Oil and Gas Exploration Valuation and the Value of Waiting

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The timing flexibility of investments in oil and gas assets can potentially add value. In this article, we examine the value of waiting in exploration projects and propose a real option–based valuation method using least-squares Monte Carlo simulation. We show that the dynamics of the oil and gas prices have a large impact on the value of the option to wait, especially for projects with long lead times and durations. The uncertainty in the forward price curve is modeled using a two-factor stochastic price process. The article also presents the valuation method in the form of MATLAB functions and routines that can be used as an efficient test and analysis platform using the industry-standard input formats.

Introduction

Most projects in the oil and gas industry have embedded waiting options—commonly called “deferral options.” The decision makers can wait, learn from the evolution of technical and economic uncertainties, and exercise the investment at the most favorable moment. In the exploration business, investment decisions are often dominated by the uncertainty of finding commercial hydrocarbon volumes. However, in many cases, oil price volatility may generate significant value through waiting options for exploration opportunities.

As an example of a waiting option, assume that we own some volume of crude oil that we would like to sell at the market spot price. If we have no flexibility in timing, the value of the crude oil would be the volume times the current spot price. On the other hand, we may have the flexibility to defer the sale of oil until the moment when the price may be higher and the oil may be more valuable. The expected value of the sales with the timing flexibility will be at least as high as the value of sales without this flexibility. Thus, it may be worthwhile to invest in such flexibility.

In the example described above, the value of the trading decision was realized immediately. The economic conditions that were the basis for the decision were the actual spot oil prices at the time the sale was made. This is quite different from the common petroleum exploration and production project scenario where the cash flows generated by the investment

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decision often extend 20 to 30 years into the future. Given these extended time horizons, basing the investment decision on the current economic conditions is unlikely to maximize value creation from the investment. For such valuations, using the current spot oil price as the decision basis is suboptimal.

The short-term/long-term (STLT) price model introduced by Schwartz and Smith (2000) includes mean reversion in short-term price deviations and uncertainty in the long-term equilibrium level to which prices revert. It provides several advantages over more basic models and is still simple enough to be communicated to corporate decision makers who are generally not experts in financial modeling or option theory. The balance between realism and ease of communication of the model has led us to choose this model in favor of one-factor models, in which it is assumed that only one source of uncertainty contributes to the uncertainty in prices, or other multifactor models where two or more factors contribute to the uncertainty in prices (see Cortazar and Schwartz [2003], Geman [2000], and Schwartz [1997] for two- and three-factor price models).

Waiting options for project execution have been studied, for example, in Smith and McCardle (1999) and Schwartz and Smith (2000). These authors show that the investment decision for the projects with long lead times and durations is not sensitive to the variations in the short-term factor, implying that the current price levels are not the only piece of information required for valuation of long-term projects. The waiting options have also been studied in works of Paddock et al. (1988), Bjerkusund and Ekern (1990), Dixit (1992), Laughton and Jacoby (1993), Dixit and Pindyck (1994), and Dias (2004).

We formulate the exploration waiting option using the least-squares Monte Carlo (LSM) framework (Longstaff and Schwartz 2001). We propose using the forward1 price curve—derived from the parameters and state variables of the two-factor price process—to construct the cash flow profile and estimate the exercise value at each stage. The continuation value is estimated using a regression line on the outcomes of in-the-money paths for the next stage. This application of LSM algorithm is consistent with the structure of the valuation problem as the decision makers usually study the forward curve and its effect on value before drilling a prospect.2

The valuation algorithm has been coded in MATLAB functions and subroutines and can be readily shared and implemented. We believe that sharing the codes can go a long way in creating a consistent analysis platform for a variety of similar problems. Our work contributes to the literature of petroleum asset valuation by formulating the exploration waiting options using the LSM simulation method. Furthermore, this article utilizes the insights from the two-factor price model. The next section elaborates on the definition of value, and the following section discusses the price model used in the valuation. In subsequent sections, the exploration waiting option is explained and the option valuation algorithm is presented. The Analysis of Results section presents the results of computations and illustrates general trends. The last section summarizes the article and concludes.

1In order to avoid confusion, we are using the term “forward prices” instead of “futures prices.” Thus, forward prices are the futures contracts traded on public exchanges such as New York Mercantile Exchange (NYMEX).

2The convergence properties of the LSM method are discussed in Moreno and Navas (2003), Clement et al. (2002), and Stentoft (2004).
Definition of Value

In a corporation funded with publicly traded securities, or one mandated to act as if it were, “value” is the financial market value of the assets involved. Net present value (NPV) is intended to estimate such values when they are not directly observable. Following from this, the value of an investment opportunity is the value of the risky cash flows from the investment in a broad and liquid market. In this framework, it is the market’s beliefs about market uncertainties that are important and these beliefs should be reflected in the assumptions and models used for future oil prices. Thus, we do not use and calibrate models with the goal of forecasting the future oil price. Rather, we use and calibrate price models in an attempt to capture the market’s view of future price uncertainties. 3

The market does not really care about whether the firm considering the investment has an optimistic or a pessimistic view on future oil prices. What is important is the market’s belief, and these beliefs are encoded in the forward prices in a well-developed and liquid market. Furthermore, as the traded forward prices include the market’s view of the future oil price uncertainty, these prices should not be risk discounted. Doing so amounts to accounting for the same uncertainty twice. 4

As mentioned above, when using the forward curve for the oil price, the cash flows should not be risk discounted for the oil price uncertainty; this uncertainty is already built into the forward prices. The same argument applies to any other market uncertainty included in the cash flows. What about private uncertainties such as reserves and production? As argued by Smith and Nau (1995), one should use the experts’ assessed probabilities—that is, the experts’ view on the private uncertainties—for these. Furthermore, if these private uncertainties are dependent on market uncertainties, the probabilities should be assessed as conditional probabilities on contemporaneous market state.

One challenge in using the futures markets to calibrate the probabilistic price process is that the maturities of the exchange-traded futures and options contracts are much shorter than the time horizons of the projects we are evaluating. Though the projects may take 10 years to develop and produce 20 years or more, the forward contracts and options on the forwards are currently traded only 8 years out. 5 Thus, we need to somehow extrapolate beyond the data in the derivatives market. In performing this extrapolation, it is important to remember that we are not attempting to forecast what oil prices will be in the future. Instead, we are asking what an oil futures or options contract maturing in say, 10 years, would trade for today. In this article, we extrapolate using the STLT price model, estimating

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3The majority of asset pricing theory is built on the assumption that financial markets are efficient (frictionless). This is not because anyone believes that real markets are efficient (lacking transaction costs, information asymmetries, barriers to the quick equilibration of prices). Rather, it is because there is currently no complete approach for handling these issues, and the conjecture is that their effect on valuation is small compared with other types of approximation errors, such as those in the cash flows.

4If the firm’s beliefs about market uncertainties are not well aligned with the market’s beliefs, or the company chooses to use, say, a very conservative (relative to the forward curve) oil price, the firm’s value assessments will not be representative of the market value of the cash flows. This, in turn, will result in investments that will not maximize shareholder value.

5Crude oil futures are traded in two major commodity exchange markets, the New York Mercantile Exchange (NYMEX) and the Intercontinental Exchange (ICE). Although the trade volumes for futures and options on futures amount to several millions per month, there may still be concerns about the liquidity—that is, to what extent the prices really reflect the broad view of many participants—of long-maturity contracts.
its parameters using the available market data and assuming these estimates hold going forward.

Short-Term/Long-Term Oil Price Model

Thinking of oil prices in terms of the two-factor process is insightful. The first factor represents temporary changes in short-term prices (resulting from, for example, unusual weather or a supply disruption) that are not expected to persist. Changes in the second factor, the equilibrium level, are fundamental and are expected to persist (e.g., a major technological breakthrough in fracking technology). The mean-reversion coefficient describes the rate at which the short-term deviations are expected to disappear.

Crude oil is traded in a worldwide market with many participants. In this fairly liquid market, oil is priced through both spot contracts (for immediate delivery) and future contracts (for delivery in a specific time in the future). We assume that the market is efficient so that the prices reflect the collective idea of the participants about the trend in the supply and demand of crude oil. As a result, studying the spot and near-maturity future contracts can provide insight about the short-term behavior of the prices. Likewise, far-maturity future contracts can provide insight about the long-term dynamics of the oil prices. Methods for incorporating forward information in the parameters of the STLT price process are discussed in Schwartz and Smith (2000), Lucia and Schwartz (2002), and Jafarizadeh and Bratvold (2012).

In the STLT process, the spot price at time $t$, $S_t$, includes a short-term and a long-term component, $\chi_t$ and $\xi_t$, respectively, where $\ln(S_t) = \xi_t + \chi_t$ and

\[
\begin{align*}
    d\chi_t &= -\kappa \chi_t dt + \sigma_\chi dz\chi \\
    d\xi_t &= \mu\xi dt + \sigma_\xi dz\xi.
\end{align*}
\]

In the above equations, $dz\xi$ and $dz\chi$ are correlated increments of standard Brownian motion process with $dz\xi dz\chi = \rho_{\chi\xi} dt$. The short-term component of the two-factor process, $\chi_t$, follows an Ornstein-Uhlenbeck process with mean-reversion coefficient $\kappa$ and volatility $\sigma_\chi$; the long-term component, $\xi_t$, follows a Brownian motion with trend $\mu\xi$ and volatility $\sigma_\xi$. Figure 1 shows the mean and confidence levels for spot oil ($e^{\xi_t}+\chi_t$) and equilibrium ($e^{\xi_t}$) prices generated by the STLT price process with the parameter values in Table 1. The parameters are calibrated using the forward prices and implied volatilities of options on forward prices downloaded on October 1, 2013, from the New York Mercantile Exchange.
Figure 2. Decision tree representing the main decisions and uncertainties relevant for an exploration opportunity.

(NYMEX). We use the method presented by Jafarizadeh and Bratvold (2012) to calibrate the parameters for the STLT process using observed market information.

Let $\lambda_\xi$ and $\lambda_\chi$ be, respectively, the risk premiums for the long-term and short-term factors, then the futures price $F_{t,T}$ observed at time $t$ with maturity $T$ can explicitly calculated based on $\xi_t$, $\chi_t$, and the parameters of the STLT process (Eydeland and Wolyniec 2003; Schwartz and Smith 2000).

$$
\ln(F_{t,T}) = e^{-\kappa(T-t)}\chi_t + \xi_t + (\mu_\xi - \lambda_\xi)(T-t) - (1 - e^{-\kappa(T-t)})\frac{\lambda_\chi}{\kappa} + \frac{1}{2}
$$

$$
\left( (1 - e^{-2\kappa(T-t)}) \frac{\sigma_\chi^2}{2\kappa} + \sigma_\xi^2 (T-t) + 2(1 - e^{-\kappa(T-t)}) \frac{\rho_{\xi\chi} \sigma_\xi \sigma_\chi}{\kappa} \right).
$$

(3)

Waiting Option in Exploration Valuation

Oil and gas exploration is essentially investment in uncertain assets. It is uncertain whether oil or gas will be discovered by drilling a specific prospect. If hydrocarbons are indeed discovered, the production rates and future value generation are still uncertain. One can assess the potentials of a prospect using the available information and drawing on past experiences; however, the available information is never enough to remove the uncertainty about subsurface features. This lack of knowledge leads us to use probabilistic models for existence of hydrocarbons, volume, and reservoir characteristics.

Drilling exploration wells is in fact investing in new information. All exploration wells, irrespective of their “success” or “failure” outcome, generate new information and reduce the uncertainty about relevant aspects of the subsurface. Most exploration wells are plugged and abandoned once the target depth is reached and tests are completed. The decision to further appraise the area and possibly develop the field comes later when interpretation of results indicate an economically extractable oil or gas volume. In a capital-constrained

Figure 3. Simplified exploration tree.
environment, one should drill the exploration wells with the highest value potentials. In other words, the decision whether to drill an exploration well or not should be made based on its expected value, calculated using all of the available information.

Decision trees can be used to communicate the decision structure and calculate the expected value of an exploration decision. Figure 2 shows a typical exploration well decision structure where the decision of whether to drill an exploration well is followed by the uncertainty on the existence of hydrocarbons and whether the accumulation is commercially extractable. If the discovered volume is commercial, the company has to decide whether to develop the field using available technological solutions or to farm out and let others assume ownership. Development usually takes years to complete, and even as production starts the production rates and market prices of the hydrocarbons are uncertain. The semicircles on the production rate and market price uncertainties indicate that these are often modeled as continuous probability density functions. Finally, for each path through the tree there will be a payoff, usually depicted as a monetary value.

In common practice, the exploration decision trees are simplified even further; continuous distributions for production rates or volume may be replaced by discrete approximations and the decision about whether to develop or not may be explicitly considered in the uncertainty outcomes and end-node values. The decision tree in Figure 3 represents such a simplified formulation.

To solve the decision tree in Figure 3, we need to have probability estimates for chance of success, volume, and recovery rate. Geological data, usually combined with expert knowledge and insights from similar situations, are used to assess probability of success and in-place volume. Furthermore, most companies employ complex reservoir models that inform the assessment of the probability distribution for recoverable volume, using assumptions about production strategy and an array of probabilistic inputs. Finally, the NPVs at the end-nodes of the tree are calculated using an economic model that determines cash flows by considering the production rates, forward oil prices, tax rates, as well as royalties and bonuses.

Table 2 shows the cash flow calculation for the case of discovery in a typical exploration project. In this scenario, production of hydrocarbons starts after 2 years of construction and development—referred to as the lead time. Revenue is generated by selling oil at each

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### Table 1
Parameter values used in the STLT price process

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi_0$</td>
<td>4.5</td>
</tr>
<tr>
<td>$\mu_\xi$</td>
<td>$-0.50%$</td>
</tr>
<tr>
<td>$\sigma_\xi$</td>
<td>10%</td>
</tr>
<tr>
<td>$\chi_0$</td>
<td>0.11</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.83</td>
</tr>
<tr>
<td>$\sigma_\chi$</td>
<td>29%</td>
</tr>
<tr>
<td>$\rho_{\xi\chi}$</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Calibrated to implied information from Futures and Options downloaded on October 1, 2013, from NYMEX.

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6Depending on the reservoir characteristics, an optimal production plan is devised. This production plan requires making decisions about the number of injection and production wells, well locations, and the drainage strategy. Furthermore, the production plan will also affect the development concept and schedule.
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<tbody>
<tr>
<td>Reserve (total recoverable volume, MM bbl)</td>
<td>100</td>
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<tr>
<td>Project duration (years)</td>
<td>10</td>
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<tr>
<td>Production as a percentage of remaining reserve</td>
<td>10%</td>
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<tr>
<td>Variable OPEX (USD/bbl)</td>
<td>50</td>
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<tr>
<td>Fixed OPEX (USD million)</td>
<td>20</td>
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<tr>
<td>Project start years</td>
<td>2015</td>
<td></td>
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<tr>
<td>Lead time (years)</td>
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<tr>
<td>CAPEX (USD million)</td>
<td>800</td>
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<td>Year</td>
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<tr>
<td>Oil forward curve</td>
<td>97.51</td>
<td>99.09</td>
<td>99.59</td>
<td>99.80</td>
<td>100.06</td>
<td>100.10</td>
<td>100.11</td>
<td>100.33</td>
<td>100.57</td>
<td>101.18</td>
<td>101.57</td>
<td>101.82</td>
<td>102.02</td>
</tr>
<tr>
<td>Production</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>10.00</td>
<td>9.00</td>
<td>8.10</td>
<td>7.29</td>
<td>6.56</td>
<td>5.90</td>
<td>5.31</td>
<td>4.78</td>
<td>4.30</td>
<td>3.78</td>
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<tr>
<td>Production revenue</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>998.00</td>
<td>900.54</td>
<td>810.81</td>
<td>729.80</td>
<td>658.16</td>
<td>593.36</td>
<td>537.27</td>
<td>485.50</td>
<td>437.83</td>
<td>385.64</td>
</tr>
<tr>
<td>OPEX</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>400.00</td>
<td>390.00</td>
<td>381.00</td>
<td>372.00</td>
<td>363.00</td>
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<td>345.00</td>
<td>336.00</td>
<td>327.00</td>
<td>318.00</td>
</tr>
<tr>
<td>CAPEX</td>
<td>0.00</td>
<td>800.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Production cash flow</td>
<td>0.00</td>
<td>-800.00</td>
<td>0.00</td>
<td>598.00</td>
<td>510.54</td>
<td>429.81</td>
<td>357.80</td>
<td>295.16</td>
<td>239.36</td>
<td>192.27</td>
<td>149.50</td>
<td>110.83</td>
<td>67.64</td>
</tr>
</tbody>
</table>
year's average forward price. Next, fixed and variable operating expenditures\(^7\) (OPEX) are deducted. To simplify the model, we omitted the calculation of taxes and royalties. We further assumed that after one year of data collection and preparing, the capital expenditures (CAPEX) will be spent in year 2015. The project has 2 years' lead time, and production lasts for 10 years. The parameters used for building the declining production are also shown in Table 2.

Investing in exploration opportunities usually has flexibility in timing; the company can start drilling within a time frame (determined by exploration licenses or leases) beyond which the option to explore expires. Figure 4 shows the exploration option over two time steps. Initially, and as depicted in the first decision node, the company has three available alternatives: either to drill for oil, to walk away from this exploration opportunity, or to continue (wait) until the next time step to observe the new price curve. If the decision is to wait, the same decision structure occurs at the second time step but now with new price curves and NPVs.

Decision trees are great for structuring and communicating decisions. Unfortunately, they grow exponentially with the number of decisions and uncertainties and quickly become too big to be of much use. The next section illustrates how dynamic programming can overcome this "curse of dimensionality."

**Option Valuation Model**

A risk-neutral decision maker should choose the alternative (to drill, wait, or walk away) with the highest expected value. If they decide to wait, they face same alternatives at the next decision time step. In oil and gas exploration, drilling decisions are usually made once a year and can be postponed for many years, possibly until the end of the exploration lease. In this article, we model this investment waiting option using a modified version of the LSM method.

At each decision point, it is optimal to drill if the expected value of drilling is higher than the expected value of waiting or walking away (drop). Likewise, if the expected value of waiting is higher, then it would be optimal to continue to the next decision point.

\(^7\)Fixed operating expenditures are costs such as tariffs or labor costs that need to be paid regardless of the production level. Conversely, variable OPEX is tied to production level, such as primary processing costs, and is usually expressed as a per barrel unit costs.
In the end, the investment option value would be the aggregate value under a series of optimal decisions. In principle, the value of this real option could be calculated using one of several methods including binomial lattices, binomial trees, or the least-squares Monte Carlo approach. For this problem, it would be quite straightforward to use binomial lattices and trees if the underlying price process had been represented as a geometric Brownian motion or a one-factor mean reversion process. However, because we believe that the STLT price process is a more realistic representation of the uncertainty in future oil prices, and it is significantly more complicated to implement the STLT process on a lattice/tree, we use the LSM simulation algorithm for this work.

The expected value of drilling at each decision point is calculated as

\[ C_{ij}(\xi_{ij}, \chi_{ij}) = \hat{E} \left( \text{Continuation} | \xi_t = \xi_{ij}, \chi_t = \chi_{ij} \right) = \alpha_1 \xi_{ij} + \alpha_2 \xi_{ij}^2 \]

\[ + \alpha_3 \chi_{ij} + \alpha_4 \chi_{ij}^2 + \alpha_5 \xi_{ij} \chi_{ij}. \]  

In the equation above,  \( \hat{E} \) is the approximate conditional expectation and \( \alpha_k \)'s, \( k = 1, \ldots, 5 \), are regression coefficients. The coefficients are calculated by applying the least-squares regression technique on cross sectional information of the simulated paths. Note that in Eq. (4) we could as well use the spot and equilibrium prices as state variables—for example, by replacing \( \xi_t \) with \( e^{(\xi_t + \chi_t)} \) and \( \chi_t \) with \( e^{\xi_t} \). The modified equation will result in the same option value as with Eq. (4). The valuation algorithm is as follows:

- Simulate \( n \) independent paths for \( \xi_t \) and \( \chi_t \), use Eq. (3) to build a forward curve at \( t \) for each simulated path, then use a cash flow model like Table 2 to calculate the net present value of success and failure, \( \text{NPV}_s^i \) and \( \text{NPV}_f^i \), respectively. If the option period consists of \( T \) time steps, then simulate \( \{\xi_1, \ldots, \xi_T\}, \{\chi_1, \ldots, \chi_T\} \) and calculate \( \{\text{NPV}_s^{i1}, \ldots, \text{NPV}_s^{iT}\} \) and \( \{\text{NPV}_f^{i1}, \ldots, \text{NPV}_f^{iT}\} \) where \( i = 1, \ldots, n \).
- At the end of the license period, the management should either drill (if the expected value is positive) or relinquish (if the expected value of drilling is zero or negative),
Table 3
Brief description of developed MATLAB functions

<table>
<thead>
<tr>
<th>Function name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PriceSS</td>
<td>Generates two Ntrial × T matrices for the long- and short-term factors of the oil price. The input arguments to this function are the parameters of the STLT process, number of simulation trials Ntrial, number of time steps Nstep, and the length of time until option’s maturity T.</td>
</tr>
<tr>
<td>Futures</td>
<td>Generates a forward curve in 1 × (Lead + Years) vector of the futures prices.</td>
</tr>
<tr>
<td>Pdecline</td>
<td>Generates a production profile in a 1 × Years vector by using the inputs decline rate DeclineRate and production years Years.</td>
</tr>
<tr>
<td>NPVFutures</td>
<td>Calculates the NPV of a project using inputs such as Capex, Opex, VariableOpex, Lead, Years, and the discount rate r.</td>
</tr>
<tr>
<td>NPVGen</td>
<td>Generates an Ntrial × T matrix of NPV values.</td>
</tr>
<tr>
<td>OneTree</td>
<td>Returns the expected value resulting from the repetitive decisions.</td>
</tr>
<tr>
<td>EXPWait</td>
<td>Calculates the value of an exploration project with and without the waiting option by calling the functions above and applying the LSM simulation algorithm. This function accepts the input arguments used in all of the functions above.</td>
</tr>
</tbody>
</table>

for each simulated price path set:

\[
F_{iT} = \begin{cases} 
E (\text{Drill} | \xi_t = \xi_i T, \chi_t = \chi_i T) & \text{if } E (\text{Drill} | \xi_t = \xi_i T, \chi_t = \chi_i T) > 0 \\
0 & \text{if } E (\text{Drill} | \xi_t = \xi_i T, \chi_t = \chi_i T) \leq 0
\end{cases}
\]

for i = 1, ln

Apply backward induction; for j = T − 1, . . . , 1 (solve the associated dynamic programming problem).

- Based on estimated values from later time steps \( F_{i,j+1}, i = 1, \ldots, n \), calculate the regression constants \( \alpha_k, k = 1, \ldots, 5 \) and calculate the continuation value.

\[
C_{ij} (\xi_{ij}, \chi_{ij}) = \alpha_1 \xi_{ij} + \alpha_2 \xi_{ij}^2 + \alpha_3 \chi_{ij} + \alpha_4 \chi_{ij}^2 + \alpha_5 \xi_{ij} \chi_{ij}
\]

- Calculate the payoff from optimal decision. Set

\[
F_{ij} = \begin{cases} 
E (\text{Drill} | \xi_t = \xi_{ij}, \chi_t = \chi_{ij}) & \text{if } E (\text{Drill} | \xi_t = \xi_{ij}, \chi_t = \chi_{ij}) > C_{ij} (\xi_{ij}, \chi_{ij}) e^{-r} \\
0 & \text{if } E (\text{Drill} | \xi_t = \xi_{ij}, \chi_t = \chi_{ij}) \leq C_{ij} (\xi_{ij}, \chi_{ij}) e^{-r}
\end{cases}
\]

\( i = 1, \ldots, n \), and \( r \) is the risk-free discount rate for one subperiod.

- Set Option Value = \( (F_{11} + 1t F_{1n}) / n \).

MATLAB Functions

MATLAB programming language provides a suitable framework for implementing this algorithm. The function EXPWait implements the LSM algorithm described in the previous section.
section (a brief description of MATLAB functions is also shown in Table 3). We assume that the investment decisions are made yearly; thus, the number of time steps in the algorithm is the number of years until option maturity. The EXPWait function subsequently calls other functions to simulate long- and short-term factors of spot prices (PriceSS function), generate forward curve for each pair of simulated long- and short-term factors (Futures function), generate future NPV values (NPVGen function), and calculate the expected value from a decision tree (OneTree function). The procedure is as follows:

- The function PriceSS is called. This function generates two matrices of the size $N_{trial} \times T$ in which $N_{trial}$ is the number of simulation trials and $T$ is the length of time until license expiry. This function assumes time steps of one year in length and uses the discrete version of the STLT price model to simulate $\xi_t$ and $\chi_t$ (Jafarizadeh and Bratvold 2012).

$$
\xi_t = \xi_{t-1} + \mu_\xi \Delta t + \sigma_\xi \varepsilon_\xi \sqrt{\Delta t}
$$

$$
\chi_t = \chi_{t-1} e^{-\kappa \Delta t} - (1 - e^{-\kappa \Delta t}) \frac{\lambda_\kappa}{\kappa} + \sigma_\chi \varepsilon_\chi \sqrt{\frac{1 - e^{-2\kappa \Delta t}}{2\kappa}}
$$

$$
\varepsilon_\chi = \varepsilon_\xi \rho_{\xi\chi} + \varepsilon \sqrt{1 - \rho_{\xi\chi}^2},
$$

where $\varepsilon_\xi$ and $\varepsilon$ are independent normal random variables. The simulated results are stored in matrix variables long and short

$$
\text{long} = \begin{bmatrix}
\xi_{1,1} & \cdots & \xi_{1,T} \\
\vdots & \ddots & \vdots \\
\xi_{N_{trial},1} & \cdots & \xi_{N_{trial},T}
\end{bmatrix}
$$

$$
\text{short} = \begin{bmatrix}
\chi_{1,1} & \cdots & \chi_{1,T} \\
\vdots & \ddots & \vdots \\
\chi_{N_{trial},1} & \cdots & \chi_{N_{trial},T}
\end{bmatrix}
$$

- The function NPVGen is called to generate a $N_{trial} \times T$ matrix of NPV oil values using production profiles, project characteristics (cash flow properties and production years), and forward curves generated from each pair of $\xi_{ij}$ and $\chi_{ij}$. The results are stored in the matrix variable NPVmat.

$$
\text{NPVmat} = \begin{bmatrix}
\text{NPV}_{1,1} & \cdots & \text{NPV}_{1,T} \\
\vdots & \ddots & \vdots \\
\text{NPV}_{N_{trial},1} & \cdots & \text{NPV}_{N_{trial},T}
\end{bmatrix}
$$

- The LSM algorithm works backwards; it calculates the optimal decision at the option’s maturity $T$, stores the optimal values in the vector ValueVec, and uses backward induction to calculate the optimal values in earlier time steps.

- At option’s maturity, if expected value of drilling $E(\text{drill})$ is positive, then the project value along each path will be $E(\text{drill})$; otherwise, the decision makers will
relinquish the exploration license.

\[ \text{ValueVec}_T = \begin{cases} E(\text{drill})_{1,T} \text{ if } E(\text{drill})_{1,T} > 0 \\ 0 \text{ if } E(\text{drill})_{1,T} \leq 0 \\ \vdots \\ E(\text{drill})_{N_{\text{trial}},T} \text{ if } E(\text{drill})_{N_{\text{trial}},T} > 0 \\ 0 \text{ if } E(\text{drill})_{N_{\text{trial}},T} \leq 0 \end{cases} \]

• Moving backwards in time, the maximum value is calculated for each time step. We have used the MATLAB solver "\"" to calculate the regression coefficients \( \alpha_i \) based on the in-the-money paths at each time step. We assume the first \( b \) paths are in-the-money, then the regression coefficients at step \( i \) are calculated as follows:

\[
\begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_5 \end{bmatrix} = \begin{bmatrix} \xi_{1,i} & \xi_{2,i} & \chi_{1,i} & \chi_{2,i} & \xi_{1,i}\chi_{1,i} \\ \vdots \\ \xi_{b,i} & \xi_{2,b,i} & \chi_{b,i} & \chi_{2,b,i} & \xi_{b,i}\chi_{b,i} \end{bmatrix} \begin{bmatrix} ValueVec_{1,i+1} \\ \vdots \\ ValueVec_{b,i+1} \end{bmatrix} .
\]

• The continuation value at step \( i \) is calculated as follows:

\[
\text{Continuation}_i = \begin{bmatrix} \xi_{1,i} & \xi_{2,i} & \chi_{1,i} & \chi_{2,i} & \xi_{1,i}\chi_{1,i} \\ \vdots \\ \xi_{b,i} & \xi_{2,b,i} & \chi_{b,i} & \chi_{2,b,i} & \xi_{b,i}\chi_{b,i} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_5 \end{bmatrix} .
\]

• The option value at step \( i \) is also calculated as follows:

\[
\text{ValueVec}_i = \begin{cases} E(\text{drill})_{1,i} \text{ if } E(\text{drill})_{1,i} > Continuation_{1,i} \\ 0 \text{ if } E(\text{drill})_{1,i} \leq Continuation_{1,i} \\ \vdots \\ E(\text{drill})_{b,T} \text{ if } E(\text{drill})_{b,i} > Continuation_{b,i} \\ 0 \text{ if } E(\text{drill})_{b,i} \leq Continuation_{b,i} \end{cases} .
\]

• In the end, the option value would be the average of the discounted elements of \( \text{ValueVec}_1 \).

**Analysis of Results**

We have applied the MATLAB functions explained in the previous section to a number of valuation problems. In order to verify our model, we reconstructed some of the problems discussed in earlier works. Schwartz and Smith (2000) discussed the value of short- and long-term investments in oil properties (including the waiting option) as functions of the factors of STLT price process. They modeled the valuation as a discrete-time, infinite-horizon dynamic programming model that, in principal, is similar to the model we discuss in this article. They showed that the value of the short-term investment is sensitive to the value of both components of the STLT process, whereas the value of the long-term investment is only sensitive to the long-term component. This example is also repeated in

\[12\] The backslash \( \backslash \) solver gives \( m \) unknowns for \( n \) system of equations when \( n = m \). If \( n > m \) this function uses the linear least-squares regression to estimate \( m \).
Hahn and Dyer (2011), who employed both the LSM method and a binomial lattice to solve the problem and arrived at identical conclusions. Though this is an example of development waiting option, we can use the model to represent the problem discussed by Schwartz and Smith (2000) by assigning a probability of 1 to the chance of success. The results revealed consistency with Schwartz and Smith (2000) and Hahn and Dyer (2011). An exploration version of the long- and short-term investments is discussed in more detail below.

We considered two hypothetical exploration opportunities. The first is a representative of a large oil prospect, with 20% chance of finding mean 100 million barrels of oil equivalent (MMboe)\textsuperscript{13} and the potential to produce oil for 25 years in case of discovery in an exponentially declining production that declines 8% per year. The second project is a representative of a small oil prospect, with 20% chance of finding mean 40 MMboe and the potential to produce oil for only 5 years in case of discovery. The production exponentially declines with the rate of 40% per year. Furthermore, the development costs for the long project adds up to $800 million and the development cost for the short project is $400 million.\textsuperscript{14}

We use annual time steps and simulated 10,000 price paths. Figure 5 displays the results for the two projects, showing the value of exploration with waiting option as a function of the short- and long-term volatility (\(\sigma_x\) and \(\sigma_\xi\), respectively). The results show that sensitivity of the project values to variations in the short- and long-term factors is quite different. The value of the small prospect is sensitive to variability in both the short- and long-term factors; its value increases as the volatility of the factors increase. In contrast, the value of the large prospect is almost independent of the changes in the short-term volatility; the value changes only slightly as \(\sigma_x\) varies. This insensitivity for the large prospect is a result of long lead time and long production lifetime, because these tend to dampen the effect of the short-term variations.

A detailed understanding of a project’s future cash flows helps in better interpretation of the associated waiting option. If drilling a prospect results in a commercial discovery, then further investments are required to appraise and develop the field. The cash flow profile of this development project is often front-loaded; meaning that large revenues appear when production ramps up, and late cash flows shrink due to production decline. This implies that projects with smaller recoverable volumes, shorter lead times for development, and a steeper production decline are more sensitive to variability of oil prices in the near future. On the other hand, projects with larger recoverable volumes, long lead times for development, and a flat cash flow profile tend to be less sensitive to short-term variability of oil prices.

Because project values are discounted, cash flows in the near future are in general more important than the cash flows farther in future. However, the effect of varying discount rate on project values should be more dramatic for smaller prospects with less recoverable volumes and a shorter production period in case of discovery. Figure 6 shows the value sensitivity of the two prospects to varying discount rate (ranging from 1 to 10% annual discount rate). The solid lines show the value of prospects with waiting option and the dashed lines represent the value of prospects without the waiting option (i.e., the decision maker has to make the drill or drop decision at only one point in time). The curves show that a larger discount rate decreases the option value. However, the effect of heavy discounting is more pronounced for the smaller prospect.

\textsuperscript{13}We previously argued that volume uncertainty (conditional on discovery) is represented by a continuous probability distribution. To simplify the valuations, this continuous distribution is replaced by three representative values (using, for example, Swanson’s discretization) or by the mean value.

\textsuperscript{14}The parameter values used in this analysis are available in the accompanying file defer.mat. The file contains parameter values for the two-factor price process, discount rate, production decline parameters, and cash flow elements.
It may be insightful to study the effect of “fast-track” development or accelerated production on the value of the waiting option. Figure 7 displays the composition of value for projects with various lead times (defined as the number of years required for construction and development) and production years. All discounting is at 5% annual rate. In order to keep the comparison meaningful, the $z$-axis shows the ratio of the option value to the total project value:

$$\frac{\text{Value with Waiting Option} - \text{Value without Waiting Option}}{\text{Value with Waiting Option}}.$$  

Projects with short production lives are assumed to have a steeper decline curve and lower development costs. Although there is no straightforward relationship between these parameters, we have applied a deterministic relationship in order to facilitate the discussions and show the extent of analysis. We defined the development costs and decline rates as a
Figure 6. Sensitivity of project value with respect to discount rate.

function of the production years, as shown below:

\[ \text{decline rate} = \frac{2}{\text{Production years}} \]

\[ \text{Development cost} = 400 + (\text{Production years} - 5) \frac{500}{25} \]

The results show that the value of waiting is a larger proportion of the total value when we have long lead times and production years. For a number of very large prospects where the sum of lead time and production years in case of discovery exceeds 35 years, almost all value is option value. Smaller prospects, on the other hand, have comparably less valuable waiting options.\(^{15}\)

\(^{15}\)It should be noted that assumptions behind the valuations and analyses have led to these conclusions and are not applicable to all prospects. In order to draw meaningful insights, one should model the project-specific details and run the models.
Figure 7. Project values for various lead times and production years.

It is important to assess the scope of “outcome realization” in valuation of exploration opportunities; in other words, the length of time to realize the bulk of return on an investment decision. A project with early outcome realization will be sensitive to short-term variability, whereas a project with late outcome realizations will not.

Conclusions
This article presents a modified LSM algorithm for analysis of waiting options in exploration projects. We also modeled the dynamics of oil prices using a two-factor process. The waiting options in exploration industry may be detailed and complex. Therefore, traditional option pricing methodologies like lattice or decision tree may become difficult to implement. Furthermore, building an LSM model from scratch will also be a burden and is prone to modeling mistakes. The MATLAB implementation of the valuation algorithm can be used as a ready-made, transparent, and useful methodology to support investment decisions.

Although we discussed waiting options in the context of oil and gas exploration decisions, the algorithm can be applied in other valuation challenges after some modifications. For example, field development decisions for discovered resources have embedded waiting options and are sensitive to dynamics of prices. For these problems, we set $P_e = 1$ because there is no uncertainty about finding hydrocarbons. We may also extend the repetitive portion of the decision model to include more uncertainties and consider various development alternatives. Moreover, the production profiles for such projects could be generated by more sophisticated reservoir models.

Another extension of the model would be toward having more comprehensive cash flow models. For example, the cash flows may come from a model that considers complex tax and royalty regulations, environmental tariffs, or contract specifications.

We believe that the valuation framework in this article is also applicable to problems outside the oil and gas industry; for example, in mineral mining projects (metals, coal, etc.) or utility projects (hydroelectric infrastructure, power generation, etc.). In many instances, it seems that the benefits of evaluating waiting options outweigh the efforts.
References


Appendix

A List of Input Arguments in EXPWait Function

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity</td>
<td>The annual capacity of production infrastructure. When production is above</td>
</tr>
<tr>
<td></td>
<td>this limit, the excess oil will be produced the next year</td>
</tr>
<tr>
<td>Capex</td>
<td>The development capital costs in case of discovery</td>
</tr>
<tr>
<td>Chi0</td>
<td>Short-term factor of the oil price at time 0</td>
</tr>
<tr>
<td>DeclineRate</td>
<td>The annual decline rate of the exponentially declining production</td>
</tr>
<tr>
<td>Lead</td>
<td>Lead time of the development project in case of discovery</td>
</tr>
<tr>
<td>NPVdry</td>
<td>The net present value of all costs in case of dry hole</td>
</tr>
<tr>
<td>Opex</td>
<td>Fixed operating expenditure per year for the development project in case of</td>
</tr>
<tr>
<td></td>
<td>discovery</td>
</tr>
<tr>
<td>Pg</td>
<td>Probability of finding oil or gas in a prospect</td>
</tr>
<tr>
<td>T</td>
<td>Time to maturity for waiting option</td>
</tr>
<tr>
<td>VariableOpex</td>
<td>Variable operating expenditure as a per barrel cost</td>
</tr>
<tr>
<td>Volume</td>
<td>Total recoverable reserve in a prospect (this figure is usually multiplied</td>
</tr>
<tr>
<td></td>
<td>by the probability of finding oil in order to show the expected recoverable</td>
</tr>
<tr>
<td></td>
<td>reserve)</td>
</tr>
<tr>
<td>Xi0</td>
<td>Long-term factor of the oil price at time 0</td>
</tr>
<tr>
<td>Years</td>
<td>Total years of oil production in the development project in case of discovery</td>
</tr>
<tr>
<td>kappa</td>
<td>Coefficient of mean reversion in the STLT process</td>
</tr>
<tr>
<td>lambda</td>
<td>Risk premium for the short-term factor of the STLT process</td>
</tr>
<tr>
<td>mu</td>
<td>The trend of the long-term factor in the STLT process</td>
</tr>
<tr>
<td>r</td>
<td>Discount rate</td>
</tr>
<tr>
<td>rho</td>
<td>Correlation coefficient in the STLT process</td>
</tr>
<tr>
<td>sigmaChi</td>
<td>Volatility of the short-term factor in the STLT process</td>
</tr>
<tr>
<td>sigmaXhi</td>
<td>Volatility of the long-term factor in the STLT process</td>
</tr>
</tbody>
</table>

Biographical Sketches

Babak Jafarizadeh is a senior analyst in Statoil. His work and research interests are mainly in the fields of investment decision analysis, portfolio theory, and real option valuations in the upstream oil & gas industry. He obtained a Ph.D. degree in petroleum investment and decision analysis from the University of Stavanger and also holds B.S. and M.S. degrees in industrial engineering from Iran.

Reidar Brumer Bratvold is Professor of Petroleum Investment and Decision Analysis at the University of Stavanger. His research interests include decision analysis, project valuation, portfolio analysis, real-option valuation, and behavioral challenges in decision making. Prior to academia, he spent 15 years in the industry in various technical and management roles. Bratvold spent his early working years as a roughneck in the North Sea and as a seaman in the Norwegian merchant marine. He is a coauthor of the SPE book Making Good Decisions. Bratvold is an associate editor for the SPE Economics & Management journal and has twice served as an SPE Distinguished Lecturer. He is a Fellow and board member in the Society of Decision Professionals and a member of the Norwegian Academy of Technological Sciences for his work in petroleum investment and decision analysis. He holds a Ph.D. in petroleum engineering and an M.Sc. in mathematics, both from Stanford University, and has business and management science education from INSEAD and Stanford University.