac{PMF}{ER} \]

where \(PMF\) is the exogenously determined price of imports in foreign currency. Because of the existence of various lags and delays (see, for example, Junz and Rhomberg, 1973), we assumed a partial adjustment mechanism

\[ \Delta QIM_t = \gamma (QIM^*_t - QIM_{t-1}) \]

where \(QIM^*_t\) is the desired volume of exports. Thus our estimating equation includes \(QIM_{t-1}\) as an additional variable.

As in the case of exports, there are problems of definition as well as measurement. Some researchers have suggested that the income variable should not include things like services, transport, construction and so on as these are not importables. Others have put forward the point that one should allow for the effects of changes in the composition of demand by including as separate arguments different types of expenditures with different marginal propensities to import. (See, for example, Barker, 1970). To put these suggestions into practice would have meant enlarging our model and, at the same time, possibly introducing severe multicollinearity through the correlation of the components of final expenditure. Admittedly the level of aggregation is most probably at the expense of possible aggregation errors through changes in the composition of demand over time. The price indices are again another source of ambiguity. For example, instead of using the GDP price deflator, it has been suggested that an index of domestic price of import substitutes be used, since the weighting of the various prices in the index would allow for their individual composition in total imports, especially in view of import controls on certain goods. Morgan (1975) suggest that tariff changes should be included as a separate price variable. This is because the abolition of tariffs, which is very effective, may result in a big reduction in imports which is not fully captured by the corresponding fall in the import price index. However, the empirical findings on this variable are not decisive, with Humphrey (1979) finding a significant effect for nominal tariffs and Whitley (1979) failing to find any such effect.

The estimated equation as reported in equation (A9) in Appendix A is fairly satisfactory in terms of explanatory power, with a standard error of only 0.034 and an \(R^2\) of 0.862, given the aggregate nature of this equation. All the coefficients are significant at the 5 per cent confidence level and
bear the a priori expected signs. The coefficient of adjustment of actual to equilibrium volume of exports $\gamma$ is 0.26, which reveals a fairly low response due, perhaps, to the existing lags and delays. The long-run mean income elasticity of demand for imports is 1.74, which is well inside the range of 1.0 (Beenock and Warburton, 1980) to 2.20 (Humphrey, 1976) obtained by earlier studies for total imports \(^{(12)}\). The lag on the relative price term turned out to be of the second order, indicating that it takes about two quarters for price or exchange rate changes to affect imports. As in the case of exports, it is difficult to meaningfully compare our results with other studies because of the level of aggregation we adopted \(^{(13)}\).

With government expenditure and adjustment to factor cost projected exogenously, equations (1), (2), (10), (11), (12) and (13) determine the level of aggregate demand in the economy. We turn next to its implications for employment and unemployment.

3.3 Employment and Unemployment

In the UK it can be said that the number of school leavers and married women entering the labour force, and the number of people leaving it through retirement, change only slowly. Therefore the supply of labour (LS) can be projected exogenously and changes in unemployment are mainly determined by changes in employment. Thus we estimate an equation for employment (and not unemployment). This is in common with most large models of the UK economy (see Bank of England, 1979; LBS, 1981 and NIESR, 1979) which have behavioural equations for employment (disaggregated to various extents in the different models), with registered unemployment being then determined by some kind of technical equation \(^{(14)}\). In our case, however, unemployment ($U$) is simply determined as a residual given employment ($EMP$) and the exogenous labour supply:

\[(14) \quad U = \text{LS} - EMP\]

Employment is postulated to be primarily determined by lagged adjustment to changes in output. It also follows a negative trend. The estimated equation is in log-linear form:

\[(15) \quad \Delta \ln EMP = E(\Delta \ln EMP_{-1}, \Delta \ln QY, \text{TREND71}) \quad E_{1,2} > 0; E_{3} < 0\]

The preferred equation is reported as equation (A10) in Appendix A. It is fairly satisfactory statistically, with a goodness of fit given by an $R^2$ of 0.623 and standard error of 0.003.

The coefficients are all significant at the 5 per cent level and bear the a priori expected signs. However, the residuals are in some cases quite big, implying that over these periods the equation does not perform very satisfactorily. Higher lags for $\Delta QY$ were tried, but were not found to be
significant. The high value of the coefficient on the lagged dependent variables implies that lagged employment is an important determinant of current employment.

3.4 **INCOMES: The Wage Rate**

To analyze income from employment we first look at one of its components, namely, wages. It was only in the late sixties that the Phillips curve, as expounded by Phillips (1958) and amended by Lipsey (1960) came to be seriously questioned. The simple relationship between the rate of wage inflation and the level of excess demand in the labour market and the rate of price inflation was then extended to take price expectations into account in what has been termed the 'price-augmented Phillips curve'\(^{(15)}\). Henry et al (1976) have experimented with different versions of this function, but could not obtain a negative relationship between wage inflation and unemployment for the UK. A Phillips curve similar to the one in our theoretical model and not very dissimilar to the one in Parkin (1970) and Henry et al (1976) was estimated, but was very much less satisfactory than the real wage model, which we report here in terms of both single-equation residuals and overall simulation of the model.

The real wage model is based on the work by Sargan (1964). Trade Unions are supposed to bargain for real wages, but can only make money wage claims in the light of expected price movements\(^{(16)}\). Thus there is a target money wage as given by:

\[
\frac{W}{W_{-1}} = \left[ \frac{P}{P_{-1}} \right]^e \left[ \frac{(W/P)_d}{(W_{-1}/P_{-1})} \right]^\Pi
\]

where \(W\) is money wages, \(P\) prices, \(d\) denotes the desired value and \(e\) the expected value of a variable. Assuming that desired real wage grows at about the same rate of growth of the economy (and can thus be simply made a function of time), and that the expected change in prices in the period ahead is equal to the actual change in prices in this period, then taking logs equation (16) becomes

\[
\ln \frac{W}{W_{-1}} = \ln \frac{P_{-1}}{P_{-2}} - \Pi \ln \frac{W_{-1}}{P_{-1}} + \Pi t
\]

where \(t\) is a time trend. This is the basic equation and Sargan includes an unemployment term - which could be interpreted as indicating the state of demand in the labour market or the bargaining strength of unions - as well as a dummy variable.

We did not find any significant unemployment effect - implying the absence of any direct labour demand effect on wages - and our version also includes the retention ratio as an extra explanatory variable. This follows closely
recent work done at the National Institute (17) (although they have now replaced real wages with average earnings, which apparently performs even better). Thus it is assumed that real wages grow at a trend rate in the long run, but deviate from that trend in the short run due to changes in the retention ratio (RR) and expected consumer price inflation:

\[ \Delta \ln WR = W (\Delta \ln WR_{-1}, \Delta \ln PC_{-1}, \Delta \ln RR_{-1}, \ln (PC/WR)_{-1}, \text{TREND71}) \]

\[ W_1, z, 5 > 0; W_3, 4 < 0. \]

Here, PC is consumer prices and the retention ratio is defined thus:

\[ RR = \left[ 1 - \frac{NIC}{WS} - \frac{TP}{WS} \right] \]

where NIC is employees' national insurance contributions, TP is taxes of the personal sector and WS is wages and salaries.

Our estimated wage equation ((A12) in Appendix A), which is very similar to that reported in Henry, Karakitsas and Savage (1982), performs very well indeed given the well known difficulty of estimating a satisfactory wage equation for the UK. A measure of the goodness of fit is provided by a standard error of 0.015 and an R² of 0.597; the latter is quite high given that the equation is estimated in changes. All the coefficients bear the expected signs and are significant at the 5 per cent confidence level. The absence of any direct labour demand effect is worth commenting upon. Both the level and the percentage of unemployment were tried in changes as well as in levels (including the lags of these variables), and in each case the coefficient was uncomfortably insignificant and in some cases a perverse sign was obtained as well. Equation (A12) is, in fact, homogenous in terms of real wages. This can be seen by rewriting the equation in terms of \( \ln WR \):

\[ (1 - 1.2499L + 0.1974 L^2 + 0.3409 L^3)\ln WR = (0.0188 - 0.2475 (1-L)\ln RR_{-1} + (0.6396 - 0.3512L)\ln PC_{-1} + 0.0015 \text{TREND71}) \]

where L is the lag operator. To solve for the stationary equilibrium we set \( L = 1 \) throughout. We then see that the sum of the coefficients on \( \ln WR \) and on \( \ln PC_{-1} \) are both equal to 0.2884, so that the equilibrium form of the above equation is

\[ \ln WR = \frac{(0.0188 + 0.2884 \ln P + 0.0015 \text{TREND71})}{0.2884} \]

Thus the equilibrium process is in terms of the level of real wages and a trend.
With wage rate and employment already determined, income from employment as well as total personal income (TPY) are determined by the following equations:

\[
\begin{align*}
(19) & \quad AE = WR \cdot AH \\
(20) & \quad WS = AE \cdot EMP + \overline{WRES} \\
(21) & \quad TPY = WS + \overline{OPY}
\end{align*}
\]

where \( AE \) = average earnings, \( WRES \) = residual to make up wages and salaries and \( \overline{OPY} \) = other personal income. Total personal disposable income (\( YD \)) is obtained by subtracting taxes (\( T \)) from total personal income:

\[
(22) \quad YD = TPY - T
\]

Taxes in turn are made up of personal taxes (\( TP \)) and other taxes (\( OT \)), with the former assumed to be a simple function of total personal income and the latter including employees' national insurance contributions:

\[
\begin{align*}
(23) & \quad TP = T(TPY) \\
(24) & \quad T = TP + OT
\end{align*}
\]

The modelling of a detailed tax structure is beyond the scope of this study; for our purposes, however, the above specification is considered adequate.

Given personal disposable income, to get real personal disposable income (which, as we have seen, is an important determinant of consumption), we need an estimate of the consumer price index. This takes us to a discussion of the various price structures in the model.

**Prices**

Domestic prices play a key role in any model through their influence on relative prices and hence on the balance of payments and the exchange rate, on the real wage and real output and employment, as well as the monetary sector. Also, the rate of inflation is, by itself, a policy target of paramount importance.

We estimate an equation for the rate of change of prices (\( RP \)) using the GDP deflator as the price index. The indices for consumer prices (\( PC \)) and export prices (\( PX \)) are then linked to \( P \) through simple regressions. Our price inflation equation is one commonly found in the literature (see, for example, Lipsey and Parkin, 1970) and is not very different from that in our theoretical model. We consider a market where prices are set as a mark-up on current costs, which is influenced in the short run by changes in labour productivity which may result from random fluctuations in output. The current costs are assumed to be made up simply of labour costs and the costs of raw materials, including imported inputs. Thus the rate of inflation, \( RP \), is determined by the rate of change of wages (\( RWWR \)), the rate of change of sterling import prices (\( RMP \)) and the rate of change of productivity (\( RPRO \)):
Productivity is defined as the ratio of output to employment, i.e.
\[ PRO = \frac{QY}{EMP} \]
and its rate of change, like all the rates of change in this equation, is defined over a four quarter period. Thus, for example,
\[ RPRO_{-1} = \frac{(PRO_{-1} - PRO_{-5})}{PRO_{-5}} \]

The RP equation presented as equation (A20) in Appendix A is quite satisfactory with an R\(^2\) of 0.945 and a standard error of 0.015. All the coefficients bear the a priori expected signs with significant t statistics, except for the productivity term (lagged once) which has a rather low t value. The lag on the import price term turned out to be two quarters and the coefficient is very low, indicating that there are long lags involved before a rise in import prices can be fully reflected in domestic prices. The rise in import prices may be caused by an increase in world prices or in the exchange rate, since PM is equal to the ratio of PMF to the exchange rate. As has been shown by Lipsey and Parkin (1970), the underlying theory for such a price equation implies that the coefficients on RWR and RPRO should be equal in absolute value, and that the coefficients on RWR and RMP should sum to unity. In our model, the long run equation for prices can be written as
\[ RP = 0.52 \times RWR + 0.15 \times FMP - 0.22 \times RPRO \]

As is quite common for the UK (see, for example, Lipsey and Parkin, 1970; Goldstein, 1974 and NIESR, 1979), our equation does not have these properties. Nevertheless, the equation is well determined statistically, with the main determinants of price inflation being wage inflation (which, to some extent, reflects trade union power) and increases in sterling import prices. With the GDP deflator thus determined, consumer prices are then linked to it through the following equation:
\[ (26) \quad PC = PC(P) \quad PC_1 > 0 \]

This should be interpreted simply as a technical relationship between the two variables, rather than an equation for the determination of consumer prices. (PC\(_{-1}\) is also included in equation (A23) in Appendix A in order to make \(\chi^2(1)\) insignificant).

Export prices are also linked to P in a similar way, although the exogenously determined world price of exports (WPX) is also included as an important factor. Thus,
\[ (27) \quad PX = PX(P, WPX) \quad PX_{1,2} > 0 \]

This simple formulation also reflects the fact that exporters are more concerned with the sterling price of competing world exports than with
domestic costs or competing domestic goods as argued by Winters (1976)\(^{(18)}\). With the price of imports in foreign currency (PMF) projected exogenously, as we saw earlier, the last price index, the price of imports in domestic currency (PM) is converted into sterling by the exchange rate (ER).

Up to now, the differences between the theoretical and empirical model have not been substantial. The next sector that we turn to, however, is much more disaggregated than the theoretical counterpart. This, it is felt, is more appropriate for the problem at hand.

### 3.6 THE MONETARY SECTOR

When building up the monetary sector of our model, we had to bear in mind that it should enable us to examine, in a satisfactory way, situations where the authorities have an interest rate target as well as situations where they attempt to set the rate of growth of the money supply within a predetermined range. As we mentioned earlier, here we consider the stock of money as an intermediate target which can only be indirectly influenced by the authorities, and not as a policy 'instrument' as we did in the theoretical exercises in chapter 2. In this section we also clarify the links between the monetary and the international sectors.

The control of the money supply in the UK is much more complicated than using the interest rate to slide up or down a demand for money function\(^{(19)}\). The monetary target used in the UK is the sterling component of the broadly-defined money stock, sterling M3 (SM3). Savage's (1980) interpretation of the Bank of England's technique of controlling the money supply is as follows. A forecast is made of the rate of growth of the money stock for unchanged policies on interest rates and so on from forecasts of its main components. "The authorities then simultaneously employ a number of different policy instruments in an attempt to ensure that future movements in the main components of the money supply - that is to say, the public sector borrowing requirement, sales of public debt to the non-bank private sector, the volume of bank lending to the private sector and external flows into the private sector - are consistent with the announced range of tolerance for the rate of growth of sterling M3." (pp. 47-48).

We model all these main components mentioned by Savage except 'external flows into the private sector', which is partly projected exogenously. The monetary sector of our model consists of four behavioural equations which determine the demand for financial assets and four behavioural equations to determine interest rates along with a number of identities. It is constructed around three main identities - the government budget constraint, the definition of the money supply and the balance sheet of the banking system. This framework seems to have now been widely accepted for modelling...
the monetary sector of a UK macroeconomic model. In that respect our monetary sector is rather similar to that of NIESR (1979), although the latter is much more disaggregated. However, as will become more apparent when we look at the various behavioural equations, the arguments in our functions are quite different from those of the NIESR model.

The government budget constraint has now been given its due recognition as an important item in any macroeconomic model and can be written as

\[
(28) \quad \text{PSBR} = \text{DC} + \text{DB} - \text{DCBL} + \text{DSBLG} + \text{DEF}
\]

This identity stipulates that the public sector borrowing requirement (PSBR), plus official purchases of commercial bills by the Bank of England (DCBL), must be financed by the issue of notes and coins (DC), by sales of public sector debt to the non-bank private sector (DB), by borrowing from the banks (DSBLG) and by borrowing from overseas (including a decrease in official reserves) (DEF).

The PSBR is the excess of government expenditure over its receipts and can simply be written as

\[
(29) \quad \text{PSBR} = (QG \cdot P) - T - (QAFC \cdot P) + \text{PDRES}
\]

where PDRES is a residual to take into account some less important items including current and capital transfers. Identities (28) and (29) shed some light on the important links between fiscal policy and the money supply and, together with identity (30), on the links between the money supply and the balance of payments situation.

\[
(30) \quad \text{DEF} = \text{DFR} - \text{DOLG} - \text{D$BLG}
\]

Identity (30) reveals the relationship between external finance of the public sector (DEF) and the balance for official financing (change in foreign reserves, (DFR)), where DOLG is change in overseas lending to the public sector and D$BLG is change in bank lending to the public sector in foreign currencies. We look at the other items of identity (28) below.

We turn now to the second central identity in the monetary sector, namely, the definition of the money supply, sterling M3. The change in sterling M3 (DSM3) is defined as the sum of the change in notes and coin held by the public (DC), the change in sterling deposits of the UK private sector (DSDP) and the change in sterling deposits of the public sector (PSDG):

\[
(31) \quad DSM3 = DC + DSDP + PSDG
\]

The M3 definition of the money supply can be linked to its sterling components thus:

\[
(32) \quad DM3 = DSM3 + D$D
\]

where D$D is the change in UK residents deposits in other currencies, including valuation changes.

The change in the narrowly defined money stock M1 (DM1) is given by

\[
(33) \quad DM1 = DC + DCA
\]
where DCA is change in sterling sight deposits (current account) of the UK private sector. These deposits are only part of the items on the liabilities side of the UK banks balance sheet, which is the third central identity in our monetary sector. The full list of liabilities (in changes) is as follows:

change in sterling deposits of private sector (DSDP)
+ change in sterling deposits of public sector (DSDG)
+ change in sterling deposits of overseas sector (DSOD)
+ change in overseas currency deposits (DOCD)
+ change in net non-deposit liabilities (DNDL).

On the assets side we have:

change in sterling bank lending to private sector (DSBLP)
+ change in sterling bank lending to public sector (DSBLG)
+ change in sterling bank lending to overseas sector (DSBLO)
+ change in overseas currency assets (DOCA)

where DOCD is made up of the change in deposits in other currencies of the private, public and overseas sectors, and DOCA accounts for the change in bank lending in other currencies to the private, public and overseas sectors.

Since assets must be equal to liabilities, we have:

\[(34) \quad DSDP + DSDG + DSOD + DOCD + DNDL \]
\[= DSBLP + DSBLG + DSBLO + DOCA \]

Sterling deposits of the private sector can be subdivided into sight deposits and time deposits (DSTDP):

\[(35) \quad DSDP = DCA + DSTDP \]

Thus identity (31) can be rewritten as

\[(36) \quad DSM3 = DC + DCA + DSTDP + DSDG \]

or, using the M1 identity (33), as

\[(37) \quad DSM3 = DM1 + DSTD \]

where DSTD is the change in sterling time deposits of the private and public sectors; i.e.

\[(38) \quad DSTD = DSTDP + DSDG \]

Using identities (35) and (38) we can now rewrite the banks balance sheet (34) as:

\[(39) \quad DCA + DSTD + DSOD + DNDL = DSBLP + DSBLG + DSBLO + DNOC \]

where DNOC is the change in net overseas currency assets (DOCA - DOCD).

We are now in a position to see how these identities, together with the behavioural equations to be discussed below, build up an explanation of sterling M3. Identity (28) can be rewritten to show that sterling bank lending to the public sector is the residual of public sector finance:

\[(40) \quad DSBLG = PSBR - DC - DB + DCFL - DEF \]
This, as explained by Mayes and Savage (1980), "corresponds to actual public financing practice, since any excess of public spending over revenue (including receipts from net sales of public debt to the private and overseas sectors) is automatically met by borrowing from the banks." (p.6).

With purchases of commercial bills by the Bank of England projected exogenously, and the public sector deficit and external finance of the sector determined by identities (29) and (30), this leaves notes and coin in circulation and debt sales to the private sector to be determined endogenously.

3.6.1 Demand for currency

Given that in the UK it is a well established fact that the authorities do not attempt to influence the amount of notes and coin in circulation, but issue tender to satisfy the needs of the private sector, we take notes and coin held by the non-bank private sector (DC) to be purely demand determined. It it postulated that DC depends on nominal consumers' expenditure on goods and services (CE), as well as the interest rate on seven-day deposits (RD) and a time trend (TREND63) to take into account institutional changes.

Thus we have

\[ DC = DC(CE, RD, TREND63) \]

The estimated equation is presented as equation (A30) in Appendix A. A measure of the goodness of fit is provided by a standard error of 0.086 and an \( R^2 \) of 0.512. The latter is quite satisfactory given that DC measures the change in notes and coin held by the private sector. All the coefficients are significant at the 5 per cent level and bear the a priori expected signs. The significance of the interest rate effect is interesting because it implies that, although DC is believed to be demand determined, the authorities can, to some extent, influence that demand through changes in interest rates (20). This is important if, as is presently the case, the authorities are trying to achieve some kind of monetary targets.

3.6.2 Sales of public sector debt to non-bank private sector

One of the main monetary instruments at the disposal of the authorities is interest rates, be it under an interest rate regimen where some interest rate is used as an intermediate target or under a monetary aggregate regimen where interest rates are used to keep the money stock on target, or indeed in a single-stage strategy of conducting monetary policy where interest rates are used to influence the ultimate targets directly. In all these regimens interest rates are used to influence, amongst other things, the sales of public debt to the private sector. It is known that the Bank of England, through its influence on the short term rates in the market, can induce changes in the long rates and hence affect the demand for gilt-
edged securities. The so-called 'cashier's theory' postulates that the demand for gilts is positively related to the level of interest rates, as well as negatively related to the changes in interest rates. If interest rates are expected to fall, then it is easier for the authorities to sell gilts than when interest rates are expected to increase or to continue to move upwards after an initial upward trend; the reason being simply the fear of capital loss as interest rates go up and prices of gilts fall.

Our estimated equation is of the form:

\[ (42) \quad DB = DB (RCL, (RCL-RCL_{-1}), FW) + DB_1,3 >0 ; DB_2 <0 \]

The demand for public sector debt by the non-bank private sector is thus postulated to depend on the level of the consol rate \((RCL)\) and the change in consol rate, as well as the stock of financial wealth \((FW)\). The latter variable represents the constraint facing the private sector in allocating its financial wealth amongst the various assets. The estimated equation (equation (A31) in Appendix A) is well determined statistically with an \(R^2\) of 0.858 and a standard error of 0.366. All the coefficients are significant except for the lagged financial wealth term, which turned out to be negative. However, the sum of the coefficients on the two financial wealth terms \((FW \text{ and } FW_{-1})\) is positive, giving the expected long-run effect. The main reason for including the lagged financial wealth term is that it brings down the \(\chi^2(2)\) value to 2.95, which becomes insignificant at the 5 per cent level implying the dynamic specification of the reported equation is better than the one without the \(FW_{-1}\) term. The estimated equation, therefore, gives a lot of support to the cashier's theory. One would, however, also expect the demand for public debt to depend on the yields on substitutable financial assets. A number of interest rates, both domestic and foreign, were tried, but were either not found to be significant or had a perverse sign, or both.

The stock of financial wealth which appears in equation (42) is simply defined as the sum of the broadly defined stock of money \((M3)\) and the amount of public sector debt outstanding \((B)\), thus:

\[ (43) \quad FW = M3 + B \]

Identity (37), which defines changes in sterling M3, is the next main identity we look at. To determine DSM3 we need to explain DMI and DSTD.

### 3.6.3 Demand for money M1

As we noted earlier, the simple notion of sliding up or down a demand for money schedule does not represent the basis of controlling the money supply in the UK. Given that in the 1970's the stability of demand for money functions for the UK has been called into question, this makes this policy
even more unattainable. Nevertheless, the narrowly defined money stock M1 is generally believed to be demand determined in the UK.

A lot of work has been done on the theoretical and empirical aspects of the demand for money. Although there have been some newer developments (see, for example, Niehans, 1978 and Akerlof and Milbourne, 1980), the theories as expounded in the work of Keynes (1936), Baumol (1952), Tobin (1956, 1958) and Friedman (1956) still form the basis of most empirical work on the demand for money. As far as the M1-definition is concerned, the function is usually written as

\[ M^* = M(P, y, r, \hat{P}) \]

i.e. desired cash balances \((M^*)\) is made a function of the price level \((P)\) to reflect variations in the purchasing power of money, real income \((y)\) to indicate anticipated transactions, interest rates \((r)\) to proxy the opportunity cost of holding money, and the expected rate of inflation \((\hat{P})\) to capture the degree of substitutability between money and real assets (Boughton, 1979).

Given the absence of money illusion and the fact that we are concerned with the real purchasing power of money, we can rewrite (44) as

\[ M^*/P = m^* = M(y, r, \hat{P}) \]

Assuming a partial adjustment of the form

\[ m - m_{-1} = \sigma(m^* - m_{-1}) \]

where \(\sigma\) reflects the speed of adjustment, we obtain a dynamic specification with \(m_{-1}\) as an extra regressor, i.e.

\[ (M1/P) = M1(QY, RP_{-1}, RLA, (M1/P)_{-1}) \quad M_{1,4} > 0, M_{2,3} < 0 \]

Thus our demand for M1 function is a standard one depending on real income \((QY)\), the rate of change of prices \((RP)\) and a short-term interest rate, namely, the local authority rate \((RLA)\). The latter is generally used in the UK as the representative rate for short term assets. Our estimated equation as reported in equation (A38) in Appendix A is quite satisfactory, with an \(R^2\) of 0.927 and a standard error of 0.318. All the coefficients are statistically significant with expected signs. The long-run elasticity with respect to income is 0.75, which is nearer the top end of the range of 0.39 (Mills, 1978) to 1.32 (Boughton, 1979) found by earlier studies. The long-run interest elasticity of money M1 is -0.39, which is again well within the range of -0.06 (Hacche, 1974) to -1.05 (Goodhart and Crockett, 1970) found by earlier studies using the local authority rate. We may note that we also tried a long rate \((RCL)\) but the results were not as good (statistically) as the one reported here. Finally, the implied coefficient of adjustment \(\sigma\) turned out to be very low, indicating that only 12 per cent of any discrepancy between desired and actual money balances is made up in the first quarter. Thus lags play a very important role in the demand for
money function.

3.6.4 Sterling time deposits of private and public sectors

During the period considered in this study (1963 to 1980) it can be said that banks in the UK behaved as oligopolists in both credit and deposit markets, setting their interest rates in line with short-term money market rates and the minimum lending rate (Spencer and Mowl, 1978; Moore and Threadgold, 1982). Both bank lending and bank deposits were, to a large extent, demand determined, although banks could restrict loans to some extent through non-price factors. "Once credit was granted, the amount of funds obtained through retail deposits, after adjustment for the reserve assets and special deposits which had to be held against them, was reconciled with the quantity of credit demanded by changing other portfolio items." (Moore and Threadgold, 1982, p.4). Thus in our model we will assume that sterling time deposits of private and public sectors (DSTD) are obtained as a residual by banks rearranging their portfolios. Therefore, we can rewrite the banks' balance sheet, identity (39), as

\[ DSTD = DSBLP + DSBLG + DSBLO + DNOCA - DCA - DSOD - DNDL \]

With sterling bank lending overseas (DSBLO), net other currency assets (DNOCA), sterling deposits of overseas sector (DSOD) and net non-deposit liabilities (DNDL) assumed exogenous, and sterling sight deposits (DCA) and sterling bank lending to the public sectors (DSBLG) determined by identities (33) and (40) respectively, we need to explain sterling bank lending to the private sector (DSBLP) in order to determine DSTD and hence DSM3 from (37).

3.6.5 Sterling bank lending to the private sector

Since in the UK it is generally recognised that changes in bank lending are a major source of changes in the money stock (see, for example, Moore and Threadgold, 1980), it is rather surprising, given the importance attached to control of the money supply in recent years, how little work has been done on the determinants of bank lending. It is usually assumed that the Bank of England can influence the rate of growth of bank lending by inducing changes in short term rates, in particular MLR, over which they had direct control until recently. As MLR increases, borrowing rates increase and so effectively put a brake on bank lending. This effect was supposed to have been made possible by the Competition and Credit Control (CCC) reforms of 1971. However, the reforms were followed by the much criticised monetary expansion of 1972-73 "which raised doubts as to whether the interest-elasticity of the demand for credit was sufficiently high to enable the new system of attempting to regulate bank lending by the cost of credit rather than by lending requests and advances to operate smoothly." (Savage, 1979, p.48). In an attempt to restrain monetary expansion without raising interest rates
to 'unacceptable' levels, a new direct control, the Supplementary Special Deposits Scheme (generally known as the 'corset') was introduced in December 1973. This was supposed to discourage banks from accommodating increases in bank lending by imposing a rising marginal cash reserve requirement on banks if their interest bearing liabilities exceeded a certain limit. Banks, however, soon found ways of lending (for example, guaranteeing acceptance credits drawn up by companies) which effectively evade the corset (26).

We will assume that bank lending to the private sector is mainly demand determined. Because we estimate an equation for total sterling bank lending, it should include factors which determine both corporate and personal bank lending. As far as bank lending to the corporate sector is concerned, it has been suggested by Moore and Threadgold (1980) that its main determinants are the components of the companies' working capital needs. These are employment costs, raw material costs and stockbuilding. In our study, however, only stockbuilding turned out to be significant among these variables. This may be due to the aggregate nature of our equation. We should perhaps note, though, that we did not use the same definitions as they did (due to lack of data at the time).

We also include a 'round-tripping' variable to take account of the fact that, when possible, individuals and firms borrow from the banks to lend at a profit on the money markets. We define this variable as the difference between the rate of interest charged on loans - proxied by the clearing banks base rate (CBR) (27) - and the money market rate - proxied by the local authority rate (RLA). The own rate of interest (CBR) was tried on its own as well, but no significant coefficient could be obtained, so that it was dropped from the equation. Consumer durable expenditure (QCDE) is usually financed by borrowing of some sort and so we would expect it to affect bank lending as well. A dummy variable was also included in an attempt to catch any effect of CCC on bank lending, but this was not significant and did not improve the power of the equation. Another dummy variable (LPER) to take into account periods when direct controls on bank lending were operative in the UK, was also entered in the equation. Thus our estimated equation is of the form

\[(49) \quad (\text{DSBLP/P}) = \text{DL(QDS, QCDE, (CBR-RLA), LPER)} \quad D_{1,2,4} >0; \quad \text{DL}_3 <0\]

As reported in equation (A42) in Appendix A, the explanatory power of the equation is quite satisfactory with a R² of 0.519, given that the dependent variable is the change in sterling bank lending and the well known difficulty of estimating this variable, especially in aggregate form. The coefficients are all significant and bear the a priori expected signs. Thus bank lending to the private sector is mainly determined by consumer durable expenditure.
and stockbuilding, unanticipated changes in the latter being usually financed by the use of overdraft facilities. Although, in common with Moore and Threadgold (1982) and NIESR (1979), we did find some support for round-tripping, we could not obtain a significant own interest rate effect\(^{(28)}\). This to some extent confirms the findings of the survey on bank lending by Hotson (1979), which showed that empirical work does not reveal a marked response of bank lending to interest rate changes in the short run, although there is some response in the longer run. This is perhaps not too surprising as a very high proportion of bank lending is to the corporate sector whose borrowing is mainly determined by the components of company working capital needs (Moore and Threadgold, 1980, 1982). Thus the ability of the Bank of England to control the rate of growth of the money supply by influencing bank lending via changes in interest rates is quite limited. The significance of the dummy variable (LPER) indicates, though, that supply side effects appear to have been important in the early 1970s.

3.7 Interest Rates

We have so far taken interest rates as given. However, only one of the interest rates in the model, namely, the minimum lending rate (MLR) is regarded as a policy instrument; the others are determined endogenously and are influenced either directly or indirectly by MLR.

3.7.1 Consol Rate

In the UK open market operations are concentrated in the discount market rather than in the gilt-edged market, the aim of the authorities being to influence short-term rates and hence the whole structure of interest rates. In fact, open market operations in long-term debt are quite restrained (Artis and Lewis, 1981, p.63). Thus we would expect the long-term rate of interest, that is, the 2\(\frac{1}{2}\) per cent consol rate (RCL) to depend on, amongst other things, the short-term rate. Consols are supposed to provide a hedge against inflation; therefore, we would also expect RCL to be influenced by the expected rate of inflation. We proxy the expected rate of inflation by the actual rate of change of prices (RP). Thus, in common with most econometric models, we do not model how agents form their expectations explicitly and it could be that this explains the poor performance of this equation, as expectations play a very important role in determining the term structure of interest rates (see, for example, Goodhart, 1975). However, modelling expectations is a very difficult problem indeed, partly because data on expectations is quite scarce (see, for example, Ormerod, 1979). When modelling the long rate, income is usually included as a regressor to account for the fact that, as economic activity expands, this stimulates the demand for loans which is expected to put upward pressure on
RCL. However, we could not obtain this effect. Thus our preferred estimated equation (which is in differenced form) is a term structure relationship with the long rate depending on the short rate and inflation only.

\[ \Delta RCL = RCL(\Delta RLA, \Delta RP) \quad RCL_{1,2} > 0 \]

As can be seen from equation (A43) in Appendix A, the standard error is quite big at 0.529, although the R\(^2\) is perhaps acceptable at 0.322. As we will see below, this equation does not simulate well at all, although both RLA and RP simulate satisfactorily indicating that the equation itself is not very satisfactory. However, all kinds of variants in linear and log-linear forms were tried, including income, as well as a foreign rate as regressors, but no improvement was possible. The latter (yield on U.S. long-term government bond) was included to test the hypothesis that the UK long rate should bear a relationship with the long rate overseas. Higher lags for ARLA and ARP were also tried, but were found to be insignificant. Perhaps some lag polynomials, like the Almon lags in NIESR (1979), are needed to get better results.

### 3.7.2 Local authority rate and certificate of deposit rate

Following NIESR (1979), the local authority rate (RLA) is set equal to the certificate of deposit rate (CDR):

\[ RLA = CDR \]

and the latter is in turn determined by what could be called an inverted supply curve:

\[ CDR = MLR + CDR \left( \left( \frac{DSBLP}{P} \right)_{-1} \right) \quad CDR_{1} > 0 \]

Banks bid for wholesale deposits by adjusting the rate that they offer on these deposits. The higher the demand for advances, the higher the amount of wholesale deposits needed for banks to balance their balance sheets, and hence the higher the rate has to be set in excess of the rate of return on reserve assets (proxied by MLR). It must be said, though, that this specification "does less than justice to the theory that the rate moves in accordance with disequilibrium in the wholesale money market." (Mayes and Savage, 1980, p.8). The estimated equation is reported as equation (A44') in Appendix A and is very satisfactory, with an R\(^2\) of 0.967. The lags on the advances terms turned out to be of one and two quarters.

### 3.7.3 7-day deposits rate

The rate on 7-day deposits (RD) is assumed to be set in relation to the level of short-term market rates, thus

\[ RD = RD(RLA, RD_{-1}) \quad RD_{1,2} > 0 \]

Equation (A45) in Appendix A reveals that the estimates are quite satisfactory with an R\(^2\) of 0.964.
3.7.4 Clearing banks' base rate

The remaining interest rate in the model, the base rate of the clearing banks (CBR), is simply assumed to follow the minimum lending rate:

\[(53) \quad CBR = CBR(\text{MLR, MLR}_1) \quad CBR_{1,2} > 0\]

This is reported as equation (A46) in Appendix A, and is again quite satisfactory.

We have so far discussed the real and monetary sectors of the model. This means that we are only left with the foreign sector in order to close the model.

3.8 THE FOREIGN SECTOR

The change in foreign reserves (balance for official financing, DFR) is made up of the current balance (CB) and capital movements (DKF) thus:

\[(54) \quad DFR = CB + DKF\]

The current balance in turn is simply the difference between exports and imports of goods and services (explained by equations (12) and (13) respectively) plus a residual element (CBRES), i.e.

\[(55) \quad CB = PX.QEX - PM.QIM + CBRES\]

This leaves capital movement to be determined.

3.8.1 Capital movements

As with the rest of the model (except for the monetary sector), we do not disaggregate capital flows into short-term and long-term flows, but estimate total capital movements. This is unlike most studies of the UK capital account (see, for example, Argy and Hodjera, 1973; Hutton, 1977; and Beesock and Bell, 1979) which concentrate on the short end of the market. However, we assume that total capital movements depend on similar variables to those in the above studies, namely, interest rate and expectations of exchange rate movements. We have already mentioned that expectations in general are very difficult to model, and exchange rate expectations is no exception. The latter is very important in determining capital flows as it affects the yield upon realisation of the various interest rates.

Capital inflows are sometimes seen as reflecting the financing of current balance flows, so that the change in the current balance is sometimes also included as an argument in the equation (see, for example, Branson and Hill, 1971, and Fausten, 1975). The level of the current account can also be included as a regressor, but the expected sign on the coefficient is ambiguous (see, for example, Coghlan, 1981, p.115).

The above approach, which is the one we have adopted, is of course very different from the so-called monetary models (see, for example, Kouri and Porter, 1974 and Coghlan, 1979, 1981), which view capital flows as providing the mechanism by which to remove an excess demand for money, and thus include
variables which determine either the demand or the supply of money. We should perhaps also mention that modern portfolio theory (see, for example, Corner and Mayes, 1981) also include appropriate measures of relative risk as determinants of capital flows.

Our preferred estimated equation is of the form:

$$DKF = DKF ((RLA-RED)_{-1}, ER_{-1}, ACB) \quad DKF_1 > 0 \; ; \; DKF_2, 3 < 0$$

Thus capital movements are assumed to be determined, amongst other things, by domestic interest rates relative to foreign rates ($RLA-RED$) where the interest rate on euro-dollar deposits in London ($RED$) is used as the exogenously projected foreign rate. As can be seen from equation (A49) in Appendix A, the equation is satisfactory in terms of explanatory power, with an $R^2$ of 0.494, but the standard error is quite high at 0.692. In fact, in some periods the residuals are quite big, indicating that the equation does not perform satisfactorily in these periods. The lag on the interest rate term turned out to be of three quarters, indicating a rather slow response of capital movements to changes in relative interest rates. The exchange rate ($ER$), as well as the rate of change of the exchange rate, were tried, but the coefficient on the latter was not significantly different from zero. $ER$ as well as $ER_{-1}$ turned out to be significant, and although the contemporaneous term is positive, the sum of the coefficients on the two terms is still negative as expected. We could not obtain significant effects for both $CB$ and $ACB$ in the equation, but found that they were both significant when included on their own. We chose to leave $ACB$ in because it gave slightly better results statistically; also, this variable has been used successfully in a number of studies (see, for example, Coghlan, 1981 and Hoffman, 1980). Although it is very difficult to compare our results with earlier studies because of the level of aggregation we use, we should perhaps note that, in contrast to, for instance, Hutton (1977) and Beenstock and Bell (1979), we found a fairly large interest sensitivity of capital flows (although with a lag of three quarters).

### 3.8.2 The effective exchange rate

To model the capital account of the balance of payments, we also need an explanation of the determination of the exchange rate. Here, most of the large models of the UK economy follow the so-called (pseudo) reduced-form approach, where the exchange rate is regressed on variables which are thought to affect the balance of payments. As explained by Cuthbertson et al (1981), this approach causes not just a problem of estimation but, more importantly, introduces an element of arbitrariness in the analysis. This is because "given the structural specification of the reduced form is in principle unique, but the practice of selecting an endogenous set of
variables to appear on the RHS depends on a number of arbitrary decisions about which markets to consider. and the decisions whether to represent equilibrium in each market in terms of prices (or interest rates) or quantities." (p.23). For these reasons we chose not to follow this approach, but instead to follow the 'reaction-function' approach.

Our equation for the rate of change of the exchange rate (RER) is based on the work by Coghlan (1981) and could be interpreted as a reaction function which tries to explain the behaviour of the authorities. However, since 1977 monetary targets seem to have carried more weight with the authorities than exchange rate considerations, which implies that the exchange rate should not be considered as an intermediate target which could be forecasted from a reaction function. While this may have been true for some time since 1977, the authorities seem presently to pursue both monetary and exchange rate objectives, as the Governor of the Bank of England pointed out in a recent speech. This is in line with our work, as we consider different regimens of operating monetary policy including an exchange rate and monetary aggregate regimen.

Relative prices are important determinants of the exchange rate because of their impact on competitiveness and domestic inflation. Thus an increase in the dollar price of imports (PM$) could be negated by an increase in the exchange rate, whilst the impact of an increase in the price of exports (PX) on competitiveness could be reduced by a decrease in the exchange rate, albeit at the cost of higher inflation. A worsening of the current balance would call forth a fall in the exchange rate and so we would expect CB to affect RER positively, whilst we would also expect a positive interest rate effect.

Thus our preferred estimated equation is of the form:

\[ (57) \quad RER = RER(RMP$, RXP, (RΔA-RED)_{-1}, (CB/P), RER_{-1}) \]

\[ RER_{1,3,4,5} > 0; RER_{2} < 0; \]

where \( RER = (ER-ER_{-4})/ER_{-4}, \)

\[ RMP$ = (PM$-PM$_{-4})/PM$_{-4}, \]

and \( RXP = (PX-PX$_{-4})/PX$_{-4}. \) \( RER_{-1} \) is also included as a regressor to allow for lags in adjustment.

As can be seen from equation (A50) in Appendix A, the estimates are quite satisfactory. An indication of the goodness of fit is provided by a standard error of 0.026 and a \( R^2 \) of 0.89. All the coefficients bear the a priori expected sign and are statistically significant. The lag on the interest rate term turned out to be of two quarters, but the interesting point to note is that we did not get the usual 'perverse' sign on interest rates (33). Equation (A50) implies that there is a rather strong reaction
of the exchange rate to variations in competitiveness, thus confirming the findings of Coghlan (1981) and Hoffman (1980). The response to changes in the current balance, on the other hand, seems to be quite small. This is in contrast to Coghlan (1981) who found a stronger response, but more in line with Hoffman (1980) who failed to get a positive response. The coefficient on the lagged dependent variable turned out to be 0.28, indicating a smooth adjustment of the exchange rate to changes in import and export prices and to changes in the current balance, as well as in relative interest rates.

3.9 Simulation of the model
Up to now we have been concerned with the structure and properties of the individual equations in the model. It is a well know fact that, although single equations may have very good statistical fit, the model as a whole may not perform as well in tracking the historical data. Thus we turn our attention to the simulation properties of the model as a whole. This is also important because the simulations are used as a base for the optimization exercises in chapter 4.

A number of criteria have been used to judge how the model tracks the movement of some key variables (see, for example, Pindyck and Rubinfeld, 1976). The most common criterion is the Root Mean-Square Simulation Error (RMSE), which provides a measure of the difference between the actual path of a variable and its simulated one, and is defined as

\[
RMSE = \left[ \frac{1}{T} \sum_{t=1}^{T} (Y_t^S - Y_t^A)^2 \right]^{\frac{1}{2}}
\]

where \( Y_t^A \) is actual value taken by a certain variable, \( Y_t^S \) is simulated value for that variable, and \( T \) is the number of periods used for simulation. The criterion which we use here, however, is the RMSE expressed in percentage form thus:

\[
RMSE\% = \left[ \frac{1}{T} \sum_{t=1}^{T} \left( \frac{Y_t^S - Y_t^A}{Y_t^A} \right)^2 \right]^{\frac{1}{2}}
\]

This, unlike RMSE, penalizes large errors more, irrespective of their sign, and thus does not suffer from the problem of negative errors cancelling out positive ones. The other criterion which we use is how well the model tracks down turning points in some key endogenous variables. This second criterion is considered by many researchers to be even more important than RMSE\%. To test the sensitivity of the model to the initial period of simulation, the model was first simulated over the five years starting at 1972(2), and then the simulation time horizon was moved to the last four
years of the sample period, that is, from 1977(1) to 1980(4) - which is the period over which the simulations reported below are based. No significant change in the performance of the model was found, which implies that it is not sensitive to the initial period of simulation.

Overall, the simulation performance of the model is satisfactory; the RMSE% are quite low and figures (1-9) below show that most turning points are tracked quite satisfactorily. (In all these figures, S1M1 gives the simulated path and RUNFILE the actual path). Thus the general long-run behaviour of the actual series seems to be reproduced by the simulated series. However, there are some variables which do not perform satisfactorily in terms of RMSE%, as well as in the ability to track turning points. These are employment, the consol rate, stockbuilding and capital movements. We should note that, as we have seen, these variables do not perform very well as single equations, so that it is not too surprising that they are not well tracked in the simulations.

We will only present figures for some key variables, including most of the variables in the monetary sector, as these are the most important ones for the problem at hand. In spite of the level of aggregation, the results for the real sector are quite satisfactory. As can be seen from figure 1, the tracking performance of GDP is reasonable. Although it consistently under-predicts - except for the final year when it over-predicts - the errors in prediction are quite low, a measure of which is provided by the RMSE% which stands at only 1.5 per cent. The main endogenous components of GDP all give relatively low RMSE%, with consumers' expenditure yielding the lowest value at 1.9 per cent and fixed investment the highest at 5.0 per cent, and with imports and exports giving values of 4.3 per cent and 3.3 per cent respectively. Since prices play a key role in any macro model, it is comforting to see from figure 2 that the GDP deflator - around which the other prices indices in the model are built - is tracked rather well and has a RMSE% of only 1.8 per cent. The wage rate is usually very difficult to predict in the UK, as this relationship is quite unstable (Ormerod, 1979, p.121). However, as can be seen from figure 3, it is predicted with reasonable precision in our model, and has a RMSE% of only 3.5 per cent. The exchange rate is consistently under-predicted; a possible explanation is that it is partly determined by the rate of change of export prices which is, in fact, over-predicted. The reason for this is that the latter is determined by a technical relationship rather than a behavioural one which is, of course, less satisfactory. However, apart from the last two quarters of 1977 and the last three quarters of 1980, it follows the historical path reasonably well and has a RMSE% of 5.8 per cent, which is again quite satisfactory given that this is another relationship
FIGURE 1

GROSS DOMESTIC PRODUCT (QY)

RMSE% = 1.5%

FIGURE 2

GDP deflator (P)

RMSE% = 1.8%

104
FIGURE 3
WAGE RATE (WR)
(RMSE% = 3.5%)

FIGURE 4
EXCHANGE RATE (ER)
(RMSE% = 5.8%)
which researchers find very difficult to model. The errors in the simulation for the periods mentioned above may be due to the fact that, since October 1977, the authorities have been putting a lot of emphasis on monetary targets, and when there has been a sharp conflict between the money supply and the exchange rate objectives - as in 1977 and in 1979/80 - the Bank of England seems to have given priority to the money supply objective. In those circumstances forecasting the exchange rate by means of a reaction function may no longer be appropriate.

Figures (5-8) show the four main elements of the monetary sector. The monetary aggregate M3 in figure 5 seems to be the least well predicted variable of the four, with a RMSE% of 10.8 per cent, although M3 tends to move towards its actual value by the end of the simulation period. A careful look at the components of M3 reveals that, in fact, the errors in the latter compound to inflate that in the former. Bank lending in particular - which accounts for a high percentage of changes in the money supply - is generally over-predicted, as can be seen from figure 6, and has a RMSE% of 7.8 per cent. Sales of public sector debt in figure 7 are quite well predicted and have a RMSE% of only 2.4 per cent, and although it is also generally slightly over-predicted, this is not enough to offset the effect of bank lending on the money supply. The demand for money M1 - which is part of the broader aggregate M3 - is quite well predicted, as can be seen from figure 8, and has a RMSE% of only 2.8 per cent. We also present a figure for the local authority rate, as this is considered as an alternative intermediate target in this study. Figure 9 shows that RLA is predicted reasonably well with all the turning points tracked successfully, although the RMSE% is 8.1 per cent.

Although our model simulates satisfactorily, it is instructive to investigate whether or not it responds to changes in certain policy variables in a way which is consistent with empirical observation as well as economic theory. One useful way of investigating the dynamic behaviour of the model is to examine its dynamic multipliers. (see, for example, Klein, 1974). For a non-linear model such as ours this is done as follows: the model is first simulated using actual (historical) values for the exogenous variables to obtain the 'control solution'. A new 'disturbed solution' is then obtained by giving specific values to the exogenous variables and simulating again. The ratio of the difference between, on the one hand, the 'control' and 'disturbed' solution, and on the other hand the actual and 'disturbed' exogenous variables, then gives the dynamic multipliers of the system. The two policy instruments in our model are the level of government expenditure (QG) and the minimum lending rate (MLR). We run two exercises; the first one is an expansionary fiscal policy whereby there is a step increase
FIGURE 5
MONEY (M3)
(RMSE% = 10.8%)

FIGURE 6
STERLING BANK LENDING (SBLP)
(RMSE% = 7.8%)
FIGURE 9
LOCAL AUTHORITY RATE (RLA)
(RMSE% = 8.1%)
in QG of £100 million over the entire simulation period (1977(1)-1980(4)).
In the second exercise we look at the effect of a contractionary monetary policy in the form of a step increase in MLR of 1 per cent, again over the same period.

The dynamic multipliers of government expenditure (QG) for some key endogenous variables are presented in table 1a below. Although the shock itself is one of £100 million increase in QG, the figures are adjusted so as to show the impact of a £1 billion increase so that the multipliers can be read directly from the table. We also show the results of the exercise in a schematic way in figure 10. Although this does not show the dynamic response of the model to exogenous impulses, it nevertheless adds to an understanding of the various links in the model. (This will be helpful in interpreting the results of the exercises in chapter 4). The impact multiplier (first period change) of a unit increase in government expenditure is 1.08 for nominal GDP and 0.89 for real GDP. In our model, the direct impacts include the direct 'fiscal impact' of an increase in QG on aggregate expenditure, as well as the direct 'portfolio impact' on consumption and fixed and inventory investment due to the change in the supply of public debt on interest rates. The feedback responses of the initial change in income include positive responses on consumption, fixed and inventory investment and imports, and negative responses on these variables (except imports) due to induced rises in interest rates. As can be seen from table 1a, these feedback effects are rather small, except for imports, so that the increase in income is sustained throughout the whole period. There is some crowding out as higher interest rates bring down consumers' expenditure (but not enough to bring investment down), but this only starts to occur after a period of two and a half years. There are, in fact, two other feedback effects on consumption present in the model and they both come about via real personal disposable income; firstly, there is the effect of increased tax payments due to increased income and, secondly, the effect of higher prices on QYD. (This is not withstanding the direct inflationary effect on QCE). Because of the cost push nature of our price equation, however, the effect of an increase in government expenditure on inflation is pretty small, so that this last effect is rather minimal. The increase in public sector borrowing from the banking sector, and the increase in bank lending to the private sector through increased economic activity, result in an increase in the money stock M3. The increase in sales of public sector debt and the slowdown in bank lending to the private sector due to the fall in consumer durable expenditure (after 2½ years), as well as higher interest rates, is not enough to prevent M3 from increasing throughout the whole of the simulation period.
### Table 1a

GOVERNMENT EXPENDITURE MULTIPLIERS

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<th>Period</th>
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<th>QKP</th>
<th>QIM</th>
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<td>0.03</td>
<td>0.02</td>
<td>0.35</td>
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<td>0.02</td>
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<td>0.02</td>
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<td>0.02</td>
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### Table 1b

INCREASE IN MLR

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<th>QEX</th>
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</thead>
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<td>-30</td>
<td>-4</td>
<td>10</td>
<td>-76</td>
</tr>
</tbody>
</table>
FIGURE 10
INCREMENT IN QG
Table 1b shows the results of the second exercise. (A schematic representation is given in figure 11). Here, the results cannot be interpreted as dynamic multipliers because of the different units of measurement involved. A 1 per cent increase in MLR results in a decrease of £50 million in nominal GDP and £41 million in real GDP in the first period. The increase in MLR of 1 per cent per quarter implies an increase in all the domestic rates in the model with resultant decreases in consumption, fixed and inventory investment, and hence output and employment. The higher interest rates also result in capital inflows, which in turn brings about an appreciation of the exchange rate. This implies that exports come down but imports also decrease due to the fall in economic activity, so that there is, in fact, an improvement in the current balance. This latter result is also due to the fall in sterling price of imports due to the appreciation of the exchange rate. Thus domestic inflation in general improves, which also brings down wages. Lower economic activity and high interest rates also result in a decrease in bank lending to the private sector, and with bank lending to the public sector down as well, this implies a decrease in the money stock M3.

3.10 The derivation of the 'efficient' Phillips Curve

In the previous section an insight into the dynamic properties of the model was gained by simulating the model and working out certain dynamic multipliers. In this section, we will attempt to further evaluate these properties using a different technique.

Given a certain model, economists are often concerned with the question of the trade-off relationships between inflation and unemployment implicit in the model. In a dynamic setting, we have to deal with a three-dimensional space, with inflation, unemployment and time as the axes. By manipulating the instruments at their disposal, we know that the authorities can reach certain points in this space, but not all of them. For instance, for any time period they may not be able to reduce inflation in that period without increasing unemployment in the same or another period, or inflation itself in another period. This trade-off over time is usually different from one model to another, so that it would be instructive to work out this particular dynamic property of our model. This kind of analysis cannot be made using the simulation and dynamic multiplier techniques of the last section, and we have to resort to optimal control techniques. These techniques can help us to investigate, in an efficient way, the trade-off relationships between inflation and unemployment over time implicit in the model as a whole. The optimization framework used in this section (and in the next chapter) is presented in Appendix B.
The important point about this efficient or Pareto optimal Phillips curve is that it takes into account not only the interrelationships that exist between inflation and unemployment in the model, but also the preferences of the policy makers in terms of their two targets (U and CP) and instruments (MLR and QG). Also, since the evaluation of the trade-off between U and CP is made on a point of the 'efficient frontier', then that point cannot be dominated (Chow, 1981, p.111).

The exercises that follow are exactly similar to those in Henry, Karakitsos and Savage (1982). The initial (basic) optimization which is then used as a benchmark, uses the objective function as represented in table 1 in chapter 4, except that the CB target is now omitted. The optimization horizon is over the four years from 1977(1) to 1980(4). Four more separate optimizations were carried out with the weight on inflation first reduced by 50 per cent from 1 to 0.5, and then increased successively by 50, 100 and 200 per cent to 1.5, 2.0 and 3.0 respectively. Thus attention is first directed away from inflation towards unemployment, and then this is reverted progressively towards inflation and away from unemployment(35). Figure 12 provides a numerical approximation to the 'efficient Phillips curve' of our model. The shape of the locus indicates that as the relative importance of inflation in the cost function is increased, the average values of both CP and the rate of unemployment (UP) decrease at first and then the decrease in inflation is only at the expense of higher unemployment. We should perhaps note that our model does not contain a Phillips curve as such, but instead has a real wage specification in which there are no direct labour demand effects, so that a negative slope is not necessarily to be expected. This feature of an optimum becomes more apparent in figure 13, which reveals the dynamic adjustments in inflation and unemployment. The four lines labelled (1) to (4) show the inflation unemployment trade-off for each of the four years. The points are generated by changing the target structure in the same way as figure 12, so that the line labelled (1), for example, represents the trade-off that results after one year of the optimization, line (2) after 2 years and so on. The interesting result, which is also found by Henry, Karakitsos and Savage (1982) for the National Institute model, is that the efficient Phillips curve rotates over time. The striking feature of our model, though, (and this is not apparent in the National Institute model although they also get a kink in the locus) is that there seems to be an optimum point; we can reduce both inflation and unemployment until that optimum is reached, after which we have to trade-off one target for the other. Moreover, this optimum only becomes apparent after the second year. The shape of the loci in figures 12 and 13 is due not only to the specification of the model, but depends also on the fact that optimization
FIGURE 12
CHANGE IN WEIGHT ON INFLATION

FIGURE 13
CHANGE IN WEIGHT ON INFLATION
generates the optimal instrument-mix to reduce both inflation and unemploy-
ment according to their weighting in the cost function. 
As we increase the relative importance of inflation by changing the target 
structure of the objective function, we observe that there is a gradual 
appreciation of the exchange rate which brings about the fall in inflation. 
The exchange rate appreciation comes about through an increase in MLR. 
We also repeated these exercises, but this time reducing the weights on 
unemployment by 50 per cent from 150 to 75, and then increasing the weights 
successively by 50, 100 and 200 per cent to 225, 300 and 450 respectively. 
As can be seen from figures 14 and 15 (which are built in the same way as 
figures 12 and 13), as the priority of reducing unemployment is increased 
the cost in terms of inflation gets higher and higher over time, until in 
the fourth year, when inflation is running at very high levels (at 19.35 
per cent in the last quarter of the optimization horizon when the weight 
on U has been increased by 200 per cent), both inflation and unemployment 
increase after a certain point. Thus when inflation is running at very 
high levels, it is very difficult to decrease unemployment even by increas-
ing its weight in the objective function to very high levels. Again, this 
is the result of the optimal instrument mix generated by optimal control 
with the given target structure of the objective function and the given 
specification of the model.
NOTES

(1) RPC is defined as \((PC_{-\theta})/PC\), where PC is consumer prices.

(2) A liquid assets variable was tried, but that equation did not perform as well as the above equation. Real financial wealth was also tried instead of liquid assets, but was found to be insignificant. Had either of these two variables been included in the preferred equation, this would probably have favoured regimens involving a monetary aggregate - see chapter 2.

(3) The hypothesis is that the restricted transformed equation and the unrestricted transformed equation are equally valid; see Sargan (1964) and Hendry and Srba (1978) and Appendix A. This test of the dynamic specification of the equation was performed for all the equations, although it is not always discussed in the text.

(4) In large models of the UK economy, like LBS (1981), separate equations are also estimated for consumption of drink and tobacco, energy, and petrol. This kind of disaggregation is certainly not necessary for our purposes.

(5) See, for example, Junankar (1972) p.20, Mayes (1981), p.120.

(6) There are, of course, alternative approaches to investment behaviour. See Mayes (1981) for a survey. We opted for this one because it is the most common one used in models of the UK economy.

(7) For example, Hines and Caterphores (1970), using a modified flexible accelerator model, found lags of nine quarters on interest rates and six quarters on output, in explaining investment demand in the manufacturing industry.

(8) The cost function is not usually specified. An exception is Mills (1962), but his study concentrates on the micro side.

(9) Enoch (1978) provides a useful study of this problem.

(10) The value of UK export demand elasticities ranges from 0.6 (Hutton and Minford, 1975) to 1.0 (Duffy and Renton, 1970). For a tabular summary of these elasticities, see Arestis and Hadjimatheou (1982b, p.86).

(11) This sort of disaggregation is done by, for example, LBS (1981) and NIESR (1979). The latter's income elasticities for (i) and (ii) are around 0.5 (Mayes, 1981, p.360), which is lower than our value of 0.65.

(12) For a tabular summary of these elasticities, see Arestis and Hadjimatheou (1982b, p.81).

(13) For instance, NIESR (1979) have separate equations for goods and services and LSB (1981) distinguish between imports of fuels, basic materials and manufactures.
(14) This is to reflect the fact that an increase in employment does not imply an equal decrease in the number of registered unemployed, since some of the newly employed may not have been previously registered as unemployed.

(15) See Friedman (1968) and Phelps (1968).

(16) This derivation follows Henry et al (1976).

(17) For this particular equation, see Henry, Karakitsos and Savage (1982).

(18) Again, the lagged dependent variable is included in the estimated equation to bring down the $\chi^2$ statistic.


(20) This effect is absent in NIESR (1979), as their currency equation does not include any interest rate term. LBS (1981), on the other hand, include a long term rate (RCL) in their equation.

(21) See, for example, Artis and Lewis (1981), p.76.

(22) Proxied by the actual rate of inflation.

(23) See, for example, Artis and Lewis (1981).

(24) Mills (1978) actually uses total final expenditure of GDP as the income variable.

(25) For a tabular summary of these elasticities, see Artis and Lewis (1981), p.18.

(26) The corset, which was meant for temporary use anyway, was finally abolished in 1980.

(27) CBR is actually the base rate plus 2 per cent, as this is what the lending rate is on average.

(28) Moore and Threadgold (1982) found a significant effect for the real rate of interest. Their study is concerned with bank lending to the corporate sector only and is thus, strictly speaking, not comparable to ours.

(29) The structure of the relationships for all the interest rates in the model (except RCL) is heavily influenced by NIESR (1979).

(30) The coefficient was not actually imposed at unity, but was estimated (its value turned out to be 0.99) because we could not handle this restriction on 'GIVE'.


(32) Speech delivered at the Lord Mayor's dinner to the bankers and merchants of the City of London on 15 October 1981; see chapter 1.
(33) This is partly due to the interest rates we selected. When we tried the US treasury bill rate instead of RED as the foreign rate, we did get a perverse sign on (RLA-RUS) or (RTB-RUS) where RTB is the domestic rate on treasury bills. The perverse sign would reverse the direction of causality in the exchange rate equation in that interest rates would now depend on actual and desired exchange rates.

(34) This does not seem to be the case any longer, see chapter 1.

(35) For more details of how the locus is constructed, see Henry, Karakitsos and Savage (1982).
4.1 Introduction

In the theoretical exercises of chapter (2), we examined the monetary instrument problem with the aid of the following four regimens:

(i) Fixed Exchange Rate, Interest Rate Policy
(ii) Fixed Exchange Rate, Money Supply Policy
(iii) Flexible Exchange Rate, Money Supply Policy
(iv) Flexible Exchange Rate, Interest Rate Policy.

We investigated which of these four regimens best stabilized an economy perturbed by various domestic and foreign stochastic disturbances. In this chapter, we are interested in an empirical evaluation of the results we obtained there. The analysis permits us to relax the constraints underlying regimens (i) to (iv) and thus examine the discretionary counterparts of these regimens as well. The optimization framework used in this study is discussed in appendix B. Here, we will simply note that the optimization problem is to minimize an objective function subject to the constraint imposed by the model. The latter was described in the last chapter, so that our immediate task is to specify a satisfactory objective function. Sections 4.3 to 4.6 then describe the various pegging and discretionary regimens and section 4.7 examines the response of the system to various perturbations with a summary of all the results in section 4.8.

4.2 The Objective Function

The difficulty in specifying an objective function is that we need to know the policy makers desired values for the targets and instruments as well as their relative priorities, and this information is not readily available. Here we follow the approach of Westcott et al (1981). Table 1 shows the initial specification of the objective function which will remain unchanged for all the regimens that we examine except that other targets will be added. The three targets we consider are current balance (CB), inflation (CP) and unemployment (U), and the two instruments used to achieve these targets are the minimum lending rate (MLR) and government expenditure (QG). Recall that in the theoretical exercises of chapter 2 we had income instead of unemployment, and the change in foreign reserves instead of the current balance in the objective function. However, the chosen targets in this chapter are more commonly used in the control literature (see, for example, Currie and Karakitsos, 1980; Westcott et al, 1981). In any case, since U
### Table 1

**INITIAL SPECIFICATION OF THE OBJECTIVE FUNCTION**

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<td>U</td>
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<td>QG</td>
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<tr>
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<td>(%)</td>
<td>(m)</td>
<td>(%)</td>
<td>(Ebn)</td>
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is in fact related to income, its usage should not make much difference to our results. Also, the use of CB enables us to examine systems other than purely flexible exchange rates.

The specification of the objective function should reflect the priorities of the policy makers and the latter will determine the optimal paths of the targets and instruments. The $R_t$-line shows changes in each variable which the policy makers rate of equal priority, taking a one per cent reduction in inflation as the standard. For example, a reduction of 71 thousand in unemployment has equal priority to a 1 per cent reduction in inflation. The $x_t$ line indicates the levels at which this judgement is made (e.g. 12.91 per cent for inflation and 1.72 million for unemployment). For the targets, these figures are the average simulated values while they are the average of actual values for the instruments. The third line in table 1 indicates the desired values ($x_t^d$) for both targets and instruments and, in the case of targets, are well outside easy reach except perhaps for the current balance. The next line indicates the priorities ($P_t = 12.91/R_t$) of the policy makers at levels $x_t$, where 12.91 is the value of CP used as the standard. The last line represents the weights which correspond to the priorities determined in the $P_t$-line. In order to avoid unnecessary fluctuations in the instruments, we also have in the objective function the first difference of the instruments (DMLR and DQG) with zero desired values and appropriate weighting (see Westcott et al, 1981, p.46).

3 An Optimal Fully Discretionary Regimen

We first discuss an optimal fully discretionary regimen (OPTD), that is, one in which there are no intermediate targets and the instruments are directed straight at the ultimate targets. The assumptions underlying this run can be obtained from table 1. This basic optimization will provide a sort of benchmark which we will refer to frequently below. Therefore, we will describe it in some detail.

The optimal policy mix in this fully discretionary regimen entails expansionary fiscal policy throughout the four year horizon, involving an increase in government expenditure (QG) from its historical trajectory by £1.2 billion on average over the four year period of optimization, plus contractionary monetary policy in the first three years of the optimization, involving an increase in MLR over its historical path followed by expansionary monetary policy in the form of a decrease in MLR in the final year of the optimization.

The consequences of these policies are a decrease in inflation, a reduction in unemployment and an improvement in the current balance. The results are
summarized in table 2. The beneficiary effect on inflation only starts in
the last quarter of the first year of the optimization run and is on average
0.87 percentage points lower over the four years than the historical path,
with the most influential effect being felt in the third year. The effects
on unemployment follow the same pattern in that it only starts falling in
the fourth quarter, and the major effects are again felt in the third year
with unemployment being lower on average by 133 thousand over the four years.
The current balance is still in deficit although there is a significant
improvement and this comes about not through an increase in real exports,
which actually decrease (average -£81 million), but through falling infla-
tion. These results are consistent with our findings in the discussion of
the 'efficient Phillips curve' in the last chapter. Recall that we found
that so long as the relative weight of inflation in the cost function is not
too high, both unemployment and inflation could be reduced simultaneously by
employing QG and MLR efficiently. The important point to note here is that,
given that inflation in our model is of the cost-push type and wage inflation
is not affected by demand pressures, government expenditure cannot be used to
influence inflation in any significant way, but may be directed instead
towards unemployment without much fear of creating inflation. It is then
left for monetary policy to influence inflation by its impact on the exchange
rate (these properties are confirmed by the simulation exercises in the last
chapter).
Thus the increase in aggregate demand via increased government expenditure
(average +£1184 million) causes an increase in income (average +£563 million)
and a decrease in unemployment, but also increases real imports (average
+£367 million). However, the higher interest rates result in an inflow of
capital which puts pressure on the exchange rate to appreciate (average
+0.051). This implies that the sterling price of imports decreases and this
more than offset the higher propensity to import (average +£367 million)
because of increased income, producing as a result an improvement in the
current balance. Lower sterling import prices also mean lower domestic
prices and hence a lower wage rate (average -0.028) which further depresses
domestic inflation. Thus the transmission mechanism of monetary policy is
mainly through the impact of the exchange rate on activity and prices.
Although interest rates affect consumers' expenditure, fixed and inventory
investment as well as demand for currency and debt sales, their main role is
to influence the path of the exchange rate. (These links in the model are
shown in figures 10 and 11 in chapter 3). As can be seen from figure 1, the
optimal fiscal-monetary mix of the fully discretionary regimen leads to a
<table>
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<th>Instruments</th>
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(OPTD - S1MD) where OPTD = Optimisation , S1MD = Simulation

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</table>
rightward shift of IS curve, together with a leftward followed by a rightward shift of the LM curve as monetary policy reverses to a more expansionary stance, resulting in an increase in income from point A to B. Point B is the four year average new level of income. The CB curve also shifts rightward to indicate an improvement in the current balance.

4.4 Rules and Discretion
As we saw in chapter (2), the rules of regimens (i) to (iv) imply that we are operating monetary policy as a two-stage strategy where at one level the optimal target paths of some intermediate variables are calculated and, at the other level, the instruments are manipulated in order to achieve those target paths for the intermediate variables. Pegging regimens implies that once a rule has been found, it is kept unchanged for a rather long period of time even though new information is available; the instruments respond to the new information only in so far as to bring the intermediate variables back on course if they happen to be off target. Using the techniques of optimal control, we need not dichotomize decision taking into two levels, but instead look at the whole process in one optimization run; that
is, when looking at the regimens involving rules, we need not do an optimization run to find out what the rules should be and then optimize again to steer the economy towards achieving these rules. What optimal control enables us to do is to find the optimal instrument-mix aimed at not just the ultimate targets, but at the intermediate targets as well. Thus we avoid the possibility of diverting attention away from the ultimate targets, as well as the problem of 'recursiveness' emphasised by Bryant (1980) whereby a two-stage dichotomization would not take the simultaneity between the intermediate target and the endogenous variables into account. (See chapter 2 above). This analysis also enables us to drop the assumption made in the theoretical exercises that the intermediate targets were actually totally met, but recognize instead that they are targets to aim for (together with the ultimate targets).

The theoretical regimens (i) to (iv) all involved 'rules' of some kind or another, but these were not arbitrarily defined rules but optimally derived ones. Recall that given the assumptions that we made there, the optimal policy was to choose the instruments in each of the four regimens so that the autoregressive structure of the model was completely destroyed. These crucial assumptions were that (i) all the coefficients in the model were deterministic, (ii) there were as many instruments as targets, and (iii) control was costless. At the empirical level, it is very easy to get rid of the last two assumptions, but not the first one. In this study we do not attempt to tackle the problem of non-deterministic coefficients. This problem is a very complicated one for a non-linear model such as ours, and only approximate solutions are available (see Chow, 1976; Karakitsos, Rustem and Zarrop, 1981). In the empirical exercises the use of optimization techniques enables us to derive the optimal rules without the restrictive assumptions (ii) and (iii). (This is clearly revealed in table 1).

In each of the four pegging regimens, the optimal rule is defined as that rule which, given the policy makers objectives (as expressed in the objective function) and the model in use, would satisfy these objectives as closely as possible in terms of achieving the lowest value of the cost function (see Karakitsos and Rustem, 1981). As compared to the optimal fully discretionary regimen of the last section, each of the four 'rules' or pegging regimens has two additional constraints. As is explained in appendix B, this restriction implies that the total cost in the discretionary policy is never greater than that for the rule. As we will find out below, however, it does not necessarily follow that each single target has to perform better under the discretionary policy. For each of the regimens the nearer the optimal paths of the target variables (inflation, unemployment and current balance) to
their desired values, the better it makes that particular regimen. If we assume that the preferences of the policy makers, as embodied in the objective function, do not change over the optimization horizon, then we can compare the different regimens. To do this we must keep the same time horizon for all the regimens and this is taken to be the four years from 1977 (1) to 1980 (4). Also, values for the exogenous variables in these four years of the optimization horizon are assumed to be known for all the regimens. We also keep the same objective function throughout (except, of course, for the two extra constraints), so that the assumptions regarding monetary and fiscal policies in all the regimens are those determined in the optimal fully discretionary regimen; thus government expenditure and MLR keep the same desired values and weights in all the regimens.

4.5 The Pegging Regimens

The four pegging regimens we examine are similar to those in chapter 2. The exchange rate and interest rate pegging regimen (OPT1) is similar to regimen (i) in our theoretical exercises. We want to derive simultaneously an optimal rule for the exchange rate as well as one for the interest rate. The interest rate we have chosen as the intermediate target is the short term local authority rate (RLA). To obtain the optimal rule, we require that the first derivative of the objective function with respect to the change in the exchange rate (DER) and the change in RLA (DRLA) to be equal to zero. To the original objective function (as in table 1) we thus add DER and DRLA with desired values of zero for all sixteen periods of the optimization run. DER and DRLA are then highly penalized except for the first period (weight of zero on both variables for the first period) until the paths of these two variables are as close to their desired values of zero as possible. Note that it is not possible to have both DER and DRLA exactly on target because they both carry high penalty weights implying that we have imposed two further restrictions to the objective function. This applies to all the pegging regimens we will be examining. This will give us the optimal values for ER and RLA. We now reproduce the optimal rule by replacing DER and DRLA in the objective function with ER and RLA with desired values of 0.6 and 8.5 for all sixteen periods, which are the optimal values derived above, and again increase the weights on the two variables until ER and RLA are as close to their desired values as possible. (This is only done in order to enable us to examine at a later stage the discretionary counterparts of this regimen). This is achieved by having weights of $10^7$ and $10^3$ on ER and RLA respectively.

The results are given in table 3, and show that the optimal rule calls for an expansionary monetary policy with an annual rate of growth of sterling M3 of 20 per cent on average over the four years but with big quarterly fluctuations, and
## TABLE 3

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<th>FOREIGN RESERVES PEG</th>
<th>EXCHANGE RATE PEG</th>
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<td>QEX, m</td>
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<td>QG, m</td>
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<table>
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<td>QG, m</td>
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All the values in this table (and similar ones below) are the difference between optimal and simulated ones. The year value is the average of four quarters.
a fiscal policy which is contractionary in the first and final years, but expansionary in the middle two years of the optimization period\(^{(1)}\). Although table 3 shows that there are big changes in MLR from its historical values, the optimal path of MLR itself is quite stable at around 7.6 per cent. The consequences of this optimal rule policy are that inflation is higher than it would otherwise be (especially in the fourth year) and unemployment is better in the middle two years, but much worse afterwards. In the final quarter of the fourth year unemployment is, in fact, 411 thousand more than it would otherwise have been. Also, there is a deterioration in the current balance.

The next regimen we examine, the exchange rate and monetary aggregate pegging regimens (OPT2), is similar to regimen (ii) in our theoretical exercises. We derive simultaneously an optimal rule for the exchange rate and the rate of growth of sterling M3. Again we keep the same assumptions about monetary and fiscal policies as in the optimal fully discretionary regimen. This time, we initially add to the objective function (as described in table 1) the terms DER and DGSM3 with desired values of zero for the whole of the optimization horizon, where GSM3 is the rate of growth of sterling M3 and is defined as

\[
GSM3 = \frac{SM3 - SM3_{-4}}{SM3_{-4}}
\]

Thus we require the first derivative of the objective function with respect to the change in the exchange rate and the change in the rate of growth of the money supply to be equal to zero to obtain the optimal rule. The same procedure as with the previous regimen is repeated, so that we end up with two extra constraints in the objective function, namely ER and GSM3 with desired values of 0.625 and 12.84 and weights of \(10^5\) and 100 respectively\(^{(2)}\).

The results of imposing this pegging regimen are reported in table 3 and show that keeping the rate of growth of the money supply on target implies that, compared to the simulated values, there is a reduction in GSM3 in the first two years followed by a rise in the last two years of the optimization. As far as MLR is concerned, however, except for the first part of the first year, the optimal path is much lower than the actual path. Fiscal policy is contractionary for the whole of the optimization horizon with government expenditure being on average £1.16 billion lower than the historical values. The consequences of these policies are a decrease in inflation in the first three years compared to the simulated values, but this then reverses itself and inflation increases to 15.15 per cent (a rise of 1.29 per cent) by the end of the fourth year. The same pattern emerges for the current balance,
which improves in the first two and a half years but deteriorates thereafter. As far as unemployment is concerned, it is much higher throughout the four years of the optimization and runs at 2.56 million (an increase of 212 thousand) at the end of the fourth year.

This brings us to the next regimen, the foreign reserves and monetary aggregate pegging regimen (OPT3), which is similar to regimen (iii) in chapter 2, except that we derive an optimal foreign reserves target instead of assuming a purely flexible exchange rate system. Once more the same procedure is repeated so that the objective function for this regimen includes DFR and GSM3 with desired values of 0.84 and 15.16 and weights of $10^8$ and 250 respectively. As reported in table 3, having a pegged rate of growth of the money supply implies this time that GSM3 is lower than its simulated values in the first seven quarters but higher thereafter. Quite big fluctuations in MLR are, however, needed to keep GSM3 on target with MLR being higher than its historical values almost throughout the whole of the optimization horizon. Fiscal policy is contractionary in the first year, but then reverses to a quite expansionary one thereafter, so that on average over the four years government expenditure is about £930 million higher than its historical value.

The consequences of this optimal policy mix in order to achieve the optimal rule are that inflation is lower than it would otherwise be (average -0.44%), but only starts to come down in the second quarter of the second year, whilst unemployment is higher than its simulated value for the first one and a half years and thereafter performs much better, especially in the fourth year when unemployment is reduced on average by about 250 thousand. There is also an improvement in the current balance in the first two years, and in the last two years the deterioration is minimal.

The last pegging regimen (OPT4) involves an optimal foreign reserves target together with an optimal interest rate target and is thus similar to regimen (iv) in the theoretical exercises. This implies that the objective function in table 1 has an additional two constraints, namely DFR and RIA, with desired values of 0.43 and 17.0 and weights of $3 \times 10^4$ and 300 respectively.

As can be seen from table 3, keeping the rate of interest on target implies very big fluctuations in the rate of growth of the money supply, which is above its simulated values for the first three years but lower for the fourth year, although MLR is above its historical value for the whole of the optimization horizon. Fiscal policy follows the same pattern in that government expenditure is very much above its historical path for the first three years and lower in the final year. Again the fluctuations are quite big and this is certainly not a very desirable feature.

The consequences of this optimal policy mix are that inflation is lower than it would otherwise be, although the beneficial effect only starts in the second
quarter of the second year. Unemployment is better throughout the whole optimization period except for the last two quarters. As for the current balance, it deteriorates in the first three years but improves in the final year of the optimization.

We turn now to a comparison of the four pegging regimens. This is done in terms of costs in table 4 and in terms of the paths of inflation and unemployment in these regimens in figures 2 and 3. Looking at the current balance first, table 4 indicates that the 'best' regimen amongst the pegging regimens in terms of minimum costs is the foreign reserves and monetary aggregate. Turning to inflation, we see that in terms of costs again the foreign reserves and monetary aggregate regimen fares better. From figure 2 we see that the worst regimen is definitely the exchange rate and interest rate pegging regimen (OPT1) (and this is confirmed in table 4), but there is no clear cut dominant regimen over the full four years of the optimization. However, OPT3 is clearly the 'best' regimen over the last two and a half years and is not much worse than the exchange rate and monetary aggregate regimen (OPT2) which dominates most of the earlier one and a half years. At the end of the optimization period inflation in OPT3 is falling and runs at 12.87 per cent compared to 13.73 per cent for the foreign reserves and interest rate regimen (OPT4), 15.15 per cent for OPT2 and 16.39 per cent for OPT1. As we saw in our discussion on the optimal fully discretionary regimen, the main channel through which monetary policy works in this model is through the exchange rate; the role that interest rates play, although they affect demand for public debt and consumers' expenditure and investment to some extent, is mainly in influencing the path of the exchange rate. In the foreign reserves and monetary aggregate pegging regimen there is, in fact, an appreciation of the exchange rate comparable to that in the optimal fully discretionary regimens; this explains the superiority of this regimen over the other pegging regimens where the exchange rate is kept fixed. Its superiority over OPT4 (where ER is also allowed to appreciate) is due to the fact that in the latter both fiscal and monetary policies are more expansionary, so that inflationary pressures are that much higher.

As far as unemployment is concerned, table 4 shows that it is OPT4 this time which is the 'best' regimen. However, figure 3 shows that the choice is a much more difficult one. Although unemployment is much lower in OPT4 than in the other three pegging regimens over the first three years, it is very much on an upward trend and increases from 1.06 million (its lowest level in the optimization horizon) in the third quarter of the second year to 2.52 million at the end of the optimization period. Over the last year of the
Table 4

CONTRIBUTIONS OF ULTIMATE TARGETS TO COSTS: PEGGING REGIMENS

<table>
<thead>
<tr>
<th>REGIMEN</th>
<th>CB</th>
<th>CF</th>
<th>U</th>
<th>Total contributions of ultimate targets</th>
<th>Total</th>
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<td>1931</td>
<td>5841</td>
<td>14578</td>
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<td>Exchange Rate &amp; Monetary Aggregate Regimen (OPT2)</td>
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<td>1354</td>
<td>2278</td>
<td>4218</td>
<td>6240</td>
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<td>1246</td>
<td>1673</td>
<td>3395</td>
<td>10626</td>
</tr>
<tr>
<td>Foreign Reserves &amp; Interest Rate Regimen (OPT4)</td>
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<td>1303</td>
<td>1374</td>
<td>4261</td>
<td>17204</td>
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<tr>
<td>Fully Discretionary Regimen (OPT0)</td>
<td>276</td>
<td>1161</td>
<td>1496</td>
<td>2933</td>
<td>3901</td>
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Optimization, it is OPT3 which dominates, although once more (as is indeed the case in all four regimens) the trend is upwards. At the end of the fourth year, unemployment in OPT3 is 2.09 million as opposed to 2.52 million in OPT4, 2.56 million in OPT2 and 2.76 million in OPT1. The pattern of unemployment in the different regimens follows quite closely, as expected, the course of government spending in that particular regimen. As can be seen from table 3, OPT4 has the more expansionary fiscal stance over the first three years and has the lowest unemployment over that period, whereas over the final year of the optimization it is OPT3 which has the more expansionary fiscal stance and thus experiences the lowest unemployment.

The fact that the foreign reserves and monetary aggregate pegging regimen gives the least fluctuations in unemployment in the four pegging regimens (although it is quite big from 1.25 million in the beginning to 2.09 million at the end of the optimization horizon), and unemployment is running at the lowest level in
Figure 2

PATHS OF INFLATION IN THE FOUR PEGGING REGIMES
Figure 3

PATHS OF UNEMPLOYMENT IN THE FOUR PEGGING REGIMENS

![Graph showing paths of unemployment in the four pegging regimens.](image-url)
this regimen at the end of the optimization period, and at the same time produces the best performance in inflation, would probably give this regimen the edge over the other three if we have to choose amongst the pegging regimens. This is certainly so in terms of contributions of ultimate targets to costs, as can be seen from table 4. How does it compare, though, with the fully discretionary regimen? As we have shown in Appendix B, a regimen involving rules leads to generally higher costs than one involving discretionary policy. Thus, as can be seen from table 4, OPT3 leads to a contribution to costs of the ultimate targets of 3395 as compared to only 2933 in OPTD. However, what we may be concerned with is how the individual targets compare in the two regimens. Again, in terms of costs all three targets perform better in OPTD. Figures 4 and 5 plot the paths of inflation and unemployment in the two regimens, and reveal that CP is much lower in OPTD than in OPT3 over the whole of the optimization horizon; U is also lower except for the final year of the optimization. Table 4 shows that OPTD also dominates the other pegging regimens in terms of total cost contribution of ultimate targets as well as the contribution of individual targets to costs, except for unemployment in the foreign reserves and interest rate pegging regimen. The paths of unemployment in OPT4 and OPTD are shown in figure 5a and show that it is only in the final year of the optimization that U in the discretionary regimen is lower than in OPT4. This is due, as we have seen, to the very high level of government expenditure in the first three years of OPT4.

Discretionary Regimens

Thus an optimal fully discretionary regimen is in general preferable to one involving rules. OPTD, however, is only one extreme possibility in the whole spectrum involving discretionary policy. Even if we assume an intermediate target strategy for conducting monetary policy, we need not restrain ourselves to the pegging regimens only. In fact, relaxing the constraints on the intermediate targets so that variations in them are allowed from one period to the other, allows us to examine the discretionary counterparts of the pegging regimens. Figure 6, which is similar to figure 22.2 in Bryant (1980, p.413) shows the alternative regimens available to the policy makers. For each of the four discretionary regimens we examine eight possibilities where we gradually relax the weights on the two intermediate targets whilst keeping the same ratio of the weights (and hence the relative priorities) between these targets. The rest of the objective function is kept
Figure 4

INFLATION IN OPT3 AND OPTD
Figure 5

UNEMPLOYMENT IN OPT3 AND OPTD
Figure 5a

UNEMPLOYMENT IN OPT4 AND OPTD
unchanged for all of these exercises. The first part of table 5 shows that a weight of $10^7$ on ER and $10^3$ on RLA are required to achieve the exchange rate and interest rate pegging regimen (OPT1). In the first discretionary regimen (second column), the weights on ER and RLA in the objective function amount to only 50 per cent of those in the pegging regimen, but the ratio of the weights between the two targets is the same at $10^6$. Similarly, in the next discretionary regimen, the weights amount to only 25 per cent of those in OPT1 and so on until we get to the final discretionary regimen which has zero weights on both ER and RLA. This last regimen is, of course, exactly the same thing as the fully discretionary regimen (OPTD) we discussed earlier. Table 5 also shows the average over the four years of the optimization horizon of inflation and the rate of unemployment (UP). In OPT1 the local authority rate is pegged at 8.5 per cent, but as we mentioned before, there is some variation from quarter to quarter. As we progress from the optimal rule to the fully discretionary policy the variation in RLA increases. We calculate a measure of the deviation from this RLA rule as

$$\text{Var} (\text{RLA}) = \frac{1}{16} \sum_{t=1}^{16} (\text{RLA}_t - 8.5)^2$$
<table>
<thead>
<tr>
<th>WEIGHTS</th>
<th>EXCHANGE RATE AND INTEREST REGIMENS</th>
<th>EXCHANGE RATE AND MONETARY AGGREGATE REGIMENS</th>
<th>FOREIGN RESERVES AND MONEY AGGREGATE REGIMENS</th>
<th>FOREIGN RESERVES AND INTEREST RATE REGIMENS</th>
</tr>
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<tr>
<td>OPT1</td>
<td>ER = 0.6; RLA = 8.5</td>
<td>ER = 0.625; GSM3 = 12.84</td>
<td>DFR = 0.84; GSM3 = 15.16</td>
<td>DFR = 0.9; RLA = 17.0</td>
</tr>
<tr>
<td>OPT2</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>OPT3</td>
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<td></td>
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<tr>
<td>OPT4</td>
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<td></td>
</tr>
<tr>
<td>X</td>
<td>100 50 25 12.5 1 0.5 0.25 0.1 0</td>
<td>100 50 25 12.5 1 0.5 0.25 0.1 0</td>
<td>100 50 25 12.5 1 0.5 0.25 0.1 0</td>
<td>100 50 25 12.5 1 0.5 0.25 0.1 0</td>
</tr>
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<td>ER</td>
<td>2.5 1.25</td>
<td>2.5 1.25</td>
<td>2.5 1.25</td>
<td>2.5 1.25</td>
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<tr>
<td>GSM3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RLA</td>
<td>0.06 0.11 0.18 0.3 0.01 0.008 1.78 3.81 11.50 45.70</td>
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</tr>
<tr>
<td>eviation</td>
<td>0.62 0.82 1.14 1.78 12.64 16.99 20.48 23.32 25.36</td>
<td>0.54 0.80 1.13 1.53 5.82 8.67 12.26 16.31 21.37</td>
<td>7.40 12.51 16.93 28.23 99.26 100 90.95 76.54 63.29</td>
<td>2.37 3.24 4.50 6.92 49.83 67.00 80.76 91.94 100</td>
</tr>
<tr>
<td>eviation</td>
<td>0.14 0.25 0.3 0.66 1.97 3.71 8.33 25.16 100</td>
<td>2.50 3.72 5.30 7.15 27.20 40.56 56.92 76.30 100</td>
<td>2.50 3.72 5.30 7.15 27.20 40.56 56.92 76.30 100</td>
<td>2.50 3.72 5.30 7.15 27.20 40.56 56.92 76.30 100</td>
</tr>
<tr>
<td>GSM3</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
This variance is then standardized with respect to the value obtained in the fully discretionary regimen which has the biggest variance, and is then shown in the row labelled '% deviation from RLA rule'. The whole exercise is repeated for the other three regimens but, of course, in the regimens involving a monetary aggregate, we are interested in the deviation from GSM3 rule.

Figures 7a and 7b correspond to the exchange rate and interest rate regimens. On the vertical axis we plot the percentage deviation from the interest rate rule (last but one row of table 4) against inflation in figure 7a and unemployment in figure 7b on the horizontal axis. Therefore movement towards 100 along the vertical axis corresponds to increasing deviation from the optimal rule of 8.5 per cent for RLA. Figure 7a clearly implies that there is a trade-off between the achievement of the optimal rule and a reduction in inflation. Figure 7b, however, shows that although there is another such trade-off, this time between the achievement of the optimal rule and a reduction in unemployment, as we get nearer to the optimal rule this trade-off actually disappears and unemployment falls. Similarly, figures 8a and 8b correspond to the exchange rate and monetary aggregate regimens, so that the vertical axis measures the percentage deviation from the optimal rule of 12.84 per cent growth in sterling M3. The two figures show that this time there is a definite trade-off between the achievement of the optimal rule for GSM3 and the reduction of either inflation or unemployment. The same conclusion emerges for the foreign reserves and monetary aggregate regimens as depicted in figures 9a and 9b which measure, on the vertical axis, the percentage deviation from the optimal rule of 15.16 per cent growth in sterling M3. Things are, however, much more complicated in the foreign reserves and interest rates regimens. This time the variance does not increase monotonically as we move from the pegging towards the fully discretionary regimen. The biggest variance in RLA is in the discretionary regimen (OPT41) where the weights are at only 0.5 per cent of those in the pegging regimen; it then decreases as we move in either direction towards the pegging or the fully discretionary regimen. So we standardize the variance with respect to the value in OPT41. Plotting the percentage deviation from the optimal rule of 17 per cent for RLA on the vertical axis of figures 10a and 10b reveals that there does not seem to be any trade-off of the kind found in the other regimens. In fact, in the case of unemployment, as we move towards the optimal rule it actually decreases. However, if we plot on the vertical axis the percentage of the weights on DFR and RLA as given in the first row of table 5, then figure 10a reveals that, as we progressively increase the weights on DFR and RLA (that is, as we move from the fully discretionary regimen towards the optimal rule)(6), inflation increases at first rather slowly
EXCHANGE RATE AND INTEREST RATE REGIMENS

% Deviation from RLA Rule 100

FIGURE 7a

12.0 13.6 CP(%) 6.7 7.5 UP(%) (Rule) 0

EXCHANGE RATE AND MONETARY AGGREGATE REGIMENS

% Deviation from GSM3 Rule 100

FIGURE 8a

12.0 13.0 CP(%) 6.7 7.9 UP(%) (Rule) 0

FOREIGN RESERVES AND MONETARY AGGREGATE REGIMENS

% Deviation from GSM3 Rule 100

FIGURE 9a

12.0 12.5 CP(%) 6.7 7.1 UP(%) (Rule) 0
FOREIGN RESERVES AND INTEREST RATE REGIMENS

FIGURE 10a

 Deviation from RLA Rule 100

 FIGURE 10b

 Deviation from RLA Rule 100

FIGURE 10a'

 Discretion

 FIGURE 10b'

 Discretion

12.0 12.8 CP(%) 6.2 6.9 UP(%)
and then quite sharply. Thus, if the percentage weights represent a measure of the degree of discretion pursued, then there does seem to be a trade-off between the achievement of the optimal rule and the reduction of inflation (7). In the case of unemployment, as figure 10b shows, the earlier conclusion still holds.

So we have found that in general, but not always, the economy performs better in terms of both inflation and unemployment as we move towards the more discretionary policy. This leads us to a comparison of the discretionary regimens themselves.

Of course, we can only compare discretionary regimens which allow the same degree of discretion. Thus we look at the regimens in which the weights on the intermediate targets are only 0.1 per cent of the weights in their corresponding pegging regimens, (i.e. OPT13, OPT23, OPT33 and OPT43). The same conclusions reached below apply to the other discretionary regimens. Table 6 presents the comparison in terms of costs, and figures 11 and 12 plot the paths of inflation and unemployment respectively in these regimens.

As far as the current balance is concerned, table 6 shows that in terms of costs the 'best' discretionary regimen is the foreign reserves and interest rate regimen (OPT43), with the exchange rate and interest rate one (OPT13) being the worse. As for inflation, we see that in terms of costs again OPT13 is the worse regimen and it is the exchange rate and monetary aggregate regimen (OPT23) which performs better; however, there is no big difference between this and the foreign reserves and monetary aggregate regimen (OPT33). This is confirmed by figure 11, which shows that for almost the whole of the four year horizon (except for the first three quarters of the first year when inflation is not much different in the four regimens) OPT23 leads to lower inflation than in any of the three other discretionary regimens, although there is little to choose between this regimen and OPT33.

At the end of the optimization period, inflation in OPT23 runs at 12.70 per cent compared to 12.74 per cent in OPT33, 12.91 per cent in OPT43 and 13.96 per cent in OPT13. The reason why these two regimens employing monetary aggregate targets give rise to fairly similar paths for inflation is that, given the large degree of discretion enjoyed in these two regimens, the paths of interest rates and the exchange rate, which provide the main channels through which monetary policy works in the model, are quite similar. Recall that interest rates have a powerful effect on the exchange rate which in turn affects inflation. Although the exchange rate does appreciate in the other two regimens, the extent of the appreciation is less, especially in OPT13, and thus inflation is higher in these regimens. This
result is consistent with our earlier findings in the comparison of the pegging regimens of the dominance of the foreign reserves and monetary aggregate regimen in its performance of inflation. There the exchange rate and monetary aggregate regimen did not do well because the exchange rate was pegged and was not allowed to appreciate sufficiently.

Turning to unemployment, table 6 shows that OPT23 is once more the dominant regimen in terms of costs, although again there is not much to choose between this and OPT33. This is confirmed in figure 12, where the paths of unemployment in these two regimens involving a monetary aggregate target almost coincides with each other, with OPT23 slightly better in the first year but slightly worse thereafter. The reason OPT13 is the 'worst' regimen in terms of unemployment is that government expenditure is so much lower (average £7.83 billion) than in the other three regimens (average £8.76 billion for OPT43 and £8.89 billion in the other two), that the effect of lower interest rates (average of RLA is 11.52 per cent as opposed to 14.95 per cent in OPT23, 15.11 per cent in OPT33 and 14.94 per cent in OPT43) is not sufficient to overcome this deficiency in aggregate demand.

So if we allow for a fair amount of discretion in operating financial policy then, unlike in the case where we have pegging regimens, there is not much to choose between an exchange rate and monetary aggregate regimen and a foreign reserves and monetary aggregate regimen which dominate the other two.
Figure 11: Inflation Paths in the Discretionary Regimes

PLOT: 1 = CP (OPT13)...
PLOT: 2 = CP (OPT23)...
PLOT: 3 = CP (OPT33)...
PLOT: 4 = CP (OPT43)...


1977:1 I
1977:2 I
1977:3 I
1977:4 I
1978:1 I
1978:2 I
1978:3 I
1978:4 I
1979:1 I
1979:2 I
1979:3 I
1979:4 I
1980:1 I
1980:2 I
1980:3 I
1980:4 I
* The paths of $U$ in OPT23 and OPT33 are almost identical
regimens. However, this does not mean that 'domestic' and 'external' monetary policy could be examined separately, as we have found there are substantial differences between the exchange rate and interest rate discretionary regimen and the foreign reserves and interest rate discretionary regimen. So it seems to be true that even under discretionary policy the choice of the optimal regimen to operate monetary policy is not a trivial problem indeed.

4.7 Optimal Response of System to Shocks
Up to now all of the optimization runs were of the open-loop, deterministic type. We turn now to an approximation of a closed loop stochastic system by perturbing the system with a number of disturbances in turn. The shocks we will be considering are
(a) a domestic demand disturbance,
(b) a foreign demand disturbance,
(c) a domestic price disturbance, and
(d) a foreign price disturbance.
Monetary shocks were also examined but their impact on the model was very small and we do not present the results here. This is due to the specification of the model as, although there is a significant monetary sector in the model, the feedback to the real sector is only through interest rates. As we have seen, the effect of interest rates on the real sector is not very big and their main impact is on the exchange rate through their influence on capital flows. Thus the effect is felt mainly on prices, and unemployment is barely affected. In the regimens involving an interest rate peg even this effect becomes minimal. We also tried a shock in interest rates (instead of shocks in money demand or bank lending) but found, as expected, that in the regimens operating interest rate targets the optimal result was such as to nullify this change in interest rates, with the result that the impact was almost nil. In the other two regimens there was, of course, some impact.
We should perhaps note that the exercises that follow are not exactly similar to those we undertook in chapter 2. There we assumed that the key variables in the economy were at their desired levels and we investigated how the various regimens would cope in bringing the economy back to its original path after the shocks. Here, however, the target variables are still far from their desired values when the economy is perturbed by the shocks, so that we would expect the response to be different. What we are looking for then is which regimen would take us nearer to the desired levels of the ultimate targets after the shocks. The response of the system to the shock
relies only on fiscal and monetary policies as we allow for only two instru­
ments QG and MLR. Moreover, since we are here dealing with the pegging 
regimens, there is an additional constraint on MLR (because of its link with 
RLA) which severely restricts its use in those regimens involving an 
interest rate peg, which implies that the optimal response would have to 
rely very heavily on QG. This, of course, need not be the most effective 
way of dealing with the shocks as, for instance, some form of incomes 
policy, together with fiscal and monetary policies, might deal better with a 
shock in wages.

Another point to note with respect to chapter 2 is that there we looked at 
the various regimens in two models which differed only in the specification 
of their absorption and price equations. Due to the space and time con­
straints, we do not do the same in the exercises below. However, we conject-
ure that the most important results of this study would hold in any model and 
so this extra exercise would not have made any significant contribution to 
the thesis.

4.7.1. Domestic Demand Disturbance

As in chapter 2, the first shock we consider is one in domestic demand although, 
as we have seen, the assumptions underlying the runs are somewhat different. 
The domestic demand disturbance we consider is of the form of a shock in 
consumers' expenditure. After optimizing to get the optimal rules, we 
perturbed the system by a shock of £1 billion in QCE in the first quarter of 
the optimization period and then optimized again. The difference between 
the two optimizations, which is reported in table 7, for the four pegging 
regimens, represents the optimal response to this shock of the instruments 
and their impact on the endogenous variables and is therefore an approxima­
tion to a closed loop stochastic system. Since QCE is an endogenous variable, 
the disturbance is not only in the first quarter but is distributed in a 
declining fashion throughout the whole of the optimization horizon, as can be 
seen from table 7. The reason the shock is distributed differently in the 
four regimens, except for the first quarter of the first year, is because 
QCE affects and is affected by other variables in the model so that its path 
depends on how the system reacts to the shock.
The initial impact of a rise in consumers' expenditure is to increase income 
in the first quarter of the optimization, which reduces unemployment in that 
quarter. The rise in economic activity produces an increase in imports 
which leads to a slight depreciation of the exchange rate. This, however, 
only slightly stimulates exports so that the current balance deteriorates in 
the first quarter. The rise in consumer durable expenditure also causes a 
surge in bank lending which, if unchecked, would cause the rate of growth of 
the money supply to be blown off target in those regimens employing a target
<table>
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<tr>
<th>Variables</th>
<th>OPT1: ER + RLA Rule</th>
<th>OPT1C: Rule + Shock in OCE (OPT1C - OPT1)</th>
<th>OPT2: ER + GS3 Rule</th>
<th>OPT2C: Rule + Shock in OCE (OPT2C - OPT2)</th>
<th>OPT3: DFR + GS3 Rule</th>
<th>OPT3C: Rule + Shock in OCE (OPT3C - OPT3)</th>
<th>OPT4: DFR + RLA Rule</th>
<th>OPT4C: Rule + Shock in OCE (OPT4C - OPT4)</th>
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<td>Years</td>
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<td>CB, m</td>
<td>18 37 -25 8 0.4</td>
<td>100 98 31 -16 13</td>
<td>103 98 36 11 62</td>
<td>-9 5 3 -7 -2</td>
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<td>CR, %</td>
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<td>39 39 30 8 24</td>
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<td>-220 -86 -34 -6 -4</td>
<td>-204 -79 -15 -10 -77</td>
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<td>4 -7 -6 3 -2</td>
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<td>-0.0003 0.0001 -0.00016 -0.00025 -0.00031</td>
<td>0.30 -0.02 0.01 -0.01 0.07</td>
<td>2.23 0.61 -0.09 -0.03 0.63</td>
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<td>0.33 -0.04 -0.02 -0.01 0.06</td>
<td>0.30 -0.02 0.01 -0.01 0.07</td>
<td>2.23 0.61 -0.09 -0.03 0.63</td>
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<td>14 -12 4 -4 5</td>
<td>12 -8 0.6 7 -1</td>
<td>86 49 33 35 51</td>
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<td>1054 451 163 49 429</td>
<td>1037 428 135 17 404</td>
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for GSM3. One way of checking this is through selling public sector debt. Interest rates cannot go up in the regimens involving interest rate pegs and if they do in the other regimens they would put pressure on the exchange rate to appreciate, which is not permissible in the regimens involving exchange rate pegs. Thus, the role that this instrument can play is very restricted and can only move freely in the foreign reserves and monetary aggregate pegging regimen. An equally important determinant of sales of public debt is the stock of financial wealth and, under monetary aggregate pegs, this cannot grow by much so that DB, as can be seen from table 7, is much less in those regimens (OPT2C and OPT3C). Thus the response in the policy instruments is allowed from the very first quarter and government expenditure is reduced in all four regimens so that the rise in aggregate demand, as well as the growth in the money supply, is checked. In fact, the fall in QQ in the first quarter in OPT3C is so big that income actually decreases in that quarter, so that the initial impact in this regimen is different from the others, with imports decreasing and the current balance improving. The consequences of the optimal fiscal-monetary response are that falling incomes cause unemployment to increase (especially in OPT2C and OPT3C) and, at the same time, cause a decrease in imports which, because the exchange rate and hence exports do not change significantly, imply generally an improvement in the current balance. Inflation is no real problem in any of the four regimens. As far as the current balance is concerned, it is almost impossible to make a judgement by looking at the figures in table 7. This is because, unlike CP or U where it is obvious that a decrease is to be preferred, in the case of CB which has a desired value of zero in the cost function, an increase or decrease may be preferable depending on whether CB is in deficit or in surplus. One way of making a comparison, however, is to take the difference in the cost contribution of CB in the original and perturbed solutions. This reveals that, in fact, it is the foreign reserves and interest rate pegging regimen which deals better with the shock. As we have already mentioned, inflation barely changes due to the shock. As for unemployment, as in the deterministic case it is the foreign reserves and interest rate pegging regimen (OPT4C) which is the preferable one. As we have seen, the decrease in government expenditure in OPT1C and OPT4C is much smaller than in OPT2C and OPT3C, so that we would expect lowe
unemployment in the two former regimens. This is in spite of the fact that interest rates are generally lower in OPT2C and OPT3C but, as we have already noted, the effect of interest rates on real variables is not very big in our model.

It should be emphasized that the large swings in QG and the small impact on the ultimate targets are due to the fact that we are looking at regimens involving rules, which means that the relative weights in the objective function are very much biased in favour of the intermediate targets and not enough weights are placed on the ultimate targets. To check this, we repeated the exercises for the same discretionary counterparts of the pegging regimens which we analyzed earlier (OPT13, OPT23, OPT33 and OPT43) and, as expected, the effects on the target variables are more pronounced, although the differences amongst the various regimens are not very marked. The increase in QG is also very much lower. This is because the relative weights of the ultimate targets, intermediate targets and instruments in the discretionary regimens are much more balanced.

4.7.2 Foreign Demand Disturbance

The next shock we consider was not examined in chapter 2, because the simplicity of the model employed there did not permit such an analysis. It is a foreign demand disturbance which takes the form of a shock in world demand (WD) of £200 million in the first quarter of the optimization period. Since WD is an exogenous variable, the shock amounts to an increase in the first quarter with no further increases in the other periods. The results are shown in table 8. The immediate effect of a step increase in world trade is to boost exports (by around £210 million in the first year in all four regimens) and income with favourable effects on unemployment. Although there is a rise in imports due to the increase in economic activity, there is still an improvement in the current balance as the main impact of the shock in WD is on exports. Since inflation is actually falling in all four regimens, the exchange rate need not appreciate significantly in those regimens where it is allowed to, which implies that interest rates (when allowed to vary) do not have to change by any significant amount either. This means that the optimal response to the shock comes mainly through fiscal policy and QG can increase without affecting inflation too much. Since a surplus in the current balance is undesirable, as CB has a desired value of zero in the objective function, to offset the huge expansion in exports the optimal response is an increase in QG; this at the same time brings unemployment down even further. The increase in QG is much more restrained in the two regimens involving a monetary aggregate peg (OPT2WD and OPT3WD) because of its impact on PSBR, and hence on GSM3. However, the
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<td>154 27 17 175 53</td>
<td>79 -7 0.1 100 43</td>
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<td>-49 -5 18 23 -8</td>
<td>177 13 8 -155 11</td>
<td>227 100 24 -190 40</td>
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<td>593 -85 -96 -82 8</td>
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<td>163 18 13 -33 40</td>
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<tr>
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<td>128 37 28 -21 9</td>
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**Table 1:**

Foreign Demand Misspecification

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<td>226 -356 -412 -477 -230</td>
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<td>128 37 28 -21 9</td>
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impact of the shock in WD on exports is in the first quarter only, so that continued fiscal expansion would boost imports up and this time cause a deterioration in CB; therefore the fiscal expansion is short lived and is, in fact, followed by a contractionary effect of varying degree in the four regimens.

Comparing the four regimens with respect to the current balance we find that, in terms of cost difference, it is the foreign reserves and interest rate pegging regimen which again gives the best performance. As we have seen, inflation is actually falling in all the four regimens, although by very small amounts. As for unemployment, there is a decrease in the first three years followed by an increase in the final year in all four regimens (except in OPT3WD where it starts to increase after the second year). The biggest fall in unemployment on average over the four years is in OPT1WD, as this is the regimen involving the more expansionary fiscal stance overall.

4.7.3 Domestic Price Disturbance

We analyze next the consequences of a shock of 0.1 in the wage rate in the first quarter of the optimization period. This exercise is similar to that in chapter 2, except for the underlying assumptions mentioned above. The results are presented in table 9. The initial effect of the shock is to cause a direct increase in inflation over the first two years. In the first year inflation increases rather rapidly, because in each quarter there is a further shock in WR of even higher magnitude. The increase in WR also causes a big increase in personal disposable income and real personal disposable income (though to a much lesser extent in the latter because of the increase in inflation), which stimulates consumption in spite of the inflation effect on QCE. The initial optimal response of the system is a decrease in government expenditure in all four regimens, which is big enough to bring income down but at the same time causes an increase in unemployment. The decrease in economic activity helps bring imports down which, with the initial small increase in exports, brings about an improvement in the current balance. Since the impact on inflation starts to die away in the second year and inflation actually starts falling thereafter, government expenditure is then allowed to expand, rather dramatically in some regimens, in an attempt to bring unemployment down again. As income picks up, though, there is a surge in imports, which means that the current balance soon deteriorates. As a result there is a depreciation of the exchange rate which is, of course, rather minimal in the regimens involving an exchange rate peg (OPT1W and OPT2W) and more substantial in the other two, but yet not enough to stimulate exports in any significant way.
### Table 2

**Domestic Price Disturbance**

<table>
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<tr>
<th>Variables</th>
<th>OPT1: ER + RLA Rule</th>
<th>OPT2: ER + GSM3 Rule</th>
<th>OPT3: DFR + GSM3 Rule</th>
<th>OPT4: DFR + RLA Rule</th>
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<tr>
<td>CB,m</td>
<td>286 316 -335 -387 -30</td>
<td>98 -13 -61 -15 2</td>
<td>71 36 -114 -97 -26</td>
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<td>4.33 0.49 -3.63 -1.93 -0.19</td>
<td>4.33 0.63 -3.53 -1.91 -0.12</td>
<td>4.33 0.60 -3.57 -1.93 0.15</td>
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<td>15 37 5 -17 10</td>
<td>3 21 3 6 13</td>
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<td>QY,m</td>
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<td>18 15 -15 7</td>
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<td>-0.005 -0.012 -0.010 -0.003</td>
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<td>GR,m</td>
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<td>PSNB,m</td>
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<td>-110 249 291 90 118</td>
<td>-280 -29 -90 296 97</td>
<td>-133 5 157 62 22</td>
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Using the cost difference criterion, we find that the foreign reserves and interest rate pegging regimen provides once more the best external (current balance) stability in the face of domestic price disturbance. Although there is an initial increase in inflation, all four regimens manage to actually bring it down by the end of the fourth year. There is no big difference amongst the four regimens, although OPT1W comes out slightly on top. This is due partly to the fact that there is a slight depreciation of the exchange rate in all the four regimens, but that in OPT1W is almost zero. In all the four regimens there is an initial increase in unemployment due to the cut in QG. As this policy is reversed to an expansionary one, however, unemployment actually decreases in all but OPT4W, where we have seen the increase in QG and hence aggregate demand is quite small. Over the four years, OPT1W and OPT2W give rise to the lowest increases in unemployment.

4.7.4 Foreign Price Disturbance

The last disturbance we consider was not examined in chapter 2 because the model did not allow for it, and takes the form of a shock of 0.1 in the foreign price of imports (PMF) in the first quarter of the optimization period. Since PMF is an exogenous variable, the shock is not a continuous one but a once and for all; the consequences of the shock are presented in table 10.

The immediate impact of an increase in PMF is to increase the sterling price of imports and hence domestic inflation - CP rises by as much as 1.54 per cent in OPT1PF, 1.75 per cent in OPT2PF, 1.65 per cent in OPT3PF and 1.41 per cent in OPT4PF in the third quarter of the first year, although the averages over the year are less as can be seen from table 10. This feeds on to wages and back again on to domestic prices. The increase in consumer prices implies a decrease in real personal disposable income which results in a decrease in consumers' expenditure. The optimal response of the system, as with the previous disturbance in domestic prices, is to decrease government expenditure by varying degrees in all four regimens. As QG decreases income falls and unemployment increases. The decrease in economic activity results in a decrease in imports, which is not enough to improve the current balance because the main impact of the increase in PMF is to increase the sterling price of imports. The deterioration of the current balance results in a slight depreciation of the exchange rate, which is, of course, only minimal in those regimens involving exchange rate pegs. Although this depreciation results in a slight increase in exports, this is again not enough to compensate for the
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<tr>
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<td>0.38</td>
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<td>-0.23</td>
<td>0.18</td>
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<td>-464</td>
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### Table 10

**FOREIGN PRICE DISTURBANCE**
increase in the nominal value of imports. The increase in the sterling price of imports due to the increase in PMF is, of course, short lived, so that fiscal policy can be reversed into an expansionary one (again of different degrees in the four regimens). This implies that income and employment pick up, although there is still a deterioration in the current balance throughout almost the whole of the four years.

As far as the current balance is concerned, a comparison of the four pegging regimens, on the cost difference criterion, reveals that, as with the other disturbances, it is the foreign reserves and interest rate pegging regimens which fare better. Comparing the four regimens with respect to inflation, we find that it is brought under control in all the regimens by the end of the fourth year, with the exchange rate and interest rate regimen once more being the preferable one in terms of generating less inflation on average over the optimization horizon. This is partly due to the fact that the exchange rate barely depreciates in this regimen, although it does (though by not much) in the other regimens.

As for unemployment, only the exchange rate and monetary aggregate regimen produces a decrease in U on average over the four years. In all the regimens, unemployment initially increases due to the contractionary fiscal policy but then decreases as QG increases; however, in OPT2PF the increase in QG is rather bigger than in the other regimens. This is because, as we have seen, the expansion in QG and hence PSBR is needed to check the fall in GSM3 in this regimen. Thus, when there is a shock in the foreign price of imports, there is no one pegging regimen which performs best with respect to all three targets, with the exchange rate and interest rate regimen performing better in terms of the current balance and inflation and the exchange rate and monetary aggregate regimen in terms of unemployment.

### 4.8 Summary of Results

One very important result of this chapter seems to be that generally discretionary regimens perform better than those involving optimal rules of one kind or another in terms of all three targets of inflation, unemployment and the current balance. Also there seems generally, but not always, to be a trade-off between the achievement of the optimal rule and a reduction in either inflation or unemployment, the exception being in the foreign reserves and interest rate regimens where a move towards the optimal rule involves a reduction in unemployment. We have also found that, under discretionary policy, there is not much difference between an exchange rate and monetary aggregate regimen and a foreign reserves and a monetary aggregate regimen, with both dominating the other two regimens in terms of unemployment and
inflation. There are, however, substantial differences between the exchange rate and interest rate discretionary regimen and the foreign reserves and interest rate discretionary regimen, with the latter giving the best and the former the worst performance in terms of the current balance. Thus, even under discretionary policy the choice of the monetary instrument is not a trivial problem. A ranking of the pegging regimens in both the deterministic and stochastic cases is shown in table 11. This is based on the actual cost contribution in the deterministic case and on the cost difference between the perturbed and original solutions in the stochastic case. The table shows that in no one case is there one regimen that dominates the others in terms of all three targets. In the deterministic case, regimens involving a monetary aggregate peg seems to perform generally better with the 'best' one being the foreign reserves and monetary aggregate regimen. In the stochastic runs we found that, in the case of domestic demand disturbances, the foreign reserves and monetary aggregate pegging regimen again performs better in terms of inflation, but it is dominated by the foreign reserves and interest rate pegging regimen on the unemployment and current balance counts. It would be

Table 11

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<td>Foreign Reserves and Monetary Aggregate</td>
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<td>CB CP U</td>
<td>CB CP U</td>
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<td>Foreign Reserves and Interest Rate</td>
<td>CB CP U</td>
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<td>1 4 4</td>
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</table>

interesting to compare this result with that obtained in chapter 2. Recall that there we found that a regimen involving a monetary aggregate peg
(whatever the external operating regime) fared better in terms of all three objectives. This was because in these regimens the interest rate was allowed to move to eliminate the excess demand in the goods market, and thus prevent output and prices from rising. A choice between the two pegging regimens involving a monetary aggregate peg turned out to depend upon certain parameters in the model. In the empirical exercises, however, the foreign reserves and monetary aggregate pegging regimen is clearly preferable to the exchange rate and monetary aggregate one. The difference in these results can be explained as follows: we have already seen that the two sets of exercises assume different initial paths of the economy when the latter is perturbed; also, the empirical runs allow the instruments to vary in response to the shocks, which is not possible in the theoretical exercises. Therefore, the excess demand can also be diminished via a decrease in government expenditure in the empirical exercises and need not depend on changes in the interest rate or exchange rate only. In our model, government expenditure has a much bigger impact on economic activity than do interest rates, and since the foreign reserves and interest rate pegging regimen responds by the lowest decrease in QG, it out-performs the other regimens in terms of unemployment.

In the case of foreign demand disturbances (this was not examined in chapter 2 because of the particular specification of the model used there), table 11 reveals that it is the exchange rate and monetary aggregate pegging regimen which this time dominates in terms of inflation but gives the worst performance in unemployment.

Turning to the case of domestic price disturbances, table 11 confirms the result obtained in chapter 2, that an exchange rate and interest rate pegging regimen gives the best protection against inflation. However, in the empirical exercises, this regimen fares best in terms of unemployment as well. Once more the difference in the results can be explained in terms of the response of the instruments in the empirical runs, with government expenditure decreasing initially in all four regimens and then increasing quite sharply in the exchange rate and interest rate regimen, with obvious implications for unemployment.

This same regimen is also the 'best' one as far as inflation is concerned in the face of a foreign price shock, but does very poorly in terms of unemployment. (Again, the particular specification of the model in chapter 2 did not permit this shock to be examined there). Thus in the stochastic runs, as in chapter 2, it is very difficult to argue in favour of any one particular regimen. In the deterministic runs, however, the foreign reserves and monetary aggregate regimens comes out on top in terms of the overall contributions of the ultimate targets to costs. We should perhaps note that these results depend not just on the model in use, but also on the specification of the
objective function, which in the case of the stochastic runs also influences the response of the system to the shocks.
NOTES

(1) The policy mix in the pegging regimens can also be explained in terms of the IS-LM curves of figure 1, although we do not present these figures.

(2) Of course the optimal value for ER need not be the same as that in the exchange rate and interest rate optimal rule; this reiterates the point which is central to this study, namely, that the optimal monetary instrument problem and the issue of the optimal foreign exchange market intervention policy (or 'domestic' and 'external' monetary policy) have to be analyzed concurrently.

(3) We do not present a figure for the paths of CB in the different regimens as this is much more difficult to interpret.

(4) The paths of inflation and unemployment are also lower throughout almost the whole of the optimization horizon in OPTD than in these regimens, although we do not present these figures.

(5) There are, of course, eight other intermediate possibilities which we chose to ignore in order to keep the study to manageable length; in any case it can be argued that these possibilities are of lesser importance.

(6) Note that this time, because the vertical axis measures the percentage weights and not the percentage variances, as we progress upwards we move towards the pegging regimen.

(7) We also plotted the percentage weights against inflation/unemployment for the other regimens (although we do not present the figures here) and all of the earlier conclusions hold.

(8) We do not present the results for the discretionary regimens, as it is very difficult to distinguish between the effects in the various regimens because of the degree of discretion involved. Also, the results reported are comparable to those in chapter 2, but not the discretionary ones.

(9) As with the last disturbance, we repeated the exercise with the same four discretionary regimens and again found the effects on the target variables to be more pronounced, with inflation and unemployment falling by much bigger amounts in all the four regimens, with the response in QG being smaller aided by the response in interest rates. Once more, however, there is no significant difference amongst the regimens. This applies to the other two disturbances as well.
The standard treatment of the monetary instrument choice problem in an open economy (surveyed in chapter 1) is to assume a particular external operating regime and then to conduct the analysis in a similar way to that for a closed economy. This, however, restricts the analysis to one involving a choice between 'money' and interest rates for a given external operating regime or one between 'money' and the exchange rate under the assumption of perfect mobility of capital. It is argued here that as domestic and external aspects of monetary policy are inherently interlinked, the decisions about the choice of the domestic and external operating regimes have to be made as one integrated package of policy and not in isolation. In fact one cannot, strictly speaking, even define a domestic open market operation without specifying the external operating regime; moreover, the converse is also true, we need to define the domestic operating regime in order to analyze the consequences of an exchange market intervention action (see chapter 1, and Bryant, 1980). Thus we have examined the instrument choice problem within a framework in which 'domestic' and 'external' monetary policy are analyzed concurrently. Although this framework for monetary policy seems to be the obvious one to use, it has not been given its due importance in the literature, notable exceptions being Henderson (1979) and Bryant (1980). Henderson (1979), though provides a graphical analysis of only two of the four possible pegging regimes - each involving a pair of intermediate 'domestic' and 'external' targets - within this framework. Although Bryant (1980) provides a discussion of these pegging regimens as well as their discretionary counterparts, he does not attempt to compare these alternative regimens. This is precisely what we have done in this thesis using the techniques of optimal control theory.

Another problem with the literature is that it seems to be confined to models where prices are fixed, so that the objectives of policy are restricted to that of income stability (Sparks, 1979;
Henderson, 1979) or income and external stability (Turnovsky, 1978). We therefore avoid this restrictive assumption and deal with an objective function incorporating income, price and external stability. Chapter 2 provided the theoretical analysis based on the approach mentioned above, and analyzed the relative merits of different financial regimens (involving a pair of intermediate 'domestic' and 'external' monetary targets) for stabilization purposes within an optimal control framework. Our analysis can thus be considered as providing a general framework which embodies the work of Poole (1970), Turnovsky (1978), Henderson (1979) and others as special cases. Thus, for example, Poole (1970) considers the closed economy analogue of the problem with prices assumed fixed; Turnovsky's (1978) analysis is for a given external operating regime and also assume fixed prices, whilst Henderson (1979), as we have seen, examines only two of the four possible pegging regimens within the model again with prices assumed fixed. However, this does not mean that the results obtained in these studies are merely duplicated in our analysis. As it turns out, our analysis reveals the conditions under which some of their more general results would hold.

For instance, recall that Poole (1979) found that in the case of domestic demand disturbances, a money supply policy would be preferable to an interest rate policy as far as income stability is concerned; moreover, Turnovsky (1978) found that this would hold under both fixed and flexible exchange rates. By contrast, our analysis has shown that this result is only unambiguously confirmed in the case of a flexible exchange rate. Under a fixed exchange rate, the rankings depended upon the relative sizes of certain parameters in the model. We also found that a regimen involving an interest rate peg need not result in complete insulation from disturbances in the monetary sector (as obtained by Poole, 1970, Turnovsky, 1978 and Sparks, 1979), or indeed be preferable to one involving a monetary aggregate peg. Our analysis also revealed that the simple graphical results of Henderson (1979) would only hold if certain, not necessarily evident assumptions, are made.
In an attempt to take the covariances between the disturbances into account (which the studies above, apart from Poole, 1970, ignore), we also looked at cases domestic demand disturbances appeared together with one of the other disturbances in turn. We found that when domestic demand disturbances appear together with domestic monetary disturbances, the case for a monetary aggregate peg is diminished, but is enhanced if the former appears together with either domestic price disturbances or foreign monetary disturbances. We should perhaps note though, that these results depend on the assumptions made about the covariances between these disturbances.

The most important result that emerged from the theoretical analysis, and this is confirmed in the empirical analysis, is that there is no one regimen that dominates the others in all the cases considered. To get these results, we assumed that the objective of policy makers was to minimize a loss function subject to a linear dynamic model with additive autoregressive errors. Some simplifying assumptions were made, namely non-stochastic parameters, zero costs associated with the instruments and equal number of instruments and targets, so that the optimal policies are linear feedback control laws with the three targets fluctuating about their desired values each period. These assumptions (apart from the first one), as well as the assumption that the intermediate targets can actually be met are, however, relaxed in the empirical exercises in chapter 4.

For these empirical exercises, we use the econometric model of the U.K. economy developed in chapter 3. Although our empirical estimates of the structural equations in the model are quite acceptable, we certainly do not make the claim that it is the best way to model the U.K. economy or that it is 'better' than existing models. We simply argue that its representation of the working of the economy especially its monetary sector - is adequate and not misleading to the problem at hand. Although it is quite probable that some of our results are model specific, we conjecture that the most general (and most important) ones would hold for any model. We should stress that our results are not to be regarded as providing specific
policy conclusions but simply giving us an idea of the kind of exercises that could be undertaken in real policy making.

The model is a structural one, and is estimated using quarterly seasonally adjusted data for the period 1963(1) to 1980(4). Apart from the monetary sector, which is fairly disaggregated, the rest of the model is basically similar to the theoretical one. It could be described as of Keynesian tradition, and seeks to explain the components of aggregate demand, the main determinants of which are real output, interest rates, real personal disposable income, world trade, relative prices and the exchange rate. The latter variable as well as capital movements, are also determined endogenously. The two policy instruments in the model are government expenditure and MLR. However, since inflation in the model is of the cost-push variety, and wage inflation is not influenced by demand pressure, the fiscal instrument cannot be expected to influence inflation in any significant way. Thus government expenditure can be directed towards the unemployment objective without much fear of creating inflation. It is then left for monetary policy to influence the path of inflation through its impact on the exchange rate. In this model changes in the money supply have only an indirect effect on activity and prices, the transmission mechanism being modelled through interest rates which affect consumers' expenditure as well as fixed and inventory investment directly, with resultant effects on output and employment. Nevertheless, interest rates are more predominant in terms of their effect on capital flows and hence on the effective exchange rate which in turn affect exports. Changes in the exchange rate also result in changes in the sterling price of imports and in domestic inflation in general; this then feeds into wages and back again onto domestic prices. In the monetary sector, interest rates affect demand for currency as well as debt sales to the private sector and hence the stock of money. The links between fiscal and monetary policy is emphasised through the government budget constraint, which also provides a further link between the foreign and monetary sectors. The government budget constraint is in fact used in
conjunction with the portfolio behaviour of the commercial banks and the non-bank private sector, to build up an explanation of sterling M3. This structure enables us to appreciate the problems involved in controlling the rate of growth of the money supply in the U.K. and to understand why in chapter 4, we could not hit this intermediate target exactly. (The other possible intermediate targets considered could not also be met exactly, but the extent of the deviations were less).

Given this estimated model, we were in a position to examine the outcomes of alternative financial regimens and hence evaluate empirically the theoretical results of chapter 2. Here we should note that to the best of our knowledge, there is no other empirical study which uses the same framework for monetary policy that we use to analyze the problem of the monetary instrument in an open economy. Among the work done using control theory, Campbell (1979) favours a money supply target while Pindyck and Roberts (1974) as well as Craine and Havenner (1981) prefer an interest rate target. For the U.K., Arestis (1976) using dynamic multipliers also favours an interest rate target. However, because these studies take the external operating regime as given, their choice of monetary policy is restricted to one between 'money' and interest rates only. In fact as we have seen, because 'domestic' and 'external' monetary policy should be analyzed concurrently, the choice is a much wider one. Even if we stick to the pegging regimens there are four such possibilities to be taken into account. If we consider the discretionary counterparts of the pegging regimens as well, then as can be seen from figure 6 in chapter 4, the number of possibilities is in fact limitless due to the various degrees of discretion that we can opt for. By providing an empirical analysis of the pegging regimens as well as some of their discretionary counterparts, chapter 4 takes off where Bryant's (1980) analysis ends. The latter as we have seen, provides a discussion of these regimens but no theoretical or empirical comparison of the alternative regimens.
In order to examine these regimens, we make use of an optimal control framework which employs an objective function depicting the desires of the policy makers, to yield optimal paths for the target variable as well as the policy variables. Of course the information about the policy makers desired values for the targets and instruments as well as their relative priorities is not readily available so that the objective function is only an approximation of their desires. It is thus assumed, that the policy makers set desired paths for the current balance, inflation and unemployment and then try to achieve these targets by manipulating government expenditure and MLR, which are the two instruments available to them in this model. Thus, we rely on fiscal and monetary policies only, which it must be said might not be the most efficient way of achieving the targets. All the instruments and targets carry penalty costs to deviations in their paths from the desired ones; this applies to the intermediate targets as well. Recall that in chapter 2 monetary policy was conceived as being a two stage process which involved decisions at two levels, one choosing the target paths of some intermediate targets which would be best for stabilization purposes, and the other manipulating the instruments to achieve those paths for the intermediate target variables. Using the techniques of optimal control, in chapter 4 we could avoid this sort of dichotomization and derive the optimal instrument mix aimed at both the ultimate and the intermediate targets. Thus when looking at the pegging regimens, we did not have to optimize to obtain the optimal rules and then optimize again to steer the economy towards achieving these rules.

In the deterministic case we considered a comparison of the four pegging regimens; this exercise revealed that as far as inflation and the current balance are concerned, the foreign reserves and monetary aggregate pegging regimen outperformed the rest but did not do as well on the unemployment issue. In terms of the total contributions of the ultimate targets to costs, however, this regimen is definitely superior to the other pegging regimens. This latter criterion is in fact quite acceptable as it gives us the regimen that will lead to the least costs given our objectives and their
relative priorities, as embodied in the loss function.

We also examined the optimal responses of the pegging regimens to certain shocks. In chapter 2, we examined the relative merits of the four pegging regimens in bringing the economy back to its original (desired) path, after being perturbed by certain shocks. In the empirical exercises of chapter 4, however, what we were interested in, was how the economy would respond to certain shocks from a position which is still far from the desired path. Looking first at the case of domestic demand disturbances, recall that in the theoretical exercises the overall conclusion was that a regimen involving a monetary aggregate peg fared better in terms of price, output and external stability. However, a choice between the fixed exchange rate and money supply policy (regimen (ii)) and a flexible exchange rate and money supply policy (regimen (iii)) turned out to depend upon certain parameters in the model. In the empirical exercises we found that the foreign reserves and monetary aggregate pegging regimen (the equivalent of regimen (iii)) is clearly preferable to the exchange rate and monetary aggregate pegging regimen (the equivalent of regimen (ii)) in this case. Although the former is confirmed to be the 'superior' regimen as far as inflation is concerned, it is the foreign reserves and interest rate pegging regimen which has the edge on the unemployment count. A possible explanation of this difference in the results (apart from the point mentioned above), is that in the empirical exercises the instruments are allowed to vary in response to the shocks and this is not possible in the theoretical exercises. Thus government expenditure is allowed to decrease in response to the shock in domestic demand, with the decrease being lowest in the foreign reserves and interest rate pegging regimen with obvious implications for unemployment.

The other disturbance which we analyze in both the theoretical and empirical exercises, is a shock in domestic prices. In chapter 2, we found that regimens involving a flexible exchange rate regimens (iii) and (iv) fared better as far as income stability is concerned,
but also gave the least price stability. The empirical exercises in chapter 4 confirmed the exchange rate and interest rate pegging regimen as providing the best protection against inflation but, however, revealed that it also gives the best performance in unemployment. Again this difference in the results could be explained in terms of the response of the instruments in the empirical exercises, where government expenditure decreases initially in all the four regimens, but then increases quite sharply in the exchange rate and interest rate pegging regimens. The other two shocks analyzed in chapter 4, namely foreign demand and foreign price shocks, again did not reveal the overall superiority of any one pegging regimen. The exchange rate and monetary aggregate regimen performed best in terms of inflation and worse in terms of unemployment in the case of foreign demand disturbance, with the converse being true for the foreign price shock. The particular specification of our model in chapter 2 did not permit an examination of these two shocks there.

The analysis in chapter 2 was restricted to a comparison of the pegging regimens, but this was relaxed in chapter 4 to enable us to examine the discretionary counterparts of these regimens as well. A very important conclusion that emerged was that even if policy makers decide to use some kind of intermediate target strategy, they should not adhere rigidly to the optimal rule, but should allow the paths of the intermediate targets to alter in response to new information as it becomes available. This follows from our results, that in general as we move from the pegged towards the more discretionary regimens, the economy perform much better almost throughout the whole of the optimization horizon. In fact in most cases, there seems to be a trade off between the achievement of the optimal rule and a reduction in either unemployment or inflation. We should note though, that even under discretionary policy instrument choice is not a trivial matter as there are sometimes substantial differences amongst the regimens, especially with those involving an interest rate policy. Here, however, it is difficult to choose between a foreign reserves and monetary aggregate discretionary regimen and an exchange rate and monetary aggregate discretionary
regimen with the former getting the edge slightly over the latter in terms of costs and both dominating the other regimens.

Recall that in chapter 2, we could not make a firm conclusion on the question of strategy choice. In chapter 4 we found, (and this is another important conclusion of this study) that a fully discretionary regimen which involve no intermediate targets at all, that is one where the instruments are directed towards the ultimate targets only, is the best policy to follow in terms of giving rise to the lowest contribution of ultimate targets to costs amongst all the regimens considered. Thus, the path of inflation in this fully discretionary regimen is much lower for almost throughout the whole of the optimization horizon than in the other regimens. The same holds true for unemployment except for the foreign reserves and interest rate pegging regimen where government expenditure is much higher - and possibly too high - than in the other regimens.

We will conclude by raising some points regarding the limitations of this study and suggest some possible lines that further research may take in this field. As far as the model is concerned, we were constrained to the use of only two policy instruments namely, government expenditure and MLR. Here a more detailed specification of the public sector would increase the number of instruments available and to some extent perhaps, bring the analysis nearer to real policy making. The model is, except for the monetary sector, an aggregated one; disaggregation could perhaps lead to an improvement in some equations, but it will certainly lead to a more accurate examination of the impacts of the instruments on the various objectives. This study assumed a deterministic world where the coefficient of the behavioural equations in the model as well as its dynamic specification are perfectly known. Of course, in practice this is not so and as is well known uncertainty poses quite a fundamental problem in policy decisions. If the policy makers are faced with a number of competing models each of whom appear to be plausible, they can make use of all the models by adopting some kind of minimax strategy. Here the policy
makers would minimize the maximum loss which would arise from applying the optimal policies based on the wrong model (see, Karakitsos, Miller and Rustem, 1981). The minimax approach could also be used to enable the policy makers to choose the optimal regimen when there is no one regimen which dominate the others in all respect as we found in this study. However, the computer software needed to do these kind of exercises is not yet available.

Our analysis throughout, has assumed that the authorities could successfully implement any external financial regime that suited their objectives. However, we know that the world economies are in fact interdependent so that one country cannot, for example, just fix its exchange rate vis-à-vis another country without some kind of agreement with the latter. Taking this into account, would probably have quite substantial implications for the issue at hand. Thus further research could well be directed towards this important aspect of policy making under interdependence.
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<th>Author</th>
<th>Year</th>
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<td>TURNOVSKY, S. J.</td>
<td>1975</td>
<td>'Optimal Choice of Monetary Instrument in a Linear Economic Model with Stochastic Coefficients'</td>
<td>Journal of Money, Credit and Banking, 6, Feb., pp. 53-80</td>
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All the behavioural equations that follow have been estimated by ordinary least squares using the computer programme 'GIVE'. As explained by Hendry and Srba (1978), GIVE estimates three sets of equations:

(1) the structural equation as specified by the model builder,
(2) a restricted transformed equation which allows for first order autoregressive errors, and
(3) an unrestricted transformed equation which ignores the restrictions in (2).

An alphabetical listing of the variables with data source appears at the end. Most of the data (mainly seasonally adjusted) for the period 1970(1) to 1980(4) were obtained from the National Institute and were then traced back for the period 1963(1) to 1969(4) from the relevant statistical sources. The prefixes 'Δ' and 'ln' indicate a change and the natural logarithm of a variable respectively, while the suffix '−i' shows the lag attached to a variable. A bar over a variable name denotes an exogenous item. 't-values' appear in parentheses below the estimated coefficients. The following statistics are also reported:

\[ \rho \] : the first-order autoregressive parameter (where applicable),
\[ R^2 \] : the multiple correlation coefficient,
\[ DW \] : the Durbin-Watson statistic,
\[ s \] : the standard error of the equation,
\[ \chi^2 (k) \] : likelihood-ratio test of validity of autoregressive restrictions (\( k = \text{no. of restrictions} \))
GOODS MARKET

GDP at factor cost

(A1) \( QY = QCE + QKP + QDS + QG + QEX - QIM - QAPC \)

Nominal GDP

(A2) \( GDP = P \cdot QY \)

Consumers' expenditure

(A3) \( QCE = 0.3134 + 0.2773 \cdot QYD - 1.3742 \cdot \frac{PC - PC_{-4}}{PC_{-4}} \)
\( - 0.0359 \cdot RLA + 0.7014 \cdot QCE_{-1} \)
\( (0.98) \quad (4.52) \quad (3.35) \)
\( (4.13) \quad (8.18) \)

\( \rho = -0.38755 \), \( R^2 = 0.985 \), \( DW = 2.510 \),
\( (3.05) \)

\( s = 0.198 \), \( \chi^2 (3) = 1.03 \)
1963(1) - 1980(4)

(A4) \( QCDE = QCE - QCO \)

(A5) \( CE = PC \cdot QCE \)

Fixed investment

(A6) \( QKP = 0.2461 + 0.0571 \cdot (QY - QY_{-1}) - 0.0431 \cdot (RCL - RCL_{-1}) \)
\( (2.70) \quad (2.11) \quad (2.43) \)
\( + 0.0235 \cdot IF_{-1} + 0.8999 \cdot QKP_{-1} \)
\( (1.86) \quad (22.03) \)

\( R^2 = 0.967 \), \( DW = 2.33 \), \( s = 0.089 \),
\( \chi^2 (3) = 2.31 \)
1963(1) - 1980(4)

* The \( R^2 \) corresponds to the equation without \( \rho \). This applies to similar equations below.
Stockbuilding

\[(A7) \quad QDS = 0.4438 - 0.0313 RCL_{-2} + 0.1424 (QY_{-1} - QY_{-2}) \]
\[+ 0.1249 (QY_{-2} - QY_{-3}) \]
\[+ 0.1424 (QY - QY) \]
\[\rho = 0.57, \quad R^2 = 0.23, \quad DW = 1.10, \quad (4.77)\]
\[s = 0.264, \quad \chi^2(2) = 1.80 \]
\[1963(1) - 1980(4)\]

Exports of goods and services

\[(A8) \quad QEX = 4.0765 + 4.3196 WD + 3.0658 RPX - 3.4628 RPX_{-1} \]
\[+ 1.5468 ER \]
\[\rho = 0.57, \quad R^2 = 0.956, \quad DW = 2.310, \quad s = 0.224, \quad (2.40)\]
\[\chi^2(3) = 3.94 \]
\[1970(1) - 1980(4)\]

Imports of goods and services

\[(A9) \quad QIM = -2.8965 + 0.2017 QY + 0.8223(P_{PMF/ER})_{-2} \]
\[+ 0.6376 QIM_{-1} \]
\[\rho = 0.57, \quad R^2 = 0.862, \quad DW = 2.320, \quad s = 0.264, \quad (1.95)\]
\[\chi^2(2) = 1.92 \]
\[1971(1) - 1980(4)\]
Employment and unemployment

\[ \ln \left( \frac{\text{EMP}}{\text{EMP}_{-1}} \right) = 0.0011 + 0.7608 \ln \left( \frac{\text{EMP}}{\text{EMP}_{-2}} \right) \]
\[ + 0.0884 \ln \left( \frac{\text{QV}}{\text{QY}_{-1}} \right) - 0.00008 \text{TREND71} \]
\[ (1.18) \quad (6.90) \quad (3.27) \quad (2.16) \]

\[ \rho = -0.33658 \quad , \quad R^2 = 0.623 \quad , \quad DW = 1.918 \quad , \quad (2.09) \]

\[ s = 0.003 \quad , \quad \chi^2 (1) = 0.472 \]

1971(2) - 1980(4)

\[ \text{U} = \text{LS} - \text{EMP} \]

INCOMES

Wage rate

\[ \Delta \ln \text{WR} = 0.0188 + 0.5383 \Delta \ln \text{WR}_{-1} + 0.3409 \Delta \ln \text{WR}_{-2} \]
\[ + 0.3512 \Delta \ln \text{PC}_{-1} - 0.2475 \Delta \ln \left( 1 - \frac{\text{NIC}}{\text{WS}} - \frac{\text{TP}}{\text{WS}} \right)_{-1} \]
\[ + 0.2884 \ln \left( \frac{\text{PC}}{\text{WR}} \right)_{-1} + 0.0015 \text{TREND71} \]
\[ (2.26) \quad (3.54) \quad (2.04) \quad (1.75) \quad (2.68) \quad (2.99) \]

\[ R^2 = 0.597 \quad , \quad DW = 2.275 \quad , \quad s = 0.015 \quad , \quad (2.99) \]

\[ \chi^2 (2) = 4.09 \]

1971(2) - 1980(4)

Average earnings

\[ \text{AE} = \text{WR} \cdot \frac{\text{AH}}{} \]

Wages and salaries

\[ \text{WS} = \text{AE} \cdot \text{EMP} + \text{WRES} \]
Total personal income

\[(A15) \quad TPY = WS + OPY\]

Total personal disposable income

\[(A16) \quad YD = TPY - T\]

Taxes

\[(A17) \quad T = TP + OT\]

Personal taxes

\[(A18) \quad TP = 0.1361 \ TPY\]

Real personal disposable income

\[(A19) \quad QYD = YD / PC\]

PRICES

Inflation rate

\[(A20) \quad RP \equiv (P - P_{-4}) / P_{-4} = 0.0156 + 0.3143 (WR - WR_{-4}) / WR_{-4} \]

\[\quad (1.98) \quad (5.64)\]

\[\quad + 0.0799 (PM_{-2} - PM_{-6}) / PM_{-6} \quad (3.74)\]

\[\quad - 0.1195 (PRO_{-1} - PRO_{-5}) / PRO_{-5} \quad (1.19)\]

\[\quad + 0.4611 (P_{-1} - P_{-5}) / P_{-5} \quad (6.60)\]

\[R^2 = 0.945, \quad DW = 2.014, \quad s = 0.015, \quad \chi^2 (3) = 4.34\]

1971(1) - 1980(4)
where

\[(A21) \quad \text{PRO} = \frac{QY}{EMP} \]

**GDP price deflator**

\[(A22) \quad P = (RP \cdot P_{-4}) + P_{-4} \]

**Consumption deflator**

\[(A23) \quad PC = 0.0159 + 0.4483 P + 0.5596 PC_{-1} \]

\[(1.72) \quad (5.43) \quad (6.28) \]

\[\rho = 0.67222, \quad R^2 = 0.999, \quad DW = 0.916, \quad (5.17)\]

\[s = 0.009, \quad \chi^2(1) = 0.88 \]

1963(1) - 1980(4)

**Exports deflator**

\[(A24) \quad PX = -0.0175 + 0.1458 P + 0.1865 \overline{WPX} + 0.7230 PX_{-1} \]

\[(2.62) \quad (2.06) \quad (6.18) \quad (8.79) \]

\[\rho = 0.60461, \quad R^2 = 0.999, \quad DW = 1.005, \quad (5.36)\]

\[s = 0.010, \quad \chi^2(2) = 4.44 \]

1963(1) - 1980(4)

**Relative price of exports**

\[(A25) \quad RPX = \frac{PX}{\overline{WPX}} \]

**Imports deflator**

\[(A26) \quad PM = \frac{PMF}{ER} \]

**THE MONETARY SECTOR**

\[(A27) \quad DSBLG = PSBR - DC - DB + DCBL - DEF \]
(A28) \[ \text{PSBR} = (\bar{Q}_G \cdot P) - T - (\bar{Q}_AFC \cdot P) + \text{PDRES} \]

(A29) \[ \text{DEF} = DFR - DOLG - D$BLG \]

Demand for currency

(A30) \[ \text{DC} = 0.0053 + 0.0053 CE - 0.0143 RD + 0.0036 \text{TREND63} \]
\[
(0.27) \quad (1.99) \quad (3.49) \quad (3.45)
\]

\[ \rho = -0.23812 \quad , \quad R^2 = 0.512 \quad , \quad DW = 2.464 \quad , \quad (1.98) \]
\[ s = 0.086 \quad , \quad \chi^2(2) = 1.97 \]
1963(1) - 1980(4)

Sales of public sector debt to private sector

(A31) \[ \text{DB} = -0.7629 + 0.0419 RCL - 0.2908 (RCL - RCL_{-1}) \]
\[
(3.92) \quad (2.12) \quad (3.98)
\]
\[ + 0.1946 FW - 0.1811 FW_{-1} \]
\[
(2.12) \quad (1.89)
\]

\[ R^2 = 0.858 \quad , \quad DW = 1.959 \quad , \quad s = 0.366 \quad , \quad \chi^2(2) = 2.95 \]
1963(1) - 1980(4)

Financial wealth

(A32) \[ \text{FW} = M3 + B \]
where

(A33) \[ M3 = DM3 + M3_{-1} \]

(A34) \[ B = DB + B_{-1} \]
and

(A35) \[ DM3 = DSM3 + D$D \]

(A36) \[ DSM3 = DM1 + DSTD \]
(A37) \[ SM3 = DSM3 + SM3_{-1} \]

**Demand for money M1**

(A38) \[
\frac{M1}{P} = 1.4718 + 0.0696 QY - 0.0886 RLA \\
(1.75) \quad (3.78) \quad (7.62) \\
- 1.9895 R_{-1} + 0.8826 \left(\frac{M1}{P}_{-1}\right) \\
(2.72) \quad (22.72)
\]

\[ \rho = -0.41778, \quad R^2 = 0.927, \quad DW = 2.723, \quad s = 0.318, \quad \chi^2 (3) = 2.03 \]

1963(1) - 1980(4)

(A39) \[ DM1 = M1 - M1_{-1} \]

(A40) \[ DSTD = DSBLP + DSBLG + DSBLO + DNOCA - DCA \\
- DSOD - DNDL \]

(A41) \[ DCA = DM1 - DC \]

**Sterling bank lending to private sector**

(A42) \[
\frac{DSBLP}{P} = 0.5281 QDS + 1.4264 QCDE \\
(2.66) \quad (9.63) \\
- 0.2599 \left(\frac{CBR}{RLA}\right) - 0.7662 LPER \\
(2.53) \quad (4.08)
\]

\[ R^2 = 0.602, \quad DW = 1.600, \quad s = 0.519, \quad \chi^2 (4) = 7.52 \]

1970(1) - 1980(4)
INTEREST RATES

Consol rate

\[(A43) \quad \Delta RCL = 0.0305 + 0.2538 \Delta RLA + 5.4901 \Delta RP\]
\[\begin{align*}
(0.46) & \quad (5.39) & \quad (1.71) \\
\end{align*}\]

\[R^2 = 0.322, \quad DW = 1.914, \quad s = 0.529, \quad \chi^2 (2) = 0.841\]

1963(1) - 1980(4)

Local authority rate

\[(A44) \quad RLA = CDR = 0.1021 + 0.9864 MLR + 0.3239 (DSBLP/P)_{-1}\]
\[\begin{align*}
(0.25) & \quad (23.33) & \quad (2.32) \\
\end{align*}\]

\[+ 0.3245 (DSBLP/P)_{-2}\]
\[\begin{align*}
(2.34) \\
\end{align*}\]

\[\rho = 0.5659, \quad R^2 = 0.967, \quad DW = 0.964, \quad s = 0.541, \quad \chi^2 (2) = 0.297\]

1964(1) - 1980(4)

7-day deposit rate

\[(A45) \quad RD = -1.6249 + 0.7099 RLA + 0.2263 RD_{-1}\]
\[\begin{align*}
(4.40) & \quad (16.17) & \quad (4.41) \\
\end{align*}\]

\[\rho = 0.6421, \quad R^2 = 0.964, \quad DW = 0.824, \quad s = 0.483, \quad \chi^2 (1) = 1.43\]

1963(1) - 1980(4)

Clearing banks' base rate

\[(A46) \quad CBR = 1.9251 + 0.6904 MLR + 0.3178 MLR_{-1}\]
\[\begin{align*}
(16.25) & \quad (19.01) & \quad (8.70) \\
\end{align*}\]

\[R^2 = 0.989, \quad DW = 1.666, \quad s = 0.351, \quad \chi^2 (1) = 0.31\]

1963(1) - 1980(4)
FOREIGN SECTOR

\[(A47) \quad \text{DFR} = \text{CB} + \text{DKF} \]

\[(A48) \quad \text{CB} = \text{PX} \cdot \text{QEX} - \text{PM} \cdot \text{QIM} + \text{CBRES} \]

Capital movements

\[(A49) \quad \text{DKF} = 0.1713 (\text{RLA} - \overline{\text{RED}})_3 + 15.3113 \text{ER} \]
\[(3.13) \quad (3.18)\]
\[- 15.4545 \text{ER}_1 - 0.8026 (\text{CB} - \text{CB}_1) \]
\[(3.24) \quad (3.25)\]

\[\rho = 0.35847, \quad R^2 = 0.494, \quad DW = 1.378\]
\[(1.95)\]

\[s = 0.692, \quad \chi^2(3) = 5.66\]

1971(1) - 1980(4)

Effective exchange rate

\[(A50) \quad (\text{ER} - \text{ER}_{-4})/\text{ER}_{-4} = 0.5783 (\text{PM}$ - \text{PM}$_{-4})/\text{PM}$_{-4} \]
\[(6.19)\]
\[- 0.7454 (\text{PX} - \text{PX}_{-4})/\text{PX}_{-4} \]
\[(4.34)\]
\[+ 0.0046 (\text{RLA} - \overline{\text{RED}})_2 \]
\[(2.06)\]
\[+ 0.0310 (\text{CB}/\text{P}) \]
\[(1.90)\]
\[+ 0.2796 (\text{ER} - \text{ER}_{-5})/\text{ER}_{-5} \]
\[(2.41)\]

\[\rho = 0.85324, \quad R^2 = 0.885, \quad DW = 1.725\]
\[(7.27)\]

\[s = 0.026, \quad \chi^2(4) = 9.73\]

1971(1) - 1980(4)
## Glossary of Variables with Data Source

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<td>Total personal disposable income, nominal</td>
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APPENDIX B

AN OPTIMIZATION FRAMEWORK

In order to study the monetary instrument problem in the context of the UK economy, we employ the model developed in chapter 3 within an optimal control framework using the packages developed by the Imperial College PROPE Group (see Rustem and Zarrop, 1979). Policy-making within that framework can be considered as follows. It is assumed that the policy makers believe that their dynamic model is a fair approximation to how the economy works. In order to control it, they set desired paths for some target variables (e.g. inflation, unemployment, growth, balance of payments etc.) and they try to achieve their targets as closely as possible by judicious use of their instruments like government expenditure, minimum lending rate etc. For example, the authorities might try to achieve zero unemployment and zero inflation together with balance of payments equilibrium. However, this does not mean that these desired paths are feasible given the way the economy works. Thus the dynamic model constitutes a constraint to the authorities attempt to steer the economy towards achieving their objectives. In the context of optimal control then, the problem of policymakers can be seen as the "simultaneous determination of the optimal paths of a vector of controls (instruments) over a given time horizon to achieve a desired vector of target variables." (Currie and Karakitsos, 1980, p.3)

In those cases where some of the targets are unattainable, the authorities are assumed to trade off their objectives to allow at least some of them to be achieved. Thus the authorities are supposed to be able to rank their targets in the order in which they want them most satisfied, which implies that they attach
different penalty costs to deviations of the optimal from the desired
paths of the different targets. It is also assumed that they have
desired paths for the instruments as well, and in most of the cases
we analyze, for intermediate targets. It is not the case that a
change in the paths of the instruments or the intermediate targets
can be considered as a costless or unimportant exercise. For example,
in the case of instruments, a decrease in government expenditure is
usually considered to be politically undesirable, whilst an increase
in MLR may create anxiety in financial markets. As for intermediate
targets, a strict adherence to a monetary target, for instance, may
be very unpopular in a recession. Therefore, it is assumed that the
instruments and the intermediate targets also carry penalty costs
to deviations in their optimal from their desired paths. All the
preferences of the authorities can be put in terms of an objective
function (1)

\[(B1) \quad J(Y, U)\]

where

\[(B2) \quad Y \equiv \{ y'(1), \ldots, y'(t), \ldots, y'(T) \}'\]

and

\[(B3) \quad U \equiv \{ u'(1), \ldots, u'(t), \ldots, u'(T) \}'\]

are the vectors of endogenous variables and policy instruments
respectively, and \(y(t)\) denotes the vector of endogenous variables
at time \(t\), \(1 \leq t \leq T\), with the instruments written in the same
form.

Formally, the optimization problem in the deterministic case
can be cast as the minimization of (B1) subject to the constraint
imposed by the model written in stacked form as

\[(B4) \quad F(Y, U) = 0\]
Thus the optimization problem can be represented as

\( \text{(B5) } \min \{ J(Y,U) \mid (Y,U) \in R \} \)

where \( R \) is the feasible region defined by the equations of the model, i.e.

\( \text{(B6) } R = \{ (Y,U) \mid F(Y,U) = 0 \} \).

This problem of designing optimal policies in a deterministic world has been tackled by Rustem and Zarrop (1979, 1981) and the algorithm which they developed is "basically a hill-climbing method which uses quasi-Newton descent directions to find the minimum of a quadratic objective function of endogenous variables and policy instruments subject to the constraint implied by the econometric model." (Westcott et. al., 1981, p.44). The optimal path thus obtained by the PROPE algorithm is the open-loop\(^2\) deterministic optimum, and most of the results reported in chapter 4 are of this type. However, we also approximate a closed-loop\(^3\) stochastic system by perturbing the system with certain disturbances and optimizing again. To obtain these results, we use the model described in chapter 3 and the objective function in chapter 4.

**Rules and discretion**

In chapter 4 we examine regimens involving optimal rules as well as their discretionary counterparts. A rule involves adding a further constraint to the objective function. As explained by Karakitsos and Rustem (1981), if \((Y,U)\) satisfy the rule as well as the model \(\text{(B4)}\), then the feasible region for \((Y,U)\) becomes restricted to a subset of the feasible region \(R\) in \(\text{(B6)}\). "In this sense, the rule can be seen as a further restriction on \(R\) even in a stochastic environment." (Karakitsos and Rustem, 1981, p.9)

In a stochastic environment, the model can be written as
(B8) \[ F(Y,U,\varnothing) = 0 \]

where \( \varnothing \) is a vector of random disturbances. A discretionary policy can then be obtained as a solution to the following problem

(B9) \[ \min E\{J(Y,U) \mid F(Y,U,\varnothing) = 0\} \]

If we have a rule then \((Y,U)\) should also satisfy the restriction of that rule. Let \( \Gamma \) denote this set of values. In (B9) if we further impose the restriction \((Y,U) \in \Gamma\), we have the inequality

(B10) \[ \min E\{J(Y,U) \mid F(Y,U,\varnothing) = 0\} \leq \min E\{J(Y,U) \mid F(Y,U,\varnothing) = 0, (Y,U) \in \Gamma\} \]

In the deterministic case the relationship \( \Gamma \cap R \subseteq R \) amounts to the same results. Hence

(B11) \[ \min \{J(Y,U) \mid (Y,U) \in R\} \leq \min \{J(Y,U) \mid (Y,U) \in R \cap \Gamma\} \]

This does not depend on the objective function which need not even be known for this proposition to hold. What (B10) and (B11) imply is that the total cost in the discretionary policy is less or equal to that for the rule; it does not imply, however, as we show in chapter 4, that each single target performs better under the discretionary policy.

NOTES

(1) This section follows Westcott et. al. (1981).

(2) In an open-loop or 'no-feedback' policies, the paths of the instruments are set at the beginning of the optimization period and are followed without any regard to future events (see, for example, Chow, 1975).

(3) In closed-loop or 'feedback' policies, future paths of the instruments will depend on observations to be made in the future regarding the results of current policy (Chow, 1975).