The Embeddedness of Organizational Performance: Multiple Membership Multiple Classification Models for the Analysis of Multilevel Networks

Mark Tranmer°
University of Manchester (UK)

Francesca Pallotti
University of Greenwich

Alessandro Lomi
University of Italian Switzerland
Social Network Analysis Research Centre

° Corresponding Author: mark.tranmer@manchester.ac.uk

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Abstract

We develop a Multiple Membership Multiple Classification (MMMC) model for analysing variation in the performance of organizational sub-units embedded in a multilevel network. The model postulates that the performance of organizational sub-units varies across network levels defined in terms of: (i) direct relations between organizational sub-units; (ii) relations between organizations containing the sub-units, and (iii) cross-level relations between sub-units and organizations. We demonstrate the empirical merits of the model in an analysis of inter-hospital patient mobility within a regional community of health care organizations. In the empirical case study we develop, organizational sub-units are departments of emergency medicine (EDs) located within hospitals (organizations). Networks within and across levels are delineated in terms of patient transfer relations between EDs (lower-level, emergency transfers), hospitals (higher-level, elective transfers), and between EDs and hospitals (cross-level, non-emergency transfers). Our main analytical objective is to examine the association of these interdependent and partially nested levels of action with variation in waiting time among EDs – which is an ED nodal variable, and is one of the most commonly adopted and accepted measures of ED performance. We find evidence that variation in ED waiting time is associated with various components of the multilevel network in which the EDs are embedded. Before allowing for various characteristics of EDs and the hospitals in which they are located, we find, for the null models, that most of the network variation is at the hospital level. After adding these characteristics to the model, we find that hospital capacity and ED uncertainty are significantly associated with ED waiting time. We also find that the overall variation in ED waiting time is reduced to less than a half of its estimated value from the null models, and that a greater share of the residual network variation for these models is at the ED level and cross level, rather than the hospital level. This suggests that the covariates explain some of the network variation, and shift the relative share of residual variation away from hospital networks. We discuss further extensions to the model for more general analyses of multilevel network dependencies in variables of interest for the lower level nodes of these social structures.

1. INTRODUCTION

Interest in the analysis of multilevel networks has been growing rapidly in recent years (Snijders et al., 2013; Wang et al., 2013). Despite such interest, the general view persists that: "This area of network modelling remains thoroughly underdeveloped." (Snijders, 2011: 137). This seems to be particularly the case in the study of formal organizations, whose nested hierarchical structure makes the analysis of multilevel networks unavoidable (Lomi et al., 2014). Nevertheless, the multiple levels spanned by networks within and between organizations are typically considered independent and analysed separately as independent levels of action (Borgatti and Foster, 2003; Kilduff and Tsai, 2003). As Moliterno and Mahony concluded in their extensive review of the literature (2011: 444): "[W]hile some recent network scholarship has begun considering multiple levels of analysis, the majority of scholarship in this area has examined single- and within-level network structures and relationships."

For this reason, extant studies of organizational networks are generally unable to deliver on their central promise to provide a bridge across multiple structural levels of action (Contractor, Wasserman and Faust, 2006). This is particularly the case in the study of interorganizational relations, where the nodes are individual organizations, characterized by an internal structure with multiple hierarchical levels (DiMaggio, 1986). In this paper, we present a new model for the analysis of multilevel networks - a Multiple Membership Multiple Classification (MMMC) model – which addresses this problem directly. A MMMC model was recently applied to the analysis of (single level) social network and group dependencies (Tranmer et al., 2014). We present here, for the first time, an extension of this model for the analysis of multilevel networks. To establish the empirical value of the MMMC model in this context, we present an illustrative application to the empirical analysis of interorganizational relations – a natural multilevel setting (DiMaggio, 1986; Laumann, Galaskiewicz, and Marsden, 1978).

Our work extends existing research in at least two ways. Firstly, like studies based on Hierarchical Linear Models (HLMs), our study spans multiple levels of analysis. Unlike HLMs, however, the model we propose takes explicitly into account dependencies induced by network ties within and between structural levels through multiple affiliations. In this way, our study contributes to research in this area by extending available statistical models for multilevel systems to the analysis of multilevel social networks. Secondly, like recent studies based on

Multilevel Exponential Random Graph Models (MERGMs), we are analysing multilevel network data (Wang et al., 2013). However, unlike MERGMs, whose target of inference is multilevel network structure as defined by ties within and across levels, the focus of the MMMC model is on variation in outcomes associated with attributes of the lower-level nodes across and between the levels of a multilevel network. More specifically, the MMMC model focuses on the way in which multilevel network dependencies are associated to variations in a behavioural dependent variable defined for nodes at the lowest level, rather than on the presence or absence of ties among such nodes. Also, unlike MERGMs, the models we propose are not restricted to binary networks, but are applicable to a broader range of weighted networks.

MMMC models have only recently been applied to network data (Tranmer et al., 2014). To our knowledge, however, they have not been applied to multilevel networks. Here, we specify the MMMC for a multilevel network and illustrate its application to data that we have collected on Emergency Department (ED) waiting times − a generally accepted measure of ED operational performance (Horwitz, Green and Bradley, 2009; Lambe et al., 2003). The network we examine is multilevel because it implicates multiple interdependent levels of action. The first level is defined in terms of transfers of emergency patients between EDs (lower-level nodes). The second level is defined in terms of transfers of elective patients between hospitals (higher-level nodes) containing the ED units. Finally, the third level involves transfer of non-emergency patients between EDs and hospitals (cross-level relations). The empirical example illustrates in practice how the MMMC that we have developed may be adopted to address recent calls to develop multilevel approaches to the analysis of intra and interorganizational networks (Baker and Faulkner, 2002; Brass et al., 2004). The multilevel data structure that we will be analysing in the empirical part of the paper is described, schematically, in **Figure 1.**

<p align="center">**----- Insert Figure 1 about here -----**</p>

Our analysis focuses on variations in ED waiting times. Waiting times are an important measure of how well EDs respond to patient needs, and are commonly used as an indicator of the timeliness, efficiency, safety and patient-centeredness of emergency care (Horwitz, Green, and Bradley, 2009). The illustrative example we present as an application of the MMMC model for multilevel networks is almost ideal for at least two reasons: (i) detailed information on the

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various forms of patient transfers is publicly available, and (ii) the attention paid by health authorities in assessing health care outcomes makes the data particularly reliable.

We organize the article as follows. In Section two, we outline the motivation for developing statistical models for multilevel network dependencies. In the third section, we review methods and models for single level networks, including network autocorrelation models, and multilevel approaches via the Multiple Membership (MM) Model. We outline their conceptual similarities and differences. In the fourth section, we introduce the MMMC model for the analysis of multilevel networks, explaining how it is an extension of the single level network MM model. We explain how the parameter estimates from the MMMC model provide information about multilevel network dependencies in a lower level nodal dependent variable. In the fifth section, we describe the research design of our example, and the approach that we adopt for the estimation and evaluation of the model. In the sixth section we present the results of the analysis, and discuss their possible interpretation and implications. We conclude the article with a reflection on the limitations, general usefulness, and applicability of MMMC models to more general studies of multilevel network dependencies.

2. THE MULTILEVEL STRUCTURE OF INTERORGANISATIONAL NETWORKS

The recent interest in models for multilevel networks is driven in part by advances in statistical modelling (Wang et al., 2013) and in part by the rediscovery of the classic theoretical insight that social networks connect multiple levels of action (Boorman and White, 1976; White, Boorman and Breiger, 1976). Statistical models to investigate how structural levels of action may be (de)coupled have only recently become available.

Interorganizational communities provide an almost ideal setting for exploring the joint implications of these parallel trends. This is the case because organizations may be represented as network nodes with an internal structure characterized by multiple levels of action (Simon, 1996). This determines a recognized need for the development of rigorous approaches to the analysis of multilevel networks generated by interorganizational relations (Aguinis et al., 2011; Mathieu and Chen, 2010; Oh, Labianca, and Chung, 2006; Rousseau, 1985; 2011).

In his work on the interorganizational field of US Resident theatres, DiMaggio sets the stage for the development of models for multilevel networks that we present in this paper (1986: 363):

"The insight that organizations consist of individuals and subunits with quite different agendas and objectives is exceptionally difficult to capture in an interorganizational framework, where the smallest units of analysis are organizational nodes in networks. Indeed, network imagery militates toward treating and talking about organizations as if they were unitary actors with constant and uncontested objective functions."

We explain how this "exceptional" difficulty may be addressed by considering organizations as the macro level in a multilevel network for which the internal structure of "organizational nodes" represents the micro level. This allows us to represent in one model the way in which organizational behaviour is affected by: (i) network relations between organizations; (ii) networks of relations between sub-units contained within organizations, and finally (iii) networks connecting units to organizations across-levels.

3. MODELS FOR CROSS-SECTIONAL NETWORK DEPENDENCIES

Here, we review existing approaches for analysing single level and multilevel networks, focusing on the cross-sectional case. We identify the targets of inference for such analyses. We especially focus on models for network dependence. We explain the similarities and differences of these approaches, starting with the single-level network, before considering multilevel networks.

3.1 Single level networks.

A single level network may be defined as a set of nodes for which connections exist that could be undirected or directed. Attributes often exist for each node, and one or more of these may be regarded as a dependent, y, variable. Other node attributes may comprise a set of explanatory variables, **X**. When the target of inference is the network structure, in the context of social selection or social influence, we can investigate it with an Exponential Random Graph Model (ERGM), for which the application, software and literature are now all well established (Lusher, Koskinen and Robins, 2013). ERGMs can be fitted with or without attribute information, to assess whether such attributes are associated with the network tie structure.

Alternatively, network dependence on nodal variables might be the focus of the study, where the target of inference could be the extent to which the relationship between nodal dependent and explanatory variables is associated with the network connections, or where we wish to allow for network connections when estimating the relationship between the nodal dependent and

explanatory variables. Well established models for such cases are network autocorrelation models.

3.2 Network Autocorrelation Models.

Network autocorrelation models originally developed from spatial autocorrelation models - see, for example, Ord (1975), Doreian (1980). The model formulations are identical for the spatial and network cases, using spatial or network data respectively. The names of these models differ in the literature, but we will refer to two important cases below as the network effects model and the network disturbances model; these are the names used by Leenders (2002). We assume that the dependent variable is interval scale in all models defined below. For a single level network, the network effects (NE) model is defined as:

$$y = \rho \mathbf{W}y + \mathbf{XB} + \varepsilon$$
$$\varepsilon \sim N(0, \sigma^2) \qquad (1)$$

In (1), y is an attribute of each node of the network, that we regard as a dependent variable, and $\mathbf{X}$ is a set of explanatory variables, also attributes of each node, which we wish to relate to the dependent variable. $\mathbf{W}$ is a set of weights summarising network connections. These are usually standardised to sum to 1 across each row of $\mathbf{W}$ (Leenders, 2002). As an example, consider a network with 10 nodes, for which the first node of the network, corresponding to the first row of $\mathbf{W}$, has three connected nodes. In the absence of other information, we can define their weights equally as 1/3, summing to 1 across row 1. The seven unconnected nodes to node 1 (including the self-connection), have weights of 0. The $\mathbf{W}$ can also be defined for unequal weights, such as where connections are based on valued ties. The diagonal of $\mathbf{W}$ is assumed to be zero to disallow self-connections (loops).

The parameters of model (1) are ρ, the coefficients $\mathbf{B}$ for any explanatory variables, and the variance of the errors, σ^2. In this model, y appears on both sides of the equation. $\mathbf{W}$ provides information about the other nodes to which each node of the network is connected. Substantively, this model is useful when we think that there is a direct connection of the values of y for connected nodes when estimating the value of y for each node. In other words, using an ego-neighbourhood as an example, to determine whether the average value of the dependent variable of the alters in the ego-neighbourhood is associated with the value of the dependent

variable for the ego. The average strength of such a connection, conditional on the other explanatory variables in the model, is estimated by the (auto)correlation parameter, ρ. The   are differences in the predicted values of y from their observed values, which are assumed to be Normally distributed with mean zero and variance ^{2}. Model (1) can be fitted with or without the inclusion of nodal explanatory variables, **X.**

An alternative formulation to the network effects model, using the same data input as (1), is the network disturbances (ND) model:

$$y = \mathbf{XB} + $$
$$= \quad \mathbf{W} \quad + $$
$$\sim N(0, \; ^2) \quad (2)$$

The first line of (2) looks like a standard Ordinary Least Squares (OLS) regression model. However, the second line indicates how the network connections are taken into account in the error part of this model. The model parameters are the regression coefficients ρ, the coefficients **B,** and the variance of the error, ^{2}.

Model (2) takes into account the network through the error,   , which we assume is a vector of network random effects. Model (2) is useful for situations where we hypothesise there is an omitted variable, for which network structure exists, or network heterogeneity in y that cannot be explained by the explanatory variables alone. Model (2) is also useful when we want to estimate the regression for y on **X** given the fact that the network nodes are connected, and where we might regard the network autocorrelation as a nuisance to be taken into account in a model-based approach; in other words where we cannot assume the errors are independent. If we adopt such an approach, the model coefficients and their standard errors will be estimated having taken the network structure into account, unlike OLS. In model (2), ρ is a measure of the extent to which the error,   , of the focal node is correlated with values of the errors for connected nodes, conditional on the explanatory variables in the model, **X**. If there is no network structure to these errors, ρ is estimated as zero, and (2) reduces to a standard OLS model (Also true for model (1) when ρ is estimated as zero). Where network structure exists in the errors, ρ will typically be positive, and provides a measure of the average network autocorrelation of the error terms.

Single level NE and ND models may be fitted in software such as `R` (R core team, 2013), using the `SNA` (Butts, 2008) or `SPDEP` (Bivand et al 2005) packages.

3.3 Multiple Membership Models.

Multilevel modelling is a common technique for investigating variations in response variables in structured populations (Snijders and Bosker, 2012). Tranmer et al. (2014) explain that an MMMC model, a type of multilevel model, is useful for analysing network dependencies in a single level network nodal variable, when other groups in the population such as areas, or schools, exist alongside the network. They show how an MMMC model may be used to compare network, school and area variations in educational performance.

When we have one set of network subgroups, such as the ego-neighbourhoods that exist in a single level network, the MMMC reduces to a Multiple Membership (MM) model (3) because there is only one classification (the ego-neighbourhoods).

$$y_i = \mathbf{x}_i' \quad + \sum_{j=1}^{n} w_{i,j} u_j + e_i$$

$$i = 1, ..., n \; ; j = 1, ..., n.$$
$$u_j \sim N(0, \sigma_u^2) \quad e_i \sim N(0, \sigma_e^2)$$
$$\mathrm{Cov}(u_j, e_i) = 0 \qquad\qquad (3)$$

In Model (3), y_i is a node level dependent variable for each of the n nodes, $\mathbf{x}_i$ is a matrix of node level attributes, which we assume are explanatory variables. $\mathbf{x}_i$ includes a constant, and β is the vector of the regression coefficients of the constant and the explanatory variables. For clarity, and to reflect our empirical example, we define the network subgroups here as ego-neighbourhoods, and assume that there are no isolates, such that there are n ego-neighbourhoods with at least one alter in the network. However, the model could be also defined and used for other network subgroups, such as cliques, or for situations where some network nodes are isolates. The weight that is given to each node for their ego-neighbourhood membership is $w_{i,j}$, where $w_{i,j}$ is zero if j is not an alter in i's ego-neighbourhood. $w_{i,j}$ is also zero for i = j   (to

disallow loops). The $w_{i,j}$ are elements of the n x n weight matrix $\mathbf{W}$, which is derived from the n x n adjacency matrix, $\mathbf{A}$. The weights, $w_{i,j}$ sum to 1 for each individual, i, as was the case for the network autocorrelation models defined earlier. In other words the n x n weight matrix $\mathbf{W}$ is row-standardised, where each row sums to 1, for networks without isolates. If the network did include isolates, the rows of $\mathbf{W}$ corresponding to those isolates would have zero weights, $w_{i,j}$, for all columns of $\mathbf{W}$. As is typical in multilevel modelling, the random effects at the individual and network levels are assumed to be uncorrelated: $\mathrm{Cov}(u_j, e_i) = 0$. The between-ego-neighbourhood variance component, σ_u^2, allows us to assess the extent to which the nodal dependent variable varies between the ego-neighbourhoods, and by extension, how much variation in the nodal dependent variable is within these ego-neighbourhoods.

3.4 Similarities and differences of ND and MM models.

Models (1), (2) and (3) all take into account network dependencies in cross-sectional network data. Tranmer et al. (2014; Table 4) compared ND models (2) with MM models (3) in an empirical analysis of academic performance, given information about networks and schools. They found similar estimated model coefficients, standard errors, and goodness of fit for both approaches. These authors compared the ND and MM models because both models include the network information in the random effects (error) part of the model. However, the way in which the ND and MM models are parameterised and estimated is very different. Estimation of parameters in ND models involves the inversion of the matrix of network weights for the whole network, $\mathbf{W}$, that are derived from the network adjacency matrix, whereas this weight matrix does not need to be inverted to estimate parameters for the MM model. For the MM model, the weights for each ego are used to associate the random effects only from alters in their ego-neighbourhood. The MM model therefore implies no correlation of the nodal dependent variable for two ego-neighbourhoods with no nodes in common.

Both the ND and MM models provide estimates of explanatory variable coefficients and their standard errors for the fixed part of the model. In addition, some information about network dependency is estimated. In the ND model (2), $\hat{\rho}$ gives an estimate of the average correlation of the errors for connected nodes, given the explanatory variables. In the MM model (3) the

estimated value of the variance component ^2_u gives a measure of the extent to which the errors co-vary on average between the different ego-neighbourhoods in the network. If there is no network correlation or variation between ego-neighbourhoods in the errors,   in model (2) and ^2_u in model (3) would both be estimated as zero.

As Tranmer et al. (2014) show, model (3) may be extended to a MMMC model to include an additional level (classification) for school (or area), allowing the relative share of variation in academic performance of students due to networks and due to schools (or areas) to be assessed. Moreover, as the authors discuss in their 2014 paper, model (3) could be extended to include random coefficients – allowing, for example, network subgroup variation to be different for girls and boys with respect to academic performance.

3.5 Multilevel Networks.

A multilevel network can be defined for two levels, as a series of level 1 nodes and their connections, a series of level 2 nodes and their connections, and often also the cross-level network between level 1 nodes and level 2 nodes. Attribute information is often available for the level 1 and level 2 nodes.

An example is Lazega et al's (2008) "linked design" approach. Here, the level 1 nodes are individual cancer researchers, connected by advice seeking; the level 2 nodes are the laboratories in which they work, connected by resource sharing. The cross level network in Lazega et al's data is the affiliation of individual researchers to laboratories, where each researcher is affiliated to one laboratory. These affiliations of individuals to groups are typical in multilevel modelling (for example pupils in schools, or individuals in areas). It is straightforward to handle them in a multilevel model by including a group level above the individual. Level 1 node attributes for researchers include age, gender, research impact score, level 2 node attributes of laboratories include size, expenditure, speciality. In Lazega et al's (2008) data, there is usually one researcher affiliated to each laboratory, with a few cases where more than one researcher is affiliated. If there was exactly one researcher affiliated to every laboratory, the data could at first appear to be a multiplex network. However the network can still be regarded as multilevel because of the nature of the connections, which include a set of connections for level 1 nodes (researchers) and a set of connections for level 2 nodes (laboratories).

Wang et al. (2013) developed and applied their MERGM approach to Lazega et al's (2008) data. Their targets of inference were various aspects of the multilevel network structure. When the targets of inference are multilevel network dependencies in a level 1 nodal variable for the level 1 network, the level 2 network and the cross-level network, we can extend the MM model (3) to include three sets of classifications for these networks, making it a Multiple Membership Multiple Classification (MMMC) Model.

4. THE MMMC MODEL FOR MULTILEVEL NETWORKS.

We provide a general description of the MMMC model for investigating variations in a dependent variable for the level 1 nodes across the various components of the multilevel network. We focus on the case of a two-level network where we assume none of the network nodes is an isolate.

MMMC models may be fitted when the targets of inference are the sources of variation in a level 1 node dependent variable, y_i, across the three components of the multilevel network, and at the individual level, before and after the inclusion of nodal explanatory variables.

We can assess these sources of variation from the estimated variances of the random effects in the MMMC model. Moreover, when an MMMC model is fitted with a set of explanatory variables, we estimate their coefficients and standard errors taking into account the complex multilevel network structure of the data in the analysis.

The structure of a multilevel network with two levels can be defined as a set of n_1 nodes at level 1. For each of these level 1 nodes, we have a nodal dependent variable, y_i. We can represent this network by an n_1 by n_1 adjacency matrix. At level 2 we have a network of n_2 nodes. Sometimes there will be more than one level 1 node contained in each level 2 node. However, a special case is where there is exactly one level 1 node contained in each level 2 node; either through data availability, or in the population. When this is the case, $n_1 = n_2 = n$, and, the cross-level network will also be of dimension n by n. We will define and discuss the MMMC model in Equation (4) for this special case.

The level 1 network, level 2 network, and cross-level network can each be represented as a non-symmetric valued n by n matrix. We define these three matrices as $\mathbf{A}_1$, $\mathbf{A}_2$ and $\mathbf{A}_C$, respectively. Each row of $\mathbf{A}_1$, indexed by $i=1,...,n$, of these matrices contains the (one-step) ego-

neighbourhood information for each level 1 network node, where the row corresponds to the ego, and each non-zero element in that particular row represents an alter in that ego-neighbourhood. Similarly, the rows of $\mathbf{A}_2$ and $\mathbf{A}_C$ are also egos and the non zero elements of those rows are the alters in the ego-neighbourhoods. For each row, the total number of non-zero elements is the ego-neighbourhood size (that is, the number of alters in the ego-neighbourhood). If $\mathbf{A}_1$, $\mathbf{A}_2$ and $\mathbf{A}_C$ are binary matrices, the sum of each row i of each of these matrices, n_{1i}, n_{2i} and n_{Ci} is the total number of alters for the ego-neighbourhood of each level 1 node, level 2 node, and for the cross-level nodes, respectively. If $\mathbf{A}_1$, $\mathbf{A}_2$ and $\mathbf{A}_C$ are valued matrices, the sum of each row of these matrices gives the sum of the tie values in that ego-neighbourhood, and we may find that there are much greater tie values for some of the alters in a particular ego-neighbourhood than others, which we should take into account in the definition of the weights used in the model.

We can think of the alters of an ego-neighbourhood as the members of ego's group, and can assign weights for ego-neighbourhood membership. Usually, these will be obtained from row-standardised matrices, derived from $\mathbf{A}_1$, $\mathbf{A}_2$ and $\mathbf{A}_C$, as discussed above (in 3.3). To illustrate how network weights can be assigned for a multilevel network in the context of our empirical example, **Table 1** shows an example row of patient transfer data, for cases where at least one patient is transferred.

- Insert Table 1 about here -

In this example, ED 1 transfers patients to seven other EDs. Hospital 1, in which ED 1 is contained, transfers patients to six other hospitals. There are two hospitals, including Hosptial 1, to which patients are transferred from ED 1. In total, 73 patients are transferred from (ego) ED 1 to seven other (alter) EDs as emergency transfers, with the number transferred ranging from 1 to 51. ED weights $w_{i,j}$ can be calculated for the alters of each focal ED by dividing the number of patients transferred to that alter by the total number of patients transferred in the ego-neighbourhood, so that these weights sum to 1 across the row of the resulting weight matrix. Hospital 1 makes a total of 24 elective transfers of patients to six other hospitals in its hospital ego net, and weights $w_{i,k}$ can be calculated as for the EDs. Again, these sum to 1 across the row. For the cross-level network, ED 1 transfers 130 non-emergency patients to two hospitals, the first of which, to where the majority (129) of patients are transferred, is the hospital in which the ED

is located. The weights $w_{i,l}$ can hence be calculated, again summing to 1 across the row. The cross level weights could be alternatively defined to include only hospitals other than the hospital that contains the ED.

For the model for multilevel network dependencies, we adapt the MMMC model approach of Tranmer et al (2014), as defined in Equation (4) below:

$$y_i = \mathbf{x'}_i \beta + \sum_{j=1}^{n} w_{i,j} u_j + \sum_{k=1}^{n} w_{i,k} \nu_k + \sum_{l=1}^{n} w_{i,l} \lambda_l + e_i$$

$$i = 1,..,n \; ; \; j = 1,..,n \; ; \; k = 1,..,n \; ; \; l = 1,..,n$$

$$e_i \sim N(0, \sigma_e^2) \quad u_j \sim N(0, \sigma_u^2) \quad \nu_k \sim N(0, \sigma_\nu^2) \quad \lambda_l \sim N(0, \sigma_\lambda^2)$$

$$\text{Cov}(u_j, e_i) = \text{Cov}(\nu_k, e_i) = \text{Cov}(\lambda_l, e_i) = \text{Cov}(u_j, e_i)$$

$$= \text{Cov}(u_j, \nu_k) = \text{Cov}(u_j, \lambda_l) = 0 \qquad (4)$$

The fixed part of the model is given by $y_i = \mathbf{x'}_i \beta$, where y_i is a dependent variable for each level 1 node, i, $\mathbf{x'}_i$ is a matrix of nodal explanatory variables, and a constant term, and their respective coefficients. The three terms, $\sum_{j=1}^{n} w_{i,j} u_j$, $\sum_{k=1}^{n} w_{i,k} \nu_k$ and $\sum_{l=1}^{n} w_{i,l} \lambda_l$ are sums of the random effects for the level 1 network, u_j, level 2 network, ν_k, and cross-level network, λ_l, each multiplied by the appropriate weights: $w_{i,j}$, $w_{i,k}$ and $w_{i,l}$, respectively. These weights are zero for i=j, i=k and i=l and are also zero if j,k or l are not alters of i's ego-neighbourhood. The weights could also be row-standardised on a different function to the inverse of total patient transfers, such as the inverse square root.

The random effects, u_j, ν_k, λ_l and e_i, are assumed to be uncorrelated, and each is assumed to be normally distributed with mean zero, and variances σ_u^2, σ_ν^2, σ_λ^2, and σ_e^2, respectively. The total variation in y_i, therefore has four components: level 1 network, level 2 network, cross-level network, and the individual level. The estimated parameters from this model allow us to estimate the extent and relative share of variation in y_i across the multilevel network with respect to these four components.

In an MMMC model, the variances of random effects should be scaled to reflect typical ego-neighbourhood membership, such as average size, as Tranmer et al (2014) showed. To estimate a

typical share of variation in the response, y_i, for the different network components, we can derive an average value for their corresponding variance components. For a valued network, as is the case for our patient transfers, this can be achieved by multiplying each variance estimate $\hat{\sigma}_u^2$, $\hat{\sigma}^2$ and $\hat{\sigma}^2$, by the average of the squared non-zero elements of its associated group membership weight matrix. With these scaled values, it is then possible to estimate the percentage variation in y_i for each of the four components. It is typical in multilevel modelling to first obtain these estimates with a null model that just involves the constant in the fixed part, without explanatory variables, to measure the overall extent of variation in the nodal dependent variable for each component, before then adding explanatory variables to fixed part of the full model. Usually, these explanatory variables will explain some of the variation in the nodal dependent variable, but the relative sizes of the residual variance components from the full model will also indicate where most of the unexplained variation remains. Again, for the full model we can estimate the relative share of this in the four estimated variance components, this time conditional on the inclusion of explanatory variables in the model.

Explanatory variables can be defined in terms of ego's own values, and can also be defined with respect to the alters (peers) in the neighbourhood of each focal (ego) node: for example, the average size of alter hospitals to which the ego hospital transfers patients. We calculated such peer explanatory variables for the networks of EDs and hospitals for each ego in the neighbourhood by multiplying the values of the explanatory variables for the alters in each ego-neighbourhood by the appropriate weight matrix: the ED weight matrix for ED level explanatory variables and the hospital weight matrix for hospital level explanatory variables. In the full models presented in Section 6.4, we included explanatory variables for ego's own values as well as the corresponding peer explanatory variables. In the next section, we illustrate an application of the MMMC model to Italian health data on patient transfers, where the level 1 nodal dependent variable is a measure of ED waiting time.

5. RESEARCH DESIGN
5.1 Empirical Setting and Data
We collected data on a community of hospital organizations in Lazio in 2006. Similarly to other Italian regions, Lazio is partitioned into Local Health Units (LHUs), designed to ensure

availability of, and access to, homogenous service throughout the region. 110 hospitals provide care services for the region, 56% of which are publicly owned. Fifty-seven EDs, located within hospitals, provide emergency care services in the region. Not all hospitals have EDs; for those that do, there is exactly one ED per hospital.

Patient transfer is one of the main forms of inter-hospital collaboration (Lomi and Pallotti, 2012). The specialized health care literature on interorganizational networks has long recognized the relevance of patient transfer relations for inter-hospital collaboration both in emergency and non-emergency settings (Lee et al., 2011; Iwashyna and Courey, 2011; Veinot et al., 2012). For the case of elective inter-hospital patient transfer, for example, Lomi et al. (2014) found that inter-hospital collaboration allows patients to access better care, because patients systematically flow from lower to higher quality hospitals. In a study of inter-hospital transfer of Acute Myocardial Infarction (AMI) patients, Veinot et al (2012) found that partner selection is more likely to be based on institutional routines than on consideration of quality or performance.

We focus on three different types of relation as shown in **Figure 1.** The first type involves emergency transfers of patients between pairs of EDs in our sample. For patients admitted to an ED, highly formalized care routines are in place to determine as rapidly as possible whether these patients need to be transferred, and if so, to which ED destination. Transfers between EDs typically occur because capacity, capabilities, and expertise are unevenly distributed between hospitals. Emergency patients may be transferred because the sender ED does not have the capacity to either treat the condition itself or because of complications that might arise from treatment (Bosk et al., 2011). Emergency transfers occur within 24 hours from admission to an ED. Other important aspects related to emergency transfers include the identification of an accepting physician in the receiving ED, the timely transmission of information accompanying the patient, and the set-up of a well-functioning infrastructure to support emergency patient transfers. Data on emergency transfers were collected among all 57 EDs.

The second type of relation involves the elective transfer of patients between all the hospitals. This is a second level of action. Elective patients (also referred to as in-patients) are individuals who have already acquired the status of "admitted patient" and, therefore, who have agreed to follow treatments administered by professional medical staff who are clinically responsible and legally liable for their conditions. This is an important qualification, because elective transfers

are the outcome of individual organizational decisions over which patients have surrendered control at admission. Transfers of elective patients are driven overwhelmingly by clinical and practical constraints. For example, insufficient expertise or available capacity in the sender hospital. However, a sender hospital may choose from any number of recipient hospitals for the same patient. Elective transfers follow an organizational model of coordination that is mainly based on informal arrangements and routines established between partner hospitals (Lomi et al., 2014; Veinot et al., 2012). Unlike the transfers of emergency patients, decisions to transfer hospitalized patients can take several days and are usually discussed during periodic administrative meetings.

Finally, the third type of relation shown in **Figure 1** involves patients transferred from EDs to hospitals. We call them "non-emergency transfers" because they involve patients with stabilized conditions being transferred to a "regular" hospital wards to undergo further non-emergency treatment. These cross-level transfers are meaningful and important because they signal unexpected acts of cooperation between EDs and hospitals. They are acts of cooperation because the sender ED needs to rely on and trust the receiver hospital, which needs to accept the patient. They are unexpected because – by default – patients admitted into an ED are typically retained by the hospital to which that ED belongs. For these reasons, transfers of non-emergency patients are driven mainly by lack of available capacity (i.e., staffed beds) in the sending hospital. In our sample, ED to hospital transfers generate cross-level networks. Because each ED (lower level unit) may transfer patients to multiple hospitals (higher level units), these cross-level relations define a situation consistent with the MMMC model.

The three types of patient transfers give rise to a multilevel network of relations between hospitals, between EDs located within hospitals, and between EDs and hospitals. As our nodal dependent variable is a measure of ED performance, we only considered those 57 hospitals containing an ED in our analysis, and excluded the 53 hospitals that do not have an ED. Based on these 57 cases, we constructed three 57 by 57 non-symmetric valued square matrices. The first matrix contains in the rows (columns) the EDs sending (receiving) patients, and in the matrix elements, the number of patients transferred from the row ED i to the column ED j (the level 1 network). The second matrix contains in the rows (columns), the hospitals sending (receiving) patients, and in the matrix elements, the number of patients transferred from the row

hospital i to the column hospital k (level 2 network). The third matrix is defined by the number of patients being transferred from EDs to hospitals, and is the cross-level network.

5.2 Variables and measures

The dependent variable, y_i, in this study is a measure of ED waiting time based on the time elapsed between admission and treatment of patients (Horwitz, Green, and Bradley, 2009). In our analysis, for each ED we used the percentage of patients who received treatment within 4 hours after their admission to the hospital ED. According to the Italian NHS standards, 4 hours is the maximum waiting time for emergency patients admitted to an ED (Agenzia di Sanita' Pubblica, 2006)[1]. ED waiting time is one of the most commonly used measures of ED effectiveness and performance (Horwitz, Green, and Bradley, 2009, Guttmann et al., 2011; Dunn, 2003). This is because emergency services in hospitals do not typically supply care services. Their main task is to stabilize the conditions of incoming patients so that they may be transferred as soon as possible to an appropriate unit within the hospital, or – should this not be possible - to a unit in a different hospital. Previous studies have shown that prolonged waiting time in EDs reduces the quality of care and increases the chance of adverse events for patients (Liew and Kennedy, 2003; Hoot and Aronsky, 2008; Vieth and Rodhes, 2006). Also, prolonged waiting time decreases patient satisfaction (Taylor and Benger, 2004), and increases the number of patients who leave an ED before being seen (Fernandes, Price, and Christenson, 1997). For these reasons, ED waiting time has always received attention from both administrators and policy makers as it represents an important measure of effectiveness, efficiency and safety in emergency care (Horwitz, Green, and Bradley, 2009). Details of the various explanatory variables included in our empirical models are given in **Table 2.**

<div align="center">**- Insert Table 2 about here -**</div>

We control for the effects of two broad categories of factors that may account for the variability in performance of EDs. The first category captures salient features of hospitals – and hence are

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[1] Waiting time has been adjusted to not include any time spent in EDs due to inability to find a bed for patients requiring hospitalization.

attributes of the hospitals, the level 2 network nodes. To control for the effect of the size of hospital on the performance of EDs we use the total number of staffed beds (**Size**). We also include the average percentage of occupied beds (**Capacity**) to control for the possible effect of availability of in-patient beds into which to move emergency patients. We control for the ability of hospitals to treat, on average, more complex cases by including the Case Mix Index[2] as a measure of hospital **Complexity**. Given that the geographical location of hospitals may affect the volume of patients coming to an ED, we include the variable **Capital city**. This is a binary indicator variable taking the value of 1 if the hospital is located in Rome, the capital city, and 0 otherwise.

The second category of factors refers to specific features of EDs – i.e., to the level 1 network nodes. Because higher-volume EDs may be more crowded, thus increasing the percentage of patients in the ED with long waiting times, we include the total number of admissions (**Volume**) to control for the effect of the volume of activity. Because the ability of hospitals to accommodate emergency admissions may affect the ED waiting time, we use **Retention rate** as measured by the percentage of in-patients admissions from EDs over the total number of patients admitted to EDs. We include **Uncertainty** as measured by the proportion of red-triage admissions over the total number of admissions, which gives an indication of the medical condition of patients arriving at EDs[3]. Finally, we use the variable **ED code** to control for the complexity of the care processes performed by EDs. **ED code** is a three category variable based on a classification of EDs according to their resources (e.g., human resources, technologies) and care processes, for which we created two indicator variables. The baseline category comprises the least specialized EDs (with regard to internal staff and care processes), typically providing immediate assistance to non-urgent cases, ED code = 2 is used to indicate moderately specialized

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[2] The Case Mix Index is a composite index frequently used in the specialized literature to measure the intensity of resource consumption for patients admitted to a particular hospital during a specific time frame. It measures the average severity of illness for discharged acute care inpatients. It can be used for comparing patient sets across hospitals, specialties, and departments.

[3] Triage is a way for emergency departments to prioritize patients by acuity level into categories indicating how quickly they should be seen by a health provider (National Center for Health Statistics, 2013; Schrader and Lewis, 2013). The acuity of visits is classified internationally into four categories: emergent, urgent, semi-urgent, and non-urgent. In the Italian health care system, red-triage refers to emergent cases, yellow-triage to urgent cases, green triage to semi-urgent cases, and white triage to non-urgent cases.

EDs, typically providing assistance to semi-urgent and urgent cases. ED code = 3 is used to indicate the most specialized EDs providing assistance to emergent cases.

5.3 Empirical model specification

We fitted a series of MMMC models to these data, and single-level models for baseline comparison. The results appear in Section 6. In every case, the dependent variable was the percentage of ED patients waiting less than 4 hours for treatment, standardized to have a mean of zero and a standard deviation of 1. Because this dependent variable has an approximately Normal distribution for the 57 EDs, we fitted linear MMMC models. Some models include explanatory variables in the fixed part of the model. We included these variables both as ego's own values and as the average values of the alters in the neighbourhood of each ego: peer explanatory variables.

5.4 Model estimation and interpretation

We began by fitting a series of null models to estimate the extent of variation in ED waiting time across the multilevel network, via the estimated variance components. We evaluated the models statistically using the Deviance Information Criterion (DIC) - the smallest value indicates the best model fit, having accounted for model complexity (Browne, 2009) - and substantively by calculating the estimated share of variation in ED waiting time for the three multilevel network components and for the individual ED level, as described in Section 4.

After adding explanatory variables, we also evaluated the model fit statistically, and calculated the relative share of remaining variation in waiting time not explained by the explanatory variables. We tested the statistical significance of the estimated regression coefficients in the fixed part of the model using approximate t-ratios. By fitting a single level model in addition to the MMMC models, and comparing the estimated regression coefficients from this model with the MMMC models, we were able to assess whether we would have reached the same conclusions about the explanatory variables in each case.

MMMC models can be fitted with specialist software for modelling, such as MLwiN (Rasbash et al, 2012). The model results presented here were all estimated via an MCMC algorithm (Browne,

2009) using default flat priors for the fixed effects and a chain of 100,000 samples, implemented in MLwiN. In all models, standard diffuse (gamma) priors were assumed for the variance parameters.

6. RESULTS

6.1 Orienting question

We examine the components of variation in individual ED performance, based on a measure of waiting time, associated with: (i) the patient transfer network between EDs (emergency transfers); (ii) the patient transfer network between hospitals in which EDs are contained (elective transfers), and with (iii) the cross-level patient transfer network between EDs and hospitals (non-emergency transfers). Our orienting question is thus: which one of these three networks is associated with the largest relative share of variation in ED performance? This is the question around which we organize the empirical analysis.

Our main orienting question is theoretically valuable because extant research argues that patient transfer involves a collaborative relation that is highly conductive of mutual learning between partners – and even competing hospitals (Lomi and Pallotti, 2012). In organizational research, mutual learning is typically associated with diffusion of practices, routines and experiences (Levitt and March, 1998) – which in turn then leads to correlation or co-movements in organizational behaviour and performance. The analytical objective of the models we estimate is to identify the source of variation in ED performance across the multiple network levels in which these organizational units are embedded. Our orienting question is not only theoretically valuable, but also empirically relevant. In the case of patient transfer relations, learning is mutual in the sense that both partners can potentially benefit from establishing a collaborative problem-solving arrangement. The receiver hospital learns from the sender because patients travel with a considerable amount of information about therapies and clinical procedures performed by the sender hospital. This information would be hard or even impossible to access in any other way. The sender hospital also learns about the practices of the receiver hospital that decides to accept the patient and assure continuity of care. The sender hospital also learns by reflecting on its own practices. Our fieldwork instructs us that transferring patients is an important opportunity to formalize knowledge that is frequently implicit in daily clinical practice. Patient transfer cannot

happen without written statements about the patient's conditions, and without detailed clinical records. Understanding the level at which this learning actually affects organizational performance is an important objective of our models.

If a significant component of variation in ED performance can be associated with network relations among EDs, then the data would provide some indication that processes of organizational learning occur at the sub-organizational level – i.e., at the level of organizational units (Ingram, 2002). Conversely, if variation in ED performance is explained by (higher level) relations among hospitals, then we might conclude that the network ties between ED units do not produce autonomous learning effects. In this case, the appropriate level at which performance differentials between sub-units should be understood would be the more aggregate organizational level. A similar reasoning holds for cross-level relations. In this case, the main cause of differences in sub-unit performance would be the ability of ED sub-units to collaborate across the boundaries of the hospitals within which they are located. Finally, if variation in ED performance is explained mainly by differences in individual attributes, rather than the networks, then we would conclude that network-based processes play only a limited role in supporting learning in this organizational community. The MMMC model is uniquely suited to adjudicate between these various sources of variation in organizational sub-unit performance.

6.2 Multilevel network structure

The smallest number of EDs to which any focal ED transferred patients to was 2, with a median of 24 and a maximum of 41. The smallest non-zero number of patients transferred between any two EDs was 1, the largest 1688. The smallest number of hospitals to which any of the 57 hospitals that included an ED transferred patients to was 1, the maximum 36, with a median of 23. The smallest number of patients transferred between any two hospitals was 1, and the largest 236. For the cross-level networks, the smallest number of hospitals from which patients were transferred from EDs to hospitals was 2, the maximum 35, and the median 15. For transferred patients, the minimum was 14, with a maximum of 11,000. As expected, for these cross-level transfers, the majority of patients were transferred from the ED of a hospital to a non-emergency ward in the same hospital as the ED.

6.3 Null MMMC Models.

We fitted a series of null models to estimate the overall variation in ED waiting time for the four components of the population. The results are shown in **Table 3.**

---- Insert Table 3 about here ----

In Table 3, M1 is a single level null model used as a baseline for comparison with the more realistically complex models. M2 includes a single-level network structure, by including random effects for the single level network of ED inter-connections. M3 includes random effects for single-level network hospital (H) inter-connections. The DIC drops for these models to values of around 160.5 compared with M1 with a DIC of about 164.7, indicating the better fit for the single-level network models as compared with the single-level model, M1. M4 includes both the level 1 and level 2 networks, and is thus a model for multilevel network dependencies with respect to the level 1 nodal dependent variable. The DIC of M4 of 159.3 does not reduce much compared with the single level network models, but it does allow us to get an estimate of the share of variation of ED waiting times at the ED network and hospital network levels. Model M5 includes level 1, level 2 and cross-level network components, and the goodness of fit is improved (DIC=144.9), compared with the models that do not include a cross-level network component. This suggests that the cross-level network is associated with variations in waiting times, and allows us to estimate the share of variation in waiting times between the ED networks, hospital networks, and cross-level networks.

The results in **Table 3** suggest that the model that includes all four components in the multilevel network (M5) has the best fit, statistically. This is a rather complex model for 57 observations in the dataset, but we note that alongside these 57 rows of data, three differing 57 x 57 matrices of weights for the ED, hospital and cross-level networks are also used in the identification of the model parameters; QAP correlations on the adjacency matrices used to generate these weights indicate that these three matrices are not highly correlated with one another.

We now consider the substantive interpretation of the null model results. **Table 4** shows the mean and median non-zero weights for the ED inter connections, hospital inter-connections and ED to hospital cross-connections.

---- Insert Table 4 about here ----

The results in **Table 5** first give the total estimated residual variance. Where networks are included in the model components, the means of the squared weights are used to give typical values of the total variance. Based on these totals we can estimate the share of variation in waiting time for the different classifications in each model. The results suggest that of the three networks, hospital networks have the largest relative share of variation in waiting time above the individual level (4-5%), but that all three levels above the individual level have a non-zero share of the variation in waiting times (ED network M2: 2-3%; cross-level network M3: 1%).

---- Insert Table 5 about here ---

6.4 Full Models

In the full models, we add explanatory variables for the two levels, as described in Section 5. Again, we fitted a single level model (M6) [an OLS regression], a model with random effects for the multilevel network of ED and hospital inter-connections (M7), and the model that also includes random effects for ED to hospital cross-connections (M8). The results are shown in **Table 6**.

---- Insert Table 6 about here ----

We begin by focusing on the coefficients in the fixed part of the model, and their respective standard errors. Of the covariates included, we found that in M6, M7, and M8 in **Table 6,** capacity (a hospital-specific measure) and uncertainty (an ED-specific measure) were the only statistically significant covariates, both having a negative association with ED waiting time. These measures were only statistically significant for ego's own values, not for their corresponding peer explanatory variables. None of the peer explanatory variables was significant. In this particular example we would have come to the same overall conclusion about the association of the explanatory variables with waiting time in terms of statistical significance in all three models, but we would not be able to investigate the nature of any multilevel network variation in waiting time unexplained by these variables using the single level model (M6).

Consistently with the results for the null models, we see that the model that includes the multilevel network inter-connections and the cross-connections (M8) is statistically the best

model in **Table 6** in terms of goodness of fit, as indicated by the values of the DIC (41.78). This large drop in the value compared with models that do not include a cross-level network component suggests we should treat this result with some caution given that the estimated percentage of cross-level network variation is small, and may be partly due to the fact that many cross level transfers from ED to Hospital are to the same hospital in which the ED is located. However, inclusion of multilevel network components either with or without the cross-level network included improves the model fit to some degree compared with the single level model; there is a small drop in the value of DIC for M7 (130.70) compared with M6 (132.93) indicating a slightly better fit for M7 than M6.

---- Insert Table 7 about here ----

As expected, the estimated total residual variation in waiting time reported i**n Table 7** is reduced to between about 28 and 44 percent of its original size after the inclusion of explanatory variables when compared with the estimated total variation for the corresponding null models. The estimated relative share of this residual variation for the different network classifications used in the models is given in **Table 7.** The remaining variation is predominantly at the individual level for these MMMC models. For the relative share at all levels above the individual, we find that all three networks are associated with some share of the residual variance (ED network (3-4%), hospital network (2%), cross level network (5%)). Not only have the covariates reduced the total residual variation in waiting times, but the relative share of this residual variation for the three network components is now different from the null models, for which hospital networks had the greatest share of network variation.

7. CONCLUSIONS

We have presented an MMMC model for the analysis of multilevel network dependencies. We have illustrated the empirical value of the model with an analysis of variations in ED waiting times in the context of multilevel patient transfer networks between hospitals (Iwashyna et al., 2009).

As summarized in Tables 5 and 7, we find evidence that variation in ED waiting time is associated with various components of the multilevel network in which the EDs are embedded. Before allowing for various characteristics of EDs and the hospitals containing them, we find in the null models that most of the network variation is at the hospital level. After adding node-specific characteristics to the model, hospital capacity and ED uncertainty are significantly associated with ED waiting time. Both associations are predictably negative. The effect of hospital capacity is negative because hospitals operating close to their full capacity may experience delays in moving patients from EDs. The effect of uncertainty is negative because EDs with a high proportion of red codes (our measure of uncertainty) may experience greater pressures to attend to emergency patients. We also find that the overall variation in ED waiting time is reduced to less than a half of its estimated value from the null models, and that a greater share of the residual network variation for these models is now at the ED and cross levels. This suggests that the covariates included in the model explain some of the network variation, and shift the relative share of residual variation away from hospital networks.

Once the MMMC models are estimated, the shrunken residuals (Snijders and Bosker, 2012) from levels above the individual could be used as a basis for comparing hospitals or EDs in a fair way in terms of waiting times, using a realistically complex method. Examples of using multilevel analysis to make fair comparisons of schools can be found in the literature, for example, Leckie and Goldstein (2009). Similar approaches to those used in Education could be made using MMMC models for these health data.

Extensions to the MMMC models presented and applied here are possible. Firstly, we could add random coefficients to the explanatory variables in the models. Substantively, this would mean that, for our empirical example, ED covariates could have different associations with waiting time in different ego-neighbourhoods of EDs; similarly for hospitals. For example, the association of ED waiting time with hospital capacity could be stronger in some networks of hospitals than in others. Secondly, where data are collected over time, it would be possible to add time as a level in the MMMC model to assess the stability of multilevel network variations. Singer and Willett (2003), and Steele (2008) give a range of examples of multilevel models for longitudinal data analysis. Thirdly, the MMMC model may be used for multilevel networks with more than two levels, if such data are available. The extent to which it is possible to identify

parameter estimates in these complex models will depend on the quality of the available data, and is the subject of on-going research. More empirical analysis is needed in this area, using a variety of multilevel network datasets.

The choice of weights as well as the choice of relations between nodes for the multilevel network will affect the results of the MMMC model analysis, as well as allowing different substantive theories to be tested. The weights we used in the analyses presented were based on the inverse of the total number of patients transferred in each ego-neighbourhood. Alternative weighting schemes could be used, such as weights proportional to the inverse square root of the total. We tried this alternative for the null models, and in comparison with the original analyses found that hospital networks still had the biggest share of network variation, and that the estimated proportion of variation in ED waiting time due to individual EDs reduced slightly. The weights could also be based on different relations to patient transfers, such as the geographical distance between pairs of hospitals (and hence pairs of EDs), or the Jaccard distance, which summarises the similarity, or difference, of each pair of hospitals in terms of whether they are, say, speciality eye hospitals, or general hospitals.

When modelling variations in waiting times for the hospital networks, we could use the model described in Equation (1) with three different relations on a single level of network nodes, rather than having to choose one relation. This enables the MMMC model to be used for a multiplex analysis; for example, to assess which of the three relations is most strongly associated with variation in waiting times in networks of hospitals. However, a multiplex network is not a multilevel network, but is instead several different relations for a single level of network nodes. Moreover, a multilevel analysis of network data does not automatically make that network multilevel. For example, de Miguel Luken and Tranmer (2010) investigated single-level ego-networks using a multilevel model.

More research is needed on the association of network structure with network dependence, both for single level networks and for multilevel networks. Firstly, to assess the performance of the MMMC model for networks of differing density: from fairly sparse to very dense. Secondly, for a given network density, certain patterns of network variation may tend to be most often associated with certain network substructures, suggesting that the MERGM and MMMC model approaches might complement one another for multilevel network analysis.

ACKNOWLEDGEMENTS

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