The structural limit state function (LSF) $G(X)=g(x_1,x_2,\ldots,x_n)$ in the RSM can be formulated using a quadratic polynomial function as follows.

$$Z = g(\mathbf{x}) = a + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} c_i x_i^2 \tag{1}$$

where x_i (i=1,2,…,n)is space variable of the LSF and a, b_i, c_i (i=1,2,…,n) are the coefficients of Eq. (1).

The computation procedure of structural reliability using the RSM is given as follows.

The mean point of the random variables is chosen as the initial point $X^1 = [x_1^1, x_2^1, \cdots, x_n^1]$ $g(x_1^1, x_2^1, \cdots, x_n^1)$ and $g(x_1^1, x_2^1, \cdots, x_i^1 \pm f\sigma_i, \cdots, x_n^1)$ will be determined, f is assigned to 3 in the first step computation and 1 in the next step. The $2n$+1 value will be obtained and the value will be used in the coefficients of the Eq. (1) to obtain the RSM. The reliability index β^k and its design test point X_D^k of the structure will be determined by using the first order second moment method (Fang *et al*. 2015) based on the response surface function (k is the number of the iteration).

If Eq. (2) can be established, then it is possible to evaluate structural reliability.

$$\left\| \beta^k - \beta^{k-1} \right\| < \varepsilon \tag{2}$$

where β^k is the reliability index of the structure. On the other hand, if Eq. (2) cannot be established, then the new design testing point X_M^k will be obtained as follows.

$$X_M^k = X^k + (X_D^k - X^k)\frac{g(X^k)}{g(X^k) - g(X_D^k)} \tag{3}$$

where X^k is the initial point where the response surface function can be computed by kth iteration. X_D^k and X_M^k are the design testing point and the interpolation point at the kth response surface function, respectively. $g(X^k)$ and $g(X_D^k)$ are the values of the LSF which are corresponding to X^k and X_D^k, respectively.

A huge amount of computational time and bigger error will be produced by using the above method if the LSF is a non-linear performance function. In addition, when the higher accuracy is required, the number of interpolation and the order of the approximation function will be increased. It has been shown that the computation becomes more difficult and complex. Thus, a new method described in Section 3 has been proposed to overcome the problem.

3. Improved GA by fitness function

In structural reliability estimation, the LSF can be transformed into the fitness function (FF) as follows.

$$F(X) = \begin{cases} c_{\max} - G(X) & if\ G(X) < c_{\max} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

where $c_{\max}$ is a constant which is the maximum value of $G(X)$ in the evolutionary process.

If the LSF is explicit, differentiable and continuous function, the new FF can be established by using the gradient of the LSF for accelerating the convergence in the evolutionary process. The new FF is given as follows.

$$F^{'}(X) = \alpha \cdot \frac{F(X) - F_{\min}(Z)}{F_{\max}(Y) - F_{\min}(Z)} + (1-\alpha)\frac{\left\|\nabla F(X) - \nabla F_{\min}(Z)\right\|}{\left\|\nabla F_{\max}(Y) - \nabla F_{\min}(Z)\right\|} \tag{5}$$

where $\alpha \in [0,1]$ is power factor and it is determined according to one's experience. $F_{\max}(Y)$ and $F_{\min}(Z)$ are the maximum value of the individual fitness in the current generation and the minimum value of the individual fitness in the previous generation, $\nabla F(X)$ is the gradient of the LSF, $\nabla F_{\max}(Y)$ and $\nabla F_{\min}(Z)$ can be determined by using Eq. (6) and Eq. (7), respectively, ||•|| is 2-norm.

Eq. (6) is given as follows

$$\nabla F_{\max}(Y) = \left| \min\left| \frac{\partial F(y_1)}{\partial y_1^1}, \frac{\partial F(y_1)}{\partial y_1^2}, \frac{\partial F(y_1)}{\partial y_1^3} \right|, \cdots, \min\left| \frac{\partial F(y_n)}{\partial y_n^1}, \frac{\partial F(y_n)}{\partial y_n^2}, \frac{\partial F(y_n)}{\partial y_n^3} \right| \right| \tag{6}$$

where $Y=[y_1,\ldots y_2]$, y_1^i (i=1, 2, 3) is ith component of y_1. If the structure is plane structure, then the third component can be omitted.

Eq. (7) is given as follows

$$\nabla F_{\min}(Z) = \left| \min\left| \frac{\partial F(z_1)}{\partial z_1^1}, \frac{\partial F(z_1)}{\partial z_1^2}, \frac{\partial F(z_1)}{\partial z_1^3} \right|, \cdots, \min\left| \frac{\partial F(z_n)}{\partial z_n^1}, \frac{\partial F(z_n)}{\partial z_n^2}, \frac{\partial F(z_n)}{\partial z_n^3} \right| \right| \tag{7}$$

where $Z=[z_1,\ldots z_2]$, z_1^i (i=1, 2, 3) is ith component of z_1.

On the other hand, if the LSF is the implicit function, the new FF is shown in Eq. (4). However, the gradient is difficult to be determined. Therefore, the gradient is substituted by one order difference as follows.

$$\nabla F(X) = F(X^b) - F(X^a) \tag{8}$$

$$\nabla F_{\max}(Y) = \max\{ F(y_1^b) - F(y_1^a), \cdots, F(y_n^b) - F(y_n^a)\} \tag{9}$$

$$\nabla F_{\min}(Z) = \min\{F(z_1^b) - F(z_1^a), \cdots, F(z_n^b) - F(z_n^a)\} \tag{10}$$

where $\bullet^b$ is the paternal chromosome and $\bullet^a$ is the progeny chromosome.

The procedure for the proposed method with incorporation of improved fitness GA into the RSM for structural reliability estimation is given as follows.

Table 1 Comparison of results for Example 1.

	RSM-MLS	RSM	IGA-RSM
The structural reliability index	2.7100	2.7112	2.7111
Error		0.04%	0.004%
The largest failure point			
u_1^*	-2.5411	-2.5725	-2.5722
u_2^*	0.9417	0.8562	0.8958
Computational time (s)	50	39	10

Step 1. The LSF is determined.

Step 2. The n+1 design testing points are selected in the variable space, while f=3 (Li and Chen 2013).

Step 3. The coefficients of Eq. (1) are computed.

Step 4. The maximum failure point of the response surface function $g(\boldsymbol{x})$ is computed by using the first order second moment method.

Step 5. The maximum failure point is given as $\boldsymbol{X}$, the new response surface function is established by using Eq. (1) and Eq. (3) whereas the new FF is established by using Eq. (4).

Step 6. $\nabla F_{\max}(Y)$ and $\nabla F_{\min}(Z)$ are computed, if

$$\left|\nabla F_{\max}(Y) - \nabla F_{\min}(Z)\right| < \varepsilon' \tag{11}$$

Then X is located as the new central point.

The 2n+1 design testing points are selected in the influence domain of the point while f=1 (Li and Chen 2013).

Step 7. The 2n+1 design testing points are substituted into Eq. (1). The coefficients of the second order response surface function $g(\boldsymbol{x})$ are determined.

Step 8. The new surface function is determined. The maximum failure point X_D^k and the structural reliability index β^k are computed by using the first order second moment method, where k is the number of the iteration.

Step 9. The Step 5 to Step 8 are repeated until Eq. (12) is satisfied.

$$\frac{\left\|X_D^{k+1} - X_D^k\right\|}{\left\|X_D^{k+1}\right\|} < \varepsilon \tag{12}$$

where ε is the required accuracy.

4. Examples

Example 1. The example in Rajashekhar and Ellingwood (1993) is used in this paper to verify the proposed method. The LSF is given as follows.

$$g(\mathbf{u}) = \exp[0.4(u_1+2)+6.2] - \exp(0.3u_2+5) - 200$$

where u_1 and u_2 are two independent random variables which are considered to obey the standard normal distribution.

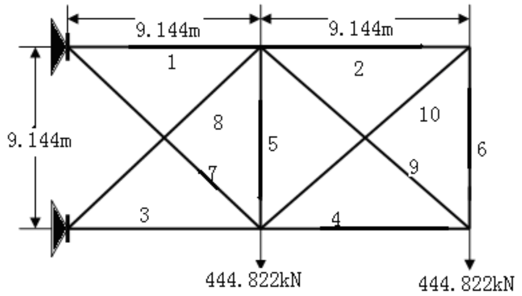

Fig. 1 10-bar truss structure

Table 2 Comparison of results for Example 2

	RSM-MLS	RSM	IGA-RSM
Reliability index	4.8083	4.8089	4.8085
The largest failure points	7.6658, 9.9949	7.6636, 9.9949	7.6651, 9.9949
	9.7302, 10.0075	9.7362, 10.0075	9.7300, 10.0075
	10.0350, 9.9949	10.0346, 9.9949	10.0354, 9.9949
	9.5916, 10.2997	9.5986, 10.2946	9.5928, 10.5981
	10.0212, 9.9855	10.0209, 9.9857	10.0212, 909855
Computational time (s)	300	240	109

The comparison of the results based on RSM using moving least squares (RSM-MLS) (Kang *et al*. 2010), classical RSM and improved GA for RSM (IGA-RSM) is shown in Table 1. The classical RSM is used as a benchmark method for comparison. It is shown from Table 1 that the computed result obtained by IGA-RSM is more accurate than RSM-MLS, while the number of iteration for IGA-RSM is the lowest among the 3 methods. The size of chromosome population is 10, cross rate is 0.3 and variation rate is 0.1. Table 1 gives the method. It is clear that the proposed method is superior in the calculation of structural reliability.

Example 2. A ten-bar truss structure is shown in Figure 1. It is widely used as an example to illustrate structural optimization design and reliability estimation. Its LSF is an implicit function with random variables as shown in Eq. (13).

$$g(\mathbf{A}) = \sigma_a - \left|\sigma(\mathbf{A})\right| \tag{13}$$

where A_i~N(64.52,1.27)(cm^2) and σ_a=172.4 MPa.

Similarly, the classical RSM is used as a benchmark method for comparison in this example. It is shown from Table 2 that the reliability index and the largest failure point obtained by IGA-RSM is more accurate than RSM-MLS, while the numbers of iteration of IGA-RSM, classical RSM and RSM-MLS are 34, 84 and 90, respectively. It is clear that the number iteration of IGA-RAM method is the lowest. It can be summarized that the IAG-RSM is superior in structural reliability estimation.