Real Time Structural Damage Assessment from Vibration Measurements

Real Time Structural Damage Assessment from Vibration Measurements

T S Maung, Hua-Peng Chen[1] and A M Alani

School of Engineering, University of Greenwich at Medway, Chatham Maritime, Kent, ME4 4TB, UK

Abstract. Damage identification in civil engineering structures using vibration measurements has become an important area of research. A reliable and cost effective method is required to detect and quantify local damage in the structures. The objective of this new proposed method is to identify structural damage in real time at a more detailed level directly from the vibration measurements. Structural damage is assumed to be associated with a proportional reduction of element stiffness matrix. Dynamics characteristics of the structure are calculated by using Newmark's numerical integration method based on measured acceleration data. The change in the stiffness matrix caused by structural damage is implemented into the equation of motion of the dynamic system. A system of governing equations is derived, where the difference between the vibration data of undamaged and damaged structures can be directly adopted. The changes in the stiffness matrix are represented by the changes in the coefficients associated with element stiffness matrixes. Those coefficients can be used as damage parameters in structural damage detection. Both the location and extent of the damage are then determined based on the inverse calculations of damage parameters of the individual elements. Finally, a numerical example is utilised to demonstrate the effectiveness of the proposed approach for on-line structural damage assessment.

1. Introduction

In recent years, structural damage detection methods based on measured vibration data have been widely utilised for assessing engineering structures. The safety of aging infrastructure and the economic considerations have been motivating factors for the development of reliable structural condition assessment methods which can be used to evaluate deterioration at the earliest stage. Objective assessment of structural conditions is not only beneficial for the cost effective maintenance strategy but also increases the overall efficiency of operation and life-span of infrastructure. It can also reduce the risk of the loss of human lives due to unexpected structural failures.

Most damage identification methods utilise the fact that any changes in structural stiffness lead to changes in modal characteristics of the structure. In structural damage detection, the measured modal parameters of the damaged structure are compared to those of the original undamaged one. A number of methods based on the measurements of natural frequencies and mode shapes for the damaged structure have been developed, such as sensitivity methods [1], modal force error methods [2], modal residual methods [3], and techniques based on genetic algorithms and neural networks [4, 5]. The

---

1. Corresponding author, E-mail: h.chen@gre.ac.uk

advantage of using the structural natural frequency shifts to detect damage is relatively straightforward. However, the change of natural frequency may not provide enough information for correct identification of the structural damage location and amount. This is because only limited number of frequencies can be measured and are available for structural damage assessment. Furthermore, the natural frequency is often not sensitive enough to initial damage in the structure. From the test study of I-40 Bridge, Farrar and Core [6] demonstrated that frequency shift is not a sensitive damaged indicator as no significant reductions of modal frequencies were observed although overall stiffness of the bridge cross section was reduced by 21%. On the other hand, reduction of stiffness due to structural damage tends to increase the curvature of the mode shapes in the vicinity of the damage. The curvature is inversely proportional to the flexural stiffness of the structure. In theory, changes in the mode shapes or mode shape curvatures could be used in damaged detection. However, detection of structural damage may not be practical if the change in stiffness is relatively small.

Some of the above mentioned methods are used to identify changes in the coefficients of the stiffness matrix, thus no information on specific damage locations in the structure can be provided. In this study, by using change in stiffness associated with element stiffness, damage of the structure can be detected at a more detailed level with relatively high accuracy. Here, the damaged stiffness matrix is directly implemented into the equation of motion for the damaged structure. The difference between the vibration data of damaged and undamaged structures are adopted into the governing equation of motion. Change in stiffness of structural elements can be represented by the coefficients associated with the stiffness matrices as damage parameters. Then damage parameters are inversely calculated in every time step from the derived equation. The identified damage parameters are then used to give the location and extent of the specific damaged elements.

Finally, a damaged cantilever beam is adopted for a numerical example to demonstrate the effectiveness of the new proposed approach. In the example, damage is simulated as a reduction of stiffness for the assumed elements. Vibration characteristics of undamaged and damaged states are generated numerically by using Newmark's time step integration method. The results from the example show that the proposed technique can correctly identify the specific location and quantity of structural damage with relatively high accuracy.

2. Basic equation for dynamic structures

In a dynamic structural system, the characteristic equation for the undamaged structure is given by

$$(\mathbf{K} - \lambda_i \mathbf{M})\boldsymbol{\phi}_i = \mathbf{0} \tag{1a}$$

Similarly, for the damaged structure it is expressed as

$$\left(\mathbf{K}^{\mathrm{d}} - \lambda_i^{\mathrm{d}} \mathbf{M}^{\mathrm{d}}\right)\boldsymbol{\phi}_i^{\mathrm{d}} = \mathbf{0} \tag{1b}$$

where $\mathbf{K}$ and $\mathbf{M}$ are the global stiffness matrix and the global mass matrix of the structure, respectively. λ_i and $\boldsymbol{\phi}_i$ are the ith eigenvalue and eigenvector, respectively. The superscript "d" denotes the quantities of the damaged structure. In damage detection, change in stiffness caused by structural damage is can be represented by $\Delta\mathbf{K}$ while the mass matrix is assumed to remain unchanged [7]. Consequently, for the damaged structure, $\mathbf{K}^{\mathrm{d}}$ and $\mathbf{M}^{\mathrm{d}}$ are expressed by

$$\mathbf{K}^{\mathrm{d}} = \mathbf{K} + \Delta\mathbf{K} \tag{2a}$$

$$\mathbf{M}^{\mathrm{d}} = \mathbf{M} \tag{2b}$$

The change in the structural stiffness matrix $\Delta \mathbf{K}$ is expressed in a simple form [8]

$$\Delta \mathbf{K} = \sum_{j=1}^{NE} \alpha_j \, \mathbf{K}^j \tag{3}$$

where *NE* is the total number of structural elements, $\mathbf{K}^j$ is the contribution of the jth element to the global stiffness matrix and α_j is the damage parameter for the jth element which ranges from 0 to -1. Any change in α_j indicates the location and extent of damage in the jth element. For example, the value of α_j will be -1 if the jth element is completely damaged. Damaged element stiffness will be completely removed in the global structural stiffness. When the jth element is no damage at all, α_j is equal to 0 and the contribution to the global structural stiffness remains unchanged.

Newmark (1959) introduced a time integration method for solving structural dynamic problems [9]. Here, Newmark's method is implemented into the formulation of a new damage detection technique. In the new method the time history is divided into a sequence of equal time steps Δt. A harmonic force is considered as loading on both undamaged and damaged structures. The equation of motion for the structural dynamics system is written as

$$\mathbf{M\ddot{U}}_i + \mathbf{C\dot{U}}_i + \mathbf{KU}_i = \mathbf{P} \sin(\omega t_i) \tag{4a}$$

$$\mathbf{M\ddot{U}}_{i+1} + \mathbf{C\dot{U}}_{i+1} + \mathbf{KU}_{i+1} = \mathbf{P} \sin(\omega t_{i+1}) \tag{4b}$$

where "i" represents the time step at t_i and "$i+1$" denotes the next time step $t_i + \Delta t$. Time dependent variable $\mathbf{U}_i$, $\mathbf{\dot{U}}_i$ and $\mathbf{\ddot{U}}_i$ are nodal displacement, velocity and acceleration, respectively. In the Newmark's method, displacement and velocity for the next time step are given by

$$\mathbf{U}_{i+1} = \mathbf{U}_i + \Delta t \mathbf{\dot{U}}_i + (0.5 - \beta)\Delta t^2 \mathbf{\ddot{U}}_i + \beta \Delta t^2 \mathbf{\ddot{U}}_{i+1} \tag{5}$$

$$\mathbf{\dot{U}}_{i+1} = \mathbf{\dot{U}}_i + \Delta t \left[(1 - \gamma)\mathbf{\ddot{U}}_i + \gamma \mathbf{\ddot{U}}_{i+1} \right] \tag{6}$$

where coefficients are taken as $\beta = \frac{1}{4}$ and $\gamma = \frac{1}{2}$. $\mathbf{U}_{i+1}$, $\mathbf{\dot{U}}_{i+1}$ and $\mathbf{\ddot{U}}_{i+1}$ can be calculated from

$$\mathbf{U}_{i+1} = \mathbf{U}_i + \Delta \mathbf{U}_i \tag{7}$$

$$\mathbf{\dot{U}}_{i+1} = \mathbf{\dot{U}}_i + \Delta \mathbf{\dot{U}}_i \tag{8}$$

$$\mathbf{\ddot{U}}_{i+1} = \mathbf{\ddot{U}}_i + \Delta \mathbf{\ddot{U}}_i \tag{9}$$

where $\Delta \mathbf{U}_i$, $\Delta \mathbf{\dot{U}}_i$ and $\Delta \mathbf{\ddot{U}}_i$ are incremental displacement, velocity and acceleration, respectively. Substituting $\mathbf{U}_{i+1}$ in equation (7) into equation (5) gives

$$\Delta \mathbf{\ddot{U}}_i = \frac{1}{\beta \Delta t^2} \Delta \mathbf{U}_i - \frac{1}{\beta \Delta t} \mathbf{\dot{U}}_i - \frac{1}{2\beta} \mathbf{\ddot{U}}_i \tag{10}$$

Similarly, substituting equation (8) into equation (6) leads to

$$\Delta \mathbf{\dot{U}}_i = \Delta t \mathbf{\ddot{U}}_i + \gamma \Delta t \Delta \mathbf{\ddot{U}}_i \tag{11}$$

From $\Delta\ddot{\mathbf{U}}_i$ in equation (10), equation (11) becomes

$$\Delta\dot{\mathbf{U}}_i = \frac{\gamma}{\beta\Delta t}\Delta\mathbf{U}_i - \frac{\gamma}{\beta}\dot{\mathbf{U}}_i + \Delta t(1 - \frac{\gamma}{2\beta})\ddot{\mathbf{U}}_i \tag{12}$$

The difference between equations (4a) and (4b) gives the incremental equilibrium equation of motion

$$\mathbf{M}\Delta\ddot{\mathbf{U}}_i + \mathbf{C}\Delta\dot{\mathbf{U}}_i + \mathbf{K}\Delta\mathbf{U}_i = \Delta\mathbf{P}_i \tag{13}$$

From $\Delta\ddot{\mathbf{U}}_i$ in equation (10) and $\Delta\dot{\mathbf{U}}_i$ in equation (12), equation (13) is rewritten as

$$\mathbf{K}^*\Delta\mathbf{U}_i = \mathbf{P}_i^* \tag{14}$$

where

$$\mathbf{K}^* = \frac{1}{\beta\Delta t^2}\mathbf{M} + \frac{\gamma}{\beta\Delta t}\mathbf{C} + \mathbf{K} \tag{15}$$

$$\mathbf{P}_i^* = \Delta\mathbf{P}_i + \left[\frac{1}{2\beta}\mathbf{M} + \Delta t\left(\frac{\gamma}{2\beta} - 1\right)\mathbf{C}\right]\ddot{\mathbf{U}}_i + \left[\frac{1}{\beta\Delta t}\mathbf{M} + \frac{\gamma}{\beta}\mathbf{C}\right]\dot{\mathbf{U}}_i \tag{16}$$

With the knowledge of system properties, $\dot{\mathbf{U}}_i$ and $\ddot{\mathbf{U}}_i$ at the initial time, $\Delta\mathbf{U}_i$ at time step *"i"* can be computed by inverting $\mathbf{K}^*$ in equation (14) as

$$\Delta\mathbf{U}_i = [\mathbf{K}^*]^{-1}\mathbf{P}_i^* \tag{17}$$

Once $\Delta\mathbf{U}_i$ is known $\Delta\ddot{\mathbf{U}}_i$ and $\Delta\dot{\mathbf{U}}_i$ can be computed from equations (10) and (12), respectively. Similarly, $\mathbf{U}_{i+1}$, $\dot{\mathbf{U}}_{i+1}$ and $\ddot{\mathbf{U}}_{i+1}$ can also be computed from equations (7), (8) and (9).

3. Inverse prediction of damage parameters

The equation of motion for the damaged structure which are subjected to the identical harmonic loading can be described as

$$\mathbf{M}\ddot{\mathbf{U}}_i^{\mathrm{d}} + \mathbf{C}\dot{\mathbf{U}}_i^{\mathrm{d}} + \mathbf{K}^{\mathrm{d}}\mathbf{U}_i^{\mathrm{d}} = \mathbf{P}\sin(\omega t_i) \tag{18}$$

Substituting damaged stiffness $\mathbf{K}^{\mathrm{d}}$ into equation (18) and assuming unchanged mass and damping for the damaged dynamic system lead to

$$\mathbf{M}\ddot{\mathbf{U}}_i^{\mathrm{d}} + \mathbf{C}\dot{\mathbf{U}}_i^{\mathrm{d}} + (\mathbf{K} + \Delta\mathbf{K})\mathbf{U}_i^{\mathrm{d}} = \mathbf{P}\sin(\omega t_i) \tag{19}$$

Evidently, the right-hand side equality of equations (4a) and (19) gives

$$\mathbf{M}\ddot{\mathbf{U}}_i^{\mathrm{d}} + \mathbf{C}\dot{\mathbf{U}}_i^{\mathrm{d}} + (\mathbf{K} + \Delta\mathbf{K})\mathbf{U}_i^{\mathrm{d}} = \mathbf{M}\ddot{\mathbf{U}}_i + \mathbf{C}\dot{\mathbf{U}}_i + \mathbf{K}\mathbf{U}_i \tag{20}$$

Simply, equation (20) can be rewritten in the following form

$$\Delta\mathbf{K}\mathbf{U}_i^{\mathrm{d}} = \mathbf{M}(\ddot{\mathbf{U}}_i - \ddot{\mathbf{U}}_i^{\mathrm{d}}) + \mathbf{C}(\dot{\mathbf{U}}_i - \dot{\mathbf{U}}_i^{\mathrm{d}}) + \mathbf{K}(\mathbf{U}_i - \mathbf{U}_i^{\mathrm{d}}) \tag{21}$$
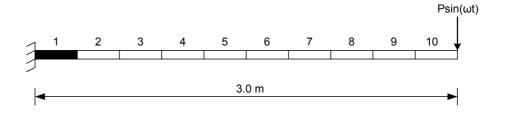
From $\Delta\mathbf{K}$ in equation (3), equation (21) becomes

$$\sum_{j=1}^{NE} \alpha_j \mathbf{K}^j \mathbf{U}_i^d = \mathbf{M}(\ddot{\mathbf{U}}_i - \ddot{\mathbf{U}}_i^d) + \mathbf{C}(\dot{\mathbf{U}}_i - \dot{\mathbf{U}}_i^d) + \mathbf{K}(\mathbf{U}_i - \mathbf{U}_i^d) \tag{22}$$

Only damage parameter α_j is an unknown vector in the above equation. Therefore, equation (22) can be expressed as a set of linear algebraic equations in the following form

$$\mathbf{A}\alpha = \mathbf{b} \tag{23}$$

in which the known coefficient matrix $\mathbf{A}$ and vector $\mathbf{b}$ are defined as

$$\mathbf{A} = \mathbf{K}^j \mathbf{U}_i^d \tag{24a}$$

$$\mathbf{b} = \mathbf{M}(\ddot{\mathbf{U}}_i - \ddot{\mathbf{U}}_i^d) + \mathbf{C}(\dot{\mathbf{U}}_i - \dot{\mathbf{U}}_i^d) + \mathbf{K}(\mathbf{U}_i - \mathbf{U}_i^d) \tag{24b}$$

In order to compute α, $\mathbf{A}$ has to be inverted. However, there are many situations where the inverse of $\mathbf{A}$ may not exist since the coefficient matrix $\mathbf{A}$ may not be square. In these cases, the inverse will be found via the Moore-Penrose pseudoinverse method. From equation (23), the damage parameters can be estimated from

$$\alpha = \mathbf{A}^\dagger \mathbf{b} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b} \tag{25}$$

where $\mathbf{A}^\dagger$ is the Moore-Penrose pseudoinverse matrix in a least squares sense, which can be calculated from the Singular Value Decomposition method. Consequently, the location and amount of damage can be determined by the obtained damage parameters at detailed level.
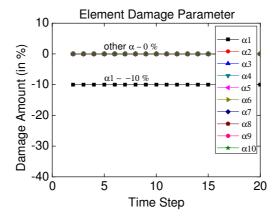
4. Numerical example

A cantilever beam with 10 elements is selected to illustrate the example. In order to understand how the procedure works, two damage cases are considered. Only the first element is assumed as 10% damage in Case 1. In Case 2, elements 1, 3 and 4 are assumed as 10%, 20% and 30% damage, respectively. Additional damage scenario is also considered in Case 2, assuming element 1 unchanged and element 3 and 4 damaged with the same amount of 5%. The undamaged and damaged structures are subjected to the same harmonic force. For simplicity, noise is not considered in this example. The location and amount of the simulated damage are then determined from the inverse calculations of the damage parameter α_j.
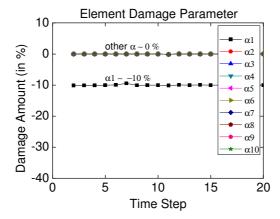

Figure 1. Cantilever beam subject to harmonic force with simulated damage Case 1 at element 1.

First, the simulated damage Case 1 is considered, as shown in Fig. 1. The proposed approach is employed to inversely identify the simulated damage in the structure. Figs. 2 and 3 show the results for inverse predictions for the simulated damage. The damaged element 1 with damage amount of 10% is clearly identified while the other elements are identified as undamaged elements. This gives an excellent agreement with the assumed simulation. From the results shown in Figs. 2 and 3, the damping of the structure has little influence on the inverse structural damage identification from the proposed approach.
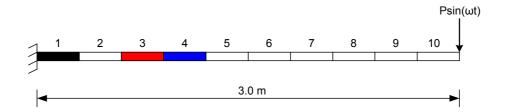

Figure 2. Inversely identified damage parameters in real time for Case 1 without damping where only element 1 damaged by 10%.
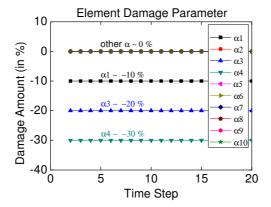

Figure 3. Inversely identified damage parameters in real time for Case 1 with damping where only element 1 damaged by 10%.

Now, the scenario with structural damage at multiple locations in various damage magnitudes is considered in simulated Case 2, as shown in Fig. 4. Figs. 5 and 6 indicate the results for inverse predictions for the simulated damage Case 2. Again, the proposed approach provides accurate inverse predictions of the simulated damage for this complex case, and the obtained results agree well with the locations and magnitudes of simulated structural damage. The results also show that the inverse predictions for the undamped structure are better and stable, comparing with those for damped structure. The damping therefore may affect the structural damage identification in cases with severe damage at multiple locations. The results in Fig. 6 also show that the damaged parameter for element 2 is slightly influenced by the damping with oscillation at the beginning of time steps due to the severe damage in the neighbor elements 1, 3 and 4.
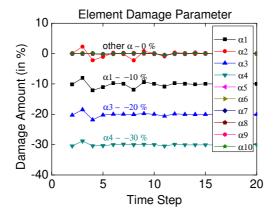
Figure 4. Cantilever beam subject to harmonic force with simulated damage Case 2 at multiple locations, i.e. elements 1, 3 and 4.
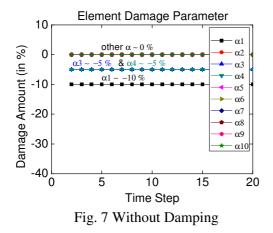

Figure 5. Inversely identified damage parameters in real time for Case 2 without damping where elements 1, 3 and 4 damaged in 10%, 20% and 30%, respectively.
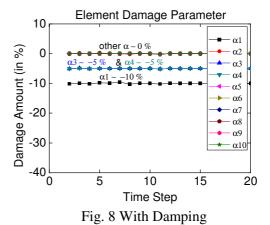

Figure 6. Inversely identified damage parameters in real time for Case 2 with damping where elements 1, 3 and 4 damaged in 10%, 20% and 30%, respectively.

For the additional damage scenario assuming both elements 3 and 4 damaged at level of 5% in the Case 2, the identified damage parameters are presented in Figs. 7 and 8 from the proposed approach without and with damping, respectively. It can be seen that the influence of surrounding damaged elements on the predictions of damage parameter at element 2 is reduced significantly and stable predictions are then obtained for all damage parameters for all elements concerned.

Fig. 7 Without Damping

Fig. 8 With Damping

Figure 7 and **8**. Inversely identified damage parameters in real time for Case 2 where elements 1, 3 and 4 are damaged in 10%, 5% and 5%, respectively.

5. Conclusion

A new method for structural damage detection in real time is presented. In this method, damage parameters are inversely calculated from the vibration measurements such as accelerations. They can be used for structural damage identification and monitoring in real time. Based on the numerical results involving the cantilever beam problems, the following conclusions are noted: (1) The proposed new method is capable of successfully detecting the damage in the structure; (2) It can be used in the case of initial damage in the structure; (3) Damage can be detected in more detail in terms of location and extent in the structure with relatively high accuracy; (4) The new proposed technique performs well and produces stable and reliable results for the three numerical examples as illustrated; (5) Damping in structures may have effect on structural damage detection if severe damage exist at multiple locations.

References

[1] Cawley P and Adams R D 1979. The location of defects in structures from measurement of natural frequency *J. Strain Analysis* **14:** 49-57.

[2] Zimmerman D C and Kaouk M 1994. Structural damage detection using a minimum rank updating theory *J. Vibration and Acoustics*, Transaction of ASME **116:** 222-31.

[3] Kabe A M 1985. Stiffness matrix adjustment using modal data *J AIAA* **23:** 1431-6.

[4] Gallego R and Rus G 2004. Identification of cracks and cavities using the topological sensitivity boundary integral equation *Comput Mech* **33**(2): 154-163.

[5] Rhim J and Lee S W 1995. A neural network approach for damage detection and identification of structures *Comput Mech* **16**(6): 437-443.

[6] Farrar C R and Cone K M. Vibration testing of the I-40 bridge before and after the introduction of damage *Proc. 13th Int. Modal Analysis Conf.* 1995 Nashville TN 203-9.

[7] Chen H P and Bicanic N 2006. Inverse damage prediction in structures using nonlinear dynamic perturbation theory *Comput Mech* **37:** 455-67.

[8] Chen H P and Bicanic N 2000. Assessment of damage in continuum structures based on incomplete modal information *Comput Struct* **74:** 559-70.

[9] Clough R W and Penzien J 1975. *Dynamics of Structures*, McGraw-Hill Book Co.