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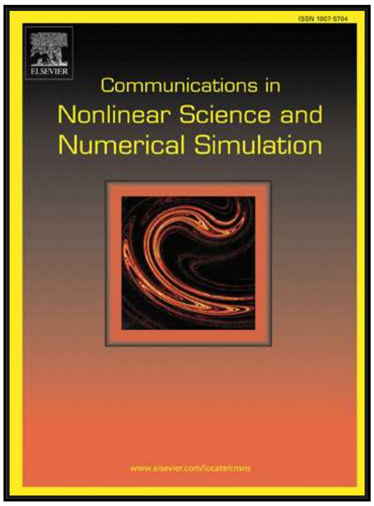

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Highlights

- We developed a discrete pest growth model with evolution of pesticide resistance.
- Two threshold conditions for extinction of pest population have been provided.
- The optimal pesticide switching times and related key factors have been discussed.
- The effects of dynamic complexity of pest population on its control were studied.

Beverton-Holt discrete pest management models with pulsed chemical control and evolution of pesticide resistance

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Abstract: Pest resistance to pesticides is usually managed by switching between different types of pesticides. The optimal switching time, which depends on the dynamics of the pest population and on the evolution of the pesticide resistance, is critical. Here we address how the dynamic complexity of the pest population, the development of resistance and the spraying frequency of pulsed chemical control affect optimal switching strategies given different control aims. To do this, we developed novel discrete pest population growth models with both impulsive chemical control and the evolution of pesticide resistance. Strong and weak threshold conditions which guarantee the extinction of the pest population, based on the threshold values of the analytical formula for the optimal switching time, were derived. Further, we addressed switching strategies in the light of chosen economic injury levels. Moreover, the effects of the complex dynamical behaviour of the pest population on the pesticide switching times were also studied. The pesticide application period, the evolution of pesticide resistance and the dynamic complexity of the pest population may result in complex outbreak patterns, with consequent effects on the pesticide switching strategies.

Keywords: Discrete model; Pest resistance; Pesticide switching; Pesticide application period; Threshold condition; Dynamic complexity

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1 Introduction

Agricultural pests are usually controlled with pesticides, a preferred method because of its efficiency. However, because of the long-term use of pesticides more than 500 species of targeted pests have developed resistance to them since 1945 [1, 2, 3]. Consequently, farmers' crop losses to pests are increasing, even though more pesticides are being used. For example, in the USA, farmers lost 7% of their crops to pest damage in the 1940s, but the percentage lost had increased to 13% by the 1980s [4].

Therefore, how to reduce or delay pest resistance to pesticides and how to use pesticides reasonably are important questions. Based on the frequency and dosage of pesticide spraying and the genetics of pest resistance, many proposals have been suggested to deal with the problem including rotation or switching between different kinds of pesticides [5], adopting an integrated pest management (IPM) strategy [6, 7, 8, 9, 10] and using other control techniques without pesticides such as leaving untreated refuges where susceptible pests can survive [5].

The forecasting of a pests population density, which can be estimated by mathematical modelling of growth trends, is an important step in the design of a pest control strategy. For example, if the density of a pest population with overlapping generations is very large, it can be treated as continuous growth. Therefore, many pest population growth trends and studies of pest control strategies have been modelled using continuous mathematical models [11, 12, 13, 14, 15, 16, 17].

Recently, we also modelled pest resistance with a continuous mathematical model and studied the optimal time for switching pesticides under three different switching strategies [18]. Moreover, we introduced the development of pesticide resistance into pest-natural enemy interaction models in which the optimal numbers of natural enemies to be released were studied in relation to the development of pest resistance [19, 20].

In the real world, the growth of most pest populations is not continuous, especially for those with non-overlapping generations, so they cannot be assumed to have continuous growth. Thus, for such pest populations and for the genetics of pest resistance, discrete mathematical models are more realistic.

Given the above, questions that arise are (1) how to model the evolution of pest resistance to a pesticide when the pest population growth is discontinuous? (2) How best to switch pesticides when aiming to eradicate a pest population? And(3)in which pest generation will pesticide switching be optimal?

To address the above questions, we developed novel discrete pest population models with impulsive chemical control and the evolution of pest resistance to pesticides. The main purpose was to address how the dynamic complexity of a pest population, development of pesticide resistance and pulsed chemical control and its spraying frequency affect optimal switching strategies, given different control aims. We have derived strong and weak threshold conditions which guarantee the extinction of the pest population, as well as an analytical formula for the optimal switching time. Further, we addressed switching strategies for a given economic injury level (EIL). Moreover, the effects of the complex dynamical behaviour of the pest population on the pesticide switching times were studied. In particular, the effects of the complex dynamics of the pest population and the pesticide application period on the pesticides' switching frequency are discussed in more detail. The main results indicated that the pesticide application period, the evolution of pesticide resistance and the dynamic complexity of the pest population may result in complex outbreak patterns, and consequently can significantly affect pesticide switching strategies.

2 Model formulation

In this section, we introduce a simple discrete pest population model with a Beverton-Holt growth function, in which the evolution of pest resistance is considered. In particular, the effects of both the frequency of pesticide applications and their cumulative number on the evolution of pest resistance are investigated.

2.1 Simple pest growth model with pesticide resistance

Throughout this study, the pest population is assumed to follow the classic Beverton-Holt model [21, 22, 23, 24, 25, 26], i.e. we have

$$P_{t+1} = \frac{aP_t}{1 + bP_t},$$

where P_t denotes the pest population size at generation t, a is the intrinsic growth rate, $b = (a-1)/K$, and K is the carrying capacity . The dynamical behaviour of the above model is completely determined by the parameter a, i.e. $a \leq 1$ means that the pest population will die out eventually, and $a > 1$ indicates that all solutions of the model with positive initial conditions will tend to its unique positive equilibrium K globally. As mentioned in the introduction, the main purpose of this study is to address how the evolution of pesticide resistance affects the success or failure of pest control when chemical control is applied. Thus, we assume $a > 1$ throughout this paper.

In the following, we divide the total pest population at generation t into two parts. Susceptible pests, very sensitive to the pesticide, are denoted by P_t^S, accounting for a proportion ω_t of the total pest population, and resistant pests, denoted by P_t^R, accounting for $1-\omega_t$ of the total pest population. This indicates that $P_t^S = \omega_t P_t$ and $P_t^R = (1-\omega_t)P_t$. Thus, ω_t may be thought of as the effectiveness of the pesticide at generation t. With increasing pest generations, the pest's resistance to the pesticide develops, and the effectiveness of the pesticide decreases, indicating that ω_t is a decreasing function of t. Therefore, the evolution of pest resistance can be described by the variable ω_t. Further, we assume that the death rates due to pesticide applications of the susceptible pests and the resistant pests are d_1 $(0 < d_1 < 1)$ and d_2 $(0 \le d_2 < 1)$, respectively. Based on these assumptions, we have the following discrete pest growth model with pesticide resistance

$$\begin{cases} P_{t+1}^S &= \frac{(1-d_1)\omega_t a P_t}{1+bP_t}, \\ P_{t+1}^R &= \frac{(1-d_2)(1-\omega_t)aP_t}{1+bP_t}. \end{cases} \tag{1}$$

Since $P_{t+1} = P_{t+1}^S + P_{t+1}^R$, the evolution of the total pest population is given by

$$P_{t+1} = \frac{[(1-d_1)\omega_t + (1-d_2)(1-\omega_t)]aP_t}{1+bP_t}. \tag{2}$$

It follows from $\omega_t = P_t^S/P_t$ that the evolution of the pest's resistance can be modelled as follows:

$$\begin{aligned} \omega_{t+1} &= \frac{P_{t+1}^S}{P_{t+1}} \\ &= \frac{(1-d_1)\omega_t}{(1-d_1)\omega_t + (1-d_2)(1-\omega_t)}. \end{aligned} \tag{3}$$

Therefore, model (1) can be written as

$$\begin{cases} P_{t+1} &= \frac{[(1-d_1)\omega_t + (1-d_2)(1-\omega_t)]aP_t}{1+bP_t}, \\ \omega_{t+1} &= \frac{(1-d_1)\omega_t}{(1-d_1)\omega_t + (1-d_2)(1-\omega_t)}. \end{cases} \tag{4}$$

It follows from $0 < d_1 < 1$ and $0 \le d_2 < 1$ that $0 < \omega_t < 1$ $(t = 1, 2, \cdots)$ holds true provided that $0 < \omega_0 < 1$.

In reality, farmers usually spray pesticide within a quite short period, and the effect of the pesticide on the pest is instantaneous, so its population density can be reduced instantaneously once the pesticide is applied. To depict this realistic control measure, we employ an impulsive difference equation based on model (4). Thus, we assume that the pesticides are applied periodically at every qth generation, then the number of pests killed at the qkth generation is $(\omega_{qk}d_1 + (1-\omega_{qk})d_2)P_{qk},\ k = 1, 2, \cdots$. Therefore, we have the following impulsive difference equation

$$\begin{cases} P_{t+1} = \frac{aP_t}{1+bP_t},\ t = 0, 1, 2, \cdots, \\ P_{qk^+} = (1 - \omega_{qk}d_1 - (1-\omega_{qk})d_2)P_{qk},\ k = 1, 2, \cdots, \\ \omega_{t+1} = \frac{(1-d_1)\omega_t}{(1-d_1)\omega_t + (1-d_2)(1-\omega_t)}, \end{cases} \tag{5}$$

where P_{qk^+} represents the number of pests after a single pesticide application at generation qk, and the initial value $P_{0^+} = P_0 > 0$. That is to say the initial density of the pest population in model (5) is chosen as the density of pests after the first pesticide spraying.

However, for simplicity, we assume that the resistant pests have near-complete resistance to the pesticide, which means that $d_2 \approx 0$ [27], so system (5) becomes

$$\begin{cases} P_{t+1} = \frac{aP_t}{1+bP_t},\ t = 0, 1, 2, \cdots, \\ P_{qk^+} = (1 - \omega_{qk} d_1) P_{qk},\ k = 1, 2, \cdots, \\ \omega_{t+1} = \frac{(1-d_1)\omega_t}{1-d_1\omega_t}. \end{cases} \tag{6}$$

Model (6) indicates that, obviously, the killing efficacy of the pesticide decreases as the resistance develops.

2.2 The effect of the frequency of pesticide applications on the evolution of pest resistance

The third equation of model (6) describes how the proportion of susceptible pests in the population develops with increasing pest generations, and thus the evolution of pest resistance with increasing time, so we call it the evolution of pest resistance equation. In reality, the frequency of pesticide applications, the pesticide application period and the dosage of the applications are also factors contributing to the pest resistance. Therefore, in order to understand the system in more detail, all of these factors should also be involved in this equation. Although achieving this was challenging, we employed the following simple method to tackle the task.

By using the general Beverton-Holt equation we extend the third equation of model (6) as follows:

$$\omega_{t+1} = \frac{(1-d_1)\omega_t}{1-d_1\omega_t^{r_k}},\ qk \le t < (k+1)q, \tag{7}$$

here at each qkth ($k = 1, 2, \cdots$) generation one pulse of pesticide is applied, and the dynamic parameter r_k, which depends on the total number of pesticide applications, was introduced to represent the effects of their period and dosage on the evolution of pest resistance. So r_k should be a function of the interval of q generations between the kth and $(k+1)$th pesticide applications, the number of pesticide applications k and the dosage D_k of the kth pesticide application for all $k = 1, 2, \cdots$. We know that the values of d_1 and d_2 strictly depend on the pesticide dosage D_k. For simplicity, we assume that the same dosage of pesticide is applied at each control event, and so, without loss of generality, we let $D_k = 1$ and d_1, d_2 represent the death rates of the pest after one unit of sprayed pesticide. Thus the simplest formula for r_k could be defined as $r_k = \frac{k+1}{q}$, i.e. $r_0 = 1/q$ for $t = 1, 2, \cdots, q-1$; $r_1 = 2/q$ for $t = q, q+1, \cdots, 2q-1$; $\cdots$; $r_k = (k+1)/q$ for $t = kq, kq+1, \cdots, (k+1)q-1$. In order to show how the spraying period and the number of pesticide applications or frequency of pesticide applications affect the development of resistance, the evolution of ω_t with four different r_k are shown in Fig.1, from which we can see that the smaller spraying period, the faster the evolution of pest resistance. This is because the smaller the spraying period the greater the number of pesticide applications, and thus faster decreases of the pest's sensitivity to the pesticide and accelerated evolution of pest resistance.

By induction, we can get the recursion formula for ω_t of equation (7) as follows:

$$\omega_t = \frac{A^t \omega_0}{M_t}, \ t = 1, 2, \cdots, \tag{8}$$

where $A = 1 - d_1,\ M_t = M_{t-1}\left(1 - A^{(t-1)r_k} Q_k M_{t-1}^{-r_k}\right),\ Q_k = d_1 \omega_0^{r_k}$ for $t = kq, kq + 1, \cdots, (k+1)q - 1$, and $M_0 = 1$.

In particular, if $r_k = 1$, i.e. the evolution of ω_t satisfies the third equation of model (4), then

$$\omega_t = \frac{A^t \omega_0}{1 - (1 - A^t)\omega_0}, \ t \geq 0. \tag{9}$$

3 Pest extinction resulting from control and the optimal time to switch pesticides

One of the main purposes of this paper is to investigate how to spray pesticides and manage the evolution of pest resistance such that the pest population will be eradicated eventually or be maintained at a density below a given value (i.e. EIL). In order to address this topic, we introduce two methods, and for each method we investigate the threshold condition which guarantees the extinction of the pest population and discuss the optimal pest generation when pesticides should be switched.

3.1 Switching pesticides with a strong threshold condition

Strong threshold condition for pest extinction: Considering the effects of pest control on the evolution of pest resistance, model (6) becomes the following periodic control model:

$$\begin{cases} P_{t+1} = \frac{aP_t}{1+bP_t}, \ t = 0, 1, 2, \cdots, \\ P_{qk^+} = (1 - \omega_{qk} d_1) P_{qk}, \ k = 1, 2, \cdots, \\ \omega_{t+1} = \frac{(1-d_1)\omega_t}{1 - d_1 \omega_t^{r_k}}. \end{cases} \tag{10}$$

where q is the period of pesticide applications and $r_k = (k+1)/q,\ P_{0^+} = P_0$.

Note that the pest resistance equation in model (10) (i.e. the third equation) is independent of the pest population growth equation (i.e. the first equation), thus ω_t can be studied independently using the formula for it given by (8).

Solving the first equation of model (10) in pulse interval $kq < t \leq (k+1)q,\ k = 0, 1, 2, \cdots$, yields

$$P_t = \frac{a^{t-qk} P_{qk^+}}{1 + b\left(\sum_{i=0}^{t-qk-1} a^i\right) P_{qk^+}}, \tag{11}$$

which means that

$$\begin{aligned} P_{(k+1)q} &= \frac{a^q P_{kq^+}}{1+bP_{kq^+}\left(\sum_{i=0}^{q-1} a^i\right)} \\ &= \frac{(1-\omega_{kq}d_1)a^q P_{kq}}{1+b(1-\omega_{kq}d_1)P_{kq}\left(\sum_{i=0}^{q-1} a^i\right)}. \end{aligned} \tag{12}$$

Denote $Y_k = P_{qk}$, then we have the following difference equation

$$Y_{k+1} = \frac{(1-\omega_{kq}d_1)a^q Y_k}{1 + b(1-\omega_{kq}d_1)\left(\sum_{i=0}^{q-1} a^i\right) Y_k}, \tag{13}$$

which is a non-autonomous Beverton-Holt difference equation, and Y_k depends on ω_{kq} or the third equation of model (10). For recent studies of non-autonomous Beverton-Holt difference equations with or without impulsive effects see [24, 25, 26, 28].

In this study, the stability of the zero solution of equation (13) is our main interest, given the practical problem of eradicating the pest population. It follows from equation (13) that the inequality

$$Y_{k+1} < (1-\omega_{kq}d_1)a^q Y_k$$

holds true for all $k = 1, 2, \cdots$. Thus, we can define the dynamic threshold value $R_0(n, T)$ as follows:

$$R_0(k, q) \doteq (1-\omega_{kq}d_1)a^q \tag{14}$$

where ω_{kq} can be calculated by (8). Therefore, if $R_0(k, q) < 1$ for all $k = 1, 2, \cdots$ (called a strong threshold condition for pest eradication), then the zero solution of equation (13) is globally asymptotically stable. This indicates that the pest population will be eradicated if the threshold value $R_0(k, q) < 1$ for all $k = 1, 2, \cdots$. The key factors including the evolution of pesticide resistance (i.e. ω_{kq}), the instantaneous killing rate (i.e. d_1), the intrinsic growth rate (i.e. a) and the period of pesticide application (i.e. q) are all involved in the formula for the threshold value, which is very dynamic. We will address the effect of the period of pesticide applications on the threshold value $R_0(k, q)$ in more detail later.

In particular, if $r_k = 1$ for $k = 1, 2, \cdots$ (i.e. $\omega(t)$ satisfies equation (4)), then

$$R_0(k, q) = \left(1 - \frac{d_1 A^{kq}\omega_0}{1-(1-A^{kq})\omega_0}\right) a^q \doteq R_0^1(k, q). \tag{15}$$

Fig.2 describes the effects of the spraying time k and the spraying period q on the threshold value $R_0(k, q)$, from which we can see that $R_0(k, q)$ is an increasing function with respect to k, and that it will reach and exceed 1 after several pesticide applications. These results confirm that the pesticide is effective at the initial stage. However, with the development of pesticide resistance, there will be an outbreak of the pest population after a certain number of pesticide sprays. Fig.2 also indicates that $R_0(k, q)$ is an increasing function with respect to period q, and the longer the spraying period, the fewer the number of times that the pesticide applications remain efficient due to the evolution of the pesticide resistance.

In Fig.3 we have plotted the solutions of model (10) for different q values to show how fast the solutions reach or exceed the given EIL and to show the effects of the pesticide application period on the density of the pest population. From Fig.3 we can see that the density of the pest population is decreasing at the first few pesticide applications, but it increases, even reaching or exceeding the EIL quickly, as the number of pesticide applications increase. This is because of the strengthening of the pest's resistance to the pesticide and the decreasing efficacy of the pesticide.

Fig.3 also shows that the longer the period of pesticide applications q (i.e. low frequency of pesticide applications), the higher the probability that there will be a pest outbreak. However, the smaller the period of pesticide applications q (i.e. higher frequency of pesticide applications), the faster the development of pest resistance, and the easier it is for the pest to reach outbreak levels. Therefore, the question is how to control pest resistance (i.e. what is the optimal generation of the pest after the start of control operations when a switch to a new type of pesticide is best) such that the pest population will die out or its density will fall below the EIL? We will address this question in the following subsection.

Justifications and the optimal time to switch pesticides: As mentioned in the introduction, pest control will fail if the pest has developed resistance to some pesticides and people repeatedly use those pesticides. If so, the density of the pest population will grow quickly (as shown in Fig.3), and even result in pest outbreaks or resurgence. Therefore, in order to control pests successfully, people usually switch from using one type of pesticide to spraying another type of pesticide to avoid or decrease the development of resistance. Thus, in order to optimally use the same type of pesticides, it is important to choose the optimal switching time with the aim of controlling the pest population economically and effectively. In the following, we will provide a method based on our model (10) to determine the optimal time for switching pesticides according to a threshold condition. We assumed that for each new type of pesticide, the evolution of pest resistance to it follows the same trend (i.e. ω follows the same equation) and has the same initial condition ω_0.

From Fig.2, we can see that $R_0(k,q)$ is increasing with respect to k and it will exceed 1 after several pesticide applications. According to the definition of $R_0(k,q)$, if the aim of pest control is to eradicate the pest population, the threshold value $R_0(k,q)$ should be below one for all $k = 1, 2, \cdots$, i.e. the strong threshold condition should be satisfied. To maintain the threshold value $R_0(k,q)$ below one, we must switch to using another kind of pesticide before the threshold value $R_0(k,q)$ reaches one. Therefore, the optimal pesticide switching tactics should be implemented at the last spraying time before $R_0(k,q)$ reaches one. Without loss of generality, we assume that the threshold value $R_0(k,q)$ will increase and exceed one unit after $k_1^{(1)}$ sprays of the same kind of pesticide, i.e.

$$k_1^{(1)} = \max\{k : R_0(k,q) \leq 1\}, \tag{16}$$

thus the optimal switching time is $k_1^{(1)}q$.

In order to determine $k_1^{(1)}$ analytically, we let $R_0(k,q) = 1$, then

$$\omega_{kq} = \frac{1 - a^{-q}}{d_1},$$

where ω_{kq} is given by (8) and $(1 - a^{-q})/d_1 \leq \omega_0$. Therefore,

$$k_1^{(1)} = \left[\left\{ k : \omega_{kq} = \frac{1 - a^{-q}}{d_1} \right\} \right],$$

and $[x]$ is defined as the greatest integer no larger than x.

In particular, $R_0(k,q) = R_0^1(k,q)$ for $r_k = 1$. In this special case, letting $R_0^1(k,q) = 1$ and solving this equation with respect to k, we can obtain the optimal switching time $k_1^{(1)}q$, where

$$k_1^{(1)} = \left[\frac{1}{q} \log_A \left(\frac{(1 - a^{-q})(1 - \omega_0)}{\omega_0(d_1 - (1 - a^{-q}))} \right) \right].$$

Thus, according to the above pesticide switching strategy, the pest population will be eradicated completely after several pesticide switches. In order to understand this strategy intuitively, we plotted some numerical simulations in Fig.4 (a), from which we can see that the pest population will be eliminated eventually, with $k_1^{(1)} = 2$. This indicates that farmers should switch to another type of pesticide after three pesticide sprays of one type of pesticide (here we assume that the first pesticide spraying is at time $t = 0$) to eliminate the pest population quickly.

3.2 Switching pesticides with a weak threshold condition

Note that if the strong threshold condition for pest eradication is satisfied, then we have

$$P_q > P_{2q} > P_{3q} > \cdots > P_{nq} > \cdots,$$

and $P_{nq} \to 0$ when n is large enough. This switching method could result in more severe environmental pollution due to the speed of switching between pesticides. Therefore, the question is how to reduce the switching frequency such that the pest population can still be eradicated or maintained at a density below the given EIL? To realize this purpose, we propose the following weak threshold condition for pest extinction.

Weak threshold condition for pest extinction: We assume that after n_i times of spraying with the ith pesticide, farmers should switch to using the $(i+1)$th pesticide, that is the ith pesticide can be used n_i times at most. For example, the first type of pesticide is sprayed at the beginning, at pest generation q, generation $2q$, $\cdots$, generation $(n_1-1)q$, and the second type of pesticide is applied at generation n_1q, generation $(n_1+1)q$, $\cdots$, generation$(n_1+n_2-1)q$, $\cdots$. Thus, all pesticides are switched at generation n_1q, generation $(n_1+n_2)q$, generation $(n_1+n_2+n_3)q$, and so on.

Denoting

$$P_{kq}^{(m)} = P_{(\sum_{i=1}^{m} n_i + k)q}, \ k = 0, 1, \cdots, n_i,$$

which is the density of the pest population at the $k+1$th pesticide spray and after $m+1$ pesticide switches. Specifically, $P_0^{(m)} = P_{(\sum_{i=1}^{m} n_i)q}$.

From (12), we have

$$\begin{aligned} P_q^{(m)} &= \frac{a^q P_{0^+}^{(m)}}{1+b\left(\sum\limits_{i=0}^{q-1} a^i\right) P_{0^+}^{(m)}} \\ &= \frac{a^q\left(1-d_1\omega_0^{(m)}\right)P_0^{(m)}}{1+b\left(\sum\limits_{i=0}^{q-1} a^i\right)\left(1-d_1\omega_0^{(m)}\right)P_0^{(m)}}, \end{aligned} \tag{17}$$

thus,

$$P_{q^+}^{(m)} = \frac{a^q\left(1-d_1\omega_0^{(m)}\right)\left(1-d_1\omega_q^{(m)}\right)P_0^{(m)}}{1+b\left(\sum\limits_{i=0}^{q-1} a^i\right)\left(1-d_1\omega_0^{(m)}\right)P_0^{(m)}},$$

and

$$\begin{aligned} P_{2q}^{(m)} &= \frac{a^q P_{q^+}^{(m)}}{1+b\left(\sum\limits_{i=0}^{q-1} a^i\right) P_{q^+}^{(m)}} \\ &= \frac{a^{2q}\left(1-d_1\omega_0^{(m)}\right)\left(1-d_1\omega_q^{(m)}\right)P_0^{(m)}}{1+b\left(\sum\limits_{i=0}^{q-1} a^i\right)\left(1-d_1\omega_0^{(m)}\right)P_0^{(m)}+b\left(\sum\limits_{i=0}^{q-1} a^i\right)a^q\left(1-d_1\omega_0^{(m)}\right)\left(1-d_1\omega_q^{(m)}\right)P_0^{(m)}}, \end{aligned}$$

$$P_{2q^+}^{(m)} = \frac{a^{2q}\left(1-d_1\omega_0^{(m)}\right)\left(1-d_1\omega_q^{(m)}\right)\left(1-d_1\omega_{2q}^{(m)}\right)P_0^{(m)}}{1+b\left(\sum\limits_{i=0}^{q-1} a^i\right)\left(1-d_1\omega_0^{(m)}\right)P_0^{(m)}+b\left(\sum\limits_{i=0}^{q-1} a^i\right)a^q\left(1-d_1\omega_0^{(m)}\right)\left(1-d_1\omega_q^{(m)}\right)P_0^{(m)}},$$

$$P_{3q}^{(m)} = \frac{a^q P_{2q^+}^{(m)}}{1+b\left(\sum_{i=0}^{q-1} a^i\right) P_{q^+}^{(m)}} = \frac{a^{3q}\prod_{i=0}^{2}\left(1-d_1\omega_{iq}^{(m)}\right)P_0^{(m)}}{1+b\left(\sum_{i=0}^{q-1} a^i\right)P_0^{(m)}\left(\sum_{k=0}^{2}\prod_{j=0}^{k}\left(1-d_1\omega_{jq}^{(m)}\right)a^{iq}\right)},$$

where $\omega_{iq}^{(m)}$ is the proportion of susceptible pests in the population at generation iq with the $(m+1)$th pesticide. By induction, we have

$$P_{n_{m+1}q}^{(m)} = \frac{a^{n_{m+1}q}\prod_{i=0}^{n_{m+1}-1}\left(1-d_1\omega_{iq}^{(m)}\right)P_0^{(m)}}{1+b\left(\sum_{i=0}^{q-1} a^i\right)P_0^{(m)}\left(\sum_{k=0}^{n_{m+1}-1}\prod_{j=0}^{k}\left(1-d_1\omega_{jq}^{(m)}\right)a^{iq}\right)}. \tag{18}$$

Due to $P_0^{(m+1)} = P_{n_{m+1}q}^{(m)}$, therefore, we have the following equation

$$P_0^{(m+1)} = \frac{a^{n_{m+1}q}\prod_{i=0}^{n_{m+1}-1}\left(1-d_1\omega_{iq}^{(m)}\right)P_0^{(m)}}{1+b\left(\sum_{i=0}^{q-1} a^i\right)P_0^{(m)}\left(\sum_{k=0}^{n_{m+1}-1}\prod_{j=0}^{k}\left(1-d_1\omega_{jq}^{(m)}\right)a^{iq}\right)}, \tag{19}$$

this is the well-known Beverton-Holt model, which has a zero equilibrium $P_1^* = 0$. It is stable provided that

$$R_0^i \doteq a^{n_i q}\prod_{j=0}^{n_i-1}\left(1-d_1\omega_{jq}^{(i-1)}\right) < 1, \text{ for all } i = 1, 2, \cdots. \tag{20}$$

Therefore, the pest population will be eradicated if condition (20) holds true. We define the above condition as the weak threshold condition for pest eradication in this paper.

Specially, if the pest has the same resistance to a different pesticide, then $n_i = n_{i+1} \doteq \tilde{n}$ and $\omega_{jq}^{(i-1)} = \omega_{jq}^{(i)} = \omega_{jq}$ for all $i = 1, 2, \cdots$. Thus,

$$R_0^i = a^{\tilde{n}q}\prod_{j=0}^{\tilde{n}-1}\left(1-d_1\omega_{jq}\right) \doteq \tilde{R}_0(\tilde{n}, q, d_1). \tag{21}$$

Note that

$$\tilde{R}_0(\tilde{n}, q, d_1) = a^q\left(1-d_1\omega_0\right)\cdot a^q\left(1-d_1\omega_q\right)\cdots a^q\left(1-d_1\omega_{(\tilde{n}-1)q}\right) = \prod_{j=0}^{\tilde{n}-1} W_j, \tag{22}$$

where $W_j = a^q\left(1-d_1\omega_{jq}\right)$, and W_j is increasing with respect to j.

Justifications and the optimal time to switch pesticides: We want to know how many times each pesticide can be sprayed or what is the optimal time for switching pesticides which can eradicate the pest population after some pesticide switches. As before, in order to eradicate the pest population we should maintain $R_0^i < 1$ for all $n_i,\ i \in \mathcal{N}$. This indicates that farmers should switch pesticides once R_0^i goes to one. Because of the complexity of R_0^i, we only focus on the special case, i.e. $\tilde{R}_0$. We assume that the threshold value $\tilde{R}_0(\tilde{n}, q, d_1)$ will exceed one after $k_2^{(2)}$ pesticide applications. From (21), we can see that $\tilde{R}_0(\tilde{n}, q, d_1)$ is an increasing function with respect to $\tilde{n}$, so

$$k_2^{(2)} = \max\{\tilde{n} : \tilde{R}_0(\tilde{n}, q, d_1) \leq 1\}, \tag{23}$$

i.e.

$$k_2^{(2)} = \left[\{\tilde{n} : \tilde{R}_0(\tilde{n}, q, d_1) = 1\}\right]. \tag{24}$$

It follows from expressions (14) and (22) that $R_0(k, q) < 1$ implies $W_k < 1$, which indicates $W_i < 1$ for all $i \leq k$, and then $\prod_{j=0}^{k} W_j < 1$. Therefore, $\tilde{R}_0(k, q, d_1) < 1$, which means that the condition $R_0(k, q) < 1$ is stronger than the condition $\tilde{R}_0(k, q, d_1) < 1$. These results confirm that $k_2^{(2)} \geq k_1^{(1)}$, i.e. the same type of pesticide can be used more times under the weak threshold condition for pest eradication than under the strong threshold condition.

Fig.4 (b) gives numerical simulations with the weak threshold condition for pest eradication. From Fig.4 (b) we can see that the pest population dies out eventually with $k_2^{(2)} = 3$ in the case of $\tilde{R}_0(\tilde{n}, q, d_1) < 1$. However, if $\tilde{R}_0(\tilde{n}, q, d_1) > 1$, then the pest population will oscillate periodically under the weak threshold condition (see Fig.5 (a)) and finally its density could exceed the given EIL.

4 Pest control with EIL as a guide

Considering the importance of reducing pollution and the cost to farmers of pest control measures, farmers usually implement them such that the density of the pest population cannot exceed the EIL. It follows from Fig.3 that if we only repeat using one kind of pesticide to control the pest, then the resistance of the pest to the pesticide is developed and the efficiency of the pesticide declines. Thus, the density of the pest population increases quickly and eventually exceeds the given EIL. Therefore, in order to control the density of the pest below the EIL, farmers usually switch to another type of pesticide before the EIL is exceeded. Therefore, we want to know what is the optimal switching time or what is the optimal frequency of one type of pesticide applications for a given EIL?

In this section, we assume that after k pesticide applications, farmers should switch to another type of pesticide. This indicates that $P_{kq} \leq EIL$ and there exists a positive integer m $(0 \leq m \leq q)$ such that $P_{kq+m} \geq EIL$.

From (12), we have

$$P_{(k+1)q} = \frac{A_k P_{kq}}{1 + B_k P_{kq}} > EIL, \tag{25}$$

where $A_k = a^q(1 - d_1\omega_{kq})$ and $B_k = bA_k(\sum_{i=0}^{q-1} a^{i-q})$. According to (25), we can get

$$(A_k - B_k EIL)P_{kq} > EIL. \tag{26}$$

It follows from $P_{kq} \leq EIL$ that

$$A_k - B_k EIL = A_k \left(1 - bEIL \sum_{i=0}^{q-1} a^{i-q}\right) > 1,$$

or

$$A_k > \frac{1}{1 - bEIL \sum_{i=0}^{q-1} a^{i-q}} > 0.$$

This indicates that q should be satisfied

$$b \sum_{i=0}^{q-1} a^{i-q} = b \sum_{i=0}^{q} \frac{1}{a^i} < \frac{1}{EIL}. \tag{27}$$

From (26), we have

$$P_{kq} A_k \left(1 - bEIL \sum_{i=0}^{q-1} a^{i-q}\right) > EIL, \tag{28}$$

thus

$$P_{kq} A_k > \frac{EIL}{1 - bEIL \sum_{i=0}^{q-1} a^{i-q}}.$$

Since $P_{kq}A_k$ is an increasing function with respect to k, we have

$$k = \left[\left\{ l \middle| P_{lq} A_l = \frac{EIL}{1 - bEIL \sum_{i=0}^{q-1} a^{i-q}} \right\}\right] + 1. \tag{29}$$

Fig.5 (b) gives the numerical simulation under this tactic of switching pesticides, from which we can see that pest control will tend towards periodic control after a certain number of pesticide switches. In reality, pest control can also tend towards periodic control under the weak threshold condition provided that the control period q is long enough (i.e. $\tilde{R}_0(\tilde{n}, q, d_1) > 1$) (see Fig.5 (a)). However, under the weak threshold condition with $\tilde{R}_0(\tilde{n}, q, d_1) > 1$, unless the switching frequency is more than with the EIL guided switching strategy, the density of the pest population will exceed the EIL.

5 The effects of dynamic complexity of the pest population on the control measures

In the previous section, we assumed that the pest population followed the classic Beverton-Holt difference equation, which means that the pest population either tends to zero or to the unique positive equilibrium if no control tactics are involved. The question now is how do the complex dynamics of the pest population affect the pest control? Also, of particular interest is how does this complexity affect the pesticide switching frequency and the optimal switching time under different switching justifications?

To address the above questions, we extended model (6) by employing the general Beverton-Holt function to describe the growth of the pest population, i.e. we have

$$\begin{cases} P_{t+1} = \frac{aP_t}{1+bP_t^m},\ t = 0, 1, 2, \cdots, \\ P_{qk^+} = (1-\omega_{qk}d_1)P_{qk},\ k = 1, 2, \cdots, \\ \omega_{t+1} = \frac{(1-d_1)\omega_t}{1-d_1\omega_t^{r_k}}. \end{cases} \tag{30}$$

where m is a positive integer.

Although we can investigate model (30) by employing the same methods as those for model (10), it is very difficult to provide the threshold conditions related to the different pesticide switching strategies. So we turn to numerical methods aiming to show how the dynamic complexity of the pest population affects the pesticide switching strategies and then how it affects the pest control. To address those questions, we first apply bifurcation analyses, as shown in Fig.6.

With m as a bifurcation parameter, bifurcation diagrams of system (30) without a pesticide switching strategy are plotted in Fig.6 (a) and with the switching strategy under the weak threshold condition in Fig.6 (b). The results indicate that system (30) may exhibit complex dynamical behaviour such as period doubling bifurcations and multiple attractors co-existing for a wide range of parameters.

In order to analyze the effects of the dynamic complexity of the pest population with the weak threshold condition guiding the pesticide switching strategy, we depict the pest population growth trends of models (6) and (30) with different control period q in Fig.7. From Fig.7 (a) and (c), we can see that one type of pesticide should be switched to another after two sprays in models (6) and (30) with control period $q = 2$ and the pest population can be eradicated after several pesticide switches. Comparing Fig.7 (a) and (c) we conclude that the density of the pest population decreases more quickly in model (30) than that in model (6). Increasing the period q from 2 to 3, it follows from Fig.7 (d) that the pest population oscillates periodically, and the switching frequency is two in model (6), and from Fig.7 (b) we can see that the pesticide switching trends become more complex, and the density of the pest population oscillates periodically with a related large amplitude in model (30) which could more easily exceed the given EIL. Thus, the dynamic complexity of the pest population may result in complex outbreak patterns, and consequently can significantly affect the pesticide switching strategies.

When choosing the EIL as the guide for the pesticide switching strategy, we focused on how the parameter m affects the EIL switching strategy. To address this question, we let m vary and fixed all other parameters in model (30), as shown in Fig.8. The main results indicate that for different values of parameter m, the pesticide switching frequencies are quite different: the larger the m value, the more frequent the need for switching, as shown in Fig.8 (a-c). In particular, each type of pesticide can be applied for about three periods ($3q$ here), and then switching should occur in the middle of the third pest control period for $m = 1$ (Fig.8(a)). If we increase m from 1 to 2, then control with each type of pesticide can be implemented for about two periods ($2q$ here), and then the switching should occur in the middle of the second control period (Fig.8(b)). Farmers should switch pesticide within one period q once $m = 3$, as shown in Fig.8(c). However, if we increase m to 4 as in Fig.8(d), then each type of pesticide can be used one more time compared with when $m = 3$. These results confirm that the dynamic complexity of the pest population can result in more complex pesticide switching strategies if the EIL guided method is employed.

6 Discussion

Pest control is an important part of agricultural management, for which chemical control by spraying pesticides is the main method. However, more and more pests have developed resistance to pesticides with the frequent use of only one or two kinds of pesticides for lengthy periods. This can lead to pest resurgence and serious losses for farmers so pest resistance management is important.

Control in pulses such as pesticide sprays or natural enemy releases is a common method for pest control in IPM and can be modelled with impulsive equations. For instance, recent studies of impulsive equations have been applied in the analysis of pulsed pest control in theory, such as the spraying of pesticides at critical times and killing pests instantly [11, 12, 13, 14, 15, 29, 30] and for biological control by releasing natural enemies at critical times [16, 17, 31, 32, 33, 34, 35, 36, 37]. The existence of high density pest populations with overlapping generations was a common assumption in those studies, which mainly focused on the effects of chemical control on the extinction or permanence of pest populations and the effects of pesticide resistance were seldom considered. However, in this paper, we developed a discrete pest population growth model which addressed pesticide resistance. Furthermore, the effects of the spraying period and the number of pesticide applications or the frequency of pesticide applications on the development of pest resistance, and consequently on the success or failure of pest control, were investigated.

In order to fight pesticide resistance and avoid pest resurgence, many principles have been proposed. Switching pesticides between two or more types is a common and effective tactic to delay or reduce the evolution of pest resistance. For instance, for controlling the peach potato aphid Myzus persicae, farmers have had to switch successively since the late 1940s from organophosphate pesticides to cyclodienes, to carbamates to pyrethroids and, finally, to neonicotinoids and now there is also resistance to the latter [38]. However, if the aim of pest control is to eradicate the pest population, what is the optimal justification for switching from one type of pesticide to another or others? How to determine the period or the frequency of pesticide application for pesticide switches? And if the aim of pest control is getting the density of the pest population below an EIL, what is the optimal time for switching pesticides? Although our results show that modelling can aid in answering such questions, it is also important for decision-makers to be aware of factors such as the biochemistry of resistance mechanisms, the extent of cross-resistance to more than one pesticide type and the likelihood of resistance to novel compounds developing, as discussed by Bass et al. [38] regarding the management of peach potato aphids. If such approaches had been used more carefully in the past, resistance by rats to anticoagulant rodenticides [39] and many other such examples of pests developing resistance to a variety of products might have been avoided or at least delayed.

To answer these questions, we provided two methods including strong and weak threshold conditions, respectively, for pest eradication to judge when we should switch pesticides if the aim is eradication of the pest population. For the former method, we provided a strong threshold condition for pest eradication, and the optimal period of pesticide application for one type of pesticide. Moreover, we investigated the optimal switching time or frequency of pesticide applications for pesticide switches. In order to maximise the utilization of a pesticide, we provided a weak threshold condition for pest eradication in a second method and we also investigated the optimal switching time and the frequency of pesticide applications between pesticide switches and discussed the advantages and disadvantages of both methods.

According to the definition of IPM, the EIL is an important threshold value for pest control. Therefore, we provided one switching method with the EIL as a switching guide and the optimal number of sprays for one type of pesticide was investigated. In order to show how the dynamic complexity of the pest population influences the pest control and pesticide switching strategies, we extended the model using the generalized Beverton-Holt function. The main results from this model indicated that the switching frequency can be significantly affected by the dynamical behaviour of the pest population, as shown in Figs.7 and Fig.8.

IPM is another tactic for fighting pest resistance, which usually controls pest populations by combining chemical control and biological control. Our future research will address questions such as how best to design IPM control tactics if the generations of pest populations do not overlap i.e. how to introduce biological control in discrete pest population growth models in theory? And what is the balance between the evolution of pest resistance and the rate of natural enemy releases?

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References

[1] Georghiou GP. Overview of insecticide resistance, in: Green, MB, LeBaron, HM, Moberg, WK (Eds): Managing resistance to agrochemicals: from fundamental research to practical strategies. American Chemical Society, Washington DC 1990; 18-41.

[2] Kotchen MJ. Incorporating resistance in pesticide masnagement: a dynamic regional approach. New York: Springer Verlag 1999; 126-135.

[3] Pesticides 101 - A Primer. http://www.panna.org/issues/pesticides-101-primer.

[4] Karaagac SU. Insecticide Resistance, In: Farzana Perveen. Insecticides and Advances in Integrated Pest Management. InTech Press 2012; 469-478.

[5] Dumont Y, Tchuenche JM. Mathematical studies on the sterile insect technique for the Chikungunya disease and Aedes albopictus. J Math Biol 2012; 65: 809-854.

[6] Flint ML. Integrated Pest Management for Walnuts, University of California Statewide Integrated Pest Management Project. Division of Agriculture and Natural Resources, second ed, University of California, Oakland, CA, Publication 1987; 3270.

[7] Onstad DW. Insect Resistance Management. Elsevier: Amsterdam; 2008.

[8] Van Lenteren JC, Woets J. Biological and integrated pest control in greenhouses. Ann Rev Ent 1988; 33: 239-250.

[9] Van Lenteren JC. Integrated pest management in protected crops, in: D.Dent (Ed.). Integrated pest management, Chapman & Hall. London 1995; 311-320.

[10] Van Lenteren JC. Measures of success in biological control of arthropods by augmentation of natural enemies. in: S.Wratten, G.Gurr(Eds.), Measures of Success in Biological Control. Kluwer Academic Publishers. Dordrecht 2000; 77-89.

[11] Liang JH, Tang SY. Optimal dosage and economic threshold of multiple pesticide applications for pest control. Math Comput Model 2010; 51: 487-503.

[12] Tan YS, Liang JH, Tang SY. The dynamical behavior of non-smooth system with impulsive control strategies. Int J Biomath 2012; 5(3): 1260018.

[13] Wang LM, Chen LS, Nieto JJ. The dynamics of an epidemic model for pest control with impulsive effect. Nonlinear Anal: Real World Appl 2010; 11: 1374-1386.

[14] Xue Y, Tang SY, Liang JH. Optimal timing of interventions in fishery resource and pest management. Nonlinear Anal: Real World Appl 2012; 13: 1630-1646.

[15] Zhang H, Chen LS, Nieto JJ. A delayed epidemic model with stage-structure and pulses for pest management strategy. Nonlinear Anal: Real World Appl 2008; 9: 1714-1726.

[16] Jiao JJ, Cai SH, Chen LS. Analysis of a stage-structured predator-prey system with birth pulse and impulsive harvesting at different moments. Nonlinear Anal: Real World Appl 2011; 12: 2232-2244.

[17] Liang JH, Tang SY, Cheke RA. An integrated pest management model with delayed responses to pesticide applications and its threshold dynamics. Nonlinear Anal: Real World Appl 2012; 13: 2352-2374.

[18] Liang JH, Tang SY, Nieto JJ, Cheke RA. Analytical methods for detecting pesticide switches with evolution of pesticide resistance. Math Biosci 2013(a); 245: 249-257.

[19] Liang JH, Tang SY, Cheke RA, Wu JH. Adaptive release of natural enemies in a pest-natural enemy system with pesticide resistance. Bull. Math Biol 2013b; 75: 2167-2195.

[20] Liang JH, Tang SY, Cheke RA, Wu JH. Models for determining how many natural enemies to release inoculatively in combinations of biological and chemical control with pesticide resistance. J Math Anal Appl 2015; 422: 1479-1503.

[21] Beverton RJH, Holt SJ. On the dynamics of exploited fish populations. Fishery Investigations Series 2, Vol. 19 Ministry of Agriculture. Fisheries and Food 1957; 1: 957.

[22] Brännström Å, Sumpter DJT. The role of competition and clustering in population dynamics. Proc R Soc B 2005; 272: 2065-2072.

[23] Geritz SAH, Kisdi E. On the mechanistic underpinning of discrete-time population models with complex dynamics. J Theor Biol 2004; 228: 261-269.

[24] Berezansky L, Braverman E. On impulsive Beverton-Holt difference equations and their applications. J Differ Equ App 2004; 10: 851-868.

[25] Kocic VL, Stutson D, Arora G. Global behavior of solutions of a nonautonomous delay logistic difference equation. J Differ Equ App 2004; 10: 1267-1279.

[26] Kocic VL. A note on the nonautonomous Beverton-Holt model. J Differ Equ App 2005; 11: 415-422.

[27] Hall RJ, Gubbins S, Gilligan CA. Invasion of drug and pesticide resistance is determined by a trade-off between treatment efficacy and relative fitness. Bull Math Biol 2004; 66: 825-840.

[28] Tang SY, Cheke RA, Xiao YN. Optimal impulsive harvesting on non-autonomous Beverton-Holt difference equations. Nonlinear Anal 2006; 65: 2311-2341.

[29] Parker FD. Management of pest populations by manipulating densities of both host and parasites through periodic releases. In: Huffaker, C. B., ed. Biological control. New York: Plenum Press, 1971.

[30] Li CT, Tang SY. The effects of timing of pulse spraying and releasing periods on dynamics of generalized predator-prey model. Int J Biomath 2012; 5(01): 1250012.

[31] Neuenschwander P, Herren HR. Biological Control of the Cassava Mealybug. Phenacoccus manihoti, by the Exotic Parasitoid Epidinocarsis lopezi in Africa. Phil Trans R Soc Lond B 1988; 318: 319-333.

[32] Tang SY, Xiao YN, Chen LS, Cheke RA. Integrated pest management models and their dynamical behaviour. Bull Math Biol 2005; 67: 115-135.

[33] Tang SY, Xiao YN, Cheke RA. Multiple attractors of host-parasitoid models with integrated pest management strategies: Eradication, persistence and outbreak. Theor Popul Biol 2008; 73: 181-197.

[34] Tang SY, Cheke RA. Models for integrated pest control and their biological implications. Math Biosci 2008; 215: 115-125.

[35] Tang SY, Xiao YN, Cheke RA. Effects of predator and prey dispersal on success or failure of biological control. Bull Math Biol 2009; 71: 2025-2047.

[36] Tang SY, Tang GY, Cheke RA. Optimum timing for integrated pest management: Modelling rates of pesticide application and natural enemy releases. J Theor Biol 2010; 264: 623-638.

[37] Tang SY, Liang JH, Tan YS, Cheke RA. Threshold conditions for integrated pest management models with pesticides that have residual effects. J Math Biol 2013; 66: 1-35.

[38] Bass C, Puinean AM, Zimmer CT, Denholm I, Field LM, Foster SP, Gutbrod O, Nauen R, Slater R, Williamson MS. The evolution of insecticide resistance in the peach potato aphid, Myzus persicae. Insect Biochemistry and Molecular Biology 2014; 51: 41-51.

[39] Greaves JH. Managing resistance to anticoagulant rodenticides: an appraisal. Pesticide Science 1995; 43: 79-82

511 **Figure Legends**

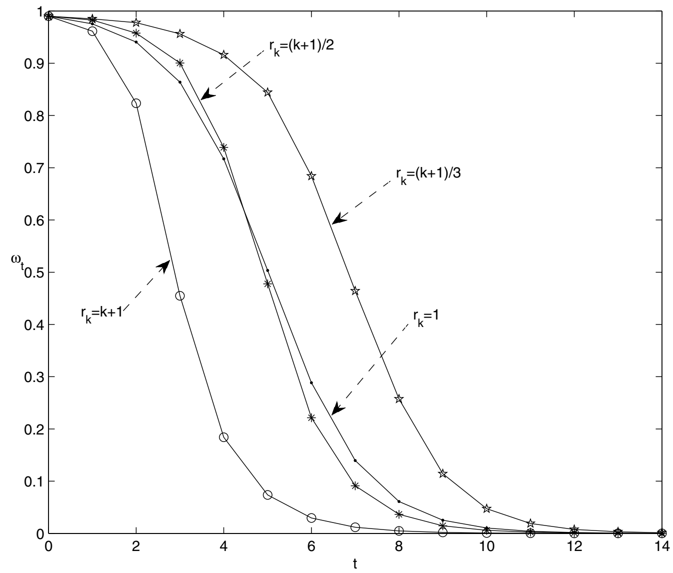

Figure 1: The effects of the frequency of pesticide applications on the evolution of ω_t with $d_1 = 0.6$. Four curves for ω_t are plotted with respect to $r_k = k+1, (k+1)/2, (k+1)/3$ and constant 1.

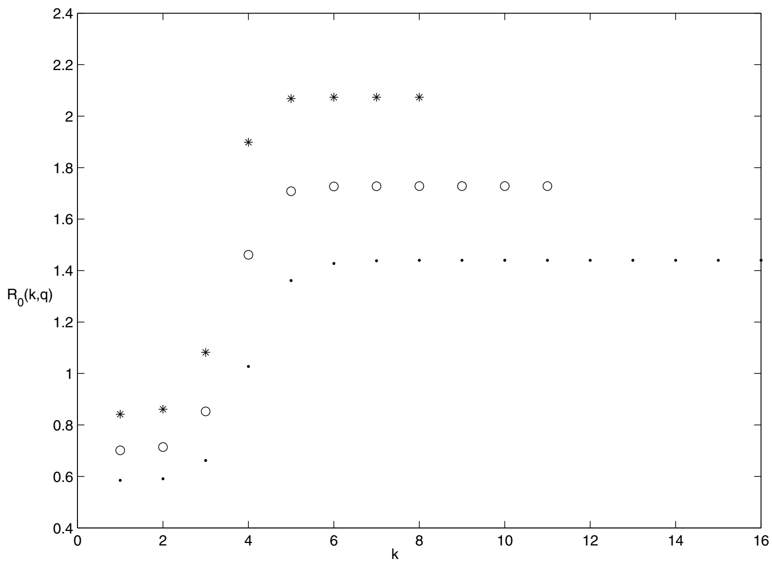

Figure 2: The effects of the period of pesticide applications on the threshold condition $R_0(k, q)$ for $q = 2(\bullet), q = 3(\circ), q = 4(*)$, respectively.

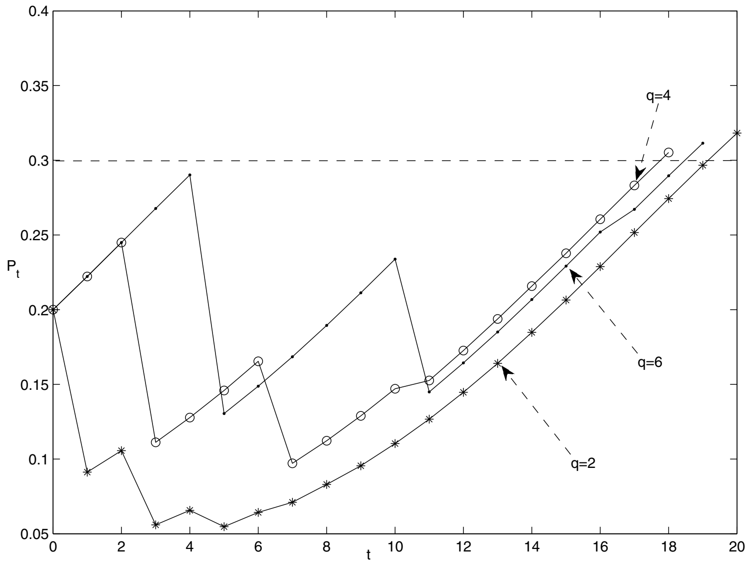

Figure 3: The effects of the period of pesticide applications on the density of the pest population predicted by model (10) for $q = 2, q = 4, q = 6$, respectively. The baseline parameter values were fixed as follows: $d_1 = 0.6, a = 1.2, b = 0.4, \omega_0 = 0.99, EIL = 0.3$ and the initial value of $P_0^+ = 0.2$.

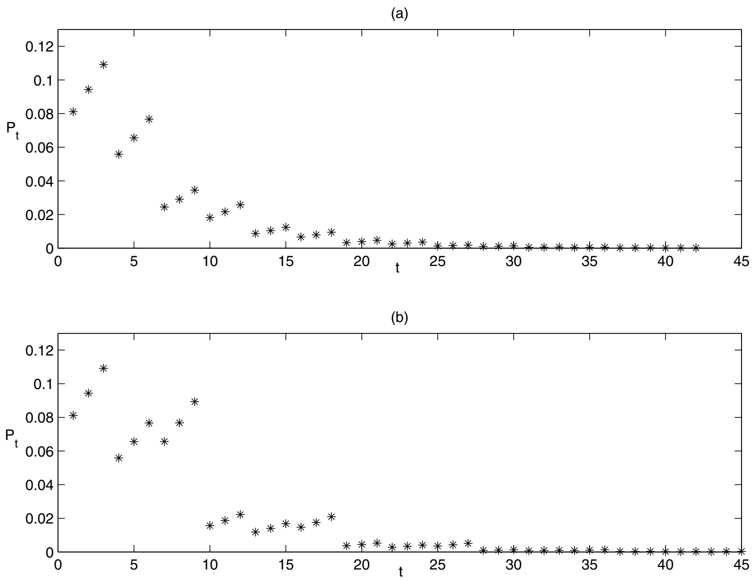

Figure 4: Illustrations of two different switching methods. The baseline parameter values are as follows: $d_1 = 0.6, a = 1.2, b = 0.4, \omega_0 = 0.99, q = 3, P_0 = 0.2$. (a) Numerical simulations of model (10) with several pesticide switches determined by the strong threshold condition; (b) Numerical simulations of model (10) with several pesticide switches determined by the weak threshold condition.

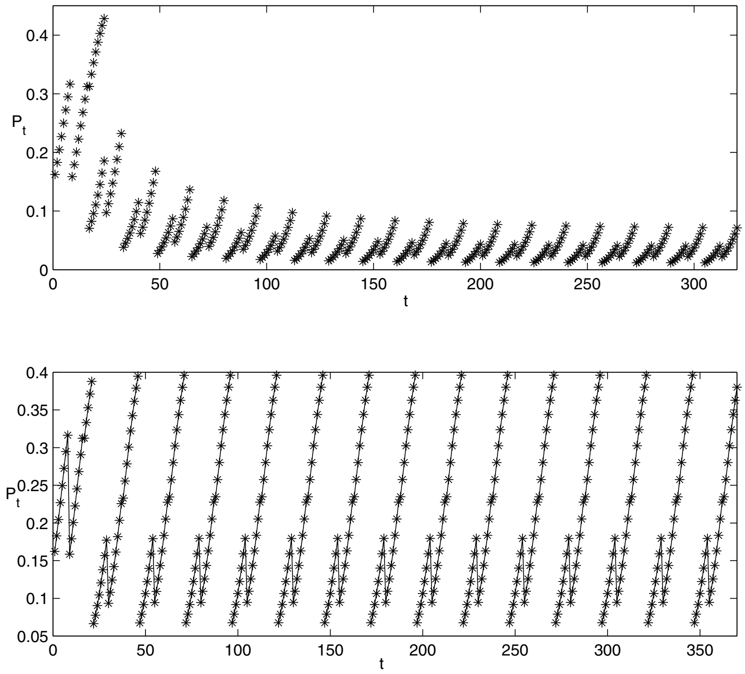

Figure 5: Illustrations of switching methods based on the weak threshold and on using the EIL-guided method. The baseline parameter values are as follows: $d_1 = 0.6, a = 1.2, b = 0.4, \omega_0 = 0.99, q = 8, P_0 = 0.2$ and $EIL = 0.4$. (a) Numerical simulations of model (10) with several pesticide switches determined by the weak threshold condition; (b) Numerical simulations of model (10) with several pesticide switches determined by the EIL guide.

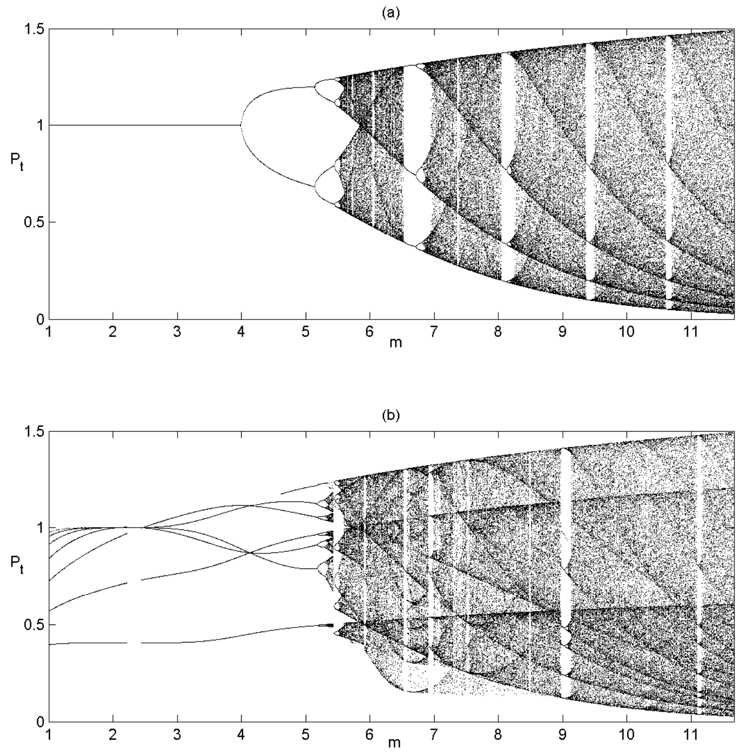

Figure 6: Bifurcation diagram for model (30) with bifurcation parameter m. The baseline parameter values are as follows: $d_1 = 0.6, a = 2, b = 1, \omega_0 = 0.99, q = 3$. (a) Pest control with no pesticide switching strategy; (b) Pest control with the switching strategy determined by the weak threshold condition.

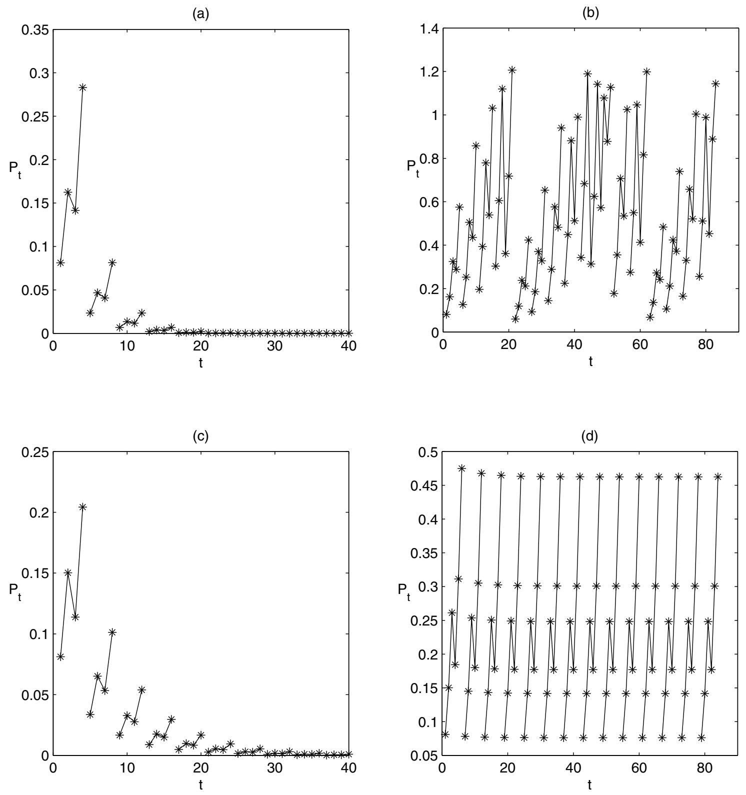

Figure 7: Illustrating the difference between model (30) and model (10) with switching strategies guided by the weak threshold condition. The baseline parameter values are as follows: $d_1 = 0.6, a = 2, b = 1, m = 5, \omega_0 = 0.99, P_0 = 0.2$. (a) Numerical simulations of model (30) with several pesticide switches which are guided by the weak threshold condition and $q = 2$; (b) Numerical simulations of model (30) with several pesticide switches which are guided by the weak threshold condition and $q = 3$; (c) Numerical simulations of model (10) with several pesticide switches which are guided by the weak threshold condition and $q = 2$; (d) Numerical simulations of model (10) with several pesticide switches which are guided by the weak threshold condition and $q = 3$.

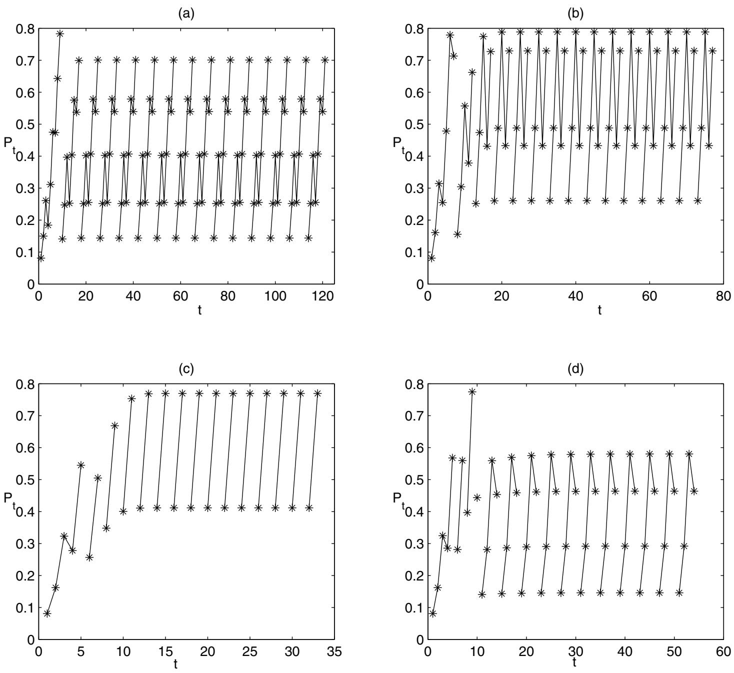

Figure 8: Illustrations of switching methods with the EIL-guided strategy for model (30) with different m. The baseline parameter values are as follows: $d_1 = 0.6, a = 2, b = 1, \omega_0 = 0.99, q = 3, P_0 = 0.2$ and $EIL = 0.8$. (a) $m = 1$; (b) $m = 2$; (c) $m = 3$; (d) $m = 4$.